Regime Switching Behaviour of the UK Equity Risk Premium

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Abstract

We apply regime-switching models to study the dynamic switching behaviour of equity risk premia. Traditionally, equity risk premia have been estimated assuming a single regime exists. Regime-switching models allow for the existence of two, or more, regimes. Three regime-switching models are employed: structural break models, threshold models and Markov regime-switching models. Both structural break models and threshold models assume that the switching mechanism is deterministic. The former allow for only a single break and the state variable is solely determined by time. Under the latter, multiple changes are allowed and the state variable is determined by an observable variable with respect to an unobserved threshold. In Markov regime-switching models, equity risk premia are allowed to switch probabilistically for each observation. This is achieved by introducing a state variable which is governed by a Markov process. To capture the co-movements among financial variables, we extend regime-switching models to a VAR framework, employing threshold autoregressive vector models and Markov regime-switching vector models. We estimate models of UK equity risk premia conditionally on the state variable which is related to business conditions. The results of non-linearity tests favour regime-switching models and suggest that regime-switching is an important characteristic of UK equity risk premia.

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Chapter 1 Introduction

The main purpose of this study is to investigate the behaviour of equity risk premia by using regime switching models. Equity risk premia refer to the difference between stock market returns and risk-free returns. They are compensation to investors who take on risky investments instead of risk-free assets. Equity risk premium behaviour may show non-linearity of different kinds. In the class of non-linear models, regime switching models have become particularly popular. These models allow the dynamic behaviour of financial time series to switch between different regimes due to changing circumstances such as financial panics, technological shocks or changes in government policy. The present study is organised into six chapters. The first and last chapters are the introduction and conclusion. Chapter 2 is a literature survey and Chapters 3 to 5 represent original empirical papers.

In Chapter 2, we review existing studies on the dynamic connection between equity risk premia and business cycles. On the one hand, the efficient market hypothesis suggests that the true equity risk premium is simply a constant. On the other hand, the consumption capital asset pricing model (CCAPM) provides a more complete framework by tying equity risk premia to the business conditions and suggests that equity risk premia change with changing business conditions. This casts doubt on the traditional assumption that equity risk premia are drawn from one stable distribution for the whole sample period since this assumption cannot accurately describe the cyclical behaviour of the data. Moreover, if equity risk premia switch from one regime to another, the long run relationship between equity risk premia and predictive variables will be time-varying and conditional on the state variable. In this case, a simple linear framework cannot capture this relationship even if it exists. The issues raised in this study provide the motivation for considering the possibility of structural breaks and regime switching in the parameters of equity risk premium models.

In Chapter 3 we consider the possibility of one, or multiple structural breaks in the parameters of equity risk premium models. Statistical tests for these structural breaks are presented, some of which rely on knowing the exact date of the breaks and some of which allow for the dates of the breaks not to be known in advance. Both multivariate and univariate models are considered. Two types of structural break test are therefore introduced: one is the tests of unit roots with structural breaks which test the null of a unit root against the alternative of break-stationarity; the other is based on predictive regression models and is used to investigate whether the long-run relationships between financial time series are stable over time. To investigate whether the relationships between equity risk premia, dividend yields, Treasury bill rates and inflation rates are subject to regime switching due to changing business conditions, we apply these structural break tests to the UK stock market. The tests clearly reject the null hypothesis of no structural break and hence suggest structural instability in equity risk premium models. In particular, they identify the break date in which breaks occur and link them to special events such as changes in policy. The results therefore suggest that regime switching is an important characteristic of equity risk premia and cannot be ignored by investors when making their asset allocation decisions.

In Chapter 4, we apply regime switching models to study the dynamic switching behaviour of the equity risk premium. Traditionally, equity risk premium models have been estimated assuming a single regime exists. Regime switching models allow for the existence of two, or more, regimes. In this study, three switching regime models are introduced: the structural break model, threshold autoregressive model and Markov switching regime model. In the structural break model, switches between regimes are solely determined by time. In the threshold autoregressive model, switches between regimes are determined by an observable variable with respect to unobserved thresholds. These thresholds can be estimated endogenously. In the Markov switching regime model, equity risk premia are allowed to switch probabilistically between different regimes. This is achieved by introducing a state variable to describe different market regimes, such as a 'good regime' and a 'bad regime'. This state variable is governed by an unobservable Markov process and is therefore a latent variable. In order to capture the dynamic co-movements among financial variables, we extend regime-switching models to a VAR framework. The Threshold autoregressive vector model and the Markov switching regime vector model are therefore introduced. Our empirical results on the UK equity risk premium are consistent with the findings of Chapter 2 and suggest structural instability in equity risk premium models.

In Chapter 5, we use non-linearity tests to analyse whether the non-linear specification associated with regime switching models makes them superior to linear models. Both portmanteau tests and specific tests are employed. The portmanteau tests are residual-based and are used to test for linearity without a specific non-linear alternative, while the specific tests examine linearity with a specific non-linear alternative. In particular, we focus on specific non-linearity tests in the context of threshold autoregressive models and Markov switching regime models. It is shown that switching regime models are useful because they provide a better fit to the data and a better explanation for the cyclical behaviour of equity risk premia. The empirical application to the UK equity risk premium supports this conclusion.

This study is structured as follows. Chapter 2 provides a literature overview on the effect of business conditions on equity risk premia. Chapter 3 considers the possibility of structural breaks in the parameters of equity risk premium models. Chapter 4 introduces regime switching models to describe the switching behaviour of equity risk premia. Chapter 5 uses non-linearity tests to analyse whether the non-linear specification model is superior to a linear model. Chapter 6 summarises and concludes by presenting the limitations of this study and possible directions for future research.

Chapter 2 Cyclical behaviour of equity risk premia

2.1 Introduction

This chapter is a review of existing studies on the dynamic connection between equity risk premia and business cycles. The efficient market hypothesis would imply that the true equity risk premium should be constant. However, the consumption-based asset pricing model provides evidence that the long-run equity risk premium can change over time. In particular, it suggests that equity risk premia change over business cycles. The issues raised in this study provide the motivation for considering the possibility of structural breaks and regime switching in the parameters of equity risk premium models.

The rest of this chapter is structured as follows. Section 2 reviews the definition of equity risk premia and then asks how large the UK equity premium is. Section 3 reviews the literature related to the assumption of a constant equity risk premium and the efficient market hypothesis (EMH). In Section 4, the consumption-based capital asset pricing model (CCAPM) is reviewed. This model suggests that equity risk premia vary over time. More precisely, it suggests that expected equity risk premia change cyclically and tend to be higher during recessions than during expansions. Section 5 reviews the empirical connection between equity risk premia and business conditions. The fact that equity risk premia change over business conditions casts doubt on the belief that mixing good and bad times together to calculate historical average can accurately describe the cyclical behaviour of future equity risk premia. In this regard, the possibility of structural breaks and regime switching in the time series of equity risk premia and in the parameters of equity risk premium models should be considered. Section 6 summarises and concludes. In particular, we identify the four main objectives in this study and organise the subsequent research into four chapters.

2.2 UK equity risk premia

With the growing interest in the development of asset pricing models and financial econometrics, new ways to estimate equity risk premia have been developed and applied over two decades. Equity risk premia play an important role in making financial decisions because they are a reflection of investors' expectations of the likely returns from holding risky equities. Welch (2000, p. 501) reported that "The equity risk premium is perhaps the single most important number in financial economics". In this chapter, we review the most common methods used in connection with the estimation of equity risk premia. This literature review starts by asking what the equity risk premium is.

Equity risk premia are defined as the difference between the rates of return from holding a risky asset and the rates of return from a risk-free asset over a given time interval. Although equity risk premia can be calculated over any time interval, the most commonly used intervals are 1 month, 1 quarter or 1 year. We can write equity risk premia by means of the following expression:

$$R_t = R_t^m - R_t^f \tag{2.1}$$

where R_t denotes the equity risk premium over the time interval t-1 to t. The subscript t is used because the return becomes known at time t. R_t^f is the risk-free rate of return over the time interval, such as the interest on Treasury bills. $R_t^m = (P_t - P_{t-1} + D_t)/P_{t-1}$ is the stock market return rate over the time interval. P_t is the price of the stock at time t, P_{t-1} is the price of the stock at time t-1 and D_t represents the dividend earned at the end of the time interval t-1 to t. An alternative measure of the equity risk premium is r_t, the log or continuously compounded equity risk premium over the time interval t-1 to t:

$$r_t = \ln(1 + R_t) \tag{2.2}$$

An annual rate of return is based on holding an asset for a whole year and simply calculating the rate of return on the asset. An annualized rate of return is based on using the returns for a period other than a year to calculate what the return would be per year if the same rate of return remained constant over a single year. In the case of monthly data, an annualised rate of return is based on holding an asset for one month. When converting a one-month rate of return to a yearly base, we assume a constant yield for each month over one year. In this case, the annualised rate of monthly return is:

$$R_t^a = (R_t + 1)^{12} - 1 \tag{2.3}$$

A continually compounded annualised rate of monthly return can be calculated by multiplying the natural log of the one-month return by 12.

$$r_t^a = 12\ln(1 + R_t) \tag{2.4}$$

Building on the theme that the equity risk premium acts as compensation to investors for the risk that they are taking, the equity risk premium for the UK stock market is typically measured as the difference between the UK stock returns and the returns on Treasury securities. We then turn to the question of how large the UK equity premium is. The Barclays Capital Equity Gilt Study (2008) reported that UK real equity returns in 2007 dropped to 1% after inflation, against 1.4% for index linked bonds. In 2006, however, UK equity returns were 11.4%, compared to -2.1% for index-linked bonds. These figures were very different from the long-run average. According to both Barclays Capital and Credit Suisse First Boston (CSFB), the average UK equity risk premium over the last 107 years (equity returns over gilts) was 4.2%, over the last 50 years was 4.9%, over the last 20 years was 1.3% and over the last 10 years was only 0.3%. To assess the performance of UK equities and Treasury bills, Figure 2.1 presents the plots for the natural logarithms of annualised UK monthly stock returns and three-month Treasury bill yields. The details of the data and statistical issues are discussed further in Chapter 3. Figure 2.2 and Figure 2.3 plot the five-year and ten-year moving averages of UK stock returns and Treasury bill yields, respectively. Figure 2.4 presents the plot for the logarithm of the annualised monthly equity risk premium on the UK FTSE All Share Index minus the three-month Treasury bill rate. These figures suggest that UK stocks perform better than Treasury bills in the long run but they are more volatile and hence riskier. Investors may face substantial losses in stock markets because financial time series may be subject to occasional, discrete changes, such as a stock market crash. For example, on Black Monday in October 1987, the UK stock market lost 26.4% of its value.

Table 2.1 supports the conclusion that equity risk premia are highly volatile. The average annualized monthly equity risk premium is 0.0444. This suggests that investors can expect an excess return of 4.44% per annual above Treasury bills yields. However, the standard error of annualised UK monthly equity risk premia is about 0.6653. This suggests that, given a 95% confidence interval, next year's equity risk premium will be between -0.6032 and 1.3532. Such a range is too wide to provide a reasonable estimate for next year's equity risk premium. One possible reason for these large standard errors is that equity risk premia may change periodically over time. As an illustration of cyclical behaviour in the UK equity risk premium, Figures 2.5 and 2.6 plot the five-year and ten-year moving averages of the UK equity risk premium, respectively. The plots reveal the trend in the data and suggest that historical equity risk premia change cyclically. The purpose of this chapter is to provide a framework to model this cyclical behaviour in the UK equity risk premium so that investors can make their asset allocation decisions on the basis of expected equity risk premia. In the literature, there are many different opinions on what the estimate of the equity risk premium is. Below we review some of these in turn.

Figure 2.1 Annualised UK monthly stock returns and Treasury bill yields
FIGURE 2.1 HERE
Figure 2.2 Five-year moving average of UK monthly stock returns and Treasury bill yields
FIGURE 2.2 HERE
Figure 2.3 Ten-year moving average of UK monthly stock returns and Treasury bill yields
FIGURE 2.3 HERE
Figure 2.4 Annualised UK monthly equity risk premia
FIGURE 2.4 HERE
Figure 2.5 Five-year moving average of UK equity risk premia
FIGURE 2.5 HERE
Figure 2.6 Ten-year moving averages of UK equity risk premia
FIGURE 2.6 HERE

Table 2. 1 Means and standard errors of equity risk premia

Frequency	Monthly	Quarterly
Mean	0.0444	0.0418
Standard error	0.6653	0.4047

2.3 Efficient Market Hypothesis (EMH) and constant equity risk premia

In the 1960s and 1970s, a number of researchers suggested that the true equity risk premium was, under the efficient market hypothesis, simply a constant. Their results also suggested that estimates of equity risk premia might be updated by investors and should converge to the true premium. In other words, the efficient market hypothesis would imply that the true equity risk premium is constant. In this section, we review the literature related to the efficient market hypothesis, perhaps one of the most controversial hypotheses in finance. Many well-known issues such as volatility, random walk hypothesis (RWH), predictability, mean reversion, speculation and anomalies are all associated with the EMH.

Recent research is divided over whether or not financial markets are efficient. The debate has become even more complicated by the fact that most financial time series have non-linear features. As a consequence, asking whether the EMH would hold in financial markets is a reasonable starting point. Fama (1970) suggested that a financial market is efficient if the prices of securities fully reflect all available information. The EMH is an important concept because it helps investors to set expected returns from holding equities and determines their resource allocation strategies. If the EMH holds, there is no way to beat the market and no one has an advantage in predicting future stock markets. All the assets traded in the market are fairly priced and no investors can consistently obtain abnormal returns (more than the market equilibrium rate of return) on their investments. In this case, investors would rather hold market portfolios than be actively engaged in trading.

Fama (1991) defined three main kinds of market efficiency, depending on the response of market prices to particular subsets of available information. The first is the weak form of efficiency, which suggests that current stock prices reflect all the information contained in historical prices. The second is the semi-strong form, which states that current stock prices fully reflect all the publicly available information and the market reacts to new arrival information efficiently by immediately and accurately incorporating it into current prices. Public information includes: past stock prices, announcements reported in a company's financial statements, such as annual reports, income statements, earnings and dividend reports, company plans, inflation and unemployment. The last category is the strong form of efficiency, which suggests that current stock prices fully reflect all existing information, whether public or private.

2.3.1 Random walk hypothesis (RWH)

The EMH is consistent with the idea that stock prices follow a random walk (Samuelson, 1965). Fama (1970) suggested that stock prices could be described by a random walk under the weak-form EMH and stock price changes were unpredictable. Let P_t denote a stock price at time *t*. P_t follows a random walk with a constant drift *r* if:

$$P_{t+1} = r + P_t + \varepsilon_{t+1} \qquad \varepsilon_{t+1} \sim i.i.d\ (0, \sigma^2) \tag{2.5}$$

where ε_{t+1} is the random disturbance term. Let Ω_t be the available information set at time *t*. Taking the expectations on both sides of Equation (2.5), we obtain:

$$r + P_t = E(P_{t+1} | \Omega_t) = E_t(P_{t+1}) \tag{2.6}$$

By the law of iterated expectation, we get:

$$E(P_{t+1} - E(P_{t+1} | \Omega_t)) = 0 \tag{2.7}$$

Equation (2.5) implies that the price of an asset is determined only by past fundamental data and therefore supports the weak form EMH. Moreover, ε_{t+1} is the random disturbance term which is assumed to be identically independently distributed (*i.i.d*) with mean 0 and variance σ^2. This says that stock price changes are also random and *i.i.d*, and stock price movements are therefore not predictable. Equation (2.7) states that the expected predicted error between the actual stock price and the expected stock price at time t+1 equals 0. This implies that the market reflects the true value of the assets and investors do not expect abnormal returns. In this case, the market is a 'fair game' with respect to the available information and the EMH is also referred to as the Rational Theory. In particular, under the EMH, expected stock price movements and equity risk premia are constant. (Samuelson, 1965). The above results imply that, if the EMH does hold, it is impossible for investors to use technique analysis to predict future markets. Active trading is pointless under the EMH and taking an extra risk is the only way to make extra returns.

2.3.2 Pros and Cons of EMH

The EMH is a controversial economic theory, which implies that stock markets are not predictable and equity risk premia are constant. In the literature, there are many pros and cons with regard to the EMH, which we discuss below in turn.

Competition in stock markets

In the 1960s, a number of researchers provided empirical evidence in favour of the EMH. These studies can be interpreted in two ways. The first maintains that the stock prices are fairly set and should reflect all available information. The second holds that the market should react to the arrival of new information efficiently by incorporating it into the stock price quickly and correctly. These two ways are based on the same simple idea: that there is competition in the stock market. This competition, which is driven by an enormous number of investors pursuing profit maximisation, results in the randomness of price movements. Stock prices change efficiently because the market participants evaluate every piece of information and therefore buy under-priced stocks and sell the over-priced ones without delay. This is why no investors can consistently beat the market and the market reflects the true value of the assets.

Information asymmetries

However, the conditions of the stock market may be inconsistent with the EMH. One common explanation for departures from the EMH is based on information asymmetries (Akerlof, 1970). The EMH claims that all participants in the market have equal access to the available information, but this does not reflect the effect of events on stock markets and is challenged by the idea of information asymmetries (see, for example, Becker, Finnerty and Friedman, 1995; Wongswan, 2005 and Ehramann and Fratzscher, 2003). This is because events such as the oil crisis of 1974 or the market crash of 2008 are accompanied by substantial uncertainty. Timmerman (2001) argued that such uncertainty could arise from the imperfect knowledge of the stock market and its updating process could be thought as one that investor would gradually have more precise estimates about the stock market when new data emerge. In this case, information asymmetry may result in the impact of shocks on the stock market remaining for a long time. For example, in the period after crashes, investors may revise their estimates of equity risk premia based on their incomplete information instead of relying on historical data. Information asymmetries may prevent stock prices from reverting to their fundamental trends immediately and may therefore give rise to risks of holding stocks. To conclude, the observed behaviour of stock markets is consistent with information asymmetries which suggest that information is distributed unevenly and different market participants may access to information which varies in quantity and quality. Certain investors may have more or better information than others and therefore may perceive the same asset differently.

Indeed, asset pricing is significantly affected by information asymmetries; see, for example, Saar (2006), Chakravarty, Sarkar and Wu, (1998), Aydogdu and Shekhar (2005), Barom, Bartram and Yadav (2006), Moerman (2006) and Chan, Menkveld and Yang (2006). The pricing of an asset is the result of a bid-ask process. The seller and the buyer make an offer for the asset according to the available information. If they access different levels of information on a transaction, a 'lemon' market may occur (Akerlof, 1970). In other words, informed investors and uninformed investors may have different beliefs about the stock market and therefore may have different market expectations. In this case, competition in the stock market may not efficiently push stock prices back to their fundamental values. Stock prices can be mispriced and may deviate from their fundamental values. Thus, it is impossible for asset prices to always fully reflect all the available information in the stock market and investors are prone to face information risk.

Behavioural Finance

Another important discrepancy between the EMH and the stock market concerns *behavioural finance*. Many researchers believe that different investors in the markets may still react to the same information differently, even when they are well informed. They think that behavioural finance can better explain many stock market anomalies, such as size effects, calendar effects, announcement based effects and insider transactions. We review the main aspects of behavioural finance below.

The EMH assumes that all investors are rational. In other words, investors will always maximize their expected utility, based on rational expectations. Behavioural Finance challenges the assumption of rationality by allowing for the possibility that investors may think irrationally. Typically, its proponents explain that irrationality may arise as a consequence of the possibility that investors may either have different expected utility functions or may fail to form rational expectations. The interpretation of these results comes from psychology and experimental economics. Cognitive psychologists believe that investors make portfolio decisions on the basis of both facts and feelings. They also suggest that attitudes guide behaviour; see for example Zanna and Rempel (1988) and Fazio (1990). Individuals may have different attitudes towards risk and therefore when faced with uncertainty may use different reference points to value gains and losses. The literature outlines three main types of attitude to risk: risk-averse, risk-neutral and risk-loving. Some relevant examples of investors' preference and behaviour resulting in different outcomes and generating irrationality are: prospect theory (Kahneman and Tversky, 1979), ambiguity aversion and risk aversion (Epstein 1999; Alary, Treich and Gollier 2010), disappointment aversion, regret and loss aversion (Kahneman and Tversky, 1979; Bell, 1982), the overconfidence effect (Fischoff and Slovic 1980; Barber and Odean, 2001 and Gervais and Odean, 2001), representativeness and conservatism (Barberis, *et al.*, 1998; Kahneman and Tversky, 1982), market over-reaction and under-reaction to new announcements (DeBondt and Thaler, 1985), herding (Huberman and Regev, 2001), psychological accounting (Tversky and Kahneman, 1981) and hyperbolic discounting (Laibson, 1997).

Financial economists have now recognised that stock prices are decided by both rational and irrational investors. Black (1986) named irrational investors as *noise traders* and believed that markets always include noise traders. Delong, Shleifer, Summers and Waldman (1990) emphasised the influence of noise traders in setting stock prices. They pointed out that irrational noise traders may have a significant impact on stock prices even though changes in investor sentiment are not related to the stock market fundamentals. Market sentiment refers to the prevailing attitude of investors to the future stock market trend. If most investors in the market expect stock prices to rise (fall), the market sentiment is bullish (bearish). An example showing the impact of investors' sentiment on stock prices is the Dot-Com Bubble in the late 1990s. NASDAQ stocks were overpriced in early 1999, but became even more overpriced by early 2000.

The above results show that different investors may have different attitudes to risk and that changes in risk aversion can significantly affect stock prices. Moreover, investors' attitudes to risk may not only differ between individuals, but also vary significantly over time. Investors revise their future risk aversion on the basis of prevailing business conditions. Campbell and Cochrane (1999) introduced the concept of *habit formation*. They suggest that falls in consumption push investors closer to their habit level and therefore investors are more risk-averse and may require more equity risk premia. Some relevant examples of time varying equity risk premia are highlighted in the literature. To begin, we look at consumption-based capital asset pricing models (CCAPM).

2.4 Consumption-based capital asset pricing models (CCAPM) and time-varying equity risk premia

In the early 1980s, researchers on asset pricing theory reported evidence that equity risk premia can change over time even in an efficient market with rational investors. The CCAPM assumes that individuals can inter-temporally smooth their consumption by trading financial assets. The intuition behind this assumption comes from the desire of individuals to smooth their consumption over time. See, for example, the permanent income hypothesis (Friedman, 1956) and life-cycle hypothesis (Modigliani and Brumberg, 1954).

The CCAPM comes from the first-order conditions for investors' optimal consumption and portfolio choice problem (Cochrane, 2001). Individuals face a choice: consume today or buy and hold financial assets in order to have an opportunity to consume more in the future. This choice can be written as a constrained optimisation problem:

$$\begin{aligned} &Max\ (u(C_t) + \beta E_t(u(C_{t+1}))) \\ &s.t \quad C_{t+1} = X_{t+1} + (X_t - C_t)(1 + R_{t+1}) \end{aligned} \tag{2.8}$$

where t and t+1 denote the current period and future period respectively, $u(C_t)$ is the utility function of an individual's current consumption C_t, X_t is the individual's current resource, $u(C_{t+1})$ is the utility function of future consumption C_{t+1}, X_{t+1} is the individual's future stochastic resource, β is the subjective discount factor and R_{t+1} is the future stochastic asset return.

The CCPAM can be expressed as,

$$p_t = E_t[\beta \frac{u'(c_{t+1})}{u'(c_t)} X_{t+1}] \tag{2.9}$$

where p_t denotes the stock price at time t . The equity risk premium required by the individual is:

$$E_t\left[r_{t+1} - r_{f,t+1}\right] = -(1 + r_{f,t+1})\operatorname{cov}_t(s_{t+1}, r_{t+1}) \tag{2.10}$$

where $s_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor, also called the inter-temporal marginal rate of substitution, $r_{f,t+1}$ denotes the risk free return at time t+1 and r_{t+1} is the asset real return at time t+1.

Equation (2.10) suggests that $\operatorname{cov}(s_{t+1}, r_{t+1})$ is crucial in determining equity risk premia. This reveals an important link between individuals' inter-temporal consumption preferences and the performance of financial assets. If business conditions are good (bad), typically when the future consumption C_{t+1} is high (low) compared to current consumption C_t, the stochastic discount factor s_{t+1} is small (large) and therefore risk-averse individuals will accept lower (higher) equity risk premia. In this case, equity risk premia may change over time as business conditions change. More precisely, they change counter-cyclically and tend to be higher during recessions than during expansions.

2.5 Cyclical behaviour of equity risk premia

2.5.1 Cyclical behaviour in stock markets

The CCAPM assumes that individuals can inter-temporally smooth their consumption by trading financial assets. This provides theoretical evidence that equity risk premia change with business conditions. Moreover, observed stock market behaviour is consistent with the idea of cyclical variation in equity risk premia. Stock markets are not static and often show cyclical behaviour. For instance, they may switch from time to time between a low-return state and a high-return state, often called a 'bear market' or a 'bull market' respectively. These two market states occur because economic patterns change. Consequently, investors need to be aware of the pattern changes and set a reasonable trading strategy to maximise their profits. It is very important for investors to identify whether a market is in a 'bull' or a 'bear' state because a bull market always presents a great opportunity for making money, while in a bear market the opposite is the case. The bear market and bull market are also called the primary market trends. Market trends reflect the general financial market movements over time. The stock market trends are classified into three major types according to their duration: primary trends, secular trends and secondary trends.

Primary trends (bear and bull markets): Primary trends are defined as the bull market and the bear market. These trends typically last between several months and a few years. A bull market is an upward trend in the stock market. Bull markets tend to be associated with a rising price or the expectation that the price will continue to rise. At such times, investors' sentiment is bullish and they are often motivated to buy. A bear market is the opposite of a bull market. It is a downward trend in the stock market and is always associated with falling prices or the expectation of future price drops. In a bear market, investors' sentiment is bearish and they are often motivated to sell. Stock market history exhibits a number of well-known primary trends. The longest lasting bull market was perhaps the bull market of the 1990s. It started in 1991 and ended in 2000 with the bursting of the technology bubble. The most famous bear market was the Great Depression. It followed the Wall Street Crash of 1929; between 1929 and 1932, the Dow Jones industrial average lost about 89% of its value.

Secular market trends (bullish and bearish markets): A secular market trend is a long-term trend and consists of a sequence of primary cycles. It usually lasts from five to twenty-five years. The secular market trend is usually either bullish or bearish. A secular bull market is one in which the prevailing trend is bullish; a secular bear market is one in which the prevailing trend is bearish. The United States stock market was in a secular bullish state between 1983 and 2007, even though the markets suffered from the crash of 1987 and the dot-com bust of 2000-2002.

Secondary Market Trends (bull corrections and bear rallies): A secondary trend is a temporary trend within a primary trend and lasts for a couple of weeks or a few months only. It is either a (bull) correction or a bear market rally. A correction is a short-term price decrease at the time of an overall bull market and a bear market rally is a short-term price increase at the time of an overall bear market. An example of a bear market rally occurred in the Dow Jones index after the stock market crash in 1929.

2.5.2 Equity risk premia and business cycles

Many researchers have investigated the dynamic connection between stock markets and business cycles. Recent empirical work has found evidence that the movements in equity risk premia and in business cycles are intertwined; see, for example, Fama (1981), Fischer and Merton (1984), Barro (1990) and Campbell and Cochrane (1999). Their results suggest that equity risk premia precede business cycles and anticipate future changes in economic activity. There are several possible explanations for this result. One is that the volatility of stock markets is higher during recessions than during expansions (Schwert (1989) and Hamilton and Lin (1996)). Higher stock market volatility during recessions raises the fundamental risk of holding stocks. Consequently, investors will expect higher returns to compensate for this higher risk. In an economic recession, this results in a higher expected return.

An alternative explanation for the cyclical equity risk premium is that an investor’s risk aversion changes with different business conditions. Brandt and Wang (2003) link this time varying risk aversion to business cycles. As explained in Section 2.3, under the information asymmetry hypothesis, a shock to the underlying economy may result in an unexpected change in the aggregate level of risk aversion which may consequently affect the size of future equity risk premia. Campbell and Cochrane (1999) used habit model to explain cyclical variation in investors' risk aversion. In their model, the falls in consumption push investors closer to their habit level and therefore investors' risk aversion is high, and vice versa. Moreover, investors' risk aversion is based on market sentiment. Market sentiment represents a common belief about future market movements. Such beliefs will affect investors' expectations about returns and hence stock prices. In general, stock markets in recessions are dominated by pessimistic investors and in expansions by optimistic investors. This implies that investors have to bear a higher risk of holding stocks in bad times than in good times. Such extra risk, in addition to fundamental risk, is called sentiment risk. It also varies cyclically. Consequently, investors may expect more equity risk premia in bad times than in good times.

The above findings show that the level of risk in stock markets is not likely to be constant because of changes in investors' risk aversion, changes in market sentiments, changes in market volatility and more importantly changes in underlying business conditions. Equity risk premia represents the compensation to investors who take the risky investments in comparison to the risk-free assets. They are the price of the risk. A higher-risk investment, thus, should be associated with a larger equity risk premium. In this case, it is reasonable to assume that equity risk premia also change over time. See for example, Campbell and Cochrane (1999), Fama and French (1989) and Lettau and Ludvigson (2001).

2.6 Conclusions

The efficient market hypothesis suggests that the equity risk premium is simply a constant. The rational investor assumption and the informational efficiency imply that there is no way to beat the market and no one has an advantage in predicting future stock markets. The CCAPM ties equity risk premia to business conditions and suggests that equity risk premia change cyclically. Behavioural finance theory improves this by introducing the possibility that some investors may think irrationally. Moreover, it points out that investors make their consumption-investment portfolio decisions on the basis of both facts and feelings. Therefore, investors when making investment decisions face uncertainty from both fundamental risk and sentiment risk. Both these risks change cyclically. Equity risk premia are compensation for investors who take the risky investment in preference to the risk-free asset. The conclusion emerges that equity risk premia change cyclically.

The fact that equity risk premia change with changing business conditions casts doubt on the belief that mixing good and bad times together to calculate the historical average. For these reasons, investors should not ignore business conditions even when making their long-term asset allocation decisions. Consequently, developing estimates of equity risk premia associated with cyclical variation should be considered. To this end, we identify four main objectives. The first is to ascertain whether equity risk premia are stationary over time, the second is to examine whether the underlying predictive structure of equity risk premium models changes over time, the third is to model the switching behaviour of equity risk premia and the fourth is to test whether the non-linear specifications associated with regime switching models make them superior to a linear model.

Appendix 2 Figures

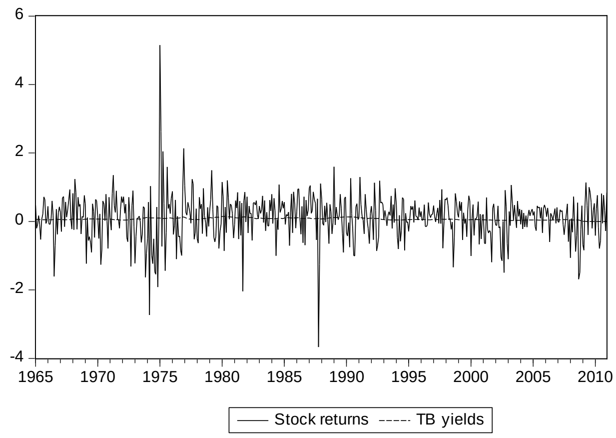


Figure 2. 1 Annualised UK monthly stock returns and Treasury bill yields

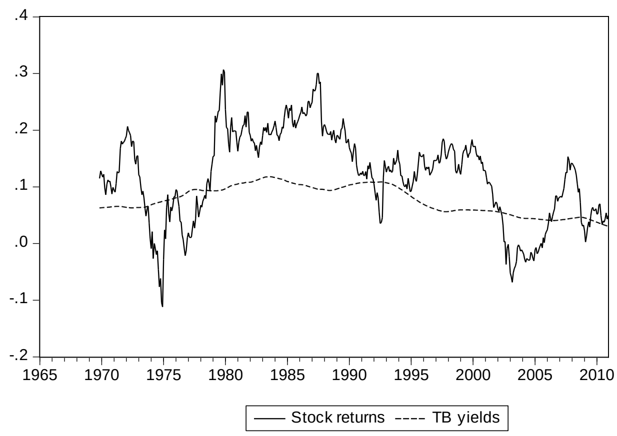


Figure 2. 2 Five-year moving average of UK monthly stock returns and TB yields

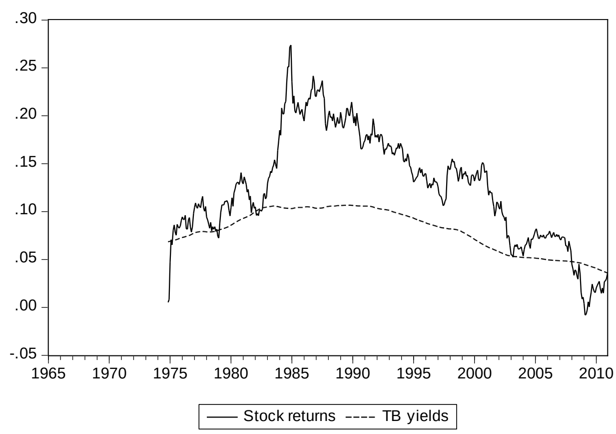


Figure 2. 3 Ten-year moving average of UK monthly stock returns and TB yields

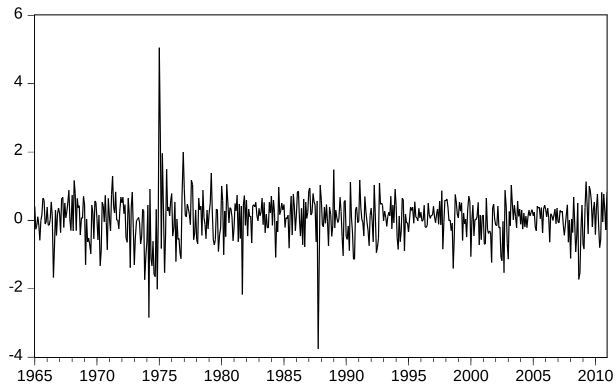


Figure 2. 4 Annualised UK monthly equity risk premia

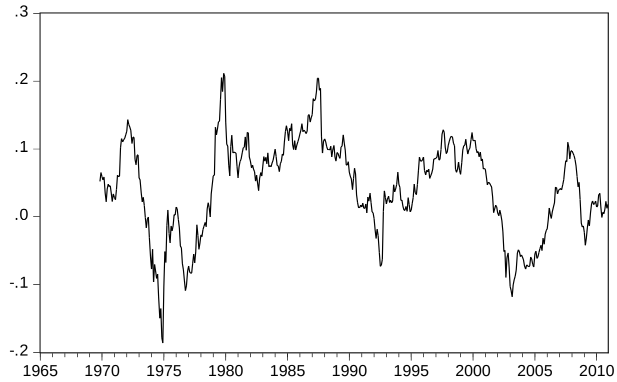


Figure 2. 5 Five-year moving average of UK equity risk premia

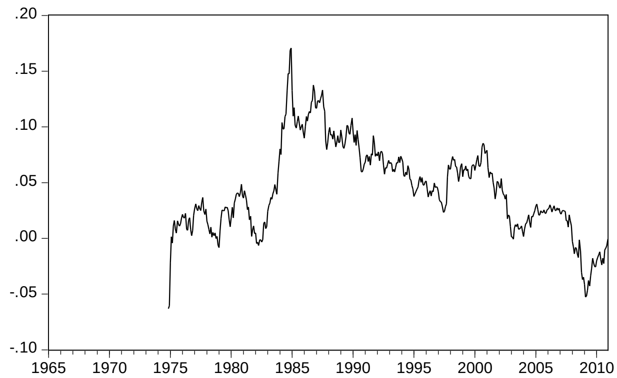


Figure 2. 6 Ten-year moving average of UK equity risk premia

Chapter 3 Structural Breaks in the UK equity risk premium: 1965-2012

3.1 Introduction

This chapter considers the possibility of structural breaks in the parameters of equity risk premium models. A structural break occurs if at least one parameter of the equity risk premium model changes at some date in the sample period. The failure to model this structural break can lead to huge forecasting errors and models becoming unreliable. The aim of this chapter is to review methods for the estimation, testing and computation of equity risk premium models which allow for structural breaks.

In recent years, great interest has been shown in testing for structural breaks in models of equity risk premia based on linear regression. In traditional approaches, it is assumed that equity risk premia are drawn from one stable distribution for the whole sample period. This relies on the *weak stationarity assumptions* that the same regression model can be applied to all observations over time. However, dynamic financial time series may deviate from these assumptions. The stock market may experience sudden crashes or upsurges due to unexpected events such as financial panics, technological shocks or changes in government policy. Although the chance of such an event is very small, it may significantly change expected (equilibrium) equity risk premia. If an event can persistently change the level of the data, it can be defined as a structural break. Because structural breaks may occur during the sample period, the probability distribution of equity risk premia depends on the sample period selected and may change over time. In this case, the traditional assumption of identically and independently distributed (IID) errors may fail to describe the behaviour of equity risk premia and is unlikely to generate a reliable forecast for equity risk premia.

Recognising the problem associated with estimating the cyclical behaviour of equity risk premia, we need to consider the possibility of structural breaks in the parameters of equity risk premium models. The key question is to find out whether structural breaks are present in the sample period. The null hypothesis is straightforward. It is to test the hypothesis that some, or all, of the parameters are stable across the sample period. The rest of this chapter is structured as follows. Section 2 is a literature review on estimating and testing equity risk premium models involving structural breaks. Section 3 discusses commonly used hypothesis tests for structural breaks with both known and unknown break dates. Both univariate and multivariate time series models are employed. The unit root tests in the presence of structural breaks can be applied to test the stationary of financial time series data. They are: the Perron (1989) test, the Zivot and Andrews (1992) test, the Perron (1997) test, the Lumsdaine and Pappell (1997) test and the Lee and Strazicich (2003) test. Structural break tests based on multivariate predictive regression models are used to investigate whether the long-run relationships between financial time series are stable over time. They are: the Chow (1960) test, the CUSUM and the CUSUMSQ tests of Brown, Durbin and Evans (1975), the Quandt-Andrews (1994) test and the Bai-Perron (1997) test. Section 4 presents some empirical applications involving the UK stock market. Section 5 summarises and concludes.

3.2 Background and Literature Review

With the growing interest in the development of asset pricing models and financial econometrics, the estimation of equity risk premia has been widely studied for over two decades. Equity risk premia, as noted above, compensate investors who prefer risky investment to buying a risk-free asset. Chapter 2 reviews the theoretical evidence for time-varying equity risk premia and suggests that they change cyclically. In this chapter, the possibility of structural breaks in the parameters of equity risk premium models is considered. We first focus on the question of whether structural breaks may occur in the equity risk premium at long horizons.

3.2.1 Why structural breaks?

Traditional approaches assume that equity risk premia are drawn from one stable distribution for the whole sample periods. Given that estimation samples often span long periods of time, this assumption ignores the fact that financial time series may change over time. Recent empirical studies have found evidence of a drop in equity risk premia. Dimson, Marsh and

Staunton (2002, 2006) calculated the average equity risk premium for 16 different countries over more than 100 years and suggested that a general decline occurred in the long-run equity risk premium. Jagannathan, McGrattan and Scherbina (2001) reported that the average excess return on the U.S. stock market was 8.9% in the 1950s and 3.98% in the 1990s.

As mentioned in Chapter 2, equity risk premia are additional returns that investors demand for taking the risky investment compared to the risk-free asset. They represent the risk for holding equities and therefore are price of risk. When equity risk premia go down, investors will ask for a lower price of risk and are therefore willing to pay higher prices for the same unit of risky assets that they choose to buy. In this case, equity risk premia should reflect not only the overall macroeconomic risk in the market but also investors' average concern about the risk involved in the investment. Building on this idea, there are two main reasons for suspecting that the underlying data generating process of equity risk premia may experience persistent changes. The first reason is that changes in macroeconomic risk may result in changes in the long-run equity risk premium. The volatility of the overall economy is important for the level of equity risk premia because economic theory implies that higher risk should be associated with higher equity risk premia. Lettau, Ludwigson and Wachter (2008) investigated the relationship between the long-run equity risk premium and the volatility of such underlying macroeconomic factors as employment, consumption and GDP growth and suggested that a fall in the volatility of the real economy could be used to explain the decline in the equity risk premium in the 1990s. Moreover, the global financial environment is a dynamic, complex and evolving system. It is far from static. Government policy, globalization, technology innovation, new forms of market organization and international financial liberalization may also result in structural changes in the financial systems and therefore in the risk of holding stocks. For example, Lucas (1976) suggested that the decisions of economic agents are affected by a change in government policy. For this reason, he concludes that any change in the policy might change the economic model. Indeed, government interventions in financial markets have been expanded for the sake of financial stability. For example, in the credit crisis of 2008, various fiscal and monetary policies have been adopted to reduce the high risk aversion in financial markets and support the financial sector in rescuing the supply of credit in the economy. This enhanced stability of the stock markets may reduce long-run risks and affect equity risk premia permanently.

The second reason for suspecting that the underlying structure in the equity risk premium may experience persistent changes is the idea of time-varying risk aversion. The changes in the aggregate level of risk aversion in stock markets can also explain the decline in the long-run equity risk premium. The traditional assumption of constant risk aversion is inconsistent with the idea of behavioural finance. It imposes a behaviour restriction on the elasticity of inter-temporal substitution and therefore may limit its ability to fit the data. Many empirical studies suggest that investors have become less risk-averse over the last twenty years. Heaton and Lucas (1999) suggest that investors' aversion to risk has significantly declined and the associated equity risk premium required by investors has declined accordingly. In particular, they attribute this shift to the increased participation rates of stock markets, which reduce the stock market risk by spreading it over a wider population. The 2007 Survey of Consumer Finances (SCF) reported that the stock market participation rates, as measured by the proportion of householders either directly or indirectly holding stocks, had risen from 23% in 1983 to about 51.7% in 2007. In fact, there are many variables that may affect the aggregate level of risk aversion, such as stock market sentiments, investor demographics, the information available to investors and the preferences for the current and future consumption. In addition, changes in the nature of investors may be permanent and dramatic. Each new generation experiences a unique political, economic, social and cultural environment which affects its attitude to financial risk. For example, Malmendier and Nagel (2009) pointed out that the risk aversion of the Great Depression generation was quite different from that of their children who had not experienced the Depression. In this case, it is possible that investors' risk aversion may be subject to permanent changes over time.

To conclude, in such a changeable economic environment, both macroeconomic risk and investors' risk aversion may experience permanent changes. Therefore, it is reasonable to assume that the underlying structure in the long-run equity risk premium may change. Before assessing the impacts of structural breaks on equity risk premium models, we review in turn some general approaches which can be employed to estimate equity risk premia.

3.2.2 Estimating equity risk premia

There is an increasing interest in the financial literature on the predictability of equity risk premia. Cochrane (1999) points out that the phenomena that equity returns, in particular, equity risk premia are predictable are 'new facts in finance'. We review two approaches which can be employed to estimate equity risk premia: the historical average equity risk premium and the predicative regression based on macroeconomic variables. In addition, we consider the issues raised by estimating the equity risk premium associated with structural breaks.

a) Historical average approach

According to Ibbotson Associates (2006), to estimate the future equity risk premium, the standard approach is to use the historical average realised excess return. To use this approach, a long historical period is required. This is because of the nature of stock markets. First, stock returns tend to slowly revert to their means in the long run, despite the short-term momentum, see Fama and French (1988a, 1988b), Poterba and Summers (1988), and Bekaert and Hodrick (1992). Given these findings, a period of abnormal high returns tends to be followed by a period of low returns. Second, stock markets tend to be very volatile. Some good years will have great returns and some bad years are likely to suffer substantial losses. Such vast fluctuations in stock markets can be confirmed by estimates of the standard deviation. Damodaran (2009), Cornell (1999) and Goetzmann and Ibbotson (2006) pointed out that the standard deviation for the US annual equity risk premium was about 21% and changes significantly according to the time interval chosen. In particular, they conclude that the standard deviation increases as the time interval decreases. In this case, a long run time series which includes good times as well as bad times is required in order to get a reasonable standard deviation and some degree of confidence in the estimates.

However, there are several disagreements that need to be addressed regarding the accuracy of the historical average equity risk premium when long horizon data are employed. First, estimates of historical averages are critically dependent on the choice of sample period and may therefore have poor out-sample performance. Researchers use historical average realised excess returns to estimate future equity risk premia. The basic assumption made in this approach is that the future will be like the past. In other words, this approach may be reasonable if returns are drawn from one stable distribution for the whole sample period. However, historical data are volatile. The estimated value of equity risk premia using historical average excess returns depends on the sample period used and may have *selection bias*. In the literature, there are many empirical results suggesting that the historical average may overestimate future expected equity risk premia. Mehra and Prescott (1985) showed that the realized return on equities in the US stock market over the Treasure-bill rate averaged about 8% per annum with a standard deviation of 16% over the period 1889-1979. They suggested that the historical average excess return was too high to be consistent with *Consumption Capital Asset Pricing Models* (CCAPM). Their results are known as the *Equity risk premium Puzzle*. Blanchard (1993) calculated the stock and bond rate for the United Kingdom, United States, German, French, Italian and Japanese markets over the period 1978-1992. He concluded that the post-1980 average equity premium was lower than the pre-1980 one. Claus and Thomas(2001) and Arnott and Bernstein (2002) suggested that future expected returns for the early 21st century were lower than past realised returns. Brown, Goetzmann and Ross (1995) pointed out that the US expected stock return was over-estimated since the market had been "lucky" over the previous 50 years. This upward bias in the sample is so called the *survivorship bias* hypothesis.

Second, the accuracy of the estimates depends on the measurement error in the observations. The standard deviation is a measure of the dispersion of a variable from its mean. It tells us how much inaccuracy there is between the historical equity risk premium and the expected realised equity risk premium. Since traditional approaches assume that equity risk premia are stable over time, the standard deviation is simply a reflection of volatility rather than of structural breaks. A standard deviation of 21% per year suggests that the measurement error in the historical data is large. This implies that equity risk premia are so changeable that history may fail to provide an accurate estimate of what next year's risk premium will be.

Third, and most importantly, stock markets may experience sudden crashes or upsurges due to unexpected events. A stock market crash is a rapid and dramatic downturn in stock prices. It may change the stock market significantly and permanently. For example, the Great Crash of 1929 and the Oil Price Shock of 1973 had permanent effects on most economic time series. In particular, no significant events or other news were reported just before the crashes or at the time that could have explained them. The crashes happened suddenly and randomly. Another example which may illustrate the persistent impact of extreme events on stock markets is the Japanese stock market crash of 1989. This crash involved a real asset bubble as well as a stock bubble. It persisted for two years and thereafter the market stagnated at a low level for more than a decade. In 2010, the Japanese market was still about 75% below its 1989 peak. Many investors have suffered significant losses, which may be impossible to recuperate in their lifetime. Moreover, there was also a big drop in confidence. The number of Japanese institutional investors who thought just before the crash that there would not be a crash to come was 90%; just afterwards, it was 30%. The foregoing examples provide evidence that stock markets may not be constant, because extreme unexpected abnormal fluctuations may significantly change them. Although the chance of such an event is very small, it may make the earlier data less relevant to today's market. In the financial literature, this phenomenon is also known as a *peso problem*. The fact that stock market may suffer from periodically catastrophic drawdowns implies that the historical average is an over-estimate of the true equity risk premium (Rietz, 1988; Schwert, 1989 and Brown, Goetzmann and Ross, 1995).

Many econometricians have investigated the effects of structural breaks on the estimation of equity risk premium models. Rietz (1988) found that the historical excess return was inconsistent with rational investor expectation, since it failed to account for possible structural breaks, such as economic disasters. Cornell (1999) emphasised the impact of permanent changes in equity risk premia on stock prices. He pointed out that a permanent decline in equity risk premia was associated with an increase in stock prices. He also concluded that historical average excess returns may lead to a double overestimation if there is a permanent decline in the equity risk premium. Pastor and Stambaugh (2001) recognised the importance of structural breaks in models of equity risk premia. If structural breaks are important, then issues such as the equity risk premium puzzle, selection bias, survivorship bias and perso problem may not be puzzles at all.

To conclude, the historical average excess return provides a perspective upon future equity risk premia. However, these simple approaches may not be robust in view of changes in equity risk premia when structural breaks occur during the sample period. Accuracy problems caused by structural breaks suggest that using the historical average for equity risk premia may result in biased estimates. In this regard, the traditional assumption of an identically and independently distributed error may be inappropriate when describing the behaviour of equity risk premiums when long horizon data are employed.

b) Predictive regression models based on macroeconomic variables

Another common way to predict the expected equity risk premium uses published information, such as prior information or other fundamental values. Stock returns tend to continue for a short period of time but in the long run revert to the mean. Fama and French (1988) saw two kinds of group stock market volatility: short-term and long-term components (see Lo and MacKinlay, 1988). On the one hand, equity risk premia are positively correlated because the risk in stock markets is likely to be persistent at short horizons. Jegadeesh and Titman (1993, 2001) suggested that current stock market performance is positively related to future performance at short intervals. Such a short-run momentum pattern is also supported by the findings of behavioural economists. Liu, Strong and Xu (1999) reported the existence of momentum in the UK stock markets and attribute it to the fact that investors tend to under-react to new information. The literature on the predictability of stock returns from past returns includes contributions by Poterba and Summers (1986, 1988), Rosenberg, Reid and Lanstein (1985), De Bandt and Thaler (1985), Fama and French (1986), Jegadeesh (1990), Lo and MacKinlay (1990a), Lehmann (1990)， French and Roll (1986), Cutler, Poterba and Summers (1990) and Jegadeesh and Titman (1993). Fama and French (1988) and Poterba and Summers (1986) pointed out that equity risk premia are auto-correlated. All these studies document the fact that future stock market performance can be forecast from past data.

On the other hand, the mean-reversion process suggests that equity risk premia tend to revert to their fundamental values in the long run (see Poterba and Summers (1988); Fama and French (1988); Siegel (1999) and Campbell (1991)). Financial econometricians believe that valuation ratios can be used as predictors of subsequent returns. This implies that the stock price should be based on fundamental values, such as the price-dividend ratio, the price-earnings ratio, the price-cash flow ratio and the book-market ratio. Basu (1977, 1983), Campbell and Shiller (1988a, 1988b) and Lamont (1998) found that stocks with higher (lower) price-earnings ratios delivered lower (higher) returns. Fama and Schwert (1977), Keim and Stambaugh (1986), Campbell (1987), Fama and French (1989) and Lewellen (2004) also reported that yield spreads on short- and long-term treasury bills and corporate bonds had good predictive power for subsequent stock returns. Rozeff (1984), Fama and French (1988) and Campbell and Shiller (1988a, 1988b) pointed out that valuation ratios, in particular, the price-dividend ratio, were negatively correlated with future long horizon returns. Such a negative correlation between stock returns and valuation ratios can be thought of as a mean-reversion of stock returns towards their fundamental values. Poterba and Summers (1988) proved that even when the stock price temporarily swung away from its market fundamentals, in the long run the negative correlation between stock returns and valuation ratios would move the stock price back to its fundamental values. Graham and Dodd (1934) believe that the valuation ratio is a key determinant of future equity risk premia. This is because that the high rate of valuation ratio implies that the stock market is undervalued, which should predict high subsequent returns.

The above studies suggest that equity risk premia could be predicted more precisely by regressing them on financial variables. However, the use of lagged financial variables to predict equity risk premia is controversial. In the 1980s and 1990s, both financial econometricians and financial historians showed that the apparent predictability of equity risk premia might be spurious. First, the view that the stock market return was predictable from valuation ratio was challenged by Shiller (1981) and LeRoy and Porter (1981). They point out that stock returns are not always at their long-run fundamental values and stock market movements are too volatile compared to their base values. Second, the predictability of equity risk premia using value ratios, such as dividend price ratios, is poor at long horizons. A large number of studies have addressed the discrepancy between the strong in-sample predictability and the weak out-of-sample predictability (Goyal and Welch (2003), Bossaerts and Hillion (1999), Butler, Grullon and Weston (2005) and Campbell and Thompson (2005)). This poor out-of-sample performance of financial variables may suggest parameter instability in the equity risk premium models. Finally, the presence of fat-tail distributions can be thought of as a mixture of two different normal distributions and therefore may be caused by unanticipated structural break; see the example of the jump diffusion model in Merton (1976).

One possible explanation for the discrepancy between the strong in-sample predictability and the weak out-of-sample performance is the fact that equity risk premia and business conditions move together. As discussed in Chapter 2, mixing bad times and good times together to estimate equity risk premia does not accurately describe the cyclical behaviour of future equity risk premia. In particular, Pesaran and Timmermann (2002) attributed the disappearance of stock return predictability to the existence of structural breaks or parameter instability. They suggest that the signs of instability in equity risk premium predictive models provide evidence that structural breaks may be present. Indeed, in such a changeable economic environment, it is not unreasonable to assume that the underlying data generating process of long-run equity risk premia may change.

The above empirical studies show that structural breaks have important implications for estimating models of equity risk premia including both the historical average approach and the predicative regression based on macroeconomic variables. The failure to model possible structural breaks correctly can lead to huge forecasting errors and unreliable models. Anomalies, such as excess stock volatility, poor out-of-sample performance, parameter instability, fat tails in the distribution, the equity risk premium puzzle and survival bias, may be simply explained by the existence of structural breaks in the underlying models. Therefore, a simple OLS approach without considering the possibility of structural breaks may not find predictability in equity risk premia, even if it exists. In this regard, we propose in this chapter to consider the question of structural break tests in estimating equity risk premia. In the following section, we review the recent developments in the relevant structural break techniques.

3.2.3 Structural breaks or outliers?

We first study the difference between structural breaks and outliers. Dynamic financial time series may deviate from the *weak stationarity assumptions*. One possible reason is that the stock market may experience sudden crashes or upsurges. Such unexpected fluctuations in the market can be due to outliers or structural breaks.

In a statistical context, an outlier is an observation that is extremely ‘high’ or ‘low’ compared with the rest of the data. When it comes to model estimation, these outliners may not only result in large residuals but also significantly change the estimation of the parameters. If an outlier can persistently change the level of the data over a long period of time, it can be defined as a structural break. Although the effect on the estimation of models is similar to that of outliers, structural breaks are different from outliers. The major difference between these two is that outliers often involve only one or a few observations, while structural breaks tend to have persistent effects on the estimated model. In other words, outliers may have only short-run effects on the level of data, while structural breaks always have long-run effects.

In the financial literature, traditionally, most financial econometricians believed that shocks had only a temporary effect on the long-run movement of financial time series and fluctuations were only temporary deviations from the long-term trend. More precisely, they thought that time series were trend stationary and could expect to return to their steady trend after a shock. Nelson and Plosser (1982) cast doubt on this view. They pointed out that shocks had a permanent effect on the long-run movement in almost all time series. Their results have been confirmed by Campbell and Mankiw (1987), Clark (1987), Cochrane (1988), Shapiro and Watson (1988), and Christiano and Eichenbaum (1989). They suggested that shocks might have temporary or permanent effects on the long-run level of a series. They also suggested that the long-run response of a series to a shock depends on its relative size and whether it is permanent. Perron (1989) re-examined the Nelson and Plosser (1982) data series and emphasised the possibility of the permanent impact of shocks on macroeconomic variables. He suggested that the Great Crash 1929 and the Oil Price Shock 1973 had a permanent effect on the economic time series. Clements and Hendry (1998) pointed out that the existence of structural breaks generated by larger shocks might be the most significant cause for the misfitted macroeconomic models. It is therefore advisable, to find out whether there are structural breaks in our sample when the long-run time series are considered.

3.2.4 Break-stationary or unit roots?

We then briefly review the literature on the unit root tests in the presence of structural breaks.

A recent development raised in economics is to be able to distinguish the unit root null from the break-stationary alternatives. This is of particular importance because failure to model possible structural breaks can bias the result of unit root tests in favour of the non-rejection of the null hypothesis of a unit root.

Testing for a unit root in time series regression is important for practical applications, for two reasons. First, the presence or the absence of a unit root is important for assessing the underlying data generating process of a time series. If a time series is non-stationary, it follows a random walk and has no tendency to convert to its equilibrium path, even in the long run. Second, the OLS method assumes that time series are stationary. If macroeconomic variables are non-stationary, the OLS estimates are biased and the test statistics diverge to infinity as the sample size increases. This phenomenon is called spurious regression. Therefore, we should check for the stationarity of each time series before building the model.

The standard approach to test the stationary of a univariate series is to use unit root tests, such as: the Dickey-Fuller (1979), the Augmented Dickey-Fuller (1981) and the Phillips-Perron (1988) tests. For the Dickey-Fuller (1979) and the Augmented Dickey-Fuller (1981) tests, they test the null hypothesis of a unit root against the alterative of being stationary. For the Phillips-Perron (1988) test, it tests the null hypothesis that a time series is stationary. However, an important issue of such tests is that they ignore the possibility of structural breaks in the data. Perron (1989) developed a modified Augmented Dickey-Fuller unit root test which allowed for an exogenously determined structural break under both the null and alterative hypotheses. He showed that the presence of structural breaks can lead to unit root test results biased towards the non-rejection of the null hypothesis of unit roots. He also suggested that the existence of a unit root in a financial time series might better be described as the existence of structural breaks. However, the assumption of known exogenous breaks has two flaws. First, Christiano (1992) argued that the unit root tests with exogenously determined break points might result in an over-rejection of the unit root hypothesis. Second, structural breaks may happen gradually and many may not be easily observable. It may be very difficult for researchers to find the correct break date for a long run of data. Instead of assuming an exogenously determined break, Zivot and Andrews (1992) allowed the break date to be determined endogenously. They performed the ADF unit root test for every possible observation, and selected the break date which yields the minimum *t*-statistic (cf. Banerjee, Lumsdaine and Stock, 1992; Perron, 1997 and Lumsdaine and Papell, 1997). Moreover, Lumsdaine and Papell (1997) pointed out that the existence of multiple structural breaks may result in the non-rejection of the unit root hypothesis. They extended the Zivot-Andrews (1992) one-break unit root test to allow for two structural breaks. However, the tests based on the Zivot-Andrews (1992) test and the Lumsdaine and Pappell (1997) test may still suffer from the problems of low power and size distortion in the presence of structural breaks. The main issue for these tests is that they assume no break under the null. Moreover, the unknown structural break points are nuisance parameters because they are not identified under the null hypothesis. As a result, the distribution of the test *t*-statistics is unreliable because of the presence of nuisance parameters. Therefore, Lee and Strazicich (2003) argue that the ability to reject such a null hypothesis does not necessarily suggest the absence of unit roots. They solve this problem by allowing for endogenous breaks, both under the null and the alternative hypotheses. Recognition of the effects of structural breaks on a financial time series provides us with a warning signal regarding the use of OLS. Moreover, even if the time series is stationary, the fact that equity risk premia change with different business conditions still raises the question whether the long-run relationships between financial time series change over time. In the next section, we review tests for structural breaks in the regression models.

3.2.5 Structural breaks in regression models

Finally, we review the issues involved in structural break tests based on regression models. Structural break tests can be applied to examine whether the parameters of the model are constant over time. In recent years, there has been an increasing interest in the task of identifying and modelling structural breaks within financial data. Many methods have been found to detect possible structural breaks and the dates of these breaks. Simple graphical methods can be used to test for the existence of structural changes; for example, a historical data plot, or an autocorrelation function plot. Clear breaks may be observable using graphical methods; otherwise, formal significance tests for the instability of parameters and for structural breaks can also be applied.

The Chow (1960) test can be applied to test for the presence of a structural break in the long-run equity risk premium model on a specific date. However, the date of the structural break is difficult to ascertain in advance and the change to a new regime may take place slowly and less obviously. The simple Chow test may fail to determine whether there are any structural changes during the sample period with unobservable switching points.

To estimate an endogenously determined switching point, Andrews (1993) and Quandt (1960) performed the Chow breakpoint test for every possible observation, and selected the break date which yielded the largest F-statistic. Quandt (1960) pointed out that the standard χ^2 critical values cannot apply if the break point is unknown, see also Andrews and Ploberger (1994). They all noted that the unknown structural break point was a nuisance parameter because it was not identified under the null hypothesis. The statistical difficulty for such a test involving nuisance parameters is that the sampling distribution of the structural break parameters has nonstandard distributions. Andrews (1993) derived the asymptotic distribution for the Sup Wald, likelihood ratio (LR) and Lagrange Multiplier (LM) statistics. Critical values (p-values) were suggested by Hansen (1997). Andrews and Ploberger (1994) took the average exponential of the Chow test sequence to improve the local power.

The above econometric literature is focused on to the case of a single break. However, multiple structural breaks may exist in many economic time series. For multiple unknown structural breaks, Brown, Durbin, and Evans (1975) proposed the standard CUSUM and CUSUMSQ tests to ascertain if there had been any structural breaks during the sample period. The CUSUM and CUSUMSQ tests are a step in the right direction for finding unknown structural break dates, but they have serious power problems (Andrews, 1993). Ploberger Kramer, and Kontrus (1989) used the local power functions of the CUSUM and CUSUMSQ tests and suggested that the CUSUM test for structural breaks has non-trivial local asymptotic power unless all changes are orthogonal to the mean regressor. Deshayes and Picard (1986) pointed out that the drawback of the CUSUMSQ test was that it had only trivial local power for the local changes which specify a one-time change in parameters.

Bai and Perron (1997) took an important step in developing a test for multiple structural breaks in linear models. In their method, the break dates are treated as unknown variables which enter in a non-linear form and are estimated simultaneously with the linear model by non-linear least squares. They also derive the asymptotic consistency, the rate of convergence and the limiting distribution for the estimators. The upper double maximum (UDMAX) and weighted double maximum (WDMAX) statistics are used to detect whether at least one break exists. A selection procedure based on a sequence algorithm is used to estimate the number of break dates in the sample.

In order to understand the importance of structural breaks in equity risk premium models, in the next section we introduce some common methods which can be used to test for possible structural breaks in the parameters of the equity risk premium model.

3.3 Structural break tests

This section reviews methods related to estimation and inference about structural changes. Both univariate and multivariate time series models are employed. The unit root tests with structural breaks can be applied to test the stationary of financial time series data, while structural break tests based on multivariate predictive regression models are used to investigate whether the long-run relationships between financial time series are stable over time.

3.3.1 Unit root tests with structural breaks

We focus first on the unit root tests which allow for the possibility of structural breaks in the data. Unit root tests can be applied to examine whether a time series is stationary. Recent research into the unit root tests has exploited developments in structural break tests. One important reason is that most approaches to detecting a unit root impose the null hypothesis of a unit root against the break stationary alternative. The other is that unit root tests which account for the possible structural breaks not only test the unit root null, but also identify the break dates when breaks are presented in the long run of data. Therefore, these tests can provide more information about the dates and characteristics of structural breaks. In particular, they can help us evaluate whether a structural change in financial time series models is related to a particular change in certain variables. Two important issues need to be considered when such tests are employed. The first one is whether the structural breaks are single or multiple. The other is whether these structural breaks are determined exogenously or endogenously. We review these two issues below.

a) Unit root tests with a single structural break

To distinguish the unit root null from one-break stationary alternatives, we introduce three commonly used tests: the Perron (1989) test, the Zivot and Andrews (1992) test and the Perron (1997) test. The Perron (1989) test assumes an exogenous determined structural break, while the Zivot and Andrews (1992) test and the Perrson (1997) test determine the break date endogenously together with the model.

Perron (1989) proposed a method for testing unit roots in the presence of an exogenous determined structural break, that is, the possible break date is fixed a priori. He considered three different specifications of the break under the alternative of stationary: A) a break in the level of a series, B) a break in the slope of a series, C) a break in both the level and the slope of the series.

The models are given below:

$$
\begin{aligned}
&\text{Model A: } \mathrm{y}_t = \alpha_0{}^A + a_1{}^A DU_t(\lambda) + \beta^A t + \rho^A y_{t-1} + \sum_{i=1}^{K} \phi_i{}^A \Delta y_{t-i} + e_t \\
&\text{Model B: } \mathrm{y}_t = \alpha_0{}^B + r^B DT_t(\lambda) + \beta^B t + \rho^B y_{t-1} + \sum_{i=1}^{K} \phi_i{}^B \Delta y_{t-i} + e_t \qquad (3.1) \\
&\text{Model C: } \mathrm{y}_t = \alpha_0{}^C + a_1{}^C DU_t(\lambda) + r^C DT_t(\lambda) + \beta^C t + \rho^C y_{t-1} + \sum_{i=1}^{K} \phi_i{}^C \Delta y_{t-i} + e_t
\end{aligned}
$$

where T_B ($1 < T_B < T$) is the break date, $DU_t(\lambda)$ is a dummy variable which captures a change in the intercept at time T_B and $DU_t(\lambda) = 1$ if $t > T_B$ and 0 otherwise, and $DT_t(\lambda)$ is a dummy variable which indicates a change in the slope at time T_B and $DT_t(\lambda) = t - T_B$ if $t > T_B$ and 0 otherwise. In general, the first and last 15% of observations are excluded from the sample period and the remaining sample is known a trimmed dataset, i.e., $\lambda = T_B/T$, $\lambda \in (0.15, 0.85)$.

Zivot and Andrews (1992) extended the Perron (1989) models to allow for an endogenous determined break, i.e., the break date T_B is endogenously selected by the test. The selection procedure is based on a sequence algorithm. It performs the ADF unit root test for every possible observation, and selects the break date which yielded the minimal *t*-statistic. The null hypothesis H_0 is tested against Models A, B and C for $\rho^i = 1$, $(i = A, B, C)$. The *t*-statistic is defined as:

$$
t_{\hat{\rho}^i} = \inf_{\lambda} t_{\hat{\rho}^i}(\lambda) \qquad (3.2)
$$

Perron (1997) proposed an alternative approach to test for the presence of a unit root with an endogenously determined structural break. He extended the Zivot and Andrews (1992) test by allowing for a structural break under both the null and alternative hypotheses and introduced the Innovational Outlier (IO_1 and IQ_2) models and the Additive Outlier (AO) model. These models are given below:

$$\begin{aligned}
&\mathrm{IO}_1 : \mathrm{y}_t = \alpha_0 + a_1 DU_t + \delta D(T_B)_t + \beta t + \rho y_{t-1} + \sum_{i=1}^{k} \phi_i \Delta y_{t-i} + e_t \\
&\mathrm{IO}_2 : \mathrm{y}_t = \alpha_0 + a_1 DU_t + rDT_t + \delta D(T_B)_t + \beta t + \rho y_{t-1} + \sum_{i=1}^{k} \phi_i \Delta y_{t-i} + e_t \qquad (3.3) \\
&\mathrm{AO}: \mathrm{y}_t = \alpha_0 + \beta t + \delta DT_t^* + \tilde{y}_t,\ \tilde{y}_t = \rho \tilde{y}_{t-1} + \sum_{i=1}^{k} \phi_i \Delta \tilde{y}_{t-i} + e_t
\end{aligned}$$

where DU_t=1 if $t > T_B$ and 0 otherwise, DT_t=t if $t > T_B$ and 0 otherwise, $D(T_B)$=1 if t=T_B+1 and 0 otherwise, and DT_t^*=t-T_B if $t > T_B$ and 0 otherwise. Here, t represents time trend and $D(T_B)$ represents the time at which the change takes place.

The IO1 model allows for an unknown shift in the intercept to take place gradually. The IO2 Model allows for an unknown shift in both the intercept and slope to take place gradually. The AO model allows for a sudden unknown shift in the slope of the trend function and therefore both segments of the trend function can be joined at the break date T_B. The test process is also a sequence algorithm which tests every observation and chooses the break date by selecting the minimum t-statistics from the ADF various tests.

b) Unit root tests with two structural breaks

Lumsdaine and Pappell (1997) pointed out that the existence of multiple structural breaks may reduce the ability of unit root tests to reject the unit root hypothesis (cf. Lumsdaine and Papell, 1997 and Maddala and Kim, 2003). To relax the assumption of a single break, we review two unit root tests which allow for multiple breaks. These two tests are: the two-break unit root test of Lumsdaine and Pappell (1997) and the minimum Lagrange Multiplier (LM) unit root test of Lee and Strazicich (2003).

Lumsdaine and Pappell (1997) applied the unit root tests which allowed for two endogenous structural breaks to re-examine for the presence of unit roots in the Nelson-Plosser (1982) data. They suggest that the unit-root test results are sensitive to the number of structural breaks that are allowed in the tests and that the one-break unit root tests may thus result in a loss of information when multiple breaks occurs. They perform a sequence of ADF tests which allow for two breaks in the deterministic trend at unknown locations. The break points are selected as the values which minimise the unit root *t*-statistic. Model A allows for two shifts in the level of the time series and Model C allows for two shifts in both level and trend. The models are given below:

$$\begin{aligned}&\text{Model A: } y_t = a_0 + \beta t + a_1 DU1_t + a_2 DU2_t + \rho y_{t-1} + \sum_{i=1}^{k}\phi_i \Delta y_{t-i} + e_t\\&\text{Model C: } y_t = a_0 + \beta t + a_1 DU1_t + a_2 DU2_t + \gamma_1 DT1_t + \gamma_2 DT2_t + \rho y_{t-1} + \sum_{i=1}^{k}\phi_i \Delta y_{t-i} + e_t\end{aligned} \quad (3.4)$$

where T_{Bi} ($1<T_{B1}<T_{B2}<T$), $i=1,2$, are the break dates, DUi_t and DTi_t are the mean and

trend shift dummy variables, respectively, $DUi_t=1$ if $t > T_{Bi}$ and 0 otherwise, and $DTi_t=t-T_{Bi}$ if $t > T_{Bi}$ and 0 otherwise.

Lee and Strazicich (2003) proposed a two-break minimum Lagrange Multiplier (LM) unit root test based on the earlier work of Schmidt and Phillips (1992). This test has three advantages. First, it allows for the presence of endogenous breaks under both the null and the alternative hypotheses. An important issue for unit root tests, such as the Zivot and Andrews (1992) and the Lumsdaine and Pappell (1997) tests, is that they assume no break under the null, that is, they test the null hypothesis of a unit root without structural breaks. Lee and Strazicich (2003) argue that the ability to reject such a null hypothesis does not necessarily suggest the absence of a unit root. Second, the Lee and Strazicich (2003) test allows for multiple structural breaks. The Perron (1997) test allows for a break under both the null and alternative hypotheses, but it only allows for a single break. Third, the unknown structural break points are nuisance parameters because they are not identified under the null hypothesis. Therefore, the unit root tests based on the Zivot and Andrews (1992) test may lose power and suffer from the problems of size distortion. Lee and Strazicich (2003) find that the distribution of the LM statistics is independent of the break points. Therefore, the LM statistics are free of nuisance parameters and are robust to any misspecification of the number of breaks. For these reasons, the problem of spurious rejection of the unit root null when there are multiple breaks does not arise for the LM unit root test of Lee and Strazicich

(2003). According to the Lagrange multiplier principle, the test statistic (Lee and Strazicich, 2003) can be obtained from the following regression:

$$\Delta y_t = \delta^{'} \Delta Z_t + \phi \tilde{S}_{t-1} + \mu_t \tag{3.5}$$

where Z_t is a vector of dummy variables, $Z_t = \left[1, t, DU1_t, DU2_t\right]$ in the model A and $Z_t = \left[1, t, DU1_t, DU2_t, DT1_t, DT2_t\right]$ in the model C, $\tilde{S}_t$ is defined as the de-trended series, i.e., residuals, $\tilde{S}_t = y_t - \tilde{\psi}_x - Z_t \tilde{\delta}$, and $\tilde{\psi}_x$ is the restricted maximum likelihood estimation (MLE), $\psi_x = y_1 - z_1 \tilde{\delta}$, y_1 and Z_1 denotes the first observations of y_t and Z_t. The null of the LM unit root test is $\phi = 0$ and the test statistic is given by:

$$\mathrm{LM}_{\tau} = t - \text{statistic for } \phi = 0 \tag{3.6}$$

Critical values are reported in Lee and Strazicich (2003).

3.3.2 Predictive regression models

In order to investigate whether the long-run relationships between equity risk premia and candidate explanatory variables are stable over time, we begin with a standard predictive regression model. In this predictive model, equity risk premia are regressed on the lagged values of the explanatory variables. The form of the simple linear predictive regression is given below.

$$R_t = c + \phi_1 x_{1,t-1} + \phi_2 x_{2,t-1} + \cdots + \phi_k x_{k,t-1} + \varepsilon_t \qquad t = 0, 1, 2, \cdots, T \tag{3.7}$$

where R_t is the equity risk premium from time $t-1$ to time t, c is a constant, $x_{1,t-1}, x_{2,t-1}, \ldots, x_{k,t-1}$ are the candidate explanatory variables observed at time t, k is the number of regressors, $\phi_1, \phi_2, \ldots, \phi_k$ are regression coefficients, ε_t is a disturbance term with mean 0 and variance σ^2, and T is the sample size.

Traditional economics assumes that the parameters of the equity risk premium model are invariant over time. We allow for the existence of structural breaks in the parameters of equity risk premium models. To understand the nature of structural breaks, we suppose that there is a possible structural break on a specific date T_1 in the sample period. In this case, the whole sample period T can be divided into two separate data sets, $t = 1, 2, \cdots, T_1$ and $t = T_1 + 1, T_1 + 2, \cdots, T$ respectively. The above model is nothing more than two separate models. They are:

$$\begin{aligned} R_{1t} &= c_1 + \phi_{11} x_{1,t-1} + \phi_{12} x_{2,t-1} + \ldots \phi_{1k} x_{k,t-1} + \varepsilon_{1t} \qquad t = 1, 2, \cdots, T_1 - 1 \\ R_{2t} &= c_2 + \phi_{21} x_{1,t-1} + \phi_{22} x_{2,t-1} + \ldots \phi_{2k} x_{k,t-1} + \varepsilon_{2t} \qquad t = T_1, T_1 + 1, \cdots, T \end{aligned} \tag{3.8}$$

where c_1 and c_2 are constants, $\{\phi_{11}, \phi_{12}, \ldots, \phi_{1k}\}$ and $\{\phi_{21}, \phi_{22}, \ldots, \phi_{2k}\}$ are regression coefficients, ε_{1t} and ε_{2t} are the disturbance terms with mean 0, variance σ_1^2 and σ_2^2 respectively, and with zero autocorrelation.

To examine the possibility that there is a structural change during the sample period, we can perform stability tests. The null hypothesis of the stability test is straightforward. It is to test the hypothesis that some of or all of parameters remain invariant in different sub-samples. Mathematically,

$$H_0: c_1 = c_2,\ \phi_{11} = \phi_{21},\ \cdots, \phi_{1k} = \phi_{2k}, \sigma_1^2 = \sigma_2^2$$

Dealing with structural breaks raises two issues. The first is whether the date of the break is known. The second is what if there are multiple structural breaks in financial time series. In the following subsections, we briefly review these two issues. Firstly, we consider the case that the date of the structural break is known.

3.3.3 Exogenous structural break tests

The most commonly used tests for structural stability are the Chow (1960) breakpoint and predictive failure tests. The basic idea of these tests is to model the regression separately for each sub-sample and to examine whether the coefficients in each linear regression remain stable for the whole sample period. The Chow tests assume the hypothesised date of the structural break is known. The null hypothesis is defined as:

$$H_0: c_1 = c_2,\ \phi_{11} = \phi_{21},\ \cdots, \phi_{1k} = \phi_{2k}$$

The Chow tests impose restrictions on the residual sum of squares. They are *F* tests and their test statistic is defined as,

$$F_{(r,T-k-1)} = \frac{\frac{(RSS_R - RSS_1 - RSS_2)}{k+1}}{\frac{(RSS_1 + RSS_2)}{T-2(k+1)}} \tag{3.9}$$

where RSS_R is the residual sum of squares for the restricted regression that covers the whole sample period, RSS_1 and RSS_2 are the residual sum of squares for each sub-sample, k is the number of regressors, r is the number of restrictions, and T is the total number of observations. An alternative expression for the Chow test is,

$$F_{(r,T-k-1)} = \frac{\dfrac{(RSS_R - RSS_U)}{k+1}}{\dfrac{RSS_U}{T-2(k+1)}} \tag{3.10}$$

where RSS_U is the residual sum of squares for a unrestricted regression that covers the whole sample period but that allows all the coefficients to shift on a specific date.

Sometimes the number of the observations of the second sub-sample may be too small ($T_2<k$). In this case, Chow proposed the Chow predictive test (1960). The F statistic is defined as,

$$F_{(T_2,T_1-k-1)} = \frac{\dfrac{(RSS_R - RSS_1)}{T_2}}{\dfrac{RSS_1}{T_1-k-1}} \tag{3.11}$$

where $T_1 = T_0$ is the number of observations for the first subsample, and $T_2 = T - T_0$ is the number of observations for the second subsample.

An important assumption made in using the Chow tests is that the hypothesised date of the structural break is known. If the breakpoint is known, the model could be simply described by two different equations, without any problem of estimation and inference. However, the date of the structural break is difficult to ascertain in advance. Structural breaks may happen gradually and many may be difficult to observe. In this situation, researchers can either choose an arbitrary break date or a break date based on some knowledge of the data.

However, different researchers make different assumptions based on their own knowledge. It is very difficult to find the correct break date for a long run of data. If a known break point is assumed, because the estimation of model coefficients is also conditioned on the value of the break point, it may result in an incorrect number of degrees of freedom and an invalid covariance matrix of estimates. Therefore, it is inappropriate to assume a structural break point. The break point should be estimated endogenously. In this case, the Chow tests may lead to misfitted models. Our interest has moved from the traditional tests which assume that the date of the structural break is known to the case where it is not known in advance.

3.3.4 Endogenous structural break tests

Several methods have been developed to identify the possible dates for structural breaks. We will introduce the CUSUM and the CUSUMSQ tests, the Quandt-Andrews structural stability test and the Bai-Perron test in this section.

a) CUSUM and CUSUMSQ tests

To find the possible date of a structural break, we can employ the CUSUM and CUSUMSQ tests of Brown, Durbin and Evans (1975). ‘The null hypothesis is that the coefficient vector is the same in every period; the alternative is simply that it is not’ (Greene 2003). Both the CUSUM and the CUSUMSQ statistics are computed using the cumulative sum of recursive residuals. The CUSUM statistics are defined as,

$$CUSUM_t = \frac{\sum_{i=k+1}^{t} w_i}{s}$$

$$\text{Where, } s^2 = (T-k)^{-1} \sum_{i=k+1}^{T} (w_i - \overline{w})^2 \qquad (3.12)$$

$$\overline{w} = (T-k)^{-1} \sum_{i=k+1}^{T} w_i$$

The CUSUMSQ statistics are defined as,

$$CUSUSQ_t = \frac{\sum_{i=k+1}^{t} w_i^2}{\sum_{i=k+1}^{T} w_i^2}$$

$$\text{Where, } w_t = \frac{\varepsilon_t}{\sqrt{v_t}} \qquad (3.13)$$

$$v_t = 1 + x_t^{'}[\sum_{i=1}^{t-1} x_i x_i^{'}]x_t$$

Here k is the number of explanatory variables excluding the constant, T is the sample size, x_t denotes a set of regressors from period 1 to period t, and w_t is the recursive residual. The advantage of the CUSUM and the CUSUMSQ tests is that they can be easily graphed. The CUSUM graph presents a plot of the CUSUM statistics over time t and a pair of 5% critical value lines. With the CUSUM test, values outside the critical lines indicate parameter instability over time. The CUSUMSQ plot includes a pair of 5% critical value bands that are not crossed if the model is structurally stable. The CUSUM and CUSUMSQ tests can be used to identify the existence of breaks, but they have serious power problems (Andrew 1993). The CUSUM test has power only for changes in the mean regressor (Ploberger Kramer, and Kontrus, 1989), while the drawback of the CUSUMSQ test is that it only has power for local changes that specify a one-time change in parameters (Deshayes and Picard, 1986). Therefore, the CUSUM and CUSUMSQ tests with good local asymptotic power may have poor global behaviour. In this situation, the Quandt-Andrews Structural Stability test is applied for testing a structural change with an unknown date.

b) Quandt-Andrews structural break test

We can use the Quandt-Andrews (1994) structural stability test to identify the date of a structural break, denoted by τ. The Quandt-Andrews test performs the Chow breakpoint test for every possible observation, and tests for a break at the point which yields the largest breakpoint *F* statistic. In general, the first and last 15% of observations are excluded from the sample period and the remaining sample is known a trimmed dataset. The null hypothesis is defined as no switching within the trimmed dataset, while the alternative hypothesis is defined as there being a structural change in parameters at a break point τ. However, the break point τ is a nuisance parameter. It is not identified under the null hypothesis. Thus, the standard x^2 critical values cannot apply in this case. Hansen (1997) offered the Critical values (*p*-values) for the Quandt-Andrews (1994) structural stability test. Three different statistics can be used: the Maximum (Max or *Sup*) statistic, the Exponential (Exp) statistic, and the Average (Ave) statistic (see Andrews, 1993 and Andrews and Ploberger, 1994). The maximum statistic of the individual Chow break point test (*F* statistic) is given as:

$$\text{Max } F = \max_{\tau_1 < \tau < \tau_2} (F(\tau)) \tag{3.14}$$

The Exp statistic is defined as:

$$\text{Exp } F = \ln\left[\frac{1}{k}\sum_{\tau=\tau_1}^{\tau_2}\exp(\frac{1}{2}F(\tau))\right] \tag{3.15}$$

The Ave statistic takes the average of the individual *F* statistics:

$$\text{Ave}F = \frac{1}{k}\sum_{\tau=\tau_1}^{\tau_2}F(\tau) \tag{3.16}$$

where τ indicates the observations in the trimmed sample period, *k* is the number of break points compared, and τ_1 and τ_2 are test sample dates for the trimming sample.

The Quandt-Andrews test is designed for testing a single unknown structural break in a linear model. In particular, it can identify at what time a possible break occurred. The next subsection discusses the Bai-Perron test which can be used for testing multiple structural breaks if the maximum number of breaks is provided.

c) Bai-Perron test

Bai and Perron (1998) proposed a test for detecting unknown multiple structural breaks in a linear regression framework. The BP model with *m* breaks is given below:

$$\begin{aligned}
R_t &= x_t'\beta + z_t'\delta_1 + u_t \quad & t &= 1,\cdots,T_1 \\
R_t &= x_t'\beta + z_t'\delta_2 + u_t \quad & t &= T_1+1,\cdots,T_2 \\
&\vdots \\
R_t &= x_t'\beta + z_t'\delta_{m+1} + u_t \quad & t &= T_m+1,\cdots,T
\end{aligned} \tag{3.17}$$

where R_t is the dependent variable at time *t*, x_t $(p \times 1)$ and z_t $(q \times 1)$ are vectors of the candidate explanatory variables observed at time t, T is the sample size, m is the number of breaks, these m breaks divide the whole sample into m+1 regimes, $T_j = (T_1, \dots, T_m)$ are unknown break points with $T_0 = 0$ and $T_{m+1} = T$, β is a vector of unchanging parameters, δ_j is a vector of parameters that change m+1 times, and u_t is a disturbance term with mean 0 and variance σ^2.

In matrix form the BP model with m breaks can be expressed as:

$$R = X\beta + \overline{Z}\delta + U \tag{3.18}$$

where $R = (R_1, \cdots, R_T)'$, $X = (x_1, \cdots, x_T)'$, $U = (u_1, \cdots, u_T)'$, $\delta = (\delta_1, \cdots, \delta_{m+1})'$, and $\bar{Z} = diag(Z_1, \dots, Z_{m+1})$ with $Z_i = \left(Z_{T_{i-1}+1}, \dots, Z_{T_i}\right)$ for $i = 1,2, \dots, m+1$.

The BP test can be applied to models of both the "pure" structural break, where all the coefficients in the regression are allowed to change across regimes, and the "partial" structural break models, where only some of coefficients may change. Model (3.3) is a partial structural break model because the parameters in β do not change across different regimes. If beta is an empty vector, then all the coefficients are subject to change and we obtain a pure structural change model:

$$R = \overline{Z}\delta + U \tag{3.19}$$

The basic idea of the BP test is that each break point is treated as an unknown parameter and is estimated together with regression coefficients. Let $\hat{\beta}(T_j)$ and $\widehat{\delta_j}(T_j)$ denote the least-squares estimates of β and δ_j.Given each m-partition $T_1, \dots, T_m$, $\hat{\beta}(T_j)$ and $\widehat{\delta_j}(T_j)$ can be obtained by minimizing the sum of squared residuals, denoted by $S_T(T_1, \cdots T_m)$:

$$S_T(T_1, \cdots T_m) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} \left[R_t - x_t^{'} \beta - z_t^{'} \delta_i \right]^2 \tag{3.20}$$

Substituting $\hat{\beta}(T_j)$ and $\widehat{\delta_j}(T_j)$ in the objective function, the estimates $\widehat{T_j} = (\hat{T}_1, \dots, \hat{T}_m)$ for break dates are chosen as follows:

$$\hat{T}_j = (\hat{T}_1, \cdots \hat{T}_m) = \arg\min_{T_1, \cdots T_m} S_T(T_1, \cdots T_m) \tag{3.21}$$

Furthermore, Bai and Perron (1998) suggested the test procedure for determining the number of breaks in the multiple structural break models. Their main idea is first to test for a single structural break. Once the first break is identified, i.e., the test rejects the null hypothesis of no structural break, this first break point divides the whole sample into two subsamples. They then reapply the test to each subsample. This sequence is repeated until the test fails to find an additional break for all subsamples. Based on this test procedure, three tests are therefore introduced by Bai and Perron (1998). They first proposes a *Sup F* type test of the null hypothesis that there is no break (*m=0*) against the alternative hypothesis of a fixed number of breaks (*m=k*). They also extend this approach to a double maximum test of the null hypothesis of no structural break versus the alternative hypothesis of an unknown number of breaks given a maximum possible number *M*. This approach includes two versions of the test: *Dmax* F_t, an equal weighted vision, and *WDmax* F_t, with weights that depend on the number of regressors and the significance level of the test. The two double maximum tests are used to test if at least one break exists. If the double maximum tests reject the null hypothesis of no break, they suggest a *Sup F* $(l+1|l)$ test of l breaks against the alternative of $l+1$ breaks to determine the number of true breaks. Some researchers prefer to replace the least squares criterion for finding the number of breaks by using information criteria. In these cases Bayesian Information Criteria (BIC) and the modified Schwarz information criteria (LWZ) can be used to determine the number of breaks.

In the next section, we consider the possibility of structural breaks in the parameters of equity risk premium models for the UK stock market. Statistical tests for these structural breaks are employed, some of which rely on knowing the exact date of the breaks and some of which allow for the dates of the breaks not to be known in advance.

3.4 Empirical Section

The main purpose of this section is to test whether structural breaks have occurred in the data generating process of the UK equity risk premium during the sample period. We therefore apply the structural break tests introduced in the previous section to the UK stock market to demonstrate the importance of testing for structural breaks in equity risk premium models over the long horizon. Both univariate and multivariate models are employed. The sample period is from January 1965 to May 2012. The data are available for quarterly and monthly frequencies. It is unlikely that the same regression model can be applied to a sample spreading over the post-war period, the 1970s stagflation, the 1990s stock market boom, and the 2007 financial crisis.

3.4.1 Motivations

The consumption-based asset pricing model (CCAPM) implies that equity risk premia change with business conditions. This provides theoretical reasons why the estimation of equity risk premium models should take account of business conditions. A number of variables have been found that appear to be predictive of equity risk premia; these variables should be related to changes in the overall macroeconomic risk, in the investor's risk aversion, or in both.

Some researchers have focused on equity price-related variables, such as dividend price ratios. Dividend price ratios or dividend yields are the percentages of dividend profits which shareholders receive from their investments relative to share prices. The literature on the predictability of equity risk premia based on dividend yields includes contributions by Ball (1978), Basu (1977, 1983), Campbell (1987), Campbell and Shiller (1988a, 1988b), Chen (1991), Cochrane (1997), Fama and French (1988) and Lamont (1998). These studies can be interpreted in two ways. One group of writers believes that dividend price ratios have the power to explain future equity risk premia. Their idea is based on the dividend discount models (Gordon, 1962), in which the equity price is the discounted value of the future expected dividend flows. The other group recommends that the dividend price ratios contain general information about the overall business conditions, the justification for this being that dividend yields critically depend on business conditions. Specifically, they are likely to be high during a recession, and low during an expansion. Chen (1991) concluded that dividend yields may fall more slowly than equity prices when business conditions are poor, resulting in an increase in the dividend price ratio.

Some have considered economic variables, such as inflation rates. Scruggs (1998) and Brandt and Wang (2003) linked the time-varying equity risk premium to the variation in inflation rates. This is because news about real economic conditions and aggregate consumption growth, which may have an effect on investors’ risk aversion, is typically correlated with news and policies affected by inflation. Based on this idea, Blanchard (1993) extended the dividend discount model and estimated equity risk premia by using dividend yields, interest rates and inflation rates. He found evidence that the equity risk premium was lower in a well-managed economy with stable and controlled inflation rates, interest rates and dividend yields than in one where these variables are volatile. This makes perfect sense, for two reasons. First, shareholders invest in equities in order to increase their future consumption. A volatile economic environment may lead investors to perceive and experience more uncertainty about future earnings, so that they express greater concern about the risk of holding equities. Economic theory holds that higher risk should be associated with a higher equity risk premium. As a result, investors may require an extra risk premium for taking the same set of risky equities during a recession rather than an expansion. Second, higher and volatile inflation rates during times of recession may reduce the real returns that investors expect to receive from an investment. The great difference between expected and actual inflation rates during a recession may also raise greater uncertainty about the investments. In such circumstances, investors may require more risk premia to compensate for the extra risk that inflation rates may increase.

Some studies have used bond-related variables, such as the short-term Treasury bill rate (Blanchard, 1993; Glosten and Jagannathan, 1989; Fama and Schwert, 1977; Keim and Stambaugh, 1986; Campbell, 1987; Fama and French, 1989 and Lewellen, 2004). These studies provide evidence that equity risk premia and interest rates move together in the long run. Similar to price-related variables and economic variables, bond-related variables also contain information about business conditions.

Other studies have addressed the importance of including lagged terms of equity risk premia. For example, Poterba and Summers (1986) suggested that monthly equity risk premia followed an first order autoregressive (AR(1)) model. This is because that the stock market performance is likely to be persistent at short horizons. As discussed in section 3.2.2, the momentum effect is consistent with both changes in business conditions and changes in investors' risk preferences. It has a significant effect on investors' expectations of return and risk attitude, and therefore on equity risk premiums.

In short, autoregressive models (AR) are among the simplest time series models. Many more complex forecasting models representing real phenomena are based on these models. To estimate equity risk premia, we extend the AR(1) model and select the lagged one-period equity risk premium, the dividend price ratio, the three-month Treasury bill rate and the inflation rate as the possible candidate predictive variables. We first assume that all parameters of explanatory variables are subject to change at each break point. In doing this, we describe simple time-series models of the equity risk premium where the current value of the premium (R_t) is based on the one-period lagged value of itself (R_{t-1}) and the one-period lagged value of the candidate macroeconomic variables (X_{t-1}):

$$R_t = c + \phi_1 R_{t-1} + \phi_2 DY_{t-1} + \phi_3 TB_{t-1} + \phi_4 RPI_{t-1} + \varepsilon_t \qquad (3.22)$$

where R_t represents the equity risk premium on the asset at time t, DY_t represents the dividend yield, TB_t represents the three-month Treasury bill rate, RPI_t denotes the inflation rate, and ε_t is the disturbance term with mean 0 and variance σ_1^2.

The main purpose of this section is to investigate whether the relationship between equity risk premia and the explanatory variables is stable over the whole sample period. To examine the possibility that there are structural breaks during the sample period, we need to perform structural break tests.

3.4.2 Data

This study uses DATASTREAM as the data source. FTSE All Share Index, Dividend Yield, Three-month Treasury Bill Yield, and Retail Price Index are collected from DATASTREAM. To assess the performance of the UK equity risk premium, we calculate these by subtracting the three-month Treasury bill yield from the UK FTSE All Share Index value-weighted rate of return. The FTSE All Share Index is a capitalisation weighted index. It is based on the market price of more than 2000 companies. It covers approximately 98% of the UK market capitalisation and is designed to measure the market performance of companies traded on the London Stock Exchange. The three-month Treasury bill rates are taken from the International Monetary Fund's (IMF) International Financial Statistics (IFS) database. The dividend yields are calculated from the FTSE All Share dividend index. The inflation rates are defined as the rate of the growth of the Retail Price Index (RPI). The data are available at the quarterly and monthly frequencies. The sample period is from January 1965 to May 2012.

Figure 3.1 Annualised UK monthly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates

FIGURE 3.1 HERE

Figure 3.2 Annualised UK quarterly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates

FIGURE 3.2 HERE

Figure 3.3 Annualised UK stock returns at the monthly and quarterly frequencies

FIGURE 3.3 HERE

Figure 3.4 De-trended UK monthly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates using HP Filter

FIGURE 3.4 HERE

Figure 3.5 De-trended UK quarterly equity risk premia, dividend yields, Treasury bill rates and inflation rates using HP Filter

FIGURE 3.5 HERE

Recalling the results of Figures 2.1-2.6 in Chapter 2, UK equities perform better than three-month Treasury bills over the long run but they are more volatile and hence riskier. To overview the performance of the UK stock market, Figures 3.1 and 3.2 present the logarithm plots of the annualised rate for UK equity risk premia, dividend yields, Treasury bill rates and inflation rates at the monthly and quarterly frequencies, respectively. Large jumps, such as November 1974, are clearly visible from the graphs. The figures also suggest that the dividend yields, three-month Treasury bill rates and inflation rates are relatively stable compared to the equity risk premia. To separate the long-term trend from the short-term fluctuations, we use the Hodrick-Prescott filter. Figures 3.4 and 3.5 plot de-trended UK equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates using the Hodrick-Prescott filter at the monthly and quarterly frequencies, respectively. Figure 3.4

suggests that the monthly equity risk premia are stationary, but the monthly dividend yields, Treasury bill rates and inflation rates go down after reaching a peak in 1975.

3.4.3 Summary statistics and Normality tests

In this section, we present summary statistics to describe the performance of equity risk premia, dividend price ratios, three-month Treasury bill rates and inflation rates for the UK stock markets. In Figure 3.6, we plot the distribution of annualised rates of monthly and quarterly equity risk premia, respectively. Figure 3.7 plots the distribution of UK annualised monthly and quarterly stock returns, respectively. Figures 3.6 and 3.7 are nearly identical, no matter which frequency is used. They suggest that most of the changes in the equity risk premium are caused by the variation in stock returns, not by the variation in three-month Treasury bill rates. Figure 3.6 also suggests that the historical equity risk premia are very widely dispersed. The annualised rates of monthly equity risk premia are more widely dispersed as the x axis runs from -400% to 520%. This huge variability is confirmed by the standard deviation of the data.

Table 3.1 reports the summary statistics and the results of the Normality test for all four variables at the monthly and quarterly frequencies. There are four findings worth noting. First, historical equity risk premia are volatile. This general result holds for both time frequencies. The standard deviations for the monthly and quarterly equity risk premia are

0.6601 and 0.4027 respectively. These figures are very high compared to their means, which are only 0.0418 and 0.0410, respectively. The standard deviation is a measure of the dispersion of a variable from its mean. A low standard deviation indicates that data tend to be very close to the mean, while a high standard deviation indicates that data spread out over a large range of values. For instance, given a 95% confidence interval, the monthly average of 0.0418 and the standard deviation of 0.6601 suggest that future months' equity risk premium will be between -1.2520 and 1.3556. Such a range is so wide that the historical average may not be a reasonable expectation for future equity risk premium.

Second, the annualised monthly equity risk premia have higher arithmetic mean at 0.0418 and higher standard deviation at 0.6601. These suggest that the annualised rate of the monthly equity risk premium is more volatile than the quarterly rate. In Table 3.1, we find that the standard deviation increases as the time interval decreases. It changes significantly depending on the time interval chosen. More precisely, it increases with the square root of time. The standard deviation reflects the volatility of the stock market and hence can be used as a measure of risk over the long term. A higher volatility suggests a greater risk and hence should be associated with a larger equity risk premium. In this case, the arithmetic average of equity risk premia tends to rise as the length of the observation interval is shortened. Our findings confirm these results and suggest that the long term investments carry less risk compared to the short term investments if there are no structural breaks. One conclusion that emerges is that the choice of investment interval is important for the expected returns and it affects investors' asset allocation decisions.

Table 3. 1 Summary statistics for EQ, DY, TB and RPI

	EQ		DY	
	Monthly	Quarterly	Monthly	Quarterly
Mean	0.0418	0.0410	0.0415	0.0417
Median	0.1094	0.1105	0.0401	0.0400
Maximum	5.0487	2.2513	0.1115	0.1107
Minimum	-3.7597	-1.3551	0.0204	0.0209
Std Dev	0.6601	0.4027	0.0124	0.0129
Skewness	0.0094	0.0911	1.1651	1.3547
Kurtosis	11.1920	7.8813	6.6101	7.5083
Jarque-Bera	1591.034 （0.0000）	188.8960 （0.0000）	437.7114 （0.0000）	217.8674 （0.0000）
Shapiro-Wilk	0.9196 （0.0000）	0.9231 （0.0000）	0.9356 （0.0000）	0.9203 （0.0000）
	TB		RPI	
	Monthly	Quarterly	Monthly	Quarterly
Mean	0.0711	0.0713	0.0596	0.0597
Median	0.0651	0.0652	0.0440	0.0445
Maximum	0.1500	0.1489	0.2382	0.2353
Minimum	0.0030	0.0038	-0.0161	-0.0139
Std Dev	0.0339	0.0336	0.0467	0.0466
Skewness	0.1350	0.1319	1.6003	1.5921
Kurtosis	2.5913	2.5842	5.4045	5.3394
Jarque-Bera	5.6885 （0.0000）	1.9102 （0.3848）	379.9434 （0.0000）	122.9419 （0.0000）
Shapiro-Wilk	0.9694 （0.0000）	0.9691 （0.0003）	0.8356 （0.0000）	0.8355 （0.0000）

Note: This table reports the summary statistics for UK equity risk premia (EQ), dividend yields (DY), three-month Treasury bill rates (TB) and inflation rates (RPI).

Figure 3.6 Distribution for UK equity risk premia at the monthly and quarterly frequencies

FIGURE 3.6 HERE

Figure 3.7 Distribution for UK stock returns at the monthly and quarterly frequencies

FIGURE 3.7 HERE

Third, stock markets may experience unexpected crashes or upsurges over short time intervals. In Table 3.1, the maximum rate for the annualised monthly equity risk premium was 504.87% in January 1975. In this month the equity returned 514.94% and the three-month Treasury bill rate was 10.07%. The lowest was -375.97% in October 1987, when equities returned -366.94% and the three-month Treasury bill rate was 9.03%. If an event can persistently change the level of the data, it can be defined as a structural break. Although the chances of such events are very small, may change expected (equilibrium) equity risk premia significantly.

Fourth, the results of the Shapiro-Wilk test and the Jarque-Bera test indicate non-normality of the data. In Table 3.1, both the Shapiro-Wilk test and the Jarque–Bera test reject the hypothesis of normality with $P<0.0001$ for the equity risk premium for both time frequencies. These Normality tests suggest the equity risk premium do not follow a normal distribution. They also suggest that there are some extreme abnormal fluctuations in our sample periods. The high value of Kurtosis in the equity risk premium suggests that the distribution of the equity risk premium has fat tails. In fact, the financial time series always exhibits a high level of Kurtosis, especially with high-frequency data (FAMA, 1965). Excess Kurtosis implies the presence of a large number of extreme observations. Such abnormal fluctuations can be considered as outliers or structural breaks. A common approach to deal with the excess volatility in stock markets employs GARCH type models. Though, these standard volatility models may help to explain the feature of volatility clustering in data, they may fail to model such a high level of Kurtosis (Polsonet and Possi, 1994). Furthermore, Zumbach (2000) pointed out that those GARCH-type models estimated by using a Quasi Maximum Likelihood method may result in biased estimates. Jondeau and Rockinger (2003) argued that the standardised residuals from GARCH models do not follow a Gaussian distribution and therefore the regression may have little explanatory power. Brigo and Mercurio (2000, 2001) solved this problem by applying a mixture of two normal distributions to describe the behaviour of stock markets. They found that a simple mixture model is adequate to capture the excess Kurtosis in stock returns. The advantage of structural break approaches is that the whole sample period can be divided into several subsamples and can be easily modelled within a simple linear framework.

To summarise, the assumption of a normal distribution fails to capture some empirical properties of the equity risk premium. In particular, it may fail to model the possibility of extreme events. If such events can persistently change the level of data, they are structural breaks. As discussed before, it is necessary to take into consideration the possibility of structural breaks when estimating these equity risk premium models. In the following section, we will investigate if the relations between equity risk premia and fundamental variables, such as dividend yields, three-month Treasury bill rates, lagged one-period equity risk premia and inflation rates are stable over the whole sample period. More precisely, we will test if the parameters of the model remain constant over time. We focus first on the unit root tests which allow for the possibility of structural breaks in the data.

3.4.4 Unit root tests

The Ordinary Least Square (OLS) assumes that time series are stationary. The long-run relationship between financial time series is complicated further by the fact that time series may be non-stationary. Indeed, the stationarity of data should be tested before we explore the long-run relationship between equity risk premia and explanatory variables. For this reason, we apply the unit root tests discussed in Section 3.3.1 to test for the stationarity of time series. These time series include: equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates.

a) Conventional unit root tests

The unit root tests can be used to determine if a time series is stationary. In this section, we apply the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. The results are reported in Table 3.2. For equity risk premia and dividend yields, both the ADF and the PP tests reject the null hypotheses of a unit root, while the KPSS tests cannot reject the null of stationarity. These results suggest that equity risk premia and dividend yields are stationary in our sample periods. However, the three-month Treasury bill rates and inflation rates are found to be non-stationary. An important issue for these conventional unit root tests is that they ignore the possibility of structural breaks in the data. Perron (1989) recognised that the presence of structural breaks may result in an under-rejection of the unit root null. He therefore concluded that the existence of unit roots in a financial time series might better be described by a structural break.

Table 3. 2 ADF, PP and KPSS unit root tests

		ADF	PP	KPSS
		H_0: has a unit root	H_0: has a unit root	H_0: is stationary
		Statistics	Statistics	Statistics
EQ	Monthly	-10.5208**	-20.8792**	0.0503
	Quarterly	-12.2998**	-12.2593**	0.0379
DY	Monthly	-2.8800*	-3.4336*	0.0922
	Quarterly	-3.2844*	-2.9492*	0.09375
RF	Monthly	-2.3817	-2.4557	0.5202**
	Quarterly	-2.6689	-2.0927	0.3449**
RPI	Monthly	-2.8057	-2.8869	0.2311**
	Quarterly	-2.3873	-2.7503	0.1524*

Note: MacKinnon (1996) one-sided *p*-values for the ADF and PP tests, ** represents significant at level 1%, * represents significant at level 5%.

b) One endogenous break unit root tests

To avoid the spurious under-rejection of unit roots in the presence of structural breaks, we test for the unit root by allowing for the possibility that structural breaks may occur in the data. This involves two goals. The first is to test whether the unit roots exists. The second is to find out whether structural breaks exist and then identify them if they exist. For these two purposes, we apply the Zivot-Andrews (1992) unit root test and the Perron (1997) unit root test to examine the null hypothesis of non-stationary against the alternative of one endogenous break-stationary.

Table 3.3 reports the results of the Zivot-Andrews (1992) and the Perron (1997) unit root tests. For UK three-month Treasury bill rates, the Zivot-Andrews test suggests that they are stationary with one break in trends at both the monthly and quarterly frequencies. Conversely, the Perron (1997) test is not able to reject the unit root hypothesis for three-month Treasury bill rates at both frequencies. As discussed in Section 3.3.1, the Perron (1997) test allows for a break under both the null and alternative hypotheses, while the Zivot-Andrews test, under the null hypothesis, only allows for a unit root without a structural break. In this case, the Perron (1997) test provides more reliable results about whether a time series is indeed stationary. Comparing with these results, we can conclude that UK Treasury bill rates at the monthly and quarterly frequencies are non-stationary even after allowing for the presence of one structural break. For inflation rates, the Zivot-Andrews and the Perron tests reject the null of a unit root for monthly data and suggest that they are break stationarity with a structural break in both the level and trend, while both these tests cannot reject the null of unit roots for the quarterly frequency.

Table 3. 3 Zivot-Andrews and Perron one-break unit root tests

Zivot-Andrews test			Monthly	Quarterly
	RF	A	1972M06	1973Q3
			-4.4929	-4.5127
		B	1979M12	1980Q2
			-4.5190*	-4.7787*
		C	1978M04	1978Q2
			-4.9649(<10%)	-5.1017*
	RPI	A	1972M05	1982Q1
			-4.2973	-3.2547
		B	1974M02	1974Q1
			-4.2425(<10%)	-3.1219
		C	1980M05	1980Q3
			-5.6668**	-4.3999
Perron test	RF	IO1	1972M05	1972Q2
			-4.4895	-4.4795
		IO2	1978M01	1977Q4
			-5.1769	-5.2360
		AO	1981M05	1981Q2
			-4.4242	-4.6853
	RPI	IO1	1972M05	1980Q2
			-4.2589	-3.2755
		IO2	1980M04	1980Q2
			-5.7350*	-4.3022
		AO	1974M01	1973Q1
			-4.1492	-2.8636

Note: ** represents significant at level 1%, * represents significant at level 5%

c) Two-break unit root tests

Because our data sample covers a long time period and many extreme events may have taken place, it is possible that the occurrence of multiple structural breaks may result in the non-rejection of the unit root hypothesis. In this case, we need to consider the possibility of two structural breaks when testing for unit roots. Lumsdaine and Papell (1997) extended the Zivot-Andrews (1992) one-break unit root test to allow for two structural breaks. The results are presented in Table 3.4. Here, Model A allows for breaks in the intercept while Model C allows for the break in the both intercept and trend. The Lumsdaine and Papell (1997) tests reject the null hypothesis of a unit root for both three-month Treasury bill rates and the inflation rates at the monthly and quarterly frequencies.

We then apply the unit root test of Lee and Strazicich (2003) to inflation rates and three-month Treasury bill rates. As discussed in Section 3.3.1, the minimum LM unit root test provides more reliable information on testing for the unit root in the data by allowing for the existence of two breaks under both the null and the alternative hypotheses. Therefore the alternatives are unambiguously break-stationary. The results are reported in Panel B, Table 3.4. The minimum LM tests find that both time series are stationary. The findings of the present investigation are consistent with the outcomes of Perron (1989) who argue that the existence of unit roots in financial time series may better be described by structural breaks. In addition, the break occurs between 1973 and 1975 in most series. This reflects a fundamental change in stock markets following the oil price shock of 1973.

There are two problems need to be noted. First, we should choose the smallest possible number of breaks. Allowing for too many breaks may lead to model misspecification because even a random walk can be explained by stationary process with many trend breaks. In this section, we only allow for two breaks. Second, outliers may affect the test results. This can best be explained with our results from Table 3.4. We find two break dates including July 1974 and February 1975 in monthly equity risk premia. Indeed, it seems unlikely that two structural breaks happen within such a short-time period. Bai (1997) solves these two problems by using iterative refinements. We will explain this in Section 3.4.8.

It can be concluded that equity risk premia and dividend yields are stationary, while that three-month Treasury bill rates and inflation rates are break-stationary. We then turn to the question of whether there are any structural breaks in multivariate equity risk premium models. In the next section, we will look at the long-run relationship between UK equity risk premia, three-month Treasury bill rates, dividend yields and inflation rates. The main purpose is to find out whether the relationship between these variable remain constant over time.

Table 3. 4 Lumsdaine and Papell and Lee and Strazicich two-break unit root tests.

Panel A: Lumsdaine and Papell (1997) unit root tests				
			Monthly	Quarterly
RF	Model A	Break	1972M05 1978M03	1976Q1 1977Q2
		Statistics	-5.7492(<10%)	-11.7817**
	Model A with Trend	Break	1972M04 1977M09	1977Q1 1979Q3
		Statistics	-5.8187(<10%)	-11.8023**
	Model C	Break	1973M06 1978M03	1977Q2 1982Q2
		Statistics	-5.8475(<10%)	-13.4263**
	Model C with Trend	Break	1981M09 1990M02	1977Q1 1982Q2
		Statistics	-5.8076(<10%)	-13.4536**
RPI	Model A	Break	1978M02 1990M07	1972Q4 1980Q1
		Statistics	-5.0244	-5.2304
	Model A with Trend	Break	1978M03 1990M06	1973Q1 1979Q4
		Statistics	-5.0405	-5.2376
	Model C	Break	1978M03 1999M11	1980Q1 2001Q3
		Statistics	-5.9471*	-6.3744 (<10%)
	Model C with Trend	Break	1978M03 1999M12	1979Q4 2001Q3
		Statistics	-5.9620*	-6.1634(<10%)
Panel B: Lee and Strazicich (2003) LM unit root tests				
RF	Model A	Break	1973M07 1976M09	1973Q2 1976Q3
		Statistics	-3.6118	3.8597
	Model C	Break	1973M05 1992M12	1978Q1 1992Q4
		Statistics	-5.1459**	-5.7962**
RPI	Model A	Break	1976M04 1980M06	1976Q1 1980Q2
		Statistics	-3.4711	-2.6819
	Model C	Break	1974M02 1982M09	1973Q4 1984Q4
		Statistics	-7.0461**	-6.1806**

Note: Critical values are reported in Lee and Strazicich (2003). The sample period is 1965-2012

3.4.5 Parameter instability in multivariate equity risk premium models

Our purpose is to find evidence for structural breaks in the relationship between equity risk premia and predictive variables. All the four predictive variables we have chosen for the equity risk premium models. They are: lagged equity risk premia, dividend price ratios, three-month Treasury bill rates and inflation rates. Both multivariate and univariate regressive models are considered. The multivariate regression includes all four explanatory variables, while the univariate regression models include only a single explanatory variable. Empirically, the multivariate model may explain a greater range of variation in the equity risk premium compared with the univariate models, but there may be some partial structural breaks which may occur only in a sub-set of regression variables. In this case, the univariate regressive models may provide more information about the dates and characterisation of particular structural breaks although they may have low predictive power. In this regard, both multivariate and univariate models need to be considered. We first estimate equity risk premia in the context of the multivariate regressive models.

Table 3.5 reports the OLS results for the full-sample multivariate models at the monthly and quarterly frequencies. The predictive power of these models is weak since their coefficients of determination R^2 are very small. The existence of structural breaks results in huge forecasting errors and suggests unreliability in the model. In Table 3.5, such small values of R^2 raise the possibility that the predictive ability of the forecasting variables with respect to the variation in equity risk premia may be changing over time.

Table 3. 5 Estimates for the multivariate regressions and Normality tests of residuals

	Monthly			Quarterly		
Variable	Coeff	Std	t-Stat	Coeff	Std	t-Stat
Cons.	-0.3350	0.1037	0.0013	-0.3619	0.1060	0.0008
EQ(-1)	0.1322	0.0416	0.0016	0.1766	0.0714	0.0144
RF(-1)	-1.7498	1.0635	0.1005	15.6777	3.1885	0.0000
DY(-1)	14.1724	3.1384	0.0000	-1.9985	1.1055	0.0723
RPI(-1)	-1.5623	0.8440	0.0647	-1.9102	0.8836	0.0319
S.E	0.6468			0.3805		
R^2	0.0465			0.1241		
Residual Normality Tests						
Kurtosis	8.9236			5.0359		
Skewness	-0.3756			-0.4805		
prob.	0.0000			0.0000		

Note: This table reports the estimate results for the multivariate regression based on lagged one-period equity risk premia (EQ(-1)), lagged one-period dividend yields (DY(-1)), lagged one-period inflation rates (RPI(-1)), and Three-month Treasury bill rates by lagged one-period (RF(-1)).

a) Outliers

Excess volatility is perhaps the most important issue in stock markets. Financial time series are commonly contaminated with outliers and structural changes due to the impact of extreme events. The existence of outliers may affect parameter estimation significantly even if the model is appropriate. This is because outliers may result in large residuals when we fit models. Least Square estimation is very sensitive to outliers and these outliers will therefore severely distort estimates. In Table 3.5, the large standard deviations and the small values for the R^2 suggest that there may be substantial volatility in our sample periods. The results of the residual normality tests are also reported in Table 3.5. These tests reject the traditional OLS assumption that the data are normally distributed. The high level of Kurtosis also supports the finding that there is substantial volatility in the data. For this reason, models estimated by OLS perform poorly. Robust estimation is a common approach to detect outliers. Table 3.6 presents the results from robust regressions for detecting outliers in models of UK equity risk premia using maximum likelihood Huber (1973) M-estimation.

Table 3. 6 Outliers in the multivariate equity risk premium model

Monthly		Quarterly	
Proportion	0.0624	Proportion	0.0582
Date	Residual	Date	Residual
1966M07	-3.5032	1966Q3	-3.2814
1973M11	-3.4219	1973Q4	-3.0305
1974M03	-5.6808	1974Q1	-3.7130
1974M08	-3.3837	1974Q3	-5.5607
1974M09	-3.4597	1974Q4	-3.6258
1974M11	-4.7115	1975Q1	5.9904
1975M01	8.7239	1976Q3	-3.1032
1975M02	3.3353	1981Q3	-3.3169
1975M04	3.5244	1987Q4	-4.7934
1975M06	-3.3104	1990Q3	-3.2381
1976M12	3.2903	2002Q3	-3.3307
1981M09	-4.4238		
1987M10	-7.2874		
2002M09	-3.0366		
2008M09	-3.5972		

Note: The residual here is the standardised robust residual.

Data cleaning techniques are often used to remove or replace the outliers from the dataset. For example, Tabachnick and Fidell (2007) used the mean of the data to replace outliers, and Tukey (1977) used the median as a replacement for outliers. However, these methods are not suitable for two reasons. First, they may reduce the spread of the population and therefore the distribution may be more leptokurtic. Consequently, this may result in an over-rejection of the null hypothesis of Normality and the likelihood of type-I error may increase. Second, an outlier that is clearly an error or noise should be excluded. However, some outliers may carry important information. For example, some outliers can be defined as structural breaks if they can persistently change the pattern of the data. In this case, omitting the impacts of such outliers may cause a loss of important information and may lead to model misspecification. In this regard, three objectives are proposed. The first is to test for the existence of structural breaks in the data. The second is to identify the exact break dates if structural breaks exist. The last is to estimate the model parameters together with the structural breaks.

b) Structural break tests

Rolling regression estimates

A common approach to assessing the stability of parameter estimates in regression models is to use rolling window regression. The model is estimated by recursive least squares (RLS) with a fixed window size. If the model is stable over time, the rolling estimates should not change too significantly. In this case, the rolling estimates can provide preliminary evidence as to whether the parameter estimates are stable over time. The size of the rolling window is fixed *a priori*. Here, we use one third of sample size. Figure 3.8 plots the rolling estimates for the lagged one-period equity risk premia, dividend yields, three-month Treasury bill rates, and inflation rates at the monthly and quarterly frequencies. The plots show that the rolling estimates for these four variables change significantly over time and therefore suggest parameter instability in the models.

Figure 3.8 Rolling regression estimates for equity risk premium models

FIGURE 3.8 HERE

Figure 3.9 CUSUMSQ tests for equity risk premium models

FIGURE 3.9 HERE

CUSMSQ tests

We then adopt the CUSUMSQ test which helps us to identify if a break took place. The CUSUMSQ tests are particularly helpful when we are not sure if, and when, a structural break may have occurred. Figure 3.9 illustrate the CUSUMSQ plots with their respective 5% critical lines for the models of UK equity risk premia based on lagged one-period equity risk premia, dividend price ratios, inflation rates and three-month Treasury bill rates at the monthly and quarterly frequencies. The movements of the CUSUMSQ statistics outside the critical lines provide the evidence of parameter instability and therefore suggest that there exist some structural breaks in the equity risk premium models. The CUSUMSQ test is a step in the right direction for testing unknown structural breaks but it only has trivial local power against the alternative in a certain direction (Andrews, 1993).

Quandt-Andrews structural break tests

To identify the date when a structural break occurs, we apply the Quandt-Andrews (1994) unknown structural break tests. The test results are reported in Table 3.7. A structural break has most likely happened around 1975 (1975q1 in the regression using quarterly data and 1975m1 using monthly data). This structural break could be explained by the 1973-1974 stock market crash. From the beginning of 1973 to the end of 1974 the UK stock market suffered one of the worst bear markets in its history. The London Stock Exchange’s FT 30 lost 73% of its value in this crash. This might be attributed to a set of major changes which occurred in the UK during the early 1970’s, such as a dramatic rise in oil prices, the high rates of inflation, the miners' strikes, the introduction of the Three-Day Working Week, the political uncertainties (two general elections were held in 1974), and a major crisis in the UK property market. The crash ended at the beginning of 1975. The stock market surged back with extremely large positive returns in January and February 1975. Over the following year, stock prices rose 150%.

Table 3. 7 Andrews-Quandt unknown breakpoint tests

		Monthly	Quarterly
Sample		1965M01 2012M05	1965Q1 2012Q1
Test sample		1972M03 2005M04	1972Q2 2005Q1
Break dates		1975M01	1975Q1
Max LR F-statistics	values	10.1396	8.6411
	Prob.	0.0000	0.0000

Note: Critical values (Prob.) are calculated using Hansen's (1997) method

Chow Test

Table 3.8 shows the results of the Chow tests (*F* statistics). The results reject the null hypothesis of no break and confirm that a break is most likely to have happened around 1975 (1975q1 for the regression using quarterly data and 1975m1 for the regression using monthly data). These results support the findings of the Quandt-Andrews tests in the last section. The Chow test assumes a known structural break date and examines whether the time series has a structural break at a specified date. However, in most cases, the date of the structural break is difficult to ascertain in advance. Structural breaks may happen gradually and many may not be easily observable. In this case, the Chow test may fail to detect possible structural breaks. The Quandt-Andrews unknown breakpoint test performs the Chow test for each observation, and selects a break at the point which yields the largest breakpoint *F* statistic. However, this test is designed to test for one break only and thus may have weaker power for multiple changes. The Bai-Perron (1998) break point test was therefore introduced to test for unknown multiple structural breaks in a linear regression framework. In the next section, we will apply the Bai-Perron test to the UK stock market.

Table 3. 8 Chow breakpoint tests

	Monthly		Quarterly	
Break date	1975M01		1975Q1	
F Statistics (Prob.)	11.1525	0.0000	10.09608	0.0000
Log LR (Prob.)	43.5374	0.0000	38.06519	0.0000
Wald Stat (Prob.)	44.6102	0.0000	40.38431	0.0000

Bai-Perron structural break tests

The results of the Bai-Perron (1998) test are shown in Table 3.9. The first and last 15% of observations are excluded from the sample period. This corresponds to a minimum 7 years and 1 month between any two successive breaks. The choice of minimum distance between two successive breaks places a limitation on the combinations of structural breaks. This refinement can reduce the possibility of selecting false break points. The Bai-Perron tests clearly reject the null of no structural break and hence reject the hypothesis of no switching in the equity risk premium model. The number of breaks is chosen by Bayesian Information Criteria (BIC). The BIC selects 2 breaks at the monthly frequency, and 1 break at the quarterly frequency. The break dates are shown in Table 3.9. The results of the Bai-Perron tests support the results already found in the Chow tests and the Quandt-Andrews tests.

These suggest that structural breaks are an important characteristic of equity risk premium models and cannot be ignored by investors when making their asset allocation decisions.

Table 3. 9 Bai-Perron breakpoint tests

Monthly	Breakpoints	0	1	**2**	3	4
	BIC	1152.009	1140.675	**1119.732**	1149.771	1178.204
	Log-Like	-556.9726	-532.2740	**-502.7709**	-498.7592	-493.9436
	RSS	235.9755	216.3535	**195.0415**	192.3105	189.0828
	No. Breaks	2				
	Breaks	1974M12	1982M06			
Quarterly	Breakpoints	0	**1**	2	3	4
	BIC	197.4687	**188.0540**	200.9795	224.7891	243.7450
	Log-Like	-83.00913	**-62.57654**	-53.31400	-49.49357	-43.24631
	RSS	26.63626	**21.45710**	19.45374	18.68295	17.48779
	No. Breaks	1				
	Breaks	1974Q4				

c) Estimations and Implications

The Bai-Perron tests reported in Table 3.9 identify two structural breaks for the multivariate equity risk premium predictive model at the monthly frequency, and one break at the quarterly frequency. The 1974 break is identified for both frequencies. This result is consistent with the findings in the unit root tests in Section 3.4.4. The 1974 break reflects a fundamental change in the stock markets following the 1973 oil price shock. The 1982 break corresponds to the change in dividend yields. Dividend yields have declined since 1982. The univariate prediction models based on dividend yields may provide more information about this break. We will discuss the details in the univariate model (Section 3.4.6).

We then split the full sample into three subsample periods for the monthly frequency, and into two subsamples for the quarterly frequency according to the results of the Bai and Perron (1997) test. For the monthly frequency, these three subsamples are: January 1965 - November 1974, December 1974 - May 1982 and June 1982 - May 2012. For the quarterly frequency, these two subsamples are: Quarter 1 1965 - Quarter 3 1974 and Quarter 4 1974-Quarter 1 2012. Table 3.10 reports the estimation results for the multivariate equity risk premium models during each subsample. The results suggest that the predictability of equity risk premia changes over time. The R^2 of 19.38% provides evidence of the predictability in the equity risk premium based on all four predictive variables for the period January 1965 to November 1974. The predictability increases dramatically to 46.57% for the second sub-period December 1974 to May 1982, but collapses to 3.63% in the third sub-sample of June 1982 to May 2012.

The results of the stepwise regressions are reported in Panel B of Table 3.10. The stepwise regressions can be used to narrow down the most significant forecasting variables. The three-month Treasury bill rates and inflation rates have significant predictive power for the equity risk premium in the first sub-sample. In the second sub-sample, however, the dividend yields, the three-month Treasury bill rates and the lagged one-month equity risk premium have predictive ability, and the inflation rates are no longer correlated. In the third sub-sample, the inflation rates and the three-month Treasury bill rates are included in the model. The Normality tests on the regression residuals are also reported in Table 3.10. We find that a simple mixture of two normal distributions can capture the excess Kurtosis in equity risk premia. The Jarque-Bera test cannot reject the Normality hypothesis for the first and the second sub-samples. However, the Kurtosis is still very high in the third sub-sample from June 1982 to May 2012. As mentioned before, the disappearance of stock market predictability may be the result of arbitrage in the market. However, there is a second possible interpretation. The small R^2 and high residual Kurtosis in the third sub-period may simply be due to structural breaks that have not been identified by the tests. For example, the rolling regression estimates plotted in Figure 3.10 suggests that there may have been a sharp change in the parameter estimates of the equity risk premium models around 2007. In fact, the global financial crisis that started in 2007 is considered to be the most devastating financial crisis since the 1929 crash. It has had a huge impact on global economics, with the collapse of large financial institutions, the downturn in stock markets and the massive bank bailout by the US Federal government. It is worth testing if this crash has had a permanent effect on the stock market. We focus only on the monthly frequency, because the candidate break of 2007 is at the end of our sample period and the quarterly frequency does not have enough observations to perform the tests. The Chow test results are reported in Table 3.11 and they reject the null hypothesis of no breakpoint over the sample period June 1982 to May 2012. The Chow tests also suggest that a break is most likely to have happened between 2007 and 2008. We then split the 1982-2012 periods into two sub-samples: July 1982-

October 2007 and November 2007- May 2012. The results of both the OLS and the Stepwise OLS are reported in Table 3.12. These results suggest that the three-month Treasury bill rates and the inflation rates can explain 17% of the variation in the equity risk premium.

The interpretation of the above results comes with a note of caution. There are many potential problems associated with structural break tests. The details on power issues in these tests are discussed further in Section 3.48. Although, in practice identifying the existence of a structural break is complicated, it is still obvious that structural breaks are an important economic characteristic of equity risk premia and cannot be ignored by investors when making their asset allocation decisions. To sum up, we find evidence of parameter instability in the multivariate equity risk premium models based on the lagged one-period equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates. To facilitate the interpretation of these results, we now examine in turn the breaks in the univariate models based on each individual forecasting variable.

Table 3. 10 OLS and Stepwise LS estimates for multivariate models with breaks

Panel A: OLS						
		Monthly			Quarterly	
Sample period		1965M01 1974M11	1974M12 1982M05	1982M06 2012M05	1965Q01 1974Q3	1974Q4 2012Q1
c	*Φ*	1.1891	-3.7579	-0.3220	1.0198	-0.4392
	SE	0.3073	0.5552	0.1287	0.3931	0.1125
	Prob.	0.0002	0.0000	0.0128	0.0139	0.0001
EQ(-1)	*Φ*	-0.1017	0.1596	0.0811	0.0262	0.0990
	SE	0.0978	0.0810	0.0524	0.1923	0.0770
	Prob.	0.3009	0.0520	0.1225	0.8923	0.2009
RF(-1)	*Φ*	-13.6452	83.3449	-0.5986	-12.8744	-1.9500
	SE	4.7028	10.1312	1.2221	5.0784	1.0958
	Prob.	0.0045	0.0000	0.6246	0.0160	0.0772
RPI(-1)	*Φ*	-3.5779	-7.6431	-2.8106	-3.3638	-2.0811
	SE	2.4037	2.9838	1.9654	2.5561	0.9537
	Prob.	0.1394	0.0122	0.1536	0.1970	0.0307
DY(-1)	*Φ*	-2.1666	-0.8726	13.8802	-0.1426	18.4759
	SE	5.4710	1.6904	4.1842	6.6714	3.6103
	Prob.	0.6928	0.6071	0.0010	0.9831	0.0000
R^2		0.1938	0.4637	0.0363	0.4124	0.1635
Panel B: Stepwise least square						
c	*Φ*	1.0220	-3.8044	-0.3156	1.0403	-0.4392
	SE	0.2386	0.5455	0.1286	0.2320	0.1125
	Prob.	0.0000	0.0000	0.0146	0.0001	0.0001
EQ(-1)	*Φ*		0.1527	0.0800		0.0990
	SE		0.0795	0.0524		0.0770
	Prob.		0.0582	0.1274		0.2009
RF(-1)	*Φ*	-12.5568	-7.6304		-13.3008	-1.9500
	SE	4.3204	2.9710		4.0664	1.0958
	Prob.	0.0044	0.0119		0.0024	0.0772
RPI(-1)	*Φ*	-3.5465		-3.3334	-3.3479	-2.0811
	SE	2.3878		1.6492	2.4382	0.9537
	Prob.	0.1402		0.0440	0.1782	0.0307
DY(-1)	*Φ*		82.0240	13.2031		18.4759
	SE		9.7608	3.9518		3.6103
	Prob.		0.0000	0.0009		0.0000
R^2		0.1858	0.4620	0.0356	0.4121	0.1635
Panel C: Residual Normality tests						
Kurtosis		4.3512	3.4707	8.8616	2.9304	4.8677
Skewness		3.3457	-0.5208	-1.2931	-0.3868	-0.5822
Normality test		0.1135	0.0863	0.0000	0.6124	0.0000

Table 3. 11 Chow test: July 1982-May 2012

Test period	1982M07 2012M05	
Break date	2007M11	
F Statistics (Prob.)	2.5542	0.0275
Log LR (Prob.)	12.9021	0.0243
Wald Stat (Prob.)	12.7709	0.0256

Table 3. 12 OLS and Stepwise LS with breaks: July 1982-May 2012

		OLS		Stepwise Least Square	
		1982M06 2007M10	2007M11 2012M05	1982M06 2007M10	2007M11 2012M05
c	*Φ*	-0.3198	0.3726	-0.3064	0.3336
	SE	0.1353	0.7446	0.1316	0.1496
	Prob.	0.0187	0.6199	0.0205	0.0300
EQ(-1)	*Φ*	0.0594	-0.0157		
	SE	0.0568	0.1579		
	Prob.	0.2963	0.9211		
RF(-1)	*Φ*	-2.8810	-11.3429	-3.0612	-11.2682
	SE	2.2454	4.7709	1.5008	4.3992
	Prob.	0.2005	0.02113	0.0205	0.0134
RPI(-1)	*Φ*	-0.4421	-5.7939	0.0422	-5.5667
	SE	3.0446	5.1058		3.7081
	Prob.	0.8847	0.2619		0.1393
DY(-1)	*Φ*	16.3364	-0.8542	15.9539	
	SE	4.9235	17.7570	4.8583	
	Prob.	0.0010	0.9618	0.0011	
R^2		0.0384	0.1757	0.0349	0.1756
Residual Normality tests					
Kurtosis		10.2224	2.3349	10.0118	-0.0031
Skewness		-1.4025	-0.2229	-1.4283	2.1536
Normality test		0.0000	0.4796	0.0000	0.4958

Note: Break date November 2007 has been tested by Chow test

3.4.6 Univariate regression models

The structural break tests based on the multivariate regressive models are statistically significant. These provide evidence of time-varying parameters in equity risk premium models. However, the structural break tests based on multivariate models alone may not provide information about the source of a particular structural break. Individual time series may response to the same shocks in a different way, such as, at different times. Also, there may be some partial structural breaks in which only a certain sub-set of model parameters are subject to change, while the others remain constant. In these cases, multivariate regression may not be able to spot such breaks. To link structural breaks to special events or changes in new policies, univariate regressive models need to be considered. In this subsection, we apply structural break tests to the univariate models based on lagged one-period equity risk premia, dividend yields, three-month Treasury bill rates, and inflation rates.

Table 3.13 reports the estimation results for the full-sample univariate regression models based on each forecasting variable. In Table 3.14, we report the results of the Bai-Perron structural break tests for each univariate model. For the monthly data, one break is identified for two of the four models examined. These two univariate models are based on three-month Treasury bill rates and inflation rates, respectively. For the model based on the lagged on-period equity risk premium, there is no break. For the dividend yield model, there are two breaks, which occurred in December 1974 and July 1982. The 1974 break is identified for all regression models. This result is consistent with the findings in the unit root tests in Section 3.4.4. The 1974 break reflects a fundamental change in the stock markets following the oil price shock of 1973. Indeed, it is not unreasonable to assume that the underlying structure of equity risk premium models, as well as the predictability of forecasting variables, may have changed after this shock. The results for the quarterly data are similar to the monthly frequency.

Table 3.15 presents the estimation results for the univariate regressive models during each sub-sample period. The break dates are identified by the structural break tests of Bai and Perron (1997) based on the results of Table 3.14. Table 3.16 displays the summary statistics for each regression variables at different time horizons, which include the full-sample period and three sub-sample periods. We illustrate the results for each univariate variable model below. The dividend yield is an important indicator of future expected returns. Fama and French (1998) suggest that the dividend yield is a useful variable for predicting future stock returns because it provides important information about the dividend income in relation to the stock price. The estimated break points for the equity risk premium model based on the dividend yields are December 1974 and July 1982. These two break points divide the whole sample into three sub-samples. As discussed before, the 1974 break reflects a fundamental change in the stock markets following the oil price shock of 1973. It had permanent effects on all our regression variables. The 1982 break was a major turning point for dividend yields. On the one hand, dividend payments declined substantially over the following 16 years. Fama and French (2001) pointed out that 67% of firms paid dividends in 1978, but only 21% paid them in 1999. On the other hand, stock prices increased significantly from 1982 to 1999. All these can lead to the drops in the level of dividend yields. The empirical hypothesis that dividend yields have declined since 1982 is consistent with our results. In Table 3.16, the average dividend yield dropped from 5.8% between 1974 and 1982 to 3.65% between 1982 and 2012. Moreover, the value of R^2 measures the predictive ability of the dividend yields for the equity risk premium. In Panel D, Table 3.15, The R^2 value is 25.57%. This suggests that the dividend yields can explain 25.57% of the variation in the equity risk premium from 1974 to 1982. However, there is no significant predictability for the first and the third sub-periods. These results suggest that the predictive power of dividend yields, with respect to the variation in equity risk premia, changes over different sub-periods. Moreover, the correlation between equity risk premia and dividend yields changes over time. In particular, we find evidence of a negative relationship between equity risk premia and dividend yields from 1965 to 1974, and of positive relation since 1974.

The structural break tests find one break, in 1974, for both the three-month Treasury bill rate model and the inflation rate model. The pattern in the predictability of equity risk premia based on these two variables is similar to that based on the dividend yields, as the R^2-value changes significantly over different subsamples. Our results suggest that inflation rates and thee-month Treasury bill rates have significant predictive power for the equity risk premium from 1965 to 1974, while there was no significant evidence for the predictability in the second sub-intervals. Despite the fact that the univariate regression models are quite noisy, there is considerable evidence that the underlying structure of equity risk premium models may change over time. To conclude, the structural break is a significant issue in estimating the long-run equity risk premium.

Table 3. 13 Estimates for univariate predictive models

	Monthly				Quarterly			
	EQ(-1)	DY(-1)	RF(-1)	RPI(-1)	EQ(-1)	DY(-1)	RF(-1)	RPI(-1)
Cons.	0.0371	-0.2386	0.0448	0.0270	0.0400	-0.2487	0.0516	0.0302
Prob.	(0.1788)	(0.0130)	(0.4885)	(0.5484)	(0.1745)	(0.0113)	(0.4606)	(0.5278)
Coeff.	0.1093	6.7497	-0.0430	0.2479	0.0888	6.9905	-0.1116	0.2240
Prob.	(0.0099)	(0.0024)	(0.9583)	(0.6767)	(0.2230)	(0.0019)	(0.8997)	(0.7229)
Std.Err	0.0276	2.2096	0.8216	0.5943	0.0726	2.2235	0.8840	0.6307
R^2	0.0117	0.0162	0.0000	0.0003	0.0079	0.0502	0.0001	0.0007

Note: This table shows estimates of the univariate predictive models based on lagged one-period equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates, respectively.

Table 3. 14 Bai-Perron Tests in the univariate models

	Monthly		Quarterly	
	No. of Breaks	Break Dates	No. of Breaks	Break Dates
DY(-1)	2	1974M12 1982M06	2	1974Q4 1982Q2
RF(-1)	1	1974M11	1	1974Q4
RPI(-1)	1	1974M09	1	1974Q4
EQ(-1)	0		0	

Table 3. 15 Estimates for univariate regression models with breaks

Panel A: Lagged one-period equity risk premium models				
	Monthly		Quarterly	
Sample period	1965M01 1974M10	1974M11 2012M05	1965Q1 1974Q3	1974Q4 2012Q1
Φ	0.1244	0.0979	0.4348	-0.0071
SE	0.0922	0.047	0.1675	0.0792
R^2	0.0154	0.0095	0.1542	0.0001
Panel B: Three-month Treasury bill rate models				
	Monthly		Quarterly	
Sample period	1965M01 1974M10	1974M11 2012M05	1965Q1 1974Q3	1974Q4 2012Q1
Φ	-15.2196	0.7840	-16.3828	0.6311
SE	3.3897	0.8327	3.4311	0.8820
R^2	0.1481	0.0020	0.3813	0.0034
Panel C: Inflation rate models				
	Monthly		Quarterly	
Sample period	1965M01 1974M10	1974M11 2012M05	1965Q1 1974Q3	1974Q4 2012Q1
Φ	-6.8869	1.0202	-7.7501	0.8458
SE	1.9691	0.6128	2.2841	0.1827
R^2	0.0954	0.0061	0.2373	0.0120

Panel D: Dividend yield models						
	Monthly			Quarterly		
Sample period	1965M01 1974M11	1974M12 1982M05	1982M06 2012M05	1965Q1 1974Q3	1974Q4 1982Q1	1982Q2 2012Q1
Φ	-11.0197	47.28801	8.8662	-7.6271	22.3101	9.3176
SE	5.3389	8.5998	3.4402	7.3674	6.7594	3.4850
R^2	0.0351	0.2557	0.0182	0.0282	0.2801	0.0571

Table 3. 16 Summary statistics in sub-samples

Sample period		EQ	DY	RF	RPI
1965M01 2012M05 (Full-sample)	Mean	0.0418	0.0415	0.0711	0.0596
	Median	0.1094	0.0401	0.0651	0.0440
	Maximum	5.0487	0.1115	0.1500	0.2382
	Minimum	-3.7597	0.0204	0.0030	-0.0161
	Std. Dev.	0.6601	0.0124	0.0339	0.0467
	Skewness	0.0094	1.1651	0.1350	1.6003
	Kurtosis	11.1920	6.6101	2.5913	5.4045
	Jarque-Bera	1591.0338	437.7114	5.6885	379.9434
	Probability	0.0000	0.0000	0.0582	0.0000
1965M01 1974M11	Mean	-0.0614	0.0447	0.0684	0.0656
	Median	0.0509	0.0422	0.0655	0.0564
	Maximum	1.2944	0.1115	0.1174	0.1681
	Minimum	-2.8441	0.0284	0.0418	0.0139
	Std. Dev.	0.7150	0.0136	0.0181	0.0335
	Skewness	-1.0545	2.1221	1.1419	1.1985
	Kurtosis	4.3702	9.5334	3.7301	4.2660
	Jarque-Bera	31.3611	300.9620	28.5048	36.4330
	Probability	0.0000	0.0000	0.0000	0.0000
1974M12 1982M05	Mean	0.1689	0.0580	0.1070	0.1409
	Median	0.1822	0.0563	0.1076	0.1406
	Maximum	5.0487	0.1107	0.1500	0.2382
	Minimum	-2.1709	0.0466	0.0443	0.0714
	Std. Dev.	0.9125	0.0080	0.0262	0.0452
	Skewness	1.7204	3.5235	-0.3710	0.4255
	Kurtosis	11.2166	22.8201	2.4973	2.3362
	Jarque-Bera	300.8050	1712.720	3.0788	4.6048
	Probability	0.0000	0.0000	0.2145	0.1000
1982M06 2007M10	Mean	0.0569	0.0365	0.0717	0.0381
	Median	0.1220	0.0374	0.0602	0.0325
	Maximum	1.4812	0.0591	0.1362	0.1035
	Minimum	-3.7597	0.0204	0.0326	0.0070
	Std. Dev.	0.5378	0.0087	0.0282	0.0195
	Skewness	-1.5288	0.0645	0.6277	1.1923
	Kurtosis	10.8461	2.2134	2.2329	4.1470
	Jarque-Bera	901.1541	8.0752	27.5044	88.9783
	Probability	0.0000	0.0176	0.0000	0.0000
2007M11 2012M05	Mean	-0.0268	0.0357	0.0147	0.0329
	Median	-0.0539	0.0334	0.0051	0.0421
	Maximum	1.1330	0.0524	0.0535	0.0535
	Minimum	-1.7297	0.0285	0.0030	-0.0161
	Std. Dev.	0.6393	0.0063	0.0181	0.0222
	Skewness	-0.4125	1.0993	1.3470	-1.2078
	Kurtosis	2.8339	3.2199	2.9174	2.8985
	Jarque-Bera	1.6233	11.1892	16.6474	13.3951
	Probability	0.4441	0.0037	0.0002	0.0012

3.4.7 Discussion and implications

The evidence found in this chapter suggests that the UK equity risk premium remained stationary over the period 1965 to 2012. However, the relationship between the equity risk premium and the forecasting variables changes over time. The structural break tests find two structural breaks in our sample period: December 1974 and June 1982. In addition, we have examined whether the recent financial crisis of 2007 has had a persistent effect on the stock market. Our results suggest that there was a breakdown in equity risk premium models in 2007. We then split our sample into four sub-samples to investigate the predictability of equity risk premia based on lagged one-period equity risk premia, lagged one-period dividend yields, lagged one-period inflation rates and Three-month Treasury bill rates lagged by one-period.

The first sub-period is from January 1965 to November 1974. This was the period of Keynesian demand management, during which the central banks operated a monetary policy and fiscal policy intended to affect the level of economic activity. Our results show that the three-month Treasury bill rates play an important role in determining the level of equity risk premia: the univariate model based on the three-month Treasury bill rates can explain 14% of the variation in monthly equity risk premia for 1965 to 1974. The second sub-sample period is from December 1974 to June 1982. These years were an anti-business period with high and increasing inflation. During this period, the oil crisis of 1973 caused a sharp increase in the price of oil and lead to high rates of inflation (the average inflation rate for this eight-year period was 0.1409). Dividend yields have strong predictability for equity risk premia during this period: the regression model based on dividend yields can explain 26% of the variation in the equity risk premium, while the multi-regression model based on all four explanatory variables can explain 47% of movements in the equity risk premium. The third period is from July 1982 to November 2007. This period was a business-friendly time of low inflation rates and low tax. The average inflation rate hovered around 3%, the average equity risk premium was 4.4% and the average dividend yield was 3.6%. In this period, the average rate of equity risk premia was much lower than the rate from 1974 to 1982. Our results are consistent with the findings of Lettau, Ludvigson and Wachter (2007). The decline in the equity risk premium can be explained by the decreasing volatility in real economic variables, such as inflation rates and interest rates. However, the predictive power for the equity risk premium model is also very low in this period. We cannot find significant evidence of the predictability in equity risk premium models based on all four forecasting variables. Ang and Bekaert (2007) point out that the predictability in stock markets is in general a short-term phenomenon because the market will eventually arbitrage it away once it is discovered. The globalisation of the markets, the integration of the international financial markets, the free-market economy, and global market capitalism further complicate the predictability of equity risk premia. These factors may even result in the disappearance of predictability in equity risk premium models based on our four candidate predictive variables in the third period. The last period which we have examined is from August 2007 to May 2012. The global financial crisis of 2007 has had a huge impact on global economics. Our estimation results suggest that three-month Treasury bill rates and inflation rates have significant predictive power for equity risk premia in this period. In fact, this period can be thought of as a Keynesian resurgence period. In 2008 and 2009, fiscal stimulus packages were widely launched across the world in order to stabilise economies over the global depression. As a result, three-month Treasury bill rates and inflation rates play a more important role in determining the equity risk premium in this period.

3.4.8 Power issues in dealing with structural breaks

This chapter examines the parameter instability in the equity risk premium models. The results of the structural break tests provide empirical evidence that the relationship between equity risk premia and candidate variables may change following structural breaks. Also, the results raise some issues in dealing with structural breaks.

Firstly, as discussed in Section 3.4.6, we should choose the smallest possible number of breaks. Allowing for too many breaks may lead to model misspecification because even a random walk can be explained by stationary process with many trend breaks. Secondly, a large number of outliers may affect the structural break test results. This can best be explained with our results from Table3.4. We find two break dates, July 1974 and February

1975, in the monthly equity risk premium. Indeed, one of them may better be described as an outlier because it seems unlikely that two structural breaks would happen within such a short time-period. Bai (1997) solved these two problems by using iterative refinements. We use Bai's (1997) method and exclude the first and last 15% of observations from the sample period to avoid the spurious breaks at the beginning and the end of the sample. This corresponds to a minimum seven years and one month between any two successive breaks. The minimum window length places a limitation on the combination of structural break points to avoid breaks occurring in consecutive years and therefore can reduce the possibility of selecting false, non-existent break points. However, this method may result in power issues if structural breaks are around or outside the boundary of test periods. In this case, the third problem for the structural break tests is the boundary issue. As discussed in Section 3.4.5, we find an additional structural break in 2007. The failure to find this break may be due to the fact it is outside the boundary. Therefore, the trimmed dataset may only have partial power and may not capture all the information contained in the whole sample period.

Lastly and most importantly, testing structural breaks in stock markets is extremely difficult because stock markets are volatile. Only a small percentage of the variation in equity risk premia can be explained by forecasting variables. This is one possible reason why structural break tests may have limited power to detect a sequence of smaller breaks even if they are actually present. Moreover, structural break tests only allow for a one-time switch at a specific date. Therefore, they may have the power to identify only extremely large breaks but may perform poorly in selecting breaks which occur gradually.

3.5 Conclusions

In this Chapter we have reviewed some widely used tests for structural breaks in the equity risk premium models.

The two most common approaches to predicting expected equity risk premia have been reviewed. The first approach uses the historical average realised returns. The second approach employs lagged financial variables, such as *fundamental values*. Both these methods fail to account for the possibility of structural breaks. Ignoring possible structural breaks in equity risk premium models can lead to huge forecasting errors and the unreliability of these models.

Therefore, tests for parameter instability and structural changes with both known and unknown break dates have been employed. Our studies have revealed that adding the structural break parameters is necessary and useful for modelling and forecasting equity risk premia. However, there are still many potential problems highlighted by the structural break literature. The Ordinary Least Square (OLS) method assumes that time series are stationary. The stationarity of data should be tested before we explore the long-run relationship between equity risk premia and explanatory variables. The unit root tests are employed to test if equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates are stationary over sample period. Our tests suggest that the equity risk premium and dividend yield are stationary, while the three-month Treasury bill rate and the inflation rates are break-stationary.

We then use the structural break tests to examine if the relationship between equity risk premia and forecasting variables are stable over time. An important limitation of the Chow test is that it assumes that the date of the break is known. In most cases, the date of the break point is difficult to ascertain in advance. Some structural breaks may happen gradually. Some structural breaks may not even be directly observable. The CUSUM test, the CUSUMSQ test and the Quandt-Andrews unknown breakpoint test can be used to find if there is an unknown structural break during our sample period. The problem with the CUSUMSQ test is that it only has trivial local power for local changes that specify a one-time change in parameters; while the Quandt-Andrews unknown breakpoint test is designed to test for one break only and thus may have weaker power for multiple changes. The Bai-Perron (BP) break point test was therefore introduced to test for unknown multiple structural breaks in a linear regression framework. The basic idea of the BP test is to identify the break points by minimising the sum of squared residuals for all observations. In the BP test, the break points are treated as unknown parameters and are estimated at the same time as the regression coefficients.

To summarise, structural break tests clearly reject the null hypothesis of no structural break and hence reject the hypothesis of no switching in the equity risk premium model of the UK stock market. The results therefore suggest that regime switching is an important characteristic of equity risk premia and should not be ignored by investors when making their asset allocation decisions. In the next Chapter, Regime Switching Models are considered. Traditionally, equity risk premium models have been estimated assuming a single regime exists. Regime switching models allow for the existence of two, or more, regimes. In addition, the Markov switching regime model of Hamilton (1989) is estimated using the entire sample period and is capable of capturing the dynamic changes in equity risk premium models even with small, frequent changes. Three different regime switching models will be introduced including the structural break model, the threshold model and the Markov switching regime model.

Appendix 3 Figures

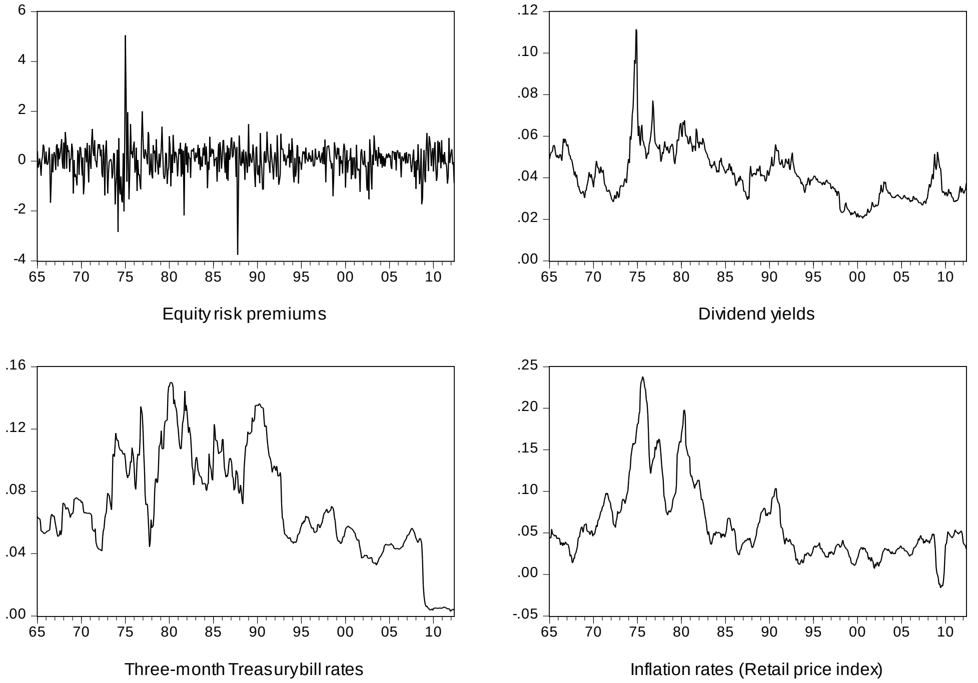


Figure 3. 1 Annualised UK monthly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates

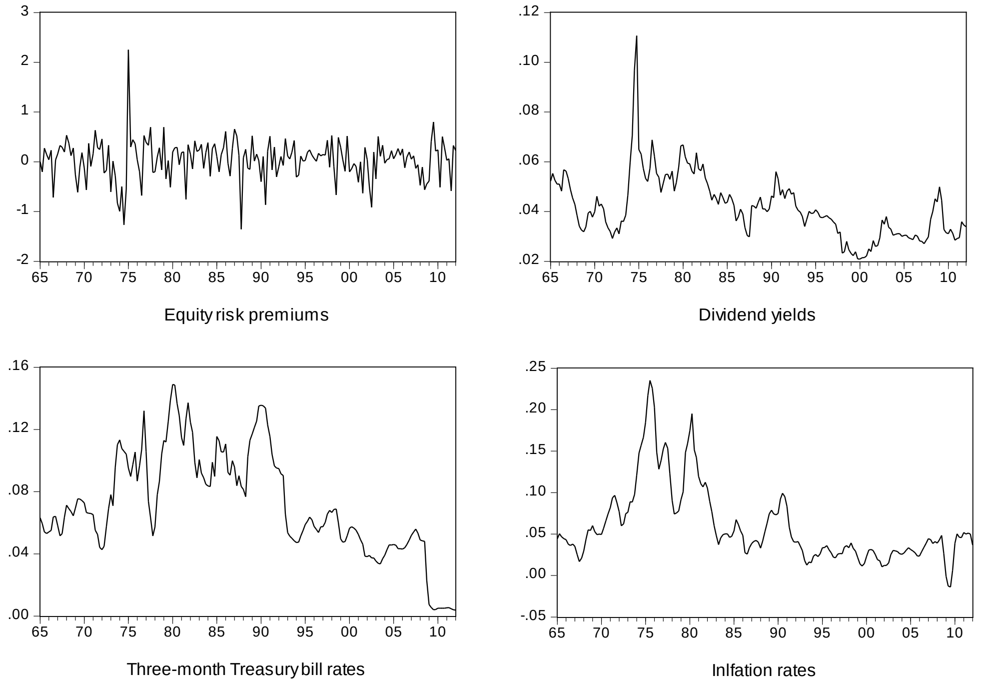


Figure 3. 2 Annualised UK quarterly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates

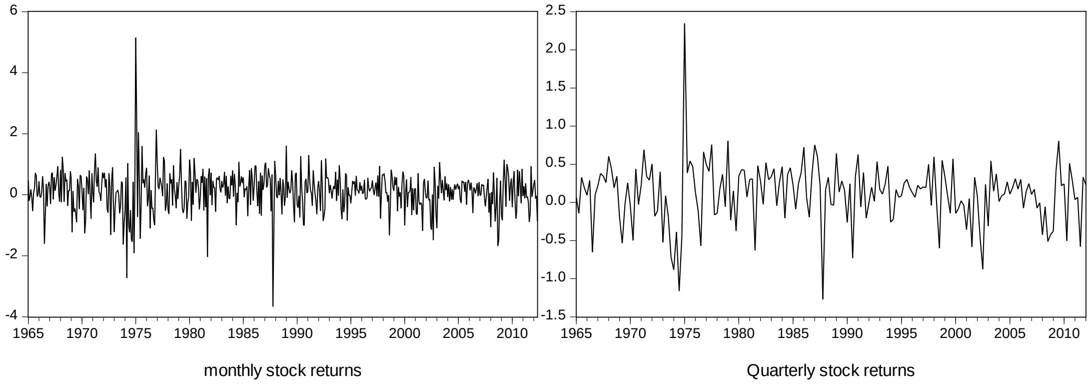


Figure 3. 3 Annualised UK monthly and quarterly stock returns

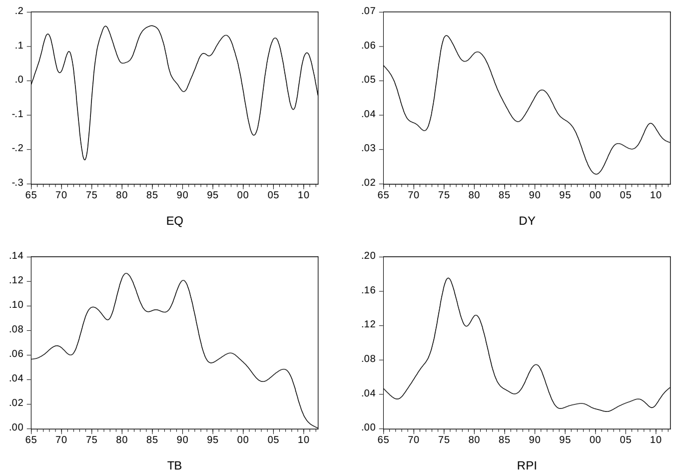


Figure 3. 4 De-trended UK monthly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates using HP Filter

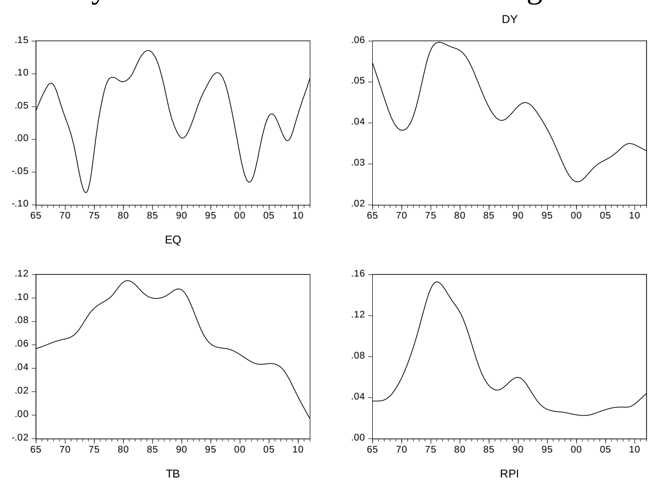
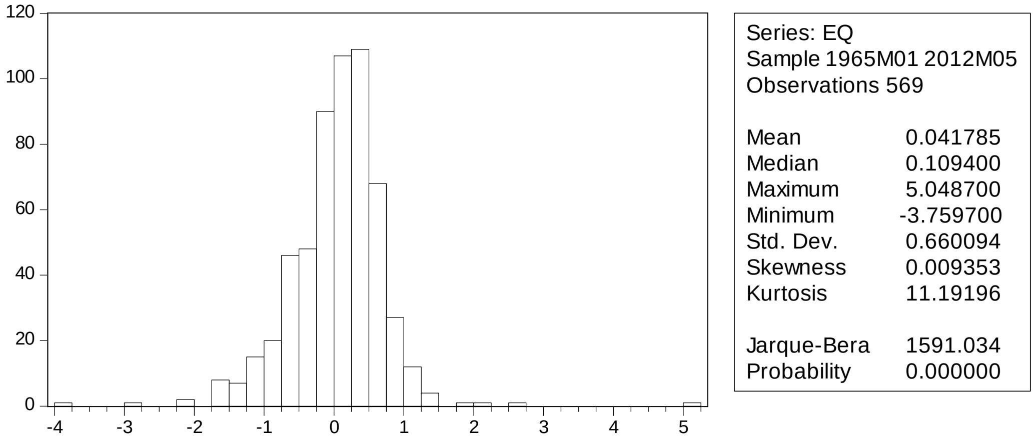
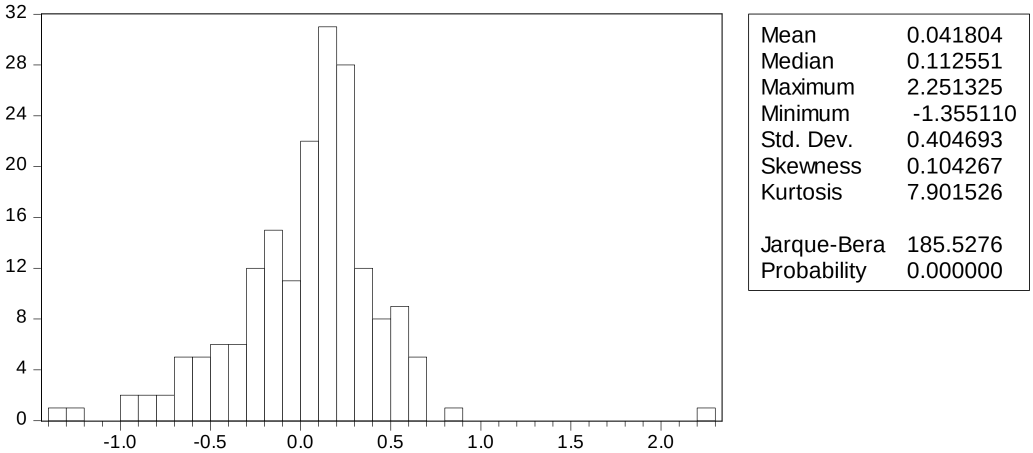


Figure 3. 5 De-trended UK quarterly equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates using HP Filter

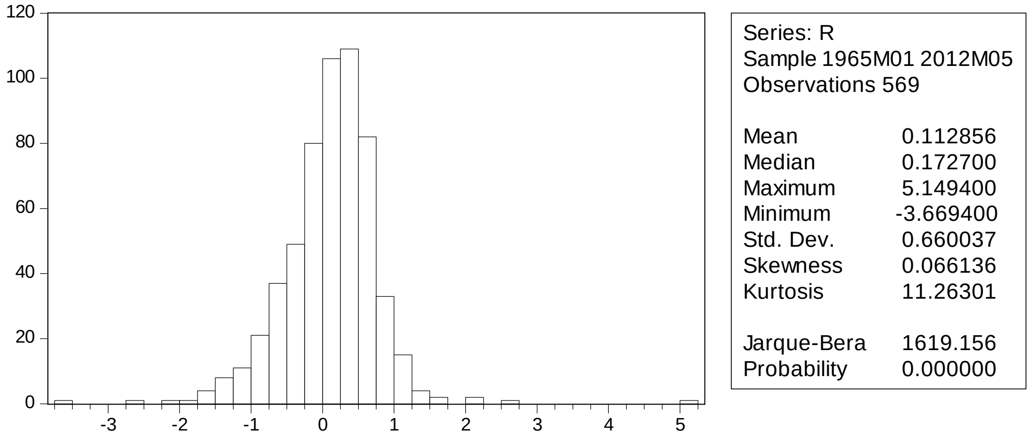


Distribution of UK monthly equity risk premia

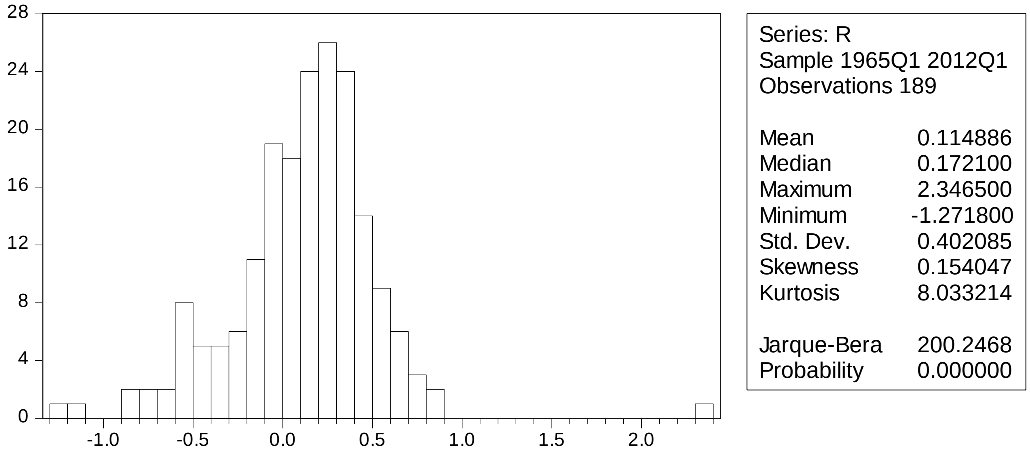


Distribution of UK quarterly equity risk premia

Figure 3. 6 Distribution of UK equity risk premia at the monthly and quarterly frequencies



Distribution of the UK monthly stock returns



Distribution of the UK quarterly stock returns

Figure 3. 7 Distribution of UK stock returns at the monthly and quarterly frequencies

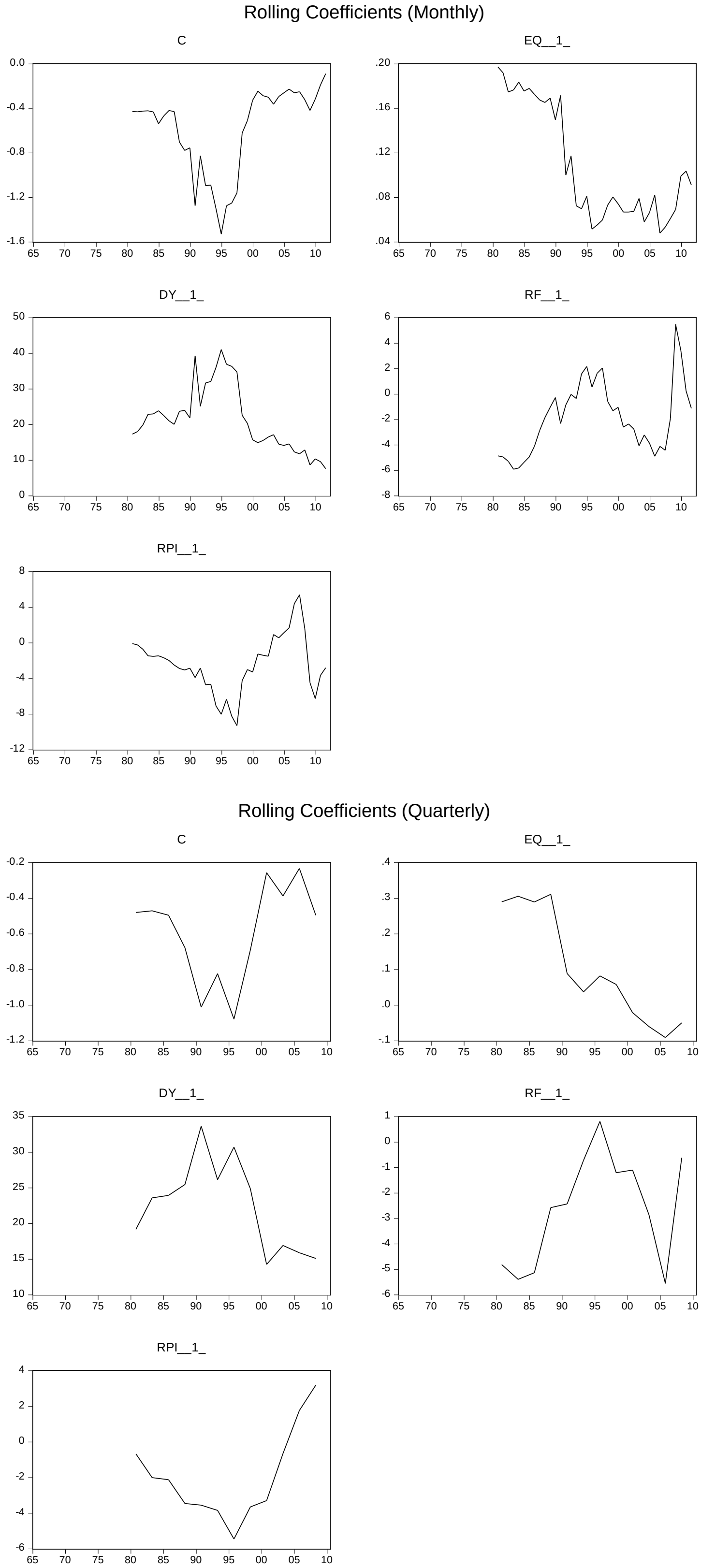
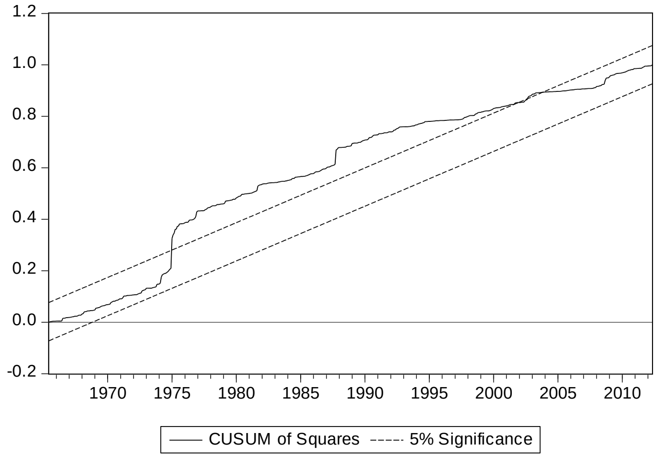
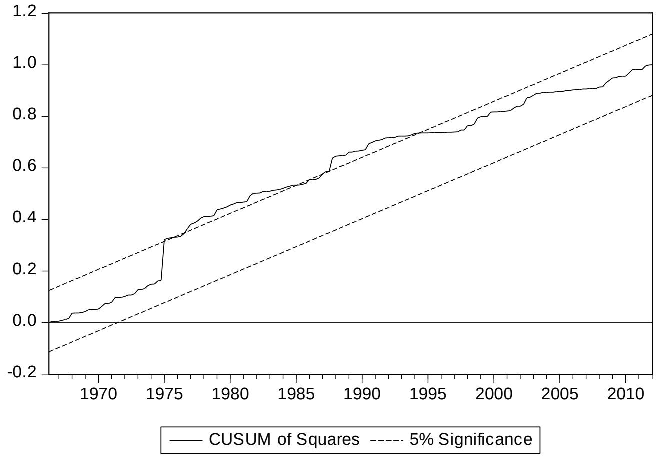


Figure 3. 8 Rolling regression estimates for equity risk premium models



Monthly



Quarterly

Figure 3. 9 CUSUMSQ tests for equity risk premium model at the monthly and quarterly frequencies

Chapter 4 Regime switching behaviour in the UK equity risk premium: 1965-2012

4.1 Introduction

This chapter studies the regime switching behaviour of equity risk premia. Traditionally, equity risk premia are estimated using a single regime model. Switching models allow for the existence of multiple regimes and assume that the data generating processes of financial time series can change across different regimes, each of which is described by different parameters.

In this chapter, three univariate regime-switching models are introduced: the structural break model, the threshold autoregressive model of Tong (1978, 1980, and 1990), and the Markov switching regime model of Hamilton (1989). Both the structural break models and the threshold autoregressive models (TAR) assume that the switching mechanism is deterministic. In the case of structural break models, allowance is made for a one-time switch only, occurring at a specific break date, and the state variable is solely determined by time. With TAR models, the state variable is determined by an observable variable with respect to an unobserved threshold and therefore each observation is allowed to switch. In contrast, Hamilton (1989) used a Markov process to model and forecast the regime-switching behaviour in financial time series. In Hamilton's model, dynamic time series are allowed to switch probabilistically between different regimes. This switching is controlled by a state variable which is not directly observable and is therefore latent. Hamilton (1989) assumed this state variable to follow a Markov process.

Univariate regime models can capture the dynamic switching behaviour in financial time series. However, they do not have the ability to capture the co-movements among multiple macroeconomic variables. To investigate the asymmetric relationships among multiple financial variables over different regimes, we extend regime switching models to a vector autoregression (VAR) framework. Threshold vector autoregressive models (TVAR) and Markov switching vector autoregressive (MS-VAR) models are employed.

These regime switching models are useful because they may provide a better fit with the data and a better explanation for the existence of different regimes. The objective of this chapter is to investigate techniques to model equity risk premia which are subject to switching regimes. In order to further understand the switching behaviour of equity risk premia, six models are considered in this study:

- Model I: Simple linear AR model.
- Model II: Structural break model.
- Model III: Threshold autoregressive models.
- Model IV: Markov switching regime models.

- Model V: Threshold vector autoregressive models.
- Model VI: Markov vector switching regime models.

The rest of this chapter is structured as follows: Section 2 is a literature review on the estimation of regime-switching equity risk premium models; Section 3 discusses the technical details of regime-switching models; Section 4 presents the empirical applications of these to the UK equity risk premium; and finally section 5 summarises and concludes.

4.2 Literature Review

It is now well known that most dynamic financial time series display non-linear behaviour. Campbell *et al.* (1997: 467) emphasised that "a natural frontier for financial econometrics is the modelling of non-linear phenomena". In order to capture the non-linear movements in the equity risk premium, regime switching models have been introduced as an alternative to the traditional linear model. These models have attracted considerable attention for three reasons. First, regime switching models can be used to model the cyclical behaviour of stock markets. Stock market trends show that over a period of time the market alternates between bull and bear markets, which in turn suggests a cyclical and regime switching behaviour in the equity risk premium. Second, extreme events may occur from time to time which cause changes in the underlying trends of stock markets. For example, there may be no advance notice of the arrival of a bear market, or a bull market, and the stock market switches randomly between these two. Third, the variation in the equity risk premium is correlated to business conditions. Equity risk premia change cyclically, and tend to be higher during recessions than during expansions. However, traditional models do not take business cycles into consideration (Lucas, 1987). The simple independently and identically distributed (i.i.d.) assumption does not account for the 'cyclical' behaviour of equity risk premia. Therefore, the traditional linear models may fail to capture the stochastic variability of equity risk premia and cannot explain what would happen if there was an unexpected shift in these data. Regime switching models are employed to help investors describe and forecast equity risk premia related to business conditions. The question, therefore, is how to describe and analyse the switching characteristics of equity risk premia.

The simplest regime switching model perhaps is the structural break model. Levy (1974) suggested using real stock returns to calculate beta coefficients separately for bull and bear markets. He assumed that the date when structural breaks happened was observable, that is, the regime that the time series was in at time t can be determined by an observable turning date. His model can be thought of as a simple dummy variable model, with a dummy variable (S_t) equal to 0 in the bear market and 1 in the bull market. The Chow (1960) test can be applied to test for the presence of a structural break if all subsamples are known. However, the date of the turning point is difficult to ascertain in advance. Quandt (1958, 1960) proposed a switching regime model with an unknown turning point. Similar regressions were also modelled by

Farley and Hinich (1970), Kim and Sigegmund (1989). While Brown, Durbin, and Evans (1975), Ploberger Kramer, and Kontrus (1989) extended one switch models to permit multiple switches, but they did not model the variable that governed this switching. Tong (1978), Tong and Lim (1980) and Tong (1990) achieved this by proposing the threshold autoregressive (TAR) model. In their model, the regime that the time series was in at time t can be determined by the value of an observable variable with unobservable thresholds, for example, a lagged value of the time series itself. The TAR model can also be thought of as a simple dummy variable model, with a dummy variable (S_t) equal to 1 if the observable variable that governed the switching is greater than a threshold value c and 0 otherwise. There have been many different applications that use TAR models to explain non-linear phenomena observed in economic time series. For example, Beaudry and Koop (1993) use the TAR framework to model GNP growth rates. Pesarn and Potter (1997) extend Beaudry and Koop (1993) work to show that US GNP is subject to floor and ceiling effects.

The structural break models and the TAR models assume that the regimes either can be observed in time or can be determined by an observable variable. In practice, we can never be sure about the regime in which time series are at a particular point t in time, that is, the regime is not actually observable. Quandt (1972) was the first to assume that the latent switching mechanism between two regimes followed a stochastic process. Blanchard and Watson (1982) proposed a stochastic bubble model to describe the regime-changing properties of stock returns. In their specification, the bubble either survived with a probability *p* or burst with the probability 1-*p* at time *t*. In other words, the stock price grew randomly and might be drawn from two different distributions which were the “survive bubble” and the “burst bubble”.

The regime switching models mentioned so far all assume that regime-switching is only affected by the current state and does not depend on the previous state. Goldfeld and Quandt (1973) were the first to introduce the Markov process to permit time-dependence. Under their assumptions, the time when a change happened was endogenous to the model and was decided by past information. They also assumed that volatility can take *k* different discrete values and that switches happen randomly. A transition matrix of the Markov process is used to calculate the probability of changes between different regimes. Cosslett and Lee (1985) calculated the Log-likelihood function for this model. Hamilton (1989, 1994, and 1996) further developed the Goldfeld and Quandt (1973) model to incorporate autoregressive elements. In his model, financial time series might switch randomly between different regimes at any time. A state variable S_t was used to indicate the regime which the financial time series was in at time *t*. The state variable was unobservable and followed a Markov process. Hamilton also used a recursive filter to make inference about the state variable conditional on information up to time *t*. Kim (1994) developed a calculation algorithm called ‘smoothed probabilities’ to infer the state variable by full information in the sample. There have been many different applications of Markov switching regime models in finance, including investigations of: the long swing in nominal exchange rates (Engel and Hamilton, 1990; Engel, 1994), the term structure of interest rates (Hamilton, 1988,1989), the identification and prediction of the US business cycle (Hamilton, 1989; Lam, 1990; Birchenhall *et al*.1999), the asset returns in the stock market (Turner, Startz, and Nelson 1990), the modelling of government expenditure (Rugemurcia, 1995), the dynamics of asset prices (Schwert, 1989,1996; Pagan and Schwert, 1990), and consumption, output and dividends dynamics (Driffill and Sola, 1998; Cecchetti, Lam and Mark,1990).

As discussed above, regime switching models can capture the dynamic switching behaviour in financial time series. They achieve this by introducing a state variable to describe different market regimes, such as a 'good regime' and a 'bad regime'. However, stock markets are often volatile. There are many financial variables that may affect the overall performance of stock markets, and these variables may interact with each other. Also, business cycles are the result of the co-movements of many macroeconomic variables. In such a framework, the univariate regime switching model may not have sufficient power to capture the co-movements in multiple financial time series. Sims (1980) introduced a VAR model which extended univariate autoregression models to the dynamic multivariate context to examine the linear inter-dependence between multiple time series. Tsay (1998) generalised the univariate threshold model to a multivariate framework. Lo and Zivot (2001) extended the SETAR model to the threshold vector autoregressive model (TVAR). In these models, the regimes are determined by the value of the threshold variable, which is one of the endogenous variables in the VAR. However, stock markets are characterised by a very high degree of volatility. As discussed in chapter 3, the predictive ability of forecasting variables with respect to the variation in the equity risk premium may change over time. Since estimation samples often span a long time-period, the threshold variable may not be observable. Also, the threshold variable may itself change within a particular time-period. Therefore, it should not be assumed that the switching mechanism is deterministic. To study the co-movements between financial time series, Krolzig (1997, 1998 and 2003) proposed the Markov switching vector autoregressive models, in which the unobservable state variable follows a Markov process.

This section has provided a literature review of regime switching models. In order to understand the importance of regime switching behaviour in equity risk premium models, we allow equity risk premia to switch between different regimes within the sample period in this chapter. Technical details of switching-regime models are introduced in the next section.

4.3 Methodology

In this section, we consider six specifications for estimating equity risk premia. Model I assumes a basic regression model without breaks, Model II uses simple structural break models, Model III introduces threshold autoregressive (TAR) models, and Model IV employs

Markov switching regime models. We then extend regime switching to a VAR framework. Two models are introduced including the Threshold Vector Autoregressive (TVAR) models and the Markov regime switching Vector Autoregressive models (MS-VAR).

4.3.1 Regime switching in univariate models

a) Model I: AR(1) with no Switching

In Model I, we assume that equity risk premia are drawn from one distribution over the sample period, i.e., no switching is allowed. Here we describe time-series models of the equity risk premium where current values of the premium are based on its lagged values. The simplest of these models is the auto-regressive of order one (AR(1)), where the current value of the equity risk premium is a function of its own value in the last period, plus a constant, plus a random shock.

The equity premium model with first order auto-regression can be defined as,

$$\begin{aligned} &R_t = c + \phi R_{t-1} + \varepsilon_t \quad t = 0,1,2,\cdots,T \\ &\varepsilon_t \sim i.i.d.N(0,\sigma^2) \end{aligned} \tag{4.1}$$

where R_t denotes the equity premium at time *t*, *T* is the sample size, *c* is a constant, ϕ is the autoregressive coefficient and ε_t is the error term at time *t*. The parameters can be estimated by maximising the log-likelihood function with respect to *c*, ϕ and σ^2.

The log likelihood function is,

$$\begin{aligned}\ln L(\theta) &= \sum_{t=1}^{T} \ln(f(R_t;\theta)) \\ &= -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\left[R_t - c - \phi R_{t-1})\right]^2\end{aligned} \tag{4.2}$$

Here θ is a parameter vector including c, ϕ and σ^2.

Maximum log-likelihood estimation is based on a set of assumptions. One important assumption is that the error terms are identically and independently distributed for the whole sample period. Under this assumption, the maximum log-likelihood estimators are equivalent to the least squares estimators. However, equity risk premia may be drawn from two or more distributions during the sample period. As such, Maximum log-likelihood estimators may not perform well and the standard linear models may be inadequate to describe the dynamic switching behaviour of equity risk premia. In the following chapter, we review three regime switching models which are commonly used to capture non-linear features in financial time series models.

b) Model II: Switching with an observable turning date

In Model II, we consider a switching model with an observable turning date. We start by assuming that we know the date when the structural break happened. Under this assumption, the whole sample period can be divided into two subsamples and the model can be described by two equations.

$$\begin{aligned} &R_t = c_1 + \phi_1 R_{t-1} + \varepsilon_{1t}\ ,\ t = 0,1,2,\cdots,t_1 \\ &\varepsilon_{1t} \sim N(0,\sigma_1^2) \\ &R_t = c_2 + \phi_2 R_{t-1} + \varepsilon_{2t}\ ,\ t = t_1+1, t_1+2,\cdots,T \\ &\varepsilon_{2t} \sim N(0,\sigma_2^2) \end{aligned} \tag{4.3}$$

where t_1 is the date when structural break happened, c_1 and c_2 are constants, ϕ_1 and ϕ_2 are the autoregressive coefficients, and $\varepsilon_{1t}$and ε_{2t} are the error terms at time t, for regime 1 and regime 2, respectively. Here, t_1 is assumed to be observable.

In order to generalise equation (4.3), we can define a dummy variable 'S_t' that is 1 if data are from regime 1, and 2 if data are from regime 2. S_t can be thought of as a state variable which indicates the regime which the time series are in at time t. Equation (4.3) can be written as more general case:

$$\begin{aligned} &R_t = c_{s_t} + \phi_{s_t} R_{t-1} + \varepsilon_{s_t} \quad t = 1,2,\cdots,T \\ &\varepsilon_{s_t} \sim N(0,\sigma_{s_t}^2) \end{aligned} \tag{4.4}$$

with

$$S_t = \begin{cases} 1 & t = 1,2,\cdots,t_1 \\ 2 & t = t_1+1,\cdots,\ T \end{cases} \tag{4.5}$$

The parameters can be estimated by maximising the log-likelihood function with respect to $c_1, c_2, \phi_1, \phi_2, \sigma_1$ and σ_2. The log-likelihood function can be written as

$$\ln L(\theta) = \sum_{t=1}^{T} \ln(f(R_t \mid S_t;\theta)) \tag{4.6}$$

Here, θ=$\{c_1, c_2, \phi_1, \phi_2, \sigma_1, \sigma_2\}$ and,

$$f(R_t \mid S_t;\theta) = \frac{1}{\sqrt{2\pi\sigma_{s_t}}}\exp\left(-\frac{[R_t - c_{s_t} - \phi_{s_t}R_{t-1}]^2}{2\sigma_{s_t}{}^2}\right) \tag{4.7}$$

The basic idea of this structural break model is to divide the whole sample period into two subsamples with an observable break date. Under this model, the one-off switching happens on a specific date and splits the whole sample into two regimes: regime 1 and regime 2. This pre-determined break date is not estimated together with the model parameters and is therefore exogenous to the model. However, as discussed in Chapter 3.3.3, the structural break model assumed observable breaks may not perform well if there are some imperfectly predictable structural changes during the sample period. Moreover, the switching mechanism in this model is only determined by time. In other words, the dummy state variable is determined by whether the time is before or after the break date. The Bai-Perron structural break tests can select break dates endogenously by data. However, as discussed in section 3.4.8, these tests have some flaws. In particular, they only allow for one-time switches and do not allow for recurrent switching within a short time period. This is the reason why structural break tests may have limited power to detect a sequence of smaller breaks even if these breaks are actually present. To solve this problem, switching regime models can be introduced in which the switching is determined by an observable variable.

c) Model III: Switching determined by an observable variable

In the threshold models, switches between different regimes are governed by an observable variable, i.e. the threshold variable, and model parameters change according to the value of this variable. A two-regime threshold model is as follows

$$R_t = \begin{cases} A_1 X_t + \varepsilon_{1t} & \text{if } q_{t-d} \le r \\ A_2 X_t + \varepsilon_{2t} & \text{if } q_{t-d} > r \end{cases} \quad t = 1, 2 \cdots T \tag{4.8}$$

where q_{t-d} is the threshold variable, *d* is the threshold lag, i.e., the delay parameter, and *r* is the threshold value. In equation (4.8), the threshold variable is assumed to be observable. It divides the whole sample into two regimes. The dependent variable R_t will be in the first regime if the threshold variable q_{t-d} is less than or equal to the threshold value *r*, while if q_{t-d} is greater than *r*, R_t will be in the second regime. In this case, although the time series R_t follows a linear process within each regime, the existence of regime switching suggests that the entire time series will have non-linear characteristics.

The self-exciting threshold autoregressive Model (SETAR) of Tong (1978, 1980, and 1990) is an extension of threshold models. It assumes that the threshold variable q_{t-d} and the predictive variable X_t are the lagged values of the time series itself, i.e., $q_{t-d} = R_{t-d}$ and $X_t = R_{t-1}$.

$$R_t = \begin{cases} c_1 + \phi_1 R_{t-1} + \varepsilon_{1t} & \text{if } R_{t-d} \le r \\ c_2 + \phi_2 R_{t-1} + \varepsilon_{1t} & \text{if } R_{t-d} > r \end{cases} \tag{4.9}$$

One feature of the SETAR model compared to the threshold model is that the threshold variable is endogenous, so that we do not need a separate state variable q_{t-d} to identify if a regime switch has occurred. Here, we consider three different cases: the case where the delay parameter and the threshold are known, the case where the threshold value is unknown, and the case where both the delay parameter and the threshold are unknown.

First, let us consider the case where the delay parameter d and the threshold value r are known *a priori*, that is, the regime at time t is also known *a priori*. For example, if $d = 1$, and $r = 0$, Equation (4.9) can simply be expressed as Equation (4.4) with:

$$S_t = \begin{cases} 1 & \text{if } R_{t-1} \leq 0 \\ 2 & \text{if } R_{t-1} > 0 \end{cases} \tag{4.10}$$

The whole sample period is divided into two regimes based on whether the threshold variable R_{t-1} is greater than or less than the threshold value 0. The state variable S_t is an indicator variable, which takes the value 0 if the observations are from the negative regime and 1 if the observations are from the positive regime. Under this structure, the above model is nothing more than two separate models. The estimation is straightforward. Each equation of Model (4.9) can be estimated separately by OLS.

Second, let us consider the case where the value of the threshold is unknown. We assume that the regime that the time series is in at time t is still determined by the value of R_{t-1}, but the threshold value r is not known.

We can re-write equation (4.4) as:

$$S_t = \begin{cases} 1 & \text{if } R_{t-1} \le r \\ 2 & \text{if } R_{t-1} > r \end{cases} \tag{4.11}$$

where r is unknown.

Since the threshold value r is unknown, the equation parameters are conditional on the value of r. Therefore, the threshold value r must be estimated together with the equation parameters. Chan (1993) showed that the conditional least square estimators of the parameters of the threshold model, including the threshold parameter, are super-consistent and asymptotically normally distributed. Following the same algorithm as Hansen (1997), the consistent estimators of Model (4.11) can be obtained by using sequential conditional least square (SCLS). The basic idea of SCLS is to minimise the sum of squared residuals with respect to the parameters. The following algorithm is typically employed. Step 1: sort the data in ascending order based on the threshold variable R_{t-1} and estimate Model (4.11), in each case setting the threshold value r to equal each observed value of R_{t-1} and in each case calculate the residual sum of squares. To put it simply, for a given value of r, we estimate the model using OLS and then repeat this approach for each potential r. Step 2: select the value of the threshold parameter r which minimises the residual sum of squares. In general, we exclude the highest and lowest 15% of observations from the search to avoid the spurious estimates at the beginning and the end of the sample (the remaining sample is termed a trimmed dataset).

Each observation within the trimmed sample can be considered as a potential candidate of the threshold parameter.

Third, the SCLS estimation method introduced above can be used where both the delay parameter d and the threshold parameter r are unknown. Recalling Equation (4.9), the regime that time series R_t is in at time t is determined according to the value of R_{t-d}. The whole sample period is divided into two regimes based on whether the threshold variable R_{t-d} is greater than or less than the threshold value r. The SCLS method allows parameters d and r can be estimated together with the equation parameters. We estimate Equation (4.9) for each possible value of d and r. The consistent estimates of d and r are the ones which yield the smallest value of the residual sum of squares.

The structural break model is a special case of threshold models, where the threshold variable q_{t-d} is time t. The basic idea of the SCLS is to transform a threshold model into a structural break model by sorting the data in ascending order based on the threshold variable. In this case, the estimation method for threshold models with an unknown threshold value is largely similar to the method of identifying unknown structural break dates discussed in Chapter 3. However, there is one important difference. In most empirical cases, the predictive variables contain the threshold variable. Therefore, sorting the data by the threshold variable may lead to a trending regressor. Hansen (1992, 2000) and Chu and White (1992) point out that the structural break tests with a trending regressor follow non-standard asymptotic distributions.

To summarise, both structural break models and threshold autoregressive models assume that regime changes are deterministic. The former allows only a single structural break, and the state variable is determined solely by time. Under the latter, multiple changes are allowed and the state variable is determined by an observable variable with respect to an unobserved threshold. However, changes in equity risk premia may not be observable: that is, the regime in which the time series are at time *t* is very difficult to observe. Moreover, the predictability of equity risk premia based on predictive variables changes over time. Therefore, the threshold variable which determines switching in financial data may also change over time. In this case, both structural break models and TAR models may not perform well if there are some imperfectly predictable structural changes during the sample period. The question addressed here is how to estimate parameters without knowing the state variable in advance; i.e., where we do not know by which regime equity risk premia are generated at time *t*. In the next section, Hamilton's (1989) Markov switching regime model will be employed to solve these issues.

d) Model IV: Switching determined by an unobservable variable

According to Hamilton (1989), the Markov switching regime model assumes that the regime switching is determined by an unobservable state variable which follows a Markov process. This unobservable state variable is a latent variable. In statistics, a latent variable is a variable that is not directly observed, but can be inferred from other observed variables by a mathematical model. In the Markov regime switching framework, the latent state variable S_t is assumed to follow a Markov process with constant transition probabilities. Under such assumptions, the state variable can be estimated along with the model parameters using Hamilton filter (1989). The Markov process and the Markov property are introduced below.

Markov (1906) defined a stochastic process as a Markov process if the probabilities of future values in a time series only depend on its most recent value and are independent of earlier periods, that is, the value of the current can capture all information for its prior.

$$Pr\left(X_{n+1} = x \middle| X_n = x_n, X_{n-1} = x_{n-1} \cdots X_1 = x_1\right) = Pr\left(X_{n+1} = x \middle| X_n = x_n\right)$$

In the Markov switching regime model, time series may change to another state, or stay in the current state at any time. The probability matrix is called the transition matrix. In this study, the transition matrix for a two state, first order Markov chain is

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}$$

where $p_{i,j=1,2}$ denotes the probability that the time series move from regime j to i. In other words, it is the probability that R_t is in the regime i conditional on which R_{t-1} is in the regime j. $p_{i,j=1,2}$ is,

$$p_{ij} = Pr(S_t = i \mid S_{t-1} = j)$$

where $p_{11} + p_{12} = 1$ and $p_{21} + p_{22} = 1$.

Under the Markov switching approach, the equity risk premium model is a combination of two different linear models, each with different coefficients. Equity risk premia may switch randomly from one regime to another. The switching mechanism is controlled by an unobservable state variable S_t, and it follows a Markov process. Recall Equation (4.4), in the case of the Markov switching regime model, the state variable S_t is still defined as

$$S_t = \begin{cases} 1 & \text{if Regime 1} \\ 2 & \text{if Regime 2} \end{cases} \tag{4.12}$$

To estimate the state variable along with the model parameters, Hamilton (1989) proposed an iterative algorithm for calculating the probability distribution of the unobservable state variable. In Hamilton's approach, the parameters can still be estimated by maximising log-likelihood function with respect to θ. Rewrite the log-likelihood function

$$\ln L(\theta) = \sum_{t=1}^{T} \ln(f(R_t \mid \Omega_{t-1}; \theta)) \tag{4.13}$$

where $\Omega_{t-1} = \{R_1, R_2, \cdots, R_{t-1}\}$ denotes all past information at time t.

Since the state variable cannot be directly observed, that is, we can never be sure about the state that the equity risk premium lies in at a particular point t, the density function of $f(R_t \mid \Omega_{t-1};\theta)$ must be expressed by summing all possible values of S_t. Here, S_t takes the value of 1 or 2.

$$\begin{aligned} f(R_t \mid \Omega_{t-1};\theta) &= \sum_{S_t=1}^{2} f(R_t, S_t \mid \Omega_{t-1};\theta) \\ &= \sum_{S_t=1}^{2} f(R_t \mid S_t, \Omega_{t-1};\theta) f(S_t \mid \Omega_{t-1};\theta) \qquad (4.14) \\ &= \sum_{S_t=1}^{2} \frac{1}{\sqrt{2\pi\sigma_{s_t}}} \exp\left(-\frac{\left[R_t - c_{s_t} - \phi_{s_t} R_{t-1}\right]^2}{2\sigma_{s_t}{}^2}\right) \Pr(S_t \mid \Omega_{t-1};\theta) \end{aligned}$$

To calculate the density equation (4.14), we need to infer probabilities of the state variable S_t conditioned on past information, that is, $Pr(S_t = 1|\Omega_{t-1};\ \theta)$ and $Pr(S_t = 2|\Omega_{t-1};\theta)$. If the state variable S_t does not depend on its past values, then the probability function of S_t conditional on θ and Ω_{t-1} is very simple. It can be defined as p, that is, $Pr(S_t = 1|\Omega_{t-1};\ \theta) = p$ and $Pr(S_t = 2|\Omega_{t-1};\theta) = 1 - p$. We can maximise the log-likelihood function to estimate the parameters with respect to $c_1, c_2,\ \phi_1, \phi_2, \sigma_1, \sigma_2$ and p. However, the state variable always displays auto-regression, i.e., S_t may depend on its own past values. In this study, we assume that the state variable at time t follows a first order Markov chain, i.e., S_t only depends on its most recent value S_{t-1}. The vector of ergodic probabilities is denoted by π. It is the unconditional probability (2×1) matrix that the equity premium at time t will be in regime 1 or regime 2.

It takes the form:

$$\pi = \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} \Pr(s_t = 1;\theta) \\ \Pr(s_t = 2;\theta) \end{pmatrix} = \begin{pmatrix} \dfrac{1-p_{22}}{2-p_{11}-p_{22}} \\ \dfrac{1-p_{11}}{2-p_{11}-p_{22}} \end{pmatrix} \tag{4.15}$$

As assumed, the state variable S_t follows a first order Markov chain. The unconditional probability function $Pr(S_t = i|\Omega_{t-1})$ can be calculate by summing all possible values of j, that is,

$$\begin{aligned} \Pr\left(S_t = i \mid \Omega_{t-1};\theta\right) &= \sum_{j=1}^{2} \Pr\left(S_t = i, S_{t-1} = j \mid \Omega_{t-1};\theta\right) \\ &= \sum_{j=1}^{2} \Pr\left(S_t = i \mid S_{t-1} = j, \Omega_{t-1};\theta\right) \Pr\left(S_{t-1} = j \mid \Omega_{t-1};\theta\right) \\ &= \sum_{j=1}^{2} \Pr\left(S_t = i \mid S_{t-1} = j\right) \Pr\left(S_{t-1} = j \mid \Omega_{t-1};\theta\right) \\ &= \sum_{j=1}^{2} p_{ij} \ \Pr\left(S_{t-1} = j \mid \Omega_{t-1};\theta\right) \end{aligned} \tag{4.16}$$

The log-likelihood function (4.13) can be written as,

$$\begin{aligned} \ln L(\theta) &= \sum_{t=1}^{T} \ln(f(R_t \mid \Omega_{t-1};\theta)) \\ &= \sum_{t=1}^{T} \ln\{\sum_{S_t=1}^{2} \frac{1}{\sqrt{2\pi\sigma_{s_t}}} \exp(-\frac{(R_t - c_{s_t} - \phi_{s_t} R_{t-1})^2}{2\sigma_{s_t}^{\ 2}}) \Pr(S_t \mid \Omega_{t-1};\theta)\} \end{aligned} \tag{4.17}$$

When we know the value of R_t at the end of time t, it is easy to update the probability function $Pr(S_t = i|\Omega_t;\theta)$ as

$$\begin{aligned}\Pr(S_t = i\,|\,\Omega_t;\theta) &= \Pr(S_t = i\,|\,\Omega_{t-1}, R_t;\theta) \\ &= \frac{\Pr(S_t = i, R_t\,|\,\Omega_{t-1};\theta)}{\Pr(R_t\,|\,\Omega_{t-1},\theta)} \\ &= \frac{f(R_t\,|\,S_t = i,\Omega_{t-1};\theta)\Pr(S_t = i\,|\,\Omega_{t-1};\theta)}{\sum_{i=1}^{2} f(R_t\,|\,S_t = i,\Omega_{t-1};\theta)\Pr(S_t = i\,|\,\Omega_{t-1};\theta)}\end{aligned} \tag{4.18}$$

Therefore, given the initial values $\theta^{(0)}=\left\{c_1^{(0)}, c_2^{(0)}, \phi_1^{(0)}, \phi_2^{(0)}, \sigma_1^{(0)}, \sigma_2^{(0)}, p_{11}^{(0)}, p_{22}^{(0)}\right\}'$, we can calculate $Pr(S_t = i|\Omega_{t-1};\theta)$ by using equation (4.16) and (4.18) iteratively. Repeating the above iterations for $t = 1,2,\cdots,T$, the estimates $\hat{\theta} = \{\hat{c}_1, \hat{c}_2, \hat{\phi}_1, \hat{\phi}_2, \hat{\sigma}_1, \hat{\sigma}_2, \hat{p}_{11}, \hat{p}_{22}\}'$ can therefore be obtained by maximising the log-likelihood function (4.17) with respect to c_1, c_2, ϕ_1, ϕ_2, σ_1, σ_2, p_{11} and p_{22}, respectively. In short, the iterative steps for Hamilton's filter are given by

Step 1: Input $\Pr(S_{t-1}|\Omega_{t-1}) = \pi$

$$\pi = \begin{pmatrix}\pi_1 \\ \pi_2\end{pmatrix} = \begin{pmatrix}\Pr(s_t = 1;\theta) \\ \Pr(s_t = 2;\theta)\end{pmatrix} = \begin{pmatrix}\dfrac{1-p_{22}}{2-p_{11}-p_{22}} \\ \dfrac{1-p_{11}}{2-p_{11}-p_{11}}\end{pmatrix}$$

Step 2: $\Pr(S_t, S_{t-1}\,|\,\Omega_{t-1}) = \Pr(S_t\,|\,S_{t-1})\Pr(S_{t-1}\,|\,\Omega_{t-1})$

Step 3: $\Pr(S_t\,|\,\Omega_{t-1}) = \sum_{s_t=1}^{2}\Pr(S_t, S_{t-1}\,|\,\Omega_{t-1})$

Step 4: $f(R_t, S_t\,|\,\Omega_{t-1}) = f(R_t\,|\,S_t, \Omega_{t-1})\Pr(S_t\,|\,\Omega_{t-1})$

Here, $f(R_t \mid S_t, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_{s_t}}} \exp(-\frac{\left[R_t - c_{s_t} - \phi_{s_t} R_{t-1}\right]^2}{2\sigma_{s_t}{}^2}$

Step 5: $f(R_t \mid \Omega_{t-1}) = \sum_{s_t=1}^{2} f(R_t, S_t \mid \Omega_{t-1})$

Step 6: Obtain estimators $\hat{\theta}$ by maximising $l(\theta) = \sum_{t=1}^{T} f(R_t \mid \Omega_{t-1})$.

Step 7: Update $\Pr(S_t | \Omega_t) = \frac{f(R_t, S_t \mid \Omega_{t-1})}{f(R_t \mid \Omega_{t-1})}$

Moreover, the future probability that equity risk premia will be in a given regime can be forecasted using ergodic probabilities π_t at time t and the transition matrix p because of Markov features. It is given by,

$$\pi_{t+1} = \pi_t P \tag{4.19}$$

While the probabilities for m-period future can be calculated by

$$\pi_{t+m} = \pi_t P^m \tag{4.20}$$

To study the asymmetric variation of financial time series associated with market conditions, three regime-switching models have been introduced: the structural break model, the TAR model and the MS-AR model. These models allow the data generating process to switch between different market regimes, where each regime is described by a different set of parameters. However, the univariate regime-switching model may not have sufficient power to capture the co-movements in multiple financial time series. Therefore, in the next section we consider multivariate extensions to the univariate-regime switching models.

4.3.2 Regime switching in multivariate models

To capture the co-movements among multiple time series through different regimes, we employ regime-switching vector autoregression. We first review a simple vector autoregression (VAR) (Sims, 1980). This model extends the univariate autoregression to a dynamic multivariate time series system to examine the linear inter-dependence between multiple time series. In a VAR, each endogenous variable is regressed as a function of the lags of every variable in the model. The basic VAR model takes the form:

$$\begin{aligned} R_t &= c + \phi R_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \Sigma) \end{aligned} \tag{4.21}$$

where R_t is the vector of dependent regime variables, c and ϕ are the constant vectors, and ε_t is the error vector. The VAR coefficients can be estimated by maximising the log-likelihood function. The log-likelihood function can be written as:

$$l(\theta) = -\frac{1}{2}\sum_{t=1}^{T} \log \det \Sigma_\theta(t) - \frac{1}{2}\sum_{t=1}^{T}(R_t - c - \phi R_{t-1})\Sigma_\theta(t)(R_t - c - \phi R_{t-1})' \tag{4.22}$$

a) Model V: Threshold vector autoregressive models

Tsay (1998) extended the univariate threshold model to a multivariate context. Lo and Zivot (2001) generalised the SETAR model to the threshold vector autoregression model (TVAR). The structure of a two regime-TVAR model is:

$$R_t = c_{s_t} + \phi_{s_t} R_{t-1} + \varepsilon_t \tag{4.23}$$

With

$$S_t = \begin{cases} 1 & \text{if } \mathrm{q}_{t-d} \le r \\ 2 & \text{if } \mathrm{q}_{t-d} > r \end{cases} \tag{4.24}$$

where R_t is the vector of dependent regime variables, q_{t-d} is the threshold variable, which is the past value of one of the endogenous variables of the regression, d is the delay parameter, and r is the threshold value.

To simplify, we shall consider the case of two regimes. If threshold variable q_{t-d}, delay parameter d and threshold value r are known *a priori,* the estimation is straightforward. Greene (2003) pointed out that VAR can be thought of as a special case of seemingly unrelated regression equations and the parameters of the VAR can be estimated separately in each regime by the least squares method. For the case of unknown delay parameters and threshold values, the sequential conditional least square (SCLS) estimation method introduced in TAR models can still be used. The consistent estimates of d and r are the ones which yield the smallest value of the Bayesian information criterion (BIC) (Schwarz, 1978):

$$BIC = -2\log L + n\log T \tag{4.25}$$

where L is the likelihood function, n is the number of parameters and T is the total sample size.

b) Model VI: Markov-switching vector autoregressive models

Krolzig (1997, 1998 and 2003) extended Hamilton's Markov switching regime models and proposed Markov switching vector autoregressive (MS-AR) models. In these extended models, the switches from one regime to another are determined by an unobservable state variable that follows a Markov process. This state variable is assumed to be common to all series in the VAR. The general form of the Markov switching regime model is:

$$\begin{cases} R_t = c_{s_t} + \phi_{s_t} R_{t-1} + z_t \delta + \varepsilon_t \\ \varepsilon_t \mid S_t \sim N(0, \Sigma_{s_t}) \end{cases} \tag{4.26}$$

where R_t is the vector of dependent regime variables, s_t is the state variable, and z_t is the vector of independent regime variables.

Model (4.26) is a 'partial' regime-dependent MS-VAR model because the parameters δ do not change across different regimes. If δ is an empty vector, then all the coefficients are subject to change and we obtain a 'pure' MS-VAR model:

$$R_t = c_{s_t} + \phi_{s_t} R_{t-1} + \varepsilon_t \tag{4.27}$$

The parameters can be computed using Hamilton's (1989) filter. This estimation method was introduced in Section 4.31 (Model IV), and we will not discuss it again.

In this section, three different regime-switching models have been introduced: the structural break models, the threshold autoregression models and the Markov switching regime models. A state variable is defined to indicate the regime in which the equity risk premium is at time *t*. In the structural break model, the state variable is determined by time *t*. In the TAR model, the state variable is determined by the past values of itself. In the MS-AR model, the state variable is a latent variable that is assumed to follows a Markov process. To capture the regime switching behaviour in multiple time series, we also review the regime switching VAR models including the TVAR models and the MS-VAR models. The next section presents empirical applications of regime switching models to the UK stock market.

4.4 Empirical Section

For the analysis, we revisit the same data that we examined in Chapter 3: equity risk premia on the UK FTSE All-Share Index, dividend price ratios, three-month Treasury bill rates and inflation rates. We first consider the case of univariate time series models, and then extend our scope to the multivariate case.

4.4.1 Model I: No switching

In the first model, we assume that equity risk premia follow an AR(1) process and there is no switching allowed. Since the error terms are normally distributed, the maximum log-likelihood estimators are equivalent to the least squares estimators. Recalling the results of Tables 3.13 in Chapter 3 (Section 3.46), we estimate the equity risk premium based on its own value in the last period.

Table 3.13 AR(1) model with no switching
TABLE 3.13 HERE

4.4.2 Model II: Structural break models

In the second model, equity risk premia are allowed to switch between different regimes and can be modelled as a combination of linear AR (1) models, each of which are characterised by a different set of coefficients. Equity risk premia are assigned to different regimes according to the specific dates at which the change takes place. These break dates can be estimated endogenously together with model parameters. Recalling the results of Table 3.15, Panel A, in Chapter 3 (Section 3.46), we split the whole sample period into four sub-samples and estimate each sub-sample separately. The results show that the relationship between the equity risk premium and forecasting variables changes over different sub-samples, and thus suggest that regime-switching is an important characteristic of the equity risk premium models. In the next section, we consider TAR models.

Table 3.15 AR(1) model with breaks
TABLE 3.15 HERE

4.4.3 Model III: TAR models

We apply the TAR model to the UK equity risk premium. The specification of the model is as in Section 4.3.1. We assume that the switching between two regimes is determined by the threshold variable, $R_{t\text{-}d}$. Equity risk premia are then assigned to different regimes according to the threshold value, r. We use the AIC and BIC information criteria to choose the optimal delay parameters and the number of regimes.

a) The case when both delay parameter d and threshold value r are known

Firstly, we consider the case where both the delay parameter d and the threshold value r are known. For simplicity, we suppose that d=1 and r=0. The observations are divided into two groups based on whether the threshold variable $R_{t\text{-}1}$ is greater than or less than the threshold value 0. Note that $R_{t\text{-}1}$ is the lagged one-month equity risk premium ($EQ_{t\text{-}1}$). The estimation is simple: Model (4.1) can be estimated by OLS separately for each equation. The results are shown in Table 4.1. Figure 4.1 plots equity risk premia against the threshold value. It is clear that the threshold divides the data into two sub-samples. However, the threshold value is difficult to ascertain in advance. As mentioned earlier, the model estimates are conditional on the value of thresholds. In this case, an assumption that the thresholds are known *a priori* may result in biased estimates.

Table 4. 1 TAR estimates in the case when threshold value is known

Regime 1: $EQ_{t-1} \leq 0$				
	Estimate	Std. Err	t-value	Pr (>|t|)
Constant	0.1374	0.0601	2.2868	0.0231
EQ_{t-1}	0.1860	0.0831	2.2401	0.0260
Residual Std. Error	0.6434			
No. of observations	236			
R^2	0.0249			
Mean	0.0416			
Standard deviation	0.6467			
Regime 2: $EQ_{t-1} > 0$				
Constant	-0.0834	0.0532	-1.5685	0.1177
EQ_{t-1}	0.3069	0.0877	3.4996	0.0005
Residual Std. Error	0.6550			
No. of observations	330			
R^2	0.0604			
Mean	0.0419			
Standard deviation	0.6705			

Figure 4.1 Equity risk premia with one known threshold
FIGURE 4.1 HERE

b) The case of unknown threshold value *r* but known delay variable *d*

Secondly, we assume that we know the delay parameter *d*, but the value of the threshold is not directly observable. The threshold value *r* can be estimated together with the model coefficients using SCLS. In general, the first and last 15% of observations are excluded from the sample period. Under the assumption that *d*=1, Table 4.2 reports the estimation results for a two-regime TAR model. This table also reports the means and standard deviations of equity risk premia for each regime. The estimated threshold value is 0.4109, which divides the whole sample period into two regimes: a 'crash' regime and a 'normal' regime. This month's equity risk premium is assigned to a 'crash' regime if last month's equity premium lies above 0.4109; otherwise it is in the 'normal' regime. For the regime with $EQ_{t-1} > 0.4109$, the average equity risk premium EQ_t is 0.0187 and the standard deviation is 0.7328. For the regime with $EQ_{t-1} < 0.4109$, these figures are 0.0499 and 0.6333, respectively. These results suggest that future equity risk premia will be lower and be more volatile if this month's equity risk premium is extremely high. Moreover, the R^2 is 0.1375 in the 'crash' regime, but only 0.0441 in the normal regime. Clearly, the predictive ability of the lagged one-month equity risk premium is higher in the 'crash' regime. This reveals a threshold asymmetry for the predictive ability of lagged equity risk premia with respect to the variation in equity risk premia.

Table 4.3 reports the results of TAR estimates with two thresholds. We therefore divide the whole sample period into three regimes according to the values of the threshold: -0.0881 and 0.4109. The means and standard errors of the equity risk premia for each regime are also reported in this table. The first regime corresponds to a 'high' regime with the average of equity risk premia being 0.0623; the third regime is the 'low' market regime where the average is 0.0187, while the second regime is the 'normal' regime. The above results reveal an uneven mean-reverting pattern in equity risk premia, raising the possibility that the lagged one-month equity risk premium affects equity risk premia significantly in the 'high' and ‘low’ market periods, whereas the effect is not significant during a ‘normal’ period. This also suggests that the past values of equity risk premium play a more important role when stock markets are in a particularly ‘good’ or ‘bad’ phase than during a ‘stable’ period.

Table 4. 2 TAR estimates in the case of one unknown threshold

Regime 1: $EQ_{t-1} \leq 0.4109$				
	Estimate	Std. Err	t-value	Pr (>|t|)
constant	0.0630	0.0332	1.8989	0.0583
EQ_{t-1}	0.1147	0.0583	1.9692	0.0496
Residual Std. Err	0.6251			
No. of observations	418			
R^2	0.0441			
Mean	0.0499			
Std	0.6333			
Regime 2: $EQ_{t-1} > 0.4109$				
Constant	-0.3771	0.1044	-3.6132	0.0004
EQ_{t-1}	0.5511	0.1199	4.5977	0.0000
Residual Std. Err	0.6855			
No. of observations	148			
R^2	0.1375			
Mean	0.0187			
Std	0.7328			

Figure 4.2 Equity risk premia with one unknown threshold

FIGURE 4.2 HERE

Figure 4.3 Equity risk premia with two unknown thresholds

FIGURE 4.3 HERE

Table 4. 3 TAR estimates in the case of two unknown thresholds

Regime 1: $EQ_{t-1} < -0.0881$				
	Estimate	Std. Err	t-value	Pr (>|t|)
constant	0.2325	0.072	3.2296	0.0015
EQ_{t-1}	0.2756	0.091	3.0299	0.0028
Residual Std. Err	0.6355			
No. of observations	198			
R^2	0.0701			
Mean	0.0623			
Std	0.1571			
Regime 2: $-0.0881 < EQ_{t-1} \leq 0.4109$				
Constant	-0.0277	0.0664	-0.4166	0.6771
EQ_{t-1}	0.3301	0.2972	1.1106	0.2672
Residual Std. Err	0.6056			
No. of observations	220			
R^2	0.0600			
Mean	0.0499			
Std	0.6333			
Regime 3: $EQ_{t-1} > 0.4109$				
Constant	-0.3771	0.1044	-3.6132	0.0004
EQ_{t-1}	0.5511	0.1199	4.5977	0.0000
Residual Std. Err	0.6855			
No. of observations	148			
R^2	0.1375			
Mean	0.0187			
Std	0.7328			

c) The case when both delay parameter d and threshold value r are unknown

In this section, we consider the case when both the delay parameter d and the threshold value r are unknown. To simplify, we will only consider the case of one threshold. We set the maximum lag order to 4, $d_{\max} = 4$. The AIC values of the TAR models for $1 \leq d \leq 4$ are reported in Table 4.4. The AIC value is smallest when $d = 2$, and EQ_{t-2} is therefore selected as the threshold variable. Table 4.5 summarises the corresponding TAR model estimates.

The TAR model relaxes the assumption that equity risk premia are drawn from a single distribution and postulate that switches between different regimes are governed by a threshold variable. However, the threshold variable may not be observable and may change over time, in which case Markov switching regime models are employed.

Table 4. 4 AIC values of TAR models

d	AIC	r
$d = 1$	1110	0.4109
$d = 2$	1052	-0.5206
$d = 3$	1082	-0.7220
$d = 4$	1111	-0.6132

Table 4. 5 TAR estimates when $d = 2$

Regime 1: $EQ_{t-2} \leq -0.5206$				
	Estimate	Std. Err	t-value	Pr (>|t|)
constant	-0.4578	0.2255	-2.0305	0.0452
EQ_{t-1}	0.0387	0.1496	0.2587	0.7965
Residual Std. Err	0.6251			
No. of observations	95			
R^2	0.0537			
Regime 2: $EQ_{t-2} > -0.5216$				
Constant	0.0690	0.0289	2.3853	0.0175
EQ_{t-1}	0.1538	0.0394	3.9070	0.0001
Residual Std. Err	0.5506			
No. of observations	470			
R^2	0.0577			

Figure 4.4 Equity risk premia with unknown threshold and delay parameter
FIGURE 4.4 HERE

4.4.5 Model IV: Markov regime-switching models

To investigate the effect of regime-switching variation on equity risk premia, we apply Markov regime-switching models to the UK stock market. Two specifications of the Markov regime-switching model are considered: Markov switching Mean-variance model and Markov switching autoregressive model.

a) Markov switching mean-variance model

In the first specification, we assume that equity risk premia are drawn from a number of different distributions with different means and different variances. Equity risk premia can switch randomly from one regime to another. The mean-variance switching model is:

$$R_t = \mu_{s_t} + \varepsilon_{s_t} \tag{4.28}$$

where s_t is the state variable, μ is the mean of the equity risk premium and R_t denotes the equity risk premium (EQ) at time t.

As discussed in Chapter 3, we should choose the smallest viable number of regimes. Table 4.6 summarises the parameter estimates for models with two, three and four regimes. We use the AIC (Akaike, 1974) or BIC (Schwarz, 1978) to determine the number of regimes. The model that generates the minimum value of AIC or BIC is selected as the optimal model. Table 4.6 shows that both the AIC and BIC favour the three-regime model. In the three-regime model, it can be seen that regime 1 is a "normal" regime characterised by a small positive mean of 0.0479 and low volatility at 0.3425. Regime 2 represents a ‘crash’ regime where the average equity risk premium is significantly negative and the volatility is very high. Panel B of Figure 4.5 suggests that Regime 2 contains the major stock market disasters including the stock market crashes of 1973, 1980, 1987 and 2008. Regime 3 can be thought of as a sustained bull regime characterised by high equity risk premia. This regime includes the bull market of 1982, the great bull market of 1990-1998 and the bull market 2002-2007.

Recalling the findings in Chapter 3, structural break tests cannot detect any breaks after 1982, while Figure 4.5 shows that there is some clear evidence that regime shifts in equity risk premia occurred during the period 1982-2007. In particular, the plots suggest that this period can be divided into four sub-periods: 1982-1992, 1992-1997, 1997-2002 and 2002-2007. In the first and third periods, equity risk premia were in ‘normal’ regimes, while in the second and fourth period they were in bull regimes. These results are consistent with stock market history. During the stock market boom period 1992 to 1997, equities kept rising and experienced tremendous growth. While in the bull rally period 2002-2007, equities made superior profits and significantly outperformed long-term bonds. As noted before, standard linear models cannot capture such dynamic changes between bull markets and bear markets and therefore cannot provide reasonable estimates with respect to these regime changes. To conclude, Markov regime switching models can better capture the dynamic switching behaviour in equity risk premia than standard linear models.

As assumed, we allow equity risk premia to switch randomly for each possible observation. The transition matrix is defined to calculate the possibility of switching for each observation. Although the turning points are not observable, we can still use the probability of switching to describe how equity risk premia switch from one regime to another. Figure 4.5 plots the filtered probability and smoothed probability for the mean-variance models. In Panel B, the first plot shows the possibility that regime 1 occurred at each date; the second plots the possibility that regime 2 occurred; and the third plots the possibility that state 3 occurred. The persistence of each regime can be analysed by looking at these three plots. Most observations in regime 1 are around 1. It shows that equity risk premia are more likely to stay in regime 1 than in regimes 2 and 3, and that regime 1 is more stable than the others. The probability of p_{11} is 0.9783, p_{22} is 0.8578 and p_{33} is 0.9447. These results suggest that the 'normal' and 'bull' regimes are more stable than the 'crash' regime. Another interesting result is that the standard error in a bull regime is the lowest one, and this is a sign that investors are optimistic about future expected returns and prefer holding stocks in a bull market. The probability of p_{23} is 0, and this suggests that the model has a zero probability of moving directly from the 'crash' regime to the 'bull' regime.

Table 4. 6 Estimates for the mean-variance Markov switching regime model

Parameters	2 regimes		3 regimes		4 regimes	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
p_{11}	0.8116	0.0707	0.9738	0.0136	0.001	0
p_{21}	0.0209		0.1422	0.0857	0.001	0
P_{31}			0.0542	0.0316	0.08	0. 0369
P_{41}					0.1955	0. 0975
p_{12}	0.1884		0.0105	0.0086	0.001	0
p_{22}	0.9791	0.0002	0.8578	0.0857	0.9496	0. 0247
P_{32}			0.001	0	0.0144	0. 0111
P_{42}					0.001	0
P_{13}			0.0157		0.8501	0. 1039
p_{23}			0		0.0504	0. 0247
P_{33}			0.9447		0.9056	0. 039
P_{43}					0.001	0
P_{14}					0.1479	
P_{24}					0.0001	
P_{34}					0	
P_{44}					0.8025	
μ_1	-0.3548	0.1952	0.0479	0.0332	0.4705	0.0321
μ_2	0.0877	0.0258	-0.3452	0.3125	0.1259	0.0298
μ_3			0.1329	0.0298	0.014	0.0274
μ_4					-0.4322	0.3378
σ_1	1.9080	0.5239	0.3425	0.0409	0.0089	0.0058
σ_2	0.2478	0.0222	2.7534	0.8987	0.066	0.0131
σ_3			0.0684	0.0136	0.3514	0.0327
σ_4					3.0306	0.9539
BIC	0.847		0.817		0.878	
AIC	0.877		0.847		0.902	

Figure 4.5 Filtered Probability and smoothed probability for the Mean-Variance Markov switching regime models

FIGURE 4.5 HERE

A clear conclusion can be drawn here. Our sample period has three regimes. Regime 1 indicates a relatively stable regime, Regime 2 indicates a bad ‘crash’ regime, and Regime 3 indicates a good 'bull' regime. The Markov switching regime model has the great advantage of allowing equity risk premia to switch between different regimes or revert to their precious state, in the same way as business cycles.

b) Markov switching autoregression models (MS-AR)

In the second specification, the MS-AR model is:

$$R_t = c_{s_t} + \phi_{s_t} R_{t-1} + \varepsilon_{s_t} \tag{4.29}$$

Table 4.7 reports the information criteria statistics. The three-regime model is selected as the optimal one. Table 4.8 reports the estimation results for the MS(3)-AR(1) model. The conclusion is similar to that using the mean-variance Markov switching regime model. There are three regimes during our sample period and equity risk premia may switch randomly among them. In Table 4.8, the average equity risk premia for these regimes are: 0.0385, -0.2981 and 0.1603, respectively. The corresponding standard errors are: 0.3498, 2.8934 and 0.0689, respectively. Therefore, regime 1 is a relatively stable regime. Regime 2 is a 'crash' regime, which contains the major stock market disasters including the oil price crash of 1973, the Black Monday crash of 1987, the tax reforms on dividends of 1997, the Dot.com crash of 2000, and the sub-prime mortgage crisis of 2007. Regime 3 is a 'bull' regime, which includes the bull markets of 1981, 1992, 1994 and 2002. These results also suggest that regime 2 is relatively volatile compared with regimes 1 and 3. Figure 4.6 plots the filtered probability and smoothed probability for this MS(3)-AR(1) model. The first, second and third plots of Figure 4.6 show the probability that regimes 1, 2 and 3 respectively may occur. Similarly to the patterns shown in Figure 4.5, equity risk premia are more likely to be in regime 1 than in regimes 2 and 3.

Table 4. 7 Information Criteria for MS-AR(1) model

No of Regimes	BIC	SIC	AIC
2	0.881	-0.413	0.888
3	0.861	-0.421	0.869
4	1.156	-0.313	1.164

Table 4. 8 Estimates for MS(3)-AR(1) model

Parameters	3 regimes	
	Estimate	Std. Error
p_{11}	0.9730	0.0148
p_{21}	0.1306	0.0833
P_{31}	0.0602	0.0341
p_{12}	0.0088	0.0072
p_{22}	0.8694	0.0833
P_{32}	0.0010	0
P_{13}	0.0183	
p_{23}	0	
P_{33}	0.9338	
μ_1 (Mean)	0.0385 (0.0424)	0.0327
μ_2 (Mean)	-0.2981(-0.3389)	0.3416
μ_3 (Mean)	0.1603 (0.1388)	0.0354
Φ_1	0.0927	0.0514
Φ_2	0.1204	0.1974
Φ_3	-0.1551	0.1149
σ_1	0.3498	0.0369
σ_2	2.8934	0.9244
σ_3	0.0689	0.0142

Figure 4.6 Filtered Probability and Smoothed Probability for MS (3)-AR (1) model
FIGURE 4.6 HERE

Markov regime-switching models can capture cyclical fluctuations in financial time series. The basic idea of these models is that bear and bull markets can be defined as different regimes. Equity risk premia are allowed to switch probabilistically between these regimes. However, Markov switching regime models are based on the univariate time series. The state variable is latent and can be inferred from other observed variables. This observed variable is truly time series itself. In this case, the univariate regime-switching model cannot be able to capture the co-movements in multiple financial time series. In the next section, we apply regime-switching to a VAR framework to explore the dynamic interactions among different financial variables.

4.4.6 Regime-switching VAR models

In this section, a vector auto regressive (VAR) model with 1 lag is employed. The model is:

$$R_t = c_{s_t} + \phi_{s_t} R_{t-1} + \varepsilon_t \tag{4.30}$$

where R_t=(EQ_t, DY_t, RF_t, RPI_t). As already mentioned in Section 3.4.1, we use the equity risk premium, the dividend price ratio, the three-month Treasury bill rate and the inflation rate as the possible candidate regressors.

a) Model Ⅴ: TVAR models

In this section, we use TVAR models to investigate the asymmetric relationships among equity risk premia, dividend price ratios, three-month Treasury bill rates and inflation rates across different regimes. Recalling Equation 4.23, we assume that the lag length and the delay parameter are both equal to 1, i.e. p=1 and d=1. Also, we assume that there is only one threshold over the sample period, and that this value divides the whole sample period into two sub-samples. As discussed in Section 4.4.5, the value of the threshold can be estimated endogenously. Panels A, B, C and D of Table 4.9 report the results of TVAR models using lagged one-month dividend yields, lagged one-month inflation rates, lagged one-month equity risk premia and three-month Treasury bill rates lagged by one-month as the threshold variables, respectively.

We first use the lagged dividend yields as the threshold variable to investigate the impact of dividend yield changes on real economic activity. The observations are divided into two regimes based on whether the threshold variable DY_{t-1} is greater than or less than the estimated threshold value of 0.0529. Regime 1 is a 'stable' regime with lower average values for the equity risk premia, dividend yields, interest rates and inflation rates, while Regime 2 is a 'volatile' regime with higher equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates. Panel A suggests that equity risk premia, three-month Treasury bill rates and inflation rates will all be high if the last period's dividend yield is extremely high. The standard error of equity risk premia in Regime 2 reported in Panel A is 0.9028, and this suggests that the equity risk premia are relatively volatile when lagged one-month dividend yields are high.

The results of Panel B show the effect of regime changes in three-month Treasury bill rates on equity risk premia, dividend price ratios and inflation rates. The estimated threshold value is 0.1042, which divides the whole sample into two regimes. Extremely high levels of three-month Treasury bill rates lagged by one-month will be accompanied by high inflation rates and high dividend yields, but regime changes in Treasury bill rates may not have a significant effect on the average values of equity risk premia. In contrast, the standard error of equity risk premia will be high in the regime where $RF_{t-1} > 0.1042$. We therefore conclude that a high three-month Treasury bill rate has little effect on the level of the equity risk premia but does have a significant effect by raising their volatility.

The results in Panel C suggest that high lagged one-month inflation rates will be associated with high dividend yields, high interest rates and low equity risk premia. Moreover, equity risk premia are extremely unstable when the inflation rates are high. Finally, we use the lagged one-month equity risk premium as the threshold variable. Panel D shows that the value of 0.2075 splits the data into two regimes: a regime with high equity risk premia and a regime with low equity risk premia. However, the results suggest that the changes in the lagged one-period equity risk premium do not have a significant effect on the dividend price ratio, the three-month Treasury bill rate and the inflation rate.

Figures 4.7 to 4.10 plot equity risk premia, dividend yields, three month Treasury bill rates and inflation rates against their threshold values, respectively. There is some clear evidence that regime shifts in the UK economy occurred during the ‘long recession’ of 1974-1982. These findings are consistent with our results in Chapter 3. Between 1974 and 1982, the oil crisis of 1973 caused a sharp increase in the price of oil and led to high rates of inflation, high interest rates, high equity risk premia and high dividend yields.

The results of Table 4.9 also imply that the predictability of forecasting variables changes over time. Indeed, TVAR models provide evidence that the relationships between equity risk premia, dividend price ratios, inflation rates and interest rates change over time. However, the underlying structure which governed the switching may not be observable and also changes over time. The assumption that the threshold variable is observable and remains unchanged over the long horizon may not be enough to capture the dynamic switching behaviour in economic models. In the next section, we apply the Markov vector autoregressive model.

Figure 4. 7 TVAR model: EQ_{t-1} is the threshold variable

FIGURE 4.7 HERE

Figure 4. 8 TVAR model: DY_{t-1} is the threshold variable

FIGURE 4.8 HERE

Figure 4. 9 TVAR model: RF_{t-1} is the threshold variable

FIGURE 4.9 HERE

Figure 4. 10 TVAR model: RPI_{t-1} is the threshold variable

FIGURE 4.10 HERE

Table 4. 9 TVAR models

Panel A: TVAR models in the case when $DY_{t\text{-}1}$ is the threshold variable						
Regime 1: $DY_{t-1} \leq 0.0529$						
	Mean	Intercept	EQ (-1)	DY(-1)	RF(-1)	RPI(-1)
EQ	0.0098 (0.5913)	-0.1213 (0.1384)	0.0839 (0.0527)	10.1068 (4.1956)*	-1.5548 (1.1930)	-3.3520 (1.2112)**
DY	0.0375 (0.0085)	0.0004 (0.0006)	-0.0003 (0.0002)	0.9632 (0.0189)***	0.0055 (0.0054)	0.0161 (0.0055)**
RF	0.0638 (0.0307)	0.0004 (0.0009)	-0.0002 (0.0004)	-0.0265 (0.0282)	1.0041 (0.0080)***	0.0090 (0.0081)
RPI	0.0448 (0.0282)	0.0021 (0.0011).	-0.0003 (0.0004)	-0.0702 (0.0343)*	0.0365 (0.0097)***	0.9599 (0.0099)***
Percentage of observation: 82.2%						
Regime 2: $DY_{t-1} > 0.0529$						
EQ	0.1901 (0.9028)	-0.9372 (0.4334)*	0.2185 (0.0684)**	20.8659 (6.7324)**	0.2058 (2.6622)	-1.2344 (1.4286)
DY	0.0601 (0.0108)	0.0080 (0.0020)***	-0.0015 (0.0003)***	0.8444 (0.0303)***	-0.0052 (0.0120)	0.0108 (0.0064).
RF	0.1048 (0.0267)	0.0024 (0.0029)	-0.0018 (0.0005)***	-0.0077 (0.0452)	0.9635 (0.0179)***	0.0081 (0.0096)
RPI	0.1279 (0.0541)	-0.0091 (0.0035)*	-7.4e-05 (0.0006)	0.1601 (0.0550)**	-0.0140 (0.0218)	1.0062 (0.0117)***
Percentage of observation: 17.8%						
Panel B: TVAR models in the case when $RF_{t\text{-}1}$ is the threshold variable						
Regime 1: $RF_{t-1} \leq 0.1042$						
	Mean	Intercept	EQ (-1)	DY(-1)	RF(-1)	RPI(-1)
EQ	0.0405 (0.5942)	-0.1496 (0.1181)	0.1187 (0.0474)*	8.4878 (3.4284)*	-1.8164 (1.3807)	-0.7394 (0.9070)
DY	0.0386 (0.0109)	0.0004 (0.0005)	-0.0006 (0.0002)**	0.9751 (0.0152)***	0.0053 (0.0061)	0.0047 (0.0040)
RF	0.0592 (0.0254)	0.0001 (0.0008)	-0.0003 (0.0003)	-0.0178 (0.0233)	1.0081 (0.0094)***	0.0031 (0.0062)
RPI	0.0478 (0.0396)	0.0002 (0.0010)	0.0007 (0.0004).	-0.0101 (0.0285)	0.0155 (0.0115)	0.9810 (0.0075)***
Percentage of observation: 80.6%						
Regime 2: $RF_{t-1} > 0.1042$						
EQ	0.0470 (0.8870)	-2.4873 (0.6840)***	0.2528 (0.0836)**	49.7185 (8.0023)***	7.5966 (5.1256)	-9.7175 (2.3000)***
DY	0.0537 (0.0110)	0.0153 (0.0030)***	-0.0018 (0.0004)***	0.6800 (0.0354)***	-0.0383 (0.0227).	0.0595 (0.0102)***
RF	0.1206 (0.0136)	0.0050 (0.0046)	-0.0025 (0.0006)***	-0.0845 (0.0543)	0.9826 (0.0348)***	0.0073 (0.0156)
RPI	0.1088 (0.0415)	-0.0031 (0.0057)	-0.0028 (0.0007)***	0.0186 (0.0666)	0.0318 (0.0426)	0.9874 (0.0191)***
Percentage of observation: 19.4%						

Table 4.9 (Continued)

Panel C: TVAR models in the case when RPI_{t-1} is the threshold variable						
Regime 1: $PI_{t-1} \leq 0.0926$						
	Mean	Intercept	EQ (-1)	DY(-1)	RF(-1)	RPI(-1)
EQ	0.0553 (0.5553)	-0.2941 (0.1283)*	0.0835 (0.0536)	11.2680 (3.7205)**	-0.8828 (1.2619)	-0.6104 (1.6955)
DY	0.0379 (0.0089)	0.0012 (0.0006)*	-0.0003 (0.0002)	0.9562 (0.0168)***	0.0028 (0.0057)	0.0050 (0.0077)
RF	0.0634 (0.0300)	0.0006 (0.0009)	7.6e-06 (0.0004)	-0.0296 (0.0251)	0.9937 (0.0085)***	0.0208 (0.0114).
RPI	0.0417 (0.0213)	0.0019 (0.0011).	8.2e-05 (0.0004)	-0.0564 (0.0308).	0.0142 (0.0105)	0.9839 (0.0141)***
Percentage of observation: 82.6%						
Regime 2: $RPI_{t-1} > 0.0926$						
EQ	-0.0226 (1.0222)	-1.4184 (0.3840)***	0.1926 (0.0660)**	25.3797 (5.8848)***	-2.9811 (2.5819)	1.6605 (1.8165)
DY	0.0585 (0.0127)	0.0079 (0.0017)***	-0.0013 (0.0003)***	0.8500 (0.0266)***	0.0121 (0.0117)	-0.0011 (0.0082)
RF	0.1074 (0.0272)	0.0021 (0.0026)	-0.0019 (0.0004)***	-0.0158 (0.0397)	0.9896 (0.0174)***	-0.0042 (0.0123)
RPI	0.1446 (0.0399)	-0.0068 (0.0032)*	-0.0003 (0.0005)	0.1241 (0.0488)*	0.0478 (0.0214)*	0.9610 (0.0151)**
Percentage of observation: 17.4%						
Panel D: TVAR models in the case when R_{t-1} is the threshold variable						
Regime 1: $EQ_{t-1} \leq 0.2075$						
EQ	0.0553 (0.6599)	-0.3396 (0.1287)**	0.2413 (0.0720)***	15.9129 (3.9575)***	-2.0717 (1.3918)	-0.6517 (1.1850)
DY	0.0418 (0.0129)	0.0021 (0.0006)***	-0.0012 (0.0003)**	0.9106 (0.0179)***	0.0115 (0.0063).	0.0058 (0.0054)
RF	0.0711 (0.0337)	0.0001 (0.0009)	-9.2e-05 (0.0005)	-0.0144 (0.0270)	1.0037 (0.0095)***	0.0061 (0.0081)
RPI	0.0584 (0.0441)	-0.0022 (0.0011)*	-0.0010 (0.0006).	0.0098 (0.0329)	0.0391 (0.0116)***	0.9797 (0.0098)***
Percentage of observation:57.7%						
Regime 2: $EQ_{t-1} > 0.2075$						
EQ	0.0233 (0.6622)	-0.3379 (0.1704)*	0.4469 (0.1018)***	8.0916 (5.1867)	-1.0080 (1.6186)	-2.5429 (1.1926)*
DY	0.0411 (0.0118)	0.0012 (0.0008)	-0.0024 (0.0005)***	0.9856 (0.0235)***	-0.0006 (0.0073)	0.0131 (0.0054)*
RF	0.0710 (0.0342)	0.0027 (0.0012)*	-0.0019 (0.0007)**	-0.0373 (0.0353)	0.9904 (0.0110)**	* 0.0024 (0.0081)
RPI	0.0612 (0.0499)	0.0012 (0.0014)	0.0023 (0.0008)**	-0.0427 (0.0431)	-0.0035 (0.0134)	0.9872 (0.0099)***
Percentage of observation:42.3%						

Note: Significant: 0 ‘***’, 0.001’**’, 0.01’*’, 0.05’.’

b) Model Ⅵ: MSVAR models

We assume that the state variable can govern the switches in the co-movements of equity risk premia, dividend price ratios, three-month Treasury bill rates and inflation rates: that is, the state variable is assumed to be common for all variables. Moreover, this state variable is latent and is assumed to follow a Markov process. Because of the computation demands of the iterative algorithm, we focus only on the mean-variance model in this section. The model is:

$$R_t = \mu_{s_t} + \varepsilon_{s_t} \tag{4.31}$$

where R_t =(EQ_t, DY_t, RF_t, RPI_t).

Table 4.10 reports the information criteria statistics for the MSVAR models with respect to the different number of regimes. The two-regime model is selected as the optimal model according to the information criteria. Table 4.11 reports the estimation results for the MSVAR mean-variance model, while Figure 4.11 plots the filtered probabilities and smoothed probabilities. The high values of P_{11} and P_{22} also show that these two regimes are extremely persistent. The results confirm the empirical findings in the TVAR models and show the existence of two regimes. In particular, we find evidence that regime shifts in the UK economy occur during the ‘long recession’ of 1974-1982. This period was characterised by high equity risk premia, high inflation rates, high interest rates and high dividend yields. In such periods, equity risk premia are more volatile.

Table 4. 10 Information Criteria for the mean-variance MSVAR model

No of Regimes	BIC	AIC
2	24.766	24.840
3	25.532	25.606

Table 4. 11 Estimates for the mean-variance MSVAR model

Parameters	EQ		DY		RF		RPI	
	Estimate	Std.	Estimate	Std	Estimate	Std	Estimate	Std
μ_1	0.0139	0.0363	0.0363	0.0004	0.0550	0.0013	0.0347	0.0009
μ_2	0.0567	0.0271	0.0525	0.0010	0.1050	0.0020	0.1117	0.0038
σ_1	0.2695	0.0197	0.0001	0.0000	0.0005	0.0003	0.0002	0.0000
σ_2	0.7758	0.0823	0.0001	0.0000	0.0006	0.0000	0.0022	0.0000
p_{11}	0.9877	0.0006						
p_{22}	0.9711	0.0122						
p_{12}	0.0123							
p_{21}	0.0289							

Figure 4.11 Filtered Probability and Smoothed Probability for the mean-variance MSVAR
FIGURE 4.11 HERE

4.5 Conclusions

In this Chapter, we have used different regime-switching models to capture the structural instability in the UK equity risk premium during the period from January 1965 to May 2012. These regime-switching models relax the assumption that equity risk premia are drawn from one distribution over the sample period, and replace it with the assumption that equity risk premia may switch from one regime to another. These models can therefore be used effectively to describe the behaviour of equity risk premia over long horizons.

Both structural break models and threshold autoregressive models assume that regime changes are deterministic. The former allow for only a single structural break and the state variable is solely determined by time. Under the latter, multiple regime changes are allowed and the state variable is determined by an observable variable with respect to an unobserved threshold. However, if we believe that the underlying switching mechanism may be unobservable, we can use a Markov switching regime framework to allow for an unobservable state variable. In this model, equity risk premia can be switched randomly for each possible observation. The latent state variable is assumed to follow a Markov process. The transition possibility matrix is defined to describe how equity risk premia switch from one regime to another. The analysis suggests that regime-switching is an important feature of equity risk premia and cannot be ignored by investors in making their asset allocation decisions.

However, regime-switching models raise the problem of unidentified nuisance parameters under the null of no switching when the log-likelihood ratio test is employed. The switching variables are nuisance parameters, and not defined under the null hypothesis. Therefore, the log-likelihood ratio test does not have a standard asymptotic distribution. In the next chapter, we will consider the question of how to test for the non-linearity in the equity risk premium.

Appendix 4 Figures

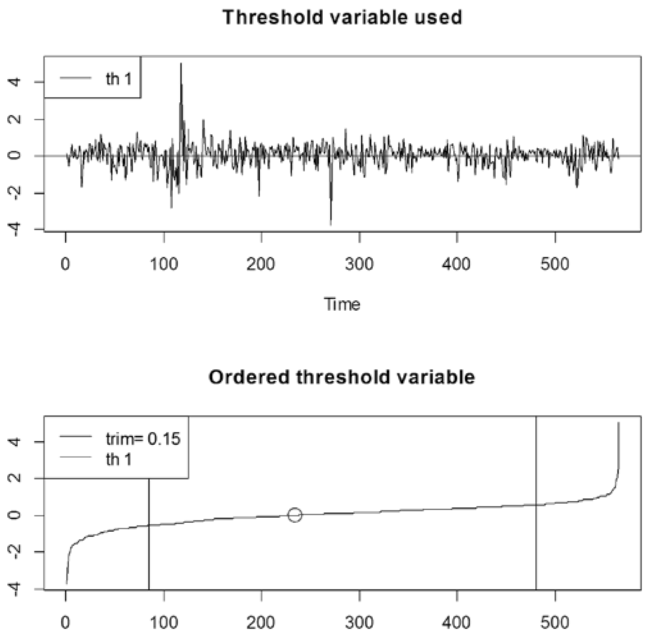


Figure 4. 1 Equity risk premia with one known threshold

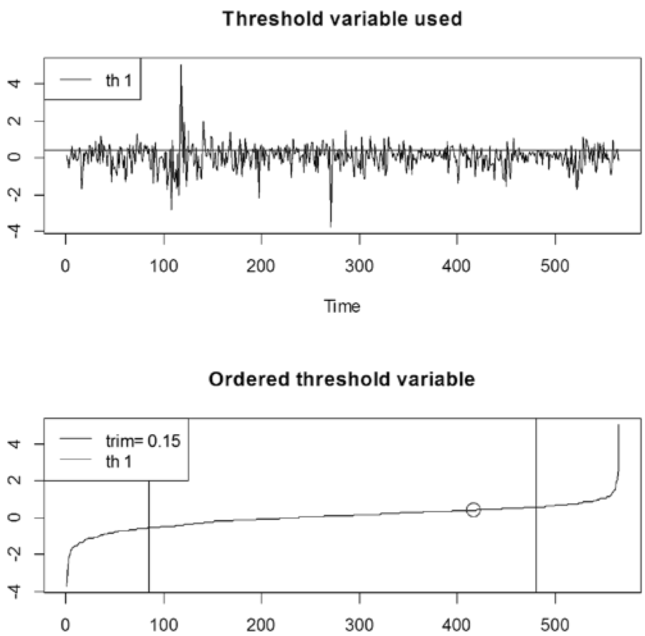


Figure 4. 2 Equity risk premia with one unknown threshold

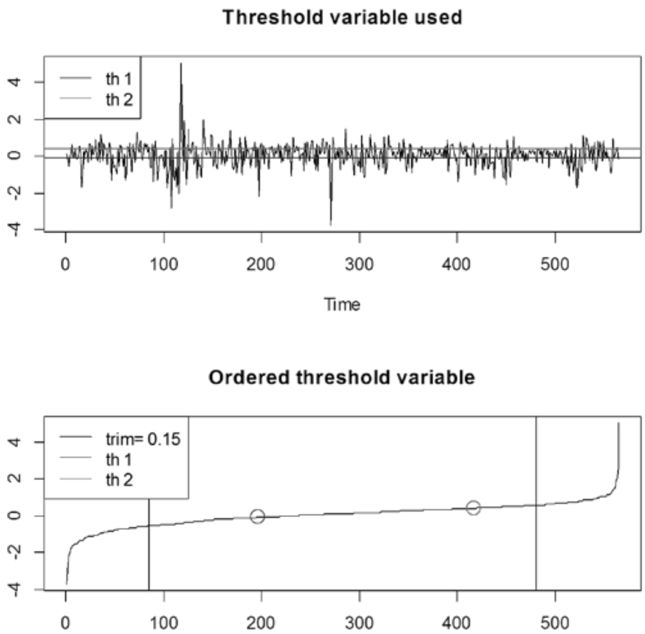


Figure 4. 3 Equity risk premia with two unknown thresholds

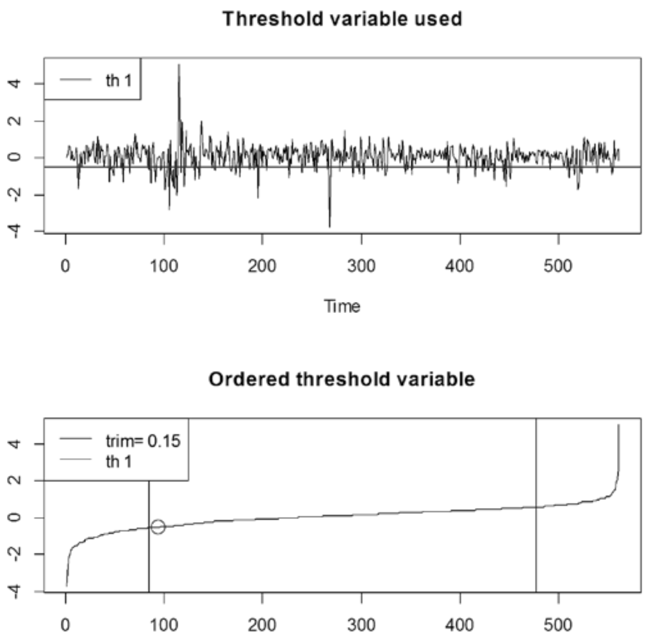


Figure 4. 4 Equity risk premia with unknown threshold variable and threshold value

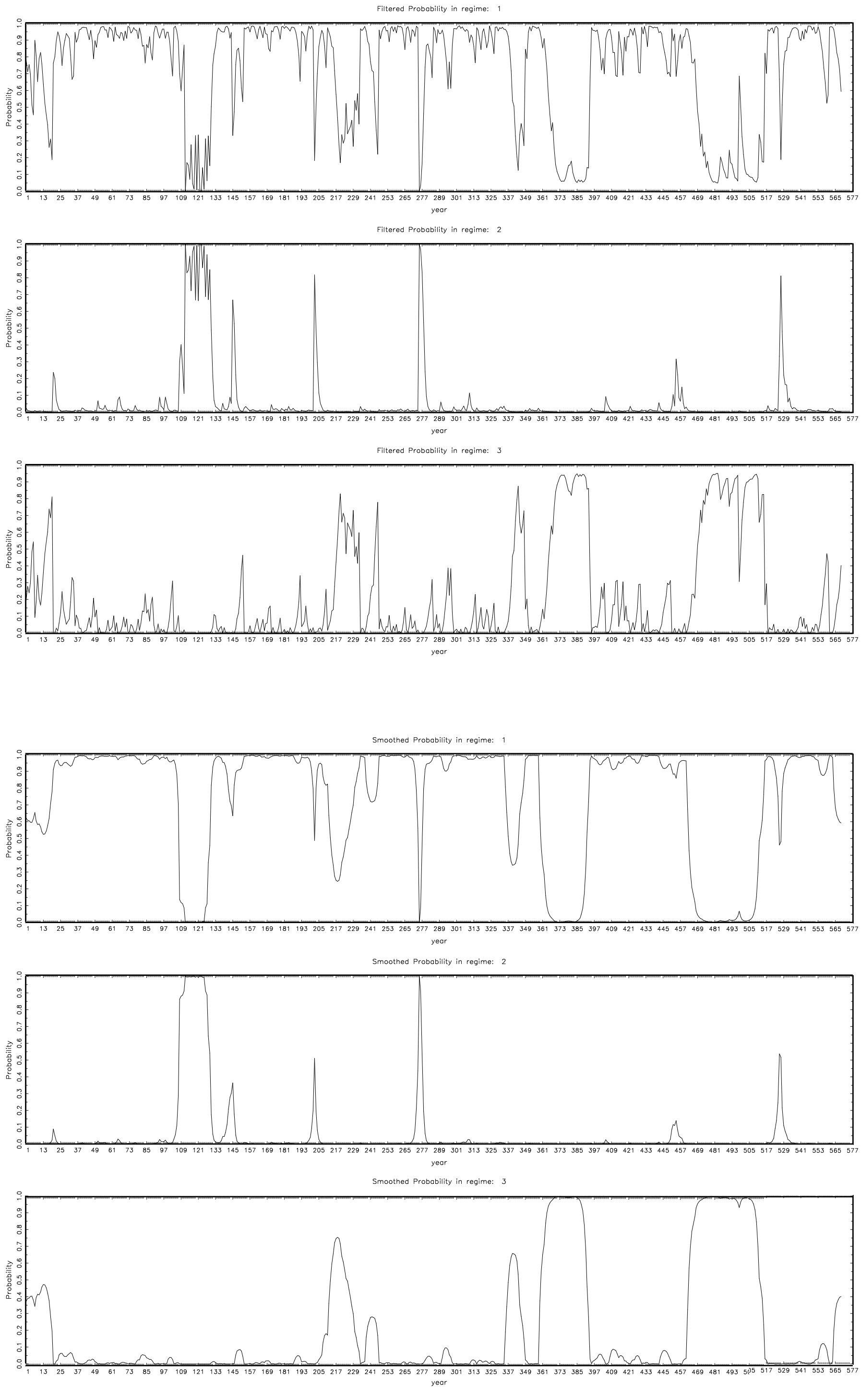


Figure 4. 5 Filtered Probability and smoothed probability for Markov regime-switching Mean-Variance model

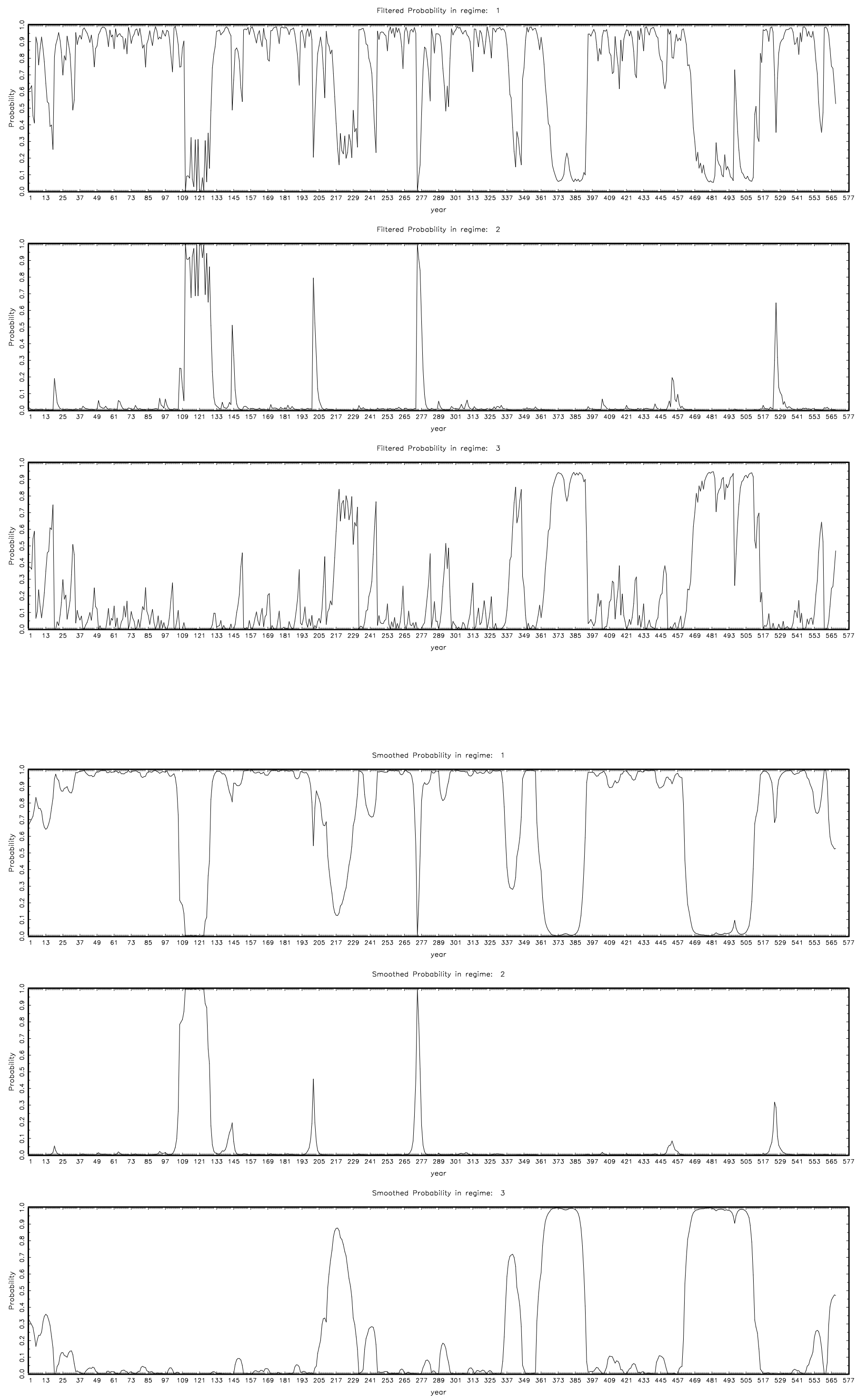


Figure 4. 6 Filtered Probability and Smoothed Probability for MS(3)-AR(1) model

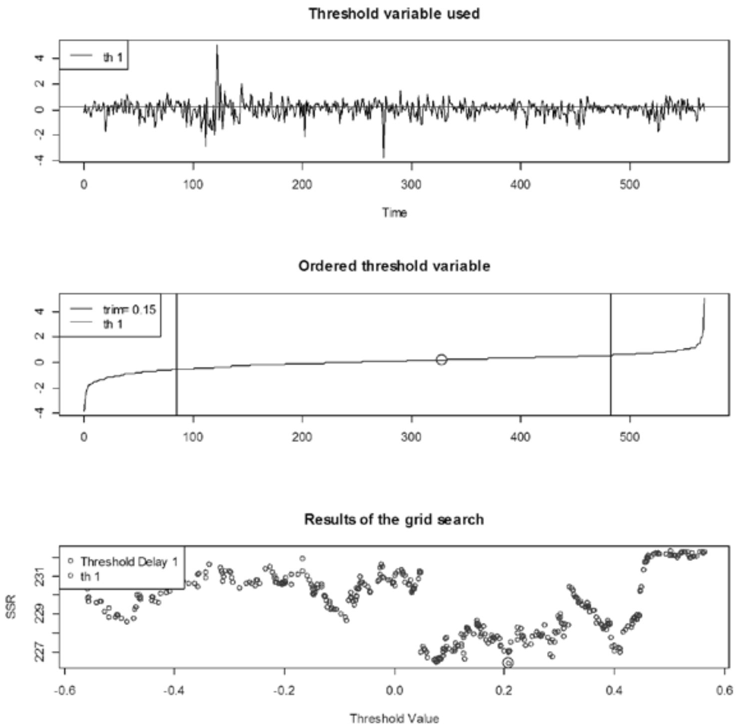


Figure 4. 7 TVAR model: Lag(EQ) is the threshold variable

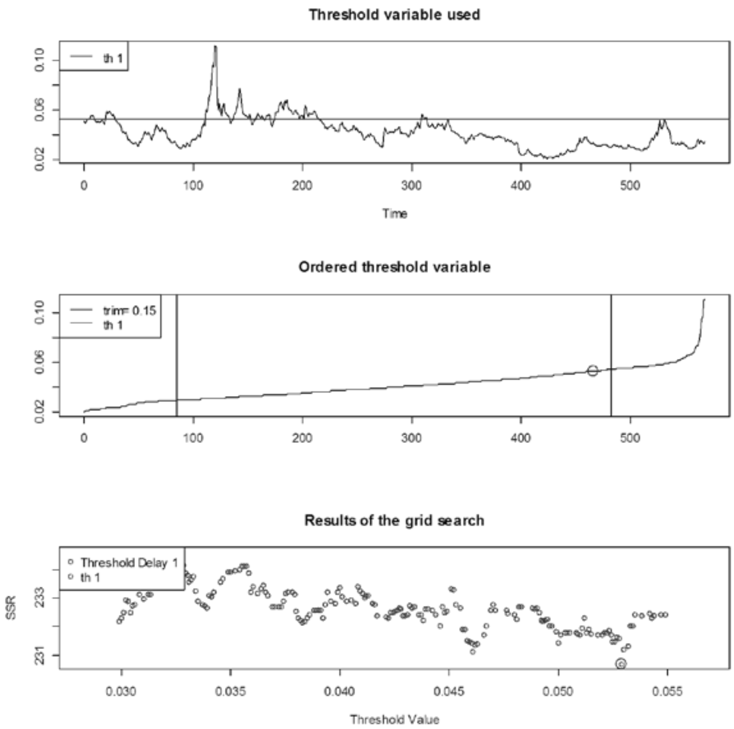


Figure 4. 8 TVAR model: Lag(DY) is the threshold variable

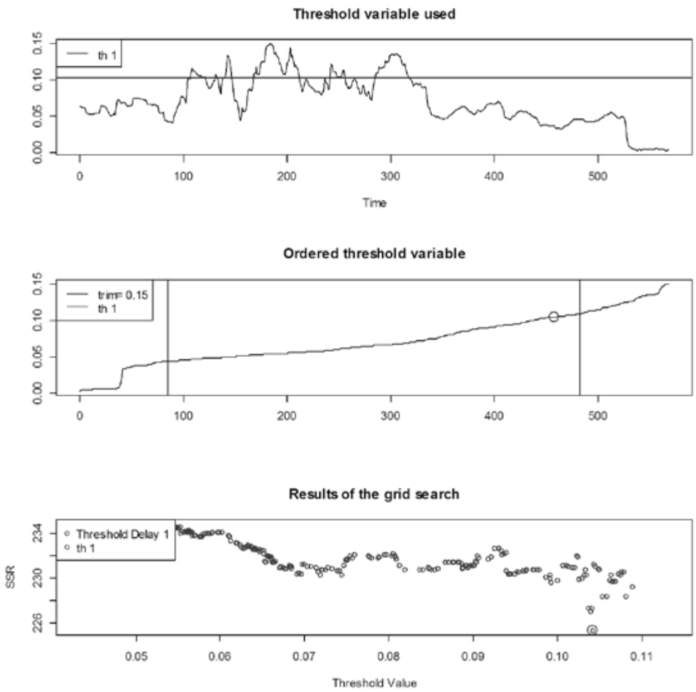


Figure 4. 9 TVAR model: Lag(RF) is the threshold variable

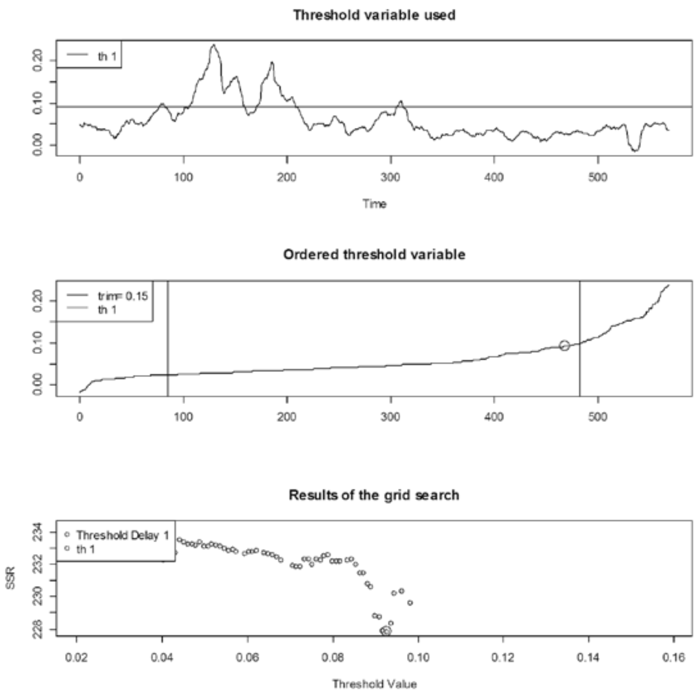


Figure 4. 10 TVAR model: Lag(RPI) is the threshold variable

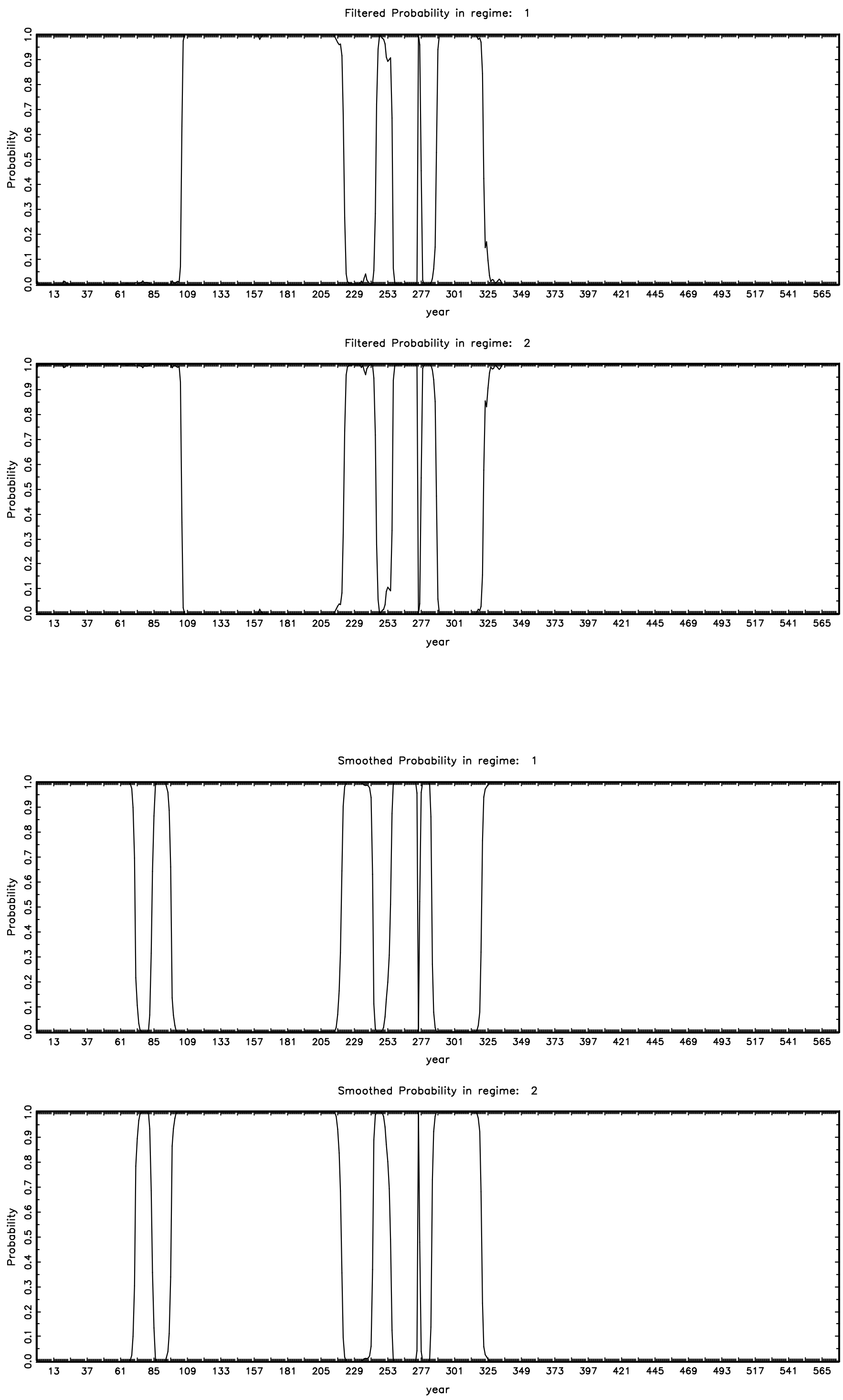


Figure 4. 11 Filtered Probability and smoothed probability for the two-regime MS-VAR mean-variance model

Chapter 5 Non-linearity Tests

5.1 Introduction

In this chapter, we apply linearity tests to detect non-linear behaviour in the equity risk premium models. In the recent literature many non-linear models have been used to analyse the dynamic behaviour of economic and financial time series. However, studies of non-linear financial time series raise the question whether non-linear specifications are superior to linear models. To avoid over-fitting the model, it is necessary to test for linearity before building a non-linear model.

Linearity tests can be used to test the null hypothesis of linearity against the alternative of non-linearity. The purpose of this chapter is to investigate some issues related to non-linearity tests subject to regime switching models. In recent years, regime-switching models have received considerable attention; these are models in which financial time series are allowed to switch among different regimes. Regime switching models are useful approaches to capture the dynamic switching behaviour in financial time series data, but they may raise a problem of unidentified nuisance parameters under the null hypothesis of no switching when the log-likelihood ratio test is employed. This chapter focuses on non-linearity tests based on regime-switching models, including threshold autoregressive models and Markov switching regime models.

The rest of the chapter is structured as follows: section 2 is a literature review on non-linearity tests; section 3 discusses commonly used tests for non-linearity including portmanteau and specific tests; section 4 presents empirical applications involving the UK FTSE All-Share Index; section 5 summarises and concludes.

5.2 Literature review

5.2.1 Brief review

The purpose of this section is to review the statistical tests which have been developed for non-linear dependence in financial time series. In particular, we focus on the non-linearity tests based on regime-switching models.

During recent years, there has been a great deal of interest in using non-linear models to analyse the dynamic behaviour of financial time series. This is because that some apparently 'random' behaviour of time series data may be produced by a simple non-linear deterministic system. A number of empirical studies have shown that most financial time series have non-linear features and cannot be simply estimated by linear models. See for example Hinich and Patterson (1985), Scheinkman and LeBaron (1989), Hsieh (1989, 1991), Pesaran and Potter (1993), De Grauwe et al. (1993), Abhyankar et al. (1995), Steurer (1995), Brooks (1996), Barkoulas and Travlos (1998), Opong et al. (1999), Kosfeld and Robe (2001), Sarantis

(2001). Phenomena such as stock market excess volatility, fat tails in the distribution, and mean reversion all indicate non-linear behaviour of financial time series. Moreover, financial markets may display changes in behaviour patterns over time. These changes may impact persistently on the behaviour of equity risk premia. In this case, structural breaks and regime switching may occur during the sample period. The traditional linear assumption may fail to capture such extreme changes and is unlikely to generate a reliable forecast for equity risk premia. In particular, the empirical results drawn from linear methods may not be robust if the time series exhibit non-linear features. However, studies in non-linear financial time series raise the question of whether non-linear models are superior to linear ones. In this case, it is important to test for linearity against non-linearity before building a non-linear model.

Many statistical tests have been developed for non-linear dependence in financial time series. In the literature, there are two main types of test for non-linearity. The first are portmanteau tests, residual-based tests and are used to test for linearity without a specific non-linear alternative. Examples of such tests are the regression equation specification error test (RESET) of Ramsey and Schmidt (1976), the BDS test of Brock, Dechert, and Scheinkman (1996) and the Tsay test (Tsay, 1986). These statistical tests provide empirical evidence for the non-linear behaviour of financial time series against the linear paradigm.

However, there are so many different types of non-linear time series model that no single test can actually determine the types of non-linearity. In this case, the second type of non-linearity test is used, the specific tests which are used to test for linearity with a specific non-linear alternative. This chapter focuses on non-linearity tests based on regime-switching models: for example, testing linearity against the alternative of self-exciting threshold autoregressive (SETAR) models (Chan 1990, 1991; Chan and Tong 1990; and Hansen 1997, 1999, 2000) and the Markov switching based non-linearity test (Hansen 1992, 1996). We first review the problem of testing for the null of linearity in the presence of regime-switching models.

5.2.2 Non-linearity tests based on unidentified nuisance parameters

As discussed in Chapter 3, economic and financial time series may suffer from structural and regime changes. It is therefore reasonable to use regime-switching models to capture non-linear behaviour in financial time series data over a long period of time. However, tests for non-linearity based on switching regime models may raise a problem of unidentified nuisance parameters under the null hypothesis of no switching. In this section, we will investigate this issue.

There are many econometric hypotheses that may involve a problem of nuisance parameters. Here, we focus on the non-linearity tests based on regime-switching models. The hypothesis of interest is whether the regime switching specification enters the linear autoregressive (AR) model. The null hypothesis is straightforward, being simply to test the hypothesis that all the parameters remain invariant in the different regimes. Two types of regime-switching models are considered: the threshold autoregressive model (TAR) and the Markov switching regime (MS-AR) model. For the TAR model, Chan and Tong (1990) and Chan (1991) point out that the threshold value is an unidentified nuisance parameter because it is not defined under the null hypothesis. For the Markov switching regime model, Hansen (1992) and Hamilton (1989) suggest that the transition probabilities are nuisance parameters because they can take any values without affecting the log-likelihood function.

The log-likelihood ratio (LR) test is the most popular hypothesis test, which can be used to evaluate the goodness-of-fit between two models. However, the LR test statistics do not have a standard asymptotic distribution in the presence of nuisance parameters. This is because the likelihood function for the test is non-quadratic with respect to the unidentified nuisance parameter at the optimum. As a result, the scores under the null hypothesis do not have a positive variance. In fact, they are identically zero and thus the information matrix of the log-likelihood function is singular. Recognising these issues, Hansen (1992, 1996) concludes that the central limit theorem (CLT) cannot be applied and therefore the log-likelihood ratio test does not have a standard χ^2 distribution.

The most convenient way to deal with this issue is to use simulation-based tests. Davidson and MacKinnon (1996) provide the theoretical framework for the bootstrap method for simulating the distribution of the log-likelihood ratio test. They also show that the size distortion of a bootstrap test is smaller than that of the corresponding conventional test based on asymptotic distribution theory. Hansen (1999) proposes a bootstrap procedure to approximate the asymptotic distribution of the non-linearity test based on threshold autoregressive models and shows that the *p*-values obtained from the bootstrap are asymptotically correct.

However, a simulation-based method, such as Monte-Carlo, may not work in the case of Markov switching regime models, for two reasons. First, Hansen (1992) points out that the likelihood ratio statistic obtained by the Monte Carlo simulation is likely to underestimate its true value, because the log-likelihood function of the Markov switching regime model tends to have multiple local optima. As a result, it is extremely difficult to obtain a Monte Carlo draw from the null model. Second, there is no asymptotic theory available in the context of Markov switching regime models. In this case, there is always a concern about whether the simulation approach can generate an accurate asymptotic approximation to the finite-sample distribution of the test statistic. Hansen (1992, 1996) proposes an alternative method to evaluate the Markov switching regime models. He uses the quasi-maximum likelihood (QML) method and finds a bound for the asymptotic distribution of the likelihood ratio statistic. The QML estimates allow for possible misspecification of the likelihood function, and are commonly used when the underlying probability distribution by which the data are generated is not fully understood. However, Hansen's approach may be conservative since it only bounds the likelihood ratio tests and cannot provide a critical value.

This section reviews issues about the non-linearity test related to regime switching models. The next section introduces the technical details for the non-linearity tests in the context of regime-switching models.

5.3 Nonlinearity tests

Regime switching models assume that time series models are non-linear. To avoid over-fitting the model, it is necessary to examine this non-linear relationship between variables. In this section, we review a number of statistical tests which have been developed to detect departures from linearity in financial time series models, including the RESET of Ramsey and Schmidt (1976), the BDS test of Brock, Deckert, and Scheinkman (1987), the threshold non-linearity tests of Hansen (1999) and the Markov switching models based non-linearity test of Hansen (1992, 1996).

5.3.1 Portmanteau tests

Portmanteau tests are used to test for linearity without a specific non-linear alternative. In this case, rejecting the null hypotheses of such tests only suggests the departure from the linear assumption, but does not specify the actual form of non-linear alternative. We review the most commonly used linearity tests in turn.

a) RESET

The RESET (regression error specification test) test examines the null hypothesis of linearity against a general specification of the non-linear alternative. The basic idea of the RESET is that if the model is truly linear then the residuals from the regression should not be correlated with the regressors included in the model or with the fitted values. Let $\hat{R}_t$ be the vector of fitted values:

$$\hat{R}_t = X_t \hat{\beta} \tag{5.1}$$

The RESET is based on the following augmented linear regression:

$$R_t = AX_t + Z_t \gamma + \varepsilon_t \tag{5.2}$$

where $Z_t = (\hat{R}_t^2, \hat{R}_t^3, \hat{R}_t^4, \cdots)$ and $\gamma = (\gamma_1, \gamma_2, \gamma_3, \cdots)$. The null hypothesis of the test is:

$$H_0 : \gamma = 0 \tag{5.3}$$

which can be tested using a standard *F*-statistic. Please note that a rejection of the null hypothesis for the RESET test suggests only that the non-linear model is appropriate, but does not specify the actual form of the non-linear alternative.

b) BDS test

The BDS test can be used to detect non-linear dependence in financial time series models. The null hypothesis of the test is that the residuals are independent and identically distributed. The idea behind this test is to check the serial correlation of the estimated residuals from a fitted linear model. To do this, we first embed the residuals $\{\hat{x}_t\}$ into *m* dimensional vectors, i.e. $x_t^{\ m} = (x_t, x_{t+1}, \cdots, x_{t+m-1})$. For $t \in N$ and $\forall \epsilon < 0$, the correlation integral at embedding dimension *m* is computed as:

$$c_{m,T}(\epsilon) = \frac{2}{(T-m+1)(T-m)} \sum \sum_{m \le s < t < T} I(x_t^m, x_t^s; \epsilon) \tag{5.4}$$

where *T* is the sample size, ϵ is an arbitrary tolerance distance, and $I(x_t^m, x_s^m; \epsilon)$ is an indicator function:

$$I(x_t^m, x_t^s; \epsilon) = \begin{cases} 1 \quad \text{if } \left| x_{t-i} - x_{s-i} \right| \le \epsilon \text{ for } i = 1, 2, \cdots, m-1 \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{5.5}$$

The BDS statistic is defined as follows:

$$b_{m,n}(\epsilon) = \frac{\sqrt{T-m+1}(c_{m,T}(\epsilon) - c_{1,T-m+1}(\epsilon)^m)}{\sigma_{m,T}(\epsilon)} \tag{5.6}$$

Under the null hypothesis, Brock, Dechert, Scheinkman and LeBaron (1997) show that the BDS test statistic converges asymptotically to the standard Normal distribution $N(0,1)$ as $m \to \infty$

The BDS test is a portmanteau test for non-linearity against a general specification of non-linearity alternatives. If the linearity null hypothesis for the BDS test is rejected, this suggests only that using any linear model would be inappropriate.

5.3.2 Specific non-linearity tests

Specific tests are used to test for linearity with a specific non-linear alternative. Two types of specific non-linearity tests are introduced: the threshold based non-linearity tests and the Markov switching regime based non-linearity tests. In Section 4.3.2, we are focusing our attention solely on models which involve two regimes. We assume that the dynamic behaviour of financial time series follows an autoregressive (AR) model in each regime.

a) Testing for threshold non-linearity

In this section, we review the non-linearity tests based on threshold autoregressive models (TAR). The null hypothesis is that the time series follows a linear autoregressive (AR) model, while the alternative hypothesis is that it follows a two-regime TAR model. However, these tests may raise the problem that the threshold value r is an unidentified nuisance parameter. To solve this problem, Hansen's (1996) bootstrap approach is introduced.

Let R_t denote an observed variable. To simplify, we assume a constant variance across two regimes, i.e. no switching in the variance.

A two regime SETAR model can be defined as:

$$\begin{aligned} R_t &= (c_1 + \phi_{1,1} R_{t-1} + \cdots + \phi_{1,p} R_{t-p})\, I(R_{t-d} \le r) \\ &\quad + (c_2 + \phi_{2,1} R_{t-1} + \cdots + \phi_{2,p} R_{t-p})\, I(R_{t-d} > r) + \varepsilon_t \end{aligned} \tag{5.7}$$

where $I(\cdot)$ is an indicator variable, $\phi_1 = (c_1, \phi_{1,1}, \cdots \phi_{1,p})$ is the autoregressive coefficient vector, when $R_{t-d} \le r$, $\phi_2 = (c_2, \phi_{2,1}, \cdots, \phi_{2,p})$ is the autoregressive coefficient vector, when $R_{t-d} > r$, p is the lag length and d is the delay parameter. In general, $d \le p$. If we set

$$x_t = (1, R_{t-1}, \cdots, R_{t-p})' \tag{5.8}$$

and

$$x_t(r) = (x_t' I(R_{t-d} \le r) \;\; x_t' I(R_{t-d} > r)) \tag{5.9}$$

Then, equation (5.7) can be simply expressed as:

$$\begin{aligned} R_t &= x_t' \phi_1 I(R_{t-d} \le r) + x_t' \phi_2 I(R_{t-d} > r) + \varepsilon_t \\ &= x_t(r)' \theta + \varepsilon_t \end{aligned} \tag{5.10}$$

where $\theta = (\phi_1', \phi_2')$

The null hypothesis is then straightforward and to test the hypothesis that all of parameters remain invariant in the two sub-samples. It is,

$$H_0 : c_1 = c_2, \phi_{11} = \phi_{21}, \cdots, \phi_{1p} = \phi_{2p} \tag{5.11}$$

For given p and d, the likelihood ratio test statistic is defined as:

$$T_n = -2 \log \left\{ \frac{L(H_0)}{L(H_1)} \right\} \tag{5.12}$$

where $L(H_0)$ is the likelihood function for the null model which is the AR(p) model, and $L(H_1)$ is that for the alternative TAR model.

However, the threshold value *r* is an unidentified nuisance parameter because it is not defined under the null hypothesis. In this case, the log-likelihood ratio test does not have a standard asymptotic distribution. Chan and Tong (1990) and Chan (1991) study the test statistics in the context of TAR models and derive a parameter-free limiting distribution for computing the probability *p*-values for the test. However, their results work only for small probability values. Hansen (1996) proposes a bootstrap procedure to approximate the asymptotic distribution of the test. The bootstrap central limit theorem shows that the probability *p*-values obtained from the bootstrap are asymptotically correct and his approach is introduced below.

Hansen (1996) uses the standard *F*-statistic defined as:

$$F_T = \sup_{r \in \Gamma}(F_T(\gamma))$$
$$F_T(r) = T(\frac{\tilde{\sigma}_T{}^2 - \hat{\sigma}_T^2(\gamma)}{\hat{\sigma}_T^2(\gamma)}) \quad (5.13)$$

where $\tilde{\sigma}_T^2$ and $\hat{\sigma}_T^2$ are the least square estimates of the variance from the null model and the alternative model, respectively. Under the normal distribution assumption, the maximum log-likelihood estimators are equivalent to the least square estimators. We can see that equation (5.13) is a function of γ. When γ is known, it is the standard *F*-statistic and its asymptotic distribution is χ^2. The problem is that γ is unknown. To solve this problem, Hansen employs the bootstrap method.

Suppose that $u_t, t = 1,\cdots,T$ is a random sample drawn from *i.i.d* $N(0,1)$, where T is the sample size. Let $y_t^* = u_t^*$ and then regress y_t^* on x_t to get the residual variance $\tilde{\sigma}_T^{*2}$ under H_0. Regress y_t^* on $x_t(r)$ to get the residual variance $\hat{\sigma}_T^{*2}$ under H_1, and thus calculate the F-statistic:

$$
\begin{aligned}
F_T^* &= \sup_{r \in \Gamma}(F_T^*(r)) \\
F_T^*(r) &= T\left(\frac{\tilde{\sigma}_T^{*2} - \hat{\sigma}_T^{*2}(r)}{\hat{\sigma}_T^{*2}(r)}\right)
\end{aligned}
\tag{5.14}
$$

The asymptotic distribution of the F-statistic can be obtained by repeated bootstrap sampling, and the p-value of the test can be calculated by counting the percentage of draws for which the simulated statistic F_T^* exceeds the observed F_T. If the probability p-value is smaller than the critical value, the null hypothesis of an AR(p) model is rejected.

b) Testing for Markov switching regime models

In this section, we review the Markov switching regime based on non-linearity tests which examine the null hypothesis of an AR(1) model against the alternative of a Markov switching regime model. These tests can involve two issues: 1) the presence of nuisance parameters, and 2) the absence of asymptotic distribution theory. In these cases, the conventional likelihood ratio tests do not have a standard χ^2 distribution. The likelihood ratio bound test developed by Hansen (1992, 1996) is therefore introduced.

As discussed in Chapter 3, the basic idea of the Markov switching regime autoregressive

(MS-AR) models is that the autoregressive parameters in each regime are determined by an unobservable variable. This unobservable variable is a latent state variable and is assumed to follow a Markov process. To simplify, we shall consider only the case of mean switching. The model can be written in the form:

$$R_t = c_1 + c_2 s_t + \phi R_{t-1} + \varepsilon_t \quad t = 1, 2, \cdots, T \tag{5.15}$$

where R_t denotes an observed variable which follows an AR(1) model with mean $c_1/(1-\phi)$ in State 0 and with mean $(c_1+c_2)/(1-\phi)$ in State 1, $\theta = (c_2, p_{11}, p_{22}, c_1, \phi, \sigma)$ is the vector of parameters, T is the sample size, and s_t is an unobservable state variable that takes as its value either 0 or 1. The switching mechanism between two regimes is determined by s_t, and it follow a first order Markov process with transition matrix:

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \tag{5.16}$$

Testing for Markov switching regime models is equivalent to testing the null hypothesis that the switching parameter equals to 0, that is, $H_0\colon c_2 = 0$. Obviously, model (5.15) is nested within an autoregressive (AR) model. This is because it can be reduced to an AR(1) process under the null hypothesis. The most popular method to test such a null is the log-likelihood ratio (LR) test. However, the likelihood cannot be affected by the values of transition probabilities p_{11} and p_{22}. In this case, these two parameters are not identified under the null, and therefore are nuisance parameters. Davies (1977, 1987) points out that the likelihood function has multiple optima with respect to nuisance parameters, and thus the conventional likelihood ratio tests do not have a standard χ^2 distribution. In this case, the first problem associated with testing for the Markov switching regime models is that p_{11} and p_{22} are nuisance parameters. Since the likelihood does not change with respect to p_{11} and p_{22}, it is reasonable to consider the likelihood ratio statistics for all possible values of these parameters. This motivated researchers to develop a supremum statistic. In fact, testing for the Markov switching parameters is similar to the case of testing for unknown structural breaks. As introduced in Chapter 2, Andrews (1993) proposes a maximum (or *Sup*) statistic for detecting an unknown structural break. In the context of Markov switching regime models, Hansen (1992, 1996) extends Andrews's test and provides a bound for the maximum of the LR process. He treats the likelihood as a function of the unknown parameters and derives a lower bound for the asymptotic distribution of the standardised likelihood ratio statistic. The second problem is the absence of asymptotic theory. The underlying probability mechanism by which the data are generated is uncertain in the context of Markov switching regime models. In this case, the quasi maximum likelihood method (QMLE) is preferred because it allows for possible mis-specification of the likelihood function.

Recognising the difficulties associated with the presence of nuisance parameters in testing for the Markov switching regime models, Hansen (1992) develops an alternative testing procedure. He uses the QMLE method and decomposes the LR surface into its mean function and a Gaussian process. In doing so, he groups the parameter vector θ into the following sub-vectors:

$$\theta = (c_2, p_{11}, p_{22}, \theta_1') = (\gamma, \theta_1') \tag{5.17}$$

where $\gamma = (c_2, p_{11}, p_{22})$ is the vector of the unknown parameters, $p' = (p_{11}, p_{22})$ is the vector of the nuisance parameters, and $\theta_1' = (c_1, \phi, \sigma)$ is a vector of the remaining parameters.

For any given vector of parameters γ, the concentrated QMLE of θ_1 can be represented as

$$\hat{\theta}_1(\gamma) = \arg\max L_T(\gamma, \theta_1) \tag{5.18}$$

Under weak regularity conditions, QMLE converges in probability to $\theta_1(\gamma)$. Mathematically speaking,

$$\hat{\theta}_1(\gamma) \xrightarrow{P} \theta_1(\gamma) \tag{5.19}$$

where $\theta_1(\gamma) = \arg\max \lim\limits_{T\to\infty} \frac{1}{T} EL_T(\gamma, \theta)$. The concentrated quasi-log-likelihood function based on $\hat{\theta}_1(\gamma)$ and $\theta_1(\gamma)$ can be expressed as

$$\begin{aligned} \hat{L}_T(\gamma) &= \hat{L}_T(\gamma, \hat{\theta}_1(\gamma)), \\ L_T(\gamma) &= L_T(\gamma, \theta_1(\gamma)), \end{aligned} \tag{5.20}$$

Let $\hat{L}_T(0, p_{11}, p_{22})$ and $L_T(0, p_{11}, p_{22})$ be the concentrated quasi-log-likelihood function under the null, the likelihood ratio processes are therefore defined as:

$$\begin{aligned} \widehat{LR}_T(\gamma) &= \hat{L}_T(\gamma) - \hat{L}_T(0, p_{11}, p_{22}) \\ LR_T(\gamma) &= L_T(\gamma) - L_T(0, p_{11}, p_{22}) \end{aligned} \tag{5.21}$$

It is possible to express the log-likelihood function as its mean and the deviation from the mean

$$\hat{L}_T(\gamma) = \hat{Q}_T(\gamma) + R_T(\gamma) \\ L_T(\gamma) = Q_T(\gamma) + R_T(\gamma) \tag{5.22}$$

where $R_T(\gamma) = E[LR_T(\gamma)]$ is the mean, and $Q_T(\gamma)$ is the deviation from the mean

$$Q_T(\gamma) = \sum_1^T q_t(\gamma) \tag{5.23}$$

$$\begin{aligned} q_t(\gamma) = {} & [l_t(\gamma, \theta_1(\gamma)) - l_t(0, p_{11}, p_{22}, \hat{\theta}_1(0, p_{11}, p_{22},)] \\ & - E[l_t(\gamma, \theta_1(\gamma)) - l_t(0, p_{11}, p_{22}, \hat{\theta}_1(0, p_{11}, p_{22},)] \end{aligned} \tag{5.24}$$

In addition, Hansen (1992) assumes

$$\frac{1}{\sqrt{T}} \left\| \hat{L}_T(\gamma) - L_T(\gamma) \right\| = o_p(1) \tag{5.25}$$

This gives that

$$\begin{aligned} \frac{1}{\sqrt{T}} \widehat{LR}_T(\gamma) &= \frac{1}{\sqrt{T}} \left[\widehat{LR}_T(\gamma) - LR_T(\gamma) \right] \\ & \quad + \frac{1}{\sqrt{T}} \left[LR_T(\gamma) - R_T(\gamma) \right] + \frac{1}{\sqrt{T}} R_T(\gamma) \\ &= O_p(1) + \frac{1}{\sqrt{T}} \left[LR_T(\gamma) - R_T(\gamma) \right] + \frac{1}{\sqrt{T}} R_T(\gamma) \\ &= O_p(1) + \frac{1}{\sqrt{T}} Q_T(\gamma) + \frac{1}{\sqrt{T}} R_T(\gamma) \end{aligned} \tag{5.26}$$

Using the fact that $R_T(\gamma) = E[LR_T(\gamma)] \le 0$ for all γ under the null hypothesis, we get:

$$\frac{1}{\sqrt{T}} \widehat{LR}_T(\gamma) \le \frac{1}{\sqrt{T}} Q_T(\gamma) + O_p(1) \tag{5.27}$$

Assuming that an empirical process central limit theorem (CLT) holds, gives:

$$\frac{1}{\sqrt{T}} Q_T(\gamma) \Rightarrow Q(\gamma) \tag{5.28}$$

where $Q_T(\gamma)$ converges weakly to $Q(\gamma)$ as $T \to \infty$, and $Q(\gamma)$ is a Gaussian process with mean zero and the covariance function $k(\gamma_1, \gamma_2)$.

From equations (5.25) and (5.26), we obtain

$$\frac{1}{\sqrt{T}}\widehat{LR}_T(\gamma) \Rightarrow Q(\gamma) \tag{5.29}$$

For any γ, we know that

$$L\hat{R}_T = sup_\gamma L\hat{R}_T(\gamma) \tag{5.30}$$

Based on equations (5.25), (5.27) and (5.28), we then have

$$P\left\{\sup_\gamma \frac{1}{\sqrt{T}} L\hat{R}_T(\gamma) \geq c\right\} \leq P\left\{\sup_\gamma \frac{1}{\sqrt{T}} Q_T(\gamma) \geq c\right\} \to P\left\{\sup_\gamma Q(\gamma) \geq c\right\} \tag{5.31}$$

In the same paper, Hansen (1992) also proposes a standardised supremum statistic. We set $V(\gamma) = k(\gamma, \gamma)$ to be the variance function, and define the sample variance estimate as

$$V_T(\gamma) = \sum_1^T q_t(\gamma)^2 \tag{5.32}$$

The standardised supremum statistic associated with the test is

$$\begin{aligned} L\hat{R}_T^* &= \sup_\gamma L\hat{R}_T^*(\gamma) \\ &= \sup_\gamma \frac{1}{\sqrt{T}} \frac{L\hat{R}_T(\gamma)}{V_T(\gamma)^{1/2}} \\ &\leq \sup_\gamma \frac{1}{\sqrt{T}} \frac{Q_T(\gamma)}{V_T(\gamma)^{1/2}} + O_p(1) \\ &\Rightarrow Q^*(\gamma) = \frac{Q(\gamma)}{V(\gamma)^{1/2}} \end{aligned} \tag{5.33}$$

where $Q^*(\gamma)$ is a Gaussian process with mean 0 and covariance function

$$k^*(\gamma_1, \gamma_2) = \frac{k(\gamma_1, \gamma_2)}{V(\gamma_1)^{1/2} V(\gamma_2)^{1/2}} \tag{5.34}$$

From equation (5.33), we then have

$$P\{L\hat{R}_T^* \geq c\} \to P\{\sup Q^*(\gamma) \geq c\} \tag{5.35}$$

Equation (5.35) implies that the critical value of $L\hat{R}_T^*$ is smaller than that of $\sup_\gamma Q^*(\gamma)$ for any given significance level. This suggests that although the asymptotic distribution for the LR statistic is unknown, we can find a bound for the standard LR statistic under the null hypothesis when T is sufficiently large. Moreover, Hansen (1996) suggests that the asymptotic distribution of this bound can be generated by a standardised Gaussian process $Q^*(\gamma)$ with mean zero and the covariance function $k^*(\gamma_1, \gamma_2)$. Therefore, we can reject the null hypothesis of linearity if the critical value of $\sup_\gamma{}^* Q(\gamma)$ is less than the significant level. However, Hansen's test only provides a bound for the standard LR test. Such a bound is not a critical value. This implies that the test based on this approach is a 'conservative' one and may under-reject.

In this section, we have reviewed the commonly used non-linearity tests in the context of regime-switching models. In the next section, we apply those tests in order to examine non-linear behaviour in the UK equity risk premium.

5.4 Empirical Section

We now present the empirical results for the non-linearity tests associated with regime-switching models. For our analysis, we revisit the same data that we examined in chapter 3 on the equity risk premium of the UK FTSE All Share Index. We test the null hypothesis of linear auto-regression against various alternative non-linear models. The simplest linear auto-regressive model is the auto-regressive of order one. Two types of non-linearity test are employed: portmanteau tests and specific tests. Portmanteau tests can be used to test departure from linearity, while specific tests associated with regime-switching models can be used to determine the number of regimes. When applied to the UK equity risk premium, all these tests reject the null hypothesis of linearity and indicate strong evidence of non-linear structure. In particular, the empirical results of threshold effect tests support the finding of threshold non-linearity and suggest an uneven mean-reversion pattern. The Markov switching regime tests favour the existence of two states and suggest that risk premia have non-linear features.

5.4.1 Scatter plots to detect non-linearity

A common approach to assessing the relationship between two continuous variables is to plot their bivariate distribution, i.e. the joint distribution of these two variables. A bivariate normal distribution suggests that two individual variables are marginally normally distributed and the relationship between them is approximately linear (Hays, 1994). In this case, the plot of the bivariate distribution provides preliminary evidence as to whether the relationship between these two variables is linear. Figure 5.1 shows the scatter plot of equity risk premia against the lagged one-period equity risk premia. For a bivariate normal distribution, the scatter plot should be approximately elliptical with decreasing density from its centre.

However, the points appear to depart from this pattern, suggesting that the model is non-linear. To visualise the non-linear relationship between the equity risk premium and the prediction variables, we plot non-parametric kernel regression curves using the Nadaraya-Watson estimators on the scatter graph. The curves in Figure 5.1 confirm the finding of non-linearity in the model and more formal significance tests are therefore required.

Figure 5.1 Scatter plots of bivariate distribution
FIGURE 5.1 HERE

5.4.2 Portmanteau non-linearity tests

a) RESET and BDS test

To avoid over-fitting the data, it is usually recommended to perform linearity tests before any further empirical analysis. We employ two commonly used portmanteau non-linearity tests: the RESET (Ramsey and Schmidt, 1976) and the BDS test (Brock, Deckert and Scheinkman, 1987).

Table 5. 1 Ramsey RESET

Variable	Coefficient	Std. Error	Std. Error	Prob.
C	-0.1263	0.1099	-1.1491	0.2510
EQ(-1)	0.0219	0.0458	0.4790	0.6321
DY(-1)	7.4307	3.5794	2.0760	0.0384
RF(-1)	-0.6779	1.0754	-0.6304	0.5287
RPI(-1)	-1.4045	0.8380	-1.6761	0.0943
FITTED^2	-1.8433	1.0968	-1.6807	0.0934

FITTED^3	6.0371	1.4869	4.0603	0.0001
	Value		Prob.	
F-statistic	13.9065		0.0000	
Likelihood ratio	28.4847		0.0000	

Table 5. 2 BDS test

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.0128	0.0033	3.8098	0.0001
3	0.0314	0.0053	5.9032	0.0000
4	0.0419	0.0063	6.6292	0.0000
5	0.0477	0.0066	7.2376	0.0000
6	0.0498	0.0063	7.8543	0.0000

We first apply the Ramsey RESET to examine whether the fit of the linear regression can be significantly improved by the additional variables $\hat{R}_t^2$ and $\hat{R}_t^3$. The results are reported in Table 5.1. The test rejects the null hypothesis that the coefficients of the additional variables are jointly zero, and therefore suggests that equation (5.36) is mis-specified. We then apply the BDS test to the estimated residuals from equation (5.36) for embedding dimensions of $m = 2,3,4,5$ and 6. The results are reported in Table 5.2. The test rejects the null hypothesis that the residuals are independent and identically distributed, and therefore provides evidence of non-linearity. However, both the RESET and the BDS tests examine the null hypothesis of non-linearity against a very general alternative hypothesis of non-linearity. As such, the rejection of these tests only suggests the existence of non-linearity and cannot identify the actual non-linear form that generated the data. In fact, there are so many different types of non-linear time series models that no single test can actually determine the type of non-linearity. In short, the linearity tests suggest that linear models are inappropriate to estimate equity risk premia. In the next section, we apply some commonly used specific tests.

5.4.4 Specific non-linearity tests

a) Tests for threshold effects

The purpose of this section is to test the null hypothesis of linearity against a specific alternative of TAR models. To determine the number of thresholds, we employ both Hansen's (1999) bootstrap approach and Chan's (1991) LR test.

Table 5.3 reports the results of the Hansen's (1999) bootstrap tests. The test statistics and corresponding *p*-values are reported. We find that the tests for the AR model against 1 threshold, and against 2 thresholds, are highly significant, but the test for 1 threshold against 2 thresholds is not. The results suggest that the null hypothesis of linearity is rejected and that a one-threshold TAR model is the best model.

Table 5.4 reports the results of the LR tests of Chan (1991). The tests examine the null hypotheses of the AR(1) model against a one-threshold TAR model, and confirm the Hansen (1999) test results above. In this case, we conclude that the linearity hypothesis is rejected and threshold effects are statistically significant in the UK equity risk premium.

Table 5. 3 Hansen's Test: AR model against threshold

Test	Statistic	P-value
Linear AR vs. 1 threshold TAR	18.4187	0
Linear AR vs. 2 threshold TAR	25.1452	0
1 threshold TAR vs. 2 threshold TAR	6.5152	0.6

Table 5. 4 Chan's Likelihood ratio test : AR model against 1 Threshold

Test	Statistic	P-value
Linear AR vs. 1 threshold TAR	18.41872	0.0038

b) Tests for Markov switching regime models

To test the null hypothesis of the AR(1) model against the alternative of two-state Markov switching regime models, we apply Hansen's (1992,1996) approach.

As discussed in Section 5.3.2, the transition probabilities p_{11} and p_{22} are nuisance parameters. Therefore, we need to evaluate the likelihood ratio statistics for each value of these parameters. Here, we know that p_{11} and p_{22} can take any values between 0 and 1, and the switching parameter μ can take any values in the set of real numbers R. To make this more practical, Hansen (1992) performs a grid search over the parameter space and calculates the likelihood statistics only with respect to these grid points. However, this method still requires a huge amount of computation. To make computation feasible, we assume that only the intercept depends on the state. In other words, the intercept term is allowed to change with regimes.

Table 5. 5 Standardised LR-test for the Markov switching mean-switching model

Standardized LR-test: 2.939	*p*-values
Grid 0	0.071
Grid 1	0.069
Grid 2	0.066
Grid 3	0.081
Grid 4	0.084

The results of Hansen's test are reported in Table 5.5. As Hansen (1996) suggests, the choice of grid does not impact the standardised LR statistics significantly, and we report only the results from Grid 3. The standardised LR-test statistic is 2.939 and the associated *p*-value is 0.081. Since Hansen's test under-rejects, we can therefore reject the null hypothesis of a single-regime AR(1) model. It can be concluded that there is strong evidence for supporting the Markov mean-switching regime model for the UK equity risk premium.

5.5 Conclusions

The main purpose of this chapter has been to test whether non-linear specifications are superior to linear ones for describing the behaviour of equity risk premia in the UK. We therefore present the methods that can be used to detect non-linear behaviour in financial time series. Two types of non-linearity tests are introduced: portmanteau tests and specific tests.

Portmanteau tests are used to test for linearity in a financial time series model without a specific non-linear alternative. Rejecting the null hypotheses in such tests suggests only a departure from the linear model assumption, not the actual form of non-linear alternative. Specific tests are used to test for linearity with a specific non-linear alternative. In this chapter, we have focused on the non-linearity tests based on two specific regime-switching models, the SETAR model and the MS-AR model. These two models can be used to describe the regime-switching behaviour in financial time series models.

However, testing for regime-switching models may involve the problem of nuisance parameters. Therefore, the conventional log-likelihood ratio test does not have a standard χ^2 distribution. To solve this problem, Hansen (1999) employs a bootstrap procedure to approximate the asymptotic distribution for the non-linearity test based on the SETAR models. He also shows that the probability *p*-values obtained from the bootstrap are asymptotically correct. The bootstrap method may provide exact inference for the tests under certain conditions. However, they may not work on tests for Markov switching regime models because of the absence of asymptotic theory. Consequently, Hansen (1992, 1996) finds a bound for the asymptotic distribution of the likelihood ratio statistic. However, as discussed in Section 4.32, his method is conservative insofar as it under-rejects the null.

To test the non-linear features in UK equity risk premia, we employ both portmanteau tests and specific tests. The empirical results of the Portmanteau tests suggest that equity risk premia in the UK show non-linear behaviour. Therefore, they may be better described by models which allow for non-linear structures. In particular, two specific regime-switching models are tested: the SETAR model and the MS-AR model. Our testing results suggest that both the SETAR model and the MS-AR model perform better than a simple linear AR model. We can conclude that regime switching is an important characteristic of equity risk premia during the period from January 1965 to May 2012. These findings are consistent with the consumption-based capital asset pricing model (CCAPM), which suggests that equity risk premia change cyclically, and tend to be higher during recessions than during expansions

Although we do find some interesting results, it is worth mentioning three potential problems with the testing of non-linearity in financial time series. The first critical issue is that there are many different types of non-linear time series models, and therefore no single test can actually determine the form of the non-linearity presented. As discussed in this chapter, portmanteau tests are used to test for linearity without a specific non-linear alternative, while specific tests examine the linearity assumption with a specific non-linear alternative. Obviously, both these tests examine only for departure from linearity and cannot actually determine the particular form of the non-linearity. For example, the SETAR test examines only the null hypothesis of linearity against the alternative of the SETAR model, while the MS-AR test examines linearity against the MS-AR model. As a result, we can only draw a conclusion about whether these two models are superior to a linear model, and cannot actually compare these two non-linear specifications. We are therefore unable to provide empirical evidence for whether the SETAR performs better than the MS-AR model.

The second critical limitation is associated with determining the number of regimes. Firstly, as discussed in Chapter 2, we should choose the smallest possible number of regimes because allowing for too many regimes may lead to model mis-specification. Secondly, except for the formal statistical tests, we should also consider the observed behaviour of time series. From a practical point of view, the latter is more important.

The third issue is associated with the testing procedure in the Markov switching regime models. As discussed in Section 5.3.2, Hansen's test may have two problems: under-rejection of the null hypothesis and huge computational demand. Therefore, in future research it will be worthwhile investigating simple, formal tests for Markov switching regime models.

Appendix 5 Figures

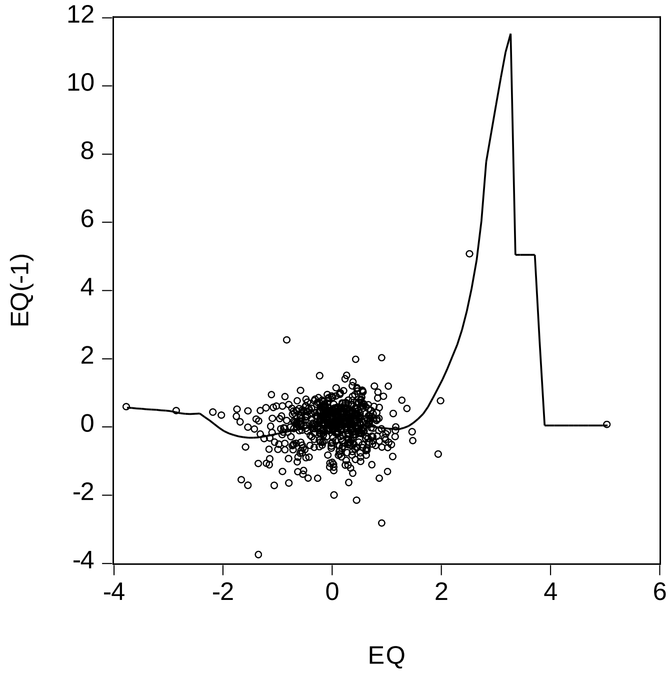


Figure 5. 1 Scatter plots of bivariate distribution

Chapter 6 Conclusions

In this study, we have estimated models of equity risk premia conditionally on state variables which are related to business conditions. These state variables can be determined endogenously. In this chapter we summarise the key findings of this research. Some limitations are identified and practical recommendations for future work are made.

6.1 Summary of findings

This study analyses regime-switching behaviour in the equity risk premium models of the UK stock market. Chapter 2 outlines four objectives. The first objective is to ascertain whether equity risk premia are stationary over time. The second is to examine whether the underlying predictive structure of equity risk premium models changes over time. The third is to model the switching behaviour of equity risk premia, and the fourth is to test whether regime switching models are superior to a linear specification. To achieve these four objectives, both univariate and multivariate time series models are estimated.

Chapter 2 provides a literature review on the existing studies of the dynamic connection between equity risk premia and business cycles, and suggests that regime switching is an important characteristic of equity risk premia. Our starting point is a dispute between the Efficient Market Hypothesis (EMH) and the Consumption Capital Asset Pricing Model

(CCAPM) over the assumption of constant equity risk premia. EMH implies that equity risk premia remain constant over time. However, this assumption is inconsistent with the observed behaviour of the stock market. First, observed stock market behaviour may be inconsistent with the assumption of constant equity risk premia. Second, information asymmetries imply that information is distributed unevenly and that investors may therefore have different stock market beliefs and expectations. Third, Behavioural Finance holds that investor sentiment plays an important role in determining the behaviour of financial markets, so that different investors may have different reactions to the same information even where they are well informed. In this case, investors face uncertainty from both fundamental risk and sentimental risk when they make investment decisions. In particular, both these risks change cyclically, with the result that equity risk premia also change cyclically. Fourth, the CCAPM ties equity risk premia to business cycles and provides theoretical evidence that these equity risk premia change over business cycles. In this case, we cast doubt on the classical assumption that identically, independently, normally distributed errors have sufficient power to capture the time-varying equity risk premia associated with cyclical variation. To sum up, the issues raised in Chapter 2 provide the motivation for considering the possibility of structural breaks and regime-switching in the parameters of equity risk premium models.

In Chapter 3, we turn to the question of whether the underlying structure of equity risk premia has experienced persistent changes and we find evidence that structural breaks have occurred in the data generating processes of UK equity risk premia. We first review two common approaches to predicting equity risk premia: historical average realised excess returns and lagged financial variables. However, these two methods assume that equity risk premia are drawn from one stable distribution over time and therefore fail to account for the possibility of structural breaks. We then investigate the reasons for suspecting that the underlying structure in equity risk premia may experience persistent changes. There are two main reasons. The first is that a fall in macroeconomic risk may result in a decline in the long-run equity risk premium. The second can be attributed to changes in the aggregate level of risk aversion in stock markets, that is, changes in investors' sentiments. Indeed, the global financial environment is in reality a dynamic, complex and ever-evolving system. Government policy, globalisation, technological innovation, new forms of market organisation and international financial liberalisation may result in structural changes in stock markets. Therefore, it is advisable to conduct structural break tests when modelling equity risk premia.

The first pre-requisite, before estimating models of the equity risk premium, is to test for the stationarity of the financial time series: equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates. In accordance with conventional unit root tests, such as the ADF test, the Philips-Perron test and the KPSS test, equity risk premia and dividend yields are found to be stationary over time. However, these stationarity tests cannot reject the null hypothesis of unit roots for three-month Treasury bill rates and inflation rates, even allowing for trends. Taking into account that the existence of unit roots in a financial time series may be better described by the presence of a single structural break we therefore apply the Zivot-Andrews (1992) and the Perron (1997) unit root tests. The former tests the null hypothesis of non-stationarity, against the alternative of one endogenous break-stationary series. The latter tests for a unit root by allowing for a structural break under both the null and the alternative hypotheses. We find monthly inflation rates to be stationary with a single structural break and the apparent non-stationarity to be an artefact of failing to model this break. Conversely, three-month Treasury bill rates are found to be non-stationary even in the presence of a structural break. However, our sample spans a long time-period during which the multiple structural breaks may have occurred, resulting in the non-rejection of the unit root hypothesis. We therefore also apply two-break versions of the multiple-break unit root tests of Lumsdaine and Papell (1997) and Lee and Strazicich (2003). Under the Lumsdaine and Papell (1997) two-break unit root tests, the null hypothesis is one of no breaks in the presence of a unit root while the alternative is of break-stationary. Under the Lagrange multiplier (LM) unit root test of Lee and Strazicich (2003), both the null and alternative hypotheses allow for the identification of multiple endogenously determined breaks. In addition, Lee and Strazicich (2003) found that the LM statistics are free of nuisance parameters and are robust to any misspecification in the number of endogenous breaks. Therefore, this test provides more reliable information for testing stationarity in the presence of multiple breaks. Our application of these two-break unit root tests rejects the null hypotheses of unit roots for three-month Treasury bill rates and inflation rates at both the monthly and quarterly frequencies. We conclude that over the period of 1965 to 2012 equity risk premia and dividend yields remained stationary, that the inflation rates were one-break stationary, and that the three-month Treasury bill rates were two-break stationary. These findings are consistent with those in Perron (1989) who argued that the existence of unit roots in financial time series may better be described in terms of stationary series with structural breaks. In most of the time-series this break occurred between 1973 and 1975, reflecting a fundamental change in stock markets following the 1973 oil price shock.

The second research objective required an examination of structural instability in financial prediction models. Here, we employ both multivariate and univariate predictive regression models. Empirically, the multivariate models can explain a greater range of variation in equity risk premia than the univariate models, but there may be some partial structural breaks occurring only in a sub-set of the regression variables. In this case, the univariate regression models may provide more information about the dates and characteristics of structural breaks even though they may have lower predictive power. Five tests are used to test for the presence of structural breaks in this study. The rolling window estimates provide preliminary evidence as to whether the parameter estimates are stable over time. The Chow test can be used to examine whether there is a structural break at a specific date. The CUSUM and CUSUMSQ tests can detect unknown structural breaks. The Quandt-Andrews unknown breakpoint test can be employed to identify one unknown break point. The Bai-Perron test can be used to detect unknown multiple structural breaks. These test results revealed that there are some underlying structural changes in the equity risk premium models. Two structural breaks are identified for the multivariate equity risk premium prediction model: 1974 and 1982. The 1974 break reflects a fundamental change in the stock markets following the 1973 oil price shock. The 1982 break corresponds to the change in dividend yields, which have declined ever since 1982. In order to examine further the parameters' instability in equity risk premium models, we then split our sample into three sub-periods and estimate each separately, employing OLS and Stepwise LS. These provide evidence of parameter instability in equity risk premium models. To facilitate the interpretation of these results, we then examine in turn the breaks in the univariate models based on each individual forecasting variable. In addition, we have looked at whether the recent financial crisis of 2007 has had a permanent effect on stock markets, and the results suggest that there was a breakdown in equity risk premium models in 2007. As discussed in Chapter 3, we attribute the failure to find this break to the boundary issue. According to the results of both multivariate and univariate regression models, we were then able to split our sample into four sub-periods:

- 1965-1974 - The Keynesian demand management period.
- 1974-1982 - The anti-business period.

- 1982-2007 - The business-friendly period.
- 2007-2012- The Keynesian resurgence.

From this, we can conclude that the predictability of equity risk premium models based on forecasting variables changes over time. Furthermore, we discuss the statistical power issues related to the structural break tests: the number of breaks, the boundary issues and the limited power for detecting breaks that occurred gradually. We then start to ask the question whether it is true that there is no structural change in the equity risk premium models over the period 1982-2007.

In Chapter 4, we therefore consider the third objective, which is to model the switching behaviour of equity risk premia by using regime-switching models. These models allow for the existence of two or more regimes and the dynamic financial switching may occur between them. We first focus on univariate time series, assuming that equity risk premia follow an AR(1) process. Three types of regime-switching models are used in this study: structural break models, threshold autoregressive (TAR) models and Markov switching autoregressive (MS-AR) models. Both structural break models and TAR models assume that the switching mechanism is deterministic. Under the former, the state variable is solely determined by time and only a one-time switch is allowed for at a specific break date. Under the latter, the state variable is determined by an observable variable with respect to an unobserved threshold and thus a switch is allowed for at each observation. In Markov switching regime models, dynamic time series are allowed to switch probabilistically for each observation. The underlying state variable is not directly observable and is therefore latent. This latent state variable is assumed to follow a Markov process. Our empirical analysis of the UK stock markets suggests that regime-switching is an important economic behaviour of the UK equity risk premium. In particular, we identify three regimes over our sample period: a 'normal' regime, characterised by a small positive mean and low volatility; a 'crash' regime where the average equity risk premium is significantly negative and the volatility is very high; and a 'bull' regime characterised by high equity risk premia. Furthermore, we find clear evidence that regime shifts in equity risk premia occurred during 1982-2007, and that this period can be divided into four sub-periods: 1982-1992 with a 'normal' regime, 1992-1997 with a 'bull' regime, 1997-2002 with a 'normal' regime and 2002-2007 with another 'bull' regime. However, business cycles are the result of the co-movements of many macroeconomic variables. The univariate regime-switching model may not capture these co-movements in multiple financial time series, and we therefore apply regime-switching to a VAR framework to explore the dynamic interactions between multiple financial variables. Two VAR models are used: threshold vector autoregressive (TVAR) models and Markov switching vector autoregressive (MS-VAR) models. We conclude that TVAR models and MS-VAR models can capture the co-movements between equity risk premia, dividend yields, three-month Treasury bill rates and inflation rates. To summarise, it has been shown that the predictive ability of equity risk premium models based on forecasting variables changes over time.

In Chapter 5, we focus on the fourth objective which is to test whether non-linear specifications are superior to linear ones. In order to answer this question and avoid over-fitting the data, we conduct linearity tests to examine the null hypothesis of linearity against the alternative of non-linearity. Two types of test are employed: portmanteau tests and specific tests. Portmanteau tests are used to test for linearity without a specific non-linear alternative, while specific tests are used to test for linearity with a specific non-linear alternative. In particular, we have investigated the issues on non-linearity tests subject to regime switching models. The TAR and MS-AR switching models are tested for non-linearity. These tests raise the problem of unidentified nuisance parameters under the null hypothesis of no switching. Under this, the log-likelihood ratio test does not have a standard asymptotic distribution. To solve this problem, we first employ Hansen’s (1999) bootstrap approach which tests the null of AR models against the alternative of self-exciting threshold autoregressive (SETAR) models. The results suggest that the SETAR model of the UK equity risk premium performs well relative to a linear AR model. We then test the Markov switching regime models using Hansen’s (1992 and 1996) likelihood ratio bound test. The results favour the two state regime-switching models. These results support the findings of Chapters 3 and 4 and suggest that regime switching does exist in the sample period. We conclude that equity risk premia have non-linear features, and that regime-switching models describe these non-linear features better than standard linear models. To summarise, structural breaks and switching regimes have important implications for estimating future equity risk premia. In particular, they suggest that even in the long-term investors should revise their investments based on short-term business conditions when making their asset location decisions.

6.2 Limitations and Remarks

Despite the efforts of this study, there remain some unresolved issues. First of all, little is still known about the impact of investor sentiment on equity risk premia. Market sentiments refer to the prevailing attitude of investors to the future stock market trend. It is a common expectation about future market movements. Even though we include the price-related variable, bond-related variables, economic variables and autocorrelation within our equity risk premium models, and suggest that all these variables are related to both fundamental risk and investors' risk attitude, we have not provided the theoretical modelling and empirical evidence on how to measure overall market sentiments and therefore quantify their effect on equity risk premia.

Second, apart from the rational investors, there are many irrational investors in stock markets. Their behaviour has not been considered in our research. A rational investor is one who maximises her expected utility according to rational expectations. Irrationality may arise as a consequence of investors having different expected utility functions or failure to form rational expectations. Cognitive biases suggest that irrational investors can be affected by short-term cyclical fluctuations even though their investment horizons are long-term in nature. Stock prices are determined by both rational and irrational investors. Therefore, irrational noise traders may have a significant impact on stock prices even though changes in investor sentiment are not related to stock market fundamentals. This is because the unpredictability of irrational noise traders' behaviour may raise greater risks of holding stocks and rational investors may require higher equity risk premia to compensate for these risks. Hence, in order to estimate equity risk premia in the market, both rational and irrational investors' behaviour should be taken into account. However, these issues are not formally tested in our work but may be worth investigating in further research.

As the empirical testing methodologies, there are several issues worthy of further development and investigation. First, as discussed in Chapter 3, structural break tests introduced in this study may have the power to identify only extremely large breaks but may not capture gradual changes in equity risk premium models. However, it is reasonable to assume that structural changes may develop over a period of time and may occur gradually. Although the Innovational outlier (IO) model introduced by Perron (1997) allows for an unknown shift to take place gradually, this model tests the null of a unit root and therefore focuses only on univariate time series. MS-AR models can capture small changes in equity risk premia by allowing them to switch randomly between different regimes. However, as discussed in Section 5.3.2, Hansen's test may have two problems: under-rejection of the null hypothesis and huge computational demand. Therefore, it would be worthwhile investigating further formal and simpler tests for Markov switching regime models.

Third, as discussed in Chapter 5, there are so many different types of non-linear time series models, and no single test can uniquely determine the form of non-linearity present in the data. The SETAR test and the MS-AR test examine only the null hypothesis of linearity. As a result, we can only draw a conclusion as to whether these two models are superior to a linear model, and cannot actually compare these two non-linear specifications. More precisely, we are unable to provide empirical evidence for whether the SETAR model can perform better than the MS-AR model. How to reconcile the results of different types of non-linearity tests remains a challenge.

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