ESSAYS ON MONETARY AND MACROPRUDENTIAL POLICIES

WITH DIFFERENT MODELS OF EXPECTATION

by

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A thesis submitted to the
University of Birmingham
for the degree of
DOCTOR OF PHILOSOPHY

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April 2016

ABSTRACT

This thesis evaluates the design of monetary and macroprudential policies. Different models of expectation are examined to get a comprehensive understanding about the work of monetary and macroprudential policies. It is begun by assuming agents are boundedly rational under Recursive Least Square (RLS) and Stochastic Gradient (SG) learning, followed by fully rational agents under rational expectation (RE).

When agents follow RLS learning, both determinacy and E-stability criteria are required to find preferred policies. We focus on the effect of habit in consumption in the design of preferred policies. We found the presence of habit in consumption enlarges both determinacy and E-stability region, under plausible policy parameters. The same methodology is then used in another model that features housing market and financial constraint. The result showed that a response to the growth of housing prices via the LTV rule may increase determinacy and E-stability. Yet, this benefit depends on the quality of housing price data and the data used in monetary policy.

We also conduct a refinement in the design of preferred economic policies by incorporating SG-stability criteria, in addition to determinacy and E-stability. The result showed that central bank's task gets more difficult since the Taylor Principle is insufficient to ensure a robust learnability of REE. In other works, we deviate from the assumption of boundedly rational agents and consider fully rational agents (RE). We examine the issue of monetary policy, banks' lending decisions and business cycles in Indonesia. This thesis completes its analysis by evaluating the role of news in the formation of agents' expectations.

JEL classification: C11, D83, E30, E52, E58

ACKNOWLEDGEMENT

Obtaining a doctoral degree is one important milestone in my life. It is not that easy as it requires a lot of efforts and sacrifices. Sometimes, I feel depressed and confused that remind me about my limitation as God's creature. Yet, I try to convert these feelings into a positive attitude by working harder and more persistent. With God's help, finally I can manage all the obstacles and arrive on this page where I have a change to recall back my journey and express my gratitude to several people who are very helpful during the completion of my Ph.D. thesis.

First and foremost, I would like to express my special appreciation to my first supervisor, Prof. Kaushik Mitra, who has given me a chance to experience a Ph.D. life by accepting me as one of his students. His intellectual guidance, patient and solitude has maintained my motivation at the highest level during my Ph.D. life. I am also in debt to my second supervisor Dr. Christoph Gortz, who is very kind and always has a time to discuss my research. His advice and suggestion on my research has been invaluable and improved the quality of this thesis significantly. My thanks also go to Dr. Federico Di Pace as my former second supervisor when I was registered in the University of St. Andrews. I still remember how he motivates me to read and learn as much as I can to find a good topic for my thesis. It was also a pleasure to meet and interact with all staffs and Ph.D. students of Department of Economics in the University of Birmingham and the University of St. Andrews. A constructive discussion during the school and Ph.D. seminar has given me valuable inputs.

I greatly appreciate the support received from Bank Indonesia for giving me the opportunity to carry out my doctoral degree and for their financial support. All supports from the Board of Governor are also greatly appreciated. My deep appreciation also goes to Prof. Ed Tower (Duke University), Prof. Kent Kimbrough (Duke University), Prof. T. Yuri M. Zagloel

(University of Indonesia), Dr. Iskandar Simorangkir (Bank Indonesia), Dr. Ramlan Ginting (Bank Indonesia), Mr. Jeffrey Kairupan (Bank Indonesia) and Mr. Tantan Heroika (Bank Indonesia), who recommend me to pursue my Ph.D. study. Thanks to my colleagues in Bank Indonesia especially to all friends in Economic Research and Monetary Policy Department and Human Resources Department who are always ready to help and give a support during a period of study. A special appreciation delivers to Dr. IGP Wira, Mr. Syachman P, and Mr. Agung for discussion and supplying data.

Finally, I would like to thank my parents, parents in law, my brothers, and my sisters for their continuous and paralleled pray and support. To the Manik family: my wife, Maria, for her unconditional love, and also for her patient and support during a period in which I am absence from many family occasion; my children, Kezia and Raffael for cheering my days through their attitude and joke, and for showing a good performance in their study. Without them, I would not be able to achieve my dream.

Birmingham, April 2016

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Chapter 1

Introduction

Understanding the way of agents form their expectation and how it relates to the economy is a key factor in the successful of modelling any economy. An early effort of modelling the expectation was begun in which researchers equate the expectation with the actual outcomes under the framework of perfect foresight. In this way, it is assumed that economic agents know not only the probability distribution of future endogenous variables, conditional on exogenous variables, but also a specific value of future exogenous variables. Given its strong assumption, this framework is viewed by researchers to be unrealistic.

The framework of perfect foresight is then relaxed by introducing a new notation called the rational expectation (RE), following Muth (1961). Under the RE, it is assumed that economic agents know the structure of the economy and all structural parameters, though they cannot observe a specific value of future exogenous variables. This framework is different from perfect foresight in terms of agents' ability to observe future values of random shocks. When they are observed, then the assumption of perfect foresight is imposed in the model. In contrast, when they are unknown, a model under the RE is assumed. Since the assumption of RE is less restrictive compared to perfect foresight, then this notation becomes a useful benchmark in recent dynamic macroeconomic models.

In the context of economic policy design, there are three possible outcomes for the equilibrium condition under the RE, i.e. determinacy, indeterminacy and non-existence of determinate equilibrium (or explosive path). Determinacy is defined as a condition where there is a unique stationary rational expectation equilibrium (REE), whilst indeterminacy is described as a condition where there is more than one stationary REE. In a situation where there is no stationary solution to the system, then we say the equilibrium goes to an explosive path. We argue that a preferred policy is the one that leads the economy to a determinate equilibrium under the RE.

Although the assumption of RE is very standard in modelling the expectation, there are researchers who view this framework as a strong assumption. The reason is that it is too ideal for economic agents to observe the true value of structural parameters. In practice, economic theory sets the model framework and describes the structure of the economy, while econometricians try to estimate the true parameter values using any data that are updated regularly. The notion where economic agents behave like econometricians who use a particular least square method to estimate the true parameter values, rather than having the RE, is known as the learning in the literature. Our interest is, given a particular economic policy, under what condition does this learning process lead to a determinate REE?

Based on the literature, there are two types of learning algorithm that are commonly used, i.e. recursive least square (RLS) learning and stochastic gradient (SG) learning. The first algorithm is different from the second one in terms of the ability of economic agents to observe the variance of state variables during the process of forming their belief. When they are all observed, then we say economic agents follow the RLS learning algorithm for estimating the structural parameters. In contrast, when they are observed with a probability of zero, then we say agents have followed the SG learning algorithm. These two different learning algorithms consequently lead to two different principles in relation to their convergence conditions to the REE. The first principle is known as E-stability. This principle states that the REE is asymptotically stable under least square learning *iff* it is expectational stable (E-stable)[1]. The second one is SG-stability. This principle states that the REE is asymptotically stable under the SG learning algorithm *iff* it is stochastic gradient stable (SG-stable)[2]. Given these two definitions, the preferred economic policy should be the one that leads the equilibrium into E-stable (under RLS learning) or SG-stable (under SG learning). A rule that leads the equilibrium into E-unstable or SG-unstable should be avoided by policy makers even if it offers higher welfare.

Although there are some earlier studies that analyse determinacy, E-stability and SG-stability issue of economic policy, there is still a gap in the literature in which stability policy has not yet been considered, that is in a model with the feature of (1) habit in consumption, (2) housing market and financial constraint. Motivated by this, we try to close this gap by evaluating determinacy and learning stability of monetary and macroprudential policies for the above macroeconomic models. In addition to this, we research on monetary policy, banks' lending decision and business cycles in Indonesia. Unlike the issue of determinacy and learning stability, the study about the Indonesian economy is carried out under the assumption of fully rational agents since we want to focus on the estimation of some structural parameters. Yet, to ensure that work is in line with the broad topic of this thesis, we also attempt to address the role of news in the formation of agents' expectation in the context of the Indonesian economy.

Concerning economic policy, there are different ways of modelling monetary policy under dynamic macroeconomic models. These include an optimal monetary policy and a simple Taylor-type interest rate rule. An optimal monetary policy is a complex monetary policy that is derived from the underlying state variables of the model where this policy is set in such a way to maximise the overall welfare, given all the frictions in the economy. In contrast, the

[1] See Evans & Honkapohja (*p.30*, 2001)
[2] See Barrucci & Landi (1997)

Taylor interest rate rule (1993) is a simple policy rule that links the level of policy rate to the deviation of inflation from its target and of output from its potential (the output gap).

Although the Taylor interest rate rule may not deliver the highest welfare to the economy, for the purpose of studying determinacy and learning stability of monetary policy, we choose this simple rule due to some reasons: Firstly, the Taylor interest rate rule is easily understood so that policy makers can easily communicate their policy to public. Secondly, policy makers, researchers and the public can well establish an effective discussion about monetary policy since they have the same level of understanding. Thirdly, this policy rule is useful in evaluating the stance of monetary policy for both developed and developing countries.

In this thesis, we consider three alternatives of the simple Taylor-type interest rate rule based on data availability, i.e. contemporaneous interest rate rule, lagged/backward looking interest rate rule and forecasted/forward looking interest rate rule. Our definition of the Taylor rule is slightly different from a standard definition earlier in which the level of policy rate does not link to the deviation of inflation and output from inflation target and potential output, but from their steady state values. In an environment of a small open economy, this standard Taylor-type interest rate rule can be extended by incorporating the exchange rate variable as one of policy rate's targets, in addition to inflation and output.

The recent US financial crisis has taught a lesson concerning ineffectiveness of monetary policy in dampening a bubble in housing prices and the financial markets. This happens as the adjustment of monetary policy through interest rate does not only affect financial markets and asset prices, but also other economic variables e.g. unemployment, investment and price level. Motivated by this, alternative targeted rules are introduced to minimise a volatility in the financial markets and asset prices. These rules are known as macroprudential policy that operates through some macroprudential instruments, for example: (1) a capital adequacy ratio

(CAR) that ensures banks hold enough money on their balance-sheets, (2) a reserve requirements that limits the size of banks' expansion in lending, (3) a leverage caps that limits the amount of loan that banks can borrow, (4) a loan to value (LTV) ratio that controls the amount of loan that borrowers can obtain, (5) a loan to value income (LTI) that puts a restriction on the size of loan relative to borrowers' income, etc.

In practice, not all the above macroprudential instruments are applied by each country. The same situation happens in Indonesia in which a particular lending parameter, i.e. the LTV ratio had not been explicitly regulated by Bank Indonesia (BI) until the beginning 2012. Before this period, this lending ratio is set by individual banks based on their level of risk appetite. Motivated by this, as mentioned earlier, we provide this thesis with a study about banks' lending decisions in Indonesia. In other parts of this thesis, we also evaluate the role of macroprudential policy in dampening a bubble in housing prices or the financial markets. To be specific, we assume that the ratio of loan to value (LTV) is operational so that it can be adjusted at any time to respond a change in the growth of housing prices or credit.

1.1. The Objective

In general, there are two objectives of this thesis: Firstly, analysing determinacy and learning stability issue of monetary and macroprudential policies. Secondly, analysing the issue of monetary policy, banks' lending decisions and business cycles in Indonesia. The first objective is achieved by examining some versions of the calibrated dynamic stochastic general equilibrium (DSGE) model. Meanwhile, the second objective is attained by estimating a specific version of DSGE model for Indonesia, where its features and specifications closely relate to the characteristics of the Indonesian economy. Although this thesis has two objectives, there is a “red line” that links them, that is the interest of studying monetary and macroprudential policies under different models of expectation.

The study about determinacy and stability issue of monetary policy is not new. There has been a bunch of previous studies about this subject either under a closed-economy or an open-economy environment. It was began by the seminal paper of Bullard & Mitra (2002) who examine determinacy and learning stability issue of a simple Taylor-type interest rate rule under a standard closed-economy New Keynesian (NK) model of Woodford (1999). Their work is then extended by other researchers in which different features and assumptions are introduced to a standard closed-economy NK model, for instance: inertia in policy rate, inertia in inflation, the existence of cost channel for monetary policy, etc. Within an open-economy framework, the study of determinacy and stability of monetary policy becomes more challenging as additional features, which are not found in a closed-economy model, have been incorporated that bring some economic policy implications.

There is still a gap in the literature where the study about determinacy and learning stability of monetary policy has not been yet carried out, i.e. in a case where habit in consumption matters. As we may know, the feature of habit in consumption is important in describing the movement of aggregate consumption and in generating the hump-shaped impulse response to demand and supply shocks. It is shown that incorporating this ad-hoc assumption in the model of consumption can help researchers in explaining the business cycle behavior. To close this gap, in chapter 2 we examine the effect of habit in consumption in the study of determinacy and E-stability issue of monetary policy. Some research questions arise:

- How do different levels of habit persistent in consumption affect the design of monetary policy when agents are boundedly rational under the recursive least square (RLS) learning rule?

- When consumption habit matters, among different alternatives of the Taylor-type interest rate rule, i.e. contemporaneous, lagged and forward looking rule, which one does offer the highest probability of leading the economy into a determinate and E-stable REE?

- If there are many policy rules that take the economy to a determinate and E-stable REE, which one among them is the most favorable?

For the first and second questions, we employ the criteria of determinacy and E-stability for choosing preferred policies. While for the third question, we use the concept of the optimal constrained interest rate rule, following Evans & McGough (2007). Unlike similar studies about the optimal policy, in our work, the optimal interest rate rule only can be chosen from determinate and E-stable policy rules. For this purpose, we assume that central bank minimises their expected loss of function in terms of inflation and output variation. Then, among all determinate and E-stable policy rules, we find a specific rule that gives a minimum loss. We do this analysis for each version of the Taylor-type interest rate rules, i.e. contemporaneous, lagged and forward looking, and compare all the results in order to choose the one that is the most advantageous. Note that Evans & McGough (2007) study the issue of the optimal constrained interest rate rule for a model of McCallum & Nelson (1999), Clarida, Gali & Gertler (2000) and Woodford (2003).

Unfortunately, a standard NK model with one good sector and two markets (i.e. consumption goods and bond market), as used in Chapter 2, is too simple to characterize the real economy. As a result, this model is difficult to explain the movement of other economic variables as shown on the empirical evidence. Given this limitation, we consider another version of NK model, where housing market is present. The reasons that motivate researchers to incorporate housing market/sector in a standard NK model can be summarised as follows: Firstly, housing investment may explain a variation in aggregate output as it is highly volatile. Secondly, the relationship between housing investment and aggregate consumption is interesting to analyse. Third, a theory that explains housing prices volatility and both propagation and amplification of shock is important to develop.

In Chapter 3, we extend the analysis of determinacy and E-stability issue of economic policy to a simple version of Iacoviello's model (2005) in which housing market and financial constraint are introduced. Since monetary policy is tied to the target of inflation and output, we need the additional targeted rule to prevent a bubble in housing prices. This additional rule is known as macroprudential policy that operates through the LTV ratio. We assume that this ratio is operational such that it gets lower when there is a tendency for a bubble in housing prices and gets higher when there is a down turn in the economy. Since macroprudential policy has been introduced, then the study about determinacy and E-stability issue of REE is evaluated for both monetary and macroprudential policies. Some relevant research questions emerge:

- Is it necessary for central bank to target housing prices or financial variable i.e. credit via the LTV rule when agents are boundedly rational and follow the RLS learning algorithm in forming their expectation?

- What is the best combination of rules, between monetary and macroprudential policies, that delivers the highest probability of getting a determinate and E-stable REE?

- Among determinate and E-stable policy rules, which one gives the optimal constrained interest rate rule?

Up to now, we have assumed that private agents follow the recursive least square (RLS) learning algorithm in forming their expectation and we look for a policy that ensures the convergence of this learning rule to rational expectation equilibrium (REE) in the long run.

Recall that E-stability principle states that a REE is asymptotically stable, under RLS learning, *iff* it is expectational stable (E-stable). Although the assumption of RLS learning makes the model more realistic, compared to the assumption of RE, such a setting is still too perfect from the view of modellers. The reason is that, under RLS learning, private agents are assumed to precisely know the variance of state variables in forming their expectation. Motivated by this, we relax this RLS learning assumption, by considering an alternative learning mechanism, namely, stochastic gradient (SG) learning, following Barrucci & Landi (1997).

SG learning is different from RLS learning in which the algorithm of the former is much simpler than that of the latter. Technically, SG learning ignores the variance of state variables used by agents in forming their forecasts. Since the algorithm of these two learning rules is different, it affects a convergence condition of private agents' belief to rational expectation equilibrium (REE). In general, there are two possible relationships concerning a convergence condition of RLS and SG learning to REE: Firstly, these two conditions are identical. This means, for a given economic policy, these learning algorithms lead private agents' belief to a unique stationary and stable REE. Secondly, a convergence condition of SG learning to REE is independent of a convergence condition of RLS learning.

In Chapter 4 of this thesis, we continue the study of determinacy and learning stability issue of economic policy in which we are uncertain about the type of learning algorithm used by agents in forming the expectation, whether it is RLS or a classic SG learning. Under this condition, we apply two learning criteria for finding robust policy rules, i.e. E-stability and SG-stability. This study is carried out using a simple version of Iacoviello's model (2005), as used in Chapter 3. A policy that leads to instability, either under RLS or SG learning, should be avoided since it leads to an inferior equilibrium. Two relevant research questions are addressed:

- How to find robust monetary and macroprudential policies that remain stable under both the RLS and the SG learning class?
- Is using more recent data in economic policy useful when the learning rule is uncertain between RLS and SG learning?

In regards to the study of monetary and macroprudential policies, it is also interesting to learn how a specific country conducts their monetary or macroprudential policy. As previously mentioned, in addition to the study of monetary and macroprudential policies under boundedly rational agents, this thesis contributes to the study of monetary policy in Indonesia under the framework of fully rational agents as discussed in Chapter 5. We use the framework of RE here as our objective is estimating some structural parameters in the model using a Bayesian technique. This chapter also equips its analysis with a study about banks' lending decisions and business cycles in Indonesia. Moreover, the role of news in the formation of agents' expectation is also evaluated. Note that under the assumption of boundedly rational agents, for instance: adaptive learning, the estimation process may become computationally expensive since we use a relatively large scale of a DSGE model.

The topic about banks' lending decisions is interesting to analyse since it shows how prudent banks are operated in Indonesia. This is motivated by the fact that the ratio of loan to value (LTV) had not been explicitly set by Bank Indonesia until the beginning of 2012. Having studied this issue, we expect that a better understanding of banks' lending decisions is established. This topic is very relevant, especially in the context of constructing a reliable macroprudential policy in Indonesia. Meanwhile, the study about monetary policy is motivated by the fact that Bank Indonesia has adopted the Inflation Targeting Framework (ITF) since 2000[3]. Under this framework, a low and stable inflation rate is targeted so that agents' expectation needs to be managed to ensure policy rate effectively reaches its target. Some research questions that try to be addressed are:

- How does Bank Indonesia conduct its monetary policy since the ITF was implemented?
- How lending parameters are set by banks and how this relates to the current setting of macroprudential policy in Indonesia?
- What are the main drivers of the Indonesian economy?

1.2. Structure of the Thesis

This thesis consists of six related chapters. In this present chapter, we describe the background and motivations for conducting a research. In this chapter, we also briefly display relevant research questions and our contributions. The core of the thesis is discussed in chapter 2 to 5, where in each chapter we study about the issues related to monetary and macroprudential policies either under the assumption of boundedly rational agents (adaptive learning) or under fully rational agents (RE). To help readers easily follow the analysis, the relevant literature, methodology and data are provided in each chapter.

In Chapter 2, we focus on the study of monetary policy that operates through a simple Taylor-type interest rate rule in an environment where consumption habit matters and agents follow RLS in forming their expectation. We begin the analysis by assuming no habit in consumption initially and use this result as the baseline. We continue the exercise by

[3] Officially, the ITF was announced by Bank Indonesia in 2005. However, Bank Indonesia had introduced this framework since 2000.

assuming habit in consumption is present and analyse how this additional assumption affects the choice concerning preferred policies, i.e. a policy rule that leads to a determinate and E-stable REE. Since preferred rules are not unique, then we adopt the concept of the optimal constrained interest rate rule to find a rule that can minimise economic volatility.

In Chapter 3, we study about monetary and macroprudential policies using the same learning framework as used in Chapter 2. Here, macroprudential policy works through the changing of loan to value (LTV) ratio and is used to dampen a bubble in housing prices or financial variable (credit). We evaluate various combinations between monetary and macroprudential policy rules that may lead the system to a unique stationary and E-stable REE. Again, since preferred policy rules may be not unique, we extend the analysis by examining the optimal constrained interest rate rule to find a combination of rules that minimises central banks' loss of function in terms of output and inflation variation.

Chapter 4 is an extension of Chapter 3. If in Chapter 3, we assume that agents follow the RLS learning algorithm, in Chapter 4 we are uncertain about the kind of learning algorithms used by agents, whether it is RLS or SG learning. In this situation, robust monetary and macroprudential policy rules are the ones that satisfy both E-stability and SG-stability criteria. Note that the SG learning algorithm differs from the RLS learning algorithm in terms of agents' ability to observe the variance of state variables in the economy. When it is fully observed, we say agents follow RLS learning, while if it is not, agents are assumed to follow SG learning.

In Chapter 5, we study about monetary policy, banks' lending decisions and business cycles in Indonesia. Unlike, earlier chapters that assume agents are boundedly rational, in this chapter we assume that agents endow with the RE. We propose and estimate a specific DSGE model that matches with the Indonesian characteristics. Finally, we summarise all the results from all chapters in Chapter 6.

Overall, this thesis reflects a good combination of methodologies between theoretical-calibrated models and empirical models based on the class of economic models. We define a theoretical-calibrated model as one which does not use data in the analysis but, instead, calibrates the parameters of the model to answer a specific research question. We follow this approach from Chapter 2 to Chapter 4. Meanwhile, we define an empirical model as one in which an economic model uses data to verify the qualitative predictions of theoretical-calibrated models. This methodology is used in Chapter 5. With respect to modelling the form of expectation, this thesis is also sufficiently complete, since it covers both the assumption of rational expectation and non-rational expectation (boundedly rational agents) in the analysis.

Chapter 2

Learning about Monetary Policy Rules when Consumption Habit Matters

2.1. Background and Motivation

It is believed that there are different economic characteristics between developed and developing countries. Among them is the motion of aggregate consumption[4]. Existing studies show that households in developed countries easily smooth their consumption as they have access to the financial markets that allow them to transfer resources across periods. Under such condition, they comfortably save (or lend) when their income is high and dis-save (or borrow) over the current consumption when it is low and below their current consumption. Unfortunately, consumption smoothing behaviour is not always possible in developing countries due to some reasons[5]: Firstly, most people still live in rural areas and work in a primary sector with a relatively low income. Secondly, even when their income is high, they mostly store it in the form of non-financial assets which are difficult to transform into a liquid asset for consumption. Thirdly, it is difficult to access different financial assets due to the absence of well-established financial markets.

In the context of a dynamic stochastic general equilibrium model (DSGE), the assumption in which households smooth their consumption by transferring resources across periods unfortunately cannot entirely explain a high persistence of aggregate consumption in developed countries, e.g. the US. In order to match the movement of aggregate consumption

[4] Agénor & Montiel (2008) make a list of characteristic features of developing countries that make them different from developed ones.
[5] See Senbeta (2011).

generated from the theoretical model with business cycle stylized facts, the feature of habit formation in consumption is required to be introduced[6]. By doing so, the motion of aggregate consumption improves significantly and a better hump shape response of consumption to exogenous shock is generated. In line with this finding, we argue that the motion of aggregate consumption in developing countries is also different from the ones in developed countries in terms of the degree of habit persistence. It is likely that consumption habit in developing countries is less persistent compared to those in developed ones.

Table 2.1
Business cycles summary of the US and Indonesia[7]
(Quarterly data, 1990 – 2013, index 2010 = 1)

	Correlation (with Real GDP)		Autocorrelation (x_t, x_{t-1})		Volatility $(\sigma_x/(\sigma_y)$	
	US	Indonesia	US	Indonesia	US	Indonesia
Real GDP	1.000	1.000	0.869	0.852	1.000	1.000
Real Consumption	0.921	0.568	0.885	0.419	0.852	1.067
Real Investment	0.952	0.684	0.896	0.563	3.091	3.899

Source: OECD, Main Indicators-Complete Data Base

We assess the above argument by evaluating quarterly macroeconomic data for Indonesia during the period of 1990 to 2013 and compare the result with data for the United States (US). We select Indonesia as a representative country for developing countries and the US as a representative country for developed countries. As reported in Table 2.1, the auto-correlation of aggregate consumption in Indonesia during the year 1990 to 2013 is of 0.419, which is lower than in the US of 0.885. As well, consumption volatility in Indonesia is much higher, with a value of 1.067, than in the US, with a value of 0.852. Note that we measure the volatility of consumption as the ratio of the standard deviation of aggregate consumption over

[6] The feature of habit formation is initially introduced by Abel (1990).

[7] All series are measured in percentage deviations from trend using a Hodrick–Prescott (HP) filter with smoothing parameter 1600.

the standard deviation of GDP in real term. Based on this empirical data, we argue that the motion of aggregate consumption in Indonesia or possibly in other developing countries are less persistent and more volatile compared to the US or other developed countries.

In what follows, we simply use the assumption of different degrees of habit parameter to characterise the motion of aggregate consumption between developed and developing countries. Note that we assume that developing countries are not different from developed ones, except in terms of their degree of consumption persistence. Although this assumption seems very strong, it enables us to understand the effect of consumption habit in the study of monetary policy, in particular for the purpose of finding the preferred policy rules.

What rules are preferred by policy makers? According to the literature, there are some common criteria for choosing preferred policy rules, for example: determinacy, learning stability, optimality, etc. A criterion of determinacy is established under the framework of a rational expectation (RE). In this way, preferred rules are ones that lead the economy to a unique stationary REE. Technically, the chosen policy rules must deliver a unique solution of RE model. Unfortunately, determinacy condition cannot be obtained all the time, as argued by Woodford (1999) and Clarida (2000). In some cases, we may expect that a particular policy rule takes the economy into an inferior equilibrium, where agents fail to coordinate towards a unique stationary REE, or follow an explosive path, where no stationary REE exists.

Although the framework of rational expectation (RE) is a very useful benchmark, many researchers view it as a strong assumption. This happens as under RE, it is assumed that economic agents understand that they are rational and know what others know. In addition, they have knowledge about the correct form of the model and all its parameters. In practice, economic theory will set the framework to describe a structure of the economy while agents behave like an econometrician who uses the RLS learning algorithm, for instance, in estimating the parameter values. This approach is known as an adaptive learning in the literature. When agents learn adaptively, it is assumed that the economy temporarily deviates from the REE. The question is that whether it will approach the REE over time? In this case, agents will update the parameter estimates towards the true value of model parameters as new data becomes available. The expectational stability or E-stability principle states that the REE is locally stable under RLS learning *iff* it is E-stable. If a policy rule leads to the REE which is not E-stable, then under RLS rule, economic agents can collaborate towards the REE with a probability of zero. Obviously, this policy rule should be avoided by policy makers, even if it offers a high welfare gain under the RE. In summary, there are two criteria for choosing the preferred policy rules when the RLS learning rule is used by economic agents, i.e. determinacy and E-stability. Under this condition, the chosen policy rules become more restrictive compared to the ones used under the framework of the RE.

The study of determinacy and E-stability, in the context of a DSGE New Keynesian (NK) model, is not new. There have been many earlier studies that try to address this issue either in an environment of a closed economy or a small open economy. In a closed economy environment, it was began by the seminal paper of Bullard & Mitra (2002) who examine determinacy and E-stability issue of simple Taylor rules in a forward looking model of Woodford (1999). Their work is then extended by incorporating the feature of inflation inertia in the Philip Curve, as discussed by Wang & Wong (2005), and policy inertia, as explored by Bullard & Mitra (2007). Further, Llosa &Tuesta (2007) study the same issue where it is assumed that the cost of monetary channel matters. In a small open economy environment, the model has brought some additional features that have implications for the analysis of determinacy and E-stability. Such implications include, but are not limited to, the issue of what the targets of central bank, i.e. producer price or consumer price, the issue of exchange rate regime and the issue of the degree of openness in trade. All these issues are discussed in Wang (2006), Bullard & Schaling (2006) and Llosa & Tuesta (2008), among others.

Although, there are many earlier studies that have tried to examine determinacy and E-stability issue of simple Taylor rules, under different structures of the model, there is still a gap in the literature where the issue of learning stability has not been explored, i.e. a case in which households endow with habit formation in consumption. Most of the foregoing studies on the NK model with habit formation in consumption and all its variations focus on the ability of the model to explain the business cycle behaviour. This means that they rely on this feature to explain the movement in aggregate consumption and to generate the hump-shaped impulse response to demand and supply shocks. Unlike these earlier works, we consider an otherwise version of NK model with the feature of habit in consumption for the study of determinacy and E-stability. We calibrate all the structural parameters using the values which are commonly used in a DSGE NK model with the feature of consumption habit.

We decide not to fix the value of habit parameter. This is due to the interest of evaluating the relationship between an interest rate rule and the degree of habit parameter in the analysis of determinacy and E-stability. This is motivated by the fact that there are different values of habit parameter used in previous studies, from no habit as reported by Woodford (1999), to a moderate value of 0.59 as reported by Smets & Wouters (2003) and to an extremely high value of 0.98 as reported by Bouakez et al. (2005).[8] In fact, from Table 2.1 earlier, we have shown empirically that there might be different degrees of consumption persistence among countries in the world.

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[8] Micro-based estimates of habit in consumption is substantially low, within a range of 0.29 - 0.5, e.g. Ravina (2011).

In addition to the issue of determinacy and E-stability, we extend the work by evaluating the optimal constrained policy rule where consumption habit matters. Unlike similar studies that derive the optimal policy rule from determinate ones, in our work, the optimal policy rule only can be chosen among determinate and E-stable policy rules. If not, the economy takes to undesired equilibrium. For this purpose, it is assumed that central bank minimises an expected loss of function in terms of the variability of inflation and output. Then, among all determinate and E-stable policy rules, we try to find the one that gives the minimum loss. We evaluate this within and between the Taylor-type class of interest rate rules and compare the results to find the most advantageous one. Our approach closely relates to the work of Evans & McGough (2007) who study the optimal constrained interest rate rule in a model of McCallum & Nelson (1999), Clarida, Gali, & Gertler (2000) and Woodford (2003). They claim that if policy makers compute the optimal unconstrained interest rate rule, within the Taylor-type class, they may lead to the rule that generates indeterminacy and/or E-instability. This problem becomes critical by the uncertainty of the structural parameters since an optimal rule that is determinate may be E-unstable under learning assumption. Unfortunately, the way that they have set the model is ad-hoc while in our work we try to derive it using a micro-founded approach as shown in Appendix 2.2.

In the following section, we present the model and discuss the parameter values employed. We start the analysis by considering a case where households are endowed with habit in consumption with no cost of monetary channel and no inertia in policy rate**.** For each type of Taylor rules considered, we present the analysis of determinacy and E-stability of REE. We extend the analysis by assuming the cost of monetary channel and policy inertia are presented in the model, subsequently. After identifying determinate and E-stable policy rules, we continue the analysis by searching for the optimal constrained policy rule for each Taylor-type rule and compare the results to find the most advantageous rule that offers the highest welfare, i.e. the lowest loss of function in terms of inflation and output variation. Finally, we conclude the analysis with a summary of our findings in the last section.

2.2. The Model

The model used is a DSGE model within the New Keynesian (NK) theoretical framework that captures the behaviour of some sectors in a closed economy environment. The economy consists of households, monopolistically competitive firms and central bank. There is a continuum of goods that enter the households' consumption basket. Households are endowed with habit in consumption and get utility from consuming habit-adjusted goods. Since habit is superficial, they are formed at the level of aggregate goods[9]. Initially, we assume that habit is external to the households and internal latter on. Habit is external when households' consumption depends on what other households are consuming and the desire to catch up to what others are consuming[10]. In contrast, habit is internal if his own consumption that influences his habit stock and does not create a consumption externality[11].

The households supply labour to a production sector but have no monopoly power to set their wages. In addition, it is assumed that the wage is flexible such that it can be set in every period. Firms set their prices in order to maximise a profit subject to demand for their goods. Prices are staggered and follow the Calvo (1983) where a fraction of θ of the firms set their prices according to rule of thumb and the remainder $(1-\theta)$ is based on current optimisation. Unlike equations in households and firms that are based on microeconomic foundations, the

[9] Habit in consumption formed in individual good is known as deep habit.
[10] See Fuhrer (2000) and Smets & Wouters (2003).
[11] See Amato & Laubach (2004) and Christiano et al. (2005).

monetary sector in the model is typically rudimentary. This sector is represented by the Taylor-type interest rate rule. Below is an explanation of each sector.

2.2.1. Households

There is a continuum household $j \in [0,1]$ whose utility consists of consumption (C) and leisure $(l = 1 - N)$. The household divides its unit of time into labour (N) and leisure (l) and gets utility from consuming habit-adjusted goods and disutility from worked hour.

$$Max\, E_0^* \sum_{t=0}^{\infty} \beta^t \left(U(C_t, H_t) - \frac{N_t^{1+\eta}}{1+\eta} \right) \qquad 2.1$$

Here, $U(C_t, H_t) = \frac{e^{g_t}(C_t - H_t)^{1-\alpha}}{1-\alpha}$ where g_t is a preference shock that follows an AR (1) process as $g_t = \rho_g g_{t-1} + u_t^g$ while $H_t = h\{{C_{t-1}}^D (C_{t-1}^o)^{1-D}\}$ is the habit stock endowed by household j[12]. Note that u_t^g is a zero mean white noise process with variance σ_g.

Parameter $D \in [0,1]$ indicates the way of habit is formed while parameter $h \in [0,1)$ denotes the habit parameter. We set $D = 0$ when habit is external and $D = 1$ when it is internal to households. Other parameters are $\beta \in (0,1)$ denoting a discount factor and $\alpha > 0$ and $\eta > 0$ corresponding to a constant coefficient of relative risk aversion of consumption and labour, respectively. Note that E_0^* represents the subjective expectation of household, conditional on information available time $t = 0$. Under rational expectations, E_t^* corresponds to mathematical expectation operator E_t evaluated using the objective distributions of the stochastic shocks, which are assumed known by the rational household. Thus, the expectation

[12] C_t^o is the average consumption.

operation is taken to describe the aggregate behaviour regardless of the true nature of the agents' expectation formation.

Household's utility function is constrained by her budget constraint as follows:

$$C_t + B_t = \left(\frac{W_t}{P_t}\right) N_t + \frac{R_{t-1}B_{t-1}}{\pi_t} + F_t \qquad 2.2$$

They earn income from wage (W) received by allocating their time to work (N) and portfolio (B) they hold at the end of time $t-1$. In addition to consumption, they leave some portfolio at the end of period t. In this model, we do not explicitly introduce money in which its role is only for the unit of account.

Maximising their utility as shown in equation (2.1), subject to the budget constraint in equation (2.2), and differentiating it with respect to C_t and B_t results in the following Euler equations:

$$\beta R_t \pi_{t+1} E_t^* \left(\frac{e^{g_{t+1}}(C_{t+1}-hC_t)^{-\alpha} - h\beta D e^{g_{t+2}}(C_{t+2}-hC_{t+1})^{-\alpha}}{e^{g_t}(C_t-hC_{t-1})^{-\alpha} - h\beta D e^{g_{t+1}}(C_{t+1}-hC_t)^{-\alpha}}\right) = 1 \qquad 2.3$$

Differentiating with respect to N_t and B_t gives the intertemporal setting the marginal rate of substitution (MRS) between leisure and consumption equal to real wage (w_t),

$$e^{g_t}(C_t - hC_{t-1})^{-\alpha} - h\beta D E_t^* e^{g_{t+1}}(C_{t+1} - hC_t)^{-\alpha} = \frac{N_t^{\eta}}{w_t} \qquad 2.4$$

Log-linearising equation (2.3) and (2.4) and using the fact that $\tilde{c}_t = \tilde{y}_t$ in equilibrium, we obtain two relevant equations as follows:

$$\tilde{y}_t = \left(\frac{h}{1+h+h^2\beta D}\right)\tilde{y}_{t-1} + \left(\frac{1+h\beta D+h^2\beta D}{1+h+h^2\beta D}\right) E_t^*\tilde{y}_{t+1} - \left(\frac{h\beta D}{1+h+h^2\beta D}\right) E_t^*\tilde{y}_{t+2} - \frac{(1-h)(1-h\beta D)}{\alpha(1+h+h^2\beta D)}(\tilde{r}_t - E_t^*\tilde{\pi}_{t+1} - \rho - g_t) \qquad 2.5$$

and,

$$\tilde{w}_t = \frac{\alpha}{(1-h)(1-h\beta D)}[(\tilde{y}_t - h\tilde{y}_{t-1}) - h\beta D(E_t^*\tilde{y}_{t+1} - h\tilde{y}_t)] + \eta\tilde{n}_t \qquad 2.6$$

Note that variable in log is represented by a small case letter, e.g. $y_t = log(Y_t)$, and its log deviation around the stead state is represented by a small letter with tilde $\tilde{y}_t = \log(Y_t) - \log(Y)$.

In addition to maximising their life time utility, a representative household j also minimises their expenditure by deciding the composition of aggregate goods (C) subject to its constraint.

$$Min \int_0^1 P_{it}C_{it}\, di \quad s.t. \ \left(\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = \bar{C}_t$$

where $i \in [0,1]$ is an index of differentiated goods i and ε is the elasticity substitution of consumption goods.

Solving this cost minimisation problem, we obtain the demand for individual good i,

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t \qquad \forall\, i \qquad 2.7$$

and consumer price index (CPI),

$$P_t = \left(\int_0^1 {P_{it}}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \qquad 2.8$$

2.2.2. Firms

Similar to households, firm $i \in [0,1]$ face two problems, i.e. minimising the cost of using any inputs and maximising the profit. Each firm has monopolistic power that comes from the differentiated goods they produce, which are imperfect substitutes. They set the price considering the demand for their products. They can set their own prices based on an optimisation as well as a rule of thumb, so there is presumably price stickiness in this sector. We follow the Calvo (1983) to model price rigidity of the firm since it is relatively more tractable and mathematically convenient. For simplicity, we also assume that production technology is the same across all goods.

The firms employ labour supplied by the households. In every period t, they minimise their cost of production consisting of wages (W_t) by choosing labour input N_{it} subject to the available production technology, i.e. Cobb-Douglas production function.

$$Min\left(\frac{W_t}{P_t}\right)N_{it} \quad s.t.\ A_tN_{it} = Y_{it} \qquad 2.9$$

A_t represents a technology and follows an autoregressive process $log(A_t) = \rho_a log(A_{t-1}) + u_t^a$ where ρ_a is an autoregressive coefficient and u_t^a is a normal distributed shock to a technology with mean zero and standard deviation σ_a. We normalize the steady-state value of technology to 1.

First order condition with respect to N_{it} gets labour demand for each firm:

$$N_t = Y_t/A_t \qquad 2.10$$

which implies the real marginal cost of the firms is as follows:

$$mc_t = \frac{w_t}{A_t} \qquad 2.11$$

Notice that i index is dropped since firms are identical.

Log-linearising equation (2.10) and (2.11) around the steady state values, we obtain:

$$\tilde{n}_t = \tilde{y}_t - \tilde{a}_t \quad 2.12$$

$$\widetilde{mc}_t = \widetilde{w}_t - \tilde{a}_t \quad 2.13$$

Combining equation (2.6), (2.12), and (2.13), we get the equation that relates real marginal cost with the output and innovation technology as:

$$\widetilde{mc}_t = \left(\frac{\alpha(1+h^2\beta D)}{(1-h)(1-h\beta D)} + \eta\right)\tilde{y}_t - \left(\frac{\alpha h}{(1-h)(1-h\beta D)}\right)\tilde{y}_{t-1} - \left(\frac{\alpha h\beta D}{(1-h)(1-h\beta D)}\right)E_t^*\tilde{y}_{t+1} - (\eta+1)\tilde{a}_t \quad 2.14$$

After choosing the level of labour input, firms simultaneously maximise their profit subject to three constraints, i.e. demand function, production function and restriction for some firms to adjust their price in each period. As previously mentioned, we follow the Calvo (1983) model where there is a fraction of firms θ that cannot reset their price based on optimisation but simply follow previous price $P_{t+s|t} = P_{t+s-1}$. There is also another fraction of firms $1-\theta$ that successfully change their price based on the optimisation behaviour P_t^{New}. In this condition, firms maximise their profit which is discounted by s-step ahead stochastic discount factor $Q_{t,t+s}$ and the probability of not being able to set the price in the future periods as follows:

$$Max\ E_0^* \sum_{s=0}^{\infty} \theta^s \left(Q_{t,t+s}\left[\left(\frac{P_{it}^{New}}{P_{t+s}} - mc_{t+s}\right)Y_{it+s}\right]\right) \quad 2.15$$

subject to the demand function for each product as follows:

$$Y_{it+s} = \left(\frac{P_{it}^{New}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \quad 2.16$$

Note that $E_t^* Q_{t,t+s}$ is the stochastic discount factor and its relation to R_t is given by $E_t^* Q_{t,t+1} = 1/R_t$. Meanwhile, parameter $\theta \in [0,1]$ represents a natural index of price stickiness and mc_t is real marginal cost.

This maximisation problem yields the price set by firms:

$$E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \frac{P_t}{P_{t+s}} \Gamma_{t+s} (P_t^{New}) Y_{it+s} = E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \frac{P_t}{P_{t+s}} \Gamma_{t+s} \left(\frac{\varepsilon}{(\varepsilon-1)} MC_{t+s} \right) Y_{it+s}$$

where $\Gamma_{t+s} = E_t^* \left(\frac{(C_{t+s} - hC_{t+s-1})^{-\alpha} - h\beta D (C_{t+s+1} - hC_{t+s})^{-\alpha}}{(C_t - hC_{t-1})^{-\alpha} - h\beta D (C_{t+1} - hC_t)^{-\alpha}} \right)$ 2.17

Log-linearising equation (2.17) around its steady state yields:

$$\tilde{p}_t^{New} = (1 - \beta\theta)(\tilde{p}_t + \widetilde{mc}_t) + \beta\theta E_t^* \tilde{p}_{t+1}^{New} \qquad 2.18$$

By using the definition of aggregate price level and the fact that all firms resetting their prices choose an identical price P_t^{New}, we have the relation between an optimal price P_t^{New} with aggregate price index P_t.

$$P_t = \left[(1-\theta) {P_t^{New}}^{1-\varepsilon} + \theta (P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \qquad 2.19$$

Again, log-linearising around the steady state gives:

$$\tilde{p}_t^{New} = \left(\frac{1}{1-\theta} \right) \tilde{p}_t - \left(\frac{\theta}{1-\theta} \right) \tilde{p}_{t-1} \qquad 2.20$$

Substituting equation (2.20) for $\tilde{p}_t^{New}$ and $\tilde{p}_{t+1}^{New}$ in (2.18), we get

$$\tilde{\pi}_t = \beta E_t^* \tilde{\pi}_{t+1} + \lambda \widetilde{mc}_t \qquad 2.21$$

where $\lambda = \frac{(1-\theta\beta)(1-\theta)}{\theta}$

Finally, we can substitute equation (2.14) into (2.21) to get the standard New Keynesian Philip Curve equation as:

$$\tilde{\pi}_t = \beta E_t^* \tilde{\pi}_{t+1} + \left(\frac{\alpha\lambda(1+h^2\beta D)}{(1-h)(1-h\beta D)} + \eta\lambda\right)\tilde{y}_t - \frac{\alpha h\lambda}{(1-h)(1-h\beta D)}\tilde{y}_{t-1} - \frac{\alpha h\beta D\lambda}{(1-h)(1-h\beta D)}E_t^*\tilde{y}_{t+1} - \lambda(\eta+1)\tilde{a}_t \quad 2.22$$

2.2.3. Central Bank

We assume that central bank uses a conventional interest rate rule that responds to inflation and output. However, when this policy rule is operated, central bank is constrained by data availability. Thus, in our model, we consider three alternatives in interest rate rule: Firstly, a case in which past data of inflation and output are used by central bank. Secondly, a case in which current data of inflation and output are not available and need to be estimated or forecasted. Thirdly, a case where current data of inflation and output are available. For each of these alternatives, we differentiate further by evaluating whether central bank does interest rate smoothing or not when conducting monetary policy. In total, there are six types of Taylor rule considered in this chapter with a specification for each rule as follows:

Responds to lagged data,

$$\tilde{r}_t = \tau_r \tilde{r}_{t-1} + \tau_\pi \tilde{\pi}_{t-1} + \tau_y \tilde{y}_{t-1} \quad 2.23$$

Responds to forward expectation,

$$\tilde{r}_t = \tau_r \tilde{r}_{t-1} + \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \quad 2.24$$

Responds to current data,

$$\tilde{r}_t = \tau_r \tilde{r}_{t-1} + \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \qquad 2.25$$

Parameter τ_r, τ_π and τ_y are non-negative values that measure the degree of responsiveness of policy rule to past interest rate, inflation and output. Note that the other three types of Taylor rule are obtained by setting $\tau_r = 0$ in equation (2.23) to (2.25), which imply no interest rate smoothing (no policy inertia).

2.2.4. Equilibrium and Market Clearing

The equilibrium of the model is a sequence of prices $\{R_t, P_t, w_t\}$ and an allocation $\{Y_t, C_t, B_t\}$ such that all first order conditions and constraints hold, and all markets are clear.

The aggregate profit from the firm is:

$$F_t = \left(\frac{P_t - MC_t}{P_t}\right) Y_t \qquad 2.26$$

where this aggregate profit is rebated to the households.

Good market is clear when,

$$C_t = Y_t \qquad 2.27$$

while for bond market, it is a zero net supply.

$$B_t = 0 \qquad 2.28$$

2.3. Methodology and Calibration

2.3.1. Methodology

We begin the analysis of each system of equations by presenting the conditions for a unique determinate REE to exist (i.e. determinacy condition). In order to do so, the system of equations is set in the form of:

$$E_t G_{t+1} = \Omega G_t + \kappa Z_t \qquad 2.29$$

where G_t is a vector of endogenous variables including its lag and Z_t is a vector of exogenous variables.

The above system is required to have the correct number of eigenvalues inside the unit circle, conditional on the number of free and predetermined variables. If that is the case, then there exists a unique stationary REE. If not, then a particular policy rule can yield several stationary REE (indeterminacy) or none at all that takes to undesirable economic outcome. Unlike a full forward looking model used by Bullard & Mitra (2002), in our model framework, we incorporate one additional feature, i.e. habit in consumption. As the result, there is at least one pre-determined variable along with the other free variables. Notice that the minimum size of Ω is 3 x 3 that encourages us to do most of determinacy analysis numerically, instead of analytically.

With regards to E-stability analysis, we follow a method developed by Evans & Honkapohja (2001). At first, it is required that the system of equations has a unique stationary REE, e.g. in terms of minimum state variable (MSV) solution. We then relax the assumption of rational expectation by using the assumption of adaptive learning. Here, we imagine that economic agents use the version of recursive least squares (RLS) algorithm in which they update model parameters when the new data becomes available. Recall that a unique stationary equilibrium under the RE assumption is not necessarily stable under RLS learning. For this to happen, we need other criteria, i.e. E-stability. Although indeterminate and explosive solution could be E-stable, we argue that such equilibrium should be avoided by policy makers[13].

In order to illustrate the concept of E-stability, consider the general class of log-stabilised model as follows:

$$S_t = B_0 + B_1 S_{t-1} + B_2 E_t^* S_{t+1} + B_3 Z_t \qquad 2.30$$

$$Z_t = \mathcal{F} Z_{t-1} + \epsilon_t \qquad 2.31$$

where S_t is an $n\,x\,1$ vector of endogenous variables, B_0 is an $n\,x\,1$ vector of constants, B_1, B_2, B_3 and $\mathcal{F}$ are $n\,x\,n$ matrices of coefficients, and Z_t is an $n\,x\,1$ vector of exogenous variables which is assumed to follow a stationary AR (1) process, so that ϵ_t is an $n\,x\,1$ vector of white noise term.

Following McCallum (1983), the MSV solution for the above system of equations takes the form[14]:

$$S_t = \bar{a} + \bar{b} S_{t-1} + \bar{c} Z_t \qquad 2.32$$

where $\bar{a}$ = 0, and $\bar{b}$ and $\bar{c}$ are given by:

$$\bar{b} = (\mathrm{I} - B_2 \bar{b})^{-1} B_1 \qquad 2.33$$

$$\bar{c} = \left(\mathrm{I} - B_2 \bar{b}\right)^{-1} (B_3 + \mathcal{F} B_2 \bar{c}) \qquad 2.34$$

[13] See Evan & Honkapohja (2001, p.219) for a discussion about learning stability of explosive solutions.
[14] The MSV solution of RE model can also be obtained using a method of Blanchard & Khan (1980). See Appendix 2.3 for a procedure of solving rational expectation model using this method.

in which matrix $(I - B_2\bar{b})^{-1}$ is assumed to be invertible. Note that equation (2.34) may yields multiple solutions for $\bar{b}$ and determinacy corresponds to a condition in which there is a unique solution for $\bar{b}$ with all eigenvalues inside the unit circle.

In order to evaluate E-stability of the MSV solution, we assume that agents have a perceived law of motion (PLM) of the form:

$$S_t = a + bS_{t-1} + cZ_t \qquad 2.35$$

We then compute the following expectation by assuming the time t information set does not include S_t.

$$E_t^* S_{t+1} = a + bS_t + c\mathcal{F}Z_t = (I + b)a + b^2 S_{t-1} + (bc + c\mathcal{F})Z_t \qquad 2.36$$

Plug in the above expectation to the original system i.e. equation (2.30), we obtain an actual law of motion (ALM) as follows:

$$S_t = B_2(I + b)a + (B_2 b^2 + B_1)S_{t-1} + (B_2 bc + B_2 c\mathcal{F} + B_3)Z_t \qquad 2.37$$

The mapping from the PLM to the ALM is then given by:

$$T(a, b, c) = [B_2(I + b)a, B_2 b^2 + B_1, B_2 bc + B_2 c\mathcal{F} + B_3] \qquad 2.38$$

The E-stability condition is characterised by the following matrix differential equations:

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c) \qquad 2.39$$

The MSV solution $(\bar{a}, \bar{b}, \bar{c})$ is the fixed point of equation (2.39). We say that a particular MSV solution $(\bar{a}, \bar{b}, \bar{c})$ is E-stable if the MSV fixed point of the differential equation (2.39) is locally asymptotically stable at that point. It can be shown that for E-stability of any MSV solution, assuming that the time t information set $(1, S'_{t-1}, Z_t)$, the eigenvalues of the matrices:

$$\bar{b}' \otimes B_2 + I \otimes B_2\bar{b} - I, \tag{2.40}$$

$$\mathcal{F}B_2 + B_2\bar{b} - I, \tag{2.41}$$

$$B_2 + B_2\bar{b} - I, \tag{2.42}$$

need to have negative real parts, where I denotes an identity matrix. If any eigenvalue of the above matrices has a positive real part, then MSV solution is not E-stable. This means a small expectation error can drive the system away from the REE. Notice that the MSV solution for $\bar{b}$ is crucial for learning stability as it affects the E-stability conditions.

After describing the concept of determinacy and E-stability, we convey a method of computing co-movement matrix, i.e. the variance-covariance matrix. The standard solution of the RE will take the form of a VAR (1) characterised by a subset of the state variables M_t, with the other variables of interest $\widehat{M}_t$ are given by specified linear function of the state variables.

We consider a generic solution of the RE model as follows:

$$M_t = \Theta M_{t-1} + \Upsilon u_t \text{ that is equivalent to equation (2.32)} \tag{2.43}$$

$$\widehat{M}_t = \Gamma M_t \tag{2.44}$$

Where parameter Θ, Υ and Γ are the relevant deep parameters. M_t is a vector of state variables and the components of u_t are exogenous white noise shocks, with mean zero and variance Σ_u. The example of disturbance u_t includes a productivity shock, a preference shock, a government spending, a monetary policy shock, etc. Since we are interested in the co-movement of M_t, then we have to compute its variance-covariance as:

$$Var\ (M_t) = EM_tM_t{}' \qquad 2.45$$

In practice, there are two convenient ways to compute the variance-covariance matrix, i.e. numerically and analytically. We do this analytically and follow the procedure as the following[15].

By using the stationary assumption, we have:

$$Var\ (M_t) = \Theta\ Var\ (M_t)\Theta' + \Upsilon\ \Sigma_u \Upsilon' \qquad 2.46$$

This can be solved for $Var\ (M_t)$ by vectorising the equation and using a formula of:

$$Vec\ (ABC) = (C' \otimes A)Vec\ (B) \qquad 2.47$$

We obtain:

$$Vec\ (Var\ (M_t)) = (I - \Theta \otimes \Theta)^{-1} Vec\ (\Upsilon \Sigma_u \Upsilon') \qquad 2.48$$

This allows us to evaluate the variance of each state variable. In addition to that, we can compute the variance-covariance of $\widehat{M}_t$ as:

$$Var\left(\widehat{M}_t\right) = \Gamma\ Var\ (M_t)\Gamma' \qquad 2.49$$

[15] See Hamilton, J.D. Time Series Analysis (pp. 264 – 265, 1994)

2.3.2. Calibration

We calibrate the parameters in the model using standard values used in a DSGE literature with a feature of habit in consumption. We begin by assuming an annual real rate of interest of 4% which implies a discount factor β of 0.99. An inverse of intertemporal substitution in consumption α is calibrated at 1.5 while an inverse of Frisch labour supply elasticity η is chosen at the value of 1. Smets & Wolters (2003) estimate parameter α around 1.39 for the euro area while Leith et al. (2012) calibrate it equal to 2. These two studies have incorporated the feature of superficial habit in consumption in their model's specification such that our choice for α falls in within this range. Meanwhile, our choice for η is in line with the findings found in Faberman (2010) and Fiorito & Zanella (2012) who estimate the value of macro Frisch elasticity between 0.6 and 1.6. Notice that all the above choices are also used in Ravenna & Walsh (2006).

For the degree of price stickiness θ, we set this parameter equal to 0.75 which is fairly standard in the DSGE literature. This implies an average duration of optimal price changes of four quarters. Concerning habit parameter h, we allow this parameter to vary. The argument is that there might be different levels of habit persistence in aggregate consumption among countries, as argued in the section of introduction. Thus, we consider three different values of habit parameter h, i.e. 0, 0.3, and 0.6 and evaluate how these values affect determinacy and E-stability of simple Taylor rules. For the purpose of computing the optimal constrained policy rule, we calibrate the persistence of taste shock ρ_g equal to 0.85 and technology shock ρ_a equal to 0.8 with standard deviation of these two shocks σ_g and σ_a are 0.35 and 0.55, respectively. Based on the existing literature, the standard deviation of taste shock and technology shock varies considerably and depends on the structure of the models and any assumptions made. Yet, our choices are in line with Smets & Wolters (2003) who estimate these values, under a 95% of confidence interval, within the range of 0.23 to 0.63 and 0.46 to 0.87, respectively. Notice that our calibrations differ in some respects from Bullard & Mitra (2002, 2007) e.g. the intertemporal elasticity substitution and the parameter relating to the degree of price stickiness, among other parameters. Fortunately, it does not affect the ability of our model to replicate their findings and conclusions, especially in a case where habit in consumption is absent in our model. All calibrated parameters are reported in Table 2.2.

Table 2.2
Calibration used in the numerical analysis
Quarterly frequency

Parameter	Description	Value
β	Discount Factor	0.99
α	Coefficient of risk aversion	1.5
η	Inverse of Frisch labour supply elasticity	1.0
θ	Degree of price stickiness	0.75
τ_y	Reaction to output	$0 \leq \tau_y \leq 10$
τ_π	Reaction to inflation	$0 \leq \tau_\pi \leq 5$
τ_r	Policy rate smoothing	$0 \leq \tau_r \leq 1$
δ	Dichotomous parameter for cost channel	0 or 1
ρ_g	AR (1) coefficient of taste shock	0.85
ρ_a	AR (1) coefficient of productivity shock	0.80
σ_g	Std. Deviation of taste shock	0.35
σ_a	Std. Deviation of productivity shock	0.55

2.4. Learning about Monetary Policy Rules

2.4.1. Responds to Lagged Data

After presenting the model's derivation, the methodology and the calibrated value of the structural parameters, then we convey the analysis to the discussion about the numerical outcomes. First, we consider a case in which lagged data are used in monetary policy. Under this condition, policy rate is adjusted based on past performance of inflation and output. This policy rule is probably the most practical one as at the time it is executed past data have been collected and are ready to be used in the analysis.

We consider lagged data in policy rule as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_{t-1} + \tau_y \tilde{y}_{t-1} \qquad 2.50$$

Before we proceed with the simulation results, we explain briefly how our numerical approach works. In all simulation exercises, we begin by setting the value of habit parameter h equal to 0 and varying the policy parameter τ_π and τ_y in the interest rate rule. When h is set at 0, it is assumed that households are not endowed with habit in consumption. In such cases, the model shrinks to the model of Bullard & Mitra (2002), i.e. a forward looking NK model. We set the upper bound of τ_π and τ_y at the value of 5 and 10, respectively, and use the value of 0 as their lower bound. We use an increment step size of 0.05 and search over a fine grid of values within 0 and 5 for τ_π and within 0 and 10 for τ_y. For each possible pair of policy parameters (τ_π, τ_y) in this grid, we evaluate whether the Eigenvalues satisfy the condition for determinacy and E-stability. If both conditions are satisfied, we plot this in the figure using a blue colour. We repeat the simulation for different values of h, i.e. 0.3 and 0.6. We do this in order to analyse the effect of consumption habit on the study of determinacy and E-stability issue of simple Taylor-type interest rate rules.

When consumption habit exists and central bank responds to past data, the system of equations is characterised by equations (2.5), (2.22) and (2.50). For the analysis of determinacy, the system of equation is set in the form of equation (2.29) by defining two artificial variables, i.e. $\tilde{\pi}_t^l$ and $\tilde{y}_t^l$ that correspond to past inflation and past output, respectively. By doing this, we obtain $G_t = \left(\tilde{y}_t, \tilde{\pi}_t, \tilde{y}_t^l, \tilde{\pi}_t^l\right)'$ as a vector of endogenous variables and $Z_t = (g_t, \tilde{a}_t)'$ as a vector of the exogenous variables. We also obtain a matrix Ω that contains relevant deep parameters, as shown in equation (2.51). Note that the exogenous disturbance processes of κ will be ignored in further analysis since the calibration of this parameter will not affect the study of determinacy and E-stability.

$$\Omega = \left\{\begin{matrix} \frac{(1-h)\eta\lambda+\alpha(\beta[(1+h)]+\lambda)}{\alpha\beta} & \frac{h-1}{\alpha\beta} & \frac{\beta(1-h)\tau_y-h\alpha(\beta+\lambda)}{\alpha\beta} & \frac{(1-h)\tau_\pi}{\alpha} \\ \frac{(\alpha+\eta[1-h])\lambda}{(h-1)\beta} & \frac{1}{\beta} & \frac{h\alpha\lambda}{\beta(1-h)} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}\right\} \qquad 2.51$$

Ideally, it would be good if we could provide analytic results concerning determinacy and E-stability of REE. Unfortunately, since the dimension of Ω is quite large, i.e. a matrix form with a size of 4 x 4, it is a bit complicated to reduce the system that allows for analytic findings[16]. This happens as there are two extra endogenous variables, i.e. $\tilde{y}_t^l$ and $\tilde{\pi}_t^l$ that are included in the system of equation. As an alternative, we consider a numerical approach instead of an analytical approach. Yet, as far as possible, we try to provide some intuitions for our numerical findings.

[16] We can only obtain the analytic result up to three dimensions by following the method conveyed by Woodford (2003) in Appendix C, Proposition C.2.

As there are two predetermined variables, the above system of equation has a unique solution only if two Eigenvalues of Ω are inside the unit circle. Under such condition, a unique stationary of REE exists which means the condition for determinacy is obtained. If there are more than two Eigenvalues of Ω inside the unit circle, then the system faces indeterminacy problem since there are multiple solutions to the model. Meanwhile, if there are less than two Eigenvalues of Ω inside the unit circle, then the system has no solution which implies non-existence of locally unique stationary equilibrium.

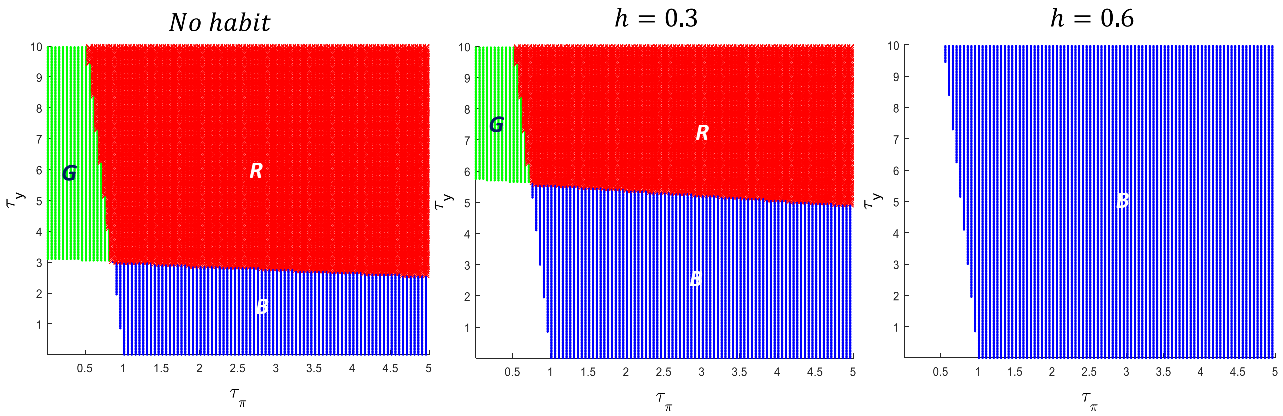
For the analysis of E-stability, we evaluate the system of equations in the form of minimum state variable (MSV) solution. We substitute equation (2.50) into equation (2.5) and reduce the system into two equations involving the endogenous variables $\tilde{y}_t$ and $\tilde{\pi}_t$. Defining $S_t = (\tilde{y}_t, \tilde{\pi}_t)'$, then the system of equations can be constructed in the form of equation (2.30) where $B_0 = 0$,

$$B_1 = \begin{Bmatrix} \frac{h\alpha+(h-1)\tau_y}{\alpha(1+h)} & \frac{(h-1)\tau_\pi}{\alpha(1+h)} \\ \frac{(h\alpha[h(\alpha+\eta)-\eta]-(h-1)(\alpha+\eta-h\eta)\tau_y)\lambda}{\alpha(h-1)(1+h)} & \frac{([h-1]\eta-\alpha)\lambda\tau_\pi}{\alpha(1+h)} \end{Bmatrix} \quad 2.52$$

$$B_2 = \begin{Bmatrix} \frac{1}{1+h} & \frac{1-h}{\alpha(1+h)} \\ \frac{(\alpha+[1-h]\eta)\lambda}{1-h^2} & \frac{\alpha\beta(1+h)+(\alpha+\eta-h\eta)\lambda}{\alpha(1+h)} \end{Bmatrix} \quad 2.53$$

The methodology for finding the MSV solution and E-stability condition can be followed as sub section 2.3.1 from equations (2.30) to (2.42), where the element of B_1 and B_2 are given by equation (2.52) and (2.53).

Figure 2.1
Determinacy and E-stability results
under lagged data rule



Blue – B : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules
Green – G : Determinate but E-unstable policy rules
Red – R : Explosive path

When central bank uses past data in their policy rule and we assume that no habit in consumption ($h = 0$), it turns out that only a subset of policy parameter space which is consistent with determinacy is also consistent with E-stability (the first panel of Figure 2.1). A determinate and E-stable REE is attained for the values of $\tau_\pi > 1$ and a low to moderate values of τ_y (< 3) which is denoted by a blue region. This implies out of this area, the economy potentially takes to an inferior equilibrium. As an illustration, the economy may be guided to a determinate but E-unstable equilibrium when central banks' reaction function to inflation τ_π is lower than 1 and to output τ_y is higher than 3 (a green area). Under such condition, economic agents fail to coordinate to achieve such a unique stationary equilibrium. As well, the economy may be lead to more than one stationary REE but these equilibriums turn out to be E-unstable under RLS learning (white colour). In an extreme case, it may be conveyed to an explosive path where there is no stationary REE (red colour). Our finding here has replicated the result of Bullard & Mitra (2002) under a full forward looking model.

We repeat the simulation for different values of habit parameter h, in particular we plot the result for h = 0.3 and h = 0.6, respectively. As discussed in the introductory section, it is likely that each country has different levels of habit persistence in consumption. For developed countries, the movement of aggregate consumption is more persistent and less volatile. In contrast, for developing countries we may expect that it becomes less persistence and more volatile. Our result shows when consumption habit becomes matter, the region of determinate and E-stable policy rule enlarges. As shown in the second panel of Figure 2.1, setting h equal to 0.3 does not only increase determinacy but also E-stability. For determinacy, this can be observed by comparing the total area of the blue and green region between the second and the first panel of Figure 2.1. While for E-stability, it can be seen from a larger region of a blue colour (only) between the second and the first panel of Figure 2.1. Based on this result, we argue when habit parameter increases and lagged data rule is used, it becomes easier for central bank to lead the economy to a determinate and E-stable REE. We confirm this argument by increasing the parameter value of h to 0.6. As illustrated in the third panel of Figure 2.1, in a situation in which habit in consumption is highly endowed by the households (h = 0.6), it turns out that a larger region of determinate policy rules is attained. In addition, all these determinate policy rules become stable under learning assumption (E-stable).

Up to now, we have considered a situation in which households are endowed with external habit in consumption. Are the results significantly different if habit is internally endowed by households? Note that when habit is internal, the system of equations is characterised by equation (2A.33) and (2A.34) in Appendix 2.2 plus equation (2.50). Using a similar procedure to external habit and assuming central bank responds to lagged data, we can show that the third panel of Figure 2.1 can be obtained by considering only a lower value of habit parameter, i.e. 0.3. This means a determinate and E-stable REE is much more easily attained when households are endowed with an internal habit rather than an external habit. A possible explanation is that no externality arises when habit is internal so that the consumption becomes less volatile.

What is the intuition behind the above findings? In general, a presence of the superficial habit in consumption increases both determinacy and E-stability. An increase in determinacy happens since the movement of aggregate consumption becomes less volatile which makes a particular response of interest rate rule on output that leads to indeterminacy problem, in an environment where consumption habit is absent, it now conveys to determinacy in an environment with consumption habit. This happens especially for a relatively high response of interest rate to output, in which the Taylor principle holds. Unfortunately, consumption smoothing only can be done at the cost of a higher volatility of inflation. As a result, a particular response of monetary policy to inflation that leads to determinacy, in an environment where no habit in consumption, it now leads to indeterminacy, in an environment with habit in consumption. This happens specifically for a relatively low response of interest rate to inflation or when the Taylor principle violates. This can be confirmed by seeing a larger white region in the second and third panel of Figure 2.1.

A phenomenon where consumption smoothing increases the volatility of inflation can be seen from the Philip Curve equation (2.22). Intuitively, when households are endowed with habit in consumption, then they try hard to smooth their consumption such that a small change in the level of consumption easily drives inflation away from its normal rate. Meanwhile, an increase in E-stability is due to a relatively stable economic condition as the result of consumption smoothing behaviour although at the expense of more volatile inflation. This situation makes agents are easily to learn the equilibrium dynamics so that the equilibrium path will not deviate too much from a unique stationary REE. As the parameter of habit increases, the region of determinacy and E-stability enlarges and collides.

2.4.2. Responds to Forward Expectation

This policy rule is commonly used by a modern central bank, specifically who adopts an inflation targeting framework (ITF). The ITF has been widely implemented not only in developed countries but also in many developing countries. Under this simple rule, monetary authority is more forward looking by responding to future expectations of inflation and output. To make it obvious, we consider a policy rule as follow:

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \qquad 2.54$$

When consumption habits matter and central bank responds to future expectations, our system of equations is characterised by equations (2.5), (2.22) and (2.54). As before, we analyse both determinacy and E-stability issue of this particular Taylor-type rule. For the analysis of determinacy, we substitute equation (2.54) into equation (2.5) and define one artificial variable of $\tilde{y}_t^l$ that associates with the variable of past output. Then, we construct the system of equations in the form of equation (2.29) in which $G_t = \left(\tilde{y}_t, \tilde{\pi}_t, \tilde{y}_t^l\right)'$ corresponds to a vector of endogenous variables including its lag and $Z_t = (\tilde{g}_t, \tilde{a}_t)'$ denotes a vector of the exogenous variables. By doing this, we obtain a matrix Ω in which its elements are shown in equation (2.55) bellow.

$$\Omega = \begin{Bmatrix} \frac{(1-h)\eta\lambda+\alpha(\beta[1+h]+\lambda)-(\alpha+[1-h]\eta)\lambda\tau_\pi}{\beta(\alpha+[h-1]\tau_y)} & \frac{(h-1)(1-\tau_\pi)}{\beta(\alpha+[h-1]\tau_y)} & -\frac{h\alpha(\beta+\lambda[1-\tau_\pi])}{\beta(\alpha+[h-1]\tau_y)} \\ \frac{(\alpha+\eta-h\eta)\lambda}{\beta(h-1)} & \frac{1}{\beta} & \frac{h\alpha\lambda}{\beta(1-h)} \\ 1 & 0 & 0 \end{Bmatrix} \qquad 2.55$$

Notice that the dimension of Ω is now 3 x 3 which is lower than a case where monetary authority responds to lagged data previously. In order to have a unique stationary REE, it is required that one Eigenvalues of Ω be inside the unit circle. Otherwise, the equilibrium will be indeterminate, if it is higher than one, and takes to an explosive path if it is less than one.

We relax the assumption of rational expectation (RE) above by assuming agents use the RLS algorithm in estimating the true parameter values of the model. E-stability of REE is then evaluated in the form of its minimum state variable (MSV) solution. We substitute equation (2.54) into (2.5) and reduce the system into two equations, involving the endogenous variables $\tilde{y}_t$ and $\tilde{\pi}_t$. Defining $S_t = (\tilde{y}_t, \tilde{\pi}_t)'$, the system of equations can be written in the form of equation (2.30) with $B_0 = 0$,

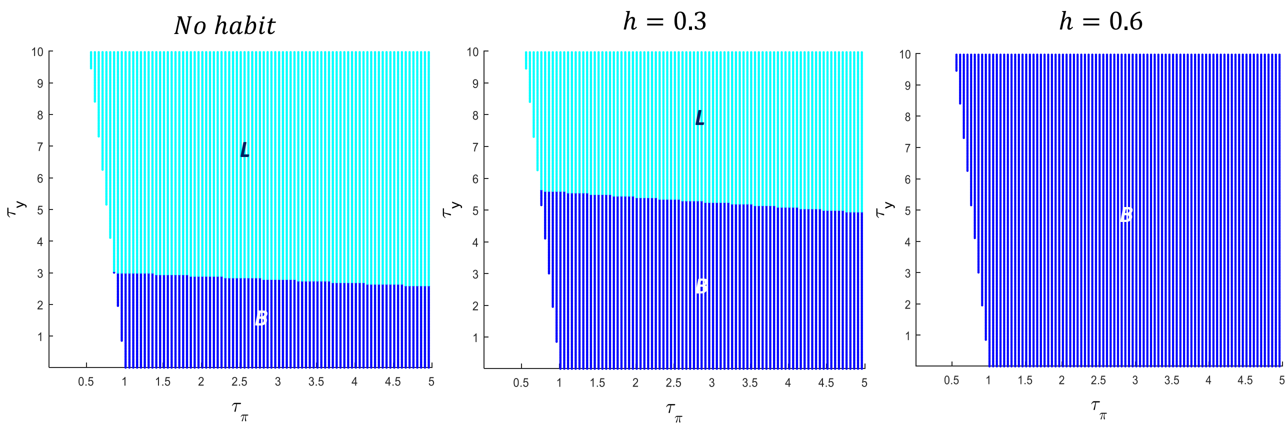
$$B_1 = \begin{Bmatrix} \frac{h}{1+h} & 0 \\ \frac{h\lambda(h[\alpha+\eta]-\eta)}{h^2-1} & 0 \end{Bmatrix} \qquad 2.56$$

$$B_2 = \begin{Bmatrix} \frac{\alpha+(h-1)\tau_y}{\alpha(1+h)} & \frac{(h-1)(\tau_\pi-1)}{\alpha(1+h)} \\ -\frac{\lambda(\alpha+[1-h]\eta)(\alpha+(h-1)\tau_y)}{\alpha(h-1)(1+h)} & \beta-\frac{\lambda(\alpha+[1-h]\eta)(\tau_\pi-1)}{\alpha(1+h)} \end{Bmatrix} \qquad 2.57$$

The methodology for finding a MSV solution and E-stability condition is the same as the previous sub-section.

When households do not endow with habit in consumption, a response to forward expectation easily leads to indeterminacy problem. As shown in the first panel of Figure 2.2, the white and light blue region correspond to a combination of policy parameters that takes the economy into indeterminate equilibrium. In details, indeterminate and E-unstable equilibrium exists when the Taylor principle is violated, i.e. $\tau_\pi < 1$, while E-stable but indeterminate equilibrium exists for the values of $\tau_\pi > 1$ and a moderate to high response to output τ_y. As mentioned previously, the only rule that should be considered by monetary authority is the one that leads to determinacy and E-stability. This can be obtained by considering the combination of policy parameters within the region of $\tau_\pi > 1$, where the Taylor principle is satisfied, and $\tau_y < 3$. Notice that unlike the response to lagged data, when monetary authority responds to forward expectation all the subset of the parameter space which is consistent with determinacy is also consistent with E-stability. Again, we successfully replicate the result of Bullard & Mitra (2002) for a case where central bank responds to forward expectation when habit in consumption is absent.

Figure 2.2
Determinacy and E-stability results under
forward expectation rule



Blue – B : Determinate and E-stable policy rules
Light Blue – L : Indeterminate but E-stable policy rules
White : Indeterminate and E-unstable policy rules

We now consider a situation in which households endow with habits in consumption (h is set at 0.3) and central bank uses future data in its interest rate rule. As shown in the second panel of Figure 2.2, the existence of habit in consumption enlarges both determinacy and E-stability within policy parameters while indeterminacy region, at the same time, get smaller. The intuition behinds this finding is similar to the case in which central bank responds to lagged data. When households smooth their consumption, output becomes less volatile while inflation gets more volatile. As a result, a particular response of interest rate to output, specifically when $\tau_\pi > 1$, that leads to indeterminacy problem in a model without the feature of consumption habit, now it leads to determinacy in the model with habit in consumption. An increase in determinacy region is then followed by an increase in E-stability region. This happens as in general the economy gets more stable such that economic agents are easier to learn this unique stationary REE. As habit parameter increases, i.e. h = 0.6, E-unstable and indeterminate region disappears and is replaced by E-stable and determinate region, as shown in the third panel of Figure 2.2. Notice that this result is similar to the third panel of Figure 2.1.

2.4.3. Responds to Contemporaneous Data

The last policy rule considered in the analysis is a policy rule that responds to contemporaneous data of inflation and output. A response to contemporaneous data seems impractical since monetary authority does not have complete information on variables, such as inflation and output, in the period where they are making a decision. However, assuming one quarter is long enough to collect all data and monetary authority uses these data instantaneously, then it becomes plausible for us to consider this policy rule in the analysis. We consider a policy rule that responds to contemporaneous data as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \qquad 2.58$$

Alternatively, we can assume that monetary authority responds to contemporaneous expectation data[17]. Either the latter or the former, it turns out that both of these policy rules give the same result for the study of determinacy and E-stability.

A procedure for analysing determinacy and E-stability condition is exactly the same as discussed earlier. Setting the system in the form of equation (2.29), we have Ω in which its elements are given by equation (2.59) below.

$$\Omega = \begin{Bmatrix} \frac{(1-h)\eta\lambda+\alpha(\beta[1+h]+\lambda)+(1-h)\beta\tau_y}{\alpha\beta} & \frac{(1-h)(\beta\tau_\pi-1)}{\alpha\beta} & -\frac{h(\beta+\lambda)}{\beta} \\ \frac{(\alpha+\eta[1-h])\lambda}{(h-1)\beta} & \frac{1}{\beta} & \frac{h\alpha\lambda}{\beta(1-h)} \\ 1 & 0 & 0 \end{Bmatrix} \quad 2.59$$

For determinacy, it is required that one Eigenvalue of Ω is inside the unit circle since only one predetermined variable includes in the model, i.e. past output $\tilde{y}_t^l$. If not, the economy takes to indeterminate equilibrium or goes to an explosive path. Meanwhile for the analysis of E-stability, the system of equations is written in the form of equation (2.30) with $B_0 = 0$,

$$B_1 = \begin{Bmatrix} \frac{h\alpha(1+\lambda\tau_\pi)}{\alpha(1+h)+\lambda\tau_\pi(\alpha+[1-h]\eta)+\tau_y(1-h)} & 0 \\ \frac{h\alpha\lambda(h[\alpha+\eta]-\eta+[1-h]\tau_y)}{(h-1)(\alpha[1+h]+\lambda\tau_\pi(\alpha+\eta[1-h])+\tau_y[1-h])} & 0 \end{Bmatrix} \quad 2.60$$

$$B_2 = \begin{Bmatrix} \frac{\alpha}{\alpha(1+h)+\lambda\tau_\pi(\alpha+\eta[1-h])+\tau_y(1-h)} & \frac{(h-1)(\beta\tau_\pi-1)}{\alpha(1+h)+\lambda\tau_\pi(\alpha+\eta[1-h])+\tau_y(1-h)} \\ -\frac{\alpha(\alpha+[1-h]\eta)\lambda}{(h-1)(\alpha[1+h]+\lambda\tau_\pi(\alpha+\eta[1-h])+\tau_y[1-h])} & \frac{(1-h)\eta\lambda+\alpha(\beta[1+h]+\lambda)+\beta(1-h)\tau_y}{\alpha(1+h)+\lambda\tau_\pi(\alpha+\eta[1-h])+\tau_y(1-h)} \end{Bmatrix} \quad 2.61$$

In an environment where there is no habit in consumption, i.e. a full forward looking NK model, responds to contemporaneous data easily leads the economy to a determinate and E-stable REE, as illustrated in Figure 2.3. E-unstable and indeterminate equilibrium are attained

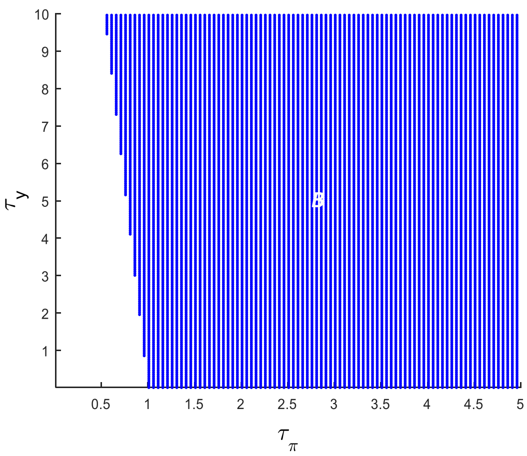
[17]A policy rule that responds to contemporaneous expectation data can be modelled as: $\tilde{r}_t = \tau_\pi E_{t-1}\tilde{\pi}_t + \tau_y E_{t-1}\tilde{y}_t$

when the Taylor principle is violated, that is an interest rate reaction function to inflation is lower than 1. In contrast, a determinate and E-stable REE exists when the Taylor principle is satisfied. Surprisingly, determinacy and E-stability region does not alter although the feature of habit in consumption is introduced with different values of habit parameter. This is an important finding since it allows us to claim the superiority of this type of policy rule over the others.

Based on the above result, there are at least two advantages of considering this simple policy rule: Firstly, it leads to a higher probability of getting determinate and E-stable policy rules in the case where the value of habit parameter is low. Secondly, it is effective for a model with parameters uncertainty, i.e. when the level of habit persistence is not easily observed. Notice that Figure 2.3 is similar to the third panel of Figure 2.1 and 2.2, above.

Figure 2.3
Determinacy and E-stability results under contemporaneous data rule

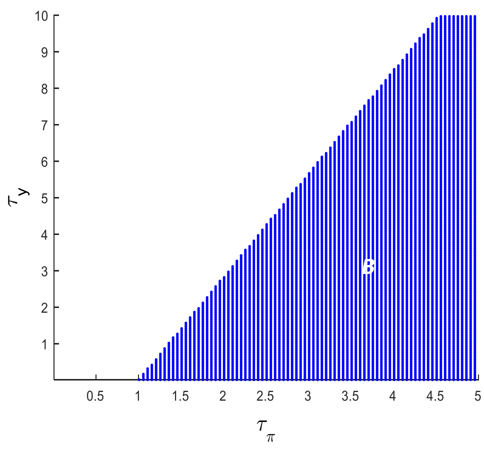
$$0 \leq h \leq 0.6$$



Blue – B : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

Figure 2.4
Determinacy and E-stability results with cost channel under various Taylor-Type policy rules

$$h = 0.6$$



Blue – B : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy

2.4.4. Cost of Monetary Channel

In earlier work, we have shown when households are endowed with habit in consumption, it becomes easier for central bank to lead the economy to a determinate and E-stable REE. Moreover, as consumption habit is highly persistent, the type of Taylor rules used by central bank becomes irrelevant since it leads to the same probability of getting a determinate and E-stable REE. Now, we slightly modify the environment of the model by assuming there is a cost of monetary channel and learn how this additional assumption makes a difference to the earlier results. In particular, we are interested in analysing the effect of the cost channel of monetary transmission, under an otherwise version of NK model with habits in consumption, on determinacy and E-stability issue of simple Taylor-type rules.

Based on the existing literature, there have been some earlier studies that have analysed the cost channel of monetary transmission. Among them is Ravenna & Walsh (2006) who provide the empirical evidence of the cost channel in the U.S. They also show the effect of cost channel in terms of monetary policy and find that a trade-off between stabilising inflation and output arises endogenously as a consequence of the cost channel. In the same spirit, we assume an endogenous cost channel for central bank. To be specific, we assume that the cost channel is present when marginal cost depends directly on the nominal interest rate. Unlike Tuesta & Llosa (2007) who employ this assumption under a full forward looking NK model, we employ it by assuming households are endowed with a high persistence of habits in consumption in an otherwise version of NK model. Since a marginal cost depends directly on the nominal interest rate, it is assumed that firms must borrow an amount of wage income from intermediaries at the gross nominal interest rate. Hence, equation (2A.11) in Appendix 2.2 needs to be changed slightly to $\varphi_t = \frac{R_t w_t}{A_t}$ in which R_t correspond to the nominal interest rate.

Following the same derivation procedure for the rest of the equations, we end up with an otherwise version of the Philip Curve equation as:

$$\tilde{\pi}_t = \beta E_t^* \tilde{\pi}_{t+1} + \left(\frac{\alpha\lambda}{(1-h)} + \eta\lambda\right)\tilde{y}_t - \frac{\alpha h\lambda}{(1-h)}\tilde{y}_{t-1} + \delta\lambda R_t - \lambda(\eta+1)\tilde{a}_t \qquad 2.62$$

where $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$

Notice that the nominal interest rate does not only appear in aggregate demand side but also in aggregate supply side. In this way, our system of equations is now characterised by equations (2.5), (2.62) and any simple policy rule adopted by central bank. As before, we analyse both determinacy and E-stability issue under a particular Taylor-type interest rate rule.

Our numerical result shows when habits in consumption are highly persistent, i.e. $h = 0.6$ and the cost channel of monetary transmission matters, different types of simple rules lead to almost similar region of determinacy and E-stability although with a smaller size compared to the case where the cost of channel is absent (see Figure 2.4)[18]. This result confirms our earlier findings in which the presence of habit in consumption has made the type of simple policy rules used becomes less relevant. The result also shows that standard instrument rules can easily induce indeterminacy and instability under RLS learning when the cost channel is present. This result is in line with the findings of Llosa & Tuesta (2007).

2.4.5. Policy Inertia

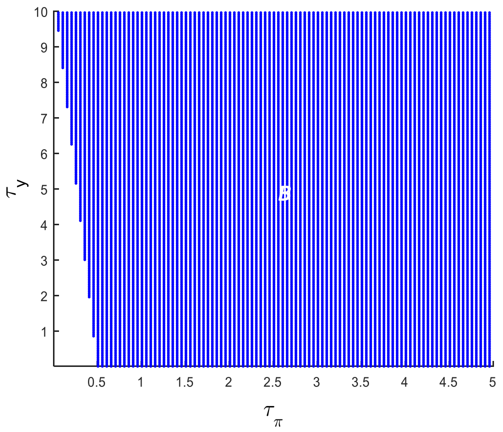
Bullard & Mitra (2007) have shown the benefit of policy inertia in leading the economy to a determinate and E-stable REE in a full forward looking New Keynesian model. We follow

[18] We obtain a similar region of determinacy and E-stability under policy parameters regardless the type of Taylor type rules adopted by central bank.

their approach but with a slightly different environment, where consumption habit matters with or without the cost channel of monetary transmission. As before, we set the habit parameter h of 0.6 and consider all policy rules in equation (2.23) to (2.25) with $\tau_r > 0$. When the cost of monetary channel is absent, an increase in the weight of policy inertia (τ_r) enlarges the region of determinacy and E-stability regardless of the type of simple rules applied by central bank (see Figure 2.5)[19]. Again, this result confirms about the irrelevant use of different simple rules in monetary policy when consumption habits matter and are highly persistent. Following Bullard & Mitra (2007), we also support the benefit of policy inertia in an increase of determinacy and E-stability region under policy parameters in a New Keynesian model with a feature of consumption habits either with or without the cost channel.

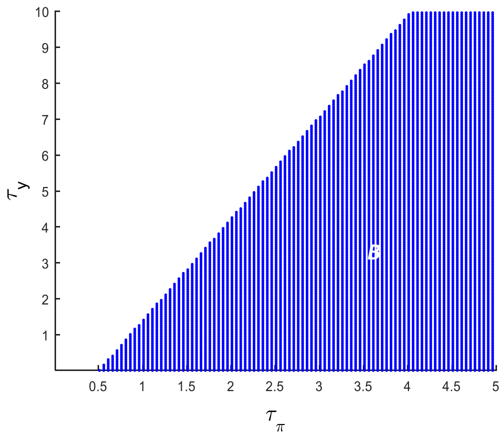
Figure 2.5
Determinacy and E-stability results with policy inertia

No cost of monetary channel
($h = 0.6$ and $\tau_r = 0.5$)



Blue - B : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

With cost of monetary channel
($h = 0.6$ and $\tau_r = 0.5$)



Blue - B : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

[19] The same result holds for current, lagged or forward expectation data in policy rule with interest rate smoothing.

2.5. The Optimal Constrained Policy Rule when Consumption Habit Matters

From the perspective of policy makers, an increase in a determinate and E-stable region in a DSGE model that features consumption habit is favourable. However, they may question which combination of policy parameters should be chosen in practise if many of them lead to a determinate and E-stable REE. We follow the concept of the optimal constrained policy rule, as discussed in Evans & McGough (2007). Under this concept, it is assumed that monetary authority has an objective to maximise the economic welfare by choosing a policy rule that ensures the variability of inflation and output at the minimum, conditional on determinacy and E-stability criteria.

We define central banks' objective function as follow:

$$Min_{\tau_\pi,\tau_y,\tau_r} Var(\tilde{\pi}_t) + \varsigma Var(\tilde{y}_t) \qquad 2.63$$

Parameter ς represents the relative weight assigned to the variance of output that reflects the importance of output volatility from the perspective of central bank. When the weight is low, more concerns are given to inflation variation relative to output variation. We set the weight of this parameter equal to 1 which means both inflation and output variations are equally important from the perspective of central bank. The reason is that households are not only concerned about price stability or inflation but also about consumption volatility by trying to smooth their consumption in each period. Meanwhile, parameter τ_π, τ_y and τ_r corresponds to monetary policy reaction function to inflation, output and inflation, respectively.

For the purpose of evaluating the optimal constrained policy, it is necessary that some disturbances are introduced to the model. We assume that two exogenous shocks may hit the economy, i.e. a preference shock and a technological shock in which they follow an AR (1) autoregressive process. As discussed in the section of the calibration, we set the persistence of preference shock equal to 0.85 and technology shock equal to 0.8 with the standard deviation of these two contemporaneous shocks are 0.35 and 0.55, respectively. Under different types of simple rule as equations (2.23) to (2.25), either with or without interest rate smoothing, we compute the variance-covariance of state variable $\tilde{\pi}_t$ and $\tilde{y}_t$ in M_t using a procedure shown in equations (2.43) to (2.49). After vectorising the variance-covariance M_t, we obtain the second moment of $\tilde{\pi}_t$ and $\tilde{y}_t$ that are used to evaluate the loss function in equation (2.63). Note that in evaluating the optimal constrained policy rule, it is assumed that households endow with habit in consumption with a persistent level of 0.6 (highly persistent). The analysis starts by assuming no cost of the monetary channel, initially, and with the cost of monetary channel, later on.

2.5.1. No Cost of Monetary Channel

We have shown earlier that using current data of inflation and output in policy rule is favourable when there is uncertainty with regards to the degree of habit in consumption. We also found that the type of simple rules used by central bank turns out to be irrelevant when habit in consumption is highly persistent. Yet, a question that remains is how to find the optimal rule? The answer relies on the kind of exogenous shock that hits the economy. As mentioned, we assume that there are two possible types of exogenous shocks, i.e. a preference shock and a technology shock. For each of these shocks, we evaluate a policy rule that offers the minimum loss either within or between the Taylor-type interest rate rules (among the contemporaneous, lagged or forward policy rules). For completeness, we also consider a case in which both of these shocks actively hit the economy.

When a positive preference shock hits the economy, among all types of policy rules considered, a rule that employs current data delivers the minimum loss. As illustrated in Table 2.3, the optimal constrained policy rule is obtained when central bank responds to contemporaneous data of inflation and output with a combination of policy parameter $\tau_\pi = 4.96$ and $\tau_y = 9.96$, respectively. This finding holds by assuming no interest rate smoothing is employed by central bank.

Table 2.3
Optimal constrained policy rule without policy inertia
(a preference shock)

	Mix A*	Mix B	Mix C
$\boldsymbol{\tau_y}$	9.96	5.21	9.96
$\boldsymbol{\tau_\pi}$	4.96	4.96	4.96
$\boldsymbol{\sigma_y}$	0.02	0.09	0.02
$\boldsymbol{\sigma_\pi}$	0.02	0.05	0.04
Loss	0.00131	0.01069	0.00161

The upper sign of * denotes a policy rule that gives the lowest loss of function
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

The loss function gets lower when policy inertia is considered in monetary policy. Under such condition, the optimal constrained policy rule is attained when central bank has accessed to current data at the combination of policy parameter $\tau_\pi = 4.96$, $\tau_y = 9.96$ and $\tau_r = 0.7$, respectively. This result is well reported in Table 2.4. Note that we set the upper bound of policy inertia at 1, such that the grid search stops when this value is reached.

Table 2.4
Optimal constrained interest rate rule with policy inertia
(a preference shock)

	Mix A*	Mix B	Mix C
$\boldsymbol{\tau_y}$	9.96	9.81	9.96
$\boldsymbol{\tau_\pi}$	4.96	1.11	4.96
$\boldsymbol{\tau_r}$	0.7	1	0.3
$\boldsymbol{\sigma_y}$	0.02	0.07	0.03
$\boldsymbol{\sigma_\pi}$	0.01	0.02	0.03
Loss	0.00069	0.00566	0.00149

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

What is the intuition behind the finding? A response of policy rate to current inflation and current output is required since a preference shock causes output and inflation move to the same direction. As an illustration, a positive preference shock increases aggregate consumption and output that cause a price of goods to rise. To dampen this increase and stabilise the economy back to its steady state level, monetary policy is required to give a response to current inflation and current output. Unfortunately, the above result should be interpreted with highest caution as it is required a very strong response of policy rate to output.

Unlike a preference shock, in a case where the disturbance comes from a technology shock, responding to output is unnecessary. This happens as a technology shock cause inflation and output move to an opposite direction. Under a technology shock, the optimal constrained policy rule holds when central bank responds to lagged data of inflation and output at the combination of policy parameter $\tau_\pi = 1.96$, and $\tau_y = 0.01$, respectively (see Table 2.5). Notice that this result is attained by assuming no policy inertia. When central bank considers inertia in policy rule, the value of loss function gets lower and the optimal constrained policy rule is obtained when they respond to current data at the policy combination of $\tau_\pi = 0.26$, $\tau_y = 0.01$ and $\tau_r = 1$ (see Table 2.6).

Table 2.5
Optimal constrained policy rule without policy inertia
(a technological shock)

	Mix A	Mix B*	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	2.06	1.96	2.81
$\boldsymbol{\sigma_y}$	0.18	0.17	0.18
$\boldsymbol{\sigma_\pi}$	0.11	0.11	0.11
Loss	0.04536	0.04194	0.04435

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.6
Optimal constrained policy rule with policy inertia
(a technological shock)

	Mix A*	Mix B	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	0.26	0.26	0.76
$\boldsymbol{\tau_r}$	1	1	1
$\boldsymbol{\sigma_y}$	0.14	0.14	0.15
$\boldsymbol{\sigma_\pi}$	0.11	0.11	0.11
Loss	0.03349	0.03350	0.03441

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

In a situation where a preference shock and a technology shock are jointly active, but no correlation between these shocks, we argue that the optimal constrained interest rule is attained when central bank adopts a forward looking interest rate rule by responding to future value of inflation and output with the combination of policy parameter $\tau_\pi = 4.96$ and $\tau_y = 0.21$, respectively (see Table 2.7). Considering policy inertia in the analysis, the loss of function attains its lowest value. Under such condition, the optimal constrained rule holds when central bank responds to forecasted data of inflation and output at the combination of $\tau_\pi = 4.96$, $\tau_y = 0.96$ and $\tau_y = 0.8$ (see Table 2.8).

Table 2.7
Optimal constrained policy rule without policy inertia
(all shocks)

	Mix A	Mix B	Mix C*
$\boldsymbol{\tau_y}$	0.11	0.51	0.21
$\boldsymbol{\tau_\pi}$	4.96	4.96	4.96
$\boldsymbol{\sigma_y}$	0.22	0.23	0.21
$\boldsymbol{\sigma_\pi}$	0.10	0.12	0.12
Loss	0.05949	0.06671	0.05767

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.8
Optimal constrained interest rate rule with policy inertia
(all shocks)

	Mix A	Mix B	Mix C*
$\boldsymbol{\tau_y}$	0.76	0.96	0.96
$\boldsymbol{\tau_\pi}$	3.16	4.31	4.96
$\boldsymbol{\tau_r}$	1	1	0.8
$\boldsymbol{\sigma_y}$	0.21	0.23	0.20
$\boldsymbol{\sigma_\pi}$	0.10	0.10	0.12
Loss	0.05515	0.06126	0.05371

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

2.5.2. Cost of Monetary Channel

We extend the analysis of the optimal constrained policy rule by considering the cost of monetary channel in the model's framework. Under this assumption, some earlier results have changed, especially for the case where a preference shock is active. When the cost channel of monetary transmission is present, whist at the same time habit in consumption is highly persistent, a high response of policy rate to output does not always required when a positive preference shock hit the economy. This happens as higher response to this variable leads to indeterminacy problem. As shown in Table 2.9, an optimal constrained interest rate rule is attained when central bank responds to current data of inflation and output at the combination of $\tau_\pi = 4.96$ and $\tau_y = 0.01$, respectively. Assuming central bank does interest rate smoothing, a lower loss of function is attained which implies a less variability of state variables or a higher economic welfare. Under such condition, the optimal rule is obtained when central bank responds to current data of inflation and output at the combination of $\tau_\pi = 4.96$, $\tau_y = 0.01$ and $\tau_r = 0.8$, respectively (see Table 2.10).

Table 2.9
Optimal constrained policy rule without policy inertia when cost channel matters
(a preference shock)

	Mix A*	Mix B	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.61
$\boldsymbol{\tau_\pi}$	4.96	4.96	4.96
$\boldsymbol{\sigma_y}$	0.07	0.1	0.06
$\boldsymbol{\sigma_\pi}$	0.10	0.09	0.13
Loss	0.01407	0.01729	0.02116

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.10

Optimal constrained policy rule with policy inertia when cost channel matters (a preference shock)

	Mix A*	Mix B	Mix C
$\boldsymbol{\tau_y}$	0.01	1.21	0.01
$\boldsymbol{\tau_\pi}$	4.96	4.96	4.96
$\boldsymbol{\tau_r}$	0.8	1	0.5
$\boldsymbol{\sigma_y}$	0.06	0.1	0.05
$\boldsymbol{\sigma_\pi}$	0.05	0.04	0.08
Loss	0.00549	0.01258	0.00875

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.11

Optimal constrained policy rule without policy inertia when cost channel matters (a technological shock)

	Mix A	Mix B*	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.36
$\boldsymbol{\tau_\pi}$	2.16	1.91	3.41
$\boldsymbol{\sigma_y}$	0.22	0.2	0.22
$\boldsymbol{\sigma_\pi}$	0.13	0.12	0.17
Loss	0.06743	0.05516	0.07488

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.12

Optimal constrained policy rule with policy inertia when cost channel matters (a technological shock)

	Mix A	Mix B*	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	0.21	0.26	0.56
$\boldsymbol{\tau_r}$	1	1	1
$\boldsymbol{\sigma_y}$	0.14	0.15	0.15
$\boldsymbol{\sigma_\pi}$	0.13	0.11	0.13
Loss	0.03653	0.03586	0.03840

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Since a response to output is not as freely as the case of no cost of monetary channel, we continue a simulation exercise by evaluating this issue in which both preference and technology shock hit the economy simultaneously while at the same time the cost of monetary channel matters. In such condition, the optimal constrained policy holds when central bank uses a backward policy rule with the combination of policy parameters $\tau_{\pi} = 4.96$ and $\tau_{y} = 0.01$, respectively (see Table 2.13). Again, considering policy inertia in the analysis helps central bank to get the lowest loss in terms of the variability of inflation and output. Under such assumption, the optimal constrained policy rule is obtained when central bank uses a forward looking interest rate rule with the combination of policy parameters $\tau_{\pi} = 4.96$, $\tau_{y} = 0.01$ and $\tau_{r} = 1$, respectively (illustrated in Table 2.14). In a case where a positive technology shock hits the economy, the result is not much different between a model that features the cost of monetary channel and a model where the cost of monetary channel is absent. The results are summarised in Table 2.11 and Table 2.12, respectively.

Table 2.13
Optimal constrained policy rule without policy inertia when cost channel matters (all shocks)

	Mix A	Mix B*	Mix C
$\boldsymbol{\tau_y}$	0.01	0.01	0.61
$\boldsymbol{\tau_\pi}$	4.71	4.96	4.96
$\boldsymbol{\sigma_y}$	0.28	0.27	0.25
$\boldsymbol{\sigma_\pi}$	0.13	0.10	0.20
Loss	0.09229	0.08165	0.09868

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

Table 2.14
Optimal constrained policy rule with policy inertia when cost channel matters
(all shocks)

	Mix A	Mix B	Mix C*
$\boldsymbol{\tau_y}$	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	2.36	3.56	4.96
$\boldsymbol{\tau_r}$	1	1	1
$\boldsymbol{\sigma_y}$	0.25	0.26	0.24
$\boldsymbol{\sigma_\pi}$	0.08	0.07	0.10
Loss	0.06818	0.07337	0.06519

The upper sign of * denotes a policy rule that gives the lowest loss.
Mix A: Current data in interest rate rule
Mix B: Lagged data in interest rate rule
Mix C: Forward looking interest rate rule

2.6. Conclusion

We have addressed two issues in this chapter: Firstly, how determinacy and expectational stability (E-stability) of rational expectation equilibrium (REE) is affected by monetary policy when consumption habit matters. Secondly, how the optimal constrained policy rule is attained when consumption habit matters and the cost of monetary channel is present or absent. Notice that the optimal rule is constrained not only by determinacy but also E-stability criteria. We focus the analysis on simple Taylor-type interest rules, i.e. contemporaneous, lagged and forward looking rule either with or without policy inertia.

We conclude that the presence of superficial habit in consumption does not only increase determinacy but also E-stability. An increase in determinacy happens as the movement of aggregate consumption is smoother which makes a particular response of policy rate to output that leads to indeterminacy, in an environment with no habit in consumption, now it leads to determinacy, in an environment where consumption habit matters. Yet, consumption smoothing behaviour can only be done at the cost of a higher volatility in inflation. As the result, a particular response of policy rate to inflation that leads to determinacy, in an environment with no habit in consumption, now it takes to indeterminacy, in an environment where consumption habit matters.

Meanwhile, an increase in E-stability is due to a relatively stable economic condition as a result of smoothing consumption behaviour. This condition makes economic agents easier to learn the equilibrium dynamic so that the equilibrium path does not deviate too much from a unique stationary REE. As habit in consumption gets persistent, a determinate and E-stable region enlarges and collides regardless of the type of simple rules employed by central bank. This result has allowed us to claim that contemporaneous policy rule is superior to other simple rules. The reason is that not only in leading the economy to a determinate and E-stable REE with higher probability but also it is immune to the model uncertainty, i.e. when the degree of consumption habit is not easily observed.

The study also shows when the cost channel of monetary transmission matter, then the probability of getting a determinate and E-stable REE reduces. This finding is valid not only in a case where consumption habit is less persistent but also when they are highly persistent. This happens as monetary policy does not only affect a demand side of the economy but also its supply side with an opposite direction. As a result, a smaller region of determinacy and E-stability is occurred. However, considering policy inertia in interest rate rule helps central bank to increase this probability, regardless the type of rules adopted.

Concerning the optimal constrained policy rule, we conclude that a decision, to choose the optimal policy relies on the assumption of exogenous shock that hits the economy. When a positive preference shock hits the economy, then responses to inflation and output are necessary to dampen economy volatility. In contrast, when a positive technology shock hits the economy, a response to output is not necessary as inflation and output moves to different direction. The result also shows that considering policy inertia in monetary policy is very useful in dampening the volatility of aggregate economy. This can be confirmed by comparing the value of loss function in which its lowest value always attains for a policy rule that incorporates policy inertia.

Overall, the above findings enrich the existing studies about determinacy and E-stability issue of simple Taylor-type policy rules. Unlike earlier studies that mostly incorporate the feature of consumption habit to improve the motion of aggregate consumption in business cycles analysis, in this chapter we consider it for the study of determinacy and E-stability. Although the model used in this chapter performs quite well in addressing some important issues concerning to determinacy, stability under learning and the optimal constrained policy rule, still, it is subject to criticism. One aspect is the way of how we set the objective function of central bank. In this work, central banks' objective function is set in an ad-hoc manner where they try to minimise the loss of function in terms of inflation and output variation. In this way, our earlier results about the optimal constrained policy rule is potentially sensitive to the way of how we model the objective function of central bank. One way to improve this limitation is by modelling central banks' objective in a way where they maximise the unconditional expectation of households' welfare. Here, the welfare function is obtained by taking a second-order approximation to the utility of the representative agents, as used in Erceg et al. (2000) and Woodford (2003), among others.

Another aspect where we believe the current model can be improved is by introducing the feature of housing assets and financial constraint. One sector with one market of goods, as we use here, is too simple to characterise a real economy. Therefore, introducing housing assets and financial constraint in the model may improve the current analysis. We follow this suggestion by analysing determinacy and E-stability issue of simple Taylor-type interest rate rules in a model where housing assets and financial constraints are present in Chapter 3.

Appendix 2.1. Notation

Parameter	Description
β	The discount factor for households
ε	The elasticity of substitution among individual goods
α	The inverse of the elasticity of intertemporal substitution in consumption
η	The inverse of the elasticity of labour supply
θ	The fraction of firms that keep their price unchanged
h	The habit parameter
τ_π	Interest rate reaction function to inflation
τ_y	Interest rate reaction function to output
τ_r	Interest rate inertia
ς	Relative weight assigned to the variance of output
ρ_g	Coefficient of preference shock AR (1) model
ρ_a	Coefficient of productivity of technology shock AR (1) model
σ_g	Standard deviation of preference shock
σ_a	Standard deviation of technology shock

Variable	Description
Y	Output
C	Consumption
N	Working Hour
B	Bond
π	Inflation
w	Real wage
F	Real lump sum profit
mc	Marginal cost
R	Gross nominal interest rate
g	Exogenous taste
A	Exogenous technology
u^g	Normally distributed shock to taste
u^a	Normally distributed shock to technology

Appendix 2.2. Model Derivation

Households

The economy is populated by a continuum of households, indexed by $j \in [0,1]$. The composite consumption good C_t that enters household's utility function is defined as an aggregate of a continuum of goods, indexed by i and of measure 1.

$$C_t = \left(\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{2A.1}$$

Household j decides the composition of the consumption basket to minimise the expenditures subject to the constraint.

$$Min \int_0^1 P_{it} C_{it}\, di \quad s.t. \ \left(\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = \bar{C}_t$$

$$\mathcal{L} \equiv \int_0^1 P_{it}\, C_{it}\, di + \psi_t \left(\bar{C}_t - \left[\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_{it}} = P_{it} - \psi_t \left(\left[\left(\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right)\right]^{\frac{1}{\varepsilon-1}} (C_{it})^{-\frac{1}{\varepsilon}} \right) = 0$$

$$C_{it} = \left(\frac{P_{it}}{\psi_t}\right)^{-\varepsilon} \left(\int_0^1 (C_{it})^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{it} = \left(\frac{P_{it}}{\psi_t}\right)^{-\varepsilon} C_t \quad \forall\, i \qquad \text{2A.2}$$

Solving for ψ_t by substituting (2A.2) into (2A.1) yields:

$$C_t = \left(\int_0^1 \left[\left(\frac{P_{it}}{\psi_t}\right)^{-\varepsilon} C_t \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_t = \left(\frac{1}{\psi_t}\right)^{-\varepsilon} \left(\int_0^1 P_{it}{}^{1-\varepsilon} di\right)^{\frac{\varepsilon}{\varepsilon-1}} C_t$$

$$\psi_t = \left(\int_0^1 P_{it}{}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \equiv P_t \qquad \text{2A.3}$$

Therefore, the demand for good i can be written as:

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t \quad \forall\, i \qquad \text{2A.4}$$

A representative household j gets utility from habit adjusted consumption goods and disutility from hours worked in each period,

$$Max\ E_0^* \sum_{t=0}^{\infty} \beta^t \left(U(C_t, H_t) - \frac{N_t^{1+\eta}}{1+\eta} \right)$$

$$S.t. \quad C_t + B_t = w_t N_t + \frac{R_{t-1} B_{t-1}}{\pi_t} + F_t \qquad \text{2A.5}$$

where,

$$U(C_t, H_t) = \frac{e^{g_t} (C_t - H_t)^{1-\alpha}}{1-\alpha}$$

$H_t \ = h\left(C_{t-1}{}^{D} \{C_{t-1}^{o}\}^{1-D}\right)$ with C_t^o is the average consumption.

$$\mathcal{L} \equiv E_t^* \sum_{t=0}^{\infty} \beta^t \left(U(C_t, H_t) - \frac{N_t^{1+\eta}}{1+\eta} + \lambda_t \left[w_t N_t + \frac{R_{t-1} B_{t-1}}{\pi_t} + F_t - C_t - B_t \right] \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t (U_C(C_t, H_t) - \lambda_t) + \beta^{t+1} U_H(C_{t+1}, H_{t+1})\ hD(C_t)^{D-1} \{C_t^o\}^{1-D} = 0$$

In equilibrium $C_t = C_t^o$ which allow us to write the above and upcoming equation as,

$$U_C(C_t, H_t) - \lambda_t + \beta U_H(C_{t+1}, H_{t+1})\ D E_t^* \frac{H_{t+1}}{C_t} = 0 \qquad \text{2A.6}$$

Note that,

$$U_C(C_t, H_t) = e^{g_t} (C_t - H_t)^{-\alpha}$$

$$U_H(C_{t+1}, H_{t+1}) = -e^{g_{t+1}} E_t^* (C_{t+1} - H_{t+1})^{-\alpha}$$

Substituting the above two equations into (2A.6) yields

$$e^{g_t} (C_t - H_t)^{-\alpha} - E_t^* e^{g_{t+1}} \beta (C_{t+1} - H_{t+1})^{-\alpha} D \frac{H_{t+1}}{C_t} = \lambda_t$$

$$e^{g_t} (C_t - hC_{t-1})^{-\alpha} - h\beta D E_t^* e^{g_{t+1}} (C_{t+1} - hC_t)^{-\alpha} = \lambda_t \qquad \text{2A.7}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = \beta^t \lambda_t - \beta^{t+1} E_t^* (\lambda_{t+1} \pi_{t+1} R_t) = 0$$

$$\lambda_t = \beta E_t^* (\lambda_{t+1} \pi_{t+1} R_t) \qquad \text{2A.8}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = \ \beta^t \left(w_t \lambda_t - N_t^{\eta} \right) = 0$$

$$w_t \lambda_t = N_t^{\eta} \qquad \text{2A.9}$$

Firms

A continuum of firms indexed by $i \in [0,1]$ operates under monopolistically competitive market with demand scheduled given by (2A.4) and a constant return to scale (CRS) production function. The firm employs labour provided by households and every period minimises its production cost.

$Min\ w_t N_{it}$

$S.t.\quad A_t N_{it} = Y_{it}$ 2A.10

$\mathcal{L} \equiv w_t N_{it} + \varphi_t (Y_{it} - A_t N_{it})$

Differentiating with respect to labour yields:

$$\frac{\partial \mathcal{L}}{\partial N_{it}} = w_t = \varphi_t A_t$$

$$\varphi_t = \frac{w_t}{A_t} \equiv mc_t \qquad 2A.11$$

We follow the Calvo (1983) model where there is a fraction of θ of firms that does not change their price and another fraction $1 - \theta$ reset their price based on optimisation. The Calvo price setting suggests $P_{it+s} = P_{it}^{New}$ that the price will be set at the previous effective price with probability θ^s $(s = 0,1,2,\ldots)$. As a result, the remainder $(1 - \theta)$ of the firms maximise the present value of their profit as follow:

$$Max\ E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\frac{P_{it}^{New}}{P_{t+s}} - mc_{t+s} \right) Y_{it+s} \right\}$$

$$S.t.\quad Y_{it+s} = \left(\frac{P_{it}^{New}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s}$$

$E_t^* Q_{t,t+s}$ is stochastic discount factor and its relation to R_t is given by $E_t^* Q_{t,t+1} = 1/R_t$

The unconstrained optimisation problem becomes,

$$V = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\frac{P_{it}^{New}}{P_{t+s}} - mc_{t+s} \right) \left(\frac{P_{it}^{New}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right\}$$

$$V = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\left[\left(\frac{P_{it}^{New}}{P_{t+s}} \right)^{1-\varepsilon} - \left(\frac{P_{it}^{New}}{P_{t+s}} \right)^{-\varepsilon} mc_{t+s} \right] Y_{t+s} \right) \right\}$$

It is possible to write P_t^{New} instead of P_{it}^{New} because all firms share the same production technology and are price takers at the labour market.

$$V = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\left[\left(\frac{P_t^{New}}{P_{t+s}} \right)^{1-\varepsilon} - \left(\frac{P_t^{New}}{P_{t+s}} \right)^{-\varepsilon} mc_{t+s} \right] Y_{t+s} \right) \right\}$$

The optimal price decision is,

$$\frac{\partial V}{P_t^{New}} = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\left[(1-\varepsilon) \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon} \left(\frac{1}{P_{t+s}} \right) + \varepsilon \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon-1} \left(\frac{1}{P_{t+s}} \right) mc_{t+s} \right] Y_{t+s} \right) \right\} = 0$$

$$\frac{\partial V}{P_t^{New}} = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon} \left(\left[(1-\varepsilon) \left(\frac{1}{P_{t+s}} \right) + \varepsilon \left(\frac{P_{t+s}}{P_t^*} \right) \left(\frac{1}{P_{t+s}} \right) mc_{t+s} \right] Y_{t+s} \right) \right\} = 0$$

$$\frac{\partial V}{P_t^{New}} = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon} \left(\left[\frac{P_t^*}{P_{t+s}} - \left(\frac{\varepsilon}{\varepsilon-1} \right) mc_{t+s} \right] Y_{t+s} \right) \right\} = 0$$

by multiplying the previous equation with a constant P_t^{New}.

$$\frac{\partial V}{P_t^{New}} = E_0^* \sum_{s=0}^{\infty} \theta^s \left\{ Q_{t,t+s} \left[\frac{P_t^{New}}{P_{t+s}} - \left(\frac{\varepsilon}{\varepsilon - 1} \right) mc_{t+s} \right] Y_{it+s} \right\}$$

Note, in a fully flexible price, i.e. $\theta = 0$, firms maximise its profit as follow:

$$Max \left(\frac{P_{it}^{New} - MC_t}{P_t} \right) Y_{it}$$

$$S.t. \quad Y_{it} = \left(\frac{P_{it}^{New}}{P_t} \right)^{-\varepsilon} Y_t$$

The unconstrained optimisation problem becomes,

$$H = \left(\frac{P_{it}^{New} - MC_t}{P_t} \right) \left(\frac{P_{it}^{New}}{P_t} \right)^{-\varepsilon} Y_t$$

$$\frac{\partial H}{P_t^{New}} = \frac{P_t^{New}}{P_t} - \left(\frac{\varepsilon}{\varepsilon - 1} \right) mc_t = 0$$

$$P_t^{New} = \mu P_t mc_t$$

$$mc_t = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon} \qquad \text{2A.12}$$

where,

$\mu = \frac{\varepsilon}{\varepsilon - 1}$ is the desired mark up in the absent of constraints on the frequency of price adjustment and all firms charge the same price, i.e. $P_t^{New} = P_t$

If price is sticky, i.e. $\theta \neq 0$, then we have,

$$0 = E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \frac{P_t}{P_{t+s}} \left\{ \frac{(C_{t+s} - hC_{t+s-1})^{-\alpha} - h\beta D(C_{t+s+1} - hC_{t+s})^{-\alpha}}{(C_t - hC_{t-1})^{-\alpha} - h\beta D(C_{t+1} - hC_t)^{-\alpha}} \right\} \left[P_t^{New} - \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_{t+s} \right] Y_{it+s}$$

where,

$$E_t^* Q_{t,t+1} = \beta E_t^* \left\{ \frac{P_t}{P_{t+1}} \left(\frac{(C_{t+1} - hC_t)^{-\alpha} - h\beta D(C_{t+2} - hC_{t+1})^{-\alpha}}{(C_t - hC_{t-1})^{-\alpha} - h\beta D(C_{t+1} - hC_t)^{-\alpha}} \right) \right\}$$

$E_t^* Q_{t,t+s} = Q_{t,t+1} Q_{t+1,t+2} \dots Q_{t+s-1,t+s}$ and $E_t^* Q_{t+s-1,t+s} = E_t^* E_{t+s1}^* Q_{t+s-1,t+s}$ are employed.

Therefore, the optimal re-set price P_t^{New} solves:

$$E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \frac{P_t}{P_{t+s}} \left\{ \frac{(C_{t+s} - hC_{t+s-1})^{-\alpha} - h\beta D(C_{t+s+1} - hC_{t+s})^{-\alpha}}{(C_t - hC_{t-1})^{-\alpha} - h\beta D(C_{t+1} - hC_t)^{-\alpha}} \right\} P_t^{New} Y_{it+s} =$$
$$E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \frac{P_t}{P_{t+s}} \left\{ \frac{(C_{t+s} - hC_{t+s-1})^{-\alpha} - h\beta D(C_{t+s+1} - hC_{t+s})^{-\alpha}}{(C_t - hC_{t-1})^{-\alpha} - h\beta D(C_{t+1} - hC_t)^{-\alpha}} \right\} \left(\left[\frac{\varepsilon}{\varepsilon - 1} \right] MC_{t+s} \right) Y_{it+s}$$

2A.13

Given the fraction θ of retailers do not change their price in period *t*; the aggregate price evolves according to:

$$P_t = \left[(1-\theta)(P_t^{New})^{1-\varepsilon} + \int_{S_t}(P_{t-1})^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t = \left[(1-\theta){P_t^{New}}^{1-\varepsilon} + \theta(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad \text{2A.14}$$

Equilibrium and Market Clearing

The equilibrium of the model is a sequence of prices $\{R_t, P_t, w_t\}$ and an allocation $\{Y_t, C_t, B_t\}$ such that all first order conditions and constraint hold, and all markets clear.

The aggregate profit from the firm is:

$$F_t = \int_0^1 \Phi_t(i)\, di$$

$$F_t = \int_0^1 \left(\frac{P_{it}}{P_t} - mc_t\right) Y_{it}\, di$$

$F_t = \int_0^1 \left(\frac{P_{it}}{P_t} - mc_t\right)\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t\, di$, by using the fact that $Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t$

$$F_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} Y_t\, di - \int_0^1 \left(\left[\frac{P_{it}}{P_t}\right]^{-\varepsilon} mc_t\right) Y_t\, di$$

$$F_t = \left(\frac{Y_t}{{P_t}^{1-\varepsilon}}\right)\int_0^1 {P_{it}}^{1-\varepsilon}di - \left(\frac{1}{P_t}\right)^{-\varepsilon} mc_t\, Y_t \int_0^1 {P_{it}}^{-\varepsilon}\, di$$

Recall, $P_t = \left(\int_0^1 {P_{it}}^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$ from 2A.3.

Thus, we can obtain the aggregate profit in real term as follows

$$F_t = \left(\frac{P_t - MC_t}{P_t}\right)Y_t, \text{ by assuming } P_t \equiv \int_0^1 P_t(i) \qquad \text{2A.15}$$

This aggregate profit will be rebate to the households.

Good market is clear when

$$C_t = Y_t \qquad \text{2A.16}$$

Steady State Level

We assume that the steady state inflation rate and production technology are zero.

Given (2A.13),

$$0 = E_0^* \sum_{s=0}^{\infty} (\beta\theta)^s \left(P^{New} - \left[\frac{\varepsilon}{\varepsilon-1}\right] MC\right) Y(i)$$

$$0 = E_0^* \sum_{s=0}^{\infty} \frac{Y(i)}{1-\beta\theta} \left(P^{New} - \left[\frac{\varepsilon}{\varepsilon-1} \right] MC \right)$$

The term is square bracket in above equation should be zero since $\frac{Y(i)}{1-\beta\theta}$ is positive.

$$P^{New} = \left(\frac{\varepsilon}{\varepsilon-1} \right) MC \qquad 2A.17$$

Given (2A.14),

$$P = \left[(1-\theta) P^{New^{1-\varepsilon}} + \theta(P)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

$$P = P^{New} \qquad 2A.18$$

Log-Linearisation around Steady State

The variable in log-linearised around its steady state is denoted by tilde.

For example $\tilde{y}_t = log(Y_t) - log(Y)$

From (2A.7),

$$[-\alpha(\tilde{c}_t - h\tilde{c}_{t-1}) + h\alpha\beta D E_t^*(\tilde{c}_{t+1} - h\tilde{c}_t)]/(1-h)(1-h\beta D) + (1-h\beta D) g_t = \tilde{\lambda}_t$$

2A.19

From (2A.8),

$$\tilde{\lambda}_t = E_t^* \tilde{\lambda}_{t+1} + (\tilde{r}_t - E_t^* \tilde{\pi}_{t+1}) - \rho \qquad 2A.20$$

where, $\rho = -ln(\beta)$

From (2A.9),

$$\tilde{w}_t + \tilde{\lambda}_t = \eta \tilde{n}_t \qquad 2A.21$$

Combining (2A.19) and (2A.20), we get Euler equation for the optimal intertemporal allocation for consumption.

$$E_t^* \Delta\tilde{c}_{t+1} = \frac{h}{1+h^2\beta D} E_t^*(\Delta\tilde{c}_t + \beta D \Delta\tilde{c}_{t+2}) + \frac{(1-h)(1-h\beta D)}{\alpha(1+h^2\beta D)} (\tilde{r}_t - E_t^* \tilde{\pi}_{t+1} - \rho - g_t)$$

Since $\tilde{c}_t = \tilde{y}_t$, the above equation can be written in terms of output variable as follows:

$$\tilde{y}_t = \left(\frac{h}{1+h+h^2\beta D} \right) \tilde{y}_{t-1} + \left(\frac{1+h\beta D+h^2\beta D}{1+h+h^2\beta D} \right) E_t^* \tilde{y}_{t+1} - \left(\frac{h\beta D}{1+h+h^2\beta D} \right) E_t^* \tilde{y}_{t+2} - \frac{(1-h)(1-h\beta D)}{\alpha(1+h+h^2\beta D)} (\tilde{r}_t - E_t^* \tilde{\pi}_{t+1} - \rho - g_t) \qquad 2A.22$$

Combining (2A.19) and (2A.21), we get the intertemporal optimality condition setting the marginal rate of substitution (MRS) between leisure and consumption equal to real wage.

$$\tilde{w}_t = \frac{\alpha}{(1-h)(1-h\beta D)}[(\tilde{y}_t - h\tilde{y}_{t-1}) - h\beta D(E_t^*\tilde{y}_{t+1} - h\tilde{y}_t)] + \eta\tilde{n}_t \qquad 2A.23$$

From (2A.10),

$$\tilde{n}_t = \tilde{y}_t - \tilde{a}_t \qquad 2A.24$$

From (2A.11),

$$\widetilde{mc}_t = \tilde{w}_t - \tilde{a}_t \qquad 2A.25$$

Combining (2A.23), (2A.24) and (2A.25) yields

$$\widetilde{mc}_t = \frac{\alpha}{(1-h)(1-h\beta D)}[(\tilde{y}_t - h\tilde{y}_{t-1}) - h\beta D(E_t^*\tilde{y}_{t+1} - h\tilde{y}_t)] + \eta(\tilde{y}_t - \tilde{a}_t) - \tilde{a}_t$$

$$\widetilde{mc}_t = \left[\frac{\alpha(1+h^2\beta D)}{(1-h)(1-h\beta D)} + \eta\right]\tilde{y}_t - \left[\frac{\alpha h}{(1-h)(1-h\beta D)}\right]\tilde{y}_{t-1} - \left[\frac{\alpha h\beta D}{(1-h)(1-h\beta D)}\right]E_t^*\tilde{y}_{t+1} - (\eta+1)\tilde{a}_t \qquad 2A.26$$

From (2A.13),

$$E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s(1 + \tilde{y}_{it+s} + \tilde{p}_t - \tilde{p}_{t+s} + \tilde{s}_t + \tilde{p}_t^{New}) = E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s\left(\frac{\varepsilon}{1-\varepsilon}\right)mc(1 + \tilde{y}_{it+s} + \tilde{p}_t - \tilde{p}_{t+s} + \tilde{s}_t + \tilde{p}_{t+s} + \widetilde{mc}_{t+s})$$

with,

$$\tilde{s}_t = \frac{\alpha}{(1-h\beta D)(1-h)}[(1+h^2\beta D)\tilde{c}_t - h\tilde{c}_{t-1} - h\beta D\tilde{c}_{t+1} - (1+h^2\beta D)\tilde{c}_{t+s} + h\tilde{c}_{t+s-1} + h\beta D\tilde{c}_{t+s+1}]$$

$$E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s(1 + \tilde{y}_{it+s} + \tilde{p}_t - \tilde{p}_{t+s} + \tilde{s}_t + \tilde{p}_t^{New}) = E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s(1 + \tilde{y}_{it+s} + \tilde{p}_t - \tilde{p}_{t+s} + \tilde{s}_t + \tilde{p}_{t+s} + \widetilde{mc}_{t+s})$$

$$E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s\tilde{p}_t^{New} = E_0^*\sum_{s=0}^{\infty}(\beta\theta)^s(\tilde{p}_{t+s} + \widetilde{mc}_{t+s})$$

$$\tilde{p}_t^{New} = (1-\beta\theta)E_t^*\sum_{s=0}^{\infty}(\beta\theta)^s(\tilde{p}_{t+s} + \widetilde{mc}_{t+s})$$

The last equation can be cast in the recursive form.

$$\tilde{p}_t^{New} = (1-\beta\theta)(\tilde{p}_t + \widetilde{mc}_t) + \beta\theta(1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_0^*(\tilde{p}_{t+s+1} + \widetilde{mc}_{t+s+1})$$

$$\tilde{p}_t^{New} = (1-\beta\theta)(\tilde{p}_t + \widetilde{mc}_t) + \beta\theta E_t^*\tilde{p}_{t+1}^{New} \qquad 2A.27$$

From (2A.14),

$$\tilde{p}_t = (1-\theta)\tilde{p}_t^{New} + \theta\tilde{p}_{t-1}$$

$$\tilde{p}_t^{New} = \left(\frac{1}{1-\theta}\right)\tilde{p}_t - \left(\frac{\theta}{1-\theta}\right)\tilde{p}_{t-1} \qquad \text{2A.28}$$

Substituting RHS of (2A.28) for $\tilde{p}_t^{New}$ and $\tilde{p}_{t+1}^{New}$ in (2A.27), we get

$$\tilde{\pi}_t = \beta(E_t^*\tilde{p}_{t+1} - \tilde{p}_t) + \lambda\widetilde{mc}_t$$

$$\tilde{\pi}_t = \beta E_t^*\tilde{\pi}_{t+1} + \lambda\widetilde{mc}_t \qquad \text{2A.29}$$

where, $\lambda = \frac{(1-\theta\beta)(1-\theta)}{\theta}$

Finally, we can substitute (2A.26) into (2A.29) to get the familiar looking NKPC equation.

$$\tilde{\pi}_t = \beta E_t^*\tilde{\pi}_{t+1} + \left(\frac{\alpha\lambda(1+h^2\beta D)}{(1-h)(1-h\beta D)} + \eta\lambda\right)\tilde{y}_t - \frac{\alpha h\lambda}{(1-h)(1-h\beta D)}\tilde{y}_{t-1} - \frac{\alpha h\beta D\lambda}{(1-h)(1-h\beta D)}E_t^*\tilde{y}_{t+1} - \lambda(\eta+1)\tilde{a}_t \qquad \text{2A.30}$$

Reduced Forms

The system of equations is characterised by (2A.22) and (2A.30) which denote investment saving (IS) and Philip Curve (PC) equation, respectively.

- $D = 0$

$$\tilde{y}_t = \left(\frac{h}{1+h}\right)\tilde{y}_{t-1} + \left(\frac{1}{1+h}\right)E_t^*\tilde{y}_{t+1} - \frac{(1-h)}{\alpha(1+h)}(\tilde{r}_t - E_t^*\tilde{\pi}_{t+1} - \rho - g_t) \qquad \text{2A.31}$$

$$\tilde{\pi}_t = \beta E_t^*\tilde{\pi}_{t+1} + \left(\frac{\alpha\lambda}{(1-h)} + \eta\lambda\right)\tilde{y}_t - \frac{\alpha h\lambda}{(1-h)}\tilde{y}_{t-1} - \lambda(\eta+1)\tilde{a}_t \qquad \text{2A.32}$$

- $D = 1$

$$\tilde{y}_t = \left(\frac{h}{1+h+h^2\beta}\right)\tilde{y}_{t-1} + \left(\frac{1+h\beta+h^2\beta}{1+h+h^2\beta}\right)E_t^*\tilde{y}_{t+1} - \left(\frac{h\beta}{1+h+h^2\beta}\right)E_t^*\tilde{y}_{t+2} - \frac{(1-h)(1-h\beta)}{\alpha(1+h+h^2\beta)}(\tilde{r}_t - E_t^*\tilde{\pi}_{t+1} - \rho - g_t) \qquad \text{2A.33}$$

$$\tilde{\pi}_t = \beta E_t^*\tilde{\pi}_{t+1} + \left(\frac{\alpha\lambda(1+h^2\beta)}{(1-h)(1-h\beta)} + \eta\lambda\right)\tilde{y}_t - \frac{\alpha h\lambda}{(1-h)(1-h\beta)}\tilde{y}_{t-1} - \frac{\alpha h\beta\lambda}{(1-h)(1-h\beta)}E_t^*\tilde{y}_{t+1} - \lambda(\eta+1)\tilde{a}_t \qquad \text{2A.34}$$

Appendix 2.3 Solving RE Model using the Method of Blanchard and Kahn (1980)

We briefly explain the procedure of solving RE model using the method of Blanchard and Kahn (1980). This method is commonly used as a standard method of solving a RE model. We begin by setting the whole system of linear equations into the state-space form as the followings:

$$\Gamma_0 \begin{bmatrix} z^s_{t+1} \\ \mathrm{E_t} z^j_{t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} z^s_t \\ z^j_t \end{bmatrix} + \Psi\epsilon_{t+1} \qquad \text{2B.1}$$

where Γ_0, Γ_1 and Ψ are matrices coefficient of the linearised stochastic model that are often composed by a combination of deep parameters. z^s_t is an $m\ x\ 1$ vector of pre-determined state variables and $\mathrm{E_t} z^j_{t+1}$ is an n x 1 vector of the jump (non-predetermined) variables with endogenous expectations of their forward-looking values. Meanwhile, ϵ_t corresponds to a vector of exogenous variables that follows a stationary process. Note that the values of jump variables at $t+1$ depend on the shock at $t+1$, while the values of pre-determined variables do not.

The above matrix system can be modified as:

$$\begin{bmatrix} z^s_{t+1} \\ \mathrm{E_t} z^j_{t+1} \end{bmatrix} = \mathrm{A} \begin{bmatrix} z^s_t \\ z^j_t \end{bmatrix} + \mathrm{B}\epsilon_{\mathrm{t+1}} \qquad \text{2B.2}$$

where $\mathrm{A} = {\Gamma_0}^{-1}\Gamma_1$ and $\mathrm{B} = {\Gamma_0}^{-1}\Psi$. At this point, we can evaluate the existence and the uniqueness of solution of the model based on the number of stable eigenvalues of matrix A. If the number of stable Eigenvalues is equal to the number of pre-determined variables m, then the solution of this RE model is exist and unique. Under this condition, the Blanchard-Kahn condition for determinacy is satisfied. If the number of stable eigenvalues is higher than pre-determined variables, then the solution is still exist but not unique (multiple solution). This condition is defined as indeterminacy condition. In contrast, if the number of stable eigenvalues is lower than pre-determined variables, then there is non-existence of locally solution of the model. Under such condition, we says the system of equations go to explosive path or do not have an equilibrium path.

Yet, identifying the existence and the uniqueness of the solution of the model is not enough. We need to find a method of getting the solution of the model. Hence, we precede the work as follows. Assuming eigenvectors of matrix A are linearly independent, matrix A is decomposed into $\mathrm{A} = M\Lambda M^{-1}$ where Λ corresponds to a diagonal matrix with the eigenvalues of Λ along its diagonal, and M is a matrix of the corresponding eigenvector. For convenient, we re-order the composition of these eigenvalues between the ones that are lower than 1 and the ones that are higher than 1, in absolute value. Having this, matrix Λ transforms into a new matrix $\bar{\Lambda}$. Similarly, the element of matrix M also requires to be re-ordered to ensure each element in the eigenvector matches with its eigenvalues in $\bar{\Lambda}$. By this process, a new matrix $\bar{\mathrm{M}}$ is formed.

$$\begin{bmatrix} z_{t+1}^{s} \\ \mathrm{E_t} z_{t+1}^{j} \end{bmatrix} = \bar{M}\bar{\Lambda}\bar{M}^{-1} \begin{bmatrix} z_t^{s} \\ z_t^{j} \end{bmatrix} + \mathrm{B}\epsilon_{\mathrm{t+1}} \qquad \text{2B.3}$$

Pre-multiplying the above system of equations by $\bar{\mathrm{M}}^{-1}$, we get a new matrix system as:

$$\begin{bmatrix} \hat{z}_{t+1}^{s} \\ \mathrm{E_t} \hat{z}_{t+1}^{j} \end{bmatrix} = \begin{bmatrix} \hat{\Lambda}_{11} & 0 \\ 0 & \hat{\Lambda}_{22} \end{bmatrix} \begin{bmatrix} \hat{z}_t^{s} \\ \hat{z}_t^{j} \end{bmatrix} + \hat{\mathrm{B}}\epsilon_{\mathrm{t+1}} \qquad \text{2B.4}$$

where $\hat{\mathrm{B}} = \bar{\mathrm{M}}^{-1}\mathrm{B}$ and the variables with a hat are the multiplication of the variable $\hat{z}_t^{i}$ and $\hat{z}_{t+1}^{i}$ $i \in \{s, j\}$ with matrix $\bar{\mathrm{M}}^{-1}$, respectively.

The strategy of finding the solution for the above system of equations is started by solving the unstable equation, followed by stable equation. Since eigenvalues of $\hat{\Lambda}_{22}$ is higher than one, then the only stable solution for the system of equations $\mathrm{E_t}\hat{z}_{t+1}^{j} = \hat{\Lambda}_{22}\hat{z}_t^{j}$ is obtained when $\hat{z}_{t+1}^{j}$ is set at 0 for all t. Using this result, it can be shown that the original variable z_t^{j} can be written in terms of the pre-determined variables z_t^{s}. After the unstable equation is solved, we continue solving for the stable one. Since eigenvalues of $\hat{\Lambda}_{11}$ is lower than one, then there is no instability problem arises for system of equations $\mathrm{E_t}\hat{z}_{t+1}^{s} = \hat{\Lambda}_{11}\hat{z}_t^{s}$. Plug in the result for unstable equation into the stable equation, it can be shown that the solution for the future values of state variables $\mathrm{E_t} z_{t+1}^{s}$ can be written in terms of pre-determined variables z_t^{s} and the exogenous variables ϵ_{t+1}. As the final step, plug in back the last result into the result for unstable equations, we obtain the solution for future values of jump variables $\mathrm{E_t} z_{t+1}^{j}$ in terms of pre-determined variables z_t^{s} and exogenous variables ϵ_{t+1}. Notice that, at this stage, all variables including forward looking or jump variables can be written as the function of the pre-determined variables and exogenous shock variables as:

$$\begin{bmatrix} z_{t+1}^{s} \\ \mathrm{E}_t z_{t+1}^{j} \end{bmatrix} = \begin{bmatrix} \widetilde{\Lambda}_{11} & 0 \\ \widetilde{\Lambda}_{12} & 0 \end{bmatrix} \begin{bmatrix} z_t^{s} \\ z_t^{j} \end{bmatrix} + \begin{bmatrix} \widetilde{\mathrm{B}}_1 \\ \widetilde{\mathrm{B}}_2 \end{bmatrix} \epsilon_t \qquad 2B.5$$

So far, the methodology of solving the RE model has assumed matrix Γ_0 is invertible. Unfortunately, this is not always a case. When matrix Γ_0 is non-invertible, we need to adopt the Scur or QZ method to decompose matrices Γ_0 and Γ_1, following Klein (2000) and Sims (2002).

Consider the matrix system as Equation (2B.1). This matrix system can be re-written as:

$$\mathrm{QTZ}' \begin{bmatrix} z_{t+1}^{s} \\ E_t z_{t+1}^{j} \end{bmatrix} = \mathrm{Q}S\mathrm{Z}' \begin{bmatrix} z_t^{s} \\ z_t^{j} \end{bmatrix} + \Psi \epsilon_{t+1} \qquad 2B.6$$

where matrix Q and Z have a property as follows: $\mathrm{QQ}' = \mathrm{Q}'\mathrm{Q} = \mathrm{I} = \mathrm{ZZ}' = \mathrm{Z}'Z$.

T and S are the upper triangular matrices where s_{ii} and t_{ii} are the diagonal elements of matrices S and T, respectively. The eigenvalues of the system are given by $\lambda_{ii} = \frac{s_{ii}}{t_{ii}}$ .We remove Q by multiplying both side of the last equation with Q'. From here, the procedure will be similar to Blanchard and Kahn (1980) as explained previously.

Chapter 3

Learning About Monetary and Macroprudential Rules

3.1. Background and Motivation

In Chapter 2, we have studied determinacy and E-stability issue of Taylor-type interest rate rules in an otherwise version of New Keynesian (NK) model with the feature of habit formation in consumption. We have seen when consumption habit matters, it eases the task of central bank to lead the economy to unique stationary rational expectation equilibrium (REE)[20]. As the parameter of habit increases, the type of simple rules used by central bank becomes less important since it delivers almost similar region of determinate and E-stable policy rules within plausible policy parameters. Unfortunately, a standard NK model with one sector and one market of goods, i.e. consumption goods, as used in earlier chapter, is too simple. This causes the model is less realistic and unable to explain the movement of other economic variables as shown in the empirical evidence. Thus, we add a complexity in this chapter by introducing durable goods, i.e. housing, in addition to consumption goods, and the feature of financial friction in the model.

According to the existing literature, there are some reasons why researchers want to bring housing market and/or housing sector in a standard NK model: Firstly, housing investment is highly volatile and potentially can be used to explain the variability of aggregate output. Secondly, the co-movement property between housing investment and aggregate consumption is interested to study. Rather than moving into an opposite direction, empirical data shows

[20] With a slight abuse of notation, the terms "unique stationary rational expectation equilibrium (REE)" and "determinate equilibrium", are used interchangeably throughout this thesis. As well, the term "E-stability" and "learnability".

that housing investment tends to have a positive correlation with aggregate consumption. *Third*, a theory that explains the volatility of housing prices and a theory that explains both propagation and amplification of shocks are required to develop.

The recent financial crisis has also given a good lesson about the importance of understanding housing and the financial markets to prevent crisis in the future. It is believed that this crisis was born in a housing sector which grew in a financial sector and had an impact to a real sector. This situation gets more difficult to manage as the financial system becomes more complex and interconnected to each other due to a financial innovation, leading to the expansion of systemic risk, especially through the mortgage market. As the result, it is argued that a traditional monetary policy that mainly focuses on inflation and output is not sufficient to avoid similar crisis in the future.

The design of prevention and recovery policies has led to the discussion about two important issues: Firstly, the benefit of responding housing prices and/or financial variables (e.g. credit growth). Secondly, the most effective policy rule to dampen the volatility of housing prices and/or financial variables. In regards to the first issue, some researchers suggest that it is necessary for central bank to lean against surges in asset prices bubbles in order to avoid macroeconomic and financial instability[21]. While others believe that central bank should react to asset prices only to the extent that they contain information about future inflation and output growth. Greenspan (2002) explains that, since it is very difficult to identify a bubble before its existence is confirmed by the bursting, the Federal Reserve does not directly react to financial imbalances. Meanwhile, Bernanke & Gertler (1999) suggest that it is unnecessary for monetary policy to respond to changes in asset prices since rules that directly target asset

[21] See Cecchetti et al. (2000) and Borio & Lowe (2002).

prices might have undesirable side effects of stifling the beneficial impact of the technology boom.

Concerning the second issue, there is unsettled agreement among researchers whether bubbles in asset prices should be handled using monetary policy, via a modified Taylor-type interest rate rule, or an explicit macroprudential policy. For example, Kannan et al. (2012) combine the macroprudential instrument with an augmented interest rate rule that also reacts to nominal credit growth by incorporating a financial sector into the model of Iacoviello & Neri (2010). They find a strong monetary reaction to credit growth and houses prices increase macroeconomic stability. In contrast, Angelini et al. (2011) advise about the benefit of introducing macroprudential policy that differs from monetary policy especially when there is a financial or a housing market shock. In their model set up, they do not only set a clear distinction between macroprudential and monetary policies but also have assumed that there is a different institution assigned to it. Given this assumption, they evaluate the interaction between monetary and macroprudential policies both under and without coordination. The result shows that a cooperative central bank will lend a hand to the macroprudential authority, working for broader objectives that not just price stability in order to improve overall economic stability.

Nevertheless, most of the existing studies about macroprudential policy have assumed that agents have a rational expectation (RE). Under this framework, it is assumed that economic agents are rational and know what others know in precise manner. As well, they have knowledge about the correct form of the economic model including all the structural parameters. It is worth to note that a RE framework differs from a perfect foresight framework in the sense that the first one has assumed that agents do not know the future values of random exogenous shocks to the economy while the second one does. This means under the RE framework agents know the probability of distribution of future exogenous variables, conditional on exogenous variables, but they do not know precisely the specific values of future exogenous variables.

Although a RE is a useful benchmark, it turns out that it is too strong assumption. As mentioned in Chapter 2, in practice economic theory will set the frameworks to describe the structure of the economy while agents behave like an econometrician by using the recursive learning least square (RLS) algorithm to estimate the parameter values. This approach is known as RLS learning in the literature. Under RLS learning, agents update the parameter estimates towards the true value of model parameters as new data becomes available. Recall that the E-stability principle states that the REE is locally stable under RLS *iff* it is E-stable. If a policy rule leads to the REE that is not E-stable, then under RLS learning, economic agents can collaborate with a zero probability. Such policy rule should be avoided by policy makers, even if it offers a high welfare gains under the RE assumption.

In this chapter, we enrich the existing studies about determinacy and E-stability of REE in an environment where housing market is present and macroprudential policy interacts with monetary policy. We analyse the interaction between monetary and macroprudential policies that may lead the economy to unique stationary and learnable equilibrium[22]. From here, the analysis is extended by evaluating the optimal constrained policy rule by assuming some exogenous shocks hit the economy, i.e. housing demand shock and technological shock. Unlike similar studies about optimal policy, in our work, the decision for choosing optimal policy is not only be constrained by determinacy criterion but also E-stability criterion since economic agents do not possess a rational expectation (RE).

[22] The term “unique stationary and learnable equilibrium” can also be written as “a determinate and E-stable rational expectation equilibrium (REE)” where the word “E-stable” stands for expectational stability.

The model used is based on a simple version of Iacoviello (2005) where housing assets do not enter the production function. As well, no entrepreneurs and physical capital investment are introduced to the model. From the perspective of learning literature, our work closely relates to Wei (2012) who studies about learnability of REE in the framework of NK model with housing market but it is different from ours in two aspects: Firstly, Wei (2012) has assumed that policy rate reacts directly to the movement of housing prices or nominal credit, whilst in our work we explicitly model an operational macroprudential policy via the changing of the ratio of loan to value (LTV). Secondly, we assume that macroprudential policy rule reacts to the growth of housing prices or nominal credit while Wei (2012) has assumed that interest rate responds to the percentage deviation of housing prices and nominal credit from its steady state level.

Our motivation to consider the growth of housing prices or nominal credit comes from the fact that observing the growth of these variables is putatively easier and practical than identifying the percentage deviation of housing prices and credit from its steady state level. Several authors have argued in favour of targeting growth over the gap on the grounds that the gap is difficult to observe in real time e.g. McCallum (2001), Orphanides (2002), and Orphanides & Williams (2006). Recently, Sim (2013) have analysed the welfare consequences of responding the growth rate of output and the output gap in a standard DSGE NK model with both nominal and real friction. He concludes that a response to the growth rate of output, which is easier to observe than the gap, is often welfare improving, with the gains from growth-targeting larger under the same sets of circumstances under which gap-targeting is costly.

Meanwhile, a requirement to explicitly model a macroprudential policy is in line with some recent studies that focus on the stabilisation of asset prices and financial imbalances. Among others is Antipa et al. (2010) who argue that macroprudential policies would have been effective in smoothing the past credit cycle and in reducing the intensity of the recession. Another is Angelini et al. (2011) who found that macroprudential policies are most helpful to counter financial shocks that lead the credit and asset prices booms. There are also some studies that show ineffective use of policy rate in handling a bubble in asset prices. Kohn (2010) claims that an aggressive use of central bank policy rate to address the built-up of financial imbalances, or to deflate an incipient bubble in some asset categories, is view by some as too blunt a response, as other sectors of the economy will be adversely affected as a consequence. Gelain et al. (2013) also suggest that a direct response to either housing prices or credit growth in the central bank interest rate rule has a serious drawback of substantially magnifying the volatility of inflation. Under the same spirit as the above mentioned literature, we consider an operational macroprudential policy, based on the Taylor-type automatic rule. To be specific, we consider rules for the loan to value (LTV) ratio that react inversely to the growth of housing prices or nominal credit. These rules can be a simple illustration of how a macroprudential policy could work in practice.

This chapter is structured as follows. Section 3.2 describes the model. It explains the behaviour of households (patient and impatient households), firms (wholesalers and retailers), and central bank who conducts both monetary and macroprudential policies. Section 3.3 discusses the methodology and the calibration for parameters used in the model. Section 3.4 explains the result of our study about determinacy and E-stability for various combinations of monetary and macroprudential policy rules. Section 3.5 discusses the optimal constrained policy rule. The final section concludes this chapter.

3.2. The Model

We use a simple version of Iacoviello (2005) in our analysis in which asset market fluctuations can have real effects to the economy through a credit channel. Similar to Wei (2012), we assume that housing assets do not enter the production function, and we do not model any entrepreneurs and physical capital investment. Yet, the model is different by which policy instrument is used by central bank to respond to housing prices or nominal credit. In our model, it is assumed that there is an active macroprudential policy that works through the LTV rule as a way to moderate housing prices or credit growth. Meanwhile, Wei (2012) has used policy rate to dampen the variability in housing prices, while the LTV is set at a fix ratio and does not respond to economic conditions, following Iacoviello (2005).

The model features patient and impatient households, wholesaler and retailer firms, and central bank that conducts both monetary and macroprudential policies. Households supply their labour to wholesaler firms and get utility from consumption goods and housing service. Housing is the main vehicle for accumulating wealth in the economy. Patient and impatient households are savers and borrowers, respectively. Borrowers are credit constrained and need collateral to obtain loans. We assume that savers can lend to borrowers directly, but the amount is controlled by central bank through the LTV ratio. Wholesaler firms convert households labour into intermediate goods in which retailer firms convert it further into final goods. We remark central bank follows the Taylor-type rule for setting interest rate and the LTV rule.

3.2.1. Patient Households

A representative of patient households (savers) maximises her utility by choosing consumption, housing service, labour hours and loan to lend as follows:

$$Max\ E_0^* \sum_{t=0}^{\infty} \beta_s^t \left(\ln C_{s,t} + j_t \ln H_{s,t} - \frac{N_{s,t}^{\eta}}{\eta} \right) \qquad 3.1$$

where the symbol E_t^* represents the subjective expectation of savers conditional on information available at time t. Under the rational expectation, E_t^* corresponds to a mathematical operator E_t evaluated using the objective distributions of the stochastic shocks which are assumed known by the rational households. Note that this subjective expectation is assumed to be equal among savers, borrowers, firms and central bank which imply there is no heterogeneity in expectation in the economy. Parameter $\beta_s \in (0,1)$ governs the patient discount factor while $C_{s,t}$, $H_{s,t}$ and $N_{s,t}$ represent consumption, housing stock and working hours of savers at time t, respectively. Parameter $\eta > 0$ denotes the inverse of labour supply elasticity, while j_t is a time-varying weight of housing in the utility. We assume $\log(j_t) = \rho_j \log(j_{t-1}) + \varepsilon_{jt}$ where ρ_j is the autoregressive coefficient and ε_{jt} is a normally distributed shock to housing demand with standard deviation σ_j. We normalize the steady-state of housing preference j to 1.

Saver's utility function is constrained by her budget constraint as follows:

$$C_{s,t} + D_{s,t} + q_t\left(H_{s,t} - H_{s,t-1}\right) = \frac{R_{t-1}D_{s,t-1}}{\pi_t} + w_{s,t}N_{s,t} + F_t \qquad 3.2$$

where $D_{s,t}$ denotes bank deposits, R_t is the gross return from deposits, q_t is the real price of housing in units of consumption and $w_{s,t}$ is saver's real wage rate. F_t is real lump-sum profits received from the firms.

The first order conditions for this optimisation problem are:

$$\frac{1}{C_{s,t}} = \beta_s E_t^* \left(\frac{R_t}{\pi_{t+1} C_{s,t+1}} \right) \qquad 3.3$$

$$w_{s,t} = \left(N_{s,t}\right)^{\eta-1} C_{s,t} \qquad 3.4$$

$$\frac{j_t}{H_{s,t}} = \frac{q_t}{C_{s,t}} - \beta_s E_t^* \frac{q_{t+1}}{C_{s,t+1}} \qquad 3.5$$

where equation (3.3) is a conventional Euler equation, equation (3.4) denotes the labour supply condition and equation (3.5) represents the intertemporal condition for housing, in which, at the margin, benefit for consuming housing equate costs in terms of consumption.

Since we are interested in the equilibrium dynamic around the steady state, the above non linear optimality conditions are then log-linearised around the steady state values to obtain the following equations:

$$\tilde{c}_{s,t} = E_t^* \tilde{c}_{s,t+1} - (\tilde{r}_t - E_t^* \tilde{\pi}_{t+1}) \qquad 3.6$$

$$\tilde{w}_{s,t} = \tilde{c}_{s,t} + (\eta - 1)\tilde{n}_{s,t} \qquad 3.7$$

$$\tilde{q}_t = \tilde{c}_{s,t} + \beta_s E_t^* \left(\tilde{q}_{t+1} - \tilde{c}_{s,t+1}\right) - (1 - \beta_s)\left(\tilde{h}_{s,t} - \tilde{j}_t\right) \qquad 3.8$$

Note that a variable in log-linearised form is denoted by tilde.

3.2.2. Impatient Households

A representative of impatient households (borrowers) maximises the following utility function.

$$Max\ E_0^* \sum_{t=0}^{\infty} \beta_b^t \left(\ln C_{b,t} + j_t \ln H_{b,t} - \frac{N_{b,t}^{\eta}}{\eta} \right) \qquad 3.9$$

where $\beta_b \in (0,1)$ is impatient discount factor and $C_{b,t}$, $H_{b,t}$ and $N_{b,t}$ corresponds to consumption, housing stock and working hours of borrowers at time t, respectively. Borrower faces two constraints when maximising her utility, i.e. the budget constraint and the collateral constraint as shown below:

$$C_{b,t} + \frac{R_{t-1} D_{b,t-1}}{\pi_t} + q_t \left(H_{b,t} - H_{b,t-1} \right) = D_{b,t} + w_{b,t} N_{b,t} \qquad 3.10$$

$$E_t^* \frac{R_t}{\pi_{t+1}} D_{b,t} = m_t E_t^* q_{t+1} H_{b,t} \qquad 3.11$$

where $D_{b,t}$ denotes bank loans and m_t represents the loan to value (LTV) ratio. Notice that equation (3.11) is the channel via which asset market fluctuations can have real effects in the economy. This borrowing constraint pegs a loan that borrowers can obtain to the present discounted value of their housing holdings. In a standard model of Iacoviello (2005), the ratio of LTV is constant which is not affected by economic conditions, while in our model set up this ratio is assumed to be operationally adjusted to respond economic conditions, following Rubio et al. (2014) among others.

The optimality condition of impatient households is characterised by a decision to choose the optimal consumption, labour supply, loan to borrow and housing service as follows:

$$\frac{1}{C_{b,t}} = \beta_b E_t^* \left(\frac{R_t}{\pi_{t+1} C_{s,t+1}} \right) + \varphi_t R_t \qquad 3.12$$

$$w_{b,t} = \left(N_{b,t} \right)^{\eta - 1} C_{b,t} \qquad 3.13$$

$$\frac{j_t}{H_{b,t}} = \frac{1}{C_{b,t}} q_t - \beta_b E_t^* \frac{1}{C_{b,t+1}} q_{t+1} - \varphi_t m_t E_t^* (q_{t+1} \pi_{t+1}) \qquad 3.14$$

where φ_t denotes the multiplier on the borrowing constraint. These first order conditions can be interpreted analogously to those of savers previously.

The above optimality conditions are then log-linearised around their steady state values to get the following equations:

$$\tilde{c}_{b,t} + \tilde{r}_t + \tilde{\varphi}_t = \beta_b R\left(E_t^* \tilde{c}_{b,t+1} + E_t^* \tilde{\pi}_{t+1} + \tilde{\varphi}_t\right) \quad 3.15$$

$$\widetilde{w}_{b,t} = \tilde{c}_{b,t} + (\eta - 1)\tilde{n}_{b,t} \quad 3.16$$

$$\frac{j}{H_b}\left(\tilde{h}_{b,t} - \tilde{j}_t\right) = \frac{\beta_b q}{C_b} E_t^*\left(\tilde{q}_{t+1} - \tilde{c}_{b,t+1}\right) - \frac{q}{C_b}\left(\tilde{q}_t - \tilde{c}_{b,t}\right) + \varphi m q \pi E_t^*(\tilde{\pi}_{t+1} + \tilde{q}_{t+1} + \widetilde{m}_t + \tilde{\varphi}_t)$$

3.17

Combining equation (3.15) and (3.17) yields:

$$\tilde{q}_t - (1 - m\beta_s)\tilde{c}_{b,t} = \gamma_c E_t^* \tilde{q}_{t+1} - \beta_b (1 - m) E_t^* \tilde{c}_{b,t+1} - m\beta_s E_t^*(\tilde{r}_t - \tilde{\pi}_{t+1}) - (1 - \gamma_c)\left(\tilde{h}_{b,t} - \tilde{j}_t\right) + (\gamma_c - \beta_b)\widetilde{m}_t \quad 3.18$$

where $\gamma_c = 1 - \frac{C_b}{q h_b}$ comes from the steady state relation.

In addition to the above optimality conditions, the equilibrium dynamics is also characterised by borrower's budget constraint and financial constraint where after log-linearising them around their steady state values can be written as:

$$\frac{C_b}{Y}\tilde{c}_{b,t} + \frac{qH_b}{Y}\left(\tilde{h}_{b,t} - \tilde{h}_{b,t-1}\right) + \frac{D_b}{\beta_s Y}\left(\tilde{r}_{t-1} + \tilde{d}_{b,t-1} - \tilde{\pi}_t\right) = \frac{D_b}{Y}\tilde{d}_{b,t} + \frac{1-\alpha}{X}(\tilde{y}_t - \tilde{x}_t) \quad 3.19$$

$$\tilde{r}_t + \tilde{d}_{b,t} = \widetilde{m}_t + \tilde{q}_{t+1} + \tilde{h}_{b,t} + E_t^* \tilde{\pi}_{t+1} \quad 3.20$$

3.2.3. Firms

The wholesaler firms operate under perfect competition and flexible price to produce a homogenous intermediate good Y_t. The intermediate goods are produced according to the following Cobb-Douglas production function:

$$Y_t = A_t {N_{s,t}}^{\alpha} {N_{b,t}}^{1-\alpha} \qquad 3.21$$

where $\alpha \in [0,1]$ measures the relative size of different types of households in terms of labour, while $N_{s,t}$ and $N_{b,t}$ correspond to labour of patient households (savers) and impatient households (borrowers), respectively. Given the above specification of production function, it is assumed that saver's labour and borrower's labour are imperfect substitutes. Meanwhile, A_t governs an exogenous technology variable that follows an autoregressive process $\log(A_t) = \rho_a \log(A_{t-1}) + \varepsilon_{a,t}$ where ρ_a is the autoregressive coefficient and $\varepsilon_{a,t}$ is a normally distributed shock to the technology with a standard deviation σ_a. We normalize the steady-state value of technology A to 1.

From optimality condition of wholesaler firms, labour demand is determined by the following equations:

$$w_{s,t} = \alpha \frac{1}{X_t}\left(\frac{Y_t}{N_{s,t}}\right) \qquad 3.22$$

$$w_{b,t} = (1-\alpha) \frac{1}{X_t}\left(\frac{Y_t}{N_{b,t}}\right) \qquad 3.23$$

Log-linearised the above production function and optimality conditions yields:

$$\tilde{y}_t = \tilde{a}_t + \alpha \tilde{n}_{s,t} + (1-\alpha)\tilde{n}_{b,t} \qquad 3.24$$

$$\tilde{y}_t = \frac{1}{\alpha}\left(\tilde{x}_t + \widetilde{w}_{s,t} + \tilde{n}_{s,t}\right) \qquad 3.25$$

$$\tilde{y}_t = \frac{1}{1-\alpha}\left(\tilde{x}_t + \widetilde{w}_{b,t} + \tilde{n}_{b,t}\right) \quad 3.26$$

A continuum of retailers indexed by z then buys intermediate goods produced by wholesaler firms at the wholesale price P_t^w in a competitive market. They transform it into differentiated goods $Y_t(z)$ and sell it at $P_t(z)$. In this way, the mark-up on price X_t arises in which the value is determined by $X_t = P_t/P_t^w$. It is also assumed that retailers have a monopolistic power and, when allowed to, will set their prices to maximise their profit, subject to demand for each differentiated product $Y_t(z)$, following Bernanke et al. (1999). The final goods Y_t^f consist of a collection of differentiated goods $Y_t(z)$ which is bundled by final goods bundlers using the following technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad 3.27$$

where $\varepsilon > 1$ is the elasticity of substitution between individual final goods.

The final good bundler chooses $Y_t(z)$ to minimise their input costs, resulting in demand for each retailer.

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f \quad 3.28$$

The price index is then given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{1}{1-\varepsilon}} \quad 3.29$$

The price-setting problem for the retailer is a standard Calvo-setting. Each retailer sells its good at price $P_t(z)$ and $(1-\theta) \in [0,1]$ is the probability of being able to change the sale price in every period.

The optimal re-set price $P_t^{New}(z)$ solves:

$$E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}} - \frac{X}{X_{t+k}} \right) Y_{t+k}(z) = 0 \qquad 3.30$$

where $\Lambda_{\mathrm{t,k}} = \frac{C_{s,t}}{C_{s,t+k}} \frac{P_t}{P_{t+k}}$; $X = \varepsilon/(\varepsilon - 1)$ and $1/X_t = mc_t$ (mark-up is the inverse of marginal cost). Given the fraction θ of retailers do not change their price in period t, the aggregate price evolves according to:

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^{New})^{1-\varepsilon}]^{1/(1-\varepsilon)} \qquad 3.31$$

The last two conditions, i.e. equation (3.30) and (3.31) can be combined to generate a standard New Keynesian Philip curve (in log-linearised version) as:

$$\tilde{\pi}_t = \beta_s E_t^* \tilde{\pi}_{t+1} - \lambda \tilde{x}_t \qquad 3.32$$

where $\lambda = \frac{(1-\theta)(1-\theta\beta_s)}{\theta}$ and $\tilde{x}_t = -\widetilde{mc}_t$

3.2.4. Central Bank

We assume that central bank conducts both monetary and macroprudential policies. For monetary policy, they use a conventional Taylor-type interest rate rule that responds to inflation and output. However, when they are operated, central bank is constrained by data availability. We consider two alternatives in conducting monetary policy: Firstly, a case in which current data of inflation and output are available. Secondly, a case in which current data of inflation and output are not available and required to be estimated or forecasted. To make it clear, we consider some variations of Taylor-type interest rate rule as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \qquad 3.33$$

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \quad 3.34$$

In this case, τ_π and τ_y are monetary reaction function to inflation and output, respectively. All variables are log-linearised around the steady state values and denoted by tilde.

In addition to monetary policy, central bank conducts macroprudential policy via the LTV rule that reacts inversely to economic condition. The LTV rule is used to dampen the increase of housing prices or credit growth. However, we are unclear on the benefit of responding these two economic variables: should central bank respond to housing prices growth or credit growth in the first place, when there is a bubble in the financial markets driven by housing market. Similarly to monetary policy, when conducting macroprudential policy, central bank is constrained by the availability of housing prices or credit data. Therefore, some alternatives of the LTV rules considered are as follows:

$$\tilde{m}_t = -\tau_q \Delta \tilde{q}_t \quad 3.35$$

$$\tilde{m}_t = -\tau_q E_t^* \Delta \tilde{q}_{t+1} \quad 3.36$$

$$\tilde{m}_t = -\tau_d \Delta \tilde{d}_{b,t} \quad 3.37$$

$$\tilde{m}_t = -\tau_d E_t^* \Delta \tilde{d}_{b,t+1} \quad 3.38$$

where τ_q and τ_d are the LTV reaction function to the growth of housing prices and credit, respectively. Again, all variables are presented in a log-linearised form around their steady state values (denoted by tilde).

In a standard model, the LTV ratio is a fixed parameter which is not affected by economic conditions. However, we can think of regulation of LTV ratio as a way to moderate housing price bubbles and credit booms. When the LTV ratio is high, the collateral constraint is less tight. And since the constraint is binding, borrowers will borrow as much as they are allowed to. Lowering the LTV ratio tightens the constraint and therefore reduces the loans that borrowers can obtain. Recent studies in macroprudential policy has proposed the Taylor-type rules for the LTV ratio so that it reacts inversely to variables such as the growth rates of GDP, credit, the credit to GDP ratio or house prices. Here, we assume that there exists a macroprudential Taylor-type rule for LTV ratio that reacts to the growth of housing prices or credit.

3.2.5. Equilibrium and Market Clearing

The equilibrium of the model is a sequence of prices $\{q_t, R_t, P_t, X_t, w_{s,t}, w_{b,t}\}$ and an allocation $\{Y_t, C_{s,t}, C_{b,t}, D_{s,t}, D_{b,t}, H_{s,t}, H_{b,t}\}$ such that all first order conditions and constraints hold, and all markets are clear.

The aggregate profit from retailers is:

$$F_t = \left(\frac{P_t - P_t^W}{P_t}\right) Y_t \qquad 3.39$$

where it will be rebated to savers since it is assumed that retailers are owned by them.

Meanwhile, the market of consumption goods, housing and bonds are clear when[23]:

$$C_{s,t} + C_{b,t} = Y_t \qquad 3.40$$

$$H_{s,t} + H_{b,t} = H = 1 \qquad 3.41$$

$$D_{s,t} + D_{b,t} = 0 \qquad 3.42$$

[23] As in Iacoviello (2005), total output can be approximated by $Y_t^f = \int_0^1 Y_t(z)\, dz \approx Y_t$

Log-linearising equations (3.40) to (3.42) around the steady state values yields:

$$\frac{C_s}{Y}\tilde{c}_{s,t} + \frac{C_b}{Y}\tilde{c}_{b,t} = \tilde{y}_t \quad 3.43$$

$$\tilde{h}_{s,t} = -\frac{H_b}{H_s}\tilde{h}_{b,t} \quad 3.44$$

$$\tilde{d}_{s,t} = -\tilde{d}_{b,t} \quad 3.45$$

3.3. Methodology and Calibration

3.3.1. Methodology

The methodology for learning analysis follows the standard approach of Evans & Honkapohja (2001)[24]. We take log-linear approximation of the equilibrium conditions around the steady state, and reduce the economic model into a system of linear dynamic equations. The linearised system consists of eight expectational dynamic equations (see Appendix 3.2 for details derivation).

We write the system as:

$$S_t = B_0 + B_1 S_{t-1} + B_2 E_t^* S_{t+1} + B_3 z_t \quad 3.46$$

where $S_t = \left(\tilde{y}_t, \tilde{\pi}_t, \tilde{x}_t, \tilde{q}_t, \tilde{d}_{b,t}, \tilde{h}_{b,t}, \tilde{r}_t\right)'$ is a vector of endogenous state variables and $z_t = (\tilde{j}_t, \tilde{a}_t)$ is a vector of exogenous state variables that follows AR (1) process, i.e. $z_t = \mathcal{F} z_{t-1} + e_t$. For simplicity, we assume that $\mathcal{F}$ is known where all its roots lie inside the unit circle. We also remark e_t as an exogenous white noise shock with the variance Σ_e.

[24] We use the same methodology in Chapter 2. Yet, to make it clear to readers, we explain it once again here.

For the analysis of the recursive least square (RLS) learning, we assume that economic agents have the perceived law of motion (PLM) as:

$$E_t^* S_t = a_t + b_t S_{t-1} + c_t z_t \qquad 3.47$$

The above PLM is consistent with the minimum state variable (MSV) solution of the system. As Bullard & Mitra (2002), we focus on this class of PLMs when examining the E-stability properties of the economy.

Given the PLM, agents form their forecasts as:

$$E_t^* S_{t+1} = a_t + b_t S_t + c_t \mathcal{F} z_t \qquad 3.48$$

Any given PLM gives an actual law of motion (ALM) that describes the temporary equilibrium value of S_t. This is obtained by substituting $E_t^* S_{t+1}$ into equation 3.46.

For PLM estimates a_t, b_t and c_t, we obtain ALM as:

$$S_t = B_o + B_2(I + b) + (B_2 b^2 + B_1) S_{t-1} + (B_2(bc + c\mathcal{F}) + B_3) z_t \qquad 3.49$$

We need to slightly modify the representation of our system of equations. We introduce the notation $w_t' = (1, S_{t-1}', z_t')$ for all state variables including the exogenous state variables and $\varphi_t' = (a_t, b_t, c_t)$ for the parameters.

We can re-write the previous PLM at time t as:

$$S_t = \varphi_t' w_t \qquad 3.50$$

and the ALM at time t as:

$$S_t = T(\varphi_t)' w_t \qquad 3.51$$

where,

$$T(\varphi_t)' = \{B_o + B_2(I + b), (B_2 b^2 + B_1), (B_2[bc + c\mathcal{F}] + B_3)\} \qquad 3.52$$

The MSV solution is given by the fixed point of *T*, that is, $\bar{\varphi}' = (\bar{a}, \bar{b}, \bar{c})$ where $\bar{a} = 0$; $\bar{b} = (I - B_2\bar{b})^{-1}B_1$ and $\bar{c} = (I - B_2\bar{b})^{-1}(B_3 + \mathcal{F}B_2\bar{c})$, by the assumption $(I - B_2\bar{b})$ is invertible.

Under this current setting, the RLS learning algorithm for estimating and updating a_t, b_t, c_t is given by:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(S_{t-1} - \varphi'_{t-1} w_{t-1})' \qquad 3.53$$

with $\gamma > 0$ is a small gain parameter and $\mathcal{R}_t$ is symmetric and positive definite matrix of second moments of the state variables.

Substituting in the ALM, we can write:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(T(\varphi_{t-1})' w_{t-1} - \varphi'_{t-1} w_{t-1})'$$

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(w'_{t-1}[T(\varphi_{t-1}) - \varphi_{t-1}])$$

where,

$$\mathcal{R}_t = \mathcal{R}_{t-1} + \gamma[w_{t-1} w'_{t-1} - \mathcal{R}_{t-1}] \qquad 3.54$$

Marcet & Sargent (1989a) show that the associated ordinary differential equation (ODE) is the vectorised version of the following ODE:

$$\frac{d\varphi}{d\tau} = \mathcal{R}^{-1} M_w [T(\varphi) - \varphi]$$

$$\frac{dR}{d\tau} = M_w - \mathcal{R}$$

where $M_w = diag(1, M_s, M_z)$ is a symmetric and positive definite matrix of second moments of the state variables used by agents in forming their forecasts.

Latter, they can show that the local stability of REE is entirely determined by the local stability, at the same point, of the following small ODE:

$$\frac{dvec\varphi}{d\tau} = vec(T(\varphi) - \varphi)$$

Let $L(\varphi) = dvec\big(T(\varphi)\big)/dvec\varphi$. The relevant Jacobian is:

$$J^{LS}(\varphi) = \frac{dvec(T(\varphi)-\varphi)}{dvec\varphi} = L(\varphi) - I \qquad 3.55$$

The local asymptotic stability of REE $\bar{\varphi}$ under least squares learning is determined by the stability of the matrix $J^{LS}(\bar{\varphi})$. The least squares algorithm converges locally to the REE *iff* the real parts of the eigenvalues of $J^{LS}(\bar{\varphi})$ are strictly negative.

For determinacy analysis, the system is slightly modified and re-written as:

$$E_t G_{t+1} = \Omega G_t + \Gamma z_t \qquad 3.56$$

where G_t represents a vector of endogenous state variables of the model including its lag and z_t governs a vector of exogenous state variables. The REE is determinate if the number of stable Eigenvalues of Ω is equal to the number of predetermined variables of the system. If the number of stable Eigenvalues is higher than predetermined variables, then we have multiple equilibria or indeterminate equilibria. In contrast, if the number of stable Eigenvalues is lower than predetermined variables, then there is non-existence of locally unique stationary equilibrium. Note that the existence of locally unique stationary equilibrium is equivalent to the existence of unique solution to the model, for instance in terms of minimum state variable (MSV).

After describing the concept of determinacy and E-stability, we convey a method of computing co-movement matrix, i.e. the variance-covariance matrix for evaluating the optimal constrained policy rules[25]. The standard solution of a RE usually takes the form of a VAR (1), obey by a subset of variables $\mathcal{W}_t$, often called the state variables (consist of endogenous and exogenous state variables), with the other variables of interest $\widehat{\mathcal{W}}_t$ is given by specified linear function of state variables.

Consider a generic solution of a RE that takes the form as follow,

$$\mathcal{W}_t = \Theta\mathcal{W}_{t-1} + \Upsilon e_t \qquad 3.57$$

$$\widehat{\mathcal{W}}_t = L\mathcal{W}_t \qquad 3.58$$

where $\mathcal{W}_t$ is vector of state variables and Θ, Υ and L are the relevant deep parameters. The component of e_t is a white noise shock with mean zero and the variance Σ_e. Since, we are interested in the co-movement of $\mathcal{W}_t$, then we can compute its variance-covariance matrix as:

$$Var(\mathcal{W}_t) = E\mathcal{W}_t\mathcal{W}_t' \qquad 3.\ 59$$

By using stationary assumption, we have:

$$Var(\mathcal{W}_t) = \Theta\, Var(\mathcal{W}_t)\Theta' + \Upsilon\Sigma_e\Upsilon' \qquad 3.60$$

where after vectorising, we obtain:

$$Vec\big(Var(\mathcal{W}_t)\big) = (I - \Theta \otimes \Theta)^{-1} Vec(\Upsilon\Sigma_e\Upsilon') \qquad 3.\ 61$$

This allows us to evaluate the second moment of each state variable. We can also compute the variance-covariance of $\widehat{\mathcal{W}}_t$ using:

$$Var\big(\widehat{\mathcal{W}}_t\big) = L\, Var(\mathcal{W}_t)L' \qquad 3.62$$

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[25] The same concept is used in Chapter 2, we explain again for clarity.

3.3.2. Calibration

In the model reduced forms, there are at least seven systems of equations that make the analysis of E-stability and determinacy issue of policy rules is not easy to be conducted analytically. As an alternative, we follow a numerical approach. We calibrate structural parameters of the model to conform to standard values used in the literature, in particular Iacoviello (2005), or with empirically estimated values in recent research. The discount factor β is set at 0.99 for patient households (savers) and at 0.98 for impatient households (borrowers). The elasticity of substitution across final goods ε is set at 4, a value which is commonly used in the literature. The inverse of the elasticity of labour supply η is set to 1.01, which makes the labour supply curve virtually flat. The fraction of the firm that keeps their price unchanged θ is given a value of 0.75 which corresponds to the average price duration of about one year (or 4 quarters). The wage income of saver's labour α is set at 0.5, which implies savers and borrowers are at the same age or have the same experience. At the steady state, the borrower's down payment rate is set at 0.1 which implies the steady-state LTV ratio m is of 0.9.

To check the robustness of our numerical result, we consider an alternative calibration for some parameters. We experience with a lower LTV ratio at 0.8 in steady state and a lower value of borrower's discount factor at 0.96 (this implies borrowers becomes more impatient). Although these alternative values change the required policy parameters that lead to a determinate and E-stable REE, the differences are not significant. Given this, our baseline results continue to hold. Therefore, for the rest of this chapter, we only report our numerical results generated under our baseline calibration.

Table 3.1
Calibration used in numerical analysis
Quarterly frequency

Parameter	Description	Value
β_s	Discount factor for patient households (savers)	0.99
β_b	Discount factor for impatient households (borrowers)	0.98
ε	The elasticity of substitution across final goods	4
η	The inverse of elasticity labour supply	1.01
θ	Degree of price stickiness	0.75
α	Saver's wage income	0.5
m	Steady state LTV ratio	0.9
τ_y	Monetary policy reaction function to output	4
τ_π	Monetary policy reaction function to inflation	5
τ_q	LTV rule reaction function to housing prices growth	1.2
τ_b	LTV rule reaction function to credit growth	1.2
ρ_j	AR (1) coefficient of a housing demand shock	0.9
ρ_a	AR (1) coefficient of a technology shock	0.8
σ_j	Std. Deviation of a housing demand shock	0.05
σ_a	Std. Deviation of a technology shock	0.02

For the purpose of computing the optimal constrained policy rules, we set the persistence of a housing demand ρ_j and a technology ρ_a at 0.9 and 0.8, respectively. In refer to the existing literature, the range for the standard deviation of a housing demand shock and a technology shock is sufficiently large. This happens as it depends on the way of how we de-trend the data, model the structure of economic model and the kind of data are used in the estimation. A common finding is that the standard deviation of housing demand shock is usually higher than technology shock. For example, Iacoviello & Neri (2010) estimate the standard deviation of a housing demand shock at 0.046 and a housing technology shock at 0.019. Meanwhile, Kannan et al. (2012) calibrate this value at 2.5% for the standard deviation of a housing demand shock and at 1.5% for the standard deviation of a TFP (technology) shock. In our numerical simulation, we follow Iacoviello & Neri (2010) by calibrating the standard deviation of a housing demand shock σ_j at 0.05 and a technological shock σ_a at 0.02. We summary all these calibrated parameters in Table 3.1.

3.4. Learning about Monetary and Macroprudential Policy Rules

In each scenario, we begin the analysis by assuming macroprudential policy responds to the growth of housing prices, initially, and to the growth of credit, afterwards. Macroprudential policy operates through the LTV rule that reacts inversely to the growth of housing prices or to the growth of credit. Due to data availability, there are two options available in conducting this LTV rule: Firstly, a case in which current data of housing prices and nominal credit are accurately available. Secondly, a case where current data of these variables are not available and need to be estimated or forecasted.

Meanwhile, monetary policy works in a conventional fashion by responding to inflation and output, but without policy inertia. As a case of the LTV rule, it is assumed that central bank is constrained by the availability of data when conducting monetary policy. As a result, there are two possible alternatives to conduct monetary policy: Firstly, a case in which current data of inflation and output are accurately available. Secondly, a case where current data of these variables are not available and need to be forecasted. In total, there are eight possible combinations between monetary and macroprudential policy rules. We discuss each of these policy combinations in the following sub-section.

3.4.1. Current Data in Interest Rate and LTV Rules

In this scenario, it is assumed that all current data, i.e. inflation, output and housing prices are accurately available to central bank. When this is a case, a combination of policy rules can be set as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \text{ and } \widetilde{m}_t = -\tau_q \Delta \tilde{q}_t \qquad 3.63$$

where central bank adopts a conventional interest rate rule that responds to current inflation and output and the LTV rule that reacts inversely to current growth of housing prices.

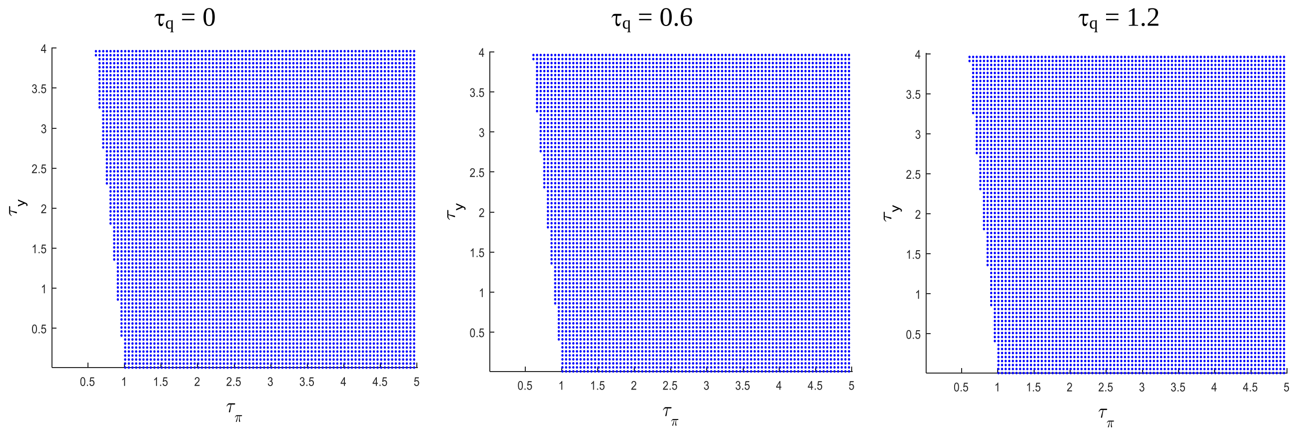
Before we proceed with the simulation result, we briefly explain how our numerical approach works. In all simulation exercises, we begin by setting the LTV reaction function parameter τ_q (when responds to housing prices) or τ_d (when responds to credit latter) at zero and varying monetary policy reaction function τ_π and τ_y. When τ_q or τ_d is set at zero, it is assumed that only monetary policy actively responds to economic conditions while macroprudential policy is assumed to be unresponsive. We set the upper bound of τ_π and τ_y at 5 and 4 respectively, while their lower bound is constrained at zero. These parameter ranges are commonly used in both theoretical and empirical works. We use an increment step size of 0.05 and search over a find grid of values between those ranges. For each possible pair of monetary policy parameter (τ_π , τ_y) in this grid, we evaluate whether their Eigenvalues satisfy the condition for determinacy and E-stability. If both conditions are satisfied, we indicate this in the figures using a blue/dark colour.

We experiment with different values of macroprudential policy parameter τ_q and τ_d within the range of 0 and 1.2. We follow this approach as we want to analyse the benefit of giving a response to the growth of housing prices or credit via the LTV rule, conditional on monetary policy rules. Our decision to fix the upper bound of τ_q and τ_d at 1.2 is realistic. As an illustration, setting τ_q and τ_d at 1 implies a one percent increase in the growth of housing prices or the growth of credit lead to a one percent decrease in the ratio of LTV from its steady state value. In other words, a 10 percent increase in the growth of housing prices or the growth of credit leads to a 10 percent decrease in the ratio of LTV from its steady state value.

Wei (2012) used a similar upper bound value of a reaction function to housing prices or credit growth, i.e. $\tau_q = 1$, though with a different interpretation. In his model, it is assumed that central bank responds to the percentage deviation of housing prices from its steady state level by adjusting their interest rate policy. This implies one percent (positive) deviation of housing prices from its steady state value leads to one percent (positive) deviation of nominal interest rate from its steady state value. However, given a large variation in housing prices, one for one response in the interest rate seems excessive and unrealistic. Further to this, a decision to use a policy rate in dampening a boom in housing prices may give undesirable impact on the other economic variables as it can contradict to the objective of exchange rate stabilisation, price stabilisation, investment attractiveness, etc. Therefore, introducing macroprudential policy via the LTV rule and allowing it to respond to economic condition makes the model set up more realistic.

Having considered this, we discuss the simulation result in detail. When all current data are accurately available, it turns out that, given the combination of τ_y and τ_π, varying the value τ_q within the range 0 and 1.2 does not give any significant changes in the region of determinate and E-stable policy rules. As illustrated in Figure 3.1, after simulating for three different values of τ_q, i.e. 0, 0.6 and 1.2, we come to the conclusion that a response to current growth of housing prices via the LTV rule is neither stabilising nor less stabilising if central bank has responded to current data of inflation and output (through interest rate rule) and remains consistent with the Taylor Principle.

Figure 3.1
Stability results of REE under current data in interest rate rule and current growth of housing prices in the LTV Rule



Blue/Dark : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

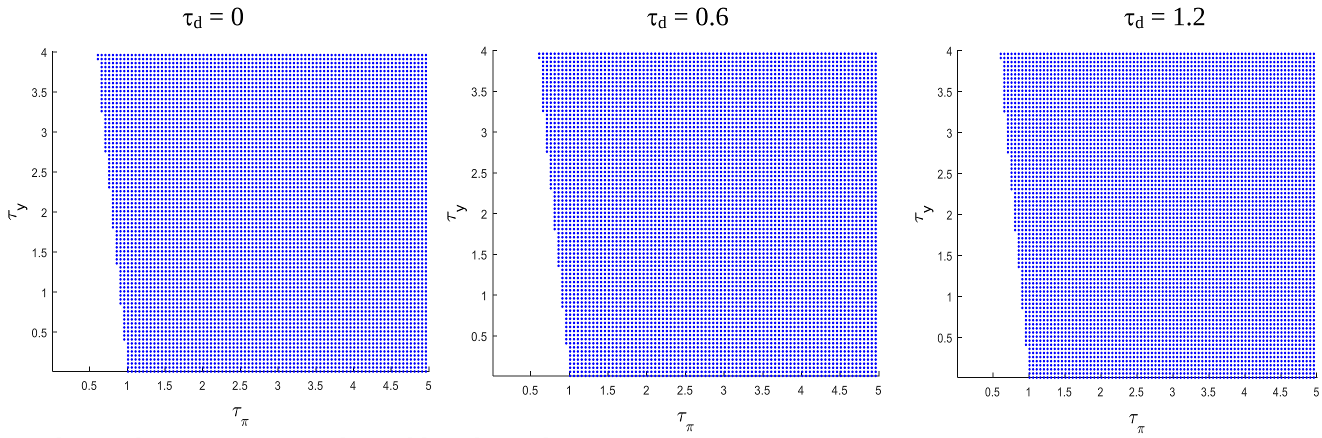
The above finding is similar to Wei (2012), who concludes a response to housing prices is redundant if central bank has access to current data of inflation and output, whilst the Taylor principle is satisfied. The difference is that Wei (2012) has used monetary policy for stabilising housing prices volatility through the adjustment of interest rate, while in our work we use macroprudential policy through the changing of LTV ratio. In addition, Wei (2012) assumes that central bank responds to the percentage deviation of housing prices from its steady state level, while in our work it is assumed that the LTV rule responds to the growth of housing prices. In practice, we argue that identifying housing prices growth is much easier than observing the percentage deviation of housing prices from its steady state value.

If a response to current growth of housing prices via the LTV rule is not beneficial for central bank in a case where they have responded to current data of inflation and output through policy rate, what will happen if it is assumed that this LTV rule responds to current growth of credit. Will similar result be attained? We consider an alternative LTV rule that interacts with monetary policy rule as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \text{ and } \tilde{m}_t = -\tau_d \Delta \tilde{d}_{b,t} \quad 3.64$$

It is assumed that current growth of credit is available along with current data of inflation and output. Although, having access to current data of inflation and output is a strong assumption, having access to current data of credit and housing prices seems realistic. It is common nowadays that firms or relevant institutions routinely collect the data of credit and housing prices. In fact, many central banks has a role in banking supervision, in addition to monetary authority, such that having access to the recent financial data e.g. credit seems plausible. In empirical work, the data of property loan is also commonly used by researchers as a leading indicator for housing prices development.

Figure 3.2
Stability results of REE under current data in interest rate rule
and current growth of credit in the LTV Rule



Blue/Dark : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

As illustrated in Figure 3.2, varying the value of macroprudential parameter τ_d, within the interval 0 to 1.2, does not alter the region of determinacy and E-stability. This finding is exactly the same as a case where the LTV rule responds to the growth of housing prices.

Given this result, we can reinstate our previous argument that a response to the growth of housing prices or the growth of nominal credit via the LTV rule is neither stabilising nor less stabilising when central bank has used current data of inflation and output in interest rate rule and sticks to the Taylor principle.

3.4.2. Current Data in Interest Rate Rule and Forecasted Data in LTV Rule

In practise, having access to all current data, is a strong assumption. Therefore, a previous result is established under unrealistic assumption and needs to be relaxed. We precede the analysis by considering an interaction between monetary and macroprudential policies where central bank adopts a combination of policy rules as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \text{ and } \tilde{m}_t = -\tau_q E_t^* \Delta \tilde{q}_{t+1} \qquad 3.65$$

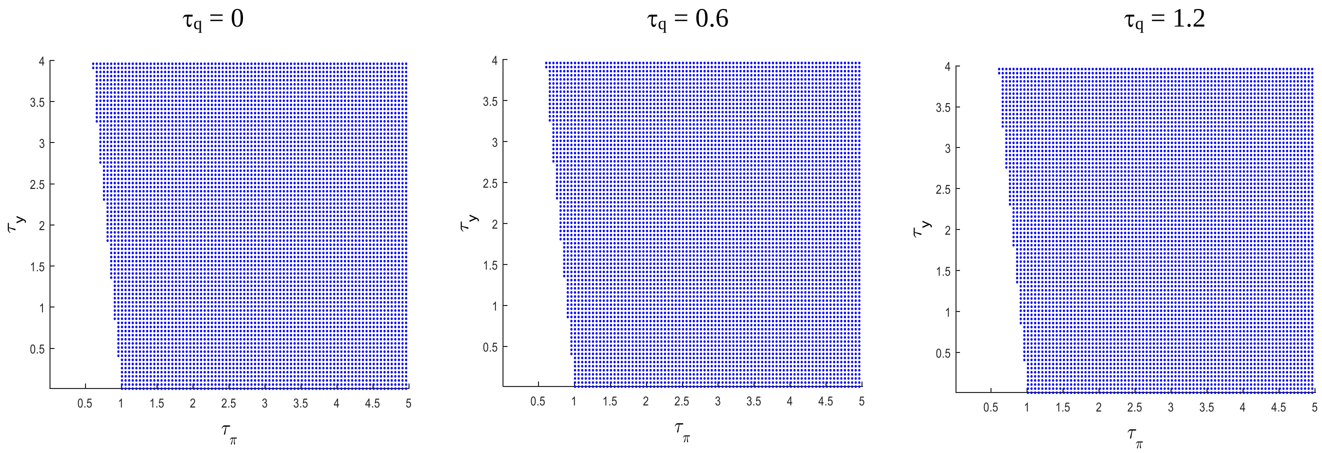
Under the above setting, central bank adopts a conventional Taylor-type interest rate rule that responds to current data of inflation and output, but the LTV rule reacts inversely to forecasted growth of housing prices[26].

The result shows that the region of determinacy and E-stability, within plausible policy parameters, is unaffected when the LTV rule responds to forecasted growth of housing prices (illustrated in Figure 3.3). As before, we plot this result for three different values of τ_q, i.e. 0, 0.6 and 1.2 and come to a conclusion that a response to forecasted growth of housing prices is unnecessary, especially when central bank have used current data of inflation and output in its monetary policy. Combining this finding with the earlier one (where all current data are available) we argue that, among these three data, i.e. inflation, output and housing prices, the

[26] In our work, forecasted data is equivalent to forward looking or future expectation data.

accuracy of inflation and output data turns out to be the most important one. This implies in a situation where monetary and macroprudential policies are conducted by two different institutions, for instance, the quality of data owned by monetary authority is more important relative to the quality of data owned by macroprudential authority.

Figure 3.3
Stability results of REE under current data in interest rate rule
and forecasted growth of housing prices in the LTV Rule



Blue/Dark : Determinate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

Will a similar result be attained when central bank reacts to forecasted growth of credit instead of forecasted growth of housing prices? We consider a combination of policy rules in which a conventional Taylor-type interest rate rule responds to current data of inflation and output while the LTV rule responds to forecasted growth of credit. Explicitly, a combination of rules considered is as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \text{ and } \tilde{m}_t = -\tau_d E_t^* \Delta \tilde{d}_{b,t+1} \qquad 3.66$$

Surprisingly, a response to forecasted growth of credit makes the system easily faces indeterminacy and instability problem under RLS learning. As an illustration, within the range 0 to 0.05 for τ_d, a response to credit growth via the LTV rule is neither stabilising nor destabilising so that determinacy and E-stability region is very similar to the case no response is given to credit growth. However, once this macroprudential parameter reaches 0.06 or more, it causes the equilibrium path easily goes to indeterminate and E-unstable REE. Based on this finding, we argue that the LTV rule that reacts to the growth of credit is less useful in general compared to the growth of housing prices. We will confirm this last argument by evaluating another type of the LTV rule below.

3.4.3. Forecasted Data in Interest Rate Rule and Current Data in LTV Rule

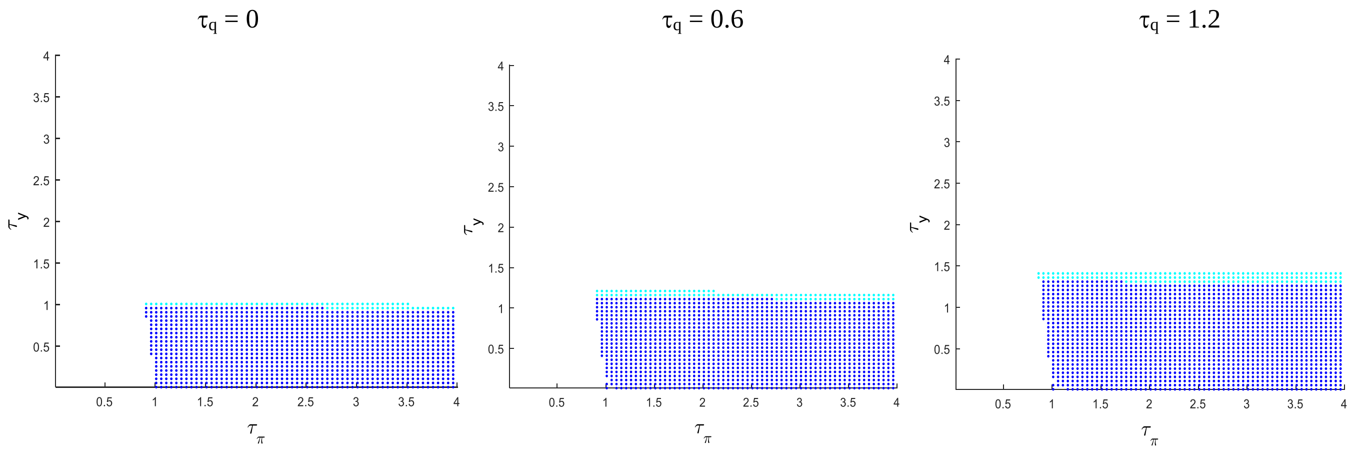
In practice, current data such as inflation and output have a delay until they are officially published by statistic agency or relevant institutions. The same limitation, however, needs not to apply for the data of housing prices and credit as they are more frequently updated, especially after the recent US financial crisis. Having access to current data for housing prices and credit may give central bank potential information gains. We investigate if such gain exists. To do so, we consider another combination of monetary and macroprudential rules as follows:

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \text{ and } \widetilde{m}_t = -\tau_q \Delta \tilde{q}_t \qquad 3.67$$

where the underlined assumption is that central bank cannot observe current value of inflation and output, but does observe current growth of housing prices. Under this condition, central bank must use lagged values of output and inflation to forecast their future values. In Figure 3.4, we plot a series of three two-dimensional plots, each with a different macroprudential policy parameter τ_q.

The first plot of Figure 3.4 simulates a condition where central bank uses forecasted data in interest rate rule while the ratio of LTV is assumed to be constant, i.e. $\tau_q = 0$. Under this circumstance, as shown in Chapter 2, we easily face indeterminacy problem, especially when the Taylor principle is violated ($\tau_\pi < 1$) or when monetary policy reaction function to output is too strong ($\tau_y > 1$). Fortunately, a response to current growth of housing prices through the LTV rule has increased the probability of leading the economy to a determinate and E-stable REE. As an illustration, the increase of the LTV reaction function parameter τ_q from 0 to 0.6 has significantly lifted up the line that splits the parameters space into determinate and indeterminate (also, E-stable and E-unstable) which means there are more parameter choices are available for central bank via policy rate to respond to output. The shifting increases as parameter τ_q reaches its upper bound value of 1.2.

Figure 3.4
Stability results of REE under forecasted data in interest rate rule
and current growth of housing prices in the LTV rule



Blue/Dark : Determinate and E-stable policy rules
Green/Grey : Indeterminate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

In general, a response to current growth of housing prices via the LTV rule does not change the fundamental picture. Yet, it offers more flexibility to central bank via policy rate in responding output. As the parameter of the LTV reaction function increases, the region of τ_y, τ_π combination that leads to a determinate and E-stable REE gradually enlarge, and the region that leads to E-stable but indeterminate REE gradually moves up. As a result, the task of central bank in taking the economy to a determinate and E-stable REE becomes easier to obtain. This is certainly a desirable situation for central bank.

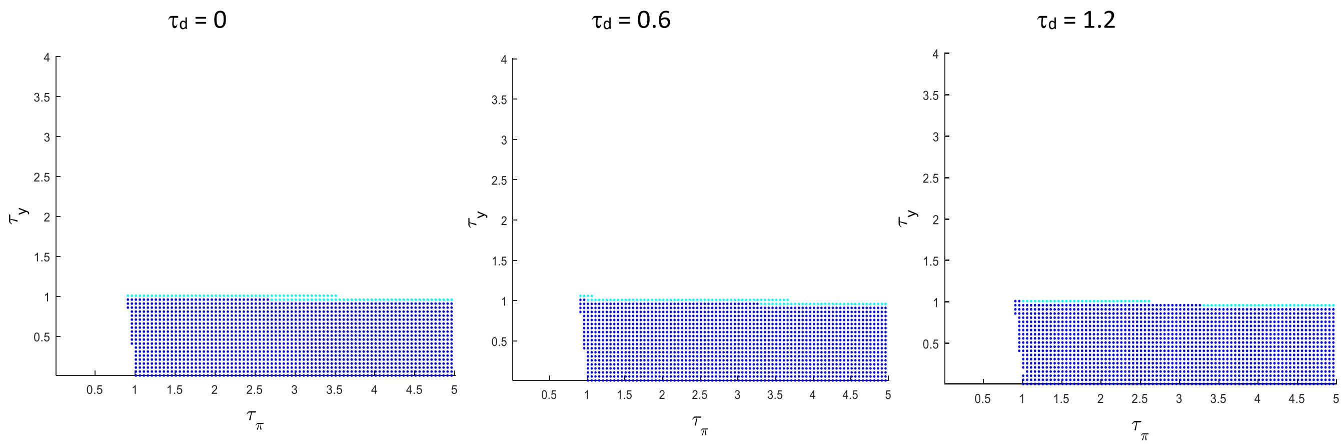
What will happen when the LTV rule responds to current growth of credit instead of current growth of housing prices in which central bank does not have accessed to current data of inflation and output when conducting monetary policy? We consider a combination of monetary and macroprudential rule as follows:

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1}, \text{ and } \tilde{m}_t = -\tau_d \Delta \tilde{d}_{b,t} \qquad 3.68$$

We can show that the previous result cannot be attained anymore. When central bank uses forecasted data in interest rate rule, a response to current credit growth via the LTV rule does not give a significant impact. This is well illustrated in Figure 3.5 where we have produced three plots of stability results of REE for different values of macroprudential parameter τ_d. Again, as the value of τ_d increases, the line that splits the parameter space into determinate and indeterminate (as well, E-stable and E-unstable) region slightly shifts up, indicating that a stronger response of LTV rule to current credit growth minimises the requirement for policy rate reaction function parameter to output. Unfortunately, the line movement is so small and hardly to be seen. Under this condition, we conclude that, with empirically plausible parameter values for τ_d, a response to credit growth is neither more stabilising nor less stabilising when central bank uses forecasted data in monetary policy.

The last result confirms about the difference between responding to housing prices and to credit via the LTV rule. Many earlier studies that employ a model variant of Iacoviello (2005) had never made a clear distinction between responding to housing prices and to financial variable (i.e. credit) when macroprudential policy is introduced. Using E-stability and determinacy criteria, we can show that a response to the growth of housing prices via the LTV rule potentially gives higher gains compared to a response to the growth of credit. This result holds especially in a situation where current data of inflation and output are not available, at the time of conducting monetary policy. We confirm the last finding once again using another alternative of policy combinations in which the LTV rule reacts to forecasted growth of housing prices or of credit while interest rate rule responds to forecasted data of inflation and output.

Figure 3.5
Stability results of REE under forecasted interest rate rule
and current growth of credit in the LTV rule



Blue/Dark : Determinate and E-stable policy rules
Green/Grey : Indeterminate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

3.4.4. Forecasted Data in Interest Rate Rule and LTV Rule

In a situation where current data for housing prices or nominal credit are not available, central bank needs to make forecasts using past data. We consider a case in which central bank initially responds to forecasted growth of housing prices and, subsequently, to forecasted growth of credit. Concerning monetary policy, it is assumed that current data for inflation and output are also not available, resulting in central bank has to use forecasts for these variables in policy rate. To make it obvious, we consider a combination of policy rules between monetary and macroprudential policy as follows:

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1}, \text{ and } \tilde{m}_t = -\tau_q E_t^* \Delta \tilde{q}_{t+1} \qquad 3.69$$

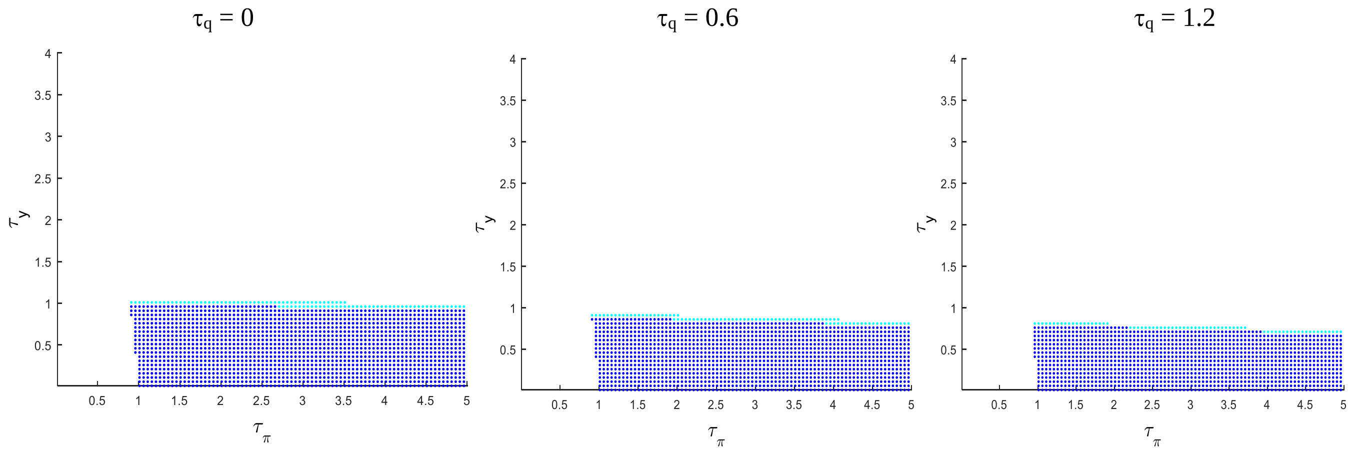
We present the numerical result in Figure 3.6. When central bank reacts to forecasted growth of housing prices through the LTV rule, determinacy and E-stability region, under plausible policy parameter τ_y and τ_π, is gradually reduced. A determinate and E-stable region in the third panel ($\tau_q = 1.2$) shrinks by around a quarter of the one in the first panel ($\tau_q = 0$). Given this, we observe a trade-off in policy reactions now. If the LTV reaction function to forecasted growth of housing prices is strong, a response to output via policy rate must be reduced to maintain stability under learning. In this situation, policy makers can observe that a response to forecasted growth of housing prices via the LTV rule is, to some extent, equivalent to forecasted output via interest rate rule; if it reacts strongly to one, it must behave more mildly to the other.

A reduction of determinacy and E-stability region is faster when central bank adopts the LTV rule that reacts inversely to forecasted growth of credit as follows:

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \text{ and } \tilde{m}_t = -\tau_d E_t^* \Delta \tilde{d}_{b,t+1} \qquad 3.70$$

In the above setting, a small reaction to credit growth via the LTV rule (for example: $\tau_d > 0.05$) has caused the equilibrium path to fall easily to indeterminate and E-unstable region. As a result, it is not easy for central bank to find a combination of policy parameters that leads the economy to a determinate and E-stable REE. Within the policy range $0 \leq \tau_d \leq 0.05$, we may find some combinations of determinate and E-stable policy rules. However, once this parameter value is sufficiently strong (higher than 0.05), we face a problem of indeterminacy and instability under RLS learning. This result confirms our previous finding concerning the advantage of responding to the growth of housing prices compared to the growth of credit when central bank uses the LTV ratio as their macroprudential instrument. When the growth of credit is mainly driven by the growth of housing prices, it is necessary for central bank to give a response to the growth of housing prices, at the first place, instead of credit growth.

Figure 3.6
Stability results of REE under forecasted data in interest rate rule
and forecasted growth of housing prices in LTV Rule



Blue/Dark : Determinate and E-stable policy rules
Green/Grey : Indeterminate and E-stable policy rules
White : Indeterminate and E-unstable policy rules

In general, the results from the above study are quite intuitive. For agents who learn adaptively, using accurate and updated data is essential for the purpose of estimating the economic model. Here, we follow the argument pointed out by Wei (2012) who studies the same issue under slightly different assumptions in modelling a policy used to tackle a boom in housing prices. We claim that the availability of current data for housing price growth improves the estimation by adding another dimension of certainty. In contrast, when this data is not available, the level of uncertainty is increased in the estimation. The presence of more accurate data in the estimation helps agents to learn the REE more easily. Therefore, for the REE to be learned, more restrictions must be imposed on policy parameters when there is less accurate data available, and vice versa. With regards to the appropriate variable that should be targeted by the LTV rule, we claim that a response to the growth of housing prices gives better outcomes compared to the growth of credit. This occurs because central bank's decision on the ratio of LTV has an immediate effect on the level of nominal credit, such that a too-early response to credit growth leads to potentially undesirable results. Under this condition, the most appropriate data to be responded to by the LTV rule is housing prices in general.

3.5. The Optimal Constrained Policy Rule

In the previous section, we have demonstrated that a response to the growth of housing prices through the LTV rule, in addition to inflation and output via interest rate rule, gives different results compared to a response to the growth of credit. Responding to the growth of credit via the LTV rule easily leads to indeterminate and E-unstable equilibrium that must be hindered by policy makers; whilst responding to the growth of housing prices may offer a benefit under certain circumstances. One of these situations is when central bank does not possess current data for inflation and output, but does observe current data of housing prices growth. In this case, giving a response to housing prices growth via the LTV rule is stabilising. However, when they cannot observe current data of housing prices growth, responding to it does not improve stability under learning. The advantage of giving a response to the growth of housing prices via the LTV rule turns out to be insignificant if central bank has accessed to current data of inflation and output and uses them to conduct monetary policy.

Unfortunately, determinate and E-stable policy rules are not enough from the perspective of the policy makers. They are also interested in finding the optimal policy rule that leads the whole economy to the highest welfare, i.e. a rule that places the economy in the most stable situation. Motivated by this, we follow the concept of Evans & McGough (2007), as used in Chapter 2, by evaluating various combinations of policy rules, that are determinate and E-stable, of which they may lead the equilibrium to the highest welfare. For this purpose, we assume that central bank sets an ad-hoc objective function by minimising the sum of inflation, output and housing prices variation. A combination of policy parameters is then evaluated to find the one that minimises this loss of function. We do this analysis for all combinations of interest rate and the LTV rule. It should be noticed that in our setting, it is assumed that central bank is concerned not only with inflation and output volatility, but also with that in respect of housing prices.

Based on the existing literature, there have been some earlier studies that analyse the issue of optimal macroprudential and (or) monetary policies in a model with the feature of housing market/sector. Among them are Kannan et al. (2012) and Rubio & Gallego (2013). Kannan et al. (2012) examines the Taylor-type interest rate rule that reacts to inflation, output and the growth rate of credit with a macroprudential policy instrument based on the LTV[27]. They conclude that a strong monetary reaction to accelerator mechanisms that push up credit growth and housing prices can dampen business cycles volatility. In addition, they show that a macroprudential instrument, specifically designed to dampen credit market cycles, can offer a stabilisation benefit in the case a financial sector or a housing demand shock hit the economy. They also suggest that it is irrelevant to use this macroprudential instrument when a

[27] Instead of affecting credit level, macroprudential instrument influences a lending rate.

productivity shock hits the economy. Therefore, they suggest the importance of identifying the source of the shock of the housing prices volatility when accessing policy optimally.

Meanwhile, Rubio & Gallego (2013) evaluate the implication of macroprudential and monetary policies for business cycles, welfare, and financial stability. They set a macroprudential rule based on LTV ratio that responds inversely to the deviation of output and housing prices from its steady state values. This macroprudential policy rule interacts with a traditional inflation-targeting Taylor rule with policy inertia. They conclude that introducing macroprudential policy via LTV rule minimises the effects of booms to the economy by controlling the amount of loan given. In addition, they can show that the LTV rule set in this way is welfare improving and suggest that the optimal LTV rule should give a higher response to housing prices than to output deviations. Unfortunately, when a shock comes from the supply-side of the economy, they found monetary and macroprudential policies may enter into a conflict. This finding, again, confirms about the importance of identifying the source of disturbances in the study of the optimal policy. Notice that the way of how we set a macroprudential rule is similar to Rubio & Gallego (2013), among others.

Nevertheless, the above mentioned literature have assumed that agents endow with the rational expectation (RE). Under this framework, agents have knowledge about the exact structure of the economy and the correct form of an agent's behaviour. It turns out this assumption is too strong, as discussed extensively in Chapter 2. The RE assumption also faces a difficulty in producing large swings in housing prices and household debt as observed in the empirical evidence. To overcome this limitation, Gelain et al. (2013) introduce a simple moving average forecast rule for a subset of households that can significantly magnify the volatility and the persistence of house prices and household debt relative to otherwise similar model with fully rational expectations. Having success of depicting the empirical evidence, then they evaluate various policy actions that might be useful to dampen the resulting excess volatility, including: responding to the growth of housing prices and credit through interest rate rule, reducing the ratio of loan to value (LTV) and controlling the ratio of loan to income (LTI) in collateral, in addition to LTV ratio. Among these alternatives, they find that controlling the ratio of LTI is the most effective instrument for dampening overall excess volatility in the model economy. They also argue that while an interest rate response to house price growth or credit growth can stabilise some economic variables, it can significantly magnify the volatility of others, particularly inflation.

In the same spirit as Gelain et al. (2013), we study the issue of optimal policy in an environment where agents in the economy are not fully rational. It is assumed that agents cannot observe the true parameters of the model and use the recursive least square (RLS) learning rule to update the parameter estimates towards the true value of model parameters as new data becomes available. Recall that E-stability principle states that rational expectation equilibrium (REE) is locally stable under RLS *iff* it is E-stable. If a policy rule leads to REE that is not E-stable, then under RLS learning, economic agents can collaborate with a zero probability. Given this restriction, we remark that the chosen optimal rule must satisfy two criteria, i.e. determinacy and E-stability. This approach is known as the optimal constrained policy rule in the literature.

For finding the optimal constrained policy rule, we consider a case in which macroprudential rule responds to housing price growth via the LTV rule while monetary policy operates in a conventional way by responding to inflation and output. The LTV rule that responds to credit growth is not considered since we have seen earlier that such rule tends to lead to economy to an inferior condition. We assume that the objective function of central bank is maximising the economic welfare by minimising the loss function in terms of inflation, output and housing prices volatility as follows:

$$\min_{\tau_\pi,\tau_y,\tau_q} Var\,(\tilde{\pi}_t) + \varsigma Var\,(\tilde{y}_t) + Var\,(\tilde{q}_t) \qquad 3.71$$

where ς represents a relative weight assigned to the variance of output that shows the importance of output volatility from the view of central bank. We set the value of this parameter ς equal to 0.1 since central bank is assumed to focus more on assets and price stabilisation. Meanwhile, parameter τ_π and τ_y represent monetary reaction function to inflation and output, respectively and parameter τ_q is the LTV reaction function to housing prices growth. Note that this loss function is consistent with studies that make a second-order approximation of utility of individuals and it differs from the standard case by incorporating housing prices.

After explaining the way of how central bank sets their objective, for the purpose of analysing the optimal policy, some disturbances needs to be introduced in the model. Here, we assume only two exogenous shocks that may hit the economy, i.e. a housing demand shock and a technological shock that follow an AR (1) autoregressive process. As previously discussed in the section of calibration, we set the persistence of housing demand shock at 0.9 and of technology shock at 0.8 with the standard deviation of 0.02 and 0.05, respectively. Then, given various combinations of interest rate and the LTV rule, as equation (3.63), (3.65), (3.67) and (3.69), we compute the variance-covariance of state variables $\tilde{y}_t, \tilde{x}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_{b,t}, \tilde{h}_t$ and $\tilde{r}_t$ in $\mathcal{W}_t$ using a procedure in equation (3.57) to (3.61). After vectoring the variance-covariance $\mathcal{W}_t$, the second moment of $\tilde{y}_t, \tilde{\pi}_t$ and $\tilde{q}_t$ are obtained where these values will be used further for evaluating the loss function in equation (3.71). In what follows, we analyse and report the optimal constrained policy rule based on the assumption of an exogenous shock that is present, starting from a housing demand shock, followed by a technology shock, and a case where these two shocks jointly hit the economy.

3.5.1. A Housing Demand Shock

When a positive housing demand shock hits the economy, among all types of policy combination, the lowest loss of function is obtained when central bank uses forecasted data for inflation and output in monetary policy and current data for housing price growth in the LTV rule with a combination of policy parameter $\tau_y = 0.06$, $\tau_\pi = 1.11$ and $\tau_q = 1.2$, respectively (see Table 3.2). This is an interesting finding since the optimal policy can be attained even when current data for inflation and output is not available and needs to be estimated. The result reported in Table 3.2 also shows that the quality of housing prices data matters for obtaining the optimal constrained policy when a housing demand shock hits the economy. When current data for housing prices growth is available, then responding to it is beneficial since it brings down the value of a loss function relatively to a case in which no response is given. Recall that a lower loss implies higher efficiency which means less variability of inflation, output and housing prices. Notice that a benefit of responding to current growth of housing prices through the LTV rule holds regardless the quality of data used in monetary policy. As the LTV reaction function to current growth of housing prices gets higher, then a lower loss is attained. The grid search for finding the optimal constrained policy stops when the LTV reaction function parameter τ_q reaches its upper bound at 1.2 while monetary reaction function to inflation τ_π and output gap τ_y will be constrained at 5 and 4, respectively.

Table 3.2
Optimal constrained policy rule
(a housing demand shock)

	Mix A	Mix B	Mix C*	Mix D
τ_y	0.01	0.01	0.06	0.06
τ_π	1.06	1.11	1.11	1.11
τ_q	1.2	0	1.2	0
σ_y	0.009	0.010	0.012	0.012
σ_π	0.004	0.003	0.006	0.006
σ_q	0.013	0.014	0.011	0.012
Loss	0.00019	0.00021	0.00018	0.00018

The upper sign of * denotes a combination of policy rule that gives the lowest loss.
Mix A: Current data in interest rule and current growth of housing prices in LTV rule
Mix B: Current data in interest rate rule and forecasted growth of housing prices in LTV rule
Mix C: Forecasted data in interest rate rule and current growth of housing prices in LTV rule
Mix D: Forecasted data in interest rate rule and forecasted growth of housing prices in LTV rule

Now, we turn to the case in which current data for housing prices growth is not available and needs to be forecasted in the LTV rule. In a situation where a housing demand shock hits the economy, the LTV rule that reacts to forecasted growth of housing prices do not give any advantage compared to the case where the LTV rule is set at a fix ratio (illustrated in Table 3.2, labelled by “Mix B” and “Mix D”). This finding is valid regardless the kind of data used in monetary policy by central bank. Intuitively, forecasted data creates another uncertainty for policy makers which potentially lead to higher volatility when they do some mistakes on forecasting. Under this condition, a combination of policy parameters that leads to the highest welfare is obtained when no response is given to forecasted growth of housing prices.

What is the intuition behind this finding? As discussed by Iacoviello (2005), when a positive housing demand shock hits the economy, housing prices will increase. The rise in housing prices increases the borrowing capacity of debtors, allowing them to spend and consume more. In addition, when demand rises, consumer price increases that reduces the real value of their outstanding debt obligations, positively affecting their net worth. Given that borrowers have a higher propensity to spend than lenders, the net effect on demand is positive, and acts as a powerful amplification mechanism. As an accelerator mechanism of demand shock works well here, then the LTV rule that reacts inversely to housing prices growth may effectively dampen housing boom. However, as shown in Table 3.2, the effectiveness of this adjustment depends critically on the quality of housing prices data its self.

3.5.2. A Technology Shock

We have seen when central bank owns current data accurately, it becomes easier for them to lead the economy to a determinate and E-stable REE. We can also show that when a housing demand shock hits the economy, the benefit of responding to the growth of housing prices via the LTV rule depends on the quality of housing prices data itself; when current growth of housing prices is observed, then responding to it is beneficial, regardless the quality of data used in interest rate rule.

Table 3.3
Optimal constrained policy rule
(a technological shock)

	Mix A*	Mix B	Mix C	Mix D
$\boldsymbol{\tau_y}$	0.26	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	4.96	1.81	2.86	2.41
$\boldsymbol{\tau_q}$	1.2	0	1.2	0
$\boldsymbol{\sigma_y}$	0.028	0.028	0.029	0.028
$\boldsymbol{\sigma_\pi}$	0.002	0.003	0.002	0.003
$\boldsymbol{\sigma_q}$	0.007	0.008	0.008	0.008
Loss	0.00013	0.00015	0.00015	0.00015

The upper sign of * denotes a combination of policy rule that gives the lowest loss.
Mix A: Current data in interest rule and current growth of housing prices in LTV rule
Mix B: Current data in interest rate rule and forecasted growth of housing prices in LTV rule
Mix C: Forecasted data in interest rate rule and current growth of housing prices in LTV rule
Mix D: Forecasted data in interest rate rule and forecasted growth of housing prices in LTV rule

When a technology shock becomes the source of uncertainty, a response to the growth of housing prices through the LTV rule is not necessary if forecasted data of housing prices growth is used. As shown in Table 3.3, labelled by “Mix B” and “Mix D”, when a technology shock is active, a combination of policy parameters that offers the highest welfare is obtained when the LTV rule reaction function parameter τ_q is set at zero with parameter $\tau_\pi = 1.81$ and $\tau_y = 0.01$ (for “Mix B”) and $\tau_\pi = 2.41$ and $\tau_y = 0.01$ (for “Mix D”). Unfortunately, when current data for the growth of housing prices is used in the LTV rule, the result suggests that it is beneficial to give a response to the growth of housing prices. Having this, we become ambiguous about the requirement of responding housing prices when technology shock hits the economy. Note that, among all policy rule combinations, using current data in both interest rate and the LTV rule is the most advantageous policy combination as it offers the lowest loss with a combination of policy parameters $\tau_y = 0.26$, $\sigma_\pi = 4.96$ and $\tau_q = 1.2$, respectively (labelled by “Mix A” in Table 3.3).

3.5.3. All Shocks

When both a housing demand shock and a technology shock are active in the economy, a response to the growth of housing prices via the LTV rule is required to find the optimal constrained policy as long as current data of housing prices growth is used (see “Mix A” and “Mix C” in Table 3.4). Again, we remark the importance of having current and valid data for the LTV rule in order to get the optimal policy. However, when forecasted data of housing prices growth is used in the LTV rule, then responding to it does not lead to the most stable economic condition. This can be confirmed from “Mix B” and “Mix D” of policy combinations as can be seen in Table 3.4. Among all possible policy rule combinations, using current data in both monetary and macroprudential policies is the most advantageous policy rules as it offers the lowest loss in terms of the variability of inflation, output and housing prices with a combination of policy parameters $\tau_\pi = 1.31$, $\tau_y = 0.01$ and $\tau_q = 1.2$, respectively.

Table 3.4
Optimal constrained policy rule
(all shocks)

	Mix A*	Mix B	Mix C	Mix D
$\boldsymbol{\tau_y}$	0.01	0.01	0.01	0.01
$\boldsymbol{\tau_\pi}$	1.31	1.31	1.66	1.66
$\boldsymbol{\tau_q}$	1.2	0	1.2	0
$\boldsymbol{\sigma_y}$	0.028	0.028	0.028	0.028
$\boldsymbol{\sigma_\pi}$	0.004	0.005	0.005	0.005
$\boldsymbol{\sigma_q}$	0.016	0.017	0.016	0.017
Loss	0.00035	0.00038	0.00037	0.00039

The upper sign of * denotes a combination of policy rules that give the lowest loss.
Mix A: Current data in interest rule and current growth of housing prices in LTV rule
Mix B: Current data in interest rate rule and forecasted growth of housing prices in LTV rule
Mix C: Forecasted data in interest rate rule and current growth of housing prices in LTV rule
Mix D: Forecasted data in interest rate rule and forecasted growth of housing prices in LTV rule

3.6. Conclusion

This chapter tries to address three important issues in relation to the recent crisis: Firstly, the benefits for central bank to target asset prices (i.e. housing prices) or financial variables (i.e. credit). Secondly, the appropriate rules to be used in both monetary and macroprudential policies by assumption agents do RLS learning in forming their expectation. Thirdly, the optimal monetary and macroprudential policies to stabilise the economy. Unlike similar studies that use a welfare perspective only, we have addressed the last issue from the perspective of determinacy and E-stability criteria combined with a welfare perspective.

In general, responding to the growth of housing prices via the LTV rule is favourable than to the growth of credit. Yet, the benefit of responding to the growth of housing prices depends on the quality of this data itself. When central bank has accessed to current data of inflation and output and uses them in monetary policy, then responding to the growth of housing prices via the LTV rule becomes irrelevant. However, when they do not have recent data of inflation and output in monetary policy, then responding to current growth of housing prices via the LTV rule is beneficial. A stronger reaction of the LTV rule on current growth of housing prices increases the probability of leading the economy to a determinate and E-stable REE. But, when current data of housing prices is also not available, then using forecasted growth of housing prices in the LTV rule, in addition to forecasted data in monetary policy, is undesirable since it shrinks the region of determinate and E-stable policy rules. The same conclusion does not emerge for a case in which central bank responds to credit growth via the LTV rule. Under such condition, the equilibrium easily falls to indeterminacy and instability problem under learning.

Finally, a decision to choose the optimal constrained policy rule depends on the type of shocks that hits the economy and the type of housing prices data observed in the LTV rule. When a housing demand shock hits the economy, responding to current growth of housing prices dampen the economy volatility, whilst responding to forecasted growth of housing prices is not necessary. This argument is valid for any types of a standard Taylor rule used in monetary policy. In contrast, when a technology shock hits the economy, a benefit of responding to the growth of housing prices via the LTV rule is ambiguous and should be taken with caution. Unlike similar studies that have used the assumption of rational expectation, our study concerning the optimal constrained policy rule has highlights the valueadded for adaptive learning analysis by adding one more criteria, i.e. E-stability, in addition to determinacy, when choosing the optimal policy rule.

Although, the assumption of recursive least square (RLS) learning has made the framework of the model becomes more plausible. Still, there are some researchers who view it as a strong assumption. Under this type of learning algorithm, it is assumed that economic agents can observe the variance of the state variables and use them in forming their forecasts. Yet, in practise, it is hard to observe the variance of all state variables when agents do learning process. Therefore, in the next chapter we consider a situation in which economic agents may deviate from a standard RLS learning rule such that we are uncertain about the type of learning rules used by agents in the economy.

Appendix 3.1. Notation

Parameter	Description
β_s	The discount factor for patient households (savers)
β_b	The discount factor for impatient households (borrowers)
J	Steady-state weight of housing in utility function
η	The inverse of the elasticity of labour supply
ε	The elasticity of substitution across final goods
m	Steady-state of the loan to value (LTV) ratio
α	Savers wage income/ Labour share for savers
X	Steady-state mark up
θ	The fraction of firms that keep their price unchanged
τ_π	Interest rate reaction function to inflation
τ_y	Interest rate reaction function to output
τ_q	LTV rule reaction function to house-price growth
τ_b	LTV rule reaction function to credit growth
ρ_j	Coefficient of housing demand shock AR (1) model
ρ_a	Coefficient of productivity or technology AR (1) model
σ_j	Standard deviation of housing demand shock
σ_a	Standard deviation of technology shock

Variable	Description
Y	Output
C_s	Saver Consumption
C_b	Borrower Consumption
H_s	Saver Housing Stock
H_b	Borrower Housing Stock
N_s	Saver Working Hour
N_b	Borrower Working Hour
j	Exogenous housing demand shock
D_s	Saver's bank deposit
D_b	Borrower's loan
q_t	Real housing prices
π	Inflation
w_s	Saver real wage
w_b	Borrower real wage
F	Real lump-sum profit
m	Loan to value (LTV) ratio
A	Exogenous technology shock
X	Mark up
mc	Marginal cost

R	Gross interest rate
u_j	Normally distributed shock to housing demand shock
u_a	Normally distributed shock to technology

Appendix 3.2. Model Derivation

Patient Households

A representative of patient households (saver) maximises their utility by choosing consumption, housing, labour hours and credit.

$$Max\ E_0^* \sum_{t=0}^{\infty} \beta_s^t \left(ln\, C_{s,t} + j_t\, ln\, H_{s,t} - \frac{N_{s,t}^{\eta}}{\eta} \right)$$

$$S.t.\ \ C_{s,t} + D_{s,t} + q_t\left(H_{s,t} - H_{s,t-1}\right) \leq \frac{R_{t-1}D_{s,t-1}}{\pi_t} + w_{s,t}N_{s,t} + F_t \qquad \text{3A.1}$$

$$\mathcal{L} \equiv E_0^* \sum_{t=0}^{\infty} \beta_s^t \left(ln\, C_{s,t} + j_t\, ln\, H_{s,t} - \frac{N_{s,t}^{\eta}}{\eta} + \lambda_{s,t} \left[\frac{R_{t-1}D_{s,t-1}}{\pi_t} + w_{s,t}N_{s,t} + F_t\ - C_{s,t} - D_{s,t} - q_t\left(H_{s,t} - H_{s,t-1}\right) \right] \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_{s,t}} \equiv \beta_s^t \left(\frac{1}{C_{s,t}} - \lambda_{s,t} \right) = 0$$

$$\frac{1}{C_{s,t}} - \lambda_{s,t} = 0 \qquad \text{3A.2}$$

$$\frac{\partial \mathcal{L}}{\partial H_{s,t}} \equiv \beta_s^t \left(\frac{j_t}{H_{s,t}} - \lambda_{s,t} q_t \right) + \beta_s^{t+1} E_t^* \lambda_{s,t+1} q_{t+1} = 0$$

$$\frac{j_t}{H_{s,t}} - \lambda_{s,t} q_t + \beta_s E_t^* \left(\lambda_{s,t+1} q_{t+1} \right) = 0 \qquad \text{3A.3}$$

$$\frac{\partial \mathcal{L}}{\partial N_{s,t}} \equiv \beta_s^t \left(-N_{s,t}^{\eta-1} + \lambda_{s,t} w_{s,t} \right) = 0$$

$$\lambda_{s,t} = \frac{N_{s,t}^{\eta-1}}{w_{s,t}} \qquad \text{3A.4}$$

$$\frac{\partial \mathcal{L}}{\partial D_{s,t}} \equiv \beta_s^t \left(-\lambda_{s,t} \right) + E_t^* \left(\frac{\beta_s^{t+1} \lambda_{s,t+1} R_t}{\pi_{t+1}} \right) = 0$$

$$\lambda_{s,t} = \beta_s E_t^* \left(\frac{\lambda_{s,t+1} R_t}{\pi_{t+1}} \right) \qquad \text{3A.5}$$

Combining (3A.2) and (3A.5), we get a standard Euler equation as:

$$\frac{1}{C_{s,t}} = \beta_s E_t^* \left(\frac{R_t}{\pi_{t+1} C_{s,t+1}} \right) \qquad \text{3A.6}$$

Substituting (3A.2) into (3A.4) yields the standard labour-leisure trade-off, equating the marginal disutility of an additional unit of labour to the marginal utility received from additional consumption.

$$w_{s,t} = N_{s,t}^{\eta-1} C_{s,t} \qquad 3A.7$$

Last, combining (3A.2) with (3A.3) to get:

$$\frac{j_t}{H_{s,t}} = \frac{q_t}{C_{s,t}} - \beta_s E_t^* \left(\frac{q_{t+1}}{C_{s,t+1}}\right) \qquad 3A.8$$

This equation equates the purchase price of housing to the pay-off (the marginal rate of substitution between housing and non-durable consumption), plus the expected resale value.

Impatient Households

A representative of impatient households (borrowers) maximises their utility function by choosing consumption, housing, labour hours and credit.

$$Max\ E_0^* \sum_{t=0}^{\infty} \beta_b^t \left(ln\, C_{b,t} + j_t\, ln\, H_{b,t} - \frac{N_{b,t}^{\eta}}{\eta} \right)$$

$$S.t.\ \ C_{b,t} + \frac{R_{t-1} D_{b,t-1}}{\pi_t} + q_t\left(H_{b,t} - H_{b,t-1}\right) \le D_{b,t} + w_{b,t} N_{b,t} \qquad 3A.9$$

$$R_t D_{b,t} \le m_t E_t^*\left(q_{t+1} H_{b,t} \pi_{t+1}\right) \qquad 3A.10$$

$$\mathcal{L} \equiv E_0^* \sum_{t=0}^{\infty} \beta_b^t \Big(ln\, C_{b,t} + j_t\, ln\, H_{b,t} - \frac{N_{b,t}^{\eta}}{\eta} + \lambda_{b,t} \Big[D_{b,t} + w_{b,t} N_{b,t} - C_{b,t} - \frac{R_{t-1} D_{b,t-1}}{\pi_t} - q_t\left(H_{b,t} - H_{b,t-1}\right) \Big] + \varphi_t \left[m_t E_t^*\left(q_{t+1} H_{b,t} \pi_{t+1}\right) - R_t D_{b,t} \right] \Big)$$, where $\varphi_t = \lambda_{b,t} \psi_t$

$$\frac{\partial \mathcal{L}}{\partial C_{b,t}} \equiv \beta_b^t \left(\frac{1}{C_{b,t}} - \lambda_{b,t} \right) = 0$$

$$\frac{1}{C_{b,t}} - \lambda_{b,t} = 0 \qquad 3A.11$$

$$\frac{\partial \mathcal{L}}{\partial H_{b,t}} \equiv \beta_b^t \left(\frac{j_t}{H_{b,t}} - \lambda_{b,t} q_t + \varphi_t m_t E_t^*[q_{t+1} \pi_{t+1}] \right) + \beta_b^{t+1} E_t^*\left(\lambda_{b,t+1} q_{t+1}\right) = 0$$

$$\frac{j_t}{H_{b,t}} - \lambda_{b,t} q_t + \varphi_t m_t E_t^*[q_{t+1} \pi_{t+1}] + \beta_b E_t^*\left(\lambda_{b,t+1} q_{t+1}\right) = 0 \qquad 3A.12$$

$$\frac{\partial \mathcal{L}}{\partial N_{b,t}} \equiv \beta_b^t \left(-N_{b,t}^{\eta-1} + \lambda_{b,t} w_{b,t} \right) = 0$$

$$\lambda_{b,t} = \frac{N_{b,t}^{\eta-1}}{w_{b,t}} \qquad 3A.13$$

$$\frac{\partial \mathcal{L}}{\partial D_{b,t}} \equiv \beta_b^t \left(\lambda_{b,t} - \varphi_t R_t \right) - E_t^* \left(\frac{\beta_b^{t+1} \lambda_{b,t+1} R_t}{\pi_{t+1}} \right) = 0$$

$$\lambda_{b,t} - \varphi_t R_t = \beta_b E_t^* \left(\frac{\lambda_{b,t+1} R_t}{\pi_{t+1}}\right) \qquad \text{3A.14}$$

From (3A.11) and (3A.14), we obtain a consumption Euler equation for impatient households adjusted to capture the borrowing constraint as:

$$\frac{1}{C_{b,t}} = \beta_b E_t^* \left(\frac{R_t}{\pi_{t+1} C_{b,t+1}}\right) + \varphi_t R_t \qquad \text{3A.15}$$

Combining (3A.11) and (3A.13) yields the standard labour-leisure trade-off for borrower:

$$w_{b,t} = N_{b,t}^{\eta-1} C_{b,t} \qquad \text{3A.16}$$

Finally, substituting (3A.11) into (3A.12), we obtain:

$$\frac{j_t}{H_{b,t}} = \frac{q_t}{C_{b,t}} - \beta_b E_t^* \left(\frac{q_{t+1}}{C_{b,t+1}}\right) - \varphi_t m_t E_t^* [q_{t+1} \pi_{t+1}] \qquad \text{3A.17}$$

This equation equates the marginal utility of non-durable consumption to shadow value of durable housing service.

Wholesalers

The wholesalers operate under perfect competition and flexible price to produce homogenous goods Y_t. These intermediate goods are produced using the following production function:

$Y_t = A_t {N_{s,t}}^{\alpha} {N_{b,t}}^{1-\alpha}$, where A_t is a temporary shock that follows:

$log(A_t) = \rho_a \, log(A_{t-1}) + u_{at}$

After intermediate goods are produced, retailers purchase them at the wholesale price P_t^w, and transform them into final goods and sell them at the price P_t. We denote the mark-up of final over intermediate goods as $X_t = P_t / P_t^w$.

The wholesalers maximise her real profit subject to its production function as follows:

$Y_t / X_t - w_{s,t} N_{s,t} - w_{b,t} N_{b,t}$

$$S.t \quad Y_t = A_t {N_{s,t}}^{\alpha} {N_{b,t}}^{1-\alpha} \qquad \text{3A.18}$$

Substituting the constraint into the profit function, we obtain unconstraint profit maximisation problem as:

$V_w \equiv \; Y_t / X_t - w_{s,t} N_{s,t} - w_{b,t} N_{b,t} \quad + \lambda_{w,t} \left(A_t {N_{s,t}}^{\alpha} {N_{b,t}}^{1-\alpha} - Y_t\right)$

First order condition with respect to patient household's labour and impatient household's labour gives:

$$\frac{\partial V_w}{\partial N_{s,t}} \equiv -w_{s,t} + \alpha\lambda_{w,t}A_t {N_{s,t}}^{\alpha-1} {N_{b,t}}^{1-\alpha} = 0$$

$$w_{s,t} = \alpha\lambda_{w,t}\, Y_t / N_{s,t} \qquad \text{3A.19}$$

$$\frac{\partial V_w}{\partial N_{b,t}} \equiv -w_{b,t} + (1-\alpha)\lambda_{w,t}A_t {N_{s,t}}^{\alpha} {N_{b,t}}^{-\alpha} = 0$$

$$w_{b,t} = (1-\alpha)\lambda_{w,t}\, Y_t / N_{b,t} \qquad \text{3A.20}$$

We solve $\lambda_{w,t}$ by substituting (3A.19) and (3A.20) back into profit maximisation function to get:

$$Y_t/X_t - \alpha\lambda_{w,t}Y_t - (1-\alpha)\lambda_{w,t}Y_t = 0$$

$$\lambda_{w,t} = 1/X_t \equiv mc_t \qquad \text{3A.21}$$

where, mc_t is the marginal cost or the inverse of the mark up X_t.

Retailers

There is a continuum of retailers indexed by z who operates under monopolistically competitive market. Retailer z buys intermediate goods in a competitive market, differentiates it at no cost into $Y_t(z)$, and sells it at $P_t(z)$. Total final goods are aggregated from each type z final good as:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

The retailer chooses $Y_t(z)$ to minimise its costs subject to the above production function.

$$Min \int_0^1 P_t(z)Y_t(z)$$

$$S.t \;\; Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\mathcal{L} \equiv \int_0^1 P_t(z)Y_t(z) + \lambda_{f,t}\left(Y_t^f - \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)$$

$$\frac{\partial\mathcal{L}}{\partial Y_t(z)} \equiv P_t(z) - \lambda_{f,t}\left[\left(\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{1}{\varepsilon-1}} Y_t(z)^{\frac{-1}{\varepsilon}}\right] = 0$$

$$Y_t(z) = \left(\frac{P_t(z)}{\lambda_{f,t}}\right)^{-\varepsilon} Y_t^f \quad \forall\, z \qquad \text{3A.22}$$

Solving for $\lambda_{f,t}$ by substituting (3A.22) into the production function yields:

$$Y_t^f = \left(\frac{1}{\lambda_{f,t}}\right)^{-\varepsilon} \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{\varepsilon}{\varepsilon-1}} Y_t^f$$

$$\lambda_{f,t} = \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{1}{1-\varepsilon}} \equiv P_t$$

The price index is then given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{1}{1-\varepsilon}} \qquad 3A.23$$

The price-setting problem for the retailer is a standard Calvo setting. Each retailer sells its good at price $P_t(z)$, and $(1-\theta) \in [0,1]$, is the probability of being able to change the sale price in every period.

The optimal pricing decision is:

$$Max E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left[\left(\frac{P_t^{New}(z) - P_{t+k}^w}{P_{t+k}}\right) Y_{t+k}(z)\right]$$

$$S.t.\ Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f$$

$$\Phi_{\mathrm{t}}(z) \equiv E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left[\left(\frac{P_t^{New}(z) - P_{t+k}^w}{P_{t+k}}\right)\left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}^f\right]$$

$$\Phi_{\mathrm{t}}(z) \equiv E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left[\left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{1-\varepsilon} - \left(\frac{P_t^*(z)}{P_{t+k}}\right)^{-\varepsilon}\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\right] Y_{t+k}^f$$

$$\frac{\partial \Phi_{\mathrm{t}}(z)}{\partial P_t^{New}} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \Bigg[(1-\varepsilon)\left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon}\left(\frac{1}{P_{t+k}}\right) +$$
$$\varepsilon \left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon-1}\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\left(\frac{1}{P_{t+k}}\right)\Bigg] Y_{t+k}^f = 0$$

$$\frac{\partial \Phi_{\mathrm{t}}(z)}{\partial P_t^{New}} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon} \left[\left(\frac{1}{P_{t+k}}\right) - \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-1}\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\left(\frac{1}{P_{t+k}}\right)\right] Y_{t+1}^f = 0$$

$$\frac{\partial \Phi_{\mathrm{t}}(z)}{\partial P_t^{New}} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon} \left[\left(\frac{1}{P_{t+k}}\right) - \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{P_{t+k}}{P_t^{New}(z)}\right)\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\left(\frac{1}{P_{t+k}}\right)\right] Y_{t+1}^f = 0$$

$$\frac{\partial \Phi_{\mathrm{t}}(z)}{\partial P_t^{New}} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}}\right)^{-\varepsilon} \left[\left(\frac{P_t^{New}(z)}{P_{t+k}}\right) - \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\right] Y_{t+1}^f = 0$$

$$\frac{\partial \Phi_{\mathrm{t}}(z)}{\partial P_t^{New}} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left[\left(\frac{P_t^{New}(z)}{P_{t+k}}\right) - \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{P_{t+k}^w}{P_{t+k}}\right)\right] Y_{t+k}^f(z) = 0$$

Therefore, the optimal re-set price $P_t^{New}(z)$ solves:

$$E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}} - \frac{X}{X_{t+k}}\right) Y_{t+k}(z) = 0 \qquad 3A.24$$

where $X = \varepsilon/(\varepsilon - 1)$ is the steady-state mark-up and $Y_t \approx Y_t^f$.

Given the fraction θ of retailers do not change their price in period t, the aggregate price evolves according to:

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^{New})^{1-\varepsilon}]^{1/(1-\varepsilon)} \qquad ` \qquad 3A.25$$

Equilibrium and Market Clearing

The equilibrium of the model is a sequence of prices $\{q_t, R_t, P_t, X_t, w_{s,t}, w_{b,t}\}$ and an allocation $\{Y_t, C_{s,t}, C_{b,t}, D_{s,t}, D_{b,t}, H_{s,t}, H_{b,t}\}$ such that all first order conditions and constraint hold, and all markets are clear.

The aggregate real profit from retailers is:

$$F_t = \int_0^1 \Phi_{\mathrm{t}}(z)\, dz$$

$$F_t = \int_0^1 \left(\frac{P_t(z) - P_t^W}{P_t}\right) Y_t(z)\, dz$$

$F_t = \int_0^1 \left(\frac{P_t(z) - P_t^W}{P_t}\right)\left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f\, dz$, by using the fact that $Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f$

$$F_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{1-\varepsilon} Y_t^f\, dz - \int_0^1 \frac{P_t^W}{P_t}\left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f\, dz$$

$F_t = \left(\frac{Y_t}{{P_t}^{1-\varepsilon}}\right) \int_0^1 P_t(z)^{1-\varepsilon}\, dz - \left(\frac{P_t^W Y_t}{{P_t}^{1-\varepsilon}}\right) \int_0^1 P_t(z)^{-\varepsilon}\, dz$ since $Y_t^f \approx Y_t$

Recall, $P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{1}{1-\varepsilon}}$ from (3A.23).

Thus, we can obtain the aggregate profit in real term as

$F_t = \left(\frac{P_t - P_t^W}{P_t}\right) Y_t$, by assuming $P_t \equiv \int_0^1 P_t(z)$ 3A.26

This aggregate profit will be transferred to patient households (savers) since it is assumed that retailers are owned by them.

Goods market is clear when:

$$C_{s,t} + C_{b,t} = Y_t \qquad 3A.27$$

While housing and bond market should satisfy:

$$H_{s,t} + H_{b,t} = H = 1 \qquad 3A.28$$

$$D_{s,t} + D_{b,t} = 0 \qquad 3A.29$$

Steady State Level

We assume that the steady state of inflation rate and production technology is one in level.

Given (3A.6),

$$R_t = \frac{1}{\beta_s}$$

Given (3A.6) and (3A.10),

$$\frac{D_b}{Y} = \beta_s m \frac{qH_b}{Y}$$

Given (3A.6) and (3A.15),

$$\varphi = \frac{\beta_s - \beta_b}{C_b}$$

Given (3A.6), (3A.15) and (3A.17),

$$\frac{qH_b}{Y} = \frac{C_b}{Y}\frac{1}{1-\gamma_c}, \text{ where } \gamma_c = \beta_b + m(\beta_s - \beta_b)$$

Given (3A.18), (3A.19) and (3A.20),

$$\frac{w_b N_b}{Y} = \frac{1-\alpha}{X}$$

Given (3A.6), (3A.9), (3A.18), (3A.19) and (3A.20),

$$\frac{C_b}{Y} = \frac{(1-\alpha)/X}{1+m(1-\beta_s)/(1-\gamma_c)}$$

Given (3A.27),

$$\frac{C_s}{Y} = 1 - \frac{C_b}{Y}$$

Given (3A.28),

$$H_s + H_b = H = 1$$

Given (3A.29),

$$D_s + D_b = 0$$

Log-Linearisation around Steady State

The variable in log-linearised around its steady state is denoted by tilde e.g. $\tilde{y}_t = log(Y_t) - log(Y)$.

Savers Consumption

From (3A.6),

$$\tilde{c}_{s,t} = E_t^* \tilde{c}_{s,t+1} - (\tilde{r}_t - E_t^* \tilde{\pi}_{t+1}) \qquad 3A.30$$

From (3A.7),

$$\tilde{w}_{s,t} = \tilde{c}_{s,t} + (\eta - 1)\tilde{n}_{s,t} \qquad 3A.31$$

From (3A.8),

$$\tilde{q}_t = \tilde{c}_{s,t} + \beta_s E_t^* \left(\tilde{q}_{t+1} - \tilde{c}_{s,t+1}\right) - (1 - \beta_s)\left(\tilde{h}_{s,t} - \tilde{j}_t\right) \qquad 3A.32$$

Borrowers Consumption and Constraint

From (3A.15),

$$\tilde{c}_{b,t} + \tilde{r}_t + \tilde{\varphi}_t = \beta_b R\left(E_t^* \tilde{c}_{b,t+1} + E_t^* \tilde{\pi}_{t+1} + \tilde{\varphi}_t\right) \qquad 3A.33$$

From (3A.17),

$$\frac{j}{H_b}\left(\tilde{h}_{b,t} - \tilde{j}_t\right) = \frac{\beta_b q}{C_b} E_t^* \left(\tilde{q}_{t+1} - \tilde{c}_{b,t+1}\right) - \frac{q}{C_b}\left(\tilde{q}_t - \tilde{c}_{b,t}\right) + \varphi m q \pi E_t^* (\tilde{\pi}_{t+1} + \tilde{q}_{t+1} + \tilde{m}_t + \tilde{\varphi}_t) \qquad 3A.34$$

Combining (3A.33) and (3A.34) yields

$$\tilde{q}_t - (1 - m\beta_s)\tilde{c}_{b,t} = \gamma_c E_t^* \tilde{q}_{t+1} - \beta_b (1 - m) E_t^* \tilde{c}_{b,t+1} - m\beta_s E_t^* (\tilde{r}_t - \tilde{\pi}_{t+1}) - (1 - \gamma_c)\left(\tilde{h}_{b,t} - \tilde{j}_t\right) + (\gamma_c - \beta_b)\tilde{m}_t \qquad 3A.35$$

where at the steady state $\gamma_c = 1 - \frac{C_b}{qH_b}$

From (3A.16),

$$\tilde{w}_{b,t} = \tilde{c}_{b,t} + (\eta - 1)\tilde{n}_{b,t} \qquad 3A.36$$

From (3A.9),

$$\frac{C_b}{Y}\tilde{c}_{b,t} + \frac{qH_b}{Y}\left(\tilde{h}_{b,t} - \tilde{h}_{b,t-1}\right) + \frac{D_b}{\beta_s Y}\left(\tilde{r}_{t-1} + \tilde{d}_{b,t-1} - \tilde{\pi}_t\right) = \frac{D_b}{Y}\tilde{d}_{b,t} + \frac{1-\alpha}{X}(\tilde{y}_t - \tilde{x}_t) \qquad 3A.37$$

From (3A.10),

$$\tilde{r}_t + \tilde{d}_{b,t} = \tilde{m}_t + \tilde{q}_{t+1} + \tilde{h}_{b,t} + E_t^* \tilde{\pi}_{t+1} \qquad 3A.38$$

Intermediate Goods Firm

From (3A.18),

$$\tilde{y}_t = \tilde{a}_t + \alpha \tilde{n}_{s,t} + (1-\alpha)\tilde{n}_{b,t} \qquad 3A.39$$

From (3A.19) and using (3A.21),

$$\tilde{y}_t = \frac{1}{\alpha}\left(\tilde{x}_t + \widetilde{w}_{s,t} + \tilde{n}_{s,t}\right) \qquad 3A.40$$

From (3A.20) and using (3A.21),

$$\tilde{y}_t = \frac{1}{1-\alpha}\left(\tilde{x}_t + \widetilde{w}_{b,t} + \tilde{n}_{b,t}\right) \qquad 3A.41$$

Retailers

From (3A.21) and (3A.24), we can re-write the optimal price setting as:

$$E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} \left(\frac{P_t^{New}(z)}{P_{t+k}} - \frac{\varepsilon}{\varepsilon-1} mc_{t+k}\right) Y_{t+k}(z) = 0, \text{ where } \Lambda_{\mathrm{t,k}} = \frac{C_{s,t}}{C_{s,t+k}} \frac{P_t}{P_{t+k}}$$

$$E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} Y_{t+k}(z) \left(\frac{P_t^{New}(z)}{P_{t+k}}\right) = \left(\frac{\varepsilon}{\varepsilon-1}\right) E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \Lambda_{\mathrm{t,k}} Y_{t+k}(z) mc_{t+k}$$

Log-linearised the above equation around its steady state yields:

$$E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \tilde{p}_t^{New} = E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \left(\tilde{p}_{t+k} + \widetilde{mc}_{t+k}\right)$$

$$\tilde{p}_t^{New} = (1-\theta\beta_s) E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \left(\tilde{p}_{t+k} + \widetilde{mc}_{t+k}\right)$$

The last equation can be cast in the recursive form:

$$\tilde{p}_t^{New} = (1-\theta\beta_s)(\tilde{p}_t + \widetilde{mc}_t) + \theta\beta_s(1-\theta\beta_s) E_0^* \sum_{k=0}^{\infty} (\theta\beta_s)^k \left(\tilde{p}_{t+k+1} + \widetilde{mc}_{t+k+1}\right)$$

$$\tilde{p}_t^{New} = (1-\theta\beta_s)(\tilde{p}_t + \widetilde{mc}_t) + \theta\beta_s E_t^* \tilde{p}_{t+1}^{New} \qquad 3A.42$$

From (3A.25), `

$$\tilde{p}_t^{New} = \left(\frac{1}{1-\theta}\right)\tilde{p}_t - \left(\frac{\theta}{1-\theta}\right)\tilde{p}_{t-1} \quad 3A.43$$

Substituting (3A.43) into (3A.42) for $\tilde{p}_t^{New}$ and $\tilde{p}_{t+1}^{New}$ yields:

$$\tilde{p}_t - \tilde{p}_{t-1} = \beta_s E_t^* \tilde{p}_{t+1} - \beta_s \tilde{p}_t + \lambda \widetilde{mc}_t$$

Finally, the familiar Philip Curve equation is obtained:

$$\tilde{\pi}_t = \beta_s E_t^* \tilde{\pi}_{t+1} - \lambda \tilde{x}_t \quad 3A.44$$

where, $\lambda = \frac{(1-\theta)((1-\theta\beta_s))}{\theta}$ and $\tilde{x}_t = -\widetilde{mc}_t$

Market Clearing

From (3A.27),

$$\frac{C_s}{Y}\tilde{c}_{s,t} + \frac{C_b}{Y}\tilde{c}_{b,t} = \tilde{y}_t \quad 3A.45$$

From (3A.28),

$$\tilde{h}_{s,t} = -\frac{H_b}{H_s}\tilde{h}_{b,t} \quad 3A.46$$

From (3A.29),

$$\tilde{d}_{s,t} = -\tilde{d}_{b,t} \quad 3A.47$$

Reduced Forms

Combining (3A.31) and (3A.40) yields:

$$\tilde{c}_{s,t} = \frac{1}{\alpha}\left(\tilde{y}_t - \tilde{x}_t - \eta\tilde{n}_{s,t}\right) \quad 3A.48$$

Combining (3A.36) and (3A.41) yields:

$$\tilde{c}_{b,t} = \frac{1}{1-\alpha}\left(\tilde{y}_t - \tilde{x}_t - \eta\tilde{n}_{b,t}\right) \quad 3A.49$$

Substituting (3A.48) and (3A.49) into (3A.45) yields:

$$\left(1 - \frac{C_s}{\alpha Y} - \frac{C_b}{(1-\alpha)Y}\right)\tilde{y}_t = -\left(\frac{C_s}{\alpha Y} + \frac{C_b}{(1-\alpha)Y}\right)\tilde{x}_t - \frac{\eta C_s}{\alpha Y}\tilde{n}_{s,t} - \frac{\eta C_b}{(1-\alpha)Y}\tilde{n}_{b,t} \quad 3A.50$$

We use (3A.39) to replace $\tilde{n}_{s,t}$ in (3A.50):

$$\tilde{x}_t = -\Omega_1\tilde{y}_t + \Omega_2\tilde{n}_{b,t} + \Omega_3\tilde{a}_t \qquad \text{3A.51}$$

where,

$$\Omega_1 = \frac{1 - \frac{C_S}{\alpha Y} - \frac{C_b}{(1-\alpha)Y} + \frac{\eta C_S}{\alpha^2 Y}}{\frac{C_S}{\alpha Y} + \frac{C_b}{(1-\alpha)Y}}$$

$$\Omega_2 = \frac{\frac{\eta C_S(1-\alpha)}{\alpha^2 Y} - \frac{\eta C_b}{(1-\alpha)Y}}{\frac{C_S}{\alpha Y} + \frac{C_b}{(1-\alpha)Y}}$$

$$\Omega_3 = \frac{\frac{\eta C_S}{\alpha^2 Y}}{\frac{C_S}{\alpha Y} + \frac{C_b}{(1-\alpha)Y}}$$

Plug it back the above result into (3A.39) yields:

$$\tilde{n}_{s,t} = \Psi_1\tilde{y}_t - \Psi_2\tilde{x}_t + \Psi_3\tilde{a}_t \qquad \text{3A.52}$$

where,

$$\Psi_1 \equiv \frac{1}{\alpha} - \left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_1}{\Omega_2}$$

$$\Psi_2 \equiv \frac{1-\alpha}{\alpha\Omega_2}$$

$$\Psi_3 \equiv \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\Omega_3}{\Omega_2}\right) - \frac{1}{\alpha}$$

Given (3A.48), (3A.49), (3A.51) and (3A.52), we can replace the variable of $\tilde{c}_{s,t}$, $\tilde{c}_{b,t}$ $\tilde{n}_{b,t}$ and $\tilde{n}_{s,t}$ on some dynamic equations as follows:

From (3A.30),

$$(1-\eta\Psi_1)\tilde{y}_t - (1-\eta\Psi_2)\tilde{x}_t = (1-\eta\Psi_1)E_t^*\tilde{y}_{t+1} - (1-\eta\Psi_2)E_t^*\tilde{x}_{t+1} - \tilde{r}_t$$

$$+E_t^*\tilde{\pi}_{t+1} + \eta\Psi_3(1-\rho)\tilde{a}_t \qquad \text{3A.53}$$

From (3A.32),

$$\tilde{q}_t = (1-\eta\Psi_1)\tilde{y}_t - (1-\eta\Psi_2)\tilde{x}_t + \beta_s E_t^*\tilde{q}_{t+1} - \beta_s(1-\eta\Psi_1)E_t^*\tilde{y}_{t+1} + \beta_s(1-\eta\Psi_2)E_t^*\tilde{x}_{t+1} + (1-\beta_s)\frac{H_b}{H_s}\tilde{h}_{b,t} + \eta\Psi_3(\beta_s\rho-1)\tilde{a}_t + (1-\beta_s)\tilde{J}_t \qquad \text{3A.54}$$

From (3A.35),

$$\tilde{q}_t - \left([1-\beta_s m]\left[1-\frac{\eta\Omega_1}{\Omega_2}\right]\right)\tilde{y}_t + \left[(1-\beta_s m)\left(1+\frac{\eta_1}{\Omega_2}\right)\right]\tilde{x}_t = \gamma_c E_t^*\tilde{q}_{t+1} -$$

$$\beta_b(1-m)\left(1-\frac{\eta\Omega_1}{\Omega_2}\right)E_t^*\tilde{y}_{t+1} + \beta_b(1-m)\left(1+\frac{\eta}{\Omega_2}\right)E_t^*\tilde{x}_{t+1} - \beta_s m(\tilde{r}_t - E_t^*\tilde{\pi}_{t+1}) -$$

$$(1-\gamma_c)\tilde{h}_{b,t} + (\gamma_c - \beta_b)\tilde{m}_t + \frac{\eta\Psi_3}{\Psi_2}[(1-\beta_s m) - \beta_b(1-m)\rho]\tilde{a}_t + (1-\gamma_c)\tilde{j}_t$$

3A.55

From (3.37),

$$\left(\frac{C_b}{Y} - \frac{C_b\eta\Omega_1}{Y\Omega_2} - \left[\frac{1-\alpha}{X}\right]\right)\tilde{y}_t - \left(\frac{C_b}{Y} + \frac{C_b\eta}{Y\Omega_2} - \left[\frac{1-\alpha}{X}\right]\right)\tilde{x}_t + \frac{qH_b}{Y}\tilde{h}_{b,t} - \frac{qH_b}{Y}\tilde{h}_{b,t-1} + \frac{D_b}{Y\beta_s}\left(\tilde{r}_{t-1} + \tilde{d}_{b,t-1} - \tilde{\pi}_t\right) - \frac{D_b}{Y}\tilde{d}_{b,t} + \frac{C_b\eta\Omega_3}{Y\Omega_2}\tilde{a}_t = 0$$

3A.56

The final reduce forms are characterised by equations (3A.53), (3A.54), (3A.55), (3A.38), (3A.56) and (3A.44), along with the Taylor-type interest rate rules and macroprudential policy rules used by central bank.

Chapter 4

Robust Monetary and Macroprudential Policy Rules under Learning Rule Uncertainty

4.1. Background and Motivation

So far, we have used two criteria to pin down the preferred policy rules used by central bank when conducting both monetary and macroprudential policies. These criteria are determinacy and expectational stability (E-stability). Determinacy is used to ensure the chosen policy rule leads to unique stationary rational expectation equilibrium (REE). A rule that takes the equilibrium into non-stationary condition needs to be ruled out as it takes to an explosive path. As well, a rule that leads to multiple stationary REE (indeterminacy) should be avoided as it delivers at least two consequences: Firstly, the mechanism under which fundamental shocks propagate through the system is not easy to observe. Secondly, the existence of sunspot shocks, in addition to fundamental shocks, affects the equilibrium allocations and business cycle fluctuations.[28]

As discussed in the previous chapters, determinacy criterion is established under the assumption where economic agents, i.e. private agents and policy makers, endow with the assumption of rational expectation (RE). Under this framework, it is assumed that agents understand that they are rational and know what others know. As well, they have knowledge about the correct form of the model and all the parameters. This framework turns out to be too strong which encourages researchers to find an alternative of modelling boundedly rational agents. Among them is the application of adaptive learning in which agents follow recursive least square (RLS) learning in forming their expectation. Under this learning framework, it is

[28] See Lubik & Schorfheide (2004)

assumed that agents update their beliefs using an econometric algorithm to learn the REE (i.e. learn adaptively) as the sample size of the data set gets larger. Here, private agents nor policy makers do not know the true parameter values of the model and try to estimate them. The interest is whether this particular learning algorithm eventually takes the equilibrium path converges to unique stationary REE. The E-stability principle states that REE is asymptotically stable under RLS learning *iff* it is expectational stable (E-stable). Hence, in addition to determinacy, we consider another criterion for choosing the preferred policy rule that is E-stability.

Unfortunately, the concept of E-stability does not in general imply learnability of REE, as argued by Giannitsarou (2005) and Christev & Slobodyan (2014), among others. This is particularly true when economic agents slightly deviate from a standard RLS learning rule and use the stochastic gradient (SG) learning rule for updating their belief. The SG learning rule is different from the RLS learning rule in which the former is less efficient from econometric approach but more realistic from the view of researchers as it helps to describe the behaviour of boundedly rational agents. Technically, under the SG learning rule, economic agents are unable to observe the second moment of state variables whereby this moment will be used in forming their forecasts. In contrast, under the RLS learning rule, the variance of state variables is assumed to be known by agent precisely. As the algorithm of these two learning processes is different, then it affects a condition for the convergence of each class of these learning rules to REE. In general, a condition for the convergence of SG learning to REE is distinct from but related to the well-known stability conditions for RLS learning (E-stability).

There have been some previous studies that analyse the convergence condition of these two learning algorithms to REE and their relationships. It was started by Barruci & Landi (1997) and Heineman (2000) who conclude that E-stability is not a necessary condition for learnability if economic agents use the SG learning algorithm. This result implies that the SG learning algorithm may converge to E-unstable REE. It is further shown by Giannitsarou (2005) that E-stability is also not sufficient condition for learnability. This means there may exist E-stable REE that are not learnable by a SG learning algorithm. Note that Giannitsarou (2005) establishes her argument using an economic model with lagged endogenous variables.

Evans & Honkapohja (2001) have used a cobweb-type model to study similar issue. They conclude that SG learning converges to REE under precisely the same condition as RLS learning. Yet, they also put a concern on the lack of general results on stability and convergence of different learning algorithms. Further, Evans et al. (2010) discuss about additional conditions that ensure E-stable REE is learnable under generalized stochastic (GSG) gradient learning including SG learning. Recently, Christev & Slobodyan (2014) study learnability of E-stable REE in a standard two equations of NK model. They propose a refinement of learnability of REE that should not only E-stable but also SG-stable. In this way, the preferred policy rule is the one that is not only stable under RLS learning but also SG learning. Like E-stability principle, SG-stability principle states that REE is asymptotically stable under stochastic gradient (SG) learning *iff* it is SG-stable. If economic policy leads to a system that is SG-unstable, then it should be avoided by policy makers even if the result offers a high welfare gain under rational expectation (RE).

In summary, there are two possible relationships with regards to the convergence condition of the RLS and SG learning rule to REE: Firstly, these two conditions are identical. This implies, for a given economic policy, these learning algorithms take the system to unique stationary REE. Secondly, the convergence condition of SG learning rule is independent of the convergence condition of RLS learning. In this way, a particular economic policy that leads the system to a determinate and stable REE under the RLS (or SG) learning rule might be unstable under the SG (or the RLS) learning rule.

In the same spirit, in this chapter we conduct a refinement of learnability of REE in the context of New Keynesian (NK) model with housing market and financial constraint in which monetary policy interacts with macroprudential policy. To be specific, we extend our work in Chapter 3 by incorporating other criteria in the design of policy rule, i.e. SG-stability, in addition to determinacy and E-stability. As shown, the convergence condition of SG and RLS learning to REE is likely sensitive to the structure and the specification of economic model. Two relevant issues are addressed: Firstly, how to find robust policy rules, for both monetary and macroprudential policies, that remain stable when there is uncertainty with regards to the type of learning algorithm used by agents when updating their beliefs (between the RLS and the SG learning algorithm). Secondly, we re-examine the advantage of having more recent data in both monetary and macroprudential policies when agents' learning process is uncertain between RLS and the SG class. We remark that recent (or current) data has a better quality than forecasted data. For this purpose, we consider some variations of monetary and macroprudential policy rules as used in Chapter 3. To my knowledge, our work is the first one who studies about learnability of E-stable REE in the framework of NK model with housing market/sector. It is expected that this study enriches the existing literatures and gives a benefit for policy makers in the design of meaningful monetary and macroprudential policies.

The rest of the chapter is organized as follows: Section 4.2 represents the model used in Chapter 3. Since we have derived the model comprehensively, we only provide the reduced forms of the model in this section. Section 4.3 describes the methodology and the calibration values for the parameters in the model. Section 4.4 discusses the result of our study on robust monetary and macroprudential policy rules under learning rule uncertainty. Section 4.5 concludes this chapter.

4.2. The Model

As mentioned, we use the same model as used in Chapter 3 to find robust monetary and macroprudential policy rules. This model is based on a simple version of Iacoviello (2005) in which asset market fluctuations can have real effects to the economy through a credit channel. The model consists of patient and impatient households, wholesalers, retailers, and central bank that conducts both monetary and macroprudential policies. Households get utility from consumption goods and housing service and disutility from hourly work supplied to the wholesalers. The wholesaler firms convert this labour into intermediate goods while the retailers transform it further into final goods. The price of final goods is sticky that creates inefficiency and gives a real effect to the economy.

Housing assets are the main vehicle for accumulating wealth in the economy. Patient and impatient households are savers and borrowers, respectively. Borrowers are credit constrained and need collateral to obtain loans from savers. Savers can lend to borrowers directly but its amount is controlled by central bank through the ratio of loan to value (LTV). This LTV ratio is assumed to be operational which means if there is a tendency for a bubble in housing prices, then this ratio will be set at a lower level to bring the economy back to its stable condition. Similarly, if there is a down turn in the economy, then this ratio will be set at a higher level to stimulate the economy. Finally, central bank is assumed to follow the Taylor-type rule for setting the interest rates.

4.2.1. Structure of the Economy

We do not derive the model again here. Instead, we directly present the reduced form of the log-linearised model that characterises the structure of the economy as follows:[29]

$$(1-\eta\Psi_1)\tilde{y}_t-(1-\eta\Psi_2)\tilde{x}_t=(1-\eta\Psi_1)E_t^*\tilde{y}_{t+1}-(1-\eta\Psi_2)E_t^*\tilde{x}_{t+1}-\tilde{r}_t+E_t^*\tilde{\pi}_{t+1} \quad 4.1$$

$$\tilde{q}_t=(1-\eta\Psi_1)\tilde{y}_t-(1-\eta\Psi_2)\tilde{x}_t+\beta_sE_t^*\tilde{q}_{t+1}-\beta_s(1-\eta\Psi_1)E_t^*\tilde{y}_{t+1}+\beta_s(1-\eta\Psi_2)E_t^*\tilde{x}_{t+1}+(1-\beta_s)\frac{H_b}{H_s}\tilde{h}_{b,t}+(1-\beta_s)\tilde{j}_t \quad 4.2$$

$$\tilde{q}_t-\left([1-\beta_sm]\left[1-\frac{\eta\Omega_1}{\Omega_2}\right]\right)\tilde{y}_t+\left[(1-\beta_sm)\left(1+\frac{\eta_1}{\Omega_2}\right)\right]\tilde{x}_t=\gamma_cE_t^*\tilde{q}_{t+1}-\beta_b(1-m)\left(1-\frac{\eta\Omega_1}{\Omega_2}\right)E_t^*\tilde{y}_{t+1}+\beta_b(1-m)\left(1+\frac{\eta}{\Omega_2}\right)E_t^*\tilde{x}_{t+1}-\beta_sm(\tilde{r}_t-E_t^*\tilde{\pi}_{t+1})-(1-\gamma_c)\tilde{h}_{b,t}+(\gamma_c-\beta_b)\widetilde{m}_t+(1-\gamma_c)\tilde{j}_t \quad 4.3$$

$$\tilde{r}_t+\tilde{d}_{b,t}=\widetilde{m}_t+E_t^*\tilde{q}_{t+1}+\tilde{h}_{b,t}+E_t^*\tilde{\pi}_{t+1} \quad 4.4$$

$$\left(\frac{C_b}{Y}-\frac{C_b\eta\Omega_1}{Y\Omega_2}-\left[\frac{1-\alpha}{X}\right]\right)\tilde{y}_t-\left(\frac{C_b}{Y}+\frac{C_b\eta}{Y\Omega_2}-\left[\frac{1-\alpha}{X}\right]\right)\tilde{x}_t+\frac{qH_b}{Y}\tilde{h}_{b,t}-\frac{qH_b}{Y}\tilde{h}_{b,t-1}+\frac{D_b}{Y\beta_s}\left(\tilde{r}_{t-1}+\tilde{d}_{b,t-1}-\tilde{\pi}_t\right)-\frac{D_b}{Y}\tilde{d}_{b,t}=0 \quad 4.5$$

$$\tilde{\pi}_t=\beta_sE_t^*\tilde{\pi}_{t+1}-\lambda\tilde{x}_t \quad 4.6$$

where,

$$\Omega_1=\frac{1-\frac{C_S}{\alpha Y}-\frac{C_b}{(1-\alpha)Y}+\frac{\eta C_S}{\alpha^2Y}}{\frac{C_S}{\alpha Y}+\frac{C_b}{(1-\alpha)Y}};\ \Omega_2=\frac{\frac{\eta C_S(1-\alpha)}{\alpha^2Y}-\frac{\eta C_b}{(1-\alpha)Y}}{\frac{C_S}{\alpha Y}+\frac{C_b}{(1-\alpha)Y}};\ \Psi_1=\frac{1}{\alpha}-\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_1}{\Omega_2};\quad \Psi_2=\frac{1-\alpha}{\alpha\Omega_2};$$

[29] The economic structure are characterised by equations: (3A.53) to (3A.56), (3A.38), and (3A.44) along with monetary and macroprudential policy rules. See Appendix 3.2 in Chapter 3 for details derivation.

$\gamma_c = 1 - \frac{C_b}{qH_b}$ and $\lambda = \frac{(1-\theta)((1-\theta\beta_s))}{\theta}$

Recall that equation (4.1) corresponds to the extensive form of saver's consumption Euler equation that determines the optimal consumption choice of saver over time. Equation (4.2) describes the marginal rate of substation (MRS) between housing service and consumption goods, also in the extensive form (this equation equates the purchase price of housing to the pay-off plus the expected resale value). Equation (4.3) combines borrower's consumption Euler equation, adjusted to capture the borrowing constraint, and the marginal utility of consumption goods to shadow value of housing service. Meanwhile, equation (4.4) and (4.5) represent the borrower's financial constraint and budget constraint, respectively. Finally, equation (4.6) expresses the standard equation of Philip Curve.

In total, there are six dynamic systems of equation, excluding two equations that describe the way of how monetary and macroprudential policies are operated. This system of equations is attributed with seven endogenous state variables, i.e. output $(\tilde{y}_t)$, mark-up cost $(\tilde{x}_t)$, inflation $(\tilde{\pi}_t)$, housing prices $(\tilde{q}_t)$, credit/loan $\left(\tilde{d}_{b,t}\right)$, housing stock $\left(\tilde{h}_t\right)$ and interest rate $(\tilde{r}_t)$, plus one exogenous state variable, i.e. housing preference shock $\tilde{j}_t$. We only consider housing preference shock as the only fundamental shock in the economy since it delivers the acceleration's impact to a bubble in housing prices and the financial markets as what we have shown in Chapter 3.

After describing the structure of the model, then it is worth to spend a time to discuss about the notation of subjective expectation operator E_t^*. This notation is important since it explains the way of how economic agents, both private agents and policy makers, update their belief about future outcome, conditional on information available at time t. Under rational expectations, E_t^* corresponds to the mathematical operator E_t evaluated using the objective distributions of the stochastic shocks which are assumed known by the rational households. In our model's environment, this subjective expectation E_t^* is assumed to be equal among all economic agents that imply a structural homogeneity exhibits in the model.

The assumption of a structural homogeneity is applied here just for convenient, although under the same model, this analysis can be extended by considering the assumption of a structural heterogeneity. Under a structural heterogeneity, different agents in the economy may respond to expectation with different ways which implies both the expectations and learning rules adopted by different agents can also differ. This latter topic is dicussed comprehensively in Mitra & Honkapohja (2002). They found that the stability condition for learning is also affected by the type of heterogeneity, including structural heterogenity, although it is not always the case. For some standard models, they found that the existence of heterogeneous expectations and learning rules does not change the convergence condition to REE under homogenous expectation and learning. This means the assumption of homogenous expectation and learning algorithm is not so restrictive. However, there are also models in which heterogeneous expectations and learning rules affect the conditions for convergence of adaptive learning to REE. This result is obtained for example in a standard forward looking NK model.

Although the assumption of a structural heterogeneity is also applicable, we limit the analysis in this chapter only for a situation where the economy has structural homogeneity. This means different agents in the economy respond to the expectation in the same way and use the same learning algorithm. Nevertheless, we are uncertain about the kind of learning algorithm used by agents between the RLS and the SG learning rule. Under this condition, we need to find robust policy rules that can anticipate this uncertainty. For that purpose, we use both E-stability and SG-stability criteria when choosing preferred policy rules[30].

4.2.2. Monetary and Macroprudential Policies

Housing market volatility can have real effect to the economy through credit constraint for borrowers. In order to stabilise that condition, central bank is required to give a response to housing prices either using macroprudential or monetary policy. As other studies, we argue that managing a bubble in housing prices using macroprudential policy is more effective and realistic than using monetary policy[31]. Given this, we assume that central bank uses macroprudential policy via the ratio of LTV to stabilise a bubble in housing prices in which this ratio is assumed to be operational and can be changed at any time as a result of an increase in housing prices. In addition to credit constraint, the feature of sticky price in final goods consumption also creates other inefficiency in the economy. In this way, inflation may arises which requires central bank to consider monetary policy to stabilise the price level. We assume that central bank uses a conventional Taylor-type interest rate rule that responds to inflation and output from their steady state values, but not to any asset/financial variables.

In practice, central bank may face a constraint on data availability when conducting both monetary and macroprudential policies. Under such condition, there are two alternatives available: Firstly, when current data is available. Secondly, when current data is impossible to obtain such that it requires to being estimated or forecasted. For monetary policy, the alternatives of Taylor-type interest rate rules are as follows:

[30] Some papers use the term *"learnability of E-stable REE"*.
[31] We discuss about this argument extensively in Chapter 3.

$$r_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \qquad 4.7$$

$$\tilde{r}_t = \tau_\pi E_t^* \tilde{\pi}_{t+1} + \tau_y E_t^* \tilde{y}_{t+1} \qquad 4.8$$

where equation (4.7) refers to a contemporaneous interest rate rule and equation (4.8) corresponds to forecasted policy rule. Parameter τ_π and τ_y are the reaction function parameter of interest rate to inflation and output, respectively. While the variables with a tilde are defined in the log-linearised form around the steady state.

Central bank also conducts macroprudential policy through the LTV ratio that reacts inversely to economic condition. Since this ratio is operational, then we can view it as a rule that can be adjusted freely to counter a bubble in housing prices. In practice, when the LTV ratio is high, the collateral constraint is less tight. And since the constraint is binding, borrowers will borrow as much as they can. Lowering the LTV ratio tightens the constraint and therefore limits the loans that borrowers can obtain. Based on data availability, some alternatives of LTV rules are as follows:

$$\widetilde{m}_t = -\tau_q \Delta \tilde{q}_t \qquad 4.9$$

$$\widetilde{m}_t = -\tau_q E_t^* \Delta \tilde{q}_{t+1} \qquad 4.10$$

where equation (4.9) corresponds to the LTV rule that responds to current growth of housing prices and equation (4.10) refers to the LTV rule that reacts inversely to forecasted growth of housing prices, assuming current data of housing prices is not available. Parameter τ_q is the LTV rule reaction function to the growth of housing prices. Again, all variables are presented in log-linearised form around their steady state values (denoted by tilde).

4.3. Methodology and Calibration

4.3.1. Methodology

We begin by describing the concept of recursive least square (RLS) learning rule. This econometric concept is discussed intensively in Evans and Honkapohja (2001) and other relevant literatures. We have also discussed this concept in earlier chapters. Yet, to contrast it with the concept of SG learning rule, we explain it again.

Assume that we have some stochastic linear system of equations as follows:

$$Y_i = d'X_i + \varepsilon_i \qquad 4.11$$

where vector Y_i and X_i have a size $n\ x\ 1$ and correspond to the collection of dependent and independent variables, respectively. We fit this system of equations using time-series data within the period $i = 1, \dots, T$.

Using the formula of least square, we can obtain the value of $n\ x\ 1$ coefficient vector d that minimises the sum of residual square $\sum_{i=1}^{T} \varepsilon_i^2$ as follows:

$$d = (\sum_{i=1}^{T} X_i X_i')^{-1}(\sum_{i=1}^{T} X_i Y_i) \qquad 4.12$$

The above term can be re-written in more familiar form as:

$$d = (X'X)^{-1}X'Y \qquad 4.13$$

where Y is the $T\ x\ 1$ vector column with i-th component is Y_i and X is the $T\ x\ k$ matrix given by $X = (x_1, \dots, x_T)'$.

Motivated by Marcet & Sargent (1989b), Evans & Honkapohja (2001) have shown that vector d can also be computed using the recursive least square (RLS) formula as:

$$d_t = d_{t-1} + t^{-1}R_t^{-1}X_t(Y_t - X_t'd_{t-1}) \quad 4.14$$

$$R_t = R_{t-1} + t^{-1}(X_tX_t' - R_{t-1}) \quad 4.15$$

where d_t and R_t denote the coeficient vector and the moment matrix of X_t using data $i = 1, \dots, t$, respectively. Note that the term $(Y_t - X_t'd_{t-1})$ refers to the most recent forecast error at time t.

In order to generate the least square values that equivalent to equation (4.13), starting values for the above recursion needs to be chosen appropriately[32]. Entering these values into equation (4.14) and using data from $i = 1, \dots, t$, the usual least square coefficient vector d_t can be obtained. This result can be verified using induction process[33].

After describing the concept of RLS algorithm, we apply this concept to our learning problem in which the economy is assumed to be characterised by equations (4.1) to (4.6) and some combinations of monetary and macroprudential policy rules. Under this condition, there are seven expectational dynamic systems of equation where, for learning purposes, this system of equations can be written as:

$$S_t = B_o + B_1S_{t-1} + B_2E_t^*S_{t+1} + B_3z_t \quad 4.16$$

$S_t = \left(\tilde{y}_t, \tilde{\pi}_t, \tilde{x}_t, \tilde{q}_t, \tilde{d}_{b,t}, \tilde{h}_{b,t}, \tilde{r}_t\right)'$ is a vector of endogenous state variables while $z_t = \tilde{j}_t$ is a vector of observed exogenous state variable that follows an autoregressive (AR) process.

$$z_t = \mathcal{F}z_{t-1} + e_t \quad 4.17$$

[32] For a comprehensive explanation, see the text book *"Learning and Expectations in Macroeconomics"* by Evans and Honkapohja

[33] Consider $R_t = t^{-1}\sum_{i=1}^{t} x_i\, x_i'$ and $d_t = t^{-1}R_t^{-1}\sum_{i=1}^{t} x_i\, y_t$, this recursion process can be seen to lead to the usual least squares formula.

As discussed previously, the notation E_t^* on $E_t^* S_{t+1}$ corresponds to the mathematical operator E_t evaluated using the objective distributions of the stochastic shock which are assumed known by the rational households. This subjective expectation is assumed to be equal among agents in the economy, i.e. private agents and policy makers, which implies no heterogeneity in expectation. $\mathcal{F}$ is assumed to be invertible with all roots lie inside the unit circle. While e_t is a white noise process with zero mean and covariance matrix Σ_e. The asymptotic covariance matrix $\lim_{t\to\infty} E z_t z_t' = M_z$ is positive definite and solves the Lyapunov equation $M_z = \mathcal{F} M_z \mathcal{F}' + \Sigma_e$.

In order to evaluate the convergence condition of a particular learning algorithm to REE, we need to solve the RE model initially. As a standard solution of RE model, it has a unique RE solution of the form:

$$S_t = \bar{a} + \bar{b} S_{t-1} + \bar{c} z_t \qquad 4.18$$

which is often called the fundamentals or minimal state variable (MSV) solution. The existence of MSV solution ensures the uniqueness of the solution of RE model. Since the RE is established under the assumption that agents understand that they are rational, know what others know, and have knowledge about the correct form of the model and the true parameters, it turns out that this framework is too strong. Hence, we relax the assumption of RE and assume that all agents follow the recursive learning square (RLS) algorithm to estimate their future outcome. Under such condition, agents have a perceived law of motion (PLM) of the form $S_t = a + bS_{t-1} + cz_t$ and estimate parameters a, b and c econometrically.

At time t, the estimated PLM is given by:

$$E_t^* S_t = a_t + b_t S_{t-1} + c_t z_t \qquad 4.19$$

One step ahead of this PLM gives the forecast function as:

$$E_t^* S_{t+1} = a_t + b_t S_t + c_t \mathcal{F} z_t \qquad 4.20$$

For simplicity, we have assumed that $\mathcal{F}$ is known by agents although it can also be estimated. Any given PLM induces an actual law of motion (ALM) that gives a temporary equilibrium value of S_t. This is obtained by substituting the term $E_t^* S_{t+1}$ into equation (4.16).

For PLM estimates a_t, b_t and c_t, we obtain ALM as follows:

$$S_t = B_o + B_2(I + b) + (B_2 b^2 + B_1) S_{t-1} + (B_2(bc + c\mathcal{F}) + B_3) z_t \qquad 4.21$$

At this stage, we need to slightly modify the representation of our system of equations. We introduce the notation $w_t' = (1, S_{t-1}', z_t')$ for all state variables including exogenous state variables and $\varphi_t' = (a_t, b_t, c_t)$ for parameters.

We can re-write the PLM at time t as:

$$S_t = \varphi_t' w_t \qquad 4.22$$

and the ALM at t as:

$$S_t = T(\varphi_t)' w_t \qquad 4.23$$

where,

$$T(\varphi_t)' = \{B_o + B_2(I + b), (B_2 b^2 + B_1), (B_2[bc + c\mathcal{F}] + B_3)\} \qquad 4.24$$

The MSV of RE solution is given by the fixed point of T, that is, $\bar{\varphi}' = (\bar{a}, \bar{b}, \bar{c})$ where $\bar{a} = 0, \bar{b} = (I - B_2\bar{b})^{-1} B_1$ and $\bar{c} = (I - B_2\bar{b})^{-1}(B_3 + \mathcal{F} B_2 \bar{c})$ by assuming $(I - B_2\bar{b})$ is invertible.

Meanwhile, the RLS algorithm for estimating and updating a_t, b_t, c_t is given by:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(S_{t-1} - \varphi'_{t-1} w_{t-1})' \qquad 4.25$$

with $\gamma > 0$ is a small gain parameter and $\mathcal{R}_t$ is symmetric and positive definite matrix of second moments of the state variables. Notice that setting $\gamma = 1/t$, we have decreasing gain sequence.

Substituting in the ALM, we can write:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(T(\varphi_{t-1})' w_{t-1} - \varphi'_{t-1} w_{t-1})'$$

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}(w'_{t-1}[T(\varphi_{t-1}) - \varphi_{t-1}])$$

where,

$$\mathcal{R}_t = \mathcal{R}_{t-1} + \gamma[w_{t-1} w'_{t-1} - R_{t-1}] \qquad 4.26$$

Marcet & Sargent (1989a) show that the associated ordinary differential equation (ODE) is the vectorised version of the following ODE:

$$\frac{d\varphi}{d\tau} = \mathcal{R}^{-1} M_w [T(\varphi) - \varphi]$$

$$\frac{dR}{d\tau} = M_w - \mathcal{R}$$

We remark $M_w = diag(1, M_s, M_z)$, that is a symmetric and positive definite matrix of second moments of the state variables used by agents in forming their forecasts. They further show that the local stability of REE is entirely determined by the local stability, at the same point, of the following small ODE[34]:

[34] See Appendix 4.1 for details derivation

$$\frac{dvec\varphi}{d\tau} = vec(T(\varphi) - \varphi)$$

Assume that $L(\varphi) = dvec\big(T(\varphi)\big)/dvec\varphi$, then the relevant Jacobian is given by:

$$J^{LS}(\varphi) = \frac{dvec(T(\varphi)-\varphi)}{dvec\varphi} = L(\varphi) - I \qquad 4.27$$

The local asymptotic stability of REE $\bar{\varphi}$ under least squares learning is determined by the stability of the matrix $J^{LS}(\bar{\varphi})$: the least squares algorithm converges locally to the REE if and only if the real parts of the eigenvalues of $J^{LS}(\bar{\varphi})$ are strictly negative. These conditions are known as E-stability condition as we have applied so far in Chapter 2 and 3 previously.

Unfortunately, the above RLS learning algorithm has assumed that economic agents' perception takes the form of a forecasting model with fixed unknown parameters that they update overtime as new data become available. As a result, the above setting does not explicitly allow for parameter drift and regime switching or for model uncertainty and robustness. Therefore, Evans et al. (2013), among others, have proposed a refinement of learnability for REE by introducing more generic learning algorithm, i.e. the general stochastic gradient (GSG) learning algorithm, in which both the RLS and the SG learning algorithm are a subset of this learning algorithm.

Using the GSG learning algorithm, the value of a_t, b_t, c_t can be estimated and updated as:

$$\varphi_t = \varphi_{t-1} + \gamma\Gamma w_{t-1}(S_{t-1} - \varphi'_{t-1}w_{t-1})' \qquad 4.28$$

where $\gamma > 0$ is a small scalar gain parameter. As discussed by Evans et al. (2013), the above equation can also be viewed as an approximation to Kalman filter in which this equation serves as the maximally robust estimator when there is uncertainty about the true data-generating process and one wants to use an estimator that works well for different alternatives models.

Substituting in the ALM, we can write:

$$\varphi_t = \varphi_{t-1} + \gamma\Gamma w_{t-1}(T(\varphi_{t-1})' w_{t-1} - \varphi'_{t-1} w_{t-1})'$$

$$\varphi_t = \varphi_{t-1} + \gamma\Gamma w_{t-1}(w'_{t-1}[T(\varphi_{t-1}) - \varphi_{t-1}]) \qquad 4.29$$

which is formally a constant gain stochastic approximation or stochastic recursive algorithm. Provided a suitable stability condition is satisfied, with sufficiently small γ, the time path of equation (4.21) previously converges to a stochastic process near REE. We explicitly restrict the attention to weighting symmetric positive definite matrices Γ.

In the current case of constant-gain learning, the trajectories of the differential equation give the mean dynamics of the stochastic process. Convergence of φ_t depends, in particular, on the properties of the mapping $T(\varphi)$. For the system equation (4.21), the mean dynamics will converge locally to a fixed point $\bar{\varphi}$ of $T(\varphi)$ if $\bar{\varphi}$ is a locally stable equilibrium of the associated differential equation.

Following Barucci & Landi (1997), the associated ODE is:

$$\frac{dvec\varphi}{d\tau} = vec[\Gamma M_w(T(\varphi) - \varphi)]$$

where the relevant Jacobian is given by[35]:

$$J^{GSG}(\varphi) = \frac{dvec[\Gamma M_w(T(\varphi) - \varphi)]}{dvec\ \varphi} - I$$

$$J^{GSG}(\varphi) = \Gamma[(T(\varphi)' - \varphi')\otimes I]\frac{dvecM_w}{dvec\ \varphi} + (\Gamma M_w \otimes I).\frac{dvec(T(\varphi) - \varphi)}{dvec\ \varphi} - I$$

[35] See Appendix 4.2 for details derivation

$$J^{GSG}(\varphi) = \Gamma[(T(\varphi)' - \varphi')\otimes I]\frac{dvecM_w}{dvec\,\varphi} + (\Gamma M_w \otimes I).\,J^{LS}(\varphi) - I$$

Further, the Jacobian evaluated at $\bar{\varphi}$ will give us:

$$J^{GSG}(\varphi) = (\Gamma M_w \otimes I).\,J^{LS}(\varphi) - I, \text{ since } T(\bar{\varphi}) = \bar{\varphi} \qquad 4.30$$

Since both Γ and M_w are positive definite, their product is non singular which implies that the only equilibrium of the differential equations is the REE. GSG-stability is obtained when matrix system in equation (4.30) is stable. In other words, GSG-stability is attained if all eigenvalues of the following Jacobian matrix have negative real parts.

$$(\Gamma M_w \otimes I)\begin{pmatrix} B_2 + B_2\bar{b} & 0 & 0 \\ 0 & \bar{b}' \otimes B_2 + I \otimes B_2\bar{b} & 0 \\ 0 & 0 & \mathcal{F}B_2 + B_2\bar{b} \end{pmatrix} - I \qquad 4.31$$

In a specicial case where $\Gamma = (M_w)^{-1}$, the above GSG-stability condition is equal to E-stability condition as shown in equation (4.27). Meanwhile, when $\Gamma = I$ the above GSG-stability condition shrinks to SG-stability condition where agents are assumed to follow the SG learning rule. The SG learning rule is a good representation of how agents' boundedly rationality performs. It is simpler than the RLS learning rule so that it is much easier to follow by agents. Technically, SG-stability condition differs from E-stability condition, which governs the convergence of RLS learning, in terms of its dependency to the second moment of M_w. The fact that SG-stability depends on M_w suggests that the stability conditions depend on how the exogenous variables are measured. As point out by Evans et al. (2010)[36], SG-stability and E-stability conditions are not always the same, that is, in general neither implies the other.

[36] via their first remark on page 245.

After describing the concept of E-stability and GSG/SG-stability, we turn the attention to the concept of determinacy. Although this concept has been discussed in earlier chapters, we recall it again to build a comprehensive understanding about the topic that we discuss here. Given the model, the system of equation can be slightly modified and re-written as:

$$E_t G_{t+1} = \Omega G_t + \kappa z_t \qquad 4.32$$

where G_t represents a vector of endogenous state variables of the model including its lag and z_t governs a vector of exogenous state variables. The REE is determinate if the number of stable eigenvalues of matrix Ω is equal to the number of predetermined variables of the system. If the number of stable eigenvalues is higher than predetermined variables, then we obtain multiple equilibria (indeterminacy problem). In contrast, if the number of stable eigenvalues is lower than predetermined variables, then non-existence of locally unique stationary equilibrium is attained. The existence of locally unique stationary equilibrium is equivalent to the existence of unique solution to the model e.g. in terms of minimum state variable (MSV) solution.

4.3.2. Calibration

Most of parameters are calibrated using the same values as used in Chapter 3 in order to ensure a comparable result. These parameters and their calibrated values are as follows: the discount factor of savers β_s is 0.99 and of borrowers β_b is 0.98; the elasticity of substitution across final goods ε is 4 while the inverse of the elasticity of labour supply η is 1.01; the Calvo parameter θ is set equal to 0.75 while the saver's wage income α and the steady state of the loan to value (LTV) ratio m are set equal to 0.5 and 0.9, respectively. For the purpose of measuring the second moment of the state variables, when agents adopt a classic stochastic general equilibrium (SG) learning rule, an exogenous shock is required to be introduced in the system. As mentioned, we consider a housing demand shock since this exogenous shock has an acceleration effect to the movement of housing prices. This shock is assumed to follow an autoregressive AR (1) process as $\tilde{J}_t = \rho_j \tilde{J}_{t-1} + e_{j,t}$, where the coefficient of AR process ρ_j is calibrated at 0.9 and a white noise shock $e_{j,t}$ is assumed to have a mean zero and standard deviation σ_j at 0.05.

4.4. Robust Monetary and Macroprudential Policies

We have seen in Chapter 3 that different quality of data used in monetary and macroprudential policies may lead to different results for learnability of REE. These results are obtained by assuming agents follow the RLS learning rule in estimating their future outcome, instead of the RE. In summary, using current data in policy rule is better than using forecasted data since it may give higher probability of directing the economy to a determinate and E-stable REE. Note that, in our setting, current data are assumed to have better quality than forecasted data since they are the latest updated data available.

In this chapter, we revisit this issue again, though using more restricted definition of learnability of REE. Unlike earlier chapters that assume agents never make an error when conducting the RLS learning rule, we consider a case in which the learning process is possibly mis-specified asymptotically relative to RLS learning. The example of this situation is a case in which agents follow SG algorithm instead of RLS algorithm in estimating future outcome. Since we are unsure about the type of learning algorithm used by agents, then a preferred policy rule is the one that is robust to the specification of learning algorithm within RLS and

SG class. In other words, the chosen policy rule should not be only E-stable under RLS learning but also SG-stable under SG learning.

We start the analysis by briefly defining two policies that might be applied by central bank, i.e. macroprudential policy and monetary policy. Macroprudential policy operates through the LTV rule that reacts inversely to the movement of housing prices growth. Since there is a constraint on data availability, there are two alternatives of conducting LTV rules: (1) current data of housing prices growth are accurately available; (2) current data of housing prices growth are not available and need to be forecasted. Meanwhile, monetary policy operates through a simple Taylor-type rule that responds to inflation and output. As a case of the LTV rule, it is assumed that monetary policy is also constrained by data availability. Under such condition, there two are options available for conducting monetary policy: (1) current data of inflation and output are accurately available; (2) these data are not available and needs to be forecasted. In aggregate, there are four combinations between monetary and macroprudential policies. Notice that the LTV rule that responds to credit growth will not be considered in this analysis since we have seen earlier that this rule is inferior relatively to the LTV rule that responds to housing prices growth.

4.4.1. Current Data in Interest Rate and LTV Rules

We consider a case where interest rate and the LTV rule jointly interact as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \, \tilde{y}_t \text{ and } \tilde{m}_t = -\tau_q \Delta \tilde{q}_t \quad 4.33$$

Under this setting, it is assumed that central bank has accessed to current data of inflation, output and housing prices growth. Substituting the above two policy rules into the dynamic system of equations in equation (4.1) to (4.6), we obtain the final reduced form of equations which is characterised by seven linearised system of equations with seven endogenous state variables, i.e. $\tilde{y}_t, \tilde{x}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_{b,t}, \tilde{h}_t$ and $\tilde{r}_t$ and one exogenous variable $\tilde{j}_t$. Among the endogenous state variables, there are four predetermined state variables, i.e. $\tilde{q}_{t-1}, \tilde{d}_{b,t-1}, \tilde{h}_{t-1}$ and $\tilde{r}_{t-1}$. We put the final system of equations as equation (4.32) and follow the relevant steps for determinacy analysis. For learning analysis, we write the system as equation (4.16) and evaluate both E-stability, by checking the eigenvalues of $J^{LS}(\varphi)$ for a negative real part, and SG-stability, by checking the eigenvalues of $J^{GSG}(\varphi)$ for a negative real part. We argue that a robust policy rule is the one that is not only determinate but also E-stable and SG-stable.

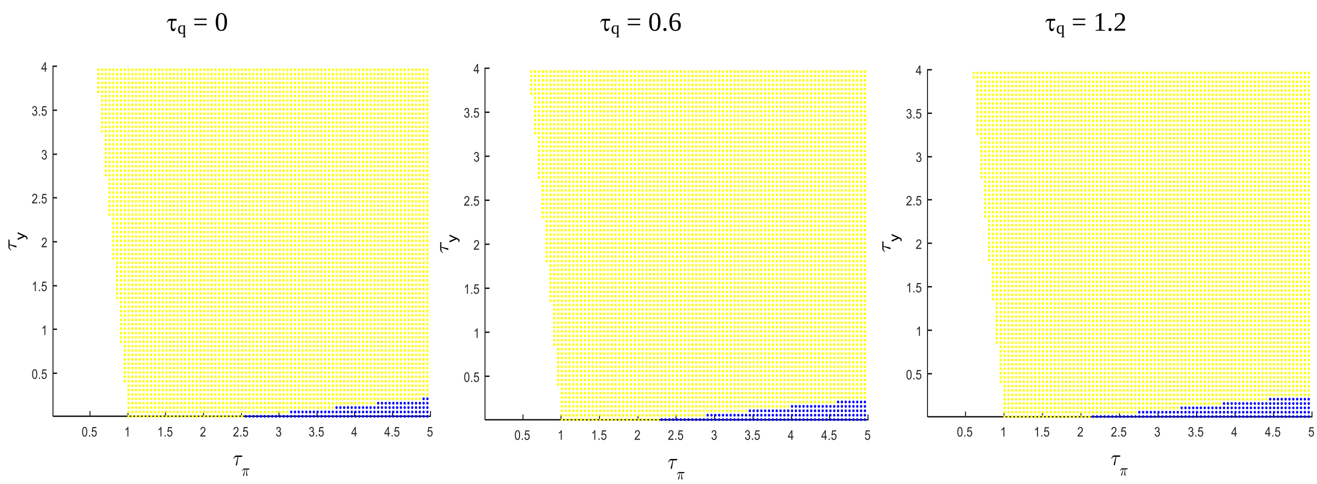
For the numerical approach, we can explain the process as follows. For each combination of interest rate and the LTV rule, we begin by assuming the LTV ratio is fixed at certain value such that it does not respond to any economic condition. This assumption implies parameter τ_q needs to be set at the value of zero since the system of equations is the form of log-linearised variables around the steady state values. Meanwhile, parameter τ_π and τ_y in policy rate are allowed to vary within a plausible range of policy parameter from 0 to 4 for τ_y and 0 to 5 for τ_π, with an increment step size is 0.05. The final numerical results are provided using two dimensional figures which make the results are easier to compare between one to another.

The same procedure is applied for different values of the LTV reaction function parameter τ_q, i.e. 0.6 and 1.2. Under this setting, the LTV rule reacts inversely to economic condition, especially to the movement of housing prices growth. A positive value of τ_q implies a percentage growth on housing prices will be dampened by a percentage reduction of the LTV ratio from its steady state value. As illustration, setting parameter τ_q equal to 1.2 implies a 10% increase in the growth of housing prices from last quarter will be tackled by a 12%

reduction in the LTV ratio from its steady state value. Since the steady state value of the LTV ratio is calibrated at 0.9, a 12% reduction causes central bank to set the LTV ratio around 0.79. This means more restrictions on the LTV ratio is imposed in order to stabilise the economic condition when housing prices bubble is observed.

After explaining the general procedure for evaluating E-stability and SG-stability and describing how our numerical task works, we arrive to a discussion about the results. When policy rate responds to current inflation and output, setting the LTV ratio at a fix value (implies τ_q is set equal to 0) is sufficient to lead the economy into E-stable REE as long as the standard Taylor principle applies[37]. This is illustrated with a yellow/grey colour in Figure 4.1. Unfortunately, this result is established under the assumption that agents follow the RLS learning rule in estimating future outcomes so that the design of policy rule depends only on E-stability criterion.

Figure 4.1
Stability results of REE under current data in interest rate rule
and current growth of housing prices in LTV rule



Blue/Dark : ESG-stable policy rules
Yellow/Grey : E-stable but SG-unstable policy rules
White : Indeterminate and E-unstable policy rules

[37] This result is discussed extensively in Chapter 3.

The way of how agents conduct their learning process can also create another uncertainty in policy design. For example, agents may follow the stochastic gradient (SG) learning algorithm, instead of the RLS learning algorithm, since it is computationally much simpler. Under such condition, we advise that a robust policy rule should not be only E-stable but also SG-stable. A policy rule that satisfies both of these criteria is called ESG-stable policy rule, whilst relevant rational expectation equilibrium (REE) that corresponds to it is called ESG-stable REE[38]. Note that we will use these terms extensively throughout this chapter.

When both SG-stability and E-stability are jointly considered, it turns out that most of E-stable policy rules become SG-unstable and only a small subset of them is SG-stable (represented by a blue/dark region in Figure 4.1). This result shows the standard Taylor principle is insufficient to ensure a robust policy rule when learning rule uncertainty arises. This finding has brought us to the conclusion that a refined criterion for learnability of REE has caused central bank is not easy to find the preferred policy rule. As the preferred rule is the one that is ESG-stable, a strong response of policy rate to inflation is required to ensure ESG-stable REE (i.e. $\tau_\pi > 2.6$). If not, central bank may take the economy to unexpected condition, e.g. indeterminate or ESG-unstable REE.

The above finding is similar to the results obtained by Christev & Slobodyan (2013). They argue that more active monetary policy is associated with SG-stability while SG-instability is concentrated in areas where τ_π is relatively low. Our work differs from them in two aspects: Firstly, in the way of how they frame the structure of the economy. Secondly, the kind of monetary policy rule is used by central bank. We frame the structure of the economy using an otherwise version of NK model with housing market and financial friction, as introduced by Iacoviello (2005). While, Christev & Slobodyan (2013) have framed it as a standard two-

[38] We use the term of ESG-stable REE and a robust learnability of REE interchangeably throughout this chapter.

equation NK model that is absent from housing market and financial constraint. In addition, we consider different combinations of monetary and macroprudential policy rules since the purpose of our study is to find a robust policy rule under learning rule uncertainty. In contrast, Christev & Slobodyan (2013) consider only one version of monetary policy rule, i.e. the expectations-based interest rate rule under commitment. In this way, central bank responds to past output, in addition to future output and future inflation.

Christev & Slobodyan (2013) found that ESG-stable policy rules associate with E-stable policy rule that delivers a high convergence speed to REE under RLS learning. In contrast, E-stable policy rule that is SG-unstable associates with a low convergence speed to REE under the same learning algorithm. Following their claim, although we do not address this issue specifically, we argue that a blue/dark region in Figure 4.1 associates with a high convergence speed to REE under RLS learning, while a region with a yellow/grey colour associates with a low convergence speed to REE under the same learning. In the following sub-section, we extend the analysis in a case where the LTV rule responds to current growth of housing prices while interest rate responds to current data of inflation and output. This experiment is illustrated in panel 2 and 3 of Figure 4.1, respectively.

In Chapter 3, we argue that it is redundant for central bank to give a response to current growth of housing prices via the LTV rule if they have used current data of inflation and output in monetary policy. However, that result holds under the assumption of the RLS learning rule in the formation of agents' expectation. When we are uncertainty about the kind of learning rule adopted by economic agents, whether it is the RLS or the SG learning algorithm, then a response to current growth of housing prices via the LTV rule is favourable for learning stability even when central bank has used current data of inflation and output in monetary policy. As can be seen in panel 2 of Figure 4.1, setting parameter τ_q equal to 0.6 has slightly increased the region of ESG-stable policy rules within policy parameter range (the blue/dark region). It follows that a lower response to inflation is required (τ_{π} > 2.3) to ensure a robust policy rule relative to a case where the LTV ratio is constant (τ_{π} > 2.6). As parameter τ_q gets higher, i.e. 1.2 (panel 3 of Figure 4.1), a lower response of policy rate to inflation is required (i.e. τ_{π} > 2.2) which implies a larger region of ESG-stability is attained within policy parameters.

In summary, there are two findings from this exercise: Firstly, the uncertainty about learning specification makes the task of central bank becomes more challenging. It is not easy now for them to direct the economy to ESG-stable REE since most of the rules that are E-stable under RLS learning turn out to be SG-unstable under SG learning. Secondly, although it still requires a relatively high response of policy rate to inflation for obtaining the desired economic condition, we conclude that the LTV rule that responds to current growth of housing prices is useful even when current data of inflation and output are used in monetary policy. This last finding is contrast to earlier results found in Chapter 3 in which a response to current growth of housing prices via the LTV rule does not improve the probability of central bank leads the economy to a determinate and E-stable REE if current data of inflation and output have been used in monetary policy. In what follows, we do a robustness check for these two findings by considering another combination between monetary policy and macroprudential policy.

4.4.2. Current Data in Interest Rate Rule and Forecasted Data in LTV Rule

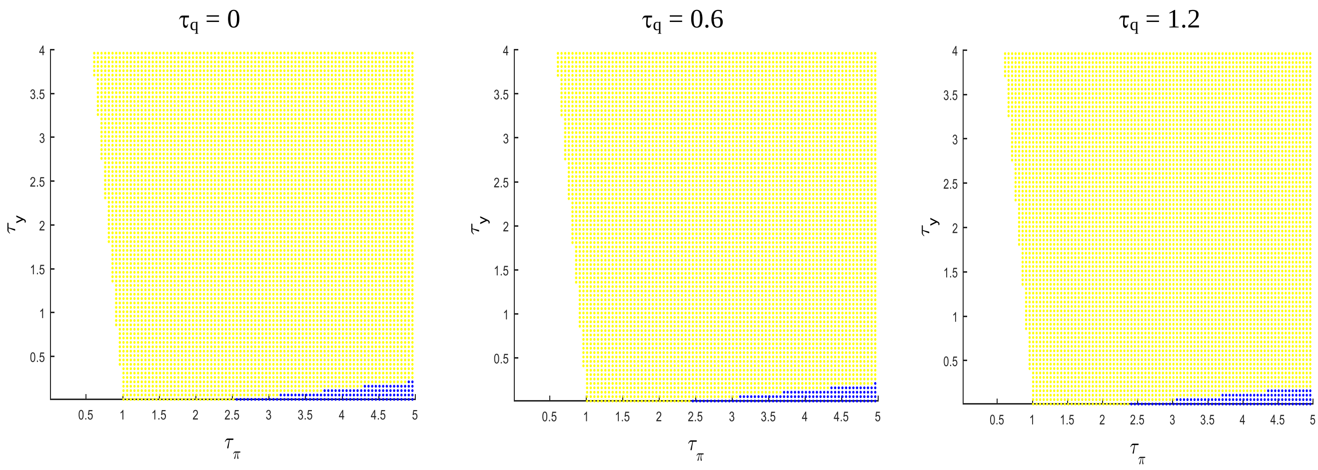
Under this setting, current growth of housing prices is not available and needs to be estimated before being used in the LTV rule. This assumption seems plausible as at the time a decision is made, most of the current data are unavailable. Meanwhile, monetary policy works in a conventional way that responds to the deviation of current inflation and output from their steady state values. This combination of policy rules can be presented as follows:

$$\tilde{r}_t = \tau_\pi \tilde{\pi}_t + \tau_y \tilde{y}_t \text{ and } \tilde{m}_t = -\tau_q \Delta E^* \tilde{q}_{t+1} \qquad 4.34$$

In the previous chapter, we have seen that a response to forecasted growth of housing prices via the LTV rule is irrelevant if central bank has used current data of inflation and output in monetary policy. This result is established under the assumption that agents in the economy follow the RLS learning algorithm in estimating future outcomes. We re-visit the same issue here, but with different environments. To be specific, we assume that agents' learning is mis-specified asymptotically relative to RLS learning such that we are uncertain about the type of learning algorithm used by agents, whether it is RLS or SG learning.

Substituting the above interest rate rule and the LTV rule into the dynamic system in equation (4.1) to (4.6), we get the reduced form of equations that is characterised by seven linearised systems of equation with seven endogenous state variables, i.e. $\tilde{y}_t, \tilde{x}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_{b,t}, \tilde{h}_t$ and $\tilde{r}_t$ and one exogenous state variable $\tilde{J}_t$. Notice that among these endogenous variables, three variables are predetermined, i.e. $\tilde{d}_{b,t-1}, \tilde{h}_{t-1}$ and $\tilde{r}_{t-1}$. As demonstrated in the previous sub-section, we present the final system of equations as equation (4.32) and follow the relevant steps for determinacy analysis. For the purpose of learning analysis, we construct the system as equation (4.16) and evaluate both E-stability and SG-stability condition. We argue that a robust policy rule is the one that is not only determinate, but also E-stable and SG-stable.

Figure 4.2
Stability results of REE under current data in interest rate rule
and forecasted growth of housing prices in LTV rule



Blue/Dark : ESG-stable policy rules
Yellow/Grey : E-stable but SG-unstable policy rules
White : Indeterminate and E-unstable policy rules

When SG-stability is considered as additional criteria in policy design, it becomes difficult for central bank to find a robust policy rule, i.e. a policy rule that is not only E-stable under RLS learning but also SG-stable under the classic SG learning. ESG-stable REE is obtained when a strong response of policy rate to inflation is imposed, as illustrated by a blue/dark region in Figure 4.2. A policy rule that is E-stable but SG-unstable should not be chosen as it takes the system to unique stationary REE with a zero probability (an area with yellow/grey colour).

From this exercise, although the effect is minimal, it can be seen that a response to forecasted growth of housing prices via the LTV rule is beneficial, especially in an environment where there is uncertainty about the type of learning algorithms used by agents. This is a useful result in which forecasted data can also be used in the LTV rule when a bubble in housing prices is observed. As illustration, setting τ_q equal to 0.6 slightly enlarges the region of EGS-stable policy rules. This can be clarify from a point where the slope of ESG-stable region (a line that splits E-stable regions into SG-stable and SG-unstable) intersects with the horizontal axis of τ_π. When the LTV ratio is constant, the minimum required value for τ_π is about 2.6 for getting ESG-stable REE. Meanwhile, when the LTV rule responds actively to forecasted growth of housing prices, e.g. by setting τ_q at 0.6, the minimum required value for τ_π reduces to around 2.5 for obtaining ESG-stable REE. This implies a slightly larger region of ESG-stable policy rules is attained when a stronger response of the LTV rule to forecasted growth of housing prices is imposed. The benefit of giving a response to forecasted growth of housing prices via the LTV rule becomes clearer when we set the LTV reaction function to housing prices growth τ_q equal to 1.2. Under this condition, the minimum required value for τ_π reduces to around 2.4 for obtaining ESG-stable of REE while at the same time policy rate is more flexible in responding to output.

Unlike a case where central bank responds to current growth of housing prices earlier, for a case where central bank responds to forecasted growth of housing prices through the LTV rule, the effect is small and hard to be seen. The reason is that there is another uncertainty that arises from using forecasted data of housing prices growth in the LTV rule. As an illustration, comparing panel 3 of Figure 4.1 and 4.2, using the same value of parameter τ_q, i.e. 1.2, a stronger monetary policy reaction function to inflation is required when forecasted data of housing prices are used in the LTV rule relative to current data of housing price are used. In panel 3 of Figure 4.2, parameter τ_π is required to be set at 2.4 (for a minimum) to obtain ESG-stable REE while in panel 3 of Figure 4.1, setting τ_π equal to 2.2 (for a minimum) is sufficient to have ESG-stable REE. We argue this small difference is important in a situation in which uncertainty arises from agents' learning algorithm. From Figure 4.1 and 4.2, we can also confirm that it is not necessary for central bank to respond to output in order to get ESG-stable REE. Instead, a strong response to inflation is required to ensure a robust policy rule that satisfies both E-stability and SG-stability.

By summarising the above findings and the findings from earlier sub-sections, we argue that a response to the growth of housing prices via the LTV rule is beneficial regardless the quality of housing prices data as long as central bank has used current data in monetary policy. Yet, the benefit is very limited. This is a nice result since in practice having access to current data of housing prices is not so easy such that most decisions by policy makers are taken using forecasted or past data.

4.4.3. Forecasted Data in Interest Rate Rule and Current Data in LTV Rule

This is the third alternative of mixing monetary and macroprudential policies. As argued several times, having access to current data is a strong assumption. In most cases, current data is not ready when a decision is made as there is a lag until it is officially published by relevant institutions. This applies not only for the data of housing prices but also for the data of inflation and output. We consider another alternative of policy combination between monetary and macroprudential policies as follows:

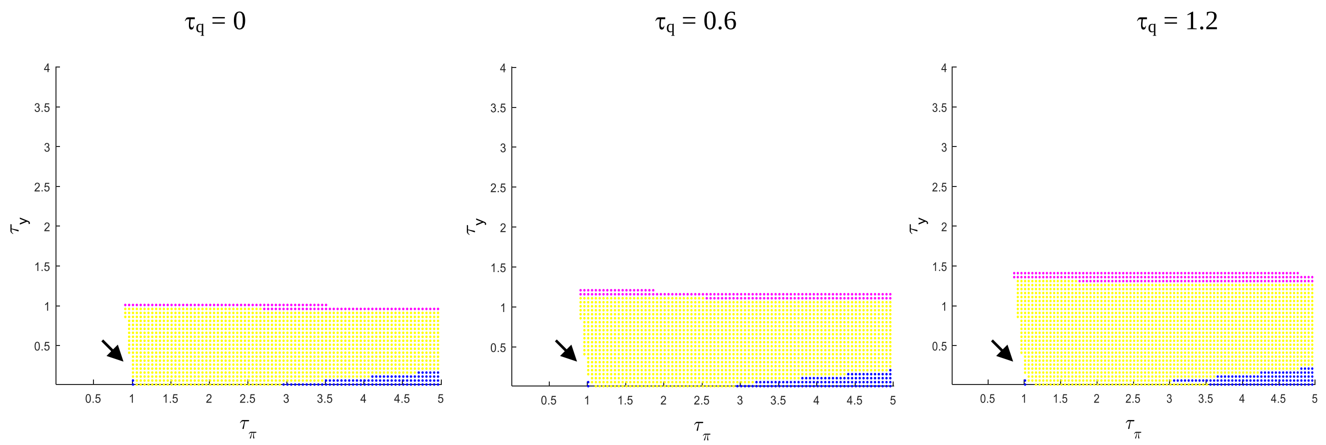
$$\tilde{r}_t = \tau_\pi E^* \tilde{\pi}_{t+1} + \tau_y E^* \tilde{y}_{t+1} \text{ and } \widetilde{m}_t = -\tau_q \Delta \tilde{q}_t \qquad 4.35$$

Under this setting, interest rate responds to forecasted deviation of inflation and output from their steady state values and the LTV rule reacts inversely to current growth of housing prices. Substituting these rules into the dynamic system of equations in equations (4.1) to (4.6), a final reduced form is obtained, which is characterised by seven linearised system of equations with seven endogenous state variables and one exogenous variable. For the analysis of determinacy, we put this final system of equations as equation (4.32) and follow relevant steps as explained in the section of methodology. Meanwhile, for the purpose of learning analysis, we write the system as equation (4.16) and evaluate both E-stability and SG-stability condition.

In Chapter 3, we have seen when central bank uses forecasted data of inflation and output in monetary policy, then a response given to current growth of housing prices via the LTV rule is favourable. When this response becomes more aggressive, then a larger region of E-stable policy rules is attained within plausible policy parameters. Notice that this result is established under the assumption that private agents and central banks follow the RLS learning rule in updating their belief. We relax the assumption of the RLS learning rule by assuming there is another type of learning algorithm that might be used by agents, i.e. SG learning. SG learning is different from RLS learning in which the first type of learning assumes that agents are unable to observe the variance of all state variables perfectly during learning process. Meanwhile, the second type of learning assumes that agents can observe the second moment of endogenous state variables precisely during learning process. Unfortunately, we are unsure about these two learning rules, which one among them is used by agents in the economy. To accommodate this uncertainty, we argue that a robust policy rule should be the one that is stable under both the RLS and the SG learning algorithm.

Adding another criterion in policy design, i.e. SG stability has caused the option for choosing the preferred policy rule becomes more restrictive. When the ratio of LTV is constant, i.e. $\tau_q = 0$, which implies no response is given to economic condition, then only a small subset of policy parameters that E-stable is also SG-stable (plotted with a blue/dark colour). This combination of policy parameters is obtained for relatively strong responses of policy rate to inflation ($\tau_\pi > 2.9$) and sufficiently low response of policy rate to output (as illustrated in panel 1 of Figure 4.3). Comparing this finding with a result produced by the first alternative of policy combination (Figure 4.1), where central bank has accessed to current data of inflation and output, we argue that a stronger reaction of policy rate on inflation is required when data used in interest rate rule is less credible. As can be seen in panel 1 of Figure 4.1, the reaction function parameter of interest rate to inflation τ_{π} only requires to be set at 2.6 (for the minimum) in order to have ESG-stable REE. While in panel 1 of Figure 4.3, this parameter needs to be set at higher value, i.e. 2.9, for a minimum. It can also be shown that most E-stable policy rules are unstable under SG-learning that causes central bank is not easy to find a robust policy rule that stable under both the SG and the RLS learning algorithm (a region with yellow/grey colour). Notice that indeterminate but E-stable policy rules turn out to be unstable under the SG learning algorithm (a region with a pink colour in Figure 4.3). While for policy parameters that are E-unstable, we do not carried out the analysis of SG-stability any more.

Figure 4.3
Stability results under forecasted data in interest rate rule
and current growth of housing prices in LTV rule



Blue/Dark : ESG-stable policy rules
Yellow/Grey : E-stable but SG-unstable policy rules
White : Indeterminate and E-unstable policy rules
Pink : Indeterminate and E-stable but SG-unstable policy rules

Is a stronger response of the LTV rule to current growth of housing prices beneficial for the whole economy when agents' learning is mis-specified asymptotically relative to RLS learning? We answer this question by repeating the numerical work for different values of parameter τ_q at 0.6 and 1.2, respectively. As shown in panel 2 and 3 of Figure 4.3, a higher response of LTV rule to current growth of housing prices, indeed, increases the region of ESG-stable policy rules. As an illustration, when the LTV reaction function to current growth of housing prices is set equal to 0.6, a more flexible response of monetary policy to output τ_y is attained for leading the system to ESG-stable REE, compared to a case where no response of the LTV rule is given to housing prices growth. At the same time, monetary reaction function to inflation τ_π does not alter. This result implies a larger ESG-stable region is obtained when LTV reaction to current growth of housing prices gets stronger. As parameter τ_q increases from 0.6 to 1.2, ESG-stable region enlarge even further although the change is relatively small such that it is hard to be seen (illustrated in panel 3 of Figure 4.3). We summarise that a response to current growth of housing prices via the LTV rule is beneficial even when current data of inflation and output are not observed.

It is also interesting to discuss about a possible ESG-stable policy rule that is attained when interest rate reaction function to inflation τ_π is set equal to 1 with no or low response of policy rate is given to output. We argue that these policy parameter combinations are important and should be considered in designing preferred policy rules that are robust to learning rule uncertainty. As shown in Figure 4.3, there are a few plots of ESG- stable policy rules that consistently appear around $\tau_\pi = 1$ although parameter τ_q has been varied within the range 0 to 1.2. If this finding is valid, then it is unnecessary for central bank gives a strong response of policy rate to inflation in order to get a robust learnability of REE. A combination of policy parameters that ensures $\tau_\pi = 1$, with a sufficiently low response to output gap, may lead to the desired REE. This is an important finding since our argument is established under the assumption that there is uncertainty about learning algorithm used by agents and a difficulty in obtaining current data in monetary policy (in fact, this is a realistic assumption). We reconfirm this last finding by considering another combination of policy parameters between monetary and macroprudential policies in the following sub-section. To be specific, we assume now that all current data are not available and need to be forecasted in both monetary and macroprudential policies.

4.4.4. Forecasted Data in Interest Rate and LTV Rule

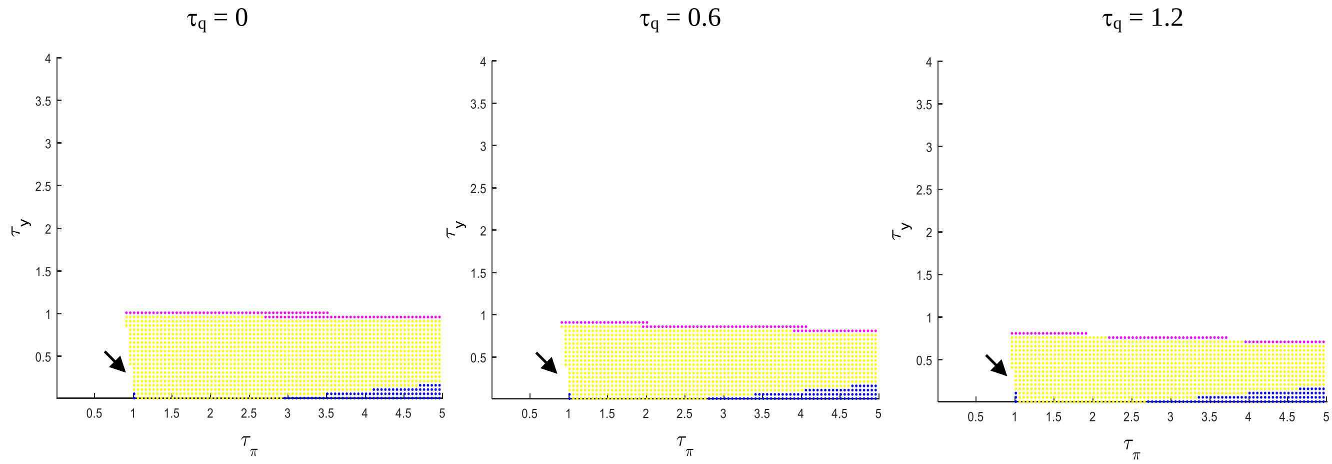
In this scenario, we assume that all relevant data, i.e. inflation, output and housing prices are not available in current period so that central bank needs to forecast them. This condition is the most realistic assumption as at the time a decision is made it is almost impossible to have access to current data. We consider a combination of policy rules as follows:

$$\tilde{r}_t = \tau_\pi E^* \tilde{\pi}_{t+1} + \tau_y E^* \tilde{y}_{t+1} \text{ and } \tilde{m}_t = -\tau_q \Delta E^* \tilde{q}_{t+1} \qquad 4.36$$

where interest rate responds to forecasted data of inflation and output while the LTV rule responds to forecasted growth of housing prices.

In earlier chapter, we have shown that a response to forecasted growth of housing prices via the LTV rule is not useful since it reduces the probability of central bank leads the economy to E-stable REE. This argument is established under the assumption that forecasted data is unreliable and invalid to be used. To some extent, this argument is too strong since it gives no room for improvement if we know that the data used in policy rules are not the recent ones. Ideally, we may expect that although data are not recent, giving a response to forecasted growth of housing prices should be beneficial, relative to a case where no response is given by the LTV rule if a bubble in housing market exists.

Figure 4.4
Stability results under forecasted data in interest rate rule
and forecasted growth of housing prices in LTV rule



Blue/Dark : ESG-stable policy rules
Yellow/Grey : E-stable but SG-unstable policy rules
White : Indeterminate and E-unstable policy rules
Pink : Indeterminate and E-stable but SG-unstable policy rules

We re-evaluate the consequence of using forecasted data in both monetary and macroprudential policies with a slightly different environment, that is the uncertainty of learning rule used by agents (between SG and RLS learning rule). Having this limitation, the preferred policy rule is the one that satisfies both E-stability and SG-stability principle. The result of this numerical simulation is reported in Figure 4.4. Similar to other scenatios, only a small subset of policy parameters that E-stable is also SG-stable (represented by a blue/dark region). Most of E-stable policy rules turn out to be not stable under the SG learning algorithm. In addition, all indeterminate but E-stable policy rule becomes SG-unstable under SG learning. Having this result, it becomes clear that the task of central bank gets more difficult when there is uncertainty with regards to the kind of learning algorithm used by agents, regardless the type of rules used in both monetary and macroprudential policies. However, the question that remains is how does this result differ from other scenarios?

As illustrated in panel 2 and 3 of Figure 4.4, the LTV rule that responds to forecasted growth of housing prices does increase ESG-stable region within plausible policy parmaters. This is an interesting result as we can show that the argument that says it is irrelevant to give a response to forecasted growth of housing prices via the LTV rule is not entirely true. An increase of the LTV reaction function parameter τ_q from 0 to 1.2 unarguably enlarges the ESG-stable region within policy parameters. As an illustration, when the LTV rule does not actively respond to economic condition, i.e. $\tau_q = 0$, it is required a strong response of policy rate to inflation ($\tau_\pi > 2.9$) in order to get a robust learnability of REE. As parameter τ_q increases from 0 to 1.2, a lower response of policy rate to inflation is required to obtain a robust learnability of REE ($\tau_\pi > 2.75$). This latter result implies a higher probability of leading the economy to ESG-stable REE.

The above numerical result also shows that a benefit of using forecasted data in the LTV rule is less significant relative to the case where current data is used in the LTV rule. Yet, it confirms our intuition whereas a response of the LTV rule to housing prices growth should be beneficial even when the data used in the LTV rule is the forecasted one. This happens since a credit channel mechanism and a housing preference shock in the model have ensured the existence of housing prices growth such that a response to the growth of housing prices via the LTV rule is required, regardless the quality of housing data owned. As an illustration, the intersection of ESG-stable region that splits ESG-stable policy with E-stable but SG-unstable policy in panel 3 of Figure 4.4 is smaller than a similar region in panel 3 of Figure 4.3. Although the difference is very small, we argue that this difference matters in the context of learning uncertainty.

The result from Figure 4.4 also confirms the existence of ESG-stable policy rules when monetary policy reaction function to inflation τ_π is equal to 1 and on output τ_y is sufficiently low. Varying the value of parameter τ_q within 0 to 1.2 does not remove the existence of these blue/dark dots around $\tau_\pi = 1$ (note that a blue/dark dot represents ESG-stable policy rule). This means a robust learnability of REE may be obtained when interest rate responds to forecasted inflation around $\tau_\pi = 1$ and to forecasted output at sufficiently low values, whilst at the same time the LTV rule responds to forecasted growth of housing prices. But this decision should be taken with care since it may easily falls to indeterminate and E-stable equilibrium (an area with a white colour) or E-stable but SG-unstable equilibrium (an area with a yellow/grey colour).

4.5. Conclusion

This chapter studies a refinement of learnability of REE in the framework of New Keynesian model with housing market and financial constraint. In addition to E-stability, we consider SG-stability criteria in the design of both monetary and macroprudential policies. Two relevant issues are addressed: Firstly, how to find robust monetary and macroprudential policy rules that remain stable even when there is uncertainty concerning the type of learning algorithms used by agents (between RLS and SG learning). Secondly, is having access to current data in monetary and macroprudential policies beneficial when the learning process is uncertain between the RLS and the SG learning algorithm.

With regards to the first issue, the results show that a refined criterion for learnability of REE has made central bank’s task becomes not easy. The standard Taylor principle is insufficient to ensure a robust learnability of REE when the uncertainty in learning rule arises between the RLS and the SG learning rule. In general, it is required a strong response of policy rate to inflation to ensure ESG-stable REE. Otherwise, central bank may take the economy to unexpected equilibrium e.g. indeterminate, E-unstable or SG-unstable REE. Concerning the second issue, we argue that the quality of data remains matter in an environment where the uncertainty in learning rule arises. Yet, this claim does not mean that there is no room for improvement if current data is not available. It can be shown that using forecasted data of housing prices growth in the LTV rule is also beneficial when bubble in housing prices is observed. However, this benefit is smaller relative to a response given to current data of housing prices growth in the LTV rule. This is an interesting finding as in reality having access to current data is almost impossible such that forecasted data is more commonly used by policy makers.

In the next chapter, we deviate from a calibrated model as we use so far (from Chapter 2 to Chapter 4) and consider an estimated model for the analysis of monetary policy. Although calibration approach helps researchers and policy makers in evaluating policy implications, under reasonable assumptions of parameters values, there are some limitations on this approach, especially when the model gets larger and complex. A calibrated model is also less powerful in characterising a specific country's economic condition since no data series is employed and entered to the model. Given these limitations, in next chapter, we focus the analysis on the estimated model by employing a Bayesian technique. A Bayesian technique is very useful in solving the limitations of the calibration approach since it offers a formal way to estimate parameters by combining prior information about parameters with the data, conditional on the restriction of the model of interest.

Appendix 4.1. E-Stability Condition

The recursive least squares algorithm is given by:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}[S_{t-1} - \varphi'_{t-1} w_{t-1}]'$$

Substituting into the ALM, we can write:

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}[T(\varphi_{t-1})' w_{t-1} - \varphi'_{t-1} w_{t-1}]'$$

$$\varphi_t = \varphi_{t-1} + \gamma \mathcal{R}_{t-1}{}^{-1} w_{t-1}[w'_{t-1}(T(\varphi_{t-1}) - \varphi_{t-1})]$$

where,

$$\mathcal{R}_t = \mathcal{R}_{t-1} + \gamma[w_{t-1} w'_{t-1} - \mathcal{R}_{t-1}]$$

Marcet & Sargent (1989) show that the associated ordinary differential equation (ODE) is the vectorised version of the following ODE:

$$\frac{d\varphi}{d\tau} = \mathcal{R}^{-1} M(\varphi)[T(\varphi) - \varphi]$$

$$\frac{dR}{d\tau} = M(\varphi) - \mathcal{R}$$

where $M(\varphi) = \lim_{t\to\infty} E w_t(\varphi)\, w_t(\varphi)'$.

They further show that the local stability of rational expectations equilibrium is entirely determined by the local stability, at the same point, of the following small ODE:

$$\frac{dvec\varphi}{d\tau} = vec(T(\varphi) - \varphi)$$

Let $L(\varphi) = dvec\big(T(\varphi)\big)/dvec\varphi$

The relevant Jacobian is given by:

$$J^{LS}(\varphi) = \frac{dvec(T(\varphi) - \varphi)}{dvec\varphi} = L(\varphi) - I$$

The local asymptotic stability of REE under least squares learning is determined by the stability of the matrix $J^{LS}(\bar{\varphi})$: the least squares algorithm converges locally to the REE *iff* the real parts of the eigenvalues of $J^{LS}(\bar{\varphi})$ are strictly negative. The conditions are also known as the E-stability condition (see Evans and Honkapohja (2001)).

Appendix 4.2. GSG-Stability Condition

The generalised stochastic gradient learning is given by:

$$\varphi_t = \varphi_{t-1} + \gamma \Gamma w_{t-1}[w'_{t-1}(T(\varphi_{t-1}) - \varphi_{t-1})]$$

Following Barucci & Landi (1997), the associated ODE of the above system is:

$\frac{dvec\varphi}{d\tau} = vec[\Gamma M_w(T(\varphi) - \varphi)]$, where $M_w = diag(1, M_s, M_z)$

And the relevant Jacobian is:

$$J^{GSG}(\varphi) = \frac{dvec[\Gamma M_w(T(\varphi)-\varphi)]}{dvec\ \varphi} - I$$

First note that:

$$\begin{aligned}
dvec[\Gamma M_w.(T(\varphi) - \varphi)] &= vec\ \Gamma d[M_w(T(\varphi) - \varphi)] \\
&= vec\ \Gamma[dM_w(T(\varphi) - \varphi) + M_w d(T(\varphi) - \varphi)] \\
&= vec\ \Gamma[I.dM_w(T(\varphi) - \varphi)] + vec[M_w d(T(\varphi) - \varphi).I] \\
&= [\Gamma(T(\varphi)' - \varphi')\otimes I].dvecM_w + (\Gamma M_w \otimes I).dvec(T(\varphi) - \varphi)
\end{aligned}$$

Using the above result, we can re-write the Jacobian matrix as:

$$\begin{aligned}
J^{GSG}(\varphi) &= \frac{dvec[\Gamma M_w(T(\varphi)-\varphi)]}{dvec\ \varphi} - I \\
&= \Gamma[(T(\varphi)' - \varphi')\otimes I].\frac{dvecM_w}{dvec\ \varphi} + (\Gamma M_w \otimes I).\frac{dvec(T(\varphi)-\varphi)}{dvec\ \varphi} - I \\
&= \Gamma[(T(\varphi)' - \varphi')\otimes I].\frac{dvecM_w}{dvec\ \varphi} + (\Gamma M_w \otimes I).\ J^{LS}(\varphi) - I
\end{aligned}$$

Further, the Jacobian evaluated at $\bar{\varphi}$ is given by:

$(\Gamma M_w \otimes I).\ J^{LS}(\varphi) - I$, since $T(\bar{\varphi}) = \bar{\varphi}$

Both Γ and M_w are positive definite. Given this property, their product is non-singular which implies the only equilibrium of the differential equation is the REE. We say matrix $J^{GSG}(\varphi)$ is stable if all its eigenvalues have negative real parts.

Appendix 4.3. Some Useful Matrix Operations

$vec(ABC) = (C' \otimes A).vec(B)$

$d(AYB) = A(dY)B$

$dvec\ y = vec\ dy$ and $d(vec\ X) = vec\ dX$

$d(AX + B) = A(dX)$

$d(AX^2) = A\big((dX)X + XdX\big)$

where dX and dy stand for the differential of matrix X and of the vector y, respectively. The products of all matrices are comformable.

Chapter 5

Monetary Policy, Lending Decision and Business Cycles in Indonesia

5.1. Background and Motivation

In earlier chapters, we have studied the issue of monetary and macroprudential policies in which economic agents deviate from the framework of a rational expectation (RE) and follow a particular learning algorithm in forming their expectation. Under this assumption, the preferred policy rule becomes more restrictive since the expected equilibrium must not only be unique (or determinate) but also stable under learning assumption. So far, we have studied the issue of learning stability in two different versions of New Keynesian (NK) model: Firstly, a standard NK model with the feature of consumption habit where agents in the economy follow the recursive least square (RLS) learning rule in making a forecast, as discussed in Chapter 2. Secondly, an otherwise version of NK model featuring housing market and liquidity constraint in which agents endow with the RLS learning algorithm when estimating the structural parameters, as studied in Chapter 3. The latter model is also used in Chapter 4 in which it is assumed that there is uncertainty with regards to the type of learning rules adopted by economic agents, whether it is RLS or SG learning.

The result from Chapter 2 shows that as the parameter of habit gets larger, determinate and E-stable policy rules, under plausible policy parameters, become easily to obtain. This happens as the feature of habit in consumption makes output less volatile, although at the cost of a slightly higher inflation, such that it becomes easier for economic agents to learn the rational expectation equilibrium (REE). Meanwhile, the result from Chapter 3 suggests that monetary authority needs to use more recent data in policy rules (either in monetary policy or macroprudential policy) to ease their work in finding determinate and E-stable policy rules. This last work is then extended, by assuming there is uncertainty with regards to the type of learning rules used by agents, as discussed in Chapter 4. The result from this chapter shows that it becomes difficult for central bank to lead the economy to a determinate and stable REE when both criteria of E-stability and SG-stability are jointly considered to find the best policy rule.

All the above analyses have been done under the assumption that the models are calibrated to those estimated in previous studies and/or those computed using long-run average data. Although the calibration approach helps researchers and policy makers in evaluating policy implications, under reasonable assumptions about parameters values, there are some limitations on this approach, especially when the model gets larger and complex e.g. for the purpose of addressing mis-specification issues and presenting more realistic features found in empirical data. Under such condition, it becomes less obvious how all deep parameters in the model are calibrated (see Beltran & Draper (2008)). A calibrated dynamic stochastic general equilibrium (DSGE) model is also less powerful in characterising a specific country's economic condition since no data series is employed and entered to the model. As well, this type of model is not always robust to alternative calibrations such that a slightly different parameter value may change the result significantly. Motivated by these limitations, in this chapter, we focus the analysis on the estimated model, instead of the calibrated one, by employing a Bayesian technique. Base on the literature, a Bayesian technique is very useful in solving the limitations in the calibration approach as it offers a formal way to estimate the parameters by combining prior information about parameters with the data, conditional on the economic model. As well, this technique offers a framework for the design of economic policy that is robust to the uncertainty around the parameters of interest (see for example: Levin et al. (2004) and Levin et al. (2005)).

A Bayesian technique is used for estimating a DSGE model for Indonesia. According to the literature, there are not many studies that try to establish a specific DSGE model for this country. The most related one is Ramayandi (2009) who examines the suitability of a simple structural small open economy model in five ASEAN economies (Indonesia, Malaysia, Philippines, Singapore, and Thailand) using the maximum likelihood estimation (MLE) approach. Motivated by this, we build and propose an alternative DSGE model for Indonesia by combining the version of a small open economy model of Gali & Monacelli (2005) with the version of housing market and financial constraint model of Iacoviello (2005). To make it more realistic, the model also incorporates the features of incomplete exchange rate pass-through, following Monacelli (2005), and the Calvo (1983) staggered price setting in domestic and import price. Our work is different from Ramayandi (2009) in the sense that the latter does not bring the feature of housing market and borrowing constraint into the model and the feature of incomplete exchange rate pass-through as used in Monacelli (2005).

On the study of DSGE model for Indonesia, there are at least four objectives that we want to achieve: Firstly, proposing an alternative DSGE model where its features and assumptions fit with the Indonesian economy characteristics. Secondly, estimating and analysing banks' lending decisions in Indonesia. This objective is motivated by the fact that the ratio of loan to value (LTV) or down payment (DP) has not been explicitly regulated by Bank Indonesia (BI) until 2012. Having knowledge about this lending parameter, we are allowed to evaluate the current setting of macroprudential policy. Thirdly, analysing the main objective of monetary policy in Indonesia through the estimation of monetary reaction function parameter. The implementation of inflation targeting framework (ITF) has raised our curiosity concerning the way of how BI conducts its monetary policy. We are interested whether BI puts higher concern on the variability of inflation, output or exchange rate. Lastly, analysing the determinant of business cycle fluctuations in Indonesia, including housing prices volatility. Recent development shows that housing prices in Indonesia has shown a significant increase, especially in some major cities in Indonesia.

In regards to the issue of banks' lending decisions, the estimation result suggests that banks in Indonesia are likely to set higher ratio of down payment (DP) for households who have only housing assets in collateral than the ones who can show both her income and housing assets when making a housing loan application. The result also shows that Bank Indonesia (BI) has consistently adopted the Taylor principle in their monetary policy which implies price stabilisation is among their main targets. As opposed to the behaviour of central bank in developed countries, BI policy rate is also occasionally used to promote economy growth but rarely used to stabilise the variability of exchange rate. Concerning the degree of economic dependency, the estimation result clarifies that the Indonesian economic dependency is not as high as predicted since the proportion of import goods in consumption goods bundle is less than 10%.

In respect to the issue of business cycles, the result confirms that there are three main drivers of the Indonesian economy, i.e. non-stationary permanent technology shock, monetary policy shock and marginal efficiency of investment (MEI) shock. The important role of non-stationary permanent technology shock is in line with Aguiar & Gopinath (2007), among others. Meanwhile, a significant contribution of monetary policy shock follows a similar result found by Aspachs-Bracons & Rabanal (2010) for the case of Spain and Euro area. As well, a large contribution of MEI shock to business cycles supports a claim argued by Justiano et al. (2010) for the case of the US during post-war period. The estimation result also shows that housing preference shock is the main disturbance of housing prices variation. Surprisingly, the contribution of monetary policy shock and loan to value (LTV) shock on the variability of housing prices is minimal.

The rest of this chapter is organized as follows. Section 5.2 briefly discusses the recent development in the implementation of macroprudential policy and the ITF in Indonesia. Section 5.3 describes our proposed open-economy DSGE model, giving proper consideration to the Indonesian's characteristics. Section 5.4 discusses the methodology and the data used in estimating the model. Section 5.5 presents the estimation results and evaluates the impacts of various structural innovations to the Indonesian economy. This section also discusses about a mechanism under which these innovations affect the Indonesian economy through the analysis of the impulse response function (IRF). Section 5.6 conducts a robustness check by evaluating the role of news shocks in the formation of the agents' expectation. The conclusion of this chapter is summarised in Section 5.7.

5.2. Macroprudential Policy and ITF in Indonesia

Since January 1st, 2014, the role of banking supervision in Indonesia has been delegated from Bank Indonesia (BI) to a new institution called the Indonesian Financial Service Authority (OJK). This delegation is formalised through the 2011 OJK Act that integrates the supervision of banking sector, non-banking financial industry and capital market to minimise frauds in financial service industries. Under this new law, the role of BI has slightly changed in which they focus now more on the financial system stability, through macroprudential policies, in addition to their role as the authority of monetary and payment system. Meanwhile, for microprudential policies (i.e. the supervision of each individual bank), which were used to be under BI's mandate, are delegated to the OJK.

As part of their role in ensuring a financial stability, since March 2012, BI has released a macroprudential regulation that controls the maximum ratio of loan to value (LTV) or down payment (DP) for housing/property loan. Currently, the ratio of LTV for property loan in Indonesia is bounded above within the range 60-90% of asset values in which the exact value depends on the types of houses, banks and credit facilities that borrowers currently have[39]. Before March 2012, BI does not explicitly control this lending parameter and gives flexibility to commercial banks in deciding the amount of loan given to borrowers based on the 5Cs principle (i.e. character, capacity, capital, collateral and condition). As the result, there is a variation with regards to the LTV ratio among banks. For a bank that has high target of credit growth, they may be very loose such that the LTV ratio can be very high. This condition may get worst when the behaviour of moral hazard occurs among banks' employees such that they try to offer a loan, as much as they can, but ignoring the quality of this loan simply to get high bonuses. Motivated by this, we are interested in estimating the ratio of LTV for housing loan among banks in Indonesian during the period 2000-2014. Although, BI has started to control the maximum ratio of this lending parameter since early 2012, we argue that there is still a room for banks to set this ratio below its maximum value. Therefore, our choice in regards to the period of observation remains valid in the estimation.

BI has also adopted the Inflation Targeting Framework (ITF) since 2000, although it is officially acknowledged in July 2005. Under this framework, a policy rate is used as an instrument to guide market's expectation. Before this period, one month BI certificate was

[39] Effective from June 18th 2015, Bank Indonesia has released a new requirement concerning the LTV ratio of housing loan. This macroprudential policy is made in an effort to maintain the momentum of economic growth in Indonesia.

used as a policy rate, instead of BI policy rate. Fortunately, the behaviour of these two interest rates is very close so that we can define them together as a policy rate in the model. The implementation of the ITF brings a curiosity about the way of how BI responds to inflation, output and exchange rate through interest rate. To address this issue, we estimate the parameters of monetary reaction function in the interest rate rule using our data sample.

5.3. Theoretical Model

The model features a small open economy model of Gali & Monacelli (2005) and a housing market and borrowing constraint model of Iacoviello (2005). Following Monacelli (2005), the feature of imperfect exchange rate pass-through is also introduced. A number of nominal and real rigidities are incorporated, i.e. the Calvo-type staggered price settings in both domestic and import consumption goods and the adjustment cost of physical capital and of housing. All these features are relevant in characterising the Indonesian economy as a developing country.

We start assuming the world economy is constituted by a continuum of small economies where each of them is populated by infinitely lived entrepreneurs, bundle firms, households and retailers, of measure one, plus a central bank. Based on its discount rate, there are two types of households in the economy, i.e. patient and impatient households. Patient households refer to a group of households that has lower discount rate than entrepreneurs, while impatient households correspond to a group of households that has higher discount rate than entrepreneurs. Households get utility from consuming a bundle of consumption goods and housing service, and disutility from supplying labour to entrepreneurs. Patient households have an option to save in domestic and foreign bonds and their role is as a lender in the economy. In contrast, impatient households only can borrow domestically without having access to foreign bond market. At the end of each period, both types of households decide how much to invest in housing, constrained by the adjustment cost of changing these assets.

Unlike households, entrepreneurs only get utility from consuming a bundle of consumption goods. They produce a homogenous good using physical capital, labour from both types of households and housing stock. Similar to impatient households, entrepreneurs can only borrow domestically without having access to foreign bond market. We assume that there is no housing goods producer in the model which means the stock of housing is always constant. This assumption seems realistic as housing stock is a slow moving variable compared to other variables in the economy. Yet, we allow for the shifting of housing ownership among entrepreneurs and households via borrowing and lending activities. At the end of each period, entrepreneurs decide how much to invest in housing and physical capital, conditional on the adjustment cost of changing these assets.

Motivated by the interest of studying banks' lending decisions in Indonesia, we assume that there is a limit on the amount of loan for entrepreneurs and impatient households. Note that in the model, we do not explicitly describe the behaviour of bank. Instead, we artificially model bank as a purely financial intermediary agent that transfers loans from lenders (patient households) to borrowers (impatient households and entrepreneurs) without taking any profit for doing this. For entrepreneurs, the amount of loans that they can borrow is bounded above by the expectation of housing prices minus some proportion of their housing assets that cannot be used as collateral. This assumption follows the recent strand of literature introduced by Kiyotaki & Moore (1997). For impatient households, we slightly deviate from the original model of Iacoviello (2005) where it is assumed that banks do not only consider future price of housing when approving the amount of housing loan but also the applicant's wage income. Given this assumption, we are allowed to analyse the importance of wage income relative to housing assets in collateral when banks make a decision about lending. Having different types of borrower, i.e. impatient households and entrepreneurs also has allowed us to learn the way of how banks treat different agents in the economy.

The way of how we model a borrowing constraint for impatient households is similar to Gelain et al. (2013), with an exception concerning the agents who can control these lending parameters. In Gelain et al. (2013), it is assumed that central bank controls the ratio of LTV and loan to income (LTI) when the borrowers want to get a loan from the lenders (patient households)[40]. Unlike their work, in our model, it is assumed that these parameters are set by an individual bank without any intervention from monetary authority. This assumption seems more realistic in the context of Indonesia, since a policy that controls the ratio of LTV or DP has not become an attention of central bank until the beginning of 2012.

Following Gali & Monacelli (2005), we assume that a bundle of goods consumed by households and entrepreneurs consists of domestic and foreign (import) goods. Considering a sufficient amount of foreign goods in aggregate consumption in Indonesia, the model allows foreign goods to enter a bundle of goods consumed. We also introduce price stickiness through retailers' activity for domestic goods and, as Monacelli (2005), import firms' activity for foreign goods. Finally, the model is closed by specifying a monetary reaction function adopted by central bank. It is assumed that the Taylor-type interest rule is employed to stabilise the Indonesian economy, given some contemporaneous exogenous shocks.

[40] The LTV ratio is the ratio of a loan to the value of an asset purchased while the LTI ratio is the ratio of a loan to wage income.

5.3.1. Entrepreneurs

A representative entrepreneur produces a homogenous good using the following production technology:

$$Y_t = e^{\Lambda_t} K_{t-1}^{\mu} H_{t-1}^{\nu} (A_t L_t')^{\sigma(1-\mu-\nu)} (A_t L_t'')^{(1-\sigma)(1-\mu-\nu)} \qquad 5.1$$

where $\mu \in (0,1)$ and $\nu \in (0,1)$ are parameters that associate with the share of capital and housing in production, respectively. Meanwhile $\sigma \in (0,1)$ corresponds to the share of patient households to the total labour. We can also think it as patient households wage share. In the above setting, we assume that there are two types of technology shock that affect the production of all goods, homogenously. These are a covariance stationary technology shock, denoted by e^{Λ_t}, and a permanent technology shock, denoted by A_t. The level of permanent technology is non-stationary with the growth rate $z_t = log(A_t/A_{t-1})$ and follows an AR (1) process:

$$z_t = \rho_z z_{t-1} + \epsilon_t^z \qquad \epsilon_t^z \sim N(0, \sigma_\epsilon^z) \qquad 5.2$$

Meanwhile, the log-linearised form of stationary shock has the following representation:

$$\Lambda_t = \rho_\Lambda \Lambda_{t-1} + \epsilon_t^\Lambda \qquad \epsilon_t^\Lambda \sim N(0, \sigma_\epsilon^\Lambda) \qquad 5.3$$

Parameter $\rho_z \in (0,1)$ and $\rho_\Lambda \in (0,1)$ correspond to the persistence parameter of permanent and neutral technology, respectively. While ϵ_t^z and ϵ_t^Λ are a white noise shock with a mean zero and variance σ_ϵ^z and σ_ϵ^Λ, subsequently. As shown in equation (5.1), in addition to labour of patient households L_t' and impatient households L_t'', housing stock H_t and physical capital K_t are used as inputs in the production function. By our notation, it is assumed that housing and physical capital are formed at the end of each period.

For both housing and physical capital, we assume that there is a quadratic cost of adjusting housing stock and capital, following Aoki et al. (2004), as follows:

$$A_{H,t} = \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{I_{H,t}}{H_{t-1}} - \delta_H\right)^2 H_{t-1} \qquad 5.4$$

$$A_{K,t} = \frac{\psi_K}{2\delta_K}\left(\frac{I_{K,t}}{K_{t-1}} - \delta_K\right)^2 K_{t-1} \qquad 5.5$$

Variables $I_{H,t}$ and $I_{K,t}$ correspond to housing and capital investment, respectively. Meanwhile, the notation of $P_{H/C}$ refers to the steady state of housing prices while ψ is the slope of adjustment cost with subscript H refers to housing and K refers to capital. This adjustment cost can be viewed as the cost of transaction, i.e. the cost of converting housing or capital purposes, and is also used in Nishiyama et al. (2009), among others. Note that, overtime, housing and capital depreciate at the rate $\delta_H \in (0,1)$ and $\delta_K \in (0,1)$, respectively.

The physical capital and housing evolve according to the following law of motion:

$$e^{\varepsilon_t^{I_K}} I_{K,t} = K_t - (1-\delta_K)K_{t-1} \qquad 5.6$$

$$I_{H,t} = H_t - (1-\delta_H)H_{t-1} \qquad 5.7$$

where $e^{\varepsilon_t^{I_K}}$ represents investment-specific technology shock, following Fisher (2006)[41]. This exogenous shock is also commonly known as marginal efficiency of investment (MEI) shock in the literature. As argued by Justiano et al. (2011), MEI shock is a good proxy for the effectiveness of financial intermediation in channelling savings' activity into productive capital in the economy. Although, we do not explicitly link this exogenous shock to financial

[41] Greenwood et al. (1998) was the first one who suggest investment specific-technology shock as an alternative to neutral technology shocks in a general equilibrium framework.

sector, the way of how this shock introduced to the model is in line with the literatures since it creates randomness and interruptions in the capital formation process.

The log-linearised form of investment-specific technology shock is assumed to follow an AR (1) process as:

$$\mathcal{E}_t^{I_K} = \rho_{I_K}\mathcal{E}_{t-1}^{I_K} + \epsilon_t^{I_K} \quad \epsilon_t^{I_K} \sim N\left(0, \sigma_\epsilon^{I_K}\right) \qquad 5.8$$

where $\rho_{I_K} \in (0,1)$ is an auto-regressive coefficient that reflects the persistence of this shock.

Entrepreneurs are allowed to borrow domestically to finance their expenditures. However, there is a limit of their borrowing activity. Since housing is one of their assets, the maximum amount of loan that entrepreneurs can obtain is bounded above by the expected price of their housing minus some proportions of this asset that cannot be used as collateral. This financial/borrowing constraint can be illustrated using the following relation:

$$B_t = (1-\chi)E_t\left(P_{H/C,t+1}H_t\pi_{\mathrm{C,t+1}}\right)e^{\mathcal{E}_t^{LTV}}/R_t \qquad 5.9$$

The variable of $P_{H/C,t}$ is the relative price of housing to the price of consumption goods (CPI), in which for future reference we simply define it as housing prices. $\pi_{C,t}$ denotes consumption price index (CPI) inflation, while R_t is gross interest rate. Parameter $\chi \in (0,1)$ represents a fraction of housing assets that is used in collateral (or down payment ratio). Given this, $(1-\chi)$ is a proxy for the ratio of loan to value (LTV) where $e^{\mathcal{E}_t^{LTV}}$ captures the variation of this ratio. Notarpietro & Siviero (2014) have used the term of a "financial shock" to characterise the variation in LTV ratio. This financial shock can be interpreted as the changes in financial regulation, following Ludvigson et al. (2013). The importance of this shock is stressed by Liu et al. (2013) in which they show that a limit in borrowing can amplify and propagate exogenous shocks only when a financial shock generates fluctuation in LTV ratio.

In line with the above literature, we argue that the financial shock is also relevant when we model financial constraint in the context of the Indonesian economy. As mentioned earlier, the LTV ratio has not become an attention of BI until 2012 that causes this lending parameter was set independently by bank, based on their individual risk appetite. Although BI did not directly control this ratio, a decision on lending may still be influenced by other factors that relate to financial regulations, for instance: the regulation that controls the ratio of non-performing loan (NPL), the ratio of loan to deposit (LDR), etc. Following the US financial crisis, on March 2012, BI had issued a new regulation that controls the ratio for LTV through BI regulation No. 15/40/DKMP. It was done to anticipate similar crisis happens in Indonesia.

In log-linearised form, an exogenous shock to the ratio of LTV is assumed to follow an AR (1) process as:

$$\mathcal{E}_t^{LTV} = \rho_{LTV}\mathcal{E}_{t-1}^{LTV} + \epsilon_t^{LTV} \qquad \epsilon_t^{LTV} \sim N(0, \sigma_\epsilon^{LTV}) \tag{5.10}$$

with $\rho_{LTV} \in (0,1)$ represents an autoregressive coefficient and ϵ_t^{LTV} is an independently and identically distributed process with mean zero and variance σ_ϵ^{LTV}. As we want to analyse the decision made by banks in lending process, then χ and the standard deviation of financial shock σ_ϵ^{LTV} are among the estimated parameters. Recall that it is assumed χ is set independently without any influence from monetary authority.

A representative entrepreneur attains utility from a flow of consumption goods C_t by maximising the following objective function:

$$Max\, E_o \sum_{t=0}^{\infty} \beta^t e^{\mathcal{E}_t^{\beta}} \ln C_t$$

Subject to equation (5.1) to (5.10) and the following flow of fund:

$$\frac{P_{Y,t}}{P_{C,t}}\frac{Y_t}{X_t} + B_t = C_t + P_{H/C,t} I_{H,t} + \frac{R_{t-1}}{\Pi_{C,t}} B_{t-1} + \frac{W_t'}{P_{C,t}} L_t' + \frac{W_t''}{P_{C,t}} L''_t + I_{K,t} + A_{K,t} + A_{H,t} \quad 5.11$$

Here, $\beta^t \in (0,1)$ stands for entrepreneur discount factor while $e^{\mathcal{E}_t^\beta}$ represents a general preference shock that affects the discount rate of entrepreneurs (determines the intertemporal substitution decisions). This contemporaneous shock can be interpreted as the change of consumer tastes due to cultural norm or society-wide unexpected events that make a shift in demand curves.

The log-linearised equation for the preference shock is assumed to follow an AR (1) process as:

$$\mathcal{E}_t^\beta = \rho_\beta \mathcal{E}_{t-1}^\beta + \epsilon_t^\beta \qquad \epsilon_t^\beta \sim N\left(0, \sigma_\epsilon^\beta\right) \quad 5.12$$

with $\rho_\beta \in (0,1)$ is an autoregressive coefficient and ϵ_t^β is an independent and identically distributed shock with mean zero and variance σ_ϵ^β.

As discussed in earlier chapters, the notation E_0 corresponds to the mathematical expectation operator evaluated using the objective distributions of the stochastic shock, which is assumed known by the rational agents. Unlike the earlier chapters, in this chapter we focus the analysis on the framework of RE as incorporating the assumption of adaptive learning to the model, whilst at the same time estimating it, makes our work is computationally expensive (it increases dimensionally of the model significantly). Yet, for future work, we may extend this analysis by considering a case where agents are boundedly rational, instead of fully rational, and estimate the model using relevant data.

In equation (5.11), W_t' and W_t'' are defined as nominal wage of patient and impatient households while L_t' and L''_t represent labour supply of patient and of impatient households, respectively. Output cannot be transformed directly into consumption goods C_t. Hence, we model retailers who can convert intermediate goods into final goods, following Bernanke et al. (1999). Retailers purchase intermediate goods from entrepreneurs at its wholesale prices P_t^w and transform into a composite of final goods whose price index is $P_{Y,t}$. By this assumption, a mark-up of final goods over intermediate goods is occurred with the amount is $X_t = \frac{P_{Y,t}}{P_{Y,t}^w}$.

By defining λ_t as the Lagrange multiplier associated with the borrowing constraint, the entrepreneur's optimal choice are characterised by consumption Euler equation, the demand for labour, physical capital and housing as the followings[42]:

$$\frac{e^{\varepsilon_t^\beta}}{C_t} = \beta E_t \left(\frac{e^{\varepsilon_{t+1}^\beta}}{\Pi_{C,t+1} C_{t+1}} \right) R_t + \lambda_t R_t \qquad 5.13$$

$$\frac{W_t'}{P_{C,t}} = \frac{\sigma(1-\mu-\nu)Y_t}{X_t L_t'} \frac{P_{Y,t}}{P_{C,t}} \qquad 5.14$$

$$\frac{W_t''}{P_{C,t}} = \frac{(1-\sigma)(1-\mu-\nu)Y_t}{X_t L_t''} \frac{P_{Y,t}}{P_{C,t}} \qquad 5.15$$

$$\frac{e^{\varepsilon_t^\beta}}{C_t} \left(\frac{1}{e^{\varepsilon_t^{I_K}}} + \Omega_{K,t} \right) = \beta E_t \left(\frac{e^{\varepsilon_{t+1}^\beta}}{C_{t+1}} \left[\frac{\mu Y_{t+1} P_{Y,t+1}}{K_t X_{t+1} P_{C,t+1}} + (1-\delta_K) \left(\frac{1}{e^{\varepsilon_{t+1}^{I_K}}} \right) + \Omega_{K,t+1} \breve{\Omega}_{K,t+1} \right] \right)$$

5.16

with $\Omega_{K,t} \equiv \frac{\psi_K}{\delta_K} \left(\frac{I_{K,t}}{K_{t-1}} - \delta_K \right)$ and $\breve{\Omega}_{K,t} \equiv \frac{1}{2} \left(\frac{I_{K,t}}{K_{t-1}} + \delta_K \right) + (1-\delta_K)$

[42] See Appendix 5.2 for details derivation

$$\frac{e^{\mathcal{E}_t^{\beta}}}{C_t}\left(P_{H/C,t}+\Omega_{H,t}\right)=\beta E_t\left(\frac{e^{\mathcal{E}_{t+1}^{\beta}}}{C_{t+1}}\left[\frac{vY_{t+1}P_{Y,t+1}}{H_tX_{t+1}P_{C,t+1}}+(1-\delta_H)P_{H/C,t+1}+\Omega_{H,t+1}\breve{\Omega}_{H,t+1}\right]+\right.$$

$$\left.(1-\chi)\lambda_t E_t\left[P_{H/C,t+1}\pi_{C,t+1}\right]e^{\mathcal{E}_t^{LTV}}\right) \qquad 5.17$$

with $\Omega_{H,t}\equiv\frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I_{H,t}}{H_{t-1}}-\delta_H\right)$ and $\breve{\Omega}_{H,t}\equiv\frac{1}{2}\left(\frac{I_{H,t}}{H_{t-1}}+\delta_H\right)+(1-\delta_H)$

As discussed in Iacoviello (2005), the above Euler equation and housing demand equations differ from a standard formulation due to the presence of λ_t. From the equation (5.13) and (5.17), λ_t can be interpreted as the increase in lifetime utility that is obtained by borrowing R_t dollars, consuming (equation [5.13]) or investing (equation [5.17]) the gains, and reducing consumption by sufficient amount in next period. Equation (5.14) and (5.15) equate the marginal product of labour with real wage for patient households and impatient households, respectively. Finally, equation (5.16) equates marginal product of capital with cost of capital.

Since we focus on the equilibrium dynamics around the steady state, the above nonlinear optimality conditions are stationerised and log-linearised around the steady state to get the following equations:

$$\beta'\tilde{c}_t=\beta E_t\tilde{c}_{t+1}-(\beta'-\beta)\tilde{\lambda}_t-\beta'\tilde{r}_t+\beta E_t\tilde{\pi}_{C,t+1}-\left(\beta\rho_\beta-\beta'\right)\mathcal{E}_t^{\beta}+\beta\rho_z z_t \qquad 5.18$$

$$\widetilde{w}_t'-\tilde{p}_C=\tilde{y}_t-\tilde{x}_t-\tilde{l}_t'-\alpha\tilde{s}_t \qquad 5.19$$

$$\widetilde{w}_t''-\tilde{p}_{C,t}=\tilde{y}_t-\tilde{x}_t-\tilde{l}_t''-\alpha\tilde{s}_t \qquad 5.20$$

$$\tilde{\imath}_{K,t}=\tilde{k}_{t-1}+\beta\left(E_t\tilde{\imath}_{K,t+1}-\tilde{k}_t\right)+\frac{1-\beta(1-\delta_K)}{\psi_K}\left(E_t\tilde{y}_{t+1}-\alpha E_t\tilde{s}_{t+1}-\tilde{k}_t-E_t\tilde{x}_{t+1}\right)+$$

$$\frac{1}{\psi_K}\{\tilde{c}_t-E_t\tilde{c}_{t+1}\}-\left(\frac{1-\rho_\beta}{\psi_K}\right)\mathcal{E}_t^{\beta}+\frac{1}{\psi_K}\left[1-\beta(1-\delta_K)\rho_{I_K}\right]\mathcal{E}_t^{I_K}-\left(\rho_z\left[\frac{\beta(1-\delta_K)}{\psi_K}-\beta\right]+1\right)z_t$$

5.21

$$\tilde{p}_{H/C,t} = \gamma_e E_t \tilde{p}_{H/C,t+1} + (1-\gamma_e)\left(E_t \tilde{y}_{t+1} - \alpha E_t \tilde{s}_{t+1} - \tilde{h}_t - E_t \tilde{x}_{t+1}\right) + m_e\left(\tilde{\lambda}_t + E_t \tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV} + E_t \tilde{c}_{t+1}\right) + \tilde{c}_t - E_t \tilde{c}_{t+1} + \left(\frac{\psi_H}{\delta_H}\right)\left[\beta\left(E_t \tilde{h}_{t+1} - \tilde{h}_t\right) - \left(\tilde{h}_t - \tilde{h}_{t-1}\right)\right] + \left[(1-m_e)\rho_\beta - 1\right]\mathcal{E}_t^{\beta} - ([\beta(1-\delta_H) - \beta\psi_H]\rho_z + \psi_H)z_t \qquad 5.22$$

with $m_e = (1-\chi)(\beta' - \beta)$ and $\gamma_e = \beta(1-\delta_H) + m_e$

The term $\tilde{s}_t$ in equation (5.19) and (5.20) corresponds to the effective term trade where its relation to domestic/producer price index and CPI, in log-linearised forms, is given by equation (5.71). We discuss this term in details when we derive the assumption of an open economy latter.

In addition to equations that are derived from the optimality conditions, the equilibrium dynamic is also characterised by stationerised and log-linearised form of the physical capital law of motion, borrowing constraint, production function and entrepreneurs' budget constraint as the followings:

$$\tilde{\imath}_{K,t} = \frac{1}{\delta_K}\left(\tilde{k}_t - (1-\delta_K)\left(\tilde{k}_{t-1} - z_t\right)\right) - \mathcal{E}_t^{I_K} \qquad 5.23$$

$$\tilde{b}_t = (1-\chi)\left(E_t \tilde{p}_{H/C,t+1} + \tilde{h}_t + E_t \tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) - \tilde{r}_t \qquad 5.24$$

$$\tilde{y}_t = \Lambda_t + \mu \tilde{k}_{t-1} + \nu \tilde{h}_{t-1} + \sigma(1-\mu-\nu)\tilde{l}'_t + (1-\sigma)(1-\mu-\nu)\tilde{l}''_t - [\mu+\nu]\gamma z_t \qquad 5.25$$

$$\frac{B}{Y}\tilde{b}_t = \frac{C}{Y}\tilde{c}_t + \frac{P_{H/C}H}{Y}\left(\tilde{h}_t - (1-\delta_H)\tilde{h}_{t-1}\right) + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{RB}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1} - \tilde{\pi}_{C,t}\right) - (1 - S' - S'')(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + \frac{P_{H/C}H}{Y}\delta_H \tilde{p}_{H/C,t} + \left[(1-\delta_H)\frac{P_{H/C}H}{Y} - \frac{RB}{Y}\right]z_t \qquad 5.26$$

with $S' = \frac{\sigma(1-\mu-\nu)+X-1}{X}$ and $S'' = \frac{(1-\sigma)(1-\mu-\nu)}{X}$

5.3.2. Households

Both types of households of mass 1 derive utility from a flow of consumption C_t^j and service from housing H_t^j and disutility from labour L_t^j. To distinguish between patient and impatient households, we use the superscripted notation j in each relevant variable in which this j is replaced by the index of " $'$ " for patient households and of " $''$ " for impatient households.

Each type of representative household maximises her utility subject to the constraints.

$$Max\, E_0 \sum_{t=0}^{\infty} {\beta^j}^t e^{\mathcal{E}_t^\beta} \left(\ln C_t^j + \varkappa e^{\mathcal{E}_t^H} \ln H_t^j - \frac{\left(L_t^j\right)^\eta}{\eta} \right)$$

Here, $\beta^j \in (0,1)$ corresponds the relevant households' type discount factor. Parameter $\varkappa \in (0,1)$ is the weight of housing services in utility and $1/(\eta - 1)$ is the labour supply elasticity where the value of η is larger than zero. We label the movement in $e^{\mathcal{E}_t^H}$ as a housing preference shock that is an exogenous shock to marginal utility of housing. According to Iacoviello & Neri (2010), there are at least two interpretations of this contemporaneous shock: Firstly, a shock that captures a shift in housing preference due to social, institutional changes and the variation of resources needed to buy a house relative to other goods. Secondly, a shock that accommodates the random changes in the factor mix required to produce home service a given housing stock.

In log-linearised form, a housing preference shock follows an AR (1) process as:

$$\mathcal{E}_t^H = \rho_H \mathcal{E}_{t-1}^H + \epsilon_t^H \qquad \epsilon_t^H \sim N(0, \sigma_\epsilon^H) \qquad 5.27$$

with $\rho_H \in (0,1)$ is the persistence parameter and ϵ_t^H is a white noise shock with mean zero and variance σ_ϵ^H.

5.3.2.1. Patient Households

Patient households choose how much to consume, work, invest in housing and lend, both domestically and internationally. They also receive the profit F_t since it is assumed retailers' firm is owned by them.

The budget constraint of the patient household is given by:

$$B'_t + \mathbb{E}_t B^*_t + \frac{W'_t}{P_{C,t}} L'_t + F_t = C'_t + P_{H/C,t} I'_{H,t} + \frac{R_{t-1}}{\Pi_{C,t}} B'_{t-1} + \frac{R^*_{t-1}}{\Pi_{C,t}} \mathbb{E}_t B^*_{t-1} + A'_{H,t} \qquad 5.28$$

where B'_t and B^*_t are real bonds denominated in domestic and foreign currency, respectively. $\mathbb{E}_t$ corresponds to the nominal bilateral exchange and R^*_t represents the gross foreign interest rate. $I'_{H,t}$ and $A'_{H,t}$ denote housing investment and the adjustment cost of changing housing stock for patient households, respectively. The law of motion of housing investment is given by:

$$I'_{H,t} = H'_t - (1 - \delta_H) H'_{t-1} \qquad 5.29$$

$$A'_{H,t} = \frac{\psi_H P_{H/C,t}}{2\delta_H} \left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H \right)^2 H'_{t-1} \qquad 5.30$$

Patient household's optimality condition is characterised by the choices that determine labour supply, consumption Euler equation and housing demand as follows:

$$\frac{W'_t}{P_{C,t}} = C'_t (L'_t)^{\eta - 1} \qquad 5.31$$

$$\frac{e^{\varepsilon^\beta_t}}{C'_t} = \beta' E_t \left(\frac{e^{\varepsilon^\beta_{t+1}}}{\Pi_{C,t+1} C'_{t+1}} \right) R_t \qquad 5.32$$

$$\frac{e^{\varepsilon^\beta_t}}{C'_t} = \beta' E_t \left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t} \frac{e^{\varepsilon^\beta_{t+1}}}{\Pi_{C,t+1} C'_{t+1}} \right) R^*_t \qquad 5.33$$

$$\frac{e^{\varepsilon_t^\beta}}{C_t'}\left[P_{H/C,t} + \Omega'_{\mathrm{H,t}}\right] = \varkappa e^{\varepsilon_t^\beta} e^{\varepsilon_t^H}\left(\frac{1}{H_t'}\right) + \beta' E_t\left(\frac{e^{\varepsilon_{t+1}^\beta}}{C_{t+1}'}\left[(1-\delta_H)P_{H/C,t+1} + \Omega'_{\mathrm{H,t+1}}\breve{\Omega}'_{\mathrm{H,t+1}}\right]\right) \quad 5.34$$

with $\Omega'_{\mathrm{H,t}} \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)$ and $\breve{\Omega}'_{\mathrm{H,t}} \equiv \frac{1}{2}\left(\frac{I'_{H,t}}{H'_{t-1}} + \delta_H\right) + (1-\delta_H)$

Equation (5.32) and (5.33) are consumption Euler equation for patient households. These two equations equate the marginal utility from consuming one unit of income in period t with discounted marginal utility from consuming the gross income obtained, R_t or R_t^* (after accounting for exchange rate), by saving the income. As well, equation (5.31) is a marginal rate of substitution between consumption and supplying labour for patient households. Whilst the last equation, i.e. equation (5.34) illustrates patient households' optimal choice for housing.

The above optimality conditions are then stationerised and log-linearised around their steady state values. Combining the relevant optimality conditions from entrepreneurs, we to obtain the following equations:

$$\tilde{y}_t = \tilde{x}_t + \eta\tilde{l}_t' + \tilde{c}_t' + \alpha\tilde{s}_t \quad 5.35$$

$$\tilde{c}_t' = E_t\tilde{c}_{t+1}' - \left(\tilde{r}_t - E_t\tilde{\pi}_{C,t+1}\right) + \left(1-\rho_\beta\right)\mathcal{E}_t^\beta + \rho_z z_t \quad 5.36$$

$$\tilde{c}_t' = E_t\tilde{c}_{t+1}' - \left(\tilde{r}_t^* - E_t\tilde{\pi}_{C,t+1}\right) + \left(1-\rho_\beta\right)\mathcal{E}_t^\beta - \Delta E_t\tilde{e}_{t+1} + \rho_z z_t \quad 5.37$$

$$\tilde{p}_{H/C,t} = \beta'(1-\delta_H)E_t\tilde{p}_{H/C,t+1} + \left(1-\beta'(1-\delta_H)\right)\left(\mathcal{E}_t^H - \tilde{h}_t'\right) + \tilde{c}_t' - \beta'(1-\delta_H)E_t\tilde{c}_{t+1}' - \frac{\psi_H}{\delta_H}\left\{\left(\tilde{h}_t' - \tilde{h}_{t-1}'\right) - \beta'\left(E_t\tilde{h}_{t+1}' - \tilde{h}_t'\right)\right\} + \beta'(1-\delta_H)\left(\rho_\beta - 1\right)\mathcal{E}_t^\beta - (\beta'[(1-\delta_H) - \psi_H]\rho_z + \psi_H)z_t \quad 5.38$$

5.3.2.2. Impatient Households

Impatient households discount the future more heavily than the patient households. Technically, they have higher discount rate than patient households that generate an incentive for them to borrow ($\beta' > \beta > \beta''$). They choose how much to consume, to work, to invest in housing and to borrow domestically. Yet, their borrowing capacity is constrained by the expected value of their housing assets (less the proportion of housing assets that cannot be used as collateral) and their wage income. In details, the constraints faced by impatient households are given by:

$$B_t'' + \frac{W_t''}{P_{C,t}} L_t'' = C_t'' + P_{H/C,t} I_{H,t}'' + \frac{R_{t-1}}{\Pi_{C,t}} B_{t-1}'' + A_{H,t}'' \qquad 5.39$$

$$I_{H,t}'' = H_t'' - (1 - \delta_H) H_{t-1}'' \qquad 5.40$$

$$A_{H,t}'' = \frac{\psi_H P_{H/C,t}}{2\delta_H} \left(\frac{I_{H,t}''}{H_{t-1}''} - \delta_H \right)^2 H_{t-1}'' \qquad 5.41$$

$$[1 - \mu^b][1 - \chi''] H_t'' E_t \left(P_{H/C,t+1} \pi_{C,t+1} \right) e^{\varepsilon_t^{LTV}} + [\mu^b] \frac{W_t''}{P_{C,t}} L_t'' = R_t B_t'' \qquad 5.42$$

Parameter $\chi'' \in (0,1)$ represents a proportion of housing assets owned by impatient households that cannot be used as collateral (down payment ratio). Under this definition, $(1 - \chi'')$ stands for the ratio of LTV where $e^{\varepsilon_t^{LTV}}$ refers to financial shocks that capture the variability in the setting of LTV ratio (this exogenous shock is assumed to be the same as the case of entrepreneurs). Parameter $\mu^b \in (0,1)$ is the weight of lending decision puts on wage income. It is assumed that these lending parameters (i.e. χ'' and μ^b) are set independently by each individual bank without control from monetary authority. Having this assumption, we are allowed to estimate these parameters empirically using some aggregate data for Indonesia.

Impatient household's optimal choice is characterised by labour supply, consumption Euler equation and housing demand as follows:

$$\frac{W_t''}{P_{C,t}}\left(e^{\varepsilon_t^{\beta}} + \mu^b \lambda_t'' C_t''\right) = C_t''(L_t'')^{\eta-1} e^{\varepsilon_t^{\beta}} \qquad 5.43$$

$$\frac{e^{\varepsilon_t^{\beta}}}{C_t''} = \beta'' E_t\left(\frac{e^{\varepsilon_{t+1}^{\beta}}}{\Pi_{C,t+1} C_{t+1}''}\right) R_t + \lambda_t'' R_t \qquad 5.44$$

$$\frac{e^{\varepsilon_t^{\beta}}}{C_t''}\left(P_{H/C,t} + \Omega_{H,t}''\right) = \varkappa e^{\varepsilon_t^{\beta}} e^{\varepsilon_t^{H}}\left(\frac{1}{H_t''}\right) + \beta'' E_t\left(\frac{e^{\varepsilon_{t+1}^{\beta}}}{C_{t+1}''}\left[(1-\delta_H)P_{H/C,t+1} + \Omega_{H,t+1}''\breve{\Omega}_{H,t+1}''\right]\right) +$$

$$[1-\mu^b][1-\chi'']\lambda_t'' E_t\left(P_{H/C,t+1}\pi_{C,t+1}\right) e^{\varepsilon_t^{LTV}} \qquad 5.45$$

with $\Omega_{H,t}'' \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I_{H,t}''}{H_{t-1}''} - \delta_H\right)$ and $\breve{\Omega}_{H,t}'' \equiv \frac{1}{2}\left(\frac{I_{H,t}''}{H_{t-1}''} + \delta_H\right) + (1-\delta_H)$

In general, the interpretation of equation (5.43) to (5.45) is analogues to the one discussed in the case of patient households with λ_t'' corresponds to the Lagrange multiplier associated with the financial constraint faced by impatient households. Similar to entrepreneurs and patient households, the above optimality conditions are then stationerised and log-linearised around the steady state. Substituting the relevant optimality conditions from entrepreneurs, we obtain the following equations:

$$(1+m_b)\left(\tilde{y}_t - \tilde{x}_t - \tilde{l}_t'' - \alpha\tilde{s}_t\right) + m_b\tilde{\lambda}_t'' - \tilde{c}_t'' = (1+m_b)(\eta-1)\tilde{l}_t'' + m_b\mathcal{E}_t^{\beta} \qquad 5.46$$

with $m_b = \mu^b(\beta' - \beta'')$

$$\beta'\tilde{c}_t'' = \beta'' E_t\tilde{c}_{t+1}'' - (\beta' - \beta'')\tilde{\lambda}_t'' - \beta'\tilde{r}_t + \beta'' E_t\tilde{\pi}_{C,t+1} - \left(\beta''\rho_\beta - \beta'\right)\mathcal{E}_t^{\beta} + \beta''\rho_z z_t$$

5.47

$$\tilde{p}_{H/C,t} = \gamma_h E_t \tilde{p}_{H/C,t+1} + (1-\gamma_h)\left(\mathcal{E}_t^H - \tilde{h}_t''\right) + m_h\left(\tilde{\lambda}_t'' + E_t \tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) + \tilde{c}_t'' - \beta''(1-\delta_H) E_t \tilde{c}_{t+1}'' + \frac{\psi_H}{\delta_H}\left\{\beta''\left(E_t \tilde{h}_{t+1}'' - \tilde{h}_t''\right) - \left(\tilde{h}_t'' - \tilde{h}_{t-1}''\right)\right\} - \left[\beta''(1-\delta_H)\left(1-\rho_\beta\right) + m_h\right]\mathcal{E}_t^\beta - (\beta''[(1-\delta_H) - \psi_H]\rho_z + \psi_H) z_t \quad 5.48$$

with $m_h = [1-\mu^b](1-\chi'')(\beta' - \beta'')$ and $\gamma_h = \beta''(1-\delta_H) + m_h$

In addition to the above equations, we also stationerise and log-linearise impatient households' financial constraint and their budget constraint that yield the following equations:

$$[1-\mu^b][1-\chi'']\frac{P_{H/C}H''}{Y}\left(E_t \tilde{p}_{H/C,t+1} + \tilde{h}_t'' + E_t \tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) + [\mu^b] s''(\tilde{y}_t - \tilde{x}_t - \alpha \tilde{s}_t) = \frac{1}{\beta'}\left(\frac{B''}{Y}\right)\left(\tilde{r}_t + \tilde{b}_t''\right) \quad 5.49$$

$$\frac{B''}{Y}\tilde{b}_t'' = \frac{C''}{Y}(\tilde{c}_t'') + \frac{P_{H/C}H''}{Y}\left(\tilde{h}_t'' - (1-\delta_H)\tilde{h}_{t-1}''\right) + \frac{RB''}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1}'' - \tilde{\pi}_{C,t}\right) + \frac{P_{H/C}H''}{Y}\delta_H \tilde{p}_{H/C,t} - S''(\tilde{y}_t - \tilde{x}_t - \alpha \tilde{s}_t) + \left[(1-\delta_H)P_{H/C}H'' - \frac{RB''}{\pi_C}\right] z_t \quad 5.50$$

5.3.3. Retailers

A continuum of retailers of mass 1, indexed by z, purchase intermediate goods Y_t from entrepreneurs at the wholesale price $P_{Y,t}^W$ in competitive market. These intermediate goods are then differentiated into $Y_t(z)$ and sell at the price $P_{Y,t}(z)$. There are no cost and labour required for this process.

It is assumed that retailers have a monopolistic power and, when allowed to, will set their prices to maximise profit, subject to demand for their differentiated product $Y_t(z)$, following

Bernanke et al. (1999). The final goods Y_t^f consist of a collection of differentiated goods $Y_t(z)$ which is bundled by final good bundlers using the following technology:

$$Y_t^f = \left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r} \qquad 5.51$$

with $\mu_t^r > 1$ captures the time-varying mark-up in domestic economy.

The bundlers try to minimise their inputs in terms of differentiated goods $Y_t(z)$. In order to do so, they set minimisation problem as follows:

$$Min \int_0^1 P_{Y,t}(z) Y_t(z)$$

Subject to the technology in equation (5.51).

The above minimisation problem yields the demand for each differentiated good $Y_t(z)$ as the followings:

$$Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} Y_t^f \qquad \forall z \qquad 5.52$$

with $P_{Y,t} = \left(\int_0^1 P_{Y,t}(z)^{-\frac{1}{\mu_t^r - 1}} dz\right)^{\mu_t^r - 1}$ corresponds to aggregate price index of final goods or producer price index (PPI).

Each retailer z chooses a sale price $P_{Y,t}(z)$ taking $P_{Y,t}^w$ and demand curve as given. The retailer follows the Calvo (1983) staggered price setting. A randomly selected fraction of firms $1 - \theta$ adjusts prices, while the remaining fraction of firms θ does not adjust. When a firm can set the price, it does so in order to maximise the present discounted value profits, $E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \Phi_{t+s}(z)$, subject to the demand for its own goods and the constraint that all demand be satisfied at the chosen price.

Retailers maximise profit which is discounted by s - step ahead stochastic discount factor $Q_{t,t+s}$ and the probability of not being able to set price in the future periods as follows:

$$Max\ E_0 \sum_{s=0}^{\infty} \theta^s\, Q_{t,t+s} \left(\frac{P_{Y,t}^{New}(z) - P_{Y,t+s}^{w}}{P_{Y,t+s}} \right) Y_{t+s}(z)$$

subject to the demand of its own goods, i.e. equation (5.52).

Note that $E_t Q_{t,t+s} = Q_{t,t+1} Q_{t+1,t+2} \cdots Q_{t+s-1,t+s}$ and $E_t Q_{t+s-1,t+s} = E_t E_{t+s-1} Q_{t+s-1,t+s}$. It is also assumed that $Y_t^f = Y_t$, following Iacoviello (2005).

A profit maximisation problem yields the equation for optimal price set by retailers as follows:

$$0 = E_0 \sum_{s=0}^{\infty} \theta^s\, Q_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}} - \left[\frac{\mu_t^r}{X_{t+s}} \right] \right) Y_{t+s}(z) \qquad 5.53$$

As a fraction $\theta \in (0,1)$ of prices stays unchanged, the aggregate price level evolves according to:

$$P_{Y,t} = \left(\theta P_{Y,t-1}{}^{-\left[\frac{1}{\mu_t^r - 1}\right]} + (1-\theta)\left(P_{Y,t}^{New}\right)^{-\left[\frac{1}{\mu_t^r - 1}\right]} \right)^{1-\mu_t^r} \qquad 5.54$$

Combining equation (5.53) and (5.54) and log-linearising it around the steady state we obtain a standard version of New Keynesian Philip Curve (NKPC) equation as follows:

$$\tilde{\pi}_{Y,t} = \beta' E_t \tilde{\pi}_{Y,t+1} - \kappa \tilde{x}_t + \kappa \tilde{\mu}_t^r \qquad 5.55$$

with $\kappa \equiv \frac{(1-\theta)(1-\beta'\theta)}{\theta}$.

Variable $\tilde{\mu}_t^r$ is an exogenous mark-up/cost push shock that captures the time varying mark-up of final goods over intermediate goods. This unanticipated shock is assumed to follow an AR (1) process as:

$$\tilde{\mu}_t^r = \rho_{\mu^r}\tilde{\mu}_{t-1}^r + \epsilon_t^{\mu^r} \qquad \epsilon_t^{\mu^r} \sim N\left(0, \sigma_\epsilon^{\mu^r}\right) \qquad 5.56$$

where $\rho_{\mu^r} \in (0,1)$ corresponds to the persistent parameter of an AR (1) process and $\epsilon_t^{\mu^r}$ is a zero mean white noise process.

5.3.4. Intertemporal Allocations

Consumers consume a composite of final consumption goods in the form of a Dixit-Stiglitz aggregator over domestic and import (foreign) goods. These different types of goods, initially, are bundled by bundler firms before it is ready to be consumed by consumers (both entrepreneurs and households). A technology that bundler firms used to combine the domestic and import goods is defined as follows:

$$C_t(j) = \left[(1-\alpha)^{\frac{1}{\varpi}} C_{D,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}} C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}} \qquad 5.57$$

where parameter $\alpha \in (0,1)$ is the share of import goods consumption in total goods consumption. This parameter is commonly viewed as an index of openness. Meanwhile, parameter ϖ refers to the elasticity substitution between domestic goods and import goods. It is assumed there is no cost and factors of production for this bundling process (e.g. labour and capital).

In addition to final consumption good bundlers, that combine domestic and import goods, there are other bundler firms where their role is to combine different types of goods produced in domestic country (or a specific foreign country i) and to combine import goods from different foreign countries i. Again, these bundlers operate without any cost and factors of production required (e.g. labour). The technology that these bundlers used is defined as follows:

$$C_{D,t}(j) = \left[\int_0^1 C_{D,t}(z,j)^{\frac{1}{\mu_t^r}} dz\right]^{\mu_t^r} \tag{5.58}$$

$$C_{F,t}(j) = \left[\int_0^1 C_{i,t}(j)^{\frac{\zeta-1}{\zeta}} di\right]^{\frac{\zeta}{\zeta-1}} \tag{5.59}$$

$$C_{i,t}(j) = \left[\int_0^1 C_{i,t}(z,j)^{\frac{1}{\mu_t^r}} dz\right]^{\mu_t^r} \tag{5.60}$$

The goods consumed ranges from $j = 0 \ldots 1$. Parameter μ_t^r denotes the time-varying mark-up in the domestic country or foreign country i while ζ is the elasticity of substitution between import goods from different foreign countries. $C_{D,t}(z,j)$ and $C_{i,t}(z,j)$ are domestic demand for a specific domestic goods and for a specific goods in foreign country i, respectively. Meanwhile, $C_{D,t}(j)$ and $C_{i,t}(j)$ are a consumption composite index, summarising the demand for each type of goods in domestic country and a specific foreign country i. $C_{F,t}(j)$ is a consumption composite index, summarising the demand for import goods from all foreign countries.

Domestic goods bundlers help consumers, i.e. households and entrepreneurs to find the best allocation of specific domestic goods. For that purpose, they minimise the expenditure for obtaining each type of domestic goods subject to its technology as follows:

$$\mathcal{L} \equiv Min \int_0^1 P_{Y,t}(z)\, C_{D,t}(z,j) dz + \varphi_t \left(C_{D,t}(j) - \left[\int_0^1 C_{D,t}(z,j)^{\frac{1}{\mu_t^r}} dz\right]^{\mu_t^r} \right)$$

The above minimisation problem yields a demand curve for each type of domestic goods as the following:

$$C_{D,t}(z,j) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} C_{D,t}(j) \qquad 5.61$$

Recall that $P_{Y,t}$ is defined as a producer price index (PPI) in domestic country in which, for simplicity, it is assumed to be equal to aggregate price index of domestic consumption goods.

Similarly, we can obtain a demand curve for each type of domestic goods in country i and demand curve for each foreign country goods as the followings:

$$C_{i,t}(z,j) = \left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} C_{i,t}(j) \qquad 5.62$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\zeta} C_{F,t}(j) \qquad 5.63$$

with $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(z)^{-\left[\frac{1}{\mu_t^r - 1}\right]} dz\right)^{\mu_t^r - 1}$ and $P_{F,t} \equiv \left(\int_0^1 {P_{i,t}}^{1-\zeta} di\right)^{\frac{1}{1-\zeta}}$ are defined as producer/domestic price index (PPI) in country i and aggregate price index for import goods, respectively.

In similar fashion, final consumption good bundlers help consumers to find the best allocation of domestic and import goods. These firms purchase domestic and foreign goods, combine them and sell to households and entrepreneurs to get profit.

They maximise their profit in perfectly competitive market as follows:

$$Max\, \Pi \equiv P_{C,t}C_t - P_{Y,t}C_{D,t} - P_{F,t}C_{F,t} \qquad \forall \text{ cohort } j$$

$$S.t \quad C_t(j) = \left[(1-\alpha)^{\frac{1}{\varpi}} C_{D,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}} C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}$$

From profit maximisation problem, we obtain a demand curve for domestic goods and import goods along with the equation for consumer price index (CPI) as follows:

$$C_{D,t}(j) = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} C_t(j) \qquad 5.64$$

$$C_{F,t}(j) = \alpha\left(\frac{P_{F,t}}{P_{C,t}}\right)^{-\varpi} C_t(j) \qquad 5.65$$

$$P_{C,t} \equiv \left[(1-\alpha){P_{Y,t}}^{1-\varpi} + \alpha {P_{F,t}}^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \qquad 5.66$$

Log-linearised CPI equation in (5.66) yields:

$$\tilde{p}_{C,t} = (1-\alpha)\tilde{p}_{Y,t} + \alpha\tilde{p}_{F,t} \Leftrightarrow \tilde{\pi}_{C,t} = (1-\alpha)\tilde{\pi}_{Y,t} + \alpha\tilde{\pi}_{F,t} \qquad 5.67$$

In the model with the feature of open economy, there is a distinction between CPI inflation $\tilde{\pi}_{C,t}$ and producer inflation (PPI inflation) $\tilde{\pi}_{Y,t}$ as the result of import goods inflation $\tilde{\pi}_{F,t}$ on the domestic economy. This is clearly illustrated in equation (5.67). In addition, as the economy of foreign country is large and behaves as a closed economy, then the foreign price coincides with the foreign currency price of foreign goods, i.e. $P^*_{F,t} = P^*_t$ and $\pi^*_{F,t} = \pi^*_t$ for all t. Given this, domestic inflation is similar to CPI inflation in the world economy.

5.3.5. Definitions and Identities

In this section, we define some definitions and identities to link the inflation, exchange rate, and the term of trade. We begin by defining the bilateral terms of trade between the domestic country and country i that represents the price of country i's goods in terms of domestic goods.

$$S_{i,t} = P_{i,t}/P_{Y,t} \quad 5.68$$

Aggregating for all country i, we obtain the effective term terms of trade (the price of imports relative to domestic currency) as:

$$S_t = \frac{P_{F,t}}{P_{Y,t}} = \left[\int_0^1 \left(S_{i,t}\right)^{1-\zeta} di\right]^{\frac{1}{1-\zeta}} \quad 5.69$$

which can be approximated by $s_t \equiv \log(S_t) \approx \int_0^1 s_{i,t}\, di$

Log-linearised it around the steady state yields:

$$\tilde{s}_t = \tilde{p}_{F,t} - \tilde{p}_{Y,t} \quad 5.70$$

Substituting equation (5.70) into (5.67) for $\tilde{p}_{F,t}$, we can find the relation among CPI inflation, producer inflation and the change of term trade as follows:

$$\tilde{\pi}_{C,t} = \tilde{\pi}_{Y,t} + \alpha \Delta \tilde{s}_t \quad 5.71$$

It is worth to note that the above relation holds independently regardless the degree of exchange rate pass-through.

Complete Exchange Rate Pass-Through

Under the above assumption, the Law of One Price (LOOP) holds on a brand level, following Gali & Monacelli (2005).

$$P_{i,t}(z) = \mathbb{E}_{i,t} P_{i,t}^{i}(z) \quad (\forall i, z \in [0,1]) \quad 5.72$$

where $P_{i,t}^{i}(z)$ represents the price of good z from country i measured in terms of country i's currency and $\mathbb{E}_{i,t}$ is the bilateral nominal exchange rate.

By integrating over all products z and all foreign countries yields:

$$P_{F,t} = \mathbb{E}_t P_t^* \qquad 5.73$$

Log-linearisation the last equation around its steady state value gives:

$$\tilde{p}_{F,t} = \tilde{p}_t^* + \tilde{e}_t \qquad 5.74$$

where $\tilde{e}_t$ is defined as the nominal effective exchange rate and $\tilde{p}_t^*$ is the log of world consumption goods price in foreign currency.

Substituting equation (5.74) into (5.70) for $\tilde{p}_{F,t}$, we can re-write the term of trade equation as:

$$\tilde{s}_t = \tilde{p}_t^* + \tilde{e}_t - \tilde{p}_{Y,t} \qquad 5.75$$

Now, let us define the effective bilateral real exchange rate as follows:

$$\mathfrak{E}_{i,t} = \frac{\mathbb{E}_{i,t} P_{C,t}^{i}}{P_{C,t}} \qquad 5.76$$

Integrating over all foreign countries and log-linearising it around the steady state yield the real effective exchange rate:

$$\tilde{q}_t = \tilde{e}_t + \tilde{p}_t^* - \tilde{p}_{C,t} \qquad 5.77$$

To link the term of trade and the real exchange rate, we combine (5.75) and (5.77) to get:

$$\tilde{q}_t = \tilde{s}_t + \tilde{p}_{Y,t} - \tilde{p}_{C,t} \qquad 5.78$$

Further, substituting equation (5.71) into the last equation for $\tilde{p}_{C,t}$, we obtain the relation between the effective bilateral exchange rate and the term of trade as the followings:

$$\tilde{q}_t = (1-\alpha)\tilde{s}_t \quad 5.79$$

Incomplete Exchange Rate Pass-Through

As mentioned in the section of introduction, this feature is incorporated into the model since we believe it makes our model fits with the characteristics of the Indonesian economy. Under incomplete pass-through the law of one price does not hold. Yet, our definition about the real effective exchange rate in equation (5.77) still holds. To link the term trade and the real exchange rate, we can combine (5.77) and (5.67) to get:

$$\tilde{q}_t = \Psi_{F,t} + (1-\alpha)\tilde{s}_t \quad 5.80$$

where $\Delta\Psi_{F,t} = \Delta\tilde{e}_t + \tilde{\pi}_t^* - \tilde{\pi}_{F,t}$

$\Psi_{F,t}$ denotes the deviation of the world price from the domestic currency price of imports, a measure of the deviations from the law of one price (LOOP gap)[43]. In line with this, we define $\tilde{\pi}_{F,t}$ as domestic currency import goods inflation while $\tilde{\pi}_t^*$ as the world goods inflation. The world goods inflation is assumed to be exogenous for domestic economy and follows an AR (1) process as:

$$\tilde{\pi}_t^* = \rho_{\pi^*}\tilde{\pi}_{t-1}^* + \epsilon_t^{\pi^*} \qquad \epsilon_t^{\pi^*} \sim N\left(0, \sigma_\epsilon^{\pi^*}\right) \quad 5.81$$

where $\rho_{\pi^*} \in (0,1)$ is the persistent parameter of an AR (1) process. A similar way of introducing exogenous world goods inflation is also seen in Matheson (2009), among others.

[43] See Monacelli (2005)

We now turn to describe the characteristic of import goods inflation $\tilde{\pi}_{F,t}$. It is assumed there are importing firms i behaves as in the Calvo's model and can set prices optimally in a forward looking manner as follows:

$$Max\ E_0 \sum_{s=0}^{\infty} \theta_F^s\, Q_{t,t+s} \left(\frac{P_{F,t}^{New}(i) - \mathbb{E}_{t+s} P_{F,t+s}^{*}(i)}{P_{F,t+s}} \right) C_{F,t+s}(i)$$

subject to $C_{F,t+s}(i) = \left(\frac{P_{F,t}^{New}}{P_{F,t}} \right)^{-\zeta} C_{F,t+s}$, which is equivalent to equation (5.63) and $E_t Q_{t,t+s} = Q_{t,t+1} Q_{t+1,t+2} \ldots Q_{t+s-1,t+s}$ and $E_t Q_{t+s-1,t+s} = E_t E_{t+s-1} Q_{t+s-1,t+s}$ are employed.

$P_{F,t}^{*}(i)$ is the foreign currency price of the imported goods (from country i), θ_F^s is the probability that the price $P_{F,t}^{New}(i)$ sets for good from country i at time t still holds s periods ahead and $Q_{t,t+s}$ is the relevant stochastic discount factor. Notice that, in general $\theta^s \neq \theta_F^s$.

The above profit maximisation problem yields the equation for optimal price set by importing firms as the followings:

$$0 = E_0 \sum_{s=0}^{\infty} \theta_F^s\, Q_{t,t+s} \left(\frac{P_{F,t}^{New}}{P_{F,t+s}} - \left(\frac{\zeta}{\zeta - 1} \right) \frac{\mathbb{E}_{t+s} P_{F,t+s}^{*}}{P_{F,t+s}} \right) C_{F,t+s} \qquad 5.82$$

As a fraction $\theta_F \in (0,1)$ of prices stays unchanged, the aggregate price level evolution follows:

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\zeta} + (1 - \theta_F) \left(P_{F,t}^{New} \right)^{1-\zeta} \right]^{1/(1-\zeta)} \qquad 5.83$$

Combining equation (5.82) and (5.83) and log-linearised it around the steady state; one can obtain an aggregate supply curve for import goods inflation as follows:

$$\tilde{\pi}_{F,t} = \beta' E_t \tilde{\pi}_{F,t+1} + \lambda_F \Psi_{F,t} \qquad 5.84$$

with $\lambda_{F,t} = \frac{(1-\theta_F)(1-\beta'\theta_F)}{\theta_F}$

From the last equation, it can be seen that import goods inflation depends on the expected import goods inflation and LOOP gap. Import goods inflation increase if the world price of goods is higher than domestic currency import goods price. Using this relationship, the world price of goods acts as the real marginal cost of imported goods that enhance import goods inflation. Note that parameter θ_F determines the degree of pass-through: $\theta_F = 0$ implies that the LOOP holds and $\theta_F > 0$ implies that pass-through is incomplete.

After describing the relationship between inflation, exchange rate and term trade, now we turn to the relationship between the terms of trade and real interest rate differential. Their relationship is obtained by equating patient households first order conditions for domestic and foreign bond holdings as the followings:

$$\beta' E_t\left(\frac{\Omega'_{C,t+1}}{\Pi_{C,t+1}}\right) R_t = \beta' E_t\left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t}\frac{\Omega'_{C,t+1}}{\Pi_{C,t+1}}\right) R_t^* \qquad 5.85$$

After log-linearisation, one can obtain the following uncovered interest parity (UIP) condition as:

$$\tilde{r}_t - \tilde{r}_t^* = E_t(\Delta\tilde{e}_{t+1}) \qquad 5.86$$

As discussed previously, it is assumed that there is no access for both entrepreneurs and impatient households to get a loan from international market, except from domestic economy. In contrast, patient households are able to share country specific risks internationally via the trading of bonds on complete security markets. This means only patient households can do lending and borrowing activity internationally. This assumption seems realistic especially in the context of Indonesia where only a small proportion of the Indonesian people has accessed to the international financial markets. In fact, only a half of the Indonesian people have accessed to domestic financial institution and it is predicted only a small portion of this number has accessed to the international financial markets[44]. The same assumption is also used in Funke & Paetz (2013) who studied about housing wealth effect for Hongkong using the framework of a two-agent, two-sector, open economy DSGE model with a housing-market sector and a borrowing constraint.

From optimality condition of patient households, it can be shown that $\beta' E_t \left(\frac{P_{C,t}}{P_{C,t+1}} \frac{\Omega'_{C,t+1}}{\Omega'_{C,t}} \right) = \mathcal{Q}_{t,t+1}$. Similarly, by accounting for the exchange rate, the characteristic of each country i in terms of domestic currency can be presented as $\beta' E_t \left(\frac{\mathbb{E}_t^i P_{C,t}^i}{\mathbb{E}_{t+1}^i P_{C,t+1}^i} \frac{{\Omega'_{C,t+1}}^i}{{\Omega'_{C,t}}^i} \right) = \mathcal{Q}_{t,t+1}$.

Equating the above two equations for both domestic and foreign country in terms of $\mathcal{Q}_{t,t+1}$ yields:

$$E_t \left(\frac{{C'_{t+1}}^i}{C'_{t+1}} \right) \left(\frac{P_{C,t+1}^i}{P_{C,t+1}} \right) \mathbb{E}_{t+1}^i = \left(\frac{{C'_t}^i}{C'_t} \right) \left(\frac{P_{C,t}^i}{P_{C,t}} \right) \mathbb{E}_t^i \qquad 5.87$$

Given that, the following identity is also hold:

$$E_t \left(\frac{{C'_{t+1}}^i}{C'_{t+1}} \right) \left(\frac{P_{C,t+1}^i}{P_{C,t+1}} \right) \mathbb{E}_{t+1}^i = \left(\frac{{C'_t}^i}{C'_t} \right) \left(\frac{P_{C,t}^i}{P_{C,t}} \right) \mathbb{E}_t^i = \left(\frac{{C'_0}^i}{C'_0} \right) \left(\frac{P_{0,t}^i}{P_{0,t}} \right) E_0^i = \Upsilon = \Upsilon_i \qquad 5.88$$

The last part is the initial consumption ratio, which is assumed to be identical for each country (symmetric initial condition). Let this ratio equal to 1, then we can obtain

$$\left(\frac{{C'_t}^i}{C'_t} \right) \left(\frac{P_{C,t}^i}{P_{C,t}} \right) \mathbb{E}_t^i = 1 \qquad 5.89$$

Thus, the condition of international risk sharing leads us to the following equation:

$$C'_t = {C'_t}^i \mathfrak{E}_{i,t} \quad \forall t \qquad 5.90$$

[44] A survey on financial access in Indonesia which is conducted by The World Bank in 2009.

Log-linearising the above equation around the steady state and integrate it over i, we obtain

$$\tilde{c}_t' = \tilde{y}_t^* + \tilde{q}_t \quad 5.91$$

As the foreign economy is exogenous to domestic economy, then there is flexibility in specifying the behaviour of foreign variables, including the world consumption $\tilde{y}_t^*$. Following Matheson (2009), we assume that the world consumption is an AR (1) process as the followings:

$$\tilde{y}_t^* = \rho_{y^*}\tilde{y}_{t-1}^* + \epsilon_t^{y^*} \quad \epsilon_t^{y^*} \sim N\left(0, \sigma_\epsilon^{y^*}\right) \quad 5.92$$

where $\rho_{y^* \in (0,1)}$ is the persistent parameter of this exogenous shock and $\sigma_\epsilon^{y^*}$ is the variance of a white noise innovation $\epsilon_t^{y^*}$.

5.3.6. Aggregation

As the model is absent from housing production, then the supply of housing stock is constant in every period. However, we allow households and entrepreneurs for the exchange of housing ownership. The total of housing stock in each period is given by:

$$\breve{H}_t = H_t + H_t' + H_t'' \equiv 1 \quad 5.93$$

In log-linearised form, we can re-write the above equation as:

$$0 = H\tilde{h}_t + H'\tilde{h}_t' + H''\tilde{h}_t'' \quad 5.94$$

where:

$$H = \frac{\phi_2}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6}, H' = \frac{\phi_3\phi_7}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6} \text{ and } H'' = \frac{\phi_4\phi_6}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6}$$

where: ϕ_2, ϕ_3, ϕ_4, ϕ_6, and ϕ_7 are deep parameters derived from steady state relation[45].

The total consumption goods is the sum of consumption goods consumed by both entrepreneurs and households (either domestic goods or importing goods). Using this definition, the total consumption goods is given by:

$$\breve{C}_t = C_t + C_t' + C_t'' \qquad 5.95$$

Log-linearised the above equation around the steady state yields:

$$\tilde{c}_t^T = \frac{1}{\phi_5+\phi_7+\phi_6}(\phi_5\tilde{c}_t + \phi_7\tilde{c}_t' + \phi_6\tilde{c}_t'') \qquad 5.96$$

where ϕ_5, ϕ_6 and ϕ_7 are deep parameters which are again derived from steady state relation.

5.3.7. Market Clearing and Equilibrium Condition

The aggregate goods market clearing for each good z requires

$$Y_t(z) = \breve{C}_{D,t}(z) + \int_0^1 C_{D,t}'^{\,i}(z)di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z) \qquad 5.97$$

where $\breve{C}_{D,t}(z) = C_{D,t}(z) + C_{D,t}'(z) + C_{D,t}''(z)$

The above equation tells that, in equilibrium, the total output produced by domestic firms is equal to the summation of domestic goods, foreign consumption goods (or export goods), physical capital investment and the cost of changing the physical capital and housing.

We follow Gali (2003) in which it is assumed government purchases a time-varying fraction of ι_t of output of each good z, financed by lump sum taxation. Consequently, aggregate goods market clearing for each good z requires:

$$(1-\iota_t)Y_t(z) = \breve{C}_{D,t}(z) + \int_0^1 C_{D,t}'^{\,i}(z)di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z) \qquad 5.98$$

[45] See Appendix 5.2 for the values of $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6$ and ϕ_7, and other steady state relations.

Substituting the relevant demand equations for each goods, i.e. equation (5.62) to (5.66) into the above equation and assuming a symmetric preference across countries, we can re-write equation (5.98) as follows:

$$(1-\iota_t)Y_t = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi}\breve{C}_t + \alpha\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi}\int_0^1\left(\frac{\mathbb{E}_{i,t}P_{F,t}^i}{P_{Y,t}}\right)^{\zeta-\varpi}\mathfrak{E}_{i,t}{}^{\varpi}C_t'^{\,i}di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

5.99

where: $S_t = \frac{P_{F,t}}{P_{Y,t}}$; $S_t^i = \frac{\mathbb{E}_t^i P_{F,t}^i}{P_{i,t}}$; $S_{i,t} = \frac{P_{i,t}}{P_{Y,t}}$; $S_t = \left(\int_0^1 S_{i,t}{}^{1-\zeta}\right)^{\frac{1}{1-\zeta}}$ and $\mathfrak{E}_{i,t} = \frac{\mathbb{E}_{i,t}P_{C,t}^i}{P_{C,t}}$

Recall that S_t is the effective term of trade of home country. S_t^i is the effective terms of trade of country i and $S_{i,t}$ is bilateral terms of trade between home economy and country i.

Further, using consumer price index (CPI) equation, i.e. $P_{C,t} = \left[(1-\alpha)P_{Y,t}{}^{1-\varpi} + \alpha P_{F,t}{}^{1-\varpi}\right]^{\frac{1}{1-\varpi}}$ (where it can be approached as $P_{C,t} = P_{Y,t}{}^{1-\alpha}P_{F,t}{}^{\alpha}$), we obtain the relation between PPI and CPI in the form of term trade as follows:

$$\frac{P_{Y,t}}{P_{C,t}} = \left(\frac{P_{Y,t}}{P_{F,t}}\right)^{\alpha} = S_t{}^{-\alpha} \qquad 5.100$$

Using equation (5.100), we replace the term $\frac{P_{Y,t}}{P_{C,t}}$ in equation (5.99) to get:

$$(1-\iota_t)Y_t = (1-\alpha)S_t{}^{\alpha\varpi}\breve{C}_t + \alpha S_t{}^{\alpha\varpi}\int_0^1\left(S_{i,t}S_t^i\right)^{\zeta-\varpi}\mathfrak{E}_{i,t}{}^{\varpi}C_t'^{\,i}di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

5.101

Defining a government expenditure shock g_t by $log(1-\iota_t) = -g_t$, we approximate the above equation around the steady state by:

$$\tilde{y}_t = (1-\alpha)\left(\frac{C}{Y}\tilde{c}_t + \frac{C'}{Y}\tilde{c}'_t + \frac{C''}{Y}\tilde{c}''_t\right) + \alpha\frac{C'}{Y}\tilde{y}^*_t + \left\{(1-\alpha)\alpha\varpi\frac{\breve{C}}{Y} + \alpha\frac{C'}{Y}[\alpha\varpi + \zeta - \varpi]\right\}\tilde{s}_t + \alpha\frac{C'}{Y}\varpi\tilde{q}_t + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{G}{Y}g_t \qquad 5.102$$

Similar to other exogenous shocks in the model, it is assumed that g_t follows an AR (1) process as follows:

$$g_t = \rho_g g_{t-1} + \epsilon^g_t \qquad \epsilon^g_t \sim N\left(0, \sigma^g_\epsilon\right) \qquad 5.103$$

with $\rho_g \in (0,1)$ is the persistent parameter and ϵ^g_t is a white noise shock with mean 0 and variance σ^g_ϵ.

From equation (5.102), we can see that domestic output depends on domestic and foreign consumption, term trade, real exchange rate, physical capital investment and government expenditure.

5.3.8. Monetary Policy

We close the model by specifying the Taylor-type interest rate rule in the log-linearised form around the steady state as follows:

$$\tilde{r}_t = \phi_r\tilde{r}_{t-1} + (1-\phi_r)\left(\phi_y\tilde{y}_t + \phi_\pi\tilde{\pi}_{C,t} + \phi_e\tilde{e}_t\right) + \epsilon^r_t \qquad \epsilon^r_t \sim N(0, \sigma^r_\epsilon) \qquad 5.104$$

where ϵ^r_t is distributed normally with mean zero and variance σ^r_ϵ.

Under the above specification, it is assumed central bank responds to the deviation of output ($\tilde{y}_t$), CPI inflation ($\tilde{\pi}_{C,t}$) and exchange rate ($\tilde{e}_t$) from its steady state value[46]. The Bayesian

[46] Lubik & Schorfheide (2007) assume that monetary reaction function responds to exchange rate depreciation $\Delta\tilde{e}_t$ in addition to output and inflation deviation from the steady state. Matheson (2010) assumes the monetary

methodology allows us to estimate the level of aggressiveness of policy function on output, inflation and exchange rate that is reflected from the estimated parameter ϕ_y, ϕ_π, and ϕ_e, respectively.

As previously mentioned when discussing about the recent development of monetary and macroprudential policies in Indonesia, the implementation of Inflation Targeting Framework (ITF) has attracted our attention about the way of how Bank Indonesia (BI) conducts its monetary policy. Ideally, we would like to see how the stance of BI alters by comparing the result before and after the implementation of ITF. Unfortunately, due to data availability, we focus the analysis on post period of the ITF and evaluate whether BI has put higher emphasis on price stabilisation than on output or on exchange rate stabilisation. For this purpose, we have considered the variable of output and exchange rate, in addition to inflation, in specifying the Taylor-type interest rate rule.

5.3.9. Shocks

In this sub section, we summarise all exogenous shocks introduced to the model and see how some of them are different to the others. In total, there are eleven exogenous shocks that influence the variables in the economy: nine domestic shocks and two foreign shocks. The domestic shocks are a monetary policy shocks ϵ_t^r, a housing preference shocks ϵ_t^H, a cost push/mark up shocks $\epsilon_t^{\mu^r}$, a covariance stationary technology shocks ϵ_t^Λ, a non-stationary permanent technology shocks ϵ_t^z, a government expenditure shocks ϵ_t^g, a general preference shocks ϵ_t^β, a marginal efficiency of investment (MEI) shocks $\epsilon_t^{I_K}$, and a LTV/financial

reaction function responds to exchange rate deviation from steady states $\tilde{e}_t$. Here, we follow Matheson (2010) when specifying monetary policy.

shocks ϵ_t^{LTV}. Meanwhile, the foreign disturbances include a world consumption shock $\epsilon_t^{y^*}$ and a world inflation shock $\epsilon_t^{\pi^*}$. Given all these shocks, we assess the contribution of different shocks to the Indonesian business cycle fluctuation. For this purpose, we will display their asymptotic variance decomposition of the main macroeconomic variables implied by the estimated model.

5.4. Methodology and Data

5.4.1. Methodology

The procedure required to estimate a Dynamic Stochastic General Equilibrium (DSGE) model using Bayesian econometric methods involves two steps:

Firstly, linearising the system of stochastic equations and solving the model by writing it into state-space representation. The model is linearised since it is built by a number of non-linear systems of equations which is derived from economic agent's optimisation behaviour in the economy. Non-linearity causes a closed analytical solution is not easy to obtain. As an alternative, the solution is approximated in the neighbourhood of a given point, i.e. the non-stochastic steady state. The solution of model is obtained when the linear stochastic different equations can be written in state-space form; that is when each endogenous variable is written in terms of its lagged (pre-determined variable) and the exogenous shock.

Secondly, estimating the model using a Bayesian method. This method is chosen since it delivers an advantage of using prior information and the data to estimate the structural parameters[47]. We argue the practicality of this method increases when the sample of data is

[47] See Beltran and Draper (2008), among others.

limited[48]. As a common knowledge, data availability and consistency is a classic problem faced by most developing countries, including in Indonesia. This happens as people in this country are still less aware about the advantage of having a long and credible time series data for policy analysis and decision making. This situation becomes more problematic when a long term time series data usually contains a structural break due to a major change in policy or a significant structural reformation. Therefore, the choice of using a Bayesian method seems very relevant since this technique gives flexibility in terms of the number of data used in the estimation. Notice that, the estimation procedure can only be executed when the solution of our RE model is unique, given the structural parameters of the model.

- **Solving the Model**

In refer to the existing literature; there are two general approaches of solving a DSGE model, i.e. global approaches and local approximation approaches. The former approach evaluates the model far away from the steady state and, sometimes, involves a switching in steady state. Yet, these global approaches are not applicable to solve the proposed model here as it is too large such that it becomes computationally expensive, given today's computer capabilities. Meanwhile, the latter approach, which is more convenient relatively to the first one, approximates the model locally around a non-stochastic steady state. This local approximation approach includes a method of (log)-linearisation, perturbation of higher order and linear-quadratic approximation.

In this chapter, we decide to analyse the model around the non-stochastic steady state by choosing the log-linearised method as the approximation methods. For this purpose, we apply

[48] See Schoot et al. (2014)

the first order terms in the Taylor series approximation (around a steady state) for each stochastic non-linear equation in the model. This method is considered since we dealt with a relatively large DSGE model that covers different economic agents' behaviour. By linearising, we simplify the problem significantly such that a solution to the model is easily to obtain. The result is not different when we linearise the model in level, instead of in logarithm form, except when interpreting the unit of measurement in the model.

After the model is log-linearised, the next step is finding a procedure to solve the rational expectation (RE) model. Based on the literature, there are different methods of solving the RE model, for example: a method of undermined coefficient, as used by Christiano (2002) among others, a method of exploiting the properties of RE errors as used by Sims (2002), and a method of splitting the variables into state and jump variables as in Blanchard & Kahn (1980). Among these three methods, the most common one is a method of splitting the variables into state and jump variables by Blanchard & Kahn (1980). Using this method, all endogenous variables are defined either as state (or pre-determined) variables or jump (or non-predetermined) variables. The existence and uniqueness of the solution is determined by comparing the number of non-explosive roots to the number of state variables. Unlike using a method of Blanchard & Khan (1980), the variables solved using a method of undetermined coefficient as used by Christiano (2002) and of expectational error as used by Sims (2002) are not necessary to be labelled as predetermined or non-predetermined variables. The difference between the last two methods is that the first one explicitly exploits the properties of the expectational errors under rational expectations while the second is not. A detail procedure for solving a rational expectation (RE) model using a method of Blanchard & Khan can be seen in appendix 2.3 in chapter 2 and will not be presented again here.

- **Bayesian Estimation**

This estimation combines the observed data with the log-linearised version of the model and the initial prior of the data to generate a set of posterior parameters. To link the observed data and the variables in the model, a measurement equation is constructed. The result of this Bayesian estimation is in the form of a distribution of each parameter in which their means are used as the estimators of each parameter in the model.

In the state-space representation, the relationship between the observed data, the solution of the log-linearised version of the model and the prior can be illustrated as follows:

$$\begin{bmatrix} z_{t+1}^{s} \\ z_{t+1}^{j} \end{bmatrix} = \begin{bmatrix} \widetilde{\Lambda}_{11} & 0 \\ \widetilde{\Lambda}_{12} & 0 \end{bmatrix} \begin{bmatrix} z_{t}^{s} \\ z_{t}^{j} \end{bmatrix} + \begin{bmatrix} \widetilde{\mathrm{B}}_{1} \\ \widetilde{\mathrm{B}}_{2} \end{bmatrix} \epsilon_{\mathrm{t}} \qquad 5.105$$

$$M_t = C \begin{bmatrix} z_{t}^{s} \\ z_{t}^{j} \end{bmatrix} + D\zeta_t \qquad 5.106$$

Equation (5.105) is the state/transition equation which describes the evolution of the endogenous variables. It is based on the linear version of the solution of RE model as we discuss earlier. Meanwhile, Equation (5.106) is the observation equation that links the observed data or observable variable M_t and some endogenous variables in the model z_t^s and z_t^j. The objective is to estimate the deep parameters in matrices $\begin{bmatrix} \widetilde{\Lambda}_{11} & 0 \\ \widetilde{\Lambda}_{12} & 0 \end{bmatrix}$ and $\begin{bmatrix} \widetilde{\mathrm{B}}_{1} \\ \widetilde{\mathrm{B}}_{2} \end{bmatrix}$, respectively.

We run the Bayesian estimation using DYNARE program of a version 4.4.3[49]. It is begun by evaluating the likelihood function of the model using the Kalman filter to obtain the log likelihood function. The log likelihood function describes the density of the observed data

[49] see Adjemian et al. (2011), http://www.dynare.org/.

given the parameters and the models. Combined with the data priors, this log-likelihood function generates the log-likelihood of the posterior as follows:

$$ln\mathcal{K}(\theta_{\mathcal{A}}|M_T,\mathcal{A}) = ln\mathcal{L}(\theta|M_T,\mathcal{A}) + ln\,p(\theta_{\mathcal{A}}|\mathcal{A})$$

The term of the left hand side refers to the log-likelihood of the posterior conditional on the observed data until period T for model $\mathcal{A}$. Meanwhile, the first term of the right hand side, i.e. $ln\mathcal{L}(\theta|M_t,\mathcal{A})$ corresponds to the likelihood function generated by Kalman filter and the second one, i.e. $\ln p(\theta|\mathcal{A})$ denotes a priori beliefs given information from the data, with $\mathcal{A}$ as the model and $\theta_{\mathcal{A}}$ as the model's parameters. In summary, there are three frameworks in Bayesian inference: Firstly, prior distribution, where we quantifies prior knowledge of parameter values. Secondly, likelihood, that is a probability of observing a data if we have a certain set of parameters values. Thirdly, posterior distribution, that is a conditional probability distribution of unknown parameters given observed data.

Unfortunately, the distribution of the posterior is composed by a complicated function of the deep parameters and is non-linear that causes it is often infeasible to derive moments of the posterior distribution analytically. To handle this problem, DYNARE has used a Monte Carlo Markov Chain (MCMC) sampling method, in particular the Random Walk Metropolis Hasting, that enables us to generates draws from posterior distribution and with those calculate numerical approximations to the distributions.

The steps of this algorithm can be described as follows:

1. Choose the initial value of θ_0 with $P(\theta|y_T) > 0$ from a starting distribution $P(\theta_0|y_T) > 0$. This initial value is often taken from the estimate at the posterior peak or from the neighbourhood around the peak.

2. Draw a candidate parameter vector θ^* from the jumping distribution: $J(\theta^*|\theta_{t-1}) = N(\theta_{t-1}, c^2 I)$ where c is a scale tuning parameter and I is the identity matrix. Without loss of generality, let assume that the candidate parameter vector θ^* is θ_{i+1} in which the probability of this parameter being accepted as parameter θ^* is $q(\theta^*|\theta_{i+1})$ and being rejected is $1 - q(\theta^*|\theta_{i+1})$.

3. Compute the acceptance ratio: $q(\theta_{i+1}|\theta_i) = min\left(1, \frac{P(\theta^*|y_T)}{P(\theta_i|y_T)}\right)$. This ratio compares the posterior given the candidate parameters θ^* to the posterior with the current parameters. If $q(\theta_{i+1}|\theta_i) = 1$ or $P(\theta^*|y_T) > P(\theta_i|y_T)$, then the chosen candidate is stored. If not, then move to the new candidate parameter using a random walk specification: $\theta^* = \theta_i + \varepsilon$ with $E[\varepsilon] = 0$.

4. Return to step 2 and repeat the procedure once again until the average acceptance ratio gets close to 25%. In practice, this standard acceptance ratio can be obtained by setting a scale factor c in the covariance matrix in such away. In addition, to minimise the effect of the initial choice of values θ_0, some proportion of the initial draw should be discarded.

DYNARE is also equipped with a tool to check the convergence of this MCMC process, following Brooks & Gelman (1998). This tool compares the variance between sequence or blocks $(\hat{B})$, and those within the replication $(\widehat{W})$ in the MCMC process. When the variance between blocks $(\hat{B})$ is closed to zero while at the same time the variance within replication is stable, then after sufficient number of replications, the convergence condition is attained.

5.4.2. Data and Its Transformation

As discussed earlier, the model features eleven exogenous shocks. The estimation is conducted with eight observable variables such that there exist as many shocks (or more) as observable variables to avoid a stochastic singularity problem[50]. The observable variables are derived from a quarterly macroeconomic data of Indonesia in which these data are initially transformed before entering to the model. Data transformation is required to ensure the observable variables align with the model-based definition. All data are transformed following Smets & Wouters (2007).

The observation period is from Q1: 2000 to Q4: 2014. We start the estimation from Q1: 2000 for two reasons: Firstly, we avoid structural break due to the Asian financial crisis in 1997-1998. Secondly, we capture the episode when Bank Indonesia began to implement an inflation targeting framework (ITF) regime. In this way, we can exclude the periods where regime changes and structural breaks were observed to avoid spurious inferences.

With regards to the data and its frequency, we use a quarterly data that covers the real gross domestic product (GDP), real consumption, real capital investment, consumption price index (CPI), relative price of housing prices index (HPI), nominal effective exchange rate (NEER), real effective exchange rate (REER), domestic interest rate and labour force[51]. Unfortunately, not all these data are reported in quarterly frequency. For example the data of labour force is published annually. As well, there are a quarterly data where its unit is recorded in annual unit instead of quarterly unit, e.g. nominal interest rate. There are also some series where its base year changes several times during a period of observation. Given all these facts and to be in line with the model-based variable definition, then all data series is required to be transformed

[50] Stochastic singularity problem happens when the number of shocks is less than that of the observables.
[51] The source of data and its description are illustrated in Appendix 5.1.

before being used in the estimation. The way of how these data series are transformed can be explained as follows:

- The series for real output (real gross domestic product), real consumption and real capital investment are supplied by the Indonesian Central Bureau of Statistic in quarterly basis[52]. To be aligned with the model-based definition, the data for capital investment is ideally obtained by subtracting gross investment with housing investment. Yet, the data for housing investment is difficult to obtain. As an alternative, we assume a fix proportion of gross investment, that is classified as physical capital investment, in each period[53]. In addition to data availability problem, we also observe that the series for output, consumption and investment data that have experienced two times change in their base year, i.e. in 2000 and 2010. For consistency, we select a 2010 as the base year and convert all old series based on a 2000 base year to new series with a 2010 base year.

  After having a consistent data series of real output, real consumption and real capital investment in a 2010 base, we follow the data transformation process as Smets & Wouters (2007). Initially, these data series are required to be seasonally adjusted. Then, they are divided by a quarterly data of labour force to obtain a new data series in per capita term, i.e. real output per capita, real consumption per capita and real capital investment per capita[54]. Note that the original series of labour force is in annually frequency. Using a linear interpolation, we can transform this series into a quarterly frequency[55].

  In order to obtain a stationer series, the data series are then put in the logarithm form and transformed it into a new series using a first difference method. Having this, the series are

52 These data series are not officially published.
53 A fix proportion of capital investment (over gross investment) is approximated by one minus the average ratio of construction sector over gross investment.
54 The number of labour force is derived from the number of population within the age of 15 and over.
55 We assume that the data of labour force grows at a linear rate throughout the year.

now in the form of $\Delta log\left(Y_t{}^{data}\right)$, $\Delta log\left(C_t{}^{data}\right)$ and $\Delta log\left(I_t{}^{data}\right)$, respectively. Finally, these data series are demeaned by subtracting their sample means before ready to be used in the estimation.

- The data series for consumer price index (CPI) is published by the Indonesian Central Bureau of Statistic in monthly frequency. Thus, it is straight forward to collect these data in a quarterly frequency by using the last month index in each quarter. Unfortunately, during observation period, the data series for CPI has experienced four times change in base year, i.e. 1996, 2002, 2007 and 2012. To have a consistent CPI data and a comparable index with housing prices index (HPI), we select a 2002 base year as a reference and convert CPI data, using a 1996, 2007 and 2012 base, into CPI data with a 2002 base. The way of how we translate CPI from one base year to another is explained as follows:

  We define M_t as the total spending required for buying a basket of consumption goods in year t. A price index is simply the spending required to purchase a basket of consumption goods in year t relative to some base year as follows:

  $I\{f\}_t = 100\left(\frac{M_t}{M_f}\right)$ where f denotes a base year f 5.107

  Using a different base year h, the above price index can be re-written as:

  $$I\{h\}_t = 100\left(\frac{M_t}{M_h}\right) \qquad 5.108$$

  Here, we assume there is no change in the type of goods in our consumption basket. In order to change a price index from one base to another base, we need to express them both in terms of the same base year. Consider a case where we want to change the price index base year f into base year h. We can convert the old price index by using the following formula:

$$I\{h\}_t = 100\left(\frac{M_t}{M_h}\right) = 100\left(\frac{M_t}{M_f}\right)\left(\frac{M_f}{M_h}\right) = I\{f\}_t\left(\frac{M_f}{M_h}\right) = \frac{I\{f\}_t}{I\{f\}_h/100} \qquad 5.109$$

As can be seen, the new price index with a h year base can be obtained by simply dividing the original index by its own value in year h divided by 100. To make it clear, we provide an example of this conversion process using our own data. Suppose we want to change a price index from a 1996 base to 2002 base. According to our data, the CPI for 2002 using the 1996 base is 262.38. Given this, we can convert all price index with a 1996 base into a price index with a 2002 base by simply dividing the former by 2.6238. If the price index on October 2003 using a 1996 is 282.4, then the price index for this period using a 2002 base is 282.4/2.6238 = 107.66. After converting all CPI data into a 2002 base year, the series are then transformed into the log CPI inflation, $log\left(\frac{CPI_t^{data}}{CPI_{t-1}^{data}}\right)$, before subtracting it with their long-run sample mean.

- The data series for nominal interest rate is taken from monthly Bank Indonesia coupon rate (SBI) which is published in the web of Bank Indonesia. Since the series is recorded in annual percentage, then we need to divide it by 400 and adding it with 1 to get a quarterly gross interest rate series as: $(R_t^{data}/400)$ +1. The new series is then put in the log form as $log\left(1+\frac{R_t^{data}}{400}\right)$ before deviating it with steady state value of interest rate in the model.
- Based on model-based definition, real housing prices is defined as the relative price of housing prices index (HPI) over consumption price index (CPI). As mentioned earlier, we choose a 2002 base year for CPI since HPI is also recorded with a 2002 base year. HPI is published quarterly by Bank Indonesia through a survey of property and residential

housing prices (SHPR). This survey is conducted as housing prices is not among the 742 commodities of CPI survey conducted by the Indonesian Central Bureau of Statistic. Since these 2 (two) surveys offer 2 (two) different data independently, then a bias in the estimation is at minimum. With regards to data transformation, the stationery series of real housing prices is obtained by setting the series into the log form as $log\left(\frac{HPI_t^{data}}{CPI_t^{data}}\right)$ and subtract it with its long-run mean. The resulting series is then interpreted as percentage deviation of real housing prices from the steady state.

- The data series for nominal effective exchange rate indexes (NEER) and real effective exchange rate index (REER) are provided in monthly averages[56]. It is published by Bank for International Settlement (BIS) using a 2010 base year (series name: broad1510). By definition, a nominal effective exchange rate (NEER) is an index of some weighted average of bilateral exchange rates. Meanwhile, a real effective exchange rate (REER) is the NEER adjusted by some measures of relative price or cost; changes in the REER thus take into account both nominal exchange rate developments and the inflation differential vis-a-vis trading partners. The stationary series are obtained by setting these data into log form as $log(reer^{data})$ and $log(neer^{data})$, respectively. The final resulting series can be interpreted as percentage deviation of nominal and real effective real exchange rate from their steady state.

5.4.3. Measurement Equations

For estimation, we need to define the observable variables to be linked with economic model variables using measurement equations. Our observable variables are taken from the result of

[56] An effective exchange rate provides a better indicator of the macroeconomic effects of exchange rates than any single bilateral rate.

data transformation as discussed in the earlier section. In details, the measurement equations are given as follows:

Real output growth:

$$\Delta log\left(Y_t^{data}\right) - \overline{\Delta log\left(Y_t^{data}\right)} = y_t^{obs} = \tilde{y}_t$$

Real consumption growth:

$$\Delta log\left(C_t^{data}\right) - \overline{\Delta log\left(C_t^{data}\right)} = c_t^{obs} = \tilde{c}_t^T$$

Real capital physical investment growth:

$$\Delta log\left(I_t^{data}\right) - \overline{\Delta log\left(I_t^{data}\right)} = i_{K,t}^{obs} = \tilde{\imath}_{K,t}$$

where Δ denotes the first-difference operator and the notation with bar corresponds to sample means. Observable variables y_t^{obs}, c_t^{obs}, $i_{K,t}^{obs}$ represent the demeaned real growth of output, consumption, and capita investment, respectively.

Consumption goods inflation:

$$log\left(\frac{CPI_t^{data}}{CPI_{t-1}^{data}}\right) - \overline{log\left(\frac{CPI_t^{data}}{CPI_{t-1}^{data}}\right)} = \pi_{C,t}^{obs} = \tilde{\pi}_{C,t}$$

Real housing prices:

$$log\left(\frac{HPI_t^{data}}{CPI_t^{data}}\right) - \overline{log\left(\frac{HPI_t^{data}}{CPI_t^{data}}\right)} = p_{H/C,t}^{obs} = \tilde{p}_{H/C,t}$$

Observable variables $\pi_{C,t}^{obs}$ and $p_{H/C,t}^{obs}$ are the demeaned CPI inflation and the demeaned real housing prices, respectively.

Real effective exchange rate:

$$log(reer^{data}) = q_t^{obs} = \tilde{q}_t$$

Nominal effective exchange rate:

$$log(neer^{data}) = e_t^{obs} = \tilde{e}_t$$

Nominal interest rate:

$$log\left(1 + \frac{R_t^{data}}{4x100}\right) - logR = r_t^{obs} = \tilde{r}_t$$

where r_t^{obs} is the detrended logged quarterly gross interest rate.

Using the above definition of our measurement equations, then the vector of observables that we use in the estimation is given by:

$$M_t = \left[y_t^{obs}, c_t^{obs}, i_{K,t}^{obs}, q_t^{obs}, e_t^{obs}, p_{H/C,t}^{obs}, \pi_{C,t}^{obs}, r_t^{obs}\right] \quad 5.110$$

5.5. Estimation

5.5.1. Calibrated Parameters

There are not many existing studies that have used a DSGE framework to estimate the Indonesian economy. Given this, some values of the calibrated parameters are taken from the existing literatures while the remaining are chosen in such way they match with the steady state of empirical data.

We start calibrating patient households' discount factor β'. We set this parameter equal to 0.9776 which implies 9.2% annual nominal interest rate (matched with a sample mean of the nominal Bank Indonesia coupon rate). We also fix the parameter of capital share in production μ equal to 0.4 which implies 60% steady state share of labour income in total output. Although this value is slightly high, our choice is still within the standard values used in the literatures e.g. Aliyev (2012), Khramov (2012), and Gadatsch (2015), among others. Capital deprecation rate δ_K is calibrated at 0.035 which implies 14% annual depreciation. We pick this value following a study by Schundeln (2012) who estimates depreciation rates of physical capital invested in manufacturing enterprises in Indonesia during the period 1985-1995 (based on firm-level data set). Based on his study, depreciation rate of physical capital in Indonesia lies within the range 8% to 14% annually. Obviously, we choose the upper bound of his estimation result.

Entrepreneurs' discount factor β is calibrated at 0.9693. This parameter is set in such a way so that a steady state ratio of physical capital over output is 0.2 (matched with the sample mean of physical capital investment over output ratio in the data). For impatient households, we choose their discount factor β'' at 0.9493. It is difficult to find a reference for this parameter using the steady state relationship, as an alternative we follow Iacoviello (2005) where we set this value below entrepreneurs and patient households discount factor to initiate borrowing activity. Following Iacoviello (2005), we also set $\varkappa$ that controls the residential housing stock over annual output at 0.1 while labour supply aversion η at 1.01. As well, we pick the value for housing share in production function v at 0.03. The parameter of housing adjustment cost ψ_H is assume to be 0 while the steady state mark up of final goods over intermediate goods X is chosen at 1.05 which implies 5% steady state mark-up of domestic goods. The reason for fixing the value of ψ_H at 0 is due to indeterminacy problem found when we set this parameter at a positive value. We decide to calibrate ϖ, the elasticity of type z domestic country goods, and ζ, the elasticity of imported goods from country i, at 1, following Monacelli & Gali

(2005). Meanwhile, housing depreciation rate δ_H is fixed at 0.01 which means 4% of annual rate, following Monacelli (2009).

Table 5.1.
Calibrated parameters[57]

Parameter	Description	Value	Targeted Variable/ Steady State
Preferences : Discount Factor			
β	Entrepreneurs	0.9693	Sample mean ratio of capital investment over output is 0.2.
β'	Patient households	0.9776	Sample mean of nominal BI rate (9.2% annual nominal interest rate)
β''	Impatient households	0.9493	Own Assumption
Other preferences parameters			
$\varkappa$	Weight on housing service	0.1	Iacoviello (2005)
η	Labour supply aversion	1.01	Iacoviello (2005)
Technology : Factors productivity			
μ	Capital share in production	0.4	A standard value used in the literatures e.g. Aliyev (2012), Khramov (2012), and Gadatsch (2015), among others.
v	Housing Share in production	0.03	Iacoviello (2005)
Other technology parameters			
ψ_H	Variable housing adjustment cost	0	Iacoviello (2005)
δ_K	Capital depreciation rate	0.035	14% annual depreciation Schundeln (2012)
δ_H	Housing depreciation rate	0.01	4% annual depreciation Monacelli (2009)
Mark-up and Elasticity			
X	Steady state gross mark-up	1.05	Iacoviello (2005)
θ	Probability domestic fixed price	0.75	Iacoviello (2005)
θ_F	Probability import fixed price	0.50	T. Matheson (2005)
ϖ	Elasticity : domestic/foreign good's bundle	1.0	Gali & Monacelli (2005)
ζ	Elasticity : among imported goods from country i	1.0	Gali & Monacelli (2005)

We also decide to calibrate the parameters of price of stickiness for domestic and import goods price as they were not well identified during estimation process. For domestic price, we set the Calvo parameter θ equal to 0.75 which implies an average duration of optimal price changes of four quarters. This value is fairly standard choice in DSGE literature. For the probability of import goods fixed price θ_F, we pick the value at lower probability, i.e. 0.5. This implies importing price is assumed to be more flexible than domestic price such that an

[57] The time unit is one quarter

average length of import price contracts is two quarters. The choice of our calibrated parameters is summarised in Table 5.1.

5.5.2. Priors Distribution

In order to be consistent with the existing literatures, the priors of structural parameters are set as close as possible to the relevant studies that estimate DSGE model featuring housing market and/or open economies using Bayesian technique.

We begin by setting a prior for the stochastic process. The standard deviations of the innovations to structural shocks are assumed to follow an inverse-gamma distribution with prior mean of 0.01 and slightly loose degrees of freedom 2, with an exception for MEI shock, financial shock, and government spending shock which are assumed to have prior mean of 0.02. This choice is fairly standard in the literatures, especially in the context of log-linearised model. For persistence parameters of shocks, we choose a beta-distribution with prior mean of 0.7 and standard deviation of 0.1. A decision to choose a beta-distribution comes from the fact that the value of these persistence parameters is bounded within zero and one. The same choice of priors, for persistence parameters of shocks, is used in Rabanal & Bracons (2010), among others.

Concerning the share of import goods in consumption goods bundles α, we set prior mean and standard deviation of this parameter equal to 0.4 and 0.1, respectively, where this prior is assumed to follow a beta-distribution. This mean value comes from the average ratio of import values over the Indonesian GDP (in constant price) during the period of observation. Notice that using the average ratio of import values over GDP to calibrate α is not entirely correct. This happens as import data in Indonesia is published as the aggregate import data that includes not only consumption goods but also durable and investment goods. Hence, instead of calibrating this parameter with the average ratio of import values over the Indonesian GDP, we let the data speaks by estimating this particular parameter using Bayesian technique.

We assume a prior mean for a wage income share of patient households σ to follow a beta-distribution with prior mean of 0.5 and standard deviation of 0.1. This mean value is within the range of comparable estimates in the literature: for example, Iacoviello (2005), using a limited information approach, estimates a wage income share of unconstrained agents of 64 percent in the US. With regards to monetary policy, we assume the standard Taylor-type policy rule that responds to the deviation of inflation, output and nominal exchange rate with inertia in policy rate. We set prior mean for monetary reaction function to inflation ϕ_π, output gap ϕ_y and nominal exchange rate ϕ_e at 1.25, 0.25 and 0.1, respectively with the same standard deviation of 0.1 for these parameters. Prior distribution of these parameters is assumed to follow an inverse gamma distribution which is quite standard in estimating policy parameters. Matheson (2009) uses the same prior means as ours when assessing the fit of small open economy DSGEs for Australia, Canada and New Zealand. For an interest rate smoothing parameter ϕ_r, we set its prior means at 0.5 with standard deviation 0.1. Since this smoothing parameter is bounded between zero and one, then we assume that the distribution of this prior follows a beta-distribution.

Table 5.2.
Prior and posterior distribution of structural parameters

Parameter		Prior				Posterior		
		Dist.	Supp.	Mean	St. Dev	Mean	5%	95%
α	Share of import con. goods	Beta	[0,1)	0.4	0.1	0.073	0.042	0.102
σ	Patient wage income share	Beta	[0,1)	0.5	0.1	0.505	0.357	0.652
μ^b	Proportion of wage income in collateral for impatient HH	Beta	[0,1)	0.5	0.1	0.533	0.370	0.687
χ	DP ratio for entrepreneurs	Beta	[0,1)	0.3	0.1	0.418	0.250	0.586
χ''	DP ratio for impatient HH	Beta	[0,1)	0.3	0.1	0.342	0.171	0.513
ψ_K	Variable capital adj. cost	Gamma	$\mathbb{R}^+$	2	0.1	2.003	1.848	2.166
ϕ_r	Taylor : interest smoothing	Gamma	$\mathbb{R}^+$	0.5	0.1	0.784	0.731	0.838
ϕ_π	Taylor : inflation	Gamma	$\mathbb{R}^+$	1.25	0.1	1.213	1.055	1.365
ϕ_y	Taylor : output	Gamma	$\mathbb{R}^+$	0.25	0.1	0.851	0.585	1.141
ϕ_e	Taylor : exchange rate	Gamma	$\mathbb{R}^+$	0.1	0.05	0.035	0.018	0.051
ρ_H	Housing Preference	Beta	[0,1)	0.7	0.1	0.749	0.630	0.887
ρ_{μ^r}	Mark up	Beta	[0,1)	0.7	0.1	0.304	0.205	0.409
ρ_z	Permanent Technology	Beta	[0,1)	0.7	0.1	0.395	0.271	0.511
ρ_Λ	Temporary Technology	Beta	[0,1)	0.7	0.1	0.630	0.474	0.785
ρ_{y^*}	World Consumption	Beta	[0,1)	0.7	0.1	0.793	0.704	0.877
$\rho_{\pi_C^*}$	World Inflation	Beta	[0,1)	0.7	0.1	0.430	0.307	0.550
ρ_g	Gov. Expenditure	Beta	[0,1)	0.7	0.1	0.497	0.373	0.632
ρ_β	General Preference	Beta	[0,1)	0.7	0.1	0.763	0.626	0.918
ρ_{I_K}	MEI	Beta	[0,1)	0.7	0.1	0.426	0.312	0.543
ρ_{LTV}	LTV	Beta	[0,1)	0.7	0.1	0.694	0.530	0.860
ϵ^H	Housing preference	Gamma	$\mathbb{R}^+$	0.01	2	0.107	0.065	0.150
ϵ^{μ^r}	Mark up/Cost Push	Gamma	$\mathbb{R}^+$	0.01	2	0.053	0.043	0.064
ϵ^z	Permanent Technology	Gamma	$\mathbb{R}^+$	0.01	2	0.002	0.002	0.003
ϵ^Λ	Temporary Technology	Gamma	$\mathbb{R}^+$	0.01	2	0.005	0.002	0.007
ϵ^{y^*}	World Consumption	Gamma	$\mathbb{R}^+$	0.01	2	0.025	0.021	0.029
$\epsilon^{\pi_C^*}$	World Inflation	Gamma	$\mathbb{R}^+$	0.01	2	0.014	0.012	0.017
ϵ^g	Gov. Expenditure	Gamma	$\mathbb{R}^+$	0.02	2	0.378	0.315	0.437
ϵ^β	General Preference	Gamma	$\mathbb{R}^+$	0.01	2	0.003	0.002	0.004
ϵ^{I_K}	MEI	Gamma	$\mathbb{R}^+$	0.02	2	0.033	0.027	0.038
ϵ^{LTV}	LTV	Gamma	$\mathbb{R}^+$	0.02	2	0.013	0.005	0.021
ϵ^r	Monetary Policy	Gamma	$\mathbb{R}^+$	0.01	2	0.002	0.002	0.003

Unlike similar studies that assume a fix value of lending parameters, in our work, we try to estimate these parameters in the context of Indonesia. This is motivated by the fact that some lending parameters, in particular the ratio of LTV, or its inverse a down payment (DP) ratio, had not been controlled by Bank Indonesia until March 2012[58]. For that purpose, we choose prior mean of down payment (DP) ratio for both entrepreneurs χ and impatient households χ'' at 0.3 with the standard deviation of 0.1. This mean value follows a beta-distribution and is in the range of comparable estimates in the literature. As illustration, Iacovielo (2005) estimates the ratio of DP in the U.S. at 0.2 for entrepreneurs and 0.4 for impatient households. As well, Notarpietro & Siviero (2015) has calibrated this parameter equal to 0.2, in line with the average for the U.S and the euro area[59]. Our choice for the prior is in the ball park of these values.

In practice, lending decision does not only depend on the amount of collateral. Banks may also consider wage income in addition to the assets owned by private agents. Since only impatient households who receive wage income the model, then we assume that these agents are constrained by not only housing assets but also wage income when applying for a housing loan. However, a question that remains: how significant banks' lending decisions is influenced by applicant's wage income. We address this issue by estimating the weight of wage income μ^b in the model. We assume priors of μ^b to follow a beta-distribution with a mean of 0.5 and standard deviation of 0.1. This choice follows Gelain et al. (2013) who calibrate this parameter equal to 0.5. Yet, under their model, it is assumed central bank has control over these lending parameters while in our works it does not. Table 5.2 summarises all priors used in the model that include prior distribution, mean and standard deviation or degree of freedom of the structural parameters.

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58 Before 2012, both micro and macroprudential policy are part of Bank Indonesia's authority. After 2012, microprudential policy is delegated to the Indonesian Financial Services Authority (OJK) while macroprudential policy remains part of Bank Indonesia's authority.

59 See Iacoviello & Neri (2010) and Calza, Monacelli & Stracca (2013) for the U.S and the euro area, respectively.

5.5.3. Posteriors Distribution

We summarise the estimation results in Table 5.2. It reports the posterior means together with 90% confidence interval obtained through draws from the posterior distribution using the Metropolis-Hasting sample algorithm. The results are based on a total of 100.000 draws and two independent chains where the Brook & Gelman (1998) convergence criteria are employed. To ensure all the estimated parameters are well identified and to see how useful the data in the estimation, we also plot the priors and the posteriors distribution for each of the estimated parameter in appendix 5.3. In general, all parameters are identified as suggested by DYNARE[60]. The parameters that are not identified have been calibrated.

Overall, our estimation results are plausible: the shapes of posterior distributions are broadly normal and the mode of posterior distribution is not excessively different from the mode calculated from the numerical optimisation of the posterior kernel. Yet, there are some parameters in which their priors are very close to their posteriors although there are still identified after checking it using a test of identification.

We start reporting the estimation results for the parameter of import goods share in consumption bundle (α), which is commonly interpreted as the degree of openness. Surprisingly, the share of import goods in consumption bundle is relatively small with posterior mean is 7.3%. The fact that the posterior distribution is different from the prior distribution implies that our data is very informative for estimating this parameter. A small proportion of import goods in consumption bundle suggests that the dependency of the Indonesian economy to foreign consumption goods is not very high. This might happen since this country is very well known for its natural resources, including agriculture products, such that most of consumption goods are supplied domestically. This result is in line with the data

[60] DYNARE applies identification analysis using the test described in Iskrev (2010).

reported by the Indonesian Central Bureau of Statistic in 2014, in which not more than 10% of the total value of import goods is classified as consumption goods. Given this fact, the model performs well in estimating this particular parameter. Notice that, this result is obtained by assuming no international trade for physical capital or investment goods. Allowing physical capital to be traded internationally may change the result.

The way of how Bank Indonesia (BI) conducts its monetary policy also attracts our attention. The implementation of Inflation Targeting Framework (ITF) since 2000 has brought the curiosity about the targets of BI. The estimation result suggests that BI consistently adopts the Taylor principle in their policy rate by raising the nominal interest at higher rate than domestic inflation. This is confirmed by looking the posterior mean of monetary reaction function to inflation ϕ_{π} that is estimated at the value of 1.21 (higher than 1). The result also suggests a relatively high response of monetary policy to output deviation from its steady state value. This is confirmed by looking the posterior mean of monetary reaction function to output ϕ_{y} which is estimated at 0.85. Having this result, we argue that the focus of BI is not only about price stabilisation but also economic growth, especially when domestic economy experiences a down turn. This result is interesting as most similar studies for developed countries report a small systematic response of policy rate to output deviation. Our finding confirms about a distinction between central bank in developed countries and developing countries, especially in the way of conducting monetary policy.

Previously, we have shown that BI does not only concern about price stabilisation but also economic condition. Yet, we are unsure whether policy rate is also used to stabilise the volatility in exchange rate. Motivated by this, we include the variable of exchange rate in the standard Taylor-type-interest rate rule. In this way, it is assumed that BI also responds to the deviation of nominal exchange rate from its steady state value. The estimation result shows that the responsiveness of policy rates to the exchange rate deviation ϕ_e takes on a lower value of 0.03. Therefore, for the case of Indonesia, it seems policy rate is rarely used as an instrument to stabilise the exchange rate. A possible explanation is that policy rate has been tied to other targets, e.g. price stabilisation. Theoretically, a country that adopts Inflation Targeting Framework (ITF) should let its exchange rate in floating. In this way, no response is necessarily given by policy rate to the exchange rate's volatility. Our estimation result is in line with this theory.

We turn our attention now to the estimation result for some lending parameters. As mentioned in earlier section, one objective of this chapter is to study the way of how banks or financial institutions carry out their lending decision. Since there are two different agents who can borrow in the economy, i.e. entrepreneurs and impatient households, we evaluate the relevant lending parameters for each of these borrowers. The estimation result suggests that banks in Indonesia are likely to set higher down payment (DP) ratio for agent who only has housing assets in collateral, but no wage income, when they decide a housing loan application. While for agent who can show both her wage income and housing assets, banks may set lower DP ratio since her wage income serves as additional guarantee. As illustrated in Table 5.2, parameter χ is estimated around 0.42 while parameter χ'' is estimated around 0.34. Recall that, these two parameters represent the ratio of DP for both entrepreneurs and impatient households, respectively. In the view of loan to value (LTV) ratio, which is an inverse of DP ratio, a higher LTV ratio is allowed to be given for impatient households, compared to entrepreneurs, since they can provide the proof of wage income in addition to the asset of housing as collateral.

Table 5.3.
Maximum loan to property sector (percentage)

Type	LTV 2012	LTV 2013			LTV 2015		
		1st home	2nd home	3rd home	1st home	2nd home	3rd home
House > 70 m2	70	70	60	50	80	70	60
House 22-70 m2	n/a	n/a	70	60	n/a	80	70
Apartment > 70 m2	70	70	60	50	80	70	60
Apartment 22-70 m2	n/a	80	70	60	90	80	70
Apartment < 21 m2	n/a	n/a	70	60	n/a	80	70
Shophouse	n/a	n/a	70	60	n/a	80	70

Source: Bank Indonesia

How the above result relates with the current LTV ratio set by Bank Indonesia (BI). As mentioned earlier, BI has controlled the LTV ratio for property loans (housing loans and apartment ownership loan) since March 2012. Currently, the LTV ratio for property loans is set within the range 60%-90%, depending on the types of property and the numbers of property owned by borrowers at the time they make a loan application. In details, the maximum loan to property sector in Indonesia is illustrated in Table 5.3.

Comparing our estimation result with the current setting of LTV ratio in Indonesia, we can show that our estimation for LTV ratio falls not far from the range 60% to 90%, i.e. 57% for entrepreneurs and 66% for impatient households. Recall that in our model set up, we assume this LTV ratio is not controlled by BI, instead we assume that banks have controlled over this ratio independently and we try to estimate it using relevant data.

The previous results have not specifically evaluated the role of wage income in lending decision. As a standard procedure, it is common nowadays banks requests for the proof of wage income in addition to housing assets in collateral. We try to address this issue by estimating the weight of wage income in banks' lending decisions relative to housing assets in collateral. The estimation results suggest that banks put roughly higher attention on wage income than housing assets in collateral. As shown in Table 5.2, the estimated weight parameter of wage income has posterior means around 0.53 which implies more than 50% of lending decision is based on wage income. This finding confirms the importance of wage income in lending decision. Concerning the incentive received by households for supplying a labour, the estimation result suggest that there is no significant difference between wage income received by patient and impatient households in Indonesia. This is reflected from posterior mean of parameter σ that falls around 0.5.

After describing the estimation results for some structural parameters, we now focus on the shock process. Overall, the estimated persistence parameters of exogenous shocks are heterogeneous, lie within the range 0.3 for a cost push shock and 0.79 for the world consumption shock. For a foreign inflation shock, the posterior mean of its persistence parameters is estimated at 0.43. Meanwhile, for a general preference shock and a housing preference shock, the estimated AR (1) coefficient are reported at 0.76 and 0.74, respectively. As well, posterior mean of the persistence parameter in the unit-root technology process and the stationary technology shock are estimated at 0.39 and 0.62, subsequently.

For the shock of government expenditure and he marginal efficiency of investment (MEI), their autoregressive coefficients are estimated at 0.49 and 0.42, respectively. As for the standard deviation of the shocks, we find that all the estimated values are to be significantly different from zero, where the less volatile one is a monetary policy white noise shock and the most volatile one is the shock of government expenditure.

5.5.4. Variance Decomposition

In this section, we assess the contribution of different contemporaneous shocks on the Indonesian economy. For this purpose, we display their asymptotic variance decomposition of the main aggregate variables implied by the model, as shown on Table 5.4. There are at least three important findings that can be summarised from this exercise: Firstly, a permanent technology shock and a monetary policy shock are unarguably important shocks since they dominantly explain the volatility of business cycle fluctuation in Indonesia. Secondly, a marginal efficiency of investment (MEI) shock is the main driver of the variability of capital investment which also matters in explaining output and consumption variation. Thirdly, a housing preference shock accounts for a massive contribution in the volatility of housing prices in Indonesia.

We start discussing the role of a permanent technology shock. As shown in Table 5.4, this exogenous contemporaneous shock accounts for more than 24% and 18% of output and consumption variation. As well, this innovation largely explains about the variability of other macro variables e.g. investment (11.34%), housing prices (10.86%), CPI inflation (19.07%), PPI inflation (18.66) and policy rate (31.93%). On the basis of this outcome, one can conclude that a permanent technology shock serves as the leading sources of the Indonesian business cycles. This finding is in line with the study by Aguiar & Gopinath (2007) who argue that emerging markets are usually characterised by fluctuation in trend growth rates that causes an innovation to trend growth rate as a source of business cycle fluctuation in emerging markets. A significant role of permanent technology shock is contrast with the role of a temporary technology shock in which the latter only contributes a small proportion of business cycle fluctuations in Indonesia, e.g. 0.35% and 0.26% of the variability of output and consumption, respectively.

Following a permanent technology shock, a monetary policy shock appears to be an important shock that drives the Indonesian business cycles. It contributes more than 17% and 19% of output and consumption volatility, respectively. A sufficiently large contribution of a monetary policy shock to the variability of consumption is line with the study by Aspachs-Bracons & Rabanal (2010) who found around 14% of consumption volatility in Spain and more than 20% in the rest of European Monetary Union (EMU) are explained by this type of innovation. This shock also well captures the variability of investment and policy rate with a contribution 7.54% and 6.02%, subsequently.

A MEI shock serves as another important shock for the Indonesian economy. It explains roughly 41% of the volatility of capital investment. Other than capital investment, a MEI shock also sufficiently matters in explaining the volatility of other aggregate variables, for instance: output (11.15%), consumption (13.34%), CPI inflation (8.97%), PPI inflation (8.75%) and interest rate (12.55%).

A significant contribution of a MEI shock is in line with the literatures, for instance: Justiano et al. (2011) and Fisher (2006), among others. In is worth to mention that the data for capital investment series in this work do not include durable consumption and change inventories, following Smets & Wouters (2003). This treatment is different from the work of Justiano et al. (2011) who combine durable consumption and change inventories in investment data series. The reason for not differentiating consumption data series into durable and non-durable consumption is due to unavailability of the Indonesian Central Bureau Agency to provide such series.

We pursue a discussion by evaluating the source of housing prices volatility in Indonesia. As mentioned earlier, the main driver of the variability of housing prices is a housing preference shock that contributes almost 70% of housing prices variation. A sufficiently large contribution of this shock in explaining the volatility of housing prices is in line with the study of Iacoviello & Neri (2010) for the US economy and of Darracq-Parries & Notarpietro (2008) for euro area. Aspachs-Bracons & Rabanal (2010) also find a very similar conclusion by using data for Spain and euro area in their analysis. Other than a housing preference shock, a permanent technology shock and a MEI shock explain roughly 10% and 5%, subsequently, of the variability of housing prices.

Table 5.4
Variance decomposition of model without news shocks – baseline model

	ϵ^{H}	$\epsilon^{\mu^{r}}$	ϵ^{Λ}	ϵ^{z}	$\epsilon^{y^{*}}$	$\epsilon^{\pi^{*}}$	ϵ^{g}	ϵ^{β}	ϵ^{I_K}	ϵ^{LTV}	ϵ^{r}
Output	11.08	13.38	0.35	24.26	2.49	9.83	8.79	1.14	11.15	0.19	17.34
Consumption	5.15	13.88	0.26	18.76	10.96	9.64	5.88	2.35	13.34	0.16	19.63
Capital Inv.	23.21	4.26	2.77	11.34	2.63	6.20	0.08	0.09	41.71	0.18	7.54
Housing prices	69.35	1.80	2.64	10.86	1.43	0.87	2.92	3.11	5.38	0.02	1.62
CPI inf.	1.28	48.45	3.25	19.07	0.48	16.30	0.22	0.21	8.97	0.01	1.76
PPI Inf.	1.20	49.58	3.69	18.66	0.36	15.60	0.16	0.20	8.75	0.01	1.80
Import Inf.	0.67	6.21	0.33	4.96	78.46	5.47	1.11	0.12	2.41	0.00	0.27
Interest Rate	5.75	6.77	2.99	31.93	0.33	28.98	2.70	1.90	12.55	0.06	6.02

The innovation of a housing preference is denoted by ϵ^{H}, a mark-up/cost push $(\epsilon^{\mu^{r}})$, a stationary technology (ϵ^{Λ}), a non-stationary technology (ϵ^{z}), a world consumption $(\epsilon^{y^{*}})$, a world inflation $(\epsilon^{\pi^{*}})$, a government expenditure (ϵ^{g}), a general preference (ϵ^{β}), a marginal efficiency of investment-MEI (ϵ^{I_K}), a loan to value/borrowing constraint (ϵ^{LTV}) and a monetary policy (ϵ^{r}). All values are presented in percentage.

Meanwhile, the role of monetary policy and LTV shocks in explaining the variability of housing prices is minimal. Together, they only contribute 1.64% of housing prices volatility. This finding shows a challenge faced by Bank Indonesia in stabilising housing prices volatility in Indonesia. The minimal role of a monetary policy shock in explaining housing prices volatility is similar to what found in Aspachs-Bracons & Rabanal (2010) for a case of Spain. With regards to CPI inflation, it is shown that mark-up/cost push shock contributes around 48% of the variability of this variable. This result is plausible as being an archipelago with more than 13.000 islands; the supply of goods in Indonesia depends heavily on infrastructures e.g. logistics and transportations. As an illustration, during rainy season, where usually characterised by unfriendly weather, there is a tendency for firms to raise their mark-up to anticipate a disturbance from supply side. Other than cost push shock, the variability of

CPI inflation is explained by a permanent technology shock (19.07%) and a world (foreign) inflation shock (16.30%). The latter is transmitted through the channel of import consumption goods.

The world inflation shock also explains the variability of import good inflation, but it is not the main driver. Most of the volatility of import goods inflation is explained by a world consumption demand shock with a contribution 78.46%. This shock is transmitted from the exchange rate channel which affects import goods inflation. Recall that import goods inflation occurs as a fraction of import firms do not adjust their import prices in every point in time but set it as a staggered fashion ala the Calvo. As the result, changes in the exchange rate are not entirely transmitted into domestic price of import due to nominal price rigidity, which implies a deviation from the law of one price assumption. In the literature, this condition is known as incomplete exchange rate pass through.

5.5.5. Historical Decomposition

Another way of evaluating the contribution of each exogenous shock to the Indonesian business cycles is by analysing the historical decomposition result. By this way, we are allowed to observe a period where the Indonesian economy experiences a recession or a boom along with the various structural shocks that may contribute to those events. We focus on the movement of output, consumption, capital investment and housing prices over the sample period, i.e. Q1-2000 to Q4-2014. As the role of a stationary technology shock is minimal, as shown from the result of variance decomposition earlier, we group this shock with a non-stationary technology shock to become one category, i.e. a technology shock. We do the same for the shock of a general preference, a government expenditure and a LTV/financial shock where we combine all these shocks into a group called the rest of the shocks. In this way, the category of shocks has reduced from eleven into eight, as reported in Figure 5.1 through Figure 5.4 bellow.

In line with the results from the variance decomposition, the shock of technology, mark-up cost, housing preference and monetary policy are the main contributors to the movement of the Indonesian economy during the period of observation. A positive effect of technology shock to the economy mainly takes place between Q3-2009 and Q4-2014, as reflected from a bar with an “orange colour” in Figure 5.1. In contrast, the effect of a technology shock to the economy before Q3-2009 is unclear as it changes from a positive to a negative effect (or vice versa) overtime. Based on its contribution, the role of a technology shock has increased since Q3-2009, compared to its role before Q3-2009.

The result from Figure 5.1 also shows that there are moments where the Indonesian economy has experienced a down turn, for instance: Q1 to Q2-2000, Q1-2001 to Q4-2002, Q2 to Q4-2003, and Q1 to Q2-2006. Notice that during period 2000-2003, the Indonesian economy has experienced a period of severe political instability, while during 2005-2006 the Indonesian economy has been underperforming due to the increase of world oil price that causes the Indonesian government to increase oil retail price domestically by reducing their subsidy on this commodity. One similarity found, during these downturn periods, is that a negative effect of monetary policy seems matter. In addition to technology and monetary policy innovations, a mark-up cost and a housing preference shock have their impacts on output volatility. A positive and large effect of a mark-up shock to the variability of output appears during Q2-2002 to Q3-2004. While a smaller effect of this shock is observed in the rest of sample periods.

Another finding is that the role of a world consumption demand shock, in explaining output volatility in Indonesia, is minimal over the sample periods. By this finding, an increase or a decrease on export does not give a significant effect to the performance of the Indonesian economy. In contrast, the effect of world inflation shock does matter to output, although it is not dominant. Note that a channel by which world inflation shock affects domestic economy is through CPI inflation.

Figure 5.1
Historical decomposition of output in Indonesian (Q-o-Q growth rate)

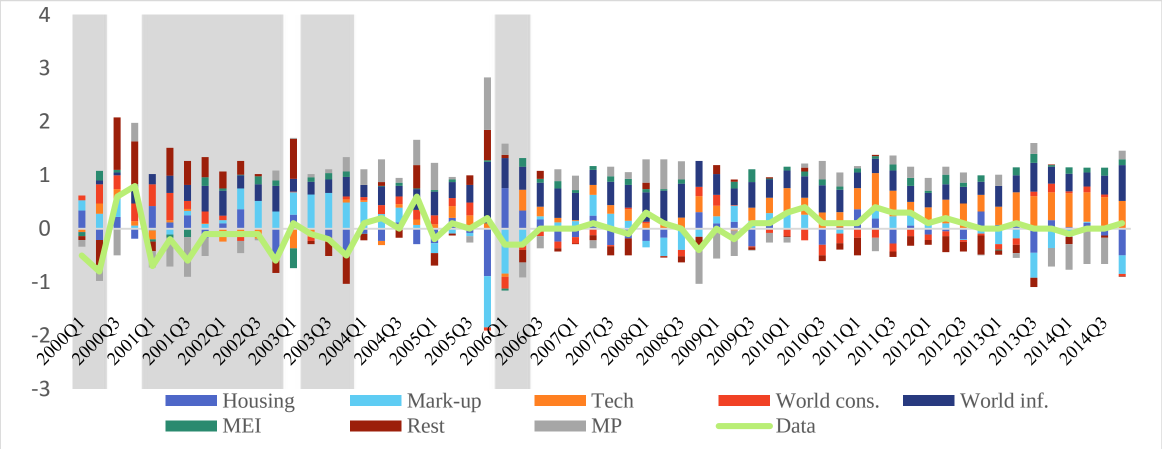
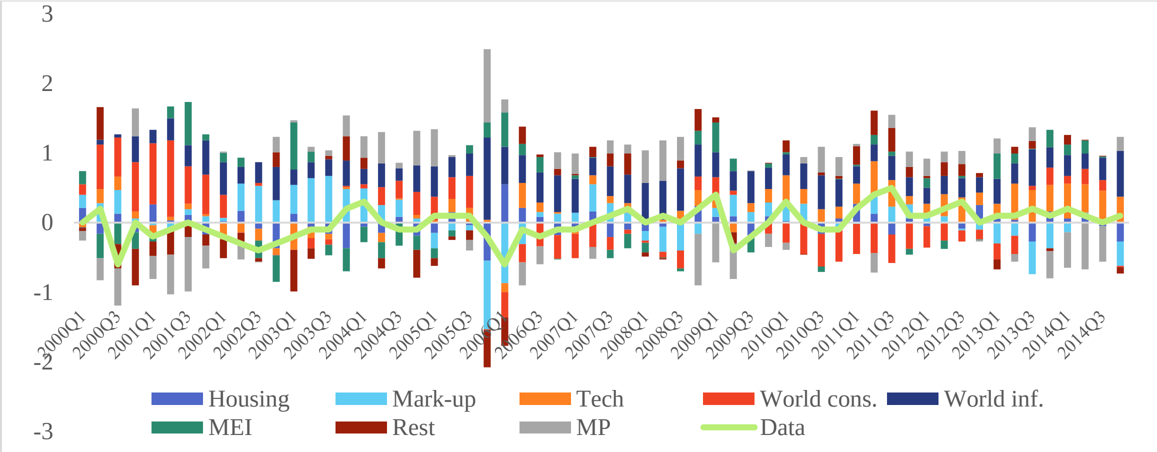


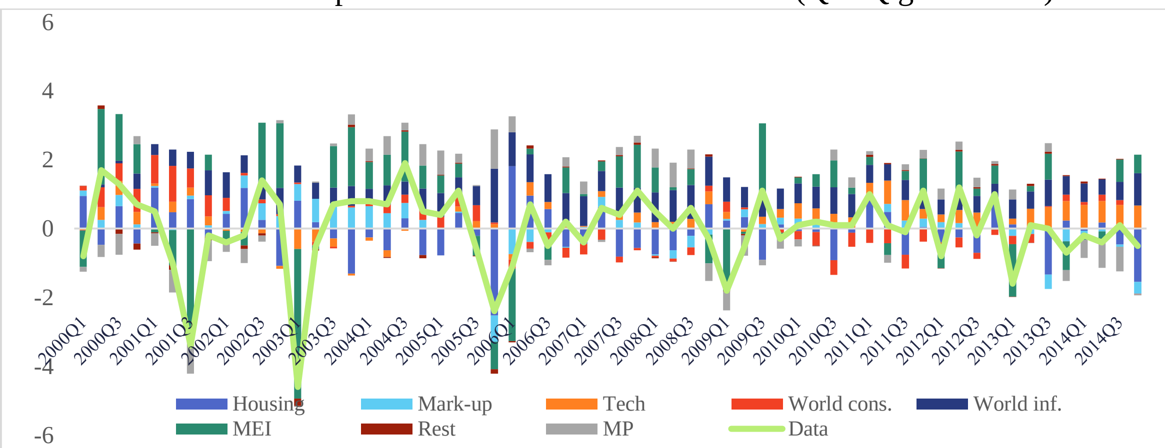
Figure 5.2 advises that the main drivers of consumption volatility are a technology shock, a monetary policy shock, a mark-up cost and a world consumption demand shock. As there is a similarity with regards to the main drivers of consumption and output, we may expect that these two variables move in the same way in the economy. Our hypothesis is confirmed by evaluating the correlation between consumption and output during sample periods that falls at positive value of 0.32 (see Appendix 5.5).

Figure 5.2
Historical decomposition of consumption in Indonesia (Q-o-Q growth rate)



The historical decomposition simulation shows that a marginal efficiency of investment (MEI) shock is the main driver of physical capital investment during the whole sample period, either with a positive effect or a negative effect (see Figure 5.3). In particular, a severe downturn on physical capital investment during Q3-2001, Q1-2003 and Q1-2006 is mostly accounted by a large negative MEI shock. Another finding is a tendency for physical capital investment to experience a downturn in the beginning of each year before increases in the next quarters.

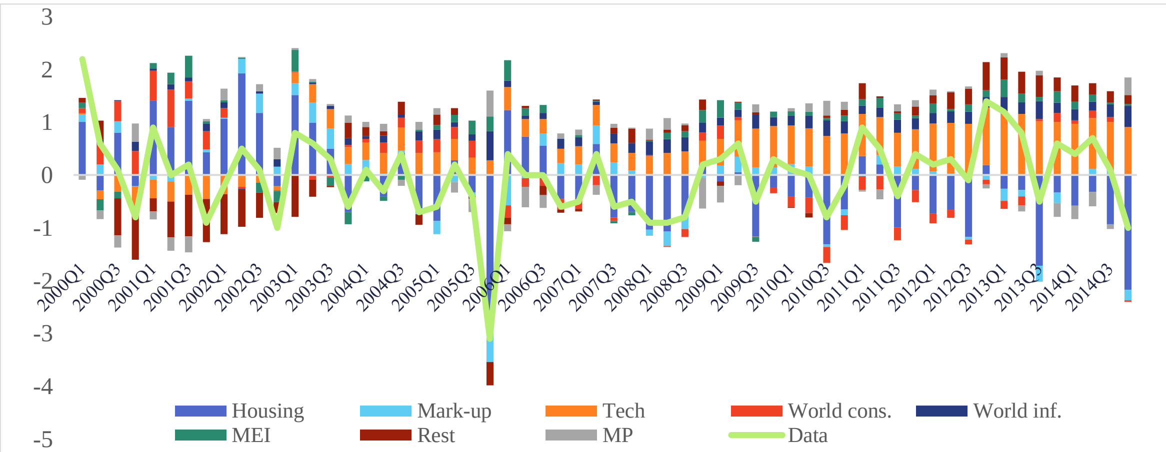
Figure 5.3
Historical decomposition of investment in Indonesia (Q-o-Q growth rate)



A result from historical decomposition simulation also advises that a housing preference shock is the main driver of housing prices volatility in Indonesia (see Figure 5.4). In particular, the severe downturn in housing prices during Q4-2005 and within Q3-2007 to Q3-2008 is mostly accounted for by large negative housing prices shocks. Notice that Q4-2005 is the period where oil retail price has increased almost 100 percent from 2,400 rupiah (12 cent) in March 2005 into 4,500 rupiah (22.5 cent) in October 2005. Meanwhile, the horizon of Q3-2007 to Q3-2008 is the period where a financial crisis arises in the US that causes a slowdown in housing demand, including in Indonesia. Note that this global financial crisis is considered by many economists to have been the worst financial crisis since the great depreciation of the 1930s.

Another important disturbance that explains the volatility of housing prices in Indonesia is a technology shock. This shock mostly gives a positive impact to the variability of housing prices starting from Q2-2003. From the historical decomposition simulation, we can also confirm that the role of a monetary policy shock is minimal in explaining the volatility in housing prices over the sample periods. This is in line with the results found when analysing the variance decomposition previsouly.

Figure 5.4
Historical decomposition of housing prices in Indonesia (Q-o-Q growth rate)

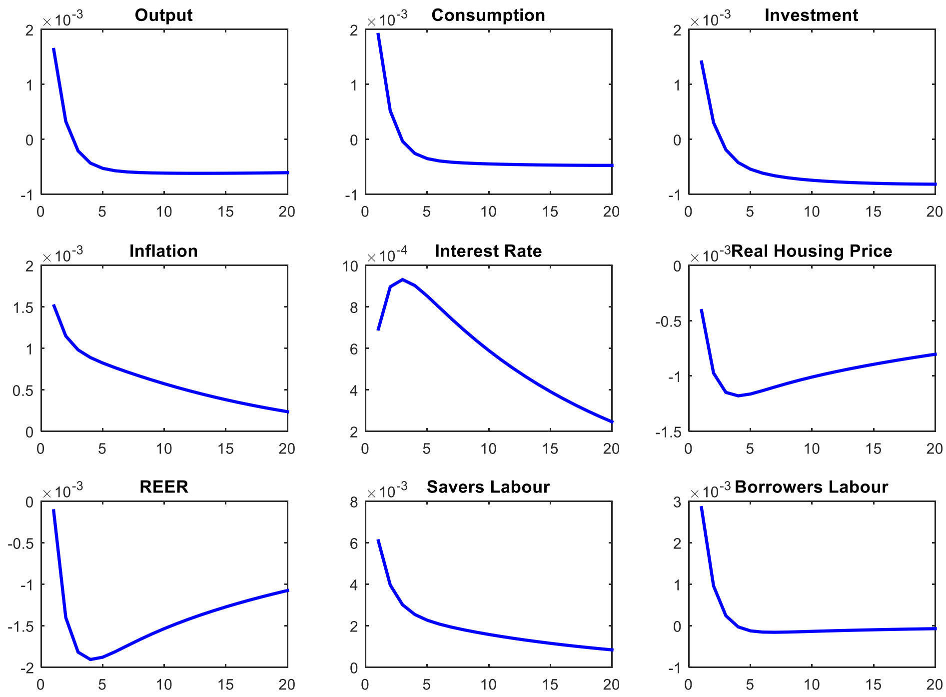


5.5.6. The Impulse Response Function

- **A Non-Stationary Permanent Technology Shock**

By the standard mechanism, a positive non-stationary permanent technology shock ϵ_t^z improves productivity. Under this condition, consumption, investment, labour and output increase with the exception concerning the housing prices (see Figure 5.5). Since output, consumption, investment and labour move to the same direction, then this contemporaneous shock successfully generates co-movement among these variables as shown in empirical data.

Figure 5.5.
Responses to a one std. deviation non-stationary technology shock



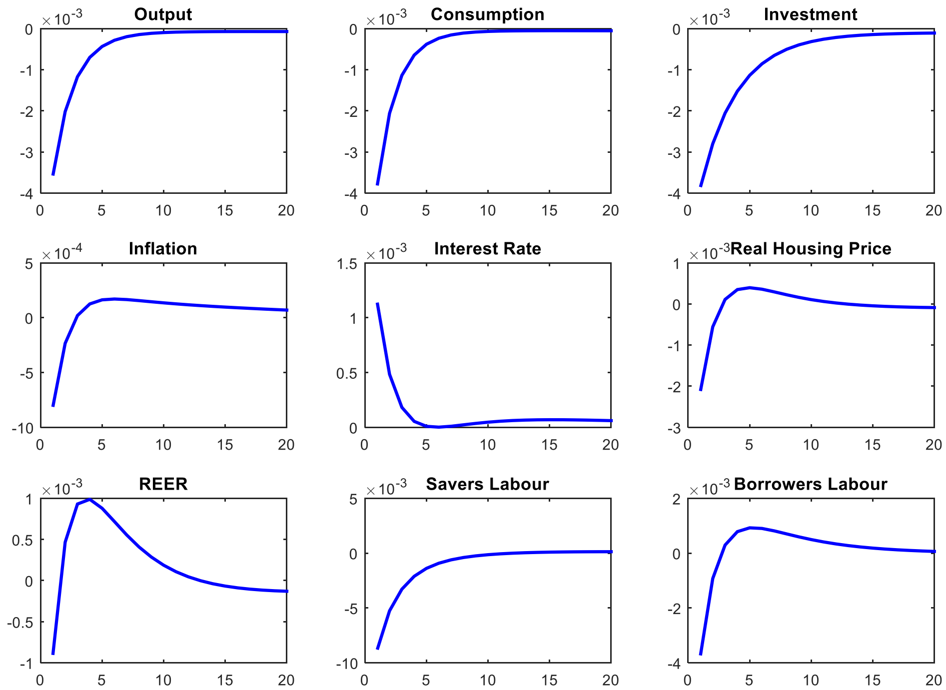
A positive permanent technology shock also triggers an increase on inflation at the early periods. This happens as people becomes optimistic about current and future economic condition. As the anticipation, BI starts to increase their policy rate slightly that makes the Indonesian currency appreciates over foreign currency. Unfortunately, this kind of shock fails to generate co-movement between consumption and housing prices as reflected from the empirical data (see Appendix 5.5). After several quarters, the effect of this exogenous shock diminishes and all variables return to their steady state level.

- **A Monetary Policy Shock**

A positive shock to interest rate ϵ_t^r indicates a monetary tightening policy. This unanticipated shock causes an immediate decline in consumption, followed by a drop in labour and output as the expected interest rate increases (as illustrated in Figure 5.6). Yet, the effect of this structural shock on consumption is not homogenous among agents. As an illustration, for patient households (savers), an increase in policy rate raises their consumption slightly above its steady state level since they get higher return from their lending activity. In contrast, borrowers (i.e. entrepreneurs and impatient households) need to reduce their consumption level since the cost of borrowing gets higher. As a reduction in borrowers' consumption is greater than an increase in lenders' consumption, in aggregate, consumption falls that leads to a decline in CPI inflation and a decrease in output latter on.

A positive monetary policy shock also causes nominal exchange rate to appreciate while investment and housing prices reduce as a decline in aggregate demand including a demand for housing. Notice that the immediate drop of consumption, investment and output reconciles a positive co-movement of these quantities in response to monetary policy shock that is generally found using VAR models (see Bernanke & Gertler, 1995 and Erceg & Levin, 2006).

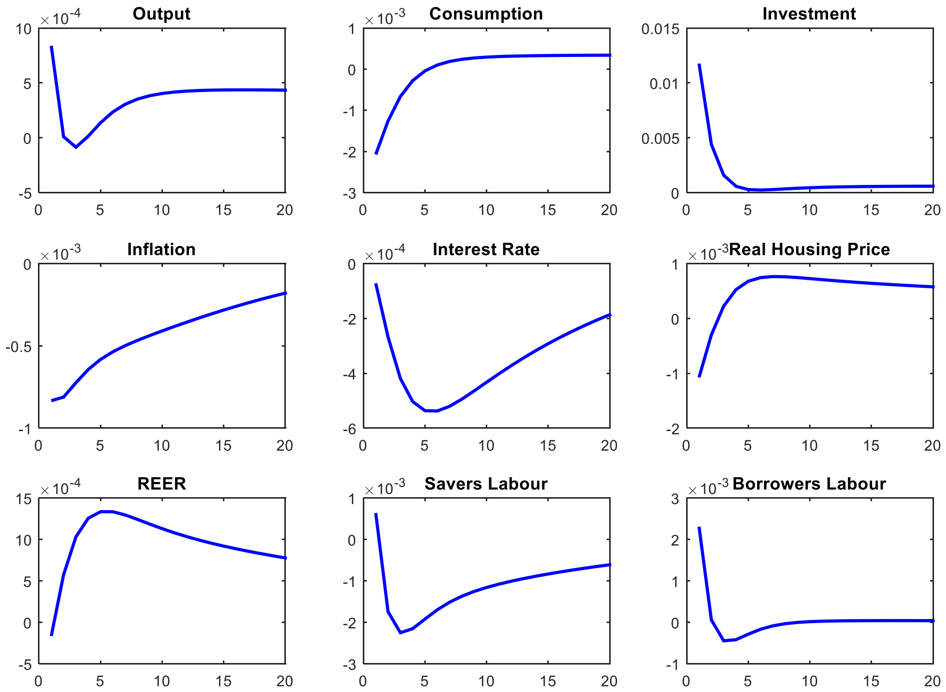
Figure 5.6.
Responses to a one std. deviation monetary policy shock



- **A Marginal Efficiency of Investment (MEI) Shock**

A positive MEI shock $\epsilon_t^{I_K}$ increases capital investment that leads to an increase in output slightly (see Figure 5.7). This innovation also causes an increase in labour for both patient and impatient households. Unfortunately, an increase on capital investment has crowded out consumption that causes this variable to decline. As the result, these two variables move in opposite direction. In summary, under a positive MEI shock, investment, output and labour move to the same direction but not consumption and housing prices. Since there is a positive correlation between output and consumption in the Indonesian business cycles (see Appendix 5.5), then our baseline model fails in generating co-movement of consumption and output.

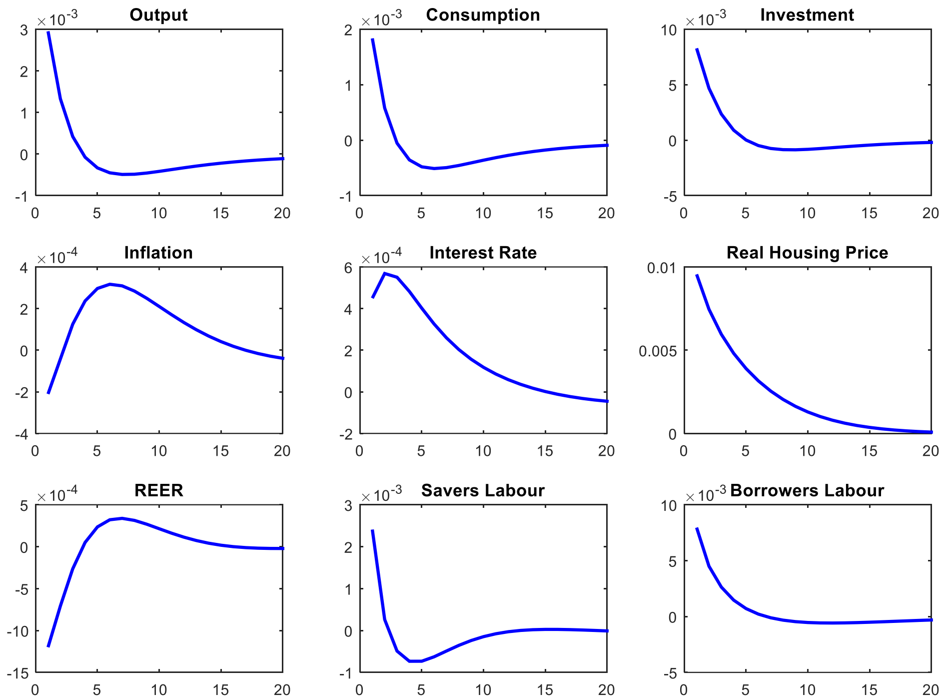
Figure 5.7.
Responses to a one std. deviation marginal efficiency of investment (MEI) shock



- **A Housing Preference Shock**

A housing-preference shock generates a disturbance that shifts housing demand. This shock can resemble effect such as temporary tax advantages to housing investment and a sudden increase in demand fuelled by optimistic consumer expectations, following Iacoviello (2005). In a simulation, a positive housing preference shock surges housing prices as a result of an increase in housing demand. A rise in housing prices gives a benefit to borrowers as the value of their housing assets in collateral increases such that more loans can be obtained. As borrowers have a higher propensity to spend than lenders, then aggregate demand increases that triggers an increase in CPI inflation. Under such condition, the real value of outstanding loan decreases that accelerates borrowing capacity of borrowers that leads to an increase in housing prices even further. As Iacoviello (2005), the accelerator mechanism of housing demand shock works very well here. Interestingly, under housing preference shock, the model works well in capturing co-movement behaviour between housing prices, consumption, investment and output as standard empirical data.

Figure 5.8.
Responses to a one std. deviation housing preference shock



5.6. The Feature of News Shocks

So far, we have estimated a specific dynamic stochastic general equilibrium (DSGE) model for Indonesia by incorporating some features that are relevant to the Indonesian economy characteristics. As well, we have evaluated the source of business cycles fluctuation in Indonesia in which some exogenous shocks are assumed to be present in the economy. Although, the model seems sensible in describing the Indonesian economy, still it could be subject to a criticism. One possibility is in the way of how the exogenous shocks are modelled.

In earlier setting, we assume that all structural shocks are to be unanticipated. Under such condition, economic agents are always be surprised by any deviation from the historical practice. Although this assumption is plausible, in reality it is unlikely to happen all the time. Agent may use and update any early information or news that they have got to anticipate future outcome. Motivated by this, we relax the assumption of unanticipated shocks by combining these shocks with the anticipated ones and evaluate their contribution to the Indonesian business cycles. According to news literature, the anticipated shocks are known as news shocks that consist of information that are useful for predicting future fundamentals but do not affect current fundamentals.

We consider an alternative version of exogenous shocks that evolves according to:

$$\epsilon_t^i = \epsilon_{t,0}^i + \epsilon_{t,news}^i \qquad \epsilon_{t,0}^i \sim N\left(0, \sigma_{\epsilon^i}^2\right) \text{ and } i \in \{z, r, I_K\} \qquad 5.118$$

The superscript i is used to differentiate types of exogenous shock, for example: z associates to a non-stationary permanent technology shock, r associates to a monetary policy shock and I_K associates to a marginal efficiency of investment (MEI) shock. The first component $\epsilon_{t,0}^i$ represents unanticipated i.i.d., zero mean, finite-variance, fundamental innovations. This shock is also known as a surprised or contemporaneous shock. Meanwhile, the second component $\epsilon_{t,news}^i$ represents news shocks component. This shock is assumed to be known to economic agents in period $t - h$, but will only materialize h quarters ahead. Given its characteristic, a "news" shock about future event can be written explicitly as:

$$\epsilon_{t,news}^i \equiv \sum_{h=1}^{H} \epsilon_{t-h}^{i,h} \qquad 5.119$$

where H is the maximum horizon where agents can memorize and anticipate advance news about future policies. In the absent of news shocks component, it turns out that $\epsilon_t^i = \epsilon_{t,0}^i$ and we label the model under such condition as the baseline model.

Following Schmitt-Grohe & Uribe (2012) and others related studies e.g. Christiano et al. (2014) and Khan & Tsoukalas (2012), we assume that news shock are driven by innovation announced four or eight quarters in advance ($h = 4\ or\ h = 8$). This implies a disturbance to the exogenous fundamentals of the economy at time t is a summation of three signals: Firstly, a signal received eight quarters in advance. Secondly, a signal received four quarters in advance. Thirdly, a surprised shock at time t itself. A signal received in four quarters in advance associates to a revised version of a signal received eight quarters in advance. Meanwhile, a signal received at time t is a revision of the sum of the signals received in eight and four quarters in advance.

We also assume that all shocks are uncorrelated and set the sum of the variance of news shocks component is, evaluated at prior means, at most one half of the variance of the corresponding unanticipated component. Note that we focus the study of news shocks only on three exogenous shocks, i.e. non-stationary permanent technology shock, monetary policy shock, and marginal efficiency of investment (MEI) shock, as these three shocks are found to be quantitatively important for the Indonesian business cycles, as shown in earlier sub sections.

According to the literatures, the study on news shocks in business cycles is not new. There have been some previous studies that analyse the role of news shocks in the economy. Among others is a seminal paper by Schmitt-Grohe & Uribe (2012) who claim the importance of anticipated shocks in explaining the US post war business cycles using a real business cycle

(RBC) model augmented with four real rigidities: internal habit formation in consumption, investment adjustment costs, variable capital utilization, and imperfect competition in labour markets, with a specific preference that governs the wealth elasticity of labour supply, following Jaimovich & Rebelo (2009). They found that anticipated shocks account for about half of predicted aggregate fluctuation in output, consumption, investment and employment. Other studies are Cochrane (1998), Hoover & Jorda (2001) and Milani & Treadwell (2012) who evaluate the role of a specific news shocks, i.e. monetary policy news shocks. As well, Mertens & Ravn (2011) and Leeper et al. (2013), who examine the role of unanticipated fiscal/tax policy shocks.

Table 5.5
Log marginal data densities of different models

Model Version	Log Marginal Likelihood
Baseline : Model without any news component	1,590.05
4 and 8 quarters ahead marginal efficiency of investment (MEI) shock	1.588.57
4 and 8 quarters ahead monetary policy shock	1,577.18
4 and 8 quarters ahead permanent technology shock	1,581.12

The marginal data density is computed using the modified harmonic mean method proposed by Gweke (1999), based on 100,000 draws for each model after discarding the first 50,000 draws.

Before evaluating the role of news shocks in the context of the Indonesian economy, one important criteria that ensures the alternative model (with news components) is better than the baseline model is the ability of the news shocks components to improve the empirical fit of the alternative model with the data. If incorporating the component of news shocks does not improve the empirical fit of the model in capturing the behaviour of empirical data, then there is no reason to evaluate the role of news shocks. For the purpose of evaluating the empirical fit of the model with the data, we need to compare the value of log marginal data densities (MDD) between the baseline model and its alternatives, as summarised in Table 5.5.

Overall, the performance of the baseline model (i.e. model without news component) is superior to alternative models. For example, comparing the value of log marginal likelihood between the baseline model and a version of model with four and eight quarters ahead of MEI shock gives a log MDD difference of 1.37. This implies “slightly evidence” in favour of baseline model over the alternative model, i.e. a model with news component in MEI shocks. Our argument here follows Jeffries (1996) who advise that a log MDD difference in the range of [1.10, 2.30] means “slightly evidence” in favour of model i over j. In terms of a Bayes Factor (BF), the range of log MDD difference within 1.10 to 2.30 is equivalent to a BF of 3 to 10, i.e. [ln 3, ln10]. If a BF falls within 10 to 100 or a log MDD difference of [2.30, 4.61], then there is “strong to very strong evidence” that model i is superior to model j. While if a BF is over 100 (a log MDD difference is over 4.61), then we can say that there is “decisive evidence” that model i is better than model j.

Comparing the value of marginal log likelihood between baseline model and a version of model with four and eight quarters ahead of monetary policy news shocks, we come to the conclusion that the baseline model is better than its alternative with a log MDD difference is equal to 12.75. This result implies that there is “decisive evidence” that baseline model is more powerful than its alternative in explaining the behaviour of empirical data. Finally, comparing the value of marginal log likelihood between baseline model and a version of model with four and eight quarters ahead of non-stationary technology news shocks, we can show there is “decisive evidence” that the baseline model is preferred with a log MDD difference of 8.82.

In summary, the above result shows that a model without news shocks component is superior to a model with news shocks component under the proposed model’s specification. Why this could be happened? We explain it using two arguments as the followings:

First, the baseline model has sufficiently accommodated all relevant features that may be important for the empirical fit of the model to data. These include nominal and real rigidities, financial constraints, and monopolistic competition in domestic markets. As the result, the baseline model has performed quite well in capturing the behaviour of empirical data, for example: co-movement behaviour among aggregate variables, as discussed in the section of the impulse response function (IRF) earlier. Incorporating news shock component into the baseline model, to improve this co-movement behaviour, seems irrelevant. In fact, introducing news shocks component to baseline model has increased model's complexity that reduces the value of log MDD.

Secondly, the baseline model may have missed a specific feature that matters for the empirical fit of the model to data, as discussed in Gortz & Tsoukalas (2011) and Gortz & Tsoukalas (2013), among others. They can show that co-movement among aggregate variables can be improved and the role of technology news shock becomes matter if the feature of nominal price and wage rigidities are introduced to the model along with the feature of financial friction that links the financial markets and real activities. Their result is obtained under the framework of two sector NK model where financial intermediaries use deposit from households and their own equity to finance the acquisitions of physical capital by capital service producers, following Gertler & Karadi (2011) and Gertler & Kiyotaki (2010).

Unfortunately, we are uncertain about the most valid argument between the above two. It might be useful to incorporate wage rigidness and other alternatives of modelling financial frictions. However, that is beyond the scope of this chapter as there are many different ways one could introduce financial frictions and it is not clear which mechanism would be the most appropriate. In order to keep the analysis simple and clear in terms of the focus on the effects of the housing market, we decide to leave this specific issue for our future work. Given this, we end the study of news shocks here and conclude that all results, under the model without news components, remains valid since incorporating news shocks component does not improve the empirical fit of the model to the Indonesian data.

5.7. Conclusion

This chapter attempts to develop an otherwise version of DSGE model for Indonesia. The proposed model is established by combining some features used in a model with housing market and borrowing constraint of Iacovielo (2005), a small open economy model of Gali & Monacelli (2005), and a model with incomplete exchange rate pass-through as Monacelli (2005). We argue that all these features are important and relevant to the Indonesian economy characteristics.

The proposed model is then estimated using a Bayesian technique. Our focus is on some deep parameters that explain about banks' lending decisions, monetary policy's objective, and the degree of economic dependency. With regards to banks' lending decisions, the estimation result suggests that banks in Indonesia are likely to set higher ratio of down payment for households who have only housing assets in collateral than the ones who can show both her income and housing assets when applying for a housing loan. This happens as the wage income serves as an additional guarantor for banks. The estimation result also shows that banks put roughly higher attention on the wage income than housing assets in collateral. All these results are obtained by assuming each bank independently set their lending ratio without control from Bank Indonesia.

Concerning the objective of monetary policy, the estimation result suggests that Bank Indonesia (BI) has consistently adopted the Taylor principle in their monetary policy which implies price stabilisation is among their main targets. The estimation result also advises that BI policy rate is not only used for the purpose of price stabilisation but occasionally used to promote economy growth, especially when the economy experiences a down turn. This finding is interesting as most similar studies in developed countries suggest about unnecessary response of interest rate to output/economic growth. In regards to the issue of exchange rate stabilisation, it shows that policy rate is rarely used to stabilise the variability of exchange rate. This is in line with a theory that says a country that adopts Inflation Targeting Framework (ITF) should let its exchange rate in floating.

As for the degree of economic dependency, the estimation result shows that the Indonesian economic dependency is not as high as predicted. This conclusion is obtained by evaluating the proportion of import goods in consumption goods bundle in which its share is less than 10%. Although, not perfectly comparable, this result is line with the data reported by the Indonesian Central Bureau of Statistic in 2014 where the share of consumption goods in total import values is not more than 10% in Indonesia.

The analysis is then extended by evaluating the source of business cycles fluctuation in Indonesia. Roughly, there are three main drivers of the Indonesian economy, i.e. non-stationary permanent technology shock, monetary policy shock and marginal efficiency of investment (MEI) shock. The important role of non-stationary permanent technology shocks is in line with the study by Aguiar & Gopinath (2007) who argue that emerging market are usually characterised by fluctuation in trend growth rates that causes an innovation to trend growth rate as a source of business cycles fluctuation in emerging market. Meanwhile, a significant contribution of monetary policy shock to business cycle fluctuations follows a similar result found by Aspachs-Bracons & Rabanal (2010) for the case of Spain and Euro area. MEI shock also serves as another important shock in the Indonesian economy. This finding supports a claim of Justiano et al. (2010) who conclude that this particular shock is the most important driver of U.S business cycle fluctuations in the post-war period.

The feature of housing market and financial constraint in the proposed model also allow us to identify the determinant of housing prices volatility. It can be shown that housing preference shock is the main disturbance of housing prices variation. This finding is in line with other similar studies in different countries, for example: the study of housing prices volatility in the US (see Iacoviello & Neri, 2008) and euro area (see Darracq-Parries & Notarpietro, 2008). Surprisingly, the contribution of monetary policy shock and loan to value (LTV) shock on the variability of housing prices is minimal that suggests a limitation of monetary policy and macroprudential policy in handling housing prices bubble in Indonesia. The estimation result also advices that CPI inflation variation is dominantly contributed by cost push shock, followed by non-stationary permanent technology shock and the world inflation shock. Note that the effect of world inflation disturbance is transmitted through the channel of import goods price that affects CPI inflation.

Up to here, the model provides estimations that access the structural parameters of the economy. In addition, the model serves well in identifying the sources of business cycle fluctuation in Indonesia. Although, the assumption of the model seems realistic, still, it is subject to criticism. One of them is the feature of the exogenous shocks that is assumed to be unanticipated by agents. Motivated by this, we consider additional features that have been found to be important in earlier work, i.e. news shocks. We limit the analysis of news shock only on the disturbance of MEI, monetary policy and non-stationary permanent technology as they dominantly influence business cycles in Indonesia.

The estimation result shows that introducing the feature of news shocks, regardless the way of how these anticipated shocks introduced to the model, either on MEI, monetary policy, or non-stationary permanent technology does not improve the empirical fit of the model in capturing the behaviour of the data. This can be seen by comparing the value of log likelihood or log marginal data density (MDD) of different models. We can show that a model with news shocks component has lower value of MDD compared to the baseline model (a model where news shocks is absent). One possible explanation is the feature of news shock does not improve co-movement behaviour among aggregate variables, as argued by Jaimovich & Rebelo (2009).

Our model describes the Indonesian economy well and results appear in line with the literature and economic intuition. However for a more detailed analysis of the indonesian economy the relevance of additional features may be considered. These features includes, but are not limited to, habit in consumption, wage stickiness and price indexation. Constructing a capital goods sector which is different than consumption goods sector also may improve the model's environment. As well, allowing the capital goods to be traded internationally may improve the estimation result as under the current setting only consumption goods can be traded internationally. The assumption of a fix housing stock is also too strong, given a significant development of housing sector in Indonesia. Again, modelling a separate housing sector in the model may take the model closer with the reality. Describing the behaviour of commercial banks explicitly, as shown in Gerali et al. (2010), is also another potential extension to the current setting. In this way, we have flexibility in introducing other macroprudential instruments than the ratio of loan to value (LTV) or down payment (DP), for instance: monetary reserve requirement, minimum capital requirement, etc. However, this goes beyond the scope of this chapter and is left for furture work.

Appendix 5.1. Data and Sources

Data	Source	Explanation
Real Gross Domestic Product (GDP)	The Indonesian Central Bureau of Statistic	Based on 2010 = 100
Real Consumption	The Indonesian Central Bureau of Statistic	Based on 2010 = 100
Real Investment	The Indonesian Central Bureau of Statistic	Based on 2010 = 100
Labour Force	The Indonesian Central Bureau of Statistic	
Interest Rate	Bank Indonesia (BI)	
Nominal Effective Exchange Rate (NEER) Index	Bank for International Settlements	BIS effective exchange rate Monthly average : 2010 = 100
Real Effective Exchange Rate (REER) Index	Bank for International Settlements	BIS effective exchange rate Monthly average : 2010 = 100
Consumer Price Index (CPI)	Bank Indonesia (BI) (Statistics Indonesia)	Based on 2002 = 100
Housing prices Index (HPI)	Bank Indonesia (BI) (Survey of Property Residential Price)	Based on 2002 = 100

Note :

The item of housing prices is not included among 742 items of goods and services surveyed by the Indonesian Central Bureau of Statistic to measure consumer price index (CPI). So far, only the item of housing contract price is included in CPI basket in Indonesia.

Survey of property residential price (SHPR) is a quarterly survey conducted by Bank Indonesia (BI) where its respondents are housing developers in some large cities in Indonesia. Currently, this survey covers 50 main housing developers in the city area of Jakarta, Bogor, Depok and Bekasi (Jabodebek) - Banten and 441 cities in the rest of Indonesia. The collected data includes price of housing in current quarter and the expectation of housing prices in the next quarter for three types of house, i.e. small (the size is less than 36 m2), medium (36-72 m2) and large (the size is larger than 70 m2). For aggregation, a weighted average method is used based on the weight of each city and the average price of housing.

Appendix 5.2. Model Derivation

A. Optimality Conditions

Entrepreneurs

$$Max\, E_o \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t^{\beta}} \ln C_t$$

$$S.t \qquad Y_t = e^{\Lambda_t} K_{t-1}^{\mu} H_{t-1}^{\nu} (A_t L'_t)^{\sigma(1-\mu-\nu)} (A_t L''_t)^{(1-\sigma)(1-\mu-\nu)} \qquad 5A.1$$

$$\frac{P_{Y,t}}{P_{C,t}} \frac{Y_t}{X_t} + B_t = C_t + P_{H/C,t} I_{H,t} + \frac{R_{t-1}}{\Pi_{C,t}} B_{t-1} + \frac{W'_t}{P_{C,t}} L'_t + \frac{W''_t}{P_{C,t}} L''_t + I_{K,t} + A_{K,t} + A_{H,t}$$

with $X_t = \frac{P_{Y,t}}{P_{Y,t}^{W}}$ 5A.2

$$(1-\chi) E_t\left(P_{H/C,t+1} H_t \pi_{C,t+1}\right) e^{\varepsilon_t^{LTV}} = R_t B_t \qquad 5A.3$$

$$e^{\varepsilon_t^{I_K}} I_{K,t} = K_t - (1-\delta_K) K_{t-1} \qquad 5A.4$$

$$I_{H,t} = H_t - (1-\delta_H) H_{t-1} \qquad 5A.5$$

$$A_{K,t} = \frac{\psi_K}{2\delta_K} \left(\frac{I_{K,t}}{K_{t-1}} - \delta_K\right)^2 K_{t-1} \qquad 5A.6$$

$$A_{H,t} = \frac{\psi_H P_{H/C}}{2\delta_H} \left(\frac{I_{K,t}}{H_{t-1}} - \delta_H\right)^2 H_{t-1} \qquad 5A.7$$

$$\mathcal{L} \equiv E_o \sum_{t=0}^{\infty} \beta^t \Bigg(e^{\varepsilon_t^{\beta}} \ln C_t + \Omega_{C,t} \Bigg[\frac{P_{Y,t}}{P_{C,t} X_t} \left(e^{\Lambda_t} A_t^{(1-\mu-\upsilon)} K_{t-1}^{\mu} H_{t-1}^{\nu} L_t'^{\sigma(1-\mu-\nu)} L_t''^{(1-\sigma)(1-\mu-\upsilon)} \right) + B_t - C_t - P_{H/C,t}(H_t - (1-\delta_H) H_{t-1}) - \frac{R_{t-1}}{\Pi_{C,t}} B_{t-1} - \frac{W'_t}{P_{C,t}} L'_t - \frac{W''_t}{P_{C,t}} L''_t - \frac{1}{e^{\varepsilon_t^{I_K}}} (K_t - (1-\delta_H) K_{t-1}) - \frac{\psi_K}{2\delta_K} \left(\frac{I_{K,t}}{K_{t-1}} - \delta_K\right)^2 K_{t-1} - \frac{\psi_H P_{H/C}}{2\delta_H} \left(\frac{I_{H,t}}{H_{t-1}} - \delta_H\right)^2 H_{t-1} \Bigg] + \lambda_t \left[(1-\chi) E_t\left(P_{H/C,t+1} H_t \pi_{C,t+1}\right) e^{\varepsilon_t^{LTV}} - R_t B_t \right] \Bigg)$$

FOCs

$$\frac{\partial \mathcal{L}}{\partial C_t} \equiv \beta^t \left(\frac{e^{\varepsilon_t^{\beta}}}{C_t} - \Omega_{C,t} \right) = 0$$

$$\frac{e^{\varepsilon_t^{\beta}}}{C_t} = \Omega_{C,t} \qquad \text{5A.8}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} \equiv \beta^t\big(\Omega_{C,t} - \lambda_t R_t\big) - \beta^{t+1} E_t\left(\frac{\Omega_{C,t+1} R_t}{\Pi_{C,t+1}}\right) = 0$$

$$\Omega_{C,t} = \beta E_t\left(\frac{\Omega_{C,t+1}}{\Pi_{C,t+1}}\right) R_t + \lambda_t R_t \qquad \text{5A.9}$$

$$\frac{\partial \mathcal{L}}{\partial L_t'} \equiv \beta^t\left(\frac{P_{Y,t}}{P_{C,t}X_t} e^{\Lambda_t} A_t^{(1-\mu-\nu)} K_{t-1}^{\mu} H_{t-1}^{\nu} \sigma(1-\mu-\nu) L_t'^{\sigma(1-\mu-\nu)-1} L_t''^{(1-\sigma)(1-\mu-\upsilon)} - \frac{W_t'}{P_{C,t}}\right)\Omega_{C,t} = 0$$

$$\frac{W_t'}{P_{C,t}} = \frac{\sigma(1-\mu-\nu)Y_t}{X_t L_t'} \frac{P_{Y,t}}{P_{C,t}} \qquad \text{5A.10}$$

$$\frac{\partial \mathcal{L}}{\partial L_t''} \equiv \beta^t\left(\frac{P_{Y,t}}{P_{C,t}X_t} e^{\Lambda_t} A_t^{(1-\mu-\nu)} K_{t-1}^{\mu} H_{t-1}^{\nu} L_t'^{\sigma(1-\mu-\nu)} (1-\sigma)(1-\mu-\nu) L_t''^{(1-\sigma)(1-\mu-\upsilon)-1} - \frac{W_t''}{P_{C,t}}\right)\Omega_{C,t} = 0$$

$$\frac{W_t''}{P_{C,t}} = \frac{(1-\sigma)(1-\mu-\nu)Y_t}{X_t L_t''} \frac{P_{Y,t}}{P_{C,t}} \qquad \text{5A.11}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} \equiv \Omega_{C,t}\left(\frac{1}{e^{\varepsilon_t^{I_K}}} + \frac{\psi_K}{\delta_K}\left(\frac{I_{K,t}}{K_{t-1}} - \delta_k\right)\right) = \beta E_t\left(\Omega_{C,t+1}\left[\frac{\mu Y_{t+1} P_{Y,t+1}}{K_t X_{t+1} P_{C,t+1}} + (1-\delta_K)\left(\frac{1}{e^{\varepsilon_{t+1}^{I_K}}}\right) + \frac{\psi_K}{\delta_K}\left(\frac{I_{K,t+1}}{K_t} - \delta_K\right)\left(\frac{I_{K,t+1}}{K_t} - \frac{1}{2}\left(\frac{I_{K,t+1}}{K_t} - \delta_K\right) + (1-\delta_K)\right)\right]\right)$$

$$\Omega_{C,t}\left(\frac{1}{e^{\varepsilon_t^{I_K}}} + \Omega_{K,t}\right) = \beta E_t\left(\Omega_{C,t+1}\left[\frac{\mu Y_{t+1} P_{Y,t+1}}{K_t X_{t+1} P_{C,t+1}} + (1-\delta_K)\left(\frac{1}{e^{\varepsilon_{t+1}^{I_K}}}\right) + \Omega_{K,t+1}\breve{\Omega}_{K,t+1}\right]\right)$$

5A.12

with $\Omega_{K,t} \equiv \frac{\psi_K}{\delta_K}\left(\frac{I_{K,t}}{K_{t-1}} - \delta_k\right)$ and $\breve{\Omega}_{K,t} \equiv \frac{1}{2}\left(\frac{I_{K,t}}{K_{t-1}} + \delta_K\right) + (1-\delta_K)$

$$\frac{\partial \mathcal{L}}{\partial H_t} \equiv \Omega_{C,t}\left(P_{H/C,t} + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{I_{H,t}}{H_{t-1}} - \delta_H\right)\right) = \beta E_t\left(\Omega_{C,t+1}\left[\frac{\upsilon Y_{t+1} P_{Y,t+1}}{H_t X_{t+1} P_{C,t+1}} + (1-\delta_H) P_{H/C,t+1} + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{I_{H,t+1}}{H_t} - \delta_H\right)\left(\frac{I_{H,t+1}}{H_t} - \frac{1}{2}\left(\frac{I_{H,t+1}}{H_t} - \delta_H\right) + (1-\delta_H)\right)\right]\right) + (1-\chi)\lambda_t E_t\big[P_{H/C,t+1}\pi_{C,t+1}\big] e^{\varepsilon_t^{LTV}}$$

$$\Omega_{C,t}\left(P_{H/C,t} + \Omega_{H,t}\right) = \beta E_t \left(\Omega_{C,t+1}\left[\frac{vY_{t+1}P_{Y,t+1}}{H_t X_{t+1} P_{C,t+1}} + (1-\delta_H)P_{H/C,t+1} + \Omega_{H,t+1}\breve{\Omega}_{H,t+1}\right]\right) + (1-\chi)\lambda_t E_t\left(P_{H/C,t+1}\pi_{C,t+1}\right)e^{\varepsilon_t^{LTV}} \qquad \text{5A.13}$$

with $\Omega_{H,t} \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I_{H,t}}{H_{t-1}} - \delta_H\right)$ and $\breve{\Omega}_{H,t} \equiv \frac{1}{2}\left(\frac{I_{H,t}}{H_{t-1}} + \delta_H\right) + (1-\delta_H)$

Patient Households

$$Max\ E_0 \sum_{t=0}^{\infty} \beta'^t e^{\varepsilon_t^\beta}\left(\ln C'_t + \varkappa e^{\varepsilon_t^H} \ln H'_t - \frac{(L'_t)^\eta}{\eta}\right)$$

$$S.t. \quad B'_t + \mathbb{E}_t B^*_t + \frac{W'_t}{P_{C,t}} L'_t + F_t = C'_t + P_{H/C,t} I'_{H,t} + \frac{R_{t-1}}{\Pi_{C,t}} B'_{t-1} + \frac{R^*_{t-1}}{\Pi_{C,t}} \mathbb{E}_t B^*_{t-1} + A'_{H,t} \qquad \text{5A.14}$$

$$I'_{H,t} = H'_t - (1-\delta_H)H'_{t-1} \qquad \text{5A.15}$$

$$A'_{H,t} = \frac{\psi_H P_{H/C,t}}{2\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)^2 H'_{t-1} \qquad \text{5A.16}$$

$$\mathcal{L} \equiv E_0 \sum_{t=0}^{\infty} \beta'^t \left\{ e^{\varepsilon_t^\beta}\left(\ln C'_t + \varkappa e^{\varepsilon_t^H} \ln H'_t - \frac{(L'_t)^\eta}{\eta}\right) + \Omega'_{C,t}\left[B'_t + \mathbb{E}_t B^*_t + \frac{W'_t}{P_{C,t}} L'_t + F_t - C'_t - P_{H/C,t}[H'_t - (1-\delta_H)H'_{t-1}] - \frac{R_{t-1}}{\Pi_{C,t}} B'_{t-1} - \frac{R^*_{t-1}}{\Pi_{C,t}} \mathbb{E}_t B^*_{t-1} - \frac{\psi_H P_{H/C,t}}{2\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)^2 H'_{t-1}\right]\right\}$$

FOCs

$$\frac{\partial \mathcal{L}}{\partial C'_t} \equiv \beta'^t \left(\frac{e^{\varepsilon_t^\beta}}{C'_t} - \Omega'_{C,t}\right) = 0$$

$$\frac{e^{\varepsilon_t^\beta}}{C'_t} = \Omega'_{C,t} \qquad \text{5A.17}$$

$$\frac{\partial \mathcal{L}}{\partial L'_t} \equiv \beta'^t \left(e^{\varepsilon_t^\beta}[-L'_t]^{\eta-1} + \Omega'_{C,t}\frac{W'_t}{P_{C,t}}\right) = 0$$

$$\frac{W'_t}{P_{C,t}} = C'_t (L'_t)^{\eta-1} \qquad \text{5A.18}$$

$$\frac{\partial \mathcal{L}}{\partial B'_t} \equiv \beta'^t\left(\Omega'_{C,t}\right) - E_t\left(\frac{\beta'^{t+1}\Omega'_{C,t+1}}{\Pi_{C,t+1}}\right) R_t = 0$$

$$\Omega'_{\mathrm{C,t}} = \beta' E_t \left(\frac{\Omega'_{\mathrm{C,t+1}}}{\Pi_{\mathrm{C,t+1}}}\right) R_t \qquad 5A.19$$

$$\frac{\partial \mathcal{L}}{\partial B^*_t} \equiv \beta'^{t}\left(\Omega'_{\mathrm{C,t}}\mathbb{E}_t\right) - E_t\left(\frac{\beta'^{t+1}\Omega'_{\mathrm{C,t+1}}}{\Pi_{\mathrm{C,t+1}}}\right)\mathbb{E}_{t+1}R^*_t = 0$$

$$\Omega'_{\mathrm{C,t}} = \beta' E_t \left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t}\frac{\Omega'_{\mathrm{C,t+1}}}{\Pi_{\mathrm{C,t+1}}}\right) R^*_t \qquad 5A.20$$

$$\frac{\partial \mathcal{L}}{\partial H'_t} \equiv \Omega'_{\mathrm{C,t}}\left(P_{H/C,t} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)\right) = \varkappa e^{\varepsilon^\beta_t} e^{\varepsilon^H_t}\left(\frac{1}{H'_t}\right) + \beta' E_t\left(\Omega'_{\mathrm{C,t+1}}\left[(1 - \delta_H)P_{H/C,t+1} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_{H,t+1}}{H'_t} - \delta_H\right)\left(\frac{I'_{H,t+1}}{H'_t} - \frac{1}{2}\left(\frac{I'_{H,t+1}}{H'_t} - \delta_H\right) + (1 - \delta_H)\right)\right]\right)$$

$$\Omega'_{\mathrm{C,t}}\left(P_{H/C,t} + \Omega'_{\mathrm{H,t}}\right) = \varkappa e^{\varepsilon^\beta_t} e^{\varepsilon^H_t}\left(\frac{1}{H'_t}\right) + \beta' E_t\left(\Omega'_{\mathrm{C,t+1}}\left[(1 - \delta_H)P_{H/C,t+1} + \Omega'_{\mathrm{H,t+1}}\breve{\Omega}'_{\mathrm{H,t+1}}\right]\right) \qquad 5A.21$$

with $\Omega'_{\mathrm{H,t}} \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)$ and $\breve{\Omega}'_{\mathrm{H,t}} \equiv \frac{1}{2}\left(\frac{I'_{H,t}}{H'_{t-1}} + \delta_H\right) + (1 - \delta_H)$

Impatient Households

$$Max\, E_0 \sum_{t=0}^{\infty} \beta''^{t} e^{\varepsilon^\beta_t}\left(\ln C''_t + \varkappa e^{\varepsilon^H_t}\ln H''_t - \frac{(L''_t)^\eta}{\eta}\right)$$

$$S.t \qquad B''_t + \frac{W''_t}{P_{C,t}}L''_t = C''_t + P_{H/C,t}I''_{H,t} + \frac{R_{t-1}}{\Pi_{\mathrm{C,t}}}B''_{t-1} + A''_{H,t} \qquad 5A.22$$

$$I''_{H,t} = H''_t - (1 - \delta_H)H''_{t-1} \qquad 5A.23$$

$$A''_{H,t} = \frac{\psi_H P_{H/C,t}}{2\delta_H}\left(\frac{I''_{H,t}}{H''_{t-1}} - \delta_H\right)^2 H''_{t-1} \qquad 5A.24$$

$$[1 - \mu^b](1 - \chi'')E_t\left(P_{H/C,t+1}H''_t\pi_{\mathrm{C,t+1}}\right)e^{\varepsilon^{LTV}_t} + [\mu^b]\frac{W''_t}{P_{C,t}}L''_t = R_t B''_t \qquad 5A.25$$

$$\mathcal{L} \equiv E_0\sum_{t=0}^{\infty}\beta''^{t}\left\{e^{\varepsilon^\beta_t}\left(\ln C''_t + \varkappa e^{\varepsilon^H_t}\ln H''_t - \frac{(L''_t)^\eta}{\eta}\right) + \Omega''_{\mathrm{C,t}}\left[B''_t + \frac{W''_t}{P_{C,t}}L''_t - C''_t - P_{H/C,t}[H''_t - (1 - \delta_H)H''_{t-1}] - \frac{R_{t-1}}{\Pi_{\mathrm{C,t}}}B''_{t-1} - \frac{\psi_H P_{H/C,t}}{2\delta_H}\left(\frac{I''_{H,t}}{H''_{t-1}} - \delta_H\right)^2 H''_{t-1}\right] + \lambda''_t\left[[1 - \mu^b](1 - \chi'')E_t\left(P_{H/C,t+1}H''_t\pi_{\mathrm{C,t+1}}\right)e^{\varepsilon^{LTV}_t} + \mu^b\frac{W''_t}{P_{C,t}}L''_t - R_t B''_t\right]\right\}$$

FOCs

$$\frac{\partial \mathcal{L}}{\partial C_t''} \equiv \beta''^t \left(\frac{e^{\varepsilon_t^\beta}}{C_t''} - \Omega_{\mathrm{C,t}}'' \right) = 0$$

$$\frac{e^{\varepsilon_t^\beta}}{C_t''} = \Omega_{\mathrm{C,t}}'' \qquad \text{5A.26}$$

$$\frac{\partial \mathcal{L}}{\partial L_t''} \equiv \beta''^t \left(e^{\varepsilon_t^\beta} [-L_t'']^{\eta-1} + \Omega_{\mathrm{C,t}}'' \frac{W_t''}{P_{C,t}} + \mu^b \lambda_t'' \frac{W_t''}{P_{C,t}} \right) = 0$$

$$\frac{W_t''}{P_{C,t}} \left(e^{\varepsilon_t^\beta} + \mu^b \lambda_t'' C_t'' \right) = C_t'' (L_t'')^{\eta-1} e^{\varepsilon_t^\beta} \qquad \text{5A.27}$$

$$\frac{\partial \mathcal{L}}{\partial B_t''} \equiv \beta''^t \left(\Omega_{\mathrm{C,t}}'' - \lambda_t'' R_t \right) - E_t \left(\frac{\beta''^{t+1} \Omega_{\mathrm{C,t+1}}''}{\Pi_{\mathrm{C,t+1}}} \right) R_t = 0$$

$$\Omega_{\mathrm{C,t}}'' = \beta'' E_t \left(\frac{\Omega_{\mathrm{C,t+1}}''}{\Pi_{\mathrm{C,t+1}}} \right) R_t + \lambda_t'' R_t \qquad \text{5A.28}$$

$$\frac{\partial \mathcal{L}}{\partial H_t''} \equiv \Omega_{\mathrm{C,t}}'' \left(P_{H/C,t} + \frac{\psi_H P_{H/C}}{\delta_H} \left(\frac{I_{H,t}''}{H_{t-1}''} - \delta_H \right) \right) - [1-\mu^b](1-\chi'') \lambda_t'' E_t \left(P_{H/C,t+1} \pi_{\mathrm{C,t+1}} \right) e^{\varepsilon_t^{LTV}} = \varkappa e^{\varepsilon_t^\beta} e^{\varepsilon_t^H} \left(\frac{1}{H_t''} \right) + \beta'' E_t \left(\Omega_{\mathrm{C,t+1}}'' \left[(1-\delta_H) P_{H/C,t+1} + \frac{\psi_H P_{H/C}}{\delta_H} \left(\frac{I_{H,t+1}''}{H_t''} - \delta_H \right) \left(\frac{I_{H,t+1}''}{H_t''} - \frac{1}{2} \left(\frac{I_{H,t+1}''}{H_t''} - \delta_H \right) + (1-\delta_H) \right) \right] \right)$$

$$\Omega_{\mathrm{C,t}}'' \left(P_{H/C,t} + \Omega_{\mathrm{H,t}}'' \right) = \varkappa e^{\varepsilon_t^\beta} e^{\varepsilon_t^H} \left(\frac{1}{H_t''} \right) + \beta'' E_t \left(\Omega_{\mathrm{C,t+1}}'' \left[(1-\delta_H) P_{H/C,t+1} + \Omega_{\mathrm{H,t+1}}'' \breve{\Omega}_{\mathrm{H,t+1}}'' \right] \right) + [1-\mu^b](1-\chi'') \lambda_t'' E_t \left(P_{H/C,t+1} \pi_{\mathrm{C,t+1}} \right) e^{\varepsilon_t^{LTV}} \qquad \text{5A.29}$$

with $\Omega_{\mathrm{H,t}}'' \equiv \frac{\psi_H P_{H/C}}{\delta_H} \left(\frac{I_{H,t}''}{H_{t-1}''} - \delta_H \right)$ and $\breve{\Omega}_{\mathrm{H,t}}'' \equiv \frac{1}{2} \left(\frac{I_{H,t}''}{H_{t-1}''} + \delta_H \right) + (1-\delta_H)$

Retailers

An implicit cost of adjusting nominal price is assumed. As well, a monopolistic competition at retail price is introduced, following Bernanke et al. (1999). Given these features, a continuum of retailers of mass 1, indexed by z, buy intermediate goods Y_t from entrepreneurs at $P_{Y,t}^W$ in a competitive market, differentiate the goods at no cost into $Y_t(z)$ and sell $Y_t(z)$ at the price $P_{Y,t}(z)$.

Final goods are assumed to be bundled by final goods bundlers at no cost using the following technology: $Y_t^f = \left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} dz \right)^{\mu_t^r}$ where $\mu_t^r > 1$ denotes the time-varying mark up over marginal cost in domestic economy.

To find an individual demand curve for $Y_t(z)$, we set a minimisation problem as follows:

$$Min \int_0^1 P_{Y,t}(z) Y_t(z) \; dz$$

$$S.t \quad Y_t^f = \left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} dz \right)^{\mu_t^r} \qquad 5A.30$$

$$\mathcal{L} \equiv \int_0^1 P_{Y,t}(z) Y_t(z) \, dz + \lambda_t^r \left(Y_t^f - \left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} dz \right)^{\mu_t^r} \right)$$

$$\frac{\partial \mathcal{L}}{\partial Y_t(z)} \equiv P_{Y,t}(z) - \lambda_t^r \left[\left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} \right)^{\mu_t^r - 1} Y_t(z)^{\frac{1}{\mu_t^r} - 1} dz \right] = 0$$

$$Y_t(z) = \left(\frac{P_{Y,t}(z)}{\lambda_t^r} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1} \right]} Y_t^f \qquad \forall z$$

Solving for λ_t^r,

$$Y_t^f = \left(\frac{1}{\lambda_t^r} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1} \right]} \left(\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu_t^r - 1} \right]} dz \right)^{\mu_t^r} Y_t^f$$

$$\lambda_t^r = \left(\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu_t^r - 1} \right]} dz \right)^{\mu_t^r - 1} \equiv P_{Y,t} \qquad 5A.31$$

$$Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1} \right]} Y_t^f \qquad \forall z \qquad 5A.32$$

Each retailer chooses a sale price $P_{Y,t}(z)$, taking $P_{Y,t}^w$ curve as given. The sale price can be changed in every period only with probability $1 - \theta$. When a price is allowed to change, retailer does so in order to maximise the present discounted value profits, $E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \Phi_{t+s}(z)$. Yet, the way that they set a sale price is subject to the demand for its own goods, $Y_{t+s}(z) = \left(\frac{P_{Y,t}^{New}(z)}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} Y_{t+s}^f$ and the constraints that all demand be satisfied at the chosen price.

Profits are discounted by s - step ahead stochastic discount factor $Q_{t,t+s}$ and by the probability of not being able to set price in the future periods.

$$Max\ E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}(z) - P_{Y,t+s}^{w}}{P_{Y,t+s}}\right) Y_{t+s}(z)$$

$$S.t \quad Y_{t+s}(z) = \left(\frac{P_{Y,t}^{New}(z)}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} Y_{t+s} \qquad 5A.33$$

where $E_t Q_{t,t+s} = Q_{t,t+1} Q_{t+1,t+2} \ldots Q_{t+s-1,t+s}$ and $E_t Q_{t+s-1,t+s} = E_t E_{t+s-1} Q_{t+s-1,t+s}$ are employed. It is assumed that $Y_t^f = Y_t$, following Iacoviello (2005).

Unconstrained maximisation problem can be written as:

$$V \equiv Max\ E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}(z) - P_{Y,t+s}^{w}}{P_{Y,t+s}}\right) \left(\frac{P_{Y,t}^{*}(z)}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} Y_{t+s}$$

$$V \equiv Max\ E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left[\left(\frac{P_{Y,t}^{New}(z)}{P_{Y,t+s}}\right)^{-\left[\frac{1}{\mu_t^r - 1}\right]} - \left(\frac{P_{Y,t}^{*}(z)}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} \left(\frac{P_{Y,t+s}^{w}}{P_{Y,t+s}}\right)\right] Y_{t+s}$$

The chosen price $P_{Y,t}^{New}(z)$ is allowed to be written as $P_{Y,t}^{New}$ because retailers share the same production technology and price takers.

The optimal price decision is given by,

$$\frac{\partial V}{P_{Y,t}^{New}} \equiv E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} \left(\frac{1}{P_{Y,t+s}} - \mu_t^r \left(\frac{P_{Y,t+s}}{P_{Y,t}^{New}(z)}\right) \left(\frac{P_{Y,t+s}^{w}}{P_{Y,t+s}}\right) \left(\frac{1}{P_{Y,t+s}}\right)\right) Y_{t+s} = 0$$

$$\frac{\partial V}{P_{Y,t}^{New}} \equiv E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}} - \mu_t^r \left(\frac{P_{Y,t+s}^{w}}{P_{Y,t+s}}\right)\right) Y_{t+s} = 0$$

$$\frac{\partial V}{P_{Y,t}^{New}} \equiv E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}} - \mu_t^r \left(\frac{P_{Y,t+s}^{w}}{P_{Y,t+s}}\right)\right) Y_{t+s}(z) = 0$$

$$0 = E_0 \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}} - \left[\frac{\mu_t^r}{X_{t+s}}\right]\right) Y_{t+s}(z) \qquad 5A.34$$

As a fraction θ of prices stays unchanged, the aggregate price level evolution follows:

$$P_{Y,t}{}^{-\left[\frac{1}{\mu_t^r-1}\right]} = \theta P_{Y,t-1}{}^{-\left[\frac{1}{\mu_t^r-1}\right]} + (1-\theta)\left(P_{Y,t}^{New}\right)^{-\left[\frac{1}{\mu_t^r-1}\right]}$$

$$P_{Y,t} = \left(\theta P_{Y,t-1}{}^{-\left[\frac{1}{\mu_t^r-1}\right]} + (1-\theta)\left(P_{Y,t}^{New}\right)^{-\left[\frac{1}{\mu_t^r-1}\right]}\right)^{1-\mu_t^r} \qquad 5A.35$$

Intertemporal Allocation

It is assumed a bundle of goods consumed by retailers and households consist of domestic and import (foreign) goods. The way of how these two goods are bundled, by the bundlers, follows a Dixit-Stieglitz aggregator as shown below:

$$C_t(j) \equiv \left((1-\alpha)^{\frac{1}{\varpi}} C_{D,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}} C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right)^{\frac{\varpi}{\varpi-1}} \qquad 5A.36$$

where

$$C_{D,t}(j) = \left(\int_0^1 C_{D,t}(z,j)^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r} \qquad 5A.37$$

$$C_{F,t}(j) = \left(\int_0^1 C_{i,t}(j)^{\frac{\zeta-1}{\zeta}} di\right)^{\frac{\zeta}{\zeta-1}} \qquad 5A.38$$

$$C_{i,t}(j) = \left(\int_0^1 C_{i,t}(z,j)^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r} \qquad 5A.39$$

The goods consumed range from $j = 0 \ldots 1$ in which $C_{D,t}(j)$ represents domestic consumption goods and $C_{F,t}(j)$ corresponds to import consumption goods. The composite of import consumption goods consists of consumption goods from all foreign country $i \in [0,1]$ while the composite of domestic consumption goods consist of differentiated goods $z \in [0,1]$ that produced domestically. The definition of $C_{i,t}(j)$ is analogues to $C_{D,t}(j)$ but it corresponds to a specific country i. As mentioned earlier, parameter μ_t^r corresponds to time-varying markup over marginal cost in the domestic country and foreign country i (assumed to be the same), while ζ is the elasticity of substitution between import goods from different foreign countries i.

Domestic consumption goods bundlers help consumers (i.e. households and entrepreneurs) to find the best allocation of specific domestic goods. For that purpose, they minimise the expenditure for obtaining each type of domestic goods subject to the technology as follows:

$$\mathcal{L} \equiv Min \int_0^1 P_{Y,t}(z)\, C_{D,t}(z,j)dz + \varphi_t \left(C_{D,t}(j) - \left[\int_0^1 C_{D,t}(z,j)^{\frac{1}{\mu_t^r}} dz \right]^{\mu_t^r} \right)$$

where for simplicity, it is assumed that the price of domestic consumption goods z is equal to producer price of goods z. From optimality condition, we obtain

$$\frac{\partial \mathcal{L}}{\partial C_{D,t}(z,j)} \equiv P_{Y,t}(z) - \varphi_t \left[\left(C_{D,t}(z,j)^{\frac{1}{\mu_t^r}} dz \right)^{\mu_t^r} C_{D,t}(z,j)^{\frac{1-\mu_t^r}{\mu_t^r}} \right] = 0$$

$$C_{D,t}(z,j) = \left(\frac{P_{Y,t}(z)}{\varphi_t} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} C_{D,t}(j) \qquad 5A.40$$

Solving for φ_t,

$$C_{D,t}(z,j) = \left(\frac{1}{\varphi_t} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} \left[\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu_t^r - 1}\right]} dz \right]^{\mu_t^r} C_{D,t}(j)$$

$$\varphi_t = \left(\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu_t^r - 1}\right]} dz \right)^{\mu_t^r - 1} \equiv P_{Y,t}$$

Thus, $C_{D,t}(z,j) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} C_{D,t}(j)$ 5A.41

where $P_{Y,t}$ is defined as the price index of domestic consumption goods which is assumed to be equal to producer price index (PPI) in domestic country, for simplicity.

Similarly, we can obtain

$$C_{i,t}(z,j) = \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\left[\frac{\mu_t^r}{\mu_t^r - 1}\right]} C_{i,t}(j) \qquad 5A.42$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\zeta} C_{F,t}(j) \qquad 5A.43$$

with $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(z)^{-\left[\frac{1}{\mu_t^r - 1}\right]} dz \right)^{\mu_t^r - 1}$ and $P_{F,t} \equiv \left(\int_0^1 {P_{i,t}}^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}$ define as producer price index (PPI) in foreign country i and aggregate price index for import consumption goods, respectively.

In similar fashion, final consumption good bundlers help consumers to find the best allocation of domestic and import goods. These firms purchase domestic and import goods, combine them and sell to households and entrepreneurs to get profits.

They maximise their profit in perfectly competitive market as follows:

$$Max\,\Pi \equiv P_{C,t}C_t - P_{Y,t}C_{D,t} - P_{F,t}C_{F,t} \qquad \forall \text{ cohort } j$$

$$S.t \quad C_t(j) = \left[(1-\alpha)^{\frac{1}{\varpi}}C_{D,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}}C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}$$

$$\mathcal{L} \equiv P_{C,t}C_t(j) - P_{Y,t}C_{D,t}(j) - P_{F,t}C_{F,t}(j) + \phi_t\left(\left[(1-\alpha)^{\frac{1}{\varpi}}C_{D,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}}C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}} - C_t(j)\right)$$

First order condition with respect to domestic and import goods consumption gives:

$$\frac{\partial\mathcal{L}}{\partial C_{D,t}(j)} \equiv -P_{Y,t} + \phi_t\left(\left(\frac{\varpi}{\varpi-1}\right)C_t(j)^{\frac{1}{\varpi}}(1-\alpha)^{\frac{1}{\varpi}}\left(\frac{\varpi-1}{\varpi}\right)C_{D,t}(j)^{-\frac{1}{\varpi}}\right) = 0$$

$$\frac{\partial\mathcal{L}}{\partial C_{D,t}(j)} \equiv C_{D,t}(j)^{\frac{1}{\varpi}} = (1-\alpha)^{\frac{1}{\varpi}}\frac{\phi_t}{P_{Y,t}}C_t(j)^{\frac{1}{\varpi}}$$

$$C_{D,t}(j) = (1-\alpha)\left(\frac{P_{Y,t}}{\phi_t}\right)^{-\varpi}C_t(j) \qquad \text{5A.44}$$

Similarly, we can obtain

$$C_{F,t}(j) = \alpha\left(\frac{P_{F,t}}{\phi_t}\right)^{-\varpi}C_t(j) \qquad \text{5A.45}$$

Solving for ϕ_t,

$$C_t(j) = \left(\left[(1-\alpha)\left(\left(\frac{P_{Y,t}}{\phi_t}\right)^{-\varpi}C_t(j)\right)^{\frac{\varpi-1}{\varpi}} + \alpha\left(\left(\frac{P_{F,t}}{\phi_t}\right)^{-\varpi}C_t(j)\right)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}\right)$$

$$C_t(j) = \left[(1-\alpha)\left(\frac{P_{Y,t}}{\phi_t}\right)^{1-\varpi} + \alpha\left(\frac{P_{F,t}}{\phi_t}\right)^{1-\varpi}\right]C_t(j)$$

$$\phi_t = \left[(1-\alpha)P_{Y,t}{}^{1-\varpi} + \alpha P_{F,t}{}^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \equiv P_{C,t} \qquad \text{5A.46}$$

We can re-write equation (5A.44) and (5A.45) as:

$$C_{D,t}(j) = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi}C_t(j) \qquad \text{5A.47}$$

$$C_{F,t}(j) = \alpha \left(\frac{P_{F,t}}{P_{C,t}}\right)^{-\varpi} C_t(j) \qquad 5A.48$$

where $P_{C,t}$ is defined as consumer price index (CPI) in domestic country (or country i from the view of foreign country).

Definitions and Identities

Term of Trade

The bilateral terms of trade between the domestic country and foreign country i represents the price of country i's goods in terms of domestic goods. This relation is defined by

$$S_{i,t} = P_{i,t}/P_{Y,t} \qquad 5A.49$$

Aggregating for all country i, we obtain the effective term terms of trade as

$$S_t = \frac{P_{F,t}}{P_{Y,t}} = \left[\int_0^1 (S_{i,t})^{1-\zeta} di\right]^{\frac{1}{1-\zeta}} \qquad 5A.50$$

which can be approximated by $s_t \equiv \log(S_t) \approx \int_0^1 s_{i,t}\, di$

Incomplete Pass-Through and Import Goods Inflation

The effective bilateral real exchange rate is given by:

$$\mathfrak{E}_{i,t} = \frac{\mathbb{E}_{i,t} P_{C,t}^{i}}{P_{C,t}} \qquad 5A.51$$

Yet, in the model's environment, it is assumed that the Law of One Price (LOOP) fails to hold, following Monacelli (2005). This is a realistic assumption as the price of goods in domestic country and in country i are not necessarily the same although the component of exchange rate has been taken into consideration. A possible argument for this condition is the existence of nominal rigidities in the price of import goods. Thus, similar to retailers, we allow importing firms to change their price only when they receive a random price change signal (the Calvo staggered contract in price setting).

With probability $1-\theta_F$, importing firms can choose its prices optimally by maximising the present discounted value of future profits as follows:

$$Max\ E_0 \sum_{s=0}^{\infty} \theta_F^s\, Q_{t,t+s} \left(\frac{P_{F,t}^{New}(i) - \mathbb{E}_{t+s} P_{F,t+s}^{*}(i)}{P_{F,t+s}}\right) C_{F,t+s}(i) \qquad 5A.52$$

$S.t.\quad C_{F,t+s}(i) = \left(\frac{P_{F,t}^{New}}{P_{F,t}}\right)^{-\zeta} C_{F,t+s}$, equivalent to equation (5A.43) where $E_t \mathcal{Q}_{t,t+s} = \mathcal{Q}_{t,t+1}\mathcal{Q}_{t+1,t+2} \dots \mathcal{Q}_{t+s-1,t+s}$ and $E_t \mathcal{Q}_{t+s-1,t+s} = E_t E_{t+s-1} \mathcal{Q}_{t+s-1,t+s}$ are employed.

$P^*_{F,t}(i)$ is the foreign currency price of the imported goods (from country i), θ_F^s is the probability that the price $P_{F,t}^{New}(i)$ set for goods from country i at time t still holds s periods ahead and $\mathcal{Q}_{t,t+s}$ is the relevant stochastic discount factor. Notice that, in general $\theta^s \neq \theta_F^s$.

The unconstrained optimisation problem is given by:

$$V \equiv E_0 \sum_{s=0}^{\infty} \theta_F^s \, \mathcal{Q}_{t,t+s} \left(\frac{P_{F,t}^{New}(i) - \mathbb{E}_{t+s} P^*_{F,t+s}(i)}{P_{F,t+s}} \right) \left(\frac{P_{F,t}^{New}}{P_{F,t}} \right)^{-\zeta} C_{F,t+s}$$

$$V \equiv E_0 \sum_{s=0}^{\infty} \theta_F^s \, \mathcal{Q}_{t,t+s} \left[\left(\frac{P_{F,t}^{New}(i)}{P_{F,t+s}} \right)^{1-\zeta} - \left(\frac{P_{F,t}^{New}}{P_{F,t}} \right)^{-\zeta} \left(\frac{\mathbb{E}_{t+s} P^*_{F,t+s}(i)}{P_{F,t+s}} \right) \right] C_{F,t+s}$$

We are allowed to write $P_{F,t}^{New}$ instead of $P_{F,t}^{New}(i)$ because all importing firms share the same production technology and price takers.

The optimal price decision is,

$$\frac{\partial V}{P_{F,t}^{New}} \equiv E_0 \sum_{s=0}^{\infty} \theta_F^s \, \mathcal{Q}_{t,t+s} \left(\frac{P_{F,t}^{New}}{P_{F,t+s}} \right)^{-\zeta} \left[(1-\zeta) \left(\frac{1}{P_{F,t+s}} \right) + \zeta \left(\frac{P_{F,t+s}}{P_{F,t}^{New}} \right) \left(\frac{1}{P_{F,t+s}} \right) \left(\frac{\mathbb{E}_{t+s} P^*_{F,t+s}}{P_{F,t+s}} \right) \right] C_{F,t+s} = 0$$

$$\frac{\partial V}{P_{F,t}^{New}} \equiv E_0 \sum_{s=0}^{\infty} \theta_F^s \, \mathcal{Q}_{t,t+s} \left(\frac{P_{F,t}^{New}}{P_{F,t+s}} \right)^{-\zeta} \left[\left(\frac{P_{F,t}^{New}}{P_{F,t+s}} \right) - \left(\frac{\zeta}{\zeta-1} \right) \left(\frac{\mathbb{E}_{t+s} P^*_{F,t+s}}{P_{F,t+s}} \right) \right] C_{F,t+s} = 0$$

where the last equation can be obtained by multiplying it with $P_{F,t}^{New}$ in both terms inside the bracket.

Therefore, the optimal re-set price $P_{F,t}^{New}$ solves:

$$0 = E_0 \sum_{s=0}^{\infty} \theta_F^s \, \mathcal{Q}_{t,t+s} \left(\frac{P_{F,t}^{New}}{P_{F,t+s}} - \left(\frac{\zeta}{\zeta-1} \right) \frac{\mathbb{E}_{t+s} P^*_{F,t+s}}{P_{F,t+s}} \right) C_{F,t+s} \qquad \text{5A.53}$$

As a fraction θ_F of prices stays unchanged, the aggregate index of import price evolves according to:

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\zeta} + (1-\theta_F) \left(P_{F,t}^{New} \right)^{1-\zeta} \right]^{1/(1-\zeta)} \qquad \text{5A.54}$$

International Risk Sharing

For simplicity, it is assumed that only savers (i.e. patient households) have accessed to foreign market bond while borrowers (impatient households and entrepreneurs) are not. From optimality condition of patient households, it can be shown that $\beta' E_t\left(\frac{P_{C,t}}{P_{C,t+1}}\frac{\Omega'_{C,t+1}}{\Omega'_{C,t}}\right) = \mathcal{Q}_{t,t+1}$. This characteristic also applies for each foreign country i. Taking into account the exchange rate, the characteristic of each country i in terms of domestic currency can be rewritten as:

$$\beta' E_t\left(\frac{\mathbb{E}_t^i P_{C,t}^i}{\mathbb{E}_{t+1}^i P_{C,t+1}^i}\frac{{\Omega'_{C,t+1}}^i}{{\Omega'_{C,t}}^i}\right) = \mathcal{Q}_{t,t+1}. \quad \text{5A.55}$$

Equating the above equations for both domestic and foreign country in terms of $\mathcal{Q}_{t,t+1}$ yields:

$$\beta' E_t\left(\frac{P_{C,t}}{P_{C,t+1}}\frac{\Omega'_{C,t+1}}{\Omega'_{C,t}}\right) = \beta' E_t\left(\frac{\mathbb{E}_t^i P_{C,t}^i}{\mathbb{E}_{t+1}^i P_{C,t+1}^i}\frac{{\Omega'_{C,t+1}}^i}{{\Omega'_{C,t}}^i}\right)$$

$$E_t\left(\frac{{C'_{t+1}}^i}{C'_{t+1}}\right)\left(\frac{P_{C,t+1}^i}{P_{C,t+1}}\right)\mathbb{E}_{t+1}^i = \left(\frac{{C'_t}^i}{C'_t}\right)\left(\frac{P_{C,t}^i}{P_{C,t}}\right)\mathbb{E}_t^i \quad \text{5A.56}$$

Given that, the following identity also holds:

$$E_t\left(\frac{{C'_{t+1}}^i}{C'_{t+1}}\right)\left(\frac{P_{C,t+1}^i}{P_{C,t+1}}\right)\mathbb{E}_{t+1}^i = \left(\frac{{C'_t}^i}{C'_t}\right)\left(\frac{P_{C,t}^i}{P_{C,t}}\right)\mathbb{E}_t^i = \left(\frac{{C'_0}^i}{C'_0}\right)\left(\frac{P_{0,t}^i}{P_{0,t}}\right)E_0^i = \Upsilon = \Upsilon_i = 1 \quad \text{5A.57}$$

The last part is the initial consumption ratio, which is assumed to be identical for each country (symmetric initial condition). Let this ratio equal to 1.

Given equation (5A.57), we can obtain

$$\left(\frac{{C'_t}^i}{C'_t}\right)\left(\frac{P_{C,t}^i}{P_{C,t}}\right)\mathbb{E}_t^i = 1$$

$$C'_t = {C'_t}^i\left(\frac{\mathbb{E}_t^i P_{C,t}^i}{P_{C,t}}\right)$$

$$C'_t = {C'_t}^i \mathfrak{E}_{i,t} \ \ \forall t \quad \text{5A.58}$$

Equilibrium and Market Clearing

Aggregate Output

The market clearing condition at the level of item z goods is:

$$Y_t(z) = e^{\Lambda_t} K_{t-1}^{\mu}(z) H_{t-1}^{\nu}(z)(A_t L'_t)^{\sigma(1-\mu-\nu)}(A_t L''_t)^{(1-\sigma)(1-\mu-\nu)} \quad \forall z$$

$$\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} Y_t = e^{\Lambda_t} K_{t-1}^{\mu}(z) H_{t-1}^{\nu}(z)(A_t L'_t)^{\sigma(1-\mu-\nu)}(A_t L''_t)^{(1-\sigma)(1-\mu-\nu)} \quad \forall z$$

which upon aggregation across the z firm becomes,

$$Y_t\Delta_t = e^{\Lambda_t}K^{\mu}_{t-1}(z)H^{\nu}_{t-1}(z)(A_tL'_t)^{\sigma(1-\mu-\nu)}(A_tL''_t)^{(1-\sigma)(1-\mu-\nu)} \qquad \text{5A.59}$$

with $\Delta_t \equiv \int_0^1 \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]} dz$ is the measure of price dispersion which can be shown (see Woodford, 2003) to follow AR(1) process given by:

$$\Delta_t = (1-\theta)\left(\frac{P^*_{Y,t}}{P_{Y,t}}\right)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]} + \theta\left(\Pi_{Y,t}\right)^{\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}\Delta_{t-1}$$

$$\Delta_t = (1-\theta)\overline{\Pi}_t^{\,-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]} + \theta\Pi_{Y,t}^{\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}\Delta_{t-1} \qquad \text{5A.60}$$

Aggregate Profits

$$F_t = \int_0^1 \Phi_t(z)\,dz$$

$$F_t = \int_0^1 \left(\frac{P_{Y,t}(z)-P^W_{Y,t}}{P_{C,t}}\right)Y_t(z)\,dz$$

$$F_t = \int_0^1 \left(\frac{P_{Y,t}(z)-P^W_{Y,t}}{P_{Y,t}}\right)\left(\frac{P_{Y,t}}{P_{C,t}}\right)\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}Y^f_t\,dz$$

by using the fact that $Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}Y^f_t$

$$F_t = \int_0^1 \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{1}{\mu^r_t-1}\right]}\left(\frac{P_{Y,t}}{P_{C,t}}\right)Y^f_t\,dz - \int_0^1 \frac{P^W_{Y,t}}{P_{C,t}}\left(\frac{P_{Y,t}}{P_{C,t}}\right)\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}Y^f_t\,dz$$

$$F_t = \left(\frac{Y_t}{P_{Y,t}^{\,-\left[\frac{1}{\mu^r_t-1}\right]}}\right)\left(\frac{P_{Y,t}}{P_{C,t}}\right)\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu^r_t-1}\right]}\,dz - \left(\frac{P^W_{Y,t}Y_t}{P_{Y,t}^{\,-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}}\right)\left(\frac{P_{Y,t}}{P_{C,t}}\right)\int_0^1 P_{Y,t}(z)^{-\left[\frac{\mu^r_t}{\mu^r_t-1}\right]}\,dz$$

since $Y^f_t = Y_t$

Recall, $P_{Y,t} = \left(\int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu^r_t-1}\right]}dz\right)^{\mu^r_t-1}$

Thus, we can obtain the aggregate profit in real term as follows:

$$F_t \equiv \left(1-\frac{1}{X_t}\right)Y_t\frac{P_{Y,t}}{P_{C,t}} \qquad \text{5A.61}$$

This aggregate profit will be rebated to savers since it is assumed that firms are owned by them.

Equilibrium

The total output of domestic firms is equal to domestic consumption and foreign consumption (or export) plus capital investment and the cost of changing capital and housing.

$$Y_t(z) = \breve{C}_{D,t}(z) + \int_0^1 {C'_{D,t}}^i(z)di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)$$

where

$$\breve{C}_{D,t}(z) = C_{D,t}(z) + C'_{D,t}(z) + C''_{D,t}(z)$$

We follow Gali (2003) in assuming that government purchases a time-varying fraction ι_t of output of each good z, financed by lump sum taxation. Consequently, aggregate goods market clearing for each good z requires:

$$(1-\iota_t)Y_t(z) = \breve{C}_{D,t}(z) + \int_0^1 {C'_{D,t}}^i(z)di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)$$

where $-log(1-\iota_t) = g_t$

$$(1-\iota_t)Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \breve{C}_{D,t} + \int_0^1 \left(\frac{P_{Y,t}^i(z)}{P_{Y,t}^i}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} {C'_{D,t}}^i di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)$$

$$(1-\iota_t)Y_t(z) = (1-\alpha)\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \int_0^1 \left(\frac{P_{Y,t}^i(z)}{P_{Y,t}^i}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left(\frac{P_{i,t}}{\mathbb{E}_{i,t}P_{F,t}^i}\right)^{-\zeta} {C'_{F,t}}^i di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)$$

Using the assumption of symmetric preferences across countries, we have:

$$(1-\iota_t)Y_t(z) = (1-\alpha)\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \int_0^1 \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left(\frac{P_{Y,t}}{\mathbb{E}_{i,t}P_{F,t}^i}\right)^{-\zeta} {C'_{F,t}}^i di + I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)$$

$$(1-\iota_t)Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left[(1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha\int_0^1 \left(\frac{P_{Y,t}}{\mathbb{E}_{i,t}P_{F,t}^i}\right)^{-\zeta} \left(\frac{P_{F,t}^i}{P_{C,t}^i}\right)^{-\varpi} {C'_t}^i di\right] + \left(\frac{P_{C,t}}{P_{Y,t}}\right)\left[I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)\right]$$

Recall $Y_t = \left(\int_0^1 Y_t(z)^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r}$

$$(1-\iota_t)Y_t = \left(\int_0^1 \left\{\left(\frac{P_{Y,t}(\mathbf{z})}{P_{Y,t}}\right)^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]} \left[(1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha \int_0^1 \left(\frac{P_{Y,t}}{\mathbb{E}_{i,t} P_{F,t}^i}\right)^{-\zeta} \left(\frac{P_{F,t}^i}{P_{C,t}^i}\right)^{-\varpi} {C'_t}^i di\right] + \left(\frac{P_{C,t}}{P_{Y,t}}\right)\left[I_{K,t}(z) + A_{K,t}(z) + \breve{A}_{H,t}(z)\right]\right\}^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r}$$

$$(1-\iota_t)Y_t = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha \int_0^1 \left(\frac{P_{Y,t}}{\mathbb{E}_{i,t} P_{F,t}^i}\right)^{-\zeta} \left(\frac{P_{F,t}^i}{P_{C,t}^i}\right)^{-\varpi} {C'_t}^i di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}(z)$$

As $\left(\int_0^1 \left(\left[\frac{P_{Y,t}(\mathbf{z})}{P_{Y,t}}\right]^{-\left[\frac{\mu_t^r}{\mu_t^r-1}\right]}\right)^{\frac{1}{\mu_t^r}} dz\right)^{\mu_t^r} = \left(\frac{1}{P_{Y,t}^{-\left[\frac{1}{\mu_t^r-1}\right]}} \int_0^1 P_{Y,t}(z)^{-\left[\frac{1}{\mu_t^r-1}\right]} dz\right)^{\mu_t^r} = 1$

$$(1-\iota_t)Y_t = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha \int_0^1 \left(\frac{\mathbb{E}_{i,t} P_{F,t}^i}{P_{Y,t}}\right)^{\zeta-\varpi} \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \left(\frac{P_{C,t}}{\mathbb{E}_{i,t} P_{C,t}^i}\right)^{-\varpi} {C'_t}^i di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

$$(1-\iota_t)Y_t = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \int_0^1 \left(\frac{\mathbb{E}_{i,t} P_{F,t}^i}{P_{Y,t}}\right)^{\zeta-\varpi} {\mathfrak{E}_{i,t}}^{\varpi} {C'_t}^i di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

$$(1-\iota_t)Y_t = (1-\alpha)\left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \breve{C}_t + \alpha \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{-\varpi} \int_0^1 \left(S_{i,t} S_t^i\right)^{\zeta-\varpi} {\mathfrak{E}_{i,t}}^{\varpi} {C'_t}^i di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

As $S_t = \frac{P_{F,t}}{P_{Y,t}}$; $S_t^i = \frac{\mathbb{E}_t^i P_{F,t}^i}{P_{i,t}}$; $S_{i,t} = \frac{P_{i,t}}{P_{Y,t}}$; $S_t = \left(\int_0^1 {S_{i,t}}^{1-\zeta}\right)^{\frac{1}{1-\zeta}}$ and $\mathfrak{E}_{i,t} = \frac{\mathbb{E}_{i,t} P_{C,t}^i}{P_{C,t}}$

with S_t is the effective term of trade of home country. S_t^i is the effective terms of trade of country i and $S_{i,t}$ is bilateral terms of trade between home economy and country i.

As $P_{C,t} = \left[(1-\alpha){P_{Y,t}}^{1-\varpi} + \alpha {P_{F,t}}^{1-\varpi}\right]^{\frac{1}{1-\varpi}}$, we can approach $P_{C,t} = {P_{Y,t}}^{1-\alpha} {P_{F,t}}^{\alpha}$ so that $\frac{P_{Y,t}}{P_{C,t}} = \left(\frac{P_{Y,t}}{P_{F,t}}\right)^{\alpha} = {S_t}^{-\alpha}$.

$$(1-\iota_t)Y_t = (1-\alpha){S_t}^{\alpha\varpi} \breve{C}_t + \alpha {S_t}^{\alpha\varpi} \int_0^1 \left(S_{i,t} S_t^i\right)^{\zeta-\varpi} {\mathfrak{E}_{i,t}}^{\varpi} {C'_t}^i di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

5A.62

Meanwhile, foreign bond market clears such that net foreign bonds evolve according to

$$\mathbb{E}_t B_t^* = \frac{R_{t-1}^*}{\Pi_{C,t}} \mathbb{E}_t B_{t-1}^* \qquad 5A.63$$

Total consumption, housing investment, domestic bonds, housing adjustment cost, and housing supply are defined as the followings:

$$\breve{C}_t = C_t + C'_t + C''_t \quad \text{5A.64}$$

$$\breve{I}_{H,t} = I_{H,t} + I'_{H,t} + I''_{H,t} \quad \text{5A.65}$$

$$\breve{A}_{H,t} = A_{H,t} + A'_{H,t} + A''_{H,t} \quad \text{5A.66}$$

$$0 = B_t + B'_t + B''_t \quad \text{5A.67}$$

$$\breve{H}_t = H_t + H'_t + H''_t \equiv 1 \quad \text{5A.68}$$

B. Steady State

$$S = \breve{H} = \mathfrak{E} = \mu^r = A = z = g = \pi_C = \pi_Y = Y^* = \pi^* \equiv 1 \quad \text{5B.1}$$

$$B^* = 0 \quad \text{5B.2}$$

Given (5A.50)

$$S = \frac{P_F}{P_Y} = 1 \Leftrightarrow P_F = P_Y \quad \text{5B.3}$$

Given (5A.46)

$$P_C = \left[(1-\alpha){P_Y}^{1-\varpi} + \alpha {P_F}^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \Leftrightarrow P_Y = P_C = P_F \quad \text{5B.4}$$

Given (5A.19)

$$R = \frac{1}{\beta'} \quad \text{5B.5}$$

Given (5A.9)

$$\Omega_C = \beta\left(\frac{\Omega_C}{\Pi_C}\right)R + \lambda R \Leftrightarrow \lambda = \frac{\beta' - \beta}{C} \quad \text{5B.6}$$

Given (5A.61)

$$F = \left(1 - \frac{1}{X}\right) Y \frac{P_Y}{P_C} \Leftrightarrow F = \left(1 - \frac{1}{X}\right) Y \qquad \text{5B.7}$$

Given (5A.28)

$$\Omega_C'' = \beta'' \left(\frac{\Omega_C''}{\Pi_C}\right) R + \lambda'' R \Leftrightarrow \lambda'' = \frac{\beta' - \beta''}{C''} \qquad \text{5B.8}$$

Given (5A.4), (5A.5), (5A.15) and (5A.23)

$$I_K = \delta_K K \qquad \text{5B.9}$$

$$I_H = \delta_H H \qquad \text{5B.10}$$

$$I_H' = \delta_H H' \qquad \text{5B.11}$$

$$I_H'' = \delta_H H'' \qquad \text{5B.12}$$

Given (5A.12)

$$\Omega_C(1 + \Omega_K) = \beta \Omega_C \left(\frac{\mu Y P_Y}{K X P_C} + (1 - \delta_K) + \Omega_K \breve{\Omega}_K\right)$$

where: $\Omega_C = \frac{1}{C}$ and $\Omega_K = \frac{\psi_K}{\delta_K}\left(\frac{I_K}{K} - \delta_k\right) = 0$

$$1 = \beta \left(\frac{\mu Y P_Y}{K X P_C} + (1 - \delta_K)\right) \Leftrightarrow 1 = \beta \left(\frac{\mu Y}{K X} + (1 - \delta_K)\right) \quad \text{since } P_C = P_Y$$

$$K = \frac{\beta \mu}{(1 - \beta(1 - \delta_K))X} Y \coloneqq \phi_1 Y \qquad \text{5B.13}$$

Given (5A.13)

$$\Omega_C\left(P_{H/C} + \Omega_H\right) = \beta \Omega_C \left(\frac{v Y P_Y}{H X P_C} + (1 - \delta_H) P_{H/C} + \Omega_H \breve{\Omega}_H\right) + (1 - \chi) \lambda P_{H/C}$$

where: $\Omega_C = \frac{1}{C}$; $\Omega_H = \frac{\psi_H}{\delta_H}\left(\frac{I_H}{H} - \delta_H\right) = 0$ and $\lambda = \frac{\beta' - \beta}{C}$

$$P_{H/C} = \beta \left(\frac{v Y P_Y}{H X P_C} + (1 - \delta_H) P_{H/C}\right) + (1 - \chi)(\beta' - \beta) P_{H/C}$$

$$P_{H/C} = \left\{\frac{\beta\nu}{(1-(1-\chi)(\beta'-\beta)-\beta(1-\delta_H))X}\right\}\frac{Y}{H} \coloneqq \phi_2\,\frac{Y}{H} \qquad \text{5B.14}$$

Given (5A.21)

$$\Omega'_{\mathrm{C}}\left(P_{H/C} + \Omega'_{\mathrm{H}}\right) = \frac{\varkappa}{H'} + \beta'\left(\Omega'_{\mathrm{C}}\left[(1-\delta_H)P_{H/C} + \Omega'_{\mathrm{H}}\breve{\Omega}'_{\mathrm{H}}\right]\right)$$

where: $\Omega'_{\mathrm{C}} = \frac{1}{C'}$ and $\Omega'_{\mathrm{H}} = \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_H}{H'} - \delta_H\right) = 0$

$$P_{H/C} = \frac{\varkappa C'}{H'} + \beta'(1-\delta_H)P_{H/C}$$

$$P_{H/C} = \left(\frac{\varkappa}{1-\beta'(1-\delta_H)}\right)\frac{C'}{H'} \coloneqq \phi_3\,\frac{C'}{H'} \qquad \text{5B.15}$$

Given (5A.29)

$$\Omega''_{\mathrm{C}}\left(P_{H/C} + \Omega''_{\mathrm{H}}\right) = \frac{\varkappa}{H''_t} + \beta''\left(\Omega''_{\mathrm{C}}\left[(1-\delta_H)P_{H/C} + \Omega''_{\mathrm{H}}\breve{\Omega}''_{\mathrm{H}}\right]\right) + [1-\mu^b](1-\chi'')\lambda'' P_{H/C}$$

where: $\Omega''_{\mathrm{C}} = \frac{1}{C''}$; $\Omega''_{\mathrm{H,t}} = \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I''_H}{H''} - \delta_H\right) = 0$ and $\lambda'' = \frac{\beta'-\beta''}{C''}$

$$P_{H/C} = \frac{\varkappa C''}{H''} + \beta''(1-\delta_H)P_{H/C} + (1-\chi'')[1-\mu^b](\beta'-\beta'')P_{H/C}$$

$$P_{H/C} = \frac{\varkappa}{1-\beta''(1-\delta_H)-(1-\chi'')[1-\mu^b](\beta'-\beta'')}\frac{C''}{H''} \coloneqq \phi_4\,\frac{C''}{H''} \qquad \text{5B.16}$$

Given (5A.3)

$$(1-\chi)P_{H/C}H = RB \Leftrightarrow B = (1-\chi)\beta' P_{H/C}H$$

$$B = (1-\chi)\beta'\phi_2 Y \qquad \text{5B.17}$$

Given (5A.10)

$$\frac{W'}{P_C} = \frac{\sigma(1-\mu-\nu)}{XL'}Y\left(\frac{P_Y}{P_C}\right)$$

$$\frac{W'}{P_C}L' = \frac{\sigma(1-\mu-\nu)}{X}Y \qquad \text{5B.18}$$

Combine (5B.7) and (5B.18) yields

$$\frac{W'}{P_C}L' + F = \left\{\frac{\sigma(1-\mu-\nu)}{X} + \left(1 - \frac{1}{X}\right)\right\} Y \coloneqq s'Y \qquad 5B.19$$

Given (5A.11)

$$\frac{W''}{P_C} = \frac{(1-\sigma)(1-\mu-\nu)}{XL''} Y \left(\frac{P_Y}{P_C}\right)$$

$$\frac{W''}{P_C}L'' = \frac{(1-\sigma)(1-\mu-\nu)}{X} Y \coloneqq s''Y \qquad 5B.20$$

Given (5A.2)

$$\frac{P_Y}{P_C}\frac{Y}{X} + B = C + P_{H/C}I_H + \frac{R}{\Pi_C}B + \frac{W'}{P_C}L' + \frac{W''}{P_C}L'' + I_K + A_K + A_H$$

where: $I_K = \delta_K K$; $P_Y = P_C$; and $A_K = A_H = 0$

$$\frac{Y}{X} + B = C + P_{H/C}\delta_H H + \frac{R}{\Pi_C}B + \frac{W'}{P_C}L' + \frac{W''}{P_C}L'' + \delta_K K$$

$$\frac{Y}{X} + B = C + \delta_H P_{H/C} H + \frac{B}{\beta'} + \frac{\sigma(1-\mu-\nu)Y}{X} + \frac{(1-\sigma)(1-\mu-\nu)Y}{X} + \delta_K K$$

$$C = \left\{\frac{(\mu+\nu)}{X} - [(1-\chi)(1-\beta') + \delta_H]\phi_2 - \delta_K\phi_1\right\} Y \coloneqq \phi_5 Y \qquad 5B.21$$

Given (5A.25)

$$[1-\mu^b](1-\chi'')P_{H/C}H'' + [\mu^b]\frac{W''}{P_C}L'' = RB''$$

$$B'' = \beta'\left\{[1-\mu^b](1-\chi'')P_{H/C}H'' + [\mu^b]\frac{W''}{P_C}L''\right\}$$

$$B'' = \beta'\left\{[1-\mu^b](1-\chi'')\phi_4\frac{C''}{Y} + [\mu^b]s''\right\} Y \qquad 5B.22$$

Given (5A.22)

$$B'' + \frac{W''}{P_C}L'' = C'' + P_{H/C}I''_H + \frac{R}{\Pi_C}B'' + A''_H$$

where: $A''_H = 0$

$$\left(\frac{\beta'-1}{\beta'}\right)\beta'\left\{(1-\chi'')[1-\mu^b]\phi_4\frac{C''}{Y}+\mu^b s''\right\}Y+s''Y=C''+\phi_4\delta_H C''$$

$$(1+\phi_4\delta_H-(\beta'-1)(1-\chi'')[1-\mu^b]\phi_4)C''=[1+(\beta'-1)\mu^b]s''Y$$

$$C''=\left(\frac{[1+(\beta'-1)\mu^b]s''}{1+\phi_4\delta_H-(\beta'-1)(1-\chi'')[1-\mu^b]\phi_4}\right)Y\coloneqq\phi_6 Y \qquad \text{5B.23}$$

Given (5A.67)

$$0=B+B'+B''\Leftrightarrow B'=-(B+B'')$$

$$B'=-\left\{(1-\chi)\beta'\phi_2 Y+\beta'\left[(1-\chi'')[1-\mu^b]\phi_4\frac{C''}{Y}+\mu^b s''\right]Y\right\}$$

$$B'=-\beta'\{(1-\chi)\phi_2+[(1-\chi'')[1-\mu^b]\phi_4\phi_6+\mu^b s'']\}Y \qquad \text{5B.24}$$

Given (5A.14)

$$B'+\mathbb{E}B^*+\frac{W'}{P_C}L'+F=C'+P_{H/C}I'_H+\frac{R}{\pi_C}B'+\frac{R^*}{\pi_C}\mathbb{E}B^*+A'_H$$

where: $A'_H=0$

$$C'=\left(\frac{W'}{P_C}L'+F\right)+\left(1-\frac{1}{\beta'}\right)B'-\delta_H P_{H/C}H'$$

$$C'=\left(\frac{W'}{P_C}L'+F\right)-\left(\frac{\beta'-1}{\beta'}\right)\beta'\{(1-\chi)\phi_2+[(1-\chi'')[1-\mu^b]\phi_4\phi_6+\mu^b s'']\}Y-\delta_H P_{H/C}H'$$

$$(1+\delta_H\phi_3)C'=(s'+(1-\beta')\{(1-\chi)\phi_2+[(1-\chi'')[1-\mu^b]\phi_4\phi_6+\mu^b s'']\})Y$$

$$C'=\frac{1}{1+\delta_H\phi_3}(s'+(1-\beta')\{(1-\chi)\phi_2+[(1-\chi'')[1-\mu^b]\phi_4\phi_6+\mu^b s'']\})\coloneqq\phi_7 Y$$

5B.25

Given (5A.68)

$$\breve{H}=1-H'-H''$$

Recall:

$$P_{H/C}=\phi_2\frac{Y}{H}=\phi_3\frac{C'}{H'}=\phi_4\frac{C''}{H''}\Longrightarrow H'=\frac{\phi_3 HC'}{\phi_2 Y}\ \text{ and }\ H''=\frac{\phi_4 HC''}{\phi_2 Y}$$

$$H = 1 - \frac{\phi_3 C'}{\phi_2 Y} H - \frac{\phi_4 C''}{\phi_2 Y} H$$

$$H\left(1 + \frac{\phi_3\phi_7}{\phi_2} + \frac{\phi_4\phi_6}{\phi_2}\right) = 1$$

$$H = \frac{\phi_2}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6} \qquad \text{5B.26}$$

$$H' = \frac{\phi_3\phi_7}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6} \qquad \text{5B.27}$$

$$H'' = \frac{\phi_4\phi_6}{\phi_2 + \phi_3\phi_7 + \phi_4\phi_6} \qquad \text{5B.28}$$

Given (5A.64)

$$\check{C} = C + C' + C''$$

$$\frac{\check{C}}{Y} = \phi_5 + \phi_7 + \phi_6 \qquad \text{5B.29}$$

Combine (5B.13) and (5B.29) yields

$$\frac{I_K}{\check{C}} = \frac{\delta_K K}{\check{C}} = \delta_K \phi_1 \frac{Y}{\check{C}} = \frac{\phi_1 \delta_K}{\phi_5 + \phi_7 + \phi_6} \qquad \text{5B.30}$$

Given (5A.62)

$$Y = \check{C} + I + G \qquad \text{5B.31}$$

C. Stationarisation and Log-Linearisation

Since some variables in the model are non-stationary, we stationarise the model following Justiano et al. (2009). All variables are then log-linearised around their steady state and denoted by tilde e.g. $\tilde{y}_t = log(\hat{Y}_t) - log(\hat{Y})$.

From (5A.1)

$$Y_t \Delta_t = e^{\Lambda_t} {A_t}^{(1-\mu-\nu)} K_{t-1}^{\mu} H_{t-1}^{\nu} L_t'^{\sigma(1-\mu-\nu)} L_t''^{(1-\sigma)(1-\mu-\nu)}$$

Stationarisation:

$$\hat{Y}_t \Delta_t = e^{\Lambda_t} \hat{K}_{t-1}^{\mu} \left(\frac{A_{t-1}}{A_t}\right)^{\mu} \hat{H}_{t-1}^{\nu} \left(\frac{A_{t-1}}{A_t}\right)^{\nu} [L_t']^{\sigma(1-\mu-\nu)} [L_t'']^{(1-\sigma)(1-\mu-\nu)}$$

Log-Linearisation (LL):

$$Y(\tilde{y}_t) = K^{\mu} H^{\nu} L'^{\sigma(1-\mu-\nu)} L''^{(1-\sigma)(1-\mu-\nu)} \{\Lambda_t + \mu \tilde{k}_{t-1} + \mu(\tilde{a}_{t-1} - \tilde{a}_t) + \nu \tilde{h}_{t-1} + \nu(\tilde{a}_{t-1} - \tilde{a}_t) + \sigma(1-\mu-\nu)\tilde{l}'_t + (1-\sigma)(1-\mu-\nu)\tilde{l}''_t\}$$

$$\tilde{y}_t = \Lambda_t + \mu \tilde{k}_{t-1} + \nu \tilde{h}_{t-1} + \sigma(1-\mu-\nu)\tilde{l}'_t + (1-\sigma)(1-\mu-\nu)\tilde{l}''_t - [\mu + \nu] z_t$$

5C.1

From (5A.18)

$$\frac{W'_t}{P_{C,t}} = C'_t (L'_t)^{\eta-1}$$

Log-linearisation (LL):

$$\frac{W'}{P_C}(\tilde{w}'_t - \tilde{p}_C) = C'(L')^{\eta-1}\{\tilde{c}'_t + (\eta-1)\tilde{l}'_t\}$$

$$\tilde{w}'_t - \tilde{p}_C = \tilde{c}'_t + (\eta-1)\tilde{l}'_t \qquad 5C.2$$

From (5A.10)

$$\frac{W'_t}{P_{C,t}} = \frac{\sigma(1-\mu-\nu)Y_t}{X_t L'_t} \frac{P_{Y,t}}{P_{C,t}}$$

As $P_{C,t} = \left[(1-\alpha){P_{Y,t}}^{1-\varpi} + \alpha {P_{F,t}}^{1-\varpi}\right]^{\frac{1}{1-\varpi}}$, we can approach it as $P_{C,t} = {P_{Y,t}}^{1-\alpha} {P_{F,t}}^{\alpha}$ so that $\frac{P_{Y,t}}{P_{C,t}} = \left(\frac{P_{F,t}}{P_{Y,t}}\right)^{-\alpha} = (S_t)^{-\alpha}$

Given $S_t = \frac{P_{F,t}}{P_{Y,t}}$, then the above equation can be re-written as:

$$\frac{W'_t}{P_{C,t}} = \frac{\sigma(1-\mu-\nu)Y_t}{X_t L'_t} S_t^{-\alpha}$$

Log-Linearisation (LL):

$$\frac{W'}{P_C}(\tilde{w}'_t - \tilde{p}_C) = \frac{\sigma(1-\mu-\nu)Y}{XL'} S^{-\alpha}\left(\tilde{y}_t - \tilde{x}_t - \tilde{l}'_t - \alpha \tilde{s}_t\right)$$

$$\tilde{w}'_t - \tilde{p}_C = \tilde{y}_t - \tilde{x}_t - \tilde{l}'_t - \alpha \tilde{s}_t \qquad 5C.3$$

Combining (5C.2) and (5C.3) yields

$$\tilde{y}_t = \tilde{x}_t + \eta \tilde{l}'_t + \tilde{c}'_t + \alpha \tilde{s}_t \qquad 5C.4$$

From (5A.27)

$$\frac{W_t''}{P_{C,t}}\left(e^{\mathcal{E}_t^\beta}+\mu^b\lambda_t''C_t''\right)=C_t''(L_t'')^{\eta-1}e^{\mathcal{E}_t^\beta}$$

Log-Linearisation (LL):

$$\frac{W''}{P_C}\left(\widetilde{w}_t''-\tilde{p}_C+\mathcal{E}_t^\beta\right)+\frac{W''\mu^b\lambda''C''}{P_C}\left(\widetilde{w}_t''+\tilde{\lambda}_t''+\tilde{c}_t''-\tilde{p}_C\right)=C''(L'')^{\eta-1}\left\{\tilde{c}_t''+(\eta-1)\tilde{l}_t''+\mathcal{E}_t^\beta\right\}$$

$$(1+\mu^b\lambda''C'')\widetilde{w}_t''+\mu^b\lambda''C''\tilde{\lambda}_t''-(1+\mu^b\lambda''C'')\tilde{p}_C-\tilde{c}_t''=(1+\mu^b\lambda''C'')(\eta-1)\tilde{l}_t''+(\mu^b\lambda''C'')\mathcal{E}_t^\beta$$

Recall $\lambda''=\frac{\beta'-\beta''}{C''}=\frac{\beta'-\beta''}{C''}$

$$[1+\mu^b(\beta'-\beta'')]\widetilde{w}_t''+[\mu^b(\beta'-\beta'')]\tilde{\lambda}_t''-[1+\mu^b(\beta'-\beta'')]\tilde{p}_C-\tilde{c}_t''=[1+\mu^b(\beta'-\beta'')](\eta-1)\tilde{l}_t''+[\mu^b(\beta'-\beta'')]\mathcal{E}_t^\beta$$

Define $\mu^b(\beta'-\beta'')\equiv m_b$

$$(1+m_b)(\widetilde{w}_t''-\tilde{p}_C)+m_b\tilde{\lambda}_t''-\tilde{c}_t''=(1+m_b)(\eta-1)\tilde{l}_t''+m_b\mathcal{E}_t^\beta \qquad 5C.5$$

From (5A.11)

$$\frac{W_t''}{P_{C,t}}=\frac{(1-\sigma)(1-\mu-v)Y_t}{X_tL_t''}\frac{P_{Y,t}}{P_{C,t}} \quad\Leftrightarrow\quad \frac{W_t''}{P_{C,t}}=\frac{(1-\sigma)(1-\mu-v)Y_t}{X_tL_t''}S_t^{-\alpha}$$

Log-linearisation (LL):

$$\frac{W''}{P_C}\left(\widetilde{w}_t''-\tilde{p}_{C,t}\right)=\frac{(1-\sigma)(1-\mu-v)Y}{XL''}S^{-\alpha}\left(\tilde{y}_t-\tilde{x}_t-\tilde{l}_t''-\alpha\tilde{s}_t\right)$$

$$\widetilde{w}_t''-\tilde{p}_{C,t}=\tilde{y}_t-\tilde{x}_t-\tilde{l}_t''-\alpha\tilde{s}_t \qquad 5C.6$$

Combining (5A.5) and (5A.6) yields

$$(1+m_b)\left(\tilde{y}_t-\tilde{x}_t-\tilde{l}_t''-\alpha\tilde{s}_t\right)+m_b\tilde{\lambda}_t''-\tilde{c}_t''=(1+m_b)(\eta-1)\tilde{l}_t''+m_b\mathcal{E}_t^\beta \qquad 5C.7$$

From (5A.12)

$$\Omega_{C,t}\left(\frac{1}{e^{\varepsilon_t^{I_K}}}+\Omega_{K,t}\right)=\beta E_t\left(\Omega_{C,t+1}\left[\frac{\mu Y_{t+1}P_{Y,t+1}}{K_t X_{t+1}P_{C,t+1}}+(1-\delta_K)\left(\frac{1}{e^{\varepsilon_{t+1}^{I_K}}}\right)+\Omega_{K,t+1}\breve{\Omega}_{K,t+1}\right]\right)$$

with $\Omega_{K,t}\equiv\frac{\psi_K}{\delta_K}\left(\frac{I_{K,t}}{K_{t-1}}-\delta_k\right)$ and $\breve{\Omega}_{K,t}\equiv\frac{1}{2}\left(\frac{I_{K,t}}{K_{t-1}}+\delta_K\right)+(1-\delta_K)$

Stationarisation:

$$\frac{1}{e^{\varepsilon_t^{I_K}}}+\frac{\psi_K}{\delta_K}\left(\frac{\hat{I}_{K,t}}{\hat{K}_{t-1}}\left[\frac{A_t}{A_{t-1}}\right]-\delta_K\right)=\beta E_t\left(\left(\frac{e^{\varepsilon_{t+1}^{\beta}}}{e^{\varepsilon_t^{\beta}}}\right)\left(\frac{\hat{C}_t}{\hat{C}_{t+1}}\left[\frac{A_t}{A_{t+1}}\right]\right)\left[\frac{\mu\hat{Y}_{t+1}}{\hat{K}_t X_{t+1}}\left[\frac{A_{t+1}}{A_t}\right]S_{t+1}^{-\alpha}+(1-\right.\right.$$

$$\left.\left.\delta_K)\left(\frac{1}{e^{\varepsilon_{t+1}^{I_K}}}\right)+\frac{\psi_K}{\delta_K}\left(\frac{\hat{I}_{K,t+1}}{\hat{K}_t}\left[\frac{A_{t+1}}{A_t}\right]-\delta_K\right)\left[\frac{1}{2}\left(\frac{\hat{I}_{K,t+1}}{\hat{K}_t}\left[\frac{A_{t+1}}{A_t}\right]+\delta_K\right)+(1-\delta_K)\right]\right]\right)$$

Log-linearisation (LL):

LHS

$$\psi_K\left(\tilde{\imath}_{K,t}+z_t-\tilde{k}_{t-1}\right)-\varepsilon_t^{I_K}$$

RHS

$$\frac{\beta\mu Y}{KX}S^{-\alpha}\left(\tilde{c}_t-E_t\tilde{c}_{t+1}+E_t\tilde{y}_{t+1}-\alpha E_t\tilde{s}_{t+1}-\tilde{k}_t-E_t\tilde{x}_{t+1}+E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}\right)+\beta(1-\delta_K)\left(\tilde{c}_t-\right.$$
$$\left.E_t\tilde{c}_{t+1}+E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}+\tilde{a}_t-E_t\tilde{a}_{t+1}-E_t\varepsilon_{t+1}^{I_K}\right)+\left(\frac{\beta}{2}\right)\left(\frac{\psi_K}{\delta_K}\right)\left(\frac{I_K}{K}\right)^2\left(\tilde{c}_t-E_t\tilde{c}_{t+1}+\tilde{a}_t-\right.$$
$$\left.E_t\tilde{a}_{t+1}+2E_t\tilde{\imath}_{K,t+1}-2\tilde{k}_t+2(E_t\tilde{a}_{t+1}-\tilde{a}_t)+E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}\right)-\left(\frac{\beta}{2}\right)\left(\frac{\psi_K}{\delta_K}\right){\delta_K}^2\left(\tilde{c}_t-E_t\tilde{c}_{t+1}+\right.$$
$$\left.(\tilde{a}_t-E_t\tilde{a}_{t+1})+E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}\right)+\beta(1-\delta_K)\left(\frac{\psi_K}{\delta_K}\right)\left(\frac{I_K}{K}\right)\left(\tilde{c}_t-E_t\tilde{c}_{t+1}+E_t\tilde{\imath}_{K,t+1}-\tilde{k}_t+\right.$$
$$\left.E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}\right)-\beta(1-\delta_K)\left(\frac{\psi_K}{\delta_K}\right)\delta_K\left(\tilde{c}_t-E_t\tilde{c}_{t+1}+\tilde{a}_t-E_t\tilde{a}_{t+1}+E_t\varepsilon_{t+1}^{\beta}-\varepsilon_t^{\beta}\right)$$

$$\tilde{c}_t-E_t\tilde{c}_{t+1}+[1-\beta(1-\delta_K)]\left(E_t\tilde{y}_{t+1}-\alpha E_t\tilde{s}_{t+1}-\tilde{k}_t-E_t\tilde{x}_{t+1}\right)+\beta\psi_K\left(E_t\tilde{\imath}_{K,t+1}-\tilde{k}_t\right)+$$
$$\left(\rho_\beta-1\right)\varepsilon_t^{\beta}-\beta(1-\delta_K)E_t\varepsilon_{t+1}^{I_K}-[\beta(1-\delta_K)-\beta\psi_K]\rho_z z_t$$

Combine LHS and RHS

$$\psi_K\left(\tilde{\imath}_{K,t}+z_t-\tilde{k}_{t-1}\right)-\varepsilon_t^{I_K}=\tilde{c}_t-E_t\tilde{c}_{t+1}+[1-\beta(1-\delta_K)]\left(E_t\tilde{y}_{t+1}-\alpha E_t\tilde{s}_{t+1}-\tilde{k}_t-\right.$$
$$\left.E_t\tilde{x}_{t+1}\right)+\beta\psi_K\left(E_t\tilde{\imath}_{K,t+1}-\tilde{k}_t\right)+\left(\rho_\beta-1\right)\varepsilon_t^{\beta}-\beta(1-\delta_K)E_t\varepsilon_{t+1}^{I_K}-[\beta(1-\delta_K)-$$
$$\beta\psi_K]\rho_z z_t$$

$$\tilde{\imath}_{K,t} = \tilde{k}_{t-1} + \beta\left(E_t\tilde{\imath}_{K,t+1} - \tilde{k}_t\right) + \frac{1-\beta(1-\delta_K)}{\psi_K}\left(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{k}_t - E_t\tilde{x}_{t+1}\right) + \frac{1}{\psi_K}\{\tilde{c}_t - E_t\tilde{c}_{t+1}\} - \left(\frac{1-\rho_\beta}{\psi_K}\right)\mathcal{E}_t^\beta + \frac{1}{\psi_K}\left[1-\beta(1-\delta_K)\rho_q\right]\mathcal{E}_t^{I_K} - \left(\rho_z\left[\frac{\beta(1-\delta_K)}{\psi_K} - \beta\right] + 1\right)z_t \qquad 5C.8$$

From (5A.13)

$$\Omega_{C,t}\left(P_{H/C,t} + \Omega_{H,t}\right) = \beta E_t\left(\Omega_{C,t+1}\left[\frac{vY_{t+1}P_{Y,t+1}}{H_tX_{t+1}P_{C,t+1}} + (1-\delta_H)P_{H/C,t+1} + \Omega_{H,t+1}\breve{\Omega}_{H,t+1}\right] + (1-\chi)\lambda_t E_t\left[P_{H/C,t+1}\pi_{C,t+1}\right]e^{\mathcal{E}_t^{LTV}}\right)$$

with $\Omega_{H,t} \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I_{H,t}}{H_{t-1}} - \delta_H\right)$ and $\breve{\Omega}_{H,t} \equiv \frac{1}{2}\left(\frac{I_{H,t}}{H_{t-1}} + \delta_H\right) + (1-\delta_H)$

Stationarisation:

$$P_{H/C,t} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}_{H,t}}{\hat{H}_{t-1}}\left[\frac{A_t}{A_{t-1}}\right] - \delta_H\right) = \beta E_t\left(\left(\frac{e^{\mathcal{E}_{t+1}^\beta}}{e^{\mathcal{E}_t^\beta}}\right)\left(\frac{\hat{C}_t}{\hat{C}_{t+1}}\right)\left[\frac{A_t}{A_{t+1}}\right]\left[\frac{\mu\hat{Y}_{t+1}}{\hat{H}_tX_{t+1}}\left[\frac{A_{t+1}}{A_t}\right]S_{t+1}^{-\alpha} + (1-\delta_H)P_{H/C,t+1} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}_{H,t+1}}{\hat{H}_t}\left[\frac{A_{t+1}}{A_t}\right] - \delta_H\right)\left[\frac{1}{2}\left(\frac{\hat{I}_{H,t+1}}{\hat{H}_t}\left[\frac{A_{t+1}}{A_t}\right] + \delta_H\right) + (1-\delta_H)\right]\right] + (1-\chi)\left(\frac{C_t}{e^{\mathcal{E}_t^\beta}}\right)\lambda_t P_{H/C,t+1}\pi_{C,t+1}e^{\mathcal{E}_t^{LTV}}\right)$$

Log-linearisation (LL):

LHS

$$P_{H/C}\left(\tilde{p}_{H/C,t} + \psi_H\left(\tilde{\imath}_{H,t} - \tilde{h}_{t-1} + z_t\right)\right)$$

RHS

$$\frac{\beta\mu Y}{HX}S^{-\alpha}\left(\tilde{c}_t - E_t\tilde{c}_{t+1} + E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + \beta(1-\delta_H)P_{H/C}\left(\tilde{c}_t - E_t\tilde{c}_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\tilde{p}_{H/C,t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + \left(\frac{\beta}{2}\right)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I_H}{H}\right)^2 P_{H/C}\left(\tilde{c}_t - E_t\tilde{c}_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + 2E_t\tilde{\imath}_{H,t+1} - 2\tilde{h}_t + 2(E_t\tilde{a}_{t+1} - \tilde{a}_t) + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) - \left(\frac{\beta}{2}\right)\left(\frac{\psi_H}{\delta_H}\right)(\delta_H)^2 P_{H/C}\left(\tilde{c}_t - E_t\tilde{c}_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + \beta(1-\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I_H}{H}\right)P_{H/C}\left(\tilde{c}_t - E_t\tilde{c}_{t+1} + E_t\tilde{\imath}_{H,t+1} - \tilde{h}_t + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) - \beta(1 -$$

$\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\delta_H P_{H/C}\Big(\tilde{c}_t - E_t\tilde{c}_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}^{\beta}_{t+1} - \mathcal{E}^{\beta}_t\Big) + (1-\chi)C\lambda P_{H/C}\Big(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}^{LTV}_t + \tilde{c}_t - \mathcal{E}^{\beta}_t\Big)$

$P_{H/C}\Big\{[1-(1-\chi)C\lambda]\big(\tilde{c}_t - E_t\tilde{c}_{t+1} + E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) - \beta(1-\delta_H)\big(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) + \beta(1-\delta_H)E_t\tilde{p}_{H/C,t+1} + \beta\psi_H\big(E_t\tilde{\imath}_{H,t+1} - \tilde{h}_t\big) + (1-\chi)C\lambda\big(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}^{LTV}_t + \tilde{c}_t\big) + \big[(1-(1-\chi)C\lambda)\rho_\beta - 1\big]\mathcal{E}^{\beta}_t - [\beta(1-\delta_H) - \beta\psi_H]\rho_z z_t\Big\}$

Combine LHS and RHS

$\tilde{p}_{H/C,t} + \psi_H\big(\tilde{\imath}_{H,t} - \tilde{h}_{t-1} + z_t\big) = [1-(1-\chi)C\lambda]\big(\tilde{c}_t - E_t\tilde{c}_{t+1} + E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) - \beta(1-\delta_H)\big(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) + \beta(1-\delta_H)E_t\tilde{p}_{H/C,t+1} + \beta\psi_H\big(E_t\tilde{\imath}_{H,t+1} - \tilde{h}_t\big) + (1-\chi)C\lambda\big(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}^{LTV}_t + \tilde{c}_t\big) + \big[(1-(1-\chi)C\lambda)\rho_\beta - 1\big]\mathcal{E}^{\beta}_t - [\beta(1-\delta_H) - \beta\psi_H]\rho_z z_t$

$\tilde{p}_{H/C,t} = [\beta(1-\delta_H) + (1-\chi)C\lambda]E_t\tilde{p}_{H/C,t+1} + [1-(1-\chi)C\lambda - \beta(1-\delta_H)]\big(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) + \tilde{c}_t - E_t\tilde{c}_{t+1} + \psi_H\left[\beta\big(E_t\tilde{\imath}_{H,t+1} - \tilde{h}_t\big) - \big(\tilde{\imath}_{H,t} - \tilde{h}_{t-1}\big)\right] + (1-\chi)C\lambda\big(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}^{LTV}_t + E_t\tilde{c}_{t+1}\big) + \big[(1-(1-\chi)C\lambda)\rho_\beta - 1\big]\mathcal{E}^{\beta}_t - ([\beta(1-\delta_H) - \beta\psi_H]\rho_z + \psi_H)z_t$

Recall

$(1-\chi)C\lambda = (1-\chi)C\left(\frac{\beta'-\beta}{C}\right) = (1-\chi)(\beta'-\beta) \equiv m_e$

$\beta(1-\delta_H) + (1-\chi)C\lambda = \beta(1-\delta_H) + m_e \equiv \gamma_e$

Therefore

$$\tilde{p}_{H/C,t} = \gamma_e E_t\tilde{p}_{H/C,t+1} + [1-\gamma_e]\big(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\big) + m_e\big(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}^{LTV}_t + E_t\tilde{c}_{t+1}\big) + \tilde{c}_t - E_t\tilde{c}_{t+1} + \left(\frac{\psi_H}{\delta_H}\right)\left[\beta\big(E_t\tilde{h}_{t+1} - \tilde{h}_t\big) - \big(\tilde{h}_t - \tilde{h}_{t-1}\big)\right] + \big[(1-m_e)\rho_\beta - 1\big]\mathcal{E}^{\beta}_t - ([\beta(1-\delta_H) - \beta\psi_H]\rho_z + \psi_H)z_t \qquad \text{5C.9}$$

where:

$\tilde{h}_t = \delta_H\tilde{\imath}_{H,t} + (1-\delta_H)\tilde{h}_{t-1} \Longrightarrow \tilde{\imath}_{H,t} = \frac{1}{\delta_H}\big(\tilde{h}_t - (1-\delta_H)\tilde{h}_{t-1}\big)$

$\tilde{\imath}_{H,t} - \tilde{h}_{t-1} = \frac{1}{\delta_H}\tilde{h}_t - \left(\frac{1}{\delta_H} - 1\right)\tilde{h}_{t-1} - \tilde{h}_{t-1} \Longrightarrow \frac{1}{\delta_H}\big(\tilde{h}_t - \tilde{h}_{t-1}\big)$

$\psi_H\big(\tilde{\imath}_{H,t} - \tilde{h}_{t-1}\big) = \left(\frac{\psi_H}{\delta_H}\right)\big(\tilde{h}_t - \tilde{h}_{t-1}\big)$

From (5A.21)

$$\Omega'_{\text{C,t}}\left[P_{H/C,t} + \Omega'_{\text{H,t}}\right] = \varkappa e^{\mathcal{E}_t^{\beta}} e^{\mathcal{E}_t^{H}} \left(\frac{1}{H'_t}\right) + \beta' E_t\left(\Omega'_{\text{C,t+1}}\left[(1-\delta_H)P_{H/C,t+1} + \Omega'_{\text{H,t+1}}\breve{\Omega}'_{\text{H,t+1}}\right]\right)$$

with $\Omega'_{\text{H,t}} \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I'_{H,t}}{H'_{t-1}} - \delta_H\right)$ and $\breve{\Omega}'_{\text{H,t}} \equiv \frac{1}{2}\left(\frac{I'_{H,t}}{H'_{t-1}} + \delta_H\right) + (1-\delta_H)$

Stationarisation:

$$P_{H/C,t} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}'_{H,t}}{\hat{H}'_{t-1}}\left[\frac{A_t}{A_{t-1}}\right] - \delta_H\right) = \varkappa\left(\frac{e^{\mathcal{E}_t^{H}}\hat{C}'_t}{\hat{H}'_t}\right) + \beta' E_t\left(\left(\frac{e^{\mathcal{E}_{t+1}^{\beta}}}{e^{\mathcal{E}_t^{\beta}}}\right)\left(\frac{\hat{C}'_t}{\hat{C}'_{t+1}}\right)\left[\frac{A_t}{A_{t+1}}\right]\left((1-\delta_H)P_{H/C,t+1} + \left[\frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}'_{H,t+1}}{\hat{H}'_t}\left[\frac{A_{t+1}}{A_t}\right] - \delta_H\right)\right]\left[\frac{1}{2}\left(\frac{\hat{I}'_{H,t+1}}{\hat{H}'_t}\left[\frac{A_{t+1}}{A_t}\right] + \delta_H\right) + (1-\delta_H)\right]\right)\right)$$

Log-linearisation (LL):

LHS

$$P_{H/C}\left\{\tilde{p}_{H/C,t} + \psi_H\left(\tilde{\imath}'_{H,t} + z_t - \tilde{h}'_{t-1}\right)\right\}$$

RHS

$$\frac{\varkappa C'}{H'}\left(\mathcal{E}_t^H + \tilde{c}'_t - \tilde{h}'_t\right) + \beta'(1-\delta_H)P_{H/C}\left(\tilde{c}'_t - E_t\tilde{c}'_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\tilde{p}_{H/C,t+1} + E_t\mathcal{E}_{t+1}^{\beta} - \mathcal{E}_t^{\beta}\right) + \left(\frac{\beta'}{2}\right)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I'_H}{H'}\right)^2 P_{H/C}\left(\tilde{c}'_t - E_t\tilde{c}'_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + 2E_t\tilde{\imath}'_{H,t+1} - 2\tilde{h}'_t + 2[E_t\tilde{a}_{t+1} - \tilde{a}_t] + E_t\mathcal{E}_{t+1}^{\beta} - \mathcal{E}_t^{\beta}\right) - \left(\frac{\beta'}{2}\right)\left(\frac{\psi_H}{\delta_H}\right)(\delta_H)^2 P_{H/C}\left(\tilde{c}'_t - E_t\tilde{c}'_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}_{t+1}^{\beta} - \mathcal{E}_t^{\beta}\right) + \beta'(1-\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I'_H}{H'}\right)P_{H/C}\left(\tilde{c}'_t - E_t\tilde{c}'_{t+1} + E_t\tilde{\imath}'_{H,t+1} - \tilde{h}'_t + E_t\mathcal{E}_{t+1}^{\beta} - \mathcal{E}_t^{\beta}\right) - \beta'(1-\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\delta_H P_{H/C}\left(\tilde{c}'_t - E_t\tilde{c}'_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}_{t+1}^{\beta} - \mathcal{E}_t^{\beta}\right)$$

$$P_{H/C}\left\{\left(\mathcal{E}_t^H + \tilde{c}'_t - \tilde{h}'_t\right) - \beta'(1-\delta_H)\left(\mathcal{E}_t^H + E_t\tilde{c}'_{t+1} - \tilde{h}'_t - E_t\tilde{p}_{H/C,t+1}\right) + \beta'\psi_H\left(E_t\tilde{\imath}'_{H,t+1} - \tilde{h}'_t\right) - \beta'[(1-\delta_H) - \psi_H]\rho_z z_t + \beta'(1-\delta_H)\left(\rho_\beta - 1\right)\mathcal{E}_t^{\beta}\right\}$$

Combine LHS and RHS

$$\tilde{p}_{H/C,t} + \psi_H\left(\tilde{\imath}'_{H,t} + z_t - \tilde{h}'_{t-1}\right) = \left(\mathcal{E}_t^H + \tilde{c}'_t - \tilde{h}'_t\right) - \beta'(1-\delta_H)\left(\mathcal{E}_t^H + E_t\tilde{c}'_{t+1} - \tilde{h}'_t - E_t\tilde{p}_{H/C,t+1}\right) + \beta'\psi_H\left(E_t\tilde{\imath}'_{H,t+1} - \tilde{h}'_t\right) - \beta'[(1-\delta_H) - \psi_H]\rho_z z_t + \beta'(1-\delta_H)\left(\rho_\beta - 1\right)\mathcal{E}_t^{\beta}$$

$$\tilde{p}_{H/C,t} = \beta'(1-\delta_H)E_t\tilde{p}_{H/C,t+1} + \left(1 - \beta'(1-\delta_H)\right)\left(\mathcal{E}_t^H - \tilde{h}'_t\right) + \tilde{c}'_t - \beta'(1-\delta_H)E_t\tilde{c}'_{t+1} - \frac{\psi_H}{\delta_H}\left\{\left(\tilde{h}'_t - \tilde{h}'_{t-1}\right) - \beta'\left(E_t\tilde{h}'_{t+1} - \tilde{h}'_t\right)\right\} + \beta'(1-\delta_H)\left(\rho_\beta - 1\right)\mathcal{E}_t^{\beta} - \left(\beta'[(1-\delta_H) - \psi_H]\rho_z + \psi_H\right)z_t \qquad \text{5C.10}$$

where:

$$\tilde{h}'_t = \delta_H \tilde{\iota}'_{H,t} + (1-\delta_H)\tilde{h}'_{t-1} \Longrightarrow \tilde{\iota}'_{H,t} = \frac{1}{\delta_H}\left(\tilde{h}'_t - (1-\delta_H)\tilde{h}'_{t-1}\right)$$

$$\tilde{\iota}'_{H,t} - \tilde{h}'_{t-1} = \frac{1}{\delta_H}\tilde{h}'_t - \left(\frac{1}{\delta_H} - 1\right)\tilde{h}'_{t-1} - \tilde{h}'_{t-1} \Longrightarrow \frac{1}{\delta_H}\left(\tilde{h}'_t - \tilde{h}'_{t-1}\right)$$

$$\psi_H\left(\tilde{\iota}'_{H,t} - \tilde{h}'_{t-1}\right) = \left(\frac{\psi_H}{\delta_H}\right)\left(\tilde{h}'_t - \tilde{h}'_{t-1}\right)$$

From (5A.22)

$$C''_t + P_{H/C,t} I''_{H,t} + \frac{R_{t-1}}{\Pi_{C,t}} B''_{t-1} + A''_{H,t} = B''_t + \frac{W''_t}{P_{C,t}} L''_t$$

$$C''_t + P_{H/C,t}(H''_t - [1-\delta_H]H''_{t-1}) + \frac{R_{t-1}}{\pi_{C,t}} B''_{t-1} + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{I''_{H,t}}{H''_{t-1}} - \delta_H\right)^2 H''_{t-1} = B''_t + \frac{W''_t}{P_{C,t}} L''_t$$

Stationarisation:

$$\hat{C}''_t + P_{H/C,t}\left(\hat{H}''_t - [1-\delta_H]\hat{H}''_{t-1}\left[\frac{A_{t-1}}{A_t}\right]\right) + \frac{R_{t-1}}{\pi_{C,t}}\hat{B}''_{t-1}\left[\frac{A_{t-1}}{A_t}\right] + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\left[\frac{\hat{I}''_{H,t}}{\hat{H}''_{t-1}}\left[\frac{A_{t-1}}{A_t}\right]\right]^2 - 2\delta_H\left[\frac{\hat{I}''_{H,t}}{\hat{H}''_{t-1}}\left[\frac{A_{t-1}}{A_t}\right]\right] + {\delta_H}^2\right)\hat{H}''_{t-1}\left[\frac{A_{t-1}}{A_t}\right] = \hat{B}''_t + \frac{\widehat{W''_t}}{P_{C,t}} L''_t$$

Log-Linearisation (LL):

$$C''(\tilde{c}''_t) + P_{H/C}H''\left(\tilde{p}_{H/C,t} + \tilde{h}''_t\right) - (1-\delta_H)P_{H/C}H''\left(\tilde{p}_{H/C,t} + \tilde{h}''_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t\right) + \frac{RB''}{\pi_C}\left(\tilde{r}_{t-1} + \tilde{b}''_{t-1} - \tilde{\pi}_{C,t} + \tilde{a}_{t-1} - \tilde{a}_t\right) + \frac{\psi_H P_{H/C}}{2\delta_H}\frac{(I''_H)^2}{H''}\left(2\tilde{\iota}''_{H,t} - \tilde{h}''_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t\right) - \psi_H I''_H P_{H/C}\tilde{\iota}''_{H,t} + \frac{\psi_H P_{H/C}}{2} I''_H\left(\tilde{h}''_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t\right) = B''\tilde{b}''_t + \frac{W''L''}{P_C}\left(\tilde{w}''_t + \tilde{l}''_t - \tilde{p}_{C,t}\right)$$

$$\frac{B''}{Y}\tilde{b}''_t = \frac{C''}{Y}(\tilde{c}''_t) + \frac{P_{H/C}H''}{Y}\left(\tilde{h}''_t - (1-\delta_H)\tilde{h}''_{t-1}\right) + \frac{RB''}{Y}\left(\tilde{r}_{t-1} + \tilde{b}''_{t-1} - \tilde{\pi}_{C,t}\right) + \frac{P_{H/C}H''}{Y}\left(\tilde{p}_{H/C,t} - (1-\delta_H)\tilde{p}_{H/C,t}\right) - \left(\frac{1}{Y}\right)\frac{W''L''}{P_C}\left(\tilde{w}''_t + \tilde{l}''_t - \tilde{p}_{C,t}\right) + \left[(1-\delta_H)\frac{P_{H/C}H''}{Y} - \frac{RB''}{Y}\right] z_t$$

Recall:

$\frac{W''L''}{P_C} = S''Y$ and $\tilde{w}''_t + \tilde{L}''_t - \tilde{p}_{C,t} = \tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t$

Therefore:

$$\frac{B''}{Y}\tilde{b}_t'' = \frac{C''}{Y}(\tilde{c}_t'') + \frac{P_{H/C}H''}{Y}\left(\tilde{h}_t'' - (1-\delta_H)\tilde{h}_{t-1}''\right) + \frac{RB''}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1}'' - \tilde{\pi}_{C,t}\right) + \frac{P_{H/C}H''}{Y}\delta_H\tilde{p}_{H/C,t} - S''(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + \left[(1-\delta_H)\frac{P_{H/C}H''}{Y} - \frac{RB''}{Y}\right]z_t \qquad \text{5C.11}$$

From (5A.29)

$$\Omega_{\mathrm{C,t}}''\left(P_{H/C,t} + \Omega_{\mathrm{H,t}}''\right) = \varkappa e^{\mathcal{E}_t^\beta}e^{\mathcal{E}_t^H}\left(\frac{1}{H_t''}\right) + \beta'' E_t\left(\Omega_{\mathrm{C,t+1}}''\left[(1-\delta_H)P_{H/C,t+1} + \Omega_{\mathrm{H,t+1}}''\breve{\Omega}_{\mathrm{H,t+1}}''\right]\right) + [1-\mu^b](1-\chi'')\lambda_t'' E_t\left(P_{H/C,t+1}\pi_{\mathrm{C,t+1}}\right)e^{\mathcal{E}_t^{LTV}}$$

with $\Omega_{\mathrm{H,t}}'' \equiv \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{I_{H,t}''}{H_{t-1}''} - \delta_H\right)$ and $\breve{\Omega}_{\mathrm{H,t}}'' \equiv \frac{1}{2}\left(\frac{I_{H,t}''}{H_{t-1}''} + \delta_H\right) + (1-\delta_H)$

Stationarisation:

$$P_{H/C,t} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}_{H,t}''}{\hat{H}_{t-1}''}\left[\frac{A_t}{A_{t-1}}\right] - \delta_H\right) = \varkappa e^{\mathcal{E}_t^H}\left(\frac{\hat{C}_t''}{\hat{H}_t''}\right) + \beta'' E_t\left(\left(\frac{e^{\mathcal{E}_{t+1}^\beta}}{e^{\mathcal{E}_t^\beta}}\right)\left(\frac{\hat{C}_t''}{\hat{C}_{t+1}''}\right)\left[\frac{A_t}{A_{t+1}}\right]\left[(1-\delta_H)P_{H/C,t+1} + \frac{\psi_H P_{H/C}}{\delta_H}\left(\frac{\hat{I}_{H,t+1}''}{\hat{H}_t''}\left[\frac{A_{t+1}}{A_t}\right] - \delta_H\right)\left[\frac{1}{2}\left(\frac{\hat{I}_{H,t+1}''}{\hat{H}_t''}\left[\frac{A_{t+1}}{A_t}\right] + \delta_H\right) + (1-\delta_H)\right]\right]\right) + [1-\mu^b](1-\chi'')\left(\frac{C_t''}{e^{\mathcal{E}_t^\beta}}\right)\lambda_t'' E_t\left(P_{H/C,t+1}\pi_{\mathrm{C,t+1}}\right)e^{\mathcal{E}_t^{LTV}}$$

Log-linearisation (LL):

LHS

$$P_{H/C}\left(\tilde{p}_{H/C,t} + \psi_H\left(\tilde{\iota}_{H,t}'' - \tilde{h}_{t-1}'' + z_t\right)\right)$$

RHS

$$\frac{\varkappa C''}{H''}\left(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t''\right) + \beta''(1-\delta_H)P_{H/C}\left(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\tilde{p}_{H/C,t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + \left(\frac{\beta''}{2}\right)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I_H''}{H''}\right)^2 P_{H/C}\left(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + \tilde{a}_t - E_t\tilde{a}_{t+1} + 2E_t\tilde{\iota}_{H,t+1}'' - 2\tilde{h}_t'' + 2(E_t\tilde{a}_{t+1} - \tilde{a}_t) + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) - \left(\frac{\beta''}{2}\right)\left(\frac{\psi_H}{\delta_H}\right){\delta_H}^2 P_{H/C}\left(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + \beta''(1-\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\left(\frac{I_H''}{H''}\right)P_{H/C}\left(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + E_t\tilde{\iota}_{H,t+1}'' - \tilde{h}_t'' + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) - \beta''(1-\delta_H)\left(\frac{\psi_H}{\delta_H}\right)\delta_H P_{H/C}\left(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + \tilde{a}_t - E_t\tilde{a}_{t+1} + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\right) + [1-\mu^b](1-\chi'')C''\lambda'' P_{H/C}\left(\tilde{c}_t'' + \tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}_t^{LTV} - \mathcal{E}_t^\beta\right)$$

$P_{H/C}\Big\{\big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t''\big) - \beta''(1-\delta_H)\big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t'' - E_t\tilde{p}_{H/C,t+1}\big) - [1-\mu^b](1-\chi'')C''\lambda''\,\big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t''\big) + \beta''(1-\delta_H)\Big(\tilde{c}_t'' - E_t\tilde{c}_{t+1}'' + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\Big) + \beta''\psi_H\big(E_t\tilde{\imath}_{H,t+1}'' - \tilde{h}_t''\big) + [1-\mu^b](1-\chi'')C''\lambda''\Big(\tilde{c}_t'' + \tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}_t^{LTV} - \mathcal{E}_t^\beta\Big) - \beta''[(1-\delta_H) - \psi_H]\rho_z z_t\Big\}$

Combine LHS and RHS

$$\tilde{p}_{H/C,t} + \psi_H\big(\tilde{\imath}_{H,t}'' - \tilde{h}_{t-1}'' + z_t\big) = \big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t''\big) - \beta''(1-\delta_H)\big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t'' - E_t\tilde{p}_{H/C,t+1}\big) - [1-\mu^b](1-\chi'')C''\lambda''\,\big(\mathcal{E}_t^H + \tilde{c}_t'' - \tilde{h}_t''\big) + \beta''(1-\delta_H)\Big(\tilde{c}_t'' - \tilde{c}_{t+1}'' + E_t\mathcal{E}_{t+1}^\beta - \mathcal{E}_t^\beta\Big) + \beta''\psi_H\big(E_t\tilde{\imath}_{H,t+1}'' - \tilde{h}_t''\big) + [1-\mu^b](1-\chi'')C''\lambda''\Big(\tilde{c}_t'' + \tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + E_t\tilde{p}_{H/C,t+1} + \mathcal{E}_t^{LTV} - \mathcal{E}_t^\beta\Big) - \beta''[(1-\delta_H) - \psi_H]\rho_z z_t$$

$$\tilde{p}_{H/C,t} = [\beta''(1-\delta_H) + [1-\mu^b](1-\chi'')C''\lambda'']E_t\tilde{p}_{H/C,t+1} + [1 - \beta''(1-\delta_H) - [1-\mu^b](1-\chi'')C''\lambda'']\big(\mathcal{E}_t^H - \tilde{h}_t''\big) + \psi_H\{\beta''\big(E_t\tilde{\imath}_{H,t+1}'' - \tilde{h}_t''\big) - \big(\tilde{\imath}_{H,t}'' - \tilde{h}_{t-1}''\big)\} + \tilde{c}_t'' - \beta''(1-\delta_H)E_t\tilde{c}_{t+1}'' + [1-\mu^b](1-\chi'')C''\lambda''\big(\tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\big) - \big[\beta''(1-\delta_H)\big(1-\rho_\beta\big) + [1-\mu^b](1-\chi'')C''\lambda''\big]\mathcal{E}_t^\beta - (\beta''[(1-\delta_H) - \psi_H]\rho_z + \psi_H)z_t$$

Recall

$$[1-\mu^b](1-\chi'')C''\lambda'' = [1-\mu^b](1-\chi'')C''\left(\frac{\beta'-\beta''}{C''}\right) = [1-\mu^b](1-\chi'')(\beta'-\beta'') \equiv m_h$$

$$\beta''(1-\delta_H) + [1-\mu^b](1-\chi'')C''\lambda'' = \beta''(1-\delta_H) + m_h \equiv \gamma_h$$

Therefore:

$$\tilde{p}_{H/C,t} = \gamma_h E_t\tilde{p}_{H/C,t+1} + (1-\gamma_h)\big(\mathcal{E}_t^H - \tilde{h}_t''\big) + m_h\big(\tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\big) + \tilde{c}_t'' - \beta''(1-\delta_H)E_t\tilde{c}_{t+1}'' + \frac{\psi_H}{\delta_H}\{\beta''\big(E_t\tilde{h}_{t+1}'' - \tilde{h}_t''\big) - \big(\tilde{h}_t'' - \tilde{h}_{t-1}''\big)\} - \big[\beta''(1-\delta_H)\big(1-\rho_\beta\big) + m_h\big]\mathcal{E}_t^\beta - (\beta''[(1-\delta_H) - \psi_H]\rho_z + \psi_H)z_t \qquad \text{5C.12}$$

where:

$$\tilde{h}_t'' = \delta_H\tilde{\imath}_{H,t}'' + (1-\delta_H)\tilde{h}_{t-1}'' \Rightarrow \tilde{\imath}_{H,t}'' = \frac{1}{\delta_H}\big(\tilde{h}_t'' - (1-\delta_H)\tilde{h}_{t-1}''\big)$$

$$\tilde{\imath}_{H,t}'' - \tilde{h}_{t-1}'' = \frac{1}{\delta_H}\tilde{h}_t'' - \left(\frac{1}{\delta_H} - 1\right)\tilde{h}_{t-1}'' - \tilde{h}_{t-1}'' \Rightarrow \frac{1}{\delta_H}\big(\tilde{h}_t'' - \tilde{h}_{t-1}''\big)$$

$$\psi_H\big(\tilde{\imath}_{H,t}'' - \tilde{h}_{t-1}''\big) = \left(\frac{\psi_H}{\delta_H}\right)\big(\tilde{h}_t'' - \tilde{h}_{t-1}''\big)$$

From (5A.2)

$$\frac{Y_t}{X_t}\frac{P_{Y,t}}{P_{C,t}} + B_t = C_t + P_{H/C,t}I_{H,t} + \frac{R_{t-1}}{\pi_{C,t}}B_{t-1} + \frac{W_t'}{P_{C,t}}L_t' + \frac{W_t''}{P_{C,t}}L_t'' + I_{K,t} + A_{K,t} + A_{H,t}$$

$$\frac{Y_t}{X_t}S_t^{-\alpha} + B_t = C_t + P_{H/C,t}(H_t - [1-\delta_H]H_{t-1}) + \frac{R_{t-1}}{\pi_{C,t}}B_{t-1} + \frac{W_t'}{P_{C,t}}L_t' + \frac{W_t''}{P_{C,t}}L_t'' + I_{K,t} + \frac{\psi_K}{2\delta_K}\left(\frac{I_{K,t}}{K_{t-1}} - \delta_K\right)^2 K_{t-1} + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{I_{H,t}}{H_{t-1}} - \delta_K\right)^2 H_{t-1}$$

Stationarisation:

$$\frac{\hat{Y}_t}{X_t}S_t^{-\alpha} + \hat{B}_t = \hat{C}_t + P_{H/C,t}\left(\hat{H}_t - [1-\delta_H]\hat{H}_{t-1}\left[\frac{A_{t-1}}{A_t}\right]\right) + \frac{R_{t-1}}{\pi_{C,t}}\hat{B}_{t-1}\left[\frac{A_{t-1}}{A_t}\right] + \frac{W_t'}{P_{C,t}}L_t' + \frac{W_t''}{P_{C,t}}L_t'' + \hat{I}_{K,t} + \frac{\psi_K}{2\delta_K}\left(\frac{\hat{I}_{K,t}}{\hat{K}_{t-1}}\left[\frac{A_t}{A_{t-1}}\right] - \delta_K\right)^2 \hat{K}_{t-1}\left[\frac{A_{t-1}}{A_t}\right] + \frac{\psi_H P_{H/C}}{2\delta_H}\left(\frac{\hat{I}_{H,t}}{\hat{H}_{t-1}}\left[\frac{A_t}{A_{t-1}}\right] - \delta_K\right)^2 \hat{H}_{t-1}\left[\frac{A_{t-1}}{A_t}\right]$$

Log-Linearisation:

$$\frac{Y}{X}S^{-\alpha}(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + B\tilde{b}_t = C\tilde{c}_t + P_{H/C}H(\tilde{p}_{H/C,t} + \tilde{h}_t) - (1-\delta_H)P_{H/C}H(\tilde{p}_{H/C,t} + \tilde{h}_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t) + \frac{RB}{\pi_C}(\tilde{r}_{t-1} + \tilde{b}_{t-1} - \tilde{\pi}_{C,t} + \tilde{a}_{t-1} - \tilde{a}_t) + \frac{W'}{P_C}L'(\tilde{w}_t' + \tilde{L}_t' - \tilde{p}_{C,t}) + \frac{W''}{P_C}L''(\tilde{w}_t'' + \tilde{L}_t'' - \tilde{p}_{C,t}) + I_K\tilde{\imath}_{K,t} + \frac{\psi_K}{2\delta_K}\frac{(I_{K,t})^2}{K}(2\tilde{\imath}_{K,t} - \tilde{k}_{t-1} + \tilde{a}_t - \tilde{a}_{t-1}) - \psi_K I_K(\tilde{\imath}_{K,t}) + \frac{\psi_K}{2}\delta_K K(\tilde{k}_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t) + \frac{\psi_H P_{H/C}}{2\delta_H}\frac{(I_{H,t})^2}{H}(2\tilde{\imath}_{H,t} - \tilde{h}_{t-1} + \tilde{a}_t - \tilde{a}_{t-1}) - \psi_H I_H P_{H/C}(\tilde{\imath}_{H,t}) + \frac{\psi_H P_{H/C}}{2}\delta_H H(\tilde{h}_{t-1} + \tilde{a}_{t-1} - \tilde{a}_t)$$

Recall:

$$\frac{W'}{P_C}L' = S'Y - F = \left(S' - \left(1 - \frac{1}{X}\right)\right)Y$$

$$\frac{W''}{P_C}L'' = S''Y$$

$$\tilde{w}_t' + \tilde{L}_t' - \tilde{p}_{C,t} = \tilde{w}_t'' + \tilde{L}_t'' - \tilde{p}_{C,t} = \tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t$$

Therefore:

$$\frac{Y}{X}(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + B\tilde{b}_t = C\tilde{c}_t + P_{H/C}H(\tilde{h}_t - (1-\delta_H)\tilde{h}_{t-1}) + P_{H/C}H(\tilde{p}_{H/C,t} - (1-\delta_H)\tilde{p}_{H/C,t}) + (1-\delta_H)P_{H/C}Hz_t + RB(\tilde{r}_{t-1} + \tilde{b}_{t-1} - \tilde{\pi}_{C,t}) - \frac{RB}{\pi_C}z_t + \left(S' - \left(1 - \frac{1}{X}\right)\right)Y(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + S''Y(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + I_K\tilde{\imath}_{K,t}$$

$$\frac{B}{Y}\tilde{b}_t = \frac{C}{Y}\tilde{c}_t + \frac{P_{H/C}H}{Y}\left(\tilde{h}_t - (1-\delta_H)\tilde{h}_{t-1}\right) + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{RB}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1} - \tilde{\pi}_{C,t}\right) - (1 - S' - S'')(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + \frac{P_{H/C}H}{Y}\delta_H\tilde{p}_{H/C,t} + \left[(1-\delta_H)\frac{P_{H/C}H}{Y} - \frac{RB}{Y}\right]z_t$$

5C.13

From (5A.9)

$$\Omega_{C,t} = \beta E_t\left(\frac{\Omega_{C,t+1}}{\Pi_{C,t+1}}\right)R_t + \lambda_t R_t \quad \Leftrightarrow \quad \frac{e^{\varepsilon_t^\beta}}{C_t} = \beta E_t\left(\frac{R_t}{\pi_{C,t+1}}\frac{e^{\varepsilon_{t+1}^\beta}}{C_{t+1}}\right) + \lambda_t R_t$$

Stationarisation:

$$\frac{e^{\varepsilon_t^\beta}}{\hat{C}_t} = \beta E_t\left(\frac{R_t}{\pi_{C,t+1}}\frac{e^{\varepsilon_{t+1}^\beta}}{\hat{C}_{t+1}}\left[\frac{A_t}{A_{t+1}}\right]\right) + \hat{\lambda}_t R_t$$

Log-linearisation (LL):

$$\frac{\varepsilon^\beta}{C}\left(\mathcal{E}_t^\beta - \tilde{c}_t\right) = \beta\left(\frac{R}{C}\right)\left(\tilde{r}_t + E_t\mathcal{E}_{t+1}^\beta - E_t\tilde{\pi}_{C,t+1} - E_t\tilde{c}_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1}\right) + \hat{\lambda}R\left(\tilde{\lambda}_t + \tilde{r}_t\right)$$

$$\beta'\tilde{c}_t = \beta E_t\tilde{c}_{t+1} - (\beta' - \beta)\tilde{\lambda}_t - \beta'\tilde{r}_t + \beta E_t\tilde{\pi}_{C,t+1} - \left(\beta\rho_\beta - \beta'\right)\mathcal{E}_t^\beta + \beta\rho_z z_t$$

5C.14

From (5A.19)

$$\Omega'_{\mathrm{C,t}} = \beta' E_t\left(\frac{\Omega'_{\mathrm{C,t+1}}}{\Pi_{\mathrm{C,t+1}}}\right)R_t \Leftrightarrow \frac{e^{\varepsilon_t^\beta}}{C'_t} = \beta' E_t\left(\frac{R_t}{\Pi_{\mathrm{C,t+1}}}\right)\left(\frac{e^{\varepsilon_{t+1}^\beta}}{C'_{t+1}}\right)$$

Stationarisation:

$$\frac{e^{\varepsilon_t^\beta}}{\hat{C}'_t} = \beta' E_t\left(\frac{R_t}{\Pi_{\mathrm{C,t+1}}}\right)\left(\frac{e^{\varepsilon_{t+1}^\beta}}{\hat{C}'_{t+1}}\left[\frac{A_t}{A_{t+1}}\right]\right)$$

Log-Linearisation (LL):

$$\frac{1}{C'}\left(\mathcal{E}_t^\beta - \tilde{c}'_t\right) = \beta'\left(\frac{R}{C'}\right)\left(\tilde{r}_t + E_t\mathcal{E}_{t+1}^\beta - E_t\tilde{\pi}_{C,t+1} - E_t\tilde{c}'_{t+1} + \tilde{a}_t - E_t\tilde{a}_{t+1}\right)$$

$$\tilde{c}'_t = E_t\tilde{c}'_{t+1} - \left(\tilde{r}_t - E_t\tilde{\pi}_{C,t+1}\right) + \left(1 - \rho_\beta\right)\mathcal{E}_t^\beta + \rho_z z_t \qquad 5C.15$$

From (5A.20)

$$\Omega'_{\mathrm{C,t}} = \beta' E_t\left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t}\frac{\Omega'_{\mathrm{C,t+1}}}{\Pi_{\mathrm{C,t+1}}}\right)R_t^* \Leftrightarrow \frac{e^{\varepsilon_t^\beta}}{C'_t} = \beta' E_t\left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t}\frac{R_t^*}{\Pi_{\mathrm{C,t+1}}}\right)\left(\frac{e^{\varepsilon_{t+1}^\beta}}{C'_{t+1}}\right)$$

Stationarisation:

$$\frac{e^{\mathcal{E}_t^{\beta}}}{C_t'} = \beta' E_t\left(\frac{\mathbb{E}_{t+1}}{\mathbb{E}_t}\frac{R_t^*}{\Pi_{\mathrm{C},t+1}}\left[\frac{A_t}{A_{t+1}}\right]\right)\left(\frac{e^{\mathcal{E}_{t+1}^{\beta}}}{C_{t+1}'}\right)$$

Log-Linearisation (LL):

$$\frac{1}{C'}\left(\mathcal{E}_t^{\beta} - \tilde{c}_t'\right) = \beta'\left(\frac{R^*}{C'}\right)\left(\tilde{r}_t^* + E_t\mathcal{E}_{t+1}^{\beta} - E_t\tilde{\pi}_{C,t+1} - E_t\tilde{c}_{t+1}' + E_t\tilde{e}_{t+1} - \tilde{e}_t + \tilde{a}_t - E_t\tilde{a}_{t+1}\right)$$

$$\tilde{c}_t' = E_t\tilde{c}_{t+1}' - \left(\tilde{r}_t^* - E_t\tilde{\pi}_{C,t+1}\right) + \left(1-\rho_{\beta}\right)\mathcal{E}_t^{\beta} - \Delta E_t\tilde{e}_{t+1} + \rho_z z_t \qquad \text{5C.16}$$

Combining (5C.15) and (5C.16), we have the following uncovered interest rate parity (UIP) condition:

$$\tilde{r}_t - \tilde{r}_t^* = \Delta E_t\tilde{e}_{t+1} \qquad \text{5C.17}$$

From (5A.28)

$$\Omega_{\mathrm{C,t}}'' = \beta'' E_t\left(\frac{\Omega_{\mathrm{C,t+1}}''}{\Pi_{\mathrm{C,t+1}}}\right)R_t + \lambda_t'' R_t \Leftrightarrow \frac{e^{\mathcal{E}_t^{\beta}}}{C_t''} = \beta'' E_t\left(\frac{R_t}{\pi_{C,t+1}}\frac{e^{\mathcal{E}_{t+1}^{\beta}}}{C_{t+1}''}\right) + \lambda_t'' R_t$$

Stationarisation:

$$\frac{e^{\mathcal{E}_t^{\beta}}}{C_t''} = \beta'' E_t\left(\frac{R_t}{\pi_{C,t+1}}\frac{e^{\mathcal{E}_{t+1}^{\beta}}}{C_{t+1}''}\left[\frac{A_t}{A_{t+1}}\right]\right) + \lambda_t'' R_t$$

Log-linearisation (LL):

$$\frac{1}{C''}\left(\mathcal{E}_t^{\beta} - \tilde{c}_t''\right) = \beta''\left(\frac{R}{\pi_C}\frac{1}{C''}\right)\left(\tilde{r}_t + E_t\mathcal{E}_{t+1}^{\beta} - E_t\tilde{\pi}_{C,t+1} - E_t\tilde{c}_{t+1}'' + \tilde{a}_t - E_t\tilde{a}_{t+1}\right) + \lambda'' R\left(\tilde{\lambda}_t'' + \tilde{r}_t\right)$$

$$\beta'\tilde{c}_t'' = \beta'' E_t\tilde{c}_{t+1}'' - (\beta' - \beta'')\tilde{\lambda}_t'' - \beta'\tilde{r}_t + \beta'' E_t\tilde{\pi}_{C,t+1} - \left(\beta''\rho_{\beta} - \beta'\right)\mathcal{E}_t^{\beta} + \beta''\rho_z z_t \qquad \text{5C.18}$$

From (5A.3)

$$(1-\chi)H_t E_t\left[P_{H/C,t+1}\pi_{C,t+1}\right]e^{\mathcal{E}_t^{LTV}} = R_t B_t$$

Log-linearisation (LL):

$$\tilde{b}_t = (1-\chi)\left(E_t\tilde{p}_{H/C,t+1} + \tilde{h}_t + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) - \tilde{r}_t \qquad \text{5C.19}$$

From (5A.25)

$$[1-\mu^b](1-\chi'')H_t^{"}E_t\big[P_{H/C,t+1}\pi_{C,t+1}\big]e^{\mathcal{E}_t^{LTV}}+[\mu^b]\frac{W_t''}{P_{C,t}}L_t''=R_tB_t''$$

Log-linearisation (LL):

$$[1-\mu^b](1-\chi'')P_{H/C}H''\big(E_t\tilde{p}_{H/C,t+1}+\tilde{h}_t''+E_t\tilde{\pi}_{C,t+1}+\mathcal{E}_t^{LTV}\big)+[\mu^b]\frac{W''L''}{P_C}\big(\widetilde{w}_t''+\tilde{L}_t''-\tilde{p}_{C,t}\big)=RB''\big(\tilde{r}_t+\tilde{b}_t''\big)$$

$$[1-\mu^b](1-\chi'')\phi_4\phi_6\big(E_t\tilde{p}_{H/C,t+1}+\tilde{h}_t''+E_t\tilde{\pi}_{C,t+1}+\mathcal{E}_t^{LTV}\big)+[\mu^b]s''\big(\widetilde{w}_t''+\tilde{L}_t''-\tilde{p}_{C,t}\big)=\frac{1}{\beta'}\left(\frac{B''}{Y}\right)\big(\tilde{r}_t+\tilde{b}_t''\big) \qquad 5C.20$$

Combine (5C.7) with (5C.20)

$$[1-\mu^b](1-\chi'')\phi_4\phi_6\big(E_t\tilde{p}_{H/C,t+1}+\tilde{h}_t''+E_t\tilde{\pi}_{C,t+1}+\mathcal{E}_t^{LTV}\big)+[\mu^b]s''(\tilde{y}_t-\tilde{x}_t-\alpha\tilde{s}_t)=\frac{1}{\beta'}\left(\frac{B''}{Y}\right)\big(\tilde{r}_t+\tilde{b}_t''\big) \qquad 5C.21$$

From (5A.4)

$$e^{\mathcal{E}_t^{I_K}}I_{K,t}=K_t-(1-\delta_K)K_{t-1}$$

Stationarisation:

$$e^{\mathcal{E}_t^{I_K}}\hat{I}_{K,t}=\widehat{K}_t-(1-\delta_K)\widehat{K}_{t-1}\left[\frac{A_{t-1}}{A_t}\right]$$

Log-linearisation (LL):

$$\tilde{\imath}_{K,t}=\frac{1}{\delta_K}\Big(\tilde{k}_t-(1-\delta_K)\big(\tilde{k}_{t-1}-z_t\big)\Big)-\mathcal{E}_t^{I_K} \qquad 5C.22$$

From (5A.68)

$$\breve{H}_t=H_t+H_t'+H_t''\equiv 1$$

Log-linearisation (LL):

$$0=H\tilde{h}_t+H'\tilde{h}_t'+H''\tilde{h}_t'' \qquad 5C.23$$

From (5A.50)

$$S_t = \frac{P_{F,t}}{P_{Y,t}}$$

Log-linearisation (LL):

$$\tilde{s}_t = \tilde{p}_{F,t} - \tilde{p}_{Y,t} \Leftrightarrow \Delta\tilde{s}_t = \tilde{\pi}_{F,t} - \tilde{\pi}_{Y,t} \qquad \text{5C.24}$$

From (5A.46)

$$P_{C,t} = \left[(1-\alpha){P_{Y,t}}^{1-\varpi} + \alpha {P_{F,t}}^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \Leftrightarrow {P_{C,t}}^{1-\varpi} = (1-\alpha){P_{Y,t}}^{1-\varpi} + \alpha {P_{F,t}}^{1-\varpi}$$

Log-linearisation (LL):

$${P_C}^{1-\varpi}(1-\varpi)\tilde{p}_{C,t} = (1-\alpha){P_Y}^{1-\varpi}(1-\varpi)\tilde{p}_{Y,t} + \alpha {P_F}^{1-\varpi}(1-\varpi)\tilde{p}_{F,t}$$

$$\tilde{p}_{C,t} = (1-\alpha)\tilde{p}_{Y,t} + \alpha\tilde{p}_{F,t} \Leftrightarrow \tilde{\pi}_{C,t} = (1-\alpha)\tilde{\pi}_{Y,t} + \alpha\tilde{\pi}_{F,t} \qquad \text{5C.25}$$

Substituting (5C.24) into (5C.25) for $\tilde{p}_{F,t}$ yields:

$$\tilde{\pi}_{C,t} = \tilde{\pi}_{Y,t} + \alpha\Delta\tilde{s}_t \qquad \text{5C.26}$$

From (5A.51)

$$\mathfrak{E}_{i,t} = \frac{\mathbb{E}_{i,t}P^i_{C,t}}{P_{C,t}}$$

Log-linearisation (LL):

$$\tilde{q}_{i,t} = \tilde{e}_{i,t} + \tilde{p}^*_t - \tilde{p}_{C,t}$$

$$\tilde{q}_t = \int_0^1 \tilde{q}_{i,t} di = \int_0^1 \tilde{e}_{i,t} + \tilde{p}^*_{C,t} - \tilde{p}_{C,t} di$$

$$\tilde{q}_t = \tilde{e}_t + \tilde{p}^*_t - \tilde{p}_{C,t} \qquad \text{5C.27}$$

To link the term trade and the real exchange rate, we combine (5C.24) and (5C.25) to get

$$\tilde{q}_t = \tilde{e}_t + \tilde{p}^*_t - \left((1-\alpha)\tilde{p}_{Y,t} + \alpha\tilde{p}_{F,t}\right)$$

$$\tilde{q}_t = \psi_F + (1-\alpha)\tilde{s}_t \qquad \text{5C.28}$$

where $\Delta\psi_{F,t} = \Delta\tilde{e}_t + \tilde{\pi}^*_t - \tilde{\pi}_{F,t}$

From (5A.53)

$$0 = E_0 \sum_{s=0}^{\infty} \theta_F^s \, Q_{t,t+s} \left(\frac{P_{F,t}^{New}}{P_{F,t+s}} - \left(\frac{\zeta}{\zeta - 1} \right) \frac{\mathbb{E}_{t+s} P_{F,t+s}^*}{P_{F,t+s}} \right) C_{F,t+s}$$

Log-linearisation

$$E_0 \sum_{s=0}^{\infty} (\beta' \theta_F)^S \left(\tilde{p}_{F,t}^{New} - \tilde{p}_{F,t+s} \right) = E_0 \sum_{s=0}^{\infty} (\beta' \theta_F)^S \left(\tilde{e}_{t+s} + \tilde{p}_{F,t+s}^* - \tilde{p}_{F,t+s} \right)$$

$$\tilde{p}_{F,t}^{New} = (1 - \beta' \theta_F) E_0 \sum_{s=0}^{\infty} (\beta' \theta_F)^S \left(\tilde{e}_{t+s} + \tilde{p}_{F,t+s}^* \right)$$

Following Monacelli (2005), the above optimality condition can be linked with LOOP gap $\psi_{F,t}$ by treating the rest of the world as an (approximately) closed economy (with goods produced in the small economy representing a negligible fraction of the world consumption baskets) which implies $\tilde{p}_{F,t}^* = \tilde{p}_t^*$ and $\tilde{\pi}_{F,t}^* = \tilde{\pi}_t^*$, for all t. Therefore, an equivalent between producer price index (CPI) and consumer price index (CPI) inflation holds in the world economy.

Using the above assumption, we can replace $\tilde{p}_{F,t+s}^*$ by $\tilde{p}_{t+s}^*$ such that we have the following relation:

$$\tilde{p}_{F,t}^{New} = (1 - \beta' \theta_F) E_0 \sum_{s=0}^{\infty} (\beta' \theta_F)^S \left(\tilde{p}_{F,t+s} + \psi_{F,t+s} \right)$$

The last equation can be cast in the recursive form.

$$\tilde{p}_{F,t}^{New} = (1 - \beta' \theta_F)\left(\tilde{p}_{F,t} + \psi_{F,t} \right) + \beta' \theta_F E_t \tilde{p}_{F,t+1}^{New} \qquad \text{5C.29}$$

From (5A.54)

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\zeta} + (1 - \theta_F)\left(P_{F,t}^{New} \right)^{1-\zeta} \right]^{1/(1-\zeta)}$$

Log-linearisation:

$$\tilde{p}_{F,t}^{New} = \left(\frac{1}{1 - \theta_F} \right) \tilde{p}_{F,t} - \left(\frac{\theta_F}{1 - \theta_F} \right) \tilde{p}_{F,t-1} \qquad \text{5C.30}$$

Substituting RHS of (5C.30) for $\tilde{p}_{F,t}^{New}$ and $\tilde{p}_{F,t+1}^{New}$ in (5C.29) yields

$$\tilde{\pi}_{F,t} = \beta' E_t \tilde{\pi}_{F,t+1} + \lambda_F \Psi_{F,t} \qquad \text{5C.31}$$

where: $\lambda_{F,t} = \frac{(1-\theta_F)(1-\beta'\theta_F)}{\theta_F}$

From (5A.58)

$C_t' = C_t'^{\,i}\mathfrak{E}_{i,t}\ \ \forall t$ given $\mathfrak{E}_{i,t} = \frac{\mathbb{E}_t P_{C,t}^i}{P_{C,t}}$

Log-linearising and integrating it over i, we get

$$\tilde{c}_t' = \tilde{y}_t^* + \tilde{q}_t \qquad 5C.32$$

From (5A.34)

$$0 = E_0 \sum_{s=0}^{\infty} \theta^s \mathcal{Q}_{t,t+s} \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}} - \left[\frac{\mu_t^r}{X_{t+s}}\right]\right) Y_{t+s}(z)$$

where $\mathcal{Q}_{t,t+s} = E_t \left(\frac{\Omega'_{C,t+s}}{\Omega'_{C,t}}\right)\left(\frac{P_{C,t}}{P_{C,t+s}}\right)$

$$E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \mathcal{Q}_{t,t+s} Y_{t+s}(z) \left(\frac{P_{Y,t}^{New}}{P_{Y,t+s}}\right) = E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \mathcal{Q}_{t,t+s} \left(\frac{\mu_{t+s}^r}{X_{t+s}}\right) Y_{t+s}(z)$$

Log-linearisation (LL):

$$E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\frac{P_Y^{New}}{P_Y}\right)\left(\tilde{p}_{Y,t}^{New} - \tilde{p}_{Y,t+s}\right) = E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\frac{\mu^r}{X}\right)\left(\tilde{\mu}_{t+s}^r - \tilde{x}_{t+s}\right)$$

$$E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\tilde{p}_{Y,t}^{New} - \tilde{p}_{Y,t+s}\right) = E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\tilde{\mu}_{t+s}^r - \tilde{x}_{t+s}\right)$$

So, combining LHS and RHS yields

$$E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s\, \tilde{p}_{Y,t}^{New} = E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\tilde{p}_{Y,t+s} - \tilde{x}_{t+s} + \tilde{\mu}_{t+s}^r\right)$$

$$\tilde{p}_{Y,t}^{New} = (1-\beta'\theta) E_0 \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\tilde{p}_{Y,t+s} - \tilde{x}_{t+s} + \tilde{\mu}_{t+s}^r\right)$$

The last equation can be cast in the recursive form

$$\tilde{p}_{Y,t}^{New} = (1-\beta'\theta)\left(\tilde{p}_{Y,t} - \tilde{x}_t + \tilde{\mu}_t^r\right) + (1-\beta'\theta)\beta'\theta \sum_{s=0}^{\infty} (\beta'\theta)^s \left(\tilde{p}_{Y,t+s+1} - \tilde{x}_{t+s+1} + \tilde{\mu}_{t+s+1}^r\right)$$

$$\tilde{p}_{Y,t}^{New} = (1-\beta'\theta)\left(\tilde{p}_{Y,t} - \tilde{x}_t + \tilde{\mu}_t^r\right) + \beta'\theta E_t \tilde{p}_{Y,t+1}^{New} \qquad 5C.33$$

Now, small digression has to be done in order to eliminate term $\tilde{p}_{Y,t}^{New}$ and $\tilde{p}_{Y,t+1}^{New}$

From (5A.35)

$$P_{Y,t}^{-\left[\frac{1}{\mu^r - 1}\right]} = \theta P_{Y,t-1}^{-\left[\frac{1}{\mu^r - 1}\right]} + (1-\theta)\left(P_{Y,t}^{New}\right)^{-\left[\frac{1}{\mu^r - 1}\right]}$$

Log-linearisation (LL):

$$P_Y{}^{-\left[\frac{1}{\mu^r-1}\right]}\left(-\left[\frac{1}{\mu^r-1}\right]\tilde{p}_{Y,t}\right) = \theta P_Y{}^{-\left[\frac{1}{\mu^r-1}\right]}\left(-\left[\frac{1}{\mu^r-1}\right]\tilde{p}_{Y,t-1}\right) + (1-\theta)P_Y^{New}{}^{-\left[\frac{1}{\mu^r-1}\right]}\left(-\left[\frac{1}{\mu^r-1}\right]\tilde{p}_{Y,t}^{New}\right)$$

$$\tilde{p}_{Y,t}^{New} = \left(\frac{1}{1-\theta}\right)\left(\tilde{p}_{Y,t} - \theta\tilde{p}_{Y,t-1}\right) \qquad \text{5C.34}$$

Substituting Eq. (5C.34) into Eq. (5C.33) yields

$$\left(\frac{1}{1-\theta}\right)\left(\tilde{p}_{Y,t} - \theta\tilde{p}_{Y,t-1}\right) = (1-\beta'\theta)\left(\tilde{p}_{Y,t} - \tilde{x}_t + \tilde{\mu}_t^r\right) + \beta'\theta\left(\frac{1}{1-\theta}\right)\left(E_t\tilde{p}_{Y,t+1} - \theta\tilde{p}_{Y,t}\right)$$

Thus, the above equation can be written as

$$\tilde{p}_{Y,t} - \theta\tilde{p}_{Y,t-1} = (1-\beta'\theta)(1-\theta)\left(\tilde{p}_{Y,t} - \tilde{x}_t + \tilde{\mu}_t^r\right) + \beta'\theta\left(E_t\tilde{p}_{Y,t+1} - \theta\tilde{p}_{Y,t}\right)$$

$$\tilde{\pi}_{Y,t} = \beta' E_t\tilde{\pi}_{Y,t+1} - \kappa\tilde{x}_t + \kappa\tilde{\mu}_t^r \qquad \text{5C.35}$$

where: $\kappa = \frac{(1-\theta)(1-\beta'\theta)}{\theta}$

From (5A.62)

$$(1-\iota_t)Y_t = (1-\alpha)S_t{}^{\alpha\varpi}\breve{C}_t + \alpha S_t{}^{\alpha\varpi}\int_0^1\left(S_{i,t}S_t^i\right)^{\zeta-\varpi}\mathfrak{E}_{i,t}{}^{\varpi}C_t'^{\,i}di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

$$(1-\iota_t)Y_t = (1-\alpha)S_t{}^{\alpha\varpi}(C_t + C_t' + C_t'') + \alpha S_t{}^{\alpha\varpi}\int_0^1\left(S_{i,t}S_t^i\right)^{\zeta-\varpi}\mathfrak{E}_{i,t}{}^{\varpi}C_t'^{\,i}di + I_{K,t} + A_{K,t} + \breve{A}_{H,t}$$

Log-linearisation (LL):

$$Y(\tilde{y}_t) = (1-\alpha)S^{\alpha\varpi}\left(C\left(\alpha\varpi\tilde{s}_t + \tilde{c}_t + \tilde{p}_{H,t} - \tilde{p}_{Y,t}\right) + C'\left(\alpha\varpi\tilde{s}_t + \tilde{c}_t' + \tilde{p}_{H,t} - \tilde{p}_{Y,t}\right) + C''\left(\alpha\varpi\tilde{s}_t + +\tilde{c}_t'' + \tilde{p}_{H,t} - \tilde{p}_{Y,t}\right)\right) + \alpha C'S^{\alpha\varpi+2(\zeta-\varpi)}\mathfrak{E}^{\varpi}\left(\int_0^1\tilde{c}_t'^{\,i}\,di + \alpha\varpi\tilde{s}_t + (\zeta-\varpi)\int_0^1\tilde{s}_{i,t}di + (\zeta-\varpi)\int_0^1\tilde{s}_t^i di + \varpi\int_0^1\tilde{q}_{i,t}di + \tilde{p}_{H,t} - \tilde{p}_{Y,t}\right) + I_K\tilde{\iota}_{K,t} + Gg_t$$

Recall $log(1-\iota_t) = -g_t$ as Gali 2003

$$Y(\tilde{y}_t) = (1-\alpha)\alpha\varpi\breve{C}\tilde{s}_t + (1-\alpha)(C\tilde{c}_t + C'\tilde{c}_t' + C''\tilde{c}_t'') + \alpha C'\left(\int_0^1\tilde{c}_t'^{\,i}\,di + \alpha\varpi\tilde{s}_t + (\zeta-\varpi)\int_0^1\tilde{s}_{i,t}di + (\zeta-\varpi)\int_0^1\tilde{s}_t^i di + \varpi\int_0^1\tilde{q}_{i,t}di\right) + I_K\tilde{\iota}_{K,t} + Gg_t$$

$$\tilde{y}_t = (1-\alpha)\left(\frac{C}{Y}\tilde{c}_t + \frac{C'}{Y}\tilde{c}_t' + \frac{C''}{Y}\tilde{c}_t''\right) + \alpha\frac{C'}{Y}\tilde{y}_t^* + \left\{(1-\alpha)\alpha\varpi\frac{\breve{C}}{Y} + \alpha\frac{C'}{Y}[\alpha\varpi + \zeta - \varpi]\right\}\tilde{s}_t + \alpha\frac{C'}{Y}\varpi\tilde{q}_t + \left(\frac{I_K}{Y}\right)\tilde{\iota}_{K,t} + \frac{G}{Y}g_t$$

$$\tilde{y}_t = (1-\alpha)\left(\frac{C}{Y}\tilde{c}_t + \frac{C'}{Y}\tilde{c}'_t + \frac{C''}{Y}\tilde{c}''_t\right) + \alpha\frac{C'}{Y}\tilde{y}^*_t + \left\{(1-\alpha)\alpha\varpi\frac{\breve{C}}{Y} + \alpha\frac{C'}{Y}[\alpha\varpi + \zeta - \varpi]\right\}\tilde{s}_t + \alpha\frac{C'}{Y}\varpi\tilde{q}_t + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{G}{Y}g_t \quad \text{5C.36}$$

From (5A.64)

$$\breve{C}_t = C_t + C'_t + C''_t$$

In steady state (SS):

$$\breve{C} = C + C' + C''$$

Log-Linearisation (LL):

$$\breve{C}\tilde{c}^T_t = C\tilde{c}_t + C'\tilde{c}'_t + C''\tilde{c}''_t$$

$$\tilde{c}^T_t = \frac{Y}{\breve{C}}(\phi_5\tilde{c}_t + \phi_7\tilde{c}'_t + \phi_6\tilde{c}''_t)$$

$$\tilde{c}^T_t = \frac{1}{\phi_5+\phi_7+\phi_6}(\phi_5\tilde{c}_t + \phi_7\tilde{c}'_t + \phi_6\tilde{c}''_t) \quad \text{5C.37}$$

D. Model Reduced Forms (Pass Through Incomplete Exchange Rate)

$$\tilde{y}_t = (1-\alpha)\left(\frac{C}{Y}\tilde{c}_t + \frac{C'}{Y}\tilde{c}'_t + \frac{C''}{Y}\tilde{c}''_t\right) + \alpha\frac{C'}{Y}\tilde{y}^*_t + \left\{(1-\alpha)\alpha\varpi\frac{\breve{C}}{Y} + \alpha\frac{C'}{Y}[\alpha\varpi + \zeta - \varpi]\right\}\tilde{s}_t + \alpha\frac{C'}{Y}\varpi\tilde{q}_t + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{G}{Y}g_t \quad \text{5D.1}$$

$$\tilde{c}^T_t = \frac{1}{\phi_5+\phi_7+\phi_6}(\phi_5\tilde{c}_t + \phi_7\tilde{c}'_t + \phi_6\tilde{c}''_t) \quad \text{5D.2}$$

$$\tilde{c}'_t = \tilde{c}'_{t+1} - \left(\tilde{r}_t - E_t\tilde{\pi}_{C,t+1}\right) + \left(1-\rho_\beta\right)\mathcal{E}^\beta_t + \rho_z z_t \quad \text{5D.3}$$

$$\tilde{\imath}_{K,t} = \tilde{k}_{t-1} + \beta\left(E_t\tilde{\imath}_{K,t+1} - \tilde{k}_t\right) + \frac{1-\beta(1-\delta_K)}{\psi_K}\left(E_t\tilde{y}_{t+1} - \alpha E_t\tilde{s}_{t+1} - \tilde{k}_t - E_t\tilde{x}_{t+1}\right) + \frac{1}{\psi_K}\{\tilde{c}_t - E_t\tilde{c}_{t+1}\} - \left(\frac{1-\rho_\beta}{\psi_K}\right)\mathcal{E}^\beta_t + \frac{1}{\psi_K}\left[1-\beta(1-\delta_K)\rho_q\right]\mathcal{E}^{I_K}_t - \left(\rho_z\left[\frac{\beta(1-\delta_K)}{\psi_K} - \beta\right] + 1\right)z_t \quad \text{5D.4}$$

$$\beta'\tilde{c}_t = \beta E_t\tilde{c}_{t+1} - (\beta'-\beta)\tilde{\lambda}_t - \beta'\tilde{r}_t + \beta E_t\tilde{\pi}_{C,t+1} - \left(\beta\rho_\beta - \beta'\right)\mathcal{E}^\beta_t + \beta\rho_z z_t \quad \text{5D.5}$$

$$\beta'\tilde{c}''_t = \beta'' E_t\tilde{c}''_{t+1} - (\beta'-\beta'')\tilde{\lambda}''_t - \beta'\tilde{r}_t + \beta'' E_t\tilde{\pi}_{C,t+1} - \left(\beta''\rho_\beta - \beta'\right)\mathcal{E}^\beta_t + \beta''\rho_z z_t \quad \text{5D.6}$$

$$\tilde{p}_{H/C,t} = \gamma_e E_t \tilde{p}_{H/C,t+1} + [1-\gamma_e]\left(E_t\tilde{y}_{t+1} - \alpha E_t \tilde{s}_{t+1} - \tilde{h}_t - E_t\tilde{x}_{t+1}\right) + m_e\left(\tilde{\lambda}_t + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV} + E_t\tilde{c}_{t+1}\right) + \tilde{c}_t - E_t\tilde{c}_{t+1} + \left(\frac{\psi_H}{\delta_H}\right)\left[\beta\left(E_t\tilde{h}_{t+1} - \tilde{h}_t\right) - \left(\tilde{h}_t - \tilde{h}_{t-1}\right)\right] + \left[(1-m_e)\rho_\beta - 1\right]\mathcal{E}_t^\beta - ([\beta(1-\delta_H) - \beta\psi_H]\rho_z + \psi_H)z_t \qquad \text{5D.7}$$

where: $\gamma_e = \beta(1-\delta_H) + m_e$ and $m_e = (1-\chi)(\beta' - \beta)$

$$\tilde{p}_{H/C,t} = \gamma_h E_t\tilde{p}_{H/C,t+1} + (1-\gamma_h)\left(\mathcal{E}_t^H - \tilde{h}_t''\right) + m_h\left(\tilde{\lambda}_t'' + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) + \tilde{c}_t'' - \beta''(1-\delta_H)E_t\tilde{c}_{t+1}'' + \frac{\psi_H}{\delta_H}\left\{\beta''\left(E_t\tilde{h}_{t+1}'' - \tilde{h}_t''\right) - \left(\tilde{h}_t'' - \tilde{h}_{t-1}''\right)\right\} - \left[\beta''(1-\delta_H)\left(1-\rho_\beta\right) + m_h\right]\mathcal{E}_t^\beta - (\beta''[(1-\delta_H) - \psi_H]\rho_z + \psi_H)z_t \qquad \text{5D.8}$$

where: $\gamma_h = \beta''(1-\delta_H) + m_h$ and $m_h = [1-\mu^b](1-\chi'')(\beta' - \beta'')$

$$\tilde{p}_{H/C,t} = \beta'(1-\delta_H)E_t\tilde{p}_{H/C,t+1} + \left(1-\beta'(1-\delta_H)\right)\left(\mathcal{E}_t^H - \tilde{h}_t'\right) + \tilde{c}_t' - \beta'(1-\delta_H)E_t\tilde{c}_{t+1}' - \frac{\psi_H}{\delta_H}\left\{\left(\tilde{h}_t' - \tilde{h}_{t-1}'\right) - \beta'\left(E_t\tilde{h}_{t+1}' - \tilde{h}_t'\right)\right\} + \beta'(1-\delta_H)\left(\rho_\beta - 1\right)\mathcal{E}_t^\beta - (\beta'[(1-\delta_H) - \psi_H]\rho_z + \psi_H)z_t \qquad \text{5D.9}$$

$$0 = H\tilde{h}_t + H'\tilde{h}_t' + H''\tilde{h}_t'' \qquad \text{5D.10}$$

where: $H = \frac{\phi_2}{\phi_2+\phi_3\phi_7+\phi_4\phi_6}$; $H' = \frac{\phi_3\phi_7}{\phi_2+\phi_3\phi_7+\phi_4\phi_6}$ and $H'' = \frac{\phi_4\phi_6}{\phi_2+\phi_3\phi_7+\phi_4\phi_6}$

$$\tilde{b}_t = (1-\chi)\left(E_t\tilde{p}_{H/C,t+1} + \tilde{h}_t + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) - \tilde{r}_t \qquad \text{5D.11}$$

$$[1-\mu^b](1-\chi'')\phi_4\phi_6\left(E_t\tilde{p}_{H/C,t+1} + \tilde{h}_t'' + E_t\tilde{\pi}_{C,t+1} + \mathcal{E}_t^{LTV}\right) + [\mu^b]s''(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) = \frac{1}{\beta'}\left(\frac{B''}{Y}\right)\left(\tilde{r}_t + \tilde{b}_t''\right) \qquad \text{5D.12}$$

$$\tilde{y}_t = \Lambda_t + \mu\tilde{k}_{t-1} + \nu\tilde{h}_{t-1} + \sigma(1-\mu-\nu)\tilde{l}_t' + (1-\sigma)(1-\mu-\nu)\tilde{l}_t'' - [\mu+v]z_t \qquad \text{5D.13}$$

$$\tilde{y}_t = \tilde{x}_t + \eta\tilde{l}_t' + \tilde{c}_t' + \alpha\tilde{s}_t \qquad \text{5D.14}$$

$$(1+m_b)\left(\tilde{y}_t - \tilde{x}_t - \tilde{l}_t'' - \alpha\tilde{s}_t\right) + m_b\tilde{\lambda}_t'' - \tilde{c}_t'' = (1+m_b)(\eta-1)\tilde{l}_t'' + m_b\mathcal{E}_t^\beta \qquad \text{5D.15}$$

where: $m_b = \mu^b(\beta' - \beta'')$

$$\tilde{\pi}_{Y,t} = \beta' E_t\tilde{\pi}_{Y,t+1} - \kappa\tilde{x}_t + \kappa\tilde{\mu}_t^r \qquad \text{5D.16}$$

where: $\kappa = \frac{(1-\theta)(1-\beta'\theta)}{\theta}$

$$\tilde{\imath}_{K,t} = \frac{1}{\delta_K}\left(\tilde{k}_t - (1-\delta_K)\left[\tilde{k}_{t-1} - z_t\right]\right) - \mathcal{E}_t^{I_K} \qquad \text{5D.17}$$

$$\frac{B}{Y}\tilde{b}_t = \frac{C}{Y}\tilde{c}_t + \frac{P_{H/C}H}{Y}\left(\tilde{h}_t - (1-\delta_H)\tilde{h}_{t-1}\right) + \frac{I_K}{Y}\tilde{\imath}_{K,t} + \frac{RB}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1} - \tilde{\pi}_{C,t}\right) - (1 - S' - S'')(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + \frac{P_{H/C}H}{Y}\delta_H\tilde{p}_{H/C,t} + \left[(1-\delta_H)\frac{P_{H/C}H}{Y} - \frac{RB}{Y}\right]z_t \qquad \text{5D.18}$$

$$\frac{B''}{Y}\tilde{b}_t'' = \frac{C''}{Y}(\tilde{c}_t'') + \frac{P_{H/C}H''}{Y}\left(\tilde{h}_t'' - (1-\delta_H)\tilde{h}_{t-1}''\right) + \frac{RB''}{Y}\left(\tilde{r}_{t-1} + \tilde{b}_{t-1}'' - \tilde{\pi}_{C,t}\right) + \frac{P_{H/C}H''}{Y}\delta_H\tilde{p}_{H/C,t} - S''(\tilde{y}_t - \tilde{x}_t - \alpha\tilde{s}_t) + \left[(1-\delta_H)P_{H/C}H'' - \frac{RB''}{\pi_C}\right]z_t \qquad \text{5D.19}$$

$$\tilde{\pi}_{C,t} = \tilde{\pi}_{Y,t} + \alpha\Delta\tilde{s}_t \qquad \text{5D.20}$$

$$\tilde{q}_t = \psi_F + (1-\alpha)\tilde{s}_t \qquad \text{5D.21}$$

$$\Delta\psi_{F,t} = \Delta\tilde{e}_t + \tilde{\pi}_t^* - \tilde{\pi}_{F,t} \qquad \text{5D.22}$$

$$\tilde{\pi}_{F,t} = \beta' E_t\tilde{\pi}_{F,t+1} + \lambda_F\Psi_{F,t} \qquad \text{5D.23}$$

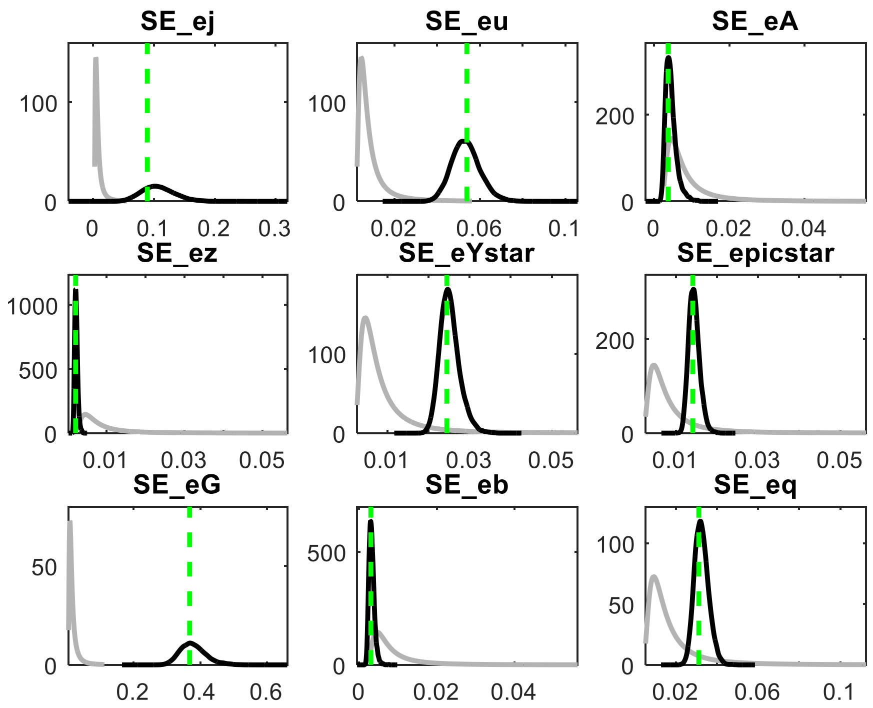
where: $\lambda_{F,t} = \frac{(1-\theta_F)(1-\beta'\theta_F)}{\theta_F}$

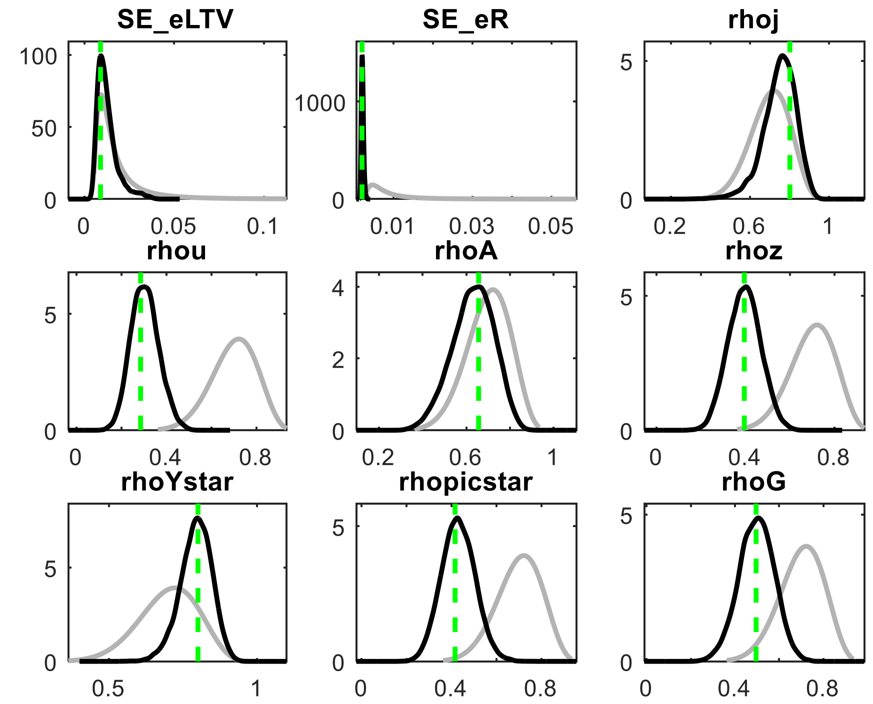
$$\tilde{c}_t' = \tilde{y}_t^* + \tilde{q}_t \qquad \text{5D.24}$$

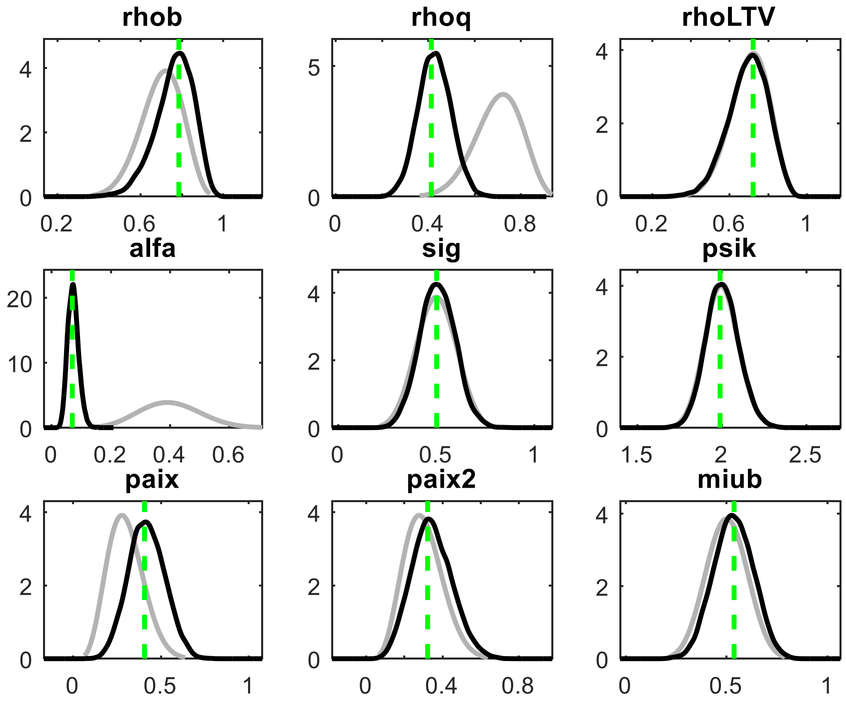
$$\tilde{r}_t = (1-\phi_r)\left(\phi_y\tilde{y}_t + \phi_\pi\tilde{\pi}_{y,t} + \phi_e\Delta\tilde{e}_t\right) + \phi_r\tilde{r}_{t-1} + \epsilon_t^r \qquad \text{5D.25}$$

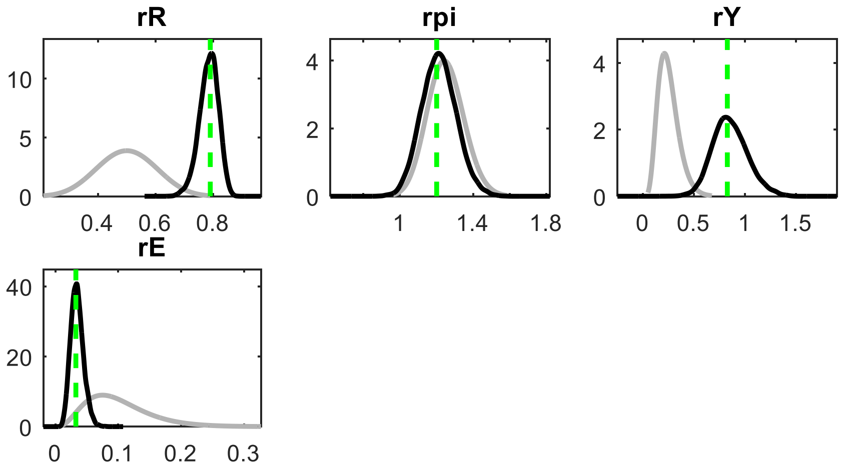
Appendix 5.3. Estimation Results: The Distribution of Priors and Posteriors of the Estimated Parameters

(light line curve is prior distribution, bold line curve is posterior distribution)



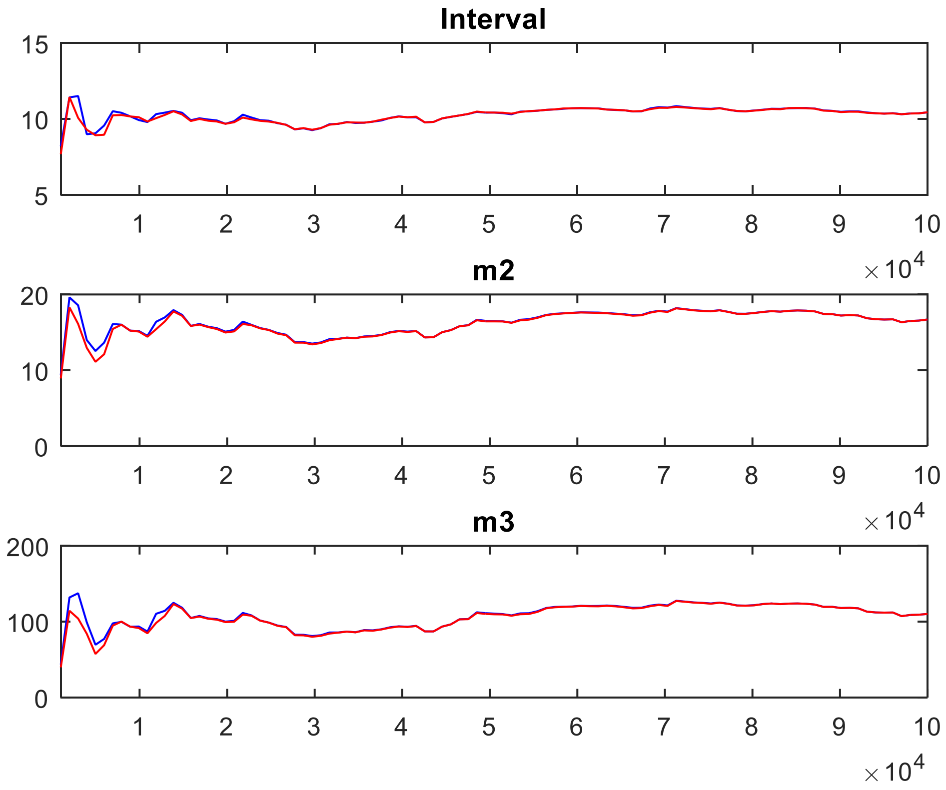



rhob
rhoq
rhoLTV
alfa
sig
psik
paix
paix2
miub
4
2
0
0.2
0.6
1
5
0
0
0.4
0.8
4
2
0
0.2
0.6
1
20
10
0
0
0.2
0.4
0.6
4
2
0
0
0.5
1
4
2
0
1.5
2
2.5
4
2
0
0
0.5
1
4
2
0
0
0.4
0.8
4
2
0
0
0.5
1

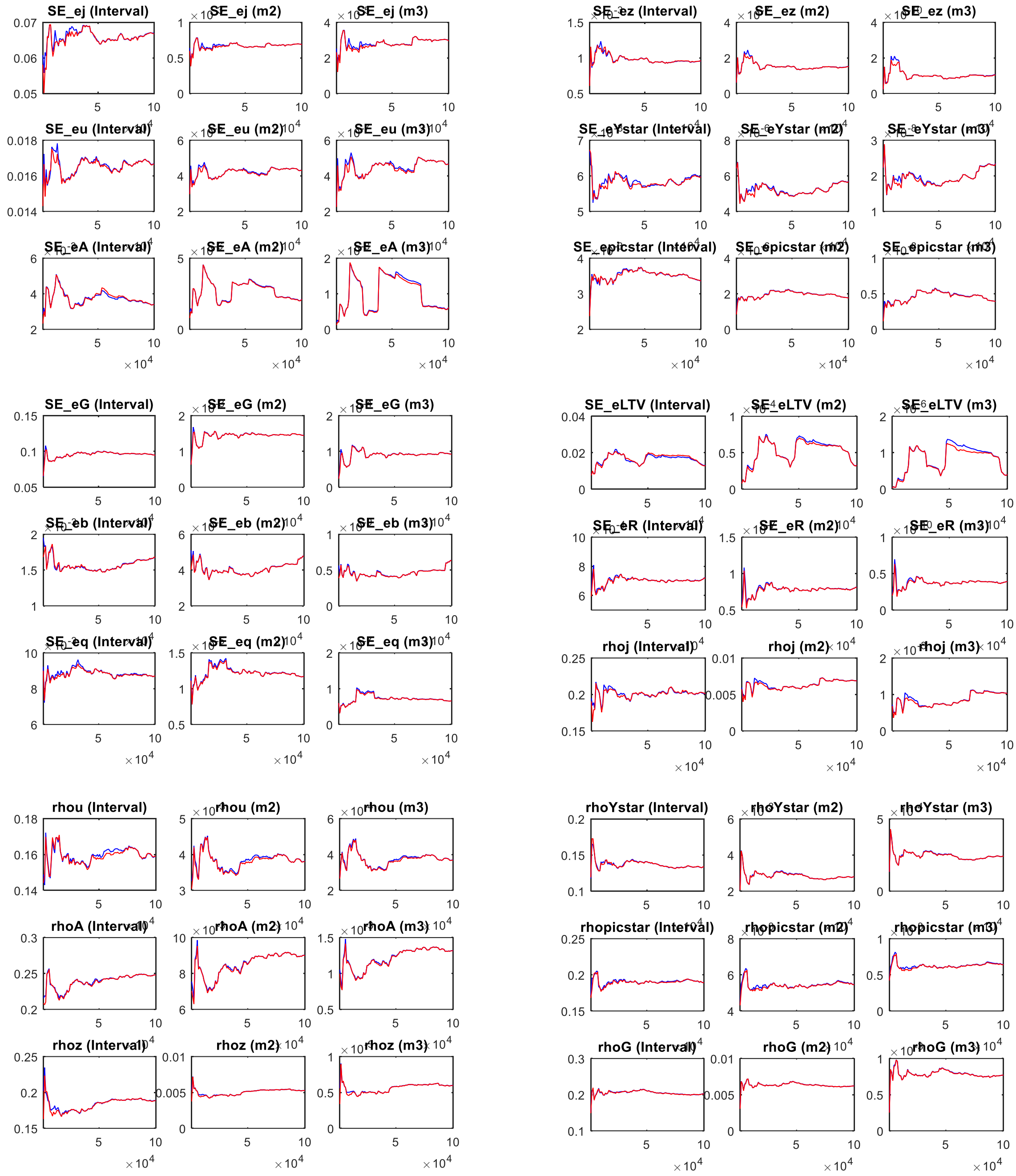

rR
rpi
rY
rE
10
5
0
0.4
0.6
0.8
4
2
0
1
1.4
1.8
4
2
0
0
0.5
1
1.5
40
20
0
0
0.1
0.2
0.3

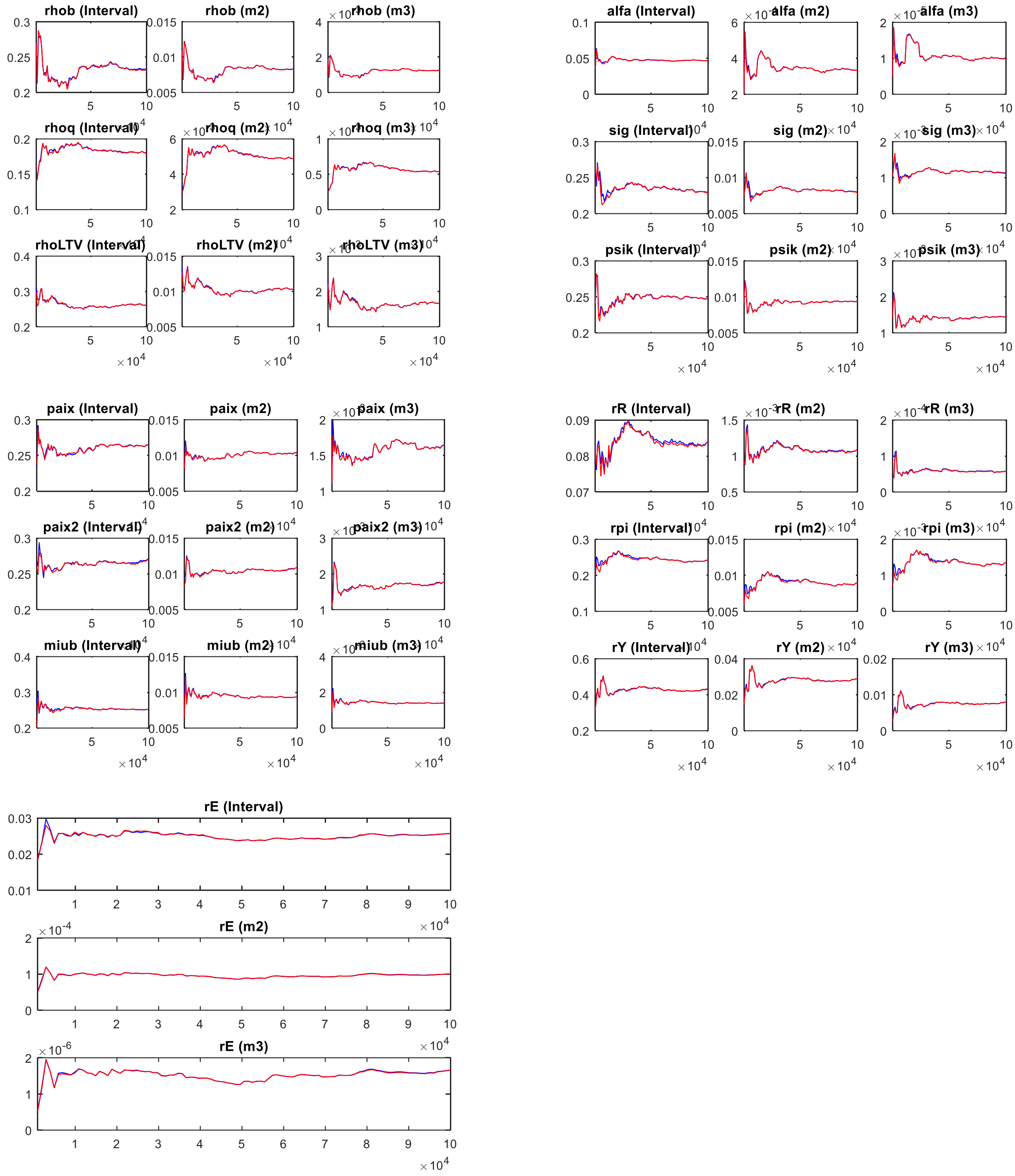
Appendix 5.4. Markov Chain Monte Carlo (MCMC) Diagnostic Check

- **Multivariate Convergence Diagnostic**



- **Univariate Convergence Diagnostic**




rhob (Interval)
rhob (m2)
rhob (m3)
rhoq (Interval)
rhoq (m2)
rhoq (m3)
rhoLTV (Interval)
rhoLTV (m2)
rhoLTV (m3)
alfa (Interval)
alfa (m2)
alfa (m3)
sig (Interval)
sig (m2)
sig (m3)
psik (Interval)
psik (m2)
psik (m3)
paix (Interval)
paix (m2)
paix (m3)
paix2 (Interval)
paix2 (m2)
paix2 (m3)
miub (Interval)
miub (m2)
miub (m3)
rR (Interval)
rR (m2)
rR (m3)
rpi (Interval)
rpi (m2)
rpi (m3)
rY (Interval)
rY (m2)
rY (m3)
rE (Interval)
rE (m2)
rE (m3)

Appendix 5.5. Moments and Correlation of Data Samples

- **Moments**

Data	Std. Deviation	Volatility*)
Output $\left(y_t^{obs}\right)$	2.46	1.00
Consumption $\left(c_t^{obs}\right)$	2.06	0.84
Capital investment $\left(i_{K,t}^{obs}\right)$	6.60	2.68
Interest rate $\left(r_t^{obs}\right)$	0.61	0.25
Inflation $\left(\pi_{C,t}^{obs}\right)$	0.80	0.33
Real housing prices $\left(q_t^{obs}\right)$	2.08	0.85

*) Volatility is measured as the ratio of std. deviation of particular variable over std. deviation of output.

- **Correlation**

Data	y_t^{obs}	c_t^{obs}	$i_{K,t}^{obs}$	r_t^{obs}	$\pi_{C,t}^{obs}$	q_t^{obs}
Output $\left(y_t^{obs}\right)$	1	0.32	-0.04	-0.48	0.03	-0.05
Consumption $\left(c_t^{obs}\right)$		1.00	0.14	-0.56	-0.17	0.11
Capital investment $\left(i_{K,t}^{obs}\right)$			1.00	-0.22	-0.18	0.02
Interest rate $\left(r_t^{obs}\right)$				1.00	0.22	-0.28
Inflation $\left(\pi_{C,t}^{obs}\right)$					1.00	-0.87
Real housing prices $\left(q_t^{obs}\right)$						1.00

Chapter 6

Concluding Remarks

The design of recovery and prevention policies has led to the discussion about the role of monetary and macroprudential policies. Prior to the US financial crisis, policy makers have assigned a strong role of monetary policy on price stabilisation. They believe that price stability is enough to ensure the economy goes to a favorable condition. Unfortunately, the recent US financial crisis has given a lesson that price stability is insufficient to ensure financial stability. A low and stable policy rate has caused asset prices, including housing prices, increases significantly that creates a bubble in the asset and the financial markets. This situation becomes more problematic as the financial system gets more complex and interconnect each other, especially through the mortgage market that causes the increase of systemic risk. Under such condition, the additional macro policy, i.e. macroprudential policy is required to avoid similar crisis in the future.

This thesis works on this ground of research interest. In particular, we examine the important of modelling agents' expectation in the design of preferred monetary and macroprudential policies. For this purpose, we model the expectation either under the assumption of boundedly rational (RLS and SG learning) or fully rational agents. Under the RLS learning algorithm, agents update their beliefs using an econometric algorithm to learn the REE as the sample size of the data set gets larger. The interest is whether this particular learning algorithm eventually takes the equilibrium path converges to a unique stationary REE. The E-stability principle states that the REE is asymptotically stable under RLS *iff* it is E-stable. Thus, in addition to uniqueness criterion, i.e. determinacy, we consider another criteria, i.e. E-stability in the design of preferred policy.

The study about economic policy design under RLS learning is begun in Chapter 2 by assuming macroprudential policy is absent in the model's structure while monetary policy operates in a conventional way by reacting to inflation and output. It is assumed that the real economy can be represented by a standard New Keynesian (NK) model with the feature of superficial habit in consumption. Our interest is how different degrees of habit persistent in consumption affects the choice of preferred monetary policy rule under this learning assumption. We can show that the presence of superficial habit in consumption does not only increase determinacy but also E-stability. An increase in determinacy happens as aggregate consumption is smoother that causes a particular response of policy rate to output that leads to indeterminacy, in an environment with no habit in consumption, now it leads to determinacy, in an environment where consumption habit matters. Meanwhile, an increase in E-stability occurs due to a relatively stable economic condition as a result of consumption smoothing behaviour. A more stable economic condition makes agents are easier to learn the equilibrium dynamic so that the equilibrium path does not deviate too much from a unique stationary REE.

As habit in consumption gets more persistent, a determinate and E-stable region, under plausible policy parameters, enlarges and collides regardless of the type of simple rules employed by central bank. This result has allowed us to claim that contemporaneous monetary policy rule is superior to other simple interest rate rules as it does not only lead the economy to a determinate and E-stable REE with a higher probability but also it is immune to parameter uncertainty (i.e. when the persistence of habit consumption is not easily observed). In our numerical exercise, we also consider a case where the cost channel of monetary transmission matters. It can be shown that the probability of getting a determinate and E-stable REE declines when this cost channel is present. This happens as monetary policy does not only affect the demand side of the economy but also its supply side with an opposite direction. This finding is valid not only for a case where consumption habit is less persistent but also when it is highly persistent. Yet, considering policy inertia in interest rate rule increases the probability of obtaining a determinate and E-stable REE.

The study about the design of monetary policy is then extended by evaluating the optimal constrained policy rule. This is motivated by the fact that there are many possible determinate and E-stable policy rules found when consumption habit is highly persistent. We can show that, in line with other studies, a decision to choose the optimal constrained policy relies on the assumption of exogenous shock that hits the economy. When a positive preference shock hits the economy, then responding to inflation and output are necessary to dampen economy volatility. In contrast, when a positive technology shock hits the economy, responding to output is not necessary as inflation and output move to an opposite direction. We can also show that considering policy inertia in monetary policy is useful for dampening the volatility of aggregate economy. Recall that the optimal policy rule is constrained in the sense that the chosen policy rule is bounded by determinacy and E-stability criteria.

Unfortunately, a standard NK model with one sector and one market of goods, i.e. consumption goods, as used in Chapter 2, is too simple. This causes the previous model is less realistic and unable to explain the movement of other economic variables as shown in the empirical data. Hence, we add a complexity in Chapter 3 by introducing durable goods, i.e. housing in addition to consumption goods. As well, we differentiate the households into two types, i.e. patient households (savers) and impatient households (borrowers) that allow us to incorporate macroprudential policy in the model’s structure. We choose the LTV ratio as our macroprudential instrument in which this ratio is assumed to be operational and used to manage the volatility in housing prices or the financial markets (credit). Meanwhile, monetary policy works in a standard way by responding to output and inflation.

In general, responding to the growth of housing prices via the changing of LTV ratio (LTV rule) is favourable than to the growth of credit. Yet, the benefit of responding to the growth of housing prices depends on the quality of this data itself and of the data used in monetary policy. When central bank has accessed to current data of inflation and output and uses them in monetary policy, then responding to the growth of housing prices via the LTV rule is irrelevant. In contrast, when current data of inflation and output are not used in monetary policy, then responding to current growth of housing prices via the LTV rule is beneficial. Under this condition, a stronger reaction of the LTV rule on current growth of housing prices enlarges the probability of central bank leads the economy to a determinate and E-stable REE. When current data of housing prices is also not available, then using forecasted housing prices growth in the LTV rule, in addition to forecasted data in monetary policy, is undesirable since it shrinks the region of determinate and E-stable policy rules.

As in Chapter 2, we also conduct the analysis of an optimal constrained policy rule in Chapter 3. It can be shown that a decision to choose the optimal constrained monetary and macroprudential policy rules depend on the assumption of shock that hits the economy and the type of housing prices data used in the LTV rule. When housing demand shock hits the economy, responding to current growth of housing prices via the LTV rule dampens the economic volatility while responding to forecasted growth of housing prices is not useful. This argument is valid for any types of standard simple interest rate rule adopted in monetary policy. In contrast, when technology shock hits the economy, a benefit of responding to the growth of housing prices via the LTV rule is ambiguous and should be taken with a caution.

Unfortunately, the concept of E-stability does not in general imply learnability of REE. This is true when economic agents slightly deviate from a standard RLS learning and use stochastic gradient (SG) learning for updating their belief. The SG learning algorithm is different from the RLS learning algorithm in which the former is less efficient from econometric approach but more realistic from the view of researchers. Technically, under the SG learning algorithm, economic agents are unable to observe the variance of state variables in which this second moment will be used in forming their forecast. In contrast, under the RLS learning algorithm, the variance of state variables is assumed to be known by agent precisely.

In Chapter 4, we conduct a refinement of learnability of REE in the context of New Keynesian (NK) model with housing market and financial constraint. To be specific, we extend our earlier work in Chapter 3 by incorporating other criteria in the design of preferred policy, i.e. SG-stability in addition to determinacy and E-stability criteria. We show that a refined criterion for learnability of REE has made central banks' task becomes not easy. The standard Taylor principle is insufficient to ensure a robust learnability of REE when the uncertainty in learning rule arises between the RLS and the SG learning rule. In general, it is required a strong response to inflation via monetary policy to ensure E-stable and SG-stable REE. Otherwise, central bank may take the economy to unexpected equilibrium e.g. indeterminate, E-unstable or SG-unstable REE.

In this chapter, we also claim that the quality of data remains matter in an environment where the uncertainty in learning rule arises. Yet, this argument does not mean that there is no room for improvement if current data is not available (current data has better quality than forecasted data). It can be shown that responding to forecasted growth of housing prices via the LTV rule is also beneficial when a bubble in housing prices is generated. Yet, this benefit is smaller compared to responding to current growth of housing prices growth. This is a nice finding since in reality having access to current data is not always so easy such that forecasted data is more realistically used by policy makers.

We continue the study about economic policy design under slightly different environments in Chapter 5. In this chapter, we deviate from a calibrated model and consider an estimated model in the analysis of monetary policy. We propose an alternative DSGE model for Indonesia by incorporating some relevant features. Following this, we estimate the model using a Bayesian technique. A Bayesian technique is chosen as it offers a formal way to estimate the parameters by combining prior information about the parameters and the data, conditional on the structure of the model. Note that we return to the framework of RE here as estimating the model using a Bayesian method while at the same time agents are assumed to be boundedly rational e.g. adaptive learning makes our work becomes computationally expensive.

In addition to the topic about monetary policy, we also provide the analysis with the study about banks' lending decisions, the degree of economic dependency and business cycles in Indonesia. From the estimation of monetary reaction function, we can show that the Taylor principle is consistently used by Bank Indonesia that makes price stability is among their main targets. We can also show that BI policy rate is occasionally used to promote economy growth, especially during economy down turn, but is rarely used to stabilise the variability of exchange rate. Concerning the issue about banks' lending decisions, it is suggested that banks in Indonesia are likely to set a higher ratio of down payment for households who have only housing assets in collateral than the ones who can show both her income and housing assets when making an application for housing loan. This happens as the wage income serves as an additional guarantor for banks. As for the degree of economic dependency, the estimation result suggests that the Indonesian economic dependency is not as high as predicted. This is reflected from the estimated share of import goods in consumption goods bundle that is less than 10%.

The analysis is then extended by evaluating the source of business cycles fluctuation. Using a result from the variance and historical decomposition, it can be shown that there are three main drivers in the Indonesian economy, i.e. non-stationary permanent technology shock, monetary policy shock and marginal efficiency of investment (MEI) shock. In regards to the variability of housing prices, the result suggests that a housing preference shock is the main disturbance of housing prices variation, followed by non-stationary technology shock. Surprisingly, the role of monetary policy shock and loan to value (LTV) shock is minimal in explaining the variability of housing prices. This exercise also advices that CPI inflation variation is dominantly contributed by cost push shock, followed by non-stationary permanent technology shock and the world inflation shock.

We also try to link this chapter with the broad topic in this thesis by addressing the issue of news in the formation of the agents' expectation. In particular, we introduce the feature of anticipated shocks, i.e. news shocks into the innovation of MEI, of monetary policy and of non-stationary permanent technology. We can show that introducing the feature of news shocks, regardless the way of how these anticipated shocks introduced to the model, either on MEI, monetary policy, or non-stationary permanent technology does not improve the empirical fit of the model to the data. This can be seen by comparing the value of log likelihood or log marginal data density (MDD) of different models in which a model with news shocks component has lower value of MDD compared to the baseline model, i.e. a model without news components. One possible explanation for this result is that the feature of news shock does not improve co-movement behaviour among aggregate variables. Having this, we argue that all the results, under a model without news shocks component, remain valid.

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