SPORADIC SIMPLE GROUPS OF LOW GENUS

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ABSTRACT

Let X be a compact connected Riemann surface of genus g and let $f : X \to \mathbb{P}^1$ be a meromorphic function of degree n. Classes of such covers are in one to one correspondence with the primitive systems, which are tuples of elements $(x_1, x_2, \cdots, x_r)$ in the symmetric group S_n taken up to conjugation and the action of the braid group, such that $x_1 \cdot x_2 \cdot \cdots \cdot x_r = 1$ and $G = \langle x_1, x_2, \cdots, x_r \rangle$ is a primitive subgroup G of S_n . This thesis is a contribution to the classification of primitive genus $g \leqslant 2$ systems of sporadic almost simple groups.

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CONTENTS

CHAPTER 1

INTRODUCTION

A connected second countable Hausdorff topological space X together with a complex structure is called a Riemann surface[27]. In other words, a Riemann surface is a one-dimensional complex manifold. Topologically, compact Riemann surfaces are homeomorphic to a sphere, a torus, or a finite number of tori joined together. The genus g of a compact Riemann surface X is defined to be number of tori which are joined together. Let $f : X \to \mathbb{P}^1$ be meromorphic function, that is, a holomorphic mapping from X to the Riemann sphere $\mathbb{P}^1$. The meromorphic function f is of degree n if the fiber $f^{-1}(p)$ for general $p \in \mathbb{P}^1$ is size n. A point $a \in \mathbb{P}^1$ is a branch point if $|f^{-1}(a)| < n$. Every meromorphic function f has finitely many branch points. Let $B = \{a_1, a_2, \cdots, a_r\}$ be the set of branch points of f and let $a_0 \in \mathbb{P}^1 \backslash B$. Denote by $\Pi_1 = \pi_1(\mathbb{P}^1 \backslash B, a_0)$, the fundamental group of $\mathbb{P}^1 \backslash B$ with the base point a_0. Let $\gamma_i \in \pi_1(\mathbb{P}^1 \backslash B, a_0)$ be corresponding to the path winding once around point a_i in the counter clockwise direction and not around any other branch points. The fundamental group Π_1 is generated by the homotopy classes of the closed paths γ_i. The homotopy lifting of paths induces an action on the fundamental group Π_1 on the fiber $f^{-1}(a_0)$ (see Section 2.1). This action gives us a homomorphism φ_f from the fundamental group Π_1 to the symmetric group S_n. The connectedness of the Riemann surface X yields that $\varphi_f(\Pi_1)$ is a transitive subgroup of S_n and this group is called the **monodromy** group of the ramified cover $f : X \to \mathbb{P}^1$ and is denoted by $\mathrm{Mon}(X, f)$. The generators $\{\gamma_1, \ldots, \gamma_r\}$ satisfy the relation $\gamma_1 \cdot \gamma_2 \cdot \ldots \cdot \gamma_r = 1$ and they are distinguished generators of the fundamental group Π_1. Thus the generators of the monodromy group $\{g_1, \ldots, g_r\}$ where

$g_i = \varphi_f(\gamma_i)$ will satisfy the same relation . Furthermore, the following statements are satisfied:

$$G = \langle g_1, \dots, g_r \rangle \tag{1}$$

$$g_1 \cdots g_r = 1, \tag{2}$$

$$\sum_{i=1}^{r} ind g_i = 2(n+g-1) \tag{3}$$

Equation 3 is called Riemann Hurwitz formula. Here $ind g_i = n -$number of orbits of (g_i) on $f^{-1}(a_0)$. Let $G =$Mon(X, f) and let $C_i = g_i^G$ be the conjugacy classes of G containing g_i. Then the set $C = \{C_1, \dots, C_r\}$ is the **ramification type** of f. In our study, we look at the set

$$N(C) = \{(g_1, \dots, g_r) : G = \langle g_1, \dots, g_r \rangle,\ \ g_i \cdots g_r = 1 \ \text{ and } \ g_i \in C_i \ \text{ for all i}\}$$

which is equivalent to the set of all monodromy homeomorphism and we call the elements of $N(C)$ Nielsen tuples. Let G be a transitive group of S_n. A genus g-system is a tuple $(g_1, \dots, g_r)$ satisfying (1), (2) and (3). If G acts primitively, then the genus g system is called a **primitive genus g system**.

We are more interested in when the meromorphic function f is indecomposable, that is, f can not written of the form $f = f_1 \circ f_2$ where degree of f_1 and f_2 more than one. Which yields the monodromy group Mon(X, f) acts primitively on fiber.

A natural question is : Which finite groups can be the monodromy groups Mon(X, f) for a fixed genus of X?

In 1990, Guralnick and Thompson [15] put forward the following conjecture: For any fixed non-negative integer g, there is a finite set $\mathscr{E}(g)$ of simple groups such that if X is a compact Riemann surfaces of genus g and $f : X \to \mathbb{P}^1$ is a meromorphic function, then the non-abelian composition factors of the mononodromy group Mon(X, f) are either alternating groups or members of $\mathscr{E}(g)$. This conjecture was established by Frohardt, Magaard [13]. As $\mathscr{E}(g)$ is finite, one would like to determine $\mathscr{E}(0)$, $\mathscr{E}(1)$ and $\mathscr{E}(2)$ explicitly. Moreover, it would be useful for future applications to determine all possibilities of how a group in $\mathscr{E}(g)$ can arise, as explicitly as possible. Let $\mathscr{E}(g)^* = (\bigcup_{(X,f)} cf(Mon(X,f)))$, it is well known that for all X, all prime p and all $n > 4$, $C_p \in \mathscr{E}(g)^*$ and $A_n \in \mathscr{E}(g)^*$. Indeed for each G which is either a C_p or A_n, there is a cover $\psi : \mathbb{P}^1 \to \mathbb{P}^1$ depending on G such that $Mon(X,f) \cong G$ (see [33, p.17]).

We call a group a genus g-group if $G = \mathrm{Mon}(X,f)$ for some (X,f) and $g(X) = g$.

Using Riemann's Existence Theorem, Guralnick and Thompson showed in [15] that the elements $\mathscr{E}(g)$ occur in a primitive monodromy action, i.e if $G \in \mathscr{E}(g)$, then $\exists (X,f)$ such that $G = \mathrm{Mon}(X,f)$ and G acts primitively on $f^{-1}(a_0)$. This fact brings the Theorem of Aschbacher and Scott into the picture.

The Fitting subgroup of the group G is denoted by $F(G)$ and it is defined to be the product of all nilpotent normal subgroups of the group G. It is the largest nilpotent normal subgroup of the group G. If a group H is perfect and $H/Z(H)$ is simple, then H is called a quasisimple. A subgroup H of the group G is called a component of G if H is a quasisimple subnormal subgroup of G. Moreover, $F^*(G) = F(G)E(G)$ is called general Fitting subgroup of the group G, where $E(G)$ is product of all components of G. Now, with these definitions we can formulate the important result of Aschbacher and Scott [1].

Theorem 1.0.1 (Aschbacher and Scott)**.** Let G be a finite group and K is a maximal subgroup of G such that $\cap_{g \in G} K^g = \{1\}$. Let P be a minimal normal subgroup of G and S be a minimal normal subgroup of P. Let $\Delta = \{S_1, \cdots, S_t\}$ be the set of G conjugates of S. Then $G = KP$ and exactly one of the following holds:

1. S of prime order
2. $F^*(G) = P \times R$ where $P \cong R$ and $K \cap P = \{1\}$
3. $F^*(G) = P$ is non-abelian, $K \cap P = \{1\}$
4. $F^*(G) = P$ is non-abelian, $K \cap P \neq \{1\} = K \cap S$
5. $F^*(G) = P$ and $K \cap P = K_1 \times \cdots \times K_t$ where $K_i = K \cap S_i \neq \{1\}, 1 \leqslant i \leqslant t$.

Shih [34] and Guralnick and Thompson [15] proved that there is no primitive genus zero systems in case 2 and 3 respectively of the Theorem 1.0.1. Aschbacher [2] studied case 4 and he showed that the general Fitting subgroup of G must be equal to $A_5 \times A_5$ in case of a genus zero system. case 5 was considered by Frohardt, Guralnick and Magaard [10] when S_i is of Lie type of rank 1. They proved that $t \leqslant 2$. Furthermore, they established that $[S_i : K] \leqslant 10000$, when S_i/K_i is point action, $t = 1$ and S_i is a classical group. It follows from this result and the results of Frohardt, Guralnick and Magaard [14] that once the actions of $[S_i : K] \leqslant 10000$, if S_i is a classical and $t = 1$. The first case of the Theorem 1.0.1 is the affine case when $F^*(G)$ is abelian group. It was first was considered by Guralnick and Thompson[15]. They proved that there are only finitely many primitive affine groups which are primitive group of genus zero and Neubauer in [32] studied and extended result to the genus one and two case. The analysis of the affine genus zero case was completed by Magaard, Shpectorov and Wang[25] they give a complete list of the primitive affine genus zero systems.In his PhD thesis Salih classified the affine primitive genus one and two systems[31]. In this thesis we are interested in final case of the Theorem 1.0.1. Our goal to determine all primitive genus zero, one and two systems for the almost simple groups G where $F^*(G)$ is a simple sporadic group.

We now briefly outline the contents of the thesis. In **Chapter 2** we review some basic background. This chapter is divided into five sections. In the first section we start with some basics in algebraic topology and collect facts on covering spaces and fundamental groups. As well, we introduce monodromy groups. Section two is devoted to the Riemann surfaces. we discuss the Riemann Existence Theorem, review the connections between the coverings of a Riemann surface and permutation groups. In section three we study the Hurwitz space $H_r^A(G)$ which is the moduli space of G-covers of the Riemann sphere $\mathbb{P}^1$, where $Inn(G) \leqslant A \leqslant Aut(G)$ and r is the number of branch points. If $A = Inn(G)$, we denote the Hurwitz space $H_r(G)^A$ by $H_r^{in}(G)$. We focus on the subset $H_r^{in}(C)$ of Hurwitz space $H_r^{in}(G)$, where C is a fixed ramification type.

Next we define Nielsen tuples. The base space O_r called configuration space, the space of branch point of f of cardinality r, is a topological space over $\mathbb{C}$. The Hurwitz space $H_r^A(G)$ is an unramified covering space of the base space O_r. As is well known, the fundamental group of the space O_r is the Artin braid group on r strands which is denoted by B_r. The braid group possesses the well known presentation on $r-1$ generators $\{Q_1, \cdots, Q_{r-1}\}$ satisfying the following relations

$$Q_iQ_{i+1}Q_i = Q_{i+1}Q_iQ_{i+1} \text{ for all } i < r-1 \tag{4}$$

and

$$Q_iQ_j = Q_jQ_i \text{ for all } i,j = 1,2,\cdots r-1 \text{ with } |i-j| \geqslant 1 \tag{5}$$

The group action of the braid group B_r on the fibers completely determines the connected components of Hurwitz space $H_r^A(G)$. In particular, the fiber of the subspace $H_r^{in}(C)$ of the Hurwitz space $H_r^{in}(G)$ are parametrized by the set $N(C)$ and the subgroup of the braid group that preserves the order of ramification type C which is defined by parabolic subgroup B. Thus the connected components $H_r^{in}(C)$ are parametrized by the $B-$orbits on the Nielsen classes $N(C)$.

Section four is devoted to describing primitive permutation groups. In the final section of this chapter we review general criteria for determining possible signatures. Our main results, Lemma 2.5.18, Lemma 2.5.19 and Lemma 2.5.20, give the complete classification of genus zero, one and two systems for all class of maximal subgroups of the group G of large index. Moreover, these lemmas together with Lemma 2.5.17 help us to prove that some sporadic simple groups are not genus zero, one and two groups.

Chapter 3 is divided into eleven sections. In the first section we explain some criteria to eliminate ramification types of sporadic simple groups. Section two, three, four and five devoted to ramification types of Mathieu groups, Janko groups, Conway groups and Higman-Sim groups, respectively, by using series of filters to reduce the set of possible ramification types. For instance, we show that the groups J_3, J_4, Co_2 and Co_1 possesses no primitive genus zero, one and two systems.

Now we will state the following Theorem:

Theorem 1.0.2. Let G be an almost simple group with $F^*(G)$ a sporadic simple group and $f : X \to \mathbb{P}^1$ be a meromorphic function where X is a compact Riemann surface of genus zero, one or two. Then G is a composition factor of $\mathrm{Mon}(X, f)$ if and only if G isomorphic to the group M_{11}, M_{12}, $M_{12}:2$, M_{22}, $M_{22}:2$, M_{23}, M_{24}, J_1, J_2, $J_2:2$, Co_3, HS, or $HS:2$

Theorem 1.0.3. Let G be any one of the groups J_3, J_4, $Co2$, $Co2:2$, Co_1, McL, $McL:2$, Suz, $Suz:2$, He, HN, $HN:2$, Fi_{22}, $Fi_{22}:2$, Fi_{23}, Fi_{24}, $Fi_{24}:2$, ON, $ON:2$, Ly, Th, Ru, $Ru:2$, B, M. Then G possesses no primitive genus zero, one and two systems.

Sections three up to section eleven give a complete proof of Theorem 1.0.3.

Chapter 4 is to provide a complete description of the braid orbits of Nielsen classes of sporadic simple groups. Firstly we present a table with the number of ramification types for each sporadic simple group for which the corresponding Nielsen classes are non-empty. This chapter is devoted to describing the GAP package MAPClASS. It is used for calculation of braid orbits. The MAPCLASS package is a modernized version of the GAP package BRAID. MAPCLASS has 17 functions. In this thesis we use two of them.

Chapter 5 contains a summary of our work.

Appendix A contains tables representing the results of our computation of primitive genus zero system in sporadic simple groups satisfying Theorem 1.0.2.

Theorem 1.0.4. Let G be a sporadic simple group. Then up to isomorphism, there exists 11 primitive genus zero groups satisfying Theorem 1.0.2. The corresponding primitive genus zero systems are enumerated in the Tables 5.4 to 5.19.

Appendix B contains tables representing the results of our computation of primitive genus one system in sporadic simple groups satisfying Theorem 1.0.2.

Theorem 1.0.5. Let G be a sporadic simple group. Then up to isomorphism, there exists 11 primitive genus one groups satisfying Theorem 1.0.2. The corresponding primitive genus one systems are enumerated in the Tables 5.20 to 5.41.

Appendix C contains tables representing the results of our computation of primitive genus two system in sporadic simple groups satisfying Theorem1.0.2.

Theorem 1.0.6. Let G be a sporadic simple group. Then up to isomorphism, there exists 11 primitive genus two groups satisfying Theorem 1.0.2. The corresponding primitive genus two systems are enumerated in the Tables 5.42 to 5.69.

CHAPTER 2

BACKGROUND

In this chapter, we review and cover some background knowledge which will be used throughout this thesis. We will start with a section on algebraic topology to collect the facts on covering space and the fundamental group that we need in the sequel.

2.1 Covering and the fundamental group

Definition 2.1.1. Let X be a topological space. A continuous map f from the interval $[0,1]$ to the space X is called a **path**; $f(0) = x_0$ is called the start (initial) point and $f(1) = x_1$ is called the end (terminal) point. In addition, if the initial and terminal points of a path are equal, then f is said to be a **loop**. We say that a loop is based at a point a_0 if its initial point is a_0.

A subset B of X is said to be **path connected** if and only if for all x and y in B there is a path from x to y in B.

Definition 2.1.2. Let f_1 and f_2 be two paths with the same initial(a_0) and terminal(b_0) points. A **homotopy** between two paths f_1 and f_2 is a continuous map $h : [0,1]^2 \to X$ such that

$$h(0,t) = f_1(t), h(1,t) = f_2(t) \; \forall t \in I \text{ and } \quad h(s,0) = a_0, h(s,1) = b_0 \; \forall s \in I.$$

Two paths f_0 and f_1 are called **homotopic**, which we denoted by $f_0 \sim f_1$, if there exists a homotopy between them. Homotopy is an equivalence relation on loops with the initial point a_0. By $[f]$ we denote the homotopy equivalence class of the loop f.

Definition 2.1.3. Let f_1 and f_2 be two paths on X with $f_1(1) = f_2(0)$. Then the **product** of f_1 and f_2 is defined by

$$(f_1 \cdot f_2)(t) = \begin{cases} f_1(2t) & \text{if} \quad 0 \leqslant t \leqslant \frac{1}{2}; \\ f_2(2t-1) & \text{if} \quad \frac{1}{2} \leqslant t \leqslant 1. \end{cases}$$

Definition 2.1.4. Let X be a path connected topological space and $x \in X$. The **fundamental group** of the space X at the base point x, is denoted by $\pi_1(X,x)$, and defined to be the set of all homotopy classes of loops f with the initial point x with respect to the product $[f_1][f_2] = [f_1 f_2]$.

Definition 2.1.5. Let X be a Hausdorff space. Then X is a **topological manifold** (**manifold**), if for each point of X there is an open neighborhood of that is homeomorphic to $\mathbb{R}^n$ for some fixed number $n \geqslant 1$.

Clearly, a manifold is connected if and only if it is a path connected. Indeed, if any two different points can be joined by a path. Moreover, connected components of a manifold are closed and open therefore they themselves are manifolds.

Definition 2.1.6. A continuous function f between two topological space X and Y is said to be a **homeomorphism** if and only if f is bijection and f^{-1} is continuous . Two topological spaces X and Y are **homeomorphic**, if there is a homeomorphism function between them.

Definition 2.1.7. A **Local Homeomorphism** is a continuous map $f : Y \longrightarrow X$ that has the following property: every point $y \in Y$ has an open neighborhood V such that f maps V homeomorphically onto $f(V)$ where $f(V)$, is open in X.

If X is a connected manifold, then the fundamental groups $\pi_1(X,x)$ and $\pi_1(X,y)$ are isomorphic, for all $x,y \in X$. We explain this statement in the following way. Since X is connected, there is a path λ from the initial point x to the terminal point y which joins these two points. Based on the fact λ^{-1} is a inverse of the path λ with $\lambda^{-1}(t) = \lambda(1-t) \forall t \in [0,1]$. Although $\lambda^{-1} \cdot \lambda \neq \lambda \cdot \lambda^{-1}$, both $\lambda^{-1} \cdot \lambda$ and $\lambda \cdot \lambda^{-1}$ are homotopic to a constant path. We take a loop class $[\gamma]$ with base point x, therefore $[\gamma]$ in $\pi_1(X,x)$. So we can follow it by a path λ^{-1} from y to x, following by a loop γ to initial point x and return to the terminal point y by using the path λ. Then we have created a loop $\lambda^{-1}\gamma\lambda$ with the base point y, such that it is an element of the fundamental group $\pi_1(X,y)$. Then we get $\pi_1(X,x)$=$\lambda\pi_1(X,y)\lambda^{-1}$. Hence, there is an isomorphism from $\pi_1(X,x)$ to $\pi_1(X,y)$ as illustrated in the Figure 2.1.

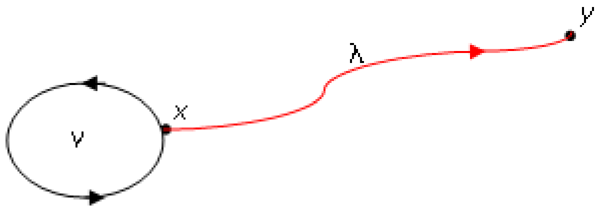

Figure 2.1:

Definition 2.1.8. Let X and Y be two topological spaces. Then Y is a **covering space** (or **cover**) of X if there exists a surjective map $f : Y \to X$ such that for every $x \in X$ there exists a path connected open neighborhood U of x, such that the inverse image of U under f is the union of disjoint open sets D_i in Y, and each D_i is mapped f homeomorphically onto U. The open neighborhood U is called an **admissible neighborhood**.

If Y is a cover of X under the surjective function f, then $f : Y \longrightarrow X$ is called the **covering map** and (Y, f) is the **covering space** of X. If X has a covering space (Y, f), then f is a local homeomorphism. The converse is not true in general, it is possible to construct a local homeomorphism which is onto but is not covering map as shown in the following example.

Example 2.1.9. Let $f : (0,10) \to \mathbb{S}^1$ be map defined by

$$f(t) = (cost, sint).$$

Then f is onto and a local homeomorphism map but $((0,10), f)$ is not a covering space of $\mathbb{S}^1$.

Definition 2.1.10. Suppose that $f : Y \to X$ is a covering map and $x \in X$. The **fiber** of x is the set given by

$$f^{-1}(x) = \{y \in Y | f(y) = x\}.$$

Let $\rho' : I \longrightarrow Y$ be a path in Y and let $f : Y \longrightarrow X$ be a cover of X. Then $f \circ \rho' : I \longrightarrow X$ is a path in X. Moreover, if ρ'_1 and ρ'_2 are two paths in Y such that $\rho'_1 {\sim} \rho'_2$, then $f \circ \rho'_1 \sim f \circ \rho'_2$. The converse raises some natural questions. If we have a path $\rho : I \longrightarrow X$ in X, does there exist a path ρ' in Y such that $f \circ \rho' = \rho$, or if ρ'_1, ρ'_2 are two paths in Y and $f \circ \rho'_1 \sim f \circ \rho'_2$ is it true $\rho'_1 \sim \rho'_2$? The answers of both these questions are positive as we will soon explains. Firstly, we will introduce new concepts in the following lemma.

Lemma 2.1.11. [35, p.181]

Given a compact metric space X, such that X is the union of a collection of open sets $\{A_i, i \in I\}$, then for every $S \subset X$, there exists a $\zeta \in \mathbb{R}$ such that if the diameter of S is less than ζ then S contained in some open set A_i. The real number ζ is called a **Lebesgues number** of open cover.

Proof. Let $x \in X$, it is clearly seen that $x \in A_i$ for some $i \in I$. Choose a real number $\varepsilon_x > 0$, such that the open ball $B(x, 2\varepsilon_x)$ of x is contained in A_i for some $i \in I$. Note that the space X is the union of the collection of open balls $\bigcup_{x \in X} B(x_i, \varepsilon_x)$. Hence $X = \bigcup_{i=1}^{n} B(x_i, \varepsilon_{x_i})$, for some finite set $\{x_1, x_2, \cdots, x_n\} \in X$, as X is a compact metric space. Now suppose that $\zeta = min\{\varepsilon_{x_1}, \varepsilon_{x_2}, \cdots, \varepsilon_{x_n}\}$ and the diameter of S is less than ζ. Then $S \subset A_i$ for some $i \in I$. Indeed, if $z \in S$, then $z \in B(x_j, \varepsilon_{x_j})$ for $j \in \{1, 2, \cdots, n\}$. Thus $S \subset B(x_j, 2\varepsilon_{x_j})$ then $S \subset A_i$, for some $i \in I$, as $B(x_j, 2\varepsilon_{x_j}) \subset A_i$. □

Definition 2.1.12. Let X and Y be two topological spaces, and $f : Y \to X$ be a covering map. Let p be a path in X. A **lift** of p is a path p' in Y such that $f \circ p' = p$.

Lemma 2.1.13. [36, p.64]

Given a covering map $f : Y \to\to X$, $y_0 \in Y$ and $x_0 = f(y_0)$. Then for any path ρ in X with initial point x_0 there is a unique path ρ' in Y with initial point y_0 such that $f \circ \rho' = \rho$.

Proof. Clearly, if ρ is in the admissible neighborhood U, then there exists a unique path ρ' with the initial point y_0 such that $f \circ \rho' = \rho$. Indeed, let D be a path component of $f^{-1}(U)$ such that $y_0 \in D$. Since each path component of $f^{-1}(U)$ is mapped topologically onto U by f, then there exists a unique path ρ' in Y with the initial point y_0 such that $f \circ \rho' = \rho$.

In case ρ is not contained in admissible neighborhood U. We can define ρ as a product of shorter paths such that each of the shorter paths is contained in admissible neighborhood U. So, apply the previous argument on each of short paths then we can find a unique path ρ' in Y with the required properties. The explanation of this argument is as follows. Assume that admissible neighborhoods $\{U_i\}$ covering X. The inverse image of admissible neighborhoods $\rho^{-1}(U)$ is open cover of the compact metric space I. Suppose that the number r large as can as possible. Let $\frac{1}{r} < \zeta$ where ζ is Lebesgues number of covering. So, the interval $I = [0, 1]$ can be divided into r subinterval as follows $[0, 1] = [0, \frac{1}{r}] \bigcup [\frac{1}{r}, \frac{2}{r}] \bigcup \cdots \bigcup [\frac{r-1}{r}, 1]$. It is clearly that ρ

maps each subinterval into admissible open neighborhoods in X. Thus, we have successfully defined a unique path ρ' over these subintervals . □

Lemma 2.1.14. [36, p.68]

Given a covering map $f : Y \to X$. Then the sets $f^{-1}(a_0), \forall a_0 \in X$, have the same cardinality.

Proof. Suppose that a_0 and a_1 are two different points in X. Let $\rho : I \to X$ be a path with initial point a_0 and terminal point a_1. We claim that there is a map $f^{-1}(a_0) \to f^{-1}(a_1)$ which is bijection. Assume that $b_0 \in f^{-1}(a_0)$, and consider the lift ρ is a path ρ' in Y with initial point b_0 so that $f \circ \rho' = \rho$. Now suppose that b_1 is a terminal point of the path ρ'. Since the uniqueness of the lift starting at the point b_0 then by previous lemma b_1 ith the only possible terminal point. Thus $b_0 \to b_1$ which is a required mapping . Similarly, by utilizing the inverse path $\overline{\rho}$, we can define a map $f^{-1}(a_1) \to f^{-1}(a_0)$. We notice that these maps are inverse each other, which implies that the map $f^{-1}(a_0) \to f^{-1}(a_1)$ is bijection. Thus for each $a_0 \in X$, $f^{-1}(a_0)$ has the same cardinality. □

We conclude from the above two lemmas, if we have a path connected space X and a covering map f, then there exists a bijection between two different fibers. Thus all fibers have the same cardinality. This cardinality is said to be the **degree** of the covering map f, it may be finite or infinite.

Next we will explain how the group $\pi_1(X,x)$ acts on the fiber $f^{-1}(x)$, via homotopy lifting. Fix a point $x \in X$ and choose a point $y_0 \in Y$ such that $y_0 \in f^{-1}(x)$. Let γ be a loop on X based at the point x, then the lift of the loop γ is a unique path $\overline{\gamma}$ in Y with the initial point y_0 (Lemma 2.1.13). Note that the terminal point of this lifted path need not be y_0, however, it must be lie in the fiber over x. The terminal point $\overline{\gamma}[1]$, consequently, depends only on the class γ in the fundamental group $\pi_1(X,x)$. This gives a right group action of the fundamental group $\pi_1(X,x)$ on the fiber $f^{-1}(x)$. This action is called the **monodromy action** of $\pi_1(X,x)$ on $f^{-1}(x)$[30].

Definition 2.1.15. Given a covering map $f : Y \to X$ of degree n, and fundamental group $\pi_1(X,x)$ based at the point x . The monodromy action of $\pi_1(X,x)$ on each fiber gives a group homomorphism

$$\rho : \pi_1(X,x) \longrightarrow S_n,$$

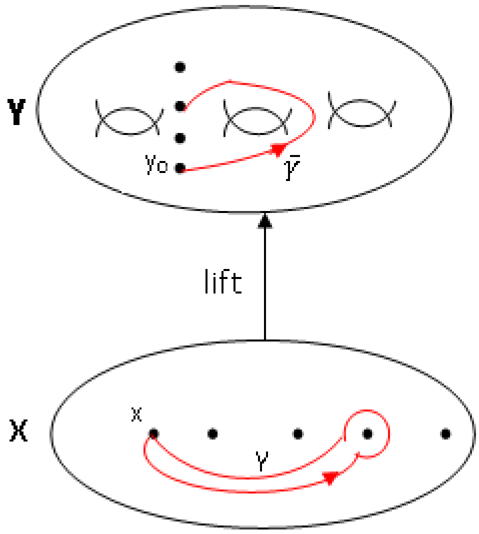

Figure 2.2:

this homomorphism is said to be **monodromy representation** of the covering map f, where S_n is the symmetric group of n points. The image of homomorphism ρ is called **monodromy group** of covering map f and denoted by $Mon(Y, f)$.

Note that, the subgroup $Mon(Y, f) \subset S_n$ is a transitive subgroup. Indeed, Y is connected and for any two any indices i and j there is an element in the monodromy group $Mon(Y, f)$ such that taking i to j.

Lemma 2.1.16. [30, p.87]

Suppose that $p : \pi_1(X, x) \to S_n$ is the monodromy representation for a path connected covering space (Y, f) of X of finite degree n. Then $Mon(Y, f)$ is a transitive subgroup of S_n.

Proof. Take two points x_i and x_j in the fiber of f over x. As Y is path connected, we can find a path β' on X with the initial point x_i and the terminal point x_j. Let $\beta = f \circ \beta'$ be the image of β' in X. Then β is a loop in X based at the point x, since both points x_i and x_j in $f^{-1}(x)$. Then we get that $p([\beta])$ is a permutation such that sends x_i to x_j. □

Moving from transitive subgroups of the fundamental group of the space to covers of the space is permitted by definition of monodromy representations. Suppose $\rho : \pi_1(X, x) \longrightarrow S_n$, is a homomorphism such that the image of the function ρ is a transitive subgroup of the symmetric group S_n. Let $K \subseteq \pi_1(X,x)$ be the subset in $\pi_1(X,x)$ of X defined by

$$K = \{[\beta] \in \pi_1(X,x) | \rho([\beta])(1) = 1\}$$

The subgroup K has index n in $\pi_1(X,x)$ such that it is correspondence with a connected covering space (Y_ρ, f_ρ) of the space X. Note that the homomorphism ρ is exactly a monodromy representation of the cover Y .

Lemma 2.1.17. Suppose that $f : Y \longrightarrow X$ is a covering map and ρ'_1, ρ'_2 are two paths in Y such that they have the same initial point. If $f \circ \rho'_1 \sim f \circ \rho'_2$, then $\rho'_1 \sim \rho'_2$

Proof. Complete proof can be found in [27] □

Definition 2.1.18. Two covering spaces (Y, f) and (Y', f') of the same space X are **isomorphic** if there exists a homeomorphism mapping $\phi : Y \to Y'$ such that $f' \circ \phi = f$.

Definition 2.1.19. Let $f : Y \longrightarrow X$ be a covering. A homomorphism $\alpha : Y \longrightarrow Y$ is said to be **deck transformation** of the covering f if $f\alpha = f$. The group formed by the set of all deck transformation is denoted by $Deck(f)$.

Let a be any point in the fiber $f^{-1}(x)$ for $x \in X$ then $\alpha(a)$ is still in fiber $f^{-1}(x)$ for any $\alpha \in Deck(f)$. This implies that the group $Deck(f)$ acts on the fiber $f^{-1}(b)$.

Now, assume that b is initial point of γ and $\widehat{\gamma}$ is a lift of γ for $\gamma \in \pi_1(X,x)$. Then γb is the end point of the path γ. Furthermore, $\alpha(b)$ and $\gamma\alpha(b)$ are initial and end points of the path $\alpha \circ \widehat{\gamma}$ respectively. The fundamental group $\pi_1(X,x)$ also acts on the fiber $f^{-1}(b)$ via monodromy action such that the initial point of the lift γ is $\alpha(b)$ and $\gamma\alpha(b)$ it's end point. By deck transformation we get $f\alpha \circ \widehat{\gamma} = f\widehat{\gamma} = \gamma$, therefor $\alpha(\gamma(b)) = \gamma\alpha(b)$. Thus the monodromy action of the fundamental group $\pi_1(X,x)$ commutes with the action of the group $Deck(f)$, and we get the following result.

Proposition 2.1.20. Suppose that $f : Y \longrightarrow X$ is covering. Then the monodromy action on the fundamental group $\pi_1(X,x)$ commutes with the action of the group $Deck(f)$ on the fiber $f^{-1}(x)$.

Proof. For proof see [36, p.68] □

Lemma 2.1.21. Let $f : Y \longrightarrow X$ be a covering, $p \in X$ and $b \in f^{-1}(p)$. Then the group

$f(\pi_1(Y,b))$ is a normal subgroup of the fundamental group $\pi_1(X,x)$ and the $Deck(f)$ isomorphic to the monodromy group G.

Proof. For a proof see [28, p.134] □

Definition 2.1.22. A covering map $f : Y \longrightarrow X$ is said to be Galois covering if $Deck(f)$ acts transitively on some fibers $f^{-1}(b)$ and Y is connected. If the degree of f is finite then we say f is finite.

The next result shows that if $f : Y \longrightarrow X$ is a Galois covering, then there is a homomorphism φ_b from the fundamental group $\pi_1(X,x)$, $x \in X$ to the group $Deck(f)$ such that φ_b is surjective and unique.

Proposition 2.1.23. Let G be a deck transformation group $Deck(f)$, where $f : Y \longrightarrow X$ is Galois covering such that for $b \in Y, x \in X$, $f(b) = x$. Let $[\gamma]$ be a loop class based on x. Then there is a unique surjective homomorphism φ_b from the fundamental group $\pi_1(X,x)$ to the group G with $\varphi_b[\gamma] = [\gamma]b$ (Recall that $[\gamma]b$ is end point of lift γ with initial point b).

Proof. For a proof see [36, p.69] □

Corollary 2.1.24. Let $[\gamma] \in \pi_1(X,x)$ and the end point of its lift is $b' \in f^{-1}(x)$. Let $\varphi_b : \pi_1(X,x) \longrightarrow Deck(f)$ with $\varphi_b[\gamma] = [\gamma]b$. Then $[\gamma].ker\varphi_b.[\gamma]^{-1} = ker\varphi_{b'}$

Proof. The lift of all elements of loop class $[\alpha] \in ker\varphi_b$ is a loop based on b. So for $[\gamma\alpha\gamma^{-1}] \in [\gamma].ker\varphi_b.[\gamma]^{-1}$, the lift for $[\gamma\alpha\gamma^{-1}]$ is a loop based on b'. Thus $[\gamma].ker\varphi_b.[\gamma]^{-1} \subseteq ker\varphi_{b'}$. On the other hand if $[\alpha'] \in ker\varphi_{b'}$ then the lift of α' is a loop based at b. Similarly, $[\gamma]^{-1}ker\varphi_{b'}[\gamma] \subseteq ker\varphi_b$. □

It is clear that form above, if the deck transformation group has trivial center, and if we pick any two points b and b' in the fiber $f^{-1}(x)$ then $b = b'$ if and only if the two homomorphism φ_b and $\varphi_{b'}$ are equal. So, the covering $f : Y \to X$ can be represented by the pair (x, φ_b). This property allows us to construct the Hurwitz spaces.

2.2 Riemann Surfaces and Riemann Existence Theorem

Definition 2.2.1. Let X be a topological space and $U \subset X$ be an open set in X. A homeomorphism $\theta : U \to V$ where $V \subset \mathbb{C}$ is said to be a **complex chart** or **simple chart** on X and the open set U is called the **domain** of the chart. If $\theta(p) = 0$ for $p \in U$, then we say the **chart centered** at p.

Assume that V and W are two open sets of the complex plane and $\theta : U \to V$ is a complex chart. If $\psi : V \to W$ is holomorphic, one to one and onto, then the composition $\theta \circ \psi : U \to W$ is also a complex chart on X[30].

Definition 2.2.2. For any two complex charts $\theta_1 : U_1 \to V_1$,$\theta_2 : U_2 \to V_2$ on X we say θ_1 and θ_2 are **compatible** if either $U_1 \cap U_2 = \emptyset$ or $\theta_2 \circ \theta_1^{-1} : \theta_1(U_1 \cap U_2) \to \theta_2(U_1 \cap U_2)$ is holomorphic.

Note that $\theta_1 \circ \theta_2^{-1}$ is holomorphic on $\theta_2(U_1 \cap U_2)$ if $\theta_2 \circ \theta_1^{-1}$ is holomorphic on $\theta_1(U_1 \cap U_2)$. The bijective function $T = \theta_2 \circ \theta_1^{-1}$ between the two charts is called the **transition function**.

Definition 2.2.3. Let X be a topological space, $U \subset X$ be an open subset of X and V be an open subset of the complex plane. A **complex atlas** on X is a collection $B = \{\theta_j : U_j \to V_j\}$ satisfying:

(1) $\bigcup_{j \in I} U_j = X$,

(2) any two charts θ_j and θ_k in B are compatible.

Furthermore, if $Y \subset X$ and $B = \{\theta_j : U_j \to V_j\}$ is a complex atlas on X then the collection $B_Y = \{\theta_j|_{Y \cap U_j} : Y \cap U_j \to \theta_j(Y \cap U_j)\}$ is an atlas on Y [30].

Let A and B be two complex atlases. If every chart in A is compatible with every chart in B, then we say A and B are **equivalent**, that means two complex atlases A and B are equivalent if and only if $A \cup B$ is also a complex atlas. An equivalence class of complex atlases on X is said to be **complex structure**.

Definition 2.2.4. A second countable Hausdorff connected topological space X together with a complex structure is called a **Riemann surface**.

Note that a Riemann surface is a 1-dimensional complex manifold. A compact Riemann surface is homeomorphic to a sphere, or a connected sum of tori.

Definition 2.2.5. Suppose that X is a Riemann surface and f is holomorphic in a punctured neighborhood of $p \in X$. We say that f has a **pole** if there exists a chart $\theta : U \to V$ with $p \in U$ such that $f \circ \theta^{-1}$ has a pole at $\theta(p)$. A function f is said to be, **meromorphic** at a point p, if f is holomorphic at p or p is a pole of f. If $f : X \to \mathbb{P}^1$ is a non-constant analytic function, then f is called a **meromorphic function**. Moreover, a pair (X, f) is called a **cover** where $f : X \to \mathbb{P}^1$ is a meromorphic function.

Proposition 2.2.6. Suppose that $f : X \to Y$ is a holomorphic map defined at $x \in X$, where X and Y are compact Riemann surfaces, . Then there exists a unique positive integer number m such that for every chart $\theta' : U' \to V'$ on Y centered at $f(x)$, there exists a chart $\theta : U \to V$ on the complex structure on X with $\theta(x) = 0$ and $\theta'(f(\theta^{-1}(z))) = z^m$.

Proof. A complete proof can be found in [30, p.44] □

The positive integer number m is called the **ramification index** of f at x and denoted by e_x. A point $x \in X$ is called a **ramified point** of f, if $e_x \geqslant 2$. The image of a ramified point of f is called a **branch point**.

Definition 2.2.7. A continuous function $f : X \longrightarrow Y$, where X and Y are compact Riemann surfaces, is called **analytic**, if for any two charts $\theta : V \longrightarrow W$ in X and $\theta' : V' \longrightarrow W'$ in Y with $f(V) \subset V'$, the map $\theta' \circ f \circ \theta^{-1} : \theta(V) \longrightarrow \theta(V')$ is holomophic.

Analytic functions between two Riemann surfaces X and Y in general is not coverings in the sense of the previous section. We can find a covering map f between two Riemann surfaces X and Y without the ramified points. In the other words, if $f : X \to Y$ is a non-constant analytic function, then we can say that f is a covering map onto its image without exceptional points $y \in Y$, where degree of f is greater than the cardinality of $f^{-1}(y)$.

Proposition 2.2.8. Let $a_1, a_2, \cdots, a_r$ be r points in Riemann sphere $\mathbb{P}^1$ and let $X = \mathbb{P}^1 - \{a_1, \cdots, a_r\}$. If $\gamma_1, \gamma_2, \cdots, \gamma_r$ are loops around the points a_i, then the fundamental group $\pi_1(X, x)$ is generated by the homotopy classes $[\gamma_i]$ of loops γ_i. Moreover, $[\gamma_1][\gamma_2]\cdots[\gamma_r] = 1$.

Proof. See [30]. □

In the above proposition we have seen that in the Riemann surface X if $f : Y \to X$ is branched at the elements a_i then the homotopy classes of each loop γ_i around a_i is $[\gamma_i]$. Which implies that, the monodromy representation ρ from the fundamental group $\pi_1(X,x)$ to the symmetric group S_n is determined by r permutation $\sigma_1, \cdots, \sigma_r \in S_n$. where $\rho(\gamma_i) = \sigma_i$ and $\sigma_1.\cdots.\sigma_r = 1$. If $\sigma = \{\sigma_1, \cdots.\sigma_r\}$, then the map $f : Y \longrightarrow \mathbb{P}^1$ is called **of type** σ.

In the previous section we understood that if $f : R \to \mathbb{P}^1$ is a meromorphic function from a Riemann surface to $\mathbb{P}^1$ in general f is not a covering if the set of branch points is non-empty. By restricting the domain to $R - f^{-1}(B)$ where $B = \{a_1, \ldots, a_r\}$ is the set of branch points, the meromorphic function f is guaranteed to be a covering. Now we turn our attention to finite coverings of the punctured sphere, that is the Riemann sphere $\mathbb{P}^1$, which is removed a finite number points with monodromy group G. Further, we look at a relationship between the elements of the fundamental group of the puncture sphere $\mathbb{P}^1 \backslash B$ and elements of the monodromy group G. According to this result we focus on the Riemann Existence Theorem . Now assume that $k(r) := \{z \in \mathbb{C} : |z| < r\}$ for $r > 0$.

Lemma 2.2.9. Let n be a natural number, Then the map f_n from $k(r^{1/n})$ to $k(r)$ mapping z to z^n is a Galois covering and the monodromy group is cyclic of order n.

Proof. A complete proof can be found in [36, p.70]. □

Lemma 2.2.10. The Galois covering $f_n : k(r^{\frac{1}{n}}) \to k(r)$ and $f : A \to k(r)$ of degree n and they are equivalent if A is connected.

Proof. A complete proof can be found in [36, p.71]. □

Suppose that $\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$ is the Riemann sphere and $B = \{b_1, \ldots, b_r\}$ is a set of branch points in $\mathbb{P}^1$. Let $D(p,r) = \{z \in \mathbb{C} : |z - p| < r\}$; $r > 0, p \in \mathbb{C}$ be a an open set centered at the point p. If $p = \infty$, then $D(p,r) = \{z \in \mathbb{C} : |z| > r^{-1}\} \cup \{\infty\}$. The Galois covering is obtained by removing the set of branch points in the Riemann sphere $\mathbb{P}^1$. In the other words, $\mathbb{P}^1 \setminus B$ is a punctured sphere and $f : R \to \mathbb{P}^1 \setminus B$ is a Galois covering and the group G is its monodromy group. Note that, for any $b \in B$ there exists $r \in R$ such that $f(r) = b$ (because f is onto). Moreover, for the two points b and r there exists open sets U_r and V_p respectively which are homeomorphic to the disc $k(r)$. The above lemma guarantees that that the covering map $f : R \to \mathbb{P}^1 \backslash B$ around each of the branch points p maps z to z^n.

Proposition 2.2.11. Let $D\backslash\{p\}$ be a punctured disc denoted by D^* and $f : R \to \mathbb{P}^1\backslash B$ be a Galois covering. Then the components of $f^{-1}(D^*)$ are permuted transitively by the monodromy group G. Moreover, if G_F is the stabilizer of F in G, where F is one component of $f^{-1}(D^*)$ then G_F is cyclic.

Proof. A complete proof can be found in [36, p.72] □

In the above proposition the generator of cyclic group is called canonical generator. For branch point b , the conjugacy class in G which containing canonical generator is denoted by C_p.

Proposition 2.2.12. Let λ be a loop based on the point p^*, $p^* \in D^*$ and let δ be another path joining two points p^* and q_0 where $q_0 = f(p); p \in R$. Then the map φ_p from the fundamental group $\pi_1(\mathbb{P}^1\backslash B, q_0)$ to the group G sends the homotopy class of $\gamma = \delta^{-1}\lambda\delta$ (representative element in $\pi_1(\mathbb{P}^1\backslash B, q_0)$) to the element in C_p.

Proof. For a proof see [36, p.73]. □

Let $b \in B$ be a branch point. In the fundamental group $\pi_1(\mathbb{P}^1\backslash B, q_0)$, there is a conjugacy class corresponding to the branch point b. We explain this statement in the following way. First we select an open disc $D(b,s)$ around the branch point b for $s > 0$ in which no point of $B \setminus \{b\}$ is contained in the disc $D(b,s)$. We draw a path δ from the base point q_0 of the fundamental group to some boundary point v of the disc $D(b,s)$. Let λ be a closed paths starting from the boundary point v winding once counter clock wise around $D(b,s)$. The set of the close path $\delta^{-1}\lambda\delta$ is a conjugacy class with respect to the branch point b where δ and v vary and it is denoted by Σ_p.

Corollary 2.2.13. Let $\varphi_b : \pi_1(\mathbb{P}^1\backslash, q_o) \to G$ be surjective homomorphism. Then $\varphi_b(\Sigma_p) = C_p$.

Note that if $\varphi_b(\Sigma_p) = C_p \neq \{1\}$ then surjective homomorphism φ_b is said to be admissible.

Corollary 2.2.14. Let $\mathbb{P}^1$ be the Riemann sphere. Let B be a finite set in $\mathbb{P}^1$. Then there is one to correspondence between

- Isomorphism classes of meromorphic function $f : X \to \mathbb{P}^1$ branched at B
- transitive equivalence classes of permutation representations $\varphi_b : \pi_1(\mathbb{P}^1\backslash B) \to S_n$.

Definition 2.2.15. Let $f : X \to \mathbb{P}^1 \backslash B$ be a finite Galois covering in which $B = \{a_1, a_2, \cdots, a_r\}$ is the set of branch points in $\mathbb{P}^1$. The **ramification type** $\overline{C} = \{C_{a_1}, C_{a_2}, \ldots, C_{a_r}\}$ of cover f is defined to be the set set of non-trivial conjugacy classes in the group G.

Theorem 2.2.16. (Riemann Existence Theorem) Assume that G is a finite group , $B \subset \mathbb{P}^1$ where $B = \{a_1, \ldots, a_r\}$. Let $\overline{C} = \{C_{a_1}, \ldots, C_{a_r}\}$ be a ramification type. Then there exists G-cover (branch-cover) of type $\overline{C}$ if and only if there exists generating tuple $(g_1, \cdots, g_r)$ of the group G with $\prod_{i=1}^{r} g_i = 1$ and $g_i \in C_{a_i}$ for $i = 1, 2, \ldots, r$.

The complete proof can found in [36]. This theorem tells us that if G is a transitive subgroup of S_n with elements $g_1, \ldots, g_r$, such that $G = \langle g_1, \ldots, g_r \rangle$, $\prod_{i=1}^{r} g_i = 1$, and $g_i \neq 1$, for $i = 1, 2, \ldots, r$, then there exists a cover map $f : X \to \mathbb{P}^1$ branched at $B = \{a_1, \ldots, a_r\}$ such that $\mathrm{Mon}(Y, f)$ is equal to G.

Theorem 2.2.17. Let X be a Riemann surface of genus g and $f : X \to \mathbb{P}$ be a meromorphic function of degree n. Then

$$2(n+g-1) = \sum_{x \in X} (e_x - 1), \tag{1}$$

where e_x is a ramification index of f at x. Equation (1) is one form of the Riemann Hurwitz formula.

Proof. A complete proof can be found [30, p.52] □

Definition 2.2.18. Suppose that G acts on a finite set Ω. The **index** of $x \in G$ on Ω is defined by

$$\mathrm{ind}(x) = |\Omega| - \mathrm{orb}(x),$$

where $\mathrm{orb}(x)$ is the number of orbits of G on Ω.

Theorem 2.2.19 (Riemann Hurwitz Theorem)**.** Let R be a Riemann surface of genus g and $f : R \to \mathbb{P}^1$ be a meromorphic function of degree n. Let $G = Mon(R, f)$ with $\delta_1, \cdots, \delta_k \in G^k$ with $\delta_1 \ldots \delta_k = 1$. Then

$$\sum_{i=1}^{k} \mathrm{ind}(\delta_i) = 2(n+g-1).$$

Proof. A complete proof can be found [30, p.58] □

Definition 2.2.20. Any tuples which satisfy the conditions laid out in Riemann Existence Theorem and Riemann Hurwitz formula are said to be **admissible**. Moreover, if $\sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_r\}$ is an admissible tuple in the G^r, where G is a transitive subgroup of the symmetric group S_n, then the pair (G, σ) is said to be a **genus g system** of degree n.

2.3 Hurwitz space

In this section we aim to formally introduce the notation for Hurwitz spaces. Most of the results and definitions in this section are taken from [36]. For the remainder of this section, we assume that the center of the group G is trivial. We denote the group of inner automorphisms of G by $Inn(G)$. The Riemann extension theorem and Corollary 2.1.24, play an important role in this section. Let $f : Y \to X$ be a Galois covering. Since $Z(G) = 1$, it follows from Corollary 2.1.24 that if we pick any two points b and b' in the fiber $f^{-1}(x)$ for $x \in X$, then $b = b'$ if and only if $\varphi_b = \varphi_{b'}$. We denote by O_r, the space of branch point sets in $\mathbb{C}$ of cardinality r. So $O_r = \{\mathbb{C}^r \backslash (a_1, \cdots, a_r) \in \mathbb{C}^r \mid \text{ there exist i and j with } a_i = a_j\}$. It is an open set of complex projective space of dimension r, and if we define the determinate to be $\prod_{i \neq j}(a_i - a_j)$, then it is the complement of the discriminate locus.

Let $A \leqslant Aut(G)$ be arbitrary but fixed. Let $B \subseteq O_r$ and $\varphi : \pi(\mathbb{P}^1 \backslash B, \infty) \to G$ be an admissible surjective homomorphism. Then two such pairs (B, φ) and (B', φ') are **A-equivalent** if $B = B'$ and $\varphi' = p \circ \varphi$ for some automorphism $p \in A$.

We define the **Hurwitz space** of G-covers to be the set of equivalence classes of the pairs (B, φ) and we denoted it by $H_r^A(G)$.

We denoted by $[B, \varphi]$, the equivalence class of the pair (B, φ). For each equivalence class $[B, \varphi]$ in the Hurwitz space $H_r^A(G)$, we identify a basis of neighborhoods as follows: Let $D_1, \ldots, D_r$ be r pairwise disjoint discs centered around the r branch points $b_1, \ldots, b_r$ in B. Let $B' = \{b'_1, \ldots, b'_r\}$ be such that $b'_i \in D_i$ and let $[B', \varphi']$ be the equivalence class of the pair (B', φ'). So the neighborhood of the equivalence classes $[B, \varphi]$ is the set of all equivalence class $[B', \varphi']$ where φ' is a composition of φ and canonical isomorphism

$$\pi_1(\mathbb{P}^1\backslash B',\infty) \to \pi_1(\mathbb{P}^1\backslash(D_1\cup\cdots\cup D_r),\infty) \to \pi_1(\mathbb{P}^1\backslash B,\infty).$$

This gives topology on $H_r^A(G)$. The Hurwitz space is denoted by $H_r^{in}(G)$ if and only if $A = Inn(G)$.

Let the tuple $\bar{g}= (g_1,\ldots,g_r) \in G^r$ be admissible and let $\rho \in A$ be any automorphism. Then the tuple $\rho(g) = (\rho(g_1),\ldots,\rho(\rho))$ is also admissible and it corresponds to another cover φ' which is also admissible such that $\varphi' = \rho\varphi$. Clearly, these two covers are A-equivalent. Hence the equivalence class of $G-$cover with the tuple $\rho(g)$ is represented by the equivalence class of the pair (B,φ).

Proposition 2.3.1. Let $H_r^A(G)$ be the Hurwitz space and $B \subseteq O_r$, then the map $\psi_A : H_r^A(G) \to O_r$ sending the equivalence class of the pair (B,φ) to the set of branch point B is a covering.

Proof. For a proof see [36, 184]. □

Note that the monodromy homomorphism is completely determined by its action on the standard generators $\{\gamma_1,\ldots,\gamma_r\}$ of the group $\pi_1(\mathbb{P}^1\backslash B,\infty)$ because $\gamma_1,\ldots,\gamma_r$ generate $\pi_1(\mathbb{P}^1\backslash B,\infty)$ and the monodromy homomorphism $\varphi : \pi_1(\mathbb{P}^1\backslash B,\infty) \to G$ is an admissible surjection. The monodromy homomorphism φ sends the generators of the group $\pi_1(\mathbb{P}^1\backslash B,\infty)$ to the group elements g_i in G such that $g_1\cdots g_r = 1$ and $\langle g_1,\ldots g_r\rangle = G$. Let

$$\mathscr{E}_r(G) = \{(g_1,\ldots g_r) : \langle g_1,\ldots g_r\rangle = G \text{ and } g_1\cdots g_r = 1\}$$

Then the group A acts on the set $\mathscr{E}_r(G)$ by sending each g_i to $\rho(g_i)$ for $\rho \in A$. The set of $A-$orbits on $\mathscr{E}_r(G)$ is denoted by $\xi_r^A(G)$ in which $\zeta_r^A(G) = \mathscr{E}_r(G)/A$. Note that $Inn(G) \simeq G/Z(G)$. If $A = Inn(G)$, then $A \simeq G/Z(G)$ but $Z(G) = 1$, therefore the set of $G-$orbits under conjugates is denoted $\xi_r^{in}(G)$.

Proposition 2.3.2. Let $\psi_A : H_r^{(A)}(G) \to O_r$ with $\psi_A([B,\varphi]) = B$ be a covering and let B_0 be fix point in O_r, such that the fiber $\psi_A^{-1}(B_0)$ contains of all equivalence classes of the pairs (B_o,φ). Then there is a bijection map between the fiber $\psi_A^{-1}(B_0)$ and the set $\mathscr{E}_r(G)^A(G)$ via sending conjugacy class of the pair (B_0,φ) to $A-$equivalence class of $(g_1,\cdots,g_r)$, where $(\varphi[\gamma_i]) = g_i$

for $i = 1 \cdots r$.

Proof. The complete proof can be found in [36, 194]. □

From the above proposition, we see that each tuple $\bar{g} = (g_1 \cdots g_r) \in \mathscr{E}_r(G)$ corresponds to a homomorphism $\varphi : (\pi \backslash B, \infty) \to G$. Furthermore, the Riemann existence theorem yields that any admissible tuple corresponds to a covering map $f : X \to \mathbb{P}^1$. Thus the space $H_r^{(A)}(G)$ is the set of equivalence classes of $G-$covers. A tuple $\bar{g} = (g_1 \cdots g_r)$ is of a ramification type $\bar{C} = (C_1, \cdots, C_r)$ if $g_i \in C_i$ for all i, then $H_r^{(A)}(C) \subseteq H_r^{(A)}(G)$ and $H_r^{(A)}(\bar{C})$ contains all equivalence class of pairs $[B, \varphi]_A$.

Definition 2.3.3. Let $\bar{C} = (C_1, \cdots, C_r)$ be a ramification type. The **Nielsen class** is defined by

$$N(\bar{C}) = \{(g_1, \dots g_r)| \;\; g_i \in C_i, \;\; \langle g_1, \dots g_r \rangle = G \;\text{ and }\; g_1 \cdots g_r = 1\}$$

Definition 2.3.4. Let r be integer number with $r \geqslant 2$. The **braid group** denoted by B_r is generated by $r-1$ generators $\{Q_1, \cdots, Q_{r-1}\}$ satisfying the following relations

$$Q_i Q_j = Q_j Q_i \;\; \text{where} \;\; |i-j| > 1 \tag{2}$$

$$Q_i Q_{i+1} Q_i = Q_{i+1} Q_i Q_{i+1} \;\;\; for \;\; i = 1, 2, \cdots, r-2 \tag{3}$$

The term braid was defined by Emil Artin. Braids forms an infinite group. The relation 3 is said to be Yang Baxter equation. Furthermore, the two relations (3) and (2) together are called braid relations. Let G be a finite group and $\bar{g} = (g_1 \cdots g_r) \in G^r$ a generating tuple of G. Then the braid group B_r acts on G^r via:

$$(g_1, \dots g_i, g_{i+1}, \cdots)^{Q_i} = (g_1, \dots g_{i+1}, g_{i+1}^{-1} g_i g_{i+1}, \cdots g_r) \;\;\; for \;\;\; i = 1 \cdots r-1. \tag{4}$$

This action is referred to as the braid action. The smallest set of tuples which contains $\bar{g} = (g_1 \cdots g_r)$ is said to be the **braid orbit** of g if it contains all image of $\bar{g}$ under B_r. Note that if $\bar{g} = (g_1 \cdots g_r) \in G^r$ is a Nielsen tuple then $\langle g_1, \dots g_r \rangle = G$ and $\prod_{i=1}^r g_i = 1$ which implies that

$$g_1 \cdots g_{i+1} \cdot g_{i+1}^{-1} \cdot g_i \cdot g_{i+1} \cdot \cdots g_r = 1 \quad \text{and}$$

$$\langle g_1, \ldots, g_{i+1}, g_{i+1}^{-1} g_i g_{i+1}, \ldots g_r = 1 \rangle = G$$

So the braid action can be restricted to an action on the set of Nielsen tuples. Assume that $C = (C_1, \cdots, C_r)$ is a ramification type of the tuple $\bar{g} = (g_1 \cdots g_r)$ where C_i is a conjugacy class of G containing g_i for $i = 1, \cdots, r$. Then the conjugacy classes C_i are permuted by the braid action. So there is a unique homomorphism $\sigma : B_r \to S_r$ with $\sigma(Q_i) = (i, i+1)$. The kernel of this homomorphism is denoted by $B^{(r)}$ and is called the pure braid group. The pure braid elements generate a pure braid group by the following relation

$$Q_{ij} = Q_{j-1} \ldots Q_{i+1} Q_i^2 Q_{i+1}^{-1} \ldots Q_{j-1}^{-1} \tag{5}$$

$$= Q_1^{-1} \ldots Q_{j-2} Q_j^2 Q_{j-2} \ldots Q_i \tag{6}$$

for $1 \leqslant i \leqslant j \leqslant r$. Because of the braid group action, we can assume that the conjugacy classes in type C are ordered in particular way. From nowon, we always assume that the same conjugacy classes in C are adjacent and form a block in C. The braids $\{Q_{ij}\}_{i,j}$ are conjugate to each other in B_r [22, p.19]. Note that the braid group B_r acts on the fiber $\psi_A^{-1}(B_0)$[proposition 2.3.2], that is, B_r acts on the set $\zeta_r^A(G)$ via the braid group action, and this action commutes with the action of $Aut(G)(Inn(G))$ on tuples.

Definition 2.3.5. Let $X = \{1, \cdots, r\}$ be a non empty set. Then $P = \{p_1, \cdots, p_s\}$ with $p_i \subseteq X$ is called a partition of X if $\bigcup_i^r p_i = X$ and p_i and p_j are disjoint set for $i \neq j$.

Definition 2.3.6. Let $X = \{1, \cdots, r\}$ be a non empty set and $P = \{p_1, \cdots, p_s\}$ be a partition of X with stabilizer S_P when S_P is a subgroup of S_r, then the fiber of S_p is called **parabolic subgroup** of B_r and it is denoted by B_p.

Let $\bar{C} = (C_1, \cdots, C_r)$ is a ramification type. For the rest of this thesis we order elements in the type $\bar{C}$ in the following way $C_i = C_j$ for $1 \leqslant i \leqslant j \leqslant r$ if and only if $i = j$. Let P be a partition of the set C, then the parabolic subgroup B_P of B_r preserves the order of conjugacy class so B_p-orbit may be shorter than the B_r-orbits as much as by the factors of $[S_r : S_P]$ which switching to the B_P-orbits may be useful for our computations.

Proposition 2.3.7. Let $\bar{C}= (C_1, \cdots, C_r)$ is a ramification type and $N(C)$ be a Nielsen class. Then there is a one to one correspondence between B_b-orbit on $N(\bar{C})$ and connected components of $H_r^{in}(G)$.

Proof. A complete proof can be found [37] □

The above proposition guarantees us that there is a one to one correspondence between B_p-orbits on the Nielsen class and connected components of $H_r^{in}(\bar{C})$. In our thesis we focus on the Hurwitz space , more precisely $H_r^{in}(\bar{C})$ of $H_r^{in}(G)$. The **MapClass** package was designed by James ,Magaard, Shpectorov and Völkein to find braid orbits for a given group and given tuple.

2.4 Primitive Permutation groups

Definition 2.4.1. Let G be a group and Ω be a non empty set. Then a right group action of G on Ω is a function $* : X \times G \longrightarrow \Omega$ satisfying the following conditions
(1) $x * e = x$ for $x \in \Omega$, where $e \in G$ is the identity;
(2) $x * (gh)$=$(x * g) * h$ for $g, h \in G$ and $x \in \Omega$.

The action of G on Ω is called transitive, if for every $x, y \in X$ there exists $g \in G$ such that $xg = y$. The group G acts faithfully on X if for any $g, h \in G$ where $g \neq h$ there exists $x \in X$ such that $xg \neq xh$. Equivalently, if $g \neq e$, then $xg \neq x$ for some x.

Definition 2.4.2. Let G be a group acting on Ω. A **block** of imprimitivity for G is a non-empty set $\Delta \subset \Omega$ such that for all $g \in G$ either $\Delta g = \Delta$ or $\Delta g \bigcap \Delta = \emptyset$. If $|\Delta| = 1$, then Δ is called a trivial block.

Lemma 2.4.3. If G is transitive on Ω and Δ is a block of imprimitivity then $\{\Delta g \mid g \in G\}$ is a partition of Ω.

Proof. [5, 142] □

Example 2.4.4. If $H < K < G$ are subgroups of G, then G acts transitively on G/H and set $\Delta = \cup Hk, k \in K$ is a block of imprimitivity.

Definition 2.4.5. A group G acting on Ω is called **primitive** if G acts on Ω transitively and there is no partition of Ω preserved by G. In other words, G is said to **act primitively** on Ω if Ω contains no nontrivial blocks. If a primitive group G acts faithfully on Ω, then G is called **primitive permutation group**.

Definition 2.4.6. Let G be a group which acts on the non empty set Ω. Then the stabilizer subgroup of $\omega \in \Omega$ is defined by

$$G_\omega = \{g \in G | \omega g = \omega\}.$$

Theorem 2.4.7. Let G acts transitively Ω . Then G is primitive if and only if G_ω is a maximal subgroup of G.

Proof. Suppose that G is primitive. Let $G_\omega \leqslant H \leqslant G$ for a subgroup H in G. Define $\Delta = \{\omega h \mid h \in H\}$. Let $g \in G$ and $\alpha \in \Delta g \bigcap \Delta$. Then $\alpha \in \Delta g$ and $\alpha \in \Delta$ which implies that $\alpha = \omega h_1 g = \omega h_2$ where $h_1, h_2 \in H$. Therefore, $\omega h_1 g h_2^{-1} = \omega$. Thus, $h_1 g h_2^{-1} \in G_\omega \leqslant H$. Hence $g = h_1^{-1}(h_1 g h_2^{-1}) h_2 \in H$ and so $\Delta g = \Delta$. Hence Δ is block. Since G is primitive, we have $\Delta = \{\omega\}$ or $\Delta = \Omega$. If $\Delta = \{\omega\}$, then $\omega h = \omega$ for each $h \in H$, which implies that $h \in G_\omega$ and hence $G_\omega = H$. If $\Delta = \Omega$, then $\omega g \in \Omega = \Delta$ for all $g \in G$. Therefore, $\omega g = \omega h$ for some $h \in H$ which implies that $\omega g h^{-1} = \omega$. So $g h^{-1} \in G_\omega \leqslant H$. Thus $g \in H$. Implying, $G = H$. Hence G_ω is a maximal subgroup of G.

Conversely, suppose that G is not primitive. Then there exists a non trivial block Δ for G. Let $\omega \in \Delta$. Then $G_\omega \leqslant G_\Delta$. Indeed for $g \in G_\omega$, $\omega g = \omega$, and so we have $\Delta g \bigcap \Delta \neq \phi$ which implies that $\Delta g = \Delta$. So if $g \in G_\Delta$ then indeed $G_\omega \leqslant G_\Delta$. Now suppose that $G_\omega = G_\Delta$ and $\alpha \in \Delta$. Since G acts transitively on Ω, then there exists $g \in G$ such that $\alpha = \omega g$. Thus $\Delta g \bigcap \Delta \neq \phi$, and it follows that $g \in G_\Delta = G_\omega$ and so $\omega g = \omega$. Then $\alpha = \omega$ and $\Delta = \{\omega\}$: a contradiction. Suppose that $G_\Delta = G$ and $\alpha \in \Omega$. Since there exists $g \in G$ such that $\alpha = \omega g$, we have $\alpha \in \Delta g = \Delta$; so $\Delta = \Omega$, a contradiction. Thus $G_\omega < G_\Delta < G$. Hence G_ω is not maximal. □

Let G be a group which acts on the non empty set Ω, then define G_ω^g by $G_\omega^g = \{g^{-1} h g | h \in G_\omega\}$. Note that if $\alpha = \omega g$ then $G_\alpha = G_\omega^g$. If G is a transitive permutation group acting on a non empty set Ω, and $\alpha \in \Omega$, then for some $g \in G$, $\alpha = \omega g$ so $G_\alpha = G_{\omega g} = G_\omega^g$. Which means that if G_ω and G_α are any two stabilizer subgroups of a transitive permutation group G, then they are conjugate in G. G_ω is maximal subgroup in G then $G_{\omega g}$ is a maximal subgroup $\forall g \in G$.

Definition 2.4.8. Assume that G is a group. A non-trivial normal subgroup N of G is called **minimal normal subgroup** if for any non-trivial normal subgroup M in G such that $M \leqslant N$ then $M = N$.

Clearly, the intersection of any two different minimal normal subgroup of the group G is trivial. Indeed, if N_1 and N_2 are two minimal normal subgroups of G, then $N_1 \cap N_2 \trianglelefteq G$, $N_1 \cap N_2 \leqslant N_1$ and $N_1 \cap N_2 \leqslant N_2$. Thus, $N_1 \cap N_2 = \{1\}$ (by minimality of N_1 and N_2). It follows that, $N_1 \leqslant C_G(N_2)$ and $N_2 \leqslant C_G(N_1)$ as $[N_1, N_2] \leqslant N_1 \cap N_2 = \{1\}$.

Definition 2.4.9. Let G be a simple group . Then L is called **almost simple group** if

$$G \leqslant L \leqslant Aut(G).$$

Let G be a non abelian simple group. Then a finite group is almost simple if and only if it is isomorphic to a group L such that $Inn(G) \leqslant L \leqslant Aut(G)$.

Let M be a maximal subgroup of an almost simple group L. Then the permutation action of the group L on the the right cosets of M via right multiplication is primitive. Thus L is a primitive subgroup of symmetric group S_n where $[L : M] = n$. To describe the maximal subgroups of S_n, we require knowledge about the maximal subgroups of all almost simple groups. If there exists a normal subgroup G in L such that G is simple, then $C_L(G)$=1.

2.5 General criteria for determining possible signatures of ramification types

Assume that $f : X \longrightarrow \mathbb{P}^1$ is a meromorphic function of degree n, where X is a compact Riemann surface of genus g. We have shown that if $\{a_1, a_2, \cdots, a_r\}$ is a set of branch points in $\mathbb{P}^1$, then the fundamental group $\pi_1(\mathbb{P}^1 - \{a_1, a_2, \cdots, a_r\}, x_0)$, where $x_0 \in \mathbb{P}^1 - \{a_1, a_2, \cdots, a_r\}$, acts transitively on the fiber $f^{-1}(x_0)$. Now, if $G = Mon(X, f)$, we are interested in the structure of the group G when the compact Riemann surface X is of genus $g \leqslant 2$ and the meromorphic function f can not be written as a composition of two homomorphic functions f_1 and f_2 where f_1 and f_2 are two functions of degree greater than or equal to two.

Definition 2.5.1. Let f be a function. Then f is called **indecomposable**, if f can not be written as a composition of two functions of degree greater than one. In the other words, f is decomposable if and only if $f = f_1 \circ f_2$ where the degree of f_1 and f_2 are both greater than one.

Theorem 2.5.2. Given a covering map $f : Y \longrightarrow X$ of a finite degree n. Then f is indecomposable if and only if the corresponding monodromy action is primitive.

Proof. A complete proof can be found in [36, p.47]. □

Definition 2.5.3. Suppose that $\{x_1, x_2, \cdots, x_r\}$ is a generating set such that

$$x_1.x_2.\cdots.x_r = 1.$$

If we set $d_i = |x_i|$, then we call $(d_1, d_2, \cdots, d_r)$ the **signature** of the tuple $(x_1, x_2, \cdots, x_r)$. To standardize matters we generally assume x_i such that $d_1 \leqslant \cdots \leqslant d_r$.

Definition 2.5.4. Assume that G is a transitive group of S_n. A genus g- system is a tuple $\bar{x} = (x_1, x_2, \cdots, x_r)$ such that for all $1 \neq x_i \in G$ $1 \leqslant i \leqslant r$, $x_1, \cdots x_r = 1$ and $G = \langle x_i | 1 \leqslant i \leqslant r \rangle$

$$\sum_{i=1}^{r} ind(x_i) \neq 2(n+g-1). \quad (7)$$

Note that a tuple $\bar{x}$ is said to be non-genus g-system if $1 \leqslant i \leqslant r$, $x_1, \cdots x_r \neq 1$, $G \neq \langle x_i | 1 \leqslant i \leqslant r \rangle$ or $\sum_{i=1}^{r} ind(x_i) = 2(n+g-1)$. Furthermore we say a group G is not of type $\bar{x}$ if and only if $\bar{x}$ is a non-genus g-system.

Theorem 2.5.5 (Ree). Assume that G acts transitively on a set of size n. If $x_1, x_2, \cdots, x_r$ are permutations generating G with $x_1.x_2.\cdots.x_r = 1$, then $O_1 + O_2 + \cdots + O_r \leqslant (r-2)n+2$ where O_i is the number of orbits of $\langle x_i \rangle$.

Ree's Theorem is a consequence of the Riemann Hurwitz formula but can be proved independent by see for example [6]. The Ree Theorem means that if $x_1, x_2, \cdots, x_r$ are permutations generating a transitive group on a set of size n, then sum of the numbers of cycles of the x_i is less than or equal to $(r-2)n+2$. We will illustrate this with the following example.

Example 2.5.6. In the Mathieu group M_{12} if $(x_1,x_2,\cdots,x_r) \in M_{12}^r$, in its action on the right cosets of maximal subgroup M_{11}. M_{12} is not of types $(2B,3A,d)$, and $(2B,4B,d)$ where d representative conjugacy classes of any order. As $(O_{2B}=8)+(O_{3A}=6)+O_d > (r-2)n+2 = 14$. So Ree's transitivity condition fails. Hence M_{12} is not of type $(2B,3A,d)$. Similarly, M_{12} can not be of type $(2B,4B,d)$.

Definition 2.5.7. Let $\bar{x} = (x_1,x_2,\cdots,x_r)$ be a tuple of elements of order $d_1,d_2,\cdots,d_r$ respectively. The **Zariski number** denoted by $A(\bar{x})$ and defined by

$$A(\bar{x}) = \sum_{i=1}^{r} \frac{d_i - 1}{d_i}.$$

Proposition 2.5.8. (**Zariski Condition**)[26]

Let G be a finite group acts transitively and faithfully on Ω. Suppose that $\bar{x} = (x_1,x_2,\cdots,x_r)$ is an admissible tuple, where $r \geqslant 3$. Then $A(\bar{x}) \geqslant \frac{85}{42}$.

Proof. A complete proof can be found in [26] □

Definition 2.5.9. Assume that G is a finite group. The **symmetric genus** of G is denoted to be $g(G)$ and defined by the smallest integer g such that G acts faithfully as automorphisms of the surface and orientably on a closed orientable surface $S_{g(G)}$ of genus $g(G)$.

The symmetric genus of G is given by $g(G) = \frac{|G|}{2}(N-2)+1$, where

$$N = \min_{\bar{x}}\{A(\bar{d}) \mid \bar{d} = signature(\bar{x}), G = \langle \bar{x} \rangle, \prod_{\bar{x}} x_i = 1\}.$$

Theorem 2.5.10. (**Marston Conder**)

A. The symmetric genus of Mathieu group M_{11} is 631 with a minimal genus action arising from $(2,4,11)$ generation of M_{11}.

B. The symmetric genus of Mathieu group M_{12} is 3169 with a minimal genus action arising from $(2,3,10)$ generation of M_{12}.

C. The symmetric genus of Mathieu group M_{22} is 34849 with a minimal genus action arising from $(2,5,7)$ generation of M_{22}.

D. The symmetric genus of Mathieu group M_{23} is 1053361 with a minimal genus action arising from $(2,4,23)$ generation of M_{23}.

E. The symmetric genus of Mathieu group M_{24} is 10200961 with a minimal genus action arising from $(3,3,4)$ generation of M_{24}.

Complete proofs can be found in [6]. In the light of this theorem, we can eliminate some signatures of possible generating sets of the group G. On the other hand, the above theorem is not enough to find the final list of possible signatures. There exist some other techniques that we will explain later.

Theorem 2.5.11. Let X be a Riemann surface of genus zero, and G be a sporadic simple group. Then there is a non-constant meromorphic function f such that G is a composition factor of monodromy group $Mon(X,f)$ if and only if $F^*(G)$ is isomorphic of one of the elements of $\{M_{11}, M_{12}, M_{22}, M_{23}, M_{24}, J_1, J_2, HS, Co_3\}$.

The complete proof can be found [24]. The ideas contained in these theorems require knowledge about fixed point ratios of primitive permutation representations of the almost simple groups. In the next definition we will define fixed point ratio and upper bound of fixed point ratio.

Definition 2.5.12. Suppose that G acts on the set Ω. Then the **fixed point ratio** of $x \in G$ on a set Ω is defined by $\{\frac{f(x)}{n}\}$ where $f(x)$ is the number of fixed points of x on Ω and $n = |\Omega|$.

In our work we are interested in almost simple groups L with $F^*(L) = G$, and $x \in L$ acts by right translation on the right cosets of some maximal subgroups M of L. The number $\mathrm{b}(G)$ is defined by

$$\mathrm{b}(G) := \mathrm{Max}\{\frac{f(x)}{n} \mid n = [L:M]; M \not\geq G,\ x \in M\}.$$

is the least upper bound for all fixed point ratios of x occurring in any transitive G-action.

Example 2.5.13. The Mathieu group M_{11} has the following conjugacy classes of maximal subgroups: $M_{10}, L_2(11), M_9.2, S_5$ and M_8[7]. Recall, the set $\bar{\mathrm{C}} = C_{g_1}\ldots, C_{g_r}$ of conjugacy classes of a group G such that $g_i \in C_{g_i}$ is called ramification type of the cover $f: X \to P$. The Mathieu group M_{11} has 10 conjugacy classes which are $C = \{1A, 2A, 3A, 4A, 5A, 6A, 8A, 8B, 11A, 11B\}$. If $M = M_{10}$ then $[M_{11}:M] = 11$ and the number of fixed point of $g_i \in C = \{2A, 3A, 4A, 5A, 6A, 8A, 8B, 11A, 11B\}$ in M_{11} acting by translation on right coset of M_{10}, is $\{3,2,3,1,0,1,1,0,0\}$. Therefore the maximum fixed point ratio of in this action is equal to $\frac{3}{11}$. Similarly, the maximal fixed point ratios on other maximal subgroups $L_2(11), M_9.2, S_5, M_8$ are $\frac{4}{12}, \frac{7}{55}, \frac{10}{66}$ and $\frac{13}{165}$ respectively. Hence the maximal fixed ratio is $\frac{4}{12} = \frac{1}{3} = b(M_{11})$.

Definition 2.5.14. A group G is called a **genus g- group** if and only if there exists a compact Riemann surface X of genus g and meromorphic function $f : X \to \mathbb{P}^1$ such that $G = Mon(X, f)$. If a group G is a composition factor of, $Mon(M, f)$, then G is said to be **genus g composition factor**.

Lemma 2.5.15. Suppose that G is a permutation group acting on n-element the set Ω. Let $x_i \in G$ then the following holds.

1. $\text{ind}(x_i) = n - \sum_{j=1}^{d_i} \dfrac{1}{d_i} f(x_i^j)$ where $d_i = |x_i|$ and $f(x_i)$ is the number of fixed points of x_i on Ω ;

2. If $G = < x_1, x_2, \cdots, x_n >$ and $x_1.x_2.\cdots.x_n = 1$, then one the following are true.

   a. $A(\bar{x}) \geqslant \dfrac{85}{42}$.

   b. G is solvable group and G is of type $(2,3,6), (2,2,d), (2,4,4), (3,3,3)$ or $(2,2,2,2)$.

   c. G of type $(2,3,3)$ and $G \simeq A_4$.

   d. G of type $(2,3,4)$ and $G \simeq S_4$.

   e. G of type $(2,3,5)$ and $G \simeq A_5$.

Proof. Complete proof can be found in [12]. □

Definition 2.5.16. Let G be a group and let

$\bar{x} = (x_1, x_2, \cdots, x_r)$ such that $G = < x_1, x_2, \cdots, x_r >$ and $x_1.x_2.\cdots.x_r = 1$

$\bar{y} = (y_1, y_2, \cdots, y_s)$ such that $G = < y_1, y_2, \cdots, y_s >$ and $y_1.y_2.\cdots.y_s = 1$

Then we say $\bar{y} < \bar{x}$ if and only if $A(\bar{y}) < A(\bar{x})$ and $\bar{y}$ is minimal in G if $\bar{y}$ is a minimal with respect to $<$.

Recall the Riemann Hurwitz formula $\sum_{i=1}^{k} \text{ind}(x_i) = 2(n+g-1)$ where a group G has a subgroup M such that $[G : M] = n$ and $\bar{x} = (x_1, x_2, \cdots, x_r)$ is a genus g-system when $x_i \in G$ acts on the right cosets of M by right multiplication . The left side of Riemann Hurwitz formula can be written as $nA(\bar{x}) - nB(\bar{x})$ where $B(\bar{x}) = (\frac{1}{n}) \sum_{i=1}^{r} \sum_{j}^{d_i-1} \frac{f(x_i^j)}{d_i}$, $f(x_i)$ are the number of fixed points of x_i and $|x_i| = d_i$. Moreover $nB(\bar{x})$ can be bounded above by $A(\bar{x})b(G)$, hence $nA(\bar{x}) - nB(\bar{x}) > nA(\bar{x})(1 - b(G))$.

The following lemma is useful, it is proved in [24].

Lemma 2.5.17. Let G be a finite group and $\bar{x}$ be minimal in G. If $\frac{f(g)}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})}$ for all (g,M), then G is not a genus zero group.

Lemma 2.5.18. Let G be a finite group and $\bar{x}$ be minimal in G. If $\frac{f(g)}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})}$ for all (g,M), then G is not a genus one group.

Proof. Suppose that $\bar{y}$ is a generating genus one system. If G is a genus one group, then the Riemann Hurwitz formula implies that $\sum_{i=1}^{k} \operatorname{ind}(y_i) = 2(n+1-1) = 2n$. In the other hand, the left side of the Riemann Hurwitz formula can be written in the form

$$\sum_{i=1}^{k} \operatorname{ind}(y_i) = nA(\bar{y}) - nB(\bar{y}) > nA(\bar{y}) - nA(\bar{y})b(G).$$

Since $b(G) = \text{Max}(\frac{f(g)}{[G:M]})$, then $nA(\bar{y}) - nB(\bar{y}) > nA(\bar{y}) - nA(\bar{x})\frac{A(\bar{y})-2}{A(\bar{x})}$
therefore we get $\sum_{i=1}^{k} \operatorname{ind}(y_i) > 2n\frac{A(\bar{y})}{A(\bar{x})}$ but $\bar{x}$ is minimal then $\frac{A(\bar{y})}{A(\bar{x})} \geqslant 1$.
Hence $\sum_{i=1}^{k} \operatorname{ind}(y_i) > 2n$. Which is impossible because by hypothesis $\sum_{i=1}^{k} \operatorname{ind}(y_i) = 2n$.
Thus G is not a genus one group. □

In the next lemma we will show that the group G does not possess a genus two system if $\frac{f(g)}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})} - \frac{1}{[G:M]}$

Lemma 2.5.19. Let G be a finite group and assume that $\bar{x}$ be minimal in G. If $\frac{f(g)}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})} - \frac{1}{[G:M]}$ for all (g,M), then G is not a genus two group.

Proof. If G were a genus two group with genus 2-system $\bar{y}$, then the Riemann Hurwitz formula would imply that $\sum_{i=1}^{k} \operatorname{ind}(y_i) = 2(n+2-1) = 2n+2$. The left hand side of the Riemann Hurwitz formula can be written in the form

$$\sum_{i=1}^{k} \text{ind}(y_i) = nA(\bar{y}) - nB(\bar{y}) > nA(\bar{x}) - nA(\bar{x})b(G).$$

where $b(G) = Max(\frac{f(g)}{[G:M]})$. By hypothesis $b(G) < \frac{A(\bar{x})-2}{A(\bar{x})} - \frac{1}{[G:M]}$. We have

$$\sum_{i=1}^{k} \text{ind}(y_i) > nA(\bar{y}) - nA(\bar{y})(\frac{A(\bar{x})-2}{A(\bar{x})} - \frac{1}{[G:M]}) = 2n\frac{A(\bar{y})}{A(\bar{x})} + A(\bar{y}).$$

Thus $\sum_{i=1}^{k} \text{ind}(y_i) > 2n\frac{A(\bar{y})}{A(\bar{x})} + A(\bar{y})$. Since $\frac{A(\bar{y})}{A(\bar{x})} \geqslant 1$ by minimality of x̄.

So, $2n+2 = \sum_{i=1}^{k} \text{ind}(y_i) > 2n + A(\bar{y}) > 2n+2$ if and only if $A(\bar{y}) \geqslant \frac{85}{42}$. which is contradiction.
Hence G is not a genus two group □

Note that if any system x̄ satisfying the condition of Lemma 2.5.19 above, then it is not a genus zero or one system i.e If $\frac{f(g)}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})} - \frac{1}{[G:M]} < \frac{A(\bar{x})-2}{A(\bar{x})}$, then by Lemma 2.5.17 and Lemma2.5.18, x̄ is not a genus zero or one system.

Lemma 2.5.20. Let G be a finite group and M a subgroup of G, then following hold

1. If $(\frac{1}{[G:M]}) \sum_{i=1}^{r} \sum_{j}^{d_i-1} \frac{f(x_i^j)}{d_i} < A(\bar{x}) - 2$, then $\bar{x}$ is not genus zero system,
2. If $(\frac{1}{[G:M]}) \sum_{i=1}^{r} \sum_{j}^{d_i-1} \frac{f(x_i^j)}{d_i} \neq A(\bar{x}) - 2$ then $\bar{x}$ is not genus one system,
3. if $(\frac{1}{[G:M]}) \sum_{i=1}^{r} \sum_{j}^{d_i-1} \frac{f(x_i^j)}{d_i} > A(\bar{x}) - 2$. then $\bar{x}$ is not genus two system,

Proof. 1. Suppose that $\bar{x}$ is a genus zero system then by the Riemann Hurwitz formula $\sum_{i=1}^{r} ind(x_i) = 2n-2$. Since

$$ind(x_i) = n - \sum_{j=1}^{d_i} \frac{f(x_i^j)}{d_i}$$

$$= n - (\sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} + \frac{f(x_i^{d_i})}{d_i})$$

$$= n - \frac{n}{d_i} - \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i}.$$

So

$$ind(x_i) = n(\frac{d_i - 1}{d_i}) - \sum_{j=1}^{d_i - 1} \frac{f(x_i^j)}{d_i},$$

therefore we get

$$\sum_{i=1}^{r} ind(x_i) = n \sum_{i=1}^{r} \frac{d_i - 1}{d_i} - \sum_{i=1}^{r} \sum_{j=1}^{d_i - 1} \frac{f(x_i^j)}{d_i}.$$

Finally, we get $\frac{1}{n} \sum_{i=1}^{r} \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} = A(\bar{x}) - 2 + \frac{2}{n}$ as $\sum_{i=1}^{r} ind(x_i) = 2n - 2$.

Hence $\bar{x}$ is a genus zero system if and only if $\frac{1}{[G:M]} \sum_{i=1}^{r} \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} = A(\bar{x}) - 2 + \frac{2}{n}$.

It is clear that $[G:M] = n \geqslant 1$. So if $(\frac{1}{[G:M]}) \sum_{i=1}^{r} \sum_{j} \frac{f(x_i^j)}{d_i} < A(\bar{x}) - 2$, then the Riemann Hurwitz formula fails. Hence the claim.

2. Suppose that $\bar{x}$ is a genus one system. Then by the Riemann Hurwitz formula $\sum_{i=1}^{r} ind(x_i) = 2n$.

   Similarly,

$$2n = \sum_{i=1}^{r} ind(x_i) = n \sum_{i=1}^{r} \frac{d_i - 1}{d_i} - \sum_{i=1}^{r} \sum_{j=1}^{d_i - 1} \frac{f(x_i^j)}{d_i}.$$

   Thus

$$\frac{1}{[G:M]} \sum_{i=1}^{r} \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} = A(\bar{x}) - 2.$$

   In the other words if

$$\frac{1}{[G:M]} \sum_{i=1}^{r} \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} \neq A(\bar{x}) - 2,$$

   then the Riemann Hurwitz formula fails. Hence $\bar{x}$ is not a genus one system

3. Suppose that $\bar{x}$ is a genus two system. Then by the Riemann Hurwitz formula $\sum_{i=1}^{r} ind(x_i) = 2n + 2$.

   So

$$\frac{1}{[G:M]} \sum_{i=1}^{r} \sum_{j=1}^{d_i-1} \frac{f(x_i^j)}{d_i} = A(\bar{x}) - 2 - \frac{2}{n}.$$

   In the other words, $\bar{x}$ is not a genus two system, if

$$(\frac{1}{[G:M]}) \sum_{i=1}^{r} \sum_{j} \frac{f(x_i^j)}{d_i} > A(\bar{x}) - 2.$$

□

Note that the above lemmas can be used to eliminate some systems, which are not of genus zero, one and two.

Lemma 2.5.21. Let $f : X \to Y$ be a Galois cover with group of deck transformation G and let A,B be two proper subgroups of G. If 1_A^G is submodule of 1_B^G, then $g(X/A) \leqslant g(X/B)$.

A complete proof of 2.5.21 can be found in [11]. If M_1 and M_2 are non-conjugate maximal subgroups of the group G affording permutation characters χ_1 and χ_2 respectively such that χ_1 lies in χ_2, then any systems eliminated as possible low genus systems in their action on the cosets of M_1, are also eliminated as potential low genus systems in their action on the cosets of M_2.

In this thesis we will determine all possible signatures of genus zero, one and two systems of sporadic simple groups. A series of filters will be used to eliminate signatures. Now we will present the main filters and typical arguments which we employ to eliminate signatures .

1. Riemann Hurwitz formula
2. Theorem 2.5.5(Ree Theorem)
3. Zariski Condition
4. For Mathieu groups $(M_{11}, M_{12}, M_{22}, M_{23}, M_{24})$ using Theorem2.5.10(Marston Conder Theorem)
5. The group algebra structure constant
6. Lemma 2.5.15
7. Lemma 2.5.17
8. Lemma 2.5.18
9. Lemma 2.5.19.
10. Lemma 2.5.21.

CHAPTER 3

POSSIBLE RAMIFICATION TYPES FOR THE SPORADIC SIMPLE GROUPS

The list of sporadic simple groups contains 26 groups. The Mathieu groups M_{11}, M_{12}, M_{22}, M_{23} and M_{24} were discovered by Mathieu $(1861, 1873)$[7]. They are the earliest sporadic simple groups to be discovered.

A **Steiner system** $S(t,k,v)$ is a finite set X of v points together with a set of k-element subset of X (called blocks and denoted by B) with the property that every t-points of the set X is contained in a unique block[8].

The automorphism group of the unique Steiner system $S(5,8,24)$ is M_{24}. The stabilizer of a point is the group M_{23} of order $|M_{24}|/24 = 10200960$. In fact M_{23} is the group of automorphisms of the Steiner system $S(4,7,23)$. The group M_{22} is the pointwise stabilizer in M_{24} of two points. $M_{22} = |M_{23}|/23 = 443520$. The pointwise stabilizer in M_{24} of three points is the group M_{21}, of order $|M_{22}|/22 = 20160$, and is isomorphic to the group $PSL_3(4)$ [7]. The Mathieu groups M_{24}, M_{23}, M_{22} together are called the large Mathieu groups. Moreover, the automorphism of the group M_{22} is a maximal subgroups of M_{24}. The embedding of the group $M_{22}:2$ in the group M_{24} has orbit shape 2+22[8].

A **binary linear code** $\mathscr{C}$ based on a finite set A is a subspace of the power set 2^A . The size of finite set A is called length of linear code $\mathscr{C}$. A triple $(\mathscr{C}, A, V)$ is a linear code over $GF(q)$ where V is a vector space over $GF(q)$, A is a basis of V and $\mathscr{C}$ subspace in V. Moreover, the number of the elements of the smallest non-empty subset in the code $\mathscr{C}$ is called the minimal weight of $\mathscr{C}$. If number of elements of every subset in $\mathscr{C}$ is even then the code is called even. The orthogonal complement of the code $\mathscr{C}$ with respect to the parity form

$$\mathscr{C}^* = \{C| \ C \in 2^A, |C \cap B| \text{ is even for all B} \in \mathscr{C}\}$$

is called dual code $\mathscr{C}^*$ of the code $\mathscr{C}$. The code $\mathscr{C}$ is called self-dual if $\mathscr{C} = \mathscr{C}^*$. A **Golay code** is a self-dual code ς of length 24 in which the minimal weight $\geqslant 8$.

The stabilizer in M_{24} of one of a Golay code word of weight 12, is the group $M \simeq M_{12}$ of order $|M_{24}|/2576 = 9540$. This fact was discovered by Frobenius using character theory. It is also known that $N_{M_{24}}(M) \simeq Aut(M_{12})$. M_{12} has a natural permutation representation of degree 12. The point sets in M_{12} in it degree 12 action is the smallest Mathieu group M_{11}. It's sharply 4-transitive permutation group on 11 points. Furthermore, the group M_{11} is one of the maximal subgroup of M_{23}.
The Janko groups J_1, J_2, J_3 and J_4 were discovered in $(1965, 1975)$[7]. The smallest Janko group J_1 was discovered by Zvonimir Janko around one hundred years after the first Mathieu group was discovered.[20]. The story of discovering of the smallest Janko group J_1 begins with the centralizer involution of group of the Ree group of Lie type. It was shown that if a is an involution of the Ree group G then $C_G(a)$ is isomorphic to external direct product $Z_2 \times PSL_2(3^n)$ and the Sylow 2-subgroups are elementary abelian groups of order eight. Conversely it has been established that all simple groups with Sylow 2-subgroup of order eight which have centralizer involution isomorphic to $Z_2 \times PSL_2(p^n)$, p and odd prime then $p = 3$, $n = 2r+1$ or $p^n = 5$. Janko showed that if G is a simple group such that G has elementary abilean Sylow 2-subgroups of order 8 and if a is an involution in G with $C(a) \cong Z_2 \times PSL_2(5)$, then G is isomorphic to smallest Janko group J_1[20]. The group J_1 has a trivial outer automorphism.
Z. Janko in [20] indicated that there are two new simple groups in which $2^{1+4} : A_5$ is a centralizer of an involution. These two new simple group are J_2 and J_3. The Janko group J_2 was found by M.Hall and D.Wales (1967) as a rank 3 permutation group on 100 points [16]. The group J_2 has non trivial outer automorphism and it is the only one of the four Janko groups that is involved in the Monster group. The Janko group J_3 was constructed by G.Higman and J.Mckay in 1969 [18]. Similarly, the largest Janko group J_4 was constructed during the proof

of the classification theorem of finite simple group. It was discovered by Z. Janko by looking for groups with an involution centralizer of the form $2^{1+12}.3.(M_{22}:2)$[21]. He showed that if a simple group G exists with involution centralizer of the form $2^{1+12}.3.(M_{22}:2)$ then G contains a subgroup of the form $2^{11}:M_{24}$. By this time, much of the proofs of the classification theorem of finite group had been completed however, studying groups with an involution centralizer remind[38]. The group J_4 has trivial outer automorphism and it is not involved in the Monster group.

A non empty set L is said to be a **lattice** if L is a finitely generated free Z-module with an integer valued bilinear form, written (a,b) for $a,b \in L$[4]. The lattice is said to be **integral** if (a,b) takes integer value. An integral lattice L is said to be a **unimodular lattice** if it is of determinate 1 or -1. The even unimodular lattice in 24-dimension Euclidean space is said to be a **Leech lattice**[4].

The Conway groups Co_1, Co_2, Co_3 can be derived from the Leech lattice. The largest Conway group Co_1 is the automorphism group of Leech lattice, modulo a center of order two, which was discovered by J.H Conway[7]. The outer automorphism group of the group Co_1 is trivial. By reducing modulo 2 and factoring out a fixed vector we get the group Co_2 which is maximal subgroup of the group Co_1. The group Co_3 is occurred as a subgroup of automorphism Leech lattice fixing a vector of "type 3". Based in the fact that the Co_3 is maximal subgroup in the largest Conway group Co_1, the group Co_3 has 2-transitive action of degree 276. The single point stabilizer of Co_3 of this action is the automorphism group of the McLaughlin group (McL) . The group (McL) was found by McLaughlin as a permutation group acting on the McLaughlin graph with 275 = 1 + 112 + 162 vertices [7]. The group Higman-Sims (HS) is a maximal subgroup of the group Co_3 which is discovered by Donald G. Higman and Charles C. Sims [7]. They derived their groups as a rank 3 primitive permutation group of degree 100. The Suzuki (Suz) group was found by M.Suzuki [7]. It also can be obtained from the Leech lattice. B. Fischer- R.L. Griess- M.P.Thorne, using the existence of Co_1 predicted the existent of a simple group with involution centralizer $2^{1+24}.Co_1$ (where 2^{1+24} denotes an extra special group of order 2^{25}) and constructed its character table. In 1980 R.L. Griess showed that this group exists and can be shown to be the automorphism group of a 196884 dimensional algebra. This is called the monster group. 20 of the sporadic simple group are involved in the Monster. The remaining six are called **pariahs**.

The Fischer groups Fi_{22},Fi_{23} and Fi_{24} are another list of the sporadic groups. They are subquotients of the Monster group. Fisher's Theorem[9] classified 3-transposition groups (a group generated by a conjugacy class of involution in which the product of any two non-commutative involutions has order three). The Fisher groups are special cases of Fisher Theorem. Further, the Fisher groups Fi_{22},Fi_{24} has non-trivial outer automorphisms.

Dieter Held in paper [17] was looking for the simple groups with the property, that this simple group containing an involution z such that centralizer of z is isomorphic to that of an involution in the group M_{24}. During his investigation he obtained the group Held (He). The group Rudvalis (Ru) , Harada-Norton (HN), Thompson (Th), Baby Monster (B), O'Nan (ON), and Lyons (Ly), are the large sporadic simple groups.

In this thesis we will compute braid orbits of Nielsen class of sporadic almost simple groups. Recall that a function f is indecomposable if f can not be written as a composition of two functions of degree greater than one. In our work we are interested in the structure of the monodromy group G of f when X is a compact Riemann surface of genus $g \leqslant 2$ and f is indecomposable meromorphic function. We recall that the monodromy group is primitive in its monodromy action on the fiber over the base point if and only if the corresponding cover of it is indecomposable. In light of this statement we are interested in finding all equivalence classes of admissible generating tuples for sporadic simple groups when the genus of the cover is 0,1 or 2. Recall definition genus g-system is defined as follows.

Definition 3.0.22. Assume that G is a transitive group of S_n. A genus g- system is a tuple $\bar{x} = (x_1, x_2, \cdots, x_r)$ such that for all $1 \neq x_i \in G$, $1 \leqslant i \leqslant r$, $x_1, \cdots x_r = 1$ and $G = \langle x_i | 1 \leqslant i \leqslant r \rangle$ and

$$\sum_{i=1}^{r} ind(x_i) = 2(n+g-1). \quad (1)$$

If G acts primitively, then the genus g system is called a **primitive genus** g **system**. Furthermore, a primitive genus g system is called a primitive low genus system if $g \leqslant 2$.

The small sporadic simple groups possess primitive permutation representation of degree $\lneq$2500 and these are stored, for example, in GAP.

We use the function **AllPrimitiveGroups(DegreeOperation,n)** to retrieve a copy of a sporadic simple group, which acts primitively on n points. We present the code and explain the following example.

Example 3.0.23. gap> `AllPrimitiveGroups(DegreeOperation,11);;`

```
[ C(11), D(2*11), 11:5, AGL(1, 11), L(2, 11), M(11), A(11), S(11) ]
gap> k:=last[6];
M(11)
```

Here we retrieve the sporadic Mathieu group M_{11} in its action on 11 points.

Each sporadic simple groups has several conjugacy classes of maximal subgroups. We record the number of conjugacy classes of maximal subgroups in the following table

Table 3.1: Number of classes of maximal subgroups of sporadic simple groups

Groups	No. Maximal Subgroups	Groups	No.Maximal Subgroups	Groups	No.Maximal Subgroups
M_{11}	5	M_{12}	7	M_{22}	7
M_{23}	6	M_{24}	9	J_1	7
J_2	9	J_3	8	J_4	11
Co_1	24	Co_2	11	Co_3	14
Fi_{22}	14	Fi_{23}	14	Fi_{24}	20
Suz	17	HS	11	McL	12
He	11	HN	14	Th	15
B	28	ON	9	Ly	9
Ru	15	M	unknown		

The GAP library stores primitive permutation groups of degree up to 2500 but some sporadic simple groups only have maximal subgroups of index more than 2500. Sometime it is convenient for us to construct permutation representations of such group of large degree. We explain how we do this via the following example.

Example 3.0.24. The smallest maximal subgroups of the Janko group J_2 are isomorphic to A_5. The degree of operation of G of this class A_5 is 10080 as 10080 = $|J2|/|A5|$. Also, there are up to conjugacy 3 distinct embeddings of $A5$ into J_2.

In GAP we can proceed as follows:

- Get J_2 and A_5 as follows

```
gap>J2 := UnderlyingGroup(TableOfMarks("J2"));
<permutation group of size 604800 with 2 generators>
gap> A5 := AlternatingGroup(5);
```

```
Alt( [ 1 ..  5 ] )
```

- Find the embeddings of A_5 into J_2 up to conjugacy.

```
gap> embs := IsomorphicSubgroups(J2,A5);;
gap> acts := List(embs,phi->Action(J2,RightCosets(J2,Image(phi)),OnRight));
<permutation group with 2 generators>, <permutation group with 2 generators>,
<permutation group with 2 generators>
```

- Check which are maximal

```
gap> for i in[1..Length(act)] do
> if Order(act[i])= Order(J2) then
> if IsPrimitive(act[i]) then Print(i)
> fi;fi;od;
3
```

- Finally, find all conjugacy class representative then find indices of representative

```
gap> cclreps := List(acts[3],G->List(ConjugacyClasses(G),Representative));;
gap> indices := List(cclreps,reps->List(reps,
g->10080-Length (Orbits(Group(g),[1..10080]))));
[ 0, 5010, 6720, 8390, 6480, 8064, 8064, 9360, 9360, 8640, 9066, 9066,
5040, 7560, 8820, 8280, 9180, 8056, 8056, 9068, 9068 ]
gap>ll := [];
[];
gap>for i in [1..Length(acts)] do
>ll[i] := [];
>for j in [2..Length(cclreps[i])] do
>Add(ll[i],rec(pos:=j,index:=indices[i][j],ord:=Order(cclreps[i][j])));
od;
od;
```

3.1 Possible Ramification Types

In this section we aim to determine all possible ramification types for sporadic simple groups of genus zero, one and two. Recall that Ree's theorem states that, if G is a transitive group on a set of size n and $x_1, x_2, \cdots, x_r$ are permutations generating G with $x_1.x_2.\cdots.x_r = 1$, then $O_1 + O_2 + \cdots + O_r \leqslant (r-2)n + 2$ where O_i is the number of orbits $\langle x_i \rangle$. In the light of Ree's theorem we can expect to eliminate certain potential ramification types from further consideration.

In our work we use the Riemann Hurwitz formula(equation 1) to identify the possible ramification types. It should be noted that the right side of Riemann Hurwitz formula (1), is easy to compute. To find left side of the Riemann Hurwitz formula we first, by using GAP compute all conjugacy class representatives of the groups. Next we compute the index for each conjugacy class representative on every type primitive permutation representation. We define a **tuple**, which is a list of permutation indices such that the sum is equal the right hand side of the Riemann Hurwitz formula. We will explain this using the following example

Example 3.1.1.

```
gap> AllPrimitiveGroups(DegreeOperation,11);;
[ C(11), D(2*11), 11:5, AGL(1, 11), L(2, 11), M(11), A(11), S(11) ]
gap> k:=last[6];
gap> reps:=List(ConjugacyClasses(k),x->Representative(x));;
gap> Ind:=List(reps,x->11-Length(Orbits(Group(x),[1..11])));
[0,8,4,6,8,8,6,8,10,10]
ll:=[];
[]
gap> for i in [2..Length(reps)] do
> Append(ll,[rec(pos:=i,index:=Ind[i],ord:=Order(reps[i]))]);
> od;
gap>ll;
[rec( index := 8, ord := 5, pos := 2 ), rec( index := 4, ord := 2, pos := 3
), rec( index := 6, ord := 4, pos := 4 ), rec( index := 8, ord := 8, pos :=
5 ), rec( index := 8, ord := 8, pos := 6 ), rec( index := 6, ord := 3, pos
```

```
:= 7 ), rec( index := 8, ord := 6, pos := 8 ), rec( index := 10, ord := 11,
pos := 9 ), rec( index := 10, ord := 11, pos := 10 ) ]
indes:=List(ll,x->x.index);
tuple:=RestrictedPartitions(20,indes);
```

[4,8,8], [4,4,4,8], [4,4,4,4,4], [6,6,8], [6,6,4,4], [8,4,8], [8,4,4,4], [8,6,6], [8,8,4], [8,4,8], [8,4,4,4], [8,6,6], [8,8,4], [8,8,4], [6,6,8], [6,6,4,4], [6,8,6], [6,8,6], [6,6,8], [6,6,4,4], [6,6,8], [6,6,8], [8,4,8], [8,4,4,4], [8,6,6], [8,8,4], [8,8,4], [8,6,6], [8,6,6], [8,8,4], [10,6,4], [10,6,4], [10,10], [10,6,4], [10,6,4], [10,10], [10,10]]

Now, we present the number of ramification types for the sporadic simple groups of genus zero, one and two system which satisfy Riemann Hurwitz formula 1.

Table 3.2: **Possible ramification type of Mathieu groups**

Groups	genus zero	genus one	genus two	total
M_{11}	79	109	155	343
M_{12}	154	256	342	752
$M_{12}:2$	16	27	7	50
M_{22}	34	43	58	135
$M_{22}:2$	177	214	255	646
M_{23}	54	55	93	202
M_{24}	162	264	308	734

For some large sporadic simple groups we will prove later that they have no ramification

Table 3.3: **Possible ramification type of Janko groups**

Groups	genus zero	genus one	genus two	total
J_1	1	0	0	1
J_2	21	45	32	98
$J_2:2$	39	51	47	137
J_3	we will prove that it has no ramification type			
J_4	we will prove that it has no ramification type			

Table 3.4: **Possible ramification type of Conway groups**

Groups	genus zero	genus one	genus two	total
Co_3	15	16	19	50
Co_2	6	4	5	15
$Co_2:2$	0	1	0	1
Co_1	5	1	0	6

Table 3.5: **Possible ramification type of Large Sporadic groups**

Groups	genus zero	genus one	genus two	total
HS	35	49	27	111
$HS:2$	86	109	106	301
McL	5	6	3	14
$McL:2$	6	10	11	27
Suz	1	10	0	11
$Suz:2$	2	9	3	14
He	0	7	1	8
$He:2$	0	8	1	9
Fi_{22}	2	12	1	15
$Fi_22:2$	4	18	13	25
Fi_{23}	0	1	0	1
Fi_{24}	we will prove that it has not ramification type			
ON	we will prove that it has no ramification type			
Th	we will prove that it has no ramification type			
Ly	we will prove that it has no ramification type			
Ru	we will prove that it has no ramification type			
B	we will prove that it has no ramification type			
M	we will prove that it has no ramification type			

In lower case next step we eliminate ramification types by using the filters that we presented in the previous chapter. We also present some additional filters.

3.1.1 The ClassStructureCharacterTable function

Let G be a group and $(g_1,g_2,\ldots,g_r)$ be a r-tuple in G with $g_1.g_2.\cdots g_r = 1$. Let $C_1,C_2,\ldots,C_r$ conjugacy class of the group G such that g_i is in C_i then the number of r-tuples $(g_1,g_2,\ldots,g_r)$ is computed by this formula

$$N(C_1,C_2,\ldots,C_r) = \frac{|C_1||C_2|\ldots|C_r|}{|G|}\sum\frac{\chi(g_1)\chi(g_2)\ldots\chi(g_r)}{\chi(1)^{r-2}} \qquad (2)$$

We use the function `ClassstructureCharacterTable` to compute the value $N(C_1,C_2,\ldots,C_r)$ for the tuples surviving the first filters . If the group algebra structure constant of any tuple is equal to zero then we remove this tuple from our candidate list. For example the Mathieu group M_{12} not of type $(2,3,10)$ and $(2,2,2,3)$ because a GAP calculation shows that structure constants of $(2,3,10)$ and $(2,2,2,3)$ are equal to zero. Thus these cases do not need to be considered.

3.1.2 Generating Group Criterion

Recall that the symmetric genus of a finite group G is the smallest integer g such that G acts faithfully as automorphisms of an oriented a closed orientable surface $S_{g(G)}$ of genus $g(G)$. The symmetric genus of G can be calculated by using the formula $g(G) = \frac{|G|}{2}(N-2)+1$, where

$$N = \min_{\bar{x}}\{A(\overline{d}) \mid \overline{d} = signature(\bar{x}), G = \langle \bar{x} \rangle, \prod_{\bar{x}} x_i = 1\}.$$

Using the GAP program, it is a straightforward exercise to calculate for all triples (x,y,z) within G such that $x \cdot y = z^{-1}$. In other words, if any triple $(x,y,z) \in G$ does not satisfy $x \cdot y \cdot z = 1$ it will be eliminated. Moreover, if the triple (x,y,z) passes this step, the second step we will examine the order of the group generated by pair (x,y) , if it does not equal to the order of group G, then we will ignore the triple (x,y,z). By using the following program we obtain above results

```
gap> AllPrimitiveGroups(DegreeOperation,11);;
gap> k:=last[6];
gap>reps:=List(ConjugacyClasses(k),x->Representative(x));;
gap> ind:=List(reps,x-> NrMovedPoints(k) -
Length(Orbits(Group(x),[1..NrMovedPoints(k)])));;
gap> elts2:=Elements(ConjugacyClass(k,reps[a]));;
gap> ff:=Filtered(elts2,x-> IsConjugate(k,x*reps[b],reps[c]^-1));;
gap> fff:=Filtered(ff,x-> Size(Group(x,reps[b])) = Size(k));;
gap> Length(fff);
```

We call this criterion the generating group criteria.

3.2 The ramification types of Mathieu groups

In this section we eliminate some ramification types of Mathieu groups by using the series of filters which were mentioned in the previous chapter and in the section 3.1.

3.2.1 The Mathieu group M_{11}

The smallest Mathieu group M_{11} is of order 7920. The classes of maximal subgroups of M_{11} are given in the first row of the following table.

Maximal subgroups	M_{10}	$L_2(11)$	$M_9 : 2$	S_5	$M_8 : S_3$
Index in G	11	12	55	66	165
N.R.T	165	150	14	12	2

The Riemann Hurwitz formula implies that the Mathieu group M_{11} has 343 ramification types. The details are given in the third row of the above table. By using the Zariski condition we can eliminate three ramification types of G in its action on the first maximal subgroups and one ramification type in its action on the third maximal subgroup.

Lemma 3.2.1. Let G be the Mathieu group M_{11} and $\bar{x} = (x_1, \cdots x_r) \in G^r$. Then $\bar{x}$ is not a genus zero, one and two-system if $A(\bar{x}) < \dfrac{95}{44}$, where $A(\bar{x}) = \sum_{i=1}^{r} \dfrac{d_i - 1}{d_i}$.

Proof. By Theorem 2.5.10 the minimal genus action is achieved by the ramification type $(2,4,11)$-generation of M_{11}. This means that if $A(\bar{x}) < \dfrac{1}{2} + \dfrac{3}{4} + \dfrac{10}{11} = \dfrac{95}{44}$, then M_{11} is not a genus zero, one and two-group. □

By using Lemma 3.2.1 we eliminate 5, 2 resp 6 ramification types of G in its action on the 1st, 2nd resp 3rd maximal subgroup. Next, by using generating group criteria we check that G is not of type $(2,6,6)$. Indeed given an element z of order six, there are conjugacy classes of pairs (x,y) of order two and six such that $xy = z^{-1}$. However, the order of the groups which are generated by conjugacy classes of the pair(x,y) are not equal to the order of the group M_{11}. It follows that the triple $(2,6,6)$ can be eliminated. Using similar considerations we eliminate 11 ramification types of first maximal subgroup ,7 ramification types of second maximal subgroup and 5 ramification types of third maximal subgroup. Finally, by using Lemma 2.5.21, we eliminate 10 ramification types of the maximal subgroup S_5 and two ramification types of maximal subgroup $M_8 : S_3$. The final list of ramification types which survive the filters is presented in the following table.

Maximal subgroups	M_{10}	$L_2(11)$	$M_9:2$	S_5	$M_8:S_3$
Indexes	11	12	55	66	165
N.R.T	146	141	2	2	0

3.2.2 The Mathieu group M_{12} and its Automorphism group

Let G be an almost simple group such that $F^*(G) = M_{12}$. The Mathieu group M_{12} has eight classes of maximal subgroups which are given in the first row in the Table 3.6. The indexes and number of ramification types are give in rows two and three respectively.

Table 3.6:

Maximal subgroups	M_{11}	$M_{10}:2$	$L_2(11)$	$M_9:S_3$	$2\times S_5$	$M_8:S_4$	$4^2:D_{12}$	$A_4\times S_3$
Indexes in G	12	66	144	220	396	495	495	1320
N.R.T	648	62	17	8	5	7	5	0

Lemma 3.2.2. Assume that G is the Mathieu group M_{12} and $(x_1,x_2,\cdots,x_r)\in G^r$. Then $\bar{x} = (x_1,\cdots x_r)$ is not genus zero, one and two-system if $A(\bar{x}) < \frac{31}{15}$, where $A(\bar{x}) = \sum_{i=1}^{r}\frac{d_i-1}{d_i}$.

Proof. This follows from the Theorem 2.5.10. Since the Mathieu group M_{12} can be generated by a triple of elements of order 2, 3 and 10 [6], which means that if $A(\bar{x}) < \frac{1}{2}+\frac{2}{3}+\frac{9}{10} = \frac{31}{15}$, then M_{12} is not genus zero, one and two-system. □

A GAP calculation shows that the group algebra structure constant of 20 ramification types of the first maximal subgroup, 9 ramification types of the second maximal subgroup and 2 ramification types of the fourth maximal subgroup in the Table3.6 above are equal to zero. Thus, all of these can not occur. By using the Zariski condition and Lemma 3.2.2 above we can eliminate 32, 16, 2 respectively 5 ramification types of G in its action on the 1st, 2nd, 3rd respectively 4th maximal subgroup. Assume that x, y and z are representatives of the conjugacy class of elements of order four. So if x, y are conjugacy classes in the same type or in different type then there are conjugacy classes of pairs (x, y) whose product is equal to z^{-1}. In both cases the order of the group $\langle x, y\rangle$ is different form the order of the Mathieu group M_{12}. Thus G is not of type $(4,4,4)$. Using similar considerations we eliminate a further 27 ramification types of the first maximal subgroups , 24 ramification types of the second maximal subgroup and 12 ramification types of the third maximal subgroup. Finally by using Lemma 2.5.21 all ramification types of the maximal subgroups $2 \times S_5$, $M_8 : S_4$, $4^2 : D_{12}$ are eliminated. So, the final list of ramification types which passed the filters we present in the following table.

Table 3.7:

Maximal subgroups	M_{11}	$M_{10} : 2$	$L_2(11)$	$M_9 : S_3$	$2 \times S_5$	$M_8 : S_4$	$4^2 : D_{12}$	$A_4 \times S_3$
Indexes	12	66	144	220	396	495	495	1320
N.R.T	569	13	3	1	0	0	0	0

Now we will describe an analysis of $Aut(M_{12})$ similar to the one above. Let G be the almost simple group $Aut(M_{12}) = M_{12} : 2$. The classes of maximal subgroups of G , their indexes, and number of ramification types is presented in the following table.

Table 3.8:

Maximal subgroups	$L_2(11) : 2$	$(2^2 \times A_5) : 2$	$H.2$	$H.2$	$S_4 \times S_3$
Indexes	144	396	495	495	1320
N.R.T	56	9	6	3	0

Similarly, by using the Zariski condition, the ClassStructureCharacterTable function and the Generating group criterion all but eight ramification types in the first maximal subgroup in Table 3.8 can be eliminated .

3.2.3 The Mathieu group M_{22} and its Automorphism group

Let G be an almost simple group such that $F^*(G) = M_{22}$. The Mathieu group M_{22} has seven classes of maximal subgroups which are shown in the first row in Table 3.9. The indexes and number of ramification types, which are satisfying the Riemann-Hurwitz formula for $g = 0, 1$ or 2 are given in the rows two and three respectively.

Table 3.9:

Maximal subgroups	$L_3(4)$	$2^4 : A_6$	A_7	$2^4 : S_5$	$2^3 : L_2(3)$	M_{10}	$L_2(11)$
Indexes in G	22	77	176	231	330	616	672
N.R.T	104	19	5	3	2	2	0

Lemma 3.2.3. Assume that G is the Mathieu group M_{22} and $(x_1, x_2, \cdots, x_r) \in G^r$. Then $\bar{x} = (x_1, \cdots x_r)$ is not genus zero, one and two-system if $A(\bar{x}) < \frac{151}{70}$, where $A(\bar{x}) = \sum_{i=1}^{r} \frac{d_i - 1}{d_i}$.

Proof. Follows from Theorem 2.5.10. □

The number of ramification types additionally satisfying the condition of Lemma 3.2.3 and the Zariski condition is shown below

Maximal subgroups	$L_3(4)$	$2^4 : A_6$	A_7	$2^4 : S_5$	$2^3 : L_2(3)$	M_{10}	$L_2(11)$
N.R.T	104	14	1	1	0	0	0

One can check in GAP that two elements x, y of order 4,in the same class or in different type such that xy is also of order 4 never generate the whole group M_{22}. Thus G is not of type $(4,4,4)$. In a similar way two , eight,one respectively one ramification type in its action on the 1st, 2nd resp 3rd and 4th maximal subgroup can be canceled. So the final list of ramification types is presented in the following table

Maximal subgroups	$L_3(4)$	$2^4 : A_6$	A_7	$2^4 : S_5$	$2^3 : L_2(3)$	M_{10}	$L_2(11)$
Indexes	22	77	176	231	330	616	672
N.R.T	102	6	0	0	0	0	0

Now we will do the similar analysis for the automorphism group of M_{22}. Let G be the almost simple group $Aut(M_{22})$. The classes of maximal subgroups of G, their indexes and the number of ramification types is presented in the following table.

Table 3.10:

Maximal subgroups	$L_3(4):2_2$	$2^4 : S_6$	$2^5 : S_5$	$2^3 : L_2(3) \times 2$	$A_6.2^2$	$L_2(11):2$
Indexes in G	22	77	231	330	616	672
N.R.T	413	157	38	35	2	1

By using the generating group criterion 121 ramification types of the first maximal subgroup in the Table 3.10 can be ruled out. Lemma 2.5.21 implies that 133 ramification types of the second maximal subgroup and all ramification types of the other maximal subgroups can be ruled out. So the final result is presented in the following table.

Maximal subgroups	$L_3(4):2_2$	$2^4:S_6$	$2^5:S_5$	$2^3:L_2(3)\times 2$	$A_6.2^2$	$L_2(11):2$
N.R.T	291	24	0	0	0	0

3.2.4 The Mathieu group M_{23}

Let G be a Mathieu group M_{23}. The classes of maximal subgroups of M_{23}, their indexes and number of ramification types are given in the following table.

Maximal subgroups	M_{22}	$L_3(4):2_2$	$2^4:A_7$	A_8	M_{11}	$2^4:(3\times A_5):2$	$23:11$
Indexes	23	253	253	506	1288	1771	40320
N.R.T	182	10	10	0	0	0	0

Lemma 3.2.4. Assume that G is the Mathieu group M_{23} and $(x_1,x_2,\cdots,x_r)\in G^r$. Then $\bar{x}=(x_1,\cdots x_r)$ is not genus zero, one and two-system if $A(\bar{x})< \dfrac{203}{92}$, where $A(\bar{x})=\sum_{i=1}^{r}\dfrac{d_i-1}{d_i}$.

Proof. Similarly, by Theorem 2.5.10. □

Lemma 3.2.4 guarantees us that 4 ramification types of the first maximal subgroup and all ramification types of the second and third maximal subgroup can be ruled out.

3.2.5 The Mathieu group M_{24}

The largest Mathieu group M_{24} is of order 244823040. Table 3.11 gives information about the classes of maximal subgroups of M_{24}.

Table 3.11:

Maximal subgroups	M_{23}	$M_{22}:2$	$2^4:A_8$	$M_{12}:2$	$2^6:(3.S_6)$	$L_3(4):S_3$	$2^6:(L_(3)\times S_3)$	$L_2(12)$	$L_2(7)$
Indexes	24	276	759	1288	1771	2024	3795	40320	1457280
b	$\frac{8}{24}$	$\frac{36}{276}$	$\frac{70}{759}$	$\frac{56}{1288}$	$\frac{91}{1771}$	$\frac{120}{2024}$	$\frac{99}{3795}$	$\frac{320}{40320}$	$\frac{960}{1457280}$

The maximal fixed point ratios in the respective actions are given in row three and the indexes of classes of maximal subgroups are given in row two in above table.

Lemma 2.5.19 implies that if $b+\frac{1}{[G:M]}<\frac{1}{85}$, then no genus system $\bar{x}=(x_1,x_2,\ldots,x_r)$ satisfies the Riemann Hurwitz formula. So the group M_{24} possesses no primitive genus zero, one and two systems in its action on the right cosets of the maximal subgroup $L_2(12)$ and $L_2(7)$. Next we present number of possible ramification types of the other maximal subgroups of M_{24}.

Table 3.12:

Maximal subgroups	M_{23}	$M_{22}:2$	$2^4:A_8$	$M_{12}:2$	$2^6:(3.S_6)$	$L_3(4):S_3$	$2^6:(L_(3)\times S_3)$
Indexes in G	24	276	759	1288	1771	2024	3795
N.R.T	697	21	6	6	0	4	0

Lemma 3.2.5. Assume that G is the Mathieu group M_{24} and $(x_1,x_2,\cdots,x_r)\in G^r$. Then $\bar{x}=(x_1,\cdots x_r)$ is not genus zero, one and two-system if$A(\bar{x})<\frac{25}{12}$, where $A(\bar{x})=\sum_{i=1}^{r}\frac{d_i-1}{d_i}$.

Proof. This follows from Theorem 2.5.10. □

Lemma 3.2.5 guarantees that 17 of the 697 ramification type of the group M_{24} acting on the cosets of M_{23} can not occur. Now by using the Generating Group Criterion, if x , y and z are representatives of conjugacy classes $3A$, $4A$ and $4C$ respectively then there are conjugacy classes of pairs (x,y) whose product is equal to z. However the order of the group $\langle x,y\rangle$ is not equal to order of the group M_{24}. Hence G is not of type $(3A,4A,4C)$. Similarly, 46 ramification type of the first maximal subgroup can be ruled out. All ramification types of other maximal subgroups are ruled out by Lemma 2.5.21.

3.3 The ramification types of Janko groups

In this section we will use series of filters to eliminate some ramification types of Janko groups.

Lemma 3.3.1. Let G be the Janko group J_1. Then G is (2,3,7)-generated.

Proof. A complete proof can be found in [39]. □

Lemma 3.3.2. Let G be the Janko group J_2. Then G is (2,3,7)-generated.

Proof. See [39]. □

Lemma 3.3.3. Let G be the Janko group J_3. Then G is (2,3,10)-generated.

Proof. See [39]. □

3.3.1 The Janko group J_1

The smallest Jonko group J_1 is of order 175560. The classes of maximal subgroups of J_1 , their indexes and their maximal fixed point ratios in the representative action are given in the following table.

Table 3.13:

Maximal subgroups	$L_2(11)$	$2^3:7:3$	$2\times A_5$	$19:6$	$11:10$	$D_6\times D_{10}$	$7:6$
Indexes	266	1045	1463	1540	1596	2926	4180
b	$\frac{10}{266}$	$\frac{10}{1045}$	$\frac{31}{1463}$	$\frac{20}{1540}$	$\frac{12}{1596}$	$\frac{46}{2926}$	$\frac{20}{4180}$

In ATLAS explain that the group A:B denotes any group having a normal subgroup of structure A for which corresponding quotient group has structure B which is a split extension or semi direct product. Let M be one of the class of maximal subgroup $2^3:7:3$, $11:10$ or $7:6$, then $b+\frac{1}{[G:M]}<\frac{1}{85}$. Using Lemma 2.5.19 implies that the group J_1 possesses no primitive genus zero, one and two system in its action on the right cosets of the maximal subgroups $2^3:7:3$, $11:10$ and $7:6$. Riemann Hurwitz formula implies that the maximal subgroup $2\times A_5$ has one ramification type $(3A,3A,3A)$ which is eliminated by the Zariski condtion.

The typical row of the character table consists mostly of character values, together with indicator and fusion information that is described in ATLAS. Usually the character values are ordinary integers, but certain algebraic irrationalities can also arise. In such cases we either print the ATLAS name for the desired irrationality in full, or just an algebraic conjugacy operator by which it can be obtained from a nearby entry in the same row. To be precise, this nearby entry can be any entry for a class in the same algebraically conjugate family as the desired one that is printed in full. In then next table we presents permutation characters of the second, fourth and sixth maximal subgroup of the group J_1.

Table 3.14:

Maximal subgroups	χ_M
$2^3:7:3$	$1a+56ab+76a+77bc+120abc+133a+209a$
$2\times A_5$	$1a+56ab+76a^2+77a^2+120abc+133a^2+209a^2$
$19:6$	$1a+56ab+76a^2+77abc+120abc+133a^2+209a^2$
$D_6\times D_{10}$	$1a+56ab+76a^3+77a^3+120a^2b^2c^2+133a^4bc+209a^4$

By Lemma 2.5.21 the group J_1 possesses no primitive genus zero, one and two system in its action on the right cosets of maximal subgroup $19:6$ and $D_6\times D_{10}$ because $1^G_{2^3:7:3}$ is submodule of $1^G_{19:6}$ and $1^G_{2\times A_5}$ is submodule of $1^G_{D_6\times D_{10}}$. Next Riemann Hurwitz formula implies that J_1 has one ramification type $(2A,3A,7A)$ of genus zero in its action on the right cosets $L_2(11)$.

3.3.2 The Janko group J_2 and its Automorphism group

Let G be an almost simple group such that $F^*(G) = J_2$. The Janko group J_2 has nine classes of maximal subgroups which are given in the first row in the Table 3.15. Note that the maximal fixed point ratio of the maximal subgroup A_5 is equal to $\frac{60}{10080}$. So Lemma 2.5.19 implies that the group J_2 possesses no primitive genus zero, one and two system in its action on the right cosets of the maximal subgroup A_5. The indexes and number of ramification types of other classes of maximal subgroups are given in the rows two and three respectively.

Table 3.15:

Maximal subgroups	U_3	$3.PGL_2(9)$	$2^{1+4}:A_5$	$2^{2+4}:(3\times S_3)$	$A_4\times A_5$	$A_5\times D_{10}$	$(L_(3)(2):2$	$5^2:D_{12}$
Indexes	100	280	315	525	840	1008	1800	2016
N.R.T	58	19	14	1	1	2	1	2

A GAP calculation shows that the group algebra structure constant of 26 ramification types of first maximal subgroup, eight ramification types of second maximal subgroup and six ramification types of third maximal subgroup of above table are equal to zero and thus are ruled out. By using the Zariski condition we can eliminate two of the 58 ramification types of J_2 acting on the cosets of $U(3)$. Let x , y and z be representative of conjugacy classes $2A$, $4A$ and $15(AorB)$ respectively then there are conjugacy classes of pair (x,y) such that whose product is equal to z. However the order group $\langle x,y\rangle$ is not equal to order of the group J_2. Hence G is not of type $(2A,4A,15AB)$. Using similar argument we can eliminate a further 23 ramification types of the first maximal subgroup,10 ramification types of the second maximal and 7 ramification types of the third maximal subgroup. In the light of Lemma 2.5.21 all ramification types of the fourth class of maximal subgroup up to eighth class of maximal subgroups can be ruled out. By the next table we show the number of ramification types of maximal subgroups of Janko group J_2 which survive the filters.

Table 3.16:

Maximal subgroups	U_3	$3.PGL_2(9)$	$2^{1+4}:A_5$	$2^{2+4}:(3\times S_3)$	$A_4\times A_5$	$A_5\times D_{10}$	$(L_(3)(2):2$	$5^2:D_{12}$
Indexes	100	280	315	525	840	1008	1800	2016
N.R.T	7	1	1	0	0	0	0	0

Now we will use similar considerations for the automorphism group of J_2. Let G be almost simple group $Aut(J_2)$. The classes of maximal subgroup of G, their indexes, and number of ramification types of maximal subgroups are presented in the following table.

Table 3.17:

Maximal subgroups	$U_3:2$	$3.A_6.2^2$	$2^{1+4}:S_5$	$H.2$	$(A_4\times A_5):2$	$(A_5\times D_{10}).2$	$(L_(3)(2):2\times 2$	$5^2:(4\times S_3)$
Indexes in G	100	280	315	525	840	1008	1800	2016
N.R.T	82	17	15	2	26	10	1	4

By using the Zariski condition, the Generating Group Criterion, the ClassStructureCharacterTable function, and Lemma 2.5.21 all but 13 ramification types in the first maximal subgroup in the above table are eliminated.

3.3.3 The Janko group J_3 and its Automorphism group

Let G be an almost simple group such that $F^*(G) = J_3$. The Janko group J_3 has eight classes of maximal subgroups which are given in the first row in the Table 3.18. The maximal fixed point ratios in the respective actions are given in row 3. The indexes of the maximal subgroups are given in row 2.

Table 3.18:

Maximal subgroups	$L_2(16):2$	$L_2(19)$	$2^4:(3\times A_5)$	$L_2(17)$	$(3\times A_6):2_2$	$3^2.(3\times 3^2):8$	$2^{1+4}:A_5$	$2^{2+4}:(3\times S_3)$
Indexes in G	6156	14688	17442	20520	23256	25840	26163	43605
b	$\frac{76}{6156}$	$\frac{96}{14688}$	$\frac{72}{17442}$	$\frac{120}{20520}$	$\frac{136}{23256}$	$\frac{80}{25840}$	$\frac{131}{26163}$	$\frac{90}{43605}$

Lemma 2.5.19 implies that no genus system $\bar{x}= (x_1, x_2, \ldots, x_r)$ satisfies the Riemann Hurwitz formula if $b + \frac{1}{[G:M]} < \frac{1}{85}$. Thus the maximal subgroup $M = L_2(16):2$ gives the only possible primitive action of G which may possess a low genus system.

Table 3.19:

Conjugacy class representative	2A	3A	3B	4A	5A	5B	6A	8A	9A	9B	9C
indexes	3040	4080	4104	4592	4924	4924	5104	5374	5472	5472	5472
Conjugacy class representative	10A	10B	12A	15A	15B	17A	17B	19A	19B		
indexes	5532	5532	5628	5740	5740	5792	5792	5832	5832		

According to the above Table 3.19, the Riemann Hurwitz formula implies that G has one ramification type $(3B, 3B, 3B)$ of genus one system, which is eliminated by the Zariski condition. Hence the group G possesses no primitive genus zero, one and two systems in its action on the right cosets of all maximal subgroups.

Let G be the almost simple group $Aut(J_3)$. The group G has seven class of maximal subgroups which are given in the table. Similarly, the maximal fixed point ratio in the respective actions are given in row 3 and the indexes of maximal subgroups are given in row 2.

Table 3.20:

Maximal subgroups	$L_2(16):4$	$2^4:(3\times A_5).2$	$L_2(17)$	$(3\times M_{10}):2$	$3^2.(3\times 3^2):8.2$	$2^{1+4}:S_5$	$2^{2+4}:(S_3\times S_3)$
Indexes	6156	17442	20520	23256	25840	26163	43605
b	$\frac{76}{6156}$	$\frac{102}{17442}$	$\frac{154}{20520}$	$\frac{136}{23256}$	$\frac{80}{25840}$	$\frac{153}{26163}$	$\frac{255}{43605}$

Lemma 2.5.19 implies that the maximal subgroup $L_2(16):4$ is the only possible primitive action of G which may possess a low genus system.

Table 3.21: Index on maximal subgroup $L_2(16):4$

Conjugacy class representative	2A	2B	3A	3B	4A	4B	5A	6A	6B	8A	8B	9A	9B	9C
indexes	3040	3078	4080	4104	4592	4594	4924	5104	5130	5374	5374	5472	5472	5472
Conjugacy class representative	10A	12A	12B	15A	17A	17B	18A	18B	18C	19A	24A	24B	34A	34B
indexes	5532	5628	5628	5740	5792	5792	5814	5814	5814	5832	5892	5892	5974	5974

According to the above table, Riemann Hurwitz formula implies that G has three ramification types $(3B,3B,3B)$, $(2B,2B,2B,2B)$, $(2B,3B,6B)$ of genus one system which are eliminated by the Zariski condition. Hence the group G possesses no primitive genus zero one and two systems in its action on the right cosets of any maximal subgroups.

3.3.4 The Janko group J_4

The largest Janko group J_4 has order 86775571046077562880. The classes of maximal subgroups of J_4 , their indexes and their maximal fixed point ratios in their representative actions are given in the following table.

Table 3.22:

M.S	$2^{11}:M_{24}$	$2^{10}:L_5(2)$	$2^{1+12}:3.M_{22}.2$	$2^{3+12}.(S_5\times L_3(2))$	$U_3(11):2$	$11^{1+2}:(5\times 2S_4)$
Ind	173067389	8474719242	3980549947	131358148251	611822174208	2716499045348352
b	$\frac{52349}{173067389}$	$\frac{285450}{8474719242}$	$\frac{194107}{3980549947}$	$\frac{1421211}{131358148251}$	$\frac{2064384}{611822174208}$	$\frac{8257536,}{2716499045348352}$
M.S	$L_2(23):5$	$L_2(23):2$	$29:28$	$43:14$	$37:12$	
Ind	530153782050816	7145550975467520	106866466805514240	144145466853949440	195440475329003520	
b	$\frac{11354112}{530153782050816}$	$\frac{454164480}{7145550975467520}$	$\frac{64880640}{106866466805514240}$	$\frac{129761280}{144145466853949440}$	$\frac{151388160,}{195440475329003520}$	

Lemma 2.5.19 implies that the group G possesses no primitive genus zero one and two systems in its action on the right cosets of any maximal subgroup.

3.4 The ramification types of the Conway groups

In this section we are going to determine all possible ramification types for the Conway groups. Firstly we start by determining the ramification types of the smallest Conway group Co_3.

3.4.1 The Conway group Co_3

Let G be a Conway group Co_3. The group G has 14 conjugacy classes of maximal subgroups which are presented in the first row of the table 3.23. The indexes and the maximal fixed point ratios in the representative actions are given in the second and third rows of the table 3.23 respectively.

Lemma 2.5.19 implies that if M be any maximal subgroup for the group Co_3 and $b+\frac{1}{[G:M]}<\frac{1}{85}$, then G possesses no primitive genus system in its action on the right cosets of the maximal

Table 3.23:

Max.Subgroups	$McL:2$	HS	$U_4(3):(2^2)_{133}$	M_{23}	$3^5:(2\times M_{11})$	$2.S_6(2)$	$U_3(5):S_3$
Indexes	276	11178	37950	48600	128800	170775	655776
b	$\frac{36}{276}$	$\frac{378}{11178}$	$\frac{750}{37950}$	$\frac{1080}{48600}$	$\frac{1120}{128800}$	$\frac{631}{170775}$	$\frac{2016}{655776}$
Max.Subgroups	$3^{1+4}:4S_6$,	$2^4.A_8$,	$L_3(4):D_{12}$	$2\times M_{12}$	$2^2.[2^7.3^2].S_3$	$S_3\times L_2(8):3$	$A_4\times S_5$
Indexes	708400	1536975	2049300	2608200	17931375	54648000	344282400
b	$\frac{1456}{708400}$	$\frac{7695}{1536975}$	$\frac{8100}{2049300}$	$\frac{7560}{2608200}$	$\frac{19215}{17931375}$	$\frac{5280}{54648000}$	$\frac{30240}{344282400}$

subgroup M. Thus the maximal subgroups McL_2, HS, $U_4(3):(2^2)_{133}$ and M_{23} are the only possible primitive actions of G which may possibly posses a low genus system.

According to the Riemann Hurwitz formula Co_3 has 50 possible ramification types of genus zero, one and two systems in its action on the right coset of the maximal subgroups $McL:2$, HS ,M_{23} and $U_4(3):(2^2)_{133}$. Given an element z of order eleven of type A or B, there are conjugacy classes of pairs (x,y) of order two of type A and three of type A respectively such that $xy=z^{-1}$. However, order of the groups which are generated by conjugacy classes of the pairs(x,y) are not equal to the order of the group $Co3$. It follows that the triple $(2A,3A,11AB)$ can be eliminated. Similarly, by the same argument 38 ramification types can be eliminated. Moreover, 7 of ramification types can be eliminated by the Zariski condition. Finally a GAP calculation shows that the group algebra structure constant of three of the ramification types is equal to zero, so these are also eliminated. Thus all 50 ramification types of $Co3$ can be eliminated except $(2B,3C,7A)$ of genus zero in its action of the right coset of the maximal subgroups $McL:2$.

3.4.2 The Conway group Co_2 and its automorphism group

Let G be an almost simple group such that $F^*(G)=Co_2$. The group Co_2 has eleven conjugacy classes of maximal subgroups which are given in the first row in the Table 3.24. Note that the maximal fixed point ratio of G actions on the maximal subgroup $2^{4+10}(S_5\times S_3)$, M_{23}, $3^{1+4}:2^{1+4}.s_5$ and $5^{1+2}:4S_4$ are equal to $\frac{34083}{3586275}$, $\frac{15360}{4147200}$, $\frac{71680}{45337600}$, $\frac{86016}{3525417662}$ respectively. So Lemma 2.5.19 implies that the group Co_2 possesses no primitive genus zero, one and two system in its action on the right cosets of these maximal subgroups. The indexes and number of ramification types of the other class of maximal subgroups are given in the rows two and three respectively.

A GAP calculation shows that the group algebra structure constant of four ramification types of

Table 3.24:

Maximal subgroups	$U_6(2):2$	$2^{10}:M_{22}:2$	McL	$2^{1+8}:S_6(2)$	$HS:2$	$(2^{1+6}\times 2^4).A_8$	$U_4(3).D_8$
Indexes in G	2300	46575	47104	46925	476928	1024650	1619200
N.R.T	13	1	0	1	0	0	0

the first maximal subgroup are equal to zero, so these will be ruled out. By using the Zariski condition 4 ramification types of the first maximal subgroup, one ramification types of the second maximal subgroup and one ramification of the fourth maximal subgroup are eliminated. Finally five ramification types in the first maximal subgroup can be eliminated by using Generating Group Criterion. Hence G possesses no primitive genus zero, one and two systems.

If $G = Aut(Co_2) = Co2 : 2$, then G possesses no primitive genus zero, one and two system because all ramification types ruled out by arguments the similar to the ones given above.

3.4.3 The Conway group Co_1

Let G be a group $Co1$. The conjugacy classes of maximal subgroups of G,their indexes and their maximal fixed point ratios in their representative actions are given in the following table.

Table 3.25:

Maximal subgroups	Indexes	fixed point ratio
Co_2	98280	$\frac{2280}{98280}$
$3.Suz:2$	1545600	$\frac{22881}{1545600}$
$2^{11}:M_{24}$	8282375	$\frac{32535}{8282375}$
Co_3	8386560	$\frac{30720}{8386560}$
$2^{1+8}.O_8(2)$	46621575	$\frac{135135}{46621575}$
$U_6(2):S_3$	75348000	$\frac{132640}{75348000}$
$(A_4\times G_2(4)):2$	688564800	$\frac{928422}{688564800}$
$2^{2+12}:(A_8\times S_3)$	2097970875	$\frac{1216215,}{2097970875}$
$2^{4+12}.(S_3\times 3S_6)$	4895265375	$\frac{1143135}{4895265375}$
$3^2.U_4(3).D_8$	17681664000	$\frac{3226080}{17681664000}$
$3^6:2M_{12}$	30005248000	$\frac{2867200}{30005248000}$
$(A_5\times J_2):2$	57288591360	$\frac{10749024}{57288591360}$
$3^{1+4}:2O_4(2):2$	165028864000	$\frac{12812800}{165028864000}$
$(A_6\times U_3(3)):2$	954809856000	$\frac{29652480}{954809856000}$
$3^{3+4}:2(S_4\times S_4)$	1650288640000	$\frac{41641600}{1650288640000}$
$A_9\times S_3$	3819239424000	$\frac{207567360}{3819239424000}$
$(A_7\times L_2(7)):2$	4910450688000	$\frac{111196800}{4910450688000}$
$(D_{10}\times(A_5\times A_5).2).2$	28873450045440	$\frac{373621248}{28873450045440}$
$5^{1+2}:GL_2(5)$	69296280109056	$\frac{111476736}{69296280109056}$
$7^2:(3\times 2A_4)$	1178508165120000	$\frac{5406720}{1178508165120000}$

Lemma 2.5.19 implies that the maximal subgroups Co_2 and $3.Suz:2$ are the only possess primitive action of G which may possess a low genus system.

Riemann Hurwitz formula implies that G has 6 possible ramification types of genus zero and one systems. All of them can be ruled out using the Zariski condition , the Generating Group

Criterion and the ClassStructureCharacterTable function.

3.5 The ramification types of the Higman-Sims group and its automorphism group

Let G be an almost simple group such that $F^*(G) = HS$. The group HS has ten conjugacy classes of maximal subgroups which are given in the first row in the table 3.26. we note that the maximal fixed point ratios of the classes of maximal subgroups $2 \times A_6.2^2$ and $5:4 \times A_5$ are equal to $\frac{2}{154}$ and $\frac{216}{36960}$ respectively. So Lemma 2.5.19 implies that the group G possesses no primitive genus zero, one and two system in its action on the right cosets of the maximal subgroup $2 \times A_6.2^2$ and $5:4 \times A_5$. The indexes and number of ramification types of the maximal subgroups are given in the rows two and three respectively.

Table 3.26:

Maximal subgroups	M_{22}	$U_3(5):2$	$L_3(4):2_1$	S_8	$2^4.S_6$	$4^3:L_3(2)$	M_{11}	$4.2^4:S_5$
Indexes	100	176	1100	1100	3850	4125	5600	5775
N.R.T	78	28	3	6	0	0	0	0

GAP calculation shows that the group algebra structure constant of 18 of ramification types of first maximal subgroup and 9 ramification types of the second maximal subgroup are equal to zero, so they can be eliminated. Furthermore, 5 ramification types of the first maximal subgroup and 2 ramification types of the second maximal subgroup will be eliminated by the Zariski condition. Given an element z of order ten of type A, there are conjugacy classes of pairs (x,y) of order two of type B and three of type A such that $xy = z^{-1}$. However, order of the groups which are generated by conjugacy classes of the pairs(x,y) are not equal to the order of the group HS. It follows that the triple $(2B,3A,10A)$ of genus zero system can be eliminated. By a similar argument 42 of the ramification types of the first maximal subgroup and 17 ramification types of the second maximal subgroup can be eliminated. Note that the permutation character of the maximal subgroup $U_3(5):2$ lies in the permutation character of the maximal subgroups $L_3(4):2_1$ and S_8. So $1^G_{U_3(5):2}$ is a submodule of $1^G_{L_3(4):2_1}$ and $1^G_{S_8}$. By using Lemma 2.5.21 we can eliminate three ramification types the third maximal subgroup and 6 ramification types of the fourth maximal subgroup. Hence we have to find braid orbits of 9 tuples of the group HS. All of these are in the action of G on the first maximal subgroup.

Let G be a group $Aut(HS)$, then G has eight classes of maximal subgroups. The maximal fixed point ratio of the maximal subgroups $H.2$ and $5:4\times S_5$ are equal to $\frac{2}{154}$, $\frac{216}{36960}$ respectively. So Lemma 2.5.19 implies that the group G possesses no primitive genus zero, one and two system in its action on the right cosets of the maximal subgroup $2\times A_6.2^2$ and $5:4\times A_5$. The other maximal subgroups are given in the first row of the table 3.27. The indexes and number of ramification types of the maximal subgroups are given in the rows two and three respectively.

Table 3.27:

Maximal subgroups	$M_{22}:2$	$L_3(4):2^2$	$S_8\times 2$	$2^5.S_6$	$4^3:(L_3(2)\times 2)$	$2^{1+6}:S_5$
Indexes	100	1100	1100	3850	4125	5775
N.R.T	78	3	6	0	0	0

By using the Zariski condition and the Generating Group Criterion all but 29 of ramification types in the first maximal subgroup in the Table 3.27 are eliminated.

3.6 The ramification types of the Fischer Groups

The Fischer groups are Fi_{22} , F_{23} and Fi_{24}. In this section we discuss why the Fischer groups do not possess primitive genus zero, one and two systems.

3.6.1 The Fischer group Fi_{22} and its automorphism group

Let G be an almost simple group such that $F^*(G)=Fi_{22}$. The group G has 12 conjugacy classes of maximal subgroups. Note that the maximal fixed point ratios of the maximal subgroups $3^{1+6}:2^{3+4}:3^2:2$, S_{10} and $M_{12}:2$ are equal to $\frac{9856}{12812800}$, $\frac{228096}{17791488}$ and $\frac{221184}{679311360}$ respectively. So Lemma 2.5.19 implies that the group G possesses no primitive low genus systems in its action on the right cosets of the maximal subgroups $3^{1+6}:2^{3+4}:3^2:2$, S_{10} and $M_{12}:2$. All other maximal subgroups and their indexes and possible number of ramification types are given in the following table.

Table 3.28:

Maximal subgroups	$2.U_6(2)$	$O_7(3)$	$O_8(2):S_3$	$2^{10}:M_{22}$	$2^6:S_6(2)$	$(2\times 2^{1+8}:U_4(2)):2$	$2_{F_4(2)}$	$2^{5+8}:(S_3\times S_6)$
Indexes	3510	14080	61776	142155	694980	1216215	1647360	3592512
N.R.T	11	1	0	10	0	0	0	0

A GAP calculation shows that the group algebra structure constant of 3 ramification types of the first maximal subgroup is equal to zero, so these can be eliminated. By using the Zariski condition two ramification types of the first maximal subgroup, one ramification type of the second maximal subgroup and 10 ramification types of the fourth maximal subgroup can be ruled out. Finally, given an element z of order eight of type A, there are conjugacy classes of pairs (x,y) of order two of type B and three of type A such that $xy = z^{-1}$. However, orders of the groups which are generated by conjugacy classes of the pairs(x,y) are not equal to the order of the group G. It follows that the triple $(2B,3A,8A)$ can be eliminated. Using similar considerations we eliminate a further 6 ramification types of the first maximal subgroup. Hence G possesses no primitive low genus systems.

Similarly, if G is the group $Fi_{22}:2$ then G has no primitive low genus system in its action on any maximal subgroup.

3.6.2 The Fischer group Fi_{23}

Let G be a Fischer group Fi_{23}. The group Fi_{23} has 14 conjugacy classes of maximal subgroups which are given in the first column in the Table 3.29. The maximal fixed point ratios in the respective actions are given in column 3. The indexes of maximal subgroups are given in column 2.

Table 3.29:

$Maximal Subgroups$	$Indexes$	$Maximal fixed point ratio$
$2.Fi_{22}$	31671	$\frac{3511}{31671}$
$O_8{}^{+}(3):S_3$	137632	$\frac{14080}{137632}$
$2^2.U_6(2).2$	55582605	$\frac{1219725}{55582605}$
$S_8(2)$	86316516	$\frac{694980}{86316516}$
$S_3 \times O_7(3)$	148642560	$\frac{1661440}{148642560}$
$2^{11}.M_{23}$	195747435	$\frac{142155}{195747435}$
$3_+{}^{1+8}.2^{1+6}.3^{1+2}.2S$	1252451200	$\frac{12812800}{1252451200}$
$3^3.[3^7].(2 \times L_3(3))$	6165913600	$\frac{15769600}{6165913600}$
S_{12}	8537488128	$\frac{17791488}{8537488128}$
$(2^2 \times 2^{1+8}).(3 \times U_4(2)).2$	12839581755	$\frac{74189115}{12839581755}$
$2^{6+8}:(A_7 \times S_3)$	16508033685	$\frac{28667925}{16508033685}$
$S_4 \times S_6(2)$	117390461760	$\frac{255752640}{117390461760}$
$S_4(4):4$	1044084577536	$\frac{35126784}{1044084577536}$
$L_2(17):2$	673496454758400	$\frac{13271040}{673496454758400}$

Lemma 2.5.19 implies that the maximal subgroups $2.Fi_{22}$, ${O_8}^+(3):S_3$ and $2^2.U_6(2).2$ are the only possible primitive actions of G which may possesses a low genus system.

Riemann Hurwitz formula implies that G has no possible ramification types of genus zero, one and two systems, in its action on the right cosets of $2.Fi_{22}$, ${O_8}^+(3):S_3$ and $2^2.U_6(2).2$. Hence G poss no primitive low genus system.

3.6.3 Fischer group Fi_{24} and its automorphism group

Let G be an almost simple group such that $F^*(G) = Fi_{24}$. The group G has 20 classes of maximal subgroups which are given in the first column in the table 3.30. The maximal fixed point ratios in the respective actions are given in column 3. The index of maximal subgroups are given in column 2.

We observe the upper bound fixed point ratio is approximately $\leqslant \frac{1}{90}$. Thus Lemma 2.5.19 implies that the group G possesses no primitive genus zero, one and two systems in its action on it's maximal subgroups. Similarly, if G is almost simple group $Fi_{24}:2$ then the upper bound for the fixed point ratios is $\leqslant \frac{1}{90}$. Hence $Fi_{24}:2$ possesses no primitive low genus system in its action for the right cosets of its 18 conjugacy classs of maximal subgroups.

Table 3.30:

$MaximalSubgroups$	$Indexes$	$Maximal fixedpointratio$
Fi_{23}	306936	$\frac{3512}{306936}$
$2.Fi_{22}:2$	4860485028	$\frac{1346788}{4860485028}$
$(3\times O_8(3):3):2$	14081405184	$\frac{1675520}{14081405184}$
$O_{10}(2)$	50177360142	$\frac{7992270}{50177360142}$
$3^7.O(3)$	125168046080	$\frac{5125120}{125168046080}$
$3^{1+10}:U_5(2):2$	258870277120	$\frac{12812800}{258870277120}$
$2^{11}.M_{24}$	2503413946215,	$\frac{93964455}{2503413946215,}$
$2^2.U_6(2):S_3$	5686767482760	$\frac{119887560}{5686767482760}$
$2^{1+12}.3U_4(3).2_2$	7819305288795	$\frac{113107995}{7819305288795}$
$3^2.3^4.3^8.(A_5\times 2A_4).2$	91122337546240	$\frac{205004800}{91122337546240}$
$(A_4\times O_8(2):3):2$	100087107696576	$\frac{375350976}{100087107696576}$
$He:2$	155717756992512	$\frac{800616960}{155717756992512}$
$3^{3+12}.(L_3(2)\times A_6)$	633363728392395	$\frac{955423755}{633363728392395}$
$2^{6+8}.(S3\times A8)$	633363728392395	$\frac{2289785355}{633363728392395}$
$(3^2:2\times G_2(3)).2$	8212275503308800	$\frac{850305600}{8212275503308800}$
$(A_5\times A_9):2$	57650174033227776	$\frac{8931326976}{57650174033227776}$
$7:6\times A_7$	11859464372549713920	$\frac{256197427200}{11859464372549713920}$
$29:14$	3091639677809511628800	$\frac{11466178560}{3091639677809511628800}$
$3^3.[3^{10}]3.GL_3(3)$	574727888823563059200	$\frac{12421693440}{574727888823563059200}$
$A_6\times L_2(8):3$	574727888823563059200	$\frac{12421693440}{574727888823563059200}$

3.7 The ramification types of the M^{c}Laughlin group and its automorphism group

Let G be an almost simple group such that $F^*(G) = M^cL$. The group G has 10 conjugacy classes of maximal subgroups. Based on the fact, that the maximal fixed point ratio on the classes of maximal subgroups $3^{1+4} : 2S_5$, $2.A_8$, M_{11}, and $5^{1+2} : 3 : 8$ is equal to $\frac{91}{15400}$, $\frac{211}{22275}$, $\frac{84}{11340}$, and $\frac{486}{299376}$ respectively, Lemma 2.5.19 guarantees that the group G possesses no primitive low genus systems in its action on the right cosets of those classes of maximal subgroups. Moreover, the six remaining classes of maximal subgroups, their indexes, and number of ramification types are given in the following table.

Table 3.31:

Maximal subgroups	$U_4(3)$	M_{22}	$U_3(5)$	$3^4 : M_{10}$	$L_3(4) : 2_2$	$2^4 : A_7$
Indexes	275	2025	7128	15400	22275	22275
N.R.T	11	2	1	0	1	1

Note that the group algebra structure constant of seven ramification types of the first maximal subgroup in the Table 3.31 are equal to zero , so they can be ruled out. Furthermore, two ramification types of the first maximal subgroup , one ramification type of the second, third, fourth and fifth maximal subgroups can be eliminated by using the Zariski condition. Finally, by using the Generating Group Criterion 2 of ramification types of the first maximal subgroup and one ramification type of the second maximal subgroup can not occur. Hence G possesses no primitive low genus system.

Now, let $G = Aut(M^cL) = M^cL : 2$, then G has eight classes of maximal subgroups. The maximal fixed point ratio on the maximal subgroups $3^{1+4} : 4S_5$, $2.s_8$, $M_{11} \times 2$ and $H.2$ is equal to $\frac{110}{15400}$, $\frac{211}{22275}$, $\frac{84}{11340}$ and $\frac{486}{299376}$ respectively. So Lemma 2.5.19 implies that the group G possesses no primitive low genus system in its action of those maximal subgroups. The other maximal subgroups, their indexes, and number of ramification types are given in the following table.

Now we will make an analysis of $M^cL : 2$ similar to that given above. By using the Generating Group Criterion and the Zariski condition all ramification types of the table above are eliminated. Hence G possesses no primitive genus zero, one and two system.

Table 3.32:

Maximal subgroups	$U_4(3):2_3$	$U_3(5):2$	$3^4:(M_{10}\times 2)$	$L_3(4):2^2$
Indexes	275	7128	15400	22275
N.R.T	22	5	0	1

3.8 Ramification types of the Suzuki group and its automorphism group

Let G be an almost simple group such that $F^*(G) = Suz$. The group $G = Suz$ has 16 conjugacy classes of maximal subgroups which are given in the first column in the table 3.33. The indexes of maximal subgroups and the maximal fixed point ratios in the respective actions are given in the columns two and three respectively.

Table 3.33:

$Maximal Subgroups$	$Indexes$	$Maximal fixed point ratio$
$G_2(4)$	1782	$\frac{162}{1782}$
$3_2.U_4(3):2_3$	22880	$\frac{480}{22880}$
$U_5(2)$	32760	$\frac{760}{32760}$
$2^{1+6}.U_4(3)$	135135	$\frac{2835}{135135}$
$3^5:M_{11}$	232960	$\frac{2560}{232960}$
$J_2:2$	370656	$\frac{4536}{370656}$
$2^{4+6}:3A_6$	405405	$\frac{2205}{405405}$
$(A_4\times L_3(4)):2_1$	926640	$\frac{2160}{926640}$
$2^{2+8}:(A_5\times S_3)$	1216215	$\frac{8505}{1216215}$
$M_{12}:2$	2358720	$\frac{8640}{2358720}$
$3^{2+4}:2(A_4\times 2^2).2$	3203200	$\frac{5320}{3203200}$
$(A_6\times A_5):2$	10378368	$\frac{4536}{10378368}$
$(3^2:4\times A_6).2$	17297280	$\frac{15120}{17297280}$
$L_3(3):2$	39916800	$\frac{34560}{39916800}$
$L_2(25)$	57480192	$\frac{6720}{57480192}$
A_7	17714880	$\frac{6720}{17714880}$

Clearly, the maximal subgroups $G_2(4)$, $3_2.U_4(3):2_3$, $U_5(2)$, $2^{1+6}.U_4(3)$, $3^5:M_{11}$ are the only subgroups where a primitive action of low genus is possible (Lemma 2.5.19). So we present the number of ramification types of those maximal subgroups in the following table

Table 3.34:

Maximal subgroups	$G_2(4)$	$3_2.U_4(3):2_3$	$U_5(2)$	$2^{1+6}.U_4(3)$	$3^5:M_{11}$
N.R.T	4	0	4	1	2

Note that the group algebra structure constant of three ramification types of the first maximal subgroup is equal to zero, so they can not occur. By using the Zariski condition one ramification type of the first maximal subgroup and four ramification types of the third maximal subgroup, one ramification type of the fourth and two ramification types of the fifth maximal subgroup will be ruled out. Hence G possesses no primitive genus low genus system in its action on the maximal subgroups.

Let $G = Aut(Suz) = Suz:2$. G has 15 classes of maximal subgroups which are given in the first column of Table 3.35. The indexes of the maximal subgroups are given in the second column and the maximal fixed point ratios are given in the third column.

According to the maximal fixed point ratio, the maximal subgroups $G_2(4):2$, $3U_4(3).(2^2)_{133}$, $U_5(2):2$, $2^{1+6}.U_4(3).2$,$3^5:(M_{11}\times 2)$ are the only ones in which G may allows a primitive action of low genus (Lemma 2.5.19). The Riemann Hurwitz formula allows 20 possible ramification types. All of them can be eliminated by using the Zariski condition and the Group Generating Criterion.

3.9 The ramification types of the Held group and its automorphism group

Let G be an almost simple group such that $F^*(G) = He$. The group $G = He$ has 10 conjugacy classes of maximal subgroups which are given in the first column in the Table 3.36. The indexes of maximal subgroups and the maximal fixed point ratios in the respective actions are given in the columns two and three respectively.

The only possible primitive actions where G may have a possible low genus systems are on the

Table 3.35:

$MaximalSubgroups$	$Indexes$	$Maximal fixed point ratio$
$G_2(4):2$	1782	$\frac{162}{1782}$
$3U_4(3).(2^2)_{133}$	22880	$\frac{480}{22880}$
$U_5(2):2$	32760	$\frac{760}{32760}$
$2^{1+6}.U_4(3).2$	135135	$\frac{2835}{135135}$
$3^5:(M_{11}\times 2)$	232960	$\frac{2560}{232960}$
$J_2:2\times 2$	370656	$\frac{4536}{370656}$
$2^{4+6}:3S_6$	405405	$\frac{2205}{405405}$
$(A_4\times L_3(4):2_3):2$	926640	$\frac{2640}{926640}$
$2^{2+8}:(S_5\times S_3)$	1216215	$\frac{8505}{1216215}$
$M_{12}:2\times 2$	2358720	$\frac{8640}{2358720}$
$3^{2+4}:2(S_4\times D_8)$	3203200	$\frac{5320}{3203200}$
$(A_6:2_2\times A_5):2$	10378368	$\frac{9408}{10378368}$
$(3^2:8\times A_6).2$	17297280	$\frac{15152}{17297280}$
$L_2(25):2$	57480192	$\frac{10080}{57480192}$
S_7	17714880	$\frac{6720}{17714880}$

classes of maximal subgroups $S_4(4):2$, $2^2.L_3(4).S_3$, and $2^6:3.S_6$. The first maximal subgroup has 8 ramification types such that all of them can be ruled out by the Generating Group Criterion. The second and third maximal subgroups have no ramification types. Hence G possesses no primitive genus systems in its action in the right cosets of maximal subgroups.

Similarly, if $G = Aut(He) = He.2$, then the only possible primitive actions of G for which a low genus systems may occur are on the classes of maximal subgroups $S_4(4):4$, $2^2.L_3(4).D_{12}$. The first maximal subgroup has eight ramification types and all of them can be ruled out. The second maximal subgroup has no possible ramification types of low genus system. Thus G possesses no primitive genus system in its action on the right cosets of maximal subgroups.

Table 3.36:

$MaximalSubgroups$	indexes	Maximal fixed point ratio
$S_4(4):2$	2058	$\frac{154}{2058}$
$2^2.L_3(4).S_3$	8330	$\frac{346}{8330}$
$2^6:3.S_6$	29155	$\frac{651}{29155}$
$2^{1+6}.L_3(2)$	187425	$\frac{945}{187425}$
$7^2:2L_2(7)$	244800	$\frac{84}{244800}$
$3.S_7$	266560	$\frac{1792}{266560}$
$7^{1+2}:(S_3\times 3)$	652800	$\frac{120}{682800}$
$S_4\times L_3(2)$	999600	$\frac{2880}{999600}$
$7:3\times L_3(2)$	1142400	$\frac{960}{1142400}$
$5^2:4A_4$	3358656	$\frac{4032}{3358656}$

3.10 The ramification types of the Rudvalis group and its automorphism group

Let G be an almost simple group such that $F^*(G) = Ru$. The group $G = Ru$ has 15 conjugacy classes of maximal subgroups which are given in the first column in the Table 3.37. The indexes of the maximal subgroups and the maximal fixed point ratios in the respective actions are given in the columns two and three respectively.

Lemma 2.5.19 implies that the maximal subgroup ${}^2F_4(2)$ is the only possible primitive action of G which may possesses a low genus system.

Riemann Hurwitz formula implies that G has no possible ramification types of genus zero , one or two systems in its action on the right cosets ${}^2F_4(2)$. Hence, G possesses no primitive genus system in its action on the right cosets of maximal subgroups.

If $G = Aut(Ru) = Ru:2$, then in similar way we can show that G possesses no primitive low genus system in its action on the right cosets of maximal subgroups.

Table 3.37: Ru

$MaximalSubgroups$	indexes	Maximal fixed point ratio
$^2F_4(2)$	4060	$\frac{92}{4060}$
$(2^6:U_3(3)):2$	188500	$\frac{980}{188500}$
$(2^2\times Sz(8)):3$	417600	$\frac{456}{417600}$
$2^{3+8}:L_3(2)$	424125	$\frac{1085}{424125}$
$U_3(5):2$	579072	$\frac{1536}{579072}$
$2.2^{4+6}:S_5$	593775	$\frac{1391}{593775}$
$L_2(25).2^2$	4677120	$\frac{3584}{4677120}$
A_8	7238400	$\frac{3840}{7238400}$
$L_2(29)$	11980800	$\frac{4160}{11980800}$
$5^2:4S_5$	12160512	$\frac{3584}{12160512}$
$3.A_6.2^2$	33779200	$\frac{7680}{33779200}$
$5^{1+2}:[2^5]$	36481536	$\frac{4608}{36481536}$
$L_2(13):2$	66816000	$\frac{4160}{66816000}$
$A_6.2^2$	101337600	$\frac{18944}{101337600}$
$5:4\times A_5$	121605120	$\frac{8736}{121605120}$

3.11 The ramification types of the large sporadic simple groups

In this section we are going to prove that the large sporadic simple groups HN, Ly, ON, Th, B and M possess no primitive genus systems in their actions on the right cosets of their maximal subgroups. Now we give tables such that the maximal subgroups are given in the first column, the indexes of the maximal subgroups, and the maximal fixed point ratios in the respective actions are given in columns two and three respectively

Table 3.38: HN

MaximalSubgroups	indexes	Maximal fixed point ratio
A_{12}	1140000	$\frac{8800}{1140000}$
$2.HS.2_3$	1539000	$\frac{7979}{1539000}$
$U_3(8):3$	16500000	$\frac{1920}{16500000}$
$2^{1+8}.(A_5 \times A_5).2$	74064375	$\frac{51975}{74064375}$
$(D_{10} \times U_3(5)).2$	108345600	$\frac{37312}{108345600}$
$5^{1+4}:2^{1+4}.5.4$	136515456	$\frac{10368}{136515456}$
$2^6.U_4(2)$	165587500	$\frac{177100}{165587500}$
$(A_6 \times A_6).D_8$	263340000	$\frac{215600}{263340000}$
$2^3.2^2.2^6.(3 \times L_3(2))$	264515625	$\frac{119625}{264515625}$
$5^2.5.5^2.4A_5$	364041216	$\frac{21504}{364041216}$
$M_{12}:2$	1436400000	$\frac{369600}{1436400000}$
$3^4:2(A_4 \times A_4).4$	2926000000	$\frac{308000}{2926000000}$
$3^{1+4}:4A_5$	4681600000	$\frac{56320}{4681600000}$

Table 3.39: Ly

MaximalSubgroups	indexes	Maximal fixed point ratio
$G_2(5)$	8835156	$\frac{7128}{8835156}$
$2.McL:2$	9606125	$\frac{15401}{9606125}$
$5^3:L_3(5)$	1113229656	$\frac{16632}{1113229656}$
$2.A_{11}$	1296826875	$\frac{34651}{1296826875}$
$5^{1+4}:4S_6$	5751686556	$\frac{299376}{5751686556}$
$3^5:(2 \times M_{11})$	13448575000	$\frac{64120}{13448575000}$
$3^{2+4}:2A_5.D_8$	73967162500	$\frac{708400}{73967162500}$
$67:22$	35118846000000	$\frac{1814400}{35118846000000}$
$37:18$	77725494000000	$\frac{2217600}{77725494000000}$

Table 3.40: *ON*

MaximalSubgroups	*Indexes*	*Maximal fixed point ratio*
$L_3(7):2$	122760	$\frac{360}{122760}$
J_1	2624832	$\frac{1344}{2624832}$
$4_2.L_3(4):2_1$	2857239	$\frac{1751}{2857239}$
$(3^2:4\times A_6).2$	17778376	$\frac{2856}{17778376}$
$3^4:2^{(1+4)}D_{10}$	17778376	$\frac{1064}{17778376}$
$L_2(31)$	30968784	$\frac{5040}{30968784}$
$4^3.L_3(2)$	42858585	$\frac{5145}{42858585}$
M_{11}	58183776	$\frac{3360}{58183776}$
A_7	182863296	$\frac{6720}{182863296}$

Table 3.41: Th

MaximalSubgroups	indexes	Maximal fixed point ratio
${}^3D_4(2):3$	143127000	$\frac{102}{1431270}$
$2^5.L_5(2)$	283599225	$\frac{3159}{283599225}$
$2^{1+8}.A_9$	976841775	$\frac{30511}{976841775}$
$U_3(8):6$	2742012000	$\frac{408}{27420120}$
$(3\times G_2(3)):2$	3562272000	$\frac{576}{35622720}$
$[3^9].2S_4$	96049408000	$\frac{3584}{960494080}$
$3^2.[3^7].2S_4$	96049408000	$\frac{3584}{960494080}$
$3^5:2S_6$	259333401600	$\frac{236544}{259333401600}$
$5^{1+2}:4S_4$	7562161990656	$\frac{2916}{7562161990656}$
$5^2:GL_2(5)$	7562161990656	$\frac{1354752}{7562161990656}$
$7^2:(3\times 2S_4)$	12860819712000	$\frac{64512}{1286081971200}$
$L_2(19):2$	13266950860800	$\frac{4902912}{13266950860800}$
M_{10}	126036033177600	$\frac{580608}{12603603317760}$
$31:15$	195152567500800	$\frac{23328}{195152567500800}$
S_5	7566216199065600	$\frac{193536}{75662161990656}$

Table 3.42: B

MaximalSubgroups	indexes	Maximal fixed point ratio
$2.(^2E_6(2)):2$	13571955000	$\frac{27081784}{13571955000}$
$2^{1+22}.Co_2$	11707448673375	$\frac{3146667615}{11707448673375}$
Fi_{23}	1015970529280000,	$\frac{4823449600}{1015970529280000}$
$2^{1+6}.L_3(2)$	1015970529280000	$\frac{4823449600}{1015970529280000}$
$2^{9+16}.S8(2)$	2613515747968125	$\frac{91161395325}{2613515747968125}$
Th	45784762417152000	$\frac{125829120}{45784762417152000}$
$2^2 \times F_4(2):2$	156849238149120000	$\frac{1609085288448}{156849238149120000}$
$2^{2+10+20}.(M_{22}2 \times S_3)$	181758140654146875	$\frac{24451988201550}{181758140654146875}$
$[2^{30}].L_5(2)$	386968944618506250	$\frac{32655623}{3538810650375}$
$S_3 \times Fi_{22}:2$	5362800438804480000	$\frac{161238689449}{335175027425280000}$
$HN:2$	7608628361513926656	$\frac{184}{1399720959}$
$O_8^+(3).S_4$	3495751397146624000	$\frac{49742}{104181510125}$
$3^{1+8}.2^{1+6}.U_4(2).2$	318113377140342784000000	$\frac{2263261}{18961034842750}$
$5:4 \times HS:2$	2341935809673986624716800	$\frac{189812697628409856}{2341935809673986624716800}$
$S_4 \times 2F4(2).2$	4816481232502590013440000	$\frac{2129309108011008}{4816481232502590013440000}$
$3^2.3^3.3^6.(S_4 \times 2S_4)$	20359256136981938176000000	$\frac{2430157994328064}{2035925613698193817600000}$
$A_5.2 \times M_{22}.2$	39032263494566443745280000	$\frac{977822987782717440}{39032263494566443745280000}$
$(S_6 \times L_3(4):2).2$	71559149740038480199680000	$\frac{7908862401183744}{7155914974003848019968000}$
$5^3.L_3(5)$	89350139381213466476937216	$\frac{4870492913664}{89350139381213466476937216}$
$5^{1+4}.2^{1+4}.A_5.4$	173115895051101091299065856	$\frac{3696704121470976}{173115895051101091299065856}$
$5^2:4S_4 \times S_5$	14426324587591757608255488000	$\frac{10629511067190951936}{14426324587591757608255488000}$
$L_2(49).2_3$	35329774500224712510013440000	$\frac{121762322841600}{35329774500224712510013440000}$
$L_2(31)$	279219185566220827404288000005	$\frac{365286968524800}{279219185566220827404288000005}$
M_{11}	524593621366973003936563200000	$\frac{243524645683200}{524593621366973003936563200000}$
$L_3(3)$	739811517312397826064384000000	$\frac{243524645683200}{739811517312397826064384000000}$
$L_2(17).2$	848607328681868094603264000000	$\frac{6899864961024}{848607328681868094603264000000}$
$L_2(11):2$	3147561728201838023619379200000	$\frac{1071508441006080}{3147561728201838023619379200000}$
$47:23$	3843461129719173164826624000000	$\frac{22}{3843461129719173164826624000000}$

If $G \simeq HN$, Ly, ON, Th, or B, then the fixed point ratios and indexes satisfy the hypothesis to Lemma 2.5.19. Thus these group do not possess primitive low genus systems.

The maximal subgroups of the Monster simple group have not been completely classified in the sense that the proof that the maximal 2-locals are exactly those given in the ATLAS exists only in the form of a preprint[29]. The other maximal subgroups were determined in [3].

Lemma 3.11.1. Let G be a group. If for all irreducible complex characters $\chi_i(g)$, $\chi_i(g)/\chi_i(1) < \frac{1}{k+1}$ with $\chi_i(g) \neq \chi_i(1)$ then $\frac{f(g)}{k} < \frac{1}{k}$, $\forall g \in G$.

A complete proof can be found in [24]. This lemma guarantees us that all fixed point ratios of all nontrivial elements of their actions on the known maximal subgroup are $< \frac{1}{100}$ except that for element $2A$ which is $< \frac{1}{44}$. So by Table 2 in [24] the only possible ramification types are $(2A,3,7)$ or $(2A,3,8)$. By Lemma 3.11.1 fixed point ratio of $2A$ is $< \frac{1}{44}$, and fixed point ratios of elements $3,7,8$ are < 230. Thus Lemma 2.5.20 suffices to rule out the Monster as a primitive low genus group.

Chapter 4

Braid Orbits on Nielsen Classes of Sporadic Simple Groups

In this chapter we provide a complete description of the braid orbits of low genus systems for the sporadic simple groups. A ramification type $\bar{\mathrm{C}}= (C_1,\ldots,C_r)$ of the group G is said to be a **Generating Type** if there exists at least one generating tuple $\bar{\mathrm{g}}= (g_1,\ldots,g_r)$ of this type in G. In the previous chapter, we used several filters to eliminate most non- generating types and were left with a small collection of possible generating types of genus zero, one and two for each sporadic simple group. Consider a generating type $\bar{\mathrm{g}}$. Finding the braid orbits $O_{\bar{\mathrm{g}}}$ of the tuple $\bar{\mathrm{g}}$ is uncomplicated. Take a first random tuple $\bar{\mathrm{t}}$ of type $\bar{\mathrm{g}}$ and begin applying the generators of the braid group to $\bar{\mathrm{t}}$ and then recording any new tuples in the list. Eventually we exhaust the orbit of $\bar{\mathrm{t}}$ and then we stop and record it. We repeat the same process for the next random tuple, and so on, until we find all the orbits. Note that the size of the braid orbits of the tuple $\bar{\mathrm{g}}$ is dependent on the type length, that is, the longer the tuple then the sum of the sizes of (the braid orbit) increases dramatically, (roughly equal to $\alpha|g_r^G|$ where $\alpha \in (0,1)$). So computing a braid orbit corresponding to a long tuple may take long time. Firstly, we present tables listing the number of generating ramification types for each sporadic simple groups for which the corresponding Nielsen class is non-empty.

Table 4.1: Number of ramification types of the Mathieu groups which passed all filters

Groups	genus zero	genus one	genus two	total
M_{11}	47	95	149	291
M_{12}	90	194	302	584
$M_{12}:2$	0	2	6	8
M_{22}	24	38	46	108
$M_{22}:2$	52	111	152	315
M_{23}	44	51	83	178
M_{24}	115	231	288	634

Table 4.2: Number of ramification types of the Janko groups which passed all filters

Groups	genus zero	genus one	genus two	total
J_1	1	0	0	1
J_2	2	3	4	9
$J_2:2$	2	7	4	13

Table 4.3: Number of ramification types which passed all filters

Groups	genus zero	genus one	genus two	total
HS	0	2	7	9
$HS:2$	4	6	19	29
Co_3	1	0	0	1

Computing braid orbits on the Nielsen tuples of given type of length three is straightforward. Firstly define double cosets.

Definition 4.0.2. Let H and K be subgroup of the groups G and $x \in G$. A double coset of H and K is the set

$$HxK = \{hxk \mid h \in H, k \in K\}.$$

Note that HxK is the union of the K-orbits on its action on the cosets of H under right multiplication. Double cosets can be used to help us to find braid orbits for the tuples of length three. The next lemma provides us with more detail.

Lemma 4.0.3. Let $C = (C_1, C_2, C_3)$ be a ramification type of length three with class representatives c_1, c_2 and c_3. Then $\langle c_1, c_2^k, (c_1 c_2^k)^{-1} \rangle$ is the generating tuple of C up to conjugation in G where k is in double coset $C_G(c_1)$ - $C_G(c_2)$ representative and $(c_1 c_2^k)^{-1}$ in C_3.

Proof. Complete proof can be found in [37]. □

For the group G the function **Find3Tuple** is defined in [37]. This function required two inputs which are the tuple(it should be of length three) and the group. If the type is a generating type then the output of this function is a list of Nielsen tuples of length one. Each tuple $\bar{g}$ represents an equivalence class of covers of $\mathbb{P}^1$ with ramification type given by $\bar{g}$.

4.1 MAPCLASS

MAPCLASS is a GAP package that is used for calculations of braid orbits and mapping class group orbits. MAPCLASS package has 17 functions. The primary functions of MAPCLASS are

- GeneratingMCOrbits(group,genus,tuple).
- AllMCOrbits(group,genus,tuple).
- GeneratinMCOrbitsCore(group,genus,tuple,partition,constant).
- AllMCOrbitsCore(group,genus,tuple,partition,constant).
- NumberGeneratingTuples(group,genus,tuple).
- TotalNumberTuples(group,genus,tuple).

In our work we used first and second above functions we will explain both of them.

4.1.1 GeneratingMCOrbits

GeneratingMCOrbits(group,genus,tuple) is the primary and most important function used in this computation. The objective of this function is to compute generating braid orbits on the Nielsen tuples of a given type. Note that this function calls a function NumberGeneratingTuples(group,genus,tuple) which finds the number of generating tuples. In this section we are going to explain how the orbits are calculated by the function GeneratingMCOrbits for a given group G, genus g, and ramification type $\bar{\mathrm{C}}= (C_1,\cdots,C_r)$. To achieve this, there are certain steps. The function first generates, for the type and genus, the action of the mapping class group generators. After that, it is calculating the total number of available tuples. This number allows us to know when we have already constructed all orbits. We require the knowledge of the variety of ways of achieving $[a_1,b_1]\ldots[a_g,b_g]c_1\ldots c_r$ for a,b and c in some finite group G, where c_i is in the conjugacy class C_i. This scheme is achievable by taking the cardinality of the set of all homomorphisms from a Fuchsian group $\Upsilon = \langle a_1,b_1,\ldots,a_g,b_g,c_1,\cdots,c_r \mid \prod_{i=1}^{g}[a_i,b_i]\prod_{i=1}^{r}c_i = 1, \rangle$ to the group G, which is denoted by $\Lambda(G,g:C_1,\ldots,C_r)$. Liebeck and Shalev[23]in Lemma

3.1, present an accurate calculation of this.

Theorem 4.1.1.

$$\Lambda(G,g:C_1,\ldots,C_r) = |G|^{2g-1}|C_1|\cdots|C_r| \sum_{\chi\in Irr(G)} (\chi(1)^{2-2g-r}(\sum_{c_i\in C_i}^{r} \chi(c_i))).$$

In particular, the number $\Lambda(G,g:C_1,\ldots,C_r)$ is exponential. The factor $|G|^{2g-1}$ should be acknowledged as the most dominant term in the expression for $\Lambda(G,g:C_1,\ldots,C_r)$.

To find orbits, we choose a random tuple that has length of $2g+r$ in which the elements of the positions $2g+1,\ldots,2g+r$ correspond with our ramification types C and hence the conjugacy class C_i contains the $2g+i$-th element. We are given access to new tuples as we successively apply generators of the mapping class group to a random chosen tuple . The already existing orbit is then compared with the new tuple. If a new tuple is not in the existing orbit then we add it. We then repeat this technique, taking tuples in our orbit, retrieved from the initial tuple. The orbit is then saved. All available tuples need to be considered, so we start taking random tuples that do not exist in our current orbits. It should be noted that the previous entire calculations were up to conjugation in G. A Minimization routine is used to check whether two tuples are conjugate or not. A limited explanation is going to be given but a more comprehensive one can be found in[19]. A tree diagram is the best way of representing the process of minimization and how it takes place. We suppose that the root of our tree is (G,x) where x is a minimal element of conjugacy class C_r. If (K,y) is a node at the level $n-1$ of our tree, the children of (K,y) would be pairs of the form $(C_K(y),m_i)$. Here m_i is is considered as the minimal element of the orbits of $(C_K(x))$ in the conjugacy class C_n. For each conjugacy class, the minimal elements that belong to the same orbit can be tracked. Until all the K are trivial or all conjugacy class are resolved, we proceed with a continuous system. Hence the tuple $g=(g_1,\ldots,g_r)$. The minimization system works in the following manner: for every g_i,a corresponding minimal element m_i is chosen from the tree. All of the g is then conjugated with $k_i\in K$ and hence taking g_i to m_i and then repeating the process with a new tuple. Elements that were previously chosen for the process of minimization will be fixed by further conjugation as we'll be conjugating by an element contained in the intersection of centralizers.

As we have seen this function require three inputs which are the group, the genus and the tuple. The tuple is a tuple of conjugacy class representatives of the given group. Although the number of orbits, length of orbits and size of centralizer etc. are outputs of this function, our requirements are the number of orbits and length of orbits. Now we give an example of a sample run of this function.

Example 4.1.2. Let G be the Mathieu group M_{11}. For genus two in the first class of maximal subgroup there is a tuple $(2A,2A,2A,2A,5A)$ of length five. The `tuple :=[ ( 3, 7)( 4,11)( 5, 9)( 6, 8), ( 1,10)( 2, 4)( 3, 5)( 8, 9), ( 1, 6)( 2, 4)( 5, 9)( 7,10), ( 2, 8)( 3, 4)( 5, 9)( 6,10), ( 1, 8, 2, 5, 7)( 3,11, 4, 9, 6) ]` is of type $(2A,2A,2A,2A,5A)$.

```
gap> GeneratingMCOrbits(G,0,Type);;
gap> Length(last);
gap>1
gap>orb:=orbits[1];;
gap>Length(orb.TableTuple);
gap>12000
```

We see that in this case the braid group has exactly one orbit of length 12000 on the Nielsen class $(2A,2A,2A,2A,5A)$. So for the tuple of long length in large groups the problem such of computing the braid orbits requires a computer with large memory and such computer may not be available. We then used the function AllMCOrbits(Group,genus,type) we will explain in next section. Now in Table 4.4 we collect the data for our calculations of braid orbits for groups of the tuples of length $\geqslant 4$.

Table 4.4: Generating braid orbits for types of length$\geqslant 4$

Groups	$RamificationTypes$	$N.Orbits$	$LengthOfOrbits$
	$(2A,2A,2A,2A,3A)$	1	2376
M_{11}	$(2A,2A,2A,2A,2A,2A)$	1	229680
	$(2A,2A,2A,2A,5A)$	1	12000
	$(2A,2A,2A,2A,3A)$	1	2376
	$(2A,2A,2A,2A,6A)$	1	12528
	$(2B,2B,2B,2B,8A)$	2	26880,26880
M_{12}	$(2B,2B,2B,3A,3A)$	2	15840,6024
	$(2B,2B,2B,2B,6B)$	2	25056,26864
	$(2B,2B,2B,2B,3B)$	2	8280,5562
	$(2B,2B,2B,2B,2B,2B)$	2	588800,332640
	$(2A,3A,3A,3A)$	2	1680,2448
M_{22}	$(2A,2A,3A,5A)$	2	1380,1500
	$(2A,2A,4A,4B)$	3	108,108,100
	$(2A,2A,2A,2A,4B)$	5	2960,13056,11232,12960,9792
	$(2A,2A,2A,2A,4A)$	4	27456,52992,52992,30912
	$(2A,2A,2A,2A,3A)$	1	21456
M_{23}	$(2A,2A,2A,2A,6A)$	1	1050336
	$(2A,2A,2A,2A,5A)$	1	732000
	$(2A,2A,2A,3A,3A)$	1	850392
	$(2A,2A,2A,2A,2A,2A)$	1	
	$(2A,2A,2A,2A,4B)$	1	72000
M_{24}	$(2A,2A,4B,6A)$	1	57023
	$(2A,2A,4B,5A)$	1	1970
	$(2A,2A,4B,8A)$	1	34944
	$(2A,2A,2A,2A,5A)$	1	342600

4.1.2 AllMCOrbits

Both generating braid orbits and non-generating braid orbits of the given groups are computed by this function. For the large groups or for the long tuples the function GeneratingMCOrbits does not work. For instance the braid orbits of the ramification types in Table 4.4 for the group M_{24} can not be found by the function GeneratingMCOrbits. Using this function is quite easy but the long time is required to compute it. Moreover we have to use several steps to find generating braid orbits. This function requires three inputs which are the group, the genus and the tuple. The number of all orbits, length of orbits and size of centralizer etc. are outputs of this function, our requirement are the number of generating orbits and length of generating orbits.

```
AllMCOrbits(group,genus,tuple)
```

The following example demonstrates how one can find generating braid orbits.

Example 4.1.3. Let G be the smallest Mathieu group M_{11} and $g = 2$. The $tuple = [(2,10)(3,4)(6,8)(7,11),(2,8,10,6)(3,7,4,11),(1,2,7)(3,8,4)(9,10,11),$
$(2,7,6,3,10,11,8,4)(5,9)]$
is of type $(2A,3A,4A,8A)$. Recall that a ramification type $\bar{C}= (C_1,\ldots,C_r)$ of the group G is said to be of generating type if there exists at least one generating tuple $\bar{g}= (g_1,\ldots,g_r)$ of this group. In the other words a ramification type $\bar{C}= (2A,3A,4A,8A)$ the will be of generating type if there exists a braid orbit O such that the tuple from it generates sporadic the simple group G.

```
gap> AllPrimitiveGroups(DegreeOperation,11);;
gap>G:=last[6];
M(11)
gap>O:=AllMCOrbits(G,0,tuple);
gap> Length(O);
4
gap>for i in [1..Length(O)]do
>Print(Length(O[i].TupleTable));
> Print("\n");
>fi;od;
951
225
12
6
```

The above program shows that there are four braid orbits of length 951,225,12,6. Now for each braid orbit we look for a group K generated by the tuple from the orbit O. Firstly we have to check that whether K is a primitive group.

```
gap>for i in [1..Length(O)]do
>h:=Group(O[i].TupleTable[1].tuple); >if Isprimitive(h)=true then ;
> Print(i);
> Print("\n");
```

```
>fi;
>od;
1
2
```

It should be noted that the first tuple from orbits one and two generate primitive groups. On the other hand the first tuple of the orbit three and four does not generate the primitive group. Next we check that the size of primitive groups is equal to the Mathieu group M_{11}.

```
gap> A:=AllPrimitiveGroups(DegreeOperation,11);
[ C(11), D(2*11), 11:5, AGL(1, 11), L(2, 11), M(11), A(11), S(11) ]
gap> h1:=Group(O[1].TupleTable[1].tuple);;
gap>for i in [1..Length(A)]do if size(A[i])=Size(h) then ;
> Print(i);
> Print("\n");
>fi;
>od;
6
gap>IsomorphismGroups(h1,A[6]);
[ (2,3)(5,11)(7,8)(9,10), (1,3,6,5)(4,9,7,11) ]-> [(2,3)(4,11)(5,6)(7,10),
(1,3,6,4)(8,11,9,10) ]
```

This means that the first tuple from orbit one generates a primitive group which isomorphic to the smallest Mathieu group M_{11}.

```
 h2:=Group(O[2].TupleTable[1].tuple);;
gap>for i in [1..Length(A)]do if size(A[i])=Size(h) then ;
> Print(i);
> Print("\n");
>fi;od;
```

So the first tuple from orbit two generates a primitive group which is not isomorphic to the group M_{11}. Hence we ignore this primitive group. Thus the ramification type $\bar{C}= (2A,3A,4A,8A)$ has one braid orbit of length 951.

CHAPTER 5

CONCLUSIONS

This thesis set out to calculate the connected components of $H^{in}(G,C)$ where G is a sporadic simple group . The total numbers of components of $H^{in}(G,C)$ are shown in the Tables

Table 5.1: Number of Components of Genus Zero

Groups	♯Ramification Type	♯comp's $r=3$	♯comp's $r=4$	♯comp's $r=5$	♯comp's $r=6$
M_{11}	47	34	12	1	-
M_{12}	90	62	25	3	-
M_{22}	24	19	5	-	-
$M_{22}:2$	52	34	16	2	-
M_{23}	44	33	10	1	-
M_{24}	115	96	18	1	-
J_1	1	1	-	-	-
J_2	2	2	-	-	-
$J_2:2$	2	2	-	-	-
Co_3	1	1	-	-	-
$HS:2$	4	4	-	-	-

Table 5.2: Number of Components of Genus one

Groups	♯Ramification Type	♯comp's $r=3$	♯comp's $r=4$	♯comp's $r=5$	♯comp's $r=6$
M_{11}	95	60	28	6	1
M_{12}	194	113	69	11	1
$M_{12}:2$	2	2	-	-	-
M_{22}	38	30	7	1	-
$M_{22}:2$	111	63	38	9	1
M_{23}	51	42	8	1	-
M_{24}	231	180	43	7	1*
J_2	3	3	-	-	-
$J_2:2$	7	6	1	-	-
HS	2	2	-	-	-
$HS:2$	6	5	1	-	-

*Ramification type of length six of Mathieu group M_{24} we were not able to compute all braid orbit.

Table 5.3: Number of Components of Genus two

Groups	♯Ramification Type	♯comp's $r=3$	♯comp's $r=4$	♯comp's $r=5$	♯comp's $r=6$
M_{11}	149	76	56	15	2
M_{12}	302	122	147	30	3
$M_{12}:2$	6	5	1	-	-
M_{22}	46	35	9	2	-
$M_{22}:2$	152	73	62	16	2
M_{23}	83	63	16	3	1
M_{24}	288	222	60	6	-
J_2	4	4	-	-	-
$J_2:2$	4	3	1	-	-
HS	7	7	-	-	-
$HS:2$	19	16	3	-	-

In fact to establish this result, firstly in Chapter 2 and the first section of Chapter 3, we presented some filters to eliminate non generating ramification types of sporadic simple groups. Moreover, we proved Lemma 2.5.18, Lemma 2.5.19 and Lemma2.5.20, that play an important role in eliminating ramification types of large sporadic simple groups. In Chapter 3 we check whether or not the sporadic simple groups possessed primitive genus g systems by using filters. For large sporadic simple groups, we found fixed point ratios and, by using Lemma 2.5.19, we showed that these groups possessed no primitive genus g-system. Note that the GAP library stores primitive permutation groups up to degree 2500. In some cases where the group possesses a permutation representation of degree < 2500, we were able to construct explicitly all the permutation representations of degree > 2500 that we needed to work with .

Secondly, we computed braid orbits of the sporadic simple groups that possess primitive genus g-systems by using the packages Braid and MapClass. However, in one case, namely $(2A,2A,2A,2A,2A,2A)$ for the large Mathieu group M_{24}, we were not able to compute all braid orbit because its Nielsen class is too big. We leave this case open but suspect that there is only one braid orbit of this type of length 12307440. We hope to be able to address it in our future work. In this thesis we compute braid orbits of Nielsen Class of sporadic simple groups, future work we will compute the braid orbits of Nielsen class of low genus systems for other almost simple groups (Classical groups).

APPENDIX A

GENUS ZERO COVERS

Appendix A contains table representing the result of our computation of primitive genus zero cover in sporadic simple groups satisfying Theorem 1.0.2. Note that N.Orbit means number of orbits, L.O means length of orbits.

Table 5.4: $M_{11}, g = 0$, Of Degree 11

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(3A,3A,8A)$	2	1	$(3A,3A,8B)$	2	1
$(3A,4A,6A)$	5	1	$(3A,4A,8A)$	3	1
$(3A,4A,8B)$	3	1	$(3A,4A,5A)$	5	1
$(4A,4A,8B)$	4	1	$(4A,4A,8A)$	4	1
$(4A,4A,6A)$	12	1	$(4A,4A,5A)$	6	1
$(2A,6A,8A)$	3	1	$(2A,6A,8B)$	2	1
$(2A,8A,8A)$	3	1	$(2A,8B,8B)$	2	1
$(2A,4A,11A)$	1	1	$(2A,4A,11B)$	1	1
$(2A,2A,3A,4A)$	1	92	$(2A,2A,4A,4A)$	1	168
$(2A,2A,2A,8A)$	1	48	$(2A,2A,2A,8B)$	1	48
$(2A,5A,8A)$	3	1	$(2A,5B,8B)$	3	1

Table 5.5: $M_{11}, g = 0$, Of Degree 12

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(3A,5A,6A)$	7	1	$(3A,6A,6A)$	12	1
$(3A,3A,8A)$	2	1	$(3A,3A,8B)$	2	1
$(3A,4A,5A)$	5	1	$(3A,4A,6A)$	5	1
$(2A,5A,11A)$	1	1	$(2A,5A,11B)$	1	1
$(2A,5A,8A)$	3	1	$(2A,5A,8B)$	3	1
$(2A,6A,11A)$	3	1	$(2A,6A,11B)$	3	1
$(2A,6A,8A)$	3	1	$(2A,6A,8B)$	3	1
$(2A,4A,11A)$	1	1	$(2A,4A,11B)$	1	1
$(2A,3A,3A,3A)$	1	63	$(2A,2A,3A,5A)$	1	100
$(2A,2A,3A,6A)$	1	92	$(2A,2A,3A,4A)$	1	156
$(2A,2A,2A,11A)$	1	33	$(2A,2A,2A,11B)$	1	33
$(2A,2A,2A,8A)$	1	48	$(2A,2A,2A,8B)$	1	48
$(2A,2A,2A,2A,3A)$	1	2376			

Table 5.6: $M_{12}, g=0$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4A,8A,8A)$	4	1	$(4A,4B,11B)$	1	1
$(4A,4B,11A)$	1	1	$(4B,4B,8B)$	4	1
$(4B,4B,6A)$	2	1	$(4B,4B,10A)$	2	1
$(4B,6B,8A)$	16	1	$(4B,6B,6B)$	38	1
$(3B,4B,8A)$	5	1	$(3B,4B,6B)$	10	1
$(4A,4B,8A)$	6	1	$(4A,4B,6B)$	8	1
$(3B,4A,4B)$	4	1	$(4B,5A,8A)$	7	1
$(4B,5A,6B)$	1	1	$(3B,4B,5A)$	7	1
$(4A,4B,5A)$	4	1	$(4B,5A,5A)$	3	1
$(3A,8A,8A)$	6	1	$(3A,4B,11B)$	1	1
$(3A,4B,11A)$	1	1	$(3A,4B,8B)$	2	1
$(3A,4B,6A)$	3	1	$(3A,4B,10A)$	3	1
$(3A,6B,8A)$	16	1	$(3A,6B,6B)$	8	1
$(3A,3A,11A)$	1	1	$(3A,3A,11B)$	1	1
$(3A,3A,6A)$	2	1	$(3A,3B,8A)$	6	1
$(3A,3B,6B)$	6	1	$(3A,5A,8A)$	4	1
$(3A,3B,5A)$	2	1	$(3A,4A,8A)$	2	1
$(3A,5A,6B)$	6	1	$(3A,5A,5A)$	6	1
$(2B,8A,11B)$	2	1	$(2B,8A,11A)$	2	1
$(2B,8A,8B)$	4	1	$(2B,6A,8A)$	4	1
$(2B,8A,10A)$	2	1	$(2B,6B,11A)$	1	1
$(2B,6B,11B)$	1	1	$(2B,6A,6B)$	8	1
$(2B,6B,10A)$	8	1	$(2B,3B,11A)$	1	1
$(2B,3B,11A)$	1	1	$(2B,3B,10A)$	2	1
$(2B,5A,11B)$	1	1	$(2B,5A,11A)$	1	1
$(2B,5A,6A)$	2	1	$(2B,5A,10A)$	2	1
$(2A,8A,8A)$	1	1	$(2A,4B,11A)$	1	1
$(2B,4B,11B)$	1	1	$(2A,6B,8A)$	6	1
$(2A,6B,6B)$	8	1	$(2A,3A,11A)$	1	1
$(2A,3A,11B)$	1	1	$(2A,5A,8A)$	3	1
$(2B,5A,6B)$	1	6	$(2B,4B,4B,4B)$	1	244
$(2B,3A,4B,4B)$	1	240	$(2B,3A,3A,4B)$	1	132
$(2B,3A,3A,3A)$	1	1	$(2A,2B,3A,3A)$	1	48
$(2B,2B,4B,8A)$	1	288	$(2B,2B,4B,6B)$	1	504
$(2B,2B,3B,4B)$	1	144	$(2B,2B,4A,4B)$	1	88
$(2B,2B,4B,5A)$	1	220	$(2B,2B,3A,8A)$	2	104,104
$(2B,2B,3A,6B)$	1	144	$(2B,2B,3A,3B)$	1	56
$(2B,2B,3A,5A)$	1	120	$(2B,2B,2B,11A)$	1	22
$(2B,2B,2B,11B)$	1	22	$(2B,2B,2B,6A)$	1	72
$(2B,2B,2B,2B,4B)$	1	7269	$(2B,2B,2B,2B,3A)$	1	2784
$(2A,2B,2B,2B,2B)$	1	2048	$(2B,2B,2B,10A)$	2	40,40
$(2A,2B,2B,8A)$	1	64	$(2A,2B,2B,6B)$	1	144
$(2A,2B,2B,3B)$	1	32	$(2A,2B,2B,5A)$	1	80
$(2A,2B,4B,4B,)$	1	80	$(2A,2B,3A,4B)$	1	66

Table 5.7: $M_{12}, g=0$, Of Degree 66

$RamificationType$	$N.Orbit$	LO	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,10A)$	2	1	$(2A,2B,2B,3B)$	1	32

Table 5.8: $M_{22}, g=0$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3A,3A,7A)$	12	1	$(3A,3A,7B)$	12	1
$(3A,3A,8A)$	16	1	$(3A,4B,5A)$	8	1
$(3A,4B,6A)$	12	1	$(3A,4A,5A)$	48	1
$(3A,4A,6A)$	24	1	$(4A,4A,4B)$	26	1
$(4A,4A,4A)$	12	1	$(2A,5A,7A)$	10	1
$(2A,5A,7B)$	10	1	$(2A,5A,8A)$	12	1
$(2A,6A,7A)$	6	1	$(2A,6A,7B)$	6	1
$(2A,6A,8A)$	12	1	$(2A,4B,11A)$	2	1
$(2A,4B,11B)$	2	1	$(2A,3A,11A)$	4	1
$(2A,3A,11B)$	4	1	$(2A,2A,3A,4B)$	1	180
$(2A,2A,3A,4A)$	1	180	$(2A,2A,2A,7A)$	3	42
$(2A,2A,2A,7B)$	3	42	$(2A,2A,2A,8A)$	4	48

Table 5.9: $M=M_{22}.2, g=0$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4D,4D,5A)$	6	1	$(4D,4D,6A)$	2	1
$(4A,4C,4D)$	3	1	$(4A,4D,6B)$	20	1
$(2C,4D,7A)$	2	1	$(2C,4D,7B)$	2	1
$(2C,4A,8B)$	3	1	$(2C,5A,6B)$	11	1
$(2C,6A,6B)$	7	1	$(2C,3A,14A)$	2	1
$(2C,3A,14B)$	2	1	$(2A,8B,8B)$	3	1
$(2C,4C,14B)$	1	1	$(2A,4C,14A)$	1	1
$(2A,6B,10A)$	10	1	$(2A,6B,14B)$	3	1
$(2A,6B,14A)$	3	1	$(2A,6B,12A)$	6	1
$(3A,4D,8B)$	4	1	$(3A,6B,4C)$	10	1
$(3A,6B,6B)$	10	1	$(3A,7B,8B)$	1	1
$(2B,7A,8B)$	1	1	$(2B,5A,10A)$	4	1
$(2B,5A,14B)$	1	1	$(2B,5A,14A)$	1	1
$(2B,5A,12A)$	1	1	$(2B,4C,11A)$	1	1
$(2B,6A,10A)$	2	1	$(2B,6A,14A)$	1	1
$(2B,6A,14B)$	1	1	$(2B,6A,12A)$	2	1
$(2B,6B,11A)$	3	1	$(2A,2A,4D,4D)$	1	128
$(2A,2A,2C,6B)$	1	156	$(2A,2B,4A,4D)$	1	94
$(2A,2A,2B,2B,3A)$	1	600	$(2A,2B,2C,5A)$	1	45
$(2A,2B,2C,6A)$	1	30	$(2A,2A,2B,10A)$	2	20
$(2A,2A,2B,14B)$	1	14	$(2A,2A,2B,14A)$	1	14
$(2A,2A,2B,12A)$	1	24	$(2A,2A,2A,2B,2C)$	1	660
$(2A,2B,3A,4C)$	1	34	$(2A,2B,3A,6B)$	1	123
$(2B,2B,4A,4A)$	1	34	$(2A,2B,2B,11A)$	1	11
$(2B,2A,3A,5A)$	1	88	$(2B,2B,3A,6A)$	1	36
$(2B,2C,3A,3A)$	1	72			

Table 5.10: $M_{22}:2, g=0$, Of Degree 77

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,4C,11A)$	1	1			

Table 5.11: $M_{23}, g=0$, Of Degree 23

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4A,4A,5A)$	104	1	$(4A,4A,6A)$	192	1
$(3A,5A,5A)$	30	1	$(3A,4A,8A)$	54	1
$(3A,4A,7B)$	15	1	$(3A,4A,7A)$	15	1
$(3A,5A,6A)$	70	1	$(3A,6A,6A)$	120	1
$(3A,3A,15B)$	4	1	$(3A,3A,15A)$	4	1
$(3A,3A,11B)$	6	1	$(3A,3A,11A)$	6	1
$(3A,3A,14B)$	4	1	$(3A,3A,14A)$	4	1
$(2A,8A,8A)$	28	1	$(2A,5A,15B)$	5	1
$(2A,5A,15A)$	5	1	$(2A,5A,11B)$	6	1
$(2A,5A,11A)$	6	1	$(2A,5A,14B)$	3	1
$(2A,5A,14A)$	3	1	$(2A,4A,23B)$	2	1
$(2A,4A,23A)$	2	1	$(2A,7B,8A)$	8	1
$(2A,7B,7B)$	4	1	$(2A,7A,8A)$	8	1
$(2A,7A,7A)$	4	1	$(2A,6A,15B)$	9	1
$(2A,6A,15A)$	6	1	$(2A,4A,11B)$	12	1
$(2A,4A,11A)$	12	1	$(2A,4A,14B)$	9	1
$(2A,4A,14A)$	9	1	$(2A,3A,3A,3A)$	1	996
$(2A,2A,4A,4A)$	1	2456	$(2A,2A,3A,5A)$	1	980
$(2A,2A,3A,6A)$	1	1428	$(2A,2A,2A,15B)$	1	90
$(2A,2A,2A,15A)1$		90	$(2A,2A,2A,11B)$	2	66,66
$(2A,2A,2A,11A)$	2	66,66	$(2A,2A,2A,14B)$	1	84
$(2A,2A,2A,14A)$	1	84	$(2A,2A,2A,2A,3A)$	1	21456

Table 5.12: $M_{24}, g = 0$, Of Degree 24

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(4B,4B,8A)$	112	1	$(4B,4B,7B)$	22	1
$(4B,4B,7A)$	22	1	$(4B,4B,4C)$	40	1
$(4B,6A,6A)$	478	1	$(4A,4B,6A)$	44	1
$(4B,5A,6A)$	158	1	$(4A,4B,5A)$	11	1
$(4B,5A,5A)$	38	1	$(3B,4B,6A)$	62	1
$(3B,4A,4B)$	4	1	$(3B,4B,5A)$	30	1
$(2A,8A,12A)$	26	1	$(2A,8A,14A)$	9	1
$(2A,8A,14B)$	9	1	$(2A,8A,11A)$	13	1
$(2A,8A,10A)$	10	1	$(2A,8A,15B)$	9	1
$(2A,8A,15A)$	9	1	$(2A,6B,8A)$	2	1
$(2A,6A,21B)$	7	1	$(2A,6A,21A)$	7	1
$(2A,6A,23B)$	5	1	$(2A,6A,23A)$	55	1
$(2A,4A,21B)$	1	1	$(2A,4A,21A)$	1	1
$(2A,7B,12A)$	4	1	$(2A,7B,14A)$	3	1
$(2A,7B,11A)$	2	1	$(2A,7B,10A)$	1	1
$(2A,7B,15B)$	2	1	$(2A,7B,15A)$	2	1
$(2A,7B,6B)$	4	1	$(2A,7A,12A)$	4	1
$(2A,7A,14B)$	3	1	$(2A,7A,11A)$	2	1
$(2A,7A,11B)$	1	1	$(2A,7A,15B)$	2	1
$(2A,7A,15A)$	2	1	$(2A,7A,6B)$	4	1
$(2A,5A,21B)$	2	1	$(2A,5A,21A)$	2	1
$(2A,5A,12B)$	3	1	$(2A,5A,14B)$	4	1
$(2A,5A,14A)$	4	1	$(2A,4C,11A)$	5	1
$(2A,4C,15B)$	4	1	$(2A,4C,15A)$	4	1
$(2A,3B,23B)$	1	1	$(2A,3B,23A)$	1	1
$(3A,4B,12A)$	26	1	$(3A,4B,14B)$	12	1
$(3A,4B,14A)$	12	1	$(3A,4B,11A)$	15	1
$(3A,4B,10A)$	18	1	$(3A,4B,15B)$	12	1
$(3A,4B,15A)$	12	1	$(3A,4B,6B)$	21	1
$(3A,6A,8A)$	96	1	$(3A,6A,7B)$	16	1
$(3A,6A,7A)$	16	1	$(3A,4C,6A)$	28	1
$(3A,4A,8A)$	10	1	$(3A,4A,7B)$	2	1
$(3A,4A,7A)$	2	1	$(3A,3A,21B)$	1	1
$(3A,3A,21A)$	2	1	$(3A,3A,23B)$	1	1
$(3A,3A,23A)$	2	1	$(3A,3A,12B)$	4	1
$(3A,5A,8A)$	28	1	$(3A,5A,7B)$	4	1
$(3A,5A,7A)$	4	1	$(3A,4C,5A)$	7	1
$(3A,3B,8A)$	14	1	$(3A,3B,7B)$	5	1
$(3A,3B,7A)$	5	1	$(3A,3B,4B)$	3	1
$(2B,4B,14B)$	8	1	$(2B,4B,14A)$	8	1
$(2B,4B,11A)$	11	1	$(2B,4B,15B)$	8	1

Table 5.13: $M_{24}, g = 0$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B, 4B, 15A)$	8	1	$(2B, 6A, 8A)$	42	1
$(2B, 6A, 7B)$	11	1	$(2B, 6A, 7A)$	11	1
$(2B, 3A, 21B)$	1	1	$(2B, 3A, 21A)$	1	1
$(2B, 3A, 23B)$	1	1	$(2B, 3A, 23A)$	1	1
$(2B, 5A, 8A)$	14	1	$(2B, 5A, 7B)$	5	1
$(2B, 5A, 7A)$	5	1	$(2A, 6A, 12B)$	19	1
$(2A, 5A, 23B)$	5	1	$(2A, 5A, 23A)$	1	1
$(2A, 2A, 4B, 6A)$	1	5730	$(2A, 2A, 4A, 4B)$	1	464
$(2A, 2A, 4B, 5A)$	1	1970	$(2A, 2A, 3B, 4B)$	1	969
$(2A, 2A, 2A, 2A, 4B)$	1	72000	$(2A, 2A, 2A, 21B)$	1	63
$(2A, 2A, 2A, 21A)$	1	63	$(2A, 2A, 2A, 23B)$	1	46
$(2A, 2A, 2A, 23A)$	1	46	$(2A, 2A, 2A, 12B)$	1	144
$(2A, 2A, 2A, 8A)$	1	1128	$(2A, 2A, 3A, 7B)$	1	224
$(2A, 2A, 3A, 7A)$	1	224	$(2A, 2A, 3A, 4B)$	1	684
$(2A, 2A, 2B, 8A)$	1	416	$(2A, 2A, 2B, 7B)$	1	98
$(2A, 2A, 2B, 7A)$	1	98	$(2A, 3A, 3A, 4B)$	1	1776
$(2A, 2B, 3A, 4B)$	1	684			

Table 5.14: $J_1, g = 0$, Of Degree 266

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A, 3A, 7A)$	7	1			

Table 5.15: $J_2, g = 0$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B, 3B, 7A)$	10	1			

Table 5.16: $J_2, g = 0$, Of Degree 280

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B, 3B, 7A)$	10	1			

Table 5.17: $J_2 : 2, g = 0$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C, 3B, 14A)$	3	1	(2C,3B,12C)	2	1

Table 5.18: $Co_3, g = 0$, Of Degree 276

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B, 3C, 7A)$	12	1			

Table 5.19: $HS : 2, g = 0$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C, 4F, 10A)$	1	1	(2D, 5C, 6C)	9	1
$(2D, 4F, 6B)$	10	1	(2D, 4F, 5C)	1	1

Appendix B

GENUS ONE COVERS

Appendix B contains table representing the result of our computation of primitive genus one cover in sporadic simple groups satisfying Theorem 1.0.2. Note that N.Orbit means number of orbits, L.O means length of orbits.

Table 5.20: $M_{11}, g = 1$, Of Degree 11

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3A,6A,6A)$	12	1	$(3A,6A,8A)$	9	1
$(3A,6A,8B)$	9	1	$(3A,8A,8A)$	3	1
$(3A,8B,8B)$	3	1	$(3A,5A,6A)$	7	1
$(3A,5A,8A)$	9	1	$(3A,5A,8B)$	9	1
$(4A,6A,6A)$	30	1	$(3A,4A,11A)$	4	1
$(3A,4A,11B)$	4	1	$(4A,6A,8A)$	18	1
$(4A,6A,8B)$	18	1	$(4A,8B,8B)$	10	1
$(4A,8A,8A)$	10	1	$(4A,8A,8B)$	10	1
$(3A,8A,8B)$	7	1	$(4A,4A,11A)$	7	1
$(4A,4A,11B)$	7	1	$(4A,5A,6A)$	31	1
$(4A,5A,8A)$	17	1	$(4A,5A,8B)$	17	1
$(4A,5A,5A)$	28	1	$(2A,6A,11A)$	3	1
$(2A,6A,11B)$	3	1	$(2A,3A,3A,3A)$	1	63
$(2A,8A,11A)$	2	1	$(2A,8A,11B)$	2	1
$(2A,8B,11A)$	2	1	$(2A,8B,11B)$	2	1
$(2A,3A,3A,4A)$	1	368	$(2A,3A,4A,4A)$	1	708
$(2A,4A,4A,4A)$	1	1328	$(2A,2A,3A,6A)$	1	156
$(2A,2A,3A,8A)$	1	160	$(2A,2A,3A,8B)$	1	160
$(2A,2A,3A,5A)$	1	100	$(2A,2A,4A,6A)$	1	472
$(2A,2A,4A,8A)$	1	304	$(2A,2A,4A,8B)$	1	204
$(2A,2A,4A,5A)$	1	500	$(2A,2A,2A,11A)$	1	33
$(2A,2A,2A,11B)$	1	33	$(2A,2A,2A,2A,3A)$	1	2376
$(2A,2A,2A,2A,4A)$	1	8832	$(2A,5A,11A)$	1	1
$(2A,5A,11B)$	1	1			

Table 5.21: $M_{11}, g = 1$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(5A,5A,5A)$	24	1	$(5A,5A,6A)$	38	1
$(5A,6A,6A)$	34	1	$(6A,6A,6A)$	36	1
$(3A,5A,11A)$	5	1	$(3A,5A,11B)$	5	1
$(3A,5A,8A)$	9	1	$(3A,5A,8B)$	9	1
$(3A,6A,11A)$	5	1	$(3A,6A,11B)$	4	1
$(3A,4A,11A)$	4	1	$(3A,4A,11B)$	4	1
$(3A,6A,8A)$	9	1	$(3A,6A,8B)$	9	1
$(3A,4A,8A)$	3	1	$(3A,4A,8B)$	3	1
$(4A,5A,5A)$	28	1	$(4A,5A,6A)$	31	1
$(4A,6A,6A)$	30	1	$(4A,4A,5A)$	6	1
$(4A,4A,6A)$	12	1	$(2A,11A,11A)$	1	1
$(2A,11B,11B)$	1	1	$(2A,8A,11A)$	2	1
$(2A,8A,11B)$	2	1	$(2A,8B,11A)$	2	1
$(2A,8B,11B)$	2	1	$(2A,8B,8B)$	2	1
$(2A,8A,8A)$	2	1	$(3A,3A,3A,3A)$	1	288
$(2A,3A,3A,5A)$	1	385	$(2A,3A,3A,6A)$	1	444
$(2A,3A,3A,4A)$	1	368	$(2A,3A,5A,5A)$	1	570
$(2A,2A,5A,5A)$	1	570	$(2A,2A,6A,6A)$	1	708
$(2A,2A,3A,11A)$	1	77	$(2A,2A,3A,11B)$	1	77
$(2A,2A,3A,8A)$	1	160	$(2A,2A,3A,8B)$	1	160
$(2A,2A,4A,5A)$	1	500	$(2A,2A,4A,6A)$	1	472
$(2A,2A,4A,4A)$	1	168,92	$(2A,2A,2A,3A,3A)$	1	8280
$(2A,2A,2A,2A,5A)$	1	12000	$(2A,2A,2A,2A,6A)$	1	12528
$(2A,2A,2A,2A,4A)$	1	8832	$(2A,2A,2A,2A,2A,2A)$	1	229680

Table 5.22: $M_{12}, g = 1$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(8B,8B,8B)$	68	1	$(4B,8B,11B)$	11	1
$(4B,8B,11A)$	11	1	$(4B,8A,8B)$	36	1
$(4B,6A,8B)$	18	1	$(4B,8B,10A)$	20	1
$(4B,6B,11B)$	27	1	$(4B,6B,11A)$	27	1
$(4B,6B,8A)$	58	1	$(4B,6A,6B)$	39	1
$(4B,6B,10A)$	39	1	$(3B,4B,11B)$	8	1
$(3B,4B,11A)$	8	1	$(3B,4B,8A)$	13	1
$(3B,4B,10A)$	10	1	$(4A,4B,11B)$	8	1
$(4A,4B,11A)$	8	1	$(4A,4B,8A)$	6	1
$(4A,4B,6A)$	10	1	$(3B,4B,6A)$	7	1
$(4A,4B,10A)$	6	1	$(4B,5A,11B)$	10	1
$(4B,5A,11A)$	10	1	$(4B,5A,8A)$	24	1
$(4B,5A,6A)$	19	1	$(4B,5A,10A)$	22	1
$(6B,8B,8B)$	124	1	$(6B,6B,8B)$	256	1
$(6B,6B,6B)$	332	1	$(3A,8B,11B)$	10	1
$(3A,8B,11A)$	10	1	$(3A,8A,8B)$	16	1
$(3A,6A,8B)$	18	1	$(3A,8B,10A)$	18	1
$(3A,6B,11B)$	16	1	$(3A,6B,11A)$	16	1
$(3A,6B,8A)$	16	1	$(3A,6A,6B)$	24	1
$(3A,6B,10A)$	28	1	$(3A,3B,11B)$	5	1
$(3A,3B,11A)$	5	1	$(3A,3B,8A)$	6	1
$(3A,3B,6A)$	4	1	$(3A,3B,10A)$	8	1
$(3A,4A,11B)$	1	1	$(3A,4A,11A)$	1	1
$(3A,4A,6A)$	3	1	$(3A,4A,10A)$	3	1
$(3A,5A,11B)$	4	1	$(3A,5A,11A)$	4	1
$(3A,5A,8A)$	4	1	$(3A,5A,6A)$	6	1
$(3A,5A,10A)$	16	1	$(3B,8B,8B)$	32	1
$(3B,6B,8B)$	50	1	$(3B,6B,6B)$	72	1
$(3B,3B,8B)$	8	1	$(3B,3B,6B)$	8	1
$(4A,8B,8B)$	44	1	$(4A,6B,8B)$	58	1
$(4A,6B,6B)$	38	1	$(3B,4A,8A)$	13	1
$(3B,4A,6B)$	10	1	$(4A,4A,8B)$	4	1
$(2B,11B,11B)$	2	1	$(2B,11A,11B)$	3	1
$(2B,11A,11A)$	3	1	$(2B,8A,11B)$	2	1
$(2B,8A,11A)$	2	1	$(2B,6A,11B)$	4	1
$(2B,6A,11A)$	4	1	$(2B,6A,8A)$	4	1
$(2B,6A,6A)$	4	1	$(2B,10A,11B)$	4	1
$(2B,10A,11A)$	4	1	$(2B,8A,10A)$	2	1
$(2B,6A,10A)$	6	1	$(2B,10A,10A)$	4	1
$(5A,8B,8B)$	66	1	$(5A,6B,8B)$	132	1
$(5A,6B,6B)$	160	1	$(3B,5A,8B)$	32	1
$(3B,5A,6B)$	44	1	$(3B,3B,5A)$	2	1
$(4A,5A,8B)$	24	1	$(4A,5A,6B)$	14	1
$(3B,4A,5A)$	7	1	$(5A,5A,8B)$	48	1

Table 5.23: $M_{12}, g = 1$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(5A,5A,6B)$	64	1	$(3B,5A,5A)$	12	1
$(4A,5A,5A)$	3	1	$(5A,5A,5A)$	6	1
$(2A,8B,11B)$	4	1	$(2A,8B,11A)$	4	1
$(2A,8A,8B)$	8	1	$(2A,6A,8B)$	4	1
$(2A,8B,10A)$	5	1	$(2A,6B,11B)$	6	1
$(2A,6B,11A)$	6	1	$(2A,6B,8A)$	6	1
$(2A,6A,6B)$	6	1	$(2A,6B,10A)$	6	1
$(2A,4A,11B)$	1	1	$(2A,4A,11A)$	1	1
$(2A,5A,11A)$	6	1	$(2A,5A,11B)$	6	1
$(2A,5A,8B)$	3	1	$(4B,4B,4B,4B)$	3	528,1056,180
$(3A,4B,4B,4B)$	1	1494	$(3A,3A,4B,4B)$	3	72,1020,132
$(3A,3A,3A,4B)$	1	792	$(3A,3A,3A,3A)$	2	132,288
$(2B,4B,4B,8B)$	1	2160	$(2B,4B,4B,6B)$	1	4000
$(2B,3B,4B,4B)$	1	936	$(2B,4A,4B,4B)$	1	972
$(2B,4B,4B,5A)$	1	1940	$(2B,3A,4B,6B)$	1	2816
$(2B,3A,3B,4B)$	1	670	$(2B,3A,4A,4B)$	1	530
$(2B,3A,4B,5A)$	1	1310	$(2B,3A,3A,8B)$	2	1310,524
$(2B,3A,4B,8B)$	1	1756	$(2B,3A,3A,6B)$	2	524,524
$(2B,3A,3A,3B)$	2	162,192	$(2B,3A,3A,4A)$	1	132
$(2B,3A,3A,5A)$	2	570,120	$(2B,2B,8B,8B)$	4	1120,784,128,96
$(2B,2B,4B,11B)$	1	396	$(2B,2B,4B,11A)$	1	396
$(2B,2B,4B,8A)$	1	672	$(2B,2B,4B,6A)$	1	552
$(2B,2B,4B,10A)$	1	540	$(2B,2B,6B,8B)$	2	1864,1864
$(2B,2B,6B,6B)$	5	2152,1524, 456,288,108	$(2B,2B,3A,11B)$	2	55,154
$(2B,2B,3A,11A)$	2	55,154	$(2B,2B,3A,8A)$	2	104,104
$(2B,2B,3A,6A)$	1	272	$(2B,2B,3A,10A)$	2	160,160
$(2B,2B,3B,8B)$	2	396,396	$(2B,2B,3B,6B)$	2	648,396
$(2B,2B,3B,3B$	2	64,72	$(2B,2B,4A,8B)$	1	672
$(2B,2B,4A,6B)$	1	504	$(2B,2B,3B,4A)$	1	144
$(2B,2B,2B,4B,4B)$	1	65472	$(2B,2B,2B,3A,4B)$	1	42000
$(2B,2B,2B,2B,8B)$	2	26880,26880	$(2B,2B,2B,3A,3A)$	2	15840,6024
$(2B,2B,2B,2B,5A)$	2	22800,9900	$(2B,2B,2B,2B,4A)$	1	12768
$(2B,2B,2B,2B,6B)$	2	25056,26864	$(2B,2B,2B,2B,3B)$	2	8280,5562
$(2A,2B,2B,2B,3A)$	1	8256	$(2A,2A,2B,2B,2B)$	1	1216
$(2B,2B,5A,8B)$	2	900,900	$(2B,2B,5A,6B)$	2	720,1440
$(2B,2B,3B,5A)$	2	300,270	$(2B,2B,4A,5A)$	1	220
$(2A,2B,2B,11A)$	1	88	$(2A,2B,2B,8A)$	1	64
$(2A,2B,2B,6A)$	1	78	$(2A,2B,2B,10A)$	1	60
$(2A,2B,4B,8B)$	1	596	$(2B,2B,5A,5A)$	4	660,180,160,40
$(2A,2B,2B,11B)$	1	88	$(2A,2B,4B,6B)$	1	894
$(2A,2B,3B,4B)$	1	165	$(2A,2B,4A,4B)$	1	120
$(2A,2B,4B,5A)$	1	500	$(2A,2B,3A,8B)$	1	372
$(2A,2B,3A,6B)$	1	584	$(2A,2B,3A,3B)$	1	108
$(2A,2B,3A,4A)$	1	66	$(2A,2B,3A,5A)$	1	330

Table 5.24: $M_{12}, g = 1$, Of Degree 12

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2A,2A,2B,8B)$	1	96	$(2A,2A,2B,6B)$	1	108
$(2A,2A,2B,5A)$	1	40	$(2A,4B,4B,4B)$	1	688,156
$(2A,3A,4B,4B)$	1	564	$(2A,3A,3A,4B)$	1	72
$(2A,3A,3A,3A)$	1	144	$(2A,2A,4B,4B)$	1	60
$(2A,3A,3A,4B)$	1	72	$(2A,2A,3A,3A)$	1	44
$(2A,2B,2B,2B,4B)$	1	12768	$(2B,2B,2B,2B,2B,2B)$	2	588800,332640

Table 5.25: $M_{12}, g = 1$, Of Degree 66

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2B,3B,11A)$	1	1	$(2B,3B,11B)$	1	1
$(2A,3A,11A)$	1	1	$(2A,3A,11B)$	1	1

Table 5.26: $M_{12}, g = 1$, Of Degree 144

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2A,3A,11A)$	1	1	$(2A,3A,11B)$	1	1

Table 5.27: $M_{12}.2, g = 1$, Of Degree 144

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2C,4C,6B)$	4	1	$(2C,4A,6A)$	2	1

Table 5.28: $M_{22}, g = 1$, Of Degree 22

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(3A,5A,5A)$	162	1	$(3A,5A,6A)$	122	1
$(3A,6A,6A)$	66	1	$(3A,3A,11B)$	18	1
$(3A,3A,11A)$	18	1	$(3A,4B,7B)$	28	1
$(3A,4B,7A)$	28	1	$(3A,4B,8A)$	26	1
$(3A,4A,7A)$	52	1	$(3A,4A,7B)$	52	1
$(3A,4A,8A)$	60	1	$(4B,4B,5A)$	14	1
$(4B,4B,6A)$	24	1	$(4B,4B,5A)$	108	1
$(4A,4B,6A)$	50	1	$(4A,4A,5A)$	158	1
$(4A,4A,6A)$	104	1	$(2A,5A,11A)$	14	1
$(2A,5A,11B)$	14	1	$(2A,6A,11A)$	10	1
$(2A,6A,11B)$	10	1	$(2A,3A,3A,3A)$	2	1680,2448
$(2A,7A,7A)$	16	1	$(2A,7A,7B)$	12	1
$(2A,7B,7B)$	16	1	$(2A,7A,8A)$	16	1
$(2A,7B,8B)$	16	1	$(2A,8A,8A)$	10	1
$(2A,2A,3A,5A)$	2	1380,1500	$(2A,2A,3A,6A)$	2	864,744
$(2A,2A,4B,4B)$	3	108,108,100	$(2A,2A,4A,4B)$	3	108,108,100
$(2A,2A,4A,4A)$	5	840,840,488,200,216	$(2A,2A,2A,11A)$	6	33
$(2A,2A,2A,11B)$	6	33	$(2A,2A,2A,2A,3A)$	2	22464,22032

Table 5.29: $M_{22}, g = 1$, Of Degree 77

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2A,4A,11A)$	2	1	$(2A,4A,11B)$	2	1

Table 5.30: $M = M_{22}.2, g = 1$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4D,4D,8A)$	8	1	$(4D,4D,7B)$	10	1
$(4D,4D,7A)$	10	1	$(4B,4D,8B)$	5	1
$(4C,4D,5A)$	20	1	$(4C,4D,6A)$	8	1
$(4A,4D,8B)$	20	1	$(4D,5A,6B)$	62	1
$(4D,6A,6B)$	51	1	$(2C,4D,11A)$	5	1
$(2C,5A,8B)$	12	1	$(2C,4B,10A)$	3	1
$(2C,4B,14B)$	2	1	$(2C,4B,14A)$	2	1
$(2C,4B,12A)$	4	1	$(2C,4C,8A)$	3	1
$(2C,4C,7A)$	2	1	$(2C,4C,7B)$	2	1
$(2C,4A,10A)$	9	1	$(2C,4A,14B)$	7	1
$(2C,4A,14A)$	7	1	$(2C,4A,12A)$	6	1
$(2C,6A,8B)$	5	1	$(2C,6B,8A)$	9	1
$(2C,6B,7B)$	14	1	$(2C,6B,7A)$	14	1
$(4B,4C,4C)$	3	1	$(4B,4C,6B)$	15	1
$(4B,6B,6B)$	49	1	$(4A,4C,4C)$	6	1
$(4A,4C,6B)$	37	1	$(3A,4D,12A)$	12	1
$(3A,4C,8B)$	13	1	$(3A,6B,8B)$	50	1
$(2B,8B,11A)$	3	1	$(2B,8A,10A)$	3	1
$(2B,8A,14A)$	2	1	$(2B,8A,14B)$	2	1
$(2B,8A,12A)$	2	1	$(2B,7B,14A)$	4	1
$(2B,7B,10A)$	2	1	$(2B,7B,14B)$	4	1
$(4A,6B,6B)$	200	1	$(2A,8B,10A)$	4	1
$(2A,8B,14B)$	6	1	$(2A,8B,14A)$	6	1
$(2A,8B,12A)$	6	1	$(3A,4D,10A)$	21	1
$(3A,4D,14A)$	9	1	$(3A,4D,14B)$	3	1
$(2B,7B,12A)$	3	1	$(2B,7A,10A)$	4	1
$(2B,7A,14A)$	3	1	$(2B,7A,14B)$	2	1
$(2B,7A,12A)$	3	1	$(2A,2C,2C,4B)$	1	50
$(2A,2C,2C,4A)$	1	108	$(2A,2C,3A,4D)$	1	378
$(2A,2A,4C,4D)$	1	200	$(2A,2A,4D,6B)$	1	1080
$(2A,2A,2C,8B)$	1	136	$(2B,2C,4D,4D)$	1	4
$(2B,2C,2C,4C)$	1	16	$(2B,2C,2C,6B)$	1	66
$(2B,2C,3A,4B)$	1	96	$(2B,2C,3A,4A)$	1	258
$(2A,2B,4D,5A)$	1	330	$(2A,2B,4D,6A)$	1	204
$(2A,2B,2C,8A)$	1	44	$(2A,2B,2C,7A)$	1	63
$(2A,2B,2C,7B)$	1	63	$(2A,2B,4B,4C)$	1	60
$(2A,2B,4B,6B)$	1	222	$(2A,2B,4A,4C)$	1	160
$(2A,2B,4A,6B)$	1	984	$(2A,2B,3A,4D)$	1	276
$(2A,3A,3A,4D)$	1	480	$(2B,2B,4D,8B)$	1	40

Table 5.31: $M_{22}.2, g=1$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,2B,2C,10A)$	1	20	$(2B,2B,2C,14B)$	1	14
$(2B,2B,2C,14A)$	1	14	$(2B,2B,2C,12A)$	1	24
$(2B,2B,4B,5A)$	1	45	$(2B,2B,4B,6A)$	1	72
$(2B,2B,4C,4C)$	1	16	$(2B,2B,4A,5A)$	1	350
$(2B,2B,4A,6A)$	1	168	$(2A,2B,2B,2C,2C)$	1	312
$(2A,2A,2B,2B,4B)$	1	1024	$(2A,2A,2B,2B,4A)$	1	4864
$(2B,2B,4C,6B)$	1	96	$(2B,2B,6B,6B)$	2	66,228
$(2B,2B,3A,8A)$	1	80	$(2B,2B,3A,7B)$	2	56,35
$(2B,2B,3A,7A)$	2	56,35	$(2A,2B,2B,2B,4C)$	1	368
$(2A,2B,2B,2B,6B)$	1	1404	$(2B,2B,2B,2B,5A)$	1	300
$(2B,2B,2B,2B,6A)$	1	432	$(2A,2A,2A,2B,4D)$	1	5232
$(2B,2B,2B,2C,3A)$	1	648	$(2A,2A,2B,2B,2B,2B)$	1	6704

Table 5.32: $M_{22}.2, g=1$, Of Degree 77

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C,4A,7A)$	2	1	$(2C,4A,7B)$	2	1
$(2C,4C,8A)$	3	1	$(2C,4C,8B)$	3	1
$(2C,5A,6B)$	11	1	$(2C,6A,6B)$	7	1
$(2C,3A,11A)$	2	1	$(2C,3A,11B)$	2	1
$(2B,2C,2C,4D)$	1	16			

Table 5.33: $M_{23}, g = 1$, Of Degree 23

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4A,5A,5A)$	396	1	$(4A,4A,8A)$	568	1
$(4A,4A,7B)$	206	1	$(4A,4A,7A)$	206	1
$(4A,5A,8A)$	170	1	$(3A,5A,7B)$	59	1
$(3A,5A,7A)$	59	1	$(3A,4A,15B)$	57	1
$(3A,4A,15A)$	57	1	$(3A,4A,11B)$	63	1
$(3A,4A,11A)$	63	1	$(3A,4A,14B)$	51	1
$(3A,4A,14A)$	51	1	$(3A,6A,8A)$	350	1
$(3A,6A,7B)$	138	1	$(3A,6A,7A)$	138	1
$(3A,3A,23B)$	6	1	$(3A,3A,23A)$	6	1
$(2A,8A,15B)$	28	1	$(2A,8A,15A)$	28	1
$(2A,8A,11B)$	28	1	$(2A,8A,11A)$	28	1
$(2A,8A,14B)$	26	1	$(2A,8A,14A)$	26	1
$(2A,5A,23A)$	6	1	$(2A,5A,23B)$	6	1
$(2A,7B,15B)$	11	1	$(2A,7B,15A)$	11	1
$(2A,7B,11B)$	12	1	$(2A,7B,11A)$	12	1
$(2A,7B,14B)$	5	1	$(2A,7B,14A)$	12	1
$(2A,7A,15B)$	11	1	$(2A,7A,15A)$	11	1
$(2A,7A,11B)$	12	1	$(2A,7A,11A)$	12	1
$(2A,7A,14B)$	12	1	$(2A,7A,14A)$	5	1
$(2A,6A,23B)$	14	1	$(2A,6A,23A)$	14	1
$(4A,6A,6A)$	1220	1	$(4A,5A,6A)$	776	1
$(2A,3A,3A,4A)$	1	11784	$(2A,2A,4A,5A)$	1	10680
$(2A,2A,4A,6A)$	1	16656	$(2A,2A,3A,8A)$	1	4352
$(2A,2A,3A,7B)$	1	1799	$(2A,2A,3A,7A)$	1	1799
$(2A,2A,2A,23B)$	2	69,69	$(2A,2A,2A,23A)$	2	69,69
$(2A,2A,2A,2A,4A)$	1	244224			

Table 5.34: $M_{24}, g = 1$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4B,4B,12A)$	484	1	$(4B,4B,14B)$	284	1
$(4B,4B,14A)$	284	1	$(4B,4B,11A)$	315	1
$(4B,4B,10A)$	300	1	$(4B,4B,15B)$	288	1
$(4B,4B,15A)$	288	1	$(4B,4B,6B)$	327	1
$(4B,6A,8A)$	1736	1	$(4B,6A,7B)$	444	1
$(4B,6A,7A)$	444	1	$(4B,4C,6A)$	510	1
$(4A,4B,8A)$	182	1	$(4B,4B,7B)$	44	1
$(4A,4B,7A)$	182	1	$(4A,4B,4C)$	22	1
$(4B,5A,8A)$	461	1	$(4B,5A,7B)$	104	1
$(4B,5A,7A)$	104	1	$(4B,4C,5A)$	147	1
$(3B,4B,8A)$	128	1	$(3B,4B,7B)$	62	1
$(3B,4B,7A)$	62	1	$(3B,4B,4C)$	38	1
$(6A,6A,6A)$	7516	1	$(4A,6A,6A)$	652	1
$(4A,4A,6A)$	32	1	$(2A,8A,21B)$	17	1
$(2A,8A,21A)$	17	1	$(2A,8A,23B)$	9	1
$(2A,8A,23A)$	9	1	$(2A,8A,12B)$	41	1
$(2A,12A,12A)$	48	1	$(2A,12A,14B)$	49	1
$(2A,14B,14B)$	31	1	$(2A,12A,14A)$	49	1
$(2A,14A,14B)$	12	1	$(2A,14A,14A)$	31	1
$(2A,7B,21B)$	3	1	$(2A,7B,21A)$	7	1
$(2A,7B,23B)$	3	1	$(2A,7B,23A)$	3	1
$(2A,7B,12B)$	11	1	$(2A,7A,21B)$	7	1
$(2A,7A,23A)$	3	1	$(2A,7A,12B)$	3	1
$(2A,11A,12A)$	11	1	$(2A,11A,14B)$	43	1
$(2A,7A,23B)$	3	1	$(2A,10A,11A)$	21	1
$(2A,11A,14A)$	22	1	$(2A,11A,11A)$	19	1
$(2A,10A,12A)$	30	1	$(2A,10A,14B)$	26	1
$(2A,10A,14A)$	26	1	$(2A,10A,10A)$	22	1
$(2A,12A,15B)$	47	1	$(2A,14B,15B)$	24	1
$(2A,14A,15B)$	24	1	$(2A,11A,15B)$	26	1
$(2A,10A,15B)$	21	1	$(2A,15B,15B)$	19	1
$(2A,12A,15A)$	47	1	$(2A,14B,15A)$	24	1
$(2A,14A,15A)$	24	1	$(2A,11A,15A)$	26	1
$(2A,10A,15A)$	21	1	$(2A,15A,15B)$	33	1
$(2A,15A,15A)$	19	1	$(2A,6B,12A)$	28	1
$(2A,6B,14B)$	33	1	$(2A,6B,14A)$	33	1
$(2A,6B,11A)$	31	1	$(2A,6B,10A)$	15	1
$(2A,6B,15B)$	27	1	$(2A,6B,15A)$	27	1
$(2A,6B,6B)$	6	1	$(2A,4C,21B)$	7	1
$(2A,4C,21A)$	7	1	$(2A,4C,23B)$	7	1
$(2A,4C,23A)$	7	1	$(2A,4C,12B)$	6	1
$(3A,8A,8A)$	238	1	$(3A,4B,21B)$	30	1
$(3A,4B,21A)$	30	1	$(3A,4B,23B)$	19	1
$(3A,4B,23A)$	19	1	$(3A,4B,12B)$	54	1

Table 5.35: $M_{24}, g = 1$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3A,6A,12A)$	412	1	$(3A,6A,14B)$	218	1
$(3A,6A,14A)$	218	1	$(3A,6A,11A)$	241	1
$(3A,6A,10A)$	210	1	$(3A,6A,15B)$	230	1
$(3A,6A,15A)$	230	1	$(3A,6A,6B)$	243	1
$(3A,4A,12A)$	20	1	$(3A,4A,14B)$	22	1
$(3A,4A,14A)$	22	1	$(3A,4A,11A)$	32	1
$(3A,4A,10A)$	14	1	$(3A,4A,15B)$	24	1
$(3A,4A,15A)$	24	1	$(3A,4A,6B)$	15	1
$(3A,7B,8A)$	64	1	$(3A,7B,7B)$	13	1
$(3A,7A,8A)$	64	1	$(3A,7A,7B)$	16	1
$(3A,7A,7A)$	13	1	$(3A,5A,12A)$	112	1
$(3A,5A,14B)$	58	1	$(3A,5A,14A)$	58	1
$(3A,5A,11A)$	50	1	$(3A,5A,10A)$	69	1
$(3A,5A,15B)$	51	1	$(3A,5A,15A)$	51	1
$(3A,5A,6B)$	66	1	$(3A,4C,8A)$	69	1
$(3A,4C,7B)$	26	1	$(3A,4C,7A)$	26	1
$(3A,4C,4C)$	10	1	$(3A,3B,12A)$	40	1
$(3A,3B,14B)$	30	1	$(3A,3B,14A)$	30	1
$(3A,3B,11A)$	27	1	$(3A,3B,10A)$	21	1
$(3A,3B,15B)$	20	1	$(3A,3B,15A)$	20	1
$(3A,3B,6B)$	10	1	$(5A,6A,6A)$	2362	1
$(4A,5A,6A)$	191	1	$(4A,4A,5A)$	8	1
$(5A,5A,6A)$	673	1	$(2A,2B,4B,4B)$	1	10852
$(4A,5A,5A)$	84	1	$(5A,5A,5A)$	138	1
$(3B,6A,6A)$	576	1	$(3B,4A,6A)$	34	1
$(3B,5A,6A)$	227	1	$(3B,4A,5A)$	22	1
$(3B,5A,5A)$	76	1	$(3B,3B,6A)$	28	1
$(3B,3B,5A)$	6	1	$(2B,8A,8A)$	92	1
$(2B,4B,21B)$	12	1	$(2B,4B,21A)$	12	1
$(2B,4B,23B)$	7	1	$(2B,4B,23A)$	7	1
$(2B,4B,12B)$	19	1	$(2B,6A,12A)$	70	1
$(2B,6A,14B)$	70	1	$(2B,6A,14A)$	70	1
$(2B,6A,11A)$	80	1	$(2B,6A,10A)$	34	1
$(2B,6A,15B)$	69	1	$(2B,6A,15A)$	69	1
$(2B,6A,6B)$	30	1	$(2B,4A,14B)$	3	1
$(2B,4A,14A)$	3	1	$(2B,4A,15B)$	3	1
$(2B,4A,15A)$	3	1	$(2B,7B,8A)$	32	1
$(2B,7B,7B)$	6	1	$(2B,7A,8A)$	32	1
$(2B,7A,7B)$	17	1	$(2B,7A,7A)$	6	1
$(2B,5A,12A)$	21	1	$(2B,5A,14B)$	20	1
$(2B,5A,14A)$	20	1	$(2B,5A,11A)$	34	1
$(2B,5A,10A)$	10	1	$(2B,5A,15B)$	27	1
$(2B,5A,15A)$	27	1	$(2B,5A,6B)$	14	1

Table 5.36: $M_{24}, g = 1$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,4C,8A)$	12	1	$(2B,4C,7B)$	4	1
$(2B,4C,7A)$	4	1	$(2B,3B,14B)$	4	1
$(2B,3B,14A)$	4	1	$(2B,3B,15B)$	3	1
$(2B,3B,15A)$	3	1	$(2A,7B,21A)$	3	1
$(2A,2A,4B,8A)$	1	19960	$(2A,2A,4B,7A)$	1	5313
$(2A,2A,4B,7A)$	1	5313	$(2A,2A,4B,4C)$	1	5104
$(2A,2A,6A,6A)$	2	84012,960	$(2A,2A,4A,6A)$	1	7044
$(2A,2A,4A,4A)$	1	384	$(2A,2A,2A,2A,6A)$	1	989280
$(2A,2A,2A,2A,4A)$	1	74496	$(2A,2B,4B,6A)$	1	86610
$(2A,2A,2A,2A,5A)$	1	342600	$(2A,2A,2A,2A,3B)$	1	75816
$(2A,2A,2A,3A,3A)$	1	35060	$(2A,2A,2A,2B,3A)$	1	85986
$(2A,2A,2A,2B,2B)$	2	10896,37506	$(2A,2A,3A,12A)$	1	3876
$(2A,2A,3A,14A)$	1	2366	$(2A,2A,3A,14B)$	1	2366
$(2A,2A,3A,11A)$	1	2926	$(2A,2A,3A,10A)$	1	2130
$(2A,2A,3A,15A)$	1	2400	$(2A,2A,3A,15B)$	1	2400
$(2A,2A,3A,6B)$	1	2397	$(2A,2A,5A,6A)$	2	48480,27330
$(2A,2A,4A,5A)$	1	2000	$(2A,2A,5A,5A)$	2	8280,340
$(2A,2A,3B,6A)$	1	6744	$(2A,2A,3B,4A)$	1	504
$(2A,2A,3B,5A)$	1	2505	$(2A,2A,3B,3B)$	1	258
$(2A,2A,2B,14A)$	1	812	$(2A,2A,2B,14B)$	1	812
$(2A,2A,2B,11A)$	1	913	$(2A,2A,2B,12A)$	1	720
$(2A,2A,2B,10A)$	1	720	$(2A,2A,2B,15A)$	1	750
$(2A,2A,2B,15B)$	1	750	$(2A,2A,2B,6B)$	1	360
$(2A,3A,3A,6A)$	1	27024	$(2A,3A,3A,4A)$	1	2172
$(2A,3A,3A,5A)$	1	8430	$(2A,3A,3A,3B)$	1	2592
$(2A,2B,3A,4A)$	1	336	$(2A,2B,3A,6A)$	1	7836
$(2A,2B,3A,5A)$	1	2775	$(2A,2B,3A,3B)$	1	552
$(2B,2B,3A,3A)$	1	290	$(2B,3A,3A,3A)$	1	3220
$(2A,2B,2B,5A)$	1	340			

Table 5.37: $J_2, g = 1$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,12A)$	8	1	$(2B,4A,7A)$	4	1

Table 5.38: $J_2, g = 1$, Of Degree 315

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,7A)$	10	1			

Table 5.39: $J_2 : 2, g = 1$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C,4C,8A)$	6	1	$(2C,4B,12B)$	5	1
$(2C,4C,24A)$	2	1	$(2C,4C,24B)$	2	1
$(2A,2C,2C,3B)$	1	141	$(2C,4A,12B)$	1	1
$(2C,4C,6B)$	5	1			

Table 5.40: $HS, g = 1$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3A,11A)$	3	1	$(2B,3A,11B)$	3	1

Table 5.41: $HS:2, g = 1$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,2C,2C,5A)$	1	1	$(2C,4F,11B)$	3	1
$(2D,4E,8B)$	10	1	$(2D,4E,7A)$	10	1
$(2D,3A,20D)$	2	1	$(2D,3A,20E)$	2	1

APPENDIX C

GENUS TWO COVERS

Appendix C contains table representing the result of our computation of primitive genus two cover in sporadic simple groups satisfying Theorem 1.0.2. Note that N.Orbit means number of orbits, L.O means length of orbits.

Table 5.42: $M_{11}, g = 2$, Of Degree 11

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(6A,6A,6A)$	36	1	$(3A,6A,11A)$	4	1
$(3A,6A,11B)$	4	1	$(3A,8A,11A)$	3	1
$(3A,8B,11B)$	5	1	$(3A,8B,11A)$	5	1
$(3A,8A,11B)$	5	1	$(3A,5A,11A)$	5	1
$(3A,5A,11B)$	5	1	$(6A,6A,8A)$	28	1
$(6A,6A,8B)$	28	1	$(8A,8A,8A)$	14	1
$(8A,8A,8B)$	14	1	$(8A,8B,8B)$	14	1
$(8B,8B,8B)$	14	1	$(6A,8A,8A)$	12	1
$(6A,8A,8B)$	28	1	$(6A,8B,8B)$	12	1
$(4A,6A,11A)$	18	1	$(4A,6A,11B)$	18	1
$(4A,8A,11A)$	11	1	$(4A,8A,11B)$	11	1
$(4A,8B,11A)$	11	1	$(4A,8B,11B)$	11	1
$(4A,5A,11A)$	18	1	$(4A,5A,11B)$	18	1
$(2A,11A,11A)$	1	1	$(2A,11B,11B)$	1	1
$(5A,5A,5A)$	24	1	$(5A,64,6A)$	34	1
$(5A,6A,8A)$	33	18	$(5A,6A,8B)$	33	1
$(5A,8A,8A)$	23	1	$(5A,8A,8B)$	23	1
$(5A,8B,8B)$	23	1	$(5A,5A,6A)$	38	1
$(5A,5A,8A)$	36	1	$(5A,5A,8B)$	36	1
$(3A,3A,3A,3A)$	1	288	$(3A,3A,3A,4A)$	1	1104
$(3A,3A,4A,4A)$	2	2128,92	$(3A,4A,4A,4A)$	1	4428
$(4A,4A,4A,4A)$	3	2880,4776,504	$(2A,3A,3A,6A)$	1	444
$(2A,3A,3A,8A)$	1	450	$(2A,3A,3A,8B)$	1	450
$(2A,3A,3A,5A)$	1	385	$(2A,3A,4A,6A)$	1	1417
$(2A,3A,4A,8A)$	1	951	$(2A,3A,4A,8B)$	1	951
$(2A,3A,4A,5A)$	1	1528	$(2A,4A,4A,6A)$	1	3016

Table 5.43: $M_{11}, g = 2$, Of Degree 11

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,4A,4A,8A)$	1	1940	$(2A,4A,4A,8B)$	1	1940
$(2A,4A,4A,5A)$	1	3150	$(2A,2A,6A,6A)$	1	1940
$(2A,2A,3A,11A)$	1	77	$(2A,2A,3A,11B)$	1	77
$(2A,2A,6A,8A)$	1	566	$(2A,2A,6A,8B)$	1	566
$(2A,2A,8A,8A)$	2	272,48	$(2A,2A,8A,8B)$	1	436
$(2A,2A,4A,11A)$	1	286	$(2A,2A,4A,11B)$	1	286
$(2A,2A,2A,3A,3A)$	1	8280	$(2A,2A,2A,4A,4A)$	1	57720
$(2A,2A,2A,3A,4A)$	1	27204	$(2A,2A,2A,2A,6A)$	1	15228
$(2A,2A,2A,2A,8A)$	1	10944	$(2A,2A,2A,2A,8B)$	1	10944
$(2A,2A,2A,2A,2A,2A)$	1	229680	$(2A,2A,2A,2A,5A)$	1	12000
$(2A,2A,5A,6A)$	1	680	$(2A,2A,5A,8A)$	1	630
$(2A,2A,5A,8B)$	1	630	$(2A,2A,5A,5A)$	1	570
$(2A,2A,8B,8B)$	2	272,48			

Table 5.44: $M_{11}, g = 2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3A,5A,11A)$	17	1	$(5A,5A,11B)$	17	1
$(5A,5A,8A)$	16	1	$(5A,5A,8B)$	16	1
$(5A,6A,11A)$	20	1	$(5A,6A,11B)$	20	1
$(5A,6A,8A)$	33	1	$(5A,6A,8B)$	33	1
$(6A,6A,11A)$	13	1	$(6A,6A,11B)$	13	1
$(6A,6A,8A)$	28	1	$(6A,6A,8B)$	28	1
$(3A,11A,11A)$	5	1	$(3A,11A,11B)$	2	1
$(3A,11B,11B)$	5	1	$(3A,8A,11A)$	5	1
$(3A,8A,11B)$	5	1	$(3A,8B,11A)$	5	1
$(3A,8B,11B)$	5	1	$(3A,8A,8A)$	3	1
$(3A,8A,8B)$	7	1	$(3A,8B,8B)$	3	1
$(4A,5A,11A)$	18	1	$(4A,5A,11B)$	18	1
$(3A,5A,8A)$	17	1	$(3A,5A,8B)$	17	1
$(4A,6A,11A)$	18	1	$(4A,6A,11B)$	18	1
$(4A,5A,8A)$	18	1	$(4A,5A,8B)$	18	1
$(4A,4A,11A)$	7	1	$(4A,4A,11B)$	7	1
$(4A,4A,8A)$	4	1	$(4A,4A,8B)$	4	1
$(3A,3A,3A,5A)$	1	1155	$(3A,3A,3A,6A)$	1	1494
$(3A,3A,3A,4A)$	1	1104	$(2A,3A,5A,5A)$	1	1850
$(2A,3A,5A,6A)$	1	2010	$(2A,3A,6A,6A)$	1	1791
$(2A,3A,3A,11A)$	1	286	$(2A,3A,3A,11B)$	1	286
$(2A,3A,3A,8A)$	1	450	$(2A,3A,3A,8B)$	1	450
$(2A,3A,4A,5A)$	1	1525	$(2A,3A,4A,6A)$	1	1417
$(2A,3A,4A,4A)$	1	708	$(2A,2A,5A,11A)$	1	385
$(2A,2A,5A,11B)$	1	385	$(2A,2A,5A,8A)$	1	630
$(2A,2A,5A,8B)$	1	630	$(2A,2A,6A,11A)$	1	352

Table 5.45: $M_{11}, g = 2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,2A,6A,11B)$	1	352	$(2A,2A,6A,8A)$	1	566
$(2A,2A,6A,8B)$	1	566	$(2A,2A,3A,3A,3A)$	1	26856
$(2A,2A,4A,11A)$	1	286	$(2A,2A,4A,11B)$	1	286
$(2A,2A,4A,8A)$	1	304	$(2A,2A,4A,8B)$	1	304
$(2A,2A,2A,3A,5A)$	1	37200	$(2A,2A,2A,3A,6A)$	1	35492
$(2A,2A,2A,3A,4A)$	1	27204	$(2A,2A,2A,2A,11A)$	1	6897
$(2A,2A,2A,3A,11B)$	1	6897	$(2A,2A,2A,2A,8A)$	1	10944
$(2A,2A,2A,2A,8B)$	1	10944	$(2A,2A,2A,2A,2A,3A)$	1	692280

Table 5.46: $M_{11}, g = 2$, Of Degree 55

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,4A,11A)$	1	1	$(2A,4A,11B)$	1	1

Table 5.47: $M_{11}, g = 2$, Of Degree 66

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,4A,11A)$	1	1	$(2A,4A,11B)$	1	1

Table 5.48: $M_{12}, g = 2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(8B,8B,11A)$	92	1	$(8B,8B,11B)$	92	1
$(8A,8B,8B)$	180	1	$(6A,8B,8B)$	104	1
$(8B,8B,10A)$	120	1	$(4B,11B,11B)$	16	1
$(4B,11A,11B)$	16	1	$(4B,11A,11A)$	16	1
$(4B,8A,11B)$	33	1	$(4B,8A,11A)$	33	1
$(4B,8A,8A)$	44	1	$(4B,6A,11A)$	22	1
$(4B,6A,11B)$	22	1	$(4B,6A,8B)$	46	1
$(4B,6A,6A)$	22	1	$(4B,10A,11B)$	26	1
$(4B,10A,11A)$	26	1	$(4B,10A,8A)$	47	1
$(4B,6A,10A)$	28	1	$(6B,8B,11B)$	150	1
$(6B,8B,11A)$	150	1	$(6B,8A,8B)$	260	1
$(6A,6B,8B)$	162	1	$(6B,8B,10A)$	186	1
$(6B,6B,11B)$	211	1	$(6B,6B,11A)$	211	1
$(6B,6B,8A)$	256	1	$(6A,6B,6B)$	230	1
$(6B,6B,10A)$	268	1	$(3A,11B,11B)$	12	1
$(3A,11A,11A)$	12	1	$(3A,8A,11B)$	10	1
$(3A,8A,11A)$	10	1	$(3A,8A,8A)$	6	1
$(3A,6A,11B)$	15	1	$(3A,6B,11A)$	15	1
$(3A,6B,8A)$	18	1	$(3A,6A,6A)$	14	1
$(3A,10A,11B)$	14	1	$(3A,10A,11A)$	14	1
$(3A,10A,8A)$	18	1	$(3A,6A,10A)$	24	1
$(3A,10A,10A)$	14	1	$(3B,8B,11B)$	28	1
$(3A,8B,11A)$	28	1	$(3B,8A,8B)$	52	1
$(3B,6A,8B)$	28	1	$(3B,8B,10A)$	32	1
$(3B,6B,11B)$	40	1	$(3B,6B,11A)$	40	1
$(3B,6B,8A)$	50	1	$(3B,6A,6B)$	36	1
$(3B,6B,10A)$	44	1	$(3B,3B,11B)$	7	1

Table 5.49: $M_{12}, g=2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3B,3B,11A)$	7	1	$(3B,3B,8A)$	8	1
$(3B,3B,10A)$	4	1	$(4A,8B,11B)$	33	1
$(4A,8B,11A)$	33	1	$(4A,8A,8B)$	36	1
$(4A,6A,8B)$	46	1	$(4A,8B,10A)$	47	1
$(4A,6A,11B)$	27	1	$(4A,6B,11A)$	27	1
$(4A,6A,6B)$	39	1	$(4A,6B,10A)$	39	1
$(3B,4A,11B)$	8	1	$(3B,4A,11A)$	8	1
$(3B,4A,8A)$	5	1	$(3B,4A,6A)$	7	1
$(3B,4A,10A)$	10	1	$(4A,4A,11A)$	1	1
$(4A,4A,11B)$	1	1	$(4A,4A,6A)$	2	1
$(4A,4A,10A)$	2	1	$(5A,8B,11A)$	72	1
$(5A,8B,11A)$	2	1	$(5A,8A,8B)$	120	1
$(5A,6A,8B)$	100	1	$(5A,8B,10A)$	112	1
$(5A,6B,11B)$	100	1	$(5A,6B,11A)$	100	1
$(5A,6B,8A)$	132	1	$(5A,6A,6B)$	132	1
$(5A,6B,10A)$	156	1	$(3B,5A,11B)$	19	1
$(3B,5A,11A)$	19	1	$(3B,5A,8A)$	22	1
$(3B,5A,6A)$	22	1	$(3B,5A,10A)$	28	1
$(4A,5A,11B)$	10	1	$(4A,5A,11B)$	10	1
$(4A,5A,8A)$	7	1	$(4A,5A,6A)$	19	1
$(4A,5A,10A)$	22	1	$(5A,5A,11B)$	30	1
$(5A,5A,11A)$	30	1	$(5A,5A,8A)$	48	1
$(5A,5A,6A)$	58	1	$(5A,5A,10A)$	68	1
$(2A,11A,11B)$	8	1	$(2A,8A,11B)$	4	1
$(2A,8A,11A)$	4	1	$(2A,8A,8A)$	4	1
$(2A,6A,11B)$	1	1	$(2A,6A,11A)$	1	1
$(2A,6A,8A)$	4	1	$(2A,10A,11B)$	4	1
$(2A,10A,11A)$	4	1	$(2A,10A,10A)$	6	1
$(2A,8A,10A)$	5	1	$(4B,10A,10A)$	31	1
$(4A,6B,8A)$	16	1	$(4B,4B,4B,8B)$	1	14744
$(4B,4B,4B,6B)$	1	26568	$(3B,4B,4B,4B)$	1	6075
$(4A,4B,4B,4B)$	1	6478	$(4B,4B,4B,5A)$	1	11665
$(3A,4B,4B,8B)$	1	10560	$(3A,4B,4B,6B)$	1	18276
$(3A,3B,4B,4B)$	1	3750	$(3A,4A,4B,4B)$	1	4446
$(3A,4B,4B,5A)$	1	8790	$(3A,3A,4B,8B)$	1	7824
$(3A,3A,4B,6B)$	1	11952	$(3A,3A,3B,4B)$	1	2484
$(3A,3A,4A,4B)$	1	2100	$(3A,3A,4B,5A)$	1	5760
$(3A,3A,3A,8B)$	2	2328,2328	$(3A,3A,3A,6B)$	1	2376,3744
$(3A,3A,3A,3B)$	2	774,486	$(3A,3A,3A,4A)$	1	792
$(2B,4B,8A,8A)$	1	13764	$(2B,4B,4B,11B)$	1	2431
$(2B,4B,4B,11A)$	1	2431	$(2B,4B,4B,8A)$	1	17776
$(2B,4B,4B,6A)$	1	2874	$(2B,4B,4B,10A)$	1	3240
$(2B,4B,6B,8B)$	1	22624	$(2B,4B,6B,6B)$	1	34978
$(2B,3B,4B,8B)$	1	624	$(2B,3B,4B,6B)$	1	6696

Table 5.50: $M_{12}, g = 2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3A,3A,10A)$	2	650,650	$(2B,3A,3B,8B)$	2	1477,1477
$(2B,3A,3B,6B)$	2	2088,1876	$(2B,3A,3B,3B)$	2	296,432
$(2B,3A,4A,8B)$	2	2722,1876	$(2B,3A,4A,6B)$	1	2816
$(2B,3A,3B,4A)$	1	670	$(2B,3A,4A,4A)$	1	240
$(2B,3A,5A,8B)$	1	1310	$(2B,3A,5A,6B)$	2	5850,3910
$(2B,2B,4A,11A)$	1	396	$(2B,3A,5A,4A)$	2	3720,3720
$(2B,2B,4A,6A)$	1	522	$(2B,2B,8B,11B)$	2	1078,1078
$(2B,2B,2B,3B,4B)$	1	99720	$(2B,2B,8A,8B)$	3	1944,992,384
$(2B,2B,6A,8B)$	1	25208	$(2B,2B,8B,10A)$	2	1390,1390
$(2B,2B,4B,4B,4B)$	1	424280	$(2B,2B,6B,11B)$	2	1166,1782
$(2B,2B,6B,11A)$	2	1166,1782	$(2B,2B,6B,8A)$	2	1864,1864
$(2B,2B,6A,6B)$	1	3456	$(2B,2B,6B,10A)$	2	2040,2040
$(2B,2B,3A,4B,4B)$	1	288928	$(2B,2B,3A,3A,4B)$	1	179048
$(2B,2B,5A,10A)$	1	1030	$(2B,2B,3B,11B)$	2	363,264
$(2B,2B,3B,11A)$	2	363,264	$(2B,2B,3B,8A)$	2	396,396
$(2B,2B,3B,6A)$	1	640	$(2B,2B,3B,10A)$	2	390,390
$(2B,2B,4A,11B)$	1	396	$(2B,3A,3B,5A)$	2	1080,1100
$(2B,2B,4A,8A)$	1	288	$(2B,3A,5A,5A)$	2	1360,3360
$(2B,2B,4A,10A)$	1	540	$(2B,2B,2B,4B,8B)$	1	353568
$(2B,2B,2B,4B,6B)$	1	518472	$(2B,2B,8B,11A)$	2	1078,1078
$(2B,2B,2B,4B,5A)$	1	278100	$(2B,2B,2B,3A,8B)$	2	108984,108984
$(2A,2B,4B,8A)$	1	756	$(2B,2B,2B,3A,3B)$	2	27864,28692
$(2B,2B,2B,3A,4A)$	1	42000	$(2B,2B,2B,3A,5A)$	2	93600,55200
$(2A,2B,4B,11B)$	1	539	$(2B,2B,2B,2B,11A)$	2	25168,16698
$(2B,2B,2B,2B,8A)$	1	26880	$(2B,2B,2B,2B,6A)$	1	46944
$(2A,2B,2B,2B,2B,2B)$	1	1247232	$(2B,2B,2B,2B,10A)$	2	28000,28000
$(2A,2B,2B,2B,8B)$	1	58432	$(2A,2B,2B,2B,6B)$	1	83088
$(2A,2B,2B,2B,3B)$	1	14400	$(2A,2B,2B,2B,4A)$	1	12768
$(2A,2B,2B,2B,5A)$	1	48080	$(2B,2B,5A,11B)$	2	880,495
$(2B,2B,5A,11A)$	2	880,495	$(2B,2B,5A,8A)$	2	900,900
$(2B,2B,5A,6A)$	1	1740	$(2B,2B,3A,3A,3A)$	2	33384,61520
$(2A,2B,2B,4B,4B)$	1	82656	$(2A,2B,2B,3A,4B)$	1	51828
$(2A,2B,2B,3A,3A)$	1	3106	$(2A,2A,2B,2B,3A)$	1	6444
$(2A,2A,2A,2B,2B)$	1	1520	$(2A,2A,2B,2B,4B)$	1	11488
$(2A,2B,8B,8B)$	1	2860	$(2B,2B,2B,2B,11B)$	2	25168,16698
$(2A,2B,4B,11A)$	1	539	$(2B,2B,2B,3A,6B)$	2	118368,162168
$(2A,2B,4B,6A)$	1	525	$(2A,2B,4B,10A)$	1	580
$(2A,2B,6B,8B)$	1	3894	$(2A,2B,6B,6B)$	1	5400
$(2A,2B,3A,11B)$	1	330	$(2A,2B,3A,11A)$	1	330
$(2A,2B,3A,8B)$	1	372	$(2A,2B,3A,6A)$	1	330
$(2A,2B,3A,10A)$	1	330	$(2A,2B,3B,8A)$	1	688
$(2A,2B,3B,6B)$	1	892	$(2A,2B,3B,3B)$	1	120
$(2A,2B,4A,8A)$	1	756	$(2A,2B,4A,6B)$	1	894
$(2A,2B,3B,4A)$	1	165	$(2A,2B,4A,4A)$	1	88

Table 5.51: $M_{12}, g = 2$, Of Degree 12

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,3B,4B)$	1	1148	$(2B,4A,4B,8B)$	1	5310
$(2B,4A,4B,6B)$	1	7290	$(2B,3B,4A,4B)$	1	1461
$(2B,4A,4A,4B)$	1	972	$(2B,4B,5A,8B)$	1	11985
$(2B,4B,5A,6B)$	1	18660	$(2B,3B,4B,5A)$	1	3885
$(2B,4A,4B,5A)$	1	3740	$(2B,4B,5A,5A)$	1	9185
$(2A,3A,8B,8B)$	2	5358,4552	$(2B,3A,4B,11B)$	1	1683
$(2B,3A,4B,11A)$	1	1683,4552	$(2B,3A,4B,8A)$	1	2722
$(2B,3A,4B,6A)$	1	2037,4552	$(2B,3A,4B,10A)$	1	2325
$(2B,3A,6B,8B)$	2	2476,2476	$(2B,3A,6B,6B)$	2	11208,8252
$(2B,3A,3A,11B)$	2	528,286	$(2B,3A,3A,11A)$	2	528,286
$(2B,3A,3A,8A)$	2	524,524	$(2B,3A,3A,6A)$	1	1080
$(2A,2B,5A,8B)$	1	2317	$(2A,2B,5A,6B)$	1	3258
$(2A,2B,3B,5A)$	1	2572	$(2A,2B,4A,5A)$	1	500
$(2A,2B,5A,5A)$	1	1892	$(2A,2A,2B,11B)$	1	66
$(2A,2B,2B,11A)$	1	66	$(2A,2A,2B,8A)$	1	96
$(2A,2A,2B,6A)$	1	42	$(2A,2A,2B,10A)$	1	80
$(2A,4B,4B,8B)$	1	3840	$(2A,4B,4B,6B)$	1	5640
$(2A,3B,4B,4B)$	1	1088	$(2A,4A,4B,4B)$	1	1004
$(2A,4B,4B,5A)$	1	3362	$(2A,3A,4B,8B)$	1	2460
$(2A,3A,4B,6B)$	1	3432	$(2A,3A,3B,4B)$	1	624
$(2A,3A,4A,4B)$	1	582	$(2A,3A,4B,5A)$	1	2088
$(2A,3A,3A,8B)$	1	1536	$(2A,3A,3A,6B)$	1	2112
$(2A,3A,3A,3B)$	1	426	$(2A,3A,3A,4A)$	1	360
$(2A,3A,3A,5A)$	1	1380	$(2A,2A,4B,8B)$	1	520
$(2A,2A,4B,6B)$	1	756	$(2A,2A,3B,4B)$	1	120
$(2A,2A,4A,4B)$	2	76,64	$(2A,2A,4B,5A)$	2	295,135
$(2A,2A,3A,8B)$	1	360	$(2A,2A,3A,6B)$	1	456
$(2A,2A,3A,3B)$	1	54	$(2A,2A,3A,4A)$	1	72
$(2A,2A,3A,5A)$	1	180	$(2B,2B,2B,2B,2B,3A)$	2	1745280,2524320
$(2A,2A,2A,6B)$	3	36,36,36	$(3A,3A,3A,5A)$	2	510,2360
$(2B,2B,2B,4A,4B)$	1	98496	$(2B,2B,2B,2B,2B,4B)$	2	510,7824000
$(2A,2A,2A,8B)$	1	48			

Table 5.52: $M_{12}, g = 2$, Of Degree 66

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3B,4A,4B)$	4	1	$(2B,6A,6B)$	8	1
$(2B,5A,6A)$	2	1	$(2A,4B,11A)$	1	1
$(2A,4B,11B)$	1	1	$(2A,6B,6B)$	8	1
$(2A,5A,6B)$	6	1			

Table 5.53: $M_{12}, g = 2$, Of Degree 144

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,10A)$	2	1			

Table 5.54: $M_{12}, g = 2$, Of Degree 220

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2B, 3B, 10A)$	2	1			

Table 5.55: $M_{12}.2, g = 2$, Of Degree 144

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2C, 3B, 12B)$	2	1	$(2C, 3B, 12C)$	2	1
$(2C, 3A, 12A)$	1	1	$(2C, 4A, 6C)$	4	1
$(2B, 4C, 6C)$	4	1	$(2A, 2C, 2C, 3A)$	1	22

Table 5.56: $M_{22}, g = 2$, Of Degree 22

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(3A, 5A, 7A)$	208	1	$(3A, 5A, 7B)$	208	1
$(3A, 5A, 8A)$	228	1	$(3A, 6A, 7A)$	130	1
$(3A, 6A, 7B)$	130	1	$(3A, 6A, 8A)$	124	1
$(3A, 4B, 11A)$	29	1	$(3A, 4B, 11B)$	29	1
$(3A, 4A, 11A)$	58	1	$(3A, 4A, 11B)$	58	1
$(4A, 5A, 5A)$	264	1	$(4A, 5A, 6A)$	180	1
$(4B, 5A, 5A)$	264	1	$(4B, 5A, 6A)$	180	1
$(4B, 6A, 6A)$	94	1	$(4B, 4B, 7B)$	40	1
$(4B, 4B, 7A)$	40	1	$(4B, 4B, 8A)$	36	1
$(4A, 6A, 6A)$	176	1	$(4A, 4B, 7A)$	98	1
$(4A, 4B, 7B)$	98	1	$(4A, 4B, 8A)$	74	1
$(4A, 4A, 7A)$	150	1	$(4A, 4A, 7B)$	150	1
$(4A, 4A, 8A)$	188	1	$(2A, 3A, 3A, 4B)$	2	3492,2688
$(2A, 3A, 3A, 4A)$	1	14904	$(2A, 7A, 11A)$	16	1
$(2A, 7A, 11B)$	16	1	$(2A, 7B, 11A)$	16	1
$(2A, 7B, 11B)$	16	1	$(2A, 8A, 11A)$	18	1
$(2A, 8A, 11A)$	18	1	$(2A, 2A, 3A, 7A)$	2	1820,1456
$(2A, 2A, 3A, 7B)$	2	1820,1456	$(2A, 2A, 3A, 8A)$	2	1584,1584
$(2A, 2A, 4B, 5A)$	6	780,750,750, 900,630,630	$(2A, 2A, 4B, 6A)$	6	576,352,352, 372,504,504
$(2A, 2A, 4A, 5A)$	4	1672,936 ,1672,1104	$(2A, 2A, 4A, 6A)$	4	1672,936, 1672,1104
$(2A, 2A, 2A, 2A, 4B)$	5	2960,13056, 11232,12960,9792	$(2A, 2A, 2A, 2A, 4A)$	4	27456,52992, 52992,30912

Table 5.57: $M_{22}, g = 2$, Of Degree 77

RamificationType	*N.Orbit*	*L.O*	*RamificationType*	*N.Orbit*	*L.O*
$(2A, 4B, 11A)$	4	1	$(2A, 4B, 11B)$	4	1
$(2A, 6A, 11A)$	12	1	$(2A, 6A, 11B)$	12	1

Table 5.58: $M_{22}.2, g = 2$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4D,4D,11A)$	25	1	$(4D,5A,8B)$	76	1
$(4B,4D,10A)$	34	1	$(4B,4D,14B)$	15	1
$(4B,4D,14A)$	15	1	$(4B,4D,12A)$	20	1
$(4C,4D,8A)$	21	1	$(4C,4D,7A)$	19	1
$(4C,4D,7B)$	19	1	$(4A,4D,10A)$	62	1
$(4A,4D,14B)$	40	1	$(4A,4D,14A)$	40	1
$(4A,4D,12A)$	54	1	$(4D,6A,8B)$	56	1
$(4D,6B,8A)$	92	1	$(4D,6B,7B)$	97	1
$(4D,6B,7A)$	97	1	$(2C,8A,8B)$	11	1
$(2C,7B,8B)$	15	1	$(2C,7A,8B)$	15	1
$(2C,5A,10A)$	17	1	$(2C,5A,14B)$	20	1
$(2C,5A,14A)$	20	1	$(2C,5A,12A)$	19	1
$(2C,6A,10A)$	9	1	$(2C,6A,14A)$	7	1
$(2C,6A,14B)$	7	1	$(2C,6A,12A)$	8	1
$(2C,6B,11A)$	23	1	$(4B,4B,8B)$	16	1
$(4B,6B,8B)$	68	1	$(4C,4C,5A)$	12	1
$(4C,4C,6A)$	4	1	$(4A,4C,8B)$	38	1
$(4A,6B,8B)$	187	1	$(2A,10A,10A)$	22	1
$(2A,12A,14B)$	12	1	$(2A,12A,14A)$	12	1
$(2A,12A,12A)$	18	1	$(4B,5A,6B)$	132	1
$(4B,6A,6B)$	58	1	$(5A,6B,6B)$	621	1
$(6A,6B,6B)$	406	1	$(3A,8B,8B)$	41	1
$(3A,4C,10A)$	41	1	$(3A,4C,14A)$	16	1
$(3A,4C,14B)$	16	1	$(3A,4C,12A)$	20	1
$(3A,6B,10A)$	164	1	$(3A,6B,14A)$	87	1
$(3A,6B,14B)$	87	1	$(3A,6B,12A)$	116	1
$(2B,10A,11A)$	8	1	$(2B,11A,14A)$	4	1
$(2B,11A,14B)$	4	1	$(2B,11A,12A)$	7	1
$(2A,10A,14A)$	16	1	$(2B,10A,14B)$	16	1
$(2A,14A,14A)$	10	1	$(2B,14A,14B)$	7	1
$(2A,14B,14B)$	10	1	$(2B,10A,12A)$	18	1
$(2A,2C,4B,4D)$	1	492	$(2A,2C,4A,4D)$	1	1194
$(2A,2C,2C,5A)$	1	340	$(2A,2C,2C,6A)$	1	156
$(2A,2C,3A,4C)$	1	348	$(2A,2C,3A,6B)$	1	2794
$(2A,2A,4D,8B)$	1	1376	$(2A,2A,2C,10A)$	1	320
$(2A,2A,2C,14B)$	1	266	$(2A,2A,2C,14A)$	1	266
$(2A,2A,2C,12A)$	1	288	$(2A,2A,4C,4C)$	2	132,76
$(2A,2A,6B,6B)$	3	5398,4350,312	$(2A,3A,4D,4D)$	1	3026
$(2B,2C,4C,4D)$	1	122	$(2B,2C,4D,6B)$	1	744
$(2B,2C,2C,8B)$	1	72	$(2B,2C,4B,4B)$	1	136
$(2B,2C,4A,4B)$	1	292	$(2B,2C,4A,4A)$	1	814
$(2B,2C,3A,5A)$	1	965	$(2B,2C,3A,6A)$	1	448

Table 5.59: $M_{22}.2, g = 2$, Of Degree 22

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,2B,4D,8A)$	1	412	$(2A,2B,4D,7B)$	1	476
$(2A,2B,4D,7A)$	1	476	$(2A,2B,2C,11A)$	1	121
$(2A,2B,4B,8B)$	1	328	$(2A,2B,4C,5A)$	2	305,315
$(2A,2B,4C,6A)$	1	193	$(2A,2B,4A,8A)$	1	952
$(2A,2B,5A,6B)$	2	1625,1500	$(2A,2B,3A,10A)$	2	375,375
$(2A,2B,3A,14B)$	2	196,231	$(2A,2B,3A,14A)$	2	196,231
$(2A,2B,3A,12A)$	2	318,210	$(2A,3A,3A,4C)$	1	296
$(2B,3A,3A,6B)$	3	1552,886,1968	$(2B,2B,4D,10A)$	1	210
$(2B,2B,4D,14A)$	1	98	$(2B,2B,4D,14B)$	1	98
$(2B,2B,4D,12A)$	1	120	$(2B,2B,5A,5A)$	3	480,360,90
$(2B,2B,4B,7B)$	2	91,42	$(2B,2B,4B,7A)$	2	91,42
$(2B,2B,4C,8B)$	1	104	$(2B,2B,4A,8A)$	1	264
$(2B,2B,4A,7B)$	1	350	$(2B,2B,4A,7A)$	1	350
$(2B,2B,5A,6A)$	2	315,310	$(2B,2B,6A,6A)$	3	60,160120
$(2A,2B,2B,2C,4D)$	1	3400	$(2A,2A,2B,2B,5A)$	2	8075,7650
$(2A,2A,2B,2B,6A)$	2	5172,3756	$(2A,2B,2B,3A,3A)$	2	12594,10188
$(2B,2B,6B,8B)$	2	912,288	$(2B,2B,3A,11A)$	2	55,44,44,44
$(2B,2B,2B,2C,4B)$	1	912	$(2B,2B,2B,2C,4A)$	1	2160
$(2A,2B,2B,2B,8B)$	1	2400	$(2B,2B,2B,3A,4D)$	1	4536
$(2B,2B,2B,2B,8A)$	1	768	$(2B,2B,4B,8A)$	1	120
$(2B,2B,2B,2B,7B)$	2	588,294	$(2B,2B,2B,2B,2B,2C)$	1	6144
$(2B,2B,2B,2B,7A)$	2	588,294	$(2B,3A,4B,4D)$	1	630
$(2B,3A,4A,4D)$	1	2270	$(2C,2C,2C,4D)$	1	90
$(2C,2C,3A,3A)$	1	534	$(2A,2A,4C,6C)$	2	704,1104
$(2A,2A,2B,2C,3A)$	1	13356	$(2B,4D,4D,4D)$	1	576
$(2A,2A,2A,2B,4C)$	2	4920,3384	$(2A,2A,2A,2B,6B)$	2	27954,22194
$(2A,2A,2A,2A,2B,2B)$	2	137480,113040	$(2A,2A,2A,2C,2C)$	1	5172

Table 5.60: $M_{22}:2, g = 2$, Of Degree 77

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C,4C,7A)$	2	1	$(2C,4C,7B)$	2	1
$(2C,4A,12A)$	3	1	$(2C,4A,10)$	4	1
$(2B,6B,12A)$	4	1	$(2B,6B,10A)$	1	1
$(2B,6A,11A)$	3	1	$(2B,6B,12A)$	2	1
$(2B,6B,10A)$	2	1	$(2A,4D,11A)$	1	1
$(2A,4D,11B)$	1	1	$(2A,2C,2C,6B)$	1	48
$(2A,2B,2C,5A)$	1	30	$(2A,2C,2V,4A)$	1	50

Table 5.61: $M_{23}, g = 2$, Of Degree 23

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(5A,5A,5A)$	972	1	$(4A,5A,8A)$	1484	1
$(5A,5A,5A)$	972	1	$(4A,5A,8A)$	1484	1
$(4A,5A,7B)$	622	1	$(4A,5A,7A)$	622	1
$(4A,4A,15B)$	480	1	$(4A,4A,15A)$	480	1
$(4A,4A,11B)$	564	1	$(4A,4A,11A)$	564	1
$(4A,4A,14B)$	464	1	$(4A,4A,14A)$	464	1
$(4A,6A,8A)$	564	1	$(4A,6A,7B)$	1157	1
$(4A,6A,7A)$	1157	1	$(5A,5A,6A)$	2178	1
$(5A,6A,6A)$	3416	1	$(6A,6A,6A)$	4860	1
$(3A,8A,8A)$	568	1	$(3A,5A,15B)$	159	1
$(3A,5A,15A)$	159	1	$(3A,5A,11B)$	148	1
$(3A,5A,11A)$	148	1	$(3A,5A,14B)$	150	1
$(3A,5A,14A)$	150	1	$(3A,4A,23B)$	50	1
$(3A,4A,23A)$	50	1	$(3A,7B,8A)$	272	1
$(3A,7B,7B)$	98	1	$(3A,7A,8A)$	272	1
$(3A,6A,15B)$	269	1	$(3A,6A,15A)$	269	1
$(3A,6A,11B)$	311	1	$(3A,6A,11A)$	311	1
$(3A,6A,14B)$	265	1	$(3A,6A,14A)$	265	1
$(2A,8A,23B)$	16	1	$(2A,8A,23A)$	16	1
$(2A,15B,15B)$	16	1	$(2A,15A,15B)$	24	1
$(2A,15A,15A)$	16	1	$(2A,11B,15B)$	23	1
$(2A,11B,15A)$	23	1	$(2A,11B,11B)$	18	1
$(2A,11A,15B)$	23	1	$(2A,11A,15A)$	23	1
$(2A,11A,11B)$	22	1	$(2A,11A,11A)$	18	1
$(2A,14B,15B)$	22	1	$(2A,14B,15A)$	22	1
$(2A,11B,14B)$	20	1	$(2A,11A,14B)$	20	1
$(2A,14B,14B)$	24	1	$(2A,14A,15B)$	22	1
$(2A,14A,15A)$	22	1	$(2A,11B,14A)$	20	1
$(2A,11A,14A)$	20	1	$(2A,11A,14B)$	17	1
$(2A,14A,14A)$	24	1	$(2A,7B,23B)$	8	1
$(2A,7B,23A)$	8	1	$(2A,7A,23B)$	8	1
$(2A,7A,23A)$	8	1	$(2A,3A,4A,4A)$	1	103728
$(2A,3A,3A,5A)$	1	34170	$(2A,3A,3A,6A)$	1	54918
$(2A,2A,5A,5A)$	1	30400	$(2A,2A,4A,8A)$	1	34944
$(2A,2A,4A,7B)$	1	15848	$(2A,2A,4A,7A)$	1	15848
$(2A,2A,5A,6A)$	1	48180	$(2A,2A,5A,6A)$	1	72528
$(2A,2A,3A,15B)$	1	3335	$(2A,2A,3A,15A)$	1	3335
$(2A,2A,3A,11B)$	1	4070	$(2A,2A,3A,11A)$	1	4070
$(2A,2A,3A,14B)$	1	3206	$(2A,2A,3A,14A)$	1	3206
$(2A,2A,2A,3A,3A)$	1	850392	$(2A,2A,2A,2A,5A)$	1	732000
$(2A,2A,2A,2A,6A)$	1	1050336	$(2A,2A,2A,2A,2A,2A)$	1	16463280
$(3A,3A,3A,3A)$	2	28980,8316			

Table 5.62: $M_{24}, g = 2$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(4B,8A,8A)$	3440	1	$(4B,4B,21B)$	388	1
$(4B,4B,21A)$	388	1	$(4B,4B,23B)$	245	1
$(4B,4B,23A)$	245	1	$(4B,4B,12B)$	792	1
$(4B,6A,12A)$	4734	1	$(4B,6A,14B)$	2962	1
$(4B,6A,14A)$	2962	1	$(4B,6A,11A)$	3303	1
$(4B,6A,10A)$	2642	1	$(4B,6A,15B)$	2960	1
$(4B,6A,15A)$	2960	1	$(4B,6A,6B)$	2697	1
$(4A,4B,12A)$	316	1	$(4A,4B,14B)$	242	1
$(4A,4B,14A)$	242	1	$(4A,4B,11A)$	278	1
$(4A,4B,10A)$	185	1	$(4A,4B,15B)$	268	1
$(4A,4B,15A)$	268	1	$(4A,4B,6B)$	204	1
$(4B,7B,8A)$	988	1	$(4B,7B,7B)$	253	1
$(4B,7A,7A)$	212	1	$(4B,4C,5A)$	1406	1
$(4B,5A,14B)$	820	1	$(4B,5A,14A)$	820	1
$(4B,5A,11A)$	810	1	$(4B,5A,10A)$	823	1
$(4B,5A,15B)$	806	1	$(4B,5A,15A)$	806	1
$(4B,5A,6B)$	786	1	$(4B,4C,8A)$	958	1
$(4B,4C,7B)$	300	1	$(4B,4C,7A)$	300	1
$(4B,4C,4C)$	172	1	$(3B,4B,12A)$	328	1
$(3B,4B,14B)$	264	1	$(3B,4B,14A)$	264	1
$(3B,4B,11A)$	322	1	$(3B,4B,10A)$	199	1
$(3B,4B,15B)$	234	1	$(3B,4B,15A)$	234	1
$(6A,6A,8A)$	15392	1	$(6A,6A,7B)$	4825	1
$(6A,6A,7A)$	4825	1	$(4C,6A,6A)$	3896	1
$(4A,6A,8A)$	1286	1	$(4A,6A,7B)$	414	1
$(4A,6A,7A)$	414	1	$(4A,4C,6A)$	202	1
$(4A,4A,8A)$	36	1	$(4A,4A,7B)$	15	1
$(4A,4A,7A)$	15	1	$(4A,4A,4C)$	12	1
$(2A,12A,21B)$	57	1	$(2A,12A,21A)$	57	1
$(2A,12A,23B)$	33	1	$(2A,12A,23A)$	33	1
$(2A,12A,12B)$	83	1	$(2A,14B,21B)$	28	1
$(2A,14B,21A)$	39	1	$(2A,14B,23B)$	23	1
$(2A,14B,23A)$	23	1	$(2A,12B,14B)$	61	1
$(2A,14A,21B)$	39	1	$(2A,14A,21A)$	28	1
$(2A,14A,23B)$	23	1	$(2A,14A,23A)$	23	1
$(2A,12B,14A)$	61	1	$(2A,11A,21B)$	36	1
$(2A,11A,21A)$	36	1	$(2A,11A,23B)$	22	1
$(2A,11A,23A)$	22	1	$(2A,11A,12B)$	52	1
$(2A,10A,21B)$	34	1	$(2A,10A,21A)$	34	1
$(2A,10A,23B)$	21	1	$(2A,10A,23A)$	21	1
$(2A,10A,12B)$	49	1	$(2A,15B,21B)$	33	1
$(2A,15B,21A)$	33	1	$(2A,15B,23B)$	22	1

Table 5.63: $M_{24}, g = 2$, Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,15B,23A)$	22	1	$(2A,12B,15B)$	60	1
$(2A,15A,21B)$	33	1	$(2A,15A,21A)$	33	1
$(2A,15A,23B)$	22	1	$(2A,15A,23A)$	22	1
$(2A,12B,15A)$	60	1	$(2A,6A,21B)$	23	1
$(2A,6B,21A)$	23	1	$(2A,6B,23B)$	19	1
$(2A,6B,23A)$	19	1	$(2A,6B,12B)$	40	1
$(3A,8A,12A)$	740	1	$(3A,8A,14B)$	448	1
$(3A,8A,14A)$	448	1	$(3A,8A,11A)$	436	1
$(3A,8A,10A)$	458	1	$(3A,8A,15B)$	444	1
$(3A,8A,15A)$	444	1	$(3A,8A,6B)$	414	1
$(3A,6A,21B)$	294	1	$(3A,6A,21A)$	294	1
$(3A,6A,23B)$	193	1	$(3A,6A,23A)$	193	1
$(3A,6A,12B)$	540	1	$(3A,4A,21B)$	30	1
$(3A,4A,21A)$	30	1	$(3A,4A,23B)$	22	1
$(3A,4A,23A)$	22	1	$(3A,4A,12B)$	45	1
$(3A,7B,12A)$	228	1	$(3A,7B,14B)$	130	1
$(3A,7B,14A)$	122	1	$(3A,7B,11A)$	112	1
$(3A,7B,10A)$	142	1	$(3A,7B,15B)$	115	1
$(3A,7B,15A)$	117	1	$(3A,6B,7B)$	36	1
$(3A,7A,12A)$	228	1	$(3A,7A,14B)$	122	1
$(3A,7A,14A)$	130	1	$(3A,7A,11A)$	112	1
$(3A,7A,10A)$	142	1	$(3A,7A,15B)$	117	1
$(3A,7A,15A)$	117	1	$(3A,6B,7A)$	136	1
$(3A,5A,21B)$	78	1	$(3A,5A,21A)$	78	1
$(3A,5A,23B)$	42	1	$(3A,5A,23A)$	42	1
$(3A,5A,12B)$	144	1	$(3A,4C,12A)$	192	1
$(3A,4C,14B)$	146	1	$(3A,4C,14A)$	146	1
$(3A,4C,11A)$	160	1	$(3A,4C,10A)$	99	1
$(3A,4C,15B)$	144	1	$(3A,4C,15A)$	144	1
$(3A,4C,6B)$	90	1	$(3A,3B,21B)$	18	1
$(3A,3B,21A)$	18	1	$(3A,3B,23B)$	20	1
$(3A,3B,23A)$	20	1	$(3A,3B,12B)$	36	1
$(5A,6A,8A)$	4756	1	$(5A,6A,7B)$	1343	1
$(5A,6A,7A)$	1343	1	$(5A,4C,6A)$	1323	1
$(4A,5A,8A)$	481	1	$(4A,5A,7B)$	151	1
$(4A,5A,7A)$	151	1	$(4A,4C,5A)$	80	1
$(5A,5A,8A)$	1152	1	$(5A,5A,7B)$	277	1
$(5A,5A,7A)$	277	1	$(3B,4B,6B)$	117	1
$(4C,5A,5A)$	444	1	$(3B,6A,8A)$	1070	1
$(3B,6A,7B)$	414	1	$(3B,6A,7A)$	414	1
$(3B,4C,6A)$	191	1	$(3B,4A,8A)$	102	1
$(3B,4A,7B)$	41	1	$(3B,4A,7A)$	41	1
$(3B,5A,8A)$	404	1	$(3B,5A,7B)$	142	1
$(3B,5A,7A)$	142	1	$(3B,4C,5A)$	96	1

Table 5.64: $M_{24}, g = 2$ Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(3B,3B,8A)$	32	1	$(3B,3B,7B)$	12	1
$(3B,3B,7A)$	12	1	$(3B,8A,12A)$	114	1
$(2B,8A,14B)$	103	1	$(2B,8A,14A)$	103	1
$(2B,8A,11A)$	121	1	$(2B,8A,10A)$	64	1
$(2B,8A,15B)$	119	1	$(2B,8A,15A)$	119	1
$(2B,6B,8A)$	80	1	$(2B,6A,21B)$	66	1
$(2B,6A,21A)$	66	1	$(2B,6A,23B)$	50	1
$(2B,6A,23A)$	50	1	$(2B,6A,12B)$	99	1
$(2B,4A,21B)$	4	1	$(2B,4A,21A)$	4	1
$(2B,4A,23B)$	5	1	$(2B,4A,23A)$	5	1
$(2B,4A,12A)$	48	1	$(2B,7B,14B)$	22	1
$(2B,7B,14A)$	54	1	$(2B,7B,11A)$	54	1
$(2B,7B,10A)$	22	1	$(2B,7B,15B)$	47	1
$(2B,7B,15A)$	47	1	$(2B,6B,7B)$	33	1
$(2B,7A,12A)$	48	1	$(2B,7A,14B)$	54	1
$(2B,7A,14A)$	22	1	$(2B,7A,11A)$	54	1
$(2B,7A,10A)$	22	1	$(2B,7A,15B)$	47	1
$(2B,7A,15A)$	47	1	$(2B,6B,7A)$	33	1
$(2B,5A,21B)$	24	1	$(2B,5A,21A)$	24	1
$(2B,5A,23B)$	19	1	$(2B,5A,23A)$	19	1
$(2B,5A,12B)$	36	1	$(2B,4C,12A)$	16	1
$(2B,4C,14B)$	23	1	$(2B,4C,14A)$	23	1
$(2B,4C,11A)$	24	1	$(2B,4C,10A)$	6	1
$(2B,4C,15B)$	20	1	$(2B,4C,15A)$	20	1
$(2B,3B,21B)$	1	1	$(2B,3B,21A)$	1	1
$(2A,4B,4B,4B)$	1	526208	$(2A,2A,4B,12A)$	1	49424
$(2A,2A,4B,14B)$	1	33194	$(2A,2A,4B,14A)$	1	33194
$(2A,2A,4B,11A)$	1	39149	$(2A,2A,4B,10A)$	1	28660
$(2A,2A,4B,15B)$	1	32855	$(2A,2A,4B,15A)$	1	32855
$(2A,2A,4B,6B)$	1	28341	$(2A,2A,6A,8A)$	1	181688
$(2A,2A,6A,7B)$	1	55727	$(2A,2A,6A,7A)$	1	55727
$(2A,2A,4C,6A)$	1	43352	$(2A,2A,4A,8A)$	1	15112
$(2A,2A,4A,7B)$	1	4424	$(2A,2A,4A,7A)$	1	4424
$(2A,2A,4A,4C)$	1	2384	$(2A,2A,2A,2A,8A)$	1	2173440
$(2A,2A,2A,2A,7B)$	1	675514	$(2A,2A,2A,2A,7A)$	1	675514
$(2A,2A,2A,2A,4C)$	1	468928	$(2A,2A,2A,3A,4B)$		1
$(2A,2A,2A,2B,4B)$	1	1002768	$(2A,2A,3A,21B)$	1	2947
$(2A,2A,3A,21A)$	1	2947	$(2A,2A,3A,23B)$	1	2185
$(2A,2A,3A,23A)$	1	2185	$(2A,2A,3A,12B)$	1	3876
$(2A,2A,5A,7B)$	1	16555	$(2A,2A,5A,7A)$	1	16555
$(2A,2A,5A,4C)$	1	13860	$(2A,2A,3B,8A)$	1	12528

Table 5.65: $M_{24}, g = 2$ Of Degree 24

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2A,2A,3B,7B)$	1	4669	$(2A,2A,3B,7A)$	1	4669
$(2A,2A,3B,4C)$	1	2656	$(2A,2A,2B,21B)$	1	805
$(2A,2A,2B,21A)$	1	805	$(2A,2A,2B,23B)$	1	575
$(2A,2A,2B,23A)$	1	575	$(2A,2A,2B,12B)$	1	1224
$(2A,3A,4B,6A)$	1	3876	$(2A,3A,4A,4B)$	1	29134
$(2A,3A,4B,5A)$	1	10852	$(2A,3A,3B,4B)$	1	28658
$(2A,3A,3A,8A)$	1	57840	$(2A,3A,3A,7B)$	1	17038
$(2A,3A,3A,7A)$	1	17038	$(2A,3A,3A,4C)$	1	14080
$(2A,2B,4B,6A)$	1	86610	$(2A,2B,4A,4B)$	1	5112
$(2A,2B,4B,5A)$	1	31000	$(2A,2B,3B,4B)$	1	4941
$(2A,2B,3A,8A)$	1	15072	$(2A,2B,3A,7B)$	1	5201
$(2A,2B,3A,7A)$	1	5201	$(2A,2B,3A,4C)$	1	2640
$(2A,2B,2B,8A)$	1	2080	$(2A,2B,2B,7B)$	1	763
$(2A,2B,2B,7A)$	1	763	$(2A,2B,2B,4C)$	1	208
$(3A,3A,3A,4A)$	1	118236	$(2B,3A,3A,4B)$	1	34464
$(2B,2B,3A,4B)$	1	4680	$(2B,2B,2B,4B)$	1	624
$(2A,2A,5A,8A)$	1	57000	$(2A,2A,4C,5A)$	1	13860

Table 5.66: $J_2, g = 2$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,3B,10A)$	6	1	$(2B,3B,10B)$	6	1
$(2A,5C,6B)$	1	1	$(2A,5D,6B)$	1	1

Table 5.67: $J_2:2, g = 2$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C,6C,5D)$	4	1	$(2C,4C,14A)$	9	1
$(2A,2C,2C,4A)$	1	28	$(2A,4C,12C)$	2	1

Table 5.68: $HS, g = 2$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2B,4C,20A)$	3	1	$(2B,4C,20B)$	3	1
$(2B,4C,7A)$	18	1	$(2B,6B,6B)$	20	1
$(2B,5C,6B)$	22	1	$(2B,5C,5C)$	18	1
$(2B,3A,15A)$	2	1			

Table 5.69: $HS:2, g = 2$, Of Degree 100

$RamificationType$	$N.Orbit$	$L.O$	$RamificationType$	$N.Orbit$	$L.O$
$(2C,6E,10A)$	1	1	$(2C,4F,15A)$	1	1
$(2D,6B,6E)$	8	1	$(2D,3A,20C)$	1	1
$(2D,5C,6E)$	6	1	$(2D,4D,11B)$	1	1
$(2D,4F,6A)$	5	1	$(2D,4C,12B)$	3	1
$(2D,4C,10C)$	6	1	$(2D,4C,10D)$	22	1
$(2A,4E,20E)$	3	1	$(2A,4E,20D)$	3	1
$(2A,6C,10C)$	1	1	$(2A,2D,2D,4C)$	1	268
$(2D,4F,12B)$	6	1	$(2A,4F,10C)$	2	1
$(2A,4F,10D)$	14	1	$(2A,2A,2D,6C)$	1	12
$(2A,2A,2D,4F)$	1	160			

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