A FUZZY LOGIC APPROACH TO LOCALISATION IN WIRELESS LOCAL AREA NETWORKS

by
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ABSTRACT

This thesis examines the use and value of fuzzy sets, fuzzy logic and fuzzy inference in wireless positioning systems and solutions. Various fuzzy-related techniques and methodologies are reviewed and investigated, including a comprehensive review of fuzzy-based positioning and localisation systems. The thesis is aimed at the development of a novel positioning technique which enhances well-known multi-nearest-neighbour (kNN) and fingerprinting algorithms with received signal strength (RSS) measurements. A fuzzy inference system is put forward for the generation of weightings for selected nearest-neighbours and the elimination of outliers. In this study, Monte Carlo simulations of a proposed multivariable fuzzy localisation (MVFL) system showed a significant improvement in the root mean square error (RMSE) in position estimation, compared with well-known localisation algorithms. The simulation outcomes were confirmed empirically in laboratory tests under various scenarios. The proposed technique uses available indoor wireless local area network (WLAN) infrastructure and requires no additional hardware or modification to the network, nor any active user participation. The thesis aims to benefit practitioners and academic researchers of system positioning.

Keywords: wireless positioning, localisation algorithms, fuzzy inference, RSS, kNN, WLAN.

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At the end of this phase of my journey

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TABLE OF CONTENTS

LIST OF ABBREVIATIONS

AFP	assisted fuzzy positioning
AFS	assisted fuzzy system
ANFIS	adaptive neuro-fuzzy inference system
AOA	angle of arrival
AP	access point
APL	(Johns Hopkins University) Applied Physics Laboratory
BOA	bisector of area
CA	centre-average
CID	cell identification
COA	centre of average
CoA	centre of area
COG	centre of gravity
COGS	centre of gravity method for singletons
COO	cell of origin
D2D	device-to-device communication
DGPS	differential Global Positioning System
DoA	direction of arrival
EIF	extended information filter
FAM	fuzzy associative memory
FCC	Federal Communications Commission
FCM	fuzzy C-mean
FDOA	frequency difference of arrival
FIS	fuzzy inference system

FL	fuzzy logic
FM	frequency modulation
FML	fuzzy mark-up language
FP	fingerprint
FRB	fuzzy rule base
GA	genetic algorithm
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GSM	Global System for Mobile Communications
IEEE	Institute of Electrical and Electronics Engineers
IFP	incorporated fuzzy positioning
IoT	Internet of things
IR	infrared radiation
ISO	International Organization for Standardization
kNN	k nearest neighbour
LAT	lateration
LBS	location-based service
LM	leftmost maximum
LMS	least mean squares
LOS	line-of-sight
MF	membership function
MIMO	multi-input multi-output
MISO	multi-input single-output
MOM	mean of maxima

MVFL	multiple variable fuzzy localisation
NLOS	non-line-of-sight
NN	neural network
OLS	orthogonal least square
POA	phase of arrival
Post-AFP	post-assisted fuzzy positioning
Pre-AFP	pre-assisted fuzzy positioning
QoS	quality of service
RF	radio frequency
RFID	radio frequency identification
RM	rightmost maximum
RMSE	root mean square error
RSS	received signal strength
RSSI	received signal strength indicator
SISO	single input single output
SLAM	simultaneous localisation and mapping
SNR	signal to noise ratio
SVD	singular value decomposition
SVFL	single variable fuzzy localisation
TDOA	time difference of arrival
TDOF	time difference of flight
TKS	Takagi-Sugeno
TMA	trapezoid median average
TOA	time of arrival

TOF	time of flight
TWTMA	trapezoidal weighted trapezoid median average
UWB	ultra-wide band
v.	versus
WLAN	wireless local area network
WSN	wireless sensor network
WTMA	weighted trapezoid median average

LIST OF FIGURES

LIST OF TABLES

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1 INTRODUCTION

1.1 Overview

A positioning or localisation algorithm can be defined as a set of processes to determine an unknown location of a target in a predefined space. For centuries, people relied on maps to pinpoint the location of objects in a predefined area. But with the astonishing development of global digital mapping, the technology used to determine the position of objects has changed dramatically.

In particular, the emergence of telecommunications, especially wireless telecommunications, in the last century, and its widespread incorporation into portable, handheld devices, has revolutionised the use of positioning algorithms and positioning systems. The origin of this revolution in technology may be traced to the requirements imposed by communication authorities on telecommunication service providers—primarily, the E911 rules first mandated in 1996 by the US Federal Communication Commission (FCC). The E911 required cellular network operators to incorporate techniques to determine the location of an emergency caller. In emergencies, a more accurate estimation of a caller's position aids faster response times, and improves the ability of first responders to better allocate their resources when busy with multiple or major emergencies. The E911 required two levels of implementation. The first is to identify the phone tower that conducted the call, using the so-called *cell identification technique* [1]. The second imposed a strict accuracy requirement of 50 metres, a requirement which has yet to be fully achieved. Since the majority of calls are made from mobile devices and from indoors, the commission amended its mandate in 2015 to add an accuracy requirement for the vertical position of the caller in addition to the horizontal in order to pinpoint, for example, the floor in a high-rise building from which the emergency call is made. Unfortunately, current positioning technologies implemented for outdoor environments do not work nearly so well indoors.

The E122 standards implemented in Europe in 2001 are similar to the E911. Both sets of standards stipulate the required accuracy of location estimation. In doing so, both have driven the development of location-based services (LBSs) in which the quality of service is correlated with achieved position accuracy. In addition to the impact of E911 and E122, LBS applications received a significant boost following the development of new handheld devices such as the smartphone and tablet PC [2].

The arrival of smart handheld devices brought new location-related challenges to the fore. First, the richness of the sensory information typical in these devices creates a high demand for battery power; this demand in turn limits the use of self-localisation techniques which require large processing capabilities, and therefore more power. Second, the increasing use of these devices indoors complicates the localisation task due to their dependency on the Global Positioning System (GPS) and the Global System for Mobile Communications (GSM). The GPS signal is not designed to penetrate walls and the GSM is unable to access indoor environments because it lacks prior information about the indoor network infrastructure. In addition, smartphones may be connected by another means—for example, through the wireless local area network (WLAN) infrastructure in an indoor environment.

It is evident that positioning systems are heavily dependent on many factors, including the environment, adopted technology, type of sensory information, communication protocols, available resources, time constraints, computational cost, accuracy and precision requirements, and the availability of local and global maps and grids. All these factors influence the choice of an appropriate positioning algorithm.

1.2 Research Scope and Aim

The present research project began with an investigation of WLAN environments and efforts to model these environments with the techniques collectively known as *soft computing*. At the outset, the main goal was to develop a model for a WLAN environment using a fuzzy inference-based system to handle the uncertainties pervading the environment. Later, the research scope was narrowed to focus on localisation and positioning in WLANs with service orientation. The localisation process for indoor WLAN environments addresses many detailed issues in the design of WLAN hardware, software and applications. Uncertainties in WLAN operation—including systematic errors from measurement devices, random noise from the inherent properties of the signal, and biased, uncontrolled errors due to non-line-of-sight (NLOS) obstacles—increase the need for robust models of WLAN environments. This need has been heightened by increasing demands on WLAN services in recent years. As many of these services are location-dependent, accurate location estimation has become paramount. Moreover, many services require a guaranteed level of quality of service (QoS) which, in turn, puts a greater burden on location estimation when LBSs are involved. Since service providers often suffer from limited resources within their systems, an efficient resource utilisation/allocation algorithm is necessary to guarantee a desired level of QoS. This raises several challenges for researchers: (i) eliciting the environment parameters; (ii) designing an appropriate fuzzy logic based model system using a proper rule-based approach; and (iii) accounting for the dynamic interaction of various parameters (QoS, LBS, rule-base system and WLAN) in the context of evolving wireless environments (presence of obstacles, diminishing signal strength, moving targets and partially known or unknown environments).

This research was focused on dynamic environments to enhance our understanding of inherent parameters and their interactions in WLAN environments. The research aimed to model the localisation technique(s) for indoor environments with any WLAN communication topology by using soft-computing capabilities, specifically, fuzzy logic. The novel aspect of the research lies in creating a generalised localisation model for indoor WLAN environments or enhancing already existing models with respect to QoS and resource use measures. This may be achieved through the reasoning power of fuzzy inference to reduce the burden of uncertainty associated with most available localisation systems. The proposed methodology, which is based on a fingerprinting technique, focused not only on designing, implementing and testing a fuzzy approach for the location estimation, but also on novel enhancements to the existing multi-nearest neighbour (kNN) algorithm. Specifically, a fuzzy inference system (FIS) was proposed with two input variables: the distance, which is interpolated from received signal strength; and the variation in the received signal strength (RSS). Those two inputs are used to generate weights for every selected nearest neighbour, Then, later, before calculating the estimated location, an 'outliering' step takes place to eliminate wrongly-selected nearest neighbour fingerprints.

1.3 Contribution

This thesis offers an innovative analytical and methodological approach in estimating object location in indoor wireless environments. It combines a proximity-based approach, specifically, the kNN, with analysis of the RSS fingerprint. This approach offers advantages for overcoming the inherent uncertainties of available algorithms and ill-defined cases. It also provides a setting for fuzzy set theory to tackle the uncertainties pervading the RSS due to random signal variations in indoor environments, the multipath of received signals, reflection, diffraction and other sources of noise on Wi-Fi public band frequencies.

The proposed RSS fingerprinting algorithm depends on combining the kNN algorithm with a Takagi-Sugeno (TKS) fuzzy inference system—that is, a Takagi-Sugeno FIS or TKS-FIS. In practice, this entails infusing the TKS with the distance D_j, the Euclidean or 'straight-line' distance between a target point j and the corresponding nearest-neighbour fingerprint selected by the kNN algorithm. D_j is calculated from the RSS in the collected signal space. The process allows the TKS to produce a weighting or degree of participation for every single nearest fingerprint in the determination of the target's location, rather than the averaging technique common employed in the kNN algorithm. The weighting approach guarantees at least partial participation for the nearest neighbour considered to be spatially distant from the target, based on blind RSS measurements. The use of TKS allows the algorithm to 'smooth' the value of the produced weighting in the presence of approximately ±8 dB uncertainty in the propagation model, a value determined by empirical measurements. Further, as the Euclidean distance is not sufficient to distinguish between various scenarios, an additional means for discriminating the quality of RSS was included in the proposed system. Specifically, the TKS-FIS was infused with an extra input—the variation in signal—to construct what is known as a *multivariable fuzzy localisation* (MVFL) system. The infusion of the extra data allows us to assess the level of agreement between fingerprints and targets, which indicates the summation of changes caused by a particular access point (AP). This means that a better sense of the level of association between target fingerprint and selected nearest fingerprints may be obtained when the weighting produced by the TKS is high [3].

The thesis encourages the use of statistical studies in examining localisation in particular environments. Statistical analysis of the MVFL results, using the kNN algorithm, showed a non-negligible failure average in determining the actual nearest neighbours in a signal space based on the RSS measurements. This limitation was addressed with an outliering mechanism to exclude some of the neighbours selected by the kNN algorithm. The mechanism depends on a distance function in the Euclidean space, where every kNN tuple is assumed to establish a complete triangle. Where the fingerprints under investigation form the corners of the triangle, only the triangle enclosing the smallest area is selected [4].

Additionally, this thesis offers a classification of fuzzy-based localisation principles and evaluate the performance of other approaches according to the performance criteria proposed for this research.

1.4 Thesis overview

Chapter 2 provides a comparative review of different positioning techniques for WLAN, including their historical background and terminology, the challenges faced by positioning systems, the uncertainty which pervades wireless positioning, the inherent characteristics of fuzzy systems to deal with uncertainty, and fuzzy-based methodologies linked to wireless positioning systems.

Chapter 3 introduces WLAN modelling and localisation methods, location estimation systems with their various classification for the deployed techniques, LBSs and their applications, the propagation models and, finally, the RSS-based fingerprinting method.

Chapter 4 introduces fuzzy sets and fuzzy systems by presenting fuzzy set theory and its properties for linguistic information representation, fuzzy reasoning, FISs, the construction of fuzzy rules and fuzzy arithmetic.

Chapter 5 outlines the hybridising of the fuzzy system for WLAN and indoor position estimation, the methodology employed, the kNN algorithm, the fuzzy inference combined with the kNN algorithm, the enhancement to the fuzzy inference via the multivariable approach, and the enhancement via robust statistics and the outliering algorithm.

Chapter 6 provides the experimental evaluation of the fuzzy-based positioning systems with the kNN discussed in Chapter 5. Finally, this section presents concluding remarks and possible improvements to the proposal and guidelines for future research directions.

2 BACKGROUND

2.1 Introduction

The first considerable work on wireless positioning and localisation could be traced back to the pioneering work of the Johns Hopkins University Applied Physics Laboratory (APL) in monitoring the radio transmission of Sputnik (first human-made satellite by the former Soviet Union 1957) [5]. As a consequence of this work, the satellite was approximately located along its orbit using the microwave signals emanating from the satellite and its Doppler shift effect. This work led to the appearance of the Transit system in 1961 [6] as the first satellite positioning system. Gradually, Transit was made obsolete by the emergence of the Global Positioning System (GPS) in 1996 [6], which became the most popular and widely-used positioning system [7]. Since then, with the astonishing developments in wireless technologies, several device-enabled positioning systems have emerged.

The development of (wireless) positioning technology made a giant step forward after the US Federal Communication Commission (FCC) introduced the requirements for safety services such as E911 [8], which forces cellular network operators to provide the position of wireless terminals at a defined level of accuracy. The E911 has been a significant driving force behind research into localisation technologies for almost two decades. In turn, localisation research has been central to other critical activities, such as location-sensitive billing information, fraud detection, intelligent transportation systems and enhanced network performance [1], [8], [9].

The importance of localisation in wireless sensor networks (WSNs) arises from several factors, including *inter alia* the identification and correlation of gathered data, node addressing, query management of nodes localised in a determined region, evaluation of nodes density and coverage, energy map generation, geographic routing, and object tracking.

These factors make localisation systems a key technology for the development and operation of WSNs. This chapter addresses the localisation problem, with a particular focus on fuzzy-based reasoning [10]. Although wireless positioning systems originated in the early 1950s, and matured with the emergence of several GPS and GSM-type positioning technologies, several issues motivate further developments in the field.

First, after the successful applications of satellite-LBS for outdoor environments, attention turned to indoor environments where LBS presents business opportunities to increase customer satisfaction by providing targeted information in line with customers' preferences. Second, indoor positioning techniques continue to face technical challenges that restrict the level of accuracy, including multipath due to NLOS conditions and a higher density of obstacles, which worsens signal attenuation. Third, boosted by industrial applications, a demand for millimetre and nanometre positioning has emerged. Fourth, with the development of 5G networks, it has become possible to establish multiple mobile relays. For instance, device-to-device communication (D2D), which currently exceeds 30.6 exabytes per month [11], is expected to grow strongly, prompting new collaborative architectures in positioning schemes. With exponential increases in data rates and the diversity of mobile applications, big data analytics are expected to play a vital role in future LBSs. This ultimately opens the door for new positioning algorithms to address fresh challenges not seen in previous wireless systems. For instance, technology related to *massive multiple-input multiple-output* (massive MIMO), with data rates in the order of gigabits per second, has begun to appear in Samsung and Huawei mobile products [12]. Sixth, the emergence of Internet-of-things (IoT) technology enforces the need for new system designs and architectures which support reliability, mobility and spectrum management [13].

Numerous review articles have examined wireless sensor positioning technology and techniques. For example, Liu *et al* [14] have closely surveyed the localisation techniques for the WLAN environments available in 2005. They proposed a metric to compare 20 available localisation systems, depending on the accuracy and the precision of the outcomes, in addition to the complexity and robustness of the systems.

Seco *et al* [15] have surveyed the available localisation techniques for indoor WLANs, based on the mathematical models. They proposed to group these techniques into four categories: (i) geometry-based techniques; (ii) techniques based on optimisation principles; (iii) scene-analysis-based versions; and (iv) Bayesian-based models. Their discussion focused on feasibilities and robustness under NLOS constraints and noise removal.

Mautz [16] has surveyed 13 available solutions for indoor localisation which work at centimetre and millimetre wavelengths. He imposed a geodesic point of view on all methods. With the survey focusing on optical indoor positioning systems, his proposed metric for comparison included the accuracy of localisation, signal frequency parameters, and market demand (with some financial analysis based on the cost of imaging devices).

Bensky [17] provided a comprehensive review of radio-navigation techniques with specific methods for radio distance estimation. Tahat *et al.* [18] covered more modern developments in the field of wireless positioning, with a focus on algorithms for moving receivers.

2.2 Uncertainty pervading wireless positioning systems

Uncertainty is often seen as an inherent operational aspect of any wireless system, regardless of the technology employed. However, where the types of uncertainty in sensor networks are identified and quantified, more effective and efficient data management strategies may be developed. These strategies would straightforwardly enhance the quality of the positioning systems. In this respect, several types of uncertainty are distinguishable [18]–[21]:

- *Communication uncertainty,* where a mobile sensor network exhibits intermittent connection patterns. Hence, quantifying communication uncertainty of communication links would contribute to better routing decisions.
- *Sensing uncertainty*, where sensor range and coverage are predominately affected by environmental interferences, noise and other systematic physical limitations of the sensor hardware. Accounting for such uncertainty through statistical, soft computing or other models which capture sensor behaviour would facilitate effective sensor deployment strategies.
- *Data uncertainty* due to inherent imprecision affecting sensor readings. Assigning confidence values or distributions to sensor readings would ultimately improve quality and decision-making in networked sensor systems.

In outdoor urban environments using the cellular network to estimate the position of a receiver (mobile client), the signal attenuation radio propagation model offers a means of analysis of the receiver's location. However, such a model is jeopardised by NLOS and multipath issues, which, in turn, negatively affect the positioning accuracy. Therefore, both sensing uncertainty and data uncertainty should be accounted for through appropriate uncertainty modelling. Sensing uncertainty can account for signal propagation affected by environmental constraints, as well as factors such as update rate limitation and correlated errors from the receiver's clock offset lag. Data uncertainty can account for fluctuations in sensor readings over time. Similarly, the use of odometer-like sensors such as a wheel encoder, which provides incremental position measurements, has an unbounded accumulation of estimation errors over long travelling distances; this may trigger non-negligible sensory and data uncertainties.

Accounting for such uncertainties is of paramount importance. To improve the accuracy of location estimates in network-based systems, the choice of a proper tool to deal or compensate for uncertainties is crucial to improving the accuracy of the positioning system. In this regard, one should acknowledge the capability of fuzzy logic to deal with uncertainties. Fuzzy analysis has been extensively used and successfully applied in various disciplines and at various levels. By providing a notational platform for the representation of knowledge and inductive reasoning in imprecise and uncertain circumstances, fuzzy systems are a vital field for the application of fuzzy set theory. Fuzzy sets can incorporate human knowledge, granular computing, deterministic and 'crisp' (i.e., not fuzzy) information to describe complex system behaviours without recourse to precise mathematical models, notably for the positioning problem discussed above.

2.3 Characteristics of fuzzy systems in dealing with uncertainty

Fuzzy logic (FL) introduced by Zadeh in the 1960s [22] is a form of multi-valued logic that formalises approximate reasoning. The base of FL is the fuzzy set, which is a generalisation of the classical set. FL aims to model human reasoning, which is approximate by nature, rather than precise, and permit the inference of a possible, imprecise conclusion from a collection of imprecise premises. For instance, knowing that:

IF Node A is CLOSE to Node B, THEN mobile accuracy is HIGH

AND, IF Node A is FAR from Node B THEN mobile accuracy is MEDIUM,

We want to infer the state of mobile accuracy IF Node A is VERY FAR from Node B.

The meaning of an imprecise proposition is represented as an elastic constraint on (linguistic) variables, while the inference is derived through propagation of these elastic constraints. This extends the domain of inference systems of propositional, predicate and multivalued logics. FL provides a systematic framework for dealing with fuzzy quantifiers (e.g., *very*, *high*, *most*) and enables the underlying theory to subsume both predicate logic and probability theory. In turn, this characteristic of fuzzy logic makes it possible to deal with various types of uncertainty within a single conceptual framework.

In addition to its value as a conceptual framework for approximate reasoning—using linguistic variables, fuzzy quantifiers, fuzzy rules, canonical forms and connectives—the mathematical foundation of the fuzzy logic offers an attractive platform for applying fuzzy logic in real-life applications. [23], [24].

Fuzzy set theory is a broad discipline with many sub-divisions. FL is just one topic within the discipline; many related topics are discussed in the literature, including fuzzy arithmetic, fuzzy mathematical programming and fuzzy topology [25], [26]. The development of fuzzy set theory has given rise to fuzzy estimation, fuzzy optimisation, fuzzy pattern matching and fuzzy classification. These techniques have found numerous applications in wireless positioning systems.

From a mathematical representation point of view, any linguistic variable takes values from the linguistic domain rather than from the domain of real numbers [22], [27], [28]. For example, 'distance' is a linguistic variable with values *close* and *far*. These values are used as labels for fuzzy sets, where each fuzzy set is expressed by a *membership function*, e.g. $\mu_{distance}(u)$, with a degree of association within the real number interval [0, 1].

For instance, $\mu_{distance}(u)$is expressed by the following membership function:

$$\mu_{distance}(u) = \begin{cases} 1 & if\ u \leq 20 \\ \frac{1}{u} & if\ 20 \leq u \leq 50 \\ 0 & if\ u > 50 \end{cases} \qquad (2.1)$$

An example of a rule where this can be applied is:

IF distance is “close” AND elapsed time is “short” THEN weight is “high”.

According to Alcalá-Fdez and Alonso [29], its ability to handle uncertain information in a methodical order, makes FL a wise choice for modelling the non-linear and complex systems or drawing inferences from expert-like rules. Developing fuzzy logic components, including connectives, optimal numbers of fuzzy rules, and the parameters for underlying fuzzy sets, is widely debated and several contributions in the field are available. Examples of fuzzy software available can be found at [29], and [30].

Although most applications of FL since the early 1970s are in software, several hardware related applications are noteworthy. For instance, Toga and Watanabe at Bell Laboratories developed the first fuzzy chip in 1985 [31], which served as the basis for many commercial applications in intelligent systems and expert system-related applications. Yamakawa’s fuzzy computer [32] is often seen as an important milestone in the development of sixth-generation computers, which are capable of handling data intelligently and similar to human reasoning. The development of fuzzy logic has also benefited from the development of standards, namely, the IEEE 1855–2016 standard for the fuzzy markup language FML, which has the ability to model fuzzy logic systems in a human-readable language and is executable on any hardware platform [33].

Another advantage of fuzzy systems is their ability to function as standalone applications, or fully or partially combined with other systems and techniques. Fuzzy systems may augment or hybridise other systems (e.g., neural networks, genetic algorithms (GAs), stochastic and statistical systems) yielding hybrid modes of, *inter alia*, estimation theory and control. Moreover, they are extendable to handle data representation and manipulation (e.g., arithmetic of fuzzy numbers and operations), reasoning (fuzzy implication and inferencing), statistics, classification, clustering and estimation (fuzzy Bayesian, fuzzy Kalman) [29].

2.4 Terminology and background of positioning systems

We refer to an object with an unknown position as the *target object* or, simply, the *target*. The position or location of the target object may be determined with respect to a predefined frame of reference. The frame may be defined on an absolute scale (such as the spatial Galilean frame) or on an relative scale (e.g., with respect to nearby objects). A *positioning algorithm* refers to the set of processes, steps or mathematical model(s) which establish spatial relationships between the target and measurements leading to an approximation of the target's location. Location-based systems include any system that delivers services according to the quality of location estimate required for an industrial, medical, safety or other commercial application. Typically, the positioning technology is ultimately linked to the context, sensory information and perceived environment, and the positioning algorithm employed.

Figure 2.1 (from [16]) summarises some of these technologies in terms of level of accuracy.

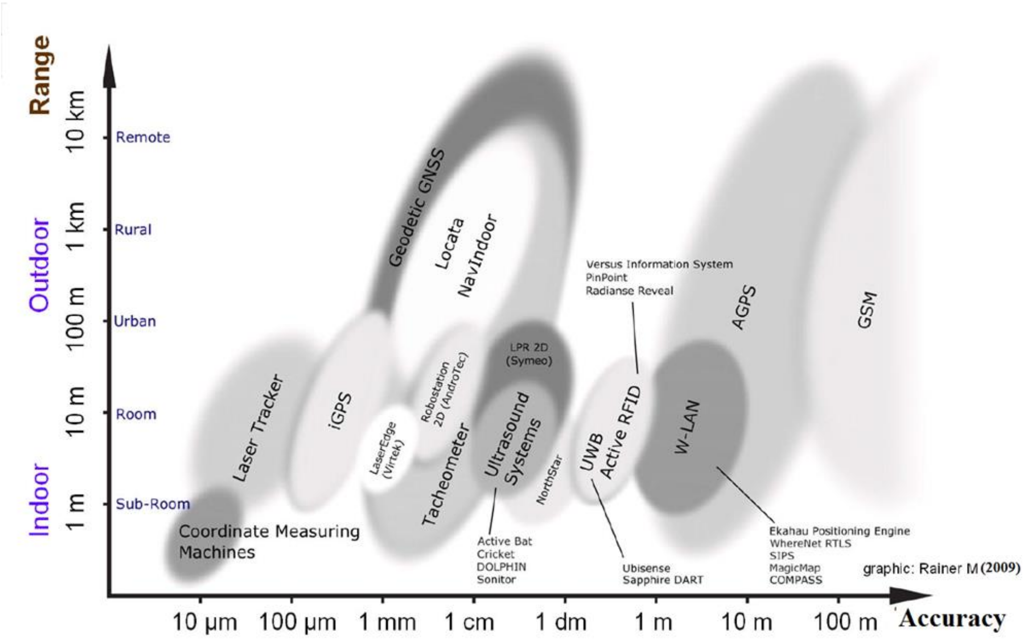

Figure 2.1 Accuracy of various positioning technologies[16].

As shown in Figure 2.2, the positioning algorithm is heavily dependent on the available resources, time constraints, computational cost, accuracy and precision requirements. Several factors contribute to the choice of the appropriate positioning algorithm (see Chapter 3).

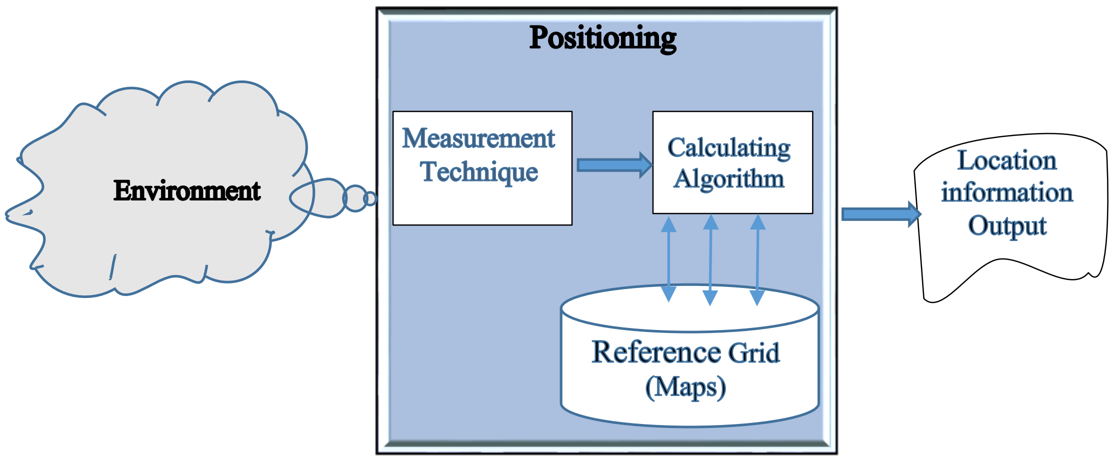

Figure 2.2 Positioning system architecture.

2.5 Challenges of positioning systems

Services based on the proliferation of mobile phones and similar devices are an everyday and increasingly important feature of contemporary lifestyle. Many of these services are location-dependent—in other words, the quality of received service relies on the quality of the acquired location. LBSs necessarily need accurate estimates of location, which in turn has fuelled close analysis of the localisation problem and its requirements.

In commerce and trade, LBSs offer strong potential for business innovation in customer services, transportation and navigation. At the same time, we have witnessed a tremendous leap in hardware industries associated with handheld devices, along with the development in software application, software engineering and cloud computing. ABI Research [34] and the FCC [2] have surveyed the business opportunities of location-dependent services, and identified a huge potential for various industries. In 2016, IBM [35] estimated that the value of indoor location-based business opportunities would reach US$ 10 billion by 2020. A recent market report predicted the global indoor location market to reach US$17 billion by 2025 [36].

Standards such as E911 [8] and E122 [37] were adopted depending on the technology involved in the services. Later, these standards tended to impose accuracy requirements on the services and the service providers. These requirements were possible to meet in many instances where the service operated in an outdoor environment. However, when the same services shift indoors, these accuracy requirements are hardly ever met.

From a technical perspective, the LBS industry has faced many challenges, which can be categorised into *logistical/regulatory* and *physical* challenges.

2.5.1 Logistical/regulatory challenges

The main concern here is the lack of clear and transparent legislation in national jurisdictions, but perhaps also in trade and technology agreements to organise the relationships between various sectors of the industry. That may include the service providers, hardware manufacturers, software developers, clients, operators and any third party involved in this enterprise, for example, governments, especially when safety and privacy concerns are detected.

Some of these concerns were addressed in the E911 and E122 rules, but they still are not mature enough to be universally accepted.

2.5.2 Physical challenges

The main physical challenges (some of which have already been mentioned above) include the following:

- The deployment environment, if mostly indoors, has different requirements to that for the outdoors. The size and characteristics of the environment, including its geographical area, internal layout and density of devices, is an issue that needs to be specifically addressed. Related issues include the network topology in the environment, homogeneity in hardware, the mobility and dynamic nature of the network component and its users, and the adaptability of the system to potential change. An important industry issue is whether the users of the provided services are willing to participate in the development cycle by sharing their resources with other users and with the system itself. And, finally, is the question of whether data processing is centralised or decentralised.

- The challenge of utilising available technologies such as GPS or GSM, where the proposed solution may be assisted by such technologies or operate standalone.
- Methodology constraints, which may include the nature of available sensory information, the means used to acquire measurements, the type of measurements, and the level of ambiguity in the measurements, particularly those mentioned in Sections 2.2 and 2.5 above. We may add to those constraints, the computational constraints, and time and labour constraints related to costs and accuracy trade-offs. An example is the fingerprinting-based technologies. These involve multi-stage processing and measurements analysis, but they lack for universal applicability to any target environment.
- Post-deployment constraints, such as quality assessment, resources access limitations, privacy issues and the ability to update and maintain sensing platforms.

2.6 Fuzzy-based methodologies linked to wireless positioning systems

A key finding of the survey analysis performed for this research relates to the level of involvement of the fuzzy-based methodology in the positioning system. From this perspective, one may distinguish two main learnings about fuzzy methodologies. First, the fuzzy methodology can be part of the core of the estimation process of the target positioning. Second, the methodology may play a secondary role in the overall positioning system where a non-fuzzy-based algorithm is employed for the estimation process and fuzzy reasoning is used as a support the decision maker.

We shall refer to the first class as incorporated fuzzy positioning (IFP), as shown in Figure 2.3, and the second class, as assisted fuzzy positioning (AFP), as shown in Figure 2.4 and Figure 2.5.

In IFP, the fuzzy system is incorporated within the positioning algorithm itself. In this class, various directions are possible, depending on how and at what level the fuzzy system has been employed.

In AFP, the fuzzy system assists the positioning algorithm to enhance the result of position estimation. For example, the pre-AFP, as shown in Figure 2.4, is used to fine-tune the measurements acquired from the environment, detect uncertainty in the readings of sensors and receivers, and eliminate noise in the signal. This class is, in particular, considered when data fusion techniques are included, and when there are more than one source in the system for a measurement .

Post-AFP is used to calculate errors or uncertainties in the location estimation and provide feedback to the position algorithm or to the user to carry out fine positioning tasks or maintain the positioning consistency, particularly when the system is combined with another estimator such as a Kalman filter.

Alternatively, it is possible to hybridise the IFP and AFP to increase the uncertainty handling features of the positioning system, for example, see [38]–[40].

Table 2.1 summarises the literature on the use of fuzzy systems for addressing localisation, based on the aforementioned classification. The literature indicates a dominance of IFP-like usage in localisation systems.

Table 2.1 System methodologies.

IFP	[21], [41]–[64]
Pre-AFP	[67]–[76]
Post-AFP	[65], [66], [77]–[81]
Hybrid	[38]–[40]

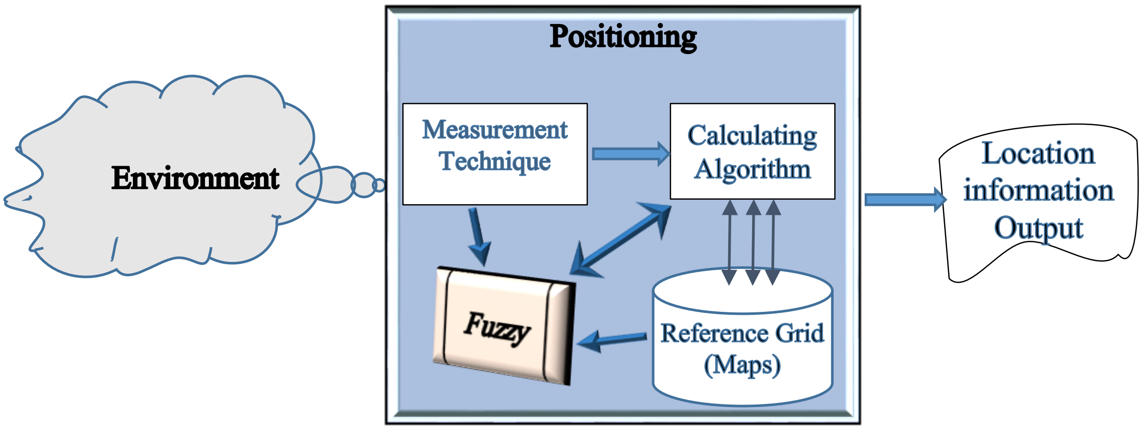

Figure 2.3 Incorporated fuzzy positioning (IFP).

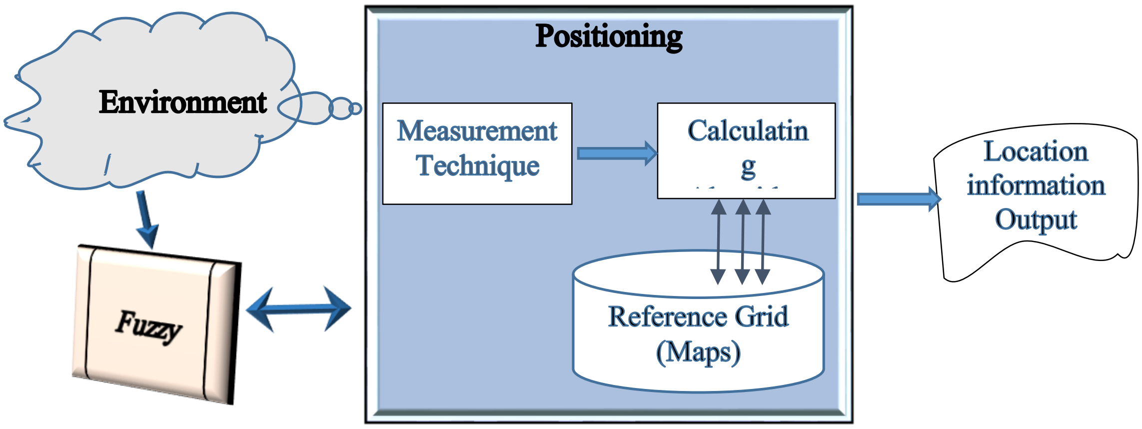

Figure 2.4 Pre-assisted fuzzy positioning (pre-AFP).

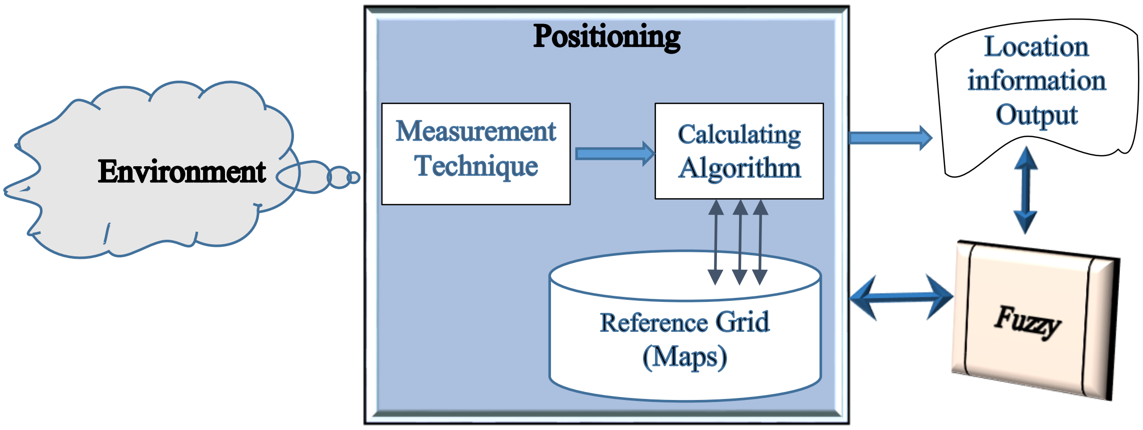

Figure 2.5 Post-assisted fuzzy positioning (post-AFP).

2.7 Historical background

Pérez-Neira *et al.* [82], [83] claim to be the first to introduce fuzzy logic into the object tracking problem. The researchers applied fuzzy logic to enhance the performance of the classical tracking system. In particular, the model-free function approximation capability of fuzzy logic was used to obtain high-resolution angle estimates from the spatial–spectral density. Their primary operation was to track the angular positions from sources' snapshot vectors. For the proposed system to obtain the distance of two close sources, the system is fed with two inputs: the first is the maximum spatial power density (periodogram); the second is the main beam normalised bandwidth. It was realised that, in comparison with their earlier work, fewer snapshots would be necessary to make a successful angle estimation. Moreover, the proposed system was able to produce an estimate for the direction of arrival (DoA) with constraints such as the angle separating two nodes less than the predefined threshold value in the data vector. The result was a robust tracking system as accurate as the minimum variance or Capon DoA estimator techniques, and with less computational requirements.

At first glance, this research may not seem directly related to the positioning problem, at least as the problem was introduced. However, insofar as it deals with angle position estimation, the research opened a promising door for the use of fuzzy logic in solving other positioning problems.

Contrary to the authors’ claims, earlier studies reveal the use of fuzzy systems to address object tracking and positioning. For example, the so-called *sketching* method was assessed experimentally in the early 1980s by Haar [55]as a system to solve the 'Layout Problem'—that is for deriving symbolic position estimates for objects from a relational scene (environment) description. To achieve this, Haar used a fuzzy relational database and inference system. In the algorithm, he employed fuzzy logic at two levels. First, a FIS was used to build a relational database among various independent objects in the environment, which, in turn, was used to construct a coarse-resolution sketch based on symbolic spatial descriptives, i.e., *left*, *right*, *above*, *below*, *distance* and *bearing*. This aimed to produce a two-dimensional estimate for the object's position in the environment. Second, the truth values were applied as a confidence interval to be associated with each symbolic descriptive rule, and utilised for error analysis at a later stage. The technique has several drawbacks, including the use of a single interval fuzzy variable and the assumption that the position of at least one fixed object must be known. In the case of an unknown object position, not fixing the position well initially may lead to poor performance because of the sequential nature of this technique. Despite such limitations, the symbolic power of fuzzy logic enhanced the sketching results and effectively leveraged a trade-off between spatial relations and coordinate positions. More interestingly, this method performed well without much prior information concerning the environment, provided a relatively good initial position was fixed.

The use of fuzzy tools in the domain of positioning and localisation has gained momentum because these tools may be easily designed and utilised.

2.8 Parametric measures and evaluations

To assess the topic rigorously, an evaluation was conducted of the performance of different positioning systems obtained from various perspectives in the literature. At the outset, it was considered whether the classification presented in Section 2.6 was sufficient. In this regard, it was difficult to allocate every proposed system to a single classification. This is because any given proposal often attempts to accommodate numerous identified deficiencies in the classical positioning system at different levels, thereby overlapping with more than one class. The evaluation of research papers undertaken for the present thesis was conducted initially from a purely statistical perspective based on the occurrence of fuzzy-related terminology in the title, keywords or abstracts of the selected papers. The evaluation was primarily conducted using two well-known scientific databases, IEEE *Xplore*® and ScienceDirect, both databases in which research into positioning technology is widely reported and discussed. The two databases also host many scientific journals in relevant fields. The evaluation continued to distinguish proposals where a fuzzy tool was employed only as an aid to the positioning objectives from those in which fuzzy tools were used to represent knowledge and manipulate it at the deepest level. Similarly, the evaluation distinguished among the various hybrid schemes between those where the fuzzy-based approach was employed with a classical approach or with an approach based on soft computing. In each instance particular interest was given to the fuzzy-based methodology employed in the underlying (fuzzy) positioning system. The results presented in Figure 2.6 and Figure 2.7 summarise the relative proportions of the main fuzzy tools employed by the identified fuzzy-based approach to tackle the positioning problem, as observed in ScienceDirect and IEEE *Xplore*®, respectively. Surprisingly, while some studies in this field [84]–[86] report the use of type-*2* fuzzy sets and systems to address the problem of localisation, neither database recorded instances of type-2 fuzzy solutions. The results shown in the histograms in Figure 2.6 and Figure 2.7 represent only the dominant methods; any technique below 1% was ignored. It should be noted further that type-2 fuzzy methods sometimes depend on the clustering method class as well, hiding a fine-grained distinction among the various clustering methods employed.

The results in both databases were similar—that is, optimisation-based approaches are dominant in fuzzy literature related to positioning systems, followed by clustering-based approaches, then the classification and rulebase approaches, and finally fuzzy arithmetic tools.

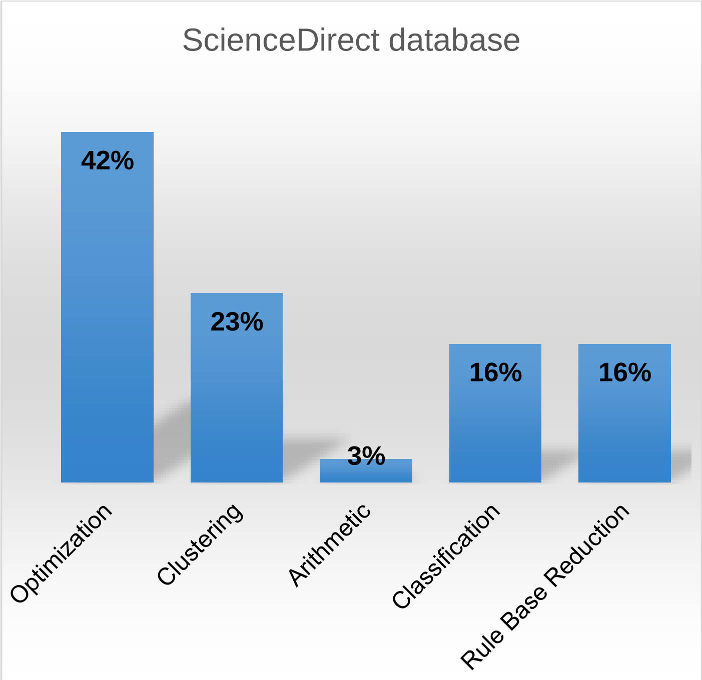

Figure 2.6 ScienceDirect fuzzy tool histogram.

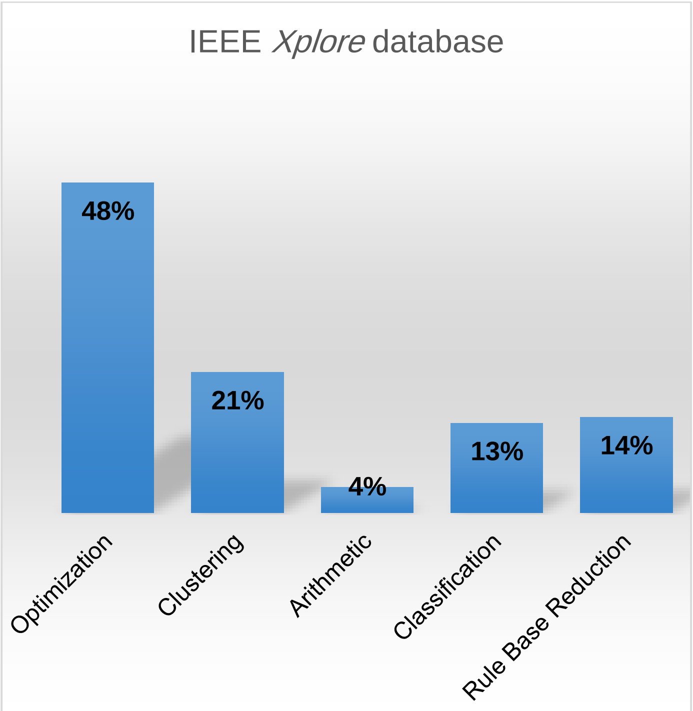

Figure 2.7 IEEE *Xplore* fuzzy tool histogram.

Another interest was to see if fuzzy systems or tools were used as the only means of location estimation, or if they were ever combined with other soft computing tools, for example, neural networks or classical estimators such as Kalman filters. The results of this investigation in ScienceDirect and IEEE *Xplore*® are summarised in the pie-charts in Figure 2.8 and Figure 2.9, respectively.

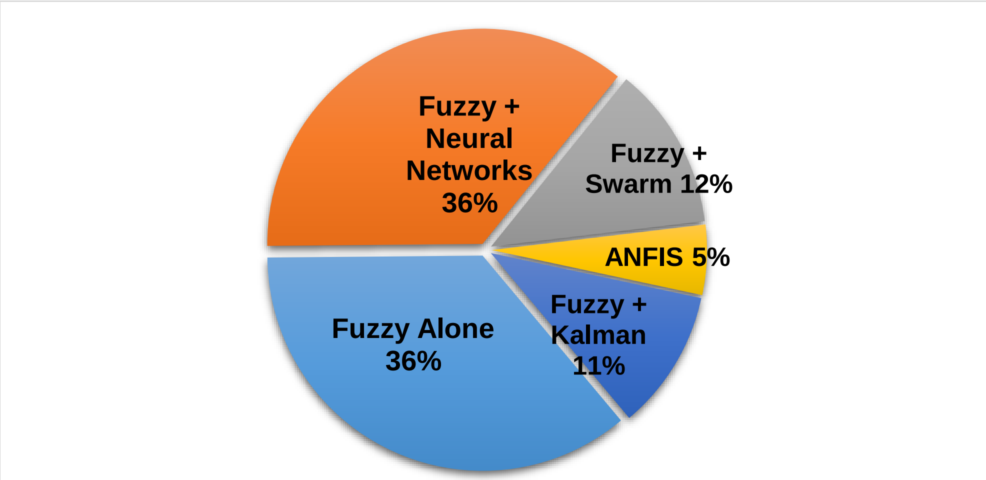

Figure 2.8 ScienceDirect pie-chart.

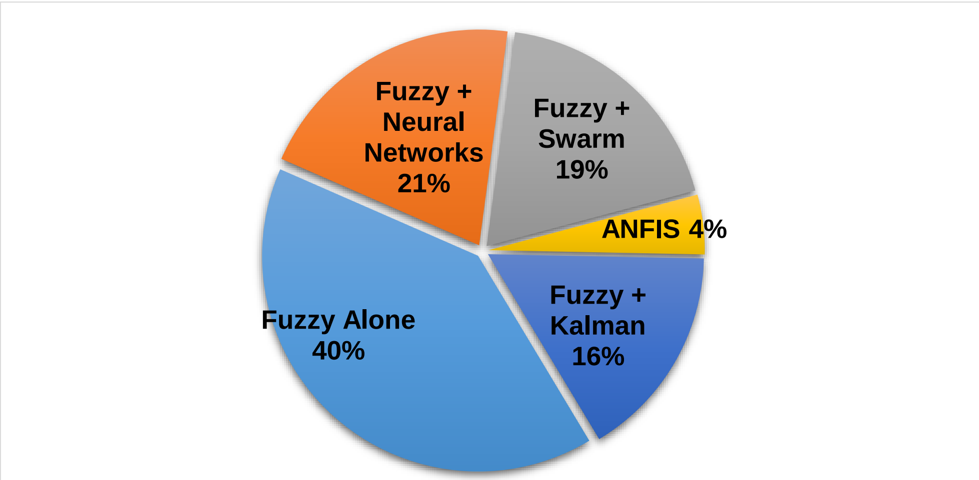

Figure 2.9 IEEE *Xplore* pie-chart.

The results from the two databases were substantial similar. This includes the dominance of fuzzy-alone-based approaches, followed by hybrid fuzzy logic and neural-network-based approaches (although these ranked equally in ScienceDirect). Next in frequency were the hybrid schemes of fuzzy tools and so-called 'swarm' optimisation, followed by fuzzy tools with Kalman filter. Finally, a small proportion of surveyed papers (fewer than 5% investigated systems based on adaptive neuro-fuzzy inference systems (ANFIS), as applied to positioning problems.

The next level of the evaluation was to introduce performance criteria to compare the proposed methodologies. These criteria and parametric measures have been broadly divided into four parts: (i) system metrics (Table 2.2); (ii) environment metrics (Table 2.3); (iii) fuzzy metrics (Table 2.4); and, (iv) positioning metrics (Table 2.5).

The comparison discriminated between the fuzzy tools employed as an augmentation to other classical positioning approaches and those instances where a fuzzy system-like approach lies at the core of the positioning technique. The performance criteria and parametric measures are very important in the evaluation to enhance system performance or overcome deficiencies observed during the implementation of the position estimation.

Unfortunately, many of these criteria were not mentioned, even implicitly, in a number of the review papers encountered. The performance metrics are summarised as follows.

2.8.1 System metrics

The system metrics explained here are summarised in Table 2.2.

Accuracy and precision

Accuracy and precision are two important performance metrics in a positioning system. Position accuracy is defined as the numerical distance (in metres or centimetres) between the actual target position and that of the estimated position. Moreover, precision tests the extent to which an estimation varies when it is repeated under the same circumstances.

Scalability

In general, positioning systems need to be scalable with respect to geographical space and the density of client users or terminal devices. A system is deemed scalable if it is able to be deployed in a larger geographical space or serve a larger client population with the same quality of service.

Robustness and adaptiveness

Robustness and adaptiveness are a measure of how much a positioning system is capable of handling unforeseen circumstances or accidental changes in the environment. These include, inter alia, malfunctioning of sensory nodes and APs, and the inclusion or exclusion of new obstacles which might increase noise and uncertainty levels on the test bed.

Cost (computation, labour, implementation)

Computationally fast and plausible algorithms to serve more localisation queries is desirable. Efficiency in energy and processing resources is also desirable, especially when the client wants to estimate a position using devices with limited capabilities. A need for professional labour for the system implementation is not considered cost-effective.

Complexity (measuring devices, mobile devices & other network components)

By complexity, we include the type of measuring instruments and required network infrastructure necessary for generating measurements or inputs to the positioning system, as well as the complexity associated with the estimation process itself.

Latency

Latency is often used to quantify the speed of response of a system for positioning queries.

Table 2.2 System metrics.

Citation	Accuracy and precision	Scalability	Robustness and adaptiveness	Cost	Complexity	Latency
[55]	low	Not tested	Works with little prior knowledge about the environment	low computation cost	complex	NaN
[77]	good (1-5 m)	Yes	Able to detect changes in the local environment	low computation cost	easy	Low
[69]	0-1 m	Yes	Not reported	low cost	easy	Low
[65]	Not reported	No	Able to detect changes in the local map if a global map is available	high computation	medium	low 1-10 s
[79]	good 0.5-20 m on each dimension	Yes	Yes	low computation cost	complex	very low
[87]	2.5 0-5 m	Yes	Yes	Low computation time	complex	medium
[44]	0-10 m	Yes	Yes	low	easy	low 1 sec
[70]	5-15 m	No	Yes	high computation and time	complex	high
[71]	0.1 - 0.8 m	No	No	low computation power	complex	low
[53]	2 - 5 cm	No	No	low computation power	complex	low
[72]	1-3 m	No	Yes	low computation power	simple	very low 5-15 ms
[38]	5-65 cm	Yes	Yes	high computation	medium	low 1-3 s
[52]		Yes	Yes	very low	simple	
[51]	0.5 - 2 m	No	No	low computation power	complex	
[50]	0.5 - 5 m	Yes	Yes	average computation power	medium	0.5 s
[59]	Not reported	Yes	Yes			1-4 s
[63]	90% accuracy	Yes	Yes	very high	medium	
[61]	1-3 m	No	Yes	low computation power	easy	
[49]	0-200 m	Yes	Yes	high computation	complex	1 s
[67]	0-200 m	Yes	No	low computation power	simple	1-2 s
[62]	0.10-80 cm	No	Yes	low computation power	simple	very low

[64]	0-0.5 m	No	Yes	low	simple	low
[56]	0.5-2 m	No	Yes	low computation power	medium	low
[42]	3-8 m	Yes	Yes	high computation power	complex	high 40-60 s
[43]	2 m with 60%	No	No	very high	complex	5-10 s
[88]	0-4 cm 98%	No	No	very high	complex	1-3 s
[89]	35-80 m	Yes	Yes	high computational power	complex	60-70 s
[73]	40 cm 86%	Yes	No	medium cost	simple	50-60 s
[21]	Not reported	Yes	No	medium cost	medium	1 s
[45]	95%	Yes	No	low cost	simple	3 s
[90]	2 m with 66%	No	No	low cost	simple	NaN
[74]	1 m with 75%	No	No	medium	medium	1 s
[46]	0.35-1.70 m	Yes	Yes	low	low	2 s
[47]	Not reported	No	Yes	high	medium	2 s
[3]	0-0.5 m	Yes	Yes	low	low	50 ms

Examination of the surveyed studies in Table 2.2 indicates that those in which fuzzy localisation techniques were employed are primarily related to mobile robotics, manufacturing, cellular systems, and indoor positioning using Wi-Fi, Bluetooth, RFID and laser scanning and vision systems. Irrespective of the applied methodology, various disciplines, expectations and technologies used commonly induce distinct accuracy and performance levels.

When compared with studies in positioning systems generally, the identified studies reveal a relatively low citation score. This suggests a lack of involvement of the fuzzy community in influencing the current ISO standards and known IEEE research groups on positioning systems. Therefore, further research should be conducted in this field to establish a benchmark.

When considering the accuracy achieved by the studies reviewed in Table 2.2, one notes an accuracy of around one centimetre. However, we should also consider the sensory range of the applied sensors. From this perspective, the range of the utilised sensors is also limited to around one centimetre to a few metres since ultrasound, Wi-Fi, and laser scanner-like sensors have an inherently limited range.

In respect of complexity, although most of the fuzzy-based positioning papers focused on low-cost sensory architectures, Table 2.2 demonstrates that they yield reasonably low to medium computational cost, with very few studies reporting high computational costs. These few studies were mainly related to methods where extra network infrastructure would be required to trigger the associated measurement method to ensure synchronisation between the emitter and receiver, e.g., in the case of *time difference of arrival* (TDOA).

Regarding scalability, it turns out that most of the surveyed studies in Table 2.2 did not consider such factors, especially when the approach applied only low-cost sensors and did not require infrastructural change. Otherwise, if additional hardware were required to run the positioning system, the scalability of the approach was trivially questioned. Similarly, approaches that subsume full or even partial knowledge of the environment to run the positioning system have limited scalability as well.

We distinguish some papers, e.g., [70], based only on simulation studies, from those based on real-time implementation. Notably, a simulation-based analysis does not necessarily justify all the constraints that can be satisfied in a real-time implementation-related work. Therefore, their outcomes should be viewed with caution.

Concerning latency, a quasi-majority of the surveyed studies in Table 2.2 reported a low latency, and only three papers reported a latency value higher than 10 seconds. In fact, the papers that revealed a high latency have been mainly linked to approaches in which an additional step for environment mapping is required. Therefore, on the basis of the complexity of the environment and frequency of activation of sensors, the mapping time can thereby substantially increase, which, in turn, increases the latency of the overall system.

2.8.2 Environment metrics

The environment metrics explained here are summarised in Table 2.3.

Map requirements

A typical localisation scheme requires prior information about the environment. This can be done through a site survey. For instance, in fingerprint-based schemes, the collected patterns are annotated with their physical or symbolic fixes manually before the positioning algorithm is initialised. Other schemas may require a geographical map to obtain absolute or relative estimates of the position.

Acquiring location fix

Some positioning systems may require the location fix from user devices, which could be obtained via GPS or other means in order to offer reasonable accuracy. Other positioning systems do not require this. Positioning systems able to maintain the same level of accuracy without the need for a location fix are more attractive.

Usage of indoor/outdoor landmarks

A feature of an ideal positioning system is its ability to process the target estimation anywhere without prior knowledge concerning the layout of the deployment environment. Numerous positioning systems, for example, the fingerprint-like approaches, require knowledge about the APs' locations to approximate a distance to the target. Similarly, navigation-based approaches require predefined locations to trace the trajectory to the destination. Therefore, from a system autonomy perspective, positioning systems without landmark requirements are considerably preferred over others.

Needs additional sensor (or hardware)

Although numerous sensors are embedded in the handheld devices such as smartphones and tablets, some advanced positioning systems, such as in some robotics and manufacturing applications, require advanced bandwidth, throughput and special sensory capabilities. Therefore, if the target receiver is not designed with such functionality, then the positioning systems may not operate properly or, at least, would not be able to deliver the expected performance in accuracy and precision.

Addressing device heterogeneity

On the basis of the same network conditions, it has been found that the accuracy of some positioning systems is significantly affected by the type of measurement device, especially those that depend on RSS or time of arrival (TOA). Consequently, device heterogeneity is addressed as another metric parameter for evaluating the positioning system.

User participation

The idea behind calibration-free positioning systems is to involve users to participate implicitly in order to construct a training database. Any user carrying a wireless device may be expected to contribute to the radio-map construction. This participation is more attractive compared to the scenario where the professional deployment personnel explicitly inputs location fingerprint data as feedback to the system. The user approach allows building a more comprehensive and denser database, as well as a scalable system, provided the minimum accuracy requirement is still achievable without this participation.

Scale

The size of the deployment environment is a fundamental issue when evaluating a positioning system. It can be addressed from two points of view. First, some large-scale networks would require massive data analysis prior to deployment of a positioning system.

On the other hand, some small-scale environments might lack the infrastructure to support the required positioning system. Second, from a business perspective, as the cost of deployment in a small environment is often much less than in a large one, this may give small-scale systems an advantage when it comes to commercialising the products.

Table 2.3 Environment metrics.

citation	Environment	Map requirements	Acquiring location fix	Usage of indoor landmark	Needs additional sensor (or hardware)	Addressing device heterogeneity	User participation	Scale
[82]	outdoor	No	No	No	Yes	No	No	large scale
[55]	indoor	Not necessary	Few	No	No	No	No	small scale
[68]	outdoor	Yes	Few	No	No	No	No	small scale
[77]	indoor	Yes	Yes	No	No	Yes	No	medium scale
[69]	outdoor	No	Yes	No	No	No	No	large scale
[65]	indoor	Yes	Yes	Yes	Sonar		No	medium scale
[79]	outdoor	No	No	No	No	Yes	No	large scale
[87]	outdoor	No	Yes		Yes, yaw-rate sensor, speedometer, DGPS	No	Yes	medium scale
[44]	outdoor	No		No	No			large scale
[70]	outdoor	No	Yes	No	No	Yes	No	large scale
[71]	indoor	Yes	Yes	Yes	Yes	Yes	No	small scale
[53]	indoor	No	No	No	Yes	No	No	very small scale
[72]	indoor	Yes	Yes	No	No	Yes	No	medium scale
[38]	indoor	No	No	Yes	Yes	No	No	small scale
[52]	indoor	No	Yes	No	No	Yes	No	medium scale

[51]	outdoor	No	Yes	No	Yes	No	No	medium scale
[50]	indoor	No	No	No	No	Yes	No	medium scale
[59]	indoor	Yes	No	No	No	No	No	small scale
[63]	indoor, outdoor	No	No	No	No	Yes	No	medium scale
[61]	indoor	No	No	No	No	No	No	small scale
[49]	outdoor	No	Yes	No	No	Yes	No	very large scale
[91]	outdoor	No	Yes	No	No	No	No	very large scale
[62]	indoor	Yes	No	Yes	Yes	Yes	Yes	medium scale
[64]	indoor	No	Yes	No	Yes	No	No	small scale
[56]	outdoor	NO	Yes	No	Yes	No	No	medium scale
[42]	outdoor	No	Yes	No	Yes	Yes	No	very large scale
[43]	indoor	Yes	No	No	No	No	No	small scale
[88]	indoor	Yes	Yes	Yes	Yes	No	Yes	very small scale
[89]	outdoor	Yes	No	No	Yes	No	No	large scale
[73]	outdoor	Yes	Yes	No	Yes	No	No	large scale
[21]	indoor	No	Yes	Yes	Yes	No	No	small scale
[45]	indoor	No	No	No	No	No	No	medium scale
[90]	outdoor	No	Yes	No	No	No	No	small scale
[74]	indoor	No	Yes	Yes	No	No	Yes	small scale
[46]	indoor	No	No	Yes	No	No	No	small scale
[47]	indoor	No	Yes	Yes	No	No	No	small scale
[3]	indoor	No	Yes	No	No	Yes	No	medium scale

Consideration of the results presented in Table 2.3 reveals that the application of fuzzy-based positioning systems is equally focused on indoor and outdoor positioning.

In respect of the requirement of environmental knowledge, the application of fuzzy systems follows roughly the development of the navigation systems, where clear differences between fully known, partially known, and fully unknown environments are evident. This shows that the proposed fuzzy-based approaches are mainly associated with mapping and modelling of the surroundings or perceived environment. Also includes the grid-based approach, polygonal approximation (such as ultrasound beam or cellular grid network modelling), integration over a travelled distance path as in odometer-like sensing, and a straight line from known beacons. Accordingly, they derive a position estimation.

Examination of the environmental knowledge constraint in the identified studies demonstrated that most such studies may be separated by subject into: (i) GPS positioning systems or differential GPS; (ii) local-based sensory strategies for proprioceptive-sensor in mobile robotics; or (iii) sensor node positioning in a large-scale WSN.

Moreover, the classification in Table 2.3 is rather subjective. For instance, one may expect all the fingerprint-based approaches, e.g., construction of a radio map using APs and RSS information, to have a 'map requirement.' However, the authors of such papers, e.g., [46], claim that the approach does not require any map-related knowledge. Therefore, reproducing authors' claims based on the environment knowledge requirements should be handled with caution.

The choices of location fix and users participation are primarily associated with the employed map-building approach. Most map-building approaches would typically require some prior knowledge of the environment, modelling structures (e.g., grid, straight line, polygonal cells, cubic cells), and the technologies employed. An example is the case of a cellular network that utilises the RSS intensity to calculate mobile positioning. In this case one requires information about the location and heights of the base stations, power, and the type of environment (e.g., rural, urban, height of buildings, wideness of streets), to tune the parameters of the radio propagation models that interpret the RSS intensity as a mobile base station distance. Similarly, to turn RSS intensity into distance or use any estimation-based technique, the use of triangulation with the Wi-Fi signal in the indoor environment would require at least the location of the AP. To apply vision-based techniques, for example, determining the target position with respect to identified beacons, a beacon approach requires knowledge of the beacon locations, type and shape. In a WSN array, the location of the target node would require knowledge of the reference nodes that may be applied to obtain the target's physical location.

We distinguish at least two types of users participation in the surveyed papers. The first follows the *crowdsourcing* approach, where users report their locations together with the observations (images, RSS). This information is then used to build mappings of the environment. The second type is employed as a training phase to generate a model for position estimation. It uses a user interface as a part of the estimation process, where the user may intervene to validate or prioritise some typical choices.

The observation of the scale result shows a quasi-majority of the indoor fuzzy-based positioning systems, which act in medium and small-scale environments, whereas the outdoor positioning systems act in medium and large-scale environments. The system described in [69], which examined a small-scale array of a WSN, is an exception; it is related to the outdoor environment but is considered small-scale.

2.8.3 Fuzzy evaluations

The fuzzy metrics explained here are summarised in Table 2.4.

Fuzzy hybridisation

This measure indicates whether the fuzzy-based approach was used alone or with (or assisted by) another approach (e.g., Kalman filter or other soft computing such as neural network, GA or ANFIS). This can be useful for researchers who are interested in the relevance of specific hybrid schemes.

Level of the implementation process

This examines how the fuzzy tool is actually implemented within the overall localisation algorithm. For example, the fuzzy-based approach was used in many cases to assign relative weights to some parameters which were employed in subsequent reasoning. Some proposals explored the universal approximation ability of fuzzy reasoning to tackle system nonlinearities, and some used fuzzy reasoning to enhance user–system interaction.

Type of inference

Fuzzy inference is a fundamental application of fuzzy set theory and fuzzy logic. The literature contains two common types of inference systems: (i) Mamdani; and (ii) Takagi–Sugeno (TKS). The Mamdani system primarily has output membership functions, whereas the TKS inference system has a crisp output. The former applies the defuzzification technique to a fuzzy output, whereas the latter applies a weighted average to compute the crisp output. The former is suitable for capturing expert knowledge, but it requires a substantial computational burden because of the defuzzification step. The latter works well with optimisation and adaptive techniques which customise dynamic nonlinear systems to the best data model. In addition, it is computationally more efficient [92].

Type of membership functions

The membership function (MF) is a very delicate point in the design of a fuzzy-based positioning system. The choice of MFs should be done after studying the effects of MF characteristics such as cardinality, normalisation, completeness and overlapping, which together give a certain robustness to the system, and where, for instance, the level of robustness may be tracked to modify the model.

Number of rules, variables, sets

Concern about the number of MFs employed in any system is heightened due to their direct impact on the whole system performance, since any fuzzy set can be expressed as a group of MFs. The task requires balancing two countervailing factors. First, the uniqueness of representation and rule firing can affect flexibility within the system, as suggested in [92]. Second, the existence of unnecessary or redundant rules and input variables decreases error rates, which emphasises the impact of input selection and rule firing on computational time [93].

Type of defuzzification

Defuzzification is a critical factor when implementing a fuzzy inference engine and variations such as execution time and instruction count, due to the high computational demands of defuzzification algorithms. Van Broekhoven and de Baets [94] have offered some guidance to the designer by choosing the most proper defuzzification method based on the application context. This approach might represent the different trade-off points in cost and accuracy. Knowing the trade-offs helps the designer to choose the defuzzification that best fits the precision requirements with minimum computational power demand, resulting in better coverage, delay and power consumption in hardware or less execution time in software implementations. Comparisons of new defuzzification techniques were reported by

Mahdiani et al.[95], namely, trapezoid median average (TMA), weighted trapezoid median average (WTMA), and trapezoidal weighted trapezoid median average (TWTMA). It would be beneficial to have these new techniques coded and tested.

Rule base construction and rule simplification

Often the applied rule generation technique suggests employing a rule simplification as a parallel action. This is justified by a reliance on the data-driven techniques in forming the rulebase, which often results in an excessive number of interpretations of the same rules. As a consequence, the system tends to be more complex and less transparent.

Data-driven techniques are founded on two alternative principles: (i) the balance between simplicity and accuracy; and (ii) the balance between linguistic interpretability and accuracy. Either principle results in the so-called 'curse of dimensionality,' which necessitates employing rulebase simplification. Some of the simplification techniques contain similarity analysis, set-theoretic similarity measures and orthogonal transformation-based methods [96].

Table 2.4 Fuzzy metrics.

Citation	Single fuzzy or combined with another SC technique	At what level of the localisation process they are implemented	Type of inference used	Type of Member ship functions	Number of rules, variable, sets	Type of defuzzification	Rule base construction and rule simplification
[82], [83]	Fuzzy LMS	Angle estimation	Mamdani	triangle	2 variables 9 rules	Centroid	
[55]	Fuzzy only	Position estimation and error measurement	Mamdani	triangle	6 variables	Centroid	
[68]	Fuzzy only	Error measurement propose solution	not used	NaN	NaN	NaN	NaN
[77]	Fuzzy	Uncertainty and confidence interval	not used	trapezoidal	3 variables	NaN	not used

[69]	Fuzzy	Enhance positioning accuracy	Mamdani approximate reasoning	singleton	3-4 variables with 12 rules	Centre or area	
[65]	Fuzzy	Location estimation	Mamdani	triangle	3	Maximum	
[79]	Fuzzy & GA & Kalman	Location estimation	Mamdani	triangle	3 variables with 30 rules	Mean of max	GA
[97]	Fuzzy & Kalman	Fine tuning Kalman filter parameters	mixed Mamdani-Sugeno	triangle		Centre of gravity	
[44]	Fuzzy	Location estimation (FIS)	Mamdani	triangle	9 variables with		
[70]	Fuzzy	Pre-AFS uncertainty with input data	Mamdani	triangle	2 variables	Maximum	No
[71]	Fuzzy	Obtain confidence interval	Mamdani	Gaussian	3 vars., 9 rules	Centroid	
[53]	Fuzzy + EIF	Improve localisation accuracy	Mamdani	triangle	3 inputs 45 rules	Max-min	GA
[72]	Fuzzy	Map building	Mamdani	Gaussian	16 input		
[38]	Fuzzy	Map building and position estimation	Mamdani	trapezoidal	6 inputs	Centre of gravity	Product norm
[52]	Fuzzy	Position estimation	Mamdani	triangle	5 inputs -3 variables 15 rules	Centroid	NAN
[51]	Fuzzy + GA + neural net	Weighting anchors	Mamdani	trapezoidal	5 variables 15 rules	Centroid	
[50]	Fuzzy	Weighting near neighbours	Mamdani	trapezoidal	4 variables 15 rules	Centroid	
[59]	Fuzzy	Position classification	Mamdani	trapezoidal	4 variables 28 rules	Centroid	Yes / tree reduction
[60]	Fuzzy + neural net	Position estimation and movement tracking	TKS	triangle	4 variables 19 rules		ANFIS
[63]	Fuzzy + machine learning	Location estimation and enhance estimation by compensating the small-scale variations	Max-min Winner rule	triangle	6 & 12 inputs with 3 to 9 linguistic terms & 2 variables 266 rules	Max	Similarit y analysis
[61]	Fuzzy	Distance estimation	TKS	triangle	2 variables		
[49]	Fuzzy	Degree of satisfaction estimation	Mamdani	trapezoidal	4 variables	Max	
[91]	Fuzzy	Enhance positioning accuracy	Mamdani	trapezoidal	1 variable	Max	No

[62]	Fuzzy	Position estimation	Mamdani	triangle	3 variables		
[64]	Fuzzy	Coordinate estimation	Mamdani	triangle	1 variable	Height	
[56]	Fuzzy	Weighting anchor	Mamdani	trapezoidal	4 variable	Centroid	
[42]	Fuzzy + Kalman	Estimate the state prediction of filter	TKS	Gaussian	6 variables		
[43]	FCM	Determine position at multi-stage clustering system			k-inputs	Centroid	
[88]	Fuzzy sets	Represent uncertainty in sensor measurements		trapezoidal	2 inputs	Max	
[89]	Fuzzy + Kalman	Tune the covariance matrix of KF	Mamdani	trapezoidal-triangle	1 input 9 rules	Weighted average	
[73]	Fuzzy	Score weight for neighbour hop	Mamdani	trapezoidal	5 inputs 15 rules	Max	
[21]	Fuzzy	More than one level, including position estimation	Mamdani	triangle	2 inputs 18 rules	Weighted average	
[45]	Fuzzy + neural net	Symbolic estimation after training phase	TKS	Gaussian	3 inputs 3 rules		FCM
[90]	Fuzzy	Location estimation	Mamdani + TKS	trapezoidal	5 rules	Weighted average	
[74]	Fuzzy	Radio propagation model noise compensation	Mamdani	trapezoidal	3 inputs 24 rules	Weighted average	
[46]	Fuzzy	Weighting nearest neighbour edges			5 inputs		
[47]	FCM	Carry out fuzzy partition			5 inputs		
[3]	Fuzzy	Position estimation and weighting kNN	TKS	trapezoidal	2 inputs 32 rules		

Consideration of the results presented in Table 2.4 raises several issues.

The demarcation of *fuzzy alone* from the hybrid approaches is subjective. Even though the classification is primarily guided by the authors' claims and scrutiny of the underlined work, it turns out that numerous fuzzy-alone papers also apply some standard methods of regression analysis, statistical means and/or standard deviations, which tends the content towards that of a hybrid approach.

A large majority of the fuzzy-alone methods unsurprisingly apply FISs as part of their core methodology. However, one can distinguish among various classes of application of fuzzy inference within the fuzzy-based positioning systems. First, on the basis of the input–output perspective, one may distinguish cases where the FIS is applied at the input level to handle the uncertainty pervading the inputs. For instance, the FIS may refine the distance measurement/estimation so the output of the fuzzy system is a refined distance measure. This measure can then be employed as an input to the core positioning estimation algorithm which uses triangulation, regression or another estimation-based strategy. From this perspective, the contribution of the FIS may be compared to the role of a filter which enhances the quality of the input of the positioning algorithm. Second is the use of a FIS to obtain a confidence estimate associated with the input parameters, e.g., confidence interval and reliability (either single-valued or functional). To be utilised in the position estimation algorithm through a weighted regression or probabilistic estimation process, the confidence estimate may be applied as complementary data to the inputs. Third are the cases where the FIS is used to estimate an entity directly related to the positioning system, e.g., the angular position of the target and *x*–*y* position of the target. In this regard, the fuzzy rules are elicited so the consequent part of the rule contains variables related to the components of the target. The latter two approaches seem to be the most dominant trends in the fuzzy optimisation systems surveyed here. In addition, a fourth approach involves cases in which the FIS or fuzzy entity is jointly employed with another estimator (Kalman filter, neural network or ANFIS). In respect of the Kalman filter, one distinguishes the cases where the FIS is applied to generate (after a defuzzification step) one (crisp) input of the standard Kalman filter. In fuzzy literature, proposals based on what is known as the 'fuzzy Kalman filter' have also been considered, in which a variance estimator under fuzzy constraints was investigated. Thus, to optimise the parameters of the FISs (e.g., number of fuzzy rules, fuzzy variables, modal values and spread of MFs, and connectives), hybridisation with a neural network or ANFIS is mainly employed. A fifth approach corresponds to cases where the localisation involves map-building, either concurrently with the estimation process or as a prior step in the localisation process. It is important to mention the emergence of fuzzy clustering-based approaches employed to identify appropriate landmarks or perform suitable pattern matching. In general, fuzzy similarity measures and case-based reasoning techniques are employed to identify the most plausible patterns and associative hypotheses.

Another result arising from Table 2.4 is that all FISs reviewed in the evaluation use reasonably few input variables and rules (less than nine variables). This is common in fuzzy literature to ensure the interpretability of the results and the computational efficiency of the implemented algorithm. Moreover, to model fuzzy input variables considering their popularity in the Mamdani-like FIS, the review shows the dominance of a trapezoidal or triangular-like MF.

Surprisingly, for position estimation, there are no reviewed studies that discuss the use of fuzzy arithmetic or a fuzzy number-based approach. This may be an area of interest in the future. We also note the inherent properties of fuzzy arithmetic where the multiplicity of its operations may result in some bias or drift that would require an automatic update.

2.8.4 Positioning evaluation

The final evaluation criterion is not considered as a measure. As shown in Table 2.5, it enumerates the positioning system properties based on the classification performed earlier—that is, the type of location information required, the nature of the localisation system (whether absolute or relative), the topology, communication technology/protocol, employed calculating algorithm, signal measurement techniques and type of environment.

Numerous positioning systems and algorithms have been proposed in the literature. However, owing to discrepancies of the employed technologies, environmental constraints and robustness of theoretical frameworks, it is still difficult to compare the performances of these systems and algorithms. Thus, we evaluate their performances categorically, in the hope this may provide some basis for future studies or guidelines for further evaluations.

Table 2.5 Positioning metrics.

Citation	Location information	Absolute or relative	Topology	Communication technology	Calculating algorithms	Measurement techniques
	"Physical " or "symbolic"		Self or remote direct or indirect	Wired or wireless Wi-Fi infrared	Geometric calculations proximity scene	
[82]	Physical	relative	remote - direct		Proximity NN	DoA
[77]	Physical	absolute	self-direct	ultrasonic	Proximity	TOF
[69]	Physical	absolute	remote-direct	GPS	Geometric-trilateration	TDoF
[65]	Physical and symbolic	relative	self-direct	Sonar	Dead reckoning	TOF
[79]	Physical	absolute	self-direct	GPS	Geometric trilateration	TOA
[87]	Physical	relative	remote-indirect	GPS	Geometric trilateration	TOA
[44]	Physical	relative	remote - direct	GSM-Radio	KNN	RSS

[70]	Physical	relative	remote-direct	GSM Radio	Geometric trilateration	TOA , TDOA
[71]	Physical	absolute	self-direct	infrared	Triangulation	TOF
[53]	Physical	absolute	self-direct	ultrasonic-infrared	Triangulation	TOF
[72]	Symbolic	relative	self-direct	WiFi	Proximity	RSS
[38]	Physical	relative & absolute	remote-direct	Sonar & laser	Proximity – dead reckoning	TOF, AOD
[52]	Symbolic	relative	remote-indirect	WiFi	Fingerprint	RSS
[51]	Physical	relative	remote-direct	ZigBee	Weighted COO	RSS
[50]	Physical	relative	remote-direct	WiFi	Proximity weighted average	SNR
[59]	Physical	relative	self-direct	WiFi	Proximity average	RSS
[60]	Symbolic	relative	self-direct	WiFi	Proximity average	RSS
[63]	Physical	absolute		WiFi	Fingerprint	RSS
[61]	Physical	relative	remote-direct	ZigBee	Proximity weighted average	RSS
[49]	Physical	absolute	remote-indirect	GSM-Radio	Generalised mean value	RSS, TOA, AOA
[91]	Physical	absolute	self-direct	GSM-Radio	Cell of origin	RSS
[62]		relative	self-direct	WiFi		RSS
[64]	Physical	relative	remote-direct	ZigBee	Fingerprint + kNN	RSS
[56]	Physical	absolute	remote-direct	ZigBee	Fingerprint + weighted average	RSS
[42]	Physical	absolute	remote-direct	GSM-Radio	Weighted average	RSS + TOA
[43]	Physical	relative	self-direct	WiFi	Fingerprint	RSS
[88]	Physical	relative	self-direct	Sonar	Triangulation	AOA
[75]	Physical	absolute	remote-direct	GSM-Radio	Triangulation	RSS
[73]	Physical	relative	remote-direct	infrared	Hop-count	RSS
[21]	Physical	relative	self-direct	Radio	Multilateration	RSS
[45]	Symbolic	relative	self-direct	WiFi	Fingerprint	RSS
[90]	Physical	relative	remote-direct	WiFi	Weighted kNN	RSS
[74]	Physical	relative	remote	WiFi	Weighted kNN	RSS

[46]	Physical	relative	remote-direct	WiFi	Weighted kNN + fingerprint	RSS
[47]	Physical	relative	self-direct	WiFi	Fingerprint	RSS
[3]	Physical	relative	Remote-direct	WiFi	Fingerprint + weighted kNN	RSS

Consideration of the results in Table 2.5 reveals that fuzzy-based approaches have been applied to various technology platforms, including mobile robotics with dead reckoning, sonar, infrared, laser, ultrasound-like sensors, cellular network using GSM, cell-ID, radio, differential GPS, and indoor environments using Wi-Fi, Bluetooth and ZigBee communication technologies. Timing-based measurements (time of flight (TOF), time difference of flight (TDOF) and TOA) and non-timing measurements (angle of arrival (AOA) and RSS) have been investigated by researchers.

The calculating algorithms range from a simple count and proximity-based calculus to complex hybridisation schemes passing through standard triangulation, multilateration, weighted average and geometric-based reasoning. Moreover, numerous map-building-related positioning systems employ a fingerprint-like strategy, as well as a nearest neighbour or kNN-like decision rule.

Concerning the location description, it is noteworthy that both symbolic and physical locations have been considered in the literature. Fuzzy reasoning often allows us to infer a symbolic description from a physical one. However, if physical and exact locations are not required, one expects higher accuracy of the fuzzy positioning system to only infer a symbolic description of the target location. Similarly, except when GPS or GSM measurements are involved, it is often sufficient to provide relative positioning of the target instead of absolutely.

2.9 Conclusion

This chapter provided a brief introduction to the localisation problem from historical and technical viewpoints. Special attention was given to the uncertainty challenges pervading the position estimation in the indoor WLAN environments, and some of the research work to overcome those uncertainties. We sought to provide a rational classification of the use of fuzzy sets in the localisation problem, as it is a proven tool for handling uncertainties. The chapter encompasses a literature review with some metric comparisons and evaluation where possible. In the evaluation purposes, four main classes were distinguished: system metrics, environment metrics, fuzzy metrics and positioning metrics. Each class was discussed in detail. but the main findings were as follows:

- Irrespective of the scale of the implemented environment in the system metrics, the accuracy of the proposed systems was enhanced in terms of the costs of complexity and computation.
- By utilising the power of reasoning and data extraction of fuzzy logic and fuzzy inference, the fuzzy-based solutions outperformed those of alternatives.
- When more variables were incorporated into the fuzzy inference, the precision level increased substantially.
- Very few studies reported or considered the rulebase simplification problem, which needs to be thoroughly investigated.
- In most positioning systems, specificity, consistency, redundancy and completeness of the rulebase was not been extensively discussed. Therefore, it is important to mention the numerous advantages of fuzzy logic in the context of mobile positioning, including its intuitive conceptual model, flexibility, ease of computation, multiple

combination modes, accommodation of logic-based reasoning, and hybridisation with other non-conventional techniques or soft computing tools.

- Generally, fuzzy logic is not universally accepted among practitioners. This is partly because of lack of awareness of its potential benefits. Concerning performance, FL requires further testing and evaluation, especially using benchmark data sets to create awareness. Another reason is its poor performance in some cases when compared with conventional positioning methods. Awareness of the context and metrics underpinning the design and application of fuzzy reasoning-based tools would provide encouragement to consider the proposal and seek further enhancements, especially when the approach requires manual tuning of critical parameters.

The next chapter focuses on the evolution of WLAN modelling and localisation techniques employed in the literature.

3 WLAN MODELLING AND LOCALISATION TECHNIQUES

3.1 WLAN Structures

WLANs became the main dominant network infrastructure after the US FCC decision in 1985 to open several bands of the radio spectrum for use without a government licence. These free and shared bands were allocated to wireless equipment such as portable laptops, microwave transmitters, mobile phones and tablet computers. To operate in these bands the equipment was required to use radio signals spread over a wide range of frequencies. This aspect makes the signal very susceptible to interference generated by the surrounding environment, and vulnerable to interception. These issues and others led the IEEE to standardise the operation of such bands with IEEE 802.11x. The IEEE 802.11x standards define the operational characteristics and the interface between the main two WLAN components, a client and a base station (or AP), where both are equipped with IEEE 802.11x transceivers.

3.2 Localisation

Terms such as geolocation, location sensing, radiolocation and localisation are applied interchangeably to describe any system embedded with an algorithm which uses wireless technologies to determine location. In general, all location estimation systems tend to have the same main components, as illustrated in Figure 3.1 [4].

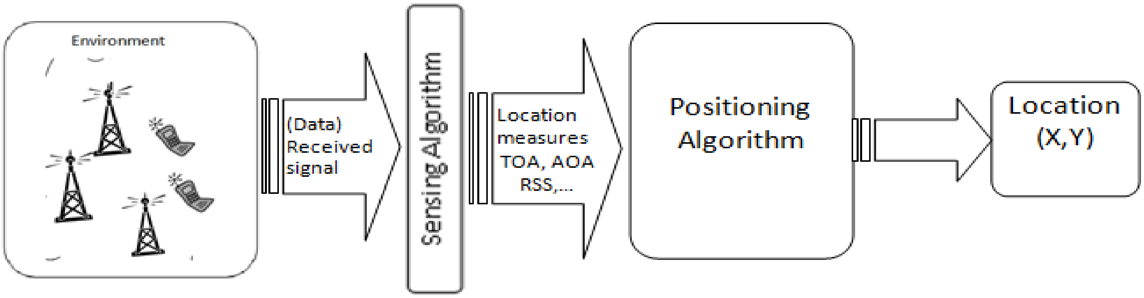

Figure 3.1 Main components of a localisation system [4].

Usually, the class of localisation system under consideration is determined by the various classifications of localisation followed by Alakhras *et al.* [3], [98]; Oussalah *et al.* [4]; and Hightower and Borriello [7]. The type of data required by each localisation system plays a key role in identifying its localisation class. In particular, the localisation algorithm is considered a *physical localisation* when the location is interpreted as a 2D or 3D coordinate position, as with GPS. If the location is interpreted linguistically (e.g., *room, corridor, lobby*), then the term *symbolic localisation* is commonly used. When the localisation system relies on a set of common reference points for all located targets, then this is referred to as an *absolute localisation*. if it relies on its own set of references, it is then considered a *relative localisation*.

When the localisation system is being implemented in an indoor environment, the localisation system is deemed an *indoor localisation*, whereas if it is implemented in outdoor areas, then it is considered an *outdoor localisation*. Some localisation systems require the clients to be able to position themselves, a feature which is termed *self-localisation*. If the node is unable to locate itself in the environment, such as in the case of sensor networks, this is referred to as *remote localisation*.

The communication medium between various localisation system components gives rise to different classes of localisation systems, for example, the environment may be fully wired, a combination of wired and wireless components, or fully wireless, as in WLANs. Moreover, the communication protocol used is another factor in determining the class of the localisation system, such as Wi-Fi, Bluetooth and GSM. The variations in the localisation system present force the need for various types of calculating algorithms and measurement techniques. The next section presents the most common localisation techniques suitable for the intended application, particularly, the Wi-Fi WLANs for indoor environments.

3.3 Location Estimation Systems

As highlighted in Chapter 2, positioning algorithms are typically dependent on the available resources, time constraints, computational cost, accuracy and precision requirements. There is no agreed favourite positioning system or algorithm across the spectrum as yet [21]. Several aspects contribute to the choice of the appropriate positioning algorithm—these will be discussed in the following sub-sections.

Physical and symbolic localisation systems are distinguished by the type of data required and the interpretation of the target location. As mentioned, symbolic localisation interprets a location using linguistic terms. However, a physical localisation system, with some additional data or infrastructure change, or both, may be enhanced to provide symbolic localisation. The resolution of physical positioning systems has implications for the definitiveness of the symbolic information that may be derived. "Purely symbolic location systems typically provide only very coarse-grained physical positions. Using them often requires multiple readings or sensors to increase accuracy—such as using multiple overlapping proximity sensors to detect someone's position within a room" [7]. However, the coarse-grained or relative positions require a vague description of position information, which provide a rational ground for applying fuzzy-like reasoning in modelling linguistic descriptions of the symbolic position information, or defining coarse-grained position information.

3.3.1 Infrastructure-based systems

Positioning techniques may be classified depending on whether they rely on the existing network infrastructure or not. The quasi-majority of positioning techniques rely on existing network infrastructure regardless of whether it is GPS, cellular or a wireless sensory array network, or any personalised network infrastructure (e.g., in case of military mobile infrastructure) [57]. Many robotics applications use an odometer-like sensor to track the position of the terminal from the initial starting location while external sensors help to build a local map of the environment [68], [70], [99], [100]. Often, handset-based positioning requires an increased processing and storage capabilities due to the highly centralised architecture at the handset, where all sensory information is gathered, processed and integrated [101]. According to Liu *et al* [14]., handset-based methods are associated with an increased cost for developing a suitable low-power, economically-integrated technology on wireless communications systems, especially when used within WSNs. Placing the burden on the infrastructure decreases the computational and power demands on the objects being located [14], which makes many more applications possible [66], [102]–[104].

3.3.2 Reference-dependent systems

Absolute and relative localisation systems rely on a set of reference points for all targets under investigation, the main difference is that absolute localisation depends on predefined reference points in geographical coordinates (longitude and latitude), sometimes known as *global localisation*. In contrast, relative localisation provides an estimation of the location about particular landmarks (street, university, bank, room)—sometimes known as *local* *localisation*. The calculating algorithms may be used to infer the absolute estimation for the target location, given many relative measurements, provided the reference landmark locations are given and fixed. Often, the reference landmarks (points) are not fixed at a precise location and usually are in a moving state, for example, in the case of mobile devices used as reference points in a WLAN environment. Such points provide inaccurate measurements, which then require uncertainty analysis to correlate the local and global views accurately.

3.3.3 Environment type

Appropriate positioning methodology depends on whether we are considering an indoor or outdoor environment. For instance, GPS is a well-recognised system for outdoor environments, but indoors the GPS signal may be unable to penetrate walls and buildings [105], which, in turn, creates a need for other positioning technology (e.g., Wi-Fi, Bluetooth, beacons, fingerprints). While the accuracy of GPS of up to about two metres is acceptable for outdoor open environments, this lack of precision may mean that the target object is actually located in another room or building when the same system is used for indoor environments. Moving objects and removing furniture can also significantly impact the level of accuracy. Indeed, in indoor environments, the radio propagation model is characterised as site-specific, non-deterministic signal fading, with severe multipath effects and diffraction and very low probability of line-of-sight (LOS) availability. Nevertheless, due to the relatively reduced scale of the indoor environment compared with the outdoor environment, the indoor environment makes it possible to conduct comprehensive site surveys to collect measurements from available anchor nodes, e.g., Wi-Fi APs, in a large set of scenarios, and construct robust maps accordingly [3], [4], [47], [106].

3.3.4 Measurement methodology

Many measurement techniques are discussed in the literature (detailed in Section 3.5), for example, AOA, TOA, phase of arrival, RSS or radiofrequency propagation model. But the selection of any technique is heavily dependent on the infrastructure and available devices. Despite the reasonable accuracy achieved by localisation systems that depend on radiofrequency propagation models in the outdoor environments, such as the GSM [6], we cannot recognise this accuracy level as reasonable if the environment type switches to the indoors. The cause of this shortfall in accuracy may be the size of the environment, or it may be because the communication protocols used in the indoor environments impose many uncertainties on the propagation model. NLOS is an important concern as a source of error, as are a number of devices and services in the environment, primarily when public band frequencies are used in WLAN environments. Further, the specifications of site parameters are not identical for all sites, a factor which makes it more challenging for a universal radiofrequency propagation model.

3.3.5 Communication protocol

The protocol used to establish communication between client nodes (targets) and servers (localisation processors) is another factor in selecting the localisation class. This may be characterised by the communication medium for the environment in question as wired or wireless. Hence, it is necessary to provide the server with accurate measurement data which include NLOS factors, signal diffraction, signal attenuation, and multipath effects—all have a direct impact on accuracy [76], [107]–[111].

3.3.6 Information processing architectures

Positioning systems may be classified depending on the amount of processing carried out at a single node. In a centralised architecture, all processing is carried out at a single node which forms the data fusion centre. This might be the mobile terminal or a base station. In contrast, in a decentralised architecture, the position of the target is carried by local anchors, providing an opportunity for various collaboration schemes. Such architectures are acknowledged for increased fault tolerance, scalability and cost efficiency, but they are also sometimes sub-optimal when data propagation results in a biased or overconfident position estimation [21], [51], [112].

3.3.7 Cooperative and non-cooperative architectures

In the context of WSN and the constant communication between the various components of the network, one distinguishes cooperative and non-cooperative architectures. In the former, the mobile may exchange information with other mobile units in order to enhance positioning accuracy. In a non-cooperative architecture, the mobiles can only exchange information with anchors or base stations [108], [113]. The cooperative architecture helps to substantially lower costs and enhance positioning accuracy, especially in the case of a mobile far from a fixed reference node when other mobiles of known locations are present in its vicinity. Nevertheless, issues may arise due to privacy concerns while exchanging information among various network units.

3.3.8 Calculating algorithm

The variations in sensory and measurement technology indicate another defining characteristic to classify the localisation systems, i.e., the selection of calculating algorithms for position estimation. The calculation algorithm may include geometric approaches (lateration, multilateration, triangulation and area calculations), proximity approaches (NN, kNN, ID-CODE), and scene analysis, which is often fingerprint-based. Those position estimating techniques each have their strengths and weaknesses. Hence, using a hybrid algorithm that does not rely on a single form of the calculating algorithm has shown better performance as a localisation system [80], [81]. This is especially so when inherent uncertainty cannot be estimated or ill-defined; in these circumstances, according to the fuzzy community, a fuzzy set-based approach has an edge in localisation [92].

3.3.9 Range-based and range-free schemes

Whether the system is range-based or range-free is also a way to distinguish between various localisation schemes, and most commonly used with WSNs. The range-based system needs to have any means for range measurement between successive nodes interpreted by distance, time or angle acquired via the aforementioned measurements (AOA, TOA, RSS). Section 3.5 provides details on those measurements, which are often used in combination [4], [42], [114], [115]. Range-free localisation includes fingerprinting where an initially constructed map or grid to actual measurement is set, and the hop count determined from each anchor using a dedicated routing protocol [116], [117]. The accuracy of range-based localisation is traditionally better than that for range-free localisation. This is so provided the mobile nodes are equipped with necessary means to obtain the required measurement (for example, directional antenna) [101], and the network dense enough with anchor nodes to obtain the location anywhere in the network at the required accuracy level without changing environments at the physical level.

3.3.10 Deterministic and non-deterministic methods

In deterministic methods, the location information is represented as a solution to an analytical or approximating problem. In this, deterministic mapping occurs without explicit accounting of any uncertainty, as opposed to probabilistic, fuzzy or statistics-based methods. Notably, among deterministic methods one encounters k-means-like matching in fingerprint association and the range intersection method [118]. Non-deterministic methods include Bayesian-like reasoning for fingerprint matching, Kalman filtering, belief propagation approaches [97], [119]–[123], joint probability distribution of a network using factorisation on a graphical model [56], [106], [124], and a variety of soft computing-related techniques [78], [125], [126]. In general, if knowledge about the distribution is available, probabilistic techniques perform better than deterministic ones and are to be preferred.

3.4 Location-Based Services (LBSs) and Applications

With the development of LBSs [6], positioning technology has actively entered the realms of almost every application, including industrial, medical, safety, transport and many commercial applications. The positioning task offers an answer to the question “Where am I?" By answering this basic question, it contributes to answering subsequent questions such as “What is nearby?”, “How can I reach that location?” and “How do I optimise my resources while achieving my task?”

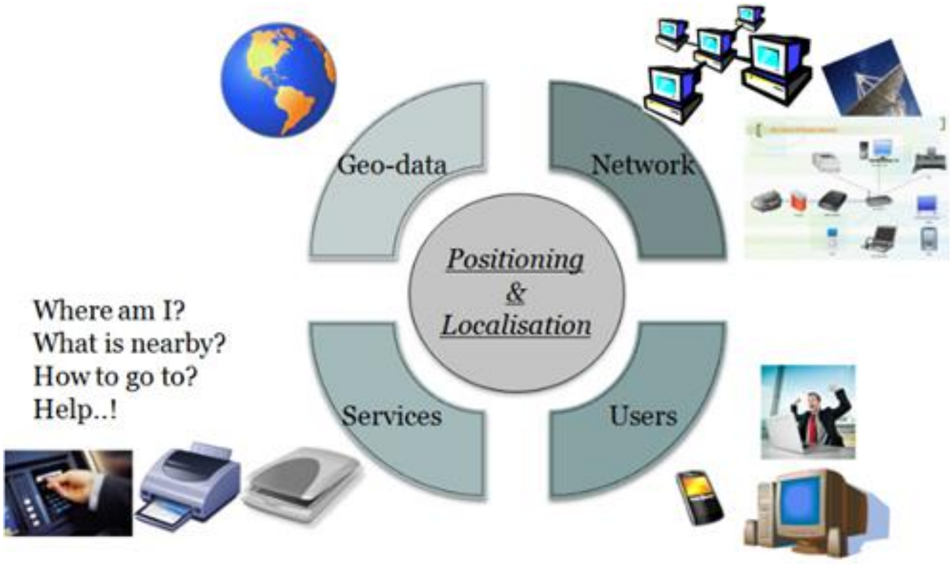

Figure 3.2 LBS system architecture [3].

As indicated in Figure 3.2, the positioning problem is ultimately linked to the context, sensory information and perceived environment, which, in turn, substantially constrain the position estimation process and the level of accuracy. For instance, some mobile location services attempt only to find out whether predefined attractions or facilities (e.g., hotels, shops, fuel stations) are in the vicinity of the user. Such services do not require the exact location of the facility as it is enough to indicate the presence or absence of a given facility within the vicinity. In network-based localisation, one seeks, for example, to identify the node responsible for the deterioration of network service or subject to an initial attack. This objective may require an exhaustive review of the activity history of all candidate nodes. In geo-data positioning, one often requires the latitude-longitude estimation of the target object, which may involve advanced state estimation techniques. Industrial robotics applications often demand very high precision in the realm of nanotechnology in order to achieve complex rendezvous tasks. Central to any positioning technology are the environmental constraints and the quality of the available prior knowledge, which also reflect the level of autonomy possessed by the device(s) to be positioned. From this perspective, one may distinguish an environment which is *fully-known*, *partially known* or *entirely unknown*. For instance, in WSNs, an emitter/receiver continuously senses its surrounding environment and searches for event occurrences. The latter may include changes in RSS, as well as other sensory information (e.g., temperature, pressure, lighting, humidity); this ultimately requires full knowledge of the nodes from which each piece of information is captured. On the other hand, autonomous systems equipped with advanced camera and wireless sensory modalities have the ability to fully map a completely unknown environment and execute complex navigation tasks. The estimation process in such a case includes both the target positioning and environment map estimation. SLAM (short for *simultaneous localisation and mapping*) techniques fall into this category [127], [128].

3.5 Measurement technology

The inability of propagation models to accurately describe the multipath parameters brings our efforts to localise a target in any indoor environment to the conventional localisation methods of triangulation, proximity measure and scene analysis. These methods each have their strengths and weaknesses, which motivates hybridisation among them in an attempt to achieve better accuracy and performance. Briefly, below, we consider the most popular algorithms; as mentioned in [14] and [129]; in the next section this discussion will be extended with RSS and fingerprinting techniques.

3.5.1 Triangulation

Typically, according to Liu *et al.*, "triangulation uses the geometric properties of the triangle to estimate the target location" [14]. It can be done via *lateration* or *angulation*; the selection depends on the environment, e.g. range, indoor/outdoor, antenna properties. Thereby, “lateration estimates the position of an object by measuring its distance from multiple known reference points,” while angulation calculates “the position via measuring the angle of bearing relative to two points with known separation” [3]. The triangulation measurements can include:

- **TOA (time of arrival):** The distance from the target to the measuring unit is directly proportional to the propagation time. A TOA measurement must be made with respect to signals from three different references. Using TOA is more suitable for the LOS environment, with the constraint that all transmitting and receiving units must be precisely *synchronised*.
- **TDOA (time difference of arrival):** The relative position can be estimated by examining the difference in time at which the signal arrives at multiple measuring units with constant distance, rather than one absolute arrival time. A correlation technique is used to estimate the TDOA at the measuring units. This technique imposes synchronisation between the measuring units but not on the target. Moreover, it results in a heavier mathematical derivation compared to the TOA technique.
- **TOF (time of flight from transmitter to receiver and back):** Although more moderate clock synchronisation than for TOA is required, it is considered to work as conventional radar, where a target responds to radar signal, and the complete round trip propagation time is calculated. However, it is still difficult for the measuring unit to know the exact delay/processing time caused by the target; for medium– and long-range systems this delay can be ignored, but not so for indoor applications. Also, it is difficult for the indoor environment to find the LOS channel between transmitter and receiver.
- **Received signal phase or phase of arrival (POA):** This assumes all transmission units emit pure sinusoidal signals with the same frequencies and zero offsets. In order to be

able to estimate the phases of signals received at a specified location, the transmitters are placed at particular locations within an imaginary cubic building. This constraint imposes extra ambiguity on indoor positioning due to the absence of LOS.

- **RSS-based (received signal strength or signal attenuation-based):** The transmitted signal power decreases with respect to travelled distance. This technique estimates the distance by measuring the RSS at a target location, and then compares it with the transmitted signal power from a particular transmitter. RSS-based methods attempt to calculate the signal path loss due to propagation. However, the path loss propagation model does not hold at all times, especially in indoor environments due to the multipath effect of signal reflection and refraction. Further, the parameters employed in these models are site-specific. The TOA and AOA signal would be affected by the multipath effect, which reduces the accuracy of the estimated location.
- **AOA (angle of arrival):** The location estimation is done via intersecting pairs of angle directions formed by the circular radius from a base station or beacon to the target. The position can be estimated with few measuring units, but it requires special directional antennas, LOS channels and synchronisation between transmission units.

3.5.2 Scene analysis

Scene analysis relies on collecting features first (fingerprints) of a scene then estimating the location of an object by matching online measurements with the closest *a priori* location fingerprints. This algorithm consists of two stages:

- **Offline:** The site survey is conducted to collect relative fingerprint information
- **Online:** Location positioning techniques are applied using current observations.

The calculating algorithms for estimating a target location in the fingerprinting technique may use:

- **Probabilistic method:** This estimates the probabilities that a target node is at a particular position, given a vector of its RSS observations and Bayesian posterior probability. The positioning is then carried out using Kalman filter or Bayesian formulas.
- **kNN (multi-nearest neighbours):** Using the RMS error or the Euclidean distance, the online measurements determine the best *k* correlations of fingerprints from the offline map. Then the position is estimated by averaging the positions of those *k* fingerprints. To achieve better accuracy, the positions of those fingerprints are associated with weights according to pre-defined weighting criteria.
- **Neural Network:** The online observations are used to localise the target as the output of the neural network weights matrix after the training and weights optimisation of the neural network is done using the offline observations.
- **Proximity:** Many current cellular networks are currently supporting this technique for all mobile handset devices with cell of origin (COO) or cell identification (CID). Traditionally, the system consists of a set of antennas with predefined locations. When an antenna detects a target object, the algorithm assumes that this object is at the same position as that antenna. The density of the network plays a significant factor in satisfying the accuracy requirements of the system.

3.6 GPS System and Outdoor Positioning

The GPS is recognised globally as a standard navigation and positioning system for outdoor environments with a metre-like accuracy where satellite coverage is good. However, in the presence of obstacles or in an indoor environment, electromagnetic waves are attenuated, reducing the efficiency of the GPS signal drastically [43]. For instance, the Global Navigation Satellite System (GNSS) signals attenuate by 20–30 dB in indoor environments. Infrared radiation (IR) technology is nowadays incorporated in most smartphones, PDAs, and TVs as a wireless positioning technology that depends on LOS between the emitting and receiving antennae, which is conditioned by the absence of interference from other sources [114]. Radiofrequency technology [130] has the advantage of penetrating obstacles and human bodies, which results in broader coverage and (relatively) reduced hardware infrastructure requirements. RF technology encompasses a wide range of sub-technologies in narrow bands (RFID, Bluetooth, WLAN-Wi-Fi and FM) and wide bands, all of which can achieve centimetre-level accuracy. ZigBee is a promising WLAN standard which supports solutions for communications in the 20 to 30-metre range, and is suitable for applications that require low power consumption and low data throughput. Ultrasound is another non-expensive technology which can work at lower frequencies. The ultrasound signals are used to estimate the position of the emitter tags from the receivers. Ultrasound signals have relatively low accuracy compared with many IR technologies and suffer from interference from reflected sources like walls, metals and obstacles [54].

The availability of cheap accelerometer and odometer sensors enabled the development of internal-mode positioning technology where the location is determined by integration over the travelled path from the initial position of the target. Obviously, over long distances, the accumulation of errors constitutes a serious handicap to such technology. However, whenever there is a possibility to update a target position using external sensors (to reduce the effect of error accumulation), the method shows promise [99]. With the availability of compass sensors in many mobile handheld devices, the use of magnetic function and map has emerged as a promising positioning technology [131].

Finally, several hybrid models that use more than one technology have emerged; in these a variety of sensor technologies are used on the same platform. For instance, many current smartphones are already embedded with an odometer sensor (internal positioning), proximity sensors, Wi-Fi and Bluetooth sensors. The variety of available sensors and measurement modalities (e.g., RSS, AOA, TOF, TDOF, CID) have led to various localisation schemes such as triangulation, trilateration, hyperbolic localisation, data matching, and many more [16]. Various commercial hybrid positioning systems are currently used in services from Combain Mobile, Navizon, Xtify, PlaceEngine, Skyhook, Devicescape, Google Maps for Mobile, and SopenBmap for application in smartphones [19].

3.7 Indoor Positioning

As mentioned in Chapter 2, the indoor location market includes indoor positioning-based services (and thus positioning systems) and solutions designed to support use cases around (*indoor*) location-based analytics. Generally, the location estimation system for the indoor environment is highly dependent on the available environment infrastructure. In many cases the algorithms and measurements are not readily applicable to every problem. In this section, we present the most common WLAN indoor localisation techniques and algorithms.

3.7.1 Radio frequency identification (RFID)

Lately, RFID has become the dominant algorithm in LBS and management. Typically, RFID–based techniques comprise the following components:

- **Transponder**: Usually it is placed on the target and represents the actual data-carrying device of an RFID system, more commonly referred to as a tag

- **Interrogator** or **reader**: Usually this contains an radiofrequency functional block (transceiver), a processing unit and a means of association to the tag (coupling). Many readers are equipped with a supplementary communication protocol mediator to allow them to communicate with the tags and forward the data received to another system
- **Application software** or **algorithm**: Plays the role of filtering collected data from the tags through the reader.

RFID is limited to the frequency range 135 kHz to 5.8 GHz, with attainable distance up to 100 m for an active RFID and 15 m for a passive RFID. It functions whenever a tag enters the interrogator range or is placed on a surface equipped with the necessary data acquisition means, which can be an electrical or magnetic field. When this range is as small as one centimetre, the RFID system is said to be in *closed-coupling* mode, which may increase power consumption. For a range of one metre the RFID system is said to be *remote coupling*, and for 1–15 m, *long coupling*. The coupling mode will affect the utilised frequency and the other hardware elements.

A limitation on localisation with RFID systems is the need for a considerable amount of infrastructure, including readers and tags to facilitate the localisation with acceptable accuracy. Also, the RFIDs are not equipped with RSS functions, which play a significant role in enhancing accuracy levels [132].

RFID tags are classified broadly into *passive* and *active* tags; the former is not equipped with a power source, the latter has some sort of power supply. Passive tags need to be in the reader's range to be activated; otherwise, the tags will not have enough power to send the signal back to the interrogator. Active tags do not need the power emitted by the reader, an advantage for the interrogator, which may use the saved power to increase the range of the device. However, the ability of tags to use saved power in transmitting the signal back to the reader is doubtful, as the tags are not equipped with the means to create the necessary high-frequency signals, merely to modulate the reader field. The feature to use saved power is linked to physically limiting the tag's ability from reaching the interrogator's range, which is optimised on the basis of frequency and the transmitting power of the interrogator [132]. In brief, RFIDs present several obstacles for solving the localisation problem.

3.7.2 Propagation models

Although the radio propagation model is the dominant method in use for outdoor localisation, radio propagation modelling for indoor environments faces many challenges arising from the signal properties inherent in the model. Intuitively, numerous signal properties cause the received signal to differ from the sent signal. The LOS is the most crucial property, as described in [133]. Among the factors involved in signal variation are: (i) free-space LOS; (ii) signal attenuation; (iii) signal absorption; (iv) noise; (v) multipath gain caused by signal reflection; (vi) signal diffraction; and (vii) scattering. Factors (v)–(vii) may be either useful or harmful to the RSS). These factors can generate a tremendous amount of ambiguity for WLAN positioning, which contributes considerably to the location estimation errors for all current localisation systems [134], [135]. The variations are known as *temporal, large-scale* and *small-scale* variations.

Temporal variation becomes evident as the measured RSS varies over a period of time while the receiver is steady at the same location. The reasons for this variation may be sudden changes in the WLAN infrastructure, moving objects in the environment, or operations of other devices on the same frequency band [134]. The issue may be managed by capturing the histogram of the RSS during the fingerprint map building in the offline stage, rather than capturing instantaneous RSS—this allows the best possible estimation of location.

Large-scale variation results from signal properties when the signal travels over long distances. This variation generates a new fingerprint signature, which is influential in determining new positions.

Small-scale variation becomes apparent when the target object moves on a wavelength scale, i.e., movements of centimetres or millimetres, depending on the communication protocol in the test environment. This variation may be managed by simply cancelling the variation and compensating for it in the calculations of the positioning algorithm. The main difficulty is how to accurately determine the compensation parameters in the online fingerprinting stage, especially in dynamic environments.

In respect of the defects listed above, the indoor radio propagation model is highly dependent on the environment and the received signal parameters considered in the model. Building a universal propagation model is a complicated and time-consuming process. instead, it may be more fruitful to formulate a model via approximation techniques and consider the most critical parameters of the signal in the environment to describe the path length from the emitter node to the target objects.

In indoor environments, the main parameter describing the path length between the emitter node and the target objects is path loss, which represents the ratio of emitted and received signal power. In this regard, the Friis transmission model can be proposed [136]:

$$L = \frac{P_r}{P_t} = g_t . g_r . (\frac{\lambda}{4\pi d})^2 \qquad (3.1)$$

where P_r and P_t are the received and transmitted power, respectively; g_r and g_t, the gain for receiving and transmitting antenna, respectively; λ, the wavelength and d, the physical path length between emitter and receiver. Signal power falls as the inverse square of the path length. Equation (3.1) can be transferred to a more familiar logarithmic scale, where the propagation loss (path loss) unit is the decibel (dB), thus:

$$PL_{dB} = G_t + G_r + 22 + 20\log(\frac{d}{\lambda}) \quad (3.2)$$

where G_r and G_t are the gain for the receiving and transmitting antennas in dB, respectively. Applying a 12.3-cm wavelength (frequency 2.44 GHz) to (3.2) yields (3.3), assuming 0 dB antenna and $G_r = G_t = 0$:

$$PL_{dB-2.44\,freespace} = 40.2 + 20\log(d) \quad (3.3)$$

It can be seen from Figure 3.3 that a decay of about 40 dB of the signal power occurs during the first metre of the journey. According to (3.3) and Figure 3.3, the best distance for the 2.44 GHz protocol with a scale of 80 dB and zero gain antenna is 50 metres.

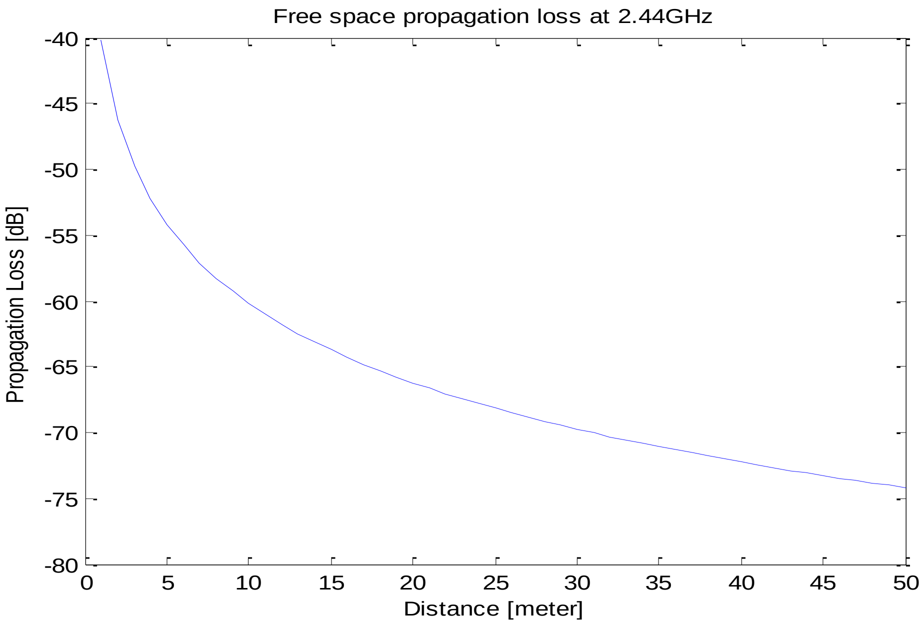

Figure 3.3 Path loss for 2.44 GHz in free space.

For indoor environments, mathematical simulation is only capable of simulating specific scenarios. Therefore, an alternative general modelling method is required. For this, the most common practice is to predict the path loss using the empirical models based on real measurements made on the target environment to collect the power loss at specific points and then perform numerical curve-fitting over the sampled data. In [136], we can find that the following equation describes the model constructed for the indoor environment using the same frequency band:

$$PL_{dB-2.4\,indoor} = 40 + 31\log(d) \pm 8 \qquad (3.4)$$

This experiment took place at an office with gypsum board inner walls. From (3.4) and Figure 3.4 the signal power decayed by 70 dB during the first metre; the best distance that a 2.4 GHz protocol achieved with a scale of 80 dB and zero gain antenna was about 20 metres.

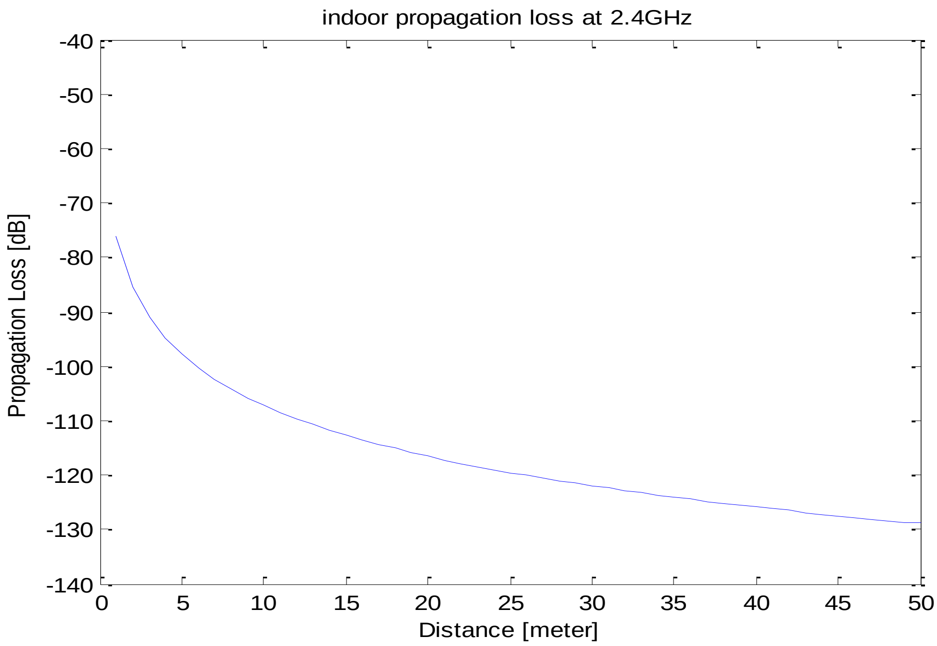

Figure 3.4 Path loss for 2.4 GHz in an indoor environment.

An interesting result from modelling WLAN indoor signal propagation using the 2.4 GHz frequency band is presented by Sadiki and Paimblanc [137], who studied the influence of the environment on the propagation model. As the environment components were altered, the experiments resulted in three different propagation models . This concept was employed in the localisation process investigated by Wu *et al.* [138]. The suggested design used ANFIS to represent the signal propagation model inside a test environment based on assembled fingerprints. The design was then tested against a curve-fitting routine prior to deployment in the localisation algorithm. Their design conferred more reliable performance than with curve fitting alone. However, the fingerprints generated were based on the simulation, not authentic environment situations, and where the testbed was a free space of 11.4 by 9.6 metres, which may not represent well signal behaviour in a larger environments. Moreover, the design was not tested empirically against unforeseen environment changes. The ANFIS tool was also used in [139] to model the path loss for a GSM-900 band system for outdoor environments.

The main limitation on soft computing tools such as ANFIS follows from not using the power of fuzzy inference functions such as various norms and defuzzification methods, and relying on standard functions only. This does not always guarantee to produce the optimal membership degree for output variables.

3.7.3 RSS and fingerprinting

The RSS-based fingerprinting technique has become the most exploited technique for indoor localisation. The technique appears easy to deploy, is more receptive to noise and can deliver very acceptable accuracy to several applications [140]. Nevertheless, the performance of the method suffers from several shortcomings which raises questions over its status as the principal choice for localisation applications on WLANs in the indoor environment. The efficiency of any fingerprinting technique is massively dependent on the density of the fingerprint map, which has to be continuously updated to meet changes in the deployment environment, such as replacement of APs or changes in environmental features. Moreover, due to the dynamic nature of indoor WLANs, obstacles can sharply reduce the RSS by blocking the LOS between APs and target objects. In addition, indoor WLANs are deployed on public band frequencies. For example, the frequencies used for Wi-Fi and infrared are commonly shared by many home devices such microwave ovens and mobile devices, or even used by service provider. This overlap produces irregular samples of the RSS during both the offline and online phases of the technique.

Fundamentally, two separate phases are carried out in building any fingerprint-based system. The goal of the first phase is to construct a fingerprint map or radio map. In essence, this is a database of the referential node tuples from the RSS vector $FP_i(RSS_{i1}, RSS_{i2}, \dots, RSS_{in})$., and its correlated physical FP_i *(x,y)*, or symbolic location. The goal of the second phase is to match the measured RSS $T_j(RSS_{i1}, RSS_{i2}, \dots, RSS_{in})$ vector with the database and forward the results to the calculating algorithm, which will produce the target's estimated position T_j *(x,y)*. These are referred to as the offline and online phases, respectively. Figure 3.5 shows a schematic of the methodology.

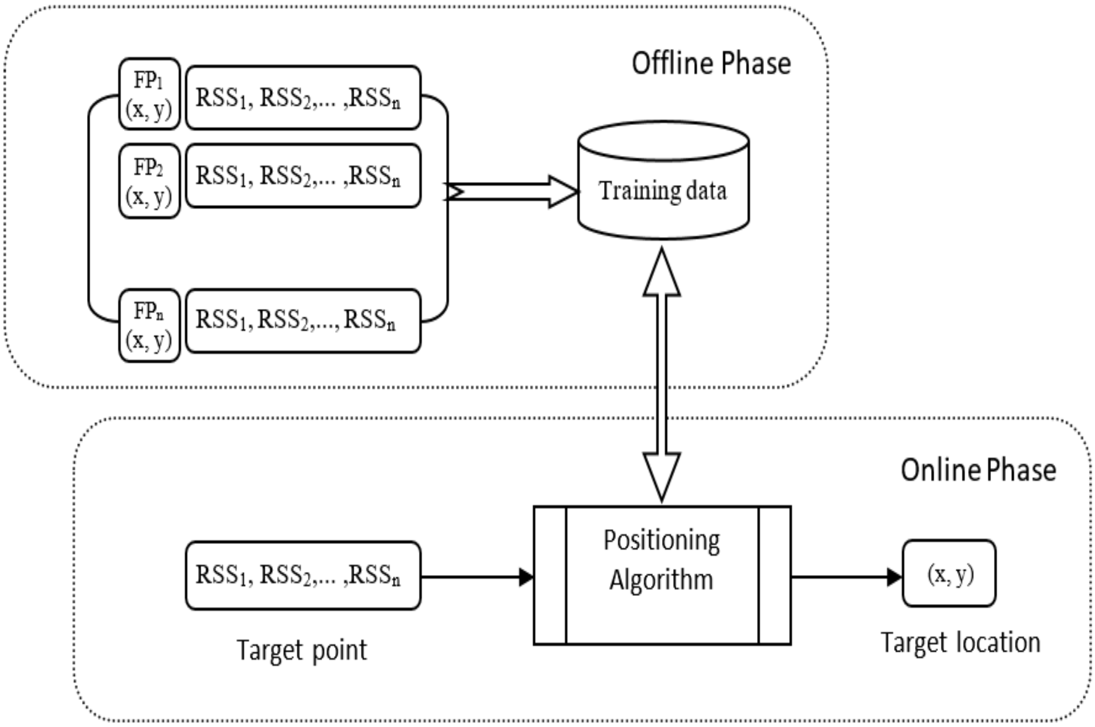

Figure 3.5 Diagram of fingerprinting localisation methodology [4].

Constructing the fingerprint map is commonly done by one of two methods: empirical observations; or via a radio propagation model.

For the empirical approach, the relative or absolute position coordinates are recorded according to manual observations of the RSS measurements after the physical network deployment on the test environment. The main limitation of this approach is the need to re-collect the observations to match the simplest changes in the test environment to meet the localisation accuracy requirements. Another limitation, highlighted in [135], is the uncertainty associated with variations in RSS values among various built-in wireless reader tags. These variations compel a need for old-fashioned and error-prone scaling for each target object and environment. Motivated by those limitations, Kjærgaard and Munk [141] proposed assembly of the fingerprint database by collecting RSS values as ratios among various combinations of APs rather than as absolute RSS values—this technique is known as *hyperbolic location fingerprinting*.

The propagation model approach computes the reference RSS for each reference fingerprint in the observation area. A deficiency of this approach is its reduced accuracy in comparison to the empirical approach, due to the complexity of the wireless radio propagation model in the indoor environment, as discussed in the next section.

Various algorithms are proposed in the literature to calculate the best matching fingerprint location to the target location [142]. Commonly, the Euclidean distance can be used to calculate the physical path length between the fingerprints and the objects to be positioned. The basic Euclidean distance operations can be done by the following equation:

$$d = \sqrt{\sum_{i=1}^{n} (RSS_{FP_i} - RSS_T)^2} \qquad (3.5)$$

where the RSS_{FP_i} is the value of fingerprint RSS assembled in the offline phase and RSS_T is the measured RSS for the target T during the online phase, where a small d indicates high similarity. Similarly, in the same paper, Gansemer *et al* [142]. presented a new enhanced Euclidean distance formula. This was adopted to normalise the distance *d* to the fingerprints only involved during the measuring phase. It is especially so in dynamic environments that not all fingerprints (base stations) are available to the target (mobile station) during the measuring phase in the corresponding test bed. The enhanced formula is:

$$d = \sqrt{\frac{1}{m}\sum_{i=1}^{n}(RSS_{FPi} - RSS_T)^2} \qquad (3.6)$$

where *m* is the number of matching (available) fingerprints during the measurement phase. Bayes rule determination (Bayesian inference) is another probabilistic approach to estimate the possible location for a target. As presented in [143] and [144] the possible location *T(x,y)*, from the reference locations set *FP(x,y)*, given the observation *RSS* vector that maximises the conditional probability $p(l_i/RSS)$, can be calculated as follows:

$$p(l_i/RSS) = \frac{p(RSS/l_i)\,p(l_i)}{p(RSS)} \qquad (3.7)$$

where $p(l_i/RSS)$ is the conditional probability of measuring the RSS at the l_i location, which may be approximated from the number of occurrences of vector $(RSS_{i1}, RSS_{i2}, \ldots, RSS_{in})$ at the location l_i, according to the database available in the offline phase.

The kNN search mechanism, mentioned earlier, is another approach used to estimate the location of a target. It was first used by Bahl and Padmanabhan [115] in their RADAR system, in which they propose multiple neighbour points (*k*) at roughly the same distance from the target point in the signal space, with the distance calculated using the same Euclidean distance rule given in (3.5). The density of the fingerprint database and the symmetrical pattern of fingerprints in the test area are arguably the main issues with this approach. Furthermore, any modification in the test environment may result in removal of some reference fingerprints from the database. As a result, the algorithm will pick a reference fingerprint that is spatially far from the target object, which decreases the accuracy of the final position estimation. In this regard, we claim that the RSS does not have to be the sole indication for the physical path length.

3.8 Conclusion

In this chapter, the background and definitions of localisation systems were discussed, with a well-defined classification of the various localisation systems. The main focus was on the environment type (e.g. the indoor environment), the networking technology (e.g. WLAN), the measurement techniques (e.g., RSS), and location estimation techniques such as the fingerprinting and kNN algorithms, including the propagation model, its inherited uncertainties and how these affect accuracy. The next chapter explains how to build a FIS to handle these uncertainties, including the fundamental idea of TKS-FIS, the inferencing principle, membership function manipulation, defuzzification processes and the rationale of using more input variables in the reasoning principle to reduce uncertainty.

4 FUZZY SETS AND FUZZY SYSTEMS

4.1 Overview

The principal idea behind the fuzzy set as it was proposed by Zadeh [22] in 1965 is a mathematical concept to represent vagueness, imprecision or incompleteness in data. The concept is beneficial in practice when the reference knowledge is constructed using verbal expressions and is based on expert knowledge or data which are naturally imprecise.

Furthermore, fuzzy logic provides a means to associate system dependencies—in other words, the inputs and outputs of any model via what is referred to as fuzzy rules—while fuzzy-inferencing offers a tool to derive implications given a group of fuzzy rules, in what is referred to as fuzzy reasoning.

Fuzzy sets differ significantly from conventional ('crisp') sets. For example, the application of fuzzy sets in a control strategy, where the controller is incorporated in the process according to a pre-constructed mathematical model of the controlled object that requires quantitative and numeric analysis.

The execution steps of any task involve manoeuvring procedures based on oral descriptions, which is a globally recognised practice. Developing a purely mathematical representation to model all the details of the verbally executed task would produce results that deviate from the execution of verbal descriptions. Yet, in automatic control strategies, the linguistic model looks very attractive because it resembles human verbal communication [145].

Generally, so-called fuzzy systems rely on expert data, which is often expressed by the construction of fuzzy rules in which input and output variables are characterised by linguistic values and *membership functions* to describe these values.

This chapter presents the main constituents of fuzzy rule-based systems, their structure, main blocks and operation principles.

4.2 Fuzzy Set

Explaining the fuzzy set first requires introducing the *crisp set*, where, for any element, the degree of belongingness is characterised by the expression:

$$m_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} \Rightarrow m_A(x) \in \{0,1\} \qquad (4.1)$$

A pictorial description of (4.1) is shown in Figure 4.1. Crisp set models answer the trivial question, "Does it belong to the set? (Y/N)", which characterises the binary nature of a classical set.

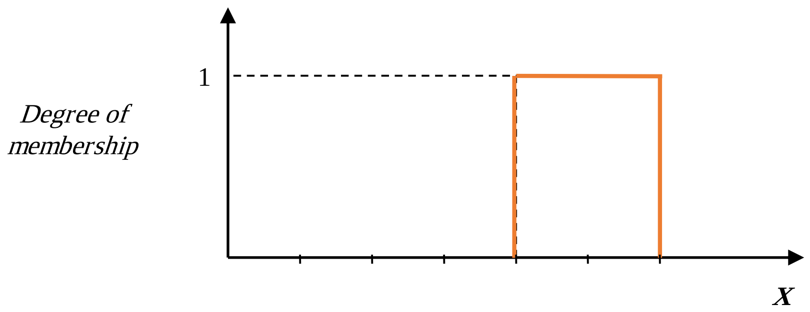

Figure 4.1 Representation of the crisp set.

In contrast, with the fuzzy set, we might rephrase the above question thus: "What is the degree of belongingness?" Intuitively the answer could be any value in the range 0–100%. In special cases, the answer 0% resembles "Does not belong at all," while 100% resembles "Fully belongs to." In these cases the fuzzy set is reduced to the classical. In other words, the classical set is a special case of the fuzzy set.

Where the answer takes a value within the range of the closed interval [0 – 100%], we may formally model this scenario according to the fuzzy literature and define the degree of belongingness on the real-number interval [0 – 1] as:

$$\mu_A(x) \in [0,1] \quad , x \in X \qquad (4.2)$$

where μA represents the membership function of the fuzzy set A, which is a degree of membership to x in the universe of discourse, $\boldsymbol{X}$, as shown in Figure 4.2.

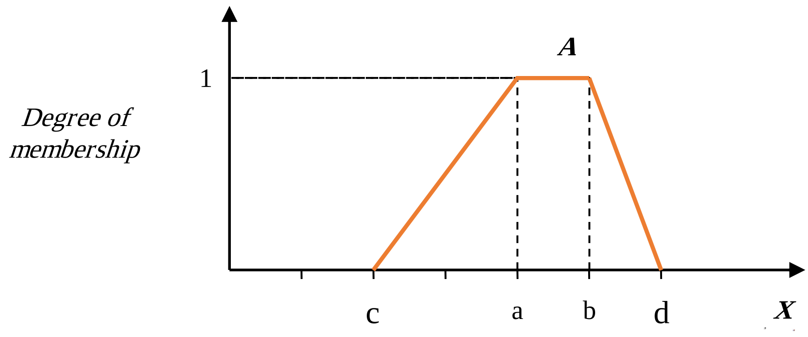

Figure 4.2 Fuzzy set A.

Elements between *a* and *b* belong *strongly* to the fuzzy set *A* with a degree of 1; formally this is expressed as $\mu_A(a) = 1$ and $\mu_A(b) = 1$. Elements less than *a* or greater than *b* belong to the same set with a gradually and linearly decaying value (smaller than 1) depending on its occurrence in the set [146], for example, $\mu_A(m) = 0.3$ for some $c < m < a$.

Figure 4.3 shows some common types of membership functions used in the literature, i.e., the trapezoid, triangular, Gaussian and singleton types of membership function. These are defined through the associated parameter values, e.g., a, b, c and d for the trapezoid membership function.

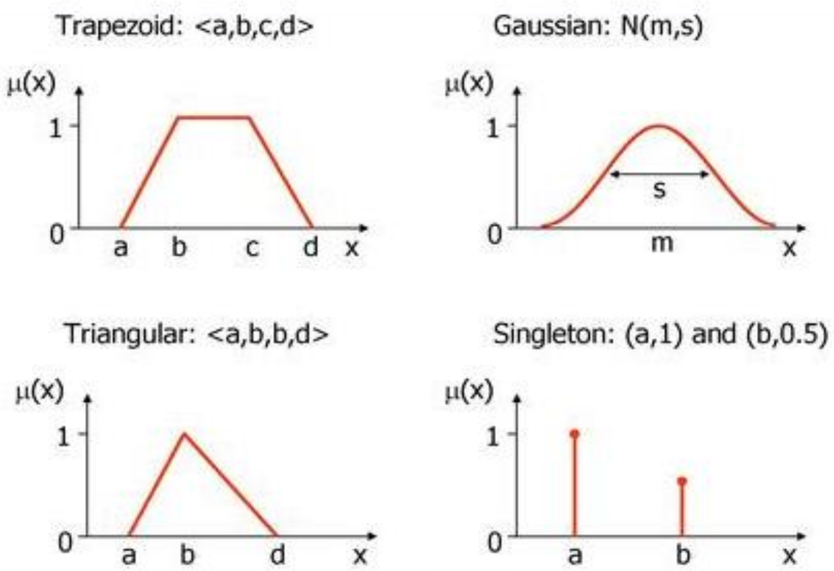

Figure 4.3 Some common parameterised membership functions.

Membership functions have many shapes. A common example of a function which produces a bell curve is based on the exponential function [147]:

$$\mu(x) = \left[\frac{-(x-x_0)^2}{2\sigma^2}\right] \qquad (4.3)$$

The model in (4.3) represents a standard Gaussian curve with a maximum value of 1, where x is the independent variable on the universe of discourse, x_0 is the position of the peak relative to the universe of discourse, and σ is the standard deviation [148]. An alternative model is:

$$\mu(x) = \left[1 + \frac{-(x-x_0)^2}{2\sigma^2}\right]^{-1} \qquad (4.4)$$

The triangular membership function is described by:

$$\mathrm{\mu(x)} = \begin{cases} 1 - \frac{\bar{\mathrm{x}}-\mathrm{x}}{\bar{\mathrm{x}}-\mathrm{x_l}} & , \ \mathrm{x_l} < \mathrm{x} < \bar{\mathrm{x}} \\ 1 - \frac{\mathrm{x}-\bar{\mathrm{x}}}{\mathrm{x_r}-\bar{\mathrm{x}}} & , \ \bar{\mathrm{x}} < \mathrm{x} < \mathrm{x_r} \end{cases} \qquad (4.5)$$

Several definitions of fuzzy membership functions useful for evaluating a fuzzy set are shown in Figure 4.4. These are:

- *Support*: Elements of the universe $\{x, \forall\, x \in X, where\ \mu(x) \neq 0\,\}$
- *Core*: The set of elements of the universe $\{x, \forall\, x \in X, where\ \mu(x) = 1\,\}$
- *α–cut*: The set of elements of the universe $\{x, \forall\, x \in X, where\ \mu(x) \geq \alpha\,\}$
- *Height*: The maximum degree of membership for a particular fuzzy set. This is usually equal to 1 for a normal fuzzy set. Sub-normal fuzzy sets indicate the occurrence of some conflict when inferred from normal fuzzy sets. It is not necessary for *height* to be 1, any fuzzy set with *height* 1 said to be normal, otherwise, a normalization process will be necessary at times.

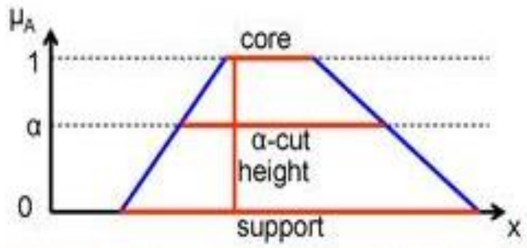

Figure 4.4 Schematic of support, core, α-cut and height of fuzzy set A.

4.2.1 Linguistic values and linguistic variables

The result of associate meanings (semantics) with a fuzzy set are [149]:

- **Linguistic variable**: The labelled domain of the fuzzy set, for example, *age*, *temperature* and *speed*
- **Linguistic value**: The labelled fuzzy-sets of the domain, for example: *young*, *old* for *age*; *slow*, *rapid*, *fast* for *speed;* and *short*, *medium*, and *tall* for height (see Figure 4.5).
- **Linguistic modifier**: Linguistic descriptions that modify the linguistic variable. Includes the following classes:

- ***Expansive*** (more or less, approximately, rather): correspond to a loss of precision or to a weakening of the original label.
- ***Restrictive*** (very, strongly, really): correspond to an increased precision or represent a reinforcement of the original label.

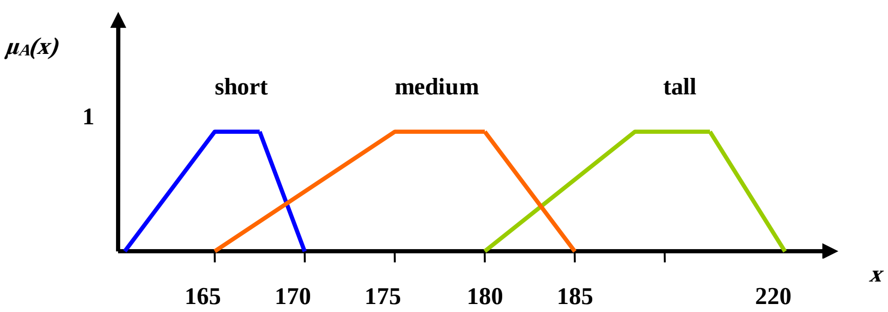

Figure 4.5 Example of fuzzy linguistic variables.

Figure 4.5 raises a question, "Is this model of human height valid for basketball players? Or jockeys? The answer is definitely *no*, since linguistic values are inherently context-dependent.

4.2.2 Fuzzy set representations

The above-mentioned representations, using the membership degree are referred to *vertical representations* [146]. Corresponding to classical set theory, a fuzzy set can be viewed as a union of its subsets, where every subset is a *level-cut* of the main fuzzy set, e.g.:

The α–level cut $\mu_{\alpha}(F) = \{u \in U: \mu(u) \geq \alpha\}, for\ 1 \geq \alpha > 0$.

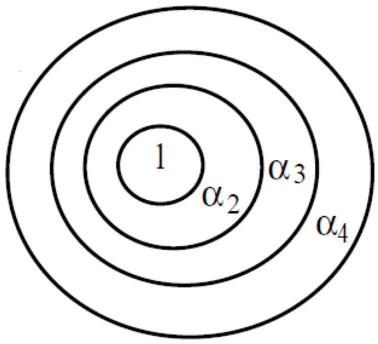

Figure 4.6 Horizontal representation of a fuzzy set.

The generated family of crisp sets $\{\mu_{\alpha} : \forall\ 1 \geq \alpha > 0\}$ constitutes the main fuzzy set and is referred to as the *horizontal representation*, as shown in Figure 4.6 with α–cut and membership grades $\alpha_1 > \alpha_2 > \ldots > \alpha_n$.

Level-cut representation is very necessary to understand the extension principle of fuzzy operators. It is reliable to retrieve *μ(x)* using its *α–cuts* as in (4.6):

$$\mu(x) = \sup_{\alpha \in [0,1]} \min\{\alpha, \mu(x)\} \tag{4.6}$$

Alternatively, a *hypercubic* representation as proposed by Kosko [147] may be used. In this scheme, a fuzzy set is generated from the power set of a crisp set. Specifically, the power set of *X* defines a hypercube of dimension |*X*|, with each dimension represents the characteristic-function of one element and can take the values 0 or 1. Then only vertices of this hypercube are valid sets. In crisp set theory, it is possible to represent a subset of X describing only the coordinates of a vertex as n-tuple composed of 0s and 1s. Accordingly, any point inside the hypercube represents a valid subset of X. Then, the vertices of the hypercube are the crisp sets, where fuzziness is 0. The centre of the cube, where every coordinate is 0.5 is the set with maximum fuzziness [147].

As shown in the example in Figure 4.7, if $U = \{A, B\} \Rightarrow |U| = 2$, then the crisp set is represented by the vertices, while the inner point represents the fuzzy set = {0.5/X + 0.4/Y}.

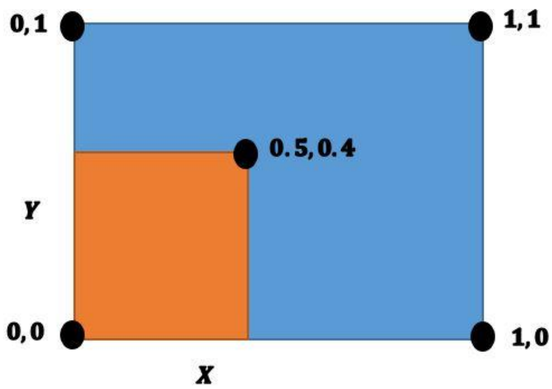

Figure 4.7 Hypercube representation of a fuzzy set.

Efficient use of fuzzy sets requires a wise selection of their representation. The aforementioned representations all belong to type-1 fuzzy sets. Type-2 fuzzy sets, proposed by Zadeh in [150], gain some attention from various researchers but relatively fewer applications were reported, and none in the WLAN localisation domain.

However, Mendel [148] elaborated on the type-2 fuzzy set and the extension principle, where a type-2 fuzzy-set is viewed as a type-1 fuzzy-set with grades of membership that are themselves fuzzy; in this case it is said to have a *3D representation*, as shown in Figure 4.8. Every value in the universe of discourse is represented by a set of grades to the same main linguistic term. In other words, the parent membership-function will be the domain of the child membership-function. This representation has added value especially when the degree of membership itself is ambiguous [148].

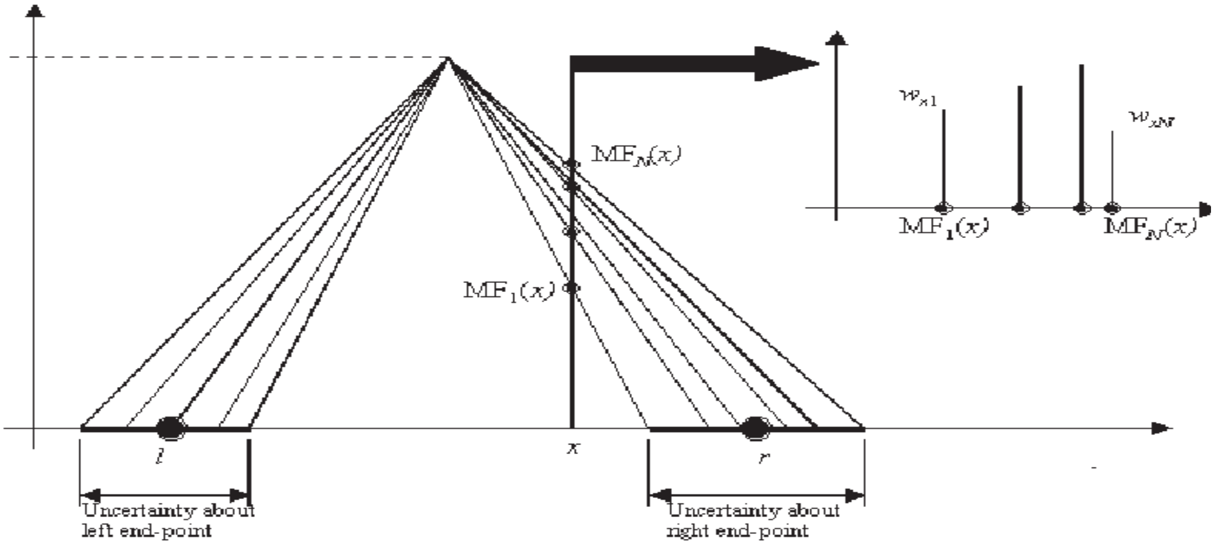

Figure 4.8 Type-2 fuzzy set representation.

4.2.3 Connectives and basic operations of fuzzy sets

It is important to know how to handle fuzzy sets; For example, if a fuzzy set describes a group of older people, and another fuzzy set describes a group of tall people, and we wish to consider an element which has a membership-degree with the older people set as $\mu_{old}(x)$ = 0.5, and a membership-degree with the tall people set as $\mu_{tall}(x)$ = 0.7. What then is the degree of membership for the same element to the older and tall people sets µold∧tall (x) = ? To find a solution, we will use conjunctions and disjunctions on fuzzy sets, as is done with crisp set operators).

Inclusion

Two principal methods may be used to determine the *inclusion* of *F* into *G*:

- ***Simple inclusion***, proposed by Zadeh (as cited in [146])

$$F \subseteq G \iff \forall u \in U\,, \qquad F(u) \leq G(u)$$

- ***Strong inclusion***, proposed by Dubois and Prade [151], *support* of F included in the *core* of G $\quad F \subseteq_s G \iff \mathrm{s(F)} \subseteq {}^{\circ}G$

Intersection and union (t-norms)

The intersection and union operators are generalised formulas for the conjunction and disjunction operators in multi-valued logic; known as *t-norm* (triangular norm) and *S-norm* (t-conorm). Briefly, the t-norm is a 2-space function, [0,1] x [0.1] to [0,1], that satisfies the following five basic axioms [152]:

- $T(0,0) = 0$
- $T(x,1) = x$
- $T(x,y) = T(y,x)$
- $T(a,b) <= T(x,y)\ if\ a <= x\ and\ b <= y$
- $T(T(x,y),z) = T\big(x,T(y,z)\big)$

The S-norm is a 2-space function, [0,1] x [0,1] to [0,1], that satisfies the following five basic axioms [152]:

- $S(1,1) = 1$
- $S(x,0) = x$
- $S(x,y) = S(y,x)$
- $S(a,b) <= S(x,y)\ if\ a <= x\ and\ b <= y$
- $S\ (S(x,y),z) = S(x,S(y,z))$

Complementation

Negation (NOT) or complementation in its simplest form can be calculated as in (4.7).

$$\mu_{-A}(x) = 1 - \mu_A(x) \qquad (4.7)$$

The following table briefly list (some) t-norm and t-conorm fuzzy connectives, as proposed by different researchers [153].

Table 4.1 Some well-known t-norm and t-conorm operators [153]

	T-norm
Drastic	$\mu_{A\wedge B}(x) = \begin{cases} \mu_A(x) & if\ \mu_B(x) = 1 \\ \mu_B(x) & if\ \mu_A(x) = 1 \\ 0 & otherwise \end{cases}$
Bounded-Gill	$\mu_{A\vee B}(x) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$
Product	$\mu_{A\wedge B}(x) = \mu_A(x) \cdot \mu_B(x)$
Yager, p≥1	$\mu_{A\wedge B}(x) = 1 - \min\{1, (P\left(\left(1-\mu_A(x)\right)^P + \left(1-\mu_B(x)\right)^P\right))^{1/P}\}$
Dubois	$\mu_{A\wedge B}(x) = \dfrac{\mu_A(x)\mu_B(x)}{\max(\mu_A(x), \mu_B(x), r)}$
Zadeh	$\mu_{A\wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\}$
	T-conorm
Drastic	$\mu_{A\vee B}(x) = \begin{cases} \mu_A(x) & if\ \mu_B(x) = 0 \\ \mu_B(x) & if\ \mu_A(x) = 0 \\ 1 & otherwise \end{cases}$
Bounded-Gill	$\mu_{A\wedge B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$
Product	$\mu_{A\vee B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$
Yager, p≥1	$\mu_{A\vee B}(x) = \min\{1, (\mu_A(x)^P + \mu_B(x)^P)^{1/P}\}$
Dubois	$\mu_{A\wedge B}(x) = \dfrac{\mu_A(x) + \mu_B(x) - \left(\mu_A(x)\mu_B(x)\right) - \min\{1-r, \mu_A(x), \mu_B(x)\}}{\max(1-\mu_A(x), 1-\mu_B(x), r)}$
Zadeh	$\mu_{A\vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

Other sets of t-norms are available in the literature. There are many different ways to define the norms in fuzzy set theory; it depends on the way we want to use them, especially as we manipulate fuzzy rules, because defining norms differently will directly affect the interpretation of specific rules. A comparative study of the various types of t-norms by Wang [153], although the comparison was limited by six criteria, proves that in particular cases the minimum is the greatest t-norm, and the drastic intersection the smallest t-norm. Similarly, the maximum operation yields the smallest t-conorm acting as a fuzzy union operation, and the drastic union is the most encompassing.

4.3 Fuzzy Inference Systems

The term *fuzzy inference system* (FIS) broadly describes a system that uses fuzzy sets and fuzzy logic to infer conclusions (outputs) from imprecise, uncertain or vague information (inputs). Various terms are used by scholars and other in disciplines, but FIS is preferable since it highlights the operational structure and knowledge foundation [154]. Fundamentally, any FIS comprises the following functional segments and definitions (see Figure 4.9) [154]:

- *Rulebase*—holds conditional statements, referred to as *fuzzy rules*
- *Database*—holds linguistic descriptions for the fuzzy rules in the form of fuzzy sets together with descriptions of fuzziness referred to as *membership functions*
- *Decision-making unit*—this is the core reasoning segment which describes the operations on fuzzy-rules. Includes operations such as *min, max and multiplication*
- *Fuzzification inference*—transforms the crisp data into fuzzy data
- *Defuzzification inference*—transforms the fuzzy data into crisp data.

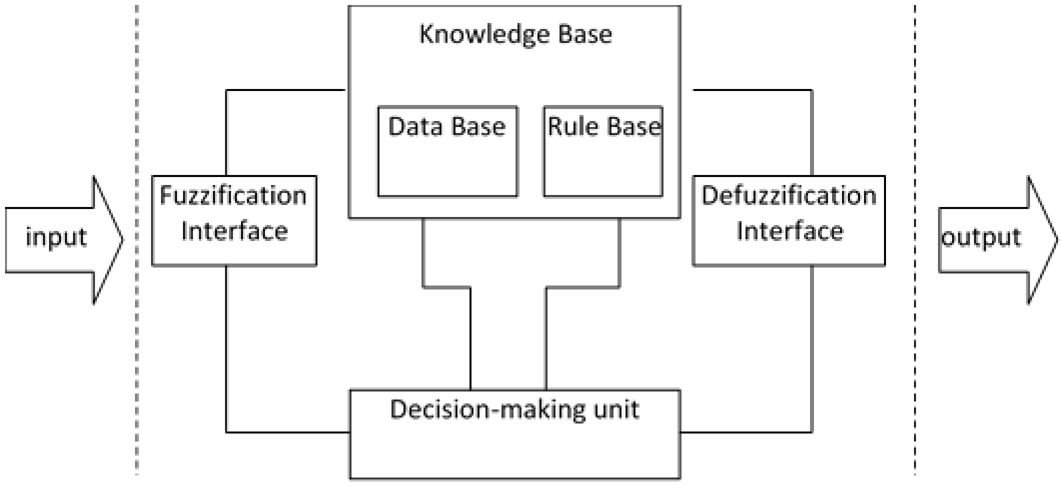

Figure 4.9 Fuzzy inference system with five functional blocks [154].

4.3.1 Fuzzy reasoning

The primary task of FIS and fuzzy norms is to help us, using combinations of certain rules, to draw conclusions based on the fuzzy knowledge given in the rulebase.

If we have a fuzzy set with certain degree of membership $\mu A(x)$, and given that $A \rightarrow B$, we need to achieve a membership degree $\mu B(y)$ of output variable y to fuzzy set B.

Example: If “temperature is low” then “heater is ON”

If we know the degree of membership for medium temperature, we can draw a weak conclusion based on the available overlap between 'low' and 'medium' fuzzy sets (Figure 4.10).

Using different fuzzy norms, and different ways to evaluate the fuzzy sets, may give quite different results. Another way of evaluating fuzzy sets can be done via their structural properties based on Boolean algebraic relations, i.e., associativity, commutativity, identity, absorption, idempotence, De Morgan's laws, and distributivity [151].

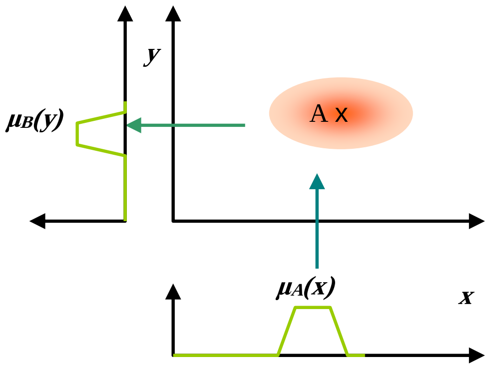

Figure 4.10 Basic fuzzy reasoning concept.

A comprehensive platform for extending standardised or non-standardised arithmetic, logical or relational operators has been created since the early appearance of fuzzy set theory by many scholars [22], [151], [155]–[158]. This solid topological foundation renders the theory and its inferencing operations capable of being embedded in any application.

4.3.2 Fuzzy implication

Broadly speaking, there are two major categories of fuzzy implications: (i) *explicit-representation-based*; and (ii) *implicit-representation-based*. Regardless of their representation style, most well-known fuzzy implication families, as introduced in [159], are:

R- and S-Implications

- S-implications: $I_S(a, b) = S(n(a), b)$ (4.8)

  where S is t-conorm, and n is strong negation.

- R-implications: $I_R(a, b) = sup\{z \in [0, 1]: T(a, z) \leq b\}$ (4.9)

  where T is t-norm.

- Reciprocal R-implications $I_C(a, b) = I_R(n(b), n(a))$ (4.10)

Non-commutative conjunctions

$$T^{*}(a,b) = \begin{cases} b, & if\ a+b>1 \\ 0, & otherwise \end{cases} \tag{4.11}$$

Exotic implications (QL-Implication)

$$I(a,b) = S(n(a), T(a,b)) \tag{4.12}$$

where S is *t-conorm*, n is a strong negation, and T is the *n-dual t-norm* of S.

Interval-based implications

Proposed by Bilgiç and Türkşen [160], and not dependent on implicit or explicit representations:

$$CNF(p \longrightarrow q) = \neg p \lor q \tag{4.13}$$

$$DNF(p \longrightarrow q) = (p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q) \tag{4.14}$$

where CNF and DNF are the normal conjunctive and normal disjunctive forms, respectively.

4.4 Fuzzy Rules

As mentioned, the fuzzy rule is a conditional statement written in the form of an *if-then* clause and used to capture human knowledge about a particular phenomenon or describe the behaviour of a system, regardless of the provided information, its completeness, vagueness or precision.

Fundamentally, the conditional has two parts corresponding to facts given (input), referred to as the *premise or antecedent part*, and the resulting experience (output), referred to as the *consequent part*. The premise and consequent parts are described in linguistic terms, which are quantified in fuzzy terms according to the description embedded in the definition of the membership-function. The number of terms in each part decides the fuzzy *If-Then rule*, as detailed below [159].

Its simplest class is single-input single-output (SISO), which is described as:

$$If\ x\ is\ A\ Then\ y\ is\ B$$

When many inputs are included in the expression, this is referred to as multiple-input single-output (MISO) [159]:

$$If\ \ x_1\ \ is\ A_1^j\ \ and\ \ x_2\ is\ A_2^k\ and\ , \dots, and\ x_n\ \ is\ \ A_n^l\ \ Then\ y_q\ \ is\ B_q^p$$

The third class is multiple-input multiple-output (MIMO), which takes the form:

$$If\ \ x_1\ \ is\ A_1^j\ \ \ and\ \ x_2\ \ is\ A_2^k\ \ and\ , \dots, and\ x_n\ \ is\ \ A_n^l\ \ Then\ y_1\ \ is\ B_1^r\ \ and\ y_2\ \ is\ B_2^s$$

The MIMO structure can be interchangeably written in the form of a MISO structure. The above if-then rule can be written as two separate rules:

$$If\ \ x_1\ \ is\ A_1^j\ \ and\ \ x_2\ \ is\ A_2^k\ \ and\ , \dots, and\ x_n\ \ is\ \ A_n^l\ \ Then\ y_1\ \ is\ B_1^r$$

$$If\ \ x_1\ \ is\ A_1^j\ \ and\ \ x_2\ \ is\ A_2^k\ \ and\ , \dots, and\ x_n\ \ is\ \ A_n^l\ \ Then\ y_2\ \ is\ B_2^s$$

The fourth class, single-input multiple-output (SIMO), is not used since it can be constructed from a group of SISO statements.

An alternative way to model the fuzzy rulebase system depends on the type of parameters in the antecedent part and the consequent part [161]. That is:

- **Mamdani rules:** Where the antecedent is a conjunction of fuzzy memberships and the consequent part is a fuzzy set [3]. For example:

$$If\ \text{“}age\ is\ young\text{”}\ and\ \text{“}car\ is\ high\ power\text{”}\ then\ \text{“}risk\ is\ high\text{”}$$

- **Sugeno rules:** Where the antecedent is a conjunction of fuzzy memberships, and the consequent part is a (crisp) real-valued function of the input variables [3]. For example:

$$If\ \text{“}age\ is\ young\text{”}\ and\ \text{“}car\ power\ is\ high\text{”}\ Then\ \text{“}risk = w_0 + w_1 * age + w_2 * power\text{”}$$

  where w_0, w_1 and w_2 are real values.

Although Mamdani's rules are easier to interpret, many modelling systems use a Sugeno rule system since it is easier to adjust and adapt to the data. Both systems have been used extensively in modelling and control. Through the use of linguistic labels and membership functions, a fuzzy if-then rule can capture the spirit of a 'rule of thumb' used in human decision-making. From another point of view, due to the qualifiers on the premise parts, each fuzzy if-then rule can be viewed as a local description of the system under consideration [3].

4.5 Fuzzy Rules Inferencing

The inferences from fuzzy rules are typically achieved through the following sequential steps, as shown in Figure 4.11:

1. **Fuzzification** of crisp inputs to fuzzy values via assigning them a degree of fulfilment to corresponding fuzzy sets according to the underlying membership functions
2. **Inference** decision drawn from the rulebase using fuzzy implications, i.e., fuzzy-set-based operators.
3. **Defuzzification** of the output to real crisp values, using approaches such as centre of gravity (COG) and mean of maxima (MOM).

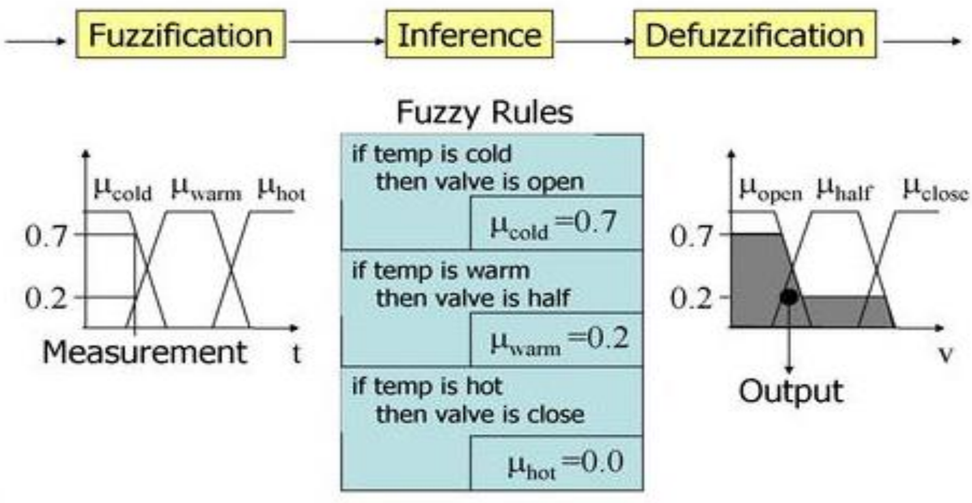

Figure 4.11 Fuzzy inference and its three main components.

4.6 Defuzzification methods

Many defuzzification methods are used in the literature. We shall mention here the best known, as described in [162].

Centre of Gravity (COG)

A very popular principle, despite its high computational cost, alternatively referred to as *centroid of the area* [163]. Generally it is expressed by the following two models, for the discrete and continuous versions, respectively:

$$u = \frac{\sum_i \mu(x) x_i}{\sum_i \mu(x_i)} \qquad (4.15)$$

$$u = \frac{\int x_i \mu(x) dx}{\int \mu(x) dx} \qquad (4.16)$$

When the membership-function is a singleton, this model can be interpolated to:

$$u = \frac{\sum_i \mu(s_i) S_i}{\sum_i \mu(s_i)} \qquad (4.17)$$

where S_i represents the location of i in U, while $\mu(s_i)$ represents the *firing strength* α_i of rule i.

Bisector of Area (BOA)

As its name suggests, the BOA chooses the x coordinate of the perpendicular line, which divides the region into two equal regions:

$$\left\{ x \;\middle|\; \int_{Max}^{x} \mu(x) dx = \int_{x}^{Min} \mu(x) dx \right\} \qquad (4.18)$$

where *Min* and *Max* represent the leftmost and the rightmost boundaries of U. The performance of such a method is not guaranteed in the case of a singleton membership-function.

Centre of Average (COA)

This method returns the crisp value, which belongs to the average of maximums of the individual centres of areas for each output membership-function:

$$y_q^{Crisp} = \frac{\sum_{i=1}^{R} b_i^q \sup_{y_q} \{\mu B_q^i(y_q)\}}{\sum_{i=1}^{R} \sup_{y_q} \{\mu B_q^i(y_q)\}} \tag{4.19}$$

where $sup\{ \mu(x) \} = hight\{ \mu(x) \}$

Max Criterion

This method returns the crisp value, which represents the maximum among all centres of areas in the output function:

$$y_q^{Crisp} \in \left\{ arg \sup_{y_q} \{\mu B_q(y_q)\} \right\} \tag{4.20}$$

where $arg \sup_x \{\mu(x)\} = \max\{ \sup_x \{\mu(x)\}$

Mean of Maxima (MOM)

This method returns the average of all objects when their membership degrees reach the maximum. The method disregards the shape of the fuzzy set, but the computational complexity is relatively good.

$$y = \frac{\sum_{i=1}^{k} \mu_i(x)}{k} \tag{4.21}$$

Leftmost Maximum (LM), and Rightmost Maximum (RM)

This method does nothing more than choose one of the maximum left or right limits of the output membership-function. Hence, it requires little computational power. The method resembles the hard-limiting functions, which is a necessary action in applications that use obstacle avoidance algorithms. Formally they are represented by the following models.

$$y_{RM} = \frac{\sup(x')}{\mu(x')} = \sup_{x \in [\mathrm{Min,Max}]} \mu(x) \tag{4.22}$$

$$y_{LM} = \frac{\inf(x')}{\mu(x')} = \sup_{x \in [\mathrm{Min,Max}]} \mu(x) \tag{4.23}$$

4.7 Semantics and Measurement of Fuzzy Sets

The firm mathematical foundations of the fuzzy set theory have enabled it to enter the realm of almost every engineering discipline and application [146], as shown in Figure 4.12.

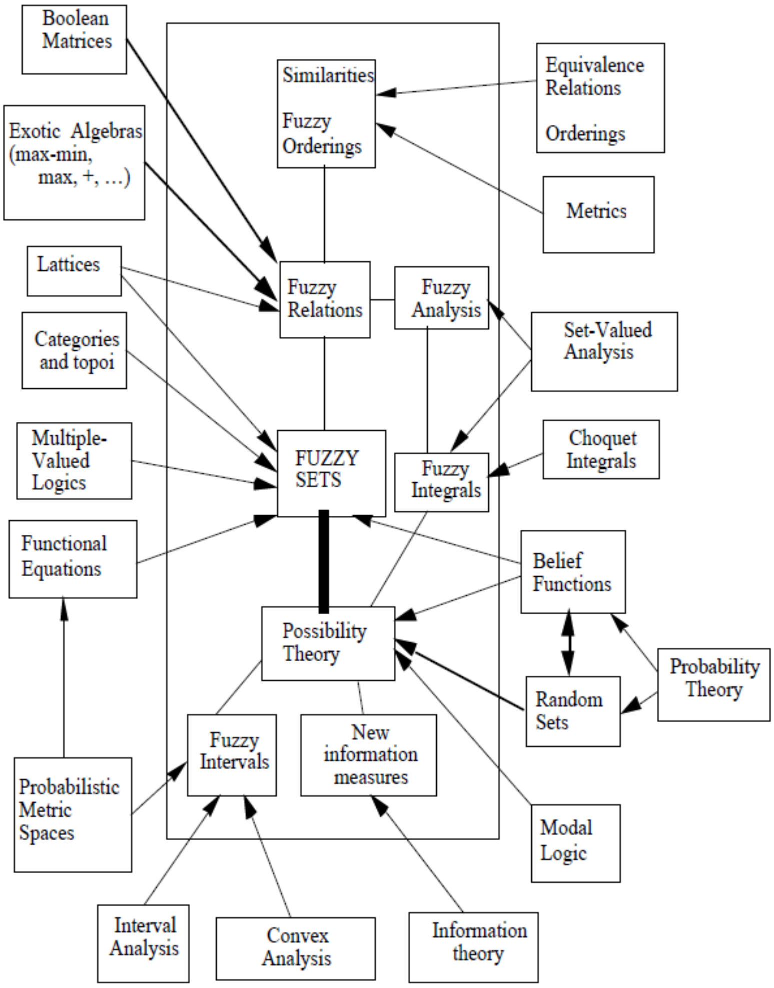

Figure 4.12 Mathematical environment of fuzzy set theory [146].

It seems necessary to question the semantics (interpretations) of fuzzy sets in order to properly understand the fuzzy set tools, especially reasoning and inferencing. These are carried out via the implication and norm operators, where each implication and norm results in a different meaning, depending on how it was defined (see fuzzy implications, t-norm, and t-conorm).

4.7.1 Meaning of membership grade

A membership grade reflects, *inter alia*, the following measures [160]:

- **Degree of Similarity:** Describes the degree of closeness of an element to some ideal element
- **Degree of Preference:** Quantifies the decision of selecting an object with respect to a certain object
- **Degree of Uncertainty:** Interprets the degree of plausibility for a certain object to obtain a certain grade.

4.7.2 Measuring membership grades

Many approaches can be used for evaluating membership-degrees [160]:

Ordinal approach

The membership-degree is outlined on an ordered-relations, such as ordering relation $\geq F$ [158]. For example, a statement like : $a > F(b)\ or\ a >_F\ b\ \Leftrightarrow \mu(a)_F > \mu(b)_F$, describes that "$a$ is more compatible with F than b". In such case the fuzzy set F is represented by the pair $\{\ (supp(x), >_F)\ \ \forall\ x \in X, where\ \mu(x) \neq 0\ \}$

Cardinal approach

The membership grade is a number, which is defined by operations on this number.

These operations are commonly one of the following alternatives [160]:

- **Distance**: When a membership function is interpreted in terms of similarity to referential set U is equipped with a distance d, then $F(u)$ can be computed by means of a decreasing function of the distance $d(u,u^*)$ of u to the prototype element u^* of F [160].

- **Frequency**: The degree of membership *F(u)* can be computed as the proportion of observations that do not rule out the situation *u*. Then the membership function can be interpreted as a *plausibility* function, *F(u)* =1 meaning that *u* is ruled out by no observation [160].
- **Cost**: *F(u)* may reflect the toll *u* would prefer for *u* being a prototype of *F*. Then, the set of membership grades is no longer the unit-interval but the non-negative reals, where 0 corresponds to full membership and non-membership is described by +∞ [160].

4.8 Constructing Fuzzy Rules (Fuzzy Rule Learning)

Fuzzy models may be built from human expertise and knowledge of the system under investigation. As systems have grown more complex it has become increasingly difficult to construct models directly from the domain knowledge of the system. This is due not only to the complexity of interactions within the system, but perhaps also an incomplete knowledge of the system operations. A fuzzy model can be used to provide a functional approximation of the relationships of the underlying system. Recently, many learning algorithms have been investigated to construct fuzzy models by developing fuzzy rules through analysis of training data. Generally, they may be classified to four major classes, as defined by Berthold and Hand [149]:

4.8.1 Constructive method

Followers of this method generally try to finding fuzzy rules by 'growing' them from singletons and then seeking to add more patterns to the same class, or 'shrink' from a general partition to a more specific space by avoiding conflicts. For example, the group of techniques used jointly in forming rules from samples, also known as Free Rules Formation algorithms, used in [96], [164]–[172].

4.8.2 Grid method

This method is based on global partitioning via predefining membership functions of the space, followed by an attempt to adapt the boundaries between different grid cells, even merging the grid cells to smaller partitions if no points are covered in the predicted cell. Fuzzy associative memory (FAM) [147] and the Higgins & Goodman Algorithm [173] are examples.

4.8.3 Adaptive method

Here, the rules are randomly initialised (or via partial expert knowledge), then one tries to optimise the rule parameters (e.g., location of a membership function over universe of discourse, number of membership functions) iteratively, using (many) well-known algorithms such as gradient descent [96] and the heuristic hill climbing algorithm [174].

4.8.4 Neuro-fuzzy method

This method injects fuzzy rules into a neural network that has a structure to hold and resemble the settings of the fuzzy rules system. Then the method employs training algorithms to fine-tune the fuzzy rule parameters. Attention needs to be given to the 'curse of dimensionality' problem.[175].

4.9 Rulebase Simplification

There are many existing simplification approaches, which are categorised in [175]:

- **Features reduction**, where the simplification is achieved via pre-processing the original training data, by reducing the number of variables and infusing them into machine learning tools
- **Similarity merging and inconsistency reduction**, which merges similar fuzzy-rules, excludes redundancy and unstimulated fuzzy-rules
- **Orthogonal transformation reduction**, which uses the matrix computation to minimise the fuzzy rules. Usually, this process uses the orthogonal least square (OLS) or singular value decomposition (SVD) principles
- **Interpolative reasoning reduction** simplifies the fuzzy rules via rejecting the rules, which are approximatable by their neighbours; also provides smart inferencing to the scattered fuzzy rules
- **Hierarchical reasoning reduction** modifies the structure of the conventional fuzzy rules models and reduces the 'curse of dimensionality'.

Choosing the proper rulebase simplification method for the problem in hand requires consideration of the following issues [175]:

- When to apply rule simplification? These methods can take place before, within or after the rule-inferencing process. Choosing 'before' will require pre-processing of the data, via feature selection or feature transformation; 'within' will require the simplification to be integrated into the training schemes; and 'after' will require the simplification to compact the fuzzy rules

- Preservation of the semantic meaning of the fuzzy rules. In keeping fuzzy linguistic definitions for the fuzzy rules, it is crucial not to destroy the understandability and completeness of the fuzzy rules
- Finally, to avoid generating a sparse rulebase. The reduction methods may result in no overlapping between rule antecedents for certain observations, and this will sometimes cause the fuzzy inference to have no rule to fire.

4.10 Fuzzy Arithmetic

Fuzzy arithmetic is yet another powerful set of tools which can help in modelling and solving engineering problems with uncertain parameters. To do so, the uncertainties in the model are expressed by fuzzy numbers, i.e. a fuzzy set defined on the real universe **R**, and the problem is solved by fuzzy arithmetic, which is a generalisation to fuzzy numbers of mathematical operators such as addition and multiplication.

4.10.1 Fuzzy numbers

The concept of fuzzy number splays a major role in formulating quantitative fuzzy sets, i.e., concepts which contain terms such as *about, probably, more or less* and *around*. For any fuzzy set to be considered a fuzzy number, the following conditions must be satisfied [176]:

- Fuzzy set is defined on the real numbers R, as universe of discourse.
- Normalised: hgt = 1
- $\exists x \in R \mid \mu(x) = 1$; there exists at least one value x where the membership grade $\mu(x) = 1$, if only core = 1 (one peak point), then it is said to be the modal value.
- Monotonic: convex and continuous.

Figure 4.13 below gives two examples of fuzzy numbers: (a) the triangular fuzzy number, *About* $\bar{x}$*;* and (b) the Gaussian fuzzy number, *About* $\bar{x}$. Fuzzy arithmetic is based on two properties of fuzzy numbers:

- Each fuzzy number can fully be represented by its α–cuts.
- α–cuts of each fuzzy number are closed intervals of real numbers $\forall\alpha \in [0,1]$.

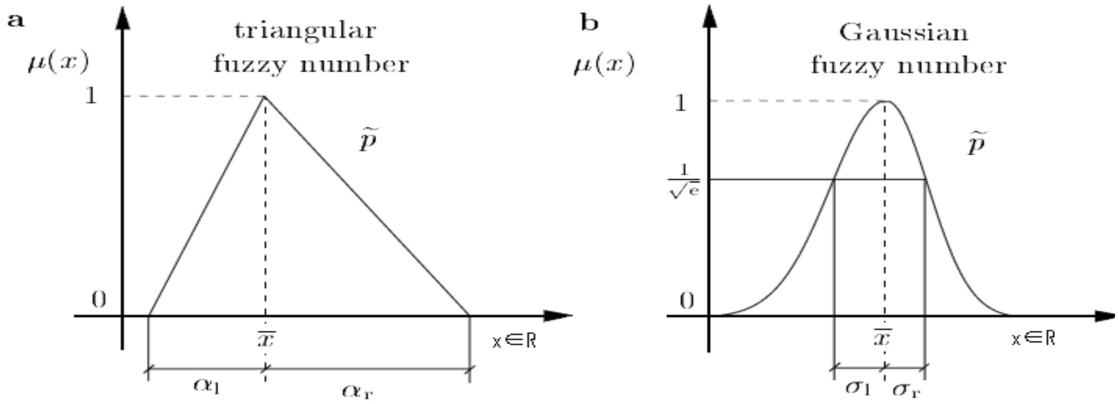

Figure 4.13 Examples of fuzzy numbers.

4.10.2 Fuzzy arithmetic operations

The abovementioned two properties of fuzzy numbers enabled us to define arithmetic operations in terms of arithmetic operations on fuzzy number α-cuts. There are two methods for implementation of fuzzy arithmetic operations, these methods differ in the way fuzzy numbers are represented.

- The first is known as L-R fuzzy number representation. The number is characterised by an ascending left and descending right, based on $X = (m, \alpha, \beta)_{LR}$. Where m is the mean value of X, α and β are left and right spreads, respectively, when m = 0, the number is the crisp value, the larger m the more fuzzy is X. This concept was proposed and studied by Dubois and Prade [151].

- The second is based on subdividing the membership degree axis into a number of equally spaced segments (discretising) [176], as shown in Figure 4.14.

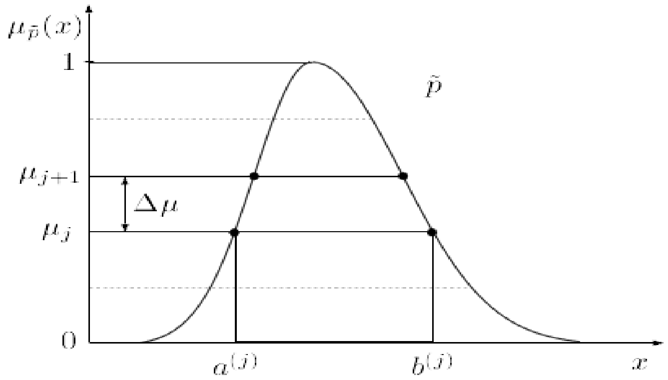

Figure 4.14 Concept of discretising a fuzzy number.

A fuzzy number considered as being approximated by a discrete fuzzy number established the basis of Zadeh's extension principle to the fuzzy arithmetic. This approach has been investigated by many researchers [150], [153], [159], [176]–[181]. It may be considered as a decomposition into intervals given by the *α–cuts* at the *α–levels* μ_{α}; where it can also be reduced to interval arithmetic to produce fuzzy arithmetic operations. The later concept was studied by Kaufman and Gupta [24].

For simplicity, only the general formula based on the extension principle of fuzzy set is mentioned here. This version is adapted from Hanss [176], who contributed heavily to fuzzy arithmetic applications. The arithmetic operation *E* on any two fuzzy sets *A* and *B* yields a fuzzy set *C*, which is defined thus:

$$\mu_c(z) = \sup_{z=E(x_1,x_2)} min\{\mu_A(x_1), \mu_B(x_2)\} \qquad \forall x_1, x_2 \in \mathcal{R} \tag{4.24}$$

where *A* and *B* are fuzzy numbers, with arbitrary membership functions $\mu_A(x_1)$ and $\mu_B(x_2)$, respectively. *E* is any of the elementary arithmetic operations (+, –, *, /).

4.11 Fuzzy Graphs

Fuzzy graphs are a promising field where fuzzy modelling theory has merged with classical graph data structure. Zadeh [172] described the primary function of a fuzzy graph is to serve as a representation of an imprecisely defined dependency. A graph G is defined as follows:

$$G = (V, E) \tag{4.25}$$

where V represents the set of vertices and E represents the set of edges, such that the edge E is a pair (x, y) of vertices in V.

A fuzzy graph is a data structure expressing the relation $R \subseteq V \times V$. Where the ordered pair *(x, y)* is defined with direction, it is called a directed graph. Alternatively, if order is not allowed, then it is said to be undirected-graph [166]. A path from *x* to *y* consists of the set of edges $a_1, a_2, \ldots$, and the edges are continuous, that is, $(x, a_1), (a_1, a_2), (a_2, a_3), \ldots, (a_n, y)$. Then *length of path* represents the number of edges along this path. Nodes a and b in G are said to be *connected* when there is a path between a and b in G. If there is a connection $\forall\, a, b \in V$ in G, then this graph is said to be a *connected graph*. Generally, any directed, connected fuzzy graph is referred to as a *network* [166].

We can think of set V as a fuzzy set. In this case, we say this graph represents a fuzzy relation of fuzzy nodes, where a membership-function of (x, y) is associated to each edge element E, and/or *vertex* V [162]. The fuzzy graph is an expression of fuzzy relations and frequently expressed as a *fuzzy matrix* [182]. See Figure 4.15.

	b_1	b_2	b_3
a_1	0.5	1.0	0.0
a_2	0.0	0.0	0.5
a_3	1.0	1.0	0.0

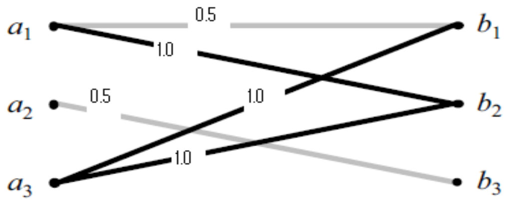

Figure 4.15 Fuzzy matrix to express the fuzzy graph.

The fuzzy matrix in Figure 4.15 can lead to the following new fuzzy sets in set B by set A.

- $\mu(a1) = \{(b1,0.5),(b2,1.0)\}$
- $\mu(a2) = \{(b3,0.5)\}$
- $\mu(a3) = \{(b1,1.0),(b2,1.0)\}$
- $\mu(a1,a2) = \{(b1,0.5),(b2,1.0),(b3,0.5)\}$

This fuzzy graph can be adapted in accordance with the results obtained using some fuzzy operations. For example, new edges (connections) may be found. This concept was investigated by McAllister [182] using the conjunction operation and projection processes. Berthold and Huber [166] pointed out an important application of fuzzy graphs, i.e., the automatic construction of fuzzy graphs from examples. "Through the concept of fuzzy graphs approximate representations of functions, contours, and sets can be derived" automatically, relaying on alpha-cuts, conjunction and disjunction operations extended to fuzzy graphs [166]. Karunambigai *et al.* [183] used fuzzy graphs in network applications, particularly to find the 'shortest path,' by representing a model based on dynamic programming.

4.12 Conclusion

This chapter provides an in-depth discussion of fuzzy set theory and how it has been used to build the FIS and its main components, and indeed leverage its advantages in positioning and position estimation problems. These advantages which are:

- Its parallel or distributed nature, which allows expression of a model of complex and nonlinear systems via a group of fuzzy rules
- Its linguistic capabilities, which allow expression of model of complex and nonlinear systems via a group of linguistic terms, interpreted as MFs. This feature capture human knowledge more easily than classical representation. The MF can also be modified using 'hedges,' which may add more precision and feasibility to the model.

In general, fuzzy set theory provides a robust model since it contains a group of rules in which a failure of one rule is not harmful to the whole system.

In the next chapter, these features are utilised to build a hybrid localisation tool for indoor WLAN environments.

5 HYBRID WLAN AND INDOOR POSITION ESTIMATION

5.1 Methodology Overview

Localisation and positioning systems have expanded quickly in response to the diverse demands of wireless networks and their location-dependent applications, as mentioned in Chapter 3. Indeed, the approach taken in location-based application development may differ in several respects, depending on: (i) the required type of location; (ii) the type of environment; (iii) type of network topology; and (iv) type of communication technology employed, cost constraints and the accepted level of uncertainty. In addition to the factors listed in (i) to (iv), the principles of estimation have also affected the development of location-dependent applications.

From the perspective of system vendors and application developers, choosing an appropriate localisation algorithm is subject to both the accuracy requirements and the costs associated with deploying an algorithm. Very often, accuracy requirements may be sacrificed for the sake of lower costs. In this regard, when the requirement for additional hardware is eliminated, the most desirable option among the many available localisation algorithms may be the fingerprinting technique.

The fingerprinting approach has two phases: (i) an offline phase, widely known as the training phase; and (ii) an online phase. The offline phase concentrates on building a localisation (fingerprints) map from the test environment. The online phase aims to produce an acceptable position for any unknown object by matching its observed RSS with surveyed references in the environment offline map. The accuracy of this technique remains under investigation. It is conditioned by the quality of the fingerprints map and the matching principle, both of which are subject to the uncertainties diffusing into the RSS during both phases. Many soft computing and machine-learning solutions have been introduced to overcome these uncertainties.

This chapter presents a fuzzy set theory-based algorithm in conjunction with the fingerprinting localisation algorithm. In particular, the Takagi-Sugeno multivariable FIS (TKS-FIS) is proposed to produce weighting factors for the fingerprints using a multi-nearest-neighbours (kNN) algorithm. This weighting mitigates the effect of uncertainties in the location estimation. In addition to the conventional multi-nearest neighbour algorithm, based on robust statistics, this chapter presents a novel outliering stage to cope with the failures of the kNN algorithm when fake neighbours are considered as genuine neighbour fingerprints.

5.2 Propagation model-based positioning

The wireless propagation model [184] explains signal behaviour over a particular path. Several factors affect wireless signal attenuation. These include fading and scattering characterised by spectrum properties, and factors characterised by the test environment, such as multipath effects from reflection, refraction and absorption due to physical characteristics of materials and random noise. These factors, in addition to unbiased errors arising from the measurement procedures, are broadly classified into three groups of signal variations (mentioned in Section 3.7): *small-scale, large-scale* and *temporal*. As a consequence, a universal propagation model that suits every environment and holds all parameters and factors to meet the required accuracy seems impossible.

Alternatives are restricted to formulations of the propagation model based on the experimental observations. Several models have been suggested in the literature. Our present focus is to pick a model that fits the intended experimental testbed. The settled model was one introduced in [136], as it was verified in a test environment very similar to the test environment proposed for this research. In this model, the signal power path loss (PL) indicator between emitted and received signal is interpreted as a function of the travelled direct path:

$$PL = -40 - 31log_{10}(d) \pm 8 \tag{5.1}$$

where PL is the path loss for the indoor environment in the 2.4 GHz frequency band, and is measured in dB; and d is the distance between emitter and receiver measured in metres (in this case the distance between AP and fingerprint). In the same vein as (5.1), [185] proposed an alternative model for indoor environments:

$$PL_{indoor} = R_0 - 10\alpha log_{10}(d) \pm \beta \tag{5.2}$$

where the values of R_0, α and β vary according to LOS and NLOS scenarios.

Considering both models, β in (5.2) and ±8 in (5.1) represent the uncertainty level which will take place in the test environment. The benefit of this uncertainty will clearly emerge when we extract the membership functions of the fuzzy system.

Figure 5.1 shows the plot of (5.2) with the addition of the maximum and minimum levels of 8 dB uncertainty (green and blue lines, respectively), and 0 dB uncertainty (red line). Based on empirical results by Oussalah *et al.* [4], α is set to be 2.1, which is appropriate for the tested indoor environment. The results of the plot (confirmed in experiments conducted at local offices with inner walls of gypsum board) show that about 70 dB of power is lost in the first metre, and that the maximum distance a 2.4 GHz system (with range of 80 dB and zero gain antennas) is able to achieve is about 20 metres [4].

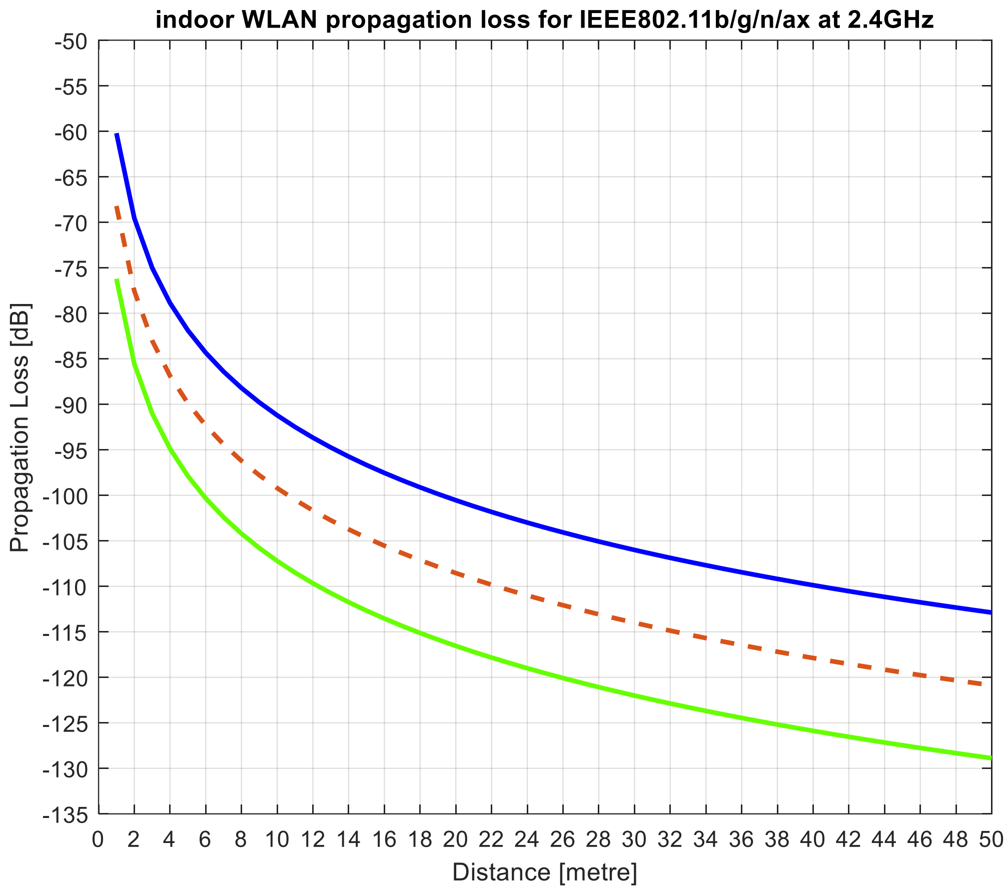

Figure 5.1 Radio propagation model [4].

5.3 RSS and Fingerprint-based Positioning

The fingerprint and RSS-based localisation approaches rely on estimating the target location from samples of the fingerprint database. The database construction takes place offline, where the measured RSS may be correlated to true ground locations prior to the deployment of the localisation algorithm. Alternatively, the true ground locations may be correlated to the RSS depending on the pre-described propagation model. This step is formally known as the offline stage, which results in tuples of RSS and their true ground correlated locations (coordinates). The coordinates are measured using available hardware devices within the network; the location estimation functions when at least three non-collinear fixed reference fingerprint points are detected.

RSS-based APs are propagation-loss equations which measure RSS values to build a signal strength map in a local area. The map may be generated using any method to measure the distance between RSS devices. A radio propagation model with positioning algorithm is always used to determine the target object position according to the RSS map [186]. Typically, the RSS values are within the interval [-40 dB, -95 dB] [3]. The industry standard defines RSS values within 256 intervals [64].

5.4 Multi-Nearest-Neighbour (kNN) algorithm

A basic nearest-neighbour algorithm is a distance-based classification algorithm first proposed by Cover and Hart [187] in 1967. They used a rulebase to classify any unknown observation to the nearest set of previously classified observations. Such rules are usually independent of the distribution of the collected sample points; thus, the error of such classification can be at least as significant as the Bayes probability. Later, this algorithm went through many enhancements, particularly with the inclusion of multiple nearest points in the distribution space and the selection of various distance functions.

The focus here is to illustrate the essential operation of the multi-nearest-neighbour (kNN) algorithm with its primary Euclidian distance function, regardless of the availability of distance functions such as the Hamming and Minkowski distance [188]. The Euclidian distance function is defined as:

$$D(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

where x is the input vector from the known data, y is the measured observation from the target to be classified, and n is the number of samples in the application. In our case, given a set of m fingerprints FP_i $(i = 1\ to\ m)$ whose data comprise the n-dimensional RSS vectors $(RSS_{i1}\ \ RSS_{i2} \dots RSS_{in})$ $(i = 1\ to\ m)$. So, given an anonymous target T whose observed

RSS vector is $(RSS_{T1}\ \ RSS_{T2} \ldots RSST_n)$, then the agreement between the i^{th} fingerprint and T can be calculated using the Euclidean distance function:

$$d_{iT} = \sqrt{\sum_{j=1}^{n} \left(RSS_{ij} - RSS_{Tj}\right)^2} \qquad (5.3)$$

Eq. 5.3 seems logical and has been favourably implemented in various indoor localisation methods, including the RADAR system [115]. However, in the event of dropping AP, which frequently happens in real applications when the signal from an AP is sharply attenuated or blocked due to a change in testbed environment, then (5.3) will produce an unpredictable result. Another choice for (5.3) can be useful if the average of the distances is calculated, thus:

$$d_{iT} = \sqrt{\frac{1}{n'} \sum_{j=1}^{n'} (RSS_{ij} - RSS_{Tj})^2} \qquad (5.4)$$

where n' is the entire number of APs detected by both the unknown target and the i^{th} fingerprint ($n' = n$ when complete coverage of all APs is detected).

Accordingly, various options may be distinguished to estimate the location of the T. They include:

- Correlating T to the jth fingerprint that returns the shortest distance:

  $$T(x, y) = AP_k(x, y) \text{ such that } d_{kT} = \min_j d_{jT} \qquad (5.5)$$

  This option agrees with the proximity approach, where each target is associated with the base station it is interacting with, and they are said to have the same location [14]. The option produces a piece of symbolic relative location information; mostly, RFID applies this localisation principle. The CID and COO employed by GSM networks are currently applying this principle too.

- Applying the triangulation and lateration principles to calculate the location of T, after turning the RSS from the signal space to a Euclidean (physical) distance, which is commonly done in some radio propagation models [143]. Briefly, considering the presence of observations from n emitters (in this case, APs), then (5.4) can be converted to this system of equations:

$$\begin{cases} \left(x_{AP_1} - x_T\right)^2 + \left(y_{AP_1} - y_T\right)^2 = R_{1T}^2 \\ \left(x_{AP_2} - x_T\right)^2 + \left(y_{AP_2} - y_T\right)^2 = R_{2T}^2 \\ \qquad\qquad\qquad . \\ \left(x_{AP_n} - x_T\right)^2 + \left(y_{AP_n} - y_T\right)^2 = R_{nT}^2 \end{cases} \tag{5.6}$$

where $R_{1T}, R_{2T}, \dots, R_{nT}$ represents the Euclidean distances from T to $AP_1, AP_2, \dots, AP_n$, respectively. The propagation model is used to infer RiT for each RSS. $\left(x_{AP_i} - x_T\right)$ represents the x,y coordinates of each APi. Several techniques can be proposed to find a solution to (5.6), such as nonlinear least square analysis. This procedure does not use the fingerprint database clearly; however, some scholars recommend using such knowledge in the wireless propagation when transforming the RSS to Euclidean distances, as in [7] and [129].

- Applying the kNN algorithm. In this regard, in correlating *T* with the fingerprint that produces the shortest distance in the fingerprint map, this condition may be loosened to consider the *k-shortest distances*. In such cases, the location is the mean of all candidate *k-nearest fingerprints*:

$$x_T = \frac{1}{k}\sum_{i=1}^{k} x_{FP_{\sigma(i)}} \quad , \quad y_T = \frac{1}{k}\sum_{i=1}^{k} y_{FP_{\sigma(i)}} \tag{5.7}$$

where xT and yT represent the estimated x,y coordinates of T, k is number of nearest-fingerprints, and $(x_{FP_{\sigma(i)}}, y_{FP_{\sigma(i)}})$ corresponds to the x,y coordinates of each candidate nearest-fingerprints, extracted from the fingerprints map. The RADAR system uses this strategy [115]. Such a principle triggers the aforementioned signal variations inherited in the RSS of the WLAN-indoor environments. As highlighted in [115], “the error vector (in physical space) corresponding to each neighbour is oriented in a different direction.” Averaging the coordinates of neighbours may enhance the accuracy of the target positioning.

Furthermore, to cope with another accumulation of signal variations, a weighted-average technique may be used to enhance system accuracy [7]:

$$x_T = \frac{1}{k}\sum_{i=1}^{k} w_i x_{FP_{\sigma(i)}} \quad , \quad y_T = \frac{1}{k}\sum_{i=1}^{k} w_i y_{FP_{\sigma(i)}} \qquad (5.8)$$

The weights wi are such that $\sum_{i=1}^{k} w_i = 1$. The modest form of the weighted-kNN is composed by determining the weights relative to the nearest fingerprint, although alternative and more advanced weighting principles, such as those that rely on fuzzy set theory, may be introduced [4].

- Applying soft computing methodologies [51], which include neural network learning principles, support vector machines and FISs [42], [64], [189], [190]. These learning and reasoning algorithms rely on the generalisation capabilities of the underlying processes. For instance, the fingerprints from the database obtained throughout in offline phase may be utilised to train the neural network and obtain an optimal weights matrix. Then the targets' RSS vectors are applied to the neural network inputs to estimate the location by discovering the optimal output from the neural network [185].

The extensive usage of fingerprinting localisation technique does not avoid remarkable dependability problems, which deserve further study. They include the:

- **Frequency dependency**. This arises from the uncertainties imposed on the RSS observations during the offline and online stages, due to the dependence on public band frequencies in the deployment of most indoor WLANs.
- **Application dependency**. The deployment of localisation algorithms usually takes place in response to LBS requests in occupied premises of dynamic indoor WLANs. The moving objects in the test environment alter the RSS due to the resultant

reflections, refractions and absorptions. This alteration may impact the RSS positively because of the created multipath, or negatively due to the occurrence of NLOS cases or increased attenuation.

- **Algorithm dependency**. The localisation algorithm comprises many sub-processes such as database construction, measurement principle, nearest fingerprints selection, and the calculating and weighting principles. Failure in any of the associated sub-processes may lead to poor performance for the final location estimation process and, therefore, failure of the whole algorithm.

5.5 Fuzzy Inference Combined kNN algorithm

To estimate the target position, either mobile or at a fixed location, in any indoor wireless environment, typically the process begins by observing the RSS powers for the target object. In our example, the APs or the fingerprints are set as references and the mobile node as a target. However, the RSS provide a coarse view of the target position in the test environment; they cannot be converted to exact and error-free distance due to the characteristics of the signal in the indoor environment, which directly affects the accuracy of a locating operation. Consequently, an algorithm which accurately estimates locations under unreliable or constrained RSS is beneficial for constructing a robust indoor positioning system. Initially, the proposed system implements the same technique adopted earlier, with the addition of a weighting principle. The weights of the corresponding k-nearest fingerprints are inferred from a TKS-FIS. Then, the final coordinates are computed by (5.8) as a weighted aggregate of most related fingerprints. The k-related fingerprints are extracted from the offline-generated fingerprints map, where the relation is defined as the shortest path to the target, as expressed in (5.4). Consequently, all distances among the target and all fingerprints in the database are computed; then, the k-closest fingerprint tuples are extracted. Each tuple consists of the ordered set FP_a, as expressed below:

$$FP_i = \{(RSS_{i1}, RSS_{i2}, \dots, RSS_{ij}), (x_i, y_i)\} \mid i \in \{1,2,\dots,k\}, j \in \{1,2,\dots,n\}$$

where *k* and *n* specify the number of nearest neighbours and the number of APs, respectively. Then RSS_{ij} denotes the RSS from the i^{th} fingerprint to the j^{th} to AP. (x_i, y_i) represents the x and y coordinates for the i^{th} fingerprint

This step aims to get vector $D =< D_1, D_2, \dots, D_i > \forall\, i \in \{1,2,\dots,k\}$, where D denotes the corresponding distance in the RSS space, and is computed as:

$$D_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(RSS_{ij} - RSS_{Tj})^2} \qquad (5.9)$$

Among the two central fuzzy inference systems; mentioned in Chapter 4, the adopted version for this work will be the TKS-FIS. The main feature of TKS fuzzy models is that they characterise the local dynamics of each fuzzy rule with a linear model. Moreover, TKS has a proven computational capability and continuous coverage to the output surface [191].

Figure 5.2 illustrates the main blocks of single-input TKS-FIS for location estimation.

This *D* vector is infused into the TKS-FIS to obtain appropriate weighting factors. The higher

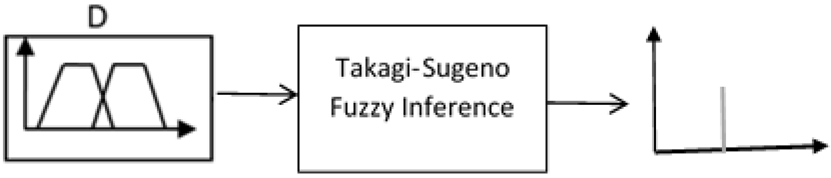

Figure 5.2 Single input TKS-FIS block diagram.

the weight, the higher the association and closer the range. The lower the weight, the weaker the association and further the range. These statements are interpreted in the TKS-FIS by a group of fuzzy *if-then* rules:

$If\ D_i\ is\ Very\ Small\ THEN\ weight\ of\ i^{th}\ fingerprint\ is\ Very\ High$

$If\ D_i\ is\ High\ \ THEN\ weight\ of\ i^{th}\ fingerprint\ is\ Very\ Low.$

The above linguistic qualifications are generated via a fuzzification process where the (crisp) inputs are converted into fuzzy sets. The fuzzy sets are characterised by their membership functions, which describe the shapes. Typically, simple parameterised models (e.g., Gaussian, triangular, trapezoidal, S-shape) are used in the literature. In our model, trapezoid membership functions are employed. The assignment of a specific (fuzzy) linguistic quantifier to the distance (in signal space) output depends on its numerical value [4], as shown in Figure 5.3.

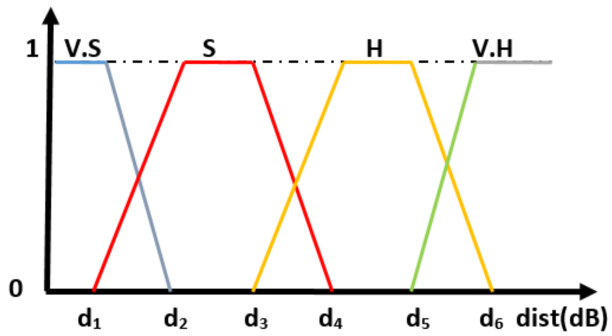

Figure 5.3 Fuzzification of input variable D.

The following rules should hold during the qualification of the fuzzy variables:

$If\ d \leq d2, then\ d\ (in\ dB) is\ classified\ as\ Very\ Small\ (VS)$

$If\ d1 \leq d \leq d4, then\ d\ is\ classified\ as\ Small\ (S)$

$If\ d_3 \leq d \leq d_6, then\ d\ is\ classified\ as\ High\ \ (H)$

$If\ \ d \geq d_5, then\ d\ is\ classified\ as\ Very\ High\ (VH)$

The decision of the limits of the membership functions d_i in Figure 5.3 follows several logical principles [4]:

1. The purity of the calculation
2. Consideration of natural and analytical features of the RSS
3. That the definitions of the membership function should hold
4. Confirming that a sufficient number of fuzzy rules are stimulated.

Indeed, the *core* and *support* of the membership function may be interpreted as the extent of the interval where the exact boundary of distance in signal space will possibly and certainly lie, respectively. This assumption matches the random-set view, where the membership function is viewed as a nested family of level α-cuts [4]. In the same direction, affirming for membership function μ_S, that $\mu_S(d) = 0.7$ is equivalent to claiming that in 70% of observed cases, the distance *d* is associated to the label *S* (small). Bilgiç and Türkşen [160] provide a more comprehensive perspective on membership function extraction, outlining five possible interpretations of the membership function and critically discussing each interpretation, as well as the constraints and elicitation principles of membership functions. As a part of the present research project, the candidate undertook a detailed study of the choice of membership functions, the defuzzification process, and the selection of appropriate t-norms[192].

From this perspective, use of our knowledge of statistics on RSS data allows us to build a bridge to the random-set interpretation of the fuzzy set. Indeed, both the uncertainty factor in radio propagation model expression (5.1) and the standard deviation of RSS values reported in other studies, (e.g., [160]) suggest that the *support* of any membership function defining the linguistic quantifiers cannot be smaller than such uncertainty.

This guarantees the fulfilment of claims 1–3 (above). On the other hand, the desire to provide a decent chance to enable multiple stimulations of fuzzy rules simulates a balanced allocation of the membership functions. By 'balanced allocation,' we mean guaranteeing fair coverage of membership functions to the universe of discourse, and not necessarily a uniform distribution. In our case, we refer to the distribution of the distance in the signal space, measured in dB, with respect to the membership-degree.

Claims 2 and 4 also force some restrictions on the simulation and testbed settings at a later time. Certainly, given that the RSS observations decline distinctly in the first metre or so (approximately 70 dB), compared with a smooth transition in the 1–50 m range, then, accordingly, we intentionally arranged the APs and fingerprints/target to be at least one metre apart in order to secure smooth coverage of the entire RSS pattern.

The distances in RSS space of d_1, d_2, d_3, d_4, d_5 and d_6 are chosen as 2, 5, 10, 15, 20 and 25 dB, respectively. The weight variable in the conclusion part of the fuzzy rules is designed as a numerical constant. Strictly speaking, the TKS-FIS forces the conclusion part of the fuzzy rule to be a non-fuzzy linear function, or constant as in the case of the zero-order Sugeno model [191]. In other words, the linguistic terms *Very Small*, *Small*, and so on, if employed in the conclusion part of the rule should have a crisp interpretation.

Table 5.1 shows the overall set of fuzzy rules employed in our fuzzy system.

Table 5.1 Fuzzy rules of single variable TKS-FIS [4].

Linguistic value	V.H	H	S	V.S
Crisp output	.05	.3	.7	1.0

For instance, some fuzzy rules can be interpreted as [4]:

$If\ D\ is\ Very\ Small\ \ then\ Weight\ is\ equal\ to\ 1$

$If\ D\ is\ Small\ then\ Weight\ is\ equal\ to\ 0.7$

$If\ D\ is\ Very\ High\ \ then\ Weight\ is\ 0.05$

The operation of the TKS-FIS can be split into the following steps [162]:

1. Fuzzification step, which converts each crisp input into a fuzzy set.
2. Inferencing the antecedents for all fuzzy rules.
3. Aggregation, which aggregates the antecedents of all fuzzy rules.
4. Defuzzification step, which converts the aggregated antecedents into a crisp value.

Lastly, the final coordinates are obtained using (5.8).

5.6 Enhancement to the Fuzzy Inference via Dual-input Approach

The new enhanced version suggests infusing the TKS-FIS with two inputs. The first input is the same as the previous section. In addition, a second input *V*, represents the *variations* of RSSs between the test-point and the nearest-neighbour associated with each emitter in the test environment. This should reflect another means for correlation between the selected nearest fingerprints—the less variation, the more spatially correlation between selected fingerprints—and thus reduce the chances for wrongly selecting a fingerprint far from the target location. Figure 5.4 shows the enhanced version of the TKS-FIS with dual inputs.

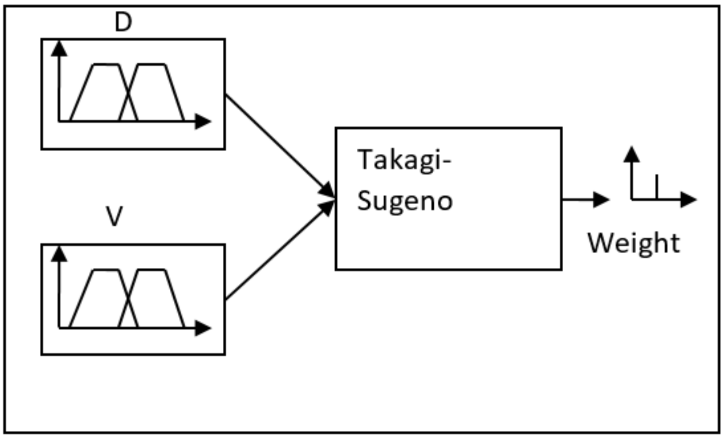

Figure 5.4 Block diagram of TKS-FIS with dual inputs [4].

More formally, the input variables D_j and V_i are expressed respectively as:

$$D_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(RSS_{ij} - RSS_{Tj})^2} \tag{5.10}$$

$$V_i = \left|\left(\max_j RSS_{ij} - \min_j RSS_{ij}\right) - \left(\max_j RSS_{Tj} - \min_j RSS_{Tj}\right)\right| \tag{5.11}$$

$$i \in \{\sigma(1), \sigma(2), \dots, \sigma(k)\}$$

The motivation for attaching the other indicator (V_i) arose from the assumption that the signal space path length is not necessarily presenting enough knowledge to identify various topologies. For example, a set of sparse points on the surface of a sphere have the measures the same distance to the centre as the grouped set of points. Rationally, attaching the variations indicator can provide information about the internal structure of the environment, which can be very fruitful, especially when the relative localisation principle is applied. Traditionally, the kNN algorithm makes a blind selection for fingerprints, regardless of their topology.

Again, quantifying the logical relation between the {D,V} set and their weight association is achieved via a group of fuzzy rules, for example:

$If\ D_i\ is\ Very\ Small\ AND\ V_i\ is\ Very\ Small\ \ THEN\ weight\ of\ i^{th}\ fingerprint\ is\ Very\ High$

$If\ D_i\ is\ High\ AND\ V_i\ is\ High\ THEN\ weight\ of\ i^{th}\ fingerprint\ is\ Very\ Low.$

$If\ D_i\ is\ High\ AND\ V_i\ is\ Small\ THEN\ weight\ of\ i^{th}\ fingerprint\ is\ Very\ Low.$

As in Section 5.5, a fuzzification step using trapezoidal member functions is initiated to obtain the fuzzy set for each of the above linguistic terms, as shown in Figure 5.3, but in this time for the dual-input case. We refer to this system as *multi-variable fuzzy localisation*. Table 5.2 briefs the overall set of fuzzy rules employed in our system.

Table 5.2 Fuzzy rules of dual-input TKS [4].

V \ D	V.H	H	S	V.S
V.H	0	0	.3	.7
H	0	0	.4	.8
S	.05	.1	.5	.9
V.S	.05	.3	.7	1

Since each fuzzy input (D, V) is denoted by the following linguistic fuzzy terms *{Very High, High, Small, Very Small}*, then a 4×4 rules matrix is generated.

Some examples of the fuzzy rules can be interpreted as [4]:

$If\ D\ is\ Very\ Small\ and\ V\ is\ Very\ Small\ then\ Weight\ is\ equal\ to$ 1.

$If\ D\ is\ Very\ Small\ and\ V\ is\ Small\ then\ Weight\ is\ equal\ to$ 0.9.

$If\ D\ is\ Small\ and\ V\ is\ Very\ Small\ then\ Weight\ is\ equal\ \ to$ 0.7.

$If\ D\ is\ High\ and\ V\ is\ High\ then\ Weight\ is\ zero$.

The continuous coverage property associated with the choice of membership functions in Figure 5.3 for TKS-FIS ensures that at least one rule is stimulated for every set of the inputs, and multiple stimulations for the fuzzy rules. This reveals the possibility of obtaining more than one answer from the same set of input data.

As in Section 5.5, the operation of the TKS-FIS is typically split into four functional operations, for example, given the observations of variables D and V, then two rules are possibly activated:

$Rule_1$: $If\ D\ is\ A_1\ and\ V\ is\ B_1\ Then\ W\ is\ C_1$.

$Rule_2$: $If\ D\ is\ A_2\ and\ V\ is\ B_2\ Then\ W\ is\ C_2$.

where A_1, B_1, A_2, B_2 stand for any of *VS, S, H, VH* fuzzy sets and C_1, C_2 represent any two weights in {0.01, 0.1, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9, 1}.

The two crisp constants *d* and *v* of the inputs *D* and *V* are fuzzified as fuzzy singletons. Using the intersection operation of membership functions with fuzzy singletons, the firing strengths are calculated:

$$\mu_{firing,1} = h_1 = \mu_{A_1}(x) \wedge \mu_{B1}(y)$$

$$\mu_{firing,2} = h_2 = \mu_{A_2}(x) \wedge \mu_{B_2}(y) \tag{5.12}$$

where the operator ∧ (AND) can be achieved using the min-norm or any alternative T-norm [162]. The rule firing strength is commonly referred to as premise membership grade or validity index at the obtained crisp constants.

Assuming E_1 as the consequence of $rule_1$ (C_1) and E_2 as the consequence of $rule_2$ (C_2) for particular observations, x and y of D and V, the final validity index of $rule_1$ and $rule_2$ is computed as:

$$E = \frac{E_1 h_1 + E_2 h_2}{h_1 + h_2} \tag{5.13}$$

Regarding defuzzification, numerous defuzzification tools are available in the fuzzy logic literature [193]. Two widely-used tools are mentioned here: the centre of area (CoA) and the centre average (CA). In the case of TKS-FIS, the defuzzification is performed by the weighted average as in (5.13). Figure 5.5 shows a diagram of this tool.

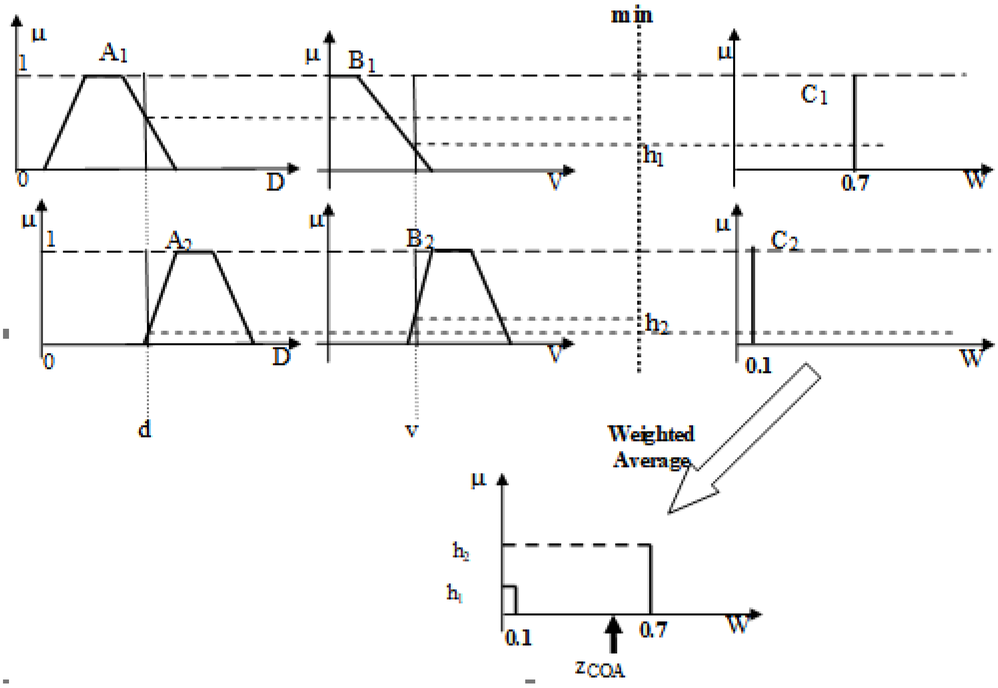

Figure 5.5 Schematic of TKF-FIS defuzzification [4].

Considering (5.7) and (5.8), the final coordinates are therefore calculated based on the coordinates of the nearest fingerprints as:

$$x_T = \begin{cases} \frac{\sum_{i=1}^{k} w_i x_{FP_{\sigma(i)}}}{\sum_{i=1}^{k} w_i}, & \text{if} \sum_{i=1}^{k} w_i \neq 0 \\ \frac{1}{k} \sum_{i=1}^{k} x_{FP_{\sigma(i)}}, & \text{otherwise} \end{cases} \tag{5.14}$$

$$y_T = \begin{cases} \frac{\sum_{i=1}^{k} w_i y_{FP_{\sigma(i)}}}{\sum_{i=1}^{k} w_i}, & \text{if} \sum_{i=1}^{k} w_i \neq 0 \\ \frac{1}{k}\sum_{i=1}^{k} y_{FP_{\sigma(i)}}, & \text{otherwise} \end{cases} \quad (5.15)$$

By analysing the performance of TKS with dual inputs V and D, we can extract the following assumptions, which are discussed in detail in [3] and [4]:

- **Assumption 1:** The result of the TKS-FIS with dual inputs can be the same as the ith nearest-fingerprint when $max(D_i, V_i) \leq d_1$ and $min(D_j, V_j) \geq d_6$, $\forall j = 1 \ldots k, j \neq i$.

  The discussion of this assumption is based on the two following items of evidence:

  - The constraint max(Di,Vi) ≤ d1 requires that both Di, and Vi are evaluated as Very Small, therefore stimulation of a unique fuzzy rule has occurred, which results in the highest weight=1 to the ith nearest-fingerprint as shown in Table 5.2
  - The constraint $min(D_j, V_j) \geq d_6, \forall j = 1 \ldots k, j \neq i$ requires that all remaining k-fingerprints are interpreted as Very High for D and V; in this case, the uniqueness of fuzzy rule stimulation results in weight=0 for all nearest fingerprints as shown in Table 5.2.

  Therefore, when (5.14) and (5.15) are implemented, the outcomes will agree with this assumption.

- **Assumption 2:** The result of the TKS-FIS with dual inputs can be the same as the conventional kNN when $min(D_j, V_j) \geq d_6$, $\forall j = 1 \ldots k$.

  The discussion of this assumption is based on the same logic as the first assumption, where constraint $min(D_j, V_j) \geq d_6, \forall j = 1 \ldots k$ requires that all k-fingerprints are interpreted as High for D and V; in this case, the uniqueness of fuzzy rule stimulation results in weight=0 for all nearest fingerprints, as shown in Table 5.2.

Therefore, when (5.14) and (5.15) are implemented the outcomes will agree with (5.7).

- **Assumption 3:** The result of the TKS-FIS with dual inputs will be the same as the conventional kNN when $max(D_i, V_i) \leq d_1$ for all k-nearest fingerprints (i = 1 ... k). The discussion of this assumption is based on the same logic as the earlier two assumptions, except that constraint $max(D_i, V_i) \leq d_1 \; \forall i = 1 \dots k$ requires that all k-fingerprints are interpreted as Very Small for D and V; in this case, the weight=1 for all nearest fingerprints, as shown in Table 5.2.

5.7 Enhancement via Robust Statistics and Outliering Algorithm

As mentioned in Sections 5.4, 5.5 and 5.6, the localisation applications for indoor environments lack a generalised efficiency. Accordingly, the analysis for each localisation principle should take place for each individual application. Then, by analysing the outcomes of the aforementioned positioning strategies that rely on the kNN algorithm, it was noted that the accuracy of kNN-based strategies is highly dependent on the quality of the picked nearest fingerprints. This is especially so when the selection methodology of the nearest fingerprints is done blindly with respect to the test environment. This means that, relying on the RSS as a lone indicator for the position is not sufficient to infer the target location, even after the addition of the signal-variation principle, as in Section 5.6. The main conclusion we have drawn after the verification of estimated locations, and the selected nearest fingerprints against the targets' actual locations, which was based on a statistical analysis, was that the kNN-based algorithms fail to pick a high-quality neighbour fingerprint with a non-negligible range of 16–19%. This equates to approximately one wrong selection for every five or six nearest fingerprints, so the likelihood of achieving a good location is reduced by almost 20%.

Therefore, an augmentation process was proposed to overcome this limitation. The proposed outliering principle is based on a sound improvisational precept from a robust statistics background [194]. The intention was to eliminate some of the wrongly-selected nearest neighbours using a distance-based triangulation mode. "The fingerprints selected by the kNN algorithm are considered only if the nearest fingerprints tend to establish a triangle of the smallest area" [4].

The area can be calculated by using either of the following techniques [195]:

- The vertices of the desired triangle, where the vertices are fetched from the fingerprints-map, thus:

$$\boldsymbol{Area} = \left|\frac{\boldsymbol{P1_x(P2_y - P3_y) + P2_x(P1_y - P3_y) + P3_x(P1_y - P2_y)}}{\boldsymbol{2}}\right| \quad (5.16)$$

  where P1, P2, P3 represent the vertices of the proposed nearest neighbours in terms of x and y coordinates.

- The sides of the desired triangle, as in Heron's formula [195], where the sides can be calculated depending on the side lengths, which can be obtained from the physical space or signal space; the advantage of the second method is its applicability for relative or absolute positionings.

$$Area = \sqrt{\omega(\omega - l_1)(\omega - l_2)(\omega - l_3)} \quad (5.17)$$

  where ω can be obtained using: $\omega = \frac{l_1 + l_2 + l_3}{2}$ (5.18)

  where l1, l2, l3 represent the side-lengths in physical-space or signal-space.

This process is repeated for every combination of k-nearest fingerprints. Formally, let $r = 3$, represent the number of vertices, and k is the number of available nearest fingerprints. Then the n combinations can be calculated as:

$$n = \binom{k}{r} = \frac{k!}{r!(k-r)!} \quad (5.19)$$

After calculating all areas for every *r* triple in the *n*-combinations space, the following claims are evaluated to consider the triple *r* valid nearest fingerprint, and ensure the optimal accuracy:

- $Area\{r_i\} \neq 0 \; \forall i = 1 \dots k$; this assumption ensures that all fingerprints in *r* are not collinear.
- $Area\{r_i\} < \theta \; \forall i = 1 \dots k$, this assumption ensures that all nearest fingerprints in *r* are spatially related where the threshold θ can be set depending on the environment setup plan.
- $Area\{r_i\} \geq \alpha \; \forall i = 1 \dots k$, this assumption ensures that all nearest fingerprints in *r* satisfy specific accuracy requirements where the threshold α can be set depending on application accuracy requirements.

At this point, the algorithm may be enhanced further by setting upper and lower thresholds for the area, which can be selected by analysing the database for the created fingerprints map. This enhancement step among other properties of the fingerprint map construction was detailed and discussed in some other works, for instance, Location Fixing and Fingerprint Matching (LFFM) for fingerprint map construction for indoor spaces [196].

6 EXPERIMENT AND SIMULATION

6.1 Introduction

This chapter describes the main scenarios and working platforms used to evaluate the proposed algorithms. The evaluation was based on simulations and experiments. The simulation scenarios based on the algorithm considered in Chapter 5 are:

- Localisation using fuzzy inference with single input, particularly the Euclidean distances between target nodes and fingerprints to achieve a weighted kNN.
- Localisation using fuzzy inference with two inputs, namely, the Euclidean distances between target nodes and fingerprints, and the signal variations between the fingerprints and the Aps.
- With augmentation to the multi-nearest-neighbour algorithm, to outlier the wrongly-selected nearest fingerprints due to the uncertainties pervading the RSS in the indoor environment.

The experimental section was limited to two experiments. The first focused on validation of the proposed multivariable fuzzy inference localisation (MVFL), by comparing the accuracy of the MVFL with alternative approaches such the traditional kNN, weighted kNN and lateration. The second experiment aimed to validate the accuracy on a different testbed and using another vendor's tools available in the market of wireless sensor networks and IoTs.

6.2 Dual Input v. Single Variable TKF-FIS and kNN Localisation

6.2.1 Simulation

To evaluate the localisation algorithm a simulation testbed was constructed to form a square region of approximately 400 m^2, where the vertices of the square represent the emitter sources; as is common in indoor WLAN environments, the emitters were stationary APs. Then the fingerprints map was formed using the model outlined in (5.1). The following constraints were imposed on the testbed environment: (i) securing a minimum distance between reference fingerprints not to be less than two metres; and (ii) constraining the testbed to produce symmetrical distribution for all reference fingerprints. These constraints were imposed to imitate the experimental test environment. To finalise the offline phase of the fingerprinting technique, a signature RSS vector was created for 64 reference fingerprints, given their x and y coordinates.

More formally, each fingerprint signature is represented by the following tuple:

$$\left\{RSS_j \ \forall\, j = 1 \dots a\right\}_i , \{x, y\}_i \ \ \forall\, i = 1 \dots n$$

where a and n represent the number of APs and the number of reference fingerprints, respectively. Then $\{RSS_j\}_i$ is the RSS for the i^{th} reference fingerprint, from the j^{th} AP. $\{x,y\}_i$ denotes the x and y coordinates of the i^{th} reference fingerprint . According to database design, a matrix of $n \times (a + 2)$ representation was used to store all fingerprint signatures. The constant 2 was used to hold the x and y coordinates for each reference fingerprint.

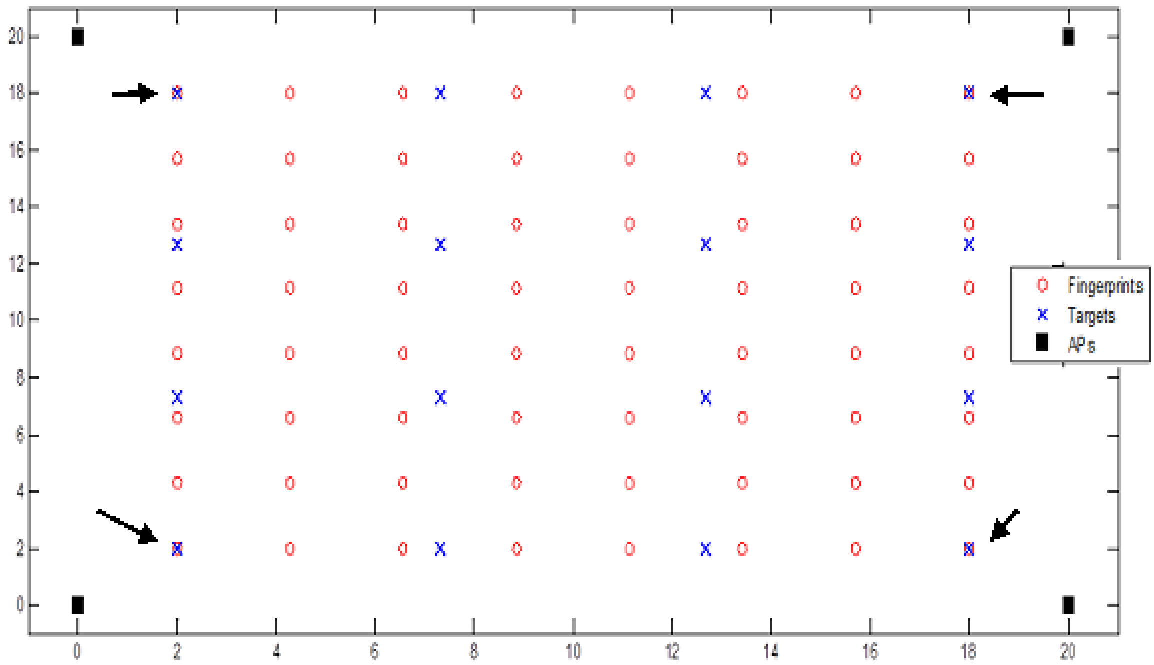

Figure 6.1 Simulation scenario for TKS-FIS using single-input and dual-input.

The same propagation model was re-used to produce a group of 16 test fingerprints. The same constraints imposed over reference fingerprints were valid here; except that the distance constraint was modified to approximately twice that of the reference fingerprints' internal distance. Moreover, some test fingerprints were forced to be concentric with some reference fingerprints, specifically, the four closest to the APs (the ones pointed by arrows in Figure 6.1). This constraint was carefully enforced to judge the behaviour of the algorithm under imperfect circumstances. The x and y coordinates of the test fingerprints were available only during the RSS production and the method verification steps while remaining protected during the other steps. Figure 6.1 shows the organisation of the reference fingerprints and APs in the simulation environment.

The following propagation formula was used during all steps of this simulation, which emulates the signal distribution pattern in the WLAN indoor space:

$$RSS_{ij} = -40 - 31log_{10}(d_{ij}) \pm \omega \tag{6.1}$$

where the symbols are as described in (5.1) and (5.2). The irregular uncertainty offset ω is proposed to estimate the 8 dB limited uncertainty factor, as in (5.1) [4].

Throughout the online phase, the k-nearest-fingerprints were extracted from the fingerprints map for every test fingerprint. Then, using the TKS-FIS described in Section 5.6, the $\{D, V\}$ tuple was infused into the proposed TKS-FIS to obtain the optimal weights, which were then used in the location estimation step with formulas (5.14) and (5.15). Figure 6.2 compiles the principal actions for the algorithm.

Algorithm 1 Fingerprint algorithm using Dual input TKS-FIS

Offline Stage :-

Create FP coordinates (MAP)
for i=1 : Nnumber of APs **do**
  Fetch the x-y coordinates for AP_i
  for j=1 : Nnumber of FPs **do**
    Fetch x-y coordinates for FP_j
    Compute the Distance d_{ij}
    Generate the RSS_{ij}
    Associate RSS_{ij} with FP_j
    Store FP tuple in the fingerprint-MAP
  end for
end for

Online Stage :-

  load FP Map
for j=1 : Nnumber of Targets **do**
  Calculate the k nearest FPs to $target_j$
  for i=1: k **do**
    Calculate V_i of $target_j$
    Calculate D_i of $target_j$
    infuse V_i and D_i to $TS - FIS$
    Collect w_i for FP_i
  end for
  Estimate location of $target_j$
  Calculate Estimation Error of $target_j$
end for

Figure 6.2 Fingerprinting with dual-input TKS-FIS algorithm.

Given knowledge of the actual position of the target from the user's perspective, the performance of the developed fuzzy positioning system can be evaluated using a standard root mean square error (RMSE) metric; namely:

$$Error = \sqrt{(X_{\mathrm{E}} - X_{T_{act}})^2 + \ (Y_{\mathrm{E}} - Y_{T_{act}})^2} \qquad (6.2)$$

where $\boldsymbol{X_{T_{Act}}}$ and $\boldsymbol{Y_{T_{Act}}}$ are the actual coordinates of target *T*.

Given the actual *x* and *y* coordinates for the test fingerprints $X_{T_{act}}$ and $Y_{T_{act}}$, this algorithm was evaluated against the estimated coordinates, X_E and Y_E, using the RMSE metric [197]. The outcomes were examined in several other localisation approaches: kNN infused with single-input TKS-FIS (Section 5.5) and [189]; conventional kNN; weighted kNN; and lateration. The comparisons are discussed briefly in the following sub-section.

6.2.2 Simulation results

To obtain a pictorial representation of the estimated locations for the test targets, we initially plotted (Figure 6.3) the estimated locations for all 16 test fingerprints using the dual-input TKS-FIS (MVFL) on the testbed against the estimated locations using the actual locations and the estimated locations of an alternative positioning algorithm, specifically the conventional kNN algorithm. The plot demonstrates the higher success of the generated dual-input TKS-FIS (MVFL) algorithm, as demonstrated by the closeness of the estimated locations to the actual test fingerprints.

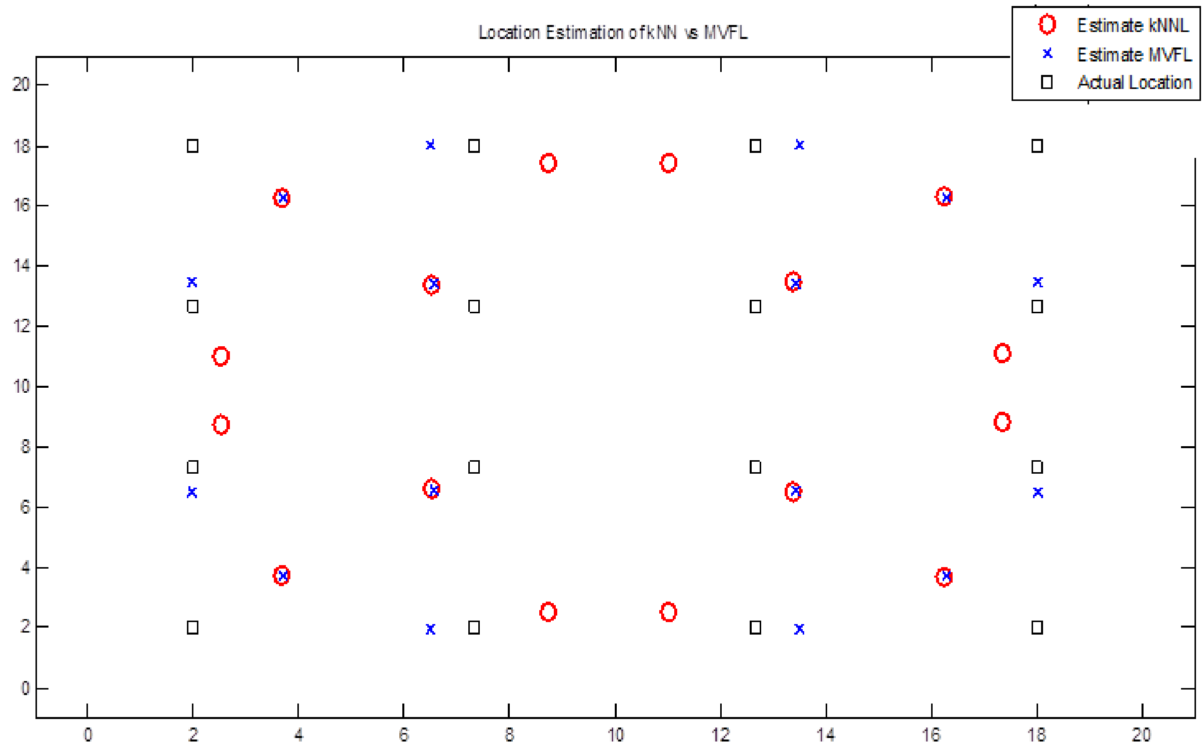

Figure 6.3 MVFL (X), test fingerprints: actual location (□); kNN estimation (O).

Considering the randomness factor ω in (6.1) and the calculation of various ω values in the range of 1–15 dB, the mean of 100 Monte Carlo simulations was calculated. This process was reproduced for the online phase, with respect to the same fingerprints map.

Figure 6.4 shows the accuracy of the dual-input TKS-FIS (MVFL) according to the RMSE along the y-axis, for each test fingerprint along the x-axis, versus well-known localisation algorithms (kNN, weighted kNN, single-input TKS-FIS and lateration).

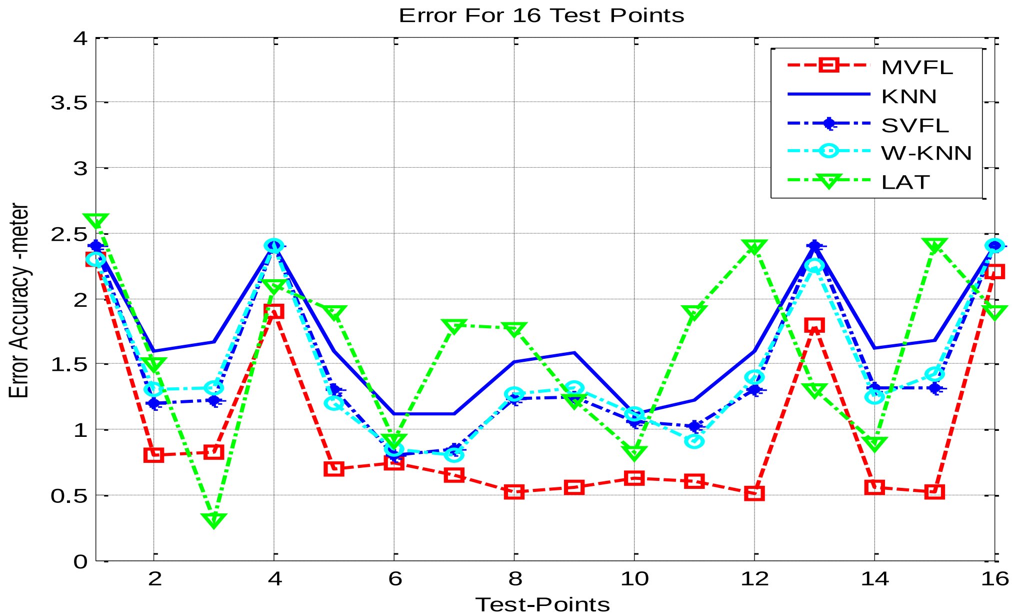

Figure 6.4 RMSE of MVFL versus other algorithms.

Despite the superior behaviour of the dual-input TKS-FIS in most circumstances over the other mentioned localisation algorithms, the performance of the dual-input TKS-FIS drops to match the single-input TKS-FIS and kNN in particular circumstances. For example, consider the *1st*, *4th*, *13th* and *16th* test fingerprints in Figure 6.4; these are the four test fingerprints collocated with the reference fingerprints (designated with arrows in Figure 6.1). This approach can be justified in light of the following discussion:

- The existence of any test fingerprint at the exact same location as the reference fingerprint does not require the exact same fingerprint signature in terms of acquired RSS due to the direct effect of the random noise implied in ω term of (6.1). However, if both points acquire the exact same RSS signature and the constraints of Assumption 1 in Section 5.6 are fully satisfied, then we may expect the algorithm to return the exact location as stored in the fingerprints map.
- Considering the fuzzification of *{D,V}* variables, and the possibility of interpreting them as *Very Small* resulting in a weight=1, then the constraints in Assumption 3 in Section 5.6 will hold, which returns the same result as the kNN version.

- The count of nearest fingerprints employed has a critical effect in determining the performance factor. In our simulation k was set to three neighbours, which was rational and showed acceptable performance in many scenarios. A decision might be made to drop this count to the minimum possible value, which is one nearest fingerprint. Of course, this requires that a minimum value of signal variations between the reference fingerprint and the test fingerprint should be detected. In this case, we expect the algorithm to pick the reference fingerprint that the test fingerprint is collocated with, as the nearest fingerprint. This course comes at a price—that of failures with the bulk of the other test fingerprints.

Later, we sought to confirm the effectiveness and robustness of the realised algorithm against sudden changes in the test environment, which can be reflected by the imposed noise intensity. To achieve this goal, the utilised propagation model (6.1) was reproduced to accommodate the change. For this purpose, the RSS value corresponding to the target fingerprint was modified to account for the noise intensity. This involved rewriting (6.1) as:

$$RSS_{iT} = -40 - 31log_{10}(d_{iT}) + \omega_{\sigma} \qquad (6.3)$$

where ω_{σ} represents a zero-mean Gaussian noise with variable standard deviation σ [4], and d_{iT} represents the physical path-length between the i^{th} AP and the test fingerprint.

In other words, this model was applied only to the test fingerprints and keeping the reference fingerprints to be generated using the model described in (6.1). Table 6.1 presents the mean RMSE for 16 test fingerprints.

Table 6.1 Dual-input TKK-FIS against other localisation algorithms' mean RMSE.

Noise level	***Mean RMSE for 16 test points***				
S dB	Dual-input TKS-FIS	Single-input TKS-FIS	W-kNN	LAT	kNN
1	0.64	0.88	0.86	0.51	1.32
2	0.89	1.41	1.47	1.60	1.69
3	0.89	1.42	1.48	1.65	1.94
4	0.93	1.43	1.53	1.71	1.98
5	0.94	1.44	1.56	1.88	2.11
6	0.95	1.47	1.61	1.93	2.14
7	0.97	1.48	1.62	1.90	2.16
8	0.98	1.48	1.66	1.91	2.21
9	0.98	1.51	1.68	2.13	2.22
10	0.99	1.52	1.71	2.42	2.27
11	1.02	1.53	1.73	2.53	2.30
12	1.03	1.57	1.74	2.52	2.37
13	1.04	1.58	1.75	2.71	2.41
14	1.04	1.62	1.78	2.77	2.44
15	1.06	1.63	1.82	2.69	2.51

The outcomes shown in Table 6.1 demonstrate the superiority of the MVFL solution over other well-known localisation algorithms. The outcomes follow a decreasing pattern in performance, which is acknowledged, against the increased noise intensity. But, interestingly, the results of MVFL based on TKS proposals have the least scattering pattern among all methods. To better understand this, Figure 6.5 presents the results of MVFL against the conventional kNN. The results reflect the confidence in solutions based on the fuzzy logic principles to exhibit more reliable behaviour than classical techniques, especially within the uncertainty constraints.

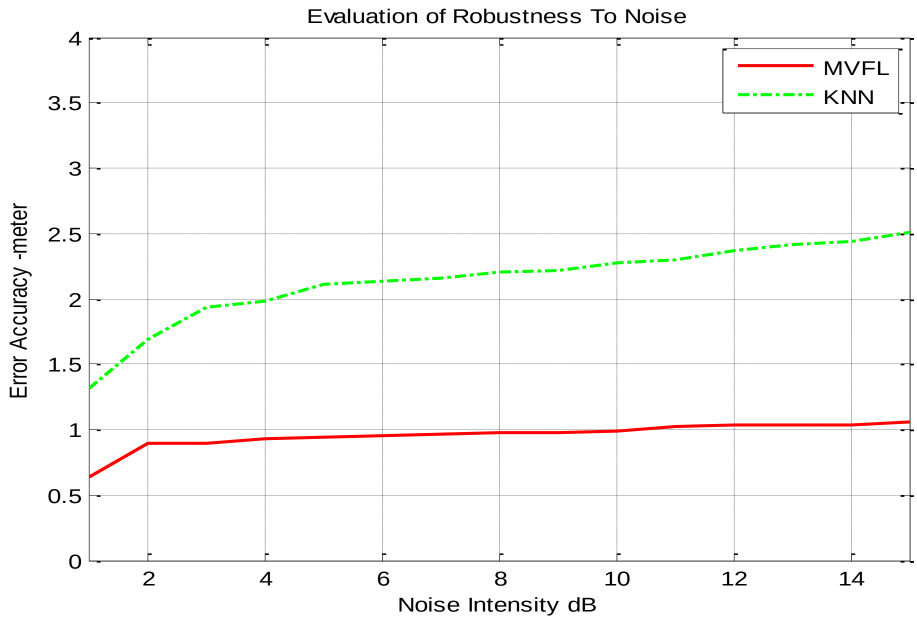

Figure 6.5 Mean RMSE plot for dual-input TKS-FIS v. kNN to express robustness.

6.3 Simulation of Dual-input TKS-FIS Localisation with Outliering Phase

In respect of the high proportion of wrongly-selected nearest fingerprints, noted in Section 5.7, we consider here the rationale behind the recommended resolution, which is the insertion of an outliering sub-process to reduce the chances for wrongly-selected fingerprints to participate largely in the location estimation. This section presents a simulation scenario to confirm the effectiveness of such a sub-process and examine its influence on the overall system behaviour. The augmented sub-process has been assessed by using the same simulation platform designed for the previous section.

According to the sequential steps listed in Figure 6.2, the insertion of the sub-process should take place during the online phase of the technique, exactly after the kNN selection step, while keeping the offline phase unchanged. This feature is of a tremendous advantage because, upon successful verification of the sub-process, it can be employed to any existing fingerprint-based localisation algorithm without the need for repeating the site survey to generate a new fingerprints database. The proposed pseudo-code for the augmented sub-process is detailed in Figure 6.6.

Algorithm 1 Outliering Algorithm

```
load FP Map
for j=1 : Nnumber of Targets  do
   Calculate the K nearest Neighbours FPs to target_j
   create S as 2K region space
   for i=1: S do
      Pick up three distinct fingerprints in S
      Calculate Area of triangle
   end for
   Select triple that yields the smallest Area
end for
```

Figure 6.6 Pseudo-code of outliering sub-process.

Against an average failure rate of 16% during implementation of the conventional nearest-fingerprinting technique, the proposed sub-process was strong enough to reduce the range of wrong estimates to approximately 5%, as shown in Figure 6.7, which gives the mean returns of 25 Monte Carlo simulations.

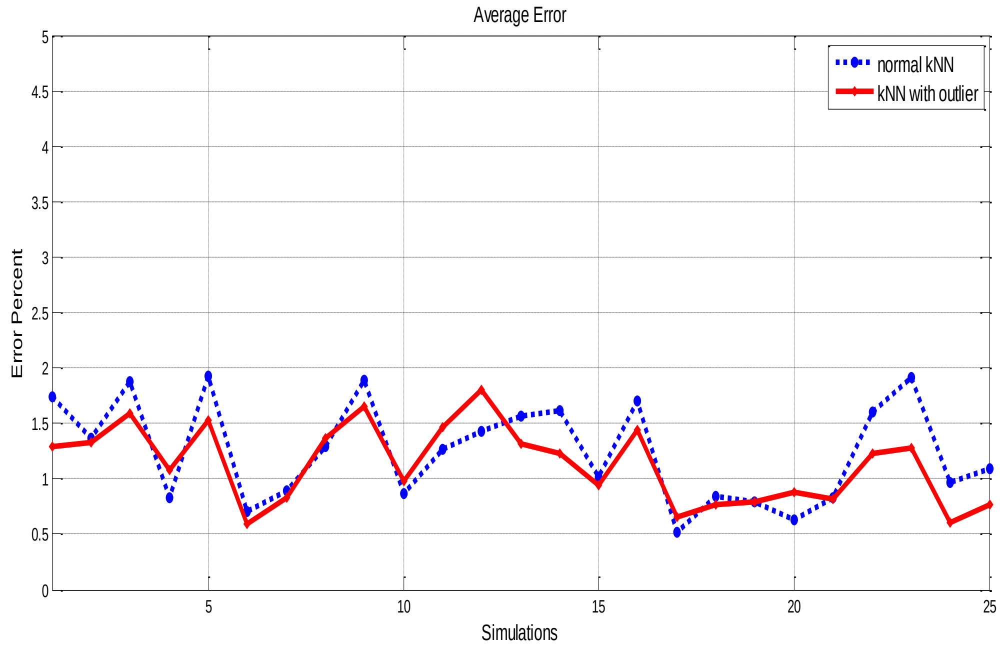

Figure 6.7 Conventional kNN v. Augmented kNN.

The application of nearest-fingerprint localisation based on area optimisation (according to the claims made in Section 5.7 with an outliering sub-process) furnishes an imperfect indication of the quality of the recently-captured nearest fingerprints. We note from Figure 6.7 that the enhanced nearest fingerprinting version may designate some true fingerprints as outlier fingerprints. This indicates the potentiality of the conventional kNN to outperform the enhanced version in some instances.

The effectiveness and robustness of the realised algorithm against sudden changes in the test environment is reflected in the imposed noise intensity or uncertainty to the RSS. Various levels of noise intensity in the range of [0 dB–15 dB] were applied to the system, and mean accuracy errors gathered (Table 6.2).

Table 6.2 Average error against noise intensities.

Noise S dB	*Average Accuracy Error (in metres)*	
	Dual-input TKS-FIS Conventional kNN	*Dual-input TKS-FIS Augmented kNN*
1	0.43	*0.53*
2	0.43	*0.58*
3	0.56	*0.58*
4	0.56	*0.61*
5	0.74	*0.62*
6	0.90	*0.65*
7	1.14	*0.69*
8	1.26	*0.71*
9	1.57	*0.98*
10	1.86	*1.18*
11	2.25	*1.41*
12	2.95	*1.60*
13	3.05	*1.69*
14	3.14	*1.60*
15	3.26	*1.90*

Considering the first four entries in Table 6.2, the conventional kNN responds to smaller noise intensities better than the augmented kNN. This shows that, in ideal cases, a smaller level of ambiguity embedded in the RSS is not very hazardous, where the RSS can still provide a sensible association with the locations. The TKS-FIS copes excellently. However, at higher noise intensities, the algorithm increases the chances of an ambiguous selection of nearest fingerprints—hence less precision is achieved in location estimation. This clarifies the more responsible behaviour of the augmented kNN, reflected in the better average accuracies in the remaining entries in Table 6.2.

While the results shown in the first four rows of Table 6.2 may be considered a flaw, they can be used to create an opportunity for prospective ideas to improve the representation of the augmented kNN, which can be done by examining the locations individually. A fundamental study on the presence of target fingerprints outside the fingerprint space is indicated. A core idea is to implement alternative measurements in area and distance. Another approach is to practise an alternative outliering algorithm for instance, to apply orientation-based outliering, especially if the supporting devices are equipped with a means of orienting such as a directional antenna.

6.4 Experiments

After the verification of the proposals using the simulation principles, two experiments were conducted to empirically verify some of the proposals.

6.4.1 Experiment 1

This experiment was arranged and set according to the floorplan shown in Figure 6.8. The experimental devices were:

- The four Transmitters were SparkLAN Multimode APs (WX-1590), as shown in Figure 6.9, and compatible with Wi-Fi standards. The transmitters were fitted to the inner part of the room ceiling, denoted by empty circles (o) in Figure 6.8.
- The receiver was a Compaq Pocket PC (iPAC-3970), as shown in Figure 6.10, equipped with a Wi-Fi LAN card (Lucenet Orinoco), which was used in the offline phase to capture the RSS of the fingerprints, and build the fingerprint map; the same receiver was used during the online phase to capture the RSS of the target fingerprints.

The testbed was an almost-square room of total area about 400 m^2, with some inner partitions to simulate the NLOS effect on some fingerprints.

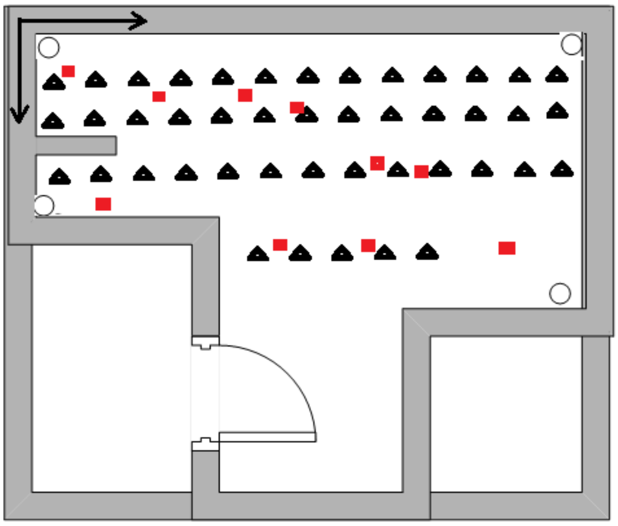

Figure 6.8 Testbed for Experiment 1.

Figure 6.9 SparkLAN (WX 1590)

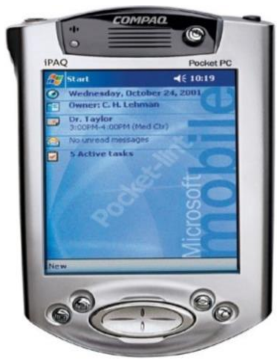

Figure 6.10 Compaq iPAC 3970.

A total of 44 points spaced no less 1.5 m apart were set as reference fingerprints and placed at the points denoted by the solid triangle in Figure 6.8.

To achieve more reliable fingerprint signatures, the RSS values were captured from each transmitter for a period of 5 seconds at every single reference fingerprint. The standard deviations for these obtained readings was computed and associated with the averaged RSS, along with the boundary conditions related to this mean RSS and the standard deviation. The boundary conditions considered in this case were limited to the number of people moving within the test environment at the moment of the RSS-capturing process, which can be expressed mathematically as:

$$RSS_{ij} = \frac{\sum_{x=1}^{B} \overline{RSS_{ij}}^{x} . \sigma_x}{\sum_{x=1}^{B} x} \qquad (6.3)$$

where *j* represents the transmitter *(AP)*, *i* represents the fingerprint, *B* represents the number of people present during this time span, $\overline{RSS_{ij}}^{x}$ represents the average *RSS* during the time span, and σ represents standard deviation during the time span.

The boundary condition assumption creates a more robust RSS and produces a more error-tolerant fingerprints map. As noted in earlier chapters, human bodies within the test environment affect the behaviour of signal propagation, regardless of the time and effort consumed to create the fingerprints-map database.

To finalise the offline phase; the accumulated RSS value is correlated with the physical *x*, *y* and *z* coordinates of the fingerprint to create the fingerprint map. The presence of a third dimension is due to the fixing of transmitters to the ceiling, whereas the fingerprints were on the floor.

To evaluate the various localisation techniques, a group of 10 test fingerprints were deployed, denoted by the solid squares in Figure 6.8. The average accuracies corresponding to each test fingerprint and each localisation technique are presented in Table 6.3.

Generally speaking, the comparisons in Table 6.3 and Table 6.4 clearly affirm the strong performance of dual-input TKS-FIS localisation. Nevertheless, some negative inferences may be drawn from the presented values. An average accuracy of 43 cm, reflected in a ratio of 1:3 against the inter-fingerprint spaces, is a respectable achievement. However, fully self-controlled robots or unmanned vehicles applying such a localisation technique would require the assistance of other sensing devices.

Table 6.3 Experiment 1 results.

Target	Mean Accuracy (in metres)				
	Dual-input TKS-FIS	**Single-input TKS-FIS**	**W-kNN**	**LAT**	**kNN**
1	0.43	0.72	0.83	0.62	1.21
2	0.46	0.61	0.71	1.52	0.74
3	0.21	0.42	0.34	1.44	0.88
4	0.46	0.53	0.77	0.39	1.43
5	0.34	0.61	0.71	0.94	1.41
6	0.28	0.54	0.63	1.33	0.94
7	0.82	1.12	1.23	1.10	2.36
8	0.17	0.48	0.46	1.08	0.91
9	0.23	0.51	0.38	0.93	0.84
10	0.93	1.55	1.31	1.02	2.74

Table 6.4 Experiment 1 relational results.

Localisation Technique	**Dual-input TKS-FIS**	**Single-input TKS-FIS**	**W-kNN**	**LAT**	**kNN**
Mean accuracy (m)	0.433	0.709	0.737	1.037	1.346
Relation to inter-fingerprint space (1.5 m)	29%	47%	49%	69%	90%
Relation to testbed space (20 m)	2%	4%	4%	5%	7%

6.4.2 Experiment 2

For the second experiment, Figure 6.11 shows the layout of the testbed, with a total of 22 fingerprints and 13 test points. Six Libelium Waspmote Wi-Fi modules (SMA5dBi 2.4 GHz Worldwide 802.11b/g) served as APs (Figure 6.12); these were installed at approximately 0.9 m above the floor level, at locations denoted by the red squares shown in Figure 6.11. We utilised a Meshlium scanner 802.15.4-PRO-AP (Figure 6.13) to capture *RSS* data from the Waspmotes. The testbed environment was a room about 12 m × 20 m, with inner storage area. It also contained some laboratory furniture and equipment. The average inter-fingerprints space was about 2 m.

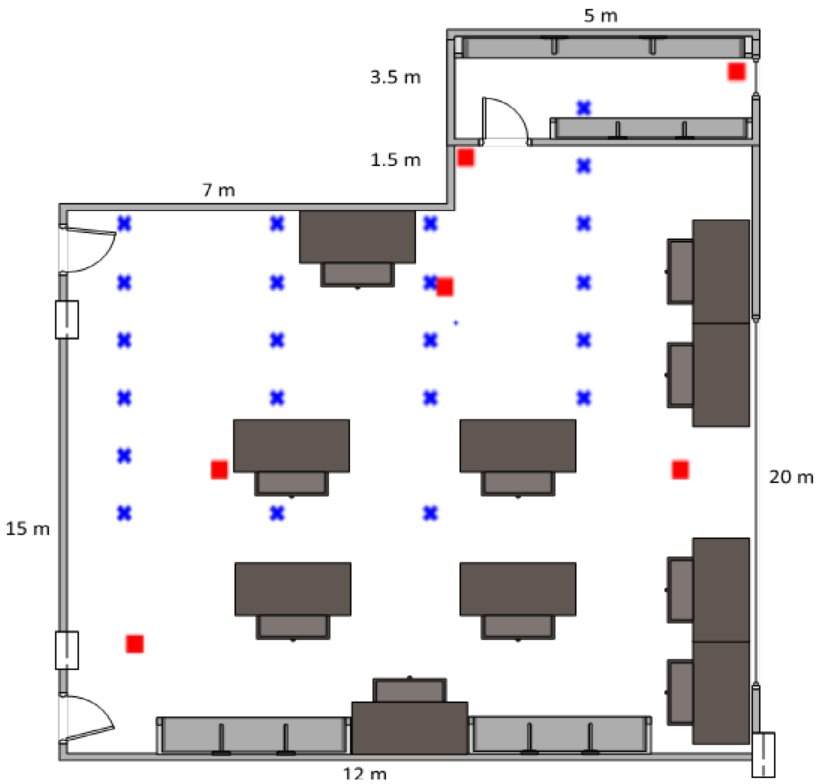

Figure 6.11 Testbed for Experiment 2.

Some constraints were added to the RSS by placing reference fingerprints in NLOS situations and blocking the *RSS* with walls and furniture. In addition, *RSS* values were captured during the daytime, with 2–4 people working in the laboratory.

As in the first experiment, the reference fingerprints were placed on the floor, at locations denoted by the blue **x** in Figure 6.11. In practice, this is to demonstrate that the algorithm works in a 3-dimensional space, with the APs and fingerprints at different elevations. However, unlike the first experiment, the RSS values here were only averaged over a time span of 3 seconds—this was to reduce the time and effort required for fingerprint collection (but at the cost of the quality of the collected RSS values and the quality of the fingerprint map).

Figure 6.12 Waspmote Wi-Fi SMA5

Figure 6.13 Meshlium scanner

To evaluate the various localisation techniques, we deployed a group of 13 test fingerprints, denoted by the empty blue circles (o) in Figure 6.14. The localisation techniques in this experiment were limited to the kNN and the weighted-kNN against the dual-input TKS-FIS or MVFL.

The accuracies corresponding to each test fingerprint and localisation technique are presented in Figure 6.15. And Figure 6.14 displays the plotting of the kNN estimates against the MVFL estimates on the testbed.

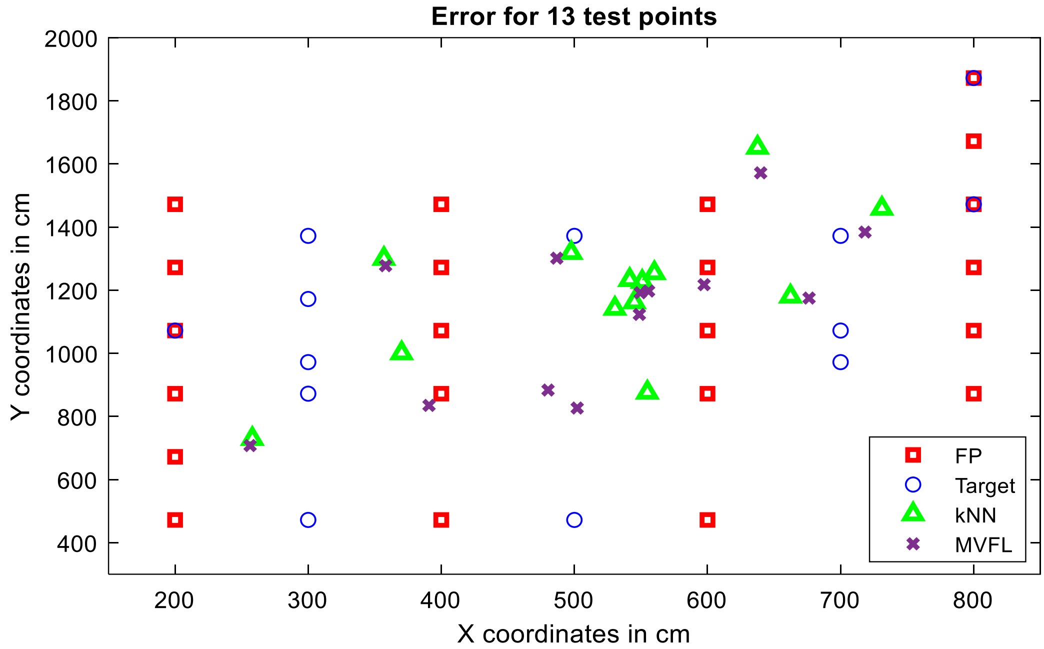

Figure 6.14 Actual locations versus estimated locations.

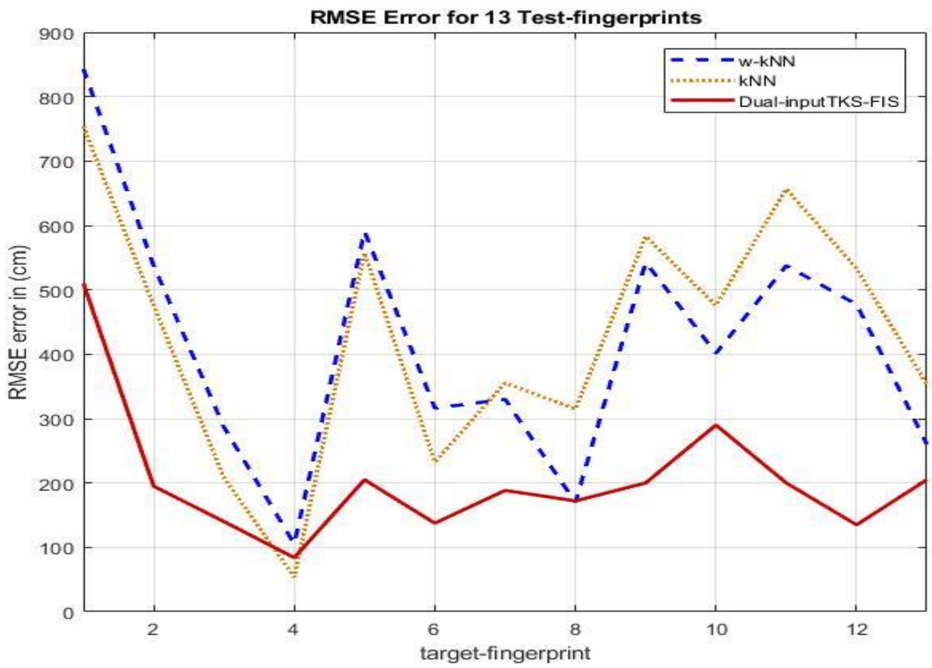

Figure 6.15 Error plot for (MVFL) dual-input TKS-FIS v. w-kNN and kNN

A test fingerprint was intentionally located inside the storage room, as shown in Figure 6.11, which has LOS to a single AP. The other test fingerprints were located on the floor to create another NLOS factor due to the presence of benches at various locations. This arrangement was expected to affect the behaviour of any RSS-based localisation algorithm.

According to Figure 6.15 and Table 6.5, the average achieved accuracy for the dual-input TKS-FIS algorithm was approximately 204 cm, despite the fact that one test fingerprint had less than a 71-cm accuracy and the majority of the others achieved accuracy around the inter-fingerprint distance. Again, these results proved that the dual-input TKS-FIS algorithm produces the best acceptable performance, confirming the feasibility of the developed localisation proposal.

Table 6.5 RMSE error in estimation for Experiment 2.

Test fingerprint	RMSE (in cm)		
	w-kNN	kNN	Dual-input TKS-FIS
T1	843	754	523
T2	537	475	190
T3	287	208	136
T4	105	54	72
T5	591	558	209
T6	316	232	152
T7	330	355	200
T8	170	315	160
T9	543	583	202
T10	401	476	290
T11	537	657	203
T12	477	533	124
T13	260	353	203
Avg.	**415**	**427**	**204**

6.5 Conclusion

This chapter covered an investigation of the proposed approach for indoor wireless localisation using dual-input TKS-FIS reasoning mechanisms referred to as MVFL. The suggestions included an enhanced version of the kNN fingerprinting algorithm via an extension with an outliering sub-process.

The primary role for the TKS-FIS was to generate weights for the two offered inputs and append these weights to the respective nearest fingerprint. The two inputs resemble the distance in signal space for each test fingerprint with respect to the transmitters, and the variation of RSSs between the target fingerprint and the nearest fingerprint. The evaluation was based on both empirical and simulation results. The outcomes were assessed against other well-known localisation approaches to confirm that the performance outcomes were within reasonable limits.

7 CONCLUDING REMARKS AND PROSPECTIVE WORK

The proposed localisation method—namely, a dual-input TKS-FIS based on fingerprinting and nearest-fingerprint algorithm—was evaluated in terms of empirical outcomes and simulation parameters, and compared with some well-known localisation methods. From this research one may reasonably confirm that the target objectives have been met, and infer the applicability of the proposed method in the localisation domain. However, these achievements do not preclude further observations and plans for refinement. Some of these were noted in the discussions in each section of this work. Among them are the dependency issues for localisation algorithms discussed in Section 5.4, viz. the frequency, application and algorithm dependencies.

It is acknowledged that RSS is irregularly spread in the 'domain of discourse,' where it is evident some power ranges in the signal spectrum are more widely used in the WLAN environments. This is particularly so in the [-65 dB, -80 dB] range, and arguably it opens the door for examining alternative means for signal power sensing. The nearest-fingerprint selection also can be re-assessed to incorporate adaptive principles to conquer the uncertainties pervading the RSS signatures. Furthermore, it is noted that the current TKS-FIS deems independence among all gathered observations. This is despite the fact that all observations attribute to the same target fingerprint, so choosing conditional independence would add more credibility to the nearest-fingerprint selection [192].

Building the fingerprint map for indoor localisation problems is labour-intensive and time-consuming. However, due to its direct influence on the accuracy of location estimation, finding a proper mechanism to construct the fingerprint map is essential if we are to improve accuracy. Therefore, a proposal has been put forward to present a fingerprint map construction technique based on the determination of location fixes and fingerprint matching. The proposal is motivated by the availability of advanced sensing capabilities in smartphones which have the potential to reduce the time and labour required for the site survey. The proposal introduces a location fixing and finger matching (LFFM) method which uses a landmark graph-based localisation approach to automatically estimate location fixes for the reference points and match them with the collected fingerprints, without requiring active user participation. The initial empirical results reveal that the LFFM is more agile than the manual fingerprint map construction method and can remarkably improve positioning accuracy [196].

We have looked at the localisation problem from both historical and technical points of view, and in accordance with the classification outlined in Chapter 2 for the application of fuzzy sets in solving localisation problems. In this respect, innovative concepts were found to be attractive for further consideration under the realm of localisation applications. Initially, other types of fuzzy systems might be explored in applications to localisation, for example, type-2 fuzzy sets, which has never been seriously examined in this field. Moreover, the reported literature on fuzzy arithmetic has shown the least representation among all features of fuzzy sets. Finally, the use of 'hedges' for modifying fuzzy membership functions is another field worth looking into, and one which would add more precision and feasibility to the fuzzy models.

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