THE STRENGTH OF MASONRY ARCHES.

By

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CONTENTS.

INTRODUCTION.

THE ARCH is usually described in Engineering text books as a curved structure, which under the action of vertical loads, exerts an inclined pressure on its supports.

It is really intermediary between a curved beam and a curved strut, approaching the former or the latter according as the bending stresses or compressive stresses are correspondingly predominant.

The THEORY of Masonry Arches has been, and is now in an unsatisfactory state owing to several reasons. Firstly, the materials of construction are generally cheap, and consequently, economy is not considered important. Secondly, up to quite a short time ago, investigators persisted in a type of theory which was

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Where the word ARCH is mentioned hereafter it refers to the hingeless voissoir arch, unless otherwise stated.

admittedly indeterminate. Thirdly, the enormous advance in the production of Iron and Steel in the Nineteenth Century gave a great impulse to the erection of structures made of these materials, and so, although engineers began to be alive to the importance of both experimental data and theory in structural design, the Masonry Arch was somewhat neglected. Fourthly, there were so many arches already built, that the dimensions of new arches were almost always based on these existing ones to, the detriment of research.

There is, therefore, up to the present practically no useful data on which to base a satisfactory theory. Empirical results are certainly necessary as the conditions under which an arch bears its load are too varied to admit of treatment by pure theory.

Although the mathematical difficulties have now been surmounted, there still remain many questions to be settled as regards practical difficulties, and until these are settled all theories of the Masonry or Voissoir Arch will be of doubtful utility.

Arch design has not a great wealth of experimental data to depend upon, (thus differing from most other engineering structures), although it has the greatest need of them.

Tests on arches have been made up to the present principally in Austria and in America, (particulars of which will be given later).

The author thinks that if a systematic test of as many types of arches and systems of loading as possible be undertaken, most of the outstanding difficulties hereinafter mentioned may be solved, and the design of arches be placed on a firm basis.

Arches have been built which are to all

appearances perfectly safe, but which according to theory cannot possess stability, and this shows that the theories are not yet in a satisfactory state, especially as efficiency and economy are becoming more and more important in the engineering world today.

CHAPTER 1.

HISTORY OF ARCHES.

The creations of man have often been foreshadowed by Nature, and the arch is no exception to this rule.

In many mountainous districts and around the sea coast, examples may be seen of natural arches formed by the erosive action of water and suchlike.

Probably crude artificial arches were constructed in clay or mud by man, at a very remote period, although, of course, these would not survive long enough to give any evidence of their existence today. Whether the advantages of an arch were discovered by accident, perhaps by some prehistoric builder who in order to save time and trouble put in a bent piece of timber when constructing his hut, and then found it more competent to bear the superincumbent weight, or whether the structure was copied from nature for "art's sake", antiquity does not disclose, but the oldest arch yet discovered was probably from its size constructed for architectural effect. ~~It is~~ THIS located in Babylonia and estimated to have been built somewhere about the year 4,000 B.C. according to Archeologists; was elliptical in form, and was constructed of burnt voissoir bricks laid in clay mortar. Its span was twenty inches.

The Chinese probably employed the arch at a very early period, for the purpose of bridging small streams, and it is known that bridges and other public works were executed in China 2,900 B.C.

Also, relics of ancient stone and brick arches have been found in Egypt. There was discovered in a structure known as Campbell's Tomb, a brick arch of of four concentric rings, having a span of eleven feet. This was supposed to have been constructed about 1540 B.C. The Egyptians, however, generally prefered the solid lintel for the purpose of spanning openings.

Remains of false arches formed by cutting out openings to a curved intrados in ordinary walls have been found in all parts of the world.

Arched sewers built about 1300 B.C. have been discovered beneath the palaces of Nimrod.

Sargon, 702-705 B.C. founded a city in Assyria the gateways of which were built with semicircular stone voissoirs of twelve to fifteen feet span, this being the first record of stone arches of any size.

The Romans apparently were the first to build arches of considerable magnitude. The earliest authentic arch, the Cloaca Maxima, was constructed about 615 B.C. It consisted of three concentric rings of stone. The span was fourteen feet. Stone bridges of fifty to seventy feet span were constructed by Aemilus Scaurus in 120 B.C. Trajan, 104 A.D. built at Alcantata in Spain a semi-circular arch of one hundred and ten feet span, which is still in existence. Many examples of Roman aquaducts may be seen today in the old Roman provinces.

With a progressive civilization the arch as the only possible structure capable of providing permanent means of communication between the banks of rivers and other openings of magnitude, became more and more utilised. It was also used to enhance the beauty of national and ecclesiastical buildings, as for example the magnificent European Cathedrals.

A notable instance of early engineering skill was a bridge built at Trezzo in Italy in 1377, but unfortunately destroyed in 1416. Its span was two hundred and fifty feet, the third largest yet constructed in stone. The first place in point of size, is held by a bridge at Plauen, Saxony, built in 1905. Its span is two hundred snd ninety five feet.

TABLE 1.

DIMENSIONS OF NOTABLE VOISSOIR ARCHES.

No.	Name, Description, etc.	Date.	Feet span.
1.	Syra. Plauen, Sax. H. Slate, 3 c.	1905	295
2.	Luxemburg, Germ. H. C.	1903	278
3.	Trezzo, Italy, H. Gran. C.	1377	251
4.	Morbego, Italy, H. Gran.3C.	1903	230
5.	Cabin John, U.S.A. H. Gran. C.	1859	220
6.	Pruth, Austria. R. Sandst. C.	1893	213
7.	Grutach, Germ. R. Sandst. C.	1901	210
8.	Isar Riv. Bavaria, H. 3 c.	1902	210
9.	Lavour, France. R. C.	1888	202
10.	Chester, Eng. H. Sandst 2 h. C.	1883	200
11.	Gour Noir, France. R. Gran. C.	1888	197
12.	Coppel, Germ. R. Sandst C.	1901	187
13.	Balloch, Mozle, Scot. R. C.	1844	180
14.	London Bridge. H. Gran. E.	1830	152
15	Grenoble, France. H. C.	1611	150
16	Ponty Prydd. H. Sandst C.	1755	140
17.	Maidenhead, R. Brick. E.	1838	128
18	Bourbonnais, R. Gran. C.	–	124
19.	Devils Bridge, Italy. H.	1000	120
20.	Avignon, France, H.	1187	103
21.	Alcantara, Sp. H.	100	100
22.	Bishop Auckland, Eng. H.	1388	100

	THICKNESS	
Feet Rise.	Cr. Feet	Spr.
59	4.9	11.2
102	4.7	7.2
88	4.0	4.0
33	4.9	7.2
57	4.2	6.2
59	6.9	10.2
52	6.6	9.2
21	3.4	4.2
90	5.4	12.5
42	4.5	7.0
53	5.7	13.8
53	5.9	8.5
90	4.5	6.0
38	4.8	10.0
54	3.2	-
35	1.5	1.5
24	5.3	7.5
6.9	2.7	3.6
60	4.5	-
51	2.4	-
50	-	-
22	1.8	-

Reference to Table.

H	Highway.
R	Railway.
C	Segmental.
3 c.	False Semi Ellipse.
E	Elliptical.
2 h.	Two Ø-hinged.
t	Thickness of Arch ring.

It will be seen from the preceding Table that the dimensions of similar arches vary greatly. The earlier arches are generally of bolder design, some of them being remarkably slender, for example, Nos. 15, 16, 18, 20, and 22. Some of the 19th. Century Bridges have arch rings of enormous thickness. This is probably due to the influence of the line of stress theories. Concrete and Reinforced Concrete Bridges of more recent date have frequently very thin arch rings, such as 2.8 feet for 165 ft. span (Concrete), or 0.6 feet for 122 feet span, (Reinforced Concrete), although Concrete is no stronger than well built stone masonry. This reduction follows upon the introduction of the Elastic Theory.

CHAPTER 2.

DESCRIPTION OF THEORIES.

There are two distinct types of theories for the design of masonry arches. The first and older theory is that which employs the "LINE OF STRESS", the second and newer, is that which is based upon the ELASTICITY of the arch ring. The former is always used for the design of voissoir arches, and the latter for either monolithic or metallic arches.

LINE OF STRESS THEORY.

Definition of the line of stress. The line of stress is the locus of the centroids of all the internal forces distributed over the cross section of the arch ring. Thus, assuming the arch to be "linear", its shape would be that of the line of stress and the direction of the resultant force in the arch ring is tengential at every point to the line of stress.

If the centroids of all the forces acting in the arch ring can be found, then the position and shape of the line of stress can be determined.

METHOD OF FAILURE OF ARCHES.

An arch may fail as follows (excluding extraneous causes).

(1) By direct crushing.

(2) By the sliding of adjacent voissoirs, (shearing).

(3) By Rotation at the edge of a joint (Bending).

CRITERIA.

(1) Crushing.

If the pressure at any point in the arch ring exceed the ultimate crushing strength of the material, the arch will yield, although its stability is not necessarily destroyed. This failure properly speaking is independent of the position of the line of stress, and

TABLE III.

Results of Arch No. 2.

Load per sq. ft.	R Extensions. Top.	R Extensions. Bottom.	L Extensions. Top.	L Extensions. Bottom.	Extensions. Average	Extensions. +Initial	Total Stress	Stress per sq.ft.	Stress per lin.ft.	Stress per l.ft. by Elast. Theory	Ratio Act/Theor	Ratio Thrust/Load.	¼ Span	Rise Crown	¾ Span	Deflec
lbs.	inches.	inches.	inches.	inches.	inches.	inches.	lbs.	lbs.	lbs.	lbs.			inches	inches	inches	inches Total
0	0.000	0.000	0.000	0.000	0.000	0.000	0	0	0	0	-	-	9.04	11.68	9.04	
102	0.002	0.007	0.001	0.007	-	0.004	5340	4730	1760	1275	1.39	17.45	-	-	-	
210	0.006	(0.018)	0.003	0.016	-	0.008	10680	9500	3550	2625	1.35	16.91	8.80	11.54	8.80	0.Ø14
210	0.000	0.000	0.000	0.000	0.000	0.008	10680	9500	3560	2625	1.35	16.91	8.80	11.54	8.80	0.14
243	0.002	0.003	0.003	0.001	0.002	0.010	13300	11080	4430	3037	1.46	18.23	8.64	11.46	8.64	0.22
276	0.003	0.004	0.005	0.003	0.004	0.012	16000	14200	5530	3450	1.55	19.30	8.56	11.38	8.56	0.30
316	0.005	0.006	0.008	0.006	0.006	0.014	18700	16600	6230	3950	1.58	19.70	8.40	11.20	8.40	0.48
355	0.008	0.008	(0.011)	0.008	0.008	0.016	21300	18800	7100	4437	1.60	20.00	8.40	11.12	8.40	0.56
390	0.011	0.011	0.011	0.011	0.011	0.019	25300	22400	8430	4875	1.73	21.60	8.40	11.04	8.40	0.64
424	0.013	0.013	0.013	0.013	0.013	0.021	28000	24190	9330	5300	1.76	22.00	8.40	11.00	8.40	0.68
460	0.015	0.016	0.015	0.016	0.0155	0.0235	31300	27700	10430	5750	1.82	22.70	8.40	11.00	8.40	0.68
496	(0.017)	0.019	0.019	0.019	0.0190	0.027	36000	32000	12000	6200	1.94	24.00	8.40	10.88	8.40	0.80
533	0.020	0.023	0.023	0.023	0.022	0.029	40000	35500	13330	6662	2.00	24.90	8.36	10.88	8.36	0.80
577	0.024	0.027	0.027	0.026	0.025	0.033	44000	39000	14670	7212	2.02	27.50	8.36	10.88	8.36	0.80
615	0.030	0.034	0.029	0.033	0.032	0.040	53200	47000	17730	7700	2.30	28170	8.32	10.84	8.32	0.84
653	0.034	0.038	(0.031)	0.037	0.036	0.044	58000	51100	19330	8162	2.35	29.50	8.32	10.84	8.32	0.84
694	0.038	0.045	0.038	0.044	0.042	0.050	66500	59000	22170	8675	2.56	32.00	8.28	10.80	8.28	0.88
735	0.045	0.050	0.044	0.050	0.047	0.055	73400	65000	24470	9200	2.65	33.10	8.24	10.80	8.24	0.88
775	0.049	0.053	0.047	0.053	0.050	0.058	77400	68500	25800	9700	2.66	33.20	8.24	10.76	8.24	0.92
816	0.051	0.058	0.049	0.056	0.053	0.061	81400	72100	27310	10200	2.67	33.20	8.20	10.76	8.20	0.92
852	0.053	0.060	0.050	0.059	0.055	0.063	83800	74000	27930	10650	2.62	32.60	8.16	10.72	8.16	0.96
886	0.056	(0.065)	0.053	(0.067)	0.058	0.066	88000	78000	29330	11075	2.63	32.70	8.16	10.64	8.16	1.04
210	-	-	-	-	-	0.008	10680	9500	3560	2625	1.35	16.91	8.60	11.04	8.60	0.64
276	(0.017)	0.008	0.009	0.007	0.008	0.016	21360	18900	7120	3450	2.06	25.80	8.56	10.96	8.56	0.72
355	0.014	0.012	0.014	0.012	0.0125	0.0205	27400	24200	9130	4437	2.06	25.80	8.48	10.96	8.48	0.72
424	0.019	0.017	0.018	0.017	0.017	0.025	34000	30000	11330	5300	2.12	26.60	8.48	10.92	8.48	0.76
496	0.024	0.022	0.023	0.022	0.022	0.030	40000	35500	13330	6200	2.13	26.70	8.40	10.88	8.40	0.80
577	0.029	0.027	0.028	(0.025)	0.027	0.035	46600	41500	15530	7212	2.15	27.00	'.32	10.84	'.32	0.84
653	0.031	0.030	0.030	0.031	0.030	0.038	50600	45000	16870	8162	2.05	25.80	8.28	10.84	8.28	0.84
735	0.034	0.033	0.033	0.033	0.033	0.041	54600	48400	18200	9200	1.98	24.70	8.28	10.84	8.28	0.84
816	0.039	0.037	0.036	0.036	0.037	0.045	60000	53000	20000	10200	1.96	24.50	8.24	10.80	8.24	0.88
886	0.044	0.040	(0.038)	0.040	0.041	0.049	65200	58000	21730	11075	1.96	24.50	8.20	10.76	8.20	0.92
210	-	-	-	-	-	0.008	10680	9500	3560	2625	1.35	16.91	8.60	11.04	8.60	0.64
276	0.005	0.004	0.005	0.004	0.045	0.0125	16600	14700	5530	2450	1.61	20.00	8.56	10.96	8.56	0.72
355	0.013	0.011	0.012	0.014	0.013	0.021	28000	24800	9330	4437	2.12	26.60	8.48	10.88	8.48	0.80
210	-	-	-	-	-	0.008	10680	9500	3560	2625	1.35	16.91	8.60	11.54	8.60	0.64
703	0.023	0.020	0.024	0.027	0.024	0.032	42600	37700	14200	8800	1.61	20.10	8.32	10.72	8.32	0.96
863	0.036	(0.028)	0.034	0.037	0.036	0.044	58600	52000	19530	10800	1.81	22.60	8.24	10.56	8124	1.22
1023	0.046	(0.035)	0.045	0.041	0.044	0.052	69400	61500	23130	12800	1.80	22.40	8.16	10.48	8.16	1.20
1210	0.045	-	0.047	-	0.044	0.052	69400	61500	23130	15000	1.54	19.10	6.40	10.08	-	1.60
3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	18.	18.	19.

EXPLANATION OF TABLE.

No load.

3060 lbs. of sand on Arch.

All apparatus on top of Arch - total 6300 lbs.

Commencement of Test. Permanent load 6300 lbs.

First series completed. Load removed to 6300 lbs. and second series begun.
Second series completed and third begun.

Third completed, fourth and final begun.

Load taken straight up to 21,000 lbs.

First cracks at springings.

FAILURE.

Column.	
1.	Number of reading.
2.	Total Load on the Arch. Obtained from Calibration of jacks.
3.	Load per square foot Total Load/ Area.
4. 5. 6. 7.	Differences from the four extensometer readings.
8.	Mean Elongations per 100 inches
9.	Total Elongations (Mean + that due to sand etc.).
10.	Total Horizontal Thrust on Arch obtained from calibration of Tie Rods in Testing Machine.
11.	Mean compressive stress per sq. ft. on cross section of Arch ring Total stress/ cross sectional area.

Column.	
12.	Stress per lineal foot of Arch ring To ta
13.	Theoretical Stress from Elastic formula 3W
14.	Ratio-Actual/Theoretical.
15.	Ratio stress per lineal ft./Load per sq. - of "Line of Stress" by Naviers Principle
16. 17. 18.	Rise of Arch at ¼ Span, Crown and ¾ Span f Deflectograph.
19.	Total Crown Deflection from Unloaded Profi
20.	Crown Deflection from Profile No.3 (Load 6
21.	Crown Deflection from Profile No.23 (Load subtracting permanent set.

depends only on the maximum intensity of the compressive stress.

(2) Sliding or Shearing.

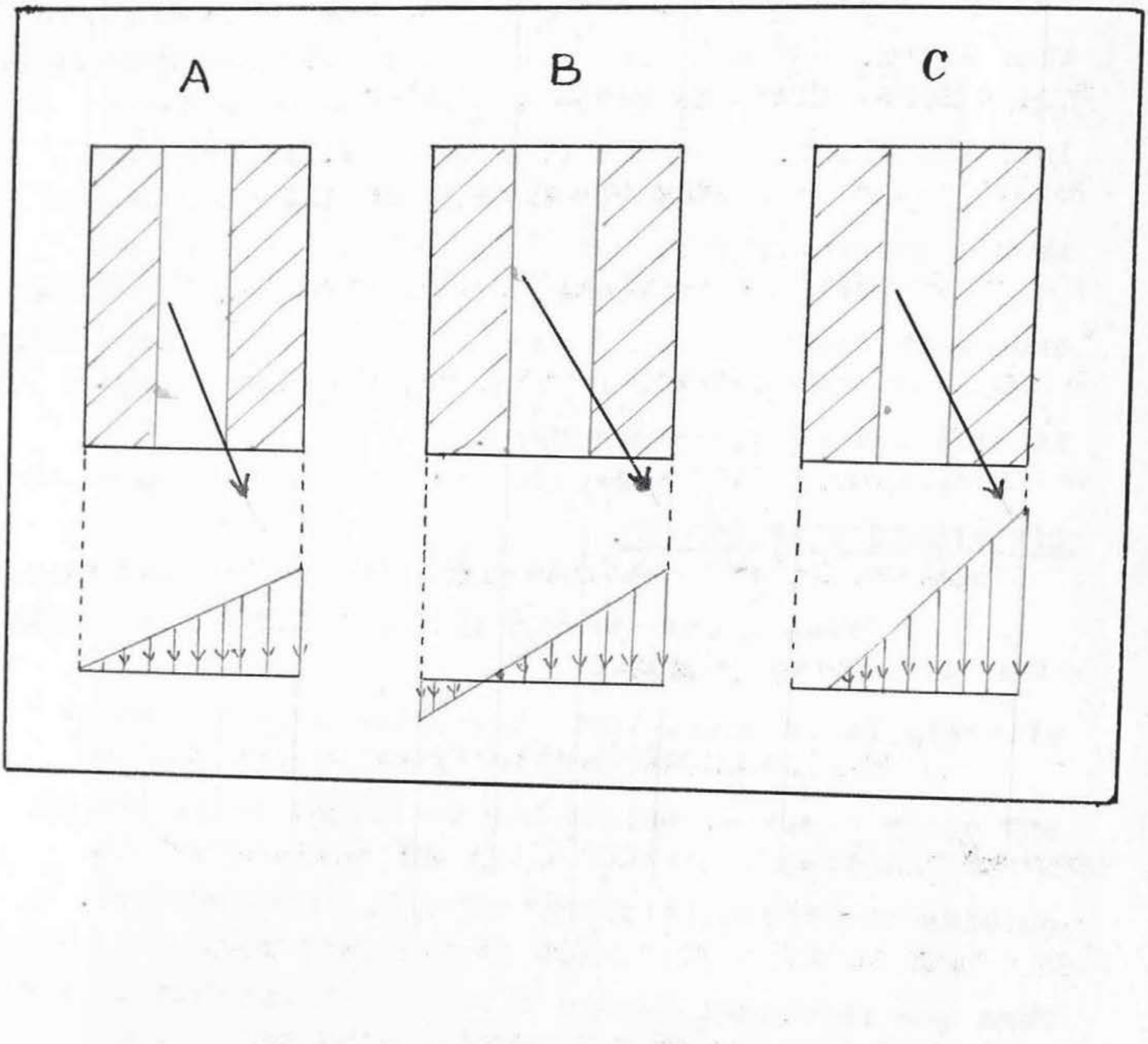
This type of failure depends upon the value of the resolved stress parallel to the plane of least shearing strength, (generally the plane of the joint). When the resolved pressure exceeds the ultimate shearing strength in that plane, then the arch will collapse. To provide against this failure the angle between the direction of resultant pressure, and the normal to the plane of least shear should be made small. This plane being almost always in voissoir arches the plane of the joints, it follows that the resultant force should not make a greater angle with the normal to the joint than that which limits the equilibrium. However, if the ratio between the maximum shear and the shear along the joint be greater than the ratio between the shearing strength of the voissoir in that plane of maximum intensity, and the shearing strength of the joint, (a possibility in brick arches), then the arch will shear in that direction across the voissoirs and independently of the joints.

(3) <u>Rotating or Bending</u>.

When the arch is subjected to a bending moment of the usual order, compression is induced, (generally speaking), in the upper side of the arch ring, and tension in the lower. These stresses must be compounded with the compression induced by "arch action". The tension may entirely disappear or some part of it may remain. If the arch ring cannot rise or fall at some other place, (that is to say, unless the line of stress passes through the extradons at two points, and the intrados at an intermediate point, or vi@a versa), this tensile stress will not endanger the stability of the arch even if the joints possess no tenacity, unless the upper or lower side, as the case may be, can crush sufficiently to enable a reverse curve to set in at that point. Then this case practically resolves itself into the first. As a matter of fact, in practice arches generally fail owing to the movement of the abutments, but as this does not concern the arch itself it will not be discussed here.

DISTRIBUTION OF STRESS.

When a body is under the action of internal stresses it is known that no tension is produced at any cross section, unless the resultant force passes outside the "middle" third" of that cross section. When the resultant passes through the boundary of the middle third the distribution of stress is shown in A. being zero at the further edge, and having a maximum value at the adjacent edge.


A
B
C

Suppose there is tensile strength at this cross section, and the resultant passes outside the middle third. Tension is now induced at the further side as shown in B. <u>But</u>, suppose that the cross section can develop no tensile strength. The equilibrium of the body is not destroyed when the resultant passes outside the middle third, this not occurring until the resultant passes entirely outside the cross section. The distribution of stress is in this casemas shown in C. The compressive strength becomes greater.

The direction and position of the line of stress represents the direction and position of the resultant force at any joint in the arch ring, so that the distribution of stress over a joint under the various conditions is as shown above. When the joints can with stand tension the distribution of stress when the line of stress departs from the middle third is as shown in B. When they are unable to withstand it the compressive strength is greater than that in B. <u>But</u> the joints will not open at a greater rate than before, (ie. no more than the compression on the opposite side will allow), and the arch is not unstable till the line of stress passes outside the cross section, or until the maximum compressive stress is greater than the ultimate crushing strength.

Nearly all authors of text books on Voissoir arch design do not state this, but say that when the line of stress deviates from the middle third, if the joints possess no tensile stress they will open entirely. This is not so as proved above.

The experimental proof of this will be seen in report of Test 1. Nevertheless the above does not sanction the departure of the line of stress in Voissoir arches from the middle third for any existing tensile strength may be destroyed, and consequently the ability of the joint to resist shearing strength may be impaired.

FACTORS OF SEFETY.

1. Against Crushing.

The theoretical factor of safety is equal to the ultimate crushing strength of the arch ring divided by the maximum compressive stress induced. Actually, this factor of safety is not of much value at present, as the ultimate crushing strength of the arch ring is never known. It is certainly not the same as the crushing strength of the voissoirs of which the arch is composed, nor yet the same as that of the material of which the joints are made.

One of the objects of this present test is to discover this quantity. It undoubtedly varies with the unsupported length of the arch ring, and with the cross sectional dimensions, for the arch ring being under compression is obviously acting as a strut, and, of course, the strength of a strut varies as cross sectional dimensions, and inversely as the length. Therefore the author thinks that the allowable stress should vary as the dimensions of the arch, in fact, it should be determined in a similar manner as the stress in struts by a modification of Gordon's or Rankine's formula. The precise form of, and the constants in such a formula can only be determined by experiment. It is certainly necessary to ascertain such fundamental facts before the design of Voissoir Arches can be placed on a satisfactory basis.

Furthermore, if the value of compressive stress induced is determined by the line of stress theory, this factor of safety is rendered still less reliable owing to the arbitrary assumptions that must necessarily be made in this theory.

(2) Against Shearing.

This also is unknown and can only be determined by experiment.

(3) Against Rotating or Bending.

This again is unsatisfactory if determined by the line of stress theory for reasons given above, but as expressed by most writers it is really only a formal quantity. However, it is not important as the limiting position of the line of stress is generally regulated (other things being equal), by the allowable compressive stress. A record of these factors of safety shows the necessity of reliable experimental data.

LINE OF STRESS THEORY.

Apart from practical and mechanical difficulties which are, of course, common to all theories of the voissoir arch, the fundamental mathematical difficulty is in finding the true position of the line of stress in the arch. It may be stated definitely, that it is impossible to surmount this difficulty by mathematical device alone, owing to the excess of unknown quantities, so that although the theory may give approximate results, it is really irrational. A possible means of partially rationalising it by experiment will be described at the end of this thesis.

Many unverified assumptions to evade this "impasse" have been made, and the chief of these will be described after the mechanical difficulties have been briefly enumerated. These refer chiefly to the external forces. The points of application, direction, and intensities of the external forces are never accurately known, except in the case of a water load, but a water load has very rarely to be dealt with.

In most cases the load is either distributed through an earth cushion, or by means of "spandrel walls", the former being used for flat arches which do not require a heavy fill.

The maximum vertical pressure due to an earth load is, of course, equal to its weight. The horizontal pressure must be assumed. Rankine's theory of Eearth pressure gives the simplest results, namely, that the greatest ratio of the horizontal and vertical components is equal to $1 \pm \sin\phi / 1 \mp \sin\phi$, ϕ being the angle of repose, that is the steepest slope at which a heap of the loose material will stand. The horizontal component may exceed the vertical if the earth is "punned" and vice versa if the earth is loose. As is generally taken to be 30 degrees the ratio resolves itself into the limiting values of 3 or 1/3.

The horizontal load is frequently neglected.

Alexander & Thomson's method of treating it by means of the Horizontal Conjugate Load Areas is probably the best (see later).

For medium sized arches the space between the arch ring and the formation level is usually taken up by longitudinal spandel walls and arches riding out over the arch ring.

The pressure exerted by these is indeterminate. It is certainly less than the actual weight.

In large arches the "spandrels" are frequently placed transversely across the arch ring at equal intervals. The forces in this case are vertical and commeasurable, but the arch ring is not so strong as the transverse walls do not increase its rigidity as a strut. Also when the load is concentrated at points in the arch ring, there must be "kinks" in the line of stress and the effect of shear is probably important.

The effect of the various systems of load distribution may be discovered by observing the difference in the results of tests of similar arch rings loaded according to these methods.

After the external forces have been assumed the line of stress can be dealt with.

In hingeless arches in order to determine the absolute and actual position of the line of stress in the arch ring, it is necessary to know the crown thrust and in order to know the crown thrust, it is necessary to know the position of the line of stress. Consequently, there is a "cul de sac" and it is at this fundamental point that the rationality of the line of stress theory ceases.

Many assumptions have been made, and the most important will now be discussed. Each one has its supporters who all believe in the superiority of their own particular theory. Some of the assumptions are apparently satisfactory, but, of course, no theory that is based on unverified assumptions can be called rational.

HYPOTHESIS OF LEAST PRESSURE.

This hypothesis is that the true line of stress is that which gives the least absolute pressure on any joint. This gives undoubtedly incorrect results. If every joint could be substituted the assumption might be correct, but then the solution would trancend the limits of mathematical evaluation.

WINKLER'S HYPOTHESIS OF LEAST SQUARES.

"For an arch ring of constant cross section that line of stress is approximately the true one which lies nearest to the axis of the arch ring as determined by the method of 'least squares' ".

The advantage possessed by this assumption is that certain conclusions can be drawn from it which agree with the elastic theory.

Its chief disadvantages are:-

(1) External forces must be vertical and uniform (of rare occurrence in practice).

(2) The cross section of the arch is not usually uniform.

(3) It gives no clue to the real shape of the line of stress. For instance, suppose a line of stress be drawn which fulfills the conditions. Reverse the direction of the offsets from the axis of the arch ring. The condition is still fulfilled, bu t the shape of the line of stress is quite different.

(4) The method is tentative, and very troublesome to apply completely.

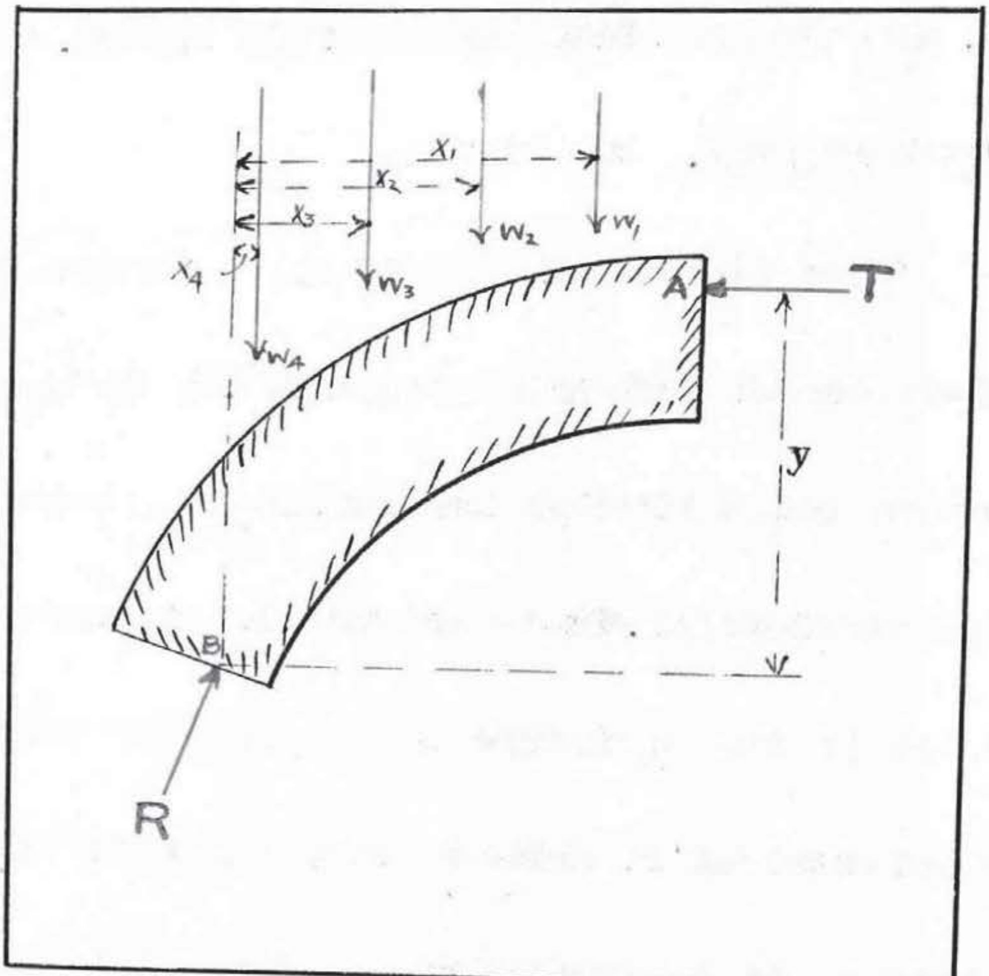
A practical interpretation of this is that if any line of stress can be constructed in the middle third of the arch ring, the true line of stress lies therein, but, unfortunately, this does not follow from Winkler's theorem, and is probably incorrect in itself.

NAVIER'S PRINCIPLE.

"The tangential stress at any point of a circle pressed by normal forces is equal to the normal intensity of pressure multiplied by the radius of curvature of the circle at that point." This is true if the arch is linear, or if the curvature of the line of stress is known, but then as it depends on the line of stress for its solution, it is impossible to determine the line of stress from it without other assumptions.

LEAST CROWN THRUST HYPOTHESIS.

This is the most common assumption. It assumes that the true line of stress is that which produces the least crown thrust consistent with equilibrium. This may



X_1
X_2
X_3
X_4
W_1
W_2
W_3
W_4
A
T
y
B
R

This may or may not be correct, but the only known method of determining is incorrect.

According to Baker, it is determined thus. For simplicity assume that the external forces are vertical and as shown in diagram.

LET

T = Crown thrust applied at A.

R = Reaction at springing applied at B. (Value and direction immaterial).

y = vertical distance between points of application of T and R.

X_1 X_2 etc = horizontal distances of w_1 w_2 (the external forces) from B.

Then, taking moments about B

$$T\,y = w_1 x_1 + w_2 x_2 \quad \text{etc.}$$

$$\text{or} \quad T = \sum \frac{wx}{y}$$

It appears from this that in order to make T smaller y should be increased. That is that the arch should have a bigger rise. This is, of course, true, but, unfortunately, it is quite irrelevant to the point in question.

The conclusion drawn by the supporters of this theory is that for a minimum crown thrust, the point of application of T should approach the extrados of the crown.

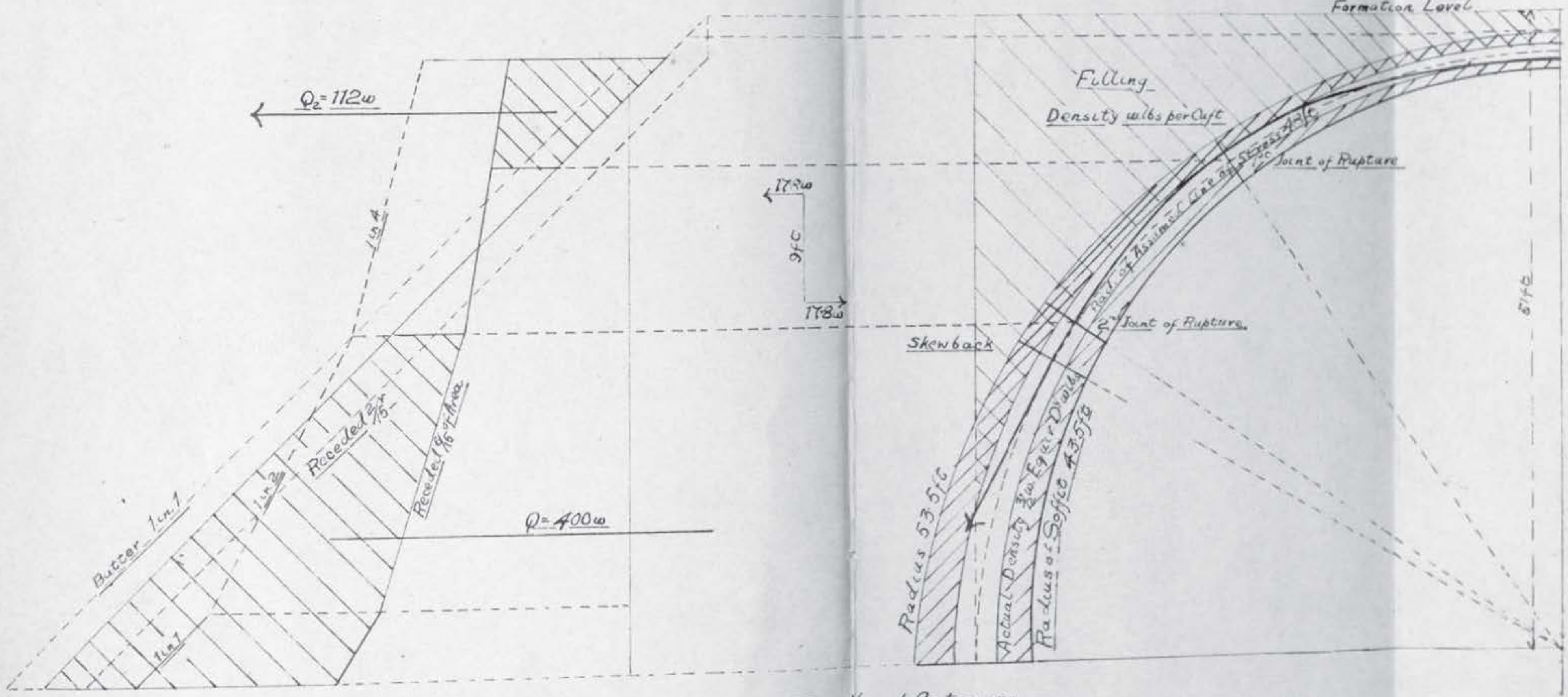
Now the thickness of an arch ring is generally small compared to the rise. Therefore instead of using y in the equation, $(a+t)$ should be substituted where a is equal to the rise and t the thickness of the arch ring.

The equation then becomes - $T = \frac{\sum wx}{a+t}$

$\sum wx$ is generally a large quantity, and the maximum and minimum values for the former y are $a+t$ and a as T cannot depart from the arch ring altogether. In practice the maximum and minimum values are probably still nearer together as the thrust T is not likely to be at the edge of the arch ring. Therefore (a+ t) does not sensibly differ from a and the value of T is only changed by a very small amount even when its point of application is shifted. from the bottom of the arch ring to the top From this it will be seen that this method is practically useless. Moreover, the line of stress according to the first equation should be as high up as possible at the crown, that is it should pass through the extrados itself at that point, - an absurd conclusion. The line of stress is assumed (to evade this absurdity) to pass through the upper limit of the middle third an absolutely arbitrary assumption, and one which quite condemns the theory. The line of stress given by this

SEGMENTAL ARCH

— *Scale — 1 inch = 10 feet* —



— Line of Stress is a Two-Nosed Catenary —

— Q_1 is Positive Horizontal Load —

— Q_2 is Neg. Horiz. Load (Neglected) —

— From Alexr & Thomn Appd Mechs —

method is of quite opposite character to that according to the two nosed Catenary method.

In all these theories when the starting point of the line of stress is assumed, the line itself is drawn by means of a force diagram and link polygon on the usual way.

ALEXANDER & THOMSON'S TWO-NOSED CATENARY THEORY.

This theory is based on Rankine's treatment of the voissoir arch. It is the most satisfactory of the line of stress theories both because as far as the mathematical part is concerned it is more rational, and because it permits the design of an arch by direct methods. Its disadvantages are that it is by no means an analytical method, and therefore it is not suitable for testing the stability of existing arches. It suffers from the same defect as the other line of stress theories in that the correctness of the position of the line of stress at the crown is not assured, but in this case it is not so serious as the shape of the line of stress is independent of its assumed position. It can also only use one system of loading, ie. vertical forces due to a fixed arrangement of weight.

N.B. If transverse spandrels are used the shape of the line of stress is more nearly polygonal and the tangents to the line of stress which meet at the centre of each transverse spandrel should be used instead of the line of stress itself.

The distribution and amount of the horizontal thrust of the abutments and the position of the "Joint of Rupture" are found by the method of Conjugate Load Areas.

N.B. This Joint of Rupture is the point where the

Horizontal load changes sign. It is not the point where the line of stress most nearly approaches the boundary of the arch ring, and it is only a Joint of Rupture in that it determines the limit of the"elastic" part of the arch ring, the portion below it being really part of the abutment.

The Conjugate Load Areas show the relation of the horizontal and vertical loads as follows. It is known that a chain or linear rib under the influence of a vertical load uniformly distributed along the span assumes the shape of a parabola, and from the properties of a parabola the following condition of equilibrium can be deduced.

"That the tangent at any point shall meet the crown tangent on the vertical through the centre of gravity of the portion of the load area from the crown back. This is a very easy way of finding the extra horizontal load due to an increased live load.

The shape of the conjugate load area for a load along the arch ring is next found as follows. The thickness of the arch ring is first taken equal to half the radius. In this way Alexander and Thomson* discovered that the horizontal area was bounded approximately by three lines of different slope, namely, (starting from the top), a line of 1 in 4 slope for the first four-ninths of the distance; then 1 in 2 for next four-ninths, and lastly 1 in 1 for the final ninth. By the equations used to determine this "treble batter line" the value of the thrust at the crown was found to be equal to $\frac{r}{3}$ and at springing $\frac{\pi r}{4}$ (in terms of earth potential r being the radius).

This extravagant arch ring and the load area corresponding to it, are now removed and an arch ring of practical thickness substituted, (say $\frac{2}{15} \frac{r}{2}$) $\mathfrak{f}$.

This is then added to the right of the "trebles batter line" as shown, the excess weight of the arch ring is also added and the line removed still further. As the thickness of a segmental arch ring is increased towards the springing this must be allowed for. It is found to be equivalent to a positive load along the arch ring, and a negative load along the formation level. The "45 degrees" line is shifted to the right a distance of $2/10.r$ and the "treble batter line" at total distance of $3/15.r$ The final horizontal load area is then as shown. The "point of rupture" is at P where the load changes sign.

Alexander and Thomson next prove the following important statement.

"That the curve which is in equilibrium under the load between itself and the formation level is a Catenary." This catenary must be transformed according to the depth of load of the crown, and the radius of curvature of the equilibrium curve at that point. It is found that this ratio is equal to the square of the ratio of transformation. When this ratio exceeds 1/3 the curve is sharpest at the vertex.

When it is less than 1/3 the vertex is flat and there are two "noses" on either side of it at a certain distance out. This latter is the only case which need be considered in the design of voissoir arches.

The proceedure now is to calculate the exact form of the transformed catenary, and to draw a "kernel" of proper shape to contain it. The width of this is increased three times, and thus the line of stress is confined to the middle third of an arch ring.

The excess weight of the arch ring is allowed for, and tables of the various quantities for different sized arches have been prepared.

This method is a very elegant and easy way of designing an arch, and gives results certainly on the safe side. The shape of the line of stress apparently is correct, but whether the line is correctly located is not certain. The loading in practice is not often distributed as the theory requires, and no real allowance is made for unsymmetrical loads, but considering the present state of ignorance about allowable stresses, and such like, it is probably the best theory to employ for Voissoir Arch design. It is very simple and very expeditious in application, in fact, with the help of the tables it is possible to design an arch of any size in half-an-hour.

ELASTIC THEORY.

This theory is seldom applied in the design of voissoir arches, as it is doubtful whether it holds at all completely for voissoir arches, and considering the present state of knowledge of the data concerning the strength of masonry, the advantages gained by its use are quite counterbalanced by the extra trouble and complication that ensue. It is the only completely rational theory of the hingeless arch yet formulated, but it labours under the same disadvantages as the others as regards practical difficulties. It is inferior to the transformed Catenary theory in one respect as it is not a direct method but simply a method of verification.

It finds the internal stresses by means of the Calculus either entirely analytically or partly by graphical methods, and can take into account the effect of axial stress on the length of the arch ring, (which is important in flat voissoir arches), and also the effect of changes of temperature which, however, are not important in masonry arches, as they probably alter only a small amount in temperature after they have been built (that is, of course, in temperate countries.)

As the theory is much too long to describe in a limited space with any degree of intelligibility it will not be further treated here.

(The discussion of the various theories is based on their representation in the following text books).

Line of Stress Theories, Baker's Masonry.

American Civil Engineers Pocket Book.

Transformed Catenary Theory. Alexander & Thomson's Applied Mechanics.

Elastic Theory. Howe's Treatise on Arches.

CHAPTER 111.

FORMER TESTS.

The most noteworthy series of tests yet completed were undertaken by the Austrian Society of Engineers in 1895. A report was published under the title of "Bericht des G ewolbe, - Auschusses" but the author has not had time to read it, so that a short epitome will be given from the Proc. Inst. C.E. Vol. CXXlV.

Tests were carried out in the following series (N. B. Voissoir arches only will be noticed).

(1) Structures of the following spans, -

(a) 4 ft. 5 in span. two brick arches.

(b) 8 ft. 10 in. span, one brick arch.

(c) 13 ft. 3 in. span, one brick arch.

The width was in each case 6 ft. 6 in. and the arches were loaded with pig iron. In series (a) the thickness was 6 in and the rise $5\frac{3}{4}$" and the bricks were built in lime-mortar. The arches withstood a load of 1435 lbs. per square foot with only slight cracks and apparently no real signs of failure.

In series (b) the load was concentrated on one half of the span. The thickness of the arch was 6 inches and the rise 10 inches. Failure occurred at a load of 884 lbs. per sq. ft. the final deflection before rupture being 2.08 inches. In series (c) 13 ft. 3 in span, thickness $5\frac{1}{2}$ inches, at 137 lbs. per sq. ft. the arch showed signs of weakness, and ruptured at 275 lbs.

(11) Span 75 ft. 6 in. one stone arch,

one brick arch.

In each case the dimensions method of building and loading were the same. The thickness was 2 ft. at the crown and 3 ft. $7\frac{1}{2}$ in. at the springings, and the radius of the arch ring was 54 ft. Portland cement-mortar was used of strength 1 to 2.6.

The stone arch failed at 709 lbs. per sq. ft. and the brick arch ruptured with a load of 638 lbs. per sq..ft. In each case the ultimate strength of the arch was by no means reached when the first crack appeared.

As the abstract given in the Proc. Inst. C.E. is very brief, it is not possible to discuss fully the results obtained. The very important point missed in these investigations was the measurement of the actual horizontal thrust in the arch. This was, of course, impossible in the larger arches, but was possible in the smaller.

A series of tests on floor arches was undertaken in America in 1891 and 1897. The arches were all composed of hollow fire-proof tiles. The results obtained are not of much use in this branch of the subject. They simply gave a comparison between the different forms of tiles.

In 1896 at the Ehinger works of the Stuttgarten Cementfabrik, a concrete arch was built and tested by dead weight. Iron girders were embedded in the arch. As the abutments gave way the test was not very successful. (Proc. Inst. C. E. Vol. CXXXlX.)

Three Monier Concrete arches were tested by a Mr Beers in 1897, but these again were only to confirm special methods of construction and not general principles. (Proc. Inst. C. E. Vol. CXXXlll.)

The Massachussets Institute of Technology possess a Hydraulic Arch Testing Apparatus, similar to that built by the author, but no results have yet been published. Provision is made for the measurement of the Horizontal thrust in the arch, and if results can be obtained from U.S.A. in time to insert in this thesis a valuable check on the author's results will be obtained.

C H A P T E R IV.

DESCRIPTION OF APPARATUS USED BY THE AUTHOR FOR ARCH TESTING.

In October 1910 Professor Dixon suggested to Messrs. Romero-Day, G. D. Agbebi, and the author, the testing of Voissoir Arches as a subject for the Fourth Year Laboratory course in Masonry.

Designs were then prepared for a segmental arch which would be of sufficient size to obtain reasonable results. For the first series the span was fixed at 10 ft. the rise 1 ft. and the width 3 ft. and it was decided that at first the arch should be loaded by means of weights. The apparatus will now be described in detail.

THE SKEWBACKS (Plate 2) & PIERS, ETC. (Plate 3.)

The skewbacks each consisted of a $9\frac{1}{2}"$ x $9\frac{1}{2}"$ x 4'0" Broad Flange I Beam to which was bolted at the correct angle by means of suitable angle plates, a steel plate 16" x 1" x 4'0". These skewbacks were connected together by two tie bars 11'8" long by $1\frac{5}{8}$" diameter upset at the ends to $1\frac{7}{8}$" and screwed. An extensometer was designed and made to measure the extension on these bars, and so to obtain the horizontal thrust due to the load on the arch. In order to dispose of frictional resistance as far as possible one of the skewbacks was made to rest on two steel rollers 1" diameter, which in turn rested on a steel plate. The whole was supported on two baulks of wood each 12" x 12" x 3'0", placed on two brickwork piers 1'0" high so as to ensure the arch being at a convenient distance above the ground.

CENTERING. (Plate 3.)

The Centering consisted of four 3" planks bolted together in pairs and tied together by tie rods, at a distance of 2'6" apart. Their upper edges were cut to a curve of 13 ft. radius. Lagging composed by 2" x 3" x 3'0" slabs was placed on the centres and slack blocks were used to support them in the usual manner.

THE ARCH. (Plate 3).

As the time available during 1910-11 was one afternoon per week, the work naturally went on very slowly and the first arch was not completed till June 1911. This arch was built of common red bricks with practically a sand joint, a very small amount of cement being added to facilitate bricklaying. The thickness of the arch ring was $4\frac{1}{2}$" or $\frac{1}{2}$ brick. As soon as the sand was dry the centres were lowered. No appreciable amount of settlement was observed as the joints were made as thin as possible.

LOADING. (Plate 3.)

It was decided to distribute the load uniformly over the arch by means of a sand "fill". A framework was therefore made of 1" boards cut to a curve to fit the extrados of the arch, and tied together securely. This was placed on the arch and filled with sand. The arch was then tested by placing half ton weights on it. A full description of the actual test will be found in Chapter V.

Owing to the lack of sufficient weights the arch could not be broken.

HYDRAULIC TESTING APPARATUS. (Plates 5 and 6.)

The author having obtained the Bowen Research Scholarship in June 1911, it was decided by Professor Dixon to continue the testing of arches. The method of testing by weights was found to be very cumbrous and unsatisfactory, so the author in the summer vacation prepared designs for a Hydraulic Arch Testing Apparatus of 70 tons capacity. Two Tangye Hydraulic Ship# Jacks each of 35 tons capacity have been acquired to produce the necessary load.

It will be seen that the height of the piers has been increased to 5 ft. Resting on these piers are two 20" x 7½" x 12'0" I Beams (B.4) connected at the bottom flanges by two pairs of 8" x 4" x 3'1½" I Beams (B.5). These latter support the Hydraulic Jacks which have been fixed firmly in an inverted position to these Beams so that they could be connected to a hydraulic gauge to enable the pressure to be read. As the Jacks have been calibrated in the 100 Ton Testing machine a fair accuracy can be obtained in measuring the applied load by reading the gauge.

Each respective ram exerts pressure on a

12" x 6" x 4'0" I Beam (B. 6) which depends by means of four $1\frac{5}{8}$" rods 14 ft. long, (two each side) from the corresponding set of beams on top of the arch.

It was obviously unsuitable to place the Jacks in an upright position on these lower beams, and then connect them by a rigid pipe to the gauge as during the test the connection would undoubtedly be broken.

<u>DISTRIBUTION OF THE LOAD</u>.

The apparatus is arranged so that the full load can be exerted on either half of the arch. The sand surcharge has been still adhered to, but a stronger and deeper box to contain it has been made. The boards of which it is composed are 2" thick, and connected together by angle irons and tie rods. The box has been hinged to allow of deflection in the arch.

Thirty wood slats 2" x 3" x 2'8" are placed transversely on the sand, and on these, longitudinally, are placed the first tier of beams (B. 3). These are twelve in number, being placed in three rows, and are of dimensions 6" x 3" x 2'3½". It will be noticed that these have been bevelled up at the ends. This is to allow any deflection in the arch to take place without hindrance. Across the centre of each set of these beams is placed a triangular cast iron beam, the upper edge of which is protected where necessary by a piece of 1¼" x 1¼" x 6" steel angle bent to 60 degrees to prevent crushing.

These triangular beams support two pairs of 12" x 5" x 3'6" I beams (B. 2) spaced 1 ft. apart. Across the centre of each pair of these beams is placed a 16" x 6" x 4'6" I beam (B. 1.) and on to the bottom flange of this are bolted two semicircular cast iron rockers 1 ft. apart, so as to rest on the beams B. 2.

These allow the arch to deflect without disturbing the topmost beams from their vertical position.

The beams B. 1. are situated exactly above the beams B. 6 and are connected to them by means of the long bars before mentioned. Across each end of the top flanges of B. 1 and each end of the bottom flanges of B. 6 are bolted two channels 4" x 2" x 1'3", face to face, the bars passing between them. The bars are screwed so that the distance between the lower beams and the jacks can be regulated by means of nuts. Thus the load can be applied either uniformly all over the arch or uniformly on either half.

<u>EXTENSOMETER:</u> (Plate 4.)

When the first arch was tested time being very limited, the extensometer was incomplete and extension was only measured on one horizontal tie bar. The extensometer has now been completed and is arranged as follows. Two cast iron rings of the form shown in Plate 4 are clamped by three set screws in each, near each end of the tie bars, the distance between them being 100 inches. Two screws are screwed parallel to the bars into the top and bottom of each ring, the heads of the screws being turned toward the middle of the bars. A small hole $\frac{1}{8}$" diameter is drilled in the head of each screw, and between each pair of screws is a measuring rod 7'0" long and $\frac{3}{4}$" diameter, tapered at the ends. Each tie bar has two of these rods supported above and below it, and free to slide on the supports. One end of the measuring rod fits into the $\frac{1}{8}$" hole in the head of one screw, while the other is about twelve inches away from the opposite screw. An inside micrometer reading to one thousahdth of an inch can be placed between this end of the bar and the screw. Thus to obtain the value of the horizontal thrust for a particular load four micrometer readings are taken, thus eliminating chance of error creeping in through inaccurate measurements. Each tie bar has been tested up to seventy ~~pounds~~ thousand pounds in the 300 Ton Testing machine, using the above extensometer.

MEASURING THE DEFLECTIONS.

In the first test the vertical deflections of the arch were found by measuring the height of the arch, at every two feet from the ground. This was inconvenient, tedious, and probably inaccurate, and in November 1911 Professor Dixon suggested using a pantograph to copy the intrados of the arch on a reduced scale. A "Deflectograph" was accordingly designed, and made by the author to give a reduction of one eighth.

The general arrangement can be seen from Plate 7.

The jointed framework rotates in a cast iron block which is supported by two pieces of angle iron from the main girder B. 4. The board on which the curves are drawn is arranged to slide up and down so that the curves may be drawn separately to avoid confusion. The diagram is produced direct on tracing cloth by a steel chisel point, the cloth being backed by carbon paper. Diagrams can be reproduced on blue prints for reference and illustration, and preliminary measurements can be taken from them without damaging the originals or introducing inaccuracies by tracing. Of course, it must be borne in mind that blue print paper expands considerably on being wetted saturated and "squeegeed". A better arrangement would be a steel point moving on a zinc sheet. The line can be made much finer and all errors due to stretching of the cloth will be avoided. An accuracy of one-hundredth of an inch in the diagram (corresponding to less than a tenth of an inch in the arch), can then be easily obtained and can be depended upon.

It is possible to measure one hundredth of an inch on the tracing cloth, but the accuracy cannot be depended upon.

The Zinc sheet has been adopted in the third test. The steel point is pressed on the board by means of a spring, and can be withdrawn when required.

The deflection of the main girder on the squeezing of the timber baulks on which the skewbacks rest will not affect the accuracy of the curves. Provision has been made to prevent the girders on top of the arch from falling and doing damage as will be seen in the photographs.

During every test the timber centreing has been kept a few inches below the arch ring, so that when the arch collapses it will not have far to fall. During the first test with the hydraulic apparatus pressure was supplied by a high pressure pump situated about twenty feet from the arch for the sake of safety, but when the arch broke it came down gently on the centres and nothing untoward happened.

A convenient platform 7 ft. from the ground has been erected round the arch, and has been made strong enough to take a pair of shear legs for lifting purposes.

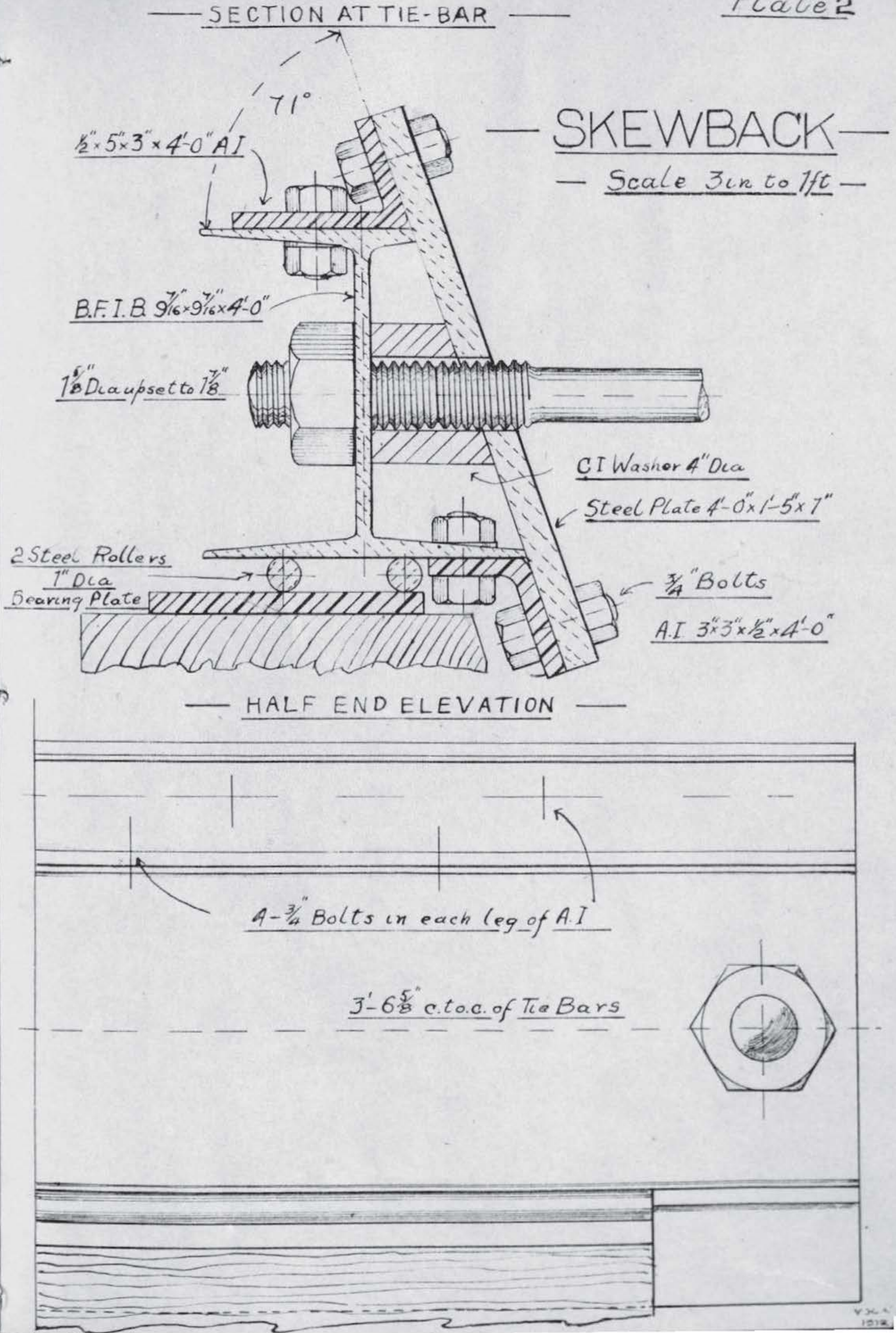
FUTURE IMPROVEMENTS DESIRABLE.

Skewbacks. Rocker bearings instead of roller bearings extending the whole width of the skew backs. The skewbacks will then be better balanced and better supported. NEW SKEWBACKS

Stronger tie bars of better steel. (Absolutely necessary.)

An additional extensometer, preferably a visible recorder.

Plate 2



SECTION AT TIE-BAR
71°
SKEWBACK
Scale 3in to 1ft
1/2"×5"×3"×4'-0" A.I
B.F.I.B. 9 7/16"×9 7/16"×4'-0"
1 5/8" Dia upset to 1 7/8"
CI Washer 4" Dia
Steel Plate 4'-0"×1'-5"×7"
2 Steel Rollers
1" Dia
Bearing Plate
3/4" Bolts
A.I 3"×3"×1/2"×4'-0"
HALF END ELEVATION
4-3/4" Bolts in each leg of A.I
3'-6 5/8" c.to.c. of Tie Bars

ARCH NO 1

Scale 1in to 1ft

FRONT ELEVATION

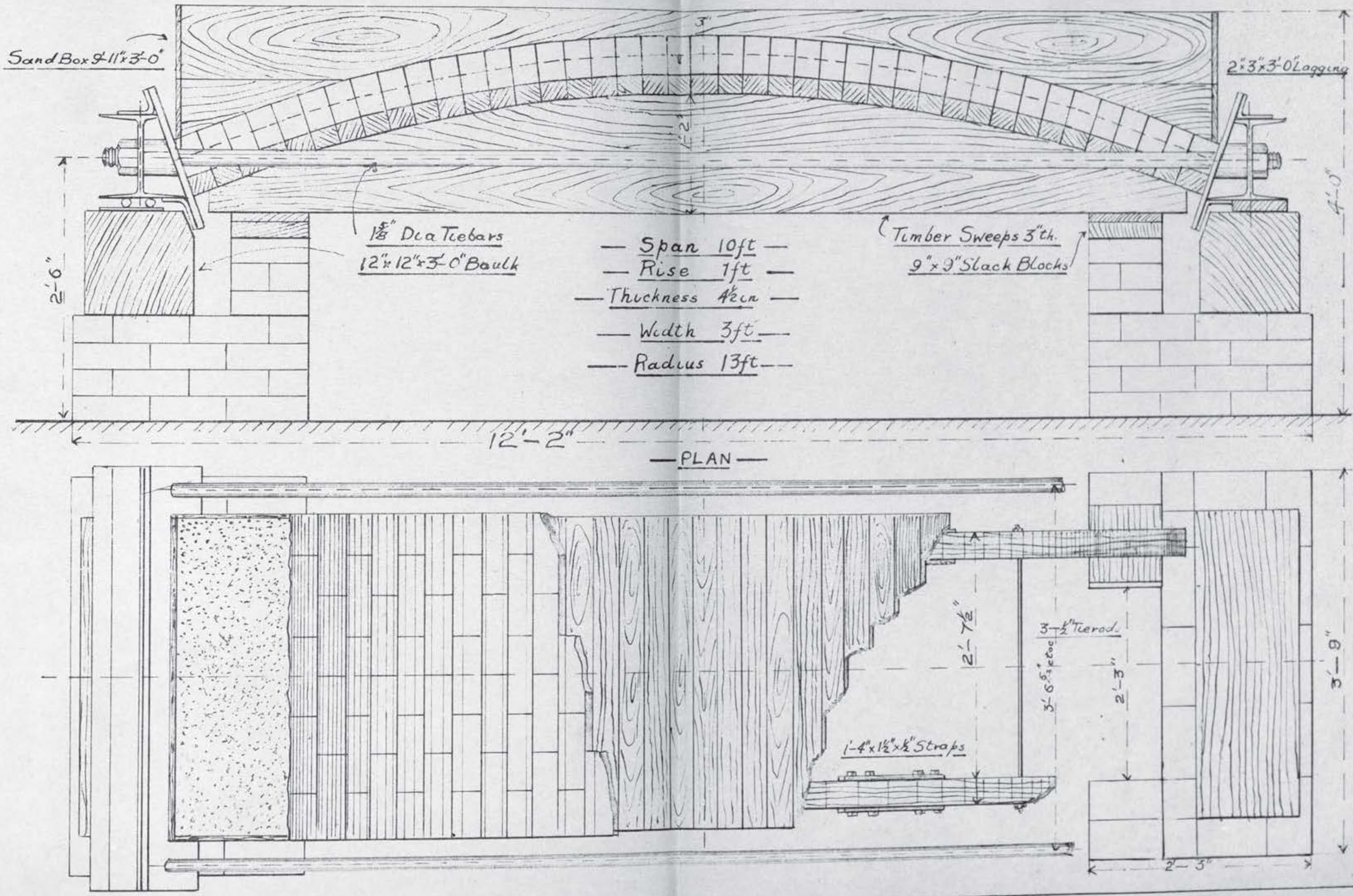
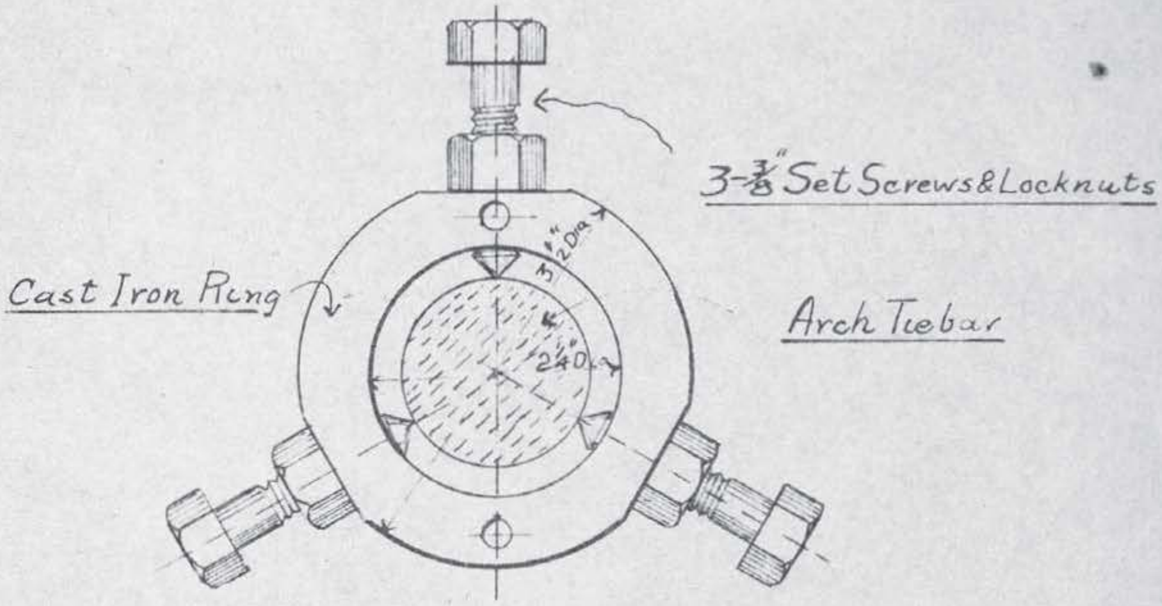


Plate 4.

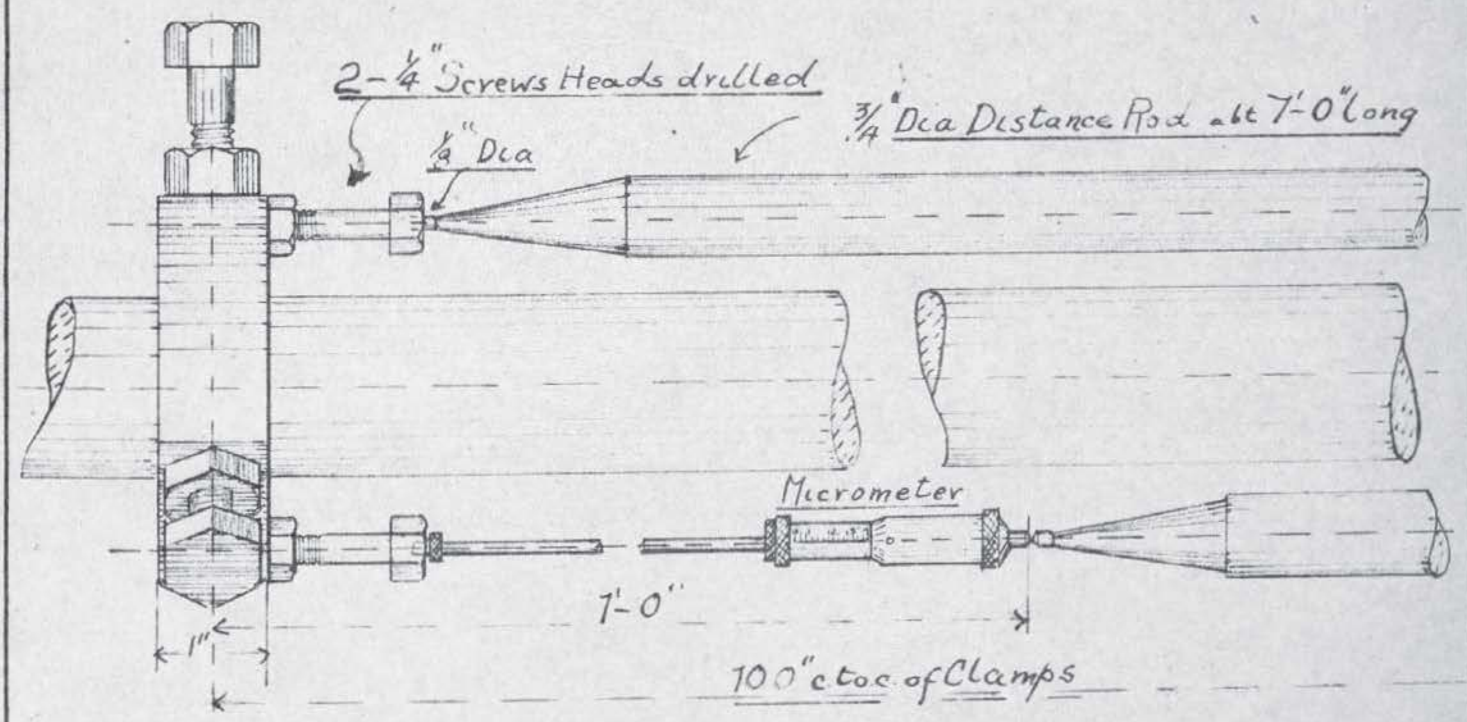
EXTENSOMETER

One Half Size

END ELEVATION

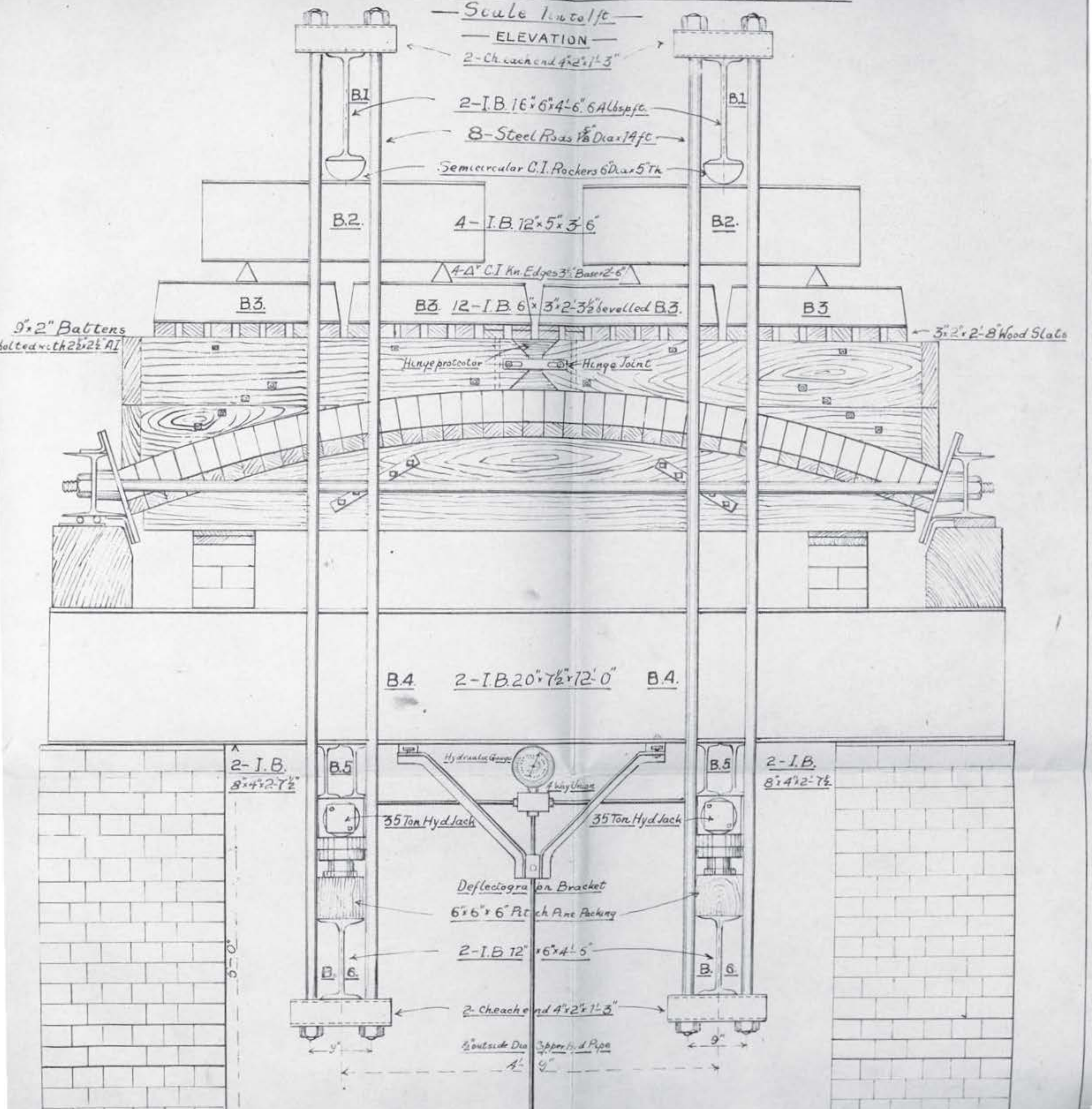


FRONT ELEVATION



HYDRAULIC ARCH TESTING APPARATUS

Plate. 5.



HYD^c ARCH TESTING APPARATUS

Scale 1in to 1ft

END ELEVATION

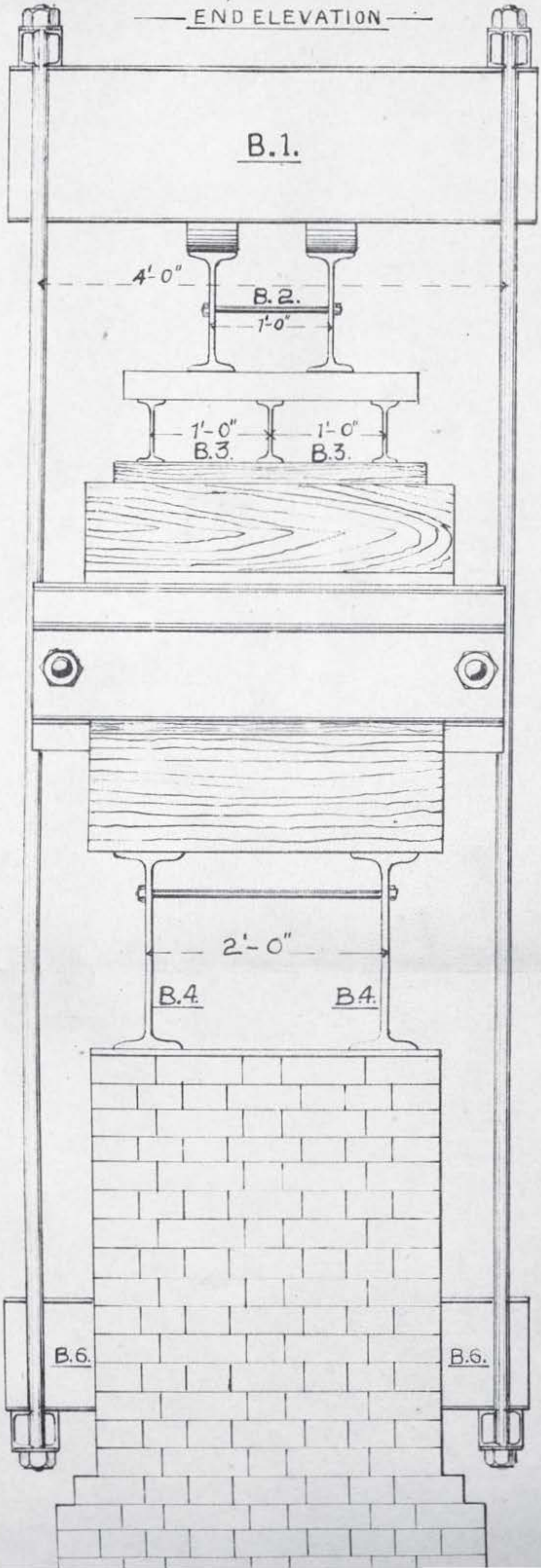
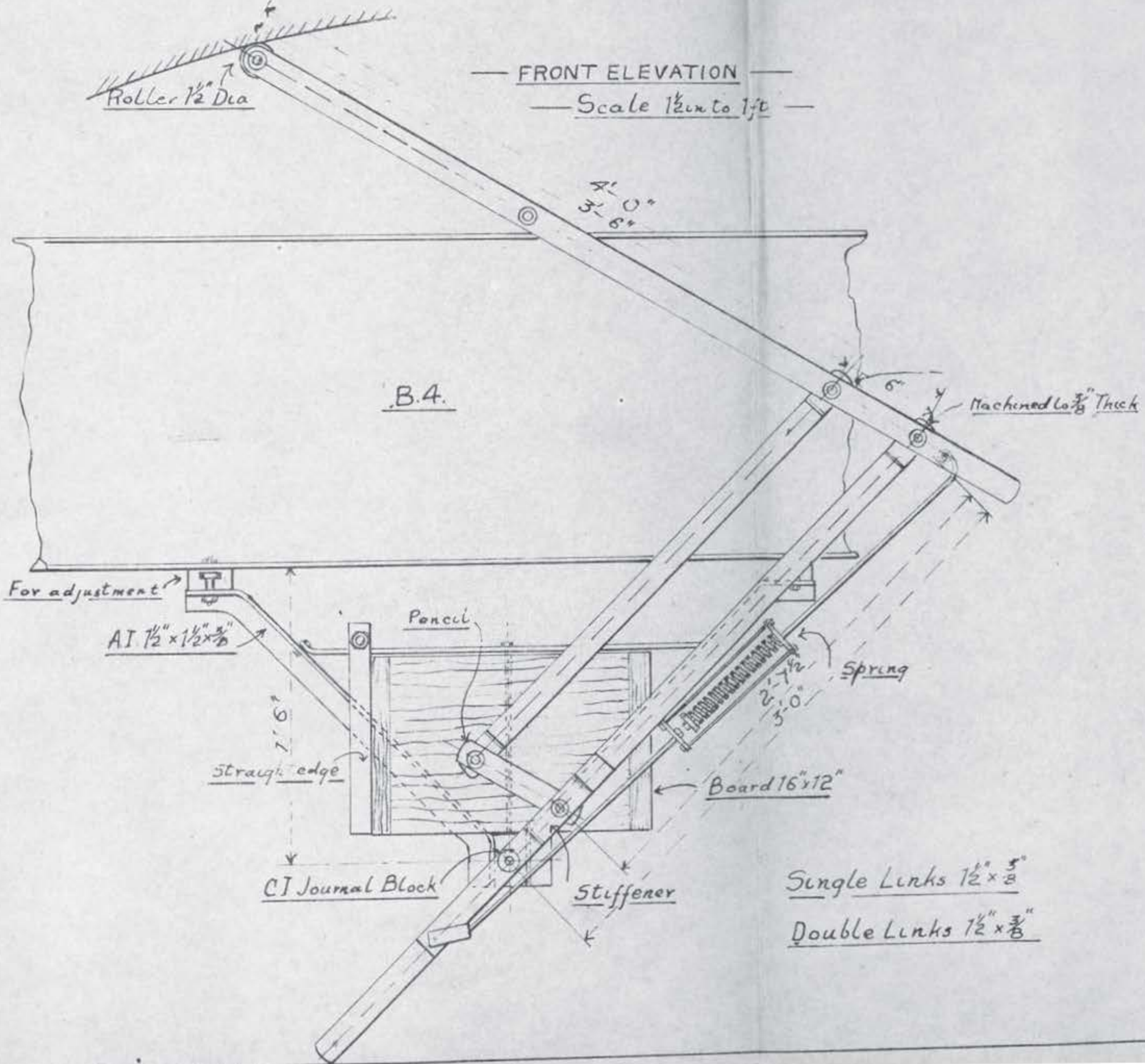


Plate. 7.

— DEFLECTOGRAPH —

— FRONT ELEVATION —

— Scale 1½ in to 1 ft —



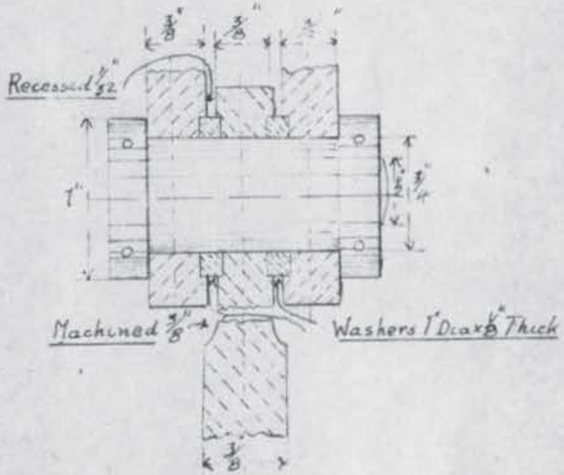
Single Links 1½" × ⅝"

Double Links 1½" × ⅜"

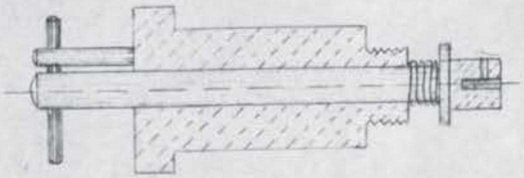
— DETAILS —

— Full Size —

— Joint —



— Pencil Pin —



— Clamp for Board —

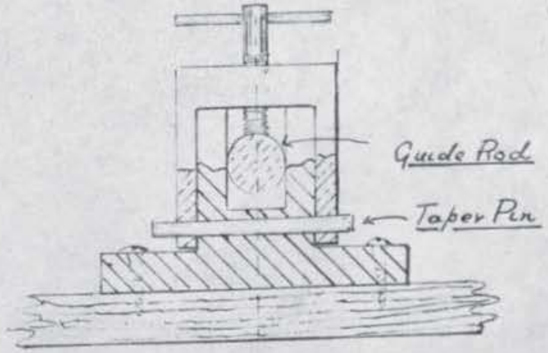
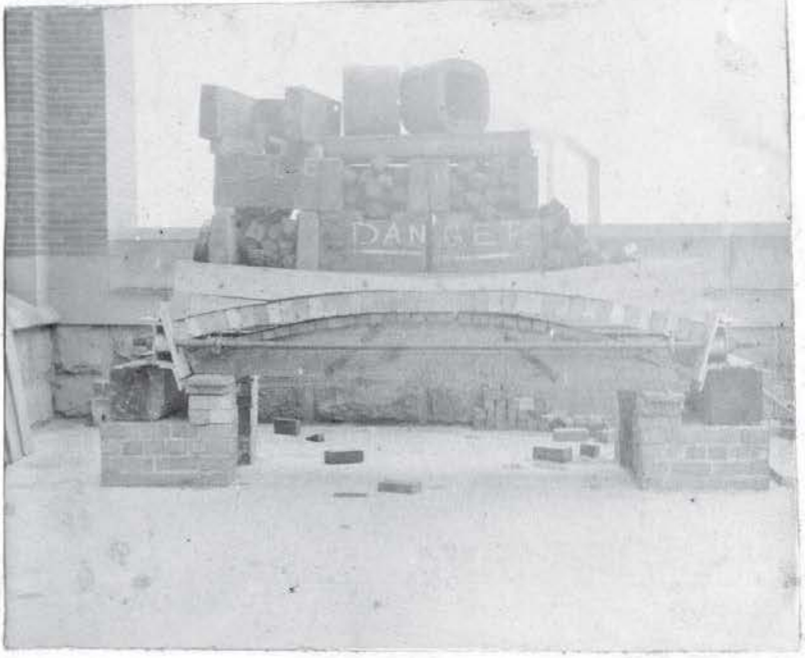
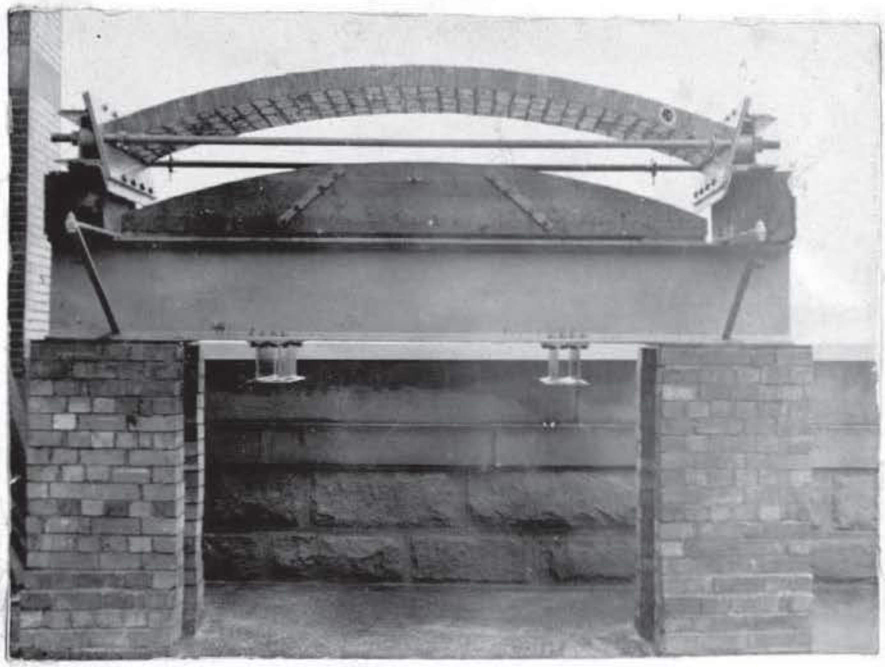


Plate 8

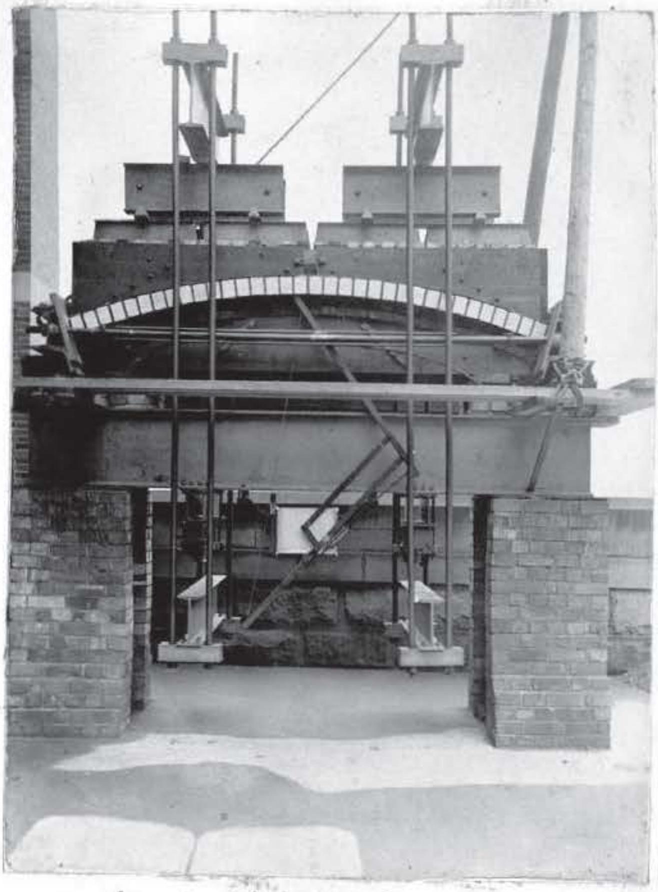


Arch No.1 — Final Stage

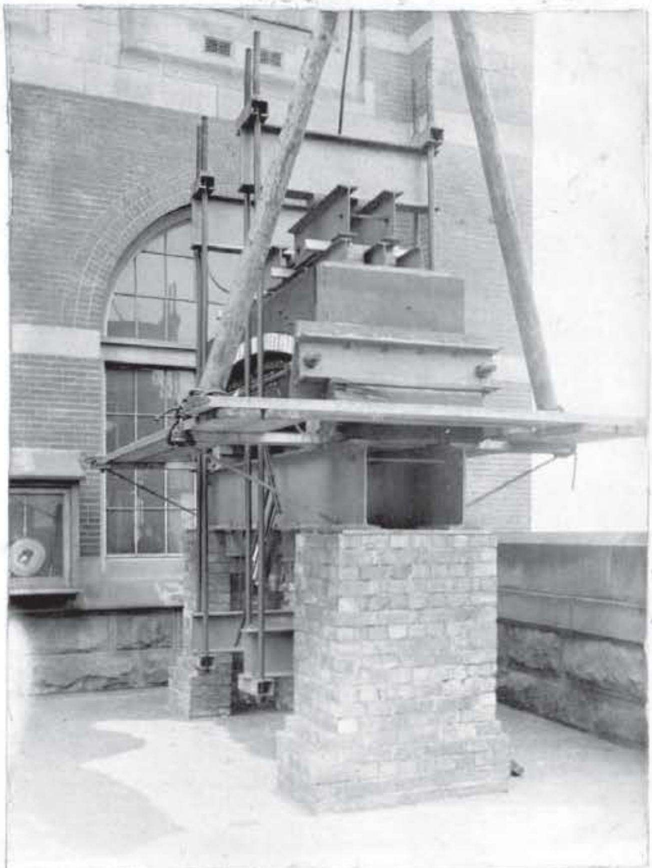


Arch No.2 First Stage

Plate 9



Arch No.2 — Completed Stage



Arch No.2 — End View

WHA
1912

CHAPTER V.

RESULT OF TESTS.

The results will not be discussed fully at this elementary stage of the work.

Data and Dimensions of Arch No. 1.

Built, - - - -	June 1911.
Tested, - -	June 1911.
Type of arch, - -	Segmental Hingeless Brick Arch
Span, - - -	10 feet.
Rise, - - -	11.68 inches.
Radius, - - -	13 feet.
Width, - - -	3 feet 1½ inches.
Angle subtended, -	42 degrees.
Thickness, - -	4½ inches (½ brick).
Material, - -	Common wire-cut red bricks.
Joints, - -	Sand, - thin.
Bond, - - -	All stretchers.
Centres removed, -	4 days after building.
Arch tested, - -	from 7 days after building.
Method of loading, -	Dead weight.
Distribution of load,	Uniform, - by sand fill.

Method of measuring deflections, - By vertical offsets every 2 ft.

Method of measuring horizontal thrust, - By Extensometer as described for Test No. 1.

Readings were taken generally at each addition of load.

The first weight was placed in the centre of the arch. This was then shifted to one side and the second weight placed next to it, and thus the symmetry in loading was preserved in each case. ~~Results of Test No. 1.~~

The quantitative results may be seen in Table 2. & curves Plate II

Discussion of Test No. 1.

The test of arch No. 1 cannot be regarded as very satisfactory. The extensometer readings cannot be relied upon as the arrangements were rather crude. The testing had to be done in a hurry and so the apparatus could not be completed. It was impossible to do repeated tests on this arch as the constant shifting of the heavy weights would have taken a very long time.

The remarkable point in this test was the enormous deflection the arch showed without collapse. The joints did not open and the bricks were not actually crushed, but were broken across owing to the 'poor' "cushioning" property of the sand joint. This "compacting" of the arch undoubtedly caused the big deflections. The horizontal thrusts recorded were rather erratic, but they agreed in being much greater than theory predicted. Their erratic nature was due to the incompleteness of the extensometer, and the extended period over which they had to be taken.

The profile of the loaded arch as seen from the tabulated deflections was very strange. The final profile shows that for four feet in the centre the arch was perfectly horizontal and flat.

Had the line of stress passed through the upper boundary of the middle third of the arch ring at the start (according to the Least Crown Thrust Theory), then the arch must have quickly become unstable as an increased uniform load causes the line of stress to move higher up in the crown.

TABLE 11.

Reference.

CALIBRATION OF EXTENSOMETER.

An extension of .0015 inches on a length of 100 inches corresponds to a force of 1,000 lbs. in each tie bar, or to a Horizontal Thrust of 2000 lbs.

(a) First sign of cracking (owing to local bending of the bricks)

(b) Test continued following day.

(c) do. do.

(d) Load hereafter extending 8 feet across the arch from the right.

Strength of bricks.

Crushing load 112,000 lbs. per brick,

- 400,000 lbs. per square foot.

TABLE 11.

TEST OF ARCH NO. 1.

No.	Load. lbs.	Extension. inches.	Load. per sq. ft.	Thrust. lbs.	Thrust. per lin. ft.	Thrust. per sq.ft.	Rise. inches.	Deflection. inches. 0	2'	4'	5'	6'	8'	10'
1	1560	0.0022	52.0	2930	976	2690	11.68	0	0	0	0	0	0	0
2	2680	0.007	89.3	9320	3106	8300	11.68	0	0.125	0	0	0.250	0.125	0
3	3800	0.009	126.6	12000	4000	10700	11.31	0	0.125	0.250	0.375	0.375	0.125	0
4	4920	0.019	164.0	25000	8333	22300	11.18	0	0.187	0.375	0.500	0.500	0.187	0
5	6040	0.022	201.3	29300	9766	26000	10.93	0	0.375	0.625	0.750	0.750	0.375	0
6	8280	0.026	276.0	34600	11533	30000	10.43	0	0.695	1.125	1.250	1.125	0.695	0
a7	10520	0.038	350.6	50600	16866	45000	9.93	0	1.195	1.625	1.750	1.625	1.195	0
b8	10520	0.038	350.6	50600	16866	45000	9.81	0	1.195	1.750	1.875	1.750	1.295	0
9	10920	0.038	364.0	50600	16866	45000	9.69	0	1.195	1.875	2.000	1.875	1.435	0
10	11420	0.038	380.6	50600	16866	45000	9.56	0	1.310	1.875	2.125	2.000	1.435	0
c 11	11420	0.038	380.6	50600	16866	45000	9.43	0	1.435	2.000	2.250	2.125	1.560	0
12	12220	0.042	407.3	56000	18666	50000	-	-	-	-	-	-	-	-
13	13020	0.042	434.0	56000	18666	50000	9.31	0	1.435	2.250	2.375	2.375	1.665	0
14	13920	0.044	464.0	58600	19533	52000	8.93	0	1.685	2.500	2.750	2.625	1.685	0
d15	11680	0.042	387.0	56000	18666	50000	8.93	0	1.685	2.500	2.750	2.625	1.685	0
16	12800	0.046	426.6	61200	20400	54800	8.81	0	1.935	2.750	2.875	2.625	1.800	0
17	13650	0.052	455.0	69200	23066	61700	8.56	0	1.935	3.125	3.125	2.875	1.935	0
18	14500	0.056	483.3	74600	24866	66500	8.31	0	2.310	3.375	3.375	3.125	2.050	0
19	15760	0.059	525.3	78600	26200	70000	8.06	0	2.435	3.625	3.625	3.250	2.050	0

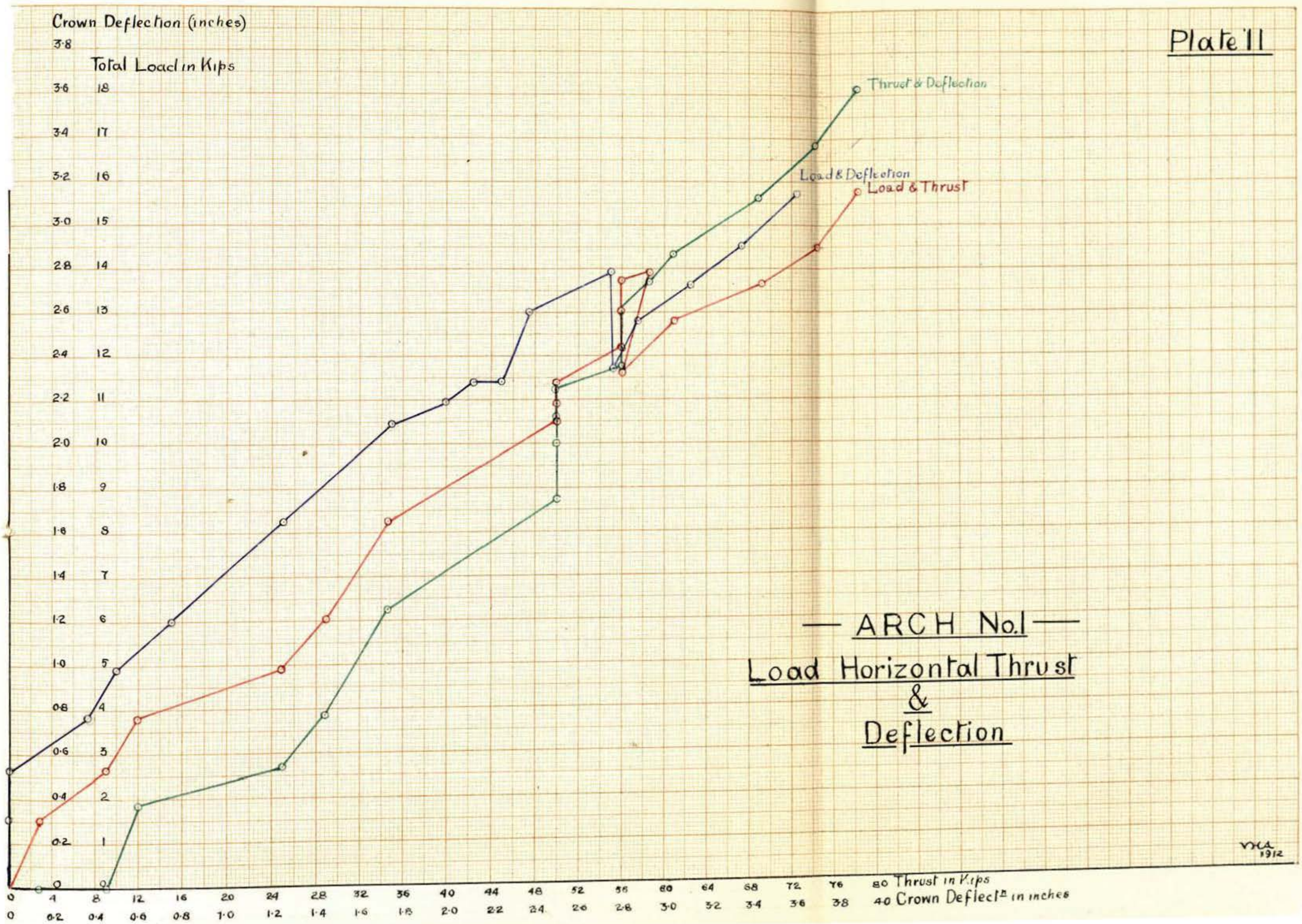


Plate 11
Crown Deflection (inches)
Total Load in Kips
Thrust & Deflection
Load & Deflection
Load & Thrust
— ARCH No.1 —
Load Horizontal Thrust & Deflection
80 Thrust in Kips
40 Crown Deflectn in inches
1912

Data and Dimensions of Arch No. 2.

Built, - - -	November 1911.
Tested, - - -	March 1912.
Type of arch, - -	Segmental Hingeless Brick Arch
Span, - - -	10 feet.
Rise, - - -	11.68 inches.
Radius, - - -	13 feet.
Width, - - -	3 feet $1\frac{1}{2}$ inches.
Angle subtended, -	42 degrees.
Thickness, - -	$4\frac{1}{2}$ inches ($\frac{1}{2}$ brick) single ring.)
Material, - -	Common wire-cut red bricks.
Joints, - - -	3 to 1 Portland Cement Mortar; no joint less than $\frac{1}{2}$ inch thick.
Bond, - - -	All stretchers.
Built, - - -	November 17th. 1911.
Centres dropped, -	December 15th. 1911.
Interval, - -	28 days.
Tested, - - -	5th. 6th. and 7th. March.
Interval, - -	109 days.
Method of loading, -	Hydraulic pressure.
Distribution of load,-	Uniform, - by sand fill.
Meathod of measuring, -	"Deflectograph".

Method of measuring Horizontal Thrust, - Complete extensometer as dexcribed.

The arch was tested in the following manner.

(1) Load taken up to about 11 tons (26,640 lbs.) at intervals of about 1,000 lbs. from 3 tons, (residual load).

(2) Load taken up to about 11 tons, (26,640 lbs.) at intervals of about 2000 lbs. from 3 tons, (rigidual (load).

(3) Load taken up to about 5 tons (10,660 lbs.) at intervals of about 2000 lbs. from 3 tons, (residual load).

(4) Load taken from 3 tons up to about 9½ tons (21100 lbs.) direct and from thence by intervals of about 4000 lbs. to the breaking load (36,300 lbs. or 16.2 tons).

Readings were taken at each addition of load.

Results of Test No. 2.

The numerical results will be found tabulated in Table 3 and plotted in Plates 12, 13, & 14.

Photographs of the fracture appear in Plate 10.

Discussion of Test No. 2.

This test was satisfactory and results have been obtained which if confirmed by other tests will be exceedingly important.

The Horizontal Thrusts recorded are extraordinary. While they do not possess the erratic nature of those recorded in Test No. 1. (owing to the improvement of the extensometer), they are absolutely at variance with the thrusts obtained by the Elastic Theory, Navier's Principle, or the parabolic theory. In every case the results are greater than those predicted and with the increasing load they diverge still further from the theoretical. It is apparently impossible that the extensometer can be at fault because in the first place four readings were taken to obtain each result, and these readings agreed very well with each other, and secondly, the tie bars were tested repeatedly both before and after the Arch Test in the 300 ton machine, <u>using the Extensometer</u>, and the readings thus obtained agreed perfectly in both cases. As all the readings were taken by the author the errors due to a different "personal equation" were eliminated. All the possible inaccuracies in the measurement of the Horizontal Thrust such as the friction of the rollers underneath the skewback, or the inability of the measuring rods to move freely in their supports, would tend to <u>reduce</u> the results. The increase in the divergence between the actual and theoretical results (with the greater loads), was <u>not</u> due to the yielding of the bars as they were not stressed up to the yield point in the arch test, and further, during calibrations in the Testing machine the yield point was passed, and the increase in length was much too large not to be noticed immediately.

However, until these results are confirmed it is not advisable to remark on them further.

There was a permanent set in the arch after the load had been taken up to about 11 tons, and removed the first time, but this deformation did not materially increase when the procedure was repeated. It was also found that the Horizontal Thrusts did not agree with each other in successive applications of load, as will be seen from the curves. This is also at present inexplainable.

The first cracks appeared when the load had reached 25,900 lbs. in the fourth series, - less than that load reached in the other tests! This may have been due to slight inaccuracies in the gauge as a different gauge was used for the final test. (N. B. the jacks were calibrated with both jacks). The cracks occurred simultaneously at the springings at each side, and were due to the crushings of the bricks.

The failure occurred when the load had reached 36,300 lbs. or 16.2 tons, the left hand end of the arch crushing completely, and the arch falling on the centres. The joints did not open to any measurable extent until the arch actually fell. The failure was gradual and quiet. A photograph of the fracture is shown. The centre part of the arch was apparently not much damaged, thus bearing out the fact that the radial thrust of the arch is greatest at the springings.

It was noticed when the arch ring was laid bare that almost continuous cracks from the springings to the crown were present. These occurred at the edges of the arch at the springings, and gradually approached each other towards the centre, thus apparently showing the pyramidical character of secondary shear due to crushing.

Thus the arch to all appearances failed by Crushing.

T A B L E III.

Results of Arch No. 2.

Load.	Load per sq. ft.	R Extensions. Top.	Bottom.	L Top.	Bottom.	Extensions. Average	+ Initial	Total Stress	Stress per sq.ft.	Stress per lin.ft.	Stress per l.ft. by Elast^{c} Theory
lbs.	lbs.	inches.	inches.	inches.	inches.	inches.	inches.	lbs.	lbs.	lbs.	lbs.
0	0	0.000	0.000	0.000	0.000	0.000	0.000	0	0	0	0
3060	102	0.002	0.007	0.001	0.007	-	0.004	5340	4730	1780	1275
6300	210	0.006	(0.018)	0.003	0.016	-	0.008	10680	9500	3550	2625
6300	210	0.000	0.000	0.000	0.000	0.000	0.008	10680	9500	3560	2625
7300	243	0.002	0.003	0.003	0.000	0.002	0.010	13300	11080	4430	3037
8300	276	0.003	0.004	0.005	0.001	0.004	0.012	16000	14200	5530	3450
9460	316	0.005	0.006	0.008	0.003	0.006	0.014	18700	16600	6230	3950
10660	355	0.008	0.008	(0.011)	0.006	0.008	0.016	21300	18800	7100	4437
11690	390	0.011	0.011	0.011	0.008	0.011	0.019	25300	22400	8430	4875
12720	424	0.013	0.013	0.013	0.011	0.013	0.021	28000	24190	9330	5300
13800	460	0.015	0.016	0.015	0.013	0.0155	0.0235	31300	27700	10430	5750
14880	496	(0.017)	0.019	0.019	0.016	0.0190	0.027	36000	32000	12000	6200
16000	533	0.020	0.023	0.023	0.019	0.022	0.029	40000	35500	13330	6662
17320	577	0.024	0.027	0.027	0.023	0.025	0.033	44000	39000	14670	7212
18450	615	0.030	0.034	0.029	0.026	0.032	0.040	53200	47000	17730	7700
19560	653	0.034	0.038	(0.031)	0.033	0.036	0.044	58000	51100	19330	8162
20810	694	0.038	0.045	0.038	0.037	0.042	0.050	66500	59000	22170	8675
22040	735	0.045	0.050	0.044	0.044	0.047	0.055	73400	65000	24470	9200
23250	775	0.049	0.053	0.047	0.050	0.050	0.058	77400	68500	25800	9700
24480	816	0.051	0.058	0.049	0.053	0.053	0.061	81400	72100	27310	10200
25560	852	0.053	0.060	0.050	0.056	0.055	0.063	83800	74000	27930	10650
26640	886	0.056	(0.065)	0.053	0.059	0.058	0.066	88000	78000	29330	11075
6300	210	-	-	-	(0.067)	-	0.008	10680	9500	3560	2625
8300	276	(0.017)	0.008	0.009	-	0.008	0.016	21360	18900	7120	3450
0660	355	0.014	0.012	0.014	0.007	0.0125	0.0205	27400	24200	9130	4437
2720	424	0.019	0.017	0.018	0.012	0.017	0.025	34000	30000	11330	5300
4880	496	0.024	0.022	0.023	0.017	0.022	0.030	40000	35500	13330	6200
7320	577	0.029	0.027	0.028	0.022	0.027	0.038	46600	41500	15530	7212
9560	653	0.031	0.030	0.030	(0.025)	0.030	0.038	50600	45000	16870	8162
2040	735	0.034	0.033	0.033	0.031	0.033	0.041	54600	48400	18200	9200
4480	816	0.039	0.037	0.036	0.033	0.037	0.045	60000	53000	20000	10200
6640	886	0.044	0.040	(0.038)	0.036	0.041	0.049	65200	58000	21730	11075
6300	210	-	-	-	0.040	-	0.008	10680	9500	3560	2625
8300	276	0.005	0.004	0.005	-	0.045	0.0125	16600	14700	5530	2450
0660	355	0.013	0.011	0.012	0.004	0.013	0.021	28000	24800	9330	4437
6300	210	-	-	-	0.014	-	0.008	10680	9500	3560	2625
1100	703	0.023	0.020	0.024	-	0.024	0.032	42600	37700	14200	8800
5900	863	0.036	(0.028)	0.034	0.027	0.036	0.044	58600	52000	19530	10800
0700	1023	0.046	(0.035)	0.045	0.037	0.044	0.052	69400	61500	23130	12600
6300	210	0.045	-	0.047	0.041	0.044	0.052	69400	61500	23130	15000
2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.

E X P L A N A T I O N OF T A B L E.

No load.

3060 lbs. of sand on Arch.

All apparatus on top of Arch - total 6300 lbs.

Commencement of Test. Permanent load 6300 lbs.

First series completed. Load removed to 6300 lbs. and second series begun.

Second series completed and third begun.

Third completed, fourth and final begun.

Load taken straight up to 21,000 lbs.

First cracks at springings.

FAILURE.

Column.

1. Number of reading.
2. Total Load on the Arch. Obtained from Calibration of jacks.
3. Load per square foot Total Load/ Area.
4. 5. 6. 7. Differences from the four extensometer readings.
8. Mean Elongations per 100 inches
9. Total Elongations (Mean + that due to sand etc.).
10. Total Horizontal Thrust on Arch obtained from calibration of Tie Rods in Testing Machine.
11. Mean compressive stress per sq. ft. on cross section of Arch ring Total stress/ cross sectional area.

Ratio Act/Theor	Ratio Thrust/ Load.	¼ Span	Rise Crown	¾ Span	Deflection at Crown.		
		inches	inches	inches	inches Total	inches minus initial Deflecn.	inches minus permanent set.
-	-	9.04	11.68	9.04			
1.39	17.45	-	-	-			
1.35	16.91	8.80	11.54	8.80	0.Ø14	0.00	-
1.35	16.91	8.80	11.54	8.80	0.14	0.00	-
1.46	18.23	8.64	11.46	8.64	0.22	0.08	-
1.55	19.30	8.56	11.38	8.56	0.30	0.16	-
1.58	19.70	8.40	11.20	8.40	0.48	0.34	-
1.60	20.00	8.40	11.12	8.40	0.56	0.42	-
1.73	21.60	8.40	11.04	8.40	0.64	0.50	-
1.76	22.00	8.40	11.00	8.40	0.68	0.54	-
1.82	22.70	8.40	11.00	8.40	0.68	0.54	-
1.94	24.00	8.40	10.88	8.40	0.80	0.66	-
2.00	24.90	8.36	10.88	8.36	0.80	0.66	-
2.02	27.50	8.36	10.88	8.36	0.80	0.66	-
2.30	28170	8.32	10.84	8.32	0.84	0.70	-
2.35	29.50	8.32	10.84	8.32	0.84	0.70	-
2.56	32.00	8.28	10.80	8.28	0.88	0.74	-
2.65	33.10	8.24	10.80	8.24	0.88	0.74	-
2.66	33.20	8.24	10.76	8.24	0.92	0.78	-
2.67	33.20	8.20	10.76	8.20	0.92	0.78	-
2.62	32.60	8.16	10.72	8.16	0.96	0.82	-
2.63	32.70	8.16	10.64	8.16	1.04	0.90	-
1.35	16.91	8.60	11.04	8.60	0.64	0.50	0.00
2.06	25.80	8.56	10.96	8.56	0.72	0.58	0.08
2.06	25.80	8.48	10.96	8.48	0.72	0.58	0.08
2.12	26.60	8.48	10.92	8.48	0.76	0.62	0.12
2.13	26.70	8.40	10.88	8.40	0.80	0.66	0.16
2.15	27.00	'.32	10.84	'.32	0.84	0.70	0.20
2.05	25.80	8.28	10.84	8.28	0.84	0.70	0.20
1.98	24.70	8.28	10.84	8.28	0.84	0.70	0.20
1.96	24.50	8.24	10.80	8.24	0.88	0.74	0.24
1.96	24.50	8.20	10.76	8.20	0.92	0.78	0.28
1.35	16.91	8.60	11.04	8.60	0.64	0.50	0.00
1.61	20.00	8.56	10.96	8.56	0.72	0.58	0.08
2.12	26.60	8.48	10.88	8.48	0.80	0.64	0.14
1.35	16.91	8.60	11.54	8.60	0.64	0.50	0.00
1.61	20.10	8.32	10.72	8.32	0.96	0.82	0.32
1.81	22.60	8.24	10.56	8124	1.12	0.98	0.48
1.80	22.40	8.16	10.48	8.16	1.20	1.06	0.56
1.54	19.10	6.40	10.08	-	1.60	1.46	0.96
14.	15.	16.	17.	18.	19.	20.	21.

Column.

12. Stress per lineal foot of Arch ring Total stress/width.

13. Theoretical Stress from Elastic formula 3WL/2R.

14. Ratio-Actual/Theoretical.

15. Ratio stress per lineal ft./Load per sq. ft. Radius of "Line of Stress" by Naviers Principle.

16.
17.
18. Rise of Arch at ¼ Span, Crown and ¾ Span from Deflectograph.

19. Total Crown Deflection from Unloaded Profile.

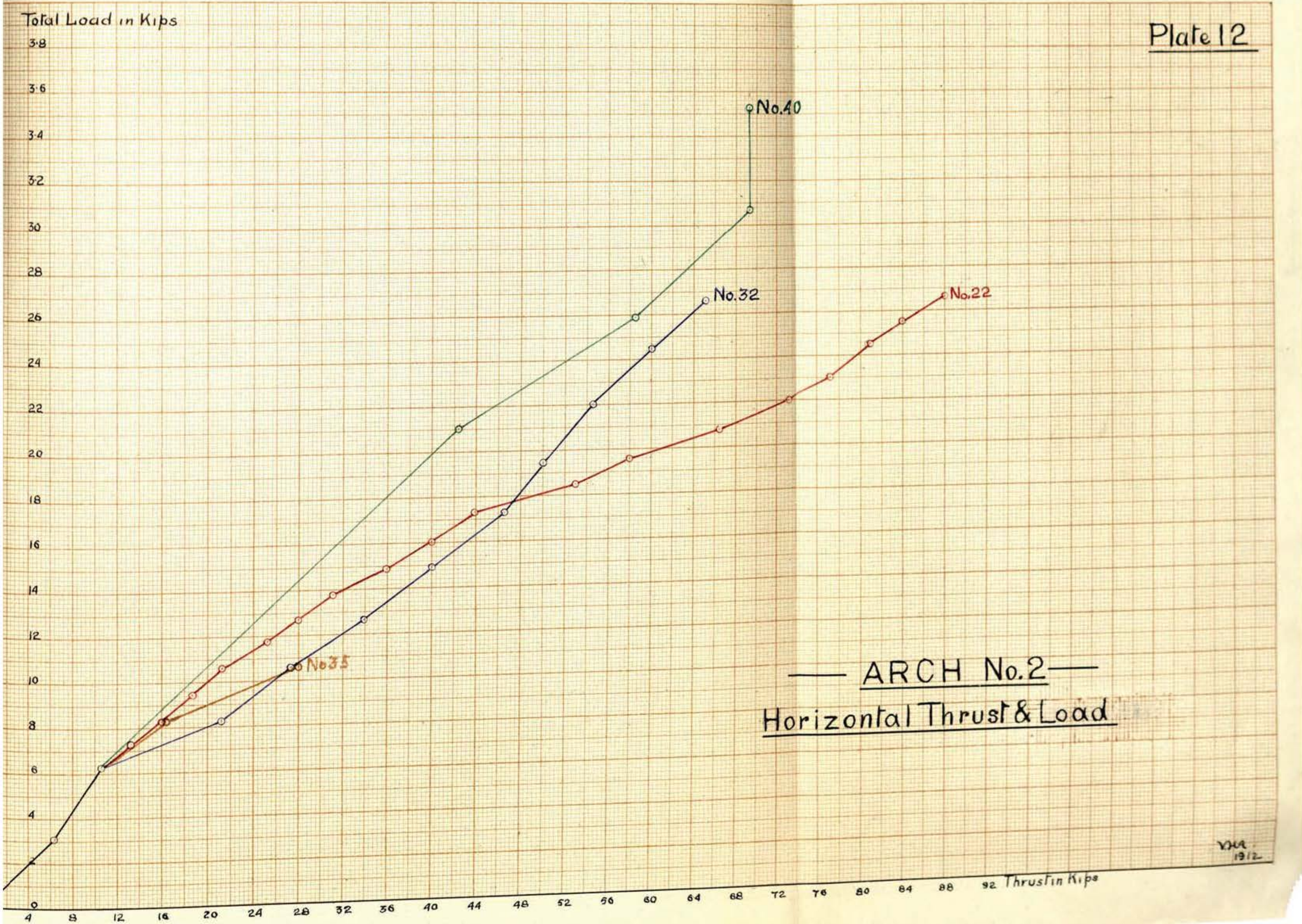
20. Crown Deflection from Profile No.3 (Load 6300 lbs.).

21. Crown Deflection from Profile No.23 (Load 6300 lbs.). subtracting permanent set.

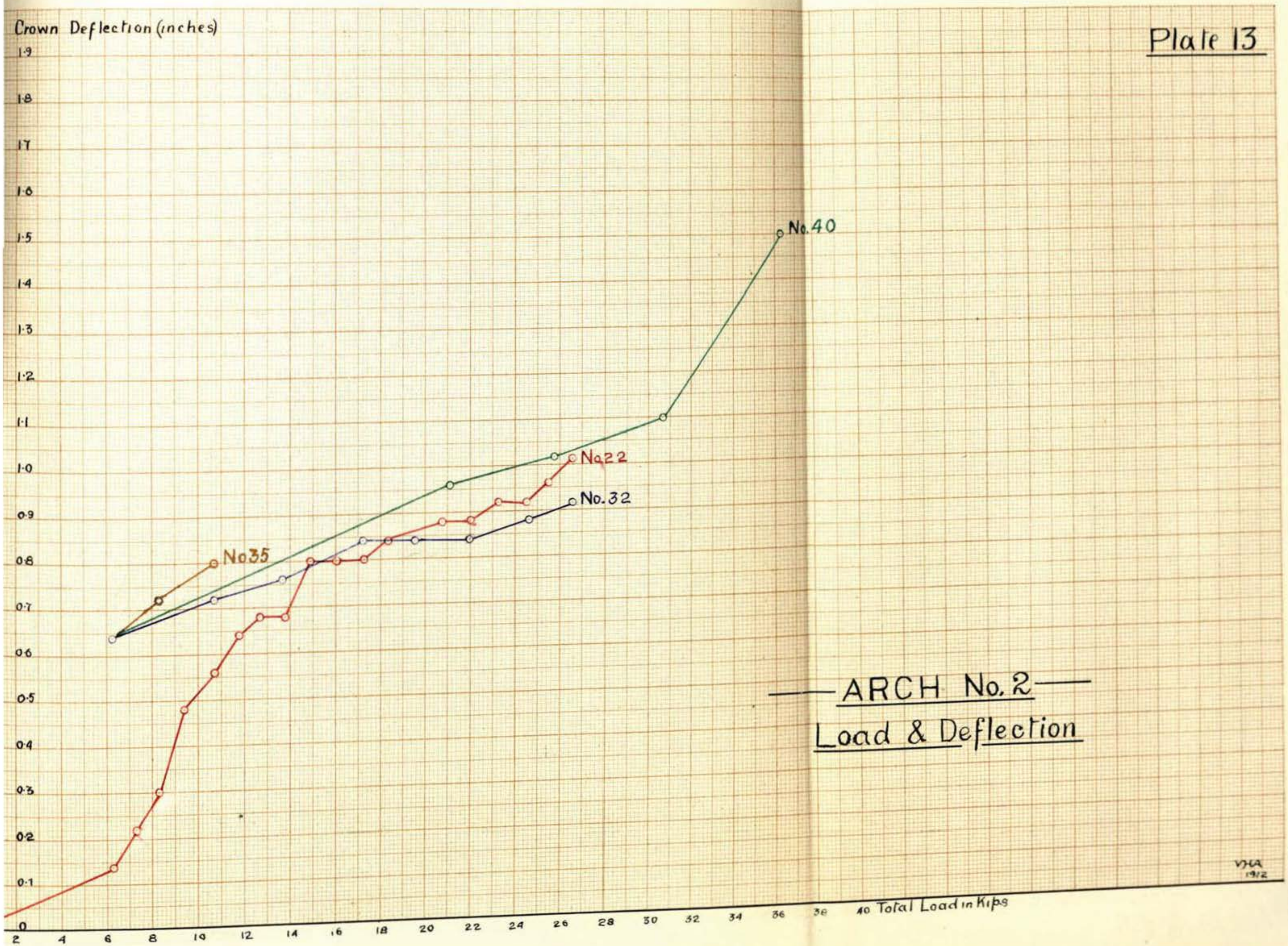
TABLE III.

SYNOPSYS OF RESULTS.

Compressive Strength of 1 brick	107000 lbs.
Compressive Strength per sq. ft.	380000 lbs.
Compressive Strength of Mortar Joint	38920 lbs.
Compressive Strength per sq. ft. of Mortar Joint	138000 lbs.
Total Load on Arch at 1st. crack.	25900 lbs.
Total Load per sq. ft.	863 lbs.
Total Horizontal Stress at 1st. crack	58600 lbs.
Total Horizontal Stress per sq. ft. do	52000 lbs.
Deflection at Crown "	1.12 inches.
Total Load on Arch at Failure	36300 lbs.
Total Load per sq ft.	1210 lbs.
Total Horizontal Stress	69000 lbs.
Total Horizontal Stress per sq. ft.	61500 lbs.
Previous Deflection before failure	1.20 inches
Percentage of Strength of one Joint	$\frac{61500}{.1380}$ 44.5%



Total Load in Kips
Plate 12
No.40
No.32
No.22
No.35
ARCH No.2
Horizontal Thrust & Load
Thrust in Kips
1912


Crown Deflection (inches)
Plate 13
No. 40
No. 22
No. 32
No 35
ARCH No. 2
Load & Deflection
40 Total Load in Kips
VHA 1912

CONCLUSION.

The only conclusion possible at this stage and with the results obtained is that a continuence of the tests is eminently desirable. It is, of course, impossible in the first test to eliminate all errors and to bring the apparatus and the procedure of testing to a completely satisfactory stage.

The preliminary tests can only indicate the directions which the research should take, and the special points which need to be elucidated. They cannot be considered in any way final, and the results so far obtained are not yet of value in the Design of Arches, but if they lead the way to a more complete research on the properties of Voissoir Arches, the preliminary work of the author will not have been in vain.

There is another branch of this subject which is quite as important as the foregoing tests on actual arches, - the optical method of the determination of the nature of stresses induced in a transparent structure by means of the properties of Polarized Light.

Alexander and Thomson in their "Applied Mechanics" describe a method of ascertaining the distribution of stress in a beam subjected to a bending moment. A brief epitome will be given.

When a ray of light enters an aelotropic transparent substance, it is usually resolved into two rays of polarised light vibrating in planes perpendicular to each other, and parallel to the directions of maximum and minimum elasticity of the substance. The position of these directions of maximum and minimum elasticity may be found by the ordinary polariscope, and from these directions the direction and position of the lines of stress may be found. Alexander and Thomson show how the distribution of stress in a glass under various bending moments may be found. The author thinks that if this method be applied to glass "arches" it may lead to some very valuable discoveries and perhaps finally solve the vexed problem of the solid hingeless arch.

An apparatus to perform these experiments would be of a fairly simple nature. A mirror polariscope would be advisable as then arches of a convenient size could be used. With the Nicol's prisms and microscope the "arches" are much too diminutive in size. The means of stressing the arch in various ways could be easily arranged. It would be possible and also very valuable to measure the applied load. The value of the crown thrust and the amount of deformation are best left to the full size test.

N.B. The results of the Third Test will be incorporated in an Appendix which will have to be sent in later.