Behaviour of Futures Markets and Implication for Portfolio Choice

by

Weifeng Zhou

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Department of Economics

Birmingham Business School

College of Social Science

University of Birmingham

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Abstract

First, we document the co-existence of the time series momentum and of the term structure factors in the global commodity futures market. We demonstrate that the strategies based on the joint time series momentum and term structure trading signal outperform time series momentum only strategies and term structure only strategies.

Second, we propose a Multivariate Volatility Regulated Kelly strategy, which imposes extra variance penalization compared to the Kelly criterion. We furthermore demonstrate the superiority of our method in relatively low correlated portfolios, relative to the fractional Kelly and full Kelly strategies. The simulation results and Chinese commodity future empirical results strongly support our method.

Third, we combine the shrinkage theory and CUSUM change point detection in order to improve the covariance estimators. The change point embedded covariance estimator can perform better than any shrinking covariance estimators in the portfolio management. We empirically test different shrinkage estimators based portfolios in global futures markets.

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Contents

Introduction

In the introduction, we provide the history and background of the futures market, review the relevant finance theories, and present the research questions, motivations, and contributions.

1. Futures Exchanges Background

The earliest recognised futures exchange is the Dojima Rice Exchange, the centre of Japan's community of rice brokers, firstly established in 1697 and officially organised in 1773. The western commodity futures market first began to trade in the 16^{th} century in England, but the futures exchange was not officially established until the London Metal and Market Exchange was founded in 1877 (Wakita, 2001).

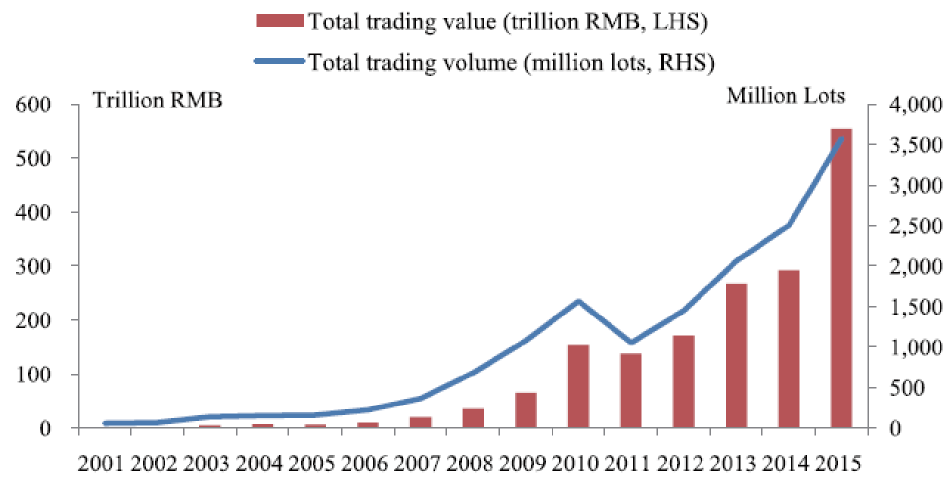
The United States is one of the earliest countries having official commodity exchanges in the West and the Chicago Board of Trade (CBOT) was established in 1848. After a network of agricultural centres had been established in Chicago, New York and other cities, the CBOT started playing a more important role. The first traded futures contracts in the United States were corn, followed by wheat and soybeans. These three basic agricultural products continuously occupy a large portion of the trades in the CBOT today (Santos, 2005).

This was then followed by cotton trades, which became the newest trend in the United States. Cotton forward contracts had been exchanged in New York since the 1850s, thus contributing to the establishment of the New York Cotton Exchange (NYCE) in 1870. Other futures contracts, such as sugar, orange juice and cocoa were developed afterwards.

In the 1970s, the futures exchange market expanded substantially. Currency futures were traded in the Chicago Mercantile Exchange (CME). The New York Mercantile Exchange (NYMEX) started providing various financial futures instruments including Treasury bonds (T-bonds), and eventually stock index futures. Exchanges, such as LME, ICE and NYMEX, offered commodity futures for precious metals including gold, silver, copper, platinum, palladium and more. Dow Jones and S&P 500 futures indexes came into being when people sought to create something in order to indicate the overall stock market (Working, 1976).

Futures trading in China has a short but high-growth history. The continuous innovation of new products and increasing liquidity pool lead to the consistent development of the futures market in mainland China. At present, it is playing an increasingly important role not only in serving the national economy but also in contributing to the futures market on a global scale. In 2016, the total trading volume of China's futures market reached 3.578 billion contracts, while the trading value increased to RMB 554.23 trillion (Shanghai institute of futures and derivatives (2016)).

Figure 1: Trading Volume and Value of China's Futures Market Between 2001 and 2015



Source: China Futures Association

The four futures exchanges, namely the Zhengzhou Commodity Exchange (ZCE, established in 1993), Dalian Commodity Exchange (DCE, established in February, 1993), Shanghai Futures Exchange (SHFE, established in 1999), and China Financial Futures Exchange (CFFEX, established in Shanghai in September, 2006) played a significant role not only in Asia but also in the world futures market. According to the statistics on the global trading volume in 2015 released by the Futures Industry Association (FIA), DCE, ZCE, SHFE and CFFEX ranked 8th, 9th, 10th, and 18th respectively.

Table 1:Top 30 Futures and Options Exchanges in 2015 by Trading Volume

2015 Ranking	2014 Ranking	Exchange	2015 (10,000 Contracts)	2014 (10,000 Contracts)	Change YOY
1	1	CME Group	353176	344277	2.58%
2	4	National Stock Exchange of India	303189	188036	61.24%
3	3	Eurex Exchange	227245	209798	8.32%
4	2	ICE	199881	221556	-9.78%
5	6	Moscow Exchange	165944	141322	17.42%
6	5	BM&F BOVESPA	135859	142048	-4.36%
7	7	Chicago Board Options Exchange	117393	132539	-11.43%
8	10	Dalian Commodity Exchange	111632	76964	45.05%
9	13	Zhengzhou Commodity Exchange	107034	67634	58.25%
10	9	Shanghai Futures Exchange	105049	84229	24.72%
11	8	NASDAQ OMX Group	104565	114745	-8.87%
12	12	Korea Exchange	79494	67777	17.29%
13	11	Bombay Stock Exchange	61489	73017	-15.79%
14	16	Johannesburg Stock Exchange	48852	29974	62.98%
15	20	Bats Exchange	39642	20199	96.26%
16	15	Japan Exchange Group	36146	30973	16.70%
17	14	HKEX	35936	31959	12.45%
18	18	China Financial Futures Exchange	34087	21758	56.66%
19	19	Taiwan Futures Exchange	26450	20241	30.67%
20	22	Miami Exchange	25261	13454	87.76%
21	17	Australian Stock Exchange	23418	24407	-4.05%
22	23	Multi Commodity Exchange	21635	13375	61.75%
23	25	Singapore Exchange	18387	12040	52.72%
24	21	Euronext	13552	14406	-5.93%
25	28	Istanbul Exchange	8888	5870	51.41%
26	26	Rosario Futures Exchange	7387	6519	13.32%
27	27	Tel Aviv Stock Exchange	6605	6405	3.13%
28	24	Metropolitan Stock Exchange of India	5799	12425	-53.32%
29	31	Tokyo Financial Exchange	4899	4090	19.77%
30	30	London Stock Exchange Group	4888	5049	-3.20%

Source: Futures Industry Association, China Futures Association

2. Features of Futures Contract

As we know today, futures contracts have evolved, as sellers and buyers begin to commit future exchanges for cash. For example, sellers would agree to offer buyers 2,000 ounces of copper at the end of December. The sellers knew the spot price of the copper, and the buyers knew how much they would pay in advance. The two parties may have exchanged written contracts in this regard, or even just a small amount of money on the basis of a 'guarantee'.

Such contracts have subsequently become increasingly popular. Banks even started using them as collateral for bank loans. They also began to change hands prior to the delivery date, thus driving the prosperity of the exchanges. If the buyer changed his mind and did not want the copper, he would sell the contract to someone who already had a demand for this item. Conversely, a seller who did not want to deliver his copper might pass his obligation on to another seller. The price would fluctuate depending on the demand and supply in the copper market. If metal factories in a country received many orders, the people owning the sell contract for copper would hold more valuable contracts due to an increasing demand; if the factory industry was experiencing a downturn period, the seller's contract would become less valuable. After a short period of time, people who had no intention of buying or selling copper entered the market. They were speculators, seeking the opportunity to buy low and sell high or sell high and buy low.

In the case of interest-rate and currency futures, namely those based on T-bonds, T-bills, Eurodollars and the five major currencies, the most significant influences are exerted by the policies and trading activities of the Federal Reserve, U.S. Treasury and foreign central banks, all of which affect interest rates.

Stock index futures are solely constructed based on the underlying stock index. They are affected by the factors influencing the stock market as a whole. Economic factors such as interest rates certainly drive price changes. Higher interest rates usually hurt the stock market. Other effects include the overall prospects for industries and political shocks, which also contribute to the stock index price.

Futures trading provides a way to establish a form of price knowledge leading to continuous price discovery. Futures prices reflect not only current cash prices, but also expectations of future prices and general economic factors.

3. Behaviour of Futures Markets

The behaviour of futures markets is affected by many factors. We can hardly disregard the anomaly of the so-called 'momentum' when referring to the features of the futures market. The relation between an asset's return and its recent relative performance history is referred to as the 'momentum' effect. Momentum is intuitively a phenomenon whereby past 'winners' will still be 'winners', while past 'losers' will still be 'losers' in the future. Momentum has first been documented in equities by Jegadeesh and Titman (1993, 2001) and Pirrong (2005) and the existence of momentum and reversals has been empirically demonstrated in futures markets. Subsequently, Miffre and Rallis (2007) find momentum in commodity futures prices. Menkhoff et al. (2012) provide evidence of momentum strategies in the foreign exchange market. Finally, Asness et al. (2013) show that momentum ubiquitously generates abnormal returns for individual stocks across the country equity index, government bonds, currencies, and commodities in several countries. Hence momentum exsits everywhere.

Another important phenomenon in finance is referred to as mean reversion, often observed in correlation with the momentum effect. Fama and French (1988) argue that a negative comovement between futures prices and risk premia can generate mean reversion in equilibrium. Poterba and Summers (1988) focus on the possibility that mean reversion results from the temporary divergences of prices from the fundamental value. In the context of futures markets, the mean reversion effect has been documented in many empirical studies. The cost of the carry model reveals a vital reason why the futures price consistently deviates from the equilibrium level. The model is very important in explaining the basic relationship of cash to futures pricing. It predicts the co-movement of spot and futures prices, which also implies mean reversion in the futures price. Bessembinder et al. (1995) indicate that term structure determines the mean reversion effect in the futures market and document the mean reverting futures prices. Several designed factor models also demonstrate the mean reversion commodity price by Schwartz (1997), Eduardo and James E. (2000), Gonzalo and Eduardo S. (2003) and Casassus and Collin-dufresne (2005).

4. Research Questions, Motivations and Contributions

Several questions have been discussed in a set of three related essays. The momentum and mean reversion are two important effects in the futures markets, as has been previously mentioned. Present literatures focus on the joint trading signal incorporating both momentum and mean-reversion in the cross-section in order to obtain a better portfolio return. However, time series momentum also exists and produces profitable strategies in the futures market by Szacmary et al. (2010), Moskowitz et al. (2012), and Baltas and Kosowski (2013). The first chapter sought to replicate the two behaviours in the time series dimension. In this chapter, we examine the significance of the time series momentum and of the time series term structure in the global futures market. We construct a combined long-short trading strategy with both the time series momentum and the term structure effect. The new strategy outperformed the time series momentum only (MOM-only) strategy and the time series term structure only (TS-only) strategy in commodity futures between 1990 and 2016.

Time series momentum has one important feature, whereby only individual asset past return and past variance is considered. Nevertheless, we all know that covariance plays an important role in risk management, asset allocation and portfolio performance. Since the seminal work of Harry (1952), mean-variance optimisation has become the most rigorous and popular method to manage portfolio weight allocation. It is therefore necessary to pose the following question: which occasion is good to use the covariance estimator and which occasion is good to use the variance estimator?

In the second chapter, we propose a Multivariate Volatility Regulated Kelly (MVRK) strategy, which imposes extra variance penalization compared to the Kelly criterion. The Kelly criterion is a covariance based strategy, while the time series momentum is a pure variance based strategy. When the correlation is not considered, the Kelly criterion is collapsed to the time series momentum. The intuition of MVRK is very simple: volatility impacts on the risk estimators that are more important than the correlation in low-correlated scenarios. MVRK implies that with the exception of very high correlated portfolios, it is better to switch more weight to variance in the covariance matrix. Our results show that the MVRK outperforms the full Kelly and fractional Kelly strategies in relatively low correlated portfolios. The simulation results show the superiority of MVRK in low correlation settings. The Chinese commodity market further provides empirical evidence of the MVRK strategy.

As indicated by the first two chapters, we believe that neither sample covariance nor variance constitute the best choice in portfolio management. Ledoit amd Wolf (2003) introduced the shrunken covariance matrix in order to replace the sample covariance. They proposed a fancy shrinkage theory suggesting the composition of the sample covariance matrix and that a highly structured estimator will give a better risk estimator. Although the shrinkage theory reduces the estimation error in the covariance matrix, the high volatility of the financial markets often breaks the covariance stability. The shrinkage method smooths the error of the sample covariance estimation. However we need to apply change point detection in order to adapt portfolio allocation when a big structure break occurs.

Change point detection in the random processes constitutes an important topic for theoretical and empirical statistics. The current body of literature has extensively studied the structural stabilities in the mean, variance and covariance of random variables, and applied those using appropriate applications in various fields. In the present chapter, the focus is predominantly placed on the structural break in the second moment.

In the third chapter, we propose an empirical approach in order to incorporate the change point detection into the shrinkage method. The main contributions are represented by three aspects. First, to the best knowledge of the authors, this chapter first applies change point detection in the shrunken covariance estimation. Second, we provide the approach combing change point with the shrinkage method in the portfolio weight allocation. Third, we empirically demonstrate that our approach improves the portfolio performance and is perfectly applicable in the global futures market.

Notice: The literature on momentum and the dataset used in the first chapter are taken from the research proposal of my PhD thesis submitted in 2015 to the Research Methods Course which is a compulsory part of my Doctoral programme. The pages 15-25, 46-47 in the thesis being quoted directly from this proposal. The second chapter, pages 48-73 in the thesis, is quoted from the paper that I have published with Ruanmin Cao, Zhenya Liu and Shixuan Wang. I wrote the chapter independently and played a core role in the full paper. My supervisor Zhenya Liu gave me advice in the completion of this paper, my senior colleagues Ruanmin Cao and ShiXuan Wang inspired me a lot in the derivation of the methodology part (see letter in page 112-113). Citation for this paper is: Cao, R.M., Liu,Z.Y., Wang, S.X. and Zhou, W.F. (2017) Multivariate Volatility Regulated Kelly Strategy: A Superior Choice in Low Correlated Portfolios. Theoretical Economics Letters ,7, 1453-1472.

Chapter 1

Times Series Momentum and Term Structure: Evidence from Commodity Futures

In this chapter, we document the co-existence of the time series momentum and term structure factors in the commodity futures market. We fail to find significant predictability of the time series term structure slope in bond, currency and equity futures. We show that the strategies based on the joint time series momentum and term structure trading signal outperform the time series momentum only strategies and the term structure only strategies. We find that the relative power between the time series momentum and the mean reversion effect is time varying and experience structure changes during extreme market periods. Our data consist of futures price and volumes for 23 commodity, 10 equity, 6 currency and 13 bond futures from January 1990 to April 2016.

1.1. Introduction

Commodity futures have become increasingly popular among investors over the past decade and have been used widely among traditional and alternative asset managers. There are two prominent phenomena existing in commodity and in the whole futures markets. One such phenomenon is mean-reversion, while the other is momentum. Both phenomena have been documented in futures markets. Several designed factor models show the mean reversion commodity price by Schwartz (1997), Eduardo and James E. (2000), Gonzalo and Eduardo S. (2003), Casassus and Collin-dufresne (2005). The term structure is the driver of the mean reversion effect in commodity prices and shows the empirical evidence of mean reversion in commodity (Bessembinder, et al., 1995) and (Erb and Harvey, 2006). Momentum has been documented in equities by Jegadeesh and Titman (1993, 2001), in futures markets by Pirrong (2005) and Miffre and Rallis (2007), in currency markets by Menkhoff et al. (2012) and everywhere by Asness et al. (2013). The strategies using the momentum trading signal are profitable in the commodity market (Miffre and Rallis, 2007) and (Fuertes, et al., 2010) and in using the term structure trading signal in the commodity market (Gorton and Rouwenhorst, 2006) and (Erb and Harvey, 2006). The strategies based on both mean reversion and momentum effects have been demonstrated to be superior to the pure momentum or the pure mean reversion strategy in equity (Balvers and Wu, 2006), currency (Serban, 2010) and the commodity futures market (Fuertes, et al., 2010).

The current body of literature focuses on the joint trading signal strategies incorporating both momentum and mean reversion from a cross sectional scenario. The trading positions are decided by signal strength in all assets and adapted in strategic and tactical asset allocation. However, momentum in time series also exists and produces profitable strategies in the futures market (Szacmary, et al., 2010), (Moskowitz, et al., 2012) and (Baltas and Kosowski, 2013). There is no direct evidence indicating that term structure in the time series can be implemented in trading strategies. We propose three guesses based on the current literature: First, the time series term structure effect exists in the global futures market, such as the time series momentum; Second, combined time series momentum and term structure strategies perform better than momentum only and term structure only strategies; Third, the structure between two effects is time varying and highly related to market states.

This chapter examines the existence of the time series term structure effect in commodity futures such as in equity, currency and bond futures. Following the framework of Moskowitz et al. (2012), we re-examine the significance of the time series momentum and of the time series term structure effect in the global futures market. Inspired by the work of Fuertes et al. (2010), we combine the trading signal of the pure time series momentum and pure time series term structure, in order to compare the strategies' performance. The trading signal of the time series momentum is simply using an instrument's own past returns while the trading signal of the time series term structure is using an instrument's own past slope of the term structure. We also provide a further analysis of the relative power between the time series momentum and term structure.

This chapter contributes to the academic literature in three ways. First, we document the existence of the time series term structure slope in commodity futures return, therefore the predictability of both time series momentum and mean reversion effects.

Nevertheless, we cannot find a significant predictability of the time series term structure slope in bond, currency and equity futures. Second, we show a combined long-short trading strategy with both the time series momentum and term structure outperforming the time series momentum only (MOM-only) strategy and the time series term structure only (TS-only) strategy in commodity futures during 1990-2016. While the time series MOM-only and TS-only strategies earn an average annualized return of 21% and 24% and a Sharpe ratio of 0.8 and 1.1, the combined double-sort strategies earn an average annualized return of 27% and a Sharpe ratio of 1.25. Third, the analysis of the time varying parameter from the rolling regression result implies that the time series momentum effect significantly increases in extreme market periods, while the time series term structure effect significantly increases during normal market periods.

The rest of the chapter is organised as follows. Section 2 presents the literature review. Sections 3 provides an overview of our dataset. Section 4 presents the methodology and analyses the predictability of the time series momentum and term structure factors. Section 5 evaluates the performance of strategies combining the time series momentum and term structure signals and analyses the parameter structure. Section 6 concludes the chapter.

1.2. Literature Review

1.2.1. Momentum Mechanism

Momentum intuitively is a phenomenon whereby past ‘winners’ will still be ‘winners’ in the future, while past ‘losers’ will still be ‘losers’ in the cross-sectional assets portfolio.

The momentum phenomenon is first documented in equity markets. Jegadeesh and Titman (1993) indicate that trading strategies, which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past, generate significant positive returns over 3 to 12 month holding periods. They find that the profitability of these ‘cross-sectional momentum’ strategies are not due to systematic risk. The results of their tests also imply that such momentum effect profits cannot be attributed to delayed stock price reactions to common factors. In their further research studies, they evaluate various explanations for the profitability of the momentum strategy as documented previously by Jegadeesh and Titman (1993).

Following their previous work, Jegadeesh and Titman (2001) successfully evidence the existence of the momentum effect by testing 8 years of historical data after 1993, suggesting that the original results were not fully due to data snooping bias. This paper also supports that momentum portfolios earning positive returns is partially consistent with behavioural models, such as CAPM (capital asset pricing model) and the Fama-French three factor model. Jegadeesh and Titman are the pioneers to prove the momentum phenomenon and lay a cornerstone for momentum research studies. They apply momentum in the equity markets and prove that this phenomenon is not occasional but consistent. Further research developments extend the study fields to various markets, such as currencies, bonds and futures.

K.Geert (1998) indicates the existence of momentum in other equity markets. They build a diversified portfolio with 12 European country equities and show that the past medium-term winners outperform the past medium-term losers by more than 1 percent per month. Sanjeev and Bhaskaran (2006) further examine momentum and reversals in international stock market indices. They find that country stock indices exhibit momentum during the first year after the portfolio formation date and reversals during the 2 subsequent years. Using a sample of 38 country indices, they find that winners outperform losers in the first 3 to 12 months after portfolio formation but underperform losers in the 2 subsequent years.

Momentum in futures markets was proved to exist in the subsequent years after the paper by Jegadeesh and Titman (2001). Futures markets have much broader and more diverse cross section investment opportunities as opposed to equities. In the US and overseas, the momentum and reversals phenomenon in futures markets including commodity and financial futures contracts are proved to be significant where momentum exhibits at short time horizon and reversals display at long time horizon. Besides, futures momentum portfolios earn positive average returns even after adjusting for risk using canonical pricing models including the CAPM and Fama-French three factor models, which indicates that standard parametric asset pricing models cannot explain the returns brought by the momentum strategies. If applying momentum strategies in futures markets, they earn positive average returns although the momentum returns will be reduced after a non-parametric risk adjustment.

Furthermore, futures momentum is correlated to, but not the subset of stock momentum (Pirrong, 2005). Pirrong empirically demonstrates the existence of momentum and reversals in futures markets. The study finds evidence of momentum in futures markets including various commodity and financial futures worldwide. Miffre and Rallis (2007) also show momentum in commodity futures prices. They use 31 US commodity futures contracts in various lookback periods to build 13 profitable strategies with a 9.38% average annual return.

Menkhoff et al. (2012) provide evidence of momentum strategies in the foreign exchange market. They also find evidence that under and subsequent overreactions explain the long-horizon momentum returns in currency, which is consistent with the equity markets. They further find that momentum returns are different from more conventional technical trading rules. The currency momentum strategies are very different from the carry trade in foreign exchange markets. The most important contribution of their paper is to indicate the significant unconditional excess returns delivered by momentum strategies in currency markets.

Momentum in fact exists everywhere. Momentum ubiquitously generates abnormal returns for individual stocks across the country equity index, government bonds, currencies, and commodities in several countries. (i.e., U.S., U.K., Continental Europe, Japan and so on). By studying the global returns to momentum and exploring its common factor structure, Asness et al. (2013) find that momentum in one asset class is positively correlated with itself in other asset classes. Note that the data in their study broadly adopts the U.S. global stock, 18 developed countries equity markets, 10 exchange rates and 27 different commodity futures across the London Metal Exchange

(LME), Intercontinental Exchange (ICE), Chicago Mercantile Exchange (CME), Chicago Board of Trade (CBOT), New York Mercantile Exchange (NYMEX), New York Commodities Exchange (COMEX), New York Board of Trade (NYBOT), Tokyo Commodity Exchange (TOCOM), 8 different futures exchanges. All these datum duration is from 1975 to 2008. The data selection maintains the diversification and representation of the samples they choose and historically proves the existence of an 'everywhere' momentum.

Although momentum strategies have historically generated high Sharpe ratios and strong positive returns in various asset markets, momentum strategies experience infrequent but strong failure in certain periods. Daniel and Moskowitz (2014) find that momentum crashes are resulted from negatively skewed returns to momentum strategies . They also find evidence that crashes occur in the 'panic' states – following market declines and when market volatility is high, and are subsequently with market 'rebounds'. They show that a conditionally high premium attached to the option-like payoffs of past losers contributes to the low ex-ante expected returns in panic states. The dynamic forecasting momentum strategy's mean and variance generates an unconditional Sharpe ratio approximately double that of the static momentum strategy. Furthermore, they show that momentum returns in panic states are correlated with, but not explained by, volatility risk. These results are robust across eight different markets and asset classes and multiple time periods.

In the process of conducting research on the cross-sectional momentum, some scholars construct the framework between the cross-sectional momentum and the time-series momentum. Lo and Mackinlay (1990) document the fact that stock returns are often positively cross auto-correlated. This result is consistent with the negative serial dependence in individual security returns and the positive autocorrelation in market indexes. They also empirically prove that the profits from a constrained investment rule are mainly due to cross effects among the securities. Especially in size-sorted portfolios, this cross effect shows a very special pattern. In their studies, the autocorrelation of stock returns is first presented. Although they did not relate the cross-sectional momentum to this phenomenon to a large extent, they indeed find the clue of momentum and the relation between momentum and time-series momentum. They provide a solid base for further studies in this area.

Afterwards, Lewellen (2002) relates the momentum phenomenon with autocorrelation in stock returns. In their studies, they report the possible factors, which cause the momentum phenomenon. Lo and Mackinlay (1990) test why there is a momentum in individual stocks on the basis of three aspects, namely the positive autocorrelation of stock returns, negative correlation between lagged returns and stock returns, and high unconditional mean relative to other stocks. They empirically find that lead-lag relations among stocks is a very crucial factor of the momentum phenomenon in stock markets. Furthermore, they provide two explanations for this result. First, the underreaction to portfolio-specific news but overreaction to macroeconomic events among investors may lead to momentum. Second, the excess covariance among stocks may also explain such a momentum phenomenon. Their evidence indicates that excess covariance is stronger than the autocorrelations, which leads to momentum profits. They extend the work of Lo and Mackinlay (1990) by including fundamental factors such as B/M, size and industry to test momentum in stock markets and the relationship between momentum and the autocorrelation of returns. However, they cannot solve the problem that blurs the explanations of momentum in B/M and size.

1.2.2. Time series momentum mechanism

Time series momentum is focusing on the positive predictability from an asset's own past returns rather than on the cross-sectional buy 'winners' and short 'losers' momentum phenomenon. Barberies et al. (1998) present a parsimonious model of investor sentiment, or of how investors form expectations of future earnings. The model they propose is motivated by a variety of psychological evidence, and in particular that, in making forecasts, people pay too much attention to the strength of the evidence they are presented with and too little attention to its statistical weight. They suppose that corporate announcements, such as those of earnings represent information that is of low strength but significant statistical weight. This assumption has yielded the prediction that stock prices underreact to earnings announcements and similar events. They have further assumed that consistent patterns of news, such as a series of good earnings announcements, represent information that is of high strength and low weight. This assumption has yielded a prediction that stock prices overreact to consistent patterns of good or bad news.

Daniel et al. (1998) propose a theory based on investor overconfidence and biased self-attribution in order to explain several of the securities returns patterns that seem anomalous from the perspective of efficient markets with rational investors. The theory is based on two premises derived from evidence in psychological studies. The first is that individuals are overconfident about their ability to evaluate securities, in the sense that they overestimate the precision of their private information signals. The second is that investors' confidence changes in a biased fashion as a function of their decision outcomes. The first premise implies overreaction to private information arrival and underreaction to public information arrival. As it is state in their paper:

"This is consistent with (1) the post-corporate event and post-earnings announcement stock price 'drift', (2) the negative long-lag autocorrelations (long-run 'overreaction'), and (3) the excess volatility of asset prices. Adding the second premise leads to (4) positive short-lag autocorrelations ('momentum'), and (5) a short-run post-earnings announcement 'drift,' and negative correlation between future stock returns and long-term measures of past accounting performance. The model also offers several untested empirical implications and implications for the corporate financial policy."

Hong and Stein (1999) model a market populated by two groups of naturally rational agents: 'newswatchers' and 'momentum traders'. They state that each newswatcher observes some private information, but fails to extract other newswatchers' information from prices. Prices underreact in the short run with gradual diffused information across the population. The underreaction indicates that the trending following strategy is profitable for momentum traders. However, if they can only implement simple (i.e., univariate) strategies, overreaction take in charge at long horizons, thus arbitrage strategy is more reasonable. They prove that any new 'behavioural' theory of asset pricing should be judged according to three criteria: 1) It should rest on assumptions about investor behaviour that are either a priori plausible or consistent with casual observation; 2) It should explain the existing evidence in a parsimonious and unified way; and 3) It should make a number of further predictions which can be subject to testing and which are ultimately validated.

Barberies, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Hong and Stein (1999) all focus on a single risky asset, therefore having direct implications for time series, rather than cross-sectional implications. However, the overall time series momentum effect across different markets has not been studied so far. Moskowitz et al. (2012) document the time series momentum in equity index, currency, commodity and bond futures for 58 liquid instruments. In their studies, they identify the features of returns that lead to their patterns and which factors are common and unique to the momentum and time-series momentum by decomposing the returns to a time series and cross-sectional momentum strategy. In fact, they further extend the framework of Lo and Mackinlay (1990) and Lewellen (2002). They historically find evidence that positive auto-covariance in futures contracts' returns is the sole reason for the majority of time series and cross-sectional momentum effects. By contrast, the contribution of serial cross-correlations and variation in mean returns to the momentum and time-series momentum effects is very small. Moreover, their studies imply that the momentum and time-series momentum have a very strong correlation since they show that the time-series momentum obtains returns related to the cross-sectional momentum. In addition, they find the persistence in returns from 1 to 12 months, which is partially reversed over longer horizons. All this evidence is consistent with the underreaction theories as well as with the delayed overreaction. Last but not least, they empirically prove the existence and consistence of the time-series momentum in extreme periods with small weights on standard risk factors, which present challenges to the random walk hypothesis and standard rational pricing models, as stipulated by current behavioural theories. Their framework documents the significant existence of the time-series momentum and provides a theoretical base for the application of such a time-series momentum strategy.

Before and after the work of Moskowitz et al. (2012), the studies of the time series momentum and its strategy performance in futures markets have been cited several times. Szacmary et al. (2010) examine the pure time series momentum strategy or trend-following trading strategy in commodity futures markets. They document that these strategies are profitable in commodity futures markets over 48 years with monthly frequency. Their results show that the time series momentum strategies earn positive mean net returns after transaction costs in at least 22 out of 28 markets over the full sample period. As they also mention in the paper, the mean returns and Sharpe ratios generally perform better than the cross-sectional momentum strategies when they are implemented in the same frameworks. In addition, the evidence in the study shows that trend-following strategies in intermediate time horizons are superior to those in short time horizons. Szacmary et al. (2010) first test the time-series momentum strategy performance in futures markets, and provide robust evidence to the application of the time-series momentum. However, in this study, only commodity futures are used and the monthly time frequency is tested. The complement of frequency and futures classes is not fully satisfied since the monthly data cannot fulfil the practice of the time-series momentum strategies and the commodities may lead to bias selection, which cannot cover the whole futures markets. In subsequent studies, Baltas and Kosowski (2013) perfect time series momentum in futures markets in all daily, weekly and monthly frequencies as well as in all classes of futures across 26 commodities, 23 equity indices, 7 currencies and 15 bonds.

Baltas and Kosowski (2013) further document the existence of strong time series momentum effects in futures markets with daily, weekly and monthly frequencies. Besides, their study implements the rebalancing frequencies in the strategies and finds that these rebalancing frequencies have low cross correlation and capture distinct returns. The rebalancing frequency and frequencies correspond to the position holding period and the lookback period, respectively. Their study also shows that in different strategies across all frequencies, the portfolio of all futures will bring a 16%-19% annualized return, above a 1.20 Sharpe ratio (before transaction costs) but with more than 10% maximum drawback. Furthermore, they empirically prove that commodity futures momentum strategies have a relatively low return, as well as a low correlation with other futures strategies. Hence, their paper robustly extends previous studies from commodity futures markets to the various classes of futures and provides evidence suggesting that the time series momentum return predictability significance depends on the early sample behaviour and does not represent a stable over time significant momentum effect. Interestingly, they find that momentum effects exist across all daily, weekly and monthly frequencies and show cross similarities. In addition, Baltas and Kosowski also prove that CTA funds are likely to implement such time series momentum strategies in practice. Finally, they cannot find any evidence to suggest that capacity constraints exert an effect on time series momentum strategies. Their works broadly test the existence of time series momentum effects and the profitability of such strategies in global futures markets, excluding the Chinese markets. They create an elegant methodology for the implementation of time series momentum strategies so that further studies can draw on the experience from their frameworks.

Baltas and Kosowski (2013) improve the time series momentum strategies by examining the role of volatility estimators and trading signals. They use 75 futures contracts from 1974 to 2013 as the data set. They prove that in average one month trading days, the Yang and Zhang (2000) estimator is 8.2 times more efficient than the standard deviation estimator, which is superior to the Parkinson (1980) estimator (5.2 times), Garman and J.Klass (1980) estimator (7.4 times) and Rogers and Satchell (1991) estimator (6.2 times), both mathematically and empirically. They also state that the range based volatility estimator is better than the return based volatility estimator when applied in the time series momentum strategies, because range estimators only use daily open, high, low and close prices, which can capture intraday information better. A range estimator such as the Yang-Zhang estimator also reduces the portfolio turnover and consequently the transaction costs for the construction and rebalancing of the portfolio.

1.2.3. Mean reversion mechanism

Mean reversion is the theory suggesting that prices and returns eventually move back towards the mean of average.

Bessembinder et al. (1995) first show that the term structure determines the mean reversion effect in futures market and document the mean reverting prices in real asset markets. The negative relationship between spot prices and the slope of the futures term structure is examined from two sources: one is the positive correlation between the implied cash flow and prices, while the other is the negative correlation between interest rate and prices. The futures contracts with different expired horizons are used to build a different term structure so that they can implement the procedure in many assets; even the reliable spot price data is elusive. Only ten futures contracts and 10 years data are applied in their procedure, which leaves the overall mean reversion effect in asset prices for further study.

Schwartz (1997) compares three models of the stochastic behaviour of commodity prices. The first model is the logarithm of commodity price one factor model, while convenience yield is added in the second model and instantaneous interest rate is further added in the third model. The mean reverting effect is assumed in all three factors. The Kalman filter method is used in order to estimate the parameters of the models in copper, gold and oil. Each stochastic model constructed in this paper implies the long term mean reversion behaviour of commodity prices. Their study reveals a strong mean reversion in the commercial commodity prices and provides several key elements in commodity asset pricing. The decomposition of each mean reversion element in a time continuous model further implies the term structure of future prices movements, which is initially strongly backwardation and finally turns upward and converges to a fixed rate of growth.

After Schwartz (1997), Eduardo and James E. (2000) develop a two-factor model of commodity prices that allows for mean-reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. Their model accommodates simple Orstein-Uhlenbeck and geometric Brownian motion price models and performs better than these two composited models. Although their work directly repeats the formula used by Rajna and Eduardo S. (1990), the model without convenience yields but short term variations and long term equilibrium price levels lead to a more transparent result.

Besides, the two factors used in their model give more ‘orthogonal’ dynamics, thus allowing a more clear analysis of the impacts of each factor.

Gonzalo and Eduardo S. (2003) develop a parsimonious three-factor model of the term structure of oil futures prices. The main contribution of their work is the implementation procedure improvement. As opposed to the complex Kalman filter method used by Schwartz (1997), the simplified parameter estimation method does not require linearity in the price expressions and is flexible with missing observations in the data. This method makes full use of all commodity linked market asset prices and can be used for other commodities as well, with the exception of oil.

Casassus and Collin-dufresne (2005) characterise a three-factor model of commodity spot prices, convenience yield and interest rates, which nests previous specifications. They show that mean reversion in spot prices can be generated by a negative correlation between time varying risk premium and spot prices. The risk premium affects the cross-section of futures prices and the spot prices affect the time-series of futures prices. Their results imply that both dependent convenience yield and time varying risk premium contribute to the mean reversion futures prices, which is naturally the term structure of commodity prices.

Erb and Harvey (2006) believe that commodity futures returns drivers are the term structure of futures prices and the roll return. They argue that several existing theoretical frameworks cannot determine asset commodity prices. The CAPM model highlights the limitations of using beta to estimate the expected return but cannot prove the commodity and the expected excess returns are zero. The insurance perspective suggests normal backwardation in commodity returns; however, not all commodity futures have a positive mean return. The hedging pressure hypothesis attempts to explain the lack of empirical support for normal backwardation. The hedging pressure hypothesis is more flexible by allowing hedgers to adopt short futures contracts. Coincidentally, this framework lacks a reliable measure, which fails to apply in practice. The theory of storage implies that futures price is dictated by storage costs, interest rate and convenience yield. But the expected convenience yield is difficult to determine for long term investors. The empirical results indicate that the term structure is an indicator of the actual commodity price behaviour.

1.2.4. Momentum and Mean Reversion

The momentum and mean reversion effects were studied together in several markets as well. The current body of literature indicates that a combination of the momentum and mean reversion portfolio will outperform the pure momentum or pure mean reversion portfolio.

Balvers and Wu (2006) consider momentum and mean reversion jointly across global equity markets. They conclude that momentum and mean reversion occur in the same assets from their empirical results. They implement a simple trading strategy that draws on the combined promise for momentum and mean reversion in 18 national stock market indexes with significant excess returns. The strategy is neither purely contrarian nor purely momentum-based, instead it uses the information of all previous price observations to endogenously aggregate the mean reversion potential with the momentum potential into a single indicator. Their work first documented that the combined indicator of momentum and mean reversion can produce a better strategy performance than single indicators.

Based on Balvers and Wu (2006), the momentum and mean reversion effects are studied in foreign exchange markets (Serban, 2010). He implements a trading strategy combining mean reversion and momentum in foreign exchange markets and shows that this combined strategy outperforms traditional foreign exchange trading strategies, such as carry trades and moving average rules with five countries data. He also finds that UIP deviations follow mean reversion and momentum.

In the meanwhile, Fuertes et al. (2010) examine the combined role of momentum and term structure signals for the design of profitable trading strategies in commodity futures markets, which fill the literature gap in momentum and mean reversion joint effects. They incorporate the trading signals by pure momentum (Jegadeesh and Titman, 1993) and pure term structure (Erb and Harvey, 2006), and form a combined signal. They show that while the individual momentum and term structure strategies perform well, the combined signals are more informative for tactically allocating wealth. They also find that the returns of these novel double-sort strategies are weakly correlated with the returns of traditional asset classes, making them attractive candidates for inclusion in well diversified portfolios.

Besides the empirical studies of joint momentum and mean reversion, Koijen et al. (2009) expand the theoretical field. They solve a dynamic asset allocation problem in which stock returns exhibit short-run momentum and long-run mean reversion. They construct an elegant continuous-time model that captures these two predictability features and derive the optimal investment strategy explicitly. Their model predicts negative hedging demands for medium-term investors, and hence find that the optimal allocation to stocks is no longer monotone in the investment horizon: it first decreases, because momentum makes stocks riskier in the short run, and subsequently increases, because mean-reversion decreases risk in the long run. Momentum substantially increases the economic value of hedging time-variation in investment opportunities. These utility gains are preserved when we impose realistic borrowing and short-sales constraints and allow the investor to trade on a monthly frequency. The most significant contribution of this paper is constructing the dynamic asset allocation model, which incorporates both the momentum effect and mean-reversion effect.

Although the momentum and mean reversion effects were documented in the main financial markets such as equity, FX and commodity, all of these studies focus on 'cross sectional' momentum as well as on the 'cross sectional' mean reversion. In other words, present literatures compose cross sectional past 'winners' and past 'losers' momentum and mean reversion indicators rather than time series indicators. This paper would present the momentum and mean reversion joint effect in another scenario, which fully depends on assets' own returns and term structures and builds a joint trading signal incorporating the time series momentum and mean reversion effects.

1.3. Data

Our data consist of futures prices and volumes for 23 commodities, 10 developed equity indexes, 6 currencies, and 13 developed government bond futures. The dataset from DataStream International spans the period January 1, 1990 to April 15, 2016. The total sample size ranges from a low of 24 contracts in the beginning to a peak of 52 contracts from 2009. Appendix A provides details on each instrument and their data source.

We focus on the most liquid contracts and the second nearest to delivery contracts after it. The return series for each instrument is constructed as follows: each day we compute the daily excess return of the most liquid contract and compound the daily returns to a cumulative return index. This calculating method guarantees the liquidity of contracts and matches a better implementable strategy, which is consistent with Moskowitz et al. (2012).

The term structure of futures contracts refers to the price curve formed by the prices of futures contracts over various expiration months. The term structure slope for each instrument is constructed as follows: each day we compute the log price gap of the most liquid contract and the contract expiring after the most liquid contract, divided by the maturity date gap between these two contracts. This is consistent with (Bessembinder, et al., 1995).

Table 1.1 presents the summary statistics of the excess returns and term structure slope on our data set. The first and second columns report the contract name and start date as well as the exchanges names. The third and fourth columns report the annual mean and volatility of return for each instrument by asset classes. The fifth and sixth columns report the annual mean and volatility of term structure for each instrument. From Table 1.1, commodity, equity and currency asset classes all have positive, zero and negative excess mean returns and mean term structure. Only bonds have significant positive excess mean returns. Very importantly, the cross-sectional variation in volatility is substantial. Commodities and equities have much larger volatilities than bonds and currencies in annual return series while term structure volatilities keep a relative low level in all instruments.

This variation in the volatility profiles is crucial for the construction of our model, which accommodates both time series momentum and term structure effects; we should adjust the variables by their realised volatility to keep them on the same scale. Besides, we should accordingly risk-adjust the position on each individual futures contract, in order to avoid the results being driven by a few dominant assets as well.

Table 1.1 Summary statistics on futures contracts. Reported are the annualized mean and annualized volatility of return and term structure of futures contracts in the full sample from Jan 1990 to April 2016.

Commodity	Data Start Date	Annual Return	Volatility	Annual mean TS	Volatility
Energy					
Light Crude Oil	NYMEX Feb-1990	1.99%	36.12%	-1.49%	1.58%
Brent Crude Oil	NYMEX Mar-2003	0.89%	23.76%	-2.54%	0.62%
Heating Oil	NYMEX May-2006	-1.50%	21.61%	-2.87%	0.50%
Natural Gas	NYMEX Dec-1990	7.98%	81.73%	-18.98%	4.33%
RBOB Gasoline	NYMEX Jan-2006	-0.08%	24.32%	0.04%	1.62%
Metals					
Copper	COMEX Jan-1990	2.34%	26.26%	2.21%	0.56%
Gold	COMEX Jan-1990	0.02%	9.39%	-1.16%	0.12%
Platinum	NYMEX Jan-1990	-0.68%	18.31%	1.42%	0.65%
Silver	COMEX Jan-1990	-3.43%	21.48%	-1.77%	0.21%
Meat					
Feeder Cattle	CME Jan-1990	0.90%	17.19%	-0.64%	1.22%
Live Cattle	CME Feb-1990	1.82%	19.12%	-2.01%	2.40%
Lean Hogs	CME Dec-2001	1.19%	34.73%	-16.69%	4.00%
Grains					
Corn	CBOT Mar-1990	0.77%	35.35%	-25.54%	2.17%
Oats	CBOT Mar-1996	0.23%	38.43%	-8.36%	3.23%
Soybean Oil	CBOT May-2006	1.99%	14.78%	-3.59%	0.34%
Soybean Meal	CBOT Jul-2006	1.71%	19.74%	2.65%	0.90%
Soybeans	CBOT Jan-1990	1.91%	25.43%	-4.67%	1.25%
Wheat	CBOT Mar-1996	1.44%	28.06%	-20.42%	1.95%
Softs					
Cocoa	ICE Mar-1990	6.49%	30.54%	-16.47%	1.20%
Coffee	ICE Mar-1990	2.11%	37.80%	-20.87%	2.29%
Cotton	ICE Mar-1990	0.69%	48.91%	-16.34%	2.47%
Orange Juice	ICE Jan-1990	2.85%	35.04%	-12.59%	1.77%
Sugar	ICE Mar-1990	-3.14%	36.54%	1.40%	3.15%

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Currency Futures	Data Start Date	Annual Return	Volatility	Annual mean TS	Volatility
AUDUSD	CME June-2009	1.33%	6.38%	2.15%	0.30%
EURUSD	CME June-2009	-0.22%	4.67%	-0.08%	0.05%
GBPUSD	CME June-2009	0.32%	4.08%	0.15%	0.03%
CADUSD	CME June-2009	-1.04%	4.26%	-0.18%	0.09%
CHFUSD	CME June-2009	-0.99%	3.77%	0.19%	0.06%
JPYUSD	CME June-2009	0.14%	3.59%	0.18%	0.03%
Bond Futures					
Australia 10yr	ASX Mar-1990	0.72%	1.20%	0.42%	0.06%
Australia 3yr	ASX Mar-1990	0.89%	1.24%	0.77%	0.10%
Canada 10yr	MX Mar-1990	1.82%	5.46%	5.50%	1.12%
Euro 2yr	Eurex Mar-1999	0.24%	1.14%	1.08%	0.12%
Euro 5yr	Eurex Mar-1999	0.75%	2.94%	2.83%	0.40%
Euro 10yr	Eurex Mar-1999	1.33%	4.65%	3.64%	0.39%
Euro 30yr	Eurex Dec-2005	1.73%	7.67%	1.65%	0.36%
Japan 10yr	TSE Mar-1990	1.55%	4.11%	5.10%	0.33%
US 2yr	CBOT Sep-92	0.45%	1.24%	1.76%	0.22%
US 5yr	CBOT Sep-92	1.58%	3.81%	5.79%	0.64%
US 10yr	CBOT Mar-90	2.13%	6.10%	8.80%	0.84%
US 30yr	CBOT Mar-90	2.73%	10.47%	8.79%	1.58%
UK 30yr	NYSE LIFFE Mar-90	1.17%	8.04%	3.79%	1.46%
Equity Futures					
SPI 200	ASX June-2006	-0.46%	11.58%	-1.09%	0.22%
CAC 40	Monep Dec-1998	-0.88%	19.13%	0.68%	0.29%
Canada 60	MX Dec-1999	0.41%	14.55%	-0.55%	0.31%
DAX	Eurex Dec-1990	5.50%	22.61%	-8.40%	0.40%
FTSE 100	NYSE-LIFFE Mar-1990	2.20%	18.31%	-3.76%	0.61%
Hang Seng	SEHK Feb-1990	3.98%	25.72%	-0.13%	0.31%
IBEX 35	MEFK Dec-2001	-0.92%	17.76%	1.38%	0.25%
Nikkei 225	CME Dec-1990	-1.85%	24.16%	-2.89%	0.41%
SMI	Eurex Nov-1990	4.70%	18.28%	1.84%	0.49%
S&P 500	CME Mar-1990	6.04%	18.02%	-4.06%	0.47%

1.4. Methodology

1.4.1. Regression Analysis and Parameter Estimation

We start from examining the time series predictability of the time series momentum and term structure factors.

We regress the excess return $r_t(i)$ for instrument i in time t on its momentum term(MOM) and term structure term(TS) in time t-1. All terms are divided by their realised volatility in order to put them in the same scale. Since Moskowitz et al. (2012) and Baltas and Kosowski (2013) document time series momentum significance in the past year and indicate that 12 months is the most profitable lookback period. Following their frameworks, we take 12 months as the momentum lagging period.

$$\frac{r_t(i)}{\sigma_{t-1}(i)} = \alpha + \beta \frac{MOM_{t-1}(i)}{\sigma_{t-2}(i)} + \gamma \frac{TS_{t-1}(i)}{\sigma_{t-2}^{TS}(i)} + \varepsilon_t(i) \tag{1.1}$$

The momentum term MOM is defined by a standard moving average (MA) of past returns over $[\mathrm{t}, \mathrm{t} - \tau]$:

$$\mathrm{MOM}_t = \int_{t-\tau}^{t} r_\mu \, d\mu \tag{1.2}$$

Where τ is the lagging period and r_u is the return at time μ.

The mean reversion term TS is defined as term structure slope:

$$TS_t = \frac{\log(F(m)) - \log(F(n))}{T} \tag{1.3}$$

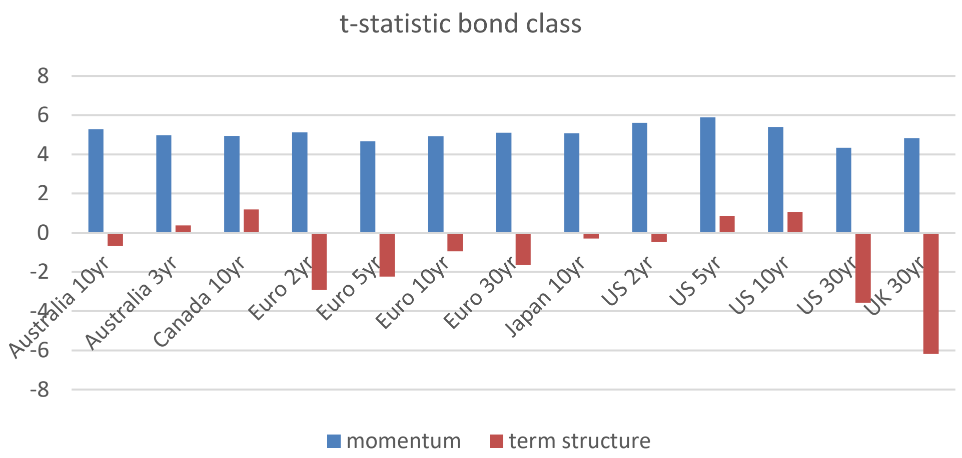
Where F(n) is the most liquid contract price at time t and F(m) is the second nearest to maturity contract price after contract n at time t, T is the contracts delivery date gap between contracts n and m.

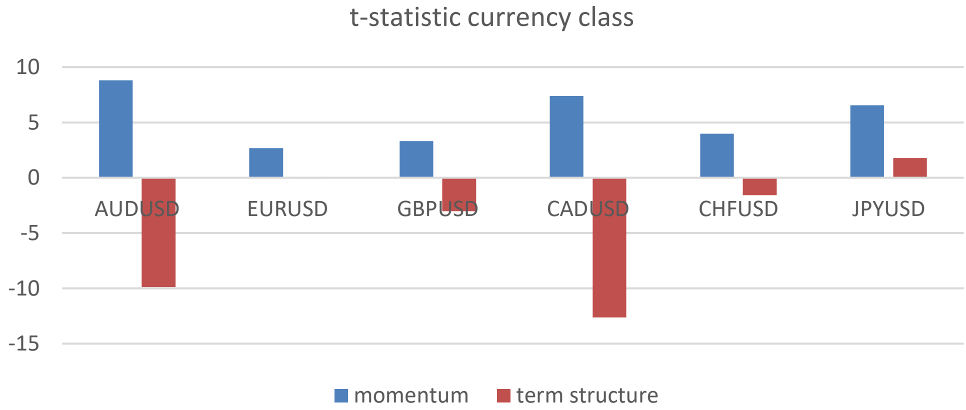
This is a simple reshape of the spot-future price parity formula: $F = Se^{(r+c-y)T}$. The spot market of futures is often highly illiquid, so we change the spot price S to the near contract price F(n) and future price F to the far contract price F(m) following Koijen et al. (2016). When TS>0, the market is in a cantango condition, investors benefit from holding a short position in the near contract; when TS<0, the market is in a backwardation condition, investors benefit from holding a long position in the near contract.

In the following study, we run a simple OLS regression on the MOM term and TS term of each instrument. The hypothesis test on β and γ tests the null hypothesis that it is equal to zero – meaning the corresponding term is not significant – versus the alternate hypothesis that the coefficient is different from zero.

We plot the t-statistics in Figure 1.1 of each instrument in the full sample, representing for estimators β and γ, which are divided by the corresponding estimated standard deviations. From Figure 1.1, we conclude the standard deviation normalized t-statistics of each β and γ. Since the sample size is very large, a t-distribution with infinitely many degrees of freedom is a normal distribution, so we compare t-statistics to the 95% confident interval values ± 1.96. Positive t-statistics of β in all instruments indicate a significant return continuation trend, thus significant time series momentum predictability. The negative t-statistics of γ in almost all instruments in the commodity class (except Brent crude oil and Gold) indicate significant reversals in the commodity class. Half of the bond and currency classes have significant negative t-statistics and none of the equity index instruments have significant t-statistics. This result leaves the argument whether term structure is an effective indicator of equity, bond and currency futures such as commodity futures.

Figure 1.1: t-statistic of momentum and term structure factors by asset class. The figure presents the t-statistics of β and γ for the linear regression $\frac{r_t(i)}{\sigma_{t-1}(i)} = \alpha + \beta \frac{MOM_{t-1}(i)}{\sigma_{t-2}(i)} + \gamma \frac{TS_{t-1}(i)}{\sigma_{t-2}^{TS}(i)} + \varepsilon_t(i)$. The dataset covers the period 1990 to 2016.





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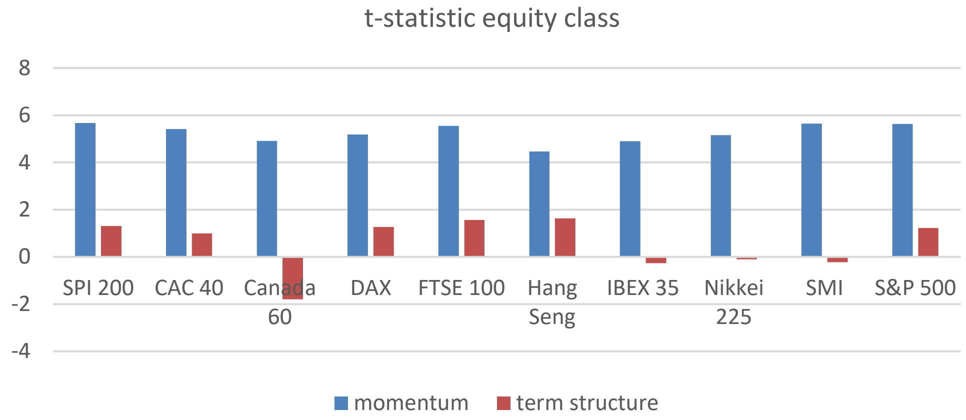
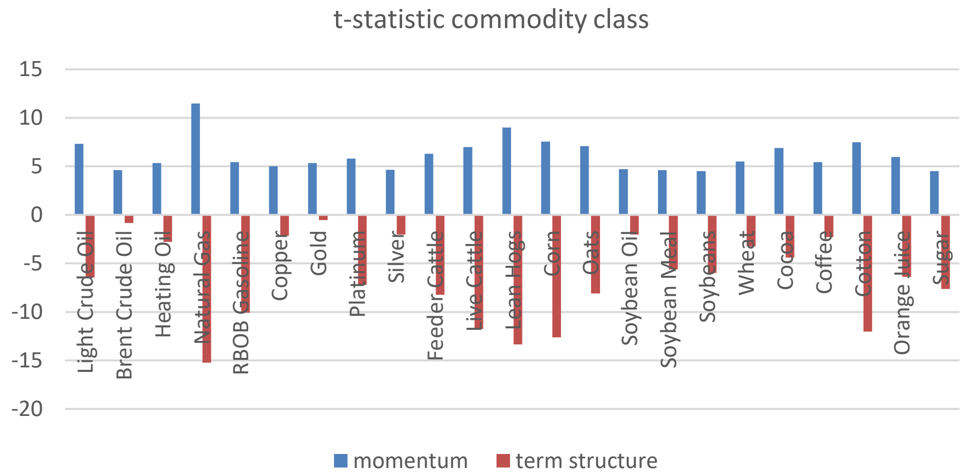




Stacking all futures contracts and dates, we run a pooled panel regression on commodity, bond, equity and currency asset class and the results are shown in Table 1.2. Table 1.2 shows the overall significance of the time series momentum and term structure in each asset class; the result is consistent with the univariate regression result in Figure 1.1. Although the univariate significance of the term structure factor is ambiguous in bond and currency futures, the significant overall coefficients of commodity, bond and currency futures from pooled OLS give us confidence to implement joint strategies in most instruments.

Table 1.2: coefficients from the pooling panel regression in four asset classes. The table reports pooled-OLS coefficients and significance in commodity, bond, currency and equity futures with full sample from Jan 1990 to April 2016.

	α	β	γ	R^2
Commodity	-5e^{-3}***	1.748***	-1.609***	0.014
Bond	2.11e^{-5}**	1.16***	-0.19***	0.005
Currency	4.1e^{-5}***	1.36***	-2.44***	0.009
Equity	9.94e^{-6}	1.08***	0.29	0.005

Now, we release the restriction of the fixed parameter of the MOM and TS factors. Using the rolling window method, we can adjust the parameters as time varies. Actually, the in-sample estimation and out-of-sample evaluation method are used in various financial markets, i.e. Goyal and Welch (2003), Lettau and Nieuwerburgh (2008) and Molodtsova and Papell (2009).

$$\frac{r_t(i)}{\sigma_{t-1}(i)} = \alpha_t + \beta_t \frac{MOM_{t-1}(i)}{\sigma_{t-2}(i)} + \gamma_t \frac{TS_{t-1}(i)}{\sigma_{t-2}^{TS}(i)} + \varepsilon_t(i) \tag{1.4}$$

We choose the window length arbitrarily, and roll the sub-sample one by one in the time series. The details of the parameter analysis are included in the next section.

1.4.2. Time Series Momentum and Term Structure Trading Strategy

We then construct the strategies based on momentum and term structure trading signals. Following Moskowitz et al. (2012) and Baltas and Kosowski (2013), the pure time series momentum strategy is investigated. For each instrument i, we consider whether the past τ months excess return is positive or negative and go long the contract if positive and short if negative. After scaling by realised volatility, we hold the position for h months. The pure time series momentum strategy results are shown in Table 1.3.

Erb and Harvey (2006) introduce a dynamic asset allocation strategy that seeks to exploit the term structure of commodity futures prices. Their strategy takes long positions in backwardated contracts and short positions in contangoed ones. Similarly to the time series momentum, we are focusing on the term structure profitability on the instrument itself rather than the portfolio. The pure time series term structure strategy return is simply the roll return of each instrument i with a holding period h.

Fuertes et al. (2010) show the low correlation between the cross sectional momentum and term structure strategies. They build the double sort strategies combining both trading signals with fixed predetermined weights. Our combined strategies based on the time series momentum and term structure allow the weight between two trading signals to vary over time. The combined trading signal strategy return is given as:

$$r_t^{com}(i) = \frac{Sign\left(\beta_t MOM_{t-1}(i) + \gamma_t TS_{t-1}(i)\right)}{\sigma_{t-1}(i)} r_t(i) \qquad (1.5)$$

1.5. Empirical Results

1.5.1. Strategies Performance

Before further implementation of this issue, we examine the average pairwise correlation between the momentum term and the mean reversion term in each asset class. The pairwise correlations are 0.02, -0.07, -0.05 and -0.08 in the commodity, bond, currency and equity sectors, respectively. These low correlations motivate our guess that joint momentum and term structure trading strategies can diversify portfolio risk and outperform single trading strategies. To compare the strategies performance, we first fix the lagging period of the momentum as 12 months in order to stay consistent with regression setting. Although various holding periods and lagging periods will give a different strategy performance, we select the most stable pair of a 12 month lagging period and 1 month holding period from Baltas and Kosowski (2013). Since the forecasting performance of the rolling scheme is sensitive to the choice of the window size (Rossi and Inoue (2012)), we use a 1 year, 3 year, 5 year and 10 year window size, and parameters ranging from high volatility to low volatility. Then, we require such window sizes as the warming data, and measure the out-of-sample parameters by rolling the initial window.

Table 1.3 lists the pure momentum, pure term structure and combined strategies performance in the commodity class and other asset classes, respectively. Out of the three strategies in the commodity asset class, the most profitable one is *combined-10Y* with an annualized return of 29.4% and Sharpe ratio of 1.56, while *MOM-1Y* is the bottom one with an annualized return of 18.6% and Sharpe ratio of 0.67. In the currency, bond and equity asset classes, the most profitable strategy is *combined-10Y* with 22.4% annual return and 0.84 Sharpe ratio while the least profitable one is *TS-5Y* with only 11.6% annual return and 0.43 Sharpe ratio. It is obvious that the pure term structure strategy performances are not as good as those in the commodity class. This is mainly attributed to the insignificant term structure factors in half of the currency and bond instruments and almost all equity instruments. This leaves the heuristic question of selecting the mean reversion factor in different asset classes.

Although the term structure factor is not significant in other classes except for commodity, we still show the implication of the tactical asset allocation on the joint trend-flowing and mean-reversion signals in the global futures market. The combined strategy always outperforms the MOM and TS strategies in commodity. The results show the superiority of combined strategies over the pure time series momentum or term structure strategies in the commodity class but the impracticability of the term structure strategies in other asset classes. The joint trading signals improve the portfolio performance in most cases.

Table 1.3: Strategies Performance. The table reports the annual return, annual volatility, maximum drawdown and annual Sharpe ratio of the pure time series momentum strategy, pure term structure strategy and combined strategy. The rolling window size is 1 year, 3 year, 5 year, and 10 year. The dataset covers Jan 1990 to April 2016

	1Y	3Y	5Y	10Y	1Y	3Y	5Y	10Y
Commodity								
Strategy	Annualized Return				Sharpe Ratio			
MOM	18.6%	19.9%	21.8%	24.1%	0.67	0.75	0.82	0.91
TS	21.1%	23.8%	24.1%	26.9%	0.89	1.05	1.11	1.35
Combined	**23.9%**	**25.2%**	**26.3%**	**29.4%**	**1.04**	**1.19**	**1.24**	**1.56**
Others								
Strategy	Annualized Return				Sharpe Ratio			
MOM	19.5%	18.1%	**18.9%**	21.2%	**0.73**	0.65	**0.69**	0.78
TS	13.6%	14.2%	11.6%	16.9%	0.51	0.55	0.43	0.62
Combined	**19.9%**	**19.2%**	16.8%	**22.4%**	0.71	**0.71**	0.62	**0.84**
	1Y	3Y	5Y	10Y	1Y	3Y	5Y	10Y
Commodity								
Strategy	Annualized Volatility				Maximum Drawdown			
MOM	23.5%	26.9%	26.6%	26.7%	-0.38	-0.32	-0.27	-0.27
TS	23.7%	22.7%	21.7%	19.7%	-0.29	-0.42	-0.38	-0.47
Combined	**22.9%**	**21.2%**	**21.2%**	**18.8%**	-0.34	-0.36	-0.33	-0.4
Others								
Strategy	Annualized Volatility				Maximum Drawdown			
MOM	26.7%	27.8%	29.4%	27.2%	-0.35	-0.4	-0.37	-0.28
TS	**26.6%**	**25.8%**	**27.0%**	27.3%	-0.71	-0.67	-0.93	-0.67
Combined	28.0%	27.1%	27.1%	**26.7%**	-0.52	-0.51	-0.59	-0.44

1.5.2. Parameter Analysis

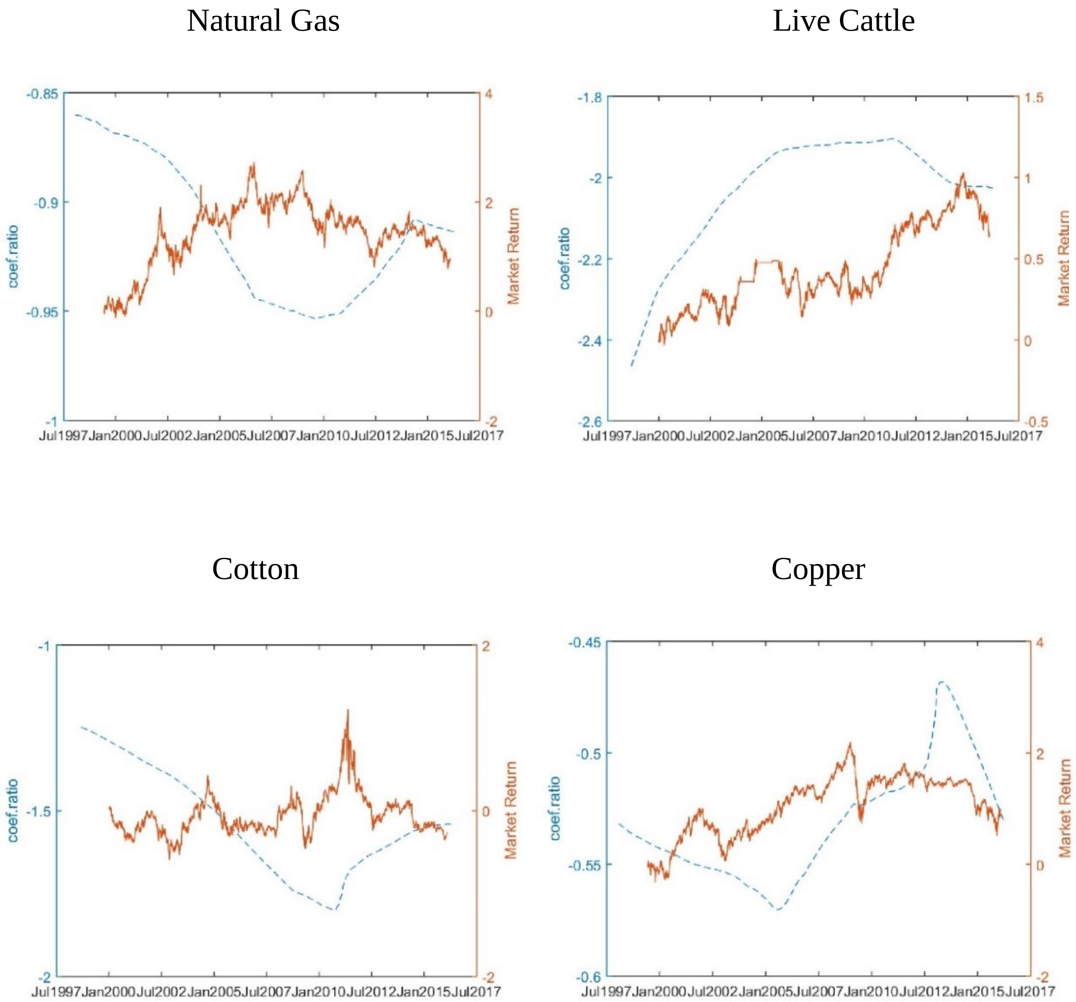
In this part, we investigate the parameter structure of the MOM and TS factors. It is important to explore the strategy efficiency in different market states and in different instruments. At first glance, we believe that the momentum effect dominates the trading signal during the extreme market periods, for instance, bull or bear market; while the term structure effect dominates in the normal periods, for example, sidewalk market. In order to test if the hypothesis is true, we plot the figures of two factors coefficients, and compare them with the market return figures.

Figure 1.2 plots the coefficient ratios between the momentum and term structure factors and market returns of five commodity instruments. The dotted line shows the coefficient ratio $\frac{\beta_t(i)}{\gamma_t(i)}$ of instrument i, and the solid line shows the market gross return of this instrument i. The momentum effect is getting stronger if the absolute value of the coefficient ratio increases, the term structure effect is getting stronger if the absolute value of the coefficient ratio decreases. The market state is either in extreme periods with a significant return trend or in normal periods with return sidewalk.

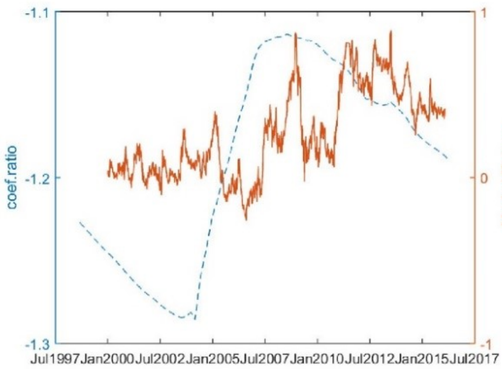
Instruments from different industries with different underlying assets have very different features; we show the figures of six instruments Natural Gas, Live Cattle, Cotton, Copper, Corn, and Oats from Energy, Meat, Soft, Metal and Grain industry, respectively. Except Corn and Oats from the Grain industry, other instruments have very similar features: the MOM effect significantly increases during the extreme market periods and the TS effect significantly increases during the normal market periods. The high correlation in the same industry leads to very similar coefficient ratio curves. This result perfectly corroborates our hypothesis only with one exception of the

Grain industry. The figures of Corn and Oats do not show the same feature like other instruments. In fact, all grain instruments show the heavy lag of coefficient power change when the market state changes. This difference in the Grain industry may be due to the under-reaction behaviour, seasonal structure changes or even the weather impacts. We leave this question to further study. Most commodity instruments except grains show the co-movement between the MOM and TS relative power and market return. This feature further confirms our estimation that the combination of both trading signals will improve the strategy performance.

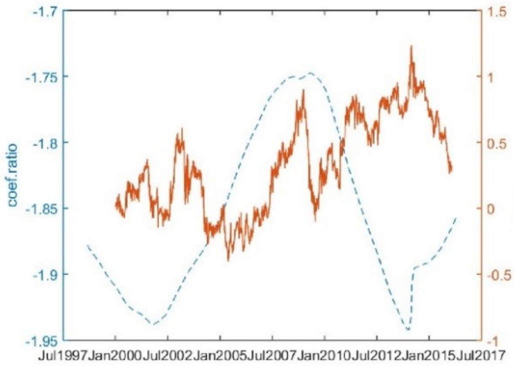
Figure 1.2: coefficient ratios between the MOM and TS factors vs. market return of six representative instruments. The blue dotted line is the coefficient ratio, the red solid line is the gross market return. The window size is 10 years.



Corn



Oats



1.6. Conclusion

In this chapter, we find the existence of the time series momentum and time series term structure in the commodity futures market with data spanning over the past 25 years. Almost all instruments are positive significant in the time series momentum and negative significant in the time series term structure slope. We fail to find the significant predictability of the term structure factor in equity, bond and currency futures. This result raises the mean reversion factor choice problem in different futures asset classes. We leave this question for future research.

In commodity futures, the individual time series MOM-only strategies earn an average 21% annual return and a 0.8 Sharpe ratio and the individual time series TS-only strategies earn an average 24% annual return and a 1.1 Sharpe ratio. The combined trading signal strategies are clearly superior to individual strategies with an average 27% annual return and a 1.25 Sharpe ratio. Our work extends the results from Fuertes et al. (2010) to another dimension, from cross sectional to time series.

Finally, the inner weight of the MOM and TS factors in the joint trading signal changes as market environment changes. The relative power of the MOM over TS experiences produces a significant increase during the extreme market periods with upward or downward trends, and moves back in the normal market periods with sidewalk returns. This feature is consistent with the intuitions of the MOM and TS factors. Investors could adjust their trading signal weights to smooth their net worth curve when the market environment changes.

Appendix A

Commodity Futures：

Our data set covers 23 commodity futures obtained from several exchanges. Cocoa, Coffee, Cotton, Orange Juice and Sugar are from the Intercontinental Exchange (ICE). Feeder Cattle, Live Cattle and Lean Hogs are from the Chicago Mercantile Exchange (CME). Corn, Oats, Soybeans, Soymeal, Soy Oil and Wheat are from the Chicago Board of Trade (CBOT). Light Crude Oil, Brent Crude Oil, RBOB Gasoline, Heating Oil, Natural Gas and Platinum are from the New York Mercantile Exchange (NYMEX). Copper, Gold and Silver are from the New York Commodities Exchange (COMEX). The data covers the period January 1990 to April 2016, with the minimum number of commodities being 15 at any point in time and all 23 commodities available after 2006.

All commodity futures we choose are from exchanges in developed countries. It avoids political risk and regulation risk at the most part. The sample also covers the most brisk traded commodities ranked by the volume of contracts traded and the annual turnover amount. All instruments in our dataset are in the top 20 in each sector (Metal, Agriculture, Energy, Meats and Softs).

Equity Index Futures：

Our data set covers 10 equity index futures, all from developed equity markets: SPI 200 (Australia), CAC 40 (France), DAX (Germany), Hang Seng (HK), TOPIX (Japan), AEX (Netherlands), IBEX 35 (Spain), SMI (Switzerland), FTSE 100 (UK) and S&P

500 (U.S). The sample covers the period January 1990 to April 2016, with the minimum number of equity indices being 6, and with all 10 indices being represented after 2006.

Price and volume data are obtained from DataStream. The returns on the country equity index futures do not include any returns on collateral from transacting in futures contracts, hence these are comparable to returns in excess of the risk-free rate or annual dividend yield. Our data follow the data used by Baltas and Kosowski (2013), deleting some relative short periods or illiquid contracts.

Currency：

Our data covers 6 currencies: the Australian dollar, AUD; the British pound, GBP; the Canadian dollar, CAD; the Japanese yen, JPY; the euro, EUR; the Swiss Franc, CHF. The data only cover the period June 2009 to April 2016 and it is all obtained from the Chicago Mercantile Exchange (CME).

Bond Futures：

Our bond futures data cover 13 contracts: 2-year, 5-year, 10-year 30-year US Treasury bill, 10-year Japanese government bond, Euro Schatz, Euro Bobl, Euro Bund, Euro Buxl, 3-year and 10-year Australia bond, 10-year UK and 10-year Canada government bond. The time spans January 1990 to April 2016, with the minimum number of bond futures being 7 at any point in time and all 13 commodities available after 2005.

Second Chapter

Multivariate Volatility Regulated Kelly Strategy: A Superior Choice in Low Correlated Portfolios

We propose a Multivariate Volatility Regulated Kelly strategy, which has extra penalization on variance compared to the Kelly criterion. The objective function is constructed and solved. We show the superiority of our method in relatively low correlated portfolios, relative to the fractional Kelly and full Kelly strategies. Our strategy reduces the short-term risk without sacrificing the growth rate to invest more in risk-free assets. Simulation results and Chinese commodity future empirical results strongly support our method.

Key words: Kelly, Low correlation, Portfolio.

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2.1. Introduction

Both the Kelly criterion and fractional Kelly strategies play important roles in asset allocation and portfolio investment. Early contributions to the theory and practice include Kelly (1956), Latane (1959), Breiman (1961), Thorp (1971) and Hakansson (1970). The Kelly criterion is widely applied in asset allocation by Browne (1997), Maclean et al. (1992), Maclean et al. (2004). Maclean et al. (2010) conclude that there are both good and bad properties of the Kelly criterion. Its main advantage, which maximizes the expected value of the logarithm of wealth period by period, is that it maximizes the limiting exponential growth rate of wealth. The main disadvantage is that its suggested wagers may be very large. Hence, the Kelly criterion can be very risky in the short term.

To overcome high risk shortcoming, Ziemba (2003) and Thorp (2006) proposed the fractional Kelly strategy: invest a proportion f of one's wealth in the Kelly portfolio and a proportion $1 - f$ in the risk-free asset. There are two key benefits of the fractional Kelly strategy: first, the volatility of a fractional Kelly portfolio is significantly less than that of the full Kelly portfolio. Second, fractional Kelly strategies are optimal with assumptions of the Merton model (Merton, 1971). In fact, theoretically, the fractional Kelly decreases volatility with a corresponding decrease in the long-run growth rate. The fractional Kelly is optimal in theory and often outperforms the full Kelly in empirical studies. Unfortunately, fractional Kelly strategies are no longer optimal when the log normality assumption is removed (Maclean, et al., 2005). Many attempts have been made to retain the optimality of fractional Kelly strategies in recent years. Extensions of fractional Kelly-like risk sensitive benchmark and the

Intertemporal Capital Asset Pricing Model (ICAPM) are used to guarantee optimality (Davis and Lleo, 2010).

In this paper, we propose a new direction for an improved Kelly strategy: a modified target function can outperform traditional Kelly or fractional Kelly strategies at both wealth growth and risk control side. Rather than maintain the optimality of fractional Kelly strategies, we are more interested in seeking better risk estimators and giving inspiration in Kelly portfolio management. We call our approach the Multivariate Volatility Regulated Kelly (MVRK) strategy, which indeed provides a modified covariance estimator that is more adapted to low correlated portfolios. MVRK is inspired by time series momentum which assumes that asset correlation can be ignored (Moskowitz, et al., 2012) and is partly connected to the covariance shrinkage method (Ledoit and Wolf, 2004).

Our results show that MVRK outperforms the full Kelly and fractional Kelly strategies in relatively low correlated portfolios. This is the main contribution of the paper. We have verified our findings in three ways. First, the theoretical intuition of MVRK is very clear and simple: volatility impacts on risk estimators are more important than correlation in low-correlated scenarios. Second, simulation results prove the advantage of MVRK in low correlation settings. Third, real data from the Chinese commodity market further support our method.

This paper is laid out as follows. In Section 2, we introduce the framework of Kelly portfolios and fractional Kelly portfolios in Merton assumptions. In Section 3, we show the methodology of the Multivariate Volatility Regulated Kelly strategy and explain the intuition compared with the full Kelly and fractional Kelly strategies. In Section 4, we simulate MVRK strategies and Kelly strategies with different correlation settings. Moreover, we show that MVRK is a better choice in low correlated portfolios. In Section 5, we use data from the Chinese commodity market and give empirical results with full Kelly, fractional Kelly and MVRK strategies. The empirical evidence further supports our method. In Section 6, we summarize the findings and make conclusions.

2.2. The Kelly Criterion Portfolio and Fractional Kelly Strategies with Risk Sensitive Control

2.2.1 The Kelly Criterion Portfolio

Assume we have a set of risky assets whose stochastic term is driven by K independent Wiener processes $W = (W_1, \dots, W_K)^T$, price dynamics then exhibit

$$\frac{dS_t}{S_t} = \mu dt + \Pi dW \quad (2.1)$$

where $S_t = (S_{1,t}, \dots, S_{N,t})^T$ is price vector of N assets at time t. μ is drift vector which has dimension of N. Π is a $N \times K$ matrix, capturing the effect of different sources of uncertainty. The independence of the coordinates of W can be assumed without loss of generality due to the matrix Π.

Referred to the property of the standard Brownian motion (Wiener Process), it is a stochastic process with Gaussian increments ($W_{t+\mu} - W_t \sim N(0, \mu)$). In the Kelly criterion, correlation and volatility are constants (like Black-Scholes Model), so there is no correlation existing between each Wiener process. If we introduce stochastic volatility into the scheme, the correlation between Wiener processes may be introduced.

Similarly, considering a portfolio policy by investing $f = (f_1, \dots, f_K)^T$ in each asset, our portfolio thus follows SDE

$$\frac{dA_t}{A_t} = (1 - f^T)rdt + f^T \mu dt + \Pi dW \tag{2.2}$$

Apply Ito's formula and define $N \times N$ covariance matrix $V = \Pi\Pi^T$

$$dlogA_t = \left(r - f^T r + f^T \mu - \frac{1}{2} f^T V f\right) dt + \Pi dW \tag{2.3}$$

The drift term is

$$D = r - f^T r + f^T \mu - \frac{1}{2} f^T V f \tag{2.4}$$

The spirit of the Kelly system then involves maximizing D with respect to f

$$\frac{\partial D}{\partial f_i} = r - \mu_i + \sum_{j \neq i}^{N} \mu_j + v_{i,i} f_i = 0 \tag{2.5}$$

Expand the portfolio to N assets, and the first order condition will give

$$f^* = (V)^{-1}(\mu - r\mathbb{I}) \tag{2.6}$$

where $\mathbb{I}$ is a $N \times N$ identity matrix. Substituting optimal f^* back in (4), the optimal return growth rate is

$$D(f^*) = r + \frac{(V)^{-1}(\mu - r\mathbb{I})^2}{2} \tag{2.7}$$

This is the explicit solution to the multivariate Kelly criterion portfolio. It is clear that the Kelly criterion maximizes the geometric growth rate. It is also clear that extremely high weights of wealth may be allocated to some assets with very high independence (low correlation) across the portfolio. Diversification is one of the most important things to consider for a portfolio manager when constructing his/her portfolios. It is reasonable to accept the assumption that assets with low correlation will be added into the portfolio. The abnormal high weights on some assets contribute to high volatility in the short run. This is also a very crucial motivation for us to propose MVRK.

If no correlation exists and $K = N$, equation (2.6) is simplified to

$$f = (Diag(V))^{-1}(\mu - r) \tag{2.8}$$

Its implication is that an investor should invest any risky asset based solely on her observation on mean return and variance. This conclusion is consistent with univariate case.

2.2.2 Fractional Kelly Strategies with Risk Sensitive Control

The fractional Kelly strategy is used to reduce this short-term risk and large wagers compared to the full Kelly strategy. We refer to betting less than Kelly as "fractional Kelly", which is simply a blend of Kelly and cash. However, this fractional Kelly is indeed a scaling of the full Kelly and reduces the risk at the expense of a lower growth rate.

We assume that asset prices are log normally distributed and that terminal portfolio wealth has a power utility function. When we introduce the relative risk aversion coefficient, the natural optimal solution from geometric Brownian motion is fractional Kelly policy. We apply Ito's lemma with the power utility function $E(\frac{A^{1+\gamma}}{1+\gamma})$, and this gives the optimal solution of the fractional Kelly.

From a fractional Kelly perspective, the risk sensitive asset model is the same as the fractional Kelly model (Davis and Lleo (2010)). Intuitively, the objective function in fractional Kelly is to add risk sensitive control to drift term D in the Kelly portfolio previously derived:

$$D - \frac{\gamma}{2} f^T V f \tag{2.9}$$

The explicit optimal solution to fractional Kelly will become

$$f^* = \frac{1}{\gamma+1} (V)^{-1} (\mu - r\mathbb{I}) \tag{2.10}$$

with the relative risk sensitive coefficient $\gamma \in (-1, \infty)$.

Substituting (2.10) into $D - \frac{\gamma}{2} f^T V f$, the optimal return growth rate is

$$D(f^*) = r + \frac{(V)^{-1}(\mu - r\mathbb{I})^2}{2(\gamma + 1)} \tag{2.11}$$

From the derivation above, we notice that the release in power utility assumptions actually breaks down the optimality of the Kelly criterion strategies. The explicit solution is not optimal only considering maximum of the portfolio growth rate and final portfolio wealth. Comparing (2.6) and (2.10), fractional Kelly strategies adjust the weight of risky assets and geometric growth rate in the same size. Hence, the reduction in the risk is reflected in the corresponding reduction in growth rate.

2.3. Multivariate Volatility Regulated Kelly

In our recent study, we found that mean-variance risk control is not always a good choice. When we invest either in low correlated portfolios or in different asset classes with relatively low correlation, the penalty of assets' own variance becomes more important than covariance.

We use the same assumptions for the MVRK as for the Kelly criterion and fractional Kelly strategies: asset prices are log normal distributed and follow a geometric Brownian motion. In the fractional Kelly strategy, we change the objective function of portfolio wealth, adding an extra risk control term. In MVRK, we also add one extra volatility control term. We now change the objective function in the Kelly criterion to

$$D - \frac{\theta}{2} f^T Diag(V) f \tag{2.12}$$

$\theta \in (-1, \infty)$ is the volatility regulation coefficient. Then the explicit solution is

$$f^* = \left(V + \theta Diag(V)\right)^{-1} (\mu - r\mathbb{I}) \tag{2.13}$$

Substituting (2.13) into (2.12), the growth rate of return now is:

$$D(f^*) = r + \frac{\left(V + \theta Diag(V)\right)^{-1} (\mu - r\mathbb{I})^2}{2} \tag{2.14}$$

Here $Diag(V)$ is the diagonal line, which is idiosyncratic risk inherent in individual asset. Put another way, $Diag(V)$ is volatility measure regardless of others' effect. The existence of the solution is explicit: $V + \theta Diag(V)$ should not be degenerate. Note that V is not invertible if $K < N$. This scenario can be true when we choose a redundant set. We call this "Multivariate Volatility Regulated Kelly" (MVRK). The implication of MVRK is clear: portfolio investment policy is restricted by an extra volatility regulation term and there is more aversion with regard to assets volatility and more weight put on the volatility regulation coefficient θ.

On the one hand, MVRK does not scale the portfolio weight, and will not affect the risky assets return, which is apparently the problem of the fractional Kelly strategy. On the other hand, MVRK can avoid extremely high weights allocated to assets and dramatically reduce the short-term risk, which is presented as the main disadvantage of the Kelly strategy.

2.4. Simulation

2.4.1 Simulation Assumptions and Settings

In this section, we will show the simulation results of the MVRK and Kelly criterion strategies. We assume that the risk free rate is zero - the fractional Kelly is equivalent to the full Kelly under this setting. Hence, we only need to compare MVRK with the full Kelly strategy[1]. The relationship between the portfolio correlation and strategy performance is the main target. Portfolio size, trading length and the variation of the volatility regulation coefficient are also considered. We state several assumptions first.

Assumption 1: Correlated asset prices follow a geometric Brownian motion.

We allow assets to have correlations in the simulation. This is an important topic in quantitative finance, as it can be applied to simulating assets held in a portfolio that are dependent on one another to determine the underlying risk of the portfolio. A geometric Brownian motion model is used to keep consistent with the full Kelly, fractional Kelly and MVRK assumptions.

Assumption 2: Portfolio returns have zero mean but fluctuate slightly.

We set the portfolio average annual return to be equal to zero and allow some assets to have positive returns and others to have negative returns. In specific markets, asset annual returns are different, i.e. S&P 500 has a 10% annual return since its inception, while some futures markets can even have a negative annual return. In this paper, the major discussing point is risk estimators, so we suppose that we always invest in a zero mean portfolio.

Assumption 3: Asset returns have a fixed volatility level.

In practice, asset annual volatility can range from very low (less than 10% in bonds) to medium (ranging from 10% to 30% in equities) and to very high (more than 30% in commodities). We show simulation results for the MVRK and Kelly strategies with fixed 15% annual volatility. We fix the return and volatility level to eliminate any disturbance to correlation effects variation on strategies. Then we give some settings for the simulation.

Simulation Setting 1: Control correlation level $\rho_{i,j}$ from low 0.1 to 0.7, with 0.1 step length.

Suppose we have a correlation matrix, denoted C.

$$C = \begin{pmatrix} 1 & \rho_{1,1} & \cdots & \rho_{1,n} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n,1} & \rho_{n,2} & \ldots & 1 \end{pmatrix}$$

Simulation Setting 2: Control the portfolio dimension as 20, 30 and 40.

Simulation Setting 3: Control trading length as 1250 trading days, 2500 trading days and 7500 trading days.

Simulation Setting 4: Simulation times = 1000.

Simulation Setting 5: Set the fraction allocated in Diag(V) as $p = \frac{\theta}{1+\theta}$, and the fraction allocated in V as $q = \frac{1}{1+\theta}$. We control the value of p from 0 to 1 with 0.01 step length, adjusting the volatility regulation coefficient from 0 to ∞.

In the case where $p = 0$, MVRK is equivalent to the full Kelly strategy which means there is no extra restriction on the idiosyncratic variance on each asset.

In the case where $p = 1$, MVRK actually collapses to a time series momentum (TSMOM) strategy. In our empirical study, it is also valid to assert that time series momentum portfolios maximize the median fortune and geometric growth with the assumption of independence across assets.

If the cross-sectional correlation is not negligible, however, the TSMOM portfolio fails. In the futures market, it might be safer to accept heterogeneity of commodities, which are easily violated in the stock market. As an indication of further research, we may change the Kelly criterion to "Correlation Regulated Kelly" in highly correlated portfolios. In this scenario, we just range the volatility regulation coefficient θ in $(-1, 0)$.

2.4.2 Simulation Results

In our experiments, we examine the relative performance of the full Kelly and MVRK strategies. The nine simulations vary asset numbers with 20, 30 and 40, and observation numbers with 1250, 2500 and 7500 successively. We compare the Sharpe ratio and Value at Risk in the two methods and show the probability that the MVRK strategies have higher Sharpe ratio and higher Value at Risk. It is not necessary to give the absolute values of Sharpe ratio and Value at Risk. For one thing, the simulation assumptions naturally give uncontrollable random effects on the portfolio, and hence on the strategy performance. For another, the primary purpose of the simulation is to compare the full Kelly and MVRK in different correlation settings.

First, we display the results with $p = 0.5$, while in this setting, the volatility regulation coefficient $\theta = 1$, which is an intermediate intensity coefficient. Under this scenario, it is very clear to overlook the relationship between correlation and strategy performance.

Table 2.1 clearly reports the correlation effects on the Kelly strategy and MVRK strategy. The monotonous decrease in the winning probability of the MVRK strategies can be observed in both the Sharpe ratio and VaR, with an increasing correlation level from 0.1 to 0.7. In different portfolio dimension and observation numbers, the monotone pattern always holds. When observation numbers increase from 1250 to 7500, the monotonous decrease in the winning probability function is more convex. Therefore, the correlation effects on the MVRK and full Kelly strategies become more significant when the observation number is larger. We fail to find significant effects of portfolio dimensions. MVRK strategies keep leading positions (winning probability exceeds 50%) in all simulations where the correlation is less than 0.4. Hence, MVRK is superior in low correlated portfolios while Kelly is better in high correlated portfolios.

Table 2.1: Simulation results of MVRK and Kelly strategies. Correlation level of the portfolio varies from 0.1 to 0.7. The table summarizes Sharperatio and VaR winning probability of MVRK strategies. $p = 0.5$.

Dimension :20	Obervation:1250		Obersvation:2500		Obersvation:7500	
Correlation\MVRK Win Prob	Sharpe ratio	VaR	Sharpe ratio	VaR	Sharpe ratio	VaR
0.1	0.74	0.73	0.83	0.82	0.97	0.96
0.2	0.61	0.62	0.68	0.68	0.86	0.86
0.3	0.54	0.54	0.58	0.60	0.70	0.72
0.4	0.50	0.51	0.47	0.50	0.52	0.56
0.5	0.38	0.40	0.35	0.39	0.30	0.36
0.6	0.27	0.30	0.20	0.24	0.10	0.14
0.7	0.15	0.19	0.07	0.10	0.001	0.01
Dimension :30	Obervation:1250		Obersvation:2500		Obersvation:7500	
Correlation\MVRK Win Prob	Sharpe ratio	VaR	Sharpe ratio	VaR	Sharpe ratio	VaR
0.1	0.70	0.69	0.77	0.78	0.89	0.88
0.2	0.58	0.59	0.64	0.64	0.77	0.76
0.3	0.54	0.51	0.55	0.56	0.62	0.62
0.4	0.48	0.47	0.47	0.48	0.46	0.49
0.5	0.40	0.40	0.35	0.37	0.29	0.31
0.6	0.30	0.30	0.19	0.24	0.09	0.12
0.7	0.13	0.17	0.04	0.08	0.002	0.01
Dimension :40	Obervation:1250		Obersvation:2500		Obersvation:7500	
Correlation\MVRK Win Prob	Sharpe ratio	VaR	Sharpe ratio	VaR	Sharpe ratio	VaR
0.1	0.69	0.68	0.75	0.75	0.90	0.90
0.2	0.60	0.60	0.64	0.64	0.80	0.79
0.3	0.57	0.58	0.61	0.60	0.74	0.74
0.4	0.52	0.54	0.60	0.60	0.64	0.64
0.5	0.50	0.53	0.49	0.50	0.51	0.53
0.6	0.39	0.41	0.36	0.40	0.31	0.37
0.7	0.29	0.31	0.17	0.22	0.04	0.09

Actually, the monotonous decreasing pattern can be observed in different MVRK strategies. In another words, with different volatility regulation coefficient θ, we can still show that MVRK strategies perform better in low correlated portfolios.

[Insert Figure 2.1 and Figure 2.2]

Figure 2.1 and Figure 2.2 plot the 3D-curves of the correlation effects on the Sharpe ratio and VaR winning probability that vary the fraction $p = \frac{\theta}{1+\theta}$ allocated in Diag(V) from 0.01 to 1 with a step of 0.01. The monotonous decreasing pattern happens in most cases except for negligibly small p (less than 0.1). The results are generally consistent with the p=0.5 results in Table 2.1, and hence consistent with the conclusion in Figure 2.1.

A very interesting finding, which is different from 20 and 30 dimension situations, is that in the 40-dimension portfolio, the MVRK strategies present very low winning probabilities with high p values even when the correlation is as small as 0.1. Looking back to the case $p = 1$, an MVRK strategy is equivalent to a TSMOM strategy and no correlation is considered at all. The sharply decreasing performance of the high-p-MVRK strategies even in low correlation implies that it may be safer not to use pure TSMOM strategies in a portfolio including large numbers of assets. In contrast, when we have very small p values, but with a high correlation, the MVRK strategies still can give higher winning probability in both Sharpe ratio and VaR. This phenomenon contrasts with our intuition: MVRK performs better in low correlated portfolios, while Kelly performs better in high correlated portfolios. However, it brings about a better risk estimator composed of a proper combination between variance and covariance estimators. The simulation results also reiterate the importance of the shrinkage method proposed by Ledoit and Wolf (2004).

2.5. Empirical Evidence from China Commodity Market

2.5.1 Data Set

We use closing prices for 37 liquid China commodity futures since Chinese commodity market now has the largest trading volume (Shanghai institute of futures and derivatives, 2016). The data spans the period from January 1, 2000 to July 11, 2016. The total sample size ranges from a low of 4 contracts at the beginning to a peak of 37 contracts from 2014.

The average pairwise correlation of return in our sample is about 26.5%. We use interest rate published by the People's Bank of China as fixed risk free rates[2]. The summary statistics are provided in Table 2.2.

Table 2.2: Summary statistics on Chinese commodity futures contracts. The annualized mean returns and volatility (standard deviation) of the futures contracts are reported in our sample from January 2000 to July 2016.

Commodity Futures Code	Annualized Return	Annualized Volatility	Commodity Futures Code	Annualized Return	Annualized Volatility
			JD	2.36%	23.84%
A	3.63%	17.99%	JM	-18.17%	20.68%
AG	-8.16%	21.80%	L	-3.98%	23.19%
AL	-2.21%	13.41%	M	2.70%	20.45%
AU	3.41%	18.71%	MA	-11.53%	19.89%
B	3.99%	19.28%	NI	-21.35%	24.19%
BB	-10.52%	32.69%	OI	-4.27%	18.94%
BU	-32.73%	24.72%	P	-6.68%	22.33%
C	2.21%	13.32%	PP	-12.60%	22.90%
CF	-0.06%	16.85%	RB	-5.74%	17.51%
CS	-26.63%	21.44%	RM	1.67%	21.47%
CU	1.18%	21.09%	RU	-1.84%	25.02%
FB	-22.75%	52.28%	SR	2.06%	18.98%
FG	-7.05%	19.75%	TA	-7.34%	22.17%
FU	1.63%	28.35%	V	-2.03%	15.35%
HC	-11.39%	19.59%	WH	3.20%	13.73%
I	-32.31%	26.49%	Y	1.82%	19.04%
IF	-1.22%	27.78%	ZC	-10.14%	15.93%
J	-17.68%	19.98%	ZN	-6.49%	22.92%

2.5.2 Strategy Construction

From (2.6), (2.10) and (2.13), investors can make portfolio decisions based on full Kelly, fractional Kelly and MVRK strategies respectively. We use past returns and calculate realized variance-covariance. Investors themselves can decide lookback periods. In our empirical study, we compare full Kelly strategies vs. MVRK strategies and fractional Kelly strategies vs. fractional MVRK strategies. Relative risk sensitive coefficient γ and volatility regulation coefficient θ vary from zero to positive infinity. We keep simulation setting 5 for MVRK, but change the step length to 0.1. Similarly, in fractional Kelly strategies, we set the fraction allocated to risky assets as $m = \frac{\gamma}{1+\gamma}$, and the fraction allocated to risk-free assets as $n = \frac{1}{1+\gamma}$. We build long-short strategies by introducing an additional weight constraint:

$$|f_1| + |f_2| + |f_3| + \cdots + |f_n| = 1 \tag{2.15}$$

We compare the annual return, Sharpe ratio, maximum drawdown, 5% value at risk and conditional 5% value at risk for these four strategies.

Table 2.3: Statistic summary of MVRK, Full Kelly, Fractional MVRK and Fractional Kelly strategies performance. The transaction cost is set as five in ten thousand per unit. $p = \frac{\theta}{1+\theta}$ is the fraction allocated Diag(V) and $m = \frac{\gamma}{1+\gamma}$ is the fraction allocated in the full Kelly strategy.

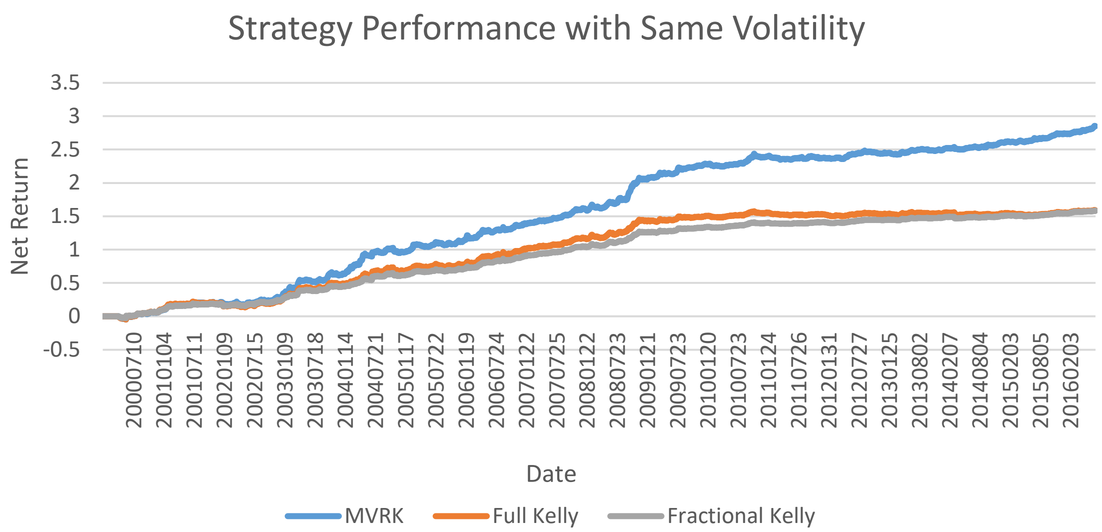
Transaction cost:0.05%	***MVRK***	***Full Kelly***	***Fractional MVRK***	***Fractional Kelly***	$p = m$
Annual return	13.86%	10.34%	12.90%	**9.73%**	0.1
Sharpe ratio	1.44	1.01	1.43	**1.01**	
Annual return	15.61%	10.34%	13.34%	9.13%	0.2
Sharpe ratio	1.60	1.01	1.60	1.01	
Annual return	16.79%	10.34%	13.04%	8.52%	0.3
Sharpe ratio	1.69	1.01	1.69	1.01	
Annual return	17.75%	10.34%	12.36%	7.92%	0.4
Sharpe ratio	1.76	1.01	1.75	1.00	
Annual return	18.57%	10.34%	11.43%	7.31%	0.5
Sharpe ratio	1.80	1.01	1.79	1.00	
Transaction cost:0.05% *Annual return*	19.35%	10.34%	10.31%	6.71%	0.6
Sharpe ratio	1.84	1.01	1.82	0.99	
Annual return	20.01%	10.34%	9.00%	6.10%	0.7
Sharpe ratio	**1.85**	1.01	**1.83**	0.98	
Annual return	20.64%	10.34%	7.56%	5.50%	0.8
Sharpe ratio	1.85	1.01	1.82	0.97	
Annual return	21.24%	10.34%	5.98%	4.89%	0.9
Sharpe ratio	1.82	1.01	1.75	0.91	
Annual return	**21.67%**	10.34%	4.35%	4.35%	1.0
Sharpe ratio	1.75	1.01			

Transaction cost:0.05%	***MVRK***	***Full Kelly***	***Fractional MVRK***	***Fractional Kelly***	$p = m$
MaximumDrawdown	8.50%	8.65%	7.04%	7.43%	0.1
5% VaR	-0.51%	-0.44%	-0.45%	-0.40%	
cVaR	-0.82%	-0.73%	-0.73%	-0.65%	
MaximumDrawdown	8.35%	8.65%	5.83%	6.21%	0.2
5% VaR	-0.53%	-0.44%	-0.42%	-0.35%	
cVaR	-0.87%	-0.73%	-0.69%	-0.58%	
MaximumDrawdown	**8.10%**	8.65%	5.13%	5.00%	0.3
5% VaR	-0.56%	-0.44%	-0.39%	-0.30%	
cVaR	-0.91%	-0.73%	-0.63%	-0.50%	
MaximumDrawdown	8.43%	8.65%	4.35%	3.87%	0.4
5% VaR	-0.60%	-0.44%	-0.35%	-0.26%	
cVaR	-0.95%	-0.73%	-0.56%	-0.43%	
MaximumDrawdown	8.89%	8.65%	3.60%	3.15%	0.5
5% VaR	-0.62%	-0.44%	-0.30%	-0.21%	
cVaR	-0.98%	-0.73%	-0.48%	-0.36%	
MaximumDrawdown	9.48%	8.65%	2.84%	2.42%	0.6
5% VaR	-0.63%	-0.44%	-0.24%	-0.17%	
cVaR	-1.01%	-0.73%	-0.40%	-0.28%	
MaximumDrawdown	10.26%	8.65%	2.04%	1.71%	0.7
5% VaR	-0.67%	-0.44%	-0.19%	-0.12%	
cVaR	-1.05%	-0.73%	-0.30%	-0.21%	
MaximumDrawdown	11.36%	8.65%	1.29%	1.03%	0.8
5% VaR	-0.69%	-0.44%	-0.12%	-0.08%	
cVaR	-1.09%	-0.73%	-0.21%	-0.13%	
MaximumDrawdown	12.74%	8.65%	0.59%	0.44%	0.9
5% VaR	-0.74%	-0.44%	-0.06%	-0.03%	
cVaR	-1.15%	-0.73%	-0.10%	-0.06%	
MaximumDrawdown	14.57%	8.65%			1.0
5% VaR	-0.80%	-0.44%			
cVaR	-1.24%	-0.73%			

From Table 2.3, as the value of p increases from 0.1 to 1.0, the volatility regulation coefficient increases from very small to very big. Even the modest MVRK strategy with $p = 0.1$ has a greater Sharpe ratio (1.44>1.01) and a smaller maximum drawdown (8.50 %< 8.65%) compared to the full Kelly strategy. When p=0.7, the MVRK strategy has the highest Sharpe ratio of 1.85 and a double annual return compared to the full Kelly strategy, with only 20% more maximum drawdown and value at risk. Moreover, if we scale the annual return to the same level, the MVRK strategies also show a better VaR and cVaR. In all other scenarios, MVRK strategies keep dominating superiority to the full Kelly strategy. Therefore, MVRK strategies always beat the full Kelly strategy in Chinese commodity markets.

The fractional Kelly strategies have a monotonously decreasing annual return and maximum drawdown when m is increasing, indicating that more weight is transferred from risky to risk free assets and that the portfolio becomes less risky at the expense of less wealth growth. Scaling the annual return to the same level for the fractional MVRK and fractional Kelly strategies, the statistics always show a higher return and a lower risk for the MVRK strategies. Even when $p \neq m$, the MVRK strategies always have a superior risk adjusted return compared to the fractional Kelly strategies. Hence, the MVRK strategies always beat the fractional strategies in Chinese commodity markets. An example with $p = m = 0.5$, the same risk adjusted strategy performance is given in Figure 2.3.

Figure 2.3: Plots of cumulative net return to the MVRK, Full Kelly and Fractional Kelly strategies. An equal volatility target is applied in three strategies.



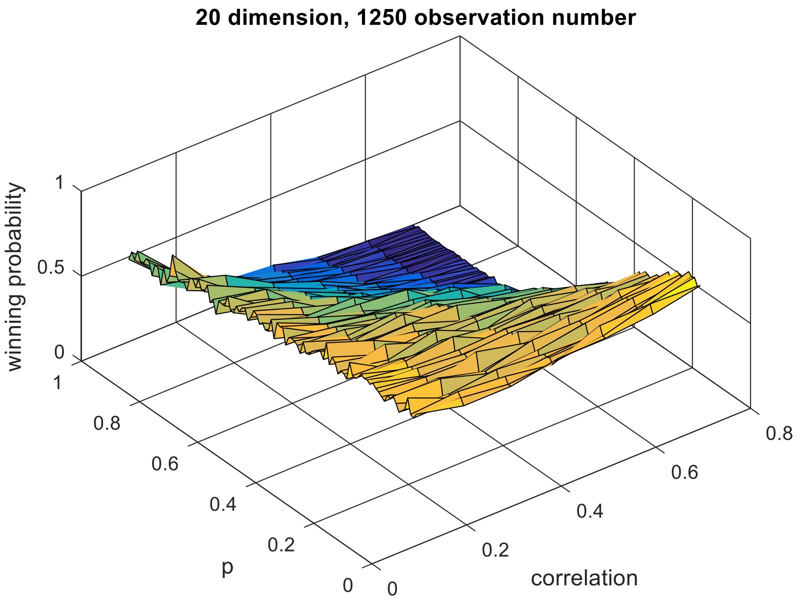
In our sample of Chinese commodity markets, the average pairwise correlation level is 22.7%, implying a relatively low correlated market. The empirical results significantly show that MVRK outperforms full Kelly and fractional Kelly in relatively low correlated portfolios.

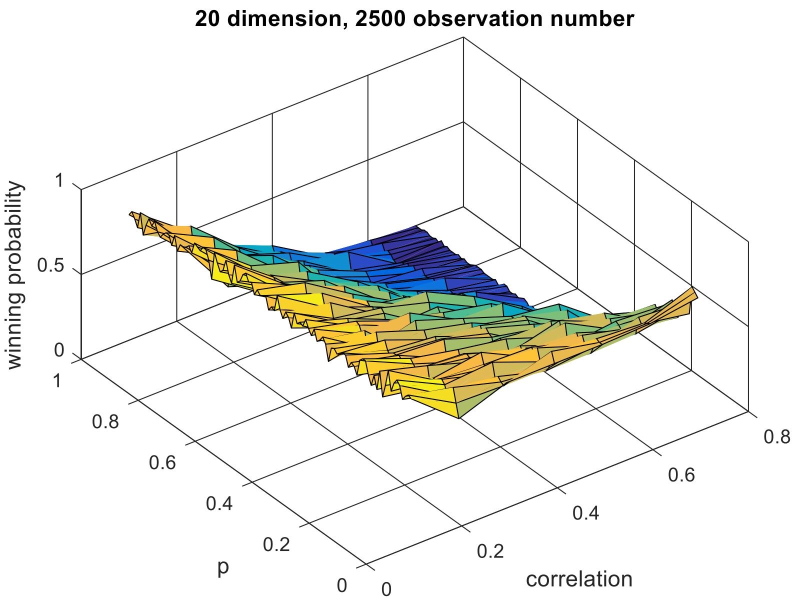
2.6. Conclusion

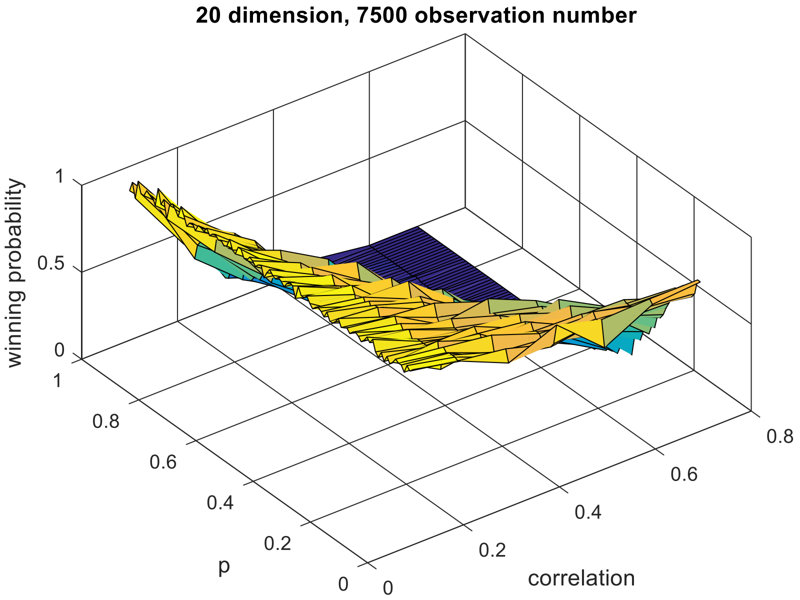
This paper proposes a modified Kelly strategy called Multivariate Volatility Regulated Kelly that outperforms the full Kelly and fractional Kelly strategies in low correlation scenarios. We claim that combining covariance and variance estimators will result in better Kelly strategies in low correlated portfolios. The simulation results show the superiority of MVRK in low correlation settings. The Chinese commodity market further provides empirical evidence of MVRK, and we believe that the global futures market can obtain a similar result.

Appendix B

Figure 2.1: Fixed dimension and observation numbers, Sharpe ratio winning probability varying transformed volatility regulation coefficient p from 0.01 to 1. The lines along the x-axis are winning probability curves in fixed p. The lines along the y-axis are winning probability curves at a fixed correlation level.



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winning probability
1
0.5
0
p
1
0.8
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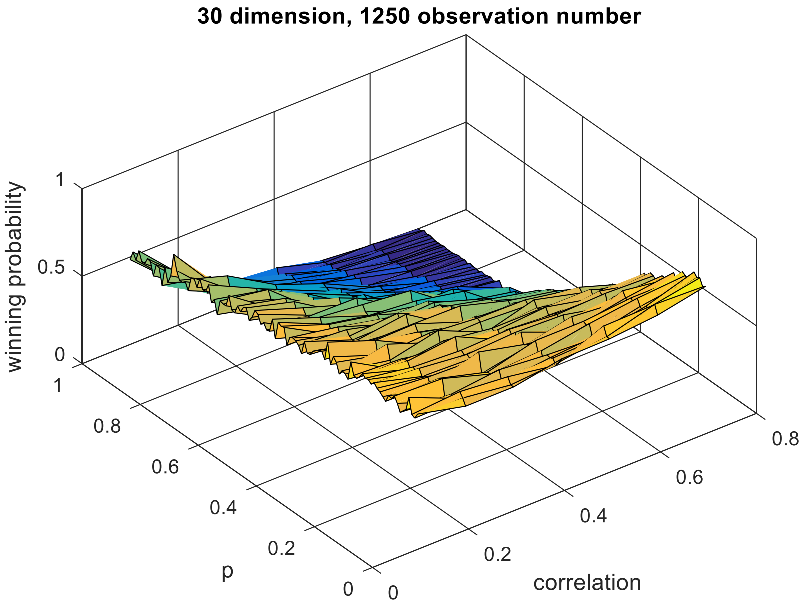
0.4
0.2
0
correlation
0
0.2
0.4
0.6
0.8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p
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0.8
0.6
0.4
0.2
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correlation
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0.2
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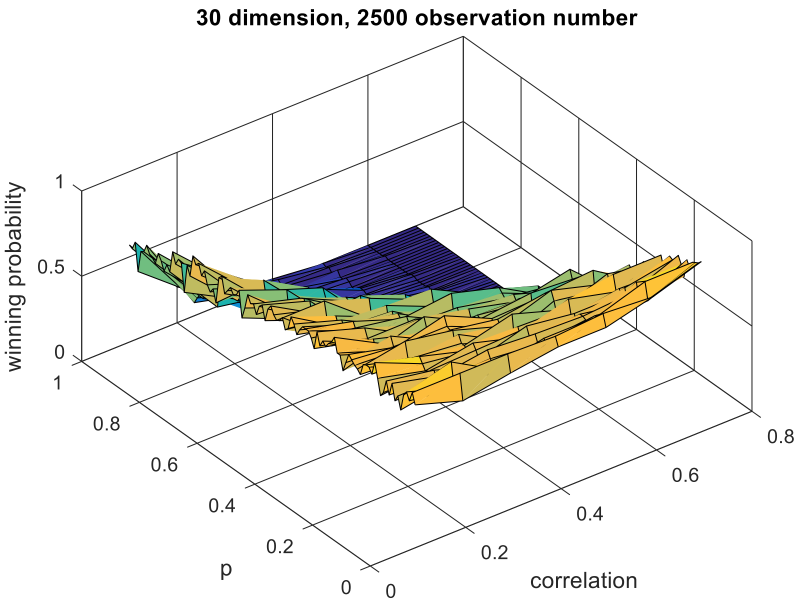
0.6
0.8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1
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p
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correlation
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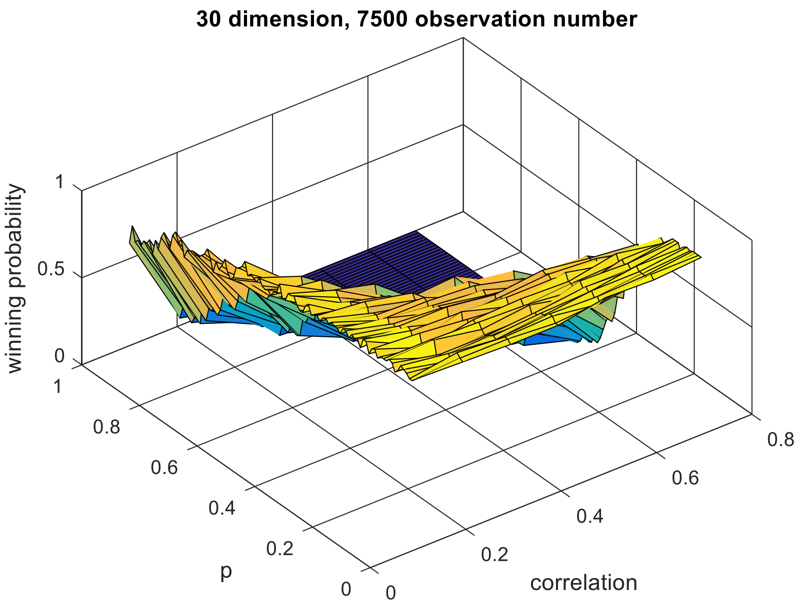
30 dimension, 7500 observation number
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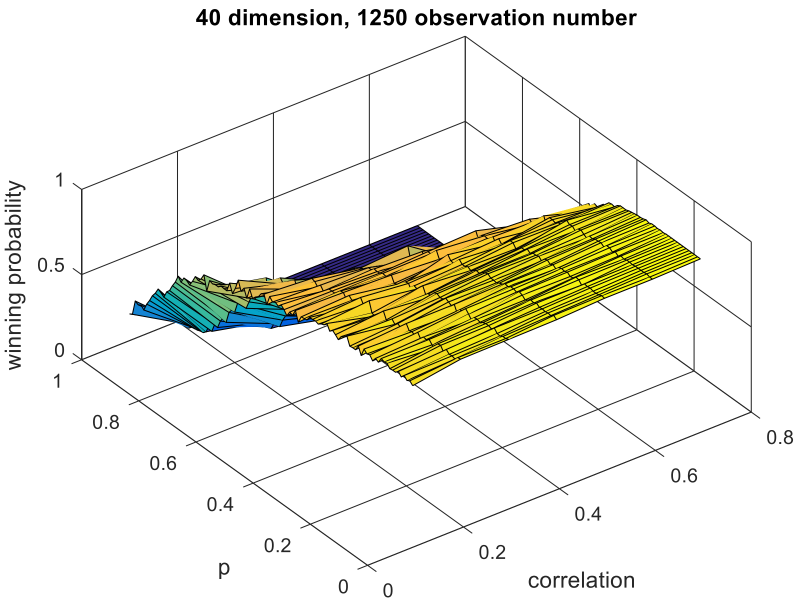
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winning probability
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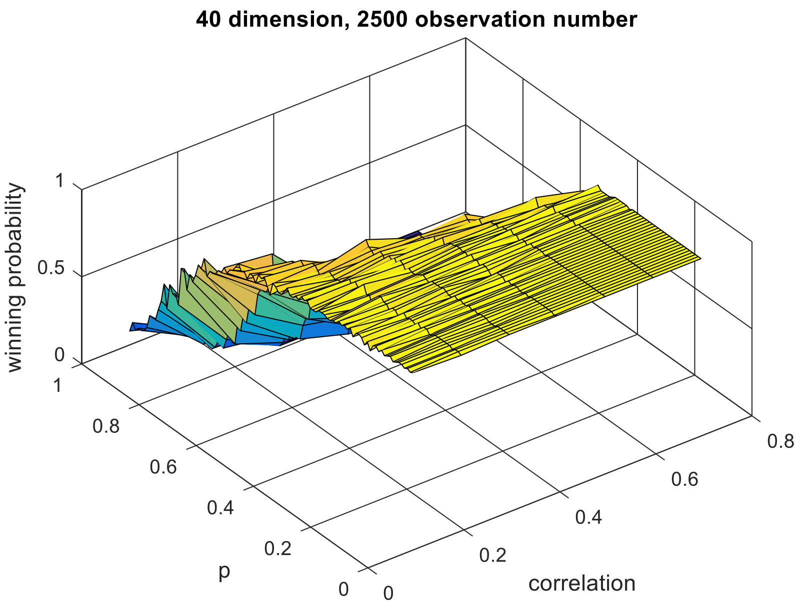
40 dimension, 2500 observation number
winning probability
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correlation
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0.8

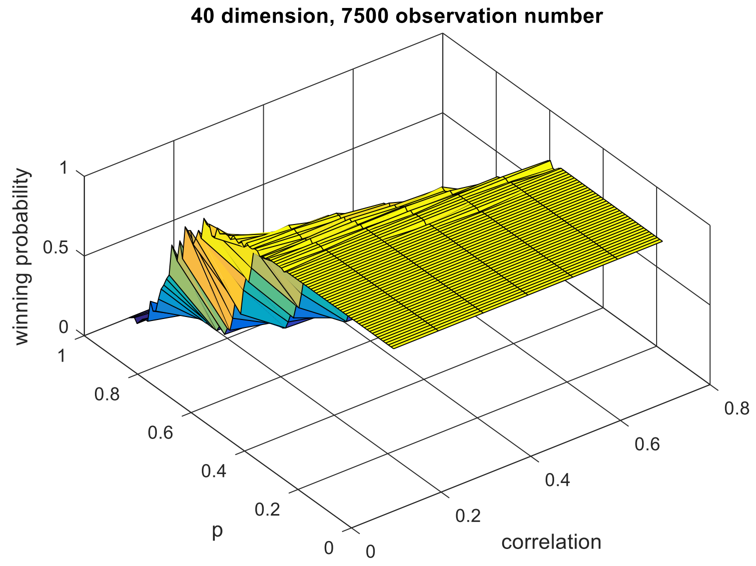
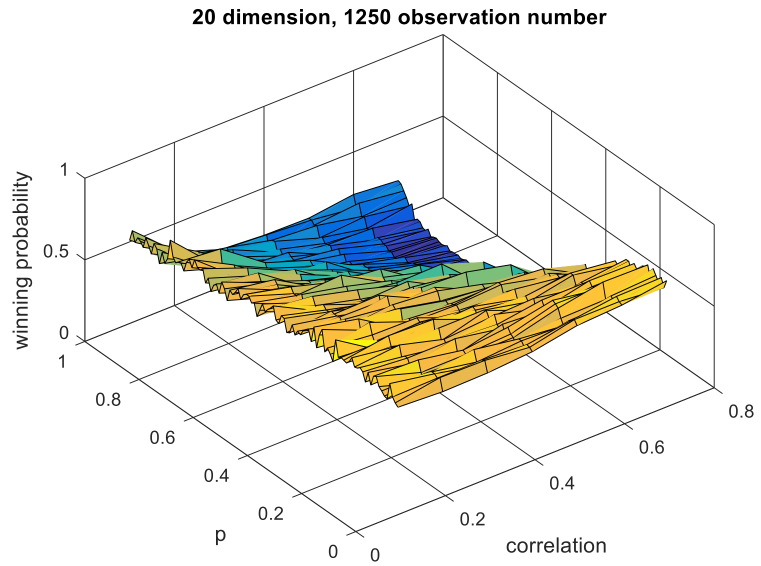
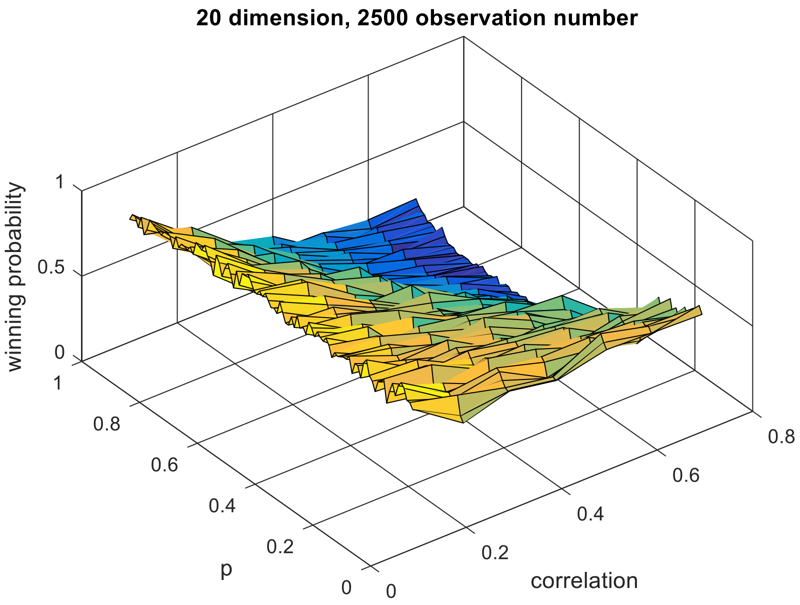


Figure 2.2: Fixed dimension and observation numbers, VaR winning probability varying transformed volatility regulation coefficient p from 0.01 to 1. The lines along the x-axis are wining probability curves in fixed p. The lines along the y-axis are wining probability curves at a fixed correlation level.

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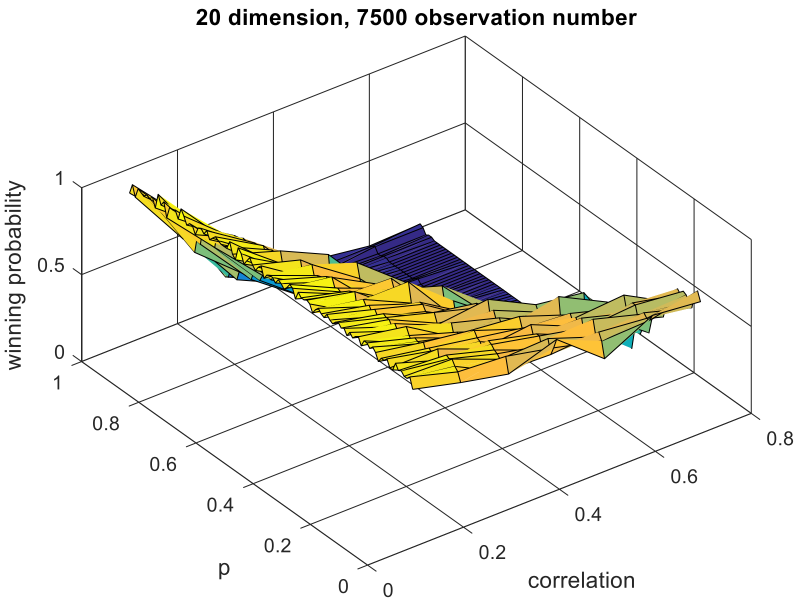
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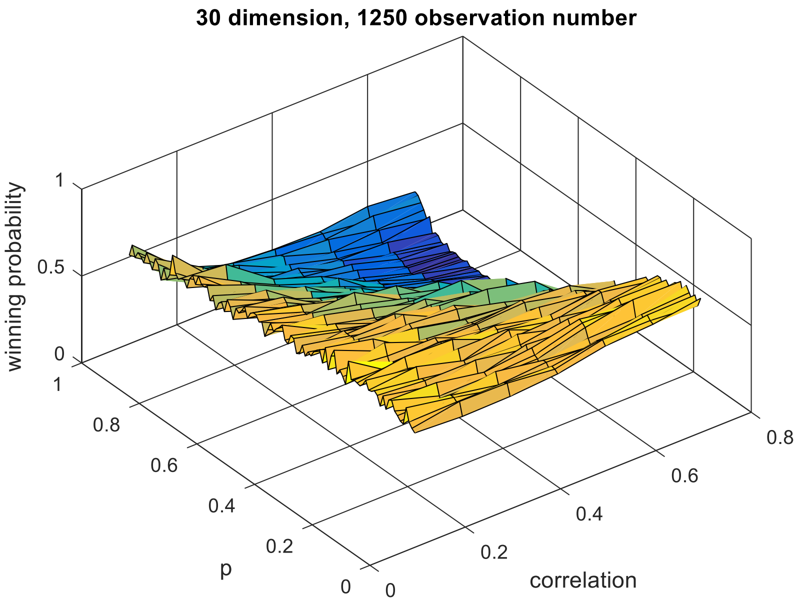
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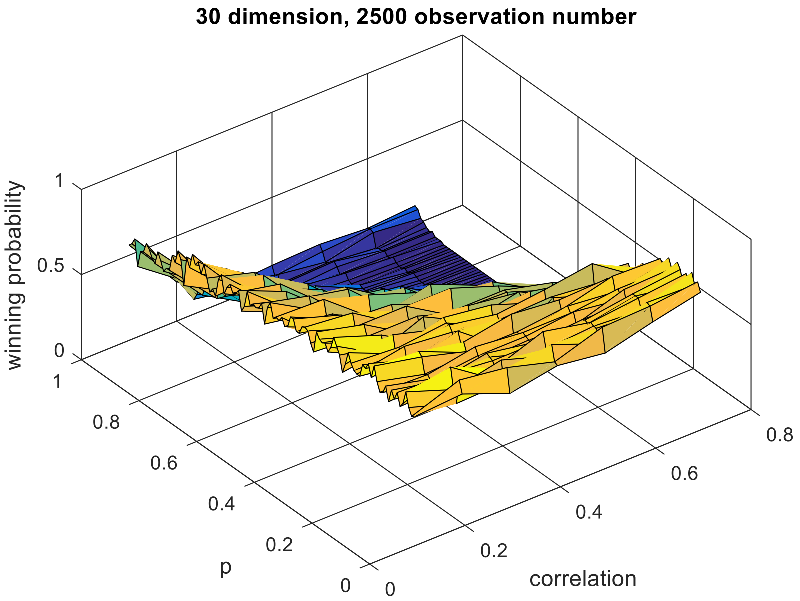
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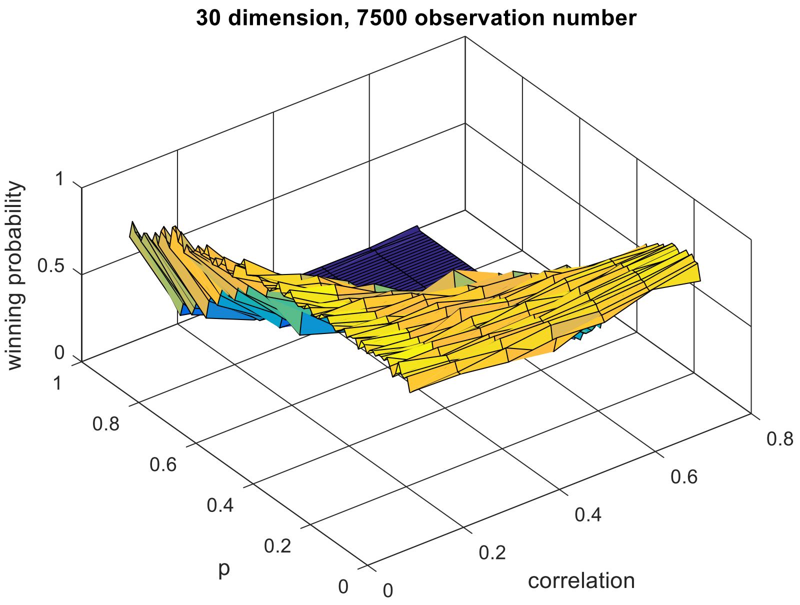
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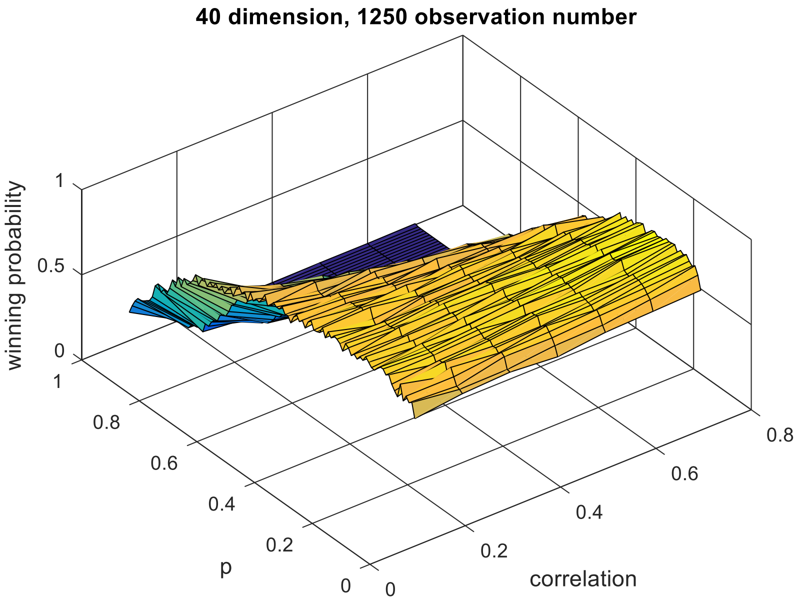
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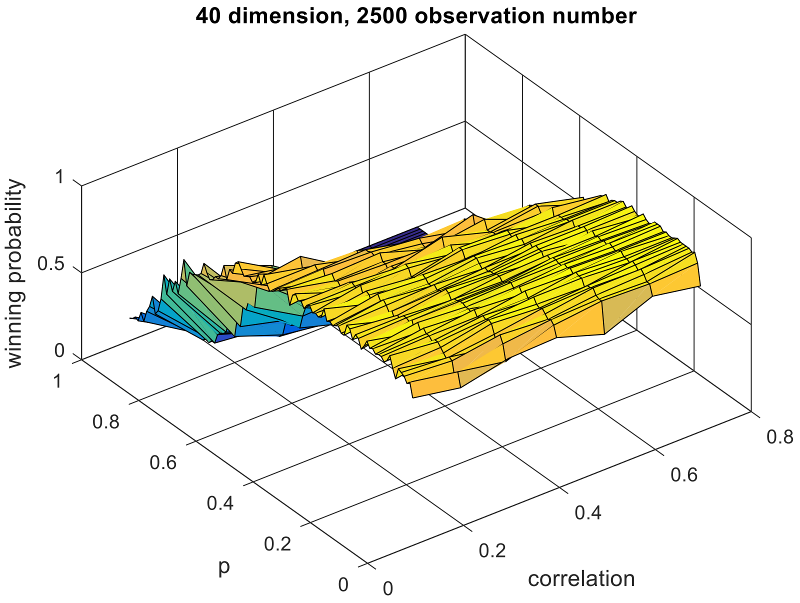
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winning probability
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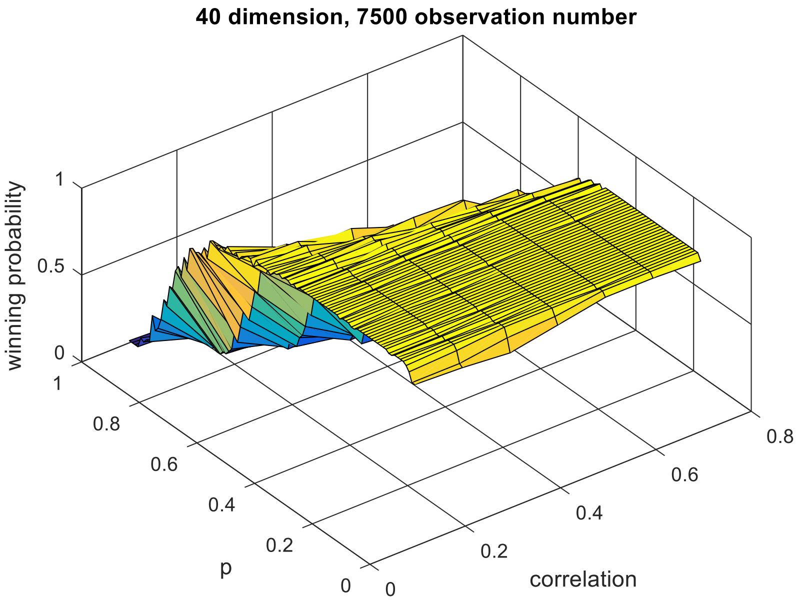
correlation


40 dimension, 7500 observation number
winning probability
1
0.5
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1
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0.6
0.4
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p
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0.4
0.6
0.8
correlation

Endnotes:

[1]When the risk free rate is positive, just scaling the same size of MVRK as for the fractional Kelly, we have a fractional MVRK. Comparing fractional MVRK and fractional Kelly strategies is always equivalent to comparing MVRK and full Kelly. As long as we show that MVRK can outperform full Kelly, we show that fractional MVRK can outperform fractional Kelly.

[2]We compute the daily excess return of the most liquid futures contract (typically the nearest or next nearest-to-delivery contract), and then compound the daily returns to a continuous return index from which we can compute returns at any horizon. Then we calculate the annualized return and volatility based on the continuous return index. The risk free rate published by the People's Bank of China (PBC or PBOC) is available at http://www.global-rates.com/interest-rates/central-banks/central-bank-china/pbc-interest-rate.aspx

Chapter 3

Change point detection in shrinking covariance matrix: does it improve portfolio performance?

In this paper, we focus on applying the change point detection method in covariance based trading strategies. We combine the shrinkage theory and CUSUM change point detection in order to improve the covariance estimators. The change point embedded covariance estimator can do better than any shrinking covariance estimators in the portfolio management. We empirically assess different shrinkage estimators based portfolios in global futures markets. With all shrinkage estimators, the change point detection enhanced portfolio outperforms the shrinkage portfolios in the commodity, equity, bond and currency sectors. The Sharpe ratio, annual return, trading turnover and maximum drawback results all indicate that the change point detection method improves the shrinkage covariance based portfolio performance.

3.1. Introduction

The covariance matrix estimation plays an important role in risk management, asset allocation and portfolio performance. The sample covariance matrix is a commonly used estimator. But when the dimension of matrix is high, the sample covariance matrix might differ significantly from the theoretical covariance matrix. This is a very common problem when more financial instruments are added into the portfolio. This situation becomes even worse when the time length T is smaller than the asset numbers K in the portfolio. The extreme amount of error results in extreme coefficients in the matrix. Hence, the mean-variance optimization will allocate biggest weight on those coefficients containing most extreme errors. To overcome this problem, Ledoit and Wolf (2003) introduced the shrunken covariance matrix in order to replace sample covariance. The advantage of the shrinkage estimator is applicable for small sample sizes and high dimensional problems. In the portfolio management aspect, the shrinkage estimation improves the mean variance optimisation portfolio performance with no restriction to Gaussian assumptions.

Although the shrinkage theory reduces the estimating error in the covariance matrix, the high volatility of financial markets often breaks the covariance stability. The shrinkage method smooths the error of the sample covariance estimation. However, we need to apply the change point detection in order to adapt portfolio allocation when a big structure break occurs. The statistical test of covariance structure breaks becomes more and more important and popular. Qu and Perron (2007) proposed a parametric method to detect changes in the multivariate volatility models, but their test is rarely applied empirically due to the fact that the parametric estimation could be burdensome along with dimensionality increasing. Aue et al. (2009) conducted non-parametric CUSUM tests in order to detect unknown breaks in the multivariate covariance structure. They also provided the completed asymptotic theory for the test. Compared with the parametric test, the CUSUM tests are not based on any multivariate volatility model so that they can be applied with high dimensions, hence more implementable in portfolio management.

We believe that combining the change point with the shrinkage method will further contribute to covariance estimation and portfolio performance. Due to the reason mentioned above, we propose an empirical approach in order to incorporate change point detection with the shrinkage method. The main contributions involve three aspects. First, to the best knowledge of the author, this chapter first applies change point detection in the shrunken covariance estimation. Second, we propose the approach combing change point with the shrinkage method in the portfolio weight allocation. Third, we empirically show that our approach improves the portfolio performance and perfectly works in the global futures market.

3.2. Literature Review

Since the seminal work of Harry (1952), the mean-variance optimisation has become the most rigorous and popular way to manage portfolio weight allocation. Estimating the covariance matrix of stock returns has always been one of the stickiest points. Errors are large when estimating the sample covariance and the most extreme coefficients in the matrix tend to take on extreme values (Jobson and Korkie (1980)). Ledoit and Wolf

(2004) proposed a fancy shrinkage theory suggesting the composition of the sample covariance matrix and a highly structured estimator will give a better risk estimator.

After the shrinkage theory was introduced into portfolio management by Ledoit and Wolf, many researchers have focused their attention on this method. Disatnik, Benninga and Summer (2007) compared empirically the various shrinkage estimators of the covariance matrices based on monthly stock return data. Their study confirmed that more sophisticated shrinkage methods do not gain significant benefit. Kwan (2008) accounted for the estimation errors in variances when shrinking the sample correlation matrix. Cao et al. (2017) combined the shrinkage theory with the Kelly criterion in low correlated portfolios.

Change point detection in the random processes is always an important topic for theoretical and empirical statistics. Extensive literature has studied the structural stabilities in the mean, variance and covariance of random variables, and applied them with appropriate applications in varied fields. In the present chapter, we consider more about the structural break in the second moment. The first attempt in testing the stability of the second moment was completed by Inclan and Tiao (1994), who proposed a cumulative sums of squares test to detect changes in the variance. Although their test is based on relatively strict assumptions, it provides a well-constructed theoretical frame for later extensions, and draws much attention among empirical researchers in the financial field.

To encounter the covariance stability, Andreou and Ghysels (2002) modelled the covariance structure of the foreign exchange market via the dynamic conditional correlation (DCC) model (Engle, 2002), and applied the likelihood ratio tests proposed by Bai and Perron (1998) in order to test the stability of coefficients in the DCC model. The paper emphasises that the breaks in the covariance structures play an important role in financial modelling, but there was no proper statistical test for testing such an issue. This fashion went through until Qu and Perron (2007) proposed a parametric method to detect changes in the multivariate volatility models and a well-established asymptotic theory. Their test obtained acknowledgements mainly from the theoretical aspects, while the test is rarely applied empirically due to the fact that the parametric estimation could be burdensome along with dimensionality increasing. The high dimension dataset is often necessary for financial and economic studies.

Limited by the computational issue, parsimonious methods are required. Later, the benefit from Aue et al. (2006) derived a strong approximation result for the cumulative sum (CUSUM) statistics in the context of the GARCH model. Aue et al. (2009) proposed non-parametric CUSUM tests in order to detect unknown breaks in the multivariate covariance structure. They also provided the completed asymptotic theory for the test. Compared with the parametric test, the CUSUM tests are not based on any multivariate volatility model so that they can be applied with high dimensions. Furthermore, in order to distinguish changes in variance and correlations, Wied et al. (2012) extended the CUSUM tests to detect changes in the correlation matrix. In the present chapter, we aim to detect unknown changes in the covariance structures of multiple assets, and the non-parametric CUSUM tests Aue et al. (2009) are applied.

Golosnoy et al. (2011) apply several CUSUM control charts in monitoring the global minimum variance portfolio (GMVP). Tobias et al. (2015) apply change point detection with Value-at-Risk of financial portfolios forecasting. Change point detection has been used in portfolio management in various literatures. In this chapter, we apply change point detection in shrunken covariance in order to improve asset allocation and overall portfolio performance.

3.3. CUSUM Change Point Detection Incorporated Strategy

3.3.1 Shrinkage covariance estimators

In this section, we consider different shrinkage covariance estimators. Generally a shrinkage method is an optimally weighted average of two existing estimators. One estimator is the sample covariance and another estimator represents the fund manager's prior view on the portfolio covariance. The formula can be shown as follows:

$$\sum_{shrink} = \sigma F + (1 - \sigma) S \tag{3.1}$$

where S is the sample covariance matrix, F is called shrinkage target, in two separate papers, Ledoit and Wolf (2003, 2004) use the single index and constant correlation model as F respectively. In practice, F can be constructed by fund managers arbitrarily according to their own portfolios. σ is the shrinkage constant, $0 < \sigma < 1$.

Constant correlation matrix shrinkage target:

The average of all the sample correlations is the estimator of the common constant correlation. This number together with the vector of sample variances implies the shrinkage target F:

$F = \bar{\rho} S + (1 - \bar{\rho}) Diag(S)$, $Diag(S)$ is the diagonal line of the covariance matrix.

$\bar{\rho} = \frac{2}{(N+1)N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij}$, average pairwise correlation.

Single index shrinkage target:

They use F as follows,

$$F = \beta\beta'\sigma_{00}^2 + \Omega_\varepsilon$$

where β are estimated from regression $X_{it} = \alpha_i + \beta_i X_{0t} + \varepsilon_{it}$, Ω_ε is the diagonal matrix containing residual variance ${\varepsilon^2}_{it}$,and σ_{00}^2 is the variance of the market returns X_{0t} (Sharpe, 1963).

Multiple factors shrinkage target:

The single index model can be extended to the multiple factor model (Bai, 2011). The shrinkage target F is estimated from $X_{it} = \alpha_i + \beta_{i1}Z_{1t} + \beta_{i2}Z_{2t} + \beta_{i3}Z_{3t} + \cdots + \varepsilon_{it}$. $Z_t = (Z_{1t}, Z_{2t}, Z_{3t} \dots Z_{kt})$ is a vector of observable factors. The F formula is:

$$F = \beta\Omega_Z\beta' + \Omega_\varepsilon$$

Diagonal matrix shrinkage target:

Cao et al. (2017) use the diagonal covariance matrix as a shrinkage target in their multivariate volatility regulated Kelly strategies (MVRK). They show this shrinkage target is superior in low correlated portfolios. MVRK simply use $F = Diag(S)$.

Sample covariance matrix:

We also use the sample covariance matrix as one benchmark estimator.

Following Ledoit and Wolf (2004), the optimal σ^* is determined by a quadratic loss function:

$$L(\sigma) = \|\sigma F + (1-\sigma)S - \sum\| \tag{3.2}$$

where estimating the asymptotic covariance $\sum$, they minimise the distance between the in sample shrinkage covariance and $\sum$ to obtain the optimal constant σ^*. In this chapter, we will inherit their framework and incorporate the change point detection method into the shrinkage system.

3.3.2 CUSUM Change Point

Considering the d-dimensional assets $y_t = [y_{1t}, y_{2t}, \cdots, y_{dt}]$ with $E[|y_t|^2] = \mu$ and $E[|y_t|^2] < \infty$, where $|.|$ denotes the Euclidean norm in R^d, the objective is to distinguish the null hypothesis from the alternative,

$$\begin{cases} H_0: \quad Cov(y_1) = \cdots = Cov(y_T) \\ H_1: \quad Cov(y_1) = \cdots = Cov(y_{k^*}) \neq Cov(y_{k^*+1}) = \cdots = Cov(y_T) \end{cases} \tag{3.3}$$

where k^* is an unknown change point. To interpret these hypotheses with our framework, we say that there is no change in the covariance structure of d-dimensional multiple assets during a certain period between t=1 and t=T, alternatively, a change occurs at an unknown date $t = k^*$. Intuitively, fund managers would adjust the position in the portfolio correspondingly. Aue et al. (2009) proposed two CUSUM tests: maximally selected self-normalised statistics $\wedge_T$ self-normalised statistics Ω_T, which are formulated as follows,

$$\wedge_T = \max_{1 \le k \le T} S_k^T \hat{\Sigma}_T^{-1} S_k \tag{3.4}$$

$$\Omega_T = \frac{1}{T} \sum_{k=1}^{T} S_k^T \hat{\Sigma}_T^{-1} S_k \tag{3.5}$$

For that,

$$S_k = \frac{1}{\sqrt{T}}\left(\sum_{t=1}^{k} vech[\tilde{y}_t\tilde{y}_t^T] - \frac{k}{T}\sum_{t=1}^{T} vech[\tilde{y}_t\tilde{y}_t^T]\right), \quad k = 1, \cdots, T$$

where $vech(.)$ is a stacking operator to stack the $d \times d$ matrix into a vector with $\Delta = \frac{d.(d+1)}{2}$ element; and the demean vector $\tilde{y}_t = y_t - y_T$ with $y_t = \frac{1}{T}\sum_{t=1}^{T} y_t$. The standardised term $\hat{\Sigma}_T$ is an estimator of the long run covariance matrix,

$$\Sigma = \sum_{j \in Z} Cov(vech[\widetilde{y_0}\tilde{y}_0^T], vech[\tilde{y}_t\tilde{y}_t^T]) \quad (3.6)$$

In this chapter, we use the Bartlett kernel and the Newy-West optimal bandwidth to estimate the long run covariance. As a result, under the null hypothesis, the asymptotic limit of two CUSUM tests is derived as below,

$$\Lambda_T \overset{D}{\rightarrow} \sup_{0 \le u \le 1} \sum_{i=1}^{\Delta} B_i^2(\mu) \quad (3.7)$$

$$\Omega_T \overset{D}{\rightarrow} \sum_{i=1}^{\Delta} \int_0^1 B_i^2(\mu) d\mu \quad (3.8)$$

where the term $\overset{D}{\rightarrow}$ represents the weakly convergence, $\Delta = \frac{d.(d+1)}{2}$, $B_1, B_2, \cdots, B_d$ denote independent Brownian bridge. Aue et al. (2009) computed the asymptotic critical values.

In order to find critical values suitable for our case, we compute the empirical critical values toward sample T and dimension K. We simulate a d-dimensional random vector with unchanged constant correlation $\rho_t = 0$ for all entities, and then compute Λ_T and Ω_T. The simulation is replicated 5000 times, and we then obtain the empirical critical values by truncating 5000 statistics at 10%, 5% and 1% significance level.

Once the empirical statistics exceed the empirical critical values, we reject the null hypothesis. The change point location k^* can be found at the point with the maximum statistics. Both tests $\wedge_T$ and Ω_T indicate the identical change location k^*,

$$k^* = \frac{1}{T} \underset{1 \le k \le T}{\operatorname{argmax}} S_k^T \hat{\Sigma}_T^{-1} S_k \tag{3.9}$$

3.3.3 Change Point Incorporating Covariance Estimator

Since the estimated covariance change over time, any covariance based trading strategies alter their weight over time as well. In the portfolio management, the basic but important assumption is: the covariance estimator remains stable over time. Under this assumption, the portfolio weight is valid. From one aspect, fund managers need to estimate covariance as accurately as they can. For example, Ledoit and Wolf (2004) proposed the shrinkage theory improving the approach in estimating the return covariance. This method tends to pull the most extreme coefficients towards more central values, thereby systematically reducing the estimation error where it matters most. From another aspect, when the current covariance structure is changed, fund managers need to take action immediately to adjust the portfolio weight. We will combine the change point schemes with several shrinkage covariance estimators, and compare their portfolio performance. The comparison shrinkage covariance estimator is listed as follows:

Shrinkage target used	*Formula*
Constant correlation matrix	$\sigma\left(\bar{\rho}S + (1-\bar{\rho})Diag(S)\right) + (1-\sigma)S$
Single index	$\sigma(\beta\beta'\sigma_{00}^2 + \Omega_\varepsilon) + (1-\sigma)S$
Multiple factors	$\sigma(\beta\Omega_Z\beta' + \Omega_\varepsilon) + (1-\sigma)S$
Diagonal matrix	$\sigma Diag(S) + (1-\sigma)S$
Sample covariance	S

Now we discuss the change point incorporating covariance estimator. The key is adjusting the covariance estimation formula once the change point is detected. We are not minimising the variance or covariance in portfolios, although the change point adjustment will reduce risk in certain periods. Our interest falls in how the CUSUM change point detection will further improve the shrinkage covariance estimator and thus the portfolio performance.

We propose a general change point incorporated covariance formula. The method is non-parametric and simple to implement. Once the change point is detected in the realised sample covariance, the CUSUM change point test gives the change point location k* in (3.9). Then we re-estimate the new sample covariance and the new shrinkage target. Then the new covariance estimator ${V_t}^*$ is defined as:

$$ {V_t}^* = {\sigma^*}_{new}F_{new} + (1 - {\sigma^*}_{new})S_{new} \quad (3.10) $$

Here we need to calculate the sample covariance matrix S_{new} after the change point k*, and sample shrinkage target F_{new}, then re-estimate the optimal shrinkage ratio ${\sigma^*}_{new}$. The covariance estimation error will be reduced by dropping the data before the change point.

This covariance estimator has a dual advantage. On the one hand, it perfectly responds to the covariance change point by re-estimating the covariance matrix. On the other hand, it does not need any extra parameter for the new estimator and is easy to merge into any covariance based trading strategies. The disadvantage mainly falls in the sample window length. If the change point is detected at a very late time, then the sample length is too short to give a non-singular covariance matrix. But this will not be a problem for a long horizon covariance estimation.

3.3.4 Test Procedure

1. *Covariance rolling window length*: We implement the change point detection using a rolling window. Because the influence of historical data decreases when the time increases, i.e. the effect of data from 10 years ago to calculate current covariance estimator is negligible. The rolling window length is set as 260 trading days, which is approximately equal to trading days in one year. We forecast the next day covariance matrix with the previous 260 days return. A different window length definitely affects the trading strategy performance. One year lookback period momentum is shown to be most profitable in the global futures market (Moskowitz, et al., 2012). Since a 12-month momentum is one of most profitable trading strategies in the global futures market, we will follow their trading signal and use the same lookback period length for the covariance estimator.

2. *Simulations of critical values*: Based on Aue et al. (2009), we simulate the dimensional samples of the normally distributed random variables representing 260 trading days. The mean of the random variables is zero, while the volatility and correlation use real data statistics. The dimension of the sample is determined by the

real data dimension. Then we repeat the above simulation 5000 times. Finally, we determine the critical value with 95% quantile of the resulting test statistics. Our simulation step is consistent with Tobias et al. (2015).

3. *Testing structure breaks in the sample covariance matrix*: We calculate the empirical statistics in each 260 window length and compare with the critical value simulated in the last step. To overcome the problem of the error change point, we propose an accuracy examination procedure. We repeat step 2 but with prior setting of change point in the 260 days sample. The pairwise correlation change level is corresponding to the real data. Then we calculate the standard deviation σ of the change point location k*. If the nearby change points deviate over $\pm\sigma$, then we do not reject H_0, and do not recalculate the covariance estimator ${V_t}^*$. If the nearby change points are in the range of $[k^* - \sigma, k^* + \sigma]$, we reject H_0, and believe there is a real change point. In Table 2, we show the change point detected percentage in terms of the whole sample.

4. *Trading signals*: The simple 260 days return moving average is used as a trading signal. We are focusing on how the change point detection improves the default portfolio performance rather than on the best choice of trading strategies. Hence we do not repeat the simulation procedures in the last three steps to calculate different trading strategy results. The trading strategy is the following mean-variance optimisation structure:

$$f = \frac{MA260}{\Sigma_{shrink}} \tag{3.11}$$

where $MA260 = (\mu_1, \mu_2, \dots, \mu_d)$is the moving average return vector, $\sum_{shrink}$ is defined in equation (3.1) and (3.10) under H_0 and H_1, respectively, and $f =$

$(f_1, f_2, \ldots, f_d)$ is the portfolio weight. We show strategy performance on MA260, MA130 and MA65 in Figure 3.1.

5. *Backtesting and portfolio performance*: Different shrinkage covariance estimators are used and compared with change point incorporated estimators. To compare the portfolio performance, we compute the portfolios' annual return, Sharpe ratio, maximum drawdown and daily turnover in all default covariance estimators and change point combined estimators.

3.4. Empirical Results

3.4.1 Data Set

In this section, we show the data set of the global futures market. Our data consist of futures prices and volumes for 25 commodities, 9 equities, 13 bonds, 6 currencies futures contracts. To construct a balanced covariance matrix, the dataset from DataStream International spans the period July 30, 2007 to September 16, 2017 for commodity, Jan 01, 2001 to September 16, 2017 for equity, Sep 09, 2005 to September 16, 2017 for bond, Jan 03, 2011 to September 16, 2017 for currency. We focus on the most liquid contracts and the second nearest to delivery contracts after it. The return series for each instrument is constructed as follows: Each day we compute the daily excess return of the most liquid contract and compound the daily returns to a cumulative return index. This calculating method guarantees the liquidity of contracts and matches a better implementable strategy, which is consistent with Moskowitz et al. (2012).

In Table 3.1, we present annualized mean return, annualized volatility, skewness, kurtosis and pairwise correlation in commodity, equity, bond and currency markets. In Table 3.2, we present the percentage of change points we detected using the commodity, equity, bond and currency data in the full sample length.

Table 3.1: Summary statistics for futures contract in four sectors. The table presents the summary statistics for the 53 futures contracts of the dataset, which are estimated using daily return series. The statistics are: annualized mean return in %, annualized volatility in %, skewness, kurtosis and pairwise correlation in commodity, equity, bond and currency markets

Commodity	Mean	Vol	Skew	Kurt	Pairwise Corr
LIGHT CRUDE OIL	-10.00%	33.80%	-0.14	4.19	
BRENT CRUDE OIL	-4.60%	31.35%	-0.08	4.81	
HEATING OIL	-0.65%	31.12%	0.01	4.71	0.21
NATURAL GAS	-26.34%	41.53%	0.02	3.28	
RBOB GASOLINE	1.59%	34.38%	-0.14	4.54	
COPPER	-3.52%	28.04%	-0.17	5.67	
GOLD	6.91%	18.88%	-0.20	6.58	
PALLADIUM	15.69%	30.22%	-0.18	4.79	
PLATINUM	-1.66%	23.59%	-0.29	5.84	
SILVER	11.62%	31.36%	-0.33	4.94	
FEEDER CATTLE	2.50%	16.52%	-0.03	5.32	
LIVE CATTLE	1.21%	16.74%	0.00	6.98	
LEAN HOGS	-3.59%	24.95%	-0.08	5.46	
CORN	2.60%	29.67%	-0.04	4.87	
OATS	1.81%	32.77%	0.05	4.29	
SOYBEAN OIL	-0.78%	24.07%	0.08	5.41	
SOYBEAN MEAL	10.85%	29.28%	-0.26	4.77	
SOYBEANS	6.21%	25.28%	-0.33	5.39	
WHEAT	-4.59%	32.91%	0.10	4.32	
COCOA	6.15%	27.79%	-0.03	4.68	
COFFEE	0.46%	31.06%	0.02	4.09	
COTTON	4.37%	28.67%	-0.13	4.50	
LUMBER	-10.95%	29.89%	0.27	3.70	
ORANGE JUICE	0.24%	32.87%	-0.07	4.29	
SUGAR	3.03%	32.49%	0.08	4.26	

Equity	Mean	Vol	Skew	Kurt	Pairwise Corr
SPI 200	3.87%	16.18%	-0.25	7.17	
CAC 40	-1.46%	22.47%	-0.18	6.10	0.39
DAX	2.92%	23.14%	-0.26	6.31	
NIKKEI 225	2.28%	24.00%	-0.23	6.18	
AEX	1.45%	10.00%	0.13	5.47	
IBEX 35	0.11%	23.17%	-0.21	5.47	
FTSE 100	0.45%	18.63%	-0.27	8.27	
S&P 500	4.42%	18.31%	-0.24	8.14	
HANG SENG	2.33%	22.60%	-0.22	5.90	

Bond	Mean	Vol	Skew	Kurt	Pairwise Corr
Australia 3 YEAR	0.25%	1.03%	0.03	6.44	
Australia 10 YEAR	0.20%	1.03%	-0.09	5.17	
EURO SCHATZ	0.42%	1.25%	-0.37	11.92	0.44
EURO BOBL	1.12%	3.69%	-1.06	11.92	
EURO BUND	2.25%	6.01%	-0.45	6.21	
EURO BUXL	3.59%	12.51%	-0.06	5.73	
Japanese 10 YEAR	0.65%	3.13%	-0.55	10.98	
UK LONG GILT	1.83%	6.83%	-0.06	5.62	
US 2 YEAR	0.35%	1.49%	-0.76	17.33	
US 5 YEAR	0.72%	3.96%	-0.38	8.81	
US 10 YEAR	1.01%	6.12%	-0.11	8.49	
US 30 YEAR	1.68%	10.56%	-0.24	5.34	
Canada 10 YEAR	1.28%	6.07%	-0.07	8.25	

Currency	Mean	Vol	Skew	Kurt	Pairwise Corr
AUD/USD	-3.65%	11.01%	-0.19	4.83	
CAD/USD	-3.13%	10.37%	0.01	4.05	0.37
CHF/USD	-1.43%	9.89%	0.23	9.69	
EUR/USD	-1.72%	9.07%	-0.03	4.78	
GBP/USD	-0.92%	8.30%	-0.25	5.51	
JPY/USD	-4.41%	9.60%	0.08	6.59	

Table 3.2: Percentage of change points detected in commodity, equity, bond and currency

Sector Quantile	**Commodity**	**Equity**	**Bond**	**Currency**
90%	38.05%	64.80%	50.78%	86.90%
95%	31.46%	49.55%	40.02%	75.27%
99%	19.25%	24.38%	26.03%	55.55%

From the data we used, we can see that the higher dimension covariance matrix has the lower percentage of rejection for the null hypothesis H_0. This relates to the sensitivity of change point detection in different dimensions. The null hypothesis H_0 is easier to be rejected in low dimension matrix, i.e. the same level change has much more influence in 5×5 matrix than 20×20 matrix.

3.4.2 Portfolio Performance Evaluation

A 12 month lookback period has been showed to be the best window length for momentum type trading strategies in global futures market Jegadeesh and Titman (2001), Moskowitz et al. (2012), Baltas and Kosowski (2013). Thus, we use 12 months as the estimation window length for both the trading signal and covariance matrix.

In Table 3.3, we show the portfolio performance of five covariance estimators and change point incorporated estimators in section 3.3. The five default estimators are the realised covariance matrix Harry (1952), constant correlation shrinkage matrix Ledoit and Wolf (2004), single index shrinkage matrix Ledoit and Wolf (2003), multiple factors shrinkage matrix Bai (2011) and diagonal covariance matrix Cao et al. (2017).

Table 3.3: Shrinkage covariance estimation portfolio performance without/with change point.

The table presents the portfolio performance in commodity, equity, bond and currency portfolios. The statistics are: excess annualized mean return in %, maximum drawdown in %, daily turnover in %, Sharpe ratio(Sharpe (1963)).

Commodity	Shrinkage Target	Covariance Estimator
	Covariance	*Covariance+Change Point*
Annual Return	3.35%	3.61%
Sharpe Ratio	0.83	0.88
Maximum Drawdown	-7.07%	-5.17%
Daily Turnover	45.36%	58.21%
	single index	*Shrinkage+Change Point*
Annual Return	4.17%	4.86%
Sharpe Ratio	0.88	0.96
Maximum Drawdown	-8.06%	-7.74%
Daily Turnover	38.79%	36.89%
	multi factor	*Shrinkage+Change Point*
Annual Return	4.13%	4.66%
Sharpe Ratio	0.88	0.96
Maximum Drawdown	-8.06%	-7.63%
Daily Turnover	39.23%	38.16%
	constant correlation	*Shrinkage+Change Point*
Annual Return	4.19%	4.54%
Sharpe Ratio	0.89	0.91
Maximum Drawdown	-8.17%	-7.80%
Daily Turnover	38.76%	37.43%
	diagonal matrix	*Shrinkage+Change Point*
Annual Return	3.96%	4.65%
Sharpe Ratio	0.87	0.96
Maximum Drawdown	-7.95%	-7.35%
Daily Turnover	40.47%	38.71%

Equity	Shrinkage Target	Covariance Estimator
	Covariance	*Covariance+Change Point*
Annual Return	1.70%	2.01%
Sharpe Ratio	0.33	0.40
Maximum Drawdown	-12.26%	-12.26%
Daily Turnover	40.36%	39.99%
	single index	*Shrinkage+Change Point*
Annual Return	1.72%	2.02%
Sharpe Ratio	0.33	0.39
Maximum Drawdown	-12.63%	-12.63%
Daily Turnover	39.85%	39.48%
	multi factor	*Shrinkage+Change Point*
Annual Return	1.83%	2.11%
Sharpe Ratio	0.34	0.40
Maximum Drawdown	-13.01%	-13.01%

Daily Turnover	39.19%	38.79%
	constant correlation	*Shrinkage+Change Point*
Annual Return	1.77%	2.05%
Sharpe Ratio	0.33	0.38
Maximum Drawdown	-13.37%	-13.37%
Daily Turnover	38.72%	38.32%
	diagonal matrix	*Shrinkage+Change Point*
Annual Return	1.72%	2.02%
Sharpe Ratio	0.34	0.40
Maximum Drawdown	-12.39%	-12.39%
Daily Turnover	40.21%	39.82%

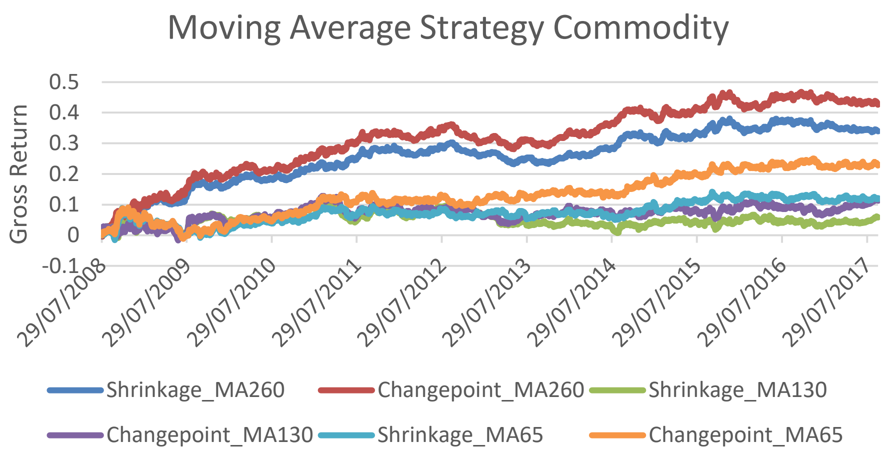
Bond	**Shrinkage Target**	**Covariance Estimator**
	Covariance	*Covariance+Change Point*
Annual Return	0.03%	0.04%
Sharpe Ratio	0.11	0.16
Maximum Drawdown	-0.95%	-0.95%
Daily Turnover	38.18%	38.84%
	single index	*Shrinkage+Change Point*
Annual Return	0.04%	0.06%
Sharpe Ratio	0.14	0.22
Maximum Drawdown	-1.13%	-1.13%
Daily Turnover	33.49%	32.96%
	multi factor	*Shrinkage+Change Point*
Annual Return	0.04%	0.05%
Sharpe Ratio	0.13	0.18
Maximum Drawdown	-1.17%	-1.18%
Daily Turnover	32.40%	32.34%
	constant correlation	*Shrinkage+Change Point*
Annual Return	0.03%	0.04%
Sharpe Ratio	0.08	0.13
Maximum Drawdown	-1.40%	-1.41%
Daily Turnover	30.36%	30.16%
	diagonal matrix	*Shrinkage+Change Point*
Annual Return	0.03%	0.05%
Sharpe Ratio	0.12	0.18
Maximum Drawdown	-1.00%	-1.18%
Daily Turnover	37.23%	32.34%

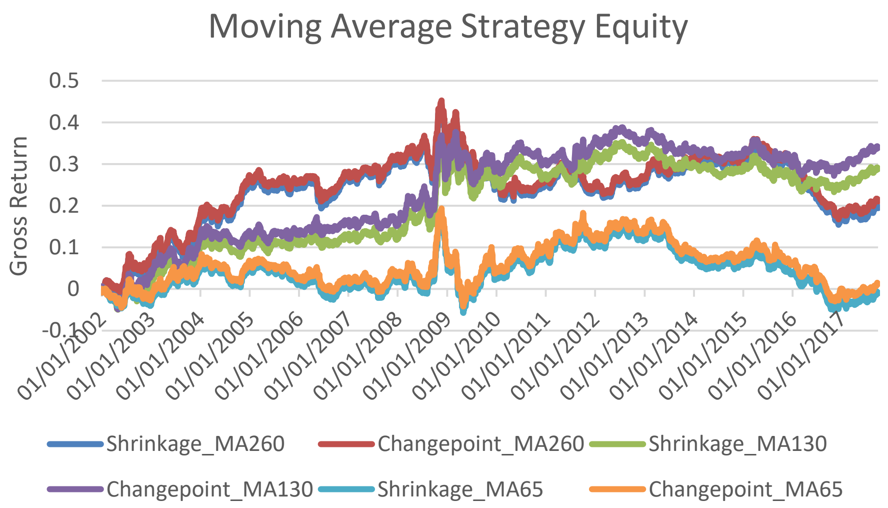
Currency	Shrinkage Target	Covariance Estimator
	Covariance	*Covariance+Change Point*
Annual Return	0.83%	0.95%
Sharpe Ratio	0.23	0.28
Maximum Drawdown	-7.05%	-6.70%
Daily Turnover	31.24%	30.52%
	single index	*Shrinkage+Change Point*
Annual Return	0.93%	0.97%
Sharpe Ratio	0.26	0.27
Maximum Drawdown	-7.10%	-6.74%
Daily Turnover	30.91%	30.33%
	multi factor	*Shrinkage+Change Point*
Annual Return	1.14%	1.06%
Sharpe Ratio	0.30	0.28
Maximum Drawdown	-7.26%	-6.94%
Daily Turnover	29.98%	29.46%
	constant correlation	*Shrinkage+Change Point*
Annual Return	1.13%	1.03%
Sharpe Ratio	0.29	0.27
Maximum Drawdown	-7.28%	-6.89%
Daily Turnover	29.61%	29.22%
	diagonal matrix	*Shrinkage+Change Point*
Annual Return	0.91%	1.00%
Sharpe Ratio	0.25	0.28
Maximum Drawdown	-7.08%	-6.75%
Daily Turnover	31.12%	30.46%

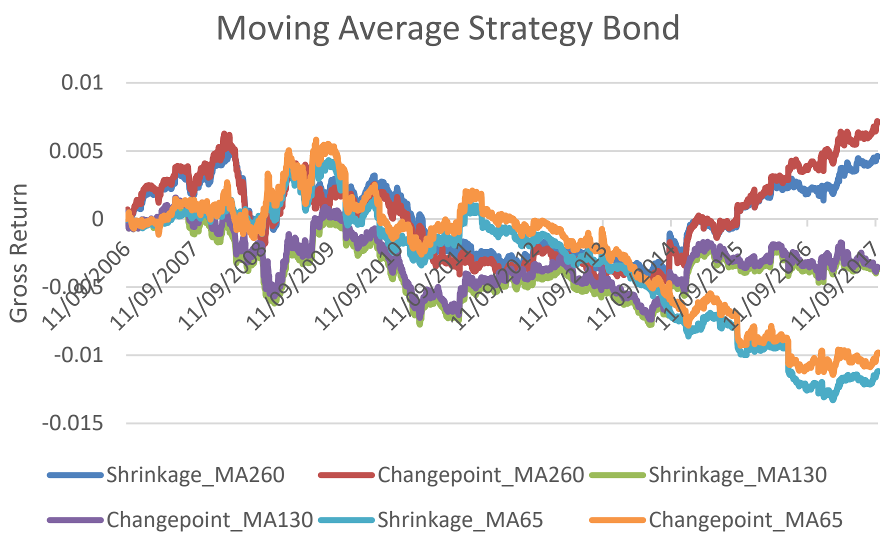
The annual return measures the profit ability of the portfolio, the Sharpe ratio measures the risk adjusted return of the portfolio, the maximum drawdown measures the maximum loss from a peak to a trough of a portfolio, the turnover measures the frequency with which assets within a portfolio are bought and sold by the managers. The annual return, Sharpe ratio and maximum drawdown are improved using the change point detection method in almost all scenarios. Another inspired point is that the daily turnover of the change point incorporated strategies are reduced in different shrinkage strategies across four futures sectors. This result indicated that the change point detection method can reduce the trading frictions. All statistics pointed out the advantage of the change point detection added shrinkage estimators. In addition, there is no significant difference in portfolio performance with different shrinkage methods. This is consistent with the conclusion in Disatnik, Benninga and Summer (2007).

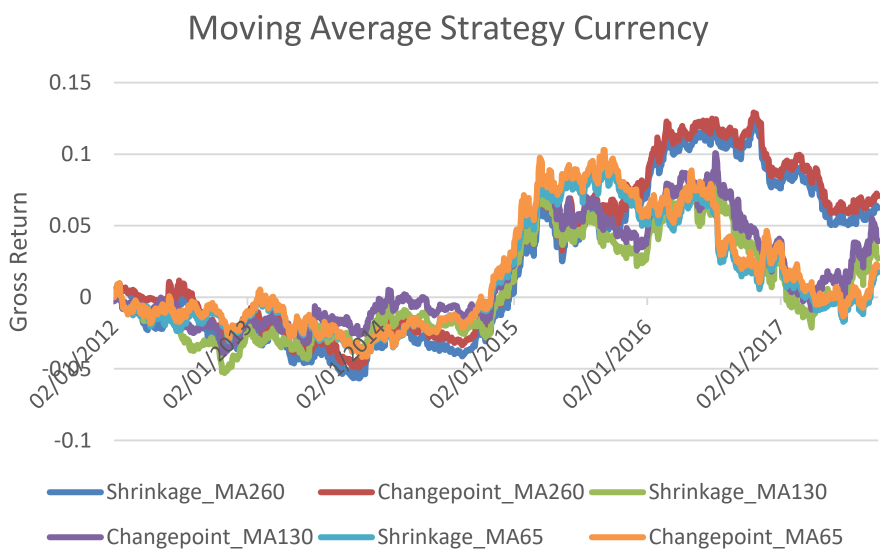
Figure 3.1: Shrinkage strategy vs. Change point embedded strategy in MA260, MA130, MA65.

Reported are the constant correlation model shrinkage strategies and change point embedded strategies in the commodity, equity, bond and currency sectors. The Red line, Dark Blue line and Orange line represent the change point strategies gross return in 260, 130, 65 days lookback period, respectively.





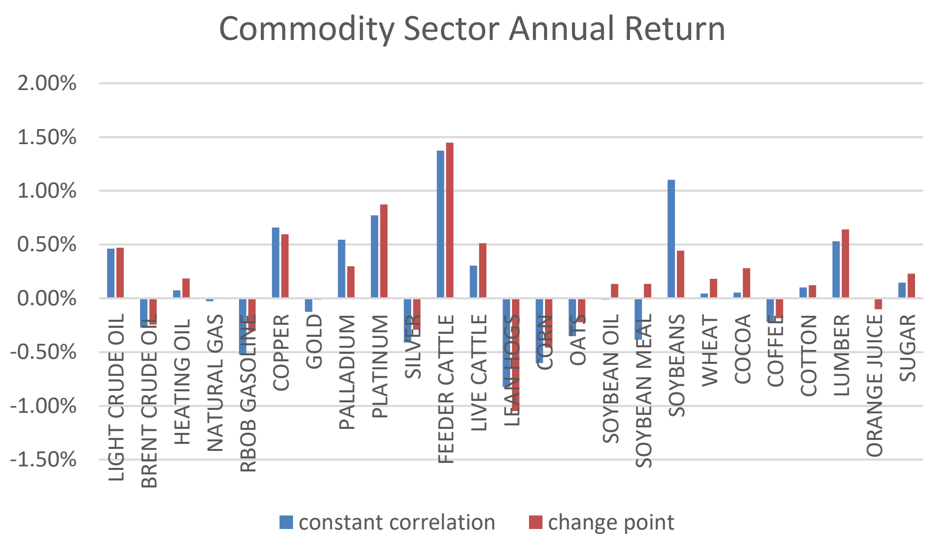


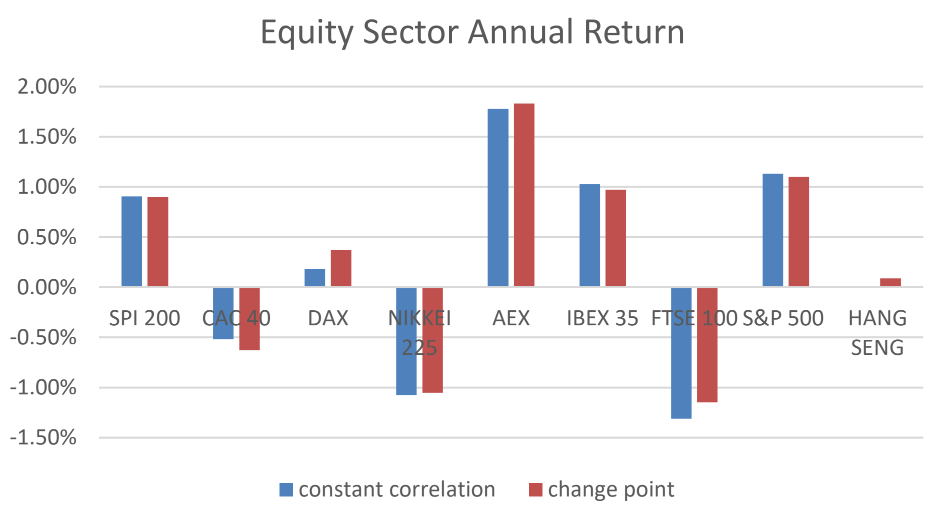


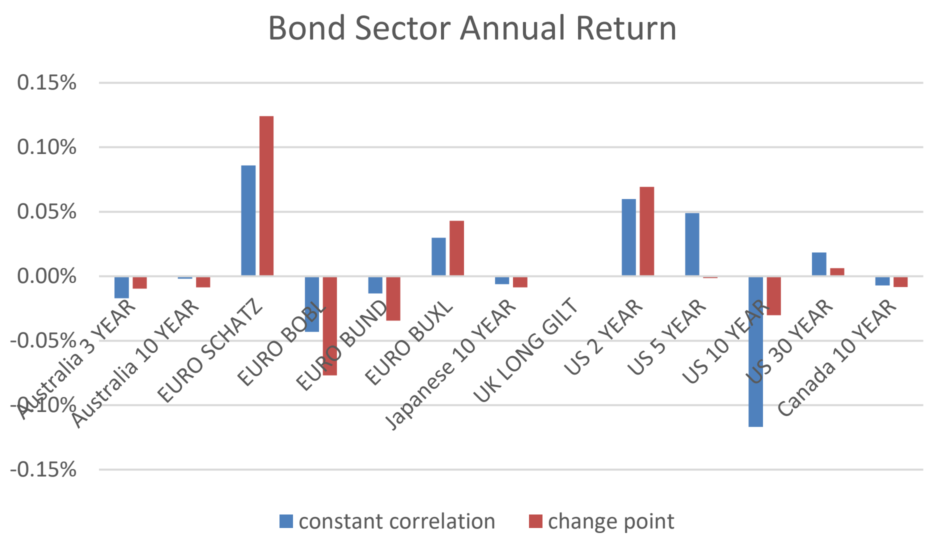
When we change the lookback period of trading signal, the change point embedded approach still improves the portfolio performance. Figure 3.1 shows the example of 260, 130, 65 days moving average trading signal strategies gross return. The lookback period heavily influenced the strategy performance, but in all lookback period strategies and in all four sectors, the change point detection enhanced the portfolio performance.

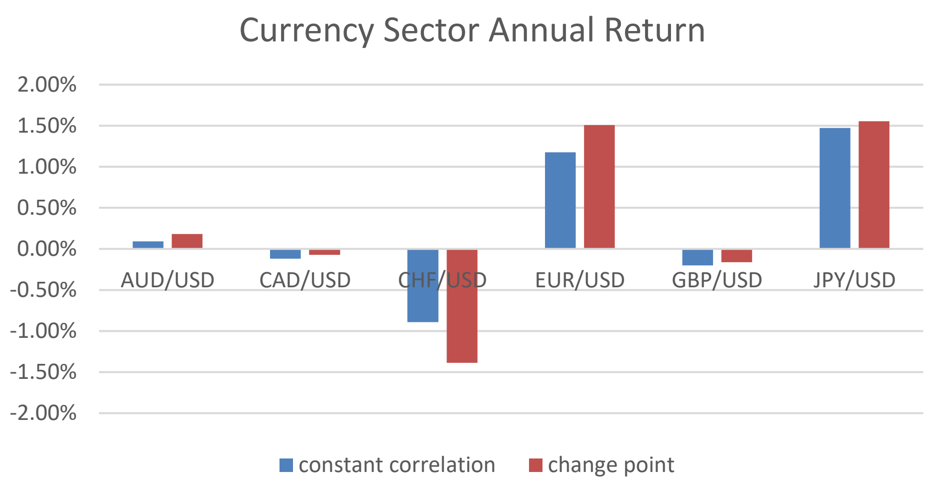
Figure 3.2: Annual mean return of all instruments in four sectors

Reported are the annual mean return of each futures instrument in %. The constant correlation shrinkage model is compared with the change point embedded shrinkage model. As we indicated before, a different shrinkage method has no significant difference in improving the portfolio performance. Here we only present the result of the constant correlation shrinkage target.









From Figure 3.2, we can hardly conclude that the change point detection method improved all instruments return in each sector. But 20 of 25 commodity, 6 of 9 equity, 6 of 13 bond, 5 of 6 currency instruments have greater annual returns with a change point incorporated shrinkage covariance estimator.

3.5. Conclusion

This chapter aims to combine change point detection with shrunken covariance estimators and examine whether the change point incorporated covariance estimator will produce a better portfolio performance. We provide a change point combined shrinkage covariance estimator formula and a test procedure of change point detection. We empirically show that the change point method improve the portfolio performance with any default shrinkage estimators, in all four sectors of global futures markets. Averagely, change point method improved the Sharpe ratio by 0.08, 0.06, 0.06 and 0.03 in commodity, equity, bond and currency sectors.

This chapter relates the popular covariance estimation method (Ledoit and Wolf, 2003) and (Ledoit and Wolf, 2004) with important issues in empirical and theoretical statistics (Aue, et al., 2009). From the portfolio management view, we propose an implementable approach to improve the performance in any covariance based portfolios. Our future study will move to combing the change point detection with random matrix method, which is another important covariance estimation method.

Conclusion, Limitations and Further Research

In the previous three chapters, several methods were discussed and applied in the futures markets, namely the time series momentum, Kelly criterion, shrinkage theory and change point detection. From an economic perspective, we illustrated the features of the global futures market and of China's commodity market revealed by these methods. From a portfolio choice perspective, we implemented the trading strategies based on these methods in order to provide the practical implications with reference to portfolio management.

In the first chapter, we find the existence of the time series momentum and time series term structure in the commodity futures market with global futures data spanning over the past 25 years. Almost all instruments are positive significant in the time series momentum and negative significant in the time series term structure slope. We demonstrate that the joint time series momentum and term structure strategies beat the MOM-only and TS-only strategies. The MOM weight increases when the commodity price exhibits a trend following feature, while the TS weight increases when the commodity price exhibits a mean reversion feature. This phenomenon is consistent with the intuition of the MOM and TS effects.

In the second chapter, we propose a modified Kelly strategy referred to as the Multivariate Volatility Regulated Kelly that outperforms the full Kelly and fractional Kelly strategies in low correlation scenarios. We claim that linearly combining the covariance and variance estimators will result in better Kelly strategies in low correlated portfolios. The simulation results show the superiority of MVRK in low correlation settings. The Chinese commodity market further provides empirical evidence of MVRK.

In the third chapter, we aim to combine change point detection with shrunken covariance estimators and examine whether the change point incorporated covariance estimator will produce a better portfolio performance. We provide a change point combined shrinkage covariance estimator formula and a test procedure for the change point detection. We empirically show that the change point method will improve the portfolio performance with any default shrinkage estimators in all four sectors of the global futures markets.

The thesis also revealed several limitations to be addressed by further research. In the first chapter, we failed to find the significant predictability of the term structure factor in equity, bond and currency futures. This result raises the mean reversion factor choice problem in different futures asset classes. It is necessary to explore the mean reversion factor based on the underlying assets. Besides, we find that not all commodity instruments perfectly coordinate the features of MOM and TS. For example, all grain instruments show the heavy lag of coefficient power change when the market state changes. This difference in the grain industry may be due to the under-reaction behaviour, seasonal structure changes or even weather impacts. This indicates that investors need to adopt more caution in the choice of trading parameters when dealing with different instruments.

In the second chapter, we reveal a weakness in the theoretical assumptions. MVRK adds the risk control term in the objective function, which is exactly the drift term in the Kelly criterion. However, we are not able to find a proper utility function in the setting of Merton (1971) in order to rebuild the theoretical structure. The optimal time-varying setting of the risk control coefficient γ and volatility regulation coefficient θ can be estimated using the shrinkage theory. This problem was solved in the third chapter.

In the third chapter, future research covers three aspects. On the one hand, the trading signals we used are a simple moving average in the current work, with more trading signals being applied in the future in order to test our approach. The change point detection method we followed is CUSUM (Aue, et al., 2009), yet more combinations between the shrinkage methods and other change point detection methods ought to be explored. On the other hand, we focus on the random matrix, which is another important covariance estimation method in the literature. It is quite interesting to examine the effect of change point detection on random matrix covariance estimators.

References

Andreou, E. & Ghysels, E., 2002. Detecting multiple breaks in finance market volatility dynamics. *Journal of Applied Econometrics*, Volume 17, pp. 579-600.

Asness, C., Moskowitz, T. J. & Pedersen, L. H., 2013. Value and momentum everywhere. *The Journal of Finance,* 68(3), pp. 929-985.

Aue, A., Berkes, I. & Horvath, L., 2006. Strong approximation for the sums of squares of augmented GARCH sequences. *Bernoulli,* 12(4), pp. 583-608.

Aue, A., Berkes, I., Horvath, L. & Reimherr, M., 2009. Break detection in the covariance structure of multivariate time series models. *The Annals of Statistics,* 37(6B), pp. 4046-4087.

Bai, J., 2011. Estimating high dimensional covariance matrices and its applications. *ANNALS OF ECONOMICS AND FINANCE,* 12(2), pp. 199-215.

Bai, J. & Perron, P., 1998. Estimating and testing linear models with multiple structural changes. *Econometrica,* 66(1), pp. 47-78.

Baltas, A. N. & Kosowski, R., 2013. *Improving time-series momentum strategies: the role of volatility estimators and trading signals.* [Online]
Available at: http://www.eea-esem.com/files/papers/eea-esem/2013/1403/BK_SigVol_v2.pdf
[Accessed 2013].

Baltas, A. N. & Kosowski, R., 2013. *Momentum strategies in futures markets and trend-following funds.* Paris, Eurofidai-Affi Paper .

Balvers, R. J. & Wu, Y., 2006. Momentum and mean reversion across national equity markets. *Journal of Empirical Finance,* 13(1), pp. 24-48.

Barberies, N., Shleifer, A. & Vishny, R., 1998. A model of investor sentiment. *Journal of Financial Economics,* Volume 49, pp. 307-343.

Bessembinder, H., Coughenour, J., Seguin, P. J. & Smoller, M. M., 1995. Mean reversion in equilibrium asset prices: evidence from the futures term structure. *The Journal of Finance,* 50(1), pp. 361-375.

Breiman, L., 1961. optimal gambling systems for favorable games. *Proceedings of the 4th Berkeley Symposium on Mathematical Statistics and Probability,* Volume 1, pp. 63-68.

Browne, S., 1997. Survival and growth with a liability: optimal portfolio strategies in continuous time. *Mathematics of Operations Research,* 22(2), pp. 468-493.

Cao, R., Liu, Z., Wang, S. & Zhou, W., 2017. Multivariate volatility regulated Kelly strategy: a superior choice in low correlated portfolios. *Theoretical Economics Letter,* Volume 7.

Casassus, J. & Collin-dufresne, P., 2005. Stochastic convenience yield implied from commodity futures and interest rates. *The Journal of Finance*, 60(5), pp. 2283-2331.

Daniel, K., Hirshleifer, D. & Subrahmanyam, A., 1998. A theory of overconfidence, self-attribution, and security market under and over-reactions. *The Journal of Finance,* Volume 53, pp. 1839-1885.

Daniel, K. & Moskowitz, T. J., 2014. *Momentum crashes.* s.l., NBER Working Paper, p. No. 20439.

Davis , M. & Lleo, S., 2010. *Fractional Kelly strategies for benchmarked asset management,* s.l.: s.n.

Davis, M. & Lleo, S., 2011. *Fractional Kelly strategies in continuous time: recent developments,* s.l.: s.n.

Disatnik, D. & Benninga, S., Summer, 2007. Shrinking the covariance matrix: simpler is better. *Journal of Portfolio Management,* 33(4), pp. 56-63.

Eduardo, S. & James E., S., 2000. Short-term variation and long-term dynamics in commodity prices. *Management Science,* 46(7), pp. 893-911.

Engle, R., 2002. Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics,* Volume 20, pp. 339-350.

Erb, C. & Harvey, C., 2006. The strategic and tactical value of commodity futures. *Financial Analysis Journal,* 62(2), pp. 69-97.

Fama, E. F. & French, K. R., 1988. Commodity futures prices: some evidence on forecast power, premiums, and the theory of storage. *The Journal of Business*, 60(1), pp. 55-73.

Fuertes, A.-M., Miffre, J. & Rallis, G., 2010. Tactical allocation in commodity futures markets: Combining momentum and term structure signals. *Journal of Banking and Finance,* 34(10), pp. 2530-2548.

Garman, M. B. & J.Klass, M., 1980. On the estimation of security price volatility from historical data. *Journal of Business,* 53(1), pp. 67-78.

Golosnoy, V., Ragulin, S. & Schmid, W., 2011. CUSUM control charts for monitoring optimal portfolio weights. *Computational Statistics and Data Analysis,* Volume 55, pp. 2991-3009.

Gonzalo, C. & Eduardo S., S., 2003. Implementing a stochastic model for oil futures prices. *Energy Economics,* 25(3), pp. 215-238.

Gorton, G. & Rouwenhorst, K., 2006. Facts and fantasies about commodity futures. *Financial Analysts Journal*, 62(2), pp. 47-68.

Goyal, A. & Welch, I., 2003. Predicting the equity premium with dividend ratios. *Management Science,* 49(5), pp. 639-654.

Hakansson, N., 1970. Optimal investment and consumption strategies under risk for a class of utility functions. *Econometrica,* Volume 38, pp. 587-607.

Harry, M., 1952. Portfolio selection. *The Journal of Finance,* 7(1), pp. 77-91.

Hong, H. & Stein, J. C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance,* Volume 6, pp. 2143-2184.

Inclan, C. & Tiao, G., 1994. Use of cumulative sums of squares for retrospective detection of changes of variance. *Journal of the American Statistical Association,* 89(427), pp. 913-923.

Jegadeesh, N. & Titman, S., 1993. Returns to buying winners and selling losers: implication for stock market efficiency. *Journal of Finance,* 48(1), pp. 65-91.

Jegadeesh, N. & Titman, S., 2001. Profitability of momentum strategies: an evaluation of alternative explanations. *The Journal of Finance,* 56(2), pp. 699-720.

Jobson, J. & Korkie, B., 1980. Estimation for Markowitz efficient portfolios. *Journal of the American Statistical Association,* Volume 75, pp. 544-554.

K.Geert, R., 1998. International momentum strategies. *The Journal of Finance,* 53(1), pp. 267-284.

Kelly, J., 1956. A new interpretation of information rate. *Bell System Technical Journal,* Volume 35, pp. 917-926.

Koijen, R. S., Moskowitz, T. J., Pedersen, L. H. & Vrugt, E. B., 2016. *Carry.* [Online]
Available at: Available at SSRN: https://ssrn.com/abstract=2298565 or http://dx.doi.org/10.2139/ssrn.2298565

Koijen, R. S., Rodriguez, J. C. & Sduelz, A., 2009. Momentum and mean-reversion in strategic asset allocation. *Management Science,* 55(7), pp. 1199-1213.

Kwan, C., 2008. Estimation error in the average correlation of security returns and shrinkage estimation of covariance and correlation matrices. *Finance Research Letter,* 5(4), pp. 236-244.

Latane, H., 1959. Criteria for choice among risky ventures. *Journal of Political Economy,* Volume 67, pp. 144-155.

Ledoit, O. & Wolf, M., 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance,* 10(5), pp. 603-621.

Ledoit, O. & Wolf, M., 2004. Honey, I shrunk the sample covariance matrix. *Journal of Portfolio Management,* 30(4), pp. 110-119.

Lettau, M. & Nieuwerburgh, S. V., 2008. Reconciling the return predictability evidence. *Review of Financial Studies,* 21(4), pp. 1607-1652.

Lewellen, J., 2002. Momentum and autocorrelation in stock returns. *The Review of Financial Studies,* 15(2), pp. 533-563.

Lo, A. W. & Mackinlay, A. C., 1990. An econometric analysis of nonsynchronous trading. *Journal of Econometrics,* 45(1), pp. 181-211.

Maclean, L., Sanegre, R., Zhao, Y. & Ziemba, W., 2004. Capital growth with security. *Journal of Economic Dynamic and Control*, 28(4), pp. 937-954.

Maclean, L., Thorp, E. & Ziemba, W., 2010. Long-term capital growth: the good and bad properties of the Kelly criterion. *Quantative Finance,* 10(7), pp. 681-687.

Maclean, L., Ziemba, W. & Blazenko, G., 1992. Growth versus security in dynamic investment analysis. *Management Science,* 38(11), pp. 1562-1585.

Maclean, L., Ziemba, W. & Li, Y., 2005. Time to wealth goals in capital accumulation and the optimal trade-off of growth versus security. *Quantitative Finance*, 5(4), pp. 343-357.

Menkhoff, L., Sarno, L., Schmeling, M. & Schrimpf, A., 2012. Currency momentum strategies. *Journal of financial economics,* Volume 106, pp. 660-684.

Merton, R., 1971. Optimal consumption and portfolio rules in a continuous time model. *Journal of Economic Theory,* Volume 3, pp. 373-413.

Miffre, J. & Rallis, G., 2007. Momentum strategies in commodity futures markets. *Journal of Banking and Finance,* 31(6), pp. 1863-1886.

Molodtsova, T. & Papell, D. H., 2009. Out of sample exchange rate predicatebility with Taylor rule fundamentals. *Journal of International Economics,* 77(2), pp. 167-180.

Moskowitz, T. J., Ooi, Y. H. & Pedersen, L. H., 2012. Time series momentum. *Journal of financial economics,* 104(2), pp. 228-250.

Parkinson, M., 1980. The extreme value method for estimating the variance of the rate of return. *Journal of Business,* 53(1), pp. 61-65.

Pirrong, C., 2005. *Momentum in futures markets.* Moscow, EFA meetings paper.

Poterba, J. M. & Summers, L. H., 1988. Mean reversion in stock prices: Evidence and Implications. *Journal of Financial Economics,* 22(1), pp. 27-59.

Qu, Z. & Perron, P., 2007. Estimating and testing structural changes in multivariate regressions. *Econometrica,* 75(2), pp. 459-502.

Rajna, G. & Eduardo S., S., 1990. Stochastic convenience yield and the pricing of oil contingent claims. *The Journal of Finance,* 45(3), pp. 959-976.

Rogers, L. & Satchell, S., 1991. Estimating variance from high, low and closing prices. *Annals of Applied Probability,* 1(4), pp. 504-512.

Rossi, B. & Inoue, A., 2012. Out-of-sample forecast tests robust to the choice of window size. *Journal of Business & Economic Statistics,* 30(3), pp. 432-453.

Sanjeev, B. & Bhaskaran, S., 2006. Macromomentum: returns predictability in international equity indices. *The Journal of Business*, 79(1), pp. 429-451.

Santos, J., 2005. Did futures markets stabilise US grain prices?. *Journal of Agricultural Economics*, 53(1), pp. 25-36.

Schwartz, S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. *The Journal of Finance,* 6, 52(3), pp. 923-973.

Serban, A. F., 2010. Combining mean reversion and momentum trading strategies in foreign exchange markets. *Journal of Banking and Finance,* 34(11), pp. 2720-2727.

Shanghai institute of futures and derivatives, 2016. *The 2016 development report on China's futures market,* Shanghai: s.n.

Sharpe, W., 1963. A simplified model for portfolio analysis. *Management Science,* 9(2), pp. 277-293.

Sharpe, W. F., 1966. Mutual fund performance. *The Journal of Business,* 39(1), pp. 119-138.

Szacmary, A. C., Shen, Q. & Sharma, S. C., 2010. Trend-following trading strategies in commodity futures: a re-examination. *Journal of Banking & Finance,* 34(2), pp. 409-426.

Thorp, E., 1971. Portfolio choice and the Kelly criterion. *Proceedings of the Business and Economics Section of the American Statistical Association,* pp. 215-224.

Thorp, E., 2006. The Kelly criterion in blackjack, sports betting and the stock market. In: *Handbook of Asset and Liability Management,*. North Holland: s.n., pp. volume 1 of Hand-book in finance.

Tobias, B., Gregor, N. W. & Dominik, W., 2015. Testing for structural breaks in correlations: does it improve value-at-risk forecasting?. *Journal of Empirical Finance,* Volume 32, pp. 135-152.

Wakita, S., 2001. Efficiency of the Dojima rice futures market in Tokugawa-period Japan. *Journal of Banking and Finance,* 25(3), pp. 535-554.

Wied, D., Kramer, D. & Dehling, H., 2012. Testing for the change in correlation at an unknown point in time using an extended functional delta method. *Econometric Theory,* Volume 28, pp. 570-589.

Working, H., 1976. Futures trading and hedging. In: *The Economics of Futures Trading.* London: Palgrave Macmillan, pp. 68-82.

Yang, D. & Zhang, Q., 2000. Drift-independent volatility estimation based on high, low, open and close prices. *Journal of Business,* 73(3), pp. 477-491.

Ziemba, W., 2003. *The stochastic programming approach to asset, liability, and wealth management.* s.l.:Research Foundation Publication,CFA Institute.