RAILWAY TRAFFIC FLOW OPTIMISATION WITH DIFFERING CONTROL SYSTEMS

by

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Abstract

Energy consumption is becoming a critical concern for modern railway design. Many technical solutions and methods such as train trajectory and driving strategy optimisation are considered and, when used in combination, optimised train trajectories, robust operating priorities and train control systems can provide better railway operation performance than using just one solution on its own.

The author of this thesis firstly describes the development of a multi-train simulator where different train control systems are simulated on a common section of high-speed line operating with four trains. The simulator is used to estimate and compare train knock-on delay performance with different control systems.

The author further demonstrates his train trajectory optimisation work. Four solution search approaches, namely Enhanced Brute Force, Dynamic Programming, Genetic Algorithm and Ant Colony Optimisation, have been implemented to find, for a specific train, the most appropriate target in order to minimise energy usage and delays.

A West Coast Main Line case study is presented in order to assess the operational impact of using optimised train trajectory and different practical train control systems. Results are presented using six different train control system configurations, combined with three different driving styles.

The results show that, by using more advanced control systems or optimal train trajectories, interactions between trains can be reduced, thereby improving
performance. This also has the effect of reducing the energy required to make a particular journey. Simple control systems, when coupled with the optimisation process, have been shown to have similar performance to the more advanced control systems. The data also shows that the algorithms achieve the objectives efficiently and accurately. The use of dynamic programming allows an objective function to be minimised with the best results and an acceptable computation time.
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List of Acronyms

ACO – Ant Colony Optimisation
ATC – Automatic Train Control
ATO – Automatic Train Operation
ATP – Automatic Train Protection
ATR – Automatic Train Regulation
ATS – Automatic Train Supervision
BF – Brute Force
CTCS – Chinese Train Control System
DE – Differential Evolution
DLR – Docklands Light Railway
DMI – Driver Machine Interface
DP – Dynamic Programming
EJTC – East Japan Train Control System
EMU – Electric Multiple Unit
ERTMS – European Rail Traffic Management System
ETCS – European Train Control System
GA – Genetic Algorithm
GOA – Grade of Automation
List of acronyms

GSM-R – Global System for Mobile Communications – Railway

LMA – Limit of Movement Authority

OHL – Overhead Line

RBC – Radio Block Centre

TSP – Travelling Salesman Problem

TVM – Transmission Voie Machine

UHF – Ultra High Frequency

VCC – Video Control Centre

VHF – Very High Frequency
Chapter 1 Introduction

1.1 Background

Railway timetables provide a fixed set of timings for trains to arrive at key point on the railway network. However, in general, trains are operated by humans, and services are commonly disturbed by external influences (Chen et al., 2012; Fan et al., 2012; Ke et al., 2012a). Such disturbances are of particular importance on high capacity railway lines, where trains are operating with short headway instead.

**Definition 1.1** (Headway): *The headway time or headway distance is the minimum time or distance between trains that the signalling system is permission, so that the following train will not be affected by the lead train.*

In railway operations, the minimum service interval between two trains that results in interference free motion is called the minimum line headway. The actual service interval between two trains should always be greater than the fixed minimum line headway to avoid interactions between trains. However, in practice, if a first train is delayed, or the station dwell time is extended, the service interval of the second train will be reduced. If the journey of the first train is disturbed, the distance between the two trains may fall below the minimum line headway distance; if this occurs the journey of the second train will also be disturbed and an interaction will occur between the two trains (Woodland, 2004). Changes in train velocity caused by these interactions will increase the energy consumption and journey time and reduce passenger comfort.
The introduction of optimised train trajectories or new operating strategies is usually considered to avoid or reduce the level of the interaction. A number of researchers have investigated using train movement simulators to analyse train operation performance (Chang et al., 20-23 April 1998; Carey, 1999; Johnson et al., 2010). Due to the complexity of the solution domain, often, metaheuristic methods such as genetic algorithms (GA) are applied to driving speed curve optimisation by searching for the best train coasting points (Bocharnikov et al., 2007; Chang et al., 1997a). Other advanced methods, such as artificial neural network and fuzzy logic, have also been employed to improve the efficiency and results of the optimisation (Acikbas et al., 2008; Hwang, 1998). However, exhaustive search (exact algorithms), such as Brute Force, provide a more straightforward approach than metaheuristics, and, importantly, they are guaranteed to find optimal solutions and to prove optimality (Bernstein et al., 2004; Caprara et al., 1995; Shasha et al., 1994). But the performance of these algorithms is not satisfactory in real time train control, as the computation time grows exponentially with the problem size (Jamal Toutouh, 2010; Hachtel et al., 1991; Lambert, 2005).

Additionally, the use of more advanced train control systems can also have a significant effect on the train operation performance. The introduction of a standard European Rail Traffic Management System (ERTMS) aims to provide interoperability in train control throughout Europe (Hicks, 30 September 2004; Daian et al., 2012). It will also allow easier cross border train movements and will reduce the cost of the system. ERTMS is composed of the European Train Control System (ETCS) and GSM-R, which is a radio based system providing communications between trains and tracks (Nemțoi et al., 2010). Three different implementation levels of ERTMS
technology exist, each providing different capacity and performance capabilities. The implementation of a particular train control system employed on a line has a direct impact on train performance in terms of journey time and energy cost, and the likelihood of disturbance occurring. Computer simulations can assist in selecting the most suitable implementation for a particular area, and provide a viable method for evaluating and analysing the performance of the system (Oh et al., 2012; Wismer, 1965).

1.2 Objectives

In this thesis consideration is given to how to minimise the impact of interactions. Four objectives are presented in this thesis, which are as follows:

(1). To develop a multi-train simulator specifically for this study, in which multiple train movements are simulated and optimised with different train control system configurations. A case study is taken into consideration, which aims to verify the accuracy of the simulator;

(2). By using the developed simulator, a number of train operations with different service intervals are simulated in order to analyse the impact of the train interactions. It is found that the interactions increase the train journey time, energy usage and thereby increase the operation cost and reduce passenger comfort;

(3). In order to reduce the effect of the interaction, an optimisation study is considered, which aims to find the most appropriate target speed series, and thus driving speed curve, to balance the minimum energy cost against the minimum delay penalty of the affected train, achieving the lowest overall cost.
In order to achieve a quick and efficient calculation, four search methods, namely Enhanced Brute Force, Dynamic Programming, Genetic Algorithm and Ant Colony Optimisation are implemented and compared;

(4). The train operating strategies also has a direct impact on train performance. In order to consider such an impact, three driving styles, namely flat-out operation, optimal operation and cautious operation, are taken into consideration in the study. Different priorities can be achieved by applying different weighting parameters to the objective function of the optimisation.

1.3 Thesis Structure

The detailed structure of the thesis is given as follows:

(1). In Chapter 1, the author presents the background, objectives and structure of the thesis. It provides a general introduction to the research work;

(2). Chapter 2 provides a review of different train control systems. Signalling systems are detailed as they are closely related to the study, followed by a description of different railway automation systems. This chapter gives the train control system background behind the multiple train movement modelling demonstrated in Chapter 4, Chapter 5 and Chapter 7. Furthermore, the European Train Control System (ETCS) is shown in detail, as it will be implemented in Chapter 7;

(3). Chapter 3 gives a review of different optimisation techniques. Two categories of optimisation algorithms, namely exact algorithms and approximate algorithms are presented. Different algorithms have their specific advantages and disadvantages when considering programming complexity, result
optimality and computation time. This chapter presents the background for the optimisation techniques that will be implemented in Chapter 6;

(4). Chapter 4 describes the development of a multi-train simulator. A literature review of traction power system modelling is provided at the beginning of the chapter. A time step based approach to vehicle movement modelling and a method to simulate multi-train running performance are described afterwards, followed by an energy-time trade-off demonstration at the end. The simulator developed will be used throughout the following case studies;

(5). Chapter 5 presents four train movement case studies. A test case study is given first in order to verify the results of the simulator against real-world data. The Chapter then shows three studies that simulate and analyse different levels of train interactions. The results gained in this Chapter will be used as the fundamental information for a train trajectory optimisation study in Chapter 6;

(6). Chapter 6 describes a train trajectory optimisation study. The work aims to find the most appropriate train trajectory to minimise a function that includes energy usage and delay for all trains. First, a method developed to achieve this aim will be described. Four algorithms are implemented, and a comparison between these algorithms is given. A description of different driving styles is presented at the end. The optimisation method and the driving styles, together with the most suit algorithm, will be used in the case study in Chapter 7;

(7). Chapter 7 presents a case study that is based on a section of the West Coast Main Line in the UK. It analyses the operational impact of using different driving styles combined with a different control system on a common railway line;
(8). Chapter 8 provides the major contribution and suggested further work of the research as a conclusion;

(9). Appendix A presents diagram of the track that is used in the case studies. Appendix B shows the paper published to Proceedings of the Institution of Mechanical Engineers, Part F Journal of Rail and Rapid Transit. The author was the primary contributor of this work as well as the lead author of the publication. Parts of the publication are expanded on in more depth in chapters 4-7.
Chapter 2  Review of Train Control Systems

2.1 Introduction

The aim of a train control system is to ensure that trains operate safely and efficiently (Greenway et al., 1974; Pollack, 1998; Sheikh et al., 1990; Chang, 1990). It is a core element of a railway system as it maintains a safe distance between two trains, controls train movements and regulates train services. As shown in Figure 2.1, the train control system is one of the sub-systems of the railway control system (Clark, 2006). It is constituted of data transmission systems, service control systems, signalling systems and train operation systems.

This chapter begins with an introduction to the key technologies that have been used in train control systems, followed by the descriptions of automatic systems and integrated train control systems.
Chapter 2 Review of Train Control Systems

Figure 2.1. The sub-system view of the railway system. (Clark, 2006)

2.2 Signalling Systems

The aim of all railway signalling systems is to prevent collisions (Bailey, 2008; Foley, 2009; Goddard, 2006). In early railway systems, trains were operated using time interval systems which used headways to protect trains.
In order to increase line capacity, schedule designers began to reduce the headway. As a result, capacity was increased, but the number of accidents was also increased. Fixed block signalling systems were created as a solution to help resolve this issue.

2.2.1 2-aspect Signalling

The basic principle of fixed block signalling is that a railway track is divided into many sections (blocks) (Goddard, 2003). Only one train is permitted to operate in one block at any one time. A signal is equipped ahead of each block which displays a red light (aspect), giving a ‘stop’ indication, if the related block is occupied by a train. The next train has to wait in front of this signal until it changes to green (proceed), which means that the front train has cleared the block, as shown in Figure 2.2. If the driver is not able to observe the signal from the sighting point, a yellow (caution) and green aspect signal can be located in between, as a repeater signal (Kerr et al., 2001).

![Figure 2.2. A 2-aspect signalling layout.](image)

The minimum interference free headway distance of a 2-aspect signalling system can be calculated by the following equation:

$$HD_2 = BD + SD + SS + OL + TL$$  \hspace{1cm} (2.1) \hspace{1cm}

where:

- \(HD\) is the headway distance, m;
BD is the braking distance, m;
SD is the sighting distance, m;
SS is the signal separation, m;
OL is the overlap length, m;
TL is the train length, m.

In practice, if the signal separation decreases, the location of the required repeater signal becomes too close to that of the main signal. Therefore, the driver may be confused by the combination of aspects displayed (Woodland, 2004).

2.2.2 3-aspect Signalling

3-aspect signalling is introduced to overcome some of the limitations of 2-aspect signalling. In 3-aspect signalling, each signal contains a red, yellow and green aspect. The block after a yellow signal provides a safe braking distance for trains, as shown in Figure 2.3. If a driver sees a green aspect, this indicates that there are at least two clear blocks ahead and that the train can continue to be driven at maximum speed until a yellow aspect is seen.

![Figure 2.3. A 3-aspect signalling layout.](image_url)

The headway distance of a 3-aspect signalling system can be calculated by the following equation:
Chapter 2 Review of Train Control Systems

\[ HD_3 = 2BD + SD + OL + TL \] (2.2)

2.2.3 4-aspect Signalling

In order to achieve greater line capacity, 4-aspect signalling was designed. In 4-aspect fixed block signalling systems, two blocks are provided for trains to brake. There are two advantages in updating from 3-aspect to 4-aspect signalling. Firstly, 4-aspect signalling provides earlier warnings, which are necessary for high speed trains, the operation of 4-aspect signalling is shown in Figure 2.4. Secondly, by shortening the length of the blocks, better block occupancy can be achieved. The use of 4-aspect signalling systems enables line capacities to be increased (Pachl, 2005).

\[ \text{Figure 2.4. A 4-aspect signalling layout.} \]

The headway distance of a 4-aspect signalling system can be calculated by the following equation:

\[ HD_4 = \frac{3}{2} BD + SD + OL + TL \] (2.3)

2.2.4 5-aspect Signalling

A 5-aspect fixed block signalling system works in a similar way to the 4-aspect system. Three blocks are provided for train braking, as shown in Figure 2.5.
The headway distance of the signalling system can be calculated by the following equation:

\[ HD_5 = \frac{4}{3} BD + SD + OL + TL \]  \hspace{1cm} (2.4)

However, when the number of aspects is increased, the driver’s task becomes increasingly complex, and the investment becomes too high.

### 2.2.5 Moving Block Signalling

The most significant advantage of moving block signalling is that the headway distance between trains can be varied according to the speed of the trains.

Figure 2.6 shows a speed diagram of a moving block signalling system. Trains are controlled to stop at a safe headway distance behind the preceding train, instead of at fixed location signals. At least a full safe braking distance needs to be maintained in the headway distance (Chang et al., 20-23 April 1998).
The headway distance of moving block signalling systems can be calculated by the following equation:

\[ HD_m = BD + SD + OL + TL \]  \hspace{1cm} (2.5)

2.3 Automation in Train Control Systems

Train speed, reliability and convenience are the key factors for ensuring the desirability of modern railways. As the complexity of the railway system increases, it is necessary to utilise automation to keep the railways running safely and efficiently.

Figure 2.7 presents the interactions and hierarchy between different automatic technologies in one place (Woodland, 2004). The figure is contained with Figure 2.1 but provides more details in railway control system part. The following sections will introduce the terminology used to describe railway automation systems.
2.3.1 Automatic Train Control

Automatic Train Control (ATC) is constituted of Automatic Train Protection (ATP) and Automatic Train Operation (ATO), as shown in Figure 2.7. It is considered as a collective term for systems automating the tasks of the train operator and driver. Due to growing traffic flow and the complexity of railway control systems, ATC has become more popular as it is able to reduce human error problems and protect trains from collision (Siahvashi et al., 2010; Kichenside et al., 1998). ATC provides more...
advanced train control strategies, therefore a number of advantages can be achieved by using an ATC system (Sasaki et al., 2005; Lockyear, 1998; Han et al., 2001):

- Increase safety;
- Reduce energy consumption;
- Reduce the amount of track-side equipment, thereby decreasing cost;
- Improve passenger comfort;
- Improve the performance of railway control systems.

ATC is suitable for application on both light rail and metro systems because the trains have similar performance characteristics. Main line railway lines are more difficult to automate fully because they include more complicated mixed traffic scenarios. Therefore, main line railways tend to be limited to applying only the ATP function (Goodman, 1999).

### 2.3.2 Automatic Train Protection

Automatic Train Protection (ATP) is a system that forces the train brakes to apply automatically if the train travels above the permitted speed or passes without authority a stop signal which could cause the train to collide with another one (Uff et al., 2001; Guo et al., 2011).

In order to provide full supervision of trains, the following data must be available to an ATP system:

- Distance to next signal at danger, or distance to next train;
- Current maximum permitted speed (line speed, permanent or temporary speed restriction);
• Distance to next speed restriction;
• Value of next speed restriction;
• Distance to the termination of speed restriction;
• Gradient;
• Actual speed of train;
• Braking performance of train;
• Maximum permitted speed of train;
• Length of train.

Some data can be transmitted to on-board systems from train control centres using balise, coded track circuits or radio communication; other data may be pre-programmed or input into systems by drivers (Leach, 1991; Tso et al., 1981). The ATP system calculates the maximum safe speed and braking curve continually during operations. If the actual train speed trends to exceed the maximum safe speed, the following actions will apply:

• Depending on the system, a warning will be displayed to the driver first;
• If the actual speed continues to increase and exceeds an acceptable level, the service brake will be applied automatically;
• If the service brake cannot reduce the speed of the train sufficiently, the emergency brake will be applied. The emergency brake will not be considered first because braking at a high rate will affect passenger comfort.

ATP can be classified into continuous ATP and intermittent ATP. A continuous ATP system equipped train is able to receive movement authorities continuously using coded track circuits, insulated conductors or radio communication. An intermittent
ATP system equipped train is only able to receive data intermittently, when it passes a beacon.

In response to the Clapham train crash in 1988, two ATP systems were developed in the UK for the Great Western main line and the Chiltern line. The systems were fitted to the trains of First Great Western, Chiltern Trains and Heathrow Express. They are intermittent ATP systems, whereby data is transmitted to the train on-board systems intermittently using loops or beacons. The Eurostar and Class 92 locomotives which operate on the Channel Tunnel Rail Link are equipped with a continuous ATP system named Transmission-Voie-Machine 430 (TVM-430), by which trains are able to update their movement authorities continually using coded track circuits (Uff et al., 2001).

### 2.3.3 Automatic Train Operation

Automatic Train Operation (ATO) is the system by which train movements are automatically controlled without the intervention of a driver (BSI, 1998; Shirai et al., 1968; Qi et al., 2011). It improves train performance by allowing ‘hands off’ operation by drivers and ensures that trains comply with required speed restrictions at all times.

An IEC working group (TC9 Working Group 40) and the European MODURBAN project classify the ATO into different levels using a Grade of Automation (GOA), as shown in Table 2.1 (International electro technical commission, 2009).
Chapter 2 Review of Train Control Systems

<table>
<thead>
<tr>
<th>GOA</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>On-sight train operation. Drivers have full responsibility and there is no system to advise on and to supervise the driver’s activities.</td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td>Non-automated train operation. The level includes ATP without ATO. Train acceleration and braking are controlled by drivers in compliance with trackside signal or in-cab signalling. Departure orders and door control are commanded manually.</td>
<td>High-speed 1, UK</td>
</tr>
<tr>
<td>Level 2</td>
<td>Semi-automated train operation. Trains are able to operate automatically from one station to the next with the protection of ATP and the supervision of drivers. Departure orders and door control are still commanded manually.</td>
<td>London Underground Victoria Line, UK</td>
</tr>
<tr>
<td>Level 3</td>
<td>Driverless train operation. No driver stays in the front cabin but an operator is necessary on board. Departure commands and door control can be commanded by the operator.</td>
<td>London Dockland Light Railway, UK</td>
</tr>
<tr>
<td>Level 4</td>
<td>Unattended train operation. No driver or operator is required. Door control and departure commands are generated automatically.</td>
<td>Downtown Line, Singapore</td>
</tr>
</tbody>
</table>

Table 2.1. Description of GOA Levels.

2.4 Integrated Train Control Systems

It is apparent that an integrated system is needed for the safe and efficient operation of a railway. Different countries and organisations aim to develop their own train control systems. For instance, the European Union developed the European Train Control System (ETCS) in the 1990s, while the Chinese Railways created the Chinese Train Control System (CTCS) and East Japan Railway produced the East Japan Train Control System (EJTC). Each control system is classified into different levels in terms of the technology that it contains, such as train detection methods and data transmission technology. A comparison between the above three control systems is shown in Table 2.2 (Han et al., 2008; Matsumo, 2005; RSSB, 2010).
Chapter 2 Review of Train Control Systems

<table>
<thead>
<tr>
<th>ETCS</th>
<th>EJTS</th>
<th>CTCS</th>
<th>Signal</th>
<th>Data transmission method</th>
<th>Track occupation detection</th>
<th>ATP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>Level 0</td>
<td>Level 0</td>
<td>Trackside signal</td>
<td>Non-electronic</td>
<td></td>
<td>Intermittent ATP</td>
</tr>
<tr>
<td>Level 1</td>
<td>Level 1</td>
<td>Level 1</td>
<td></td>
<td>Balise with/without infill (Reddy, 2001)</td>
<td>Track circuit/axle counter</td>
<td>Continuous ATP</td>
</tr>
<tr>
<td>-</td>
<td>Level 2</td>
<td>Level 2</td>
<td>In-cab signalling</td>
<td>Coded track circuit (Bachetti et al., 1998)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 2</td>
<td>-</td>
<td>Level 3</td>
<td></td>
<td>Radio communication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>Level 3</td>
<td>Level 4</td>
<td></td>
<td>Radio communication and balise</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2. Comparison of control systems in Europe, China and Japan.

The main difference between ETCS and EJTS is the transmission technology used in Level 2. ETCS uses radio communication while EJTS uses coded track circuits.

ETCS has been considered in the simulation study and will be further considered in the optimisation study in Chapter 7.

2.5 European Train Control System

The introduction of a standard European Rail Traffic Management System (ERTMS) aims to provide interoperability in train control throughout Europe. It aims to provide easier cross border train movements and to reduce the cost of train control systems (Hicks, 30 September 2004). ERTMS is composed of the European Train Control System (ETCS) and Global System for Mobile Communications-Railway (GSM-R),

19
which is a radio-based system providing communications between trains and control centres.

Figure 2.8 shows the ETCS commercial operation in Europe. The red lines represent the Level 1 applications while the green lines represent the Level 2 cases. Of the European railway network, 1% (22,100 km) has ETCS in commercial operation and approximately 13% is contracted or planned to be equipped with ETCS in the next 10 years (UIC).

![ETCS commercial operation in Europe](image)

**Figure 2.8.** ETCS commercial operation in Europe.

Several implementation levels of ERTMS technology have been defined. Different systems provide different capacity and performance capabilities, as shown in Table 2.3 (RSSB, 2010; Muttram et al., 2003; Hicks, 30 September 2004).
<table>
<thead>
<tr>
<th>System</th>
<th>ERTMS category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple aspects fixed block signalling system</td>
<td>ETCS Level 0</td>
<td>Railway line not equipped with an ETCS system.</td>
</tr>
<tr>
<td>Intermittent ATP overlay system without infill</td>
<td>ETCS Level 1 System A</td>
<td>Trains update their movement authorities utilising single spot transmission point.</td>
</tr>
<tr>
<td>Intermittent ATP overlay system with single infill</td>
<td>ETCS Level 1 System B</td>
<td>Trains update their movement authorities utilising two spot transmission points.</td>
</tr>
<tr>
<td>Continuous ATP overlay system</td>
<td>ETCS Level 2 System C</td>
<td>Trains update their movement authorities utilising radio based date transmission.</td>
</tr>
<tr>
<td>Continuous fixed block ATP in-cab system</td>
<td>ETCS Level 2 System D</td>
<td>Data defined fixed block arrangements are implemented.</td>
</tr>
<tr>
<td>Continuous moving block ATP in-cab system</td>
<td>ETCS Level 3</td>
<td>Track occupancies are determined from the last train position reports reported to the ETCS trackside.</td>
</tr>
</tbody>
</table>

**Table 2.3.** Descriptions of different integrated systems. Based on Woodland (2004).

### 2.5.1 Multiple Aspects Fixed Block Signalling System

ETCS Level 0 covers the situation when ETCS trains operate on a railway line which is not fitted with ETCS trackside equipment. In this case, the train drivers should observe the trackside signals and remain within the signed speed limits.

### 2.5.2 Intermittent ATP Overlay System without Infill

This system is described by the ERTMS Programme Team as ETCS Level 1 System A. In such a system, the multiple aspect signals are retained. Each spot transmission point (balise) is located a short distance before the signal. The balise and the signal are connected with each other via the signal adapters and the telegram coders (Lineside Electronics Unit), as shown in Figure 2.9.
When the train passes over the balise, the antenna receives movement authorities and track data via the balise according to the signal aspect. The train on-board system calculates the next braking point, the speed profile and supervises the train speed. The Limit of Movement Authority (LMA) information is displayed to the driver via the Driver Machine Interface (DMI).

**Definition 2.1** (Movement authority): The movement authority is a permission to proceed, which specifies the distance that the train is permitted to travel and data about the track ahead.

### 2.5.3 Intermittent ATP Overlay System with Single Infill

The limitation of the System A is that the train moment authority can only be updated at each balise. The signalling information that is displayed to the driver is only as good as the information available at the last balise that the train passed. Therefore, as shown in Figure 2.10, if Signal 2 is displaying a green aspect but had previously displayed a yellow aspect, the train will be supervised to stop before Signal 3, as the...
system expects Signal 3 to be red. Furthermore, the driver cannot cancel the stop command even though he sees the green aspect. This is because ATP is a fail-safe system that must not allow human intervention to reduce its effectiveness.

![Diagram of train control systems](image)

**Figure 2.10.** An intermittent ATP overlay without infill overlay.

This limitation significantly reduces the line capacity. In order to overcome it, an infill balise is placed a further distance before each signal. The location of the infill balise is usually at the sighting point in order to avoid confusion to the trackside signal aspect. These balise are also electrically connected to their relative signals via a Lineside Electronics Unit, as shown in Figure 2.11.
Figure 2.11. Diagram of an intermittent ATP overlay with single infill system.

The train is therefore able to receive an earlier update to its movement authority. The on-board system can revoke the stop command if the block ahead is cleared.

This system is described by the ERTMS Programme Team as ETCS Level 1 System B. A number of railway lines are equipped with this system throughout the world. Compared with System A, System B can mitigate the capacity reduction. More infill balise or loops can be added in order to improve line capacity, but this will significantly increase the investment and maintenance costs.

### 2.5.4 Continuous ATP Overlay System

In the continuous ATP overlay system, the line-side signals are retained. The train has an on-board radio system that allows the computer to communicate with the Radio Block Centre (RBC) using GSM-R. The balise are merely electronic position markers, as shown in Figure 2.12.
Train detection equipment, such as track circuits, are still required in this system. They send the train position to the RBC. The RBC determines the new train movement authority and radios it to the train. The on-board computer calculates its speed profile to the movement authority and the next braking point. This information is displayed to the driver. When the train passes over a balise, it receives a new position message. The on-board system determines the train position and supervises whether the current speed is correct for the distance travelled. This control system is able to provide a more frequent movement authority update.

This system is described by the ERTMS Programme Team as ETCS Level 2 System C.

### 2.5.5 Continuous Fixed Block ATP In-Cab System

In the continuous in-cab ATP system, signals are removed. A block marker is placed at each track section. The train updates its position to the RBC every time a track
section becomes clear. The balises are used as electronic position markers, as shown in Figure 2.13.

![Diagram of a continuous in-cab system.](image)

**Figure 2.13.** Diagram of a continuous in-cab system.

Here, the line side signals are no longer required. The track section can therefore be shortened significantly. The track section length in a continuous in-cab overlay is usually half of the block length in a conventional overlay system. In a 4-aspect fixed block signalling system, two blocks are provided for train braking. However, in a TVM 430 system, four track sections are provided (Turner, 1998). Therefore, better block occupations and smaller headway distances can be achieved using continuous in-cab systems.

This system is described by the ERTMS Programme Team as ETCS 2 System D. It offers potential for a significant increase in capacity.
There is a System E ERTMS Level; it is for low density application with much reduced trackside infrastructure. It is mainly considered on low frequency service railway lines and capacity is not the main reason for implementation. Therefore, System E will not be considered in this study.

### 2.5.6 Continuous Moving Block ATP In-Cab System

This system is described by the ERTMS Programme Team as ETCS Level 3. It is similar to System D except there is no requirement for trackside train detection equipment, which significantly reduces the investment and maintenance costs. Balises are installed as electronic position markers, as shown in Figure 2.14.

![Diagram of a moving block system.](image)

**Figure 2.14.** Diagram of a moving block system.

Similar to System C and System D, in a moving block system, when the train passes over the balise, the on-board computer receives a new position indicator and checks the train position and current speed (Ning, 2010). Trackside train detection equipment is removed. The train's system checks integrity and reports the completeness to the
RBC using radio signals. The RBC receives all train positions, determines the new movement authorities and sends them to the train. The on-board computer then calculates the speed profile and the next braking point and displays them to the driver.

However, this system is not described by the ERTMS Programme Team because Level 3 is not currently considered in practical discussions and there is no direct work relating to this level in Europe at the present time (UIC, 2008).

2.6 Summary

In this chapter, the author reviewed different train control systems. Section 2.1 provides a sub-system overview of the railway system. One of the sub-systems, namely the signalling system, was introduced in Section 2.2. In Section 2.3, different railway automation systems and their relationships were described. As the railway system becomes more complex, it is necessary to consider automation to ensure railway safety and efficiency. Section 2.4 introduced common train control systems that have been developed by different countries and organisations. It is apparent that an integrated train control system is needed for the design and operation of a modern railway. The European Train Control System (ETCS), whose development is funded and mandated by the European Union, is detailed in Section 2.5. It will be further considered and simulated in the case study in Chapter 7.
Chapter 3 Review of Optimisation Techniques

3.1 Introduction

Optimisation is a key topic in engineering, economics and related fields. It is the process of trying to find the best possible solution to a single- or multi-parameter problem by an objective function or a performance index within a given time limitation (Chong et al., 2008; Corne et al., 1999b; Cooper et al., 1970; Scales, 1985).

An optimisation problem has the form shown in Equation 3.1:

\[
\text{minimise } f_0(x) \quad (3.1)
\]

subject to \( g_i(x) \leq b_i, \quad i = 1, ..., m \)

\( h_j(x) = b_j, \quad j = 1, ..., l. \)

where vector \( x=(x_1, \ldots, x_n) \) is the optimisation variable to the problem; \( f_0 \) is the objective function; \( g_i \) is an inequality constraint function; \( h_j \) is an equality constraint function. A vector \( x^* \) is called optimal or a solution to the problem, if it has the smallest objective value among all vectors that satisfy the constraints, that is, for any \( z \), there is \( f_0(z) \geq f_0(x^*) \).

The objective function can also be referred to as a cost function if it is to be minimised. If it is to be maximised, the function is known as a fitness function (Hopgood, 2000).

Optimisation algorithms can be generally classified into two categories, namely exact algorithms, shown in Section 3.2, and approximate algorithms, shown in Section 3.3.
Chapter 3 Review of Optimisation Techniques

(Gramm et al., 2003; Lu, 2011; Chan et al., 2006). A brief classification is shown in Figure 3.1.

![Diagram of optimisation algorithms]

**Figure 3.1.** The sub-system view of the optimisation algorithms.

### 3.2 Exact Algorithms

The main advantage of using exact algorithms is that they are guaranteed to find optimal solutions and to prove optimality within instance dependent run time (Bernstein et al., 2004). In this section, Brute Force and Dynamic Programming will be introduced, due to their applicability to a wide variety of problems (Morton et al., 2000; Ogawa et al., 2007).

#### 3.2.1 Brute Force

**3.2.1.1 Introduction**

Brute Force is a straightforward approach for solving a problem. It is usually directly based on the statement of a problem and the definitions of concepts involved (Levitin,
2003). A brute force search enumerates all possibilities in the solution domain to find the optimum.

### 3.2.1.2 Selection sort

Selection Sort is a typical brute force method (Knuth, 1998). It is an in-place sorting algorithm, so it does not require additional storage space, therefore it is suitable to be used where computer auxiliary memory is limited. For example, a problem is assumed: given a list of $n$ orderable items, rearrange them in descending order. The selection sort algorithm works as follows:

1. The algorithm scans the entire list to find the smallest number and exchanges it with the first element of the list;
2. The algorithm scans the entire list again, but starting with the second element, to find the smallest number among the remaining $n-1$ elements and exchanges it with the second element of the list;
3. The algorithm repeats the scanning and exchanging among the remainder of the list (starting element is advancing each time), as shown in Figure 3.2 and Algorithm 3.1. After $n-1$ passes, the sorting is completed.

![Figure 3.2. A selection sort algorithm.](image)

**Algorithm 3.1** Selection sort

```plaintext
// Input: An array A[0…n-1]
// Output: An array A[0…n-1] sorted in descending order
```
The complexity of the selection sort is $O(n^2)$. Therefore, it is simple to implement but inefficient in solving problems with large lists.

### 3.2.1.3 Exhaustive Search

Many mathematical modelling problems, especially combinatorial optimisation problems, require finding an element in a solution domain that grows exponentially with an instance size. The element should maximize or minimize some desired characteristic. Exhaustive search is a brute force approach to such problems. It generates every element that satisfies the problem’s constraints in a solution domain, then finds a desired element as the optimum result (Levitin, 2003; Preneel et al., 2009). However, such a conventional brute force rapidly becomes impractical due to the processing time increases significantly when the problem is getting more complex. Therefore, an Enhanced Brute Force is proposed in this research, which will be further discussed in Section 6.4.

### 3.2.2 Dynamic Programming

#### 3.2.2.1 Introduction

Dynamic programming is a method for efficiently solving a broad range of search and discrete decision optimisation problems. It was first used and later refined by an American mathematician, Richard Bellman, in 1954 (Bellman, 1954). It uses a divide-and-conquer technique that partitions the problem into independent subproblems and
solves the subproblems recursively. The optimal solution to the original problem can then be found from the optimal solutions to each subproblem (Lew et al., 2007; Cormen et al., 2001).

### 3.2.2.2 Mathematical Presentation

A cost function can be formed in the sense that a sub-cost \( g_k(x_k, u_k) \) is incurred at each time \( k \). The total cost can therefore be written as follows:

\[
M = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k) \tag{3.2}
\]

where:

- \( k \) is the discrete time;
- \( x_k \) is the system state at time \( k \). It also contains summaries of the past information which is relevant for future optimisation;
- \( u_k \) is the system decision at time \( k \). It is associated with the state \( x_k \);
- \( N \) is the number of decisions;
- \( g_N(x_N) \) is a terminal cost incurred at the end of the process.

The aim of the dynamic programming is to select a set of decision \( U \) to minimise the cost function.

\[
U = \{u_0, u_1, u_2, \ldots, u_{N-1}\} \tag{3.3}
\]

Based on Equation 3.2, the following equations can be obtained:
Chapter 3 Review of Optimisation Techniques

\[
\begin{align*}
M_N(x_N) &= g_N(x_N) \\
\vdots \\
M_k(x_k) &= \min_{u_k \in \Delta_k} \{g_k(x_k, u_k) + M_{k+1}[f_k(x_k, u_k)]\}, \quad \text{where } k \in [0, N-1] \\
M_0(x_0) &= \min_{u_0 \in \Delta_0} \{g_0(x_0, u_0) + M_1[f_0(x_0, u_0)]\}
\end{align*}
\]  
(3.4)

where:

- \(\Delta_k\) is the feasible decision space at time \(k\).

It is important to note that \(M_0\) is obtained by the last step of the algorithm, which proceeds backward in time from \(N-1\) to 0. In dynamic programming, all the decisions being made depends on the decision made for the previous stages, that is:

\[
x_{k+1} = f_k[x_k, u_k]
\]  
(3.5)

Furthermore, let \(M^*\) denotes the optimal cost, then:

\[
M^* = \min_{u_0 \cdots u_{N-1}} [g_0(x_0, u_0) + g_1(x_1, u_1) + \cdots + g_{N-1}(x_{N-1}, u_{N-1}) + g_N(x_N)]
\]  
(3.6)

Because \(k = 0, 1, 2, \ldots, N-1\) is conditional on \(x_k\) and \(u_k\), Equation 3.6 can be formed as follows:

\[
M^* = \min_{u_0} \left[ g_0(x_0, u_0) + \min_{u_1} \left[ g_1(x_1, u_1) + \cdots + \min_{u_{N-1}} [g_{N-1}(x_{N-1}, u_{N-1}) + g_N(x_N)] \right] \right]
\]  
(3.7)

By applying Equation 3.5 in Equation 3.4 and introducing the results to Equation 3.7, the following result can be obtained:
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\[ M^* = \min_{u_0} \left[ g_0(x_0, u_0) + \min_{u_1} \left[ g_1(x_1, u_1) + \cdots + \min_{u_{N-1}} \left[ g_{N-1}(x_{N-1}, u_{N-1}) + M_N(x_N) \right] \right] \right] \]

\[ = \min_{u_0} \left[ g_0(x_0, u_0) + \min_{u_1} \left[ g_1(x_1, u_1) + \cdots + M_{N-1}(x_{N-1}) \right] \right] \]

\[ = \cdots \]

\[ = \min_{u_0 \in \Delta_0} [g_0(x_0, u_0) + M_1(x_1)] \quad (3.8) \]

The \( M^* \) can therefore be solved as

\[ M^* = M_0(x_0) \quad (3.9) \]

3.2.2.3 Elements of Dynamic Programming

An optimisation problem can be solved by dynamic programming only when the problem exhibits the characteristics of overlapping subproblems and optimal substructures (Cormen et al., 2001).

Overlapping Subproblems

Overlapping subproblems can be defined as follows:

**Definition 3.1** (Overlapping subproblems): *If an optimal solution to a given problem can be undertaken by reusing the optimal solutions to the sub-problems of the given problem, it is said that such a problem has overlapping subproblems.*

DP solves each of the smaller subproblems only once and stores the results in a table from which a solution to the given problem can be obtained. Such a procedure avoids calculating the same problem again and again and therefore reduces the computation.
Such a process is called memorisation, which is a typical top-down approach (Preneel et al., 2009; Cormen et al., 2001).

**Optimal Substructure**

It is claimed that a problem satisfies the principle of optimal substructure if the optimal solution to the problem is a combination of the optimal solutions to the subproblems. This characteristic is based on Bellman’s Principle of Optimality (Bellman, 1954):

**Definition 3.2 (Principle of optimality).** *The principle of optimality is such a property that whatever the initial conditions and choices are, the decision chosen over the remaining period must be optimal for the remaining problem, with the state resulting from the initial condition.*

Therefore, this kind of sequence problem can be solved based on the previous optimal solutions to the subsequence. Such a bottom-up approach leads DP to progress from smaller subproblems to bigger and bigger subproblems and finally solve the given problem.

### 3.2.3 Summary

In this section, the general characteristics of exact algorithms are introduced. According to different dimensions of classification, exact algorithms can be categorised into different types. To summarise, exact algorithms have the following properties:

- For a combinatorial optimisation problem with finite size instances, exact algorithms are guaranteed to find the optimal solution and prove its optimality;
• It usually takes exponential time to find the optimal solution when solving a complex problem;
• For most complex problems, the performance of using exact algorithms is not satisfactory.

3.3 Metaheuristics

3.3.1 Introduction

As introduced in the previous sections, exact algorithms, such as Dynamic Program, offer the guarantee of finding the global optimal for a given problem. However, the performance of these algorithms is not satisfactory for complex problems as the computation time grows exponentially with the problem size (Fang et al., 2011; Ahn et al., 2009). In order to overcome this drawback, the only possibility is to trade optimality for efficiency. Therefore, approximate algorithms are taken into consideration. Metaheuristics are regarded as approximate algorithms, which aim to obtain near-optimal solutions at relatively low computational cost (Portugal et al., 1994; Caldas et al., 2003). They are able to avoid getting stuck in a local solution as they are more flexible in covering a wider range of possible solutions in a shorter amount of time.

Definition 3.3 (Local solution and global solution). The principle of local solution (local optimum) is such a sub-optimal solution that has no superior neighbouring solutions, but possibly worse than a distant solution. By contrast, a global solution is such a point that is better than or equal to any of the possible solutions (Knowles et al., 2001).
Furthermore, metaheuristics are a general purpose algorithm framework that can be applied to different optimisation problems with relatively few modifications (Jared et al., 2010; Suh et al., 2011). A number of successful metaheuristics have been developed, such as Genetic Algorithm (GA) (Holland, 1992; Ho et al., 2000; Chang et al., 1997b; Kim et al., 1998), Scatter Search (Glover, 1977; Yang et al., 2010), Simulated Annealing (Kirkpatrick S. et al., 1983; Kirkpatrick et al., 1983), Tabu Search (Fred, 1986; Glover, 1990; Glover, 1989), Ant Colony Optimisation (ACO) (Dorigo et al., 2004; Dorigo et al., 1999; Renfrew et al., 2012), Differential Evolution (DE) (Storn et al., 1997; Zhong et al., 2012), etc.

Metaheuristics can be classified into single-point search or population-based search (Birattari et al., 2001). In the single-point search methods, only one single proposed solution is produced at each iteration of the algorithm, such as in Tabu Search and Simulated Annealing. On the contrary, population-based search methods, such as Ant Colony Optimisation and Genetic Algorithm, generate a group of proposed solutions called a population at each iteration. In ACO, a colony of artificial ants is used to construct solutions guided by pheromone trails and heuristic functions. In GA, the population is modified using genetic operators.

This section introduces the two metaheuristic algorithms used in this study, namely Genetic Algorithm and Ant Colony Optimization.

### 3.3.2 Genetic Algorithm

#### 3.3.2.1 Introduction

Genetic Algorithms are based on the theory of evolution. They are inspired by biological evolution processes (Goldberg, 1989; Gen et al., 1997; Michalewicz, 1994).
The algorithm presents an iterative and stochastic process that operates on a population \((p)\) of individuals. Each individual comprises one or more chromosomes, which allows each individual to represent a potential solution to a given problem and each individual is assigned with a value associated with the goodness (fitness) of the solution. The individual with higher fitness has a greater probability to be selected to generate individuals of the next population. This process runs until the population converges to an optimum solution to the problem.

### 3.3.2.2 Methodology

Genetic algorithms usually run with the following five steps (Montana et al., 1989):

1. Implement a way of encoding solutions to the problem on chromosomes;
2. Define a fitness function that returns a rating for each chromosome, so that each solution (chromosome) can be evaluated;
3. Initialise the first population of chromosomes;
4. Apply the genetic operation (iterative evolutionary process) to the chromosomes when they reproduce to alter their genomes composition. During such a process, a set of operators is adopted, that exist in nature are adopted, including selection, recombination (crossover), mutation, replacement (Jiang et al., 2001);
5. Stop the iteration when the stopping criteria is satisfied. The results will be stored.

**Algorithm 3.2 Genetic Algorithm**

// Input: Optimisation objective
// Output: An optimal solution

\[ p^0 \leftarrow \text{initialise population}() \]

\[ \text{evaluation}(p^0) \]
while \( g < \) maximum generations or terminal conditions do
\[
p^g \leftarrow \text{reproduction } p^g
\]
\[
evaluation(p^g)
\]
\[
p^{g+1} \leftarrow \text{replacement } (p^g)
\]
end while

3.3.3 Ant Colony Optimisation

3.3.3.1 Introduction
In nature, ants deposit a certain amount of pheromone on the ground to mark some favourable paths. The path with more pheromone intensity attracts other members of the colony to follow. Ant Colony Optimisation takes inspiration from such a foraging behaviour and presents a similar iterative mechanism for solving optimisation problems (Solnon, 2010).

3.3.3.2 Methodology
In ACO, a graph \( G(V,E) \) can be obtained from the set of all possible solution components \( C \), where \( V \) is the set of vertices and \( E \) is the set of edges (Dorigo et al., 2006). At each iteration, artificial ants build solutions by traversing from vertex to vertex along the edges of the graph. Each artificial ant deposits pheromone on the components (edges and vertices) that it moves along. The amount of the pheromone is associated with each possible solution component, that is, usually a component with a better solution tends to obtain more pheromone. The subsequent ants use the pheromone information as a guide towards promising regions of the search space. As shown in Figure 3.3, if ant \( k \) is assumed to be at vertex \( i \), the possibility of vertex \( j \) being selected for the next position is defined as follows:
\[ p_{ij}^k = \begin{cases} \frac{\tau_i^\alpha \eta_i^\beta}{\sum_{c \in \mathcal{N}(s^p)} \tau_c^\alpha \eta_c^\beta} & \text{if } c_{il} \in \mathcal{N}(s^p) \\ 0 & \text{otherwise} \end{cases} \]  

(3.4)

where \( \mathcal{N}(s^p) \) is the feasible neighbourhood of ant \( k \) being at vertex \( i; l \) is a vertex that has not been selected by ant \( k \) yet; \( \tau \) is the amount of pheromone deposited on the edge \((i, j)\); \( \eta \) is the desirability of the transition via edge \((i, j)\); \( \alpha \) and \( \beta \) are the parameters to control the relative importance of the pheromone versus the desirability (Dorigo et al., 2006).

![Diagram of ant colony optimisation](image)

**Figure 3.3.** Diagram of ant colony optimisation.

Each artificial ant is then able to select the next possible vertex and finally complete the traversing in the graph. Algorithm 3.3 shows the process of the ant colony optimisation. After initialisation, the algorithm iterates over three steps, as follows (Corne et al., 1999a):

1. Construction. In this step, the artificial ants construct a number of solutions by moving each vertex in the current neighbourhood consecutively until a complete solution is constructed. The rules of the vertex selection depend on
different ACO algorithms, but all of them are inspired by the foraging behaviour model of ants given in Equation 3.4;

2. Local search application. After the construction step, the local search application is taken into consideration. This application is able to improve the solutions obtained in the previous step and make the pheromone update step more purposeful;

3. Pheromone update. The algorithm reduces the pheromone values associated with bad and local solutions and increases those which are associated with good ones. The update aims to attract more ants to follow promising solutions and finally makes the optimisation process converge.

Algorithm 3.3 Ant Colony Optimisation

// Input: Optimisation objective
// Output: An optimal solution
Set parameters, initialise pheromone trails
while terminal condition not met do
  Construction
  Local search application (optional)
  Pheromone update
end while

3.3.4 Summary

This section introduced the general characteristics of metaheuristics. To summarise, the fundamental properties which characterise metaheuristic algorithms are as follows (Lu et al., 2011a):

- Metaheuristics have higher level strategies for guiding the search process;
- The aim of using metaheuristics is to explore the search space in order to find close to optimal solutions;
• Metaheuristics are approximate algorithms and are non-deterministic;
• Metaheuristics are not problem-specific, they are applicable to a wide set of different problems;
• An evaluation function relevant to the objective function is required;
• Termination criteria should usually be set.

3.4 Summary

This chapter reviewed different optimisation techniques. A general introduction of optimisation problem was given in Section 3.1. Optimisation algorithms can be generally classified into two categories, namely exact algorithms and approximate algorithms. Section 3.2 introduced two applications of the exact algorithms that are closely related to the study, namely Brute Force and Dynamic Programming. Both of these are guaranteed to find the global optimal for a given problem. However, the performance of exact algorithms is not satisfactory because the computation time grows exponentially with the problem size. Therefore, approximate algorithms were taken into consideration as they trade near-optimal solutions for efficiency. Two approximate algorithms, namely Genetic Algorithm and Ant Colony Optimisation have been described in Section 3.3. The general characteristics of optimisation techniques were introduced and the elements essential to implement and affect to an algorithm have been presented. The algorithms that are described in this chapter will be implemented in Chapter 6.
Chapter 4 Development of Multi-Train Simulator

4.1 Introduction

With the rapid development of modern computer technology, computer based simulation is becoming more important as it provides a viable and economical method of assessing different scenarios. A number of researchers have investigated using train movement simulators to evaluate railway systems. For example, Petersen and Taylor presented a general purpose model which is based on an algebraic structure to describe train movements (Petersen et al., 1982). Dessouky and Leachman developed a simulator to analyse the traffic burden on the rail networks due to the train movements from Downtown Los Angeles to San Pedro Bay Ports (Dessouky et al., 1995).

The multi-train simulator used in this study was developed in MATLAB. The simulator is able to analyse multiple train movements on different railway lines with different signalling systems, traction performance and permitted speeds. The simulated scenarios include a number of high-speed trains, which operate along a route with fixed stations and dwell time. Journey time, energy cost and train trajectory plots are output when the simulation is complete.

The theoretical description of vehicle modelling is given at first, together with a series of fundamental equations describing the physics of train motion. The vehicle movement modelling design is described in Section 4.3, which includes a discussion of energy vs time trade-offs.
4.2 Vehicle Modelling

4.2.1 The Physics of Vehicle Motion

In order to develop a train movement simulation, it is first necessary to consider the fundamental physics of train motion. The methods used to solve the dynamic movement equations have been described in earlier work (Hillmansen et al., 2007; Hsi et al., 2001; Hull et al., 2009). These equations are based on the equations of motion of the railway vehicle, subject to the constraints imposed on the vehicle by the route and driving style. The general equation of vehicle motion, known as Lomonossoff’s Equation, which is based on Newton’s Second Law, can be written as (Chen, 2008; Lu et al., 2011b):

\[ M_{tr} \frac{d^2s}{dt^2} = F_{tr} - R - F_{grad} \quad (4.1) \]

where:

- \( M_{tr} \) is the effective mass;
- \( F_{tr} \) is the traction force;
- \( F_{grad} \) is the force due to the gradient;
- \( R \) is the vehicle or train resistance.

4.2.2 The Force Due to the Gradient

The force due to the gradient shows the effect of the gradient profile and gravity acceleration. In an uphill situation, the train receives a negative gravity component acceleration against the moving direction, while in a downhill situation the train
receives a positive gravity component acceleration, as shown in Figure 4.1. Such a force can be calculated as follows:

\[ F_{\text{grad}} = M_{\text{total}} g \sin(\alpha) \]  \hspace{1cm} (4.3)

where:

- \( M_{\text{total}} \) is the tare mass plus passenger mass;
- \( \alpha \) is the slope angle.

\[ \begin{align*}
\bullet M_{\text{total}} & \text{ is the tare mass plus passenger mass;} \\
\bullet \alpha & \text{ is the slope angle.}
\end{align*} \]

\[ \begin{align*}
\text{Uphill} & \quad \text{Downhill}
\end{align*} \]

\[ \begin{align*}
\text{Direction} & \quad \text{Direction}
\end{align*} \]

\[ \begin{align*}
R & \quad R \\
F_{\text{grad}} & \quad F_{\text{grad}}
\end{align*} \]

**Figure 4.1.** Diagram of the force due to the gradient.

### 4.2.3 Effective Mass

The moment of inertia, also called the rotational inertia, is a measure of a body’s resistance to angular acceleration. It increases with the accelerated train mass and is transformed by gear ratio and wheel diameter (Wang et al., 2010). In order to improve the accuracy of the vehicle simulation, the moment of inertia should be considered in
Chapter 4 Development of Multi-Train Simulator

The calculation of the effective mass by using a rotary allowance. The effective mass can be calculated as follows:

\[ M_{tr} = M_{rs}(1 + \lambda_w) + M_l \]  

(4.4)

where:

- \( \lambda_w \) is the rotary allowance;
- \( M_l \) is the passenger load.

Equation 4.4 can be expressed in the following simple form:

\[ M_{tr} = M(1 + \lambda_w) \]  

(4.5)

The rotary allowance is constant and is usually less than 0.2. Table 4.1 shows some typical rotary allowance values (Steimel, 2007).

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Rotary allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coaches and goods carriages</td>
<td>0.03-0.04</td>
</tr>
<tr>
<td>Electric coaches</td>
<td>0.08-0.12</td>
</tr>
<tr>
<td>Electric locomotives</td>
<td>0.15-0.30</td>
</tr>
<tr>
<td>Mean value for locomotive-hauled trains</td>
<td>0.06-0.10</td>
</tr>
<tr>
<td>Electric rack railway cars (CH)</td>
<td>0.30-1.50</td>
</tr>
</tbody>
</table>

Table 4.1. Some typical rotary allowance values.

4.2.4 Modes of Movement

As presented in Figure 4.2, four movement modes are considered in a typical vehicle journey: motoring, cruising, coasting and braking (Hansen et al., 2008; Yu et al., 2004). In the motoring mode, power is used to overcome resistance and the effects of gravity so that the vehicle can achieve the required acceleration rates, as shown in Table 4.2. The train uses this mode to increase speed and move the vehicle from a low speed state to a higher speed state. In the cruising mode, power is used to keep the
train speed constant and ideally. The acceleration should be zero. In the coasting mode, power is shut down. The speed is therefore affected by resistance and the effects of gradient. The acceleration rate is usually negative unless the train is running on a steep downhill route. The selection of the coasting point plays an important part in the train control optimisation for train journey time and energy saving (Hairong et al., 2010; Wong et al., 2004). In the braking mode, the train applies the service brake or emergency brake to reduce the speed from a high level or to a standstill to a lower level in order to meet speed limits or stop at a station or signal.

![Figure 4.2. Train movement modes.](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Equations, (4.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motoring</td>
<td>$F_{\text{total}} = F_{\text{tr}} - R - F_{\text{grad}}$</td>
</tr>
<tr>
<td></td>
<td>$a = F_{\text{total}} / M_{\text{tr}}$</td>
</tr>
<tr>
<td>Cruising</td>
<td>$F_{\text{total}} = F_{\text{tr}} - R - F_{\text{grad}} = 0$</td>
</tr>
<tr>
<td></td>
<td>$a = F_{\text{total}} / M_{\text{tr}} = 0$</td>
</tr>
<tr>
<td>Coasting</td>
<td>$F_{\text{total}} = -R + F_{\text{grad}}$</td>
</tr>
<tr>
<td></td>
<td>$a = F_{\text{total}} / M_{\text{tr}}$</td>
</tr>
<tr>
<td>Braking</td>
<td>$F_{\text{total}} = F_{\text{tr}} - R - F_{\text{grad}}$</td>
</tr>
<tr>
<td></td>
<td>$a = F_{\text{total}} / M_{\text{tr}}$</td>
</tr>
</tbody>
</table>

Table 4.2. Four modes of train movement.
4.3 Vehicle Movement Modelling

4.3.1 Simulation Configurations

Section 4.2.4 introduced four movement modes: motoring, cruising, coasting and braking. A typical train journey can be represented by a sequence of these modes, such as motoring, cruising, coasting, motoring, braking, etc. The selection of the mode depends on many factors, such as train target speed, line speed limit and headway distance and the need to follow the timetable.

A number of parameters should be input into the simulator. Fixed parameters that are not changed during the modelling process are considered as constant values, such as:

- Route data, such as gradient profiles, line speed limits;
- Train traction characteristics;
- Block lengths, movement authority update intervals.

Acceleration is considered to be a dynamic parameter while train target speed is an input from the optimisation process:

1. Acceleration rate \( (\alpha) \). Changes corresponding to the train speed, gradient profile and the selection of the movement mode;
2. Train target speed \( (V_t) \). If the train trajectory needs to be optimised, target speed indicators will be given to the driver and the driver should control the train with respect to them. The speed value changes corresponding to the line speed limits and real-time train headway distance. Further details will be described in Chapter 6.
The dynamic parameters are changed from time to time and determine the amount of energy used ($E_{run}$) and journey time ($T_{run}$). Figure 4.3 shows the system input and output diagram. Table 4.3 lists all parameters that have been considered in the simulator.

**Figure 4.3.** System input and output diagram.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fixed parameters: Route data</th>
<th>Dynamic parameters: Acceleration rate ($a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration rate, ms$^{-2}$</td>
<td>1$^{st}$ balise location, km</td>
<td></td>
</tr>
<tr>
<td>Braking rate, ms$^{-2}$</td>
<td>2$^{nd}$ balise location, km</td>
<td></td>
</tr>
<tr>
<td>Resistance rate, ms$^{-2}$</td>
<td>Third station location, km</td>
<td></td>
</tr>
<tr>
<td>Gradient, %</td>
<td>Dwell time, s</td>
<td></td>
</tr>
<tr>
<td>Line speed limit, kmh$^{-1}$</td>
<td>Service interval, s</td>
<td></td>
</tr>
<tr>
<td>Rolling stock mass, tonnes</td>
<td>Sighting distance, m</td>
<td></td>
</tr>
<tr>
<td>Passenger mass, tonnes</td>
<td>Overlap, m</td>
<td></td>
</tr>
<tr>
<td>Rotary allowance</td>
<td>Safety margin, for moving block, m</td>
<td></td>
</tr>
<tr>
<td>Route length, km</td>
<td>Train length, m</td>
<td></td>
</tr>
<tr>
<td>First station location, km</td>
<td>Movement authorities update interval, s</td>
<td></td>
</tr>
<tr>
<td>Second station location, km</td>
<td>Block length, m</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.3.** Input parameters.
4.3.2 Simulation Design

For the present design the trains are simulated in sequence, that is, the simulator calculates a lead train’s movement first and then moves on to following trains. The simulator calculates the train’s dynamic parameters for each time step that forms the train trajectory, as shown in Figure 4.4 (Lu, 2011). All results such as total energy usage, journey time, train position and train speed are stored in a table, to be used at the next time step.

![Diagram](image)

**Figure 4.4.** Time step based simulation.

Assume that the train moves one time step ($\Delta T$) from Step $i$ to Step $j$ with an acceleration rate $a$, the train position ($S_j$) and speed ($V_j$) in Step $j$ can be calculated using the following equations:

$$S_j = V_i \times \Delta T + \frac{1}{2} \times a \times \Delta T^2 + S_i$$  \hspace{1cm} (4.6)
\[ V_j = V_i + a \times \Delta T \tag{4.7} \]

The total energy usage \((E_j)\) and journey time \((T_j)\) can be calculated using the following equations:

\[ P = M_{tr} \times a \times V_i \tag{4.8} \]
\[ E_j = P \times \Delta T + E_i \tag{4.9} \]
\[ T_j = T_i + \Delta T \tag{4.10} \]

where:

- \(P\) is the instantaneous power.

A smaller time step, \(\Delta T\), will result in a more accurate simulation result but a greater computation time. In order to achieve a practical simulation output, \(\Delta T\) is set as 1 s for the following simulations and optimisation studies. This accuracy is sufficient for the simulation, even though the train travels 55 meters in one second at 200 km/h.

**First train simulation**

When simulating the first train, it can be considered to run without any effects from other trains because the route is empty. Therefore, it does not need to consider the headway distance. The train is running using flat-out driving style and does not take into account the timetable. As shown in Figure 4.5, at each time step, the simulator first reads the route profiles and calculates the braking distance. In a second step, it calculates the distance to the next station and the next speed limit change location. Finally, the simulator compares these results and selects the appropriate movement mode. The simulation runs until the first train arrives at the final station. All results are stored in tables to be used for the next train’s movement simulation.
Chapter 4 Development of Multi-Train Simulator

Key:
- $t_i$ - time
- $s_i$ - train position
- $v_i$ - train speed
- $p_i$ - train engine power
- $st_i$ - station position
- $a_{ac}$ - acceleration rate
- $a_{co}$ - coasting rate
- $a_{cu}$ - cruising rate
- $a_{br}$ - braking rate

Figure 4.5. Simulation flowchart for the first train.

\[ t_i = 1, s_i = 0 \]
\[ v_i = 0, p_i = 0 \]
Following train simulation

The following trains must to ensure that the braking distance is greater than the distance to the lead train at all times. In order to achieve this purpose, movement authorities (permission to proceed) are sent to the trains. The movement authority specifies the distance that the train is permitted to travel and data about the track ahead. As shown in Figure 4.6, at each time step, the train calculates the distance to the limitation of movement authority (LMA) and compares the result with the braking distance. The final result will be considered in the movement mode selection.

![Simulation flowchart](image)

**Figure 4.6.** Simulation flowchart additional part for the following trains.
The movement authority needs to be updated at a specified time, which mainly depends on the signalling system that is used. Table 4.4 shows the movement authority updating time when different systems are used in this simulator.

<table>
<thead>
<tr>
<th>System</th>
<th>Movement authority updating time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple aspects fixed block</td>
<td>When the train is within the sighting distance of each block signal.</td>
</tr>
<tr>
<td>signalling system</td>
<td></td>
</tr>
<tr>
<td>Intermittent ATP overlay</td>
<td>When the train is running over a balise, that is, once per block.</td>
</tr>
<tr>
<td>without infill</td>
<td></td>
</tr>
<tr>
<td>Intermittent ATP overlay with</td>
<td>When the train is running over a balise, that is, twice per block.</td>
</tr>
<tr>
<td>single infill</td>
<td></td>
</tr>
<tr>
<td>Continuous ATP overlay</td>
<td>With a fixed time interval, depending on system configurations.</td>
</tr>
<tr>
<td>Continuous in-cab overlay</td>
<td>With a fixed time interval.</td>
</tr>
<tr>
<td>Moving block</td>
<td>With a fixed time interval.</td>
</tr>
</tbody>
</table>

*Table 4.4.* The movement authority updating time when different systems are used.

For instance, if two trains are running on a single line railway with a 4-aspect signalling system, the simulator decides the movement mode of the following train by the following steps:

1. If the following train is reaching in the sighting distance of a block (where the driver can observe the signal aspect in theory), the simulator will look up the position of the first train;
2. The block that is occupied by the first train can be identified. The start point of the block will be the LMA for the following train;
3. The simulator then calculates the number of the blocks between the two trains, and decides the following train’s movement mode on the basis of the result;
(4). If the following train is not approaching in the sighting distance of a block, the simulator will not look up the position of the first train, but use the past LMA to make the decision.

### 4.3.3 Energy-time Trade-off

Energy consumption is becoming a critical concern for modern railway design (Landi et al., 2008; Lin et al., 2011; Ke et al., 2012b; Ke et al., 2009). Usually, increasing the speed of the train will result in a shorter journey time. However, such performance also increases the energy consumption (and hence cost). The trade-off between energy usage and journey time is therefore taken into consideration (Freeh et al., 2007).

Section 4.3.2 describes the input and output parameters of the simulator. In Figure 4.3 the energy consumption can be expressed by the following equation:

\[
E_{\text{run}} = f(a, V_l) \quad \text{if} \quad T_{\text{min}} \leq T_{\text{run}} \leq T_{\text{max}} \tag{4.11}
\]

\[
T_{\text{run}} = g(a, V_l) \tag{4.12}
\]

where:

- \( T_{\text{max}} \) and \( T_{\text{min}} \) are the maximum and minimum permitted scheduled journey times.

Equation 4.11 and Equation 4.12 are represented. It is observed that energy consumption can be traded off against journey time. Point A corresponds to journey time priority running while Point B indicates running with energy usage priority.
Energy-time trade-off optimisation can be controlled by an objective function, which will be further discussed in Chapter 6.

### 4.4 Summary

This chapter described the development of a multi-train simulator. A literature review of the modelling of a traction power system has been given in Section 4.2. The method is based on solving the equations of motion of the railway vehicle subject to the constraints imposed on the vehicle by the route and driver style. Section 4.3 showed a time step based approach to vehicle movement modelling. A method to simulate and analyse multi-train running performance has been described in Section 4.3.2. Compared with normal single train simulations, the updating of train movement authority is required and becomes a significant part in the multi-train simulation. Section 4.3.2 also showed that the type of train control systems makes a key different on the movement authority updating. Such effect will be further discussed in Section 5.3.6 and Chapter 7. Finally, the energy-time trade-off is demonstrated in Section 4.3.3. The developed simulator will be used throughout the following case studies. In the next chapter, it will first be employed in a test case study to verify the results. Then it will be used for a number of train interaction simulations.
Chapter 5  Train Movement Simulation

5.1 Introduction

Railway timetables provide a fixed set of timings for trains to arrive key points on the railway network. The aim is for the leading train not to affect the trajectory of the following train. However, trains are operated by humans, and services are commonly disturbed by external influences, such as increased dwell times at stations, due to a larger than normal number of passengers boarding a train, or infrastructure equipment failures (D'Angelo et al., 2011; Yang et al., 2009; Ueda et al., 2005). Such disturbances are particularly influential on high capacity railway lines, where trains are operating with small headways. The choice of driving styles and the particular signalling system employed on a line have a direct impact on train performance, in terms of journey time and energy cost, and the likelihood of a disturbance occurring. Carey and Carville developed and experimented with a simulation model to predict the probability distributions of knock-on delay problems (Carey et al., 2000). This model is able to generate a reliability analysis for different train types and routes. Liu established a simulation system for multi-train operation (Liu et al., 2005). The simulation analyses train interaction performance using different signalling systems.

The simulation scenarios to be described in this chapter have been developed based on the concepts introduced in the previous chapters. Four different case studies are described, as follows:
(1) Case Study 1 considers the operation of a single train operating on a test route. The study aims to verify the accuracy of the simulator. The results are compared with previously published results (Evans, October 2007).

Based on the first case study, the following three studies aim to estimate and compare train knock-on delay performance with different service intervals and signalling systems as follows:

(2) Case Study 2 considers the operation of six trains, using a 4-aspect signalling system, with a service interval of 300 s between trains. This case study is used as a base case;

(3) Case Study 3 considers the effect of reducing the first service interval to 200 s. The knock-on delays are simulated and analysed;

(4) Case Study 4 considers the effect of further reducing the first service interval to 150 s.

5.2 Test Route Case Study

5.2.1 General Purpose

A test route case study is first considered, which presents the operation of a single train. In order to verify the simulator, the simulation results are compared with previously published data from the manufacturer of the train (Evans, October 2007).

5.2.2 Route Configurations

The test route simulation is based on a line section between Lichfield Trent Valley and Rugby stations with an intermediate stop. A single Class 390 train is assumed to
operate on this line using a 4-aspect fixed block signalling system. Figure 5.1 and Figure 5.2 show the characteristics of the route, which are taken from Railway Track Diagrams (Jacobs, 2005).

![Figure 5.1. Line speed limits data.](image1)

![Figure 5.2. Track layout of the route with distance marks.](image2)

Table 5.1 shows the input data for the case study. The intermediate stop location and stopping times are taken from the published realistic data (Evans, October 2007). The Class 390 train data comes from the results in Section 5.2.3. The block length is
optimised to fit the braking distance of the train with a 4-aspect fixed block signalling system. Other data such as overlap and sighting distance are assumed in accordance with the UK main line standard (Alston, 2000; Genner, 1997; RSSB, 2011).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train data</td>
<td>Table 5.2, Figure 5.3</td>
</tr>
<tr>
<td>Altitude and line speed limit</td>
<td>Figure 5.1</td>
</tr>
<tr>
<td>Route length, km</td>
<td>54.38</td>
</tr>
<tr>
<td>Intermediate stop, km</td>
<td>51.15</td>
</tr>
<tr>
<td>Stopping time, s</td>
<td>30</td>
</tr>
<tr>
<td>Destination station location, km</td>
<td>54.38</td>
</tr>
<tr>
<td>Block length, m</td>
<td>1200</td>
</tr>
<tr>
<td>Overlap, m</td>
<td>180</td>
</tr>
<tr>
<td>Sighting distance, m</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 5.1.** Simulation input data for case study one.

### 5.2.3 Class 390 Train Modelling

The Class 390 Pendolino Electric Multiple Unit (EMU) tilting train is considered in the simulation. The detailed traction characteristics of the train for simulation are shown in Table 5.2 (Hill, 1994a; Hill, 1994b; Kemp, 2007).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traction effort, kN</td>
<td>203.7, if $0 &lt; v &lt; 24.47$ ms$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>5095.2$/v$, if $24.47$ ms$^{-1} \leq v &lt; 62.5$ ms$^{-1}$</td>
</tr>
<tr>
<td>Braking effort, kN</td>
<td>331.3</td>
</tr>
<tr>
<td>Resistance effort, kN</td>
<td>$5.422 + 0.069v + 0.012v^2$</td>
</tr>
<tr>
<td>Maximum speed, ms$^{-1}$</td>
<td>225</td>
</tr>
<tr>
<td>Maximum power at rail, kW</td>
<td>510</td>
</tr>
<tr>
<td>Regenerative power at rail, kW</td>
<td>0, if $0 &lt; v &lt; 9.64$ ms$^{-1}$</td>
</tr>
</tbody>
</table>
Table 5.2. Simulation configurations of Class 390 Pendolino.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>kW</td>
<td>1579.02v - 15221.76, if 9.64 m/s ≤ v &lt; 24.469 m/s</td>
</tr>
<tr>
<td></td>
<td>202.74v, if 24.469 m/s ≤ v &lt; 30.23 m/s</td>
</tr>
<tr>
<td></td>
<td>6153.66 , if 30.23 m/s ≤ v &lt; 62.5 m/s</td>
</tr>
<tr>
<td>Traction drive efficiency</td>
<td>85%</td>
</tr>
<tr>
<td>Maximum torque, kN</td>
<td>204</td>
</tr>
<tr>
<td>Powered axles</td>
<td>12</td>
</tr>
<tr>
<td>Trailer axles</td>
<td>24</td>
</tr>
<tr>
<td>Train formation, cars per set</td>
<td>9</td>
</tr>
<tr>
<td>Train mass (fully seated load), t</td>
<td>509.27</td>
</tr>
<tr>
<td>Train length, m</td>
<td>220</td>
</tr>
</tbody>
</table>

Figure 5.3. Traction system characteristics of the Class 390 Pendolino.

5.2.4 Simulation Results

Figure 5.4 shows the train movement trajectory and power for the fastest possible journey. The power produced by regenerative braking is presented in negative numbers. The values used in this figure have been acquired by digitising previously published data.
Table 5.3 shows the simulation results. The regenerative braking system produces 168 kWh. However, the overall efficiency of the traction system is 85% between overhead lines and rail. Therefore, only 143 kWh returns back to the overhead line (OHL) and it is assumed that all of this power is reused elsewhere. If the train is equipped with a regenerative braking system, the energy consumption will be 1147 kWh.

<table>
<thead>
<tr>
<th>Results</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation time, s</td>
<td>1.25</td>
</tr>
<tr>
<td>Journey time of the train, s</td>
<td>1376</td>
</tr>
<tr>
<td>Energy consumption of the train, kWh</td>
<td>1004</td>
</tr>
<tr>
<td>Regenerative braking energy, kWh</td>
<td>143</td>
</tr>
</tbody>
</table>

Computer specifications: CPU=Intel Core2 Q9550 (2.83 GHz); Memory=3 GB; Operation system=Microsoft XP Professional SP3; MATLAB Version=7.11.0 (R2011b)

Table 5.3. Simulation results of the test route.
5.2.5 Simulation Comparison

Figure 5.5 shows real data from Alstom (Evans, October 2007). The train journey time and energy cost in Table 5.4 are captured from Figure 5.5 by ‘GetData Graph Digitizer’.

![Figure 5.5. Realistic train trajectory on the 54 km test operation.](image)

As seen in Figure 5.5 and Table 5.4, it is clear that the simulation results are very close to the realistic data. However, the simulated train takes 13 s less in journey time and 15 kWh less in energy. This is considered to be mainly due to the difficult in precisely modelling the driving strategy used by the driver in the real case. The simulated train continued to cruise as soon as the speed reached the line speed limit, however the driver of the real train shifted between coasting and motoring a number of times during this phase. Therefore, the average speed of the simulated train is slightly greater than the real train, which further results in a lower journey time and a higher energy consumption.
Table 5.4. Realistic data of the test route.

<table>
<thead>
<tr>
<th>Results</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journey time of the train, s</td>
<td>1396</td>
</tr>
<tr>
<td>Energy consumption of the train, kWh</td>
<td>1000</td>
</tr>
</tbody>
</table>

The comparison shows that the simulator is sufficiently accurate to be used for further analysis as the simulation results were very close to the real data. It is therefore considered that the simulator meets the design requirements and is used for train interaction simulations and further trajectory optimisation.

5.3 Train Interaction Simulations

5.3.1 General Purposes

Section 5.3.2 to Section 5.3.4 consider six trains operating, using a 4-aspect signalling system. Different service intervals between trains are considered in order to evaluate different train interaction levels. Furthermore, different signalling systems are considered in Section 5.3.5 in order to find the relationship between the train interactions and the use of different signalling systems. To avoid variance between the simulations, flat out operation is used to aid comparison.

Definition 5.1 (Flat out operation). Flat out operation is a running style by which the train is operated with its maximum possible and permitted speed at every position along a journey.

As shown in Table 5.5, there are three stopping stations along the route, located at 0 km, 13.5 km and 27.5 km. Altitude and line speed limits are simplified in order to highlight the impact of using different service intervals and signalling systems.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train data</td>
<td>Table 5.2, Figure 5.3</td>
</tr>
<tr>
<td>Altitude</td>
<td>Flat ground</td>
</tr>
<tr>
<td>Line speed limit, km/h</td>
<td>200</td>
</tr>
<tr>
<td>Route length, km</td>
<td>27.5</td>
</tr>
<tr>
<td>Intermediate stop, km</td>
<td>13.5</td>
</tr>
<tr>
<td>Dwell time, s</td>
<td>120</td>
</tr>
<tr>
<td>Destination station location, km</td>
<td>27.5</td>
</tr>
<tr>
<td>Block length, m</td>
<td>1200</td>
</tr>
<tr>
<td>Overlap, m</td>
<td>180</td>
</tr>
<tr>
<td>Sighting distance, m</td>
<td>200</td>
</tr>
<tr>
<td>Service interval, s</td>
<td>Depends on case studies</td>
</tr>
</tbody>
</table>

Table 5.5. Simulation input data for train interaction simulations.

### 5.3.2 Undisturbed Motion Case Study

The service intervals between each pair of trains are assumed at 300 s in this scenario.

Simulation results are represented in Figure 5.6. Each vertical marker on the x-axis represents a signalling block. The green, single-yellow, double-yellow and red circles in the figure indicate the signal aspect of each block at a certain time. For instance, when the second train is approaching the 7th block (point (A) in Figure 5.6), the first train is stopping in the 12th block. The signals of the 11th, 10th and 9th blocks are displaying red, single-yellow, double-yellow and green aspects respectively. Therefore, at that moment, the second train is facing a green aspect, and the signal of the 6th block is displaying a red aspect to protect the second train.
Figure 5.6. An undisturbed motion.

It is observed that there is no interaction between the trains. As shown in Table 5.6, all trains face a green aspect throughout their journeys. In other words, each of them always keeps at least three clear blocks ahead.
Table 5.6. Signal aspects of the undisturbed motion.

<table>
<thead>
<tr>
<th>Train</th>
<th>9th block</th>
<th>10th block</th>
<th>11th block</th>
<th>12th block</th>
<th>20th block</th>
<th>21st block</th>
<th>22nd block</th>
<th>23rd block</th>
</tr>
</thead>
<tbody>
<tr>
<td>First train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Second train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Third train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Fourth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Fifth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Sixth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
</tbody>
</table>

The minimum service interval which results in an undisturbed motion is called the minimum line headway time:

**Definition 5.2** (Minimum line headway time or technical headway time): The minimum line headway time indicates the minimum time interval between a pair of trains, so that the following train will not be affected by the lead train throughout the journey.

**Definition 5.3** (Minimum point headway time): The minimum point headway time for a section of a line with a station stop indicates the minimum time interval between a pair of trains, so that the following train will not be affected by the lead train throughout the journey.
Compared with the minimum headway time which is described in Section 2.2, the minimum line headway time considers the pattern of stopping trains. In Figure 5.7, a single train is shown: (1) approaching, (2) stopped, and (3) leaving a station. In step (1) the train is running at line speed as it passes Signal 1. Then it travels a distance while braking to stop at the station. After a dwell time, the train accelerates and travels far away enough to clear the overlap of Signal 4. All of the above steps must be completed before the second train can pass Signal 1; if these steps are not completed the signal will display a restrictive aspect and will therefore impact on the second train’s journey.

![Diagram showing train movement and signal aspects](image)

**Figure 5.7.** 4-aspect signal minimum line headway distance.

The minimum line headway time therefore includes train cruising time, braking time, dwell time and time for the train to clear the overlap and the train length. The minimum headway for fixed block signalling can be calculated as follows:

\[
H_{\text{min}} = SD + \frac{v^2}{2b(n-2)} + \text{Braking time} + \text{dwell time} + \text{Time for clearing OD and LD} \tag{5.1}
\]
For moving block, the minimum headway can be calculated as follows:

\[
H_{\text{min}} = \frac{SD + \text{Block}}{V} + \sum_{x_{\text{start}}}^{x_{\text{start}} + (n-2)\times \text{Block}} \frac{\Delta x}{V_{\text{act}}(x)} + \text{Dwell time}
\]

\[
+ \sum_{x_{\text{start}}}^{x_{\text{start}} + \text{OD} + \text{LD}} \frac{\Delta x}{V_{\text{act}}(x)} \tag{5.2}
\]

Signalling has a major impact on the minimum line headway time which can be achieved by railway lines. This is mainly due to the changing of block lengths. Figure 5.8 has been developed to show the relationship between the minimum line headway time and the maximum line speed with different signalling systems. The decrement of the block length from a 4-aspect to a 5-aspect signalling system is much smaller than that from a 3-aspect to a 4-aspect signalling system. The improvement in the minimum line headway time therefore reduces as the signalling system becomes more advanced.
Moving block signalling can be described as fixed block signalling with an infinite number of aspects, so that the block length approaches zero. Moving block signalling is therefore able to achieve the smallest minimum line headway time.

More advanced signalling systems have shorter minimum line headway times, however, the number of signalling locations per kilometre increases, as shown in Figure 5.9. As the number of signalling locations per kilometre increases, the driver’s task becomes increasingly complex (Weber, 1975).

**Figure 5.8.** Minimum point headway time against operating speed. (dwell time=120 s)
5.3.3 Slightly Disturbed Motion Case Study

If the service interval is below the minimum line headway time, train interactions will occur and disturbances will result. The scenario shown in Figure 5.10 shows such a disturbed motion. In this scenario, the scheduled service interval between trains remains at 300 s, however, the first train is delayed by 100 s, resulting in a 200 s service interval between the first and second train.

In such a scenario, when a preceding train stops at a station, the distance interval between it and the following train is reduced. The trajectory of the following train will be disturbed as the distance between the two trains tends towards the minimum line headway distance. If the dwell time is extended for any reason, a more significant interaction will result between the two trains.
As can be seen in Figure 5.10, interactions occur to all three following trains as they are restricted by the signalling system when they approach Station 2. For example, the journey of the second train can be described as follows:

(1) At point (a), the driver sees a double yellow signal when approaching the 9th block. The train then starts to brake;

(2) At point (b), the driver sees a single yellow signal when approaching the 10th block. The train therefore continues braking and prepares to stop before the next red signal;

(3) At point (c), the driver at first sees a red signal when approaching the 11th block. So he continues braking and the train is ready to stop before the signal. However, the signal changes to a single-yellow aspect before the train actually stops. The driver therefore accelerates the train to a low speed and then keeps on cruising;

(4) At point (d), the train stops at Station 2.
Figure 5.10. A slightly disturbed motion.

The following trains experience slighter disturbances when they approach Station 3. This is because the previously occurred disturbances reduce the speed of the trains, and further increase their headway distances. As shown in Table 5.7, the second train
receives the most significant disturbance, as it faces two yellow aspects and three double-yellow aspects. Such interactions make the second train take 906 s for its journey, which is 7% longer than the undisturbed flat-out operation time.

<table>
<thead>
<tr>
<th></th>
<th>9th block</th>
<th>10th block</th>
<th>11th block</th>
<th>12th block</th>
<th>20th block</th>
<th>21st block</th>
<th>22nd block</th>
<th>23rd block</th>
</tr>
</thead>
<tbody>
<tr>
<td>First  train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Second train</td>
<td>Green</td>
<td>Double yellow</td>
<td>Single Yellow</td>
<td>Single Yellow</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Third train</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
<td>Double yellow</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Fourth train</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Fifth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Sixth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
</tbody>
</table>

Table 5.7. Signal aspects of the slightly disturbed motion.

5.3.4 Severely Disturbed Motion Case Study

In this section, in order to consider a large disturbed motion, the service interval between the first pair of trains is reduced to 150 s, which is much less than the minimum line headway time. The following trains therefore experience severely disturbances, as shown in Figure 5.11.
Figure 5.11. A severely disturbed motion.

Similar to the situation which happens in the slightly disturbed case, all of the following trains experience disturbances when approaching stations in the large disturbed motion case. However, in this scenario, the second train faces a red aspect
when approaching the 12th block. The train therefore has to stop in front of the red signal and wait for 24 s until the first train leaves the platform and clears the 12th block. This interaction results in the second train taking 965 s for its journey, which is 14% longer than the scheduled operation time. Moreover, the acceleration section (point (A) to point (B) in Figure 5.11) results in additional energy consumption. As a result, the train uses 73.80 kWh of energy, which is almost the same as the first train. On the other hand, an undisturbed train only requires 43.36 kWh when the associated journey time is 965 s. Therefore, if the second train can reduce its speed to avoid the interactions, it can save up to 41% of energy usage.

<table>
<thead>
<tr>
<th></th>
<th>9th block</th>
<th>10th block</th>
<th>11th block</th>
<th>12th block</th>
<th>20th block</th>
<th>21st block</th>
<th>22nd block</th>
<th>23rd block</th>
</tr>
</thead>
<tbody>
<tr>
<td>First train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Second train</td>
<td>Green</td>
<td>Double yellow</td>
<td>Single Yellow</td>
<td>Red</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Third train</td>
<td>Green</td>
<td>Double yellow</td>
<td>Single Yellow</td>
<td>Single Yellow</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Fourth train</td>
<td>Green</td>
<td>Double yellow</td>
<td>Single Yellow</td>
<td>Single Yellow</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
</tr>
<tr>
<td>Fifth train</td>
<td>Green</td>
<td>Green</td>
<td>Double yellow</td>
<td>Double yellow</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Sixth train</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
</tbody>
</table>

Table 5.8. Signal aspects of the slightly disturbed motion.
5.3.5 Comparison with Different Signalling Systems

Section 5.3.4 considered a slightly disturbed motion with a 4-aspect signalling system. In order to analyse and evaluate the impact of using different signalling systems, five signalling systems are considered in this scenario. Figure 5.12 displays the results together. It is necessary to point out that each vertical marker on the x-axis does not represent a signalling block. This is because the block separations have to be optimised separately, in accordance with the different signalling systems.

Figure 5.12. Journey time against distance for different signalling systems.
Only the paths of the first two trains are presented in order to highlight the impact of using different signalling systems. Several interesting performance characteristics can be observed:

(1). For fixed block signalling systems, the performance improvement increases as more aspects are used. In this particular case study, there is a significant difference between 3-aspect and 4-aspect signalling systems; the second train comes to a complete stop when 3-aspect signalling is applied;

(2). More advanced signalling systems provide better speed control strategies, resulting in fewer disturbances compared with simpler systems. However, the difference in performance between the most advanced signalling systems is small.

5.4 Summary

This chapter presents four train movement case studies. The test case study described in Section 5.2 aimed to verify the results of the simulator against real-world data. The comparison in Section 5.2.5 showed that the simulator is accurate and meets the design requirements as it provides simulation results close to the real data available in the public domain (Evans, October 2007). In Section 5.3, six trains are simulated to operate on a single track railway. Different service intervals are assumed to evaluate different levels of train interactions. The results shows that the knock-on delay caused by the interactions will increase the train journey time and energy cost. From Section 5.3.5, it can be seem that the severity of train interactions can be reduced through the application of more advanced train signalling systems. Based on the results obtained in this chapter, the next chapter will develop a train trajectory
optimisation method. The study aims to minimise an objective function which considers the trade-off between energy usage and delay penalty.
Chapter 6 Train Trajectory Optimisation

6.1 Introduction

The train trajectory, which is conventionally controlled by the driver but, in modern train control system, maybe calculated and implemented automatically, defines the train/speed profile of the train against distance. The objective of train trajectory optimisation is to find a trajectory that minimises or maximises the performance of a given parameter or parameters (e.g. energy usage) (Lu et al., 2013; Wang et al., 2011; Su et al., 2013). The trajectory which results in the most preferable outcome, given some defined objective function, is selected as the optimal trajectory. For the same infrastructure operating the same trains, the type of control system and the objective function used for optimisation will change which solution is considered as optimal.

Previous researchers have considered different methods for train trajectory optimisation. Bocharnikov used a Genetic Algorithm to find the most appropriate upper and lower boundaries for coasting operation in order to achieve a good trade-off between energy cost and journey time (Bocharnikov et al., 2010). Artificial Neural Networks have also been proposed to find the optimal train coasting levels (Chuang et al., 2008). Other works consider the optimisation of general control (i.e., acceleration rate, target speed, cruising level, braking rate), for example, Kim and Chien presented a model for railway line simulation which uses a Simulated Annealing Algorithm to optimise the train trajectory to minimise energy consumption by considering track alignment, speed limits and schedule adherence (Kim et al., 2011). Duarte uses the
Gradient-Restoration method to determine the optimal trajectory for subway trains (Duarte et al., 1999).

In this chapter, train trajectory optimisation is discussed. Four different search methods are considered to find an optimal train target speed series to achieve the best balance between train energy consumption and delay penalty. A weighted combination of these two parameters is used as the objective function (cost function). Furthermore, different driving styles are applied to the optimisation in order to consider any particular service given by the train operating company.

### 6.2 Train Trajectory Control

Under the GB Rail Performance Regime, train operators must pay penalties if their trains disturb services run by other operators. This regime aims to provide an incentive for train operators to minimise delays.

In 2009, 4 million delay minutes were attributed to train operators (Network Rail, 2009). Operators must pay from a few pounds to over £100 per minute for every train affected, with an average of around £14 per minute (Thomas, 2008). In the thesis, £10 per minutes is used. Therefore, a serious delay which affects services can result in a significant financial loss to the operator. For short delays it may be possible to increase the speed of a train to minimise the penalty cost. However, increasing the speed of the train will also increase the energy consumption (and hence cost), and may also increase the likelihood of an interaction with the preceding train (Jiang et al., 2010; Bai et al., 2011). It is therefore appropriate to consider the balance between journey time (delay) and energy usage.
The developed simulator can control the train trajectory through a series of train target speeds. Each train receives a target speed ($VT$) when it proceeds to a station, as shown in Figure 6.1.

**Figure 6.1.** Target speed distribution.

As shown in the ‘input’ block in Figure 6.2, using the multiple train simulator, train journey time ($T_{run}$) and train trajectory can be obtained. Train energy usage ($E_{run}$) is calculated by the following equation:

$$E_{run} = \int_0^{T_{run}} u_f(t) f[v(t)]v(t)dt$$

(6.1)

where:

- $T_{run}$ is the simulated journey time;
• $u_f$ is the control signal for forward tractive effort;

• $f[v(t)]$ is the maximum tractive effort at the current vehicle speed $v(t)$.

The forward tractive effort control signal will equal zero when the power is shut down, i.e., in coasting mode or braking mode. In that case, no energy usage will be included.

\[
\begin{align*}
\dot{s} &= v \\
\dot{v} &= u_f \cdot f(v) - u_b \cdot b[v(t)] - r(v) - g(s)
\end{align*}
\] (6.2)

where

• $s$ is the train position;

• $u_b$ is the control signal for backwards braking effort;

• $b[v(t)]$ is the maximum braking effort at the current vehicle speed $v(t)$;

• $g(s)$ is the force due to the gradient;

• $r(v)$ is the resistive force at the current vehicle speed, which can be calculated by the following equation with constants $a$, $b$, $c$, known as the Davis equation (Loumiet et al., 2005). The constants are empirical and relative to the track and aero-dynamic resistance.

\[
r(v) = a + b |v| + cv^2
\] (6.3)

The boundary conditions are given as follows, where the initial conditions are:

\[
\begin{align*}
v(0) &= 0 \\
s(0) &= 0
\end{align*}
\] (6.4)

And the final conditions are:
Some constraints are imposed:

\[
\begin{align*}
\{ & v(T) = 0 \\
& s(T) = S_t \\
& v \leq v_{\text{lim}}(s) \\
& u_f \in [0, 1] \\
& u_b \in [0, 1]
\end{align*}
\]  \hspace{1cm} (6.6)

where:

- \( v_{\text{lim}}(s) \) is the train target speed or line speed limit (whichever is smaller) at the current position (\( s \)).

Table 6.1 shows the control signal values for \( u_f \) and \( u_b \) in motoring, cruising, coasting and braking modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Tractive effort control signal, ( u_f )</th>
<th>Braking effort control signal, ( u_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motoring mode</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Cruising mode</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Coasting mode</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Braking mode</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 6.1.** Forward and backwards control signal values for different tractive modes.

In order to rank different train trajectories, the outputs of the simulator have to be evaluated by an objective function. The objective function used here is a cost function, thus the train trajectory that results in the lowest evaluation value will be considered to be the optimum solution.
6.3 Cost Function

The aim of train trajectory optimisation is to find the most appropriate train target speed series to minimise a function that includes energy usage and train punctuality for all trains. The cost function to be minimised is thus:

\[ M_{opt} = \sum_{i=1}^{n} (D_{run_i} w_d F_d + E_{run_i} w_e F_e) \]  \hspace{1cm} (6.7)

where:

- \( n \) is the number of trains to be optimised;
- \( D_{run} \) is the simulated delay in second;
- \( F_d \) is the unit delay penalty cost per second;
- \( F_e \) is the unit energy cost per kWh;
- \( w_i \) and \( w_e \) are the weightings that are associated with the delay penalty and energy usage respectively.

The delay cost can be calculated from the simulated journey time using the following equations (Vanderbei, 2000):

\[ D_{run} = \sum_{j=1}^{m} \begin{cases} T_{run_j} - T_{s_j}, & \text{if } T_{run_j} \geq T_{s_j} \\ 0, & \text{if } T_{run_j} < T_{s_j} \end{cases} \]  \hspace{1cm} (6.8)

where:

- \( m \) is the number of stations;
- \( T_{s} \) is the scheduled journey time between two stations.
This method provides a general approach that can be applied to different scenarios by changing the weightings, \( w_t \) and \( w_e \). The approach is able to consider:

1. Both journey time and energy usage together as both parameters are transformed into costs;
2. A delay penalty that can be varied along a route;
3. The interactions between trains are considered by recalculating the behaviour of the second and subsequent trains based on the performance of all trains in the network, apart from the leading train.

Four search methods, namely Enhanced Brute Force, Dynamic Programming, Genetic Algorithm and Ant Colony Optimisation, are used to search for the optimum target speed series using the objective function defined by Equation (6.7).

### 6.4 Optimisation Methods

#### 6.4.1 Enhanced Brute Force

In this thesis, each potential target speed in a series is assumed as a candidate solution and has an individual solution domain. The complexity of the domain depends on the assumed search boundary and search interval. A conventional brute force search enumerates all possibilities in the solution domain to find the optimum (Preneel et al., 2009). This rapidly becomes impractical due to the processing time required for complex problems (Levitin, 2003). The enhanced brute force algorithm is able to address this problem by constraining the solution domain (Faheem, 7-10 February 2010).
Figure 6.3 shows a flowchart for the enhanced brute force algorithm suggested by the author.

1. Firstly, the method calculates a series of estimated target speeds \( V_{T_{est_1}}, V_{T_{est_2}} \ldots V_{T_{est_i}} \) by comparing the train service interval \( (T_{s_{int}}) \) with the minimum line headway time \( (HD_{t_{min}}) \), as shown in Equation (6.9). If the service interval is smaller than the minimum line headway, in theory the train has to stop somewhere along the track and wait for the leading train to move far enough ahead. Therefore the estimated journey time is the scheduled journey time \( (T_{est}) \) plus the waiting time.

\[
T_{est} = T_s + (HD_{t_{min}} - T_{s_{int}})
\]  

(6.9)

Based on Equation 4.12, the estimated target speed \( (VT) \) can be calculated using Equation 6.10.

\[
VT = g^{-1}(a, T_{est})
\]  

(6.10)

2. The estimated speed target speed can only be considered as an initial approximation as the minimum line headway time is based on an ideal station position and a flat route. However, such a result can help direct the search method and reduce the solution domain. Using this approach the search method can be constrained to only consider candidate solutions that are near to the estimated series.

Based on Equation 4.11 and Equation 4.12, the enhanced brute force method enumerates all possibilities in the reduced solution domain and a number of energy cost and journey time pairs are produced using Equation 6.11.

\[
A(D_{run}, E_{run}) = \sum_{x=V_{T1}}^{b} \sum_{y=V_{T2}}^{b} f(a, VT_1 \ldots VT_i)
\]  

(6.11)
Finally, all the energy cost and journey time pairs are evaluated using the cost function, Equation 6.7. The top ranking solution is selected as the most appropriate.

![Flowchart of the Enhanced Brute Force Algorithm approach.](image)

### 6.4.2 Dynamic Programming

As presented in Section 3.2.2, Dynamic Programming is a methodical procedure which systematically evaluates a large number of candidate solutions in a multi-step problem. It uses advanced searching approaches and will always identify the true optimal solution. Each candidate solution includes a set of sub-solutions and each sub-
solution is associated with one sequential problem step. All candidate solutions must be selected during the optimisation process. Therefore, a larger searching boundary or a smaller searching interval will result in a larger solution domain and hence a greater computational time.

A ‘transition path’ is associated with each sub-solution to indicate the decision from one problem step to the following one (Singhal et al., 2011). Furthermore, each transition path will result in a ‘transition cost’. Dynamic Programming aims to make a decision in each problem step that minimises the total cost for all the decisions made.

As shown in Figure 6.4, in this optimisation study, each target speed series can be assumed as a candidate solution which is associated with a subset of target speeds (nodes, $VT_1… VT_i$). Searching the best node at each step can be considered as a sequence problem step. Once a node has been selected, the associated train will be able to complete a part of its journey. The delay cost and energy cost will be output and evaluated using an equation to calculate the transition cost ($C$). The equation is based on the objective function, Equation (6.7).

$$C = D_{run}\cdot w_d F_d + E_{run}\cdot w_e F_e$$ (6.12)

The method will finally find the best path (target speed series) in terms of the transition cost to achieve the lowest total cost.
Figure 6.4. A diagram of the route selection of the Dynamic Program.

The algorithm has the following steps:

(1). Based on the search boundaries, search interval and objective function, the algorithm firstly generates a search map with a number of different target speeds (sub-solutions);

(2). The method then calculates the transition costs from the starting point to each of the nodes in Step 1. Typical results are as shown in Equation 6.13.

\[
\begin{align*}
C_{11} &= C_{0-11} \\
C_{12} &= C_{0-12} \\
C_{13} &= C_{0-13} \\
\vdots \\
C_{1n} &= C_{0-1n}
\end{align*}
\]  

(6.13)

By choosing the lowest transition cost, the best path to reach Step 1 from the starting point can be obtained, as shown in Equation 6.14:
Next, the method calculates the transition cost from each of the nodes in Step 2 to those in Step 3. With the addition of the previous results, the total cost of each node will be:

\[
\begin{align*}
C_{21} &= \min\{C_{11} + C_{1-21}, C_{12} + C_{12-21}, C_{13} + C_{13-31}, ..., C_{1n} + C_{1n-21}\} \\
C_{22} &= \min\{C_{11} + C_{11-22}, C_{12} + C_{12-22}, C_{13} + C_{13-32}, ..., C_{1n} + C_{1n-22}\} \\
C_{23} &= \min\{C_{11} + C_{11-23}, C_{12} + C_{12-23}, C_{13} + C_{13-33}, ..., C_{1n} + C_{1n-23}\} \\
& \vdots \\
C_{2n} &= \min\{C_{11} + C_{11-2n}, C_{12} + C_{12-2n}, C_{13} + C_{13-3n}, ..., C_{1n} + C_{1n-2n}\}
\end{align*}
\]

(6.15)

By choosing the lowest total cost, the best path to reach Step 2 from the start point can be obtained. The lowest cost will be:

\[
C_2 = \min\{C_{21}, C_{22}, C_{23}, ..., C_{2n}\}
\]

(6.16)

(4). The algorithm repeats the above process until all the nodes have been selected and the best whole path is finally obtained. The total cost will be:

\[
C_{all} = C_1 + C_2 + C_3 + \ldots + C_i
\]

(6.17)

After the calculation at each step, every node knows the best path from the start point to itself. Therefore the method only needs to consider the next path from each node, which avoids calculating the same sub-problem repeatedly, which reduces the computation time. The searching time will be:

\[
T = n + n^n(i - 1)
\]

(6.18)
6.4.3 Genetic Algorithm

An alternative approach is to use a genetic algorithm (GA), a type of metaheuristic. As presented in Section 3.3.2, compared with exact algorithms, such as Brute Force, Genetic Algorithms use a heuristic method to converge on a result, rather than calculating all the possible candidate solutions. Therefore, metaheuristics cannot be guaranteed to find the optimal solution; however, the computational cost is much smaller because these methods trade near-optimal solutions for efficiency.

A genetic algorithm is a search procedure that is based on the rules of genetics and natural selection (Cox, 2005). As a metaheuristic approach, it does not require gradient information of the objective function. The algorithm presents an iterative and stochastic process that operates on a population of individuals. Each individual comprises one or more chromosomes, which allow each individual to represent a potential solution to a given problem, where each individual is assigned a value associated with the fitness of this solution. Individuals with higher fitness have a greater likelihood of being selected to generate the individuals in the next population. The algorithm runs until the population converges to an optimum solution to the problem, or until it is stopped by a termination function.

Figure 6.5 shows a general view of the genetic algorithm. In this study, the algorithm starts by randomly generating an initial population of solutions. This is followed by applying a genetic operator that includes selection, recombination (crossover), mutation and reproduction to this population.
Figure 6.5. The diagram of the genetic algorithm.

The algorithm has the following steps:

1. Each population is made up of individuals that represent a target speed series. 20 individuals \( C_1, C_2 \ldots C_{20} \) are created in each population;

2. All target speed series are input into the multi-train simulator and the solutions are ranked using the objective function;

\[ \begin{array}{c}
\text{Generate initial population} \\
\begin{array}{cccc}
V_{T1} & V_{T2} & V_{T3} & \ldots & V_{TP} & V_{TP+1}
\end{array} \\
C_1 & 47 & 31 & 89 & \ldots & 37 & 51 & 42 \\
C_2 & 58 & 41 & 37 & \ldots & 12 & 74 & 27 \\
C_{20} & 54 & 45 & 36 & \ldots & 47 & 39 & 41 \\
\end{array} \\
\text{Evaluate population} \\
\text{Stop conditions} \quad \text{Yes} \\
\text{Select parents} \\
\text{Recombination} \\
\text{Mutation} \\
\text{Replacement} \\
\text{End} \\
\begin{array}{ccc}
52 & 49 & 37 & \ldots & 44 & 38 & 55
\end{array} \]
(3) In order to generate a population for the next generation, 20 new individuals are created using the following processes:

1. Selection. In the first step, the algorithm chooses some suitable individuals (parents) for breeding new individuals (offspring). The probability of selecting individuals is proportional to their fitness;

2. The top two ranking individuals are retained for the next generation;

3. Recombination and mutation: After selection, the next 12 ranked individuals are put into pairs and ‘crossed-over’. The parents’ chromosomes will be combined using a recombination operator to give birth to offspring. The recombination is a process that models information exchange among several individuals. It is done by randomly swapping one or more genomes between two chromosomes. The last six ranked individuals are mutated. Mutation is a process that injects new material into a population. It is done by randomly substituting a genome of a chromosome with a different genome;

4. Replacement. In the last step, the new individuals are evaluated and take the place of their parents. The genetic operation runs until termination conditions are achieved.

(4) Finally, the program returns to the second step and repeats the processes.

The algorithm runs until either of the termination conditions are achieved, namely, the cumulative change in the fitness function value is less than the function tolerance or the number of generations exceeds the maximum allowable number.
6.4.4 Ant Colony Optimisation

The fourth method is to use another heuristic approach, namely Ant Colony Optimisation (ACO). The inspiration for ant colony optimisation comes from the foraging behaviour of some ant species (Dorigo et al., 2006), as explained in Section 3.3.3. In ACO, a number of artificial ants are created to build solutions to an optimisation problem and exchange information on the solution optimality via pheromones. The ant with the best solution leaves more pheromone on the route. The route with the most pheromone attracts further ants and therefore the optimisation process converges.

**Construction graph and solution**

In this study, each target speed is assumed as one node, thus a target speed series can be considered as a set of nodes. The selection of the next target speed series from the start position can be represented as a route (edge), as shown in Figure 6.6. Choosing different routes results in different costs (COS) for train energy and delay penalty. In order to reduce the number of candidate nodes and hence computational complexity, a searching boundary and a searching interval are assumed.

It is assumed that when ant \( k \) is at start position \( i \), the possibility of route \( j \) being selected for the next position is as follows:

\[
p^k_{ij} = \frac{PHE^\beta_{ij} \cdot DES^\beta_{ij}}{\sum_{real\ allowed_k} PHE^\beta_{ir} \cdot DES^\beta_{ir}}, \quad j \in allowed_k
\]  

(6.19)

where:

- \( allowed_k \) is the feasible routes of ant \( k \) when at the starting position, \( i \);
- **PHE** is the amount of pheromone deposited for the transition from the starting position, \( i \);
- **DES** is the desirability of the transition from the starting position, \( i \) (typically \( 1/COS_{ij} \));
- \( \alpha \) and \( \beta \) are the parameters that control the influence of **PHE** and **DES**.

**Figure 6.6.** A diagram of the route selection of the Ant Colony Optimisation.

It is obvious that the cost of the route makes a key difference in the heuristic information. Each route that the artificial ants have passed can be evaluated by using Equation 6.7 as a cost function and can be further ranked by probability using Equation 6.19. Each ant at the next generation chooses the route based on a random proportional rule. In order to avoid obtaining a local optimum and achieving a fast convergence, 40% of the artificial ants will consider the routes which are close in value to the route with a high pheromone level. Therefore the nearby versus nodes will be chosen randomly to form a new route. Furthermore, 20% of the artificial ants will choose a brand new random route.
Pheromone update and termination condition

Based on the rules described above and by applying Equation 6.19, each artificial ant is able to select the next possible route to complete the journey. Afterwards, the pheromone change on each route will be updated using the following equations:

\[
\Delta PHE = \begin{cases} 
\sum_{k=1}^{m} \frac{Q}{\cos(j)}, & \text{if ant } k \text{ transits from starting position, } i, \text{ via node } j \\
D, & \text{if no ant transits from start position, } i, \text{ via node } j
\end{cases}
\]

where:

- \( Q \) is the parameter to control the influence of \( \Delta PHE \);
- \( D \) is a negative number;
- \( m \) is the number of the ants that transit from start position \( i \) via node \( j \).

The termination conditions are made by two criterions. Firstly, the number of generations exceeds the maximum allowable number. Secondly, the number of generations in which the best solution has not been changed exceeds the maximum allowable number.

6.4.5 Optimisation Methods Comparison

The previous sections have described the application of four different searching methods for the optimisation. This section shows the results and a comparison between them.

The optimisation is based on the previous slightly disturbed motion case study. The same route data, signalling system, train performance and service intervals are
retained. Some of the key parameters for the simulator, as introduced in Chapter 5, are shown in Table 5.2, Table 5.5 and Figure 5.3.

The following trains are expected to achieve an optimised total cost by applying an optimal train target speed series which is calculated by the four searching methods. Different delay penalty weighting and energy usage weighting ratios are assumed by the cost function in order to consider different driving styles, i.e. 2/8, 5/5 and 8/2. A further discussion on driving styles is provided in Section 6.5.

Figure 6.7 shows the train trajectories with 4-aspect signalling systems using the different optimisation methods. The first subfigure shows that if all trains are operating without any optimisation (i.e., flat-out running), interactions will occur. The other subfigures show that all the following trains should brake in advance to their scheduling stopping point due to the restrictive signal located on the fourteenth block. The second train experiences the strongest disturbance as it is the only train that passes a single yellow aspect (the others only pass a double yellow signal).

The trains in the other four subfigures are running with optimal target speed series. As shown in the subfigures, when the weighting ratio 5:5 is applied in the objective function, all the following trains operate at lower speeds and no interactions occur. The difference between all optimal target series and their applications (see Figure 6.10) is small. Without considering the computation time, all of the searching methods can successfully produce an optimal, or close to optimal, solution.
Figure 6.7. Train trajectories with 4-aspect signalling systems using different optimisation methods when the weighting ratio is 5:5.

Figure 6.8 and Figure 6.9 demonstrate the procedure by which the objective function output evolves with the generation using GA and ACO when the weighting ratio is 5:5. It can be observed that, due to the effect of the guidance, the algorithms obtain better solutions in each new generation. The searching finally converges to the
optimum in the 55\textsuperscript{th} and 59\textsuperscript{th} generations respectively, whilst the best individuals achieve the cost value at 1038 and 1048. Table 6.2 shows the five top individuals of the 20\textsuperscript{th} generation in a GA process. It can be seen that the optimised speed limit series is able to reduce the total cost by up to 17.92%. It is important to note that some higher ranking individuals may achieve higher total costs than the lower ranking ones. This is considered to be mainly due to using different weighting ratios (e.g. 8:2) which change the operation priority.

<table>
<thead>
<tr>
<th></th>
<th>Individual (speed limit series)</th>
<th>Cost function value</th>
<th>Total cost (£)</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.9 41.8 50.0 40.5 49.2 41.9</td>
<td>1042.5</td>
<td>208.5</td>
<td>17.9%</td>
</tr>
<tr>
<td>2</td>
<td>48.9 41.8 50.0 40.5 48.9 41.9</td>
<td>1042.9</td>
<td>208.6</td>
<td>17.9%</td>
</tr>
<tr>
<td>3</td>
<td>48.9 41.8 50.0 40.5 48.9 41.7</td>
<td>1043.15</td>
<td>208.6</td>
<td>17.9%</td>
</tr>
<tr>
<td>4</td>
<td>48.9 41.5 50.0 40.5 49.2 41.9</td>
<td>1044.16</td>
<td>208.8</td>
<td>17.8%</td>
</tr>
<tr>
<td>5</td>
<td>48.9 42.8 50.0 40.5 48.9 41.9</td>
<td>1046.09</td>
<td>209.2</td>
<td>17.6%</td>
</tr>
<tr>
<td>-</td>
<td>55.6 55.6 55.6 55.6 55.6 55.6</td>
<td>-</td>
<td>254.0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 6.2.** Top five individuals of the 20\textsuperscript{th} generation in a GA process.
Figure 6.8. The mean and minimum outputs at each generation using a GA when the weighting ratio is 5:5.

Figure 6.9. The mean and minimum outputs at each generation using ACO when the weighting ratio is 5:5.
The best result when using different weighting ratios are compared for the four optimisation algorithms in Figure 6.10. The weighting ratios range from 1:9 to 9:1. It can be observed that the enhanced brute force and dynamic programming methods both consistently identify the optimum solutions. However, it should be noted that there will be a difference in computation time between the two problems, which is dependent on the complexity of the problem.

The results produced from the enhanced brute force and dynamic programming methods can thus be used as reference values to compare the results from other search methods. It can be found that both the GA and ACO result in suboptimal solutions. It can be seen in Figure 6.10 that the GA obtains better results in comparison to the ACO for the weighting ratios range from 1:9 to 5:5. However, as the delay weighting ratio changes with a heavier weight towards delay, solutions are generated that have higher train speeds (this is experienced for weightings above 6:4). As a result, the train movements, and hence interactions, become more complex, which creates increasing problems with algorithm convergence. As less heuristic information is used by the GA, the approach can become unstable and achieve poor solutions when the weighting ratios are 8:2 and 9:1.
Figure 6.10. Cost function output against weighting ratios curves using different algorithms.

<table>
<thead>
<tr>
<th></th>
<th>Mean cost function value</th>
<th>Deviation (%)</th>
<th>Average optimisation computation time, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>No optimisation</td>
<td>1216</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Enhanced Brute Force</td>
<td>1034 (-15%)</td>
<td>0</td>
<td>117542</td>
</tr>
<tr>
<td>Dynamic Programming</td>
<td>1034 (-15%)</td>
<td>0</td>
<td>1820</td>
</tr>
<tr>
<td>Genetic Algorithm</td>
<td>1058 (-13%)</td>
<td>36.8</td>
<td>481</td>
</tr>
<tr>
<td>Ant Colony Optimisation</td>
<td>1073 (-12%)</td>
<td>9</td>
<td>654</td>
</tr>
</tbody>
</table>

Table 6.3. Computational time comparison between the four algorithms applied for 10 runs.

In this comparison study, each algorithm was run 10 times. The mean cost function value, deviation (the dispersion of the data elements from the mean) and average optimisation computation time are presented in Table 6.3. The computer is equipped
with Intel Core2 Q9550 (2.83GHz) CPU and 3 GB memory in Microsoft XP Professional SP3 and MATLAB 7.11.0.

It can be observed that the results achieved by ACO have lower deviation than those produced by the GA. This is because for the GA the heuristic information is taken from the output of individuals in each generation and the GA thus ensures that better individuals have more impact on the future search direction. Therefore, the heuristic guidance makes the results vary within a larger range. Compared with the GA, ACO relies on heuristic information that can guide the search procedure into a relatively fast convergence. Therefore, such strong heuristic information ensures ACO achieves robust results and hence smaller deviation. Since the enhanced brute force algorithm and dynamic programming are exact algorithms that produce optimal results, they are considered to have no deviation.

Compared with the metaheuristic algorithms, the enhance brute force algorithm and dynamic programming achieve better mean values but have significantly higher computation times. Especially for the enhanced brute force, the searching time efficiency is obviously sacrificed. Since each target speed in a series implies a candidate solution, the computational complexity becomes enormous. Compared with exact algorithms, both the GA and the ACO provide a more intelligent and efficient searching method.
6.5 Driving Styles

6.5.1 General Purpose

In the previous sections four methods to optimise the train trajectory have been discussed. It is important to be aware that the weighting parameters in Equation 6.7 will vary according to the driving style that a train operating company may give to a particular service, that is, the trade-off between journey time and energy usage. In this thesis, three driving styles are considered:

1. Maximum speed operation (flat-out). Trains are driven at the maximum line speed, which results in interactions between trains on busy lines. Interactions will result in increased journey times and energy costs;

2. Journey time priority operation (journey time priority). The train trajectory of the second train is optimised for journey time, taking into account the interactions between the two trains. This can be achieved by applying a large weighting parameter to the delay penalty but a smaller one to the energy usage. Such an driving style will result in an optimal journey time at a moderate energy cost;

3. Interaction aversion operation (cautious). A speed trajectory for the second train is calculated to ensure that no interactions occur. This can be achieved by applying a small weighting parameter to the delay penalty and a greater one to energy usage. It can also be achieved by applying large weighting parameters specifically to the candidate solution that will result in a disturbed operation. This driving style will result in a longer journey time with a lower energy cost.
Chapter 6 Train Trajectory Optimisation

The selection of a particular driving style will result in a different set of weightings being used in the evaluation function. Using the optimisation algorithms, the optimal train speed limits will be found in line with the evaluation function. The optimal train trajectory is therefore produced, as shown in Figure 6.11.

\[\text{Operating Priority Cost function Optimisation algorithm Optimal train trajectory Weighting parameters Optimal train target speed series Route profile Gradient, Line speed limits Instant running info.}\]

**Figure 6.11.** Flowchart of the optimisation system.

### 6.5.2 Driving Styles Comparison

Figure 6.12 shows an example of the train trajectory of a second train when using the three different driving styles. The flat-out operation train (the solid line) is running at the line speed, which is 200 km/h. The speeds of the trains using the journey time priority operation (the dashed line) and the cautious operation (the dotted line) are lower, at 178.2 km/h and 108 km/h respectively. It can be observed that the flat-out operation train experiences the strongest disturbance, followed by the journey time priority operation train. It is important to note that the journey time achieved through flat-out operation is higher than that of the journey controlled using journey time priority operation. This is because the acceleration section (from point (a) to point (b) in Figure 6.12) takes additional time. The train using cautious operation is not affected throughout its operation, as the driver will always see green aspect signals; however, this type of operation results in the longest journey time, as shown in Table 6.4.
Figure 6.12. Train trajectories for a single journey using three different driving styles.

<table>
<thead>
<tr>
<th></th>
<th>5th block</th>
<th>6th block</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, km/h</td>
<td>Signal aspect</td>
<td>Speed, km/h</td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>200</td>
<td>Yellow</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>178.2</td>
<td>Yellow</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>108</td>
<td>Green</td>
</tr>
</tbody>
</table>

Table 6.4. Signal aspects and speed of the train using the three driving styles.

6.6 Summary

In this chapter, the author described a train trajectory optimisation study. The study was aimed at finding the most appropriate train trajectory to minimise a function that includes energy usage and delay for all trains. A method developed to achieve this aim was introduced in Section 6.2. It uses a series of train target speeds to control the train trajectory. In order to rank different train target speed series, a cost function was introduced in Section 6.3. The function is combined with a delay penalty and an
energy cost, together with two weighting parameters. In order to find the best result quickly and efficiently, four search methods were introduced in Section 6.4.1 to Section 6.4.4. A comparison of the results between these methods is given in Section 6.4.5, which shows that dynamic programming is well suited to solve such an optimisation problem as it achieves the best solution in an acceptable computation time. Finally, three train driving styles were described in Section 6.5 to consider the use of different weighting parameters in the cost function. Based on the searching methods and the driving styles that have been presented in this chapter, the next chapter will describe a case study that considers a section of the West Coast Main Line in the Britain.
Chapter 7  West Coast Main Line Case Study

7.1 Introduction

In order to assess the operational impact of using optimised train trajectories and different practical train control systems, a West Coast Main Line based case study has been developed. The case study is based on the simulation framework described in Section 5.3.3. Four Class 390 Pendolino EMU tilting trains are simulated to operate on the line. The scheduled service interval between trains is normally 300 s, however, in the case study, the departure time of the first train is delayed by 100 s, resulting in a 200 s service interval between the first and second train. By estimating and comparing train knock-on delay performance, the case study is aimed at analysing the impact of using different driving styles and different train control systems on a practical railway line.

In order to achieve the objective, three scenarios are considered:

(1) All trains using the flat-out driving style;
(2) All trains using the journey time priority driving style;
(3) All trains using the cautious driving style.

Each scenario above is further combined with the following six practical ETCS system configurations, which were detailed in Section 2.5:

(1) Intermittent ATP overlay without infill;
(2) Intermittent ATP overlay with single infill;
(3) 4-aspect fixed block signalling system;
(4) Continuous ATP overlay;
(5) Continuous in-cab overlay;
(6) Moving block.

In Chapter 6, dynamic programming is considered to be a well suited searching method for train trajectory optimisation. Therefore, in this case study, it will be used to find the most appropriate train target speed series for each scenario.

7.2 Route Configuration

The case considered in this thesis is based on the West Coast Main Line in Great Britain between Rugby and Birmingham International stations. This section of the route is 35.9 km long and has an intermediate station which is located at Coventry, 18.78 km from Rugby station, as shown in Figure 7.1. The speed limit and gradient profiles of this section are shown in Figure 7.2.

Figure 7.1. Track layout of the route with distance marks.
The calculations within the case study have been based on the assumptions outlined in Table 7.1. The layout is based on the data provided in Railway Track Diagrams (Jacobs, 2005).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train data</td>
<td>Table 5.2, Figure 5.3</td>
</tr>
<tr>
<td>Altitude, m</td>
<td>Figure 7.2</td>
</tr>
<tr>
<td>Line speed limit, kmh^{-1}</td>
<td>Figure 7.2</td>
</tr>
<tr>
<td>Train control system configuration</td>
<td>Table 4.4</td>
</tr>
<tr>
<td>Route length, km</td>
<td>35.90</td>
</tr>
<tr>
<td>First station location, km</td>
<td>0</td>
</tr>
<tr>
<td>Second station location, km</td>
<td>18.78</td>
</tr>
<tr>
<td>Third station location, km</td>
<td>35.90</td>
</tr>
<tr>
<td>Dwell time, seconds</td>
<td>120</td>
</tr>
<tr>
<td>Service interval, s</td>
<td>Dependent on trains</td>
</tr>
<tr>
<td>Sighting distance, m</td>
<td>180</td>
</tr>
<tr>
<td>Overlap, m</td>
<td>180</td>
</tr>
<tr>
<td>Safety margin, for moving block, m</td>
<td>180</td>
</tr>
</tbody>
</table>
## 7.3 Simulation Result

The simulation results are presented from Section 7.3.1 to Section 7.3.4 in train sequence. Section 7.3.5 presents the simulation of the whole network (4 trains together) and summarises the findings.

### 7.3.1 The First Train

As presented in Section 4.3.2, the first train can be considered to run without any effects from other trains because the route is empty. Therefore, the motion of the first train has not been disturbed in any of the three scenarios when combined with any of the control systems. The results are shown in Table 7.2. The journey time and energy cost are the same as those of ordinary scheduled trains but a £34 delay penalty is caused due to the late departure (100 s); These can be used as reference values to compare with the results from other trains.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train length, m</td>
<td>200</td>
</tr>
<tr>
<td>Frequency of updating movement authorities, for moving block, s</td>
<td>3</td>
</tr>
<tr>
<td>Block length, m</td>
<td>Dependent on scenario</td>
</tr>
<tr>
<td>Section block, m</td>
<td>Dependent on scenario</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; balise location</td>
<td>10 m before signal</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; balise location</td>
<td>8 s at line speed before signal</td>
</tr>
</tbody>
</table>

**Table 7.1.** Simulator input data.
<table>
<thead>
<tr>
<th>Train runs with any of the control systems in any of the scenarios</th>
<th>Journey time (seconds)</th>
<th>Energy usage (kWh)</th>
<th>Delay penalty (£)</th>
<th>Energy cost (£)</th>
<th>Total cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1077</td>
<td>596</td>
<td>34</td>
<td>71</td>
<td>105</td>
</tr>
</tbody>
</table>

**Table 7.2.** Full results of the first train from the West Coast Main Line case study simulation.

### 7.3.2 The Second Train

Due to the delay which occurred to the first train, the service interval between the first and second train is reduced from 300 s to 200 s, which is below the minimum line headway time. Therefore, train interactions will occur and disturbances will result. The level of the disturbance may vary depending on the driving style and train control system that the train is implementing.

Figure 7.3 shows the delay penalty cost, energy cost and total cost of the second train when different control systems are employed for the three scenarios. It can be seen that the optimised train trajectory incurs less delay penalty and energy cost. The cautious operation train further reduces the energy costs, but increases the delay penalty.

In general, the train using the more advanced control system consistently achieves a lower total delay penalty, despite using different driving styles. This is because more advanced control system configurations are able to operate at reduced train headway distance; this results in fewer disturbances. By achieving a consistently low delay penalty, there is also less variation in energy consumption for the journey. When the
total delay penalty and energy cost are combined, the more advanced signalling systems perform better.

It can be observed that, as the train control system becomes more advanced, the delay penalty reduces. There is a significant improvement from the intermittent ATP overlay (ETCS Level 1 systems) to the continuous ATP overlay (ETCS Level 2 systems). This is because the radio transmission provided by the continuous ATP overlay can achieve more frequent movement authority operate than the balise transmission provided by the intermittent ATP overlay. Furthermore, the difference in incurred energy cost between the three driving style scenarios becomes less significant, as the more complicated train control systems take more advanced control of train movements.
Figure 7.3. Delay penalty cost, energy cost and total cost of the second train incurred using different control systems.

The train trajectories of the second train are presented in Figure 7.4. It can be seen that intermittent ATP overlay without infill, intermittent ATP overlay with single balise and 4-aspect signalling all have similar performance. In order to improve the legibility of the figure, only the 4-aspect signalling system is used to compare with the other three systems. In general, it can be observed that the flat-out operation trains experience the strongest disturbance, although they are running at the highest speeds compared with the journey time priority and the cautious operation trains. The cautious operation trains are running at lower speeds than the journey time priority.
operation trains in order to avoid the disturbance from the first train. Furthermore, it can be noted that the trains using the more advanced systems are able to operate at a higher speed in both operation scenarios because they require a smaller headway distance. The trains using the 4-aspect fixed block system have the poorest operation performance because such a system has a disadvantage in block occupation and data transmission.

Figure 7.4. Driving speed profiles of the second train using different train control system configurations.
Table 7.3 shows the full simulation results of the second train; the numbers in brackets show the percentage difference compared with flat-out operation. The results for the first train, which is unaffected by other services, are also shown. The results show that when the train switches its driving style from flat-out operation to journey time priority operation, the total cost can be reduced by up to 18% with a reduction in journey time of 2%. Meanwhile, with cautious operation, the lowest energy cost is achieved, but with higher journey time (and hence delay penalty).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Journey time (seconds)</th>
<th>Energy usage (kWh)</th>
<th>Delay penalty (£)</th>
<th>Energy cost (£)</th>
<th>Total cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaffected train, no optimisation required</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Intermittent ATP overlay without infill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1145</td>
<td>618</td>
<td>23</td>
<td>74</td>
<td>97</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1133</td>
<td>600</td>
<td>19</td>
<td>72</td>
<td>91 (-6%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1166</td>
<td>453</td>
<td>26</td>
<td>54</td>
<td>80 (-18%)</td>
</tr>
<tr>
<td>Intermittent ATP overlay with single balise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1142</td>
<td>626</td>
<td>22</td>
<td>75</td>
<td>97</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1129</td>
<td>540</td>
<td>17</td>
<td>65</td>
<td>82 (-15%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1166</td>
<td>453</td>
<td>26</td>
<td>54</td>
<td>80 (-18%)</td>
</tr>
<tr>
<td>4-aspect signalling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1145</td>
<td>619</td>
<td>23</td>
<td>74</td>
<td>97</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1128</td>
<td>540</td>
<td>17</td>
<td>65</td>
<td>82 (-15%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1166</td>
<td>453</td>
<td>26</td>
<td>54</td>
<td>80 (-18%)</td>
</tr>
<tr>
<td>Continuous ATP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1138</td>
<td>630</td>
<td>21</td>
<td>76</td>
<td>97</td>
</tr>
<tr>
<td>Journey time</td>
<td>1115</td>
<td>565</td>
<td>13</td>
<td>68</td>
<td>81</td>
</tr>
<tr>
<td>overlay</td>
<td>priority operation</td>
<td>(-2%)</td>
<td></td>
<td>(-16%)</td>
<td></td>
</tr>
<tr>
<td>--------------------</td>
<td>--------------------</td>
<td>-------</td>
<td>---</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1139 (0%)</td>
<td>489</td>
<td>21</td>
<td>59</td>
<td>80 (-16%)</td>
</tr>
<tr>
<td>Continuous in-cab</td>
<td>Flat-out operation</td>
<td>1095</td>
<td>633</td>
<td>5</td>
<td>76</td>
</tr>
<tr>
<td>Journey time</td>
<td>1092 (0%)</td>
<td>617</td>
<td>4</td>
<td>74</td>
<td>78 (-4%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1115 (+2%)</td>
<td>511</td>
<td>13</td>
<td>61</td>
<td>74 (-9%)</td>
</tr>
<tr>
<td>Moving block</td>
<td>Flat-out operation</td>
<td>1075</td>
<td>622</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>Journey time</td>
<td>1075 (0%)</td>
<td>606</td>
<td>0</td>
<td>73</td>
<td>73 (-3%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1089 (+1%)</td>
<td>534</td>
<td>7</td>
<td>64</td>
<td>71 (-5%)</td>
</tr>
</tbody>
</table>

Table 7.3. Full results of the second train from the West Coast Main Line case study simulation.

### 7.3.3 The Third Train

The service interval between the second train and the third train is 300 s as scheduled. However, due to the knock-on delay impact from the preceding train, the motion of the third train may be disturbed. The following two impacts should be taken into account when considering the disturbance to the third train:

1. The driving style and control system that the third train is using;
2. The driving style and control system that the proceeding train (the second train) is using. In different scenarios, the second train experiences different disturbances, resulting in different impacts to the third train. It is important to note that the first train does not have an impact, as it is unaffected throughout the journey.
Figure 7.5 shows the costs of the third train when different control systems are applied to the three scenarios. Compared with Figure 7.3, the delay penalties that are incurred by the third train are reduced, especially in the flat-out operation scenario and the journey time priority operation scenario. However, it is interesting to note that the total cost of the cautious operation train is higher than the journey time priority operation train when simple train control systems are implemented. This is considered to be mainly due to experiencing different impacts from the second train. In the cautious operation scenario, the second train achieves a greater journey time than it does in the journey time priority operation scenario. The greater journey time results in a smaller headway distance between the second train and the third train. Therefore, the third train in the cautious operation scenario suffers greater disturbance and hence a higher delay penalty.
Figure 7.5. Delay penalty cost, energy cost and the total cost of the third train incurred using different control systems.

The train trajectories of the third train are presented in Figure 7.6. Compared with Figure 7.4, it can be seen that the third train experiences less disturbance than the second train as no train is required to stop before the stations. Furthermore, the third train is able to run at higher speeds, especially in the flat-out operation scenario.
Figure 7.6. Driving speed curves of the third train using different train control system configurations.

Table 7.4 shows the full simulation results of the third train. The total cost can be reduced by up to 20% for a decrease in journey time of 3% when the train switches its driving style from flat-out operation to journey time priority operation. As described before, due to the slow proceeding train, the cautious operation train may achieve a
higher delay penalty cost together with a higher energy cost than the journey time priority train.

<table>
<thead>
<tr>
<th></th>
<th>Journey time (seconds)</th>
<th>Energy usage (kWh)</th>
<th>Delay penalty (£)</th>
<th>Energy cost (£)</th>
<th>Total cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unaffected train, no optimisation required</strong></td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td><strong>Intermittent ATP overlay without infill</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1123</td>
<td>618</td>
<td>16</td>
<td>74</td>
<td>90</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1094</td>
<td>561</td>
<td>5</td>
<td>67</td>
<td>72 (-20%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1158</td>
<td>458</td>
<td>24</td>
<td>55</td>
<td>79 (-12%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Intermittent ATP overlay with single balise</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1122</td>
<td>625</td>
<td>16</td>
<td>75</td>
<td>91</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1089</td>
<td>584</td>
<td>4</td>
<td>70</td>
<td>74 (-19%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1158</td>
<td>458</td>
<td>24</td>
<td>55</td>
<td>79 (-13%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4-aspect signalling</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1108</td>
<td>611</td>
<td>10</td>
<td>73</td>
<td>83</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1088</td>
<td>582</td>
<td>3</td>
<td>70</td>
<td>73 (-12%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1158</td>
<td>458</td>
<td>24</td>
<td>55</td>
<td>79 (-5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Continuous ATP overlay</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1079</td>
<td>638</td>
<td>2</td>
<td>77</td>
<td>79</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (-10%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1102</td>
<td>537</td>
<td>9</td>
<td>64</td>
<td>73 (-8%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Continuous in-cab</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1077</td>
<td>615</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
</tr>
</tbody>
</table>
Chapter 7 West Coast Main Line Case Study

<table>
<thead>
<tr>
<th></th>
<th>Cautious operation</th>
<th></th>
<th>562</th>
<th>3</th>
<th>67</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1087 (+1%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moving block</td>
<td>Flat-out operation</td>
<td>1077</td>
<td>615</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>Journey time</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>priority operation</td>
<td>(0%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cautious</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>operation</td>
<td>(0%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.4. Full results of the third train from the West Coast Main Line case study simulation.

7.3.4 The Fourth Train

The service interval between the fourth train and the third train is 300 s as scheduled. However, similar to the experience of the third train, the motion of the fourth train may be disturbed due to the knock-on delay impact from the proceeding train.

Figure 7.7 shows the costs of the fourth train when different control systems are employed for the three scenarios. Compared with Figure 7.3 and Figure 7.5, it can be observed that the delay penalties that are incurred to the fourth train are further reduced, especially when the advanced control systems such as Continuous In-Cab and Moving Block Systems are implemented. The journey time priority operation train experiences the least disturbance, as a very small delay penalty is incurred with any of the control systems. It can be seen that, when simple control systems are implemented, the total cost of the cautious operation train is not only higher than the journey time priority operation train, but also higher than the flat-out operation train in some cases.
Figure 7.7. Delay penalty cost, energy cost and total cost of the fourth train incurred using different control systems.
The train trajectories of the fourth train are presented in Figure 7.8. Compared with Figure 7.4 and Figure 7.6, it can be observed that the fourth train experiences the least impact, as the train is able to operate at approximate line speed limits in most cases. If the scheduled service interval can be extended, the disturbance can be further reduced or avoided.

**Figure 7.8.** Driving speed curves of the fourth train using different train control system configurations.
Table 7.5 shows the full simulation results of the third train. It can be observed that, in most cases, the delay penalties are 0, especially in the journey time priority operation.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Journey time (seconds)</th>
<th>Energy usage (kWh)</th>
<th>Delay penalty (£)</th>
<th>Energy cost (£)</th>
<th>Total cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaffected train, no optimisation required</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Intermittent ATP overlay</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1125</td>
<td>626</td>
<td>16</td>
<td>75</td>
<td>91</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1075 (-4%)</td>
<td>603</td>
<td>0</td>
<td>72</td>
<td>73 (-20%)</td>
</tr>
<tr>
<td>Without infill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1153 (+2%)</td>
<td>467</td>
<td>24</td>
<td>56</td>
<td>80 (-12%)</td>
</tr>
<tr>
<td>Intermittent ATP overlay with single balise</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1089</td>
<td>626</td>
<td>4</td>
<td>75</td>
<td>79</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1078 (+1%)</td>
<td>610</td>
<td>0</td>
<td>73</td>
<td>73 (-8%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1153 (+6%)</td>
<td>467</td>
<td>24</td>
<td>56</td>
<td>80 (+1%)</td>
</tr>
<tr>
<td>4-aspect signalling</td>
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<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1077</td>
<td>612</td>
<td>0</td>
<td>73</td>
<td>74</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077 (0%)</td>
<td>608</td>
<td>0</td>
<td>73</td>
<td>74 (0%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1153 (+7%)</td>
<td>467</td>
<td>24</td>
<td>56</td>
<td>80 (+8%)</td>
</tr>
<tr>
<td>Continuous ATP overlay</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat-out operation</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077 (0%)</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1077 (0%)</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
</tr>
<tr>
<td>Continuous in-cab</td>
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<tr>
<td>Flat-out operation</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077 (0%)</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
</tr>
</tbody>
</table>
7.3.5 Summary

The previous sections present the simulation result of each train in sequence. Based on the results, this section shows the simulation of the whole network (4 trains together) and summarises the findings.

Figure 7.9 shows the costs of the set of services when different control systems are employed in the three scenarios. In general, as would be expected, it is found that the trains in the journey time priority operation scenario achieve the lowest total delay penalty and total cost, and the trains in the cautious operation scenario obtain the lowest total energy cost but medium total cost.

Similar to the results shown in the previous sections, it is important to note that when a train is using a simple control system combined with journey time priority operation or cautious operation, it can achieve a similar result to a train that is using an advanced control system.

Furthermore, as described in Section 7.3.2, it can be found that the second train in the cautious operation scenario achieves less total cost than the train in the other two

<table>
<thead>
<tr>
<th>Moving block</th>
<th>Cautious operation</th>
<th>1087 (+1%)</th>
<th>562</th>
<th>3</th>
<th>67</th>
<th>71 (-1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat-out operation</td>
<td>1077</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>Journey time priority operation</td>
<td>1077 (0%)</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
<td></td>
</tr>
<tr>
<td>Cautious operation</td>
<td>1077 (0%)</td>
<td>596</td>
<td>0</td>
<td>71</td>
<td>71 (0%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.5. Full results of the fourth train from the West Coast Main Line case study simulation.
scenarios. However, if considering the whole network, it is interesting to note that the trains in the cautious operation scenario achieve higher total costs than the trains in the journey time priority operation scenario, especially when simple control systems are implemented, as shown in Figure 7.9. Such a result reduces the practical applicability of using the cautious operation on a heavily loaded railway line.

**Figure 7.9.** Delay penalty cost, energy cost and total cost of the network incurred using different control systems.

Based on the simulation results shown above, the following key findings can be summarised:
1. For those control systems that are able to communicate target speed updates to the driver in real time, the use of the optimisation process will reduce the number of interactions and incurred energy cost;

2. The Dynamic Programming algorithm has been demonstrated as an appropriate method for determining the optimum target speed profile;

3. The more advanced train control systems are able to reduce the number of interactions between trains through the provision of better control strategies;

4. Some of the benefits of more advanced control systems may be realised with simple control systems combined with driver advisory systems, coupled with an optimisation process;

5. The effect of using different driving styles is diminished when the train is using a more advanced control system;

6. Cautious operation is more suitable for the off-peak time service, where trains usually have greater service intervals. In peak time service, implementing cautious operation reduces the cost for a single train, but it may produce a greater knock-on delay impact on the following trains, resulting in a greater total cost for the network.

The above findings can be used as guidance for future works.

7.4 Summary

In this chapter, multiple-train trajectory optimisation has been discussed. The differences in journey time and energy usage were assessed for a high-speed line when six different control systems are combined with three different driving priorities have been considered. The case study was based on a section of the West Coast Main
Line in Great Britain with four Class 390 Pendolino EMU tilting train trains. The simulation configuration, including the route data and train data, has been presented in Section 7.2. The simulation result of each train is presented in sequence from Section 7.3.1 to Section 7.3.4, followed by Section 7.3.5 showing the simulation of the whole network and summarising the findings. It can be seen that the implementation of the optimisation process is able to reduce the level of interactions and hence improve operation performance. When combined with the optimisation process, the use of some simple control systems can achieve similar benefits as the use of some advanced control systems. Based on the results that are presented in previous chapters, Chapter 8 will detail the conclusions and future work.


Chapter 8  Conclusions and Future Work

8.1 General Summary of Contents

The thesis can be divided into three sections as follows:

(1) Reviews of train control systems and optimisation techniques;
(2) Multiple train simulator development and train interaction simulation;
(3) Train trajectory optimisation and case study.

In the first section, consisting of Chapter 2 and Chapter 3, the writer reviewed different train control systems and optimisation techniques. In Chapter 2, a subsystem overview of the railway system was provided, followed by an introduction to signalling systems. As the railway system becomes more complex, it is necessary to consider automation to ensure railway safety and efficiency. A number of railway automation systems and common train control systems that have been developed by different countries and organisations are described. In Chapter 3, a general introduction to the optimisation problem has been presented, which shows that optimisation algorithms can be generally classified into exact algorithms and approximate algorithms. Enhanced Brute Force, Dynamic Programming, Genetic Algorithm and Ant Colony Optimisation are then introduced.

The second section, consisting of Chapter 4 and Chapter 5, presented the development of a multiple train simulator and a train interaction simulation. In Chapter 4, a MATLAB based multi-train simulator is described, which aims to model the train movements on different railway lines with different signalling systems, traction
performance and permitted speeds. Based on the simulator, train trajectory and train disturbance can be calculated and evaluated. In Chapter 5, four case studies of the train interaction simulation are presented in order to verify the developed simulator and estimate the knock-on delay performance when different signalling systems and service intervals are implemented.

The third section, consisting of Chapter 6 and Chapter 7, was used to report on a train trajectory optimisation study and a case study based on the West Coast Main Line. In Chapter 6, a train trajectory optimisation study is presented. The work aims to find the most appropriate train trajectory to minimise an objective function that includes energy usage and delay for all trains. Using the results obtained in Chapter 6, Chapter 7 presents a case study that is based on a section of the West Coast Main Line. The aim is to estimate and analyse the impact of using different driving styles together with different train control systems on a practical railway line. It is found that the use of the optimisation process and advanced control systems reduces the number of interactions and incurred total cost.

8.2 Conclusions

The major findings and contributions are concluded by different sub-sections as follows, associated with the objectives presented in Section 1.2.

8.2.1 Multiple Train Simulator Development

Since the introduction of the railways, a number of new systems have been developed to improve safe train movement strategies. Each of them has its own advantages and disadvantages in terms of investment cost, efficiency and stability. Therefore, before
upgrading or implementing a system, it is important to choose a solution that is able to meet the requirements. The use of computer simulations can serve this purpose in a simple and economic way. In this thesis, a time step based approach to model vehicle movements and a method to simulate multiple train running performance are described. A literature review of the modelling of a traction power system was given in Section 4.2. Section 4.3 introduced the approach and the method, together with a discussion of energy-time trade-off. The major findings and contributions are concluded as follows:

1. The simulator has been developed specifically developed to achieve the objectives of the study. Compared with normal single train simulations, it is able to estimate and compare multiple train knock-on delay performance in different configurations;
2. Different railway lines, control systems, traction performance and permitted speeds are modelled associated with published data. In order to consider multiple train operations, train movement authority updating is required and becomes a significant part of the multiple train simulation;
3. The developed simulator is used throughout the study, including the train interaction simulation, the train trajectory optimisation and the West Coast Main Line case study.

8.2.2 Train Interaction Simulation

Railway timetables provide a fixed set of timings for trains to arrive at key points on the railway network. The driver should operate the train respecting the timetable and avoiding any effect on the trajectory of the following train. However, in reality, if a
train is delayed, or a station dwell time is extended for any reason, the service interval will be reduced and train interactions will occur. By using the new simulator, a number of train operation scenarios have been simulated in order to analyse the impact of the train interactions. Section 5.2 presents a test case study in order to verify the accuracy of the simulator. In Section 5.3, six trains were simulated to operate on a single railway line with different service intervals in order to evaluate different train interaction levels. The major findings and contributions are concluded as follows:

1. The result of the test route case study show that the simulator is accurate as it provides simulation results close to the real data available in the public domain (Evans, October 2007). It is therefore considered that the simulator meets the design requirements and can be used for the train interaction simulation and the trajectory optimisation;

2. The result of the train interaction simulation shows that the knock-on delay caused by the interactions will become stronger if the service interval between two trains is getting smaller. In addition, such interactions also increase the train journey time and energy cost;

3. The result of the comparison between the use of different signalling systems shows that the severity of train interactions can be reduced through the application of more advanced train signalling systems.

8.2.3 Train Trajectory Simulation

It is noted that a serious delay that affects services can result in a significant financial loss to the operator. However, increasing the speed of the train may minimise the delay penalty, but will also increase the energy consumption. Therefore, it is
appropriate to consider a trajectory optimisation that aims to find the most appropriate target speed series, and thus train trajectory, to balance the minimum energy cost against the minimum delay penalty of the affected train. A method developed to achieve this aim is introduced in Section 6.2, followed with a cost function description. In order to find the journey time priority results quickly and efficiently, four search methods were implemented and the results have been compared in Section 6.4. Section 6.5 considers the use of different driving styles. The major findings and contributions are concluded as follows:

1. The developed simulator can control the train trajectory through a series of train target speeds. Each simulated train receives a target speed when it proceeds to a station. Train journey time, energy usage and trajectory can be obtained as the output of the simulator;

2. Each train target speed series can be evaluated using the cost function. Two weighting parameters, which are associated with the delay penalty and energy usage, are used in the cost function. The parameters provide a general approach that can be applied to different scenarios;

3. Four searching methods are used to search for the optimum target speed series. All of them are demonstrated as appropriate methods. The enhanced brute force algorithm and DP are able to achieve journey time priority results without any deviation, while the GA and the ACO can only obtain sub journey time priority solutions. However, the enhanced brute force algorithm and DP require higher computation time, especially the enhanced brute force. Furthermore, due to the difference in the use of heuristic information, GA achieves higher deviation than ACO. Therefore, it is considered that the
dynamic programming algorithm is well suited to solve such an optimisation problem, as it is able to achieve the best solution in an acceptable computation time;

4. The weighting parameters in the cost function will vary according to the driving style that a train is given. Three different driving priorities are considered and compared. It can be found that the flat-out operation train experiences the strongest disturbance, followed by the journey time priority operation train and the cautious operation train. It is important to note that the journey time achieved through flat-out operation is higher than that of the journey controlled using journey time priority operation due to the poor driving strategy that the flat-out operation train provides.

8.2.4 West Coast Main Line Case Study

Based on the results obtained in Chapter 6, a West Coast Main Line case study was presented. Four trains were simulated to operate one after the other on a common railway line. Three scenarios were considered; the trains in each scenario were simulated using one driving style combined with six different control systems. The introduction and the route data are presented first, showing the simulation configurations. The results of each train and the whole network were discussed in sequence in Section 7.3. The major findings and contributions are concluded as follows:

1. The use of the optimisation process and more advanced train control systems can reduce the number of train interactions and incurred energy cost through the provision of better control strategies. The effect of using different driving
styles is diminished when the train uses a more advanced control system. Furthermore, it is important to note that when a train is using a simple control system combined with the optimisation process, it can achieve a similar benefit to a train that is using an advanced control system;

2. In the case study, all the trains in each scenario are simulated to use one driving style. Generally, it is found that implementing the cautious operation results in a reduced cost for a single train as the energy cost is reduced significantly. However, compared with the journey time priority operation, cautious operation may produce a much stronger knock-on delay impact to the following trains, resulting in a greater total cost for the network. Therefore, the cautious operation is not considered to be well suited for use in a heavy loaded service railway line.

8.3 Future Work

8.3.1 Optimisation Conjunction

This thesis has presented a number of search methods and driving styles in train trajectory optimisation. It is considered that some of them can be used in conjunction with one or more of the others.

Firstly, it was found that the enhanced brute force algorithm and DP are able to achieve optimal results but require much higher computation times, while the GA and the ACO achieve the opposite results. Therefore, it is considered that a combined searching method could be further considered in the optimisation process (Lin et al., 2008). The GA or the ACO are expected to be used producing an more accurate
estimated solution. Such an estimated solution can be used to reduce the solution domain in the enhanced brute force or the DP, thereby improving the results of the optimisation and the searching efficiency.

Secondly, in the West Coast Main Line case study, it was found that the use of the cautious operation is able to reduce the cost for a single train but may result in a greater total cost for the whole network. Therefore, it is considered that each train in a network should be able to select an appropriate driving style that minimises the total cost of the network. A new method should be developed to make the optimal selection for each train efficiently and accurately.

8.3.2 Type of Targeted Line

This research has targeted main line railways. The simulation configurations such as vehicle type, control system and route data all refer to main line railways operating with a single type of trains. As described in Chapter 5, train interactions occur more easily in heavy loaded railway lines, where trains usually have small service intervals. Therefore, it is considered that mixed traffic railway lines, such as metro railway lines, or even high capacity high-speed lines (e.g., HS2 in Great Britain), may be well suited for further studies.

In addition, some more complex scenarios that combine different route options and stopping patterns can be considered. Figure 8.1 shows a multiple platform station layout. The following four scenarios can be considered for a single train operation:

1. Stopping at the in-line platform with 60 km/h speed limit;
2. Stopping at the off-line platform with 20 km/h speed limit;
3. Non-stopping through the in-line platform with 60 km/h speed limit;
4. Non-stopping through the off-line platform with 20 km/h speed limit.

![A multiple platform station layout.](image)

**Figure 8.1.** A multiple platform station layout.

If multiple train operations are considered, the scenarios can be more complex. Furthermore, when other types of railway lines or scenarios are considered, the regulation methods will need to be verified.

### 8.3.3 Validation and Verification

The studies covered in this thesis, such as train trajectory optimisation and driving style, are based on theoretical simulations. Validation and verification needs to be considered as future work in order to verify the practical applicability. However, the simulations and case studies that have been demonstrated in the previous chapters provide useful information for further developments on train operation and energy saving.
Appendix A Track Diagram

Appendix A shows the track diagrams of the layout that have been used in the simulations (Jacobs, 2005).

Figure A.1 and Figure A.2 show the layout between Lichfield Trent Valley and Rugby stations. It has been used in the simulation described on Section 4.3.

Figure A.1. Layout between Lichfield Trent Valley and Rugby stations 1.
Figure A.2. Layout between Lichfield Trent Valley and Rugby stations 2.

Figure A.3 to Figure A.5 show the layout between Rugby and Birmingham International stations. It has been used in the simulation described on Chapter 7.

Figure A.3. Layout between Rugby and Birmingham International stations 1.
Figure A.4. Layout between Rugby and Birmingham International stations 2.

Figure A.5. Layout between Rugby and Birmingham International stations 3.
Appendix B Published Journal Paper

Publications generated to date based on the research results in this thesis are listed as follows:


The following pages show the paper that has been published in the Proceedings of the Institution of Mechanical Engineers, Part F Journal of Rail and Rapid Transit. The review of the research during the PhD viva showed that the use of the term 'optimal' in Table 2 is misleading. The correct term is 'journey time priority'.
The application of an Enhanced Brute Force Algorithm to minimise energy costs and train delays for differing railway train control systems

Ning Zhao*, Clive Roberts, Stuart Hillmansen

(Birmingham Centre for Railway Research and Education, School of Electronic, Electrical and Computer Engineering,) University of Birmingham, Edgbaston, UK

Abstract

This paper demonstrates an Enhanced Brute Force Algorithm application for optimising the driving speed curve by trading-off reductions in energy usage against increases in delay penalty. A simulator is used to compare the train operation performance with different train control system configurations when implemented on a section of high-speed line operating with two trains, including differences in journey time and train energy consumption.

Results are presented using six different train control system configurations combined with three different operating priorities. Analysis of the results shows that the operation performance can be improved by eliminating the interactions between trains using advanced control systems or optimal operating priorities. The algorithm is shown to achieve the objectives efficiently and accurately.

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configurations with intermediate levels of complexity (e.g. ETCS level 2 and 1 with in-fill) when coupled with the optimisation process have been shown to have similar performance to the more advanced control system.

**Keywords**

Optimal train control system, Brute Force, multi-train simulator, energy consumption, train delay

1 Introduction

In railway operations, the minimum service interval between two trains that results in undisturbed motion is called the minimum line headway. The actual service interval between two trains should always be greater than the fixed minimum line headway to avoid interactions between trains. However, in practice, if the first train is delayed, or the station dwell time is extended, the service interval will be reduced. If the journey of the first train is disturbed, the distance between the two trains may fall below the minimum line headway distance; if this occurs the journey of the second train will also be disturbed and an interaction will occur between the two trains [1]. Changes in train velocity caused by interactions will increase the energy consumption, journey time and reduce passenger comfort.

In this paper consideration is given to how to minimise the effect of interactions. The aim is to find the most appropriate target speed series and thus, driving speed curve, to balance the minimum energy cost against the minimum delay penalty of the affected train, achieving the lowest overall cost. Furthermore, the study aims to assist in the understanding of the different train control system configurations by employing them.
in the train interaction optimisation study. The European Train Control System (ETCS) is considered in the study. It aims to provide interoperability in train control throughout Europe and allow easier cross border train movements [2]. Different ETCS configurations exist, each providing different capacity and performance capabilities. The use of computer simulations can assist in selecting the most suitable implementation for a particular area.

A multi-train simulator has been developed specifically for this study, in which two train movements are simulated and optimised with six different train control system configurations. The simulation of a basic case study on the West Coast Mainline has been used to illustrate the key performance limitations of each system. In principle, the simulation could be extended to cover a more complex arrangement.

Due to the complexity of the solution domain, often, metaheuristic methods such as genetic algorithms (GA) are applied to driving speed curve optimisation by searching for the best train coasting points [3, 4]. Other advanced methods such as artificial neural network and fuzzy logic have also been employed to improve the efficiency and results of the optimisation [5, 6]. However, exhaustive search (exact algorithms) such as Brute Force provides a more straightforward approach than metaheuristics, and, importantly, they are guaranteed to find optimal solutions and to prove optimality [7]. But the performance of these algorithms is not satisfactory in real time train control as the computation time grows exponentially with the problem size [8]. In order to overcome this drawback, an Enhanced Brute Force searching method has been used to constrain the solution domain, therefore reducing the computational cost.
2 Optimisation Methodology

2.1 Driving Speed Curve Control

Under the UK Rail Performance Regime, train operators must pay penalties if their trains disturb services run by other operators. This regime aims to provide an incentive for the train operators to minimise delays.

In 2009, the amount of train delay attributed to train operators was 4 million minutes [9, 10]. The operators need to pay an average of approximately £10 for every minute of lateness they cause beyond their target [11]. Therefore, a serious delay that affects services can result in a significant financial loss to the operator. For short delays it may be possible to increase the speed of a train to minimise the penalty cost. However, increasing the speed of the train will also increase the energy consumption (and hence cost), and may also increase the likelihood of an interaction with the preceding train [12, 13]. It is therefore appropriate to consider the balance between journey time (and hence delay) and energy usage.

Figure 1. System input and output diagram.
The developed simulator can control the driving speed curve through a series of train target speeds, as shown in the ‘optimisation input’ block in Figure 1. Using the simulator, the train journey time, $T_{\text{run}}$, and train energy usage, $E_{\text{run}}$, can be calculated. In order to rank different train trajectories, the outputs of the simulator can be evaluated by Equation (1). The driving speed curve that results in the lowest evaluation value will be the best choice.

$$\text{Evaluation value} = \left[ (T_{\text{run}1} - T_{s1})w_{t1} + (T_{\text{run}2} - T_{s2})w_{t2} + \cdots + (T_{\text{run}i} - T_{si})w_{ti} \right] F_d + E_{\text{run}}F_e w_e \quad (1)$$

where $T_{\text{run}1}$, $T_{\text{run}2}$ and $T_{\text{run}i}$ are the simulated journey times to the first, second and final station; $T_{s1}$, $T_{s2}$ and $T_{si}$ are the scheduled journey times to the first, second and final station; $F_d$ is the unit delay penalty cost per second (in £); $F_e$ is the unit energy cost (in £) per kWh; $w_{t1}$, $w_{t2}$, $w_d$ and $w_e$ are the weightings that are associated with the delay penalty and energy usage respectively.

This method provides a generic approach that can be made applicable to different scenarios by changing the weightings. The technique is able to consider: (1) both journey time and energy usage together as both parameters are transformed into costs; and (2) a delay penalty that can be varied along a route.

2.2 Enhanced Brute Force Algorithm

In a practical situation, it is important to find an appropriate target speed series quickly and efficiently. An Enhanced Brute Force searching method has therefore been developed to serve this purpose.
A conventional Brute Force search will enumerate all possibilities in the solution domain to find the optimum. For complex problems, such as driving speed curve analysis, this rapidly becomes impractical due to the processing time required. The Enhanced Brute Force algorithm is able to address this problem by constraining the solution domain [14]. Figure 2 shows a flowchart for the Enhanced Brute Force algorithm, with the following steps:

1. In Step 1, the method initially calculates an estimated target speed series by comparing the train service interval with the minimum line headway.

2. In Step 2, the method will only consider the candidate solutions which are close in value to the estimated series, thereby reducing the solution domain. The computing time is therefore decreased significantly.

3. In Step 3, the method enumerates the candidate solutions and evaluates them using Equation (1). The top ranking solution will be selected as the most appropriate.
2.3 Operating Priorities

In the previous two sections a method to optimise the driving speed curve has been discussed. It is important to be aware that the weighting parameters in Equation (1) will vary according to the operational priority that a train operating company may give to a particular service, that is, the trade-off between journey time and energy usage. In this paper, three operating priorities are considered:
4. Maximum speed operation (flat-out). Trains are driven at the maximum line speed, which on busy lines will result in interactions between trains. Interactions will result in increased journey times and energy costs.

5. Optimal journey time operation (optimal). The driving speed curve of the second train is optimised so that the minimum journey time is achieved when the interactions between the two trains is taken into account. This will result in an optimal journey time at a moderate energy cost.

6. Interaction aversion operation (cautious). A driving speed curve for the second train is calculated to ensure that no interactions occur. This will result in a longer journey time with a lower energy cost.

The selection of a particular operating priority will result in a different set of weightings being used in the evaluation function. Using the Enhanced Brute Force Algorithm, the optimal train target speed series will be found in line with the evaluation function. The optimal driving speed curve is therefore produced, as shown in Figure 3.

**Figure 3.** Flowchart of the optimisation system.
3 West Coast Main Line Test Case

3.1 Introduction

The test case considered in this paper is based on the West Coast Main Line in the UK between Rugby and Birmingham International. This section of the route is 35.9 km long and has an intermediate station, which is located at Coventry, 18.78 km from Rugby station, as shown in Appendix A. The line speed limit and altitude profiles are shown in Figure 4. In order to analyse the effect of the interactions, the service interval is set to be lower than the minimum line speed headway. Track layout and simulation inputs are presented in Appendix B.

![Figure 4](image)

**Figure 4.** The altitude and line speed limit data for the West Coast Main Line case study section.

The simulated scenarios include two Class 390 Pendolino high-speed trains, which operate along a single track with three stations and a fixed station dwell time. Journey time cost, energy cost and speed against distance figures are outputted when the simulation is complete. The detail of the vehicle traction performance is shown in
Appendix A. Table 1 shows the descriptions of the six different train control systems that have been implemented in the simulator for the case study [15].

**Table 1.** Descriptions of the different train control systems considered.

<table>
<thead>
<tr>
<th>System</th>
<th>ERTMS ETCS category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-aspect fixed block signalling system</td>
<td>Level 0</td>
<td>For railway lines that do not use the ETCS system.</td>
</tr>
<tr>
<td>Intermittent ATP overlay without infill</td>
<td>Level 1 System A</td>
<td>Trains update their movement authorities using a single transmission point.</td>
</tr>
<tr>
<td>Intermittent ATP overlay with single infill</td>
<td>Level 1 System B</td>
<td>Trains update their movement authorities using two transmission points.</td>
</tr>
<tr>
<td>Continuous ATP overlay</td>
<td>Level 2 System C</td>
<td>Trains update their movement authorities using radio based data transmission.</td>
</tr>
<tr>
<td>Continuous in-cab overlay</td>
<td>Level 2 System D</td>
<td>As ‘continuous ATP overlay’, but short fixed blocks are used.</td>
</tr>
<tr>
<td>Moving block</td>
<td>Level 3</td>
<td>Train positions are continuously reported. Following trains are granted movement authority based on the position of the preceding train.</td>
</tr>
</tbody>
</table>

3.2 *Simulator description*

A number of researchers have investigated using train movement simulators to analyse railway operation [16-18]. In this study, a MATLAB based multi-train simulator was developed to model the train movements on different railway lines with different control systems, traction performance and permitted speeds. It is used to calculate driving speed curves and evaluate train disturbance. A typical train journey
can be represented by a sequence of modes, such as motoring, cruising, coasting, braking, etc. As shown in Figure 1, a number of parameters should be inputted into the simulator. Fixed parameters that are not changed during the modelling process are considered as constant values. The dynamic parameters, such as acceleration rate and train target speed, are changed from time to time and determine the energy usage and journey time.

As a time-step based simulator, it calculates the train dynamic parameters and selects the appropriate mode for each time step in the train operation. Lomonossoff’s Equations, which have previously been described in earlier works, are used to solve the dynamic movement equations [19-21]. Appendix A shows the details of the vehicle modelling and train movement modelling.

*Simulation result*

Figure 5 shows the total delay penalty and energy cost of the second train when different control systems are employed. It can be seen that optimal operation incurs less delay penalty and energy cost. Cautious operation further reduces energy costs, but increases delay penalty.

In general, the train using the more advanced control system consistently achieves a lower total delay penalty, despite using different operation priorities. This is because more advanced control system configurations are able to operate at reduced train headway distance; this advantage results in fewer disturbances. By achieving a consistently low delay penalty, there is also less variation in energy consumption for
the journey. When the total delay penalty and energy cost are combined, the more advanced signalling systems perform better.

It can be observed that as the train control system becomes more advanced, the delay penalty reduces. There is a significant improvement from the intermittent ATP overlay (ETCS Level 1 systems) to the continuous ATP overlay (ETCS Level 2 systems). This is because the radio transmission provided by the continuous ATP overlay can achieve more frequent movement authority control than the balise transmission provided by the intermittent ATP overlay. Furthermore, the difference in incurred energy cost between the three operating priorities becomes less significant, as the more complicated train control systems take more advanced control of train movements naturally.
Figure 5. Total delay penalty and energy cost incurred for the test case using different control systems.

From Figure 5, it can be observed that flat-out operation trains result in higher delay penalty than optimal operation trains. The reason for this can be found in Figure 6, which shows an example of the driving speed curves of a second train when using the three different operating priorities. It is can be noticed that the journey time achieved by the flat-out operation (the solid line) is higher than that of the journey controlled using optimal operation (the dashed line). This is because the acceleration section (from point (a) to point (b) in Figure 6) takes additional time. The train using a
cautious operation (the dotted line) is not affected throughout its operation, as the driver will always see green aspect signals; however, this type of operation results in the longest journey time.

**Figure 6.** An example of the driving speed curves for a single journey using three different operating priorities.

The driving speed profiles of the second train are presented in Figure 7. Intermittent ATP overlay without infill, intermittent ATP overlay with single balise and 4-aspect signalling all have similar performance. In order to improve the legibility of the figure, only 4-aspect signalling system is used to compare with the other three systems. In general, the cautious operation trains are run at lower speeds than the optimal operation trains in order to avoid the disturbance from the first train. Furthermore, the trains using the more advanced systems are able to operate at a higher speed in both operations because they require a smaller headway distance. The trains using the 4-aspect fixed block system have the poorest operation performance because such a system has a disadvantage in the block occupation and data transmission.
Figure 7. Driving speed curves of the second train using different train control system configurations.

Table 2 shows the full simulation results of the second train, the numbers in brackets show the percentage difference compared with flat-out operation. Also shown are the results for a train unaffected by other services. The results show that when the train switches its operating priority from flat-out operation to optimal operation, the total cost can be reduced by up to 16% for an increase in journey time of 2.2%. Meanwhile, with cautious operation the lowest energy cost is achieved, but with higher delay penalties.
Table 2. Full results of the second train from the West Coast Main Line case study simulation.

<table>
<thead>
<tr>
<th></th>
<th>Journey time (seconds)</th>
<th>Energy usage (kWh)</th>
<th>Delay penalty (£)</th>
<th>Energy cost (£)</th>
<th>Total cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaffected train, no optimisation required</td>
<td>1068</td>
<td>615</td>
<td>0</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Intermittent ATP overlay without infill</td>
<td>Flat-out operation</td>
<td>1137</td>
<td>616</td>
<td>13</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Optimal operation (0%)</td>
<td>1137</td>
<td>593</td>
<td>13</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>Cautious operation (1.8%)</td>
<td>1158</td>
<td>453</td>
<td>18</td>
<td>56</td>
</tr>
<tr>
<td>Intermittent ATP overlay with single balise</td>
<td>Flat-out operation</td>
<td>1133</td>
<td>618</td>
<td>13</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Optimal operation (-0.4%)</td>
<td>1128</td>
<td>530</td>
<td>11</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Cautious operation (2.2%)</td>
<td>1158</td>
<td>466</td>
<td>18</td>
<td>56</td>
</tr>
<tr>
<td>4-aspect signalling</td>
<td>Flat-out operation</td>
<td>1133</td>
<td>618</td>
<td>12</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Optimal operation (-0.7%)</td>
<td>1125</td>
<td>577</td>
<td>10</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Cautious operation (2.2%)</td>
<td>1158</td>
<td>466</td>
<td>18</td>
<td>56</td>
</tr>
<tr>
<td>Continuous ATP overlay</td>
<td>Flat-out operation</td>
<td>1125</td>
<td>631</td>
<td>9</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Optimal operation (-1.4%)</td>
<td>1109</td>
<td>575</td>
<td>8</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Cautious operation</td>
<td>1130</td>
<td>492</td>
<td>12</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>Flat-out operation</td>
<td>Optimal operation</td>
<td>Cautious operation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>-------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous in-cab</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1094 633 3.3 76 79</td>
<td>1087 608 3.3 73 76</td>
<td>1113 514 9 62 71</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4%) (-16%)</td>
<td>(-0.6%) (-4%)</td>
<td>(1.7%) (-10%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moving block</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1075 619 1 74 75</td>
<td>1075 603 1 72 73</td>
<td>1089 551 4 66 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0%) (-3%)</td>
<td></td>
<td>(1.3%) (-7%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4 Conclusion

In this paper, multiple-train driving speed curve optimisation is discussed. The differences in journey time and energy usage on a high-speed line when different control systems are combined with different driving priorities have been considered. A multiple-train simulator has been developed to achieve this purpose.

The main line test case study considers the operation of two trains. An Enhanced Brute Force searching method was developed in order to identify the most appropriate target speed series to optimise the train operation. Furthermore, three operating priorities were defined to consider the trade-off between journey time and energy usage of the second train.

This study has the following key findings:
1. For those control systems that are able to communicate target speed updates to the driver in real time, the use of the optimisation process will reduce the number of interactions and incurred energy cost.

2. The Enhanced Brute Force algorithm has been demonstrated as an appropriate method for determining the optimum target speed profile.

3. The more advanced train control systems are able to reduce the number of interactions between trains through the provision of better control strategies.

4. Some of the benefits of more advanced signalling systems, may be realised with simple signalling systems combined with driver advisory systems coupled with an optimisation process.

5. The effect of using different operation priorities is diminished when the train is using a more advanced control system.

Acknowledgements

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Appendix A: Development of the multi-train simulator

Vehicle Modelling

The Class 390 Pendalino train is considered in the simulation. The details of the traction system are shown in Table 3 [22, 23]. Figure 8 shows the traction characteristics of the train.
Table 3. Traction system of the Class 390 Pendolino.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traction effort, kN</td>
<td>203.7, if $0 &lt; v &lt; 24.47 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>5095.2/v, if $24.47 \text{ ms}^{-1} \leq v &lt; 62.5 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td>Braking effort, kN</td>
<td>331.3</td>
</tr>
<tr>
<td>Resistance effort, kN</td>
<td>$5.422 + 0.069v + 0.012v^2$</td>
</tr>
<tr>
<td>Maximum speed, kmh^{-1}</td>
<td>225</td>
</tr>
<tr>
<td>Maximum power at rail, kW</td>
<td>510</td>
</tr>
<tr>
<td>Regenerative power at rail, kW</td>
<td>$0$, if $0 &lt; v &lt; 9.64 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$1579.02v - 15221.76$, if $9.64 \text{ ms}^{-1} \leq v &lt; 24.469 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$202.74v$, if $24.469 \text{ ms}^{-1} \leq v &lt; 30.23 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$6153.66$, if $30.23 \text{ ms}^{-1} \leq v &lt; 62.5 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td>Traction driver efficiency</td>
<td>85%</td>
</tr>
<tr>
<td>Maximum torque, kN</td>
<td>204</td>
</tr>
<tr>
<td>Train mass (fully seated load), t</td>
<td>509.27</td>
</tr>
<tr>
<td>Train length, m</td>
<td>220</td>
</tr>
</tbody>
</table>

Figure 8. Traction system characteristics of the Class 390 Pendolino.
Train movement Modelling

Table 4 shows the four typical movement states for train motion, where $F_{tr}$ is the traction force; $R$ is the resistance; $F_{grad}$ is the force due to the gradient; $M$ is the train mass. Figure 9 shows the simulation flowchart.

**Table 4.** Four modes of train movement.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$F_{total}$</th>
<th>$\alpha_{total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motoring</td>
<td>$F_{total} = F_{tr} - R + F_{grad}$</td>
<td>$\alpha_{total} = F_{total}/M$</td>
</tr>
<tr>
<td>Cruising</td>
<td>$F_{total} = F_{tr} - F_{tr} + F_{grad}$</td>
<td>$\alpha_{total} = F_{total}/M = 0$</td>
</tr>
<tr>
<td>Coasting</td>
<td>$F_{total} = -(R + F_{grad})$</td>
<td>$\alpha_{total} = F_{total}/M$</td>
</tr>
<tr>
<td>Braking</td>
<td>$F_{total} = -(F_{tr} + R + F_{grad})$</td>
<td>$\alpha_{total} = F_{total}/M$</td>
</tr>
</tbody>
</table>
Figure 9. Simulation flowchart.

Appendix B: West Coast Main Line case study

Figure 10 and Table 5 show the track layout and the simulation input for the West Coast Main Line case study. The layout is based on the data provided in Railway Track Diagrams [24].
Figure 10. Track layout of the route with distance marks.

Table 5. Simulation input.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train data</td>
<td>Figure 8</td>
</tr>
<tr>
<td>Altitude and Line speed limit</td>
<td>Figure 4</td>
</tr>
<tr>
<td>Route length, km</td>
<td>35.90</td>
</tr>
<tr>
<td>Dwell time, s</td>
<td>120</td>
</tr>
<tr>
<td>Service interval, s</td>
<td>Dependent on scenario</td>
</tr>
<tr>
<td>Sighting distance, m</td>
<td>180</td>
</tr>
<tr>
<td>Overlap, m</td>
<td>180</td>
</tr>
<tr>
<td>Safety margin, for moving block, m</td>
<td>180</td>
</tr>
<tr>
<td>Train length, m</td>
<td>200</td>
</tr>
<tr>
<td>Frequency of updating movement authorities, for</td>
<td>3</td>
</tr>
<tr>
<td>moving block, s</td>
<td></td>
</tr>
<tr>
<td>Block length, m</td>
<td>Dependent on scenario</td>
</tr>
<tr>
<td>Section block, m</td>
<td>Dependent on scenario</td>
</tr>
<tr>
<td>1st balise location</td>
<td>10 m before signal</td>
</tr>
<tr>
<td>2nd balise location</td>
<td>8 s at line speed before signal</td>
</tr>
</tbody>
</table>

References


References


