AN EXPLORATION OF HIGH-ACHIEVING STUDENTS' EXPERIENCES OF LEARNING AND BEING EXAMINED IN A-LEVEL MATHEMATICS.

By

BARBARA ANNE MINARDS

A thesis submitted to the
University of Birmingham
for the degree of
DOCTOR OF PHILOSOPHY

School of Education
University of Birmingham
October 2012

ABSTRACT

In this thesis I explore high achieving students' experiences of learning and being examined in mathematics during their two-year post 16 modular course. I am a practitioner researcher and used a mixed method case study approach with interview data collected from eight students during the learning of each pure mathematics module and subsequent examination. Data was also gathered from their mathematics teachers regarding their perceptions of the students' experiences. This interview data was triangulated by questionnaire responses from the cohort of mathematics students at the end of each year. In a second strand of analysis, I developed a model from Sierpinska (1994) to analyse the nature of the demand the examination papers taken by these students. The level of challenge was found to be surprisingly consistent. Synthesis of the data showed that these high achieving students do find A-level mathematics difficult, with the difficulties remaining similar throughout their two year course. There was a significant overlap between learning and being examined and the difficulties described by the students reveal external factors such as workload, pace, memory and decision making. There very few references to mathematics as a source of difficulty, instead the majority of descriptions featured novelty.

ACKNOWLEDEMENTS

I would like to thank the many people who helped me complete this thesis: the students and teachers who gave up their time to provide the data for this study; colleagues who offered encouragement and support throughout and to Pat Perks for her invaluable help at the eleventh hour.

I could not have imagined let alone completed this thesis without the advice, encouragement and wisdom of my supervisor, Steph Prestage. From the beginning of my teaching career as one of my PGCE tutors, throughout various job changes and ofsted inspections, through my journey as a practitioner researcher, from Masters to PhD student, Steph has been a constant source of inspiration. I would like to thank her for her ongoing support, patience and guidance.

Finally, I would like to thank my beloved husband James, whose love, support and belief made this possible.

TABLE OF CONTENTS

LIST OF APPENDIX ITEMS

APPENDIX 1

APPENDIX 2

APPENDIX 3

APPENDIX 4

LIST OF FIGURES

LIST OF TABLES

GLOSSARY

Year 11	Final year of compulsory education
Year 12	First year of Sixth Form
Year 13	Second year of Sixth Form

SATs	Standard Assessment Tasks
GCSE	General Certificate of Secondary Education
GCE	General Certificate of Education
A-level	Advanced Level
AS	Advanced Subsidiary, course taken in Year 12
A2	Second year of full A-level course, taken in Year 13

ACME	Advisory Committee on Mathematics Education
CASE	Campaign for Science and Engineering in the UK
DFE	Department for Education
EDEXCEL	The UK's largest examination board
HMI	Her Majesty's Inspector
HMSO	Her Majesty's Stationary Office
LOGO	Computer programme used in teaching and learning mathematics
OCR	Oxford, Cambridge & Royal Society of Arts examination board
Ofqual	Office of qualifications and examinations regulation
Ofsted	Office for standards in education, children's services and skills
TGAT	Task Group on Assessment and Testing
UCAS	Universities and Colleges Admissions Service
UDGS	Model for learning mathematics (Hoyles, 1986): Using, Discriminating, Generalising, Synthesising
UMS	Uniform mark score

CHAPTER 1: INTRODUCTION

1.1 Origin of research question.

I am a practitioner researcher and was motivated to investigate an issue within my own institution. My dual roles of full time teacher and part time researcher influenced each aspect of my thesis from the origin of the research focus, literature searches, choice of methods, research design, data collection, analysis and write up. As described by Robson (1993) there are negative impacts of being a practitioner researcher such as limitations on time and inexperience. However there is also a wealth of positives such as insider opportunities (Pring 2000) which I took advantage of. Researching an issue within my own school meant I had access to students, teachers and relevant documents. My insights gained from being a teacher informed my research design as I knew when students and teachers were available, and conversely the parts of the academic year when time was more pressured.

From my Masters research on the experiences of re-sit GCSE mathematics students (Minards 2006), I found that these students had strongly held beliefs of what they expected from a mathematics lesson. This expectation of a 'traditional' lesson (Burton 1984) was formed from their previous experiences and was typically described as teacher led with a restricted range of activities and little opportunity for student responsibility. All of the re-sit students had the performance goal (Dweck 2000) of attaining a GCSE grade C rather than wanting to learn more mathematics. Their descriptions of previous mathematics lessons fitted with the literature on mathematics anxiety (Buxton 1981), as students often masked misunderstanding and demonstrated a helpless approach to difficulties (Dweck 2000).

I noticed that the A-level students in my own school demonstrated similar traits to those displayed by the GCSE re-sit students, despite the considerable difference in mathematical ability and prior attainment. Whilst A-level students often displayed a less anxious approach to learning mathematics, their criteria for success were very similar to those of the re-sit students. Completing questions quickly and with relative ease were seen as indicators of success and understanding mathematical topics were seldom, if ever discussed. As a teacher, I was surprised by the number of A-level students describing mathematics as 'hard' even when they began the course with an A* at GCSE and went on to achieve top A-level grades. I was struck by the paradox that whilst their achievement in mathematics was different there were many similarities in the descriptions of learning mathematics between GCSE re-sit students and high achieving A-level students.

I was interested in exploring A-level students' understanding of mathematics but I had noticed as a teacher that students are more fluent in the language of difficulty. Whilst teaching I was struck by how infrequently students talked about their understanding or mis-understanding of mathematics, but often described the difficulties they experienced. It seems that difficulty is the way that many students communicate what they don't understand. By considering descriptions of difficulty as examples of students' *not* understanding, I decided to take a deficit model to consider the understanding of A-level mathematics students. Similarly I had observed that students talked about their difficulties in specific rather than general terms. They were more able to give descriptions of difficulties with particular questions than on more general topics. As a teacher of A-level mathematics, I had frequently heard students talk about difficult questions on differentiation, yet rarely heard talk of their difficulties with the concept of differentiation. This suggested a way forward for my research and I decided to provoke descriptions of easy and difficult by asking students about their perceptions of mathematics questions. I wanted to research whether students found A-level mathematics difficult and if so, what and why. After reading some of the literature I decided to research high achieving A-level mathematics students' understanding through the language of difficulty. Taking the dictionary definition of difficulty as meaning *hard to understand* (The New Oxford Dictionary of English 1998) and by collecting data about what students described as difficult I intended to explore their understanding in mathematics.

The paradox I observed within my own school, of high attainment in a subject perceived as difficult seemed to fit within the national picture of A-level mathematics. There are more grade A and A*s awarded in mathematics than any other major A-level subject (defined as having over 20,000 entries by the Department for Education). Currently it is one of the most popular subjects and participation is increasing. On paper it appears popular and easy to achieve high grades, and yet, mathematics is seldom spoken about in this light either by students who study the subject or their teachers. Whilst there is some debate about the falling standards in both mathematics and in A-levels in general, my research does not focus on this. Instead I describe and consider issues within the current examination system as this is the only one available to the cohort of students and teachers that is the focus of my research. Since the introduction of Curriculum 2000 where all A-levels became modular (see Chapter 3 section 3.2.3) sixth form students take examinations more frequently than in the previous, linear format.[1] Because of the frequency of modular examinations, I believed that there were two aspects to the experience of A-level mathematics, learning and being examined. I wanted to explore students' responses to both aspects of their A-level experience. In order to triangulate the responses of students I also explored their teachers' perceptions of the student experiences.

In parallel with collecting data from students and their teachers I also wanted to analyse the AS and A2 examination questions. There were indications in the research, Torrance (2007) 'assessment as learning' that analysis of A-level examinations would provide insight into learning A-level mathematics. Similarly Bassett et al (2009) suggest that examination questions indicate the principles of the education system that they assess.

> *"Exam questions are never innocent things. The principles and assumptions behind them are suggestive indicators of the principles and assumptions behind an education system."* p.10 Bassett et al. (2009)

[1] Data was collected from 2008 to 2010 and my thesis describes the state of play within this time period. Currently at policy level there is debate about the reduction of modular examinations and a new National Curriculum.

1.1.1 Research Questions.

The principal questions of this research study are:

1. Do high achieving Grammar school students find learning and being examined in A-level mathematics difficult?

   1a. What are the nature of these difficulties?

   1b. What do A-level mathematics teachers consider to be students' difficulties?

   1.c What do the descriptions of difficulty reveal about the students' understanding?

2. What is the level of demand of A-level pure mathematics questions?

3. What connections are there between the level of demand of the examination questions and the descriptions of difficulty given by students and teachers?

1.2 Context of the School and features of the A-level course.

The data was collected from a mixed Grammar School in the West Midlands during the period July 2008 to June 2010. The school is selective at age eleven with ninety students in each year group. There is a large comprehensive Sixth Form and the ninety 'internal' students are joined by approximately one hundred and sixty 'external' students from other high schools. I had a choice and decided that I wanted to follow A* and A grade GCSE students who had been at the Grammar School for all of their secondary education as I was most interested in the highest achieving students' descriptions of difficulty. Year 12 students, aged 16 are in their first year of sixth form and study AS-level courses which are examined in January and June. Year 13 students are in their second year of post-16 education and study A2-levels. The AS and A2 modules combine to make an A-level qualification, for further details see Appendix 1, p.A1-2. This research focuses on the current A-level mathematics course as offered by the Oxford, Cambridge and Royal Society of Arts (OCR) examination board. This consists of three AS modules, typically taken during the first year of sixth form, and three A2 modules taken in Year 13. The six module scores are totaled to give an overall score in A-level mathematics. Each module is equally weighted and examined by one ninety-minute paper. There are four pure mathematics modules (Core 1, Core 2, Core 3 and Core 4) which are common to all A-level mathematics courses and two applied modules which can be chosen from Mechanics, Statistics or Decision. There are six possible combinations of applied modules and students can change options during their course. I decided to focus my research on the Core mathematics modules as they are common to all students. I did not choose to explore students' experiences of Further Mathematics because of the relatively small numbers of students involved and complications in the combinations and order of the modules studied.

Whilst the Core mathematics syllabus material (see Appendix 1, A1-3-15) is taught by all of the schools following the OCR A-level mathematics specification, the order and timing for each topic are decided by individual schools. The school that is the focus of this research enters all the Year 12 students for the first pure mathematics paper, Core 1, in January. Whilst this is common practice, some schools choose to enter students for all three AS modules in the June of Year 12 and this may affect the timings and order of the modules taught. In this particular school, each AS-Level subject is timetabled for four hour-long lessons per week and every group is taught by two teachers. Both teachers teach the Core 1 module in the autumn term, which equates to forty-four one-hour lessons including eight devoted to revision. In the spring term, the 'pure teacher' teaches the Core 2 module, whilst the 'applied teacher' teaches either the first Mechanics, Statistics or Decision module. This is the equivalent of twenty-eight lessons including two for revision and means that the second and third modules are taught in 64% of the time of the first module. This reduction in teaching time was noticed and commented on widely by both students and their teachers as discussed in Chapter 6.

The format for the teaching the A2 modules in Year 13 follows the same pattern as that for Year 12. Core 3 is taught by both teachers in the autumn term and from January the ‘pure teacher’ teaches Core 4 and the ‘applied teacher’ teaches the second applied module. There is one additional hour per week, where students are taught in larger groups and this fifth lesson is used to teach Core 3 and then Core 4. As explored in Chapter 7, this increase in teaching time resulted in a reduction in the frequency that pace was included in students’ descriptions of lessons. Tomlinson (2004) found that with the modular GCSE and A-level system a student would typically take 42 examinations and the equivalent of two terms of learning time would be taken up with revision and sitting examinations (Chapter 3 section 3.2.3). I analysed the teaching timetable for Year 12 and Year 13 mathematics in the school and found that in each 38 week academic year, revision for upcoming modules took five weeks and study leave during examination periods took three weeks. This was consistent with the finding of Tomlinson (2004) and additionally showed that students were at most twelve ‘learning weeks’ away from their next AS or A2 mathematics module. For further details of this analysis see Appendix 1, A1-16-17. As examinations played such a large part in students’ A-level experience, it confirmed my decision to consider the nature of the demand of A-level examination questions and students’ and teachers’ responses to these in my research.

In contrast to the other work done in the area of choosing to study mathematics beyond the compulsory requirements at age 16 (Tebbutt 1993; Matthews & Pepper 2006, 2007; Vidal Rodeiro 2007; Brown et al 2008; Mendick 2008) this research considers high achieving students. Unlike Mendick (2005) I did not consider gender issues in choices of A-levels or experiences of doing mathematics as the performance of girls within the school at both GCSE and A-level is higher than that of boys across all subjects including mathematics. Similarly I did not explore the influence of ethnicity on A-level choices and experiences because of the make-up of the school population.

The overarching theme of my research is exploring students' understanding through their description of difficulties in learning A-level mathematics. Sub themes which emerged in the design are the current regime of examinations and testing, as I began to plan for my research it seemed impossible to explore Sixth Form students' experiences without including examinations. As discussed previously, students had five sets of examinations within the research period, and were at most twelve 'learning weeks' away from their next examination. I decided to create a model to explore the demands of A-level mathematics questions and analyse students' descriptions of difficulties of these questions against the framework provided by the model. The first round of interviews with students regarding their experiences of learning the first pure AS mathematics module, Core 1 presented many references to teaching. I decided that interviewing teachers at subsequent rounds of interviews would provide useful opportunities to triangulate the students' responses. I did not ask teachers about how they taught A-level mathematics; rather I explored their perceptions of the students' experiences, particularly what students found difficult. A synthesis of the data collected from students and teachers combined with an analysis of the demands of the pure examination papers provide a thorough exploration of high ability students' difficulties in A-level mathematics and offers insights into their understanding. What follows is an overview of each chapter of my thesis.

1.3 Chapter by Chapter Overview

My interest is why high achieving students describe the whole journey of learning mathematics as difficult. As descriptions of difficulty potentially indicate a lack of understanding, in Chapter 2 I consider the literature on understanding to see how others have defined it and researched it. Initially I explore the curriculum reports to find what role understanding is expected to play in learning mathematics in English schools. The starting point is the Spens Report (1938) and the curriculum reports are considered from this through to the Cockcroft (DES,1982) report and the Cambridge Primary Review (Alexander et al., 2010). Whilst spanning over seventy years, the reports talk about the importance of understanding and offer ways for teachers to develop their students' understanding. Literature that addresses what is meant by the term understanding and how it can be described is considered next. Several models for describing mathematical understanding are explored and although there are different models with different terminology, common themes are the need for action from the learners and independence from their teachers. Whilst there are numerous models with different terminology, the role of deep understanding described as 'relational' (Skemp, 1976) prevails. Alongside the investigation into how the research suggested that understanding could be described, these models are also considered as a potential theoretical framework for my own research. The selection of one particular model (Sierpinska, 1994) is explained and the development of this for use in my research is described in Chapter 5.

In order to set the data collected from my own study into context, the factors that affect students' experiences of learning mathematics are considered. From Chapter 2 where the importance of understanding is considered, Chapter 3 explores the issues within national, school and internal contexts. National contexts include the debate about the purpose of A-level mathematics, modular examinations and the use of examination results as a measure not only of individual students' achievement, but of teachers' and schools' performance, described as 'hyper-accountability' by Mansell (2007). The descriptions of A-level mathematics are unclear in several aspects with seemingly contradictory evidence regarding participation levels and standards both over time and compared with other subjects. The national factors affected the school context of classroom practice and the issues involved in attainment grouping and 'teaching to the test' where learning is dominated by preparation for examinations, described as 'Assessment as Learning' by Torrance (2007), are explored. Finally the Internal contexts are considered, with reasons for choosing A-levels, motivation for studying defined by Dweck (2000) as learning or performance goals.

In Chapter 4 I consider different research approaches that would be suitable for a longitudinal study of a group of students that began in the final stages of their GCSE mathematics course through to their final A2 examination two years later. The overriding factors in the choice of research methods were the experience from my Masters research which suggested a qualitative approach would be most suitable and practical effects of being a practitioner researcher, as many approaches would not have been feasible with my full time teaching commitments. The research design is presented in two parts, firstly the timetable of development and use of model to classify the level of demand of examination questions and secondly the timetable of data collection from students and teachers. I considered both students' difficulties in learning each pure mathematics module and in the examination that followed. Teachers' views on students' difficulties in learning each pure mathematics module and the following examination were then collected and analysed. As I was investigating descriptions of difficulty in order to explore understanding, I needed a framework from which to analyse the responses of both students and teachers. Initially I started with dictionary definitions of easy and difficult.

> Easy "*achieved without great effort; presenting few difficulties*" p.583 The New Oxford Dictionary of English 1998

> Difficult *"needing much effort or skill to accomplish, deal with or understand."* p. 514 The New Oxford Dictionary of English 1998

However I needed a model that could be used both to analyse the verbal and written responses of students and teachers, and also to classify the demand of examination questions. In Chapter 5 the adaptation of the model of understanding by Sierpinska (1994) is described and the classifications of examination questions from AS and A2 mathematics are explored. The research considered in Chapter 2 explores different models for describing understanding. Initially I had hoped to explore the depth of understanding expected in AS and A2 mathematics examinations, and from Torrance's (2007), 'assessment as learning' this would provide insight into the level of understanding required in learning A-level mathematics. However, the difficulties in assessing understanding were numerous and significant. The way forward was suggested by the work of Sierpinska (1990) who described that understanding could be considered as the number of epistemological obstacles that are overcome. Duffin & Simpson (2000) also provided guidance with their research which described that whilst teachers could not see their students' understanding, they could look at 'external manifestations' for evidence of understanding. As a result of my reading of the literature I decided to measure the 'obstacles' as presented by the demand in an examination question that A-level mathematics students had to overcome. I describe how I created my own model, based on the model of understanding by Sierpinska (1994) to gauge the level of obstacle offered by A-level pure mathematics questions. Descriptions of the four classifications, Identification, Discrimination, Generalisation and Synthesis are provided with examples of each type of examination question. I then present an analysis of the 24 examination papers set during the research period of 2008 to 2010. The model is used to determine the nature of the demand of the questions rather than the mathematics. There could be different levels of demanding questions written on the same mathematical topic. This model offered a framework against which I analysed the students' and teachers' descriptions of difficulty. There was a tension between my analysis which showed the majority of questions of the first type of obstacle, Identification, and the perceptions of students and teachers who described the examinations as difficult. The data from the examinations throughout the research period was collected and analysed and used to set the data from students and teachers into context as described in Chapters 6 and 7.

Based on their experiences of GCSE mathematics, students decided whether to continue with learning the subject in the Sixth Form, initially with AS mathematics in Year 12 and then with A2 in Year 13. There was a widely held expectation amongst the GCSE students that AS mathematics would be significantly more difficult than at GCSE. This was despite of their high target grade profile with over 80% predicted an A or A* at GCSE. The most common reasons that students did not choose to continue with mathematics at AS was that they thought that were not good enough at the subject or that they felt it would be too difficult.

Conversely, students who had chosen to continue with mathematics did so because they enjoyed it and believed they were good at it.

In Chapter 6 students' experiences of AS mathematics throughout Year 12 were compared to their expectations in Year 11. Eight high achieving students from GCSE formed the nested case study subunits (Thomas, 2011) who were interviewed individually whilst learning each of the pure mathematics modules, Core 1 and Core 2 and after each examination. Their responses were set into context by data collected from a questionnaire for all mathematics students issued at the end of Year 12. Despite many students choosing the subject because they were good at it, over two-thirds of students felt that mathematics was their most difficult AS-level. Mathematics was commonly described as the AS subject with the highest workload and the pace and intensity of the learning was a feature in each stage of the data collection. The language of examinations dominated both students' and teachers' descriptions of learning.

The AS examinations were analysed using the model to classify the nature of the demand of the questions. Both Core 1 and Core 2 consisted of a majority of Identification type questions. The data collected from students and teachers showed a strong link between the classification and perception of difficulty. Identification questions were typically described as standard and most often rated as easy by students and teachers. The Generalisation questions were most frequently described as difficult with novelty considered to be the main source of difficulty. Once a question was considered novel, then the factors that affected the perception of difficulty were the amount of decision making in regard to the mathematical technique required and the memorisation of the appropriate rule or method. Students and teachers had strong views of what they expected from the Core 1 and Core 2 papers, they expected them to be very similar in format and question style to previous examinations.

When making their decision whether to continue to A2 mathematics, the overwhelming factor was students' AS grade and the grade they felt they could achieve at A2. As with the previous transition from GCSE to AS-level, students expected a large increase in difficulty in the next stage of learning mathematics, from AS to A2. This resulted in over a third of Year 12 students not continuing with mathematics in Year 13.

In Chapter 7, data was collected at the same stages in the year as those in the AS course. The findings were surprisingly similar to those described in Chapter 6. The students' descriptions of what they found difficult did not develop over the course of their experiences of learning or being examined in Core 3 or Core 4. Difference or novelty was again the overwhelming reason given by students and teachers for why they perceived questions to be difficult. Similarly, the language of examinations dominated the students' and teachers' descriptions of learning mathematics. During interviews that explored the difficulties in learning specific mathematical topics, trigonometry in Core 3 and vectors in Core 4, reasons given included high mark allocation or position in an examination paper.

As with the AS examinations, students expected Core 3 and Core 4 to be similar to previous papers and described repeatedly practising past examination questions as part of their learning. However, for the first time it emerged through the interview data that students were aware of the limitations of this approach as they felt it did not prepare them for novel questions. Core 3 and particularly Core 4 were widely perceived to be difficult examinations.

However, analysis using the model showed that this was not the case. Whilst the number of Identification questions decreased, there were still no questions that required the act of Synthesis, bringing previously separate concepts together to form a complete picture. The number of Generalisation questions remained few enough so that a student could still gain a grade A having omitted them. It was striking that students who went on to achieve highly in the A-level mathematics course (one A*, three A's and two B grades) not only described novel questions as difficult, but displayed a 'helpless response' (Dweck, 2000) when faced with unusual requests. Similarly, teachers described that different questions were difficult as they required students 'to think' or to 'really understand.' Chapter 8 gives my conclusions.

CHAPTER 2: UNDERSTANDING MATHEMATICS

2.1 Introduction

My interest was exploring high achieving A-level students' declarations of difficulty in mathematics. As explained in Chapter 1, I decided to consider the language of difficulty as highlighting a lack of understanding and, as understanding or the lack of it is at the heart of learning mathematics, this chapter considers how understanding is perceived as a part of learning mathematics in the literature. The majority of UK government reports this century consider the understanding of mathematics to be a major goal of learning mathematics, whether it is understanding mathematics for its own sake or understanding mathematics for a better grasp on everyday living. Alongside understanding, ways in which teachers might encourage understanding is also a significant part of these reports. What is meant by understanding and how research defines this term is explored next. Whilst there is different language used, most of the research of the last 30 years discusses the duality of deep and surface understanding (Biggs, 1999, 2007). Overwhelmingly, the curriculum reports and policy documents support the goal of deep understanding and discusses the benefits of this rather than surface understanding. In this chapter I explore this literature and, in a search for a potential framework for my research, consider various definitions and models.

2.2 Curriculum Reports

A browse through government reports with a curriculum focus over the last hundred years reveals common ambitions for the teaching of mathematics. Three reasons are often given – (a) mathematics trains the mind, (b) is part of our cultural heritage and (c) is a subject important for understanding the world. The last of these is generally emphasised in policy reports focussing on the curriculum. In 1938 The Spens Report (Spens, 1938) suggests a move away from crude utilitarian approaches to an approach more in line with the teaching of the arts.

> *"*[mathematics] *should be taught as art and music and Physical Science should be taught, because it is one of the main lines which the creative spirit of man has followed in its development."* (Spens, 1938, p.177)

Mathematics 5-11 (HMI, 1979) similarly put understanding at the heart of teaching and learning mathematics.

> *"We teach mathematics in order to help people to understand things better - perhaps to understand the jobs in which they might later be employed, or to understand the creative achievements of the human mind or the behaviour of the natural world."* (HMI, 1979, p.4)

The Cockcroft (1982) report on the teaching of mathematics from the primary years to post-16 was a response to the 1977 government report that questioned the level of mathematical attainment in children. It states that the first task of mathematics teachers was to develop the mathematical understanding of each student.

> *"the mathematics teacher has the task of enabling each pupil to develop, within his capabilities, the mathematical skills and understanding required for adult life, for employment and for further study and training,"* (DES, 1982, p.4)

This raised the question of what constituted 'appropriate understanding'. The report's answer to this indicates the students' ability to apply mathematical knowledge in unfamiliar contexts.

> *"understanding in mathematics implies an ability to recognise and to make use of a mathematical concept in a variety of settings, including some which are not immediately familiar."* (DES, 1982, p.68)

For further description, the report references Skemp (1971) and uses the definitions of 'relational' and 'instrumental' understanding (discussed in section 2.3.1). The importance of student activity in gaining, furthering and applying understanding is repeatedly stated throughout the report. Whilst not prescribing a specific style of mathematics teaching, the report recommends that at all levels, in order to support a deeper understanding of mathematics, various teaching and learning activities should be included (DES, 1982, p71).

Cockroft has remained a most significant report on the teaching and learning of mathematics and impacted on various policy reports in the following decades (HMI 1985) though assessment and testing began to dominate the agenda in the nineties (TGAT 1987; HMSO, 1989; HMSO 1993).

The Smith Report (2004) is the next significant report on the teaching and learning of mathematics and considers the 14 -19 age group, reiterating the main purpose for teaching mathematics.

> *"Mathematical training disciplines the mind develops logical and critical reasoning, and develops analytical and problem solving skills to a high degree."* (Smith, 2004, p.11)

It concludes that 'broadening and deepening' teachers' mathematical understanding is key to fully challenging and developing students' understanding (Smith, 2004, p.109). The Smith

Report also recommends that teachers should have more opportunities to consider different approaches to teaching mathematics (similar to the 1938 Spens Report), emphasising activities rather than memorisation as key to the development of students' mathematical thinking.

The Cambridge Review (Alexander et al., 2010) lists twelve aims of primary education, including 'exploring, knowing, understanding and making sense'. Understanding is valued and the role of student action is again seen as a key part to furthering understanding.

> *"learning is an interactive process and that understanding builds through joint activity between teacher and pupil and among pupils in collaboration"*
> (Alexander et al., 2010, p. 199)

From Cockcroft (DES, 1982) to the Cambridge Review (2010) the curriculum reports talk about understanding and offer ways to develop understanding. Increasing variety in pedagogy and activity on the part of students are seen as key across all phases of mathematics education. The outcomes of successful mathematics teaching are widely considered to be 'deep understanding' where students can apply their knowledge in different contexts. Literature which considers what is meant by understanding mathematics is discussed in the following section.

2.3 Understanding

As the curriculum reports consistently suggest that learning mathematics should be focussed on furthering understanding, this section explores what is meant by this term. The first group of research on understanding spans a century and offers two types of understanding. Whilst different terminology is used, these can be categorised using the labels 'surface' and 'deep'

(Marton and Säljö 1976; Entwistle 1987; Biggs 1999, 2007). The terms surface and deep are used in the literature to describe two different approaches to learning rather than understanding but I found them to be useful both here and in the following chapter. Elsewhere in the literature are more complex models of understanding and these are considered in section 2.3.2. As I wanted to consider aspects of understanding in A-level mathematics, and given the modular nature of the course, I felt that assessment would play a significant part in my research. So I consider the literature on assessing understanding in section 2.3.3.

2.3.1 Two types of understanding

Dewey (1933) uses the label 'direct' or 'apprehension' for surface understanding, where an object or concept is known in a basic and incomplete form. Deep understanding is 'indirect/ mediated' or 'comprehension' where the concept is known about with a definite and complete meaning. The path towards understanding is described as the 'relation of means-consequence' where reasons can be given for an outcome or the prevention of an outcome. The reverse path may also be adopted, where the consequences from a particular action can be accurately predicted and described. This process is seen as a problem solving one, and promotes the need for action in gaining understanding.

> *"passivity is the opposite of thought; that it is not only a sign of failure to call out judgement and personal understanding, but that that it also dulls curiosity, generates mind-wandering, and causes learning to be a task rather than a delight."* (Dewey, 1933, p.261)

The Cockcroft Report (DES, 1982) makes the distinction between rote learning and understanding clear in the descriptions of 'mechanical' and 'fluent' performance.

> *"Fluent performance is based on understanding of the routine which is being carried out; mechanical performance is performance by rote in which the necessary understanding is not present."* (DES, 1982, p.70)

It goes on to reference the work of Skemp (1976), who defines ‘Relational Understanding’ as knowing what to do and why and ‘Instrumental Understanding’ as being able to follow rules without reason. Skemp considers the former to be far more valuable and questions why so many teachers were delivering instrumental type lessons.

> *“one of the arguments which I shall use against instrumental understanding, that it usually involves a multiplicity of rules rather than fewer principles of more general application.”* (Skemp, 1976, p.21)

Whilst maintaining that relational understanding is superior to instrumental understanding, Skemp (1976) identifies advantages of both types of understanding. Instrumental understanding is considered easier to master within a specific context. Students can often get the answer more quickly and reliably than using relational methods. This leads to more immediate and obvious rewards as a student can correctly answer many similar questions and receive lots of positive feedback such as numerous ‘ticks’ on their work.

Relational understanding is described as having the advantage of being more adaptable to new tasks. Skemp also considers it easier to remember, whilst there is more to learn; the connections in addition to the separate rules, these results are longer lasting. Relational knowledge could also become a goal in itself, leaving students less reliant on external rewards.

> *“if people get satisfaction from relational understanding, they may not only try to understand rationally new material which is put before them, but also actively seek out new material and explore new areas,”* (Skemp, 1976, p. 24)

Skemp lists several reasons why instrumental understanding is used by many teachers. These include time pressures, as relational understanding often takes longer and is thought to be more difficult. Skemp considers that many mathematics syllabuses are over-burdened and called for a reduction in the amount of content covered.

There is agreement and consistency between descriptions of two types of understanding over time, from Dewey (1933), to Skemp (1976), Hiebert (1984) through to Rittle-Johnson & Alibali (1999). Hiebert (1984) identifies two separate types of knowledge about mathematics. The first is labelled as ‘form’ and covers knowledge of the symbols of mathematics and algorithms and procedures for solving problems, also described as ‘the syntax of the system’. The second is labelled ‘understanding’ which involves intuitions and ideas about how mathematics works, called “the semantics of the system”. Hiebert states that it is generally recognised that students could gain ‘form’ and ‘understanding’ independent of each other and difficulties in learning mathematics arise when students cannot make the connections between the two types of knowledge.

Three potential ‘sites’ in the learning process are identified as places where ‘form’ and ‘understanding’ can be linked. The first is when the symbols used to represent a problem are linked with references that give them meaning. The second is when a connection is made between a procedure to solve a problem and the concept underlying this procedure. Thirdly, the final opportunity to forge a link between the two types of knowledge is when the solution to a problem can be connected to the meaning, if the question had been solved in a real-life context. Thus the student would know whether their answer was reasonable. Hiebert suggests that ‘form’ was being taught better than ‘understanding’, or the making of connections between the two. This led to many students viewing mathematics as an exercise in memorising and following numerous isolated rules, with little relevance outside the classroom.

> *"the formal symbolism of arithmetic moves children away from their natural, intuitive problem-solving skills that were anchored in real-world experiences to rules of symbol manipulation, many of which have lost touch with their understandings."* (Hiebert, 1984, p.502)

Suggestions for enhancing these connections include students being given opportunities to develop their own strategies, rather than being provided with a set of seemingly new, formal rules. Additionally, the use of estimation should be promoted, so that students gain an approximate idea of the answer before actually solving a problem. However, the applications to the teaching of A-level mathematics may not be straightforward. Hiebert questions whether it would be possible for these strategies to be successful when learning more advanced algorithms, where it may be difficult to link concrete models to the procedures. The conclusions drawn are that mathematics has meaning for students only when the connections between 'form' and 'understanding' are made.

Similarly, whilst discussing mathematical knowledge rather than understanding, Rittle-Johnson & Alibali (1999) also use two categories that represent the surface 'procedural' and deep 'conceptual'.

> *"conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. We define procedural knowledge as action sequences for solving problems. These two types of knowledge lie on a continuum and cannot always be separated; however, the two ends of the continuum represent two different types of knowledge."* (Rittle-Johnson & Alibali, 1999, p.175)

Their study investigates how conceptual instruction influenced fourth and fifth grade students' problem solving skills and how instruction on problem solving influenced students' conceptual understanding. The results of this study show that whilst procedural instruction led to some gains in conceptual understanding and problem solving, the conceptual instruction led to the largest gains in students' conceptual understanding and to them successfully using a variety of problem solving strategies, particularly to novel problems.

> *"although there are reciprocal relations between conceptual and procedural knowledge, their influence on one another may not be equivalent."* (Rittle-Johnson & Alibali, 1999, p.188)

This was relevant to my study as I had observed as a practitioner that many A-level lessons focused on only procedural instruction, yet I questioned whether I could distinguish this from conceptual instruction. With the two aspects of the students' experience of A-level mathematics (see Chapter1), learning and being examined, I wanted to develop a model to analyse what was being examined in the A-level papers. The original model considered by Skemp (1976), with its two categories was enlightening in terms of the questions it raised regarding the type of obstacles presented in A-level mathematics questions. However, I felt that only having two categories would not enable me to analyse the data collected from examination papers in sufficient detail. I searched the literature for a model of mathematical understanding that when adapted to analyse examination questions would be manageable in terms of noticing differences in the written papers.

2.3.2 Models for Understanding

When considering the research on models of understanding, there was enough literature that specifically considered mathematical understanding and so I did not research models outside the context of mathematics. Some models considered a particular aspect of mathematics such as geometry (Van Hiele 1958) and algebra (Tall 1992), but as I wanted a model to use in my research of A-level pure mathematics I did not consider models that described a specific area of mathematics. In exploring peoples' models for understanding, I considered labels that would be useful in developing a model for my own data collection and analysis. When thinking about my research design, I wanted a model that would be pragmatically manageable when analysing 24 examination papers over the two year research period.

Mousley (2005) reviews the literature on mathematical understanding and categorises the different theories into three groups; 'understanding as a structured process', 'understanding as forms of knowing' and 'understanding as a process'. The three categories used by Mousley (2005) fit with those described by Pirie and Kieren (1994) who consider the literature to be grouped as understanding in terms of mental objects and the connections between them, understanding as a dynamic process and categories of understanding. In this section there are four models that I am going to share, which represent each of the categories. Along with a description of each model is a comment about the suitability for use in my research.

In grounding their models in the body of literature on mathematical understanding, the model that most often occurs in the research is that of Skemp (1976). This is also referenced in the Cockroft Report which was the start of my literature search, described in section 2.2. The two types of mathematical understanding 'relational' and 'symbolic' is an example of the 'forms of knowing' type (Mousley, 2005). Within the category of 'forms of knowing' are models with more than two types of understanding. The model considered next is one that built on the work of Skemp (1976) and included two additional categories.

Byers & Herscovics (1977) reports on the findings of discussions with a group of experienced teachers on the two types of understanding as defined by Skemp (1976). From these discussions arose two types of understanding that did not fit with Skemp's model. The first was 'intuitive' which related to solving novel problems.

> *"Intuitive Understanding is the ability to solve a problem without prior analysis of the problem."* (Byers & Herscovics, 1977, p.26)

Characteristics of this type of understanding were unorthodox methods of solving a problem, when a student could not write down or fully explain the method that they used. Byers & Herscovics (1977) conclude that as this did not require the knowledge of a rule or knowing why that method worked, it was neither instrumental nor relational understanding and justified this third category. The fourth type of understanding was 'formal' and is described as the ability to express a solution using the accepted notation and conventions.

> *"Formal Understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning."* (Byers & Herscovics, 1977, p.26)

The authors describe that this type of understanding is particularly difficult to evaluate because students may just have memorised the correct form to present their solution in. They also state that memorisation has an adverse effect on relational understanding. These four types of mathematical understanding are combined into the "Tetrahedral Model", with the four components of understanding at the vertices of the shape. The current state of understanding of an individual is described as lying at some point within the tetrahedron. It is believed that different types of understanding were prevalent during different stages of a student's mathematical instruction and that the different forms may be present alongside each other.

> *"effective learning of mathematics cannot be based on any one type of understanding. Nor, in the long run, can the different kinds of understanding be arranged in linear order."* (Byers & Herscovics, 1977, p.27)

They conclude that no single point should be the goal of mathematics teaching and recommend a spiral approach to learning.

I felt that it would be impractical for me to record, track and analyse data using a model based on the framework of the three-dimensional tetrahedron. There was also literature that expressed concerns about the validity of the four categories of Byers & Herscovics (1977). Backhouse (1978) believes that the original model of understanding outlined by Skemp (1976) was sufficient and describes the tetrahedral model as 'unnecessary and inadequate'. He reasons that Byers & Herscovics (1977) were concerned about rules instead of concepts and argued that 'form' was a way of representing a concept instead of a separate way of understanding in mathematics. He believes that to understand and apply the correct notation in solving a problem was to understand the mathematical concepts behind the notation. Thus Backhouse argues that formal understanding is a part of relational understanding. He also considers intuitive understanding to be an unnecessary addition once understanding is restricted to the understanding of concepts.

Skemp (1982) further defines mathematical understanding by re-considering 'formal understanding' as defined by Byers & Herscovics (1977) as 'symbolic understanding'. This avoids the confusion between form in the sense of a formal proof which is associated with logical understanding, and form meaning notation as in 'write in the form' which Skemp classifies as 'symbolic understanding', where symbolic refers to an entire system rather than an individual symbol.

> *"Symbolic understanding is a mutual assimilation between a symbol system and a conceptual structure, dominated by the conceptual structure. Symbols are magnificent servants, but bad masters, because by themselves they do not understand what they are doing."* (Skemp, 1982, p.61)

I did not feel that the revised model by Skemp with its third category relating to the symbols associated with mathematics was sufficient to analyse the nature of the types of demand of A-level mathematics questions. I felt that there would be considerable difficulty in identifying whether questions presented a 'symbolic' challenge as the standard nature of many A-level questions meant that memorisation could have played a significant part in interpreting the symbols involved.

The model that I consider next is one which fits the category of understanding as a 'structured process' (Mousley, 2005). The 'model of knowing' proposed by von Glasersfeld (1983) is described as a viable rather than universal construction that is individual to the person trying to understand.

> *"a matter of building up, out of available elements, conceptual structures that fit into such space as is left unencumbered by constraints"* (von Glasersfeld, 1983, p.41)

It is emphasised that understanding and the organisation of concepts are unique to an individual's experiences. The personal aspect of the von Glasersfeld (1983) model suggested it would not be appropriate to use in my research. The number of acts involved in developing mathematical understanding are described as numerous and requiring significant effort. Whilst von Glasersfeld recommends that models are developed to map the structures and concepts of mathematical topics, he recognised that these were far from being formed. Yet these models are seen as key for teachers in order to develop the understanding of their students.

> *"If the goal of the teacher's guidance is to generate understanding, rather than train specific performance, his task will clearly be greatly facilitated if that goal can be represented by an explicit model of the concepts and operations that we assume to be the operative source of subject competence."* (von Glasersfeld, 1989, p.17)

It appeared that with the variety of mathematical topics at A-level mathematics, the lack of specific models and the individual nature of this model precluded this from further consideration in my research.

Having categorised the different models into three groups, Pirie and Kieren (1994) described their own theory of mathematical understanding as within the category of a dynamic process, this is consistent with 'understanding as a process' Mousley (2005).

> *"a whole, dynamic, levelled but non-linear, transcendently recursive process."* (Pirie and Kieren, 1994, p.166)

Their model has eight levels within the 'growth of mathematical understanding' for any individual working on a specific piece of mathematics. The path through the model may not be identical for the same person working on different topics, or for different students working on the same mathematics. The model starts with 'primitive knowing', the starting place of what the student could do initially when beginning to work on a new part of mathematics. 'Image making' is where a student makes distinctions in previous knowledge and visits it in new ways. This is often provoked by an activity provided by a teacher. 'Image having' is where the student can use the image without using the object or activities that provided them with the image. 'Property noticing' means that a student can manipulate and combine images or aspects of images to construct general properties relating to the particular context within which they were working. 'Formalising' consists of the student abstracting a method from previously image-dependent knowledge based on the noticed properties. 'Observing' means that a student reflects on their abstracts and are able to verbalise these as ‘theorems’. ‘Structuring’ is when the student is aware of how their collection of theories is inter-related and attempts to justify these theorems by ‘logical or meta-mathematical argument’. Finally the eighth layer is when the student has a full, structured understanding of the concept and can create questions that may develop into a new concept. Progress through these eight levels is not seen as linear or only in one direction; instead growth in mathematical understanding is viewed as dynamic.

> *“represented by back and forth movement between levels and it is thus that we characterise understanding as a dynamic and organising process”* (Pirie and Kieren, 1994, p. 172)

Although the authors agree that there is an implication of hierarchy in the model by the use of the terminology ‘level’ and ‘layer’ they state that this is not a reflection of the level of mathematics, as movement through the model does not mean progression from low-level to high-level mathematics. Full knowing of one mathematical topic could become primitive knowing for another. This is also a feature of the Sierpinska (1994) model described in the following section.

In addition to the eight layers in the growth of mathematical understanding, the model also consists of three ‘don’t need boundaries’. Beyond each boundary the student no longer needs to connect back to previous levels of understanding and is able to work within the new level. The first occurs between ‘image making’ and ‘image having’ once a student has an image of a mathematical concept they do not need to use the objects or activities that enabled that image to be formed. The second ‘don’t need boundary’ occurs between ‘property noticing’ and ‘formalising’ because a student with the grasp of a formal mathematical concept no longer needs to use an image. The final boundary separates ‘observing’ and ‘structuring’ as once a student has acquired a mathematical structure; they do not need the meaning brought to it from any of the previous levels.

A key feature to the model that demonstrates the non-linear structure is the concept of 'folding back' the two-way movement between layers that formed the process of gaining understanding. The 'returned to' level is described as being different to the original experience of the level as it had been affected by the experiences and actions of the outer levels. This is described as a 'thicker understanding' of the returned-to level. I did not think that it would be practical for me to identify the eight layers and three boundaries of the Pirie and Kieren (1994) model within students' accounts of their learning of A-level mathematics. Nor did I feel that I could analyse the A-level examinations, which often comprised of short questions, against eight different categories. The following model for understanding was the one I adapted to use in my own study.

2.3.3 Sierpinska's Model

Sierpinska (1994) states the real life aspect of her model of understanding. It was this grounding within the context of mathematics classrooms that influenced my decision to use Sierpinska's model for understanding in my own work.

> *"It is meant to contribute to a better understanding of how real people understand mathematics in real life, not of the 'human understanding' of mathematics."* (Sierpinska, 1994, p.xv)

Sierpinska (1994) bases her model for gaining mathematical understanding on the work of Ajdukiewicz (1974) where understanding is defined independently of meaning. He introduces the concept of 'object' to describe a primitive notion or initial thought and then describes meaning as 'understanding in the same way'. However, this model only relates to expressions and Sierpinska questions how it could be applied to mathematics, where learning a concept is independent of learning its name. However the idea of relating the 'object of understanding' to another object is used as a starting point to generalise the act of mathematical understanding.

Sierpinska (1994) describes 'conceptual components' as representations based on some sort of ideas of actions or procedures that allow these procedures to become generalised and applied to abstract concepts. The importance of activity in understanding is again reinforced by the language used, the need for action is emphasised with the use of the phrase an 'act of understanding'. The 'UDGS' model for learning mathematics described by Hoyles (1986) when considering student activity when using the mathematical computer software LOGO is used by Sierpinska.

> *"**Using** where a concept/s is used as a tool for functional purposes to achieve particular goals. **Discriminating** where the different parts of the structure of a concept/s used as a tool are progressively made explicit. **Generalising** where the range of applications of the concept/s used as a tool is consciously extended. **Synthesising** where the range of application of the concept/s used as a tool is consciously integrated with other contexts of application."* (Hoyles, 1986, p.113)

Sierpinska (1990, 1994) adapts the UDGS model from learning mathematics within a computer environment to describe four basic mental operations involved in understanding: identification, discrimination, generalisation and synthesis, all of which require action by the individual trying to understand. These actions are described as "experiencing, using and applying" by Sierpinska (1990). Whilst Mousley (2005) categorises this model as of the type 'forms of knowing', Siepinska considers it to be neither a way of knowing nor a process but a consideration of understanding as 'an act'.

The first of these acts, identification, is described as when the object of understanding is singled out and recognised as an item to be understood. This may involve attaching a name to the object. The second operation, discrimination, involves identifying the difference between the object to be understood and others. Generalisation is when the object is seen as a specific case of another situation or class of objects. This act could include the awareness of the potential wider applications of the object or that one or more of the assumptions about the object was not essential (Sierpinska 1990). Generalisation is described as particularly important in mathematics with algebra described as ‘the study of the generality of our assertions’ (Sierpinska, 1994, p.59) and is reliant on having identified an object to generalise. Finally, synthesis is the search for a common link, finding similarities in generalisations and forming a complete picture from previously separate concepts. An example given is when a proof is seen not as a series of individual steps, but as the completed proof as a whole.

Abstraction is not considered as a separate act of understanding, instead it is described as the process of detaching certain features from an object. However, it is involved in each of the specified acts of understanding. The four operations do not have a rigid hierarchy and Sierpinska proposes that they develop interactively. The need for these acts to be personal to the individual aiming to understand an object is stressed. This is particularly relevant to classroom situations when students cannot be expected to synthesise the teachers’ knowledge, they can only act on their own knowledge.

> *“The methodology of “acts of understanding” is not very precise. Perhaps it cannot made more precise without loss of generality.”* (Sierpinska, 1990, p. 36)

When considering what it means to understand the concept of a convergent numerical sequence, Sierpinska (1990) describes 25 distinct acts. She concludes that there is ‘probably no end to the possible generalisations and syntheses'. Again the individual nature to the person trying to understand is re-enforced by the statement below which describes that the 25 acts are not necessarily those of a student.

> *"These acts of understanding are by no means the first ones we would expect to occur to a student. Rather, they are linked with the verbalisation, clarification and formulisation of intuitive ideas that should have developed through dealing with and discussing infinite processes (not always convergent) in many different contexts."* (Sierpinska, 1990, p.35)

Sierpinska distinguishes between understanding and knowing, and does not assume that understanding is some kind of 'good understanding'. Yet 'depth of understanding' is described as potentially being measured by the number of acts of understanding a person has experienced. However the difficulty of measuring the number of acts or indeed whether they had taken place is raised as a significant issue, particularly within the context of a classroom environment. The consideration of applying a model for understanding within a mathematics classroom was helpful in terms of my own practitioner research. These practical difficulties are considered next.

Attention is seen as a necessary condition for understanding; it is the act of noticing that there is something to understand. This is highlighted as a reason for students having difficulties in understanding mathematics, as they fail to identify the object of understanding.

> *"It is very difficult for the teacher to communicate what should be attended to: mathematics deals mainly with relations and these, in general, cannot be pointed to with a finger. ... Thus, the very object of understanding in mathematics is very hard to communicate. It is difficult to make the students identify this object and maintain an interest in it."* (Sierpinska, 1994, p.63)

This is just one of the problems that Sierpinska describes when exploring the issues in understanding within the context of the mathematics classroom. Several more are discussed and primarily centre on the importance placed on examination results, an issue explored in the following chapter.

> *"Thinking, contemplation and understanding for their own sake have not been very highly valued by the modern 'enlightened' times that are concerned mainly with 'results'"* (Sierpinska, 1994, p.69)

Sierpinska warns of the difficulty of defining an object of understanding in terms of school mathematics, and highlights the difference between an object that arose from an individual's own inquisitiveness and one that is imposed on a student by their teacher.

> *"What is, for the teacher, an 'algebraic method of solving problems' may become for the student, a mechanical procedure, a school activity that is done to comply with the requirements of the teacher and the school institution. It may have nothing to do with 'methodology' and certainly nothing with answering interesting questions. The student's activity does not always have a cognitive character; very often it is a strategic activity aiming at going through the school and graduating with as little intellectual investment as possible."* (Sierpinska, 1994, p.42)

There are many factors in addition to motivation and attention that influence and may cause difficulties in understanding in the mathematics classroom. These include the complex role that different types of language used by the teacher play. Sierpinska offers a variety of these; instructional, mathematical, symbolic, body language, and suggests that these are often intertwined. These form a sophisticated system that students are expected to translate before they can begin working on understanding the mathematical topic at hand.

The activity of understanding does not change an object; it just 'maps' it onto another object. Often in mathematics, an operation at one level becomes an object in a higher level. Sierpinska likened this to Piaget's three stages of thought 'intra' where only particular objects are considered, 'inter' which involve making connections and finding relations between objects and 'trans' where individual objects are seen as part of a whole structure (Sierpinska, 1994, p.32). Finding a balance between these three stages is considered to be the key to successful understanding. If students only work on rote learning, following model solutions they will remain at the first stage of understanding. Also, if students are asked to conceptually understand ideas without getting to use them to solve problems, they would have some limited experience of the third stage, which would lack meaning, as they had not experienced the process of arriving there.

Sierpinska observes that many students are passive in their process of understanding in mathematics lessons. They tend to accept information given to them by a teacher often without question, and answer set questions often following a model solution provided. Rarely did they ask themselves questions that were not in the textbook.

> *"mathematics has to be understood in an active way because what we have physical access to are only symbols, representations of various kind. It is necessary to scratch a little through them to get to the concepts that are hidden."* (Sierpinska, 1994, p.103)

I decided to adopt this model of understanding with its four acts as one to adapt and use for my own data analysis. The development of the model for classifying the demand of examination questions from the Sierpinska (1994) model for understanding is described in Chapter 5. This was a pragmatic decision based on the feasibility of the number of categories that I believed I could identify in both written examination questions and verbal interview responses. I was also influenced by the predominance of the context of the mathematics classroom in Sierpinska's work, I believed it provided a practical way forward to analyse the nature of the demand presented in A-level mathematics examination questions.

In the following section the issues involved in assessing understanding are considered and the implications for my own work discussed.

2.4.3 Assessing Understanding

The difficulty in observing or measuring understanding either in individual lessons, in examinations or beyond is well documented, (Byers & Herscovics, 1977; Sierpinska, 1994; Duffin & Simpson, 2000) and the role of memorisation was considered as an issue. Dewey (1933) warns against misinterpreting factual recall for understanding.

> "*It is assumed too frequently that subject matter is understood when it has been stored in memory and can be reproduced on demand."* (Dewey, 1933, p.148)

Yet some consider that memorising or rote learning of results is seen as an advantage and an important part of a students' mathematical development.

> *"At all levels understanding must carry with it a degree of remembering and it is our view that, unless students have confident recall of such results as the trigonometrical addition formulae and the derivatives and the integrals of simple algebraic and trigonometrical functions, they will lack the 'building blocks' which they need to develop their study of mathematics satisfactorily."* (DES, 1982, p.179)

Many researchers (Dewey, 1933; Skemp, 1976; von Glaserfeld, 1983; Sierpinska, 1994) consider the role that memory plays in understanding. Duffin and Simpson (2000) describe that understanding something means that there is no need to remember all the details as they can be reconstructed if needed. The difference between reconstruction and reproduction is key to their identification of whether an individual understands or does not understand. The difficulty of distinguishing between reconstruction and reproduction in examination responses is also described by Entwistle (1987) as an issue in assessing understanding.

Assessing understanding over time and within different contexts are further issues. In their report on effective teachers of numeracy, Askew et al. (1997) describe the difference between ‘continued’ understanding and that which could be demonstrated whilst completing specific tasks within a lesson.

> *“while pupils may demonstrate understanding of the mathematics within the context of the lesson, the question of the extent of continued understanding or understanding in other contexts remains open.”* (Askew et al., 1997, p.8)

Byers & Herscovics (1977) report on the findings of discussions with a group of experienced teachers on the two types of understanding as defined by Skemp (1976). Teachers described the difficulty in assessing relational understanding, and reported that the examinations students took required instrumental understanding. The style of the examinations and the need for only instrumental understanding had influenced their teaching styles. The majority of the teachers believed that they taught mathematics for understanding, yet they often reflected that they restricted their teaching to the ‘how’ and omitted the ‘why’.

The difficulty in formative assessment of understanding is also described by Sierpinska (1994). The conflict between the teacher’s and students’ view of the individual’s learning is outlined. A student may think that they have understood a topic when their teacher may see their understanding as incomplete and superficial rather than general or conceptual. However Sierpinska identifies the wider issue of how any person could judge the completeness of another’s understanding. The need to check that the individual’s understanding is not contradictory to any statement of that concept would take an infinite amount of time. Instead she concludes that it is much more straightforward to show that a person has a misunderstanding, as only one contradiction is needed. This is the reason she gave for the literature to be more concerned with errors and misunderstandings.

> *“Accounts of good understanding are rare, and those that exist are often poorly justified.”* (Sierpinska, 1994, p.113)

However, Sierpinska (1990) describes that understanding of a mathematical concept could be measured by considering the epistemological obstacles that a student had conquered.

> *"understanding a concept could be measured by the number and quality of epistemological obstacles related to it that one has overcome."* (Sierpinska, 1990, p.25)

Similarly, Duffin and Simpson (2000) consider ways in which teachers could recognise their students' understanding, and call these 'external manifestations'. These physical actions include explaining, recognising in different situations and deriving consequences.

> *"only through interpreting the physical manifestations of a learner's use of their understanding that the teacher can make any kind of judgement about the learner's existing understanding."* (Duffin and Simpson, 2000, p.419)

As described by Duffin and Simpson (2000), failure to answer a question may not be a sign of a lack of understanding, but a lack of awareness of the understanding required. Similarly, answering a question does not indicate the total understanding of a student on a particular topic, the proportion of a students' understanding used in their response remains an unknown.

These papers prompted the way forward for my research, I was not researching 'amount' of understanding, but the manifestation of understanding required to answer A-level mathematics examination questions. The model that I developed as described in Chapter 5, does not define the understanding required but classifies the nature of the obstacle provided by the examination questions.

2.5 Conclusion

The curriculum reports suggest that developing understanding is central to learning mathematics. Further, they advocate that deep understanding is more meaningful and that surface understanding is often temporary, dependent on memory and specifically relates to typical examination requests. As described by the Cockcroft Report (DES, 1982), understanding mathematics is seen as not just replicating learned methods or techniques, but applying knowledge and skills to novel contexts.

> *"a widespread belief that every boy and every girl needs to develop, while at school, an understanding of mathematics and confidence in its use."* (DES, 1982, p.245)

In contrast to the benefits of deep understanding, the advantages of surface understanding, described as 'Instrumental' by Skemp (1976) are considered to be short term. It is striking that at the time of his research over thirty years ago Skemp believed that only instrumental understanding was being tested. The nature of the assessment procedures of the time were prior to the introduction of the current modular courses at both GCSE and A-level. The frequency with which students are now assessed is the subject of a large body of literature (Tomlinson, 2004; Mansell, 2007; Matthews & Pepper, 2007; Torrance, 2007 Bassett et al., 2009) and this is explored in the following chapter. Within the context of the modular A-level course I initially questioned whether students were actually required to have a deep understanding of the mathematics or whether it was sufficient to have only 'Instumental' understanding. Guided by the work of Duffin and Simpson (2000) and Sierpinska (1990) I analysed the nature of the demands offered by the A-level mathematics examination questions as the 'external manifestations' of students' understanding.

I chose the Sierpinska (1994) model of mathematical understanding as the starting point for my model because it consisted of four categories that I believed could be successfully used to classify both descriptions of learning mathematics and examination questions. The original model described understanding within the classroom context and I was encouraged by the flexible and individual nature of the four acts of understanding. The choice of the Sierpinska model was a pragmatic decision, with the two categories of Skemp being too few to be informative and the eight of Pirie and Kieren to be too many to practically use in this context. I also decided that the models of Byers & Herscovics and von Glasersfeld were not practical to implement within my two year data collection period.

In parallel with the analysis of the A-level examination questions I also analysed students' responses to these papers. I explored how they described their difficulties in learning and being examined in mathematics and synthesised these perceptions with the nature of demands as determined by the application of my model. I also collected and analysed data from teachers whom I asked to consider what their students thought of the examination papers. I had expected that the data from the students' and teachers' descriptions of difficulties in A-level mathematics would fall within the language of the research on understanding with aspects of both deep and surface. However, there were surprisingly few references to any type of understanding by either students or teachers. Instead the context was the main feature of descriptions of difficulty. When describing learning mathematics, descriptions often featured memory and pace, when describing examination questions the external factors such as position in the paper and the number of marks allocated occurred frequently.

In the following chapter, the context that the students and teachers experienced A-level mathematics and their effects on learning are considered. These factors were described as ‘potential distractions’ by Gagné (1974), and are considered in terms of the national, school and internal contexts. Within the context of mathematics education in England the national contexts include modular examinations and the debate about the purpose of A-level mathematics. These impact on the school contexts; grouping by attainment and teaching to the test which influence the internal contexts, the views of the learners.

CHAPTER 3: CONTEXTS OF STUDENTS' EXPERIENCES

3.1 Introduction

In the previous chapter, the importance of deep understanding in learning mathematics was considered alongside descriptions of models for understanding. Given that I am researching students' experiences of A-level mathematics, I decided to describe the contexts within which these experiences occur. Although my research is concerning students in Year 11 and upwards, there is little doubt that their perceptions are formed from their previous experiences of mathematics.

> *"Whatever we say we see or observe is biased by what we already know, think, believe or wish to see."* (Sierpinska, 1994, p. xii)

Students' perceptions of the difficulties of learning mathematics could only be described within the context that they were experienced. In this chapter I explore this context within three categories, two external contexts, national and school and one internal context – the students themselves. The national contexts include the debate about the purpose of A-level mathematics, modular examinations and 'hyper-accountability' a term used by Mansell (2007) to describe a system where examination results have consequences not only for the students but for their teachers, schools and local authorities. The school contexts sit within the national ones and include grouping by attainment and the practice described by Smith (2004) as 'teaching to the test'. Finally the internal contexts are considered; these are the views of the learners and include perceptions of difficulty of mathematics, reasons to study the subject at A-level and motivational goals.

Given that I am interested in perceptions of difficulty in order to explore understanding, I also consider literature that compares mathematics with other subjects. There is a history of mathematics being described as a difficult subject (Dearing, 1996; Nardi & Steward, 2003; Matthews & Pepper, 2007; Brown et al., 2008). One of the themes that I pick up through this chapter is the paradoxical findings and outcomes in the research. One of the ongoing issues is the purpose of A-level mathematics, whether it is for the elite mathematics students or for all students wanting to support their other subjects. Against the background of A-levels in general, A-level mathematics is a particular focus for debates that centre on levels of participation and standards. There appear to be conflicting views about each aspect of post-sixteen mathematics education. It is within this complex picture that the students who are the focus of this research experience A-level mathematics.

3.2 National Context

In this section the impacts of three national contexts are explored, the first of which are the issues surrounding A-level mathematics. There are three topics that feature in the literature; the dual purpose of mathematics A-level as both a service subject and a discipline in its own right, participation rates and standards both over time and in comparison to other A-levels. A major change to the A-level system in England, 'Curriculum 2000' was the move from two year linear A-level courses to a modular format, and this has implications for each aspect of the debate surrounding A-level mathematics. Since the introduction of Curriculum 2000 students typically take 4 AS levels in Year 12 and three A2 subjects in Year 13. In section 3.2.3 the advantages and disadvantages of the modular examination system are considered.

The final national context, the culture of publishing results of high stakes testing is described in section 3.2.4.

3.2.1 A-level Mathematics: Purpose and Participation

Much has been written about the value of mathematics education and the concerns arising from the lack of suitably qualified graduates (Dearing, 1996; Roberts, 2002; Smith, 2004). The dual value of a mathematical education, both to the individual and to their role in the economy via the employment market is acknowledged in the Smith Report (2004). It is these two aspects of mathematics education, both as a subject in itself and as a key support for other disciplines that fuels the debates both about participation and standards. There appears to be no consensus about who or what mathematics A-level is for. At one end of the spectrum is the mathematically gifted student who wishes to their increase their knowledge of the subject in order to further their study of the subject at University. At the other is the student who studies mathematics to support their other A-level subjects or because it is a requirement of their chosen career or university course.

Within the debate about participation, there is a conflict between the goal of greater accessibility, in order to encourage wider participation in mathematics once it ceases to be a compulsory part of every student's education, and greater challenge in order to stretch the most able. This is just one aspect of post-sixteen mathematics education which provokes conflicting views, a situation recognised by Matthews & Pepper (2007) in their study of participation in A-level mathematics.

> *"the further one probes into the area of GCE mathematics, the less straightforward the picture becomes- in almost every corner that we have shone our torch we have found a multiplicity of views and argument."* **(**Matthews & Pepper, 2007, p.5)

Whilst there was a period of concern during the 1990's about the falling numbers of A-level mathematics students (Roberts, 2002; Smith, 2004), figures 3.1 and 3.2 show that whilst the total number of A-level entries has steadily increased from the mid 1990's, those for mathematics have increased since 2005. The data covers the time period from 1996 as this is the earliest year that data is provided by the Department for Education which includes the results of 2010, the year that concludes the data collection period of this research.

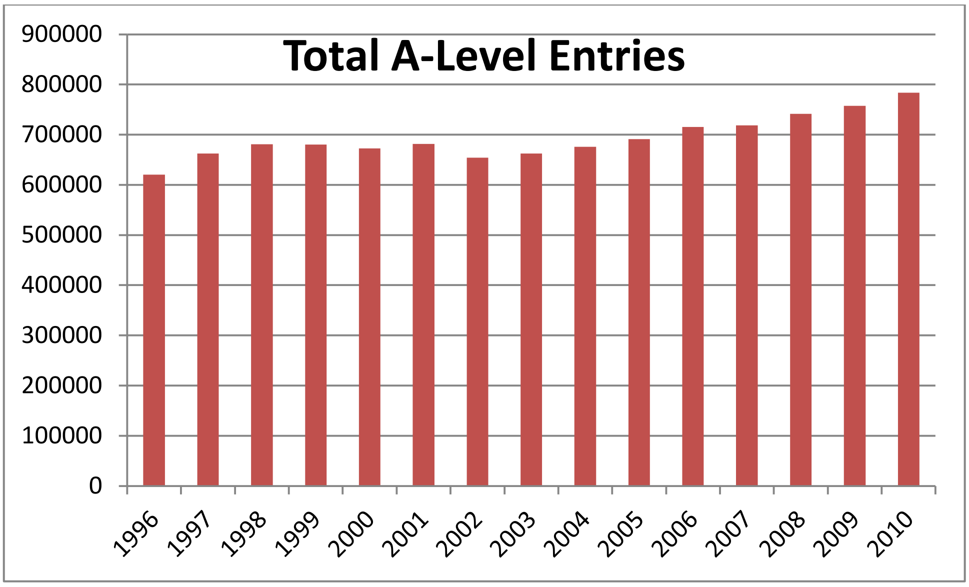


Figure 3.1: Total A-level Entries from 1996 to 2010 (Data from DFE Statistical first release SFR 02/2011).

Whilst Figure 3.1 shows the number of A-level entries, since 2002 the first cohort of students following the Curriculum 2000 specifications, these represent the number of students taking A2 modules. As described in Chapter 1 since Curriculum 2000, the A-level is comprised of AS and A2 modules.

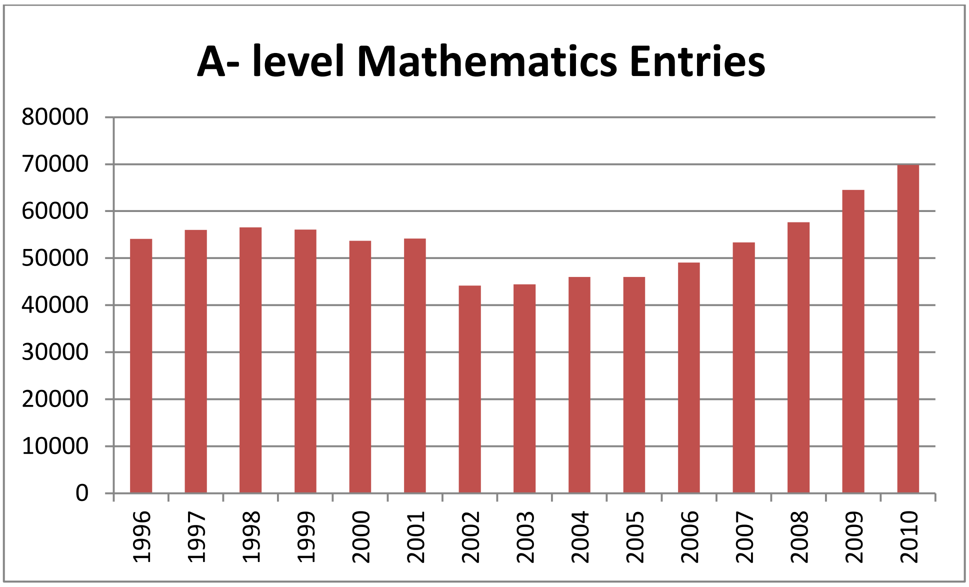


Figure 3.2: Number of A-level Mathematics Entries from 1996 to 2010 (Data from DFE Statistical first release SFR 02/2011).

Figure 3.2 shows that number of A-level mathematics entries dropped significantly in 2002. This was the first cohort of students following the modular Curriculum 2000 specification and the increase from 2005 followed changes to the specifications in 2004 (see section 3.2.3). Since 2004 the A-level mathematics syllabus has remained unchanged and the number of entries has continued to rise. In 2010 Mathematics entries again increased, taking the number of entries to 77,001 where it remains second only to English as the most popular A-level subject.

3.2.2 Standards of A-Level Mathematics

There are two main arguments about standards, the first is whether mathematics A-level has become 'easier' over time and the second is the widely held perception that mathematics is 'harder' than many other A-levels. There is no overall consensus whether the different variations of A-level mathematics syllabus from the linear to the modular courses provide equal challenge. Many researchers including Smith (2004) and Matthews & Pepper (2007) conclude that the current specifications are easier than previous versions. However, there is a minority of opinion that believe that the new specification does not represent a lowering of standards. This is the view upheld by the regulator of external qualifications in England, Ofqual (2009). The six possible combinations within the current modular specification (see section 3.2.3) present further difficulties when attempting to compare standards in mathematics, either with other subjects or over time. However, the debate about whether standards have declined in A-level mathematics is largely irrelevant to students who were either currently studying for this qualification or deciding whether to continue with their learning of mathematics post–sixteen. They can only choose between subjects offered within the existing specifications. The students who began their A-level courses in 2008 had no access to previous versions of A-level mathematics.

The second debate about standards concerns whether A-level mathematics is more difficult than other A-level subjects. This is relevant to the students in my study as it may affect their decisions whether to continue to study mathematics post-sixteen. Whilst there are a number of researchers who describe that mathematics is more difficult than other subjects (Dearing, 1996; Smith, 2004; Campaign for Science and Engineering in the UK, CASE, 2008; Ofsted, 2008) there is a surprising trend in the number of A grades achieved in A-level mathematics. The distribution of grades achieved at A-level mathematics is significantly different to that of all other popular subjects (defined as having over 20,000 entries by the Department for Education).

Figure 3.3 shows the percentage of each grade awarded in all A-level subjects from 1996 and illustrates that the number of U grades has decreased with the most significant reduction in 2002. This followed the introduction of Curriculum 2000 where many students chose to continue with only three subjects to A2 level and often did not continue with the subject of their lowest grade at AS-level. Throughout this time period, the percentage of A grades has risen from 16% to 26%.

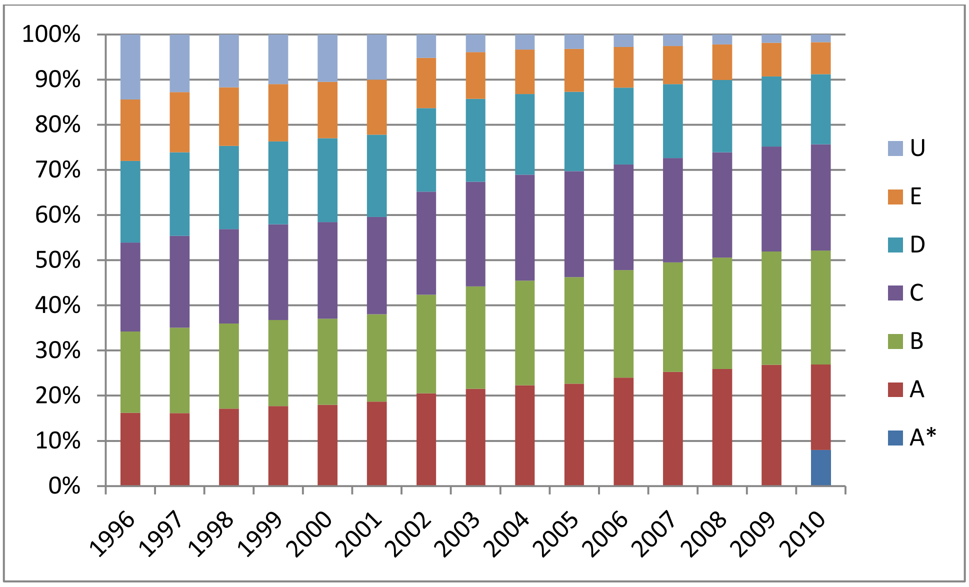


Figure 3.3: Percentage of grades awarded in all A-levels 1996-2010 (Data from DFE Statistical first release SFR 02/2011)

However, as shown in figure 3.4, when the grades awarded for A-level mathematics are considered in isolation, the percentage of A grades is considerably higher throughout the time period.

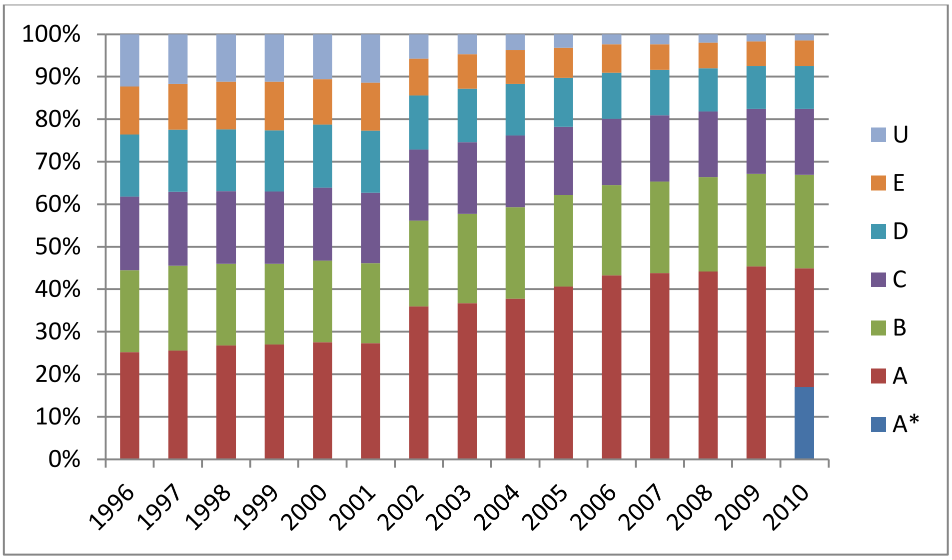


Figure 3.4: Percentage of grades awarded at A-level Mathematics 1996-2010 (Data from DFE Statistical first release SFR 02/2011).

Figure 3.4 shows that as with all A-levels, the percentage of U grades decreases and the percentage of A grades in mathematics increases over the period from 1996 to 2010. However, the percentage of A grades is significantly higher than all other popular A-levels, increasing from 25% in 1996 to 45% in 2009. Year after year the most common grade at A-level mathematics is an A. With the introduction of the A* grade for the August 2010 cohort, 8% of all entries were awarded this new grade. The trend for high attainment in mathematics continued as it has the highest percentage of A* grades of any popular subject with 17% of the highest award. This appears to contradict the literature that suggests mathematics is a difficult A-level.

The dominance of the top grades in mathematics provokes two main issues; the subject being perceived as only for a 'clever core' of students (Matthews & Pepper, 2006; 2007) and that the most able students are not sufficiently challenged by the A-level (Smith, 2004; Golding, 2007). The notion of the 'clever core' arose from studies of the average GCSE point score of students studying each A-level subject. Matthews & Pepper (2006) found that these were highest for students studying mathematics and further mathematics. The GCSE point score also rose significantly between AS and A2 level in mathematics compared to any other subject. This was consistent over several years and is documented in their final report where Matthews & Pepper (2007) describe that AS mathematics 'weeds out' the less able students.

The phenomenon of the 'clever core' is maintained by schools and colleges who either market the idea of A-level mathematics only to their highest achieving students, or those who go further and insist on a particular grade at GCSE mathematics in order to continue to AS level, a practice documented as widespread by Ofsted (2008). At the school that is the focus of this research, students must achieve a B grade from the higher tier at GCSE mathematics in order to take the subject at AS-level. Apart from Science where the requirement is also a double B grade at double award GCSE, no other subjects have such restrictions.[2] Students who are in the lowest attainment set for mathematics at GCSE are actively discouraged from considering the subject at A-level. This increased entry requirement may contribute to the widely documented perception that mathematics A-level is more difficult than most other subjects.

> *"Mathematics at advanced level is commonly perceived to be more difficult than most other subjects. Many higher-attaining Year 11 pupils indicated some anxiety about their ability to cope with A-level mathematics."* (Ofsted, 2008, p.54)

This perception of difficulty despite the dominance of top grades is part of the paradox described in the Smith Report (2004) of an A-level in mathematics that provides appropriate challenge for the most able students whilst being accessible to the large proportion of students who require mathematical skills for their future career choices.

> *"the more able students entered for A-level mathematics are insufficiently challenged and the least able are frequently overstretched."* **(**Smith, 2004, p.94)

The debate has not moved much further in subsequent years, with criticisms about the inability of the A-level qualification to achieve both these aims simultaneously. Whilst there has been improvement in the participation of A-level mathematics, there has been less progress made towards challenging the most able students. The literature suggests that achieving both greater participation and challenge within the same qualification is not possible (Golding, 2007).

---

[2] In September 2011 the school raised its Sixth Form entry requirement and all A-Level courses now require a grade B if the subject was taken at GCSE. Currently the Mathematics Department is considering whether to increase its requirement to an A grade.

Recently there was a move to adapt the grading scale of A-level to try to distinguish between the most able candidates. An A* grade is available for the first time in August 2010, for the cohort that is the focus of this research. There have not been any additional changes to the mathematics syllabus, and an A* is awarded to those students who gain an A grade overall whilst scoring at least 90% in the two compulsory Core mathematics A2 modules. As shown previously in figure 4, a higher percentage of mathematics A-level entries were awarded this top grade in comparison with all other popular A-levels. Yet the introduction of a higher grade is not considered by some (Advisory Committee on Mathematics Education: ACME, 2009) to be a solution to stretching the most able, and instead just emphasises accuracy on the existing style of questions.

However as this change to the grading system was implemented to all A-level subjects, it is unlikely to affect students' decisions of whether to continue to study mathematics in the sixth form. What may have influenced students' choice of A-levels is the perceived difficulty of each subject, although there are many questions about how the difficulty of different A-levels can be compared.

> *"Although many students rate mathematics as the most difficult subject they study, and it has a reputation of being more difficult than other A levels more generally, there is a reasonably large proportion of students who consider it to be their easiest subject…This raises the very complex question, which cannot be answered here, if anywhere, about what constitutes 'absolute difficulty'."* (Matthews & Pepper, 2007, p.20)

It would seem impossible to compare the difficulty of mathematics with that of other subjects. How would 'difficulty' be defined or measured? From whose perspective would this 'difficulty' be taken? A mathematically gifted student might find mathematics A-level easy, yet struggle with the very different demands of A-level English or art. Bassett et al. (2009)

found that there was an expectation that all A-levels should represent the same level of 'difficulty' and 'value'. The value was considered in terms of the usefulness to the individual student taking the subject.

> *"Mathematics and Media Studies are completely different qualifications with differing "worth" depending on the objective of the person doing the qualification – yet there is a belief that they must be of the same "difficulty" and have the same "value"."* **(**Bassett et al., 2009, p.22)

However, whilst the parity in 'difficulty' and 'value' across all A-level subjects is expected, this is generally not thought to be the case. From the findings of Dearing (1996) which describe that mathematics provides greater 'demands' to the 'higher risk' associated with mathematics described by Smith (2004), mathematics continues to be viewed as a difficult A-level subject. There have been calls for the greater demands of mathematics to be rewarded by increasing the 'value' of the subject. The Campaign for Science and Engineering in the UK (CASE) suggests higher Universities & Colleges Admissions Service (UCAS) points for science and mathematics A-levels as an incentive for increasing uptake of these subjects. CASE believes that the perceived difficulty of these subjects make them less appealing to schools and students who are driven by achieving high grades.

> *"The Government has not accepted the evidence that a levels vary in difficulty. But the critical point is whether or not A levels are* ***perceived*** *to vary in difficulty as that perception is what affects students' choice."* Campaign for Science and Engineering in the UK, 2008, p.1, their emphasis)

It is this perception of difficulty that is more relevant to the debate about comparable standards between subjects, particularly to students who are choosing which subjects to continue to study post-sixteen. Despite the evidence of a reduction in content and the fact that mathematics A-level had the highest percentage of A and B grade passes (66.7% in 2009, 66.9% A* to B in 2010) compared with all other major subjects, the perception remains that it is a difficult A-level.

3.2.3 Modular Examinations

Prior to Curriculum 2000, there were some modular A-level courses on offer. Whilst not widespread, there were sufficient in number to warrant research into comparisons between the linear and modular structures. Taverner & Wright (1997) found that there were very little differences in students' attitude to mathematics whether they had followed a linear or modular course. They also found no difference in the likelihood of further mathematical study and the style of teaching and learning is described as similar between the two types of courses. However, there were differences in the grades that students achieved with increased attainment in the modular courses.

> *"for students of equal prior achievement, as measured by their average GCSE score, following an A-level course which offers modular assessment allows them to gain a result half a grade higher than if they had followed a non-modular course."* **(**Taverner & Wright, 1997, p.109)

The reason given to explain this was that students were likely to re-sit modules where they had obtained a failing grade, thus improving their overall performance. This potential for higher achievement in modular A-levels was not just specific to mathematics, Dearing (1996) describes that the spreading of assessment along with the opportunity to re-sit may lead to higher grades in modular A-level courses. Since the introduction of 'Curriculum 2000' all A-Level courses are now modular, with AS modules taken in Year 12 and A2 modules taken in Year 13. Many researchers have investigated the effects of the modular system and several issues have been identified. The advantages are seen as the broadening of the post sixteen curriculum with the majority of students now studying four subjects in Year 12, rather than the three that was typical of the linear courses. There has been an increase of 29% in students taking A-level mathematics from 2001 to 2010 and in the numbers taking the subject at AS level from 2001 to 2010 of 34%. In addition to the increased participation, there were also increased pass rates. Matthews & Pepper (2007) also state that the modular format eases the transition from GCSE to A-level mathematics. Since the introduction of Curriculum 2000 (as shown in Figure 3.3) there has been an increase in the percentage of passing grades at A-level.

However the change to modular A-levels was not initially successful for mathematics. The first cohort of Curriculum 2000 who sat their A2 examinations in 2002 showed a significant drop in the number of students continuing to study mathematics for the entire A-level course (figure 3.2). In the original specification there were three pure mathematics modules and three applied modules, but there was widespread concern that too much content to cover in the AS course led to greater failure and dropout rate at the end of Year 12.

> *"There is widespread recognition that the Curriculum 2000 reforms which led to a new post-16 structure based on AS and A2 levels have been a disaster for mathematics. The original AS/A2 split simply did not work…The consequence was that the image of mathematics has suffered badly again"* (Smith, 2004, p.8)

Changes to the mathematics specifications took place in 2004. Whilst the six module structure of Curriculum 2000 was retained, the pure mathematics content from the original three modules was split between four and the requirement for applied mathematics was reduced to two modules (see Appendix 1, A1-2 for current specification). The first pure mathematics module, now labelled Core 1, was designed to help students bridge the gap between GCSE and A-level mathematics. The remaining three pure modules are labelled Core 2, Core 3 and Core 4. The four Core modules taught over two years are the focus of my research.

Whilst the content of the mathematics modules changed, the number of modules remained the same. The disadvantages of the modular A-level that feature most often in the literature are the frequency and high number of examinations, and the fragmenting of the A-level curriculum. The consequences of a large number of examinations are considered first. Tomlinson (2004) found that students who took eight GCSE and three A-level subjects would sit 42 examinations throughout their courses and would lose 'about two terms' worth' of learning as time was given to revising for and sitting examinations. Even when the A-level courses are considered in isolation, the number of examinations is still considerable. A Year 12 student studying four AS levels would sit twelve examinations in their first year of sixth form, some of these being in the first examination period in January. This pattern would be repeated in their second year with the most common choice of three A2 subjects resulting in nine modular examinations, often with re-sits of some of the AS units. The frequency of modules meant that A-level mathematics became split into four cycles centered around the January and June examination periods. Each cycle consists of learning new material, revising, sitting a practice examination and then the actual examination. Often there is a gap between beginning the next cycle as teaching time is interrupted by the examination period when students often have study leave rather than continuing with their regular timetable. This situation meant that both teaching and learning A-level mathematics became dominated by examinations, as described in Chapter 1, the A-level students were at most 12 learning weeks away from their next set of examinations.

> *"over-constrained and burdensome assessment which means that young people embarking upon A level courses have very little time to explore their chosen subjects, and find that their learning is driven by the need to undertake a high volume of regular external examinations."* (Tomlinson, 2004, p.68)

In addition to the frequency of examinations, another issue about modular A-level is the dividing up of mathematics into separate components. This outcome is described by Smith (2004, p.93) as 'splintering the unity and connectedness of mathematics' and by Bassett et al.

(2009, p.18) as ‘narrow achievement in six separate mini-courses.’ Over the twelve years that has seen A-level taught in the modular form, this splintering of the mathematics curriculum does not appear to have been overcome and the division into separate modules is seen as a threat to understanding mathematics as a whole subject with some seeing it as a ‘destructive influence’. (Kounine et al., 2008, p.18)

In September 2008, as a response to the concerns about the number of examinations taken by A-level students, most subject syllabi changed from six modules to four. Two AS modules are taken in Year 12 followed by two A2 modules taken in Year 13. However, mathematics remained one of only a few subjects that retained the six module structure. This was partly due to the additional changes that mathematics syllabuses underwent in 2004 discussed previously.

The modular system of most mathematics GCSE courses and the current AS and A2 system is described by some as ‘learning to forget’ (Nuffield Review, 2006) and this fragmenting of the curriculum into discrete modules is also described as problematic by Mansell (2007).

> *“The most pernicious effect of the modular system, however, has been to accentuate the tendency towards atomised learning, in which pupils can study a course, take an exam in it, and then move on, safe in the knowledge it will not be assessed again (unless they re-take).”* (Mansell, 2007, p.150)

In addition to the extra examinations that modular A-levels bring, there has also been a move towards a culture of re-sitting. Students have the opportunity to re-sit any of the modular examinations taken, other than those in the June sitting of their final year, without delaying their A-level result. The research of Matthews & Pepper (2007) describes the national picture where mathematics has the highest number of re-sitting of Year 12 modules, with significant increases to the overall grade achieved. The overall A-level grade awarded is based on the total score gained from each of the six modules, with no weight given to the A2 scores. This has led to some teachers and students carefully choosing ways to maxmise the overall grade achieved. The highest score on any module is automatically counted by the examination board, so there is no risk to the student and published results do not currently provide information about the original scores or number of re-sits taken.

3.2.4 Hyper-accountability

The term 'hyper-accountability' is used by Mansell (2007) to describe a system whereby examination results have many implications beyond those for the individual student who sit the examination. For teachers of A-levels these include value added measures and target grades against which an individual teacher can be judged on the performance of their classes. Since the introduction of performance related pay for teachers in 2000, the AS and A-level results of students may be used to decide if their teachers are rewarded financially.

> *"Teachers are forced not merely to pay attention to results. They live and die by them."* (Mansell, 2007, p. 14)

The actual results gained by the students in comparison to the target grades are also collated for each subject, with Heads of Department meeting with their line mangers to discuss their yearly performance relative to other subjects, their previous performances and those of the same subject nationally. The department's results may then form part of the school development plan and could be included in the School Evaluation Form (SEF), which forms the basis of the shorter Ofsted inspection. Dips in examination results can trigger a whole school or subject specific inspection and the recent Ofsted framework places a greater emphasis on students' achievement. Thus examination results have far wider consequences than for just the individual student who achieved them. In this context, Mansell (2007) argues that the drive to ensure that students achieve their target grades in examination could result in teaching that focuses on passing tests rather than furthering understanding.

> *"Thus hyper-accountability turns the pupil into a passenger in the learning experience. It encourages spoonfeeding and discourages the independence of thought which is what, surely, good teaching should foster. At base, then, hyper-accountability, in its unquestioning exhortation to teachers to ensure that all their pupils do well, no matter how much effort or thinking they put in, is anti-educational."* (Mansell, 2007, p. 226)

This is not an isolated viewpoint, with the practice of teaching for understanding being replaced with teaching of examination syllabus material described by many (Nardi & Steward, 2002b, 2003; ACME, 2005; Brown et al., 2008; Bassett et al. 2009). The following quote is typical of those that describe this practice and the reasons for it.

> *"The current generation of teachers focus on reaching quotas rather than giving students a real understanding of the subject. Professional responsibility is removed from teachers as they are restrained by targets, with a fear that failure to achieve these could put their jobs at risk."* **(**Kounine, et al., 2008, p.20)

Strikingly, teaching practices that focus on examination preparation, the results of which provide 'public measures of achievement' rather than understanding are also acknowledged by Government inspectors (Ofsted, 2006, p.18).

3.3 School Context

Within the issues of the national context described in the previous section, there are decisions taken by school and mathematics leaders regarding the teaching of the subject. These include practices such as grouping by attainment and teaching to the test. The external factors within which students experience mathematics may impact their decision to study the subject once it becomes non-compulsory, post GCSE. In their research into the reasons which influenced students' decisions whether to continue with A-level mathematics, Brown et al. (2008) found that aspects of students' prior experiences contribute to the belief in that there are 'fixed boundaries' to their mathematical learning. These external conditions include the examination structures discussed previously and the grouping structures within a school that are considered next. Brown et al. (2008) goes on to describe the negative effects that the perception of a fixed limit to their potential mathematical learning may have on students.

> *"Some students appeared to believe that there were fixed 'boundaries' for each individual person in mathematics, beyond which learning becomes extremely difficult and frustrating, and several pointed towards this personal 'fixed boundary' effect within their reasons for not continuing with mathematics."* (Brown et al., 2008, p.8)

Whilst some of the contributing factors such as the examination system are decided at a national level, the practice of attainment grouping is within each individual school's jurisdiction. The issues surrounding this practice are explored next.

3.3.1 Grouping by Attainment

The school at the centre of this study, and each of the feeder high schools that students joined the Sixth Form from, teach mathematics at GCSE in attainment groups. Thus all of the students in my study had made decisions about whether to continue learning mathematics post-sixteen based on their experiences of learning mathematics within attainment groups.

There is a body of literature that describes this system and there are two conclusions drawn, either that there are very little gains to average attainment levels (Slavin, 1990; Ireson, et al., 2005; Kutnick, P. et al., 2005) or that it disadvantages many students (Wiliam & Bartholomew, 1994; Nardi & Steward, 2003; Boaler et al., 2000). Whilst many descriptions centre on the lower expectations and limited opportunities for students placed in bottom and middle ability groups, there are also descriptions of disadvantages for those in top sets, and as my research considers high achieving students, these are discussed next.

Wiliam & Bartholomew (2004) found that whilst there were gains in attainment in GCSE mathematics grades for students placed in the 'top set' these were offset by the lower attainment of students in the middle and lower groups. They conclude that there is no overall rise in attainment at GCSE when mathematics is taught in attainment rather than mixed groups. They observed that there are differences in the teaching practices of 'top set' teachers who are often more experienced and well qualified than those teaching lower groups.

> *"teachers overestimated the capability of students in the top set, giving them work that was often too demanding, and expecting them to be able to do it quickly."* (Wiliam & Bartholomew, 2004, p.283)

These findings were replicated in the research of Ireson et al. (2005) who also describe that teachers treat the students in each set as 'homogenous' and have high expectations of all top set students with challenging work given at a fast pace. Boaler et al. (2000) also found that teachers changed their practices when teaching ability sets compared to mixed ability groups.

> *"setted lessons are often conducted as though students are not only similar, but* identical *– in terms of ability, preferred learning style and pace of learning. In setted lessons we have observed, students have been given identical work, whether or not they have found it easy or difficult, and they have all been required to complete it at the same speed."* (Boaler et al., 2000, p.640, their emphasis)

In addition to their prior experience of learning mathematics within attainment groups, I questioned whether the practice of considering students as homogeneous occurred with the teaching of A-level mathematics. This was explored in a series of interviews with students and teachers as stated in the research design in the following chapter, and the outcomes are discussed in Chapters 6 and 7.

The fast pace of the work and a competitive environment was also described by Boaler (1997) who discussed the negative effect on students learning mathematics within a top set. The top set environment was described as creating anxiety due to the pace and consistent pressure of performing at a high level. Similarly, Boaler et al. (2000) provide descriptions of students experiencing difficulties with the pace and raised expectations from being within a top attainment group.

> *"teachers raced through examples on the board, speaking quickly, often interjecting their speech with phrases such as 'Come on we haven't got much time' and 'Just do this quickly'. Set 1 lessons were also more procedural than others- with teachers giving quick demonstrations of method without explanation, and without giving the students the opportunity to find out the meaning of the different methods"* (Boaler et al., 2000, p.635)

Whilst in the minority, there are accounts of the positive effects of being in the top set. Ireson & Hallam (1999) describe advantages such as a wider range of activities, opportunities for discussion and an environment in which students are given responsibility for their choice of activity. The effect of the practice that treats the students in the top set as mathematicians with the pace of the lesson set for the highest achievers is described by Boaler et al. (2000) as having the effect of reducing the curriculum to rote learning for the majority of students.

Within the literature considered in this section, there are numerous accounts of the context of being within the top attainment group affecting students' experiences of learning. Students often refer to the fast pace of their lessons, so there is a commentary on the context of their learning in addition to their descriptions of learning mathematics. A further factor in the context of students' learning is discussed in the following section.

3.3.2 Teaching to the Test

The consequences of the national contexts, modular examinations (section 3.2.3) and the 'hyper-accountability' climate (section 3.2.4) on teaching and learning have been researched. There are numerous descriptions (Sierpinska, 1994; Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008; Torrance, 2007) of the practice of 'teaching to the test' with a focus on optimising results rather than furthering understanding. Whilst there were descriptions of this practice prior to the introduction of Curriculum 2000, the majority of research focuses on the impact of the modular A-level and GCSE systems.

> *"Links between different parts of the subject are not examined because the topics are in different modules, and as they are not examined they are not taught…The widespread perception is that best practice in teaching and learning is mutually exclusive with optimising module results."* (Golding, 2007, p.7)

Torrance (2007) describes that techniques such as sharing assessment criteria with students and providing detailed feedback on work in relation to them have become commonplace in schools and colleges. This is considered standard practice by teachers and expected by students. The modularisation of A-levels and the implementation of AS and A2 levels of Curriculum 2000 are cited as part of the move towards 'criterion-referenced assessment and competence-based assessment'. A consequence of this is described as assessment 'dominating' the learning experience of students.

In the 2006 Ofsted report investigating mathematics provision for 14-19 year olds, the dominance of the practice of answering examination questions is described. Whilst recognising that some practice was required, the report cautioned that overuse of examination questions may lead to a 'temporary boost' which would not be maintained at the next stage. Not much change had been accomplished in the following two years as documented in the 2008 Ofsted report.

> *"Pupils' learning is based too much on their acquisition of methods, rules and facts, as part of the strong focus on tests and examinations, and too little on their understanding of the underpinning concepts, on connections with their earlier learning"* (Ofsted, 2008, pp.55-6)

The reason for this lack of change in teaching strategies could be ascribed to the need to raise students' performances in national tests and examinations such as Standard Assessment Tasks: SATS, GCSEs and A-levels.

> *"*[secondary teachers] *often commented on the pressures of external assessments on them and their pupils. Feeling constrained by these pressures and by time, many concentrated on approaches they believed prepared pupils for tests and examinations, in effect, 'teaching to the test'. This practice is widespread and is a significant barrier to improvement."* (Ofsted, 2008, p.44)

In contradiction with the previous chapter, where furthering understanding is considered central to learning mathematics, here the focus appears to be achieving particular examination results. Mansell (2007) supports the observations of Ofsted and describes that repeated practice of examination questions is a quicker route to increasing test results than furthering understanding. Yet he describes that this practice can only be successful if the examination questions that students face are of a standard or typical nature. The nature of A-level mathematics questions is a focus for many researchers and there is agreement that the current style of questioning is more procedural and of a standard nature than previous specifications.

The style of modular mathematics A-level examination questions are described by the Advisory Committee on Mathematics Education (ACME, 2009) as being focused on techniques rather than problem solving. Changes in A-level question styles over time are also documented by Basset et al. (2009) who note that they have changed from fewer, long questions often with some choice of which could be answered, to examinations that require many shorter questions to be completed. The questions became more explicit, with clearer sign-posting of the method required to solve the problem, described by Bassett et al. (2009) as 'sat-nav' mathematics.

> *"If you read a map to get from A to B, you remember the route and learn about other things on the way. If you use a sat-nav you do neither of those things. The questions in the 2008 paper are heavily structured in this way and the result is that students will retain very little knowledge and develop very little understanding."* (p.12)

The situation described by Torrance (2007) as 'assessment as learning' goes further, and suggests that as examination questions become clearer and more standard, the more that understanding of principles can be replaced with repeated practice of typical requests.

> *"transparency of objectives coupled with extensive use of coaching and practice to help learners meet them is in danger of removing the challenge of learning and reducing the quality and validity of outcomes achieved. This might be characterized as a move from assessment of learning, through the currently popular idea of assessment for learning, to* assessment as learning*, where assessment procedures and practices come completely to dominate the learning experience, and 'criteria compliance' comes to replace 'learning'."* (Torrance, 2007, p.282)

The predominance of 'practising the finished product for the test' Prestage & Perks (2006, p.65) has seen many mathematics lessons taught with the aid of teaching resources from the examination boards. Several papers (Ofsted, 2006; Mansell, 2007; ACME, 2009) describe the situation at both GCSE and A-level where the textbooks used by teachers are often endorsed by a specific examination board. Schools have a choice in which examination syllabus to use and often purchase texts produced by the same board to support their teaching of the specific syllabus. This is the case in the school in this study. Many of these texts contain frequent past paper questions that promote repeated practice of examination style questions on individual topics rather than puzzles or investigations that focus on more general problem solving skills which would represent the variety suggested by Cockcroft (DES, 1982) as discussed in Chapter 2. As students frequently practise questions set by examiners as part of their learning of a module, they are less likely to be surprised by the style of questions in their actual examinations. Mansell (2007) considers the disadvantages of learning based on prescriptive examination questions to be boredom, lack of variety and challenge.

At A-level the textbooks are commonly split into two volumes, AS and A2. Instead of a single pure mathematics textbook, that contains material that may not be examined in a particular syllabus or module; there are a series of books covering the Core 1 through to Core 4 modules for each examination board. Ofsted (2006) found that few of these texts made connections between different modules or ‘promoted mathematical enquiry and understanding’ (Ofsted, 2006, p.17).

Another factor that influences A-level teaching strategies is the short amount of time in between modules. Being at most only twelve ‘learning weeks’ away, as described in Chapter 1, from an examination means that the syllabus material needs to be taught efficiently. However, there are differing opinions about the effect that this approach has on students’ experiences of learning mathematics. Many (Tomlinson, 2004; Mansell, 2007; Ofsted, 2008) describe that ‘teaching to the test’ leads to a lack of depth of understanding.

Basset et al. (2009) also describe that the frequency of the modular examinations has had a significant impact on students’ of learning mathematics, reducing the variety of their experiences. The time taken by the frequent modular examinations appears to contradict the need for the time required to provide variety, stimulate deeper understanding and enjoyment in mathematics (Kounine et al., 2008).

One of the consequences of the pressure of restricted time between modules is that only the mathematics that is tested is taught. This has further consequences, as topics or techniques that are not required in examinations are very rarely taught, they are not valued. Similarly as each module contributes the same number of marks to the overall A-level grade, there is an implication that each of the six mathematics modules are 'equivalent'. The effects of the need to maintain this equivalence and testing all that has been learned are considered by Bassett et al. (2009) as removing the opportunity of learning for its own sake. Additionally, the desire for equivalence has meant that the examination time is the same for each of the six mathematics modules. This parity becomes a paradox, as all mathematics examinations considered equal because they are each worth 72 marks and the marks gained on Core 1 count towards the overall A-level grade as much as marks gained on Core 4.

The powerful influence that the examination system has had on the learning of mathematics is documented by the following observation that a change in emphasis in teaching mathematics is dependent upon a change in the examination system.

> *"unless external assessments reflect these important processes* [using and applying mathematics], *they are unlikely to influence a significant shift in teaching and learning mathematics."* (Ofsted, 2008, pp.35-6)

This is particularly poignant given the warning given in the Cockcroft Report (DES, 1982), over twenty five years previously, where examination specifications were described as the 'greatest single factor' on the influence of mathematics teaching. Similarly the impact of the examination system has on teaching and learning is acknowledged in the request by ACME (2009) that if there were to be no negative effects for students, changes to mathematics examinations would need to be gradual. Descriptions of 'assessment as learning' are evident in every stage of education from primary through to undergraduate level. The Cambridge Review (2010) acknowledges the effects that the end of key stage 2 tests have had not only the content that is taught in Year 6 but also on the teaching styles with 'transmission' pedagogies becoming more prevalent. Similarly, Nardi & Steward (2002a) found that many Year 9 students prepared for the key stage 3 examinations by memorising solutions to common questions. Similar practices are described in accounts of learning undergraduate mathematics. When discussing courses in linear algebra Sierpinska (1994) describes that the practice of teaching to the test limits students' understanding.

> *"if they are done in the chalk-and-talk style, with little conversation with students, the students' understanding is not probed enough. Most of the questions and problems are straightforward exercises...or proofs. Neither put into question the students' understanding. In exercises they can get a correct answer by blindly applying a method shown in a model solution."* (Sierpinska, 1994, pp.116-7)

Whilst there are many accounts of the limitations of teaching to the test, there are also numerous suggestions that the practice is widespread (Tomlinson, 2004; Golding, 2007; Mansell, 2007; Torrance, 2007; Ofsted, 2006 & 2008; Bassett et al., 2009). The most frequent reason given for this practice was the emphasis placed on examination results (section 3.2.4). The importance of success in examinations was considered by Skemp (1976) and seems just as relevant to the cohort of students in my own study as it did over forty years ago.

> *"in view of the importance of examinations for future employment, one can hardly blame pupils if success in these is one of their major aims. The way pupils work cannot but be influenced by the goal for which they are working, which is to answer correctly a sufficient number of questions."* (Skemp, 1976, p. 24)

This accurately reflects the current situation with the teaching and assessment of the A-level mathematics course. The goals of achieving particular grades are not only pertinent to the individual student but to the class teacher with performance management targets and to the whole school with value added measures and league tables of examination results published nationally. The issues of motivation and goals for learning A-level mathematics are explored in section 3.4.2.

3.4 Internal Context

Students typically choose their AS-level subjects during the final year of their compulsory education, Year 11. Thus students' choice of whether to continue with their study of mathematics is made within the context of the education they have received so far. Their experiences of learning mathematics are within the factors described previously, both the national context; the debate about A-levels, modular examinations and hyper-accountability and the school context; attainment grouping and teaching to the test. Students also make decisions based on their perceptions of what they think mathematics will be like at A-level and their goals for choosing to study A-level mathematics.

3.4.1 Reasons for Choosing A-levels

There have been several investigations into the reasons why students decide to continue with their study of mathematics post-sixteen (Dick & Rallis, 1991; Tebbutt, 1993; Mendick, 2005; Brown et al., 2008; Hernandez-Martinez et al., 2008). There is significant research into the reasons why more males than females chose to study mathematics at AS level. However, female students' participation and performance in A-level mathematics is not seen to be an issue at the school that is the focus of this research. Indeed, the relatively poor performance of male students has been part of both the mathematics department and whole school development plan, and so I did not consider gender issues within my own research.

Brown et al. (2008) investigate Year 11 students' feelings about mathematics and their reasons for whether they considered taking the subject at AS level. Reasons given for students not choosing to continue with the subject are the perceived difficulty of mathematics, a lack of enjoyment in the subject along with a lack of usefulness. The main reason given is that mathematics is too difficult. Anticipated difficulty is often based on external influences such as the experiences of older siblings or friends and the messages given by teachers. The second most common reason for not continuing with mathematics is that students did not enjoy or like the subject. This reason is closely entwined with the third reason, that mathematics was boring. Brown et al. (2008) commented on the findings of Nardi and Steward (2003) who identify a "mystification through reduction" effect where students are taught to perform algorithms without the opportunity to develop the understanding of why the algorithm works. This practice fits with of teaching to the test (section 3.3.2).

As expected Brown et al. (2008) found that the proportion of students intending to continue their study of mathematics was particularly high in those who were predicted grades A* and A at GCSE. This replicated the findings of Tebbutt (1993) that prior success in the subject and the requirement for a future university course are the main reasons given for choosing A-level mathematics.

Students' choice of whether to continue to study mathematics can also be affected by their future career aspirations (Dick & Rallis, 1991). Similarly, Hernandez-Martinez et al. (2008) found that there were four 'aspirational repertoires' that were their motivation to study AS mathematics. The vast majority of students held one of these four goals: 'becoming successful, achieving personal satisfaction, vocational and idealist' and this affected their views of mathematics.

> *"In using the 'becoming successful' repertoire, students speak of an instrumental approach to their subject choices – taking AS mathematics as a high status university entry qualification, which has exchange value, in achieving success. For those using the 'vocational' repertoire, the 'use value' of mathematics in their current course becomes significant. For example mathematics is a central part of becoming and acting like an engineer, and therefore is integral to the students' imagined future as well. The other two repertoires do not necessarily speak of mathematics as 'useful', but instead may relate it as an enjoyable subject in its own right."* (Hernandez-Martinez et al., 2008, p.163)

Whilst there were links between the aspirational repertoire and socio-cultural groups, also found by Vidal Rodeiro (2007) these are not considered here as they were not a feature of my own data collection or analysis.

Within the national context of the debate on the purpose of A-level mathematics, several researchers (Vidal Rodeiro, 2007; Bassett et al., 2009; ACME, 2009) found that mathematics is considered as a high status subject. Mendick (2005) describes that it is considered to be 'the ultimate intelligence test' and this can motivate students to take the subject at A-level. Whether chosen for the further study of the subject or as a qualification required for future university or career options, the Royal Society (2008) reports how mathematics was considered to be a valuable A-level by students both as a subject in its own right and for their future prospects. This reflects the dual purpose of mathematics (section 3.2.1) as described in the Smith Report (2004). Whether students chose to continue with A-level because they want to learn more mathematics or because they want to achieve a particular grade at the subject is explored next.

3.4.2 Learning and Performance goals

There is a significant body of literature (Ames, 1992; Ames & Archer, 1988; Dweck, 2000; Pintrick, 2000; Kaplan & Middleton, 2002) which examines students' motivation to learn and considers the benefits of two types of goal. A performance or ability goal is defined as one that allows a student to demonstrate their competence and a learning or mastery goal as one that allows students to increase their competence.

> *"A performance goal is about measuring ability. It focuses students on measuring themselves from their performance, and so when they do poorly they may condemn their intelligence and fall into a helpless response. A learning goal is about mastering new things. The attention here is on finding strategies for learning. When things don't go well, this has nothing to do with the student's intellect. It simply means that the right strategies have not yet been found."* (Dweck, 2000, p.16)

The motivations of students with learning goals show similarities with the description of Skemp (1976) as discussed in Chapter 2, where the satisfaction gained from relational understanding becomes the motivation for further learning. There are also significant similarities in the two approaches to learning described as deep or surface (Marton and Säljö, 1976; Entwistle, 1987; Biggs, 1999, 2007; Lublin, 2003). The behaviours of students with learning goals closely fit the description of those with a deep approach to learning.

> "[Students] *have the intention of understanding, engaging with, operating in and valuing the subject"* (Lublin, 2003, p. 3)

Similarly, the descriptions of students with a surface approach to learning matches the descriptions of those motivated by performance goals.

> *"*[Students] *tend not to have the primary intention of becoming interested in and of understanding the subject, but rather their motivation tends to be that of jumping through the necessary hoops in order to acquire the mark, or the grade, or the qualification."* (Lublin, 2003, p.4)

Whilst using a different label for a learning goal, Ames & Archer (1988) reached similar conclusions that found students were more resilient in the face of challenge and more involved in their learning if they held a 'mastery goal'. Furthermore, students' perceptions of the goals of the class were found to influence their learning strategies.

> *"Although high-achieving students may be expected to be more knowledgeable and aware of effective learning strategies, their reported use of strategies was dependent on how they perceived the goal emphasis of the class."* (Ames & Archer, 1988, p.264)

This has implications for my research as the high ability A-level students were learning mathematics within an environment dominated by performance goals, and these are explored in Chapters 6 and 7.

The type of goal that a student holds may be particularly key in the face of difficulty and challenge. Dweck (2000) describes that there are two responses to challenge and failure, 'Mastery-Oriented' or 'Helpless'. Students who hold a learning goal often adopt a 'Mastery-Oriented' response to challenge and do not perceive they are failing if they cannot solve problems immediately. They work on strategies to tackle the problem and remain confident that they will succeed. In contrast, students who hold a performance goal frequently demonstrate a 'Helpless' response to difficulty and quickly question their ability when faced with a challenge, often blaming their lack of intelligence for the failure. Strikingly, they also lose perspective on their prior successes, abandon previously successful strategies and believe they cannot answer questions that they had been able to prior to the difficulty.

"the students showing the helpless response quickly began to doubt their intelligence in the face of failure and to lose faith in their ability to perform the task. To make matters worse, even the successes they had achieved were, in their minds, swamped by their failures." (Dweck, 2000, p.8)

This fits with the finding of Brown et al. (2008) where some students perceived that there were 'fixed cognitive boundaries' to their mathematical learning.

Whilst Ames (1992) and Turner et al. (2002) recommend that mastery goals have the most positive effect on students learning, Kaplan & Middleton (2002) note that there is significant evidence for the benefits of performance goals for particular students such as those who are older with 'high ability'. The majority of the literature suggests that there is a place for both performance and learning goals, and the following quote is typical.

"Students who were concerned about their performance and wanted to do better than others and, at the same time, wanted to learn and understand the material had an equally adaptive pattern of motivation, affect, cognition, and achievement as those just focused on mastery goals." (Pintrick, 2000, p.552)

Whilst describing both goals as 'entirely natural, desirable and necessary' Dweck (2000) warns of a potential problem when performance goals become over-emphasised.

"There are times when students need to master new tasks and acquire new skills, and there are times when they need to display and validate the skills they already have. Performance goals are indeed a critical part of achievement. The problem with performance goals arises when proving ability becomes so important to students that it drives out learning goals." (Dweck, 2000, p.152)

Midgley, Kaplan & Middleton (2001) also suggest that students thrive when both performance and learning goals are high, yet warn that the high stakes testing culture may be 'driving out' learning goals. This may be particularly key in mathematics as Pintrick (2000, p.553) describes that mathematics classrooms 'may be more performance oriented in general'.

3.5 Conclusion

In the previous chapter, furthering understanding was universally considered to be central to learning mathematics. The Cockcroft Report (DES, 1982) warned that threats to developing understanding were seen as lessons moving too quickly and the reliance on rote learning.

> *"we do not believe that it should ever be necessary in the teaching of mathematics to commit things to memory without at the same time seeking to develop a proper understanding of the mathematics to which they relate."* (DES, 1982, p.70)

In this chapter the three levels of context which effect students' experiences of learning mathematics are considered. Within the external context, at the national level the debate about the dual purpose of mathematics A-level fuels the concern over participation. Syllabuses which encourage greater numbers of students taking the subject seem to be in contradiction with the demands for an A-level course which stretches and challenges the most mathematically able students (Smith, 2004; Golding, 2007). After the concern of falling numbers of A-level mathematics entries in the mid 1990's and the dip following the implementation of the initial modular structure of Curriculum 2000, since 2005 there has been a steady increase in students taking A-level mathematics. This increase in participation combined with mathematics being the subject with the highest percentage of A grades seem to indicate that it is a popular and rewarding choice. However, it is not widely perceived in this positive light (Smith, 2004; CASE, 2008; Ofsted, 2008). Paradoxically, A-level mathematics is still perceived as a difficult subject and often marketed to only the highest achieving GCSE students, described as the 'clever core' by Matthews & Pepper (2006 & 2007).

Other national factors that impact on students' learning experience are the frequency of the modular examinations. Since the introduction of January and June sittings of AS and A2 modules in the Curriculum 2000 structure, A-level students are at most 12 learning weeks away from their next examination. This alongside the publishing of A-level results and their use in school league tables, department and teacher performance reviews has led to a system whereby a student's results have implications far beyond those of the individual who sat the examination. This system described as 'hyper-accountability' by Mansell (2007) has influenced teaching and learning and many researchers consider a tension between developing understanding and achieving highly in examinations (Tomlinson, 2004; Golding, 2007; Mansell, 2007; Torrance, 2007; Bassett et al., 2009).

These national contexts influence how mathematics is taught in schools. Headteachers and Heads of Mathematics Departments make decisions within the current climate of modular examinations and the culture of high-stakes testing. Teaching mathematics in attainment groups fits with the modular and tiered GCSE mathematics curriculum as different groups can be taught different syllabi. Yet there is a body of literature (Slavin, 1990; Ireson et al., 2005; Kutnick et al., 2005) which concludes there is no overall increase in the average achievement of students when taught in attainment groups. There are also accounts of disadvantages for students placed in either the 'bottom' or 'top' sets (Wiliam & Bartholomew, 1994; Nardi & Steward, 2003; Boaler et al., 2000). Similarly, the value placed on published examination results had led to the practice of 'teaching to the test' where only what is examined is valued and consequently this becomes what is taught (Tomlinson, 2004; Golding, 2007; Torrance, 2007; Ofsted, 2006 & 2008; ACME 2009).

The internal contexts are the views of the learners, and given their previous learning experiences, students make choices about their A-level subjects. Brown et al. (2008) describe how the external factors influence whether students continue to study mathematics beyond GCSE.

> *"The notion that there are sudden break points in mathematics, where difficulties are encountered or failures occur, is reinforced at institutional level. For example, one such boundary is created by the move from GCSE to A-Level, and the somewhat magical maturational and academic step this is perceived to be."* (Brown et al., 2008, p.8)

In addition to these external factors, other researchers consider students' motivation for further study. Whether they hold learning or performance goals (Ames, 1992; Ames & Archer,1988; Dweck, 2000; Pintrick, 2000; Kaplan & Middleton, 2002) is seen as important both for their motivation to study and particularly when faced with difficulties.

My study of high-achieving Grammar school students' experiences of A-level mathematics takes place within this complex picture of national, school and internal contexts. In the following chapter I consider different methodologies for my study and present a research design.

CHAPTER 4: METHODOLOGY

4.1 Introduction

In this chapter I consider various approaches to research design, data collection and analysis, their advantages and disadvantages, and provide reasons for the choices of methods I employed in my research. Stake (2003) suggests that issues are often chosen to be researched in terms of what can be achieved by a practitioner researcher in their particular situation. This described the formation of my own research question and design where the practicalities of being a practitioner researcher influenced all of my design decisions.

I started with the decision to research high achieving students' understanding of A-level mathematics through their descriptions of difficulties in my own school, as being a working teacher I could not take the time away from my own lessons to observe practice in other schools. Nor was it feasible to observe a sufficient number of lessons within my school as this would interfere with my own teaching commitments. Previous experiences from my own master's research on GCSE Mathematics re-sit students (Minards, 2006) showed that the qualitative data provided far more insightful information about the issues facing these students. Swan (2000) researched aspects of GCSE resit students' learning and used both quantitative and qualitative methods. He concluded that the effectiveness of the discussion activities used in the study was better measured by the qualitative data from interviews and lesson observations rather than the GCSE results of the students. Similarly, in their study of reasons why students chose not to study mathematics in the sixth form, Brown et al. (2008) used qualitative data to inform their research.

Having decided to track students in my own school through the two years of their AS and A2 courses and explore their descriptions of difficulty at several points in this two year experience, I considered the literature on longitudinal studies and in particular those studies within a qualitative research paradigm. I decided that a mixed method case study with a data collection period over two years would create a sufficient data set to explore the experiences of one cohort of A-level mathematics students. The literature on case studies led me to adopt a nested case study approach (Thomas, 2011) and this is described in detail in section 4.4.1. The mixed methods included individual interviews with students and teachers, the timings of which were influenced by the structure of the modular A-Level course. Interview data was contextualised with questionnaire data from Year 11, AS and A2 mathematics cohort. The questionnaires were completed by all students who were willing to participate at the end of each academic year; volunteer issues are considered in section 4.2.5.

A second strand of the data collection was document analysis, with the development and then use of a model to classify the level of demand in A-level mathematics questions. The documents analysed were the OCR pure mathematics examination papers from June 2007 to June 2010. From the literature on mathematical understanding discussed in Chapter 2, I developed ideas from Sierpinska (1994) to create a model to classify these examination questions. The timetable for the stages of this process are given in section 4.4.2 and detailed in the following chapter.

Several researchers (Nardi & Steward, 2003; Boaler, 1997a; 1997b; Mendick, 2005; Brown, 2008) suggest there are gender issues surrounding participation of A-level mathematics and that in the past it has been seen as a boys' subject. Mendick (2005) suggested that girls become disaffected if they are taught mathematics in a certain way. Boaler (1997a) found that girls do better when taught in mixed attainment rather than attainment groups. Brown et al. (2008) found that high achieving girls who have low confidence showed concerns that they would not be able to cope with the perceived increased demands of A-level mathematics. This fitted with the findings of Dweck (2000). However I decided not to consider gender within my research because of the context of the school. Within this school there has been a focus on boys' underachievement, as girls out-perform boys in the majority of subjects, including mathematics, at GCSE, AS and A2 level. Additionally there are a large number of girls participating and succeeding at mathematics at A-level. Similarly the makeup of the school population meant that it was not feasible to consider issues surrounding different ethnicities. In section 4.4 I present my final research design.

4.2 Methodology

4.2.1 The Role of the Practitioner-Researcher

Throughout the entire research process I had to deal with the issues related to being a practitioner-researcher; issues that impacted on my research design in relation to resources, time available and choice of research sample as well as raising particular ethical issues involved in researching my own school.

Much of the literature on practitioner research relates to the method of action research (Carr & Kemmis, 1986) with such research generally being described as 'research carried out by practitioners for the purpose of advancing their own practice' (McLeod, 1999, p.8). I rejected action research as a possible method since I was not researching my own practice to improve that practice. Allwright (2005) however extends the view of practitioner research by describing it as a ‘relationship of identity’ (p.351) between the people being investigated and the people doing the investigation, which he says influences the design and data collection as well as the ethics of the research and the knowledge gathered. He further offers practitioner research that focuses on understanding the ‘quality of life’ of rather than the ‘quality of output’ of the students (Allwright, 2005, p.353). As my research is defined to explore the nature of the experience of the A-level students rather than how well they are doing in their examinations this distinction is helpful.

During the research I was working as a fulltime mathematics teacher and carried out research with A-level students and colleagues in my own school and within a particular examination culture as described in Chapter 3. Both of these (the grammar school and examination system) were deliberate and initial choices for developing my research design as explained in Chapter 1. Robson (1993) lists the main advantages of being a practitioner researcher as ‘insider’ opportunities, which include pre-existing knowledge of the situation and people involved; practitioner opportunities, such as easier access to subjects and documents; and ‘practitioner-researcher synergy’ where practitioner insights help inform the design and analysis of data collection. Indeed these were all true in my research. I was able to ensure a high return for the Year 11 Questionnaire as the ‘insider opportunity’ of being a teacher at the school meant that I could speak to the whole year group and ask them to complete the questionnaire during assembly time. Yet the tension involved was the potential threat to the truth of the students’ responses when they were asked to describe their experiences of learning mathematics by a teacher in the school. Awareness of this issue led me to be clear about the anonymity and confidentiality of the students’ responses, particularly that the data that I collected would not be shown to other teachers at the school. Pring (2000) agrees that the main advantage of a practitioner-researcher is an understanding of the particular situation, with its social rules and practical constraints. Pring further addresses the issue of impartiality of practitioner research.

> *"Objectivity suggests that the researcher should be somewhat distant from what is being researched into. Prejudice, self-interest, familiarity, defensiveness would surely distort the research of the teacher. Who is likely to seek to falsify the very principles on which his teaching is based?"* (Pring, 2000, p.121)

In Allwright's terms I was aware during the phases of the research of developing a researcher identity with all its constraints and tensions alongside my known practitioner identity within the school. One of the steps that I took to decrease the potential risks related to familiarity for a practitioner researcher was not to collect interview data from students whom I taught. However, there were still numerous issues involved in collecting data from students that I needed to remain aware of. During the course of this research I became Deputy Head of Sixth Form with responsibility for the year group involved in this study. Whilst I did not teach any of the case study students, I was involved with all of them in a pastoral role. This added to the range of roles that I needed to balance and I chose to 'play down' my Head of Year and teacher roles as described by Bell & Nutt (2002).

> *"Since practitioner-researchers have to negotiate a range of responsibilities… decisions about emphasizing or 'playing down' the role of 'practitioner' may be an important part of such negotiation."* (Bell & Nutt, 2002, p.87)

Whilst collecting data I remained mindful of the issues and found ways to distance myself from the role of teacher. During interviews I learned to remain in a researcher role, avoided any teacher responses to issues raised and reassured participants that there were no judgements of their opinions. As with any interview situation, there were issues about power and relationships. Unequal power relations in interviews may be based on many factors including gender, race and social class.

> *"Power is most commonly assessed in terms of structural disparities between members of social groups."* (Holloway & Jefferson, 2000, p.84)

In my particular situation this referred to me interviewing students when I was Head of their year group and within this context a power difference was inevitable. Also, when I interviewed teachers, I was interviewing colleagues and there were several factors that affected the power relationships within these interview situations. As Deputy Head of Mathematics I was responsible for the performance management two of the teachers. However, there are ways to lessen the effect of unequal power relationships such as a shift in emphasis of the specific roles of the interviewer and interviewee as described by Holloway & Jefferson (2000).

> *"relational dynamics, such as understanding and respect, have the capacity to transcend structural power differences. Such relational dynamics, which draw on the deep pool of common human characteristics, does not equalize power, but makes it negotiable, rather than an inevitable effect of status differences. It shifts the emphasis from the exercise of power per se to its effects in context."* (Holloway & Jefferson, 2000, p.85)

Throughout the interviews I aimed to shift the focus away from roles within the school structure such as Head of Year and student to interviewer and interviewee. Further issues regarding interviews as a method of data collection are considered in section 4.3.3.

There were also ethical issues to consider when conducting research within the culture of my own school. At times I found it difficult to interview, analyse and write about colleagues' responses when they had generously given up their time. Understandably, students did not show a particular interest in my research, but at several points throughout the process each teacher asked about my findings. However, with limited time available to discuss non-curricular matters, colleagues were content with timescale updates rather than detailed findings. My experience as a practitioner researcher informed my ethical decisions, as they formed part of my everyday duties as a teacher and qualitative researcher. These daily decisions of an ethical nature were described by Janesick (2003).

> *“qualitative researchers, because they deal with individuals face-to-face on a daily basis are attuned to making decisions regarding ethical concerns, because this is part of life in the field.”* (Janesick, 2003, p.56)

As I was aware of the issues where I held a position of responsibility over the students whom I asked to volunteer, I ensured that at each stage of the data collection process, there was no obligation to participate, provided details of what would be involved and re-iterated that there would be no consequences if students did not participate. Futher ethical issues are considered in section 4.2.4.

4.2.2 Case Study Multi-Method Approach

The purpose of a research enquiry may be characterised into three headings ‘Exploratory, Descriptive and Explanatory’ (Robson, 1993). My research was a combination of the first two as I explored students’ understanding through their descriptions of what they considered to be difficult about learning and being examined in AS and A2 mathematics. Their responses were considered against a model for classifying the demand of mathematics examinations questions, which is described in section 4.4.2 and Chapter 5. The two types of data collection and analysis were then synthesised to form a detailed picture of the sources of difficulty and what was considered hard to understand when learning and being examined at A-level mathematics.

Robson (1993) describes three traditional research strategies whilst cautioning against the exclusive use of only one method.

> *"All methods have their strength and weaknesses. Recognizing this leads to a preference for multi-method approaches…several methods are likely to be better than any single one in shedding light on an issue."* (Robson, 1993, p.x)

The three strategies are 'experiment', where samples from a population are assigned to different experimental conditions and subjected to a planned change in one or more conditions. 'Survey', is where a relatively small amount of data is collected from a selection of individuals, often by questionnaire or structured interview. Finally, 'case study' is where detailed and intensive knowledge about a single case or small number of related cases is accumulated, usually employing a range of data collection techniques. These strategies are not necessarily distinct and a combination of these may be employed, however Robson suggests that a case study approach is most appropriate for an exploratory enquiry. As I aimed to explore students' understanding through their descriptions of difficulties in doing A-level mathematics, I wanted to achieve a 'thick description' of this situation as described by Robson and was led toward a multi method case study approach.

> *"the kind of ' thick' description provided in a well written case study report can make contact with the more implicit and informal understandings held by readers who are able to see parallels with the situation in which they work or otherwise have knowledge about."* (Robson, 1993, p.73)

A case study, or 'singularity' as described by Bassey (1995), offers the opportunity to study issues in more detail than from a typical large scale survey. A case study is not a research method per se, rather an approach which allows a single member or small group of a population to be studied via a range of evidence sources. The singularity that is the focus of this research is the September 2008 to July 2010 cohort of A-level mathematics students in a mixed Grammar School Sixth Form. The range of data collection methods within a case study to provide a valid and complete picture is described by Gillham (2000).

> *"a range of different kinds of evidence, evidence which is there in the case setting, and which has to be extracted and collated to get the best possible answers to the research questions. No one source of evidence is likely to be sufficient (or sufficiently valid) on its own."* (Gillham, 2000, p.1)

A case study allows data to be collected in depth from numerous sources, such as interviews, observations and collection of documents, which build a complete picture of the individuals studied. This enables the case to be studied in its own right, rather than as a sample from a population (Robson, 1993). This in-depth data collection from a variety of methods is the main advantage of using a case study, as the multiple sources ensures validity by triangulation. Other advantages of case studies are observing participants in their natural circumstances, without the need for a researcher to set up or control a situation. There is also the practicality that a case study fits in with small-scale research as all the effort is concentrated on one group or individual.

However, the nature and variety of the evidence gained from adopting a case study approach may cause difficulties when the data is coded. Gillham (2000) suggests that maintaining a research log is a way to help organise not only the data collected, but also ideas for future analysis. The most significant disadvantage of the case study is how far, if at all, the findings can be generalised. This issue needed to be addressed before I chose to adopt a case study approach. The case needed to be described in detail and similarities, differences and comparisons to other known cases needed to be made clear. However, whilst acknowledging the uniqueness of each individual case, Denscome (1998) illustrates the potential of this approach.

> *"Although each case is in some respects unique, it is also a single example of a broader class of things"* (Denscome, 1998, p.36)

Stake (2003) explores this issue further and identifies three types of case study, 'Intrinsic', 'Instrumental' and 'Collective' with the classifications dependent on the extent to which the individual case or the wider population was studied. Intrinsic case studies are described as those that aimed for greater understanding of a specific case and are not chosen as an example of a wider group of cases. At the other end of the spectrum are 'collective' case studies, where a number of cases are studied simultaneously in order to research a general condition within a larger population. Generalisation is seen to be at odds with gaining specific detail of individual cases. Yet in between these two categories was the 'instrumental' case study that is defined as one where a specific case is studied to further understanding of a wider situation.

> *"The case is of secondary interest, it plays a supportive role, and it facilitates our understanding of something else. The case still is looked at in depth, its contexts scrutinised, its ordinary activities detailed, but all because this helps the researcher to pursue the external interest. The case may be seen as typical or not."* (Stake, 2003, p.137)

Whilst Stake (2003) describes that many research did not fit entirely within one category, I believe that my research fits within the 'instrumental' case study. Within this type of case study Stake (2003) describes that the researcher is typically focussed on how the 'concerns of researchers and theorists are manifest in the case' (pp.140-1). In my own research the concerns were those outlined in Chapter 3; modular A-level courses, hyper-accountability, assessment as learning and the dominance of performance goals. To explore how these concerns were manifested in the cohort of A-level mathematics students within my school, I needed to decide how to gather data from this group of over one hundred students. The description of an 'embedded case study' (Yin, 2009) also described as a 'nested' case study by Thomas (2011) was helpful here and the idea of several 'subunits' is used in this research.

Eight students and four teachers were the 'subunits' used to inform the wider case of the whole cohort of students. Details about the number of students and teachers and their selection are given in section 4.4.1.

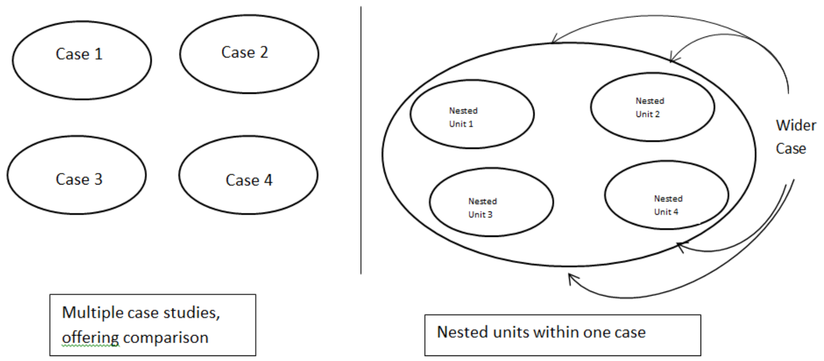


Figure 4.1 "multiple and nested case studies compared" Thomas 2011

Thomas (2011) describes nested case studies as those where the 'subunits' fit within a particular case.

> *"It is that* fitting in *in which you are interested – how does the subunit connect with the other subunits and the whole?"* (Thomas, 2011, p.152, his emphasis)

Rather than comparing and contrasting findings from multiple case studies, the data from nested case studies informs the wider case. In my research the nested units were the eight students and four teachers with the wider case being the cohort of students taking A-level mathematics at a mixed Grammar School in the West Midlands. The question of case study research having implications for the wider population is also considered by Pring (2000) who specifically describes how practitioner research could be applied in wider contexts.

> *"no one situation is unique in every respect and therefore the action research in one classroom or school can illuminate or be suggestive of practice elsewhere."* (Pring, 2000, p.131)

Again, the quality of description of the case is seen as important so that the reader can identify similarities and differences between the research and their own situation (Robson, 1993; Burns, 2000; Stake, 2003).

> *"A new case without commonality cannot be understood. Yet a new case without distinction will not be noticed. Researchers cannot know well the already known cases, the peculiarities of mind, of their readers."* (Stake, 2003, p.146)

The commonality of my research are the GCSE students who made their choices about whether to continue on to mathematics post-sixteen, and the A-level mathematics students tracked throughout Year 12 and Year 13. The distinction is that these are Grammar School students, selected for their high attainment and who go on to achieve highly in A-level mathematics.

My decision to adopt a case study approach was confirmed by the literature, parameters of the AS and A2 mathematics courses at my school and the practicalities of being a practitioner researcher which meant that many research methods were not appropriate. Therefore I did not employ any techniques involved in large-scale studies, ethnography, action research or abstract research.

4.2.3 Longitudinal Research

Initially, I questioned whether my planned research design with data collection over a two-year period would fit within the longitudinal study literature. Longitudinal is described by Ruspini (2002, p.3) as 'a rather imprecise term' which covers all collections of data over time. However, whilst concluding that there is no consensus on the minimum required time for a qualitative research study to be considered longitudinal, nor any fixed number of interviews or fieldwork hours, Saldana (2003) recommends that within an educational setting, nine months (one academic year) is the minimum time period. As my planned research exceeded this minimum period, I considered the literature on designing and analysing longitudinal studies. As with the descriptions of case studies, there are many researchers (Saldana, 2003; Menard, 2002) who describe longitudinal research as a group of methods rather than one single research approach. Menard (2002) defines three types of longitudinal research in terms of the data collection and analysis.

> *"(a) data are collected for each item or variable for two or more distinct time periods; (b) the subjects or cases analyzed are the same or at least comparable from one period to the next; and (c) the analysis involves some comparison of data between or among periods"* (Menard, 2002, p.2)

He describes three main types of longitudinal research designs, two of which involve the collection of data from the same set of cases 'prospective panel' where data is collected at several different times and 'retrospective panel' where data is collected for the cases at one time but covering the recollections of several different time periods. The third, 'repeated cross-sectional' involves collecting data from different samples of cases at each time period, with the aim that each cross section could be compared. Saldana (2003) suggests that the collection of 'baseline' data is crucial and for my own 'prospective panel' study this includes students' initial perceptions of learning mathematics, gathered by questionnaire at the end of their GCSE course along with reasons for choosing A-level mathematics as reported in Chapter 6. Saldana's advice of maintaining meticulous chronological details of all data collected was followed, with all interviews transcribed in full and questionnaire responses entered onto Excel spreadsheets. The number of participants both in the entire cohort of A-level students and the number of nested case study students and teachers meant that electronic analytical or statistical packages were not needed.

A common problem with longitudinal research is the attrition rate (Ruspini, 2002) of participants who leave the studies. Whilst there are a variety of possible reasons for this; lack of willingness to continue participation, lack of availability through moving from the area, illness or even death. Many of these factors cannot be predicted by the researcher, yet the advice that I followed was two-fold, I started the study with more participants than needed and attempted to select participants who were willing to engage over the entire research process. However, this second point led to issues with using volunteers rather than randomly selected participants and these are considered further in section 4.2.5.

A further problem with longitudinal research is described as 'panel conditioning' (Ruspini, 2002) where repeated involvement in interviews may affect participants' responses. I had anticipated that this may have occurred in my own research, particularly with the students who were interviewed six times over a two-year period. I thought that their responses may have become more detailed as they were repeatedly asked to reflect on their experiences of learning mathematics. However, as discussed in Chapters 6 and 7, this did not occur. Saldana (2003) warns of the dangers of expecting change and oversimplifying the reasons for change to an individual factor. The changes to the researcher during a longitudinal study are also not to be overlooked.

> *"Even the researcher is affected by longitudinal enterprises through personal and professional outcomes, since his or her own journey toward the destination is both processual and developmental."* (Saldana, 2003, p.14)

Ruspini (2002) outlines three types of time series analysis: ‘Temporal analysis’ which describes a trend over time; ‘Discontinuity analysis’ that additionally considers the impact of a particular event and ‘Time series regression analysis’ which compares the effects of the same changes over several time periods. My research fits within the first type of longitudinal research as I described understanding as evidenced through the descriptions of difficulty of learning and being examined in mathematics over a two-year period.

In addition to looking at the experiences of the eight ‘subunit’ students in depth, I collected information from the wider case, all the AS and A2 students, in order to provide a context for the more detailed responses of the nested unit students. Before individual research methods are considered, the ethical considerations that affected my research were explored and potential issues were addressed.

4.2.4 Ethical Issues

There are ethical issues relating to any type of research. Glesne and Peshkin (1992) lists ethical guidelines issued by the Council of the American Anthropological Association which include that the aims of an investigation should be communicated as well as possible to the ‘informant’, the right to anonymity of all ‘informants’, and safeguarding of the rights, interests and sensitivities of those studied. Robson (1993) also stresses the need to maintain confidentiality and recommends negotiating the level of involvement with the participants and sharing reports on the progress of the research with participants. I followed these guidelines throughout my research and whilst I did not want to share interim findings with participants in case it affected their future responses, I acted with the ‘common sense and responsibility’ suggested by Fontana & Frey (2003) when sharing progress with students and teachers.

I gained the ‘informed consent’ from each teacher and student before any data collection began, explaining the purpose of the research and the use of the data. This is described as the fundamental principle of ethical research by Burns (2000). I followed the policy of the school for gaining permission from students for collecting data, and under this policy, it was down to the individual students to give their consent to be involved. This consent took the form of verbal information given to all Year 11 students before they were given the option of whether to complete the questionnaire. One of the questions then asked whether students would be willing to participate further in the research by being interviewed. Students were selected from the group of 65 out of 88 questionnaire responses who replied ‘yes’ to this request. Similarly Year 12 and Year 13 mathematics students were asked whether they would complete a questionnaire and hence the numbers of responses were less than the entire cohort. The dependence on volunteers and implications for the validity of my research is considered in the following section.

4.2.5 Validity and Reliability

Unlike the tight definitions within the quantitative research paradigm, there is debate about the meanings of validity and reliability within the qualitative field. Validity in qualitative research can be considered as ‘showing your workings’, giving the research reasons for the decisions made and finding a balance between opportunism and principle (Holliday, 2002).

> *“It is something to do with it being accurate or correct or true. These are difficult (some would say impossible) things to be sure about. It is possible to recognise situations and circumstances which make validity more likely.”* (Robson, 2002, p. 170)

Robson (2002) describes these situations as rigorous and multiple data collection procedures, detailed methods and clear writing which often presents the study in stages with multiple themes. I adhered to these guidelines throughout my research, triangulating data from interview, questionnaire and written data collection sources. The need for clarity in description and explanation of findings, considered important by Janesick (2003), was a requirement which featured throughout the analysis and write-up of my research.

> *"Validity in qualitative research has to do with description and explanation and whether or not the explanation fits the description."* (Janesick, 2003, p.69)

A potential threat to the validity of my findings was my use of volunteers to participate in my research. Burns (2000) describes that the use of volunteers reduces the external validity of the findings.

> *"The problem with volunteers is that they are not likely to be a random sample of the population. They tend to be better educated, of a higher social class, more intelligent, more social, less conforming and possess a higher need for approval than non-volunteers. This means that the external validity (the confidence to generalise to the population) is reduced."* (Burns, 2000, p.18)

However, the students in my study were not a random sample, they were volunteers selected from a high achieving cohort of Grammar School sixth form mathematics students. The ethical issues outlined in the previous section meant that I needed to gain the permission of all the students and teachers who participated in my research and so I was dependent on volunteers. Following the advice from the literature of providing clarity of description of methods used, I provide details of the selected students in section 4.4.1 and was aware of the volunteer effect when analysing the data collected from these students.

The honesty in detailing the case study students' willingness to participate in the research and their academic achievements was one of the factors in achieving reliability as Robson (2002) describes.

> *"*[Reliability] *involves not only being thorough, careful and honest in carrying out the research, but also being able to show others that you have been."* (Robson, 2002, p.176)

Reliability can also be addressed in specific parts of a research process, Silverman (2000) describes the detail and care needed when transcribing tape recorded interviews.

> *"the reliability of the interpretation of transcripts may be gravely weakened by a failure to transcribe apparently trivial, but often crucial, pauses and overlaps"* (Silverman, 2000, p.187)

In order to ensure reliability, I followed this advice, not only taking care to transcribe interviews as accurately as possible, but throughout the research process I documented the procedures followed and used the categories which emerged from the data consistently. A further reliability issue arose from a researcher choosing which data to tell and in what level of detail (Stake, 2003).

> *"Most naturalistic, ethnographic, phenomenogical researchers will concentrate on describing the present case in sufficient detail so that the reader can make good comparisons."* (Stake, 2003, p.148)

I made numerous decisions about which quotes to use, and how much detail to include from each set of data collected. However, throughout the research process I selected data which was representative of the case of Grammar School students' experiences of learning and being examined in A-level mathematics.

4.3 Research Methods

4.3.1 Document Analysis

Within the context of this research, documents refer to A-level mathematics questions, most often examination papers that the cohort of students in this study sat as part of their course.

These include the two pure AS mathematics examinations Core 1 and 2 and the two A2 papers, Core 3 and 4 (copies included in Appendix 4, A4-1-13). Also included are the questions that the case study teachers and teachers brought to interviews when asked to bring questions that they found easy and difficult on specific topics, trigonometry and vectors.

Hodder (2000) explores the type, range and role of documents for research data and concludes that documents need to be interpreted within the appropriate context.

> *"[they] are prepared for personal rather than official reasons and include diaries, memos, letters, field notes and so on...Documents, closer to speech, require more contextualized interpretation."* (Hodder, 2000, p. 567)

In order to overcome the disadvantage of documents not providing reasons for responses, I adopted the approach employed by Cooper & Dunne (1998), Crisp et al. (2008) and Entwistle & Entwistle (1991) who interviewed students to gain insight into their written responses to key stage 3, GCSE and undergraduate examinations respectively. Once I had analysed the examination questions using the model described in the following chapter, I compared the analysis of the challenge in the questions with the verbal responses regarding the perceived difficulty of the questions in individual interviews with students and teachers. In order to further to triangulate the data, I collected written responses to the Core 2, 3 and 4 examination questions prior to each interview.

Bassey (1995) describes two types of studies, 'intervention' and 'non-intervention' with the analysis of the document contents fitting into the second category. Non-intervention studies are those where the existing situation, in this case mathematics examination papers, are studied without any changes being made. The analysis of content is described as more art than science by Burns (2000), and Robson (2002) describes the process as a type of common sense.

> *"content analysis is codified common sense, a refinement of ways that might be used by laypersons to describe and explain aspects of the world around them*." (Robson, 2002, p.352)

Robson (2002) distinguishes between 'manifest' and 'latent' content, the difference being whether the meaning of an item is directly present in the document or whether the researcher needs to infer or interpret the meaning. Whilst manifest content is described as more reliable, latent content is deemed appropriate if relevant to the research question.

> *"the research question should determine the type of system you are using, and it may well be that a high-inference system is appropriate. This then puts greater stress on ensuring that you can demonstrate reliability through the use of independent coders, or by some other means such as triangulating with data obtained through other sources."* (Robson, 2002, p.354)

Whilst I used latent content analysis to describe the nature of the level of challenge offered by a mathematics examination question, I triangulated this data with the written and interview responses from the students and teachers. The model achieved the 'exhaustive and mutually exclusive' qualities outlined by Robson (2002) where all of the examination questions could be categorised, and each part of an examination question fitted into only one category. In the following chapter, the development of the model for latent content analysis of mathematics examination questions is described.

The advantages of content analysis are seen by Robson (2002) as 'unobtrusive' as it is based on existing documents in this research, the AS and A2 mathematics examination papers. The documents are permanent and therefore available for reliability checks and finally, there is no cost involved as the examination papers existed prior to the research. Many of the disadvantages are not relevant to my research; partial availability of documents, bias due to the documents being written for a purpose other than the research. However the issue of causal relationships as described by Robson (2002) was relevant.

> *"Are the documents causes of the social phenomena you are interested in, or reflections of them?"* (Robson, 2002, p.358)

However, I did not research whether the type of examination questions were the source of difficulty for A-level mathematics students. Instead I explored the difficulties that students found in the process of learning and being examined in A-level mathematics. Thus the descriptions of the experiences of difficulty and not the causes of such difficulties were the focus of the research.

4.3.2 Questionnaires and Analysis

There is a large amount of literature concerning questionnaires and surveys that describe formats, question type and analysis techniques. Many questionnaires involve sampling from a large population (Denscome, 1998; Burns, 2000; Robson, 2002) however as there were only 90 students in Year 11 and 110 students who took AS mathematics in my own school I surveyed all students who agreed to participate. Gorard (2001) describes a typical questionnaire format as one that includes introduction questions on background information before the questions specific to the research.

> *"an introduction (to secure the co-operation of respondents), a question or two about the respondent (as a selection, identification or quota check to make sure you are addressing the right person), the substantive questions (about the research), and background questions (concerning respondents' personal characteristics)."* (Gorard, 2001, p.89)

Many large scale questionnaires utilise closed questions with specific answers such as multiple choice, scales of one to five or a choice of options ranging from strongly agree to strongly disagree, as described in Denscome (1998). These lead to quantitative data that can be collated and analysed statistically. Although I attempted some of this style of question in an initial study prior to my Masters research, I found that many multiple-choice questions did not provide useful information and the group size was too small to justify the use of statistical measures. As a result of this, I limited the amount of closed questions and often followed them with a 'give a reason for your choice'.

In the previous chapter, the findings of the Brown et al. (2007 & 2008) study regarding why students chose whether or not to continue with their study of mathematics post-16 were discussed, here their research design is considered. Whilst their large scale study, involving an eight page questionnaire issued to almost 2000 students from 17 schools, was significantly different to my own single school study, there were valuable insights that informed my questionnaire design. Whilst part of the questionnaire considered students experiences of the two-tier GCSE mathematics examinations, only the parts that were discussed in the 'I would rather die: attitudes of 16-year-olds towards their future participation in mathematics' Brown et al. (2007) article are included here. This section of the questionnaire included factual information such as gender and predicted grade at GCSE, closed questions such as which AS level subjects students were intending to take and whether they had considered mathematics AS-level. This question was followed by an open question 'so that students could provide any reason(s) that they wished' (Brown et al., 2007, p.22). In the discussion of the findings, the authors stated that these open responses were valuable.

> *"the free response items give additional insight into some student perceptions"*
> (Brown et al.,2007, p.22)

This re-enforced my decision to include both closed and open questions in my questionnaires. Also influential in my questionnaire design were questions where students could select words from a given list. In the Brown et al. study, this was a list of attitude words that students most associated with mathematics. I also followed the format of the instructions that did not limit the number of choices that a student could make and also permitted their own suggestions to be included. A 'scale item' is described by Burn (2000) as one that allows respondents to indicate their order of preference from a list of options. I used this to investigate the qualities that students deemed important for a person to be successful at learning mathematics. I followed the advice that the number of options should not exceed six because of the difficulty in the comparison of numerous items.

In addition to providing insight into students' motivation by learning or performance goals as described in the previous chapter, Dweck (2000) also informed my questionnaire design. Dweck suggests two approaches, 'self-theories' where questions are asked about respondents own qualities and 'other-theories' where questions are phrased about a general person. I decided to phrase questions in this latter category, and used the wording 'What do you think makes a person good at maths?' I felt that this would gather richer data as it removed the restriction of whether the individual student considered themselves to be good at mathematics, an issue that I had encountered in my Masters research. This type of question fitted with the Dweck (2000) description of a 'domain general' question, where the person as a whole, rather than a specific trait is considered. This type of question was recommended when the research covered a variety of 'social and intellectual domains' and I felt this fitted into my research into perceptions of difficulty of learning A-level mathematics. I provided a variety of options phrased in such a way that allowed students to make their own choices rather than selecting those that they felt they should. A copy of each of the three questionnaires is included in Appendix 2, A2-1-8. The issue of gaining truthful responses rather than those that participants believe will please the researcher is considered further in section 4.3.3, where interviewing techniques are discussed.

Advantages of questionnaires are that they allow data to be collected from a large number of respondents, and as they are usually self-administrated, they eliminate any personal interaction with the researcher (Burns, 2000; Robson, 2003). In order to benefit from this second point I reassured students that their responses would be anonymous and confidential, used for research purposes only and not a part of a school reporting tool. Disadvantages of a questionnaire that consist of open questions are the difficulties in coding and analysing the responses (Burns, 2000). The truth of responses cannot be guaranteed and there may be incomplete or poorly answered questions due to lack of understanding or interest of the respondents (Burns, 2000). I found that question design for the questionnaire in my Masters research was harder than anticipated; it was difficult to construct questions that led to informative responses which could be subsequently coded and analysed. However, whilst considering they might not be truthful, some of the responses to the open ended questions were insightful and I decided to include several of this type of question in the questionnaires for this research.

For this research, I designed three questionnaires. The first was given to 88 of a cohort of 89 Year 11 students at the beginning of the research period in July 2008, the second to 65 of 110 Year 12 mathematics students at the end of their AS courses in June 2009 and the third was given to 40 of 67 Year 13 students at the end of A2 in June 2010. I followed the advice of

Burns (2000) when writing the three questionnaires and used simple wording for the questions, clear presentation and appropriate response options. The ‘checklist to help avoid problems in question wording’ Robson (2003) was useful and I did not use any long, leading or double-barrelled questions. I also used an informal pre-test by asking teaching colleagues to read the first draft of each questionnaire. As I issued each questionnaire to all of the volunteers in each of the target groups: Year 11 students, Year 12 and 13 students of mathematics it would not have been feasible to ask students to complete a draft and then final version.

Each of the three questionnaires consisted of closed questions about background information including; AS levels chosen, whether continuing with mathematics next year, how much revision was planned for mathematics and how much work was done in lessons and with homework for mathematics compared to other subjects. If I had only used closed questions the reasons for the students’ choice of response would not have been apparent, and so open questions were also included. These open-ended questions formed a type of questionnaire that aimed to explain choices rather than created statistical patterns as described by Gorard (2001).

Coding of closed questions was straightforward as answers were collated for each of the response options. Similarly the scaled response questions allowed average ratings to be complied for each category. Having followed the format of the Brown et al. (2007 & 2008) questionnaire, I had several questions which gave students a choice from which they could pick any three responses to give a richer, fuller picture. Whilst this overcame the issue of students thinking that they had to pick the ‘right’ answer, it may have influenced the students’ responses to open questions. This may have meant that students' free responses were not totally independent, yet these responses were triangulated with data collected from interviews.

The coding of the open questions was far more time consuming and involved categorising a wide variety of responses into relevant headings. Although Robson (2002) warns that this process inevitably leads to the loss of some information, he describes it as necessary in order to analyse the data gathered from these open questions.

> *"try to develop a smallish set of categories (say eight to ten) into which these can be sorted. This is not an easy exercise. It is largely driven by the nature of the responses and the themes and dimensions they suggest"* (Robson, 2002, p.258)

Whilst more difficult to analyse, the responses to the open questions were insightful. These questions often asked for reasons for the responses given to previous closed questions such as choosing to study mathematics in the following year. There were also questions which asked what learning mathematics in that year had been like, and what made a person good at mathematics. For each of three questionnaires, all of the students' responses were entered onto a spreadsheet, and then key words and meanings were identified. This was an on-going process and the data was coded several times over the course of the research process as different themes emerged as appropriate categories. More details of this analysis are included in section 4.4.4.

4.3.3 Interviews and Analysis

There is a considerable body of literature describing different types of interviews and interviewing technique. The main interview types are labelled as structured, semi-structured and unstructured (Burns, 2000; Gillham, 2000; Robson, 2003). The first type is typified by the interviewer maintaining strict control over the process, a pre-determined list of questions adhered to which form a face to face version of a questionnaire. This type of questioning lends itself to the collection of quantitative data, as the tightly controlled questions lead to a fairly standard range of answers. However, it does not allow interviewees to expand on their answers or provide any additional answers, so I did not consider it appropriate for my research. Semi-structured interviews, whist maintaining a list of questions to be answered, allow the interviewee flexibility to develop their responses and speak in more detail on issues raised. The questions tend to be more open and lead to the collection of qualitative data, and this was the method that I adopted.

The advice for gathering data on personal experiences also suggested that an unstructured interview format, which allow interviewees to develop their ideas and opinions with little input from the interviewer, was most appropriate (Moore, 2000). However, my previous experiences of interviewing students suggested that this approach would not be successful. Asking teenagers about themselves as learners was problematic; students were not used to talking about their learning experiences or mathematical understanding and did not talk at length even when given the opportunity to expand on their responses. As a result of this I chose not to use the unstructured format and remained with semi-structured interviews (see Appendix 2, A2-9-15 for lists of questions and prompts).

The advantages of interviewing are described as the depth of information and flexibility in questioning that can be gained in a face-to-face situation (Burns, 2000). The validity of the data collected is ensured as it can be clarified at the time of the interview and recorded for future analysis. The disadvantages are related to the time consuming nature of the interview process, particularly the subsequent transcribing as I discovered whilst undertaking my Masters research. However, with the permission of those involved all interviews were recorded and transcribed in full (see Appendix 2, A2-16 -21 for a sample of interview transcripts). Disadvantages of semi-structured interviews include the variety of non-standard responses that are difficult to code, and interviewees may feel embarrassed or restricted by speaking in front of a recording device. The final disadvantage is the interviewer effect, which may result in participants offering answers that do not necessarily reflect their own opinion, but which please the interviewer (Burns, 2000). This was a particular issue for my research, and awareness of this led me not to ask students and teachers directly about reasons for choosing AS mathematics, difficulties or understanding. I was concerned that the answers given would have been what interviewees thought they should say; that a goal of gaining understanding of mathematical topics may have been a more correct or expected response than a goal of getting a grade A at AS-level.

Further difficulties arose when interviewing students, in addition to getting them to talk about something that they were not used to; describing why they found particular aspects of mathematics difficult, I constantly had to be aware of the type of questions asked so that students felt that they could answer honestly without any judgement. For example, when I asked for the reason that they chose not to continue with mathematics at AS or A2 level (see Appendix A2-9 October 2009 student interviews), students had to be free to say what they felt, such as it would be too challenging and require too much effort, rather than what they should say, described as ‘a socially desirable choice’ by Dweck (2000). My awareness of this issue and the need to establish rapport as described by Fontana & Frey (1994) led me to building trust with the students and teachers.

> *“Because the goal of unstructured interviewing is understanding, it becomes paramount for the researcher to establish rapport.”* (Fontana & Frey, 1994, p.367)

Trust was built throughout the two year data collection period and the interviews evolved as I established rapport (see Appendix A2-10 February 2009 through to A2-15 May 2010). As described in section 4.2.1, I learned to play down my practitioner role and emphasised my researcher role, avoiding any teacher responses to issues raised and reassuring participants that there were no judgements of their opinions. To this end I also decided not to take any notes and so not appear to give emphasis to any particular answers and to only record the interviews.

Other issues that surround the relationships involved in interviewing relate to the subsequent analysis of the interviews. The analysis of interviewees’ responses is out of their hands, and conclusions may be formed by researchers that are not in the best interest of the interviewees. As the intention of my research was to investigate the experiences of A-level mathematics students I believed that this latter point would not be an issue. During the process of choosing the candidates for interview, both students and teachers, I explained the purpose of the research and what I intended to do with the data collected. The interview process also provided the opportunity for reciprocity (Glesne & Peshkin, 1992) and I hoped that the students would gain from reflecting on themselves as learners of mathematics and from having their opinions listened to. Interviews with teachers also provided them with the time and opportunity to reflect on how they taught the pure mathematics material to A-level students.

> *"By encouraging respondents to initiate questions and comments, the interviewer breaks down the basic power dimension of the interview context by personalizing and humanizing him- or herself and empowering the respondents…when the interviewer gives respondents opportunities to introduce their own topics and concerns into the discussion, the knowledge shared and gained reflects the interests of the youth being studied as well as the interests of the researcher."* (Eder & Fingerson, 2002, p.185)

Throughout my interviews I followed these guidelines. Whilst interviewing as part of my Masters research, I discovered that I needed to improve my listening skills and find a balance between listening to the responses of the interviewees and thinking about my next line of questioning. Although I attempted a semi-structured approach, I found it difficult to decide on the correct amount of prompting, which would allow an interviewee the opportunity to develop their own response and being able to ask them for clarification of their points. When a student was not free with their responses I found it tempting to fill the gaps with my own talk, I realised that I need to give an interviewee time to think before responding, and only to offer prompts rather than my own opinions. Moore (2000) provides advice on how to encourage interviewees to talk, Robson (1993) also lists 'good listening' as one of the general skills needed by case study investigators.

> *"Good means taking in a lot of new information without bias; noting the exact words said; capturing mood and effective components; appreciating context. You need an open mind and a good memory."* (Robson, 1993, p.163)

He also lists the need for an enquiring mind, flexibility, a grasp of the issues and a lack of bias as key skills needed in order to successfully employ the case study approach. I was aware of these skills whilst I planned, conducted, transcribed and interpreted the interviews for this research. Consistency between interviews proved difficult to maintain despite trying to stick to an ordered list of questions. Interviews are in part a conversation and it was difficult to stay on track with the prepared list of questions rather than just react to an interviewee’s response. Reacting to a response may have added to the data but could also have been a distraction. During a set of interviews I may have included a question that had been provoked in one interview but not asked in the previous ones, or subsequent interviews had I felt it had been a distraction. I felt that I got better at interviewing and making decisions within the interviews over the course of the two-year data collection period. In the final year of data collection the interviews were more consistent and less conversational (For comparison see Appendix A2-16 October 2008 and A2-20 May 2010).

The case study students were interviewed individually after they had sat the pure mathematics examinations, Core 1, 2 and 3. In their study of students’ responses to mathematical test questions, Cooper & Dunne (1998) state that interviews could provide information about how students had approached questions that their written examination responses could not.

> *“What the test results cannot tell us, however, is anything about the children’s response strategy…In the context of the interview we can address this issue.”* (Cooper & Dunne, 1998, pp.123-4)

During the interviews I also discovered what meaning the students inferred from the information given in the examination, such as the order of the questions and the marks allocated. The results of the analysis of the interview data are explored in Chapters 6 and 7.

All interviews had to be fitted into lunchtimes for students and the majority of teachers, though some were conducted during mutual study periods. I was aware that I was interviewing very busy staff and students, and almost felt that I was grabbing data from what I could fit into a twenty minute lunchtime session. Consideration had to be given to how to get teachers to stop, consider and then talk about issues surrounding students' learning of mathematics during their working week. This was an issue that impacted on the quality of the interview responses; I had to balance my desire to collect rich data of considered interview responses with the time available for teachers to consider the topics raised in interviews beforehand. The frequency of the interviews was affected by how much time I felt it was reasonable to ask students and teachers to give up, and when it would be appropriate. I did not consider it reasonable for students to give up their time after school, nor during their examinations and study leave. Similarly I avoided interviewing teachers during times of higher workload, such as reporting periods. Further issues that arose from the practicalities of interviewing within a school context are discussed in the research design section.

4.4 Research Design

Once I had researched and decided on the data collection methods to be used, I decided on the number of students to form the case study. This decision was influenced by the time constraints of being a practitioner researcher, I decided that a case study of eight was feasible within the time constraints and would generate sufficient data as outlined in the table in section 4.4.2. It also allowed for 'attrition' (Ruspini, 2002) described in section 4.2.3, and when two students decided not to continue with A2 mathematics, the remaining six provided a valid set of data throughout Year 13. Dealing with the 'real-world' factors described by Boehm (1980) affected the timing and frequency of the data collection methods. Having looked at the shape of the academic years from the end of Year 11, through Year 12 and Year

13, I planned a timetable for my data collection. The restrictions mentioned in the Chapter 1, such as the modular examination structure and the resulting structure of the A-level course, and impositions on students' and teachers' time were taken into account.

4.4.1 Selecting the 'Nested Case Study' Students

I used a 'criterion sample' (Patton, 2002), also described as 'purposive' by Robson (2002), of students to follow throughout their AS-level mathematics course. In order to eliminate extra variables, such as previous experiences of GCSE mathematics, I selected students who had been at the school since Year 7. Of the 110 students doing AS level mathematics, 46 had come from through the school. As all the other feeder schools offer the modular GCSE course as opposed to the linear course that the 'internal' students take, I felt that the different course, in addition to the different teachers and school environment would have introduced too many additional variables into the sixth form students' experiences of mathematics.

There were six AS mathematics groups in the cohort of Year 12 that was the focus of this research, each taught by two different teachers. Having considered the literature on power relationships within interview situations, I discounted groups that I taught as I did not want to interview my own students. I asked the other teachers in the mathematics department whether they would be willing to participate in my research and four agreed. To preserve the anonymity of the teachers I assigned them labels, 'Teacher 1, 2, 3 and 4' and used these throughout without reference to their gender. Having identified the two groups that were taught by the teachers who had volunteered, I selected four students from each class. Whilst not studying gender differences, I chose two males and two females from each teaching group, aiming for as similar entry profile as possible. As I was interested in researching the experiences of high achieving students, I wanted to obtain a sample of eight students whom had all achieved an A* grade at GCSE, but this was not possible given the other restrictions of 'Internal' and within the two selected classes. However table 4.1 shows that there were very little differences between the uniform mark score (UMS) obtained between the A* and A grades. When writing notes from initial analysis of interview data I found it unwieldy to refer to 'Student A'. I subsequently assigned a name to each student that began with the letter of their 'label', reflected their gender and these are used throughout.

Table 4.1 Initial details of the 8 Case Study Students.

Student	GCSE UMS/600	GCSE Grade
Alice	544 (91%)	A*
Bella	552 (92%)	A*
Craig	575 (96%)	A*
Dan	551 (92%)	A*
Emma	586 (98%)	A*
Frankie	535 (89%)	A
George	530 (88%)	A
Henry	537 (90%)	A

I also used an 'Information-Oriented Selection' where cases are selected on the basis of expectations about their information content in order to maximise the usefulness of data collected from a sub-group of a population (Flyvbjerg, 2004). The first criterion was the students' initial questionnaire responses to whether they had offered to take further part in the research. Although this may have had an effect on the data I collected, as considered in section 4.2.5, I needed to have the co-operation of the students, as they were required to give up some of their free time in order to participate. In order to gain the richest possible data, I wanted to work with students who were able and willing to express their opinions, not necessarily mathematical, and would be happy to do so whilst being recorded so when there was a choice of students in each class whom fitted the 'internal', gender and grade criteria, the length and detail of their responses to the open questions on the questionnaire were also considered.

I had not considered the choice of applied mathematics module when I selected the students, as I was only collecting data about the experiences of learning and being examined in pure mathematics I did not anticipate this as an issue. However, it was raised in one set of interviews in the second year of A-level study. When the difficulties involved in learning vectors, a topic in the Core 4 module were discussed, the choice of applied module was raised by three of the four teachers. As vectors were used in the mechanics module, it was considered by these teachers to be an advantage for students to have taken this option rather than the statistics or decision modules. However, as none of the students in the case study had taken mechanics I did not feel it affected the data and this issue was not raised by any student in any stage of the data collection.

4.4.2 Timetable of Data Collection and Analysis

There were two strands of data collection and analysis that were planned and carried out in parallel. The first of these was document analysis; the development and use of the model to classify the levels of demand of mathematics examination questions. Table 4.2 shows the timetable of this development and subsequent use of the model. The details of the model and the results from the analysis of June 2007 to June 2010 AS and A2 pure mathematics papers are provided in Chapter 5.

Table 4.2 Timetable of development and use of model to classify the level of demand of examination questions

When	Method	Focus
September 2007 – August 2008	Literature search	To find appropriate model of understanding to base my model on
August 2008	Initial analysis of June 2007 Core 1, 2 and 4 papers	Development of model from Sierpinska (1994)
November 2008	Written examination analysis sheet completed by 20 Year 13 students	Testing the model against students' perceptions of easy and difficult
November 2008	Written examination analysis sheet completed by 6 mathematics teachers	Testing the model against teachers' perceptions of easy and difficult
November 2008	Analysis of Examiners' reports for June 2007 papers	Testing the model against Examiners' comments on performance of candidates
November 2008	Synthesis of written data	Refinement of Model
December 2008	Application of model to pure mathematics examination papers	Classification of January and June 2008 Core 1 to Core 4 papers
February 2009	Application of model to pure mathematics examination papers	Classification of January 2009 Core 1 to Core 4 papers
August 2009	Application of model to pure mathematics examination papers	Classification of June 2009 Core 1 to Core 4 papers
February 2010	Application of model to pure mathematics examination papers	Classification of January 2010 Core 1 to Core 4 papers
August 2010	Application of model to pure mathematics examination papers	Classification of June 2010 Core 1 to Core 4 papers

The second strand was the interview data collected from students and teachers which was put into context with questionnaire data gathered from the cohorts of Year 11, 12 and 13 students at the end of the GCSE, AS and A2 courses respectively. The interview data had two aspects, exploring students' experiences of learning A-level mathematics and exploring responses to A-level mathematics examinations. To gain a fuller picture of perceptions of difficulty of examinations, written data collection sheets were issued prior to the interviews regarding Core 2 and Core 3 and this addition is discussed in section 4.4.3. Table 4.3 shows the timetable of the interviews, written data sheets and questionnaires. The two strands of data collection and analysis were subsequently intertwined and the model provided a framework to analyse students' and teachers' descriptions of difficulty of A-level mathematics questions.

Table 4.3 Timetable of Data Collection

July 2009	Y12 Mathematics Students	65 out of 105	Questionnaire	Experiences of AS mathematics and whether continuing to A2
Nov 2009	Case Study Students	6	Interview	Learning Core 3 trigonometry
Nov 2009	Case study Teachers	4	Interview	Students' experiences of learning Core 3 trigonometry
Feb 2010	Case Study Students	6	Written examination analysis sheet	Core 3 Examination
Feb 2010	Case Study Students	6	Interview	Core 2 Examination Initial thoughts on Core 3 module
Feb 2010	Case study Teachers	4	Written examination analysis sheet	Core 3 Examination
Feb 2010	Case study Teachers	4	Interview	Core 3 Examination Students' experiences of learning Core 4
May 2010	Case Study Students	6	Interview	Learning Core 4 Vectors
May 2010	Case study Teachers	4	Interview	Students' experiences of learning Core 4 Vectors
June 2010	Y13 Mathematics Students	40 out of 67	Questionnaire	Experiences of A2 mathematics
July 2010	Case Study Students	4 out of 6	Written examination analysis sheet	Core 4 Examination
July 2010	Case study Teachers	3 out of 4	Written examination analysis sheet	Core 4 Examination

4.4.3 Changes to Research Design

> *"Much 'real world' research is messy: uncontrolled variables abound, predictor and criterion measures interact, alternative hypotheses cannot be ruled out, standard statistical measures cannot be applied without massive violation of assumptions."* (Boehm, 1980, p.498)

The 'real world issues' for this research included how to maintain the participation of the teachers and students in the data collection process. As discussed in section 4.3.3, this informed the length of each interview and rather than one long interview in each of Year 12 and Year 13, repeated shorter interviews were used balanced by the time that students and teachers were willing to give up. Students had to enjoy the interviews in order to continue to participate. It seemed awkward doing mathematics in an interview situation, so I limited this and after one interview where I asked students to talk through how easy or difficult they considered four questions I presented them with, I asked students to talk about questions they had seen previously. There were difficulties when tracking students down, arranging interview times and getting them to attend as agreed. Whilst they appeared willing to participate and give up their time, understandably it was not their priority and students needed written reminders to attend. As a result of this I did not want to impose further on the students' or teachers' time by providing them with 'work' to do outside of the interviews. Yet this meant that there was only the time within the interview for students and teachers to contemplate their answers. I was conscious about how 'in depth' or representative their responses were, as they were answering 'on the spot' and they may have expanded on or changed their answers if they had had more time to consider their response. I felt that these 'on the spot' interviews may only have given part of the picture, yet I was reluctant to impose further on the time of busy students and teachers.

It is not uncommon for the research emphasis to change during the collection and first reading of data in a qualitative research project as Robson (2002) reported.

> *"It is highly desirable that an explicit plan is prepared…in the full knowledge and expectation that aspects of this may change as the work continues"* p.184

My research design became quite flexible over the two years as one data collection informed the next. Changes that were made in subsequent interviews are outlined below along with reasons for decisions taken.

Having decided to do interviews on the perceived difficulty of the Core 1 examination without prior preparation work from students, I decided that richer and more representative data would be gained by collecting written data prior to subsequent interviews. Students were issued with a copy of the examination paper that they had sat, along with a data collection sheet that asked for each question part to be classified as 'easy' or 'difficult' with a reason for their choice. This was completed prior to the interviews regarding the Core 2 and Core 3 examinations, and allowed the interview questions to expand on these responses. Whilst I felt that this provided richer data and 'thick description' of students' perceptions of difficulty of examination questions, I could not go back and recapture the data from Core 1.

A similar situation led to my decision to collect data from teachers. Initially I had planned to collect data from students during the learning of each pure mathematics module and following each examination and from their teachers at the end of each academic year. Yet it became clear following the interviews that focused on learning and being examined in Core 1 that the views of teachers at these same points would add valuable data. However, having made this decision, I did not go back and collect data from teachers about teaching co-ordinate geometry and the Core 1 examination.

When collecting data about the difficulties of learning Core 1 mathematics, I selected a topic from this syllabus to create questions for students to discuss during interview. My choice was influenced by the Mathematics Department's scheme of work as I wanted to explore the experiences of a topic that students had been taught prior to the interviews in October 2008. This gave a choice of indices, surds and straight line co-ordinate geometry. There was more content in this latter topic and it provided a wider variety and richer source of questions within which I could explore understanding through aspects of difficulty. I wrote four questions on this topic and asked the students to place them in an order of difficulty. The questions were designed to fit each of the four classifications of demand from the model described in the following chapter. However, there were three written questions and one in diagram form. I had designed the questions to represent each classification, but had neglected the difference in presentation. When describing the level of difficulty of the questions, students were affected by the style of the questions. Some students were influenced by whether a diagram had been given or how 'wordy' the question was. It was difficult to analyse whether it was the phrasing of the question or its mathematical content that led to a question being described as 'easy' or 'difficult'. In order to overcome this in subsequent interviews, I asked students and teachers to choose their own questions to discuss. In the interviews I was then able to ask why they had selected the particular questions and explore their reasons further. I also collected their questions and was able to consider similarities and differences and analyse them using the model classifications. Asking participants to bring questions to the interviews required them to be more involved in the research project. However, this involvement was not universally accepted, one student, 'Dan' explained that he had only brought short questions as they were quicker for him to write down and bring to the interview.

When selecting the mathematical topic for the interviews on learning Core 3 and Core 4, I identified part of the syllabus that had a range of questions, both in style and difficulty level. In Core 3 the topic of numerical methods was discounted, because many questions used in lessons, from the textbook and past examination papers were very similar. The material in the 'algebra and functions' section had been covered in 'The Three Week Plan' at the end of Year 12 and had been affected by the issues of this novel way to the end of the academic year at this school. This left two topics, calculus and trigonometry. Both of these contained a significant amount of material, but the calculus covered a wider range of topics as it encompassed both differentiation and integration. As I explored what made an A2 level question difficult, I wanted students and teachers to discuss different difficulty levels within the same mathematical topic. I felt that trigonometry would allow this to happen, rather than the calculus where it could arise that the different topics within this subject could be given as the reason for difficulty. For example; differentiation of a product could have been described as easy, integration with a linear substitution as of medium difficulty and volumes of revolution could have been described as the most difficult. I did not feel that this would lead to a rich discussion or comparison in question style to further the question of 'what makes an A2 level question difficult?' and so I chose to make trigonometry the focus of this stage of data collection. I decided not to choose a mathematical topic, such as the exponential function or natural logarithms as a focus because students were not taught by topic, they were taught by the structure given in the Core 3 syllabus as described in Chapter 1.

The choice of topic to explore in Core 4 was informed by the data gathered from interview responses that considered the difficulties of the Core 3 examination and initial thoughts on the Core 4 course. Vectors had been described as a difficult topic by the majority students in these interviews and so I chose this topic to explore further in the final round of interviews regarding experiences of learning A2 mathematics.

There were several difficulties in collecting data from students during their final days of sixth form. Whilst all the students were compliant and brought two questions to their interview, it was evident that they had not spent much time looking for examples of vector questions that they found difficult. The students were open about this, some saying that their questions came from 'the first place they looked'. Interviews were harder to schedule than previous meetings. Fitting in time at break or lunch was more difficult as students had revision sessions to attend. The quality and depth of their interview responses also reflected that their focus was understandably on their examination preparation rather than thinking and talking about aspects of learning mathematics.

There were further difficulties with the final stage of data collection concerning the Core 4 examination paper. Initially I had planned to interview all the students and teachers following the completion of a written examination breakdown sheet as with Core 2 and Core 3. However there were practical issues that prevented these plans from being carried out. The students had officially left the school in the month prior to their A2 examinations and this made communication with them difficult. I felt it was not appropriate to ask them to provide data during their examinations, but they did not return to the school after their final examination. Whilst all the case study students had previously agreed to complete the Core 4 written data collection form and come into school to be interviewed for a final time, in reality once their examinations had been completed they were not motivated to do so. I decided not to go ahead with the interviews and just tried to collect written data. Having posted the data collection sheet home and followed up with an email reminder and a telephone call, I received four out of the six forms.

There were similar difficulties collecting data from the four teachers involved. After the A2 examinations had been completed, there were several changes to the usual timetable that made collecting data difficult. In addition, it appeared that teachers were less willing to give up their time to provide data towards the end of an academic year. As a result one teacher did not complete a data collection sheet.

4.4.4 Analysing Qualitative Data

The document analysis of the examination questions is explained in the following chapter. This model also provided a framework to analyse the responses of students and teachers against. In parallel with this analysis, there was repeated reading of the qualitative data gained from interviews, written examination response sheets and questionnaires in order to identify common sources of difficulty. The two types of analysis were then synthesised and the findings are reported in Chapter 6 and 7. Moore (2000) described that there was no 'right' way to analyse qualitative data, and that each researcher needed to find a method that worked for them that they could apply 'systematically and rigorously'.

> *"Qualitative analysis is a very personal process and you must find the approach that suits you best. There are some basic rules and principles, there are also some mechanical techniques that will help you to manipulate the data, but in the end, you have to work through it trying to make sense of it all."* (Moore, 2000, p.146)

Thomas (2011) believed that interpretive inquirers could not fracture situations into individual variables and instead described a process for analysis called 'constant comparative method'.

> *"defined by the simple principle of going through the data again and again (this is the constant bit), comparing each element- phrase, sentence or paragraph – with all of the other elements (this is the comparative bit)…The basic principle governing the process of constant comparison is that you emerge with themes that capture or summarise the essence (or essences) of your data."* (Thomas, 2011, p. 171)

This process was further broken down into ten steps: examining data; making an electronic copy of the raw data; making a working copy of the data, parts of which can be highlighted when read and make a list of important ideas 'temporary constructs'; re-read data in light of list and note where each idea occurs in the data; delete any ideas that were not re-enforced by the data but keep the data and note any counter examples; on rereading the data, identify a list of 'second-order constructs' that summarise the ideas in the data; re-read the data again and refine the 'second-order constructs' and once these accurately and fully reflect the data, rename them as the final ‘themes’; look to make connections between themes; map the themes and finally select key quotes from the data to illustrate the themes. It seemed to me that this was similar to grounded theory (Burns, 2000; Gillham, 2000; Robson, 2002; Charmaz, 2003; Ryan & Bernard, 2003; Corbin & Strauss, 2008) and this discussed in the following section.

Grounded theory is a strategy for data analysis and not restricted to specific methods of data collection (Charmaz, 2003). It is described as ‘systematic and inductive guidelines for collecting and analyzing data’. Similarly, Ryan & Bernard (2003) describe grounded theory as iterative in nature and the close reading of interview transcriptions was the process that I employed.

> *“by which the analyst becomes more and more “grounded” in the data and develops increasingly richer concepts and models of how the phenomenon being studies really works. To do this, the grounded theorist collects verbatim transcripts of interviews and reads through a small sample of text (usually line by line).”* (Ryan & Bernard, 2003, p.279)

The lack of preconceptions and the creation and revision of theory is described by Burns (2000) and the emergence of themes for coding from re-reading of the data by Gillham (2000).

> *“you do not start out with a priori theoretical notions (whether derived from the literature or not) – because until you get in there and get hold of your data, get to understand the context, you won’t know what theories (explanations) work best or make the most sense.”* (Gillham, 2000, p.2)

These explanations are developed as data collection and analysis are intertwined, with one informing the other. This recursive process is described by Robson (2002).

> *“the researcher is expected to make several visits to the field to collect data. The data are then analysed between visits. Visits continue until the categories found through analysis are ‘saturated’. Or, in other words, you keep on gathering information until*

> *you reach diminishing returns and you are not adding to what you already have."* (Robson, 2002, p.192)

Whilst there was a fixed end point to the collection of data in my research as the students came to the end of their A-level course, I experienced the effect of diminishing returns that Robson (2002) describes. Throughout the second year of the A-level course I found that there were fewer and fewer instances of new reasons given why mathematics was perceived as difficult. The majority of the categories used to account for difficulties described in Chapter 6 and 7 emerged in the first year of data collection and there were only two new categories found in the second year.

> *"The end results of grounded theory are often displayed through the presentation of segments of text-verbatim quotes from informants-as exemplars of concepts in theories."* (Ryan & Bernard, 2003, p.280)

In keeping with this description, I used quotes from students and teachers to illustrate my findings throughout Chapters 6 and 7.

The advantages of a grounded theory approach listed by Robson (2002) are the flexible, yet systematic nature of the strategy. It is considered to be most useful when researching in a novel context where there is no existing theoretical framework. Disadvantages include the difficulty in knowing when sufficient categories have emerged, and the potential contradiction between flexibility and the systematic development of categories. Finally, starting a research project without any preconceived ideas is seen as impossible. I was particularly aware of the danger of this final point in my own research, and took care to ensure that any assumptions about understanding and perceptions of difficulty of A-level mathematics that I may have formed as a teacher did not influence my data collection and analysis.

Grounded theory analysis consists of three categories of coding (Charmaz, 2003; Robson, 2002; Corbin & Strauss, 2008).

> *"open coding to find the categories; axial coding to interconnect them; and selective coding to establish the core category or categories"* (Robson, 2002, p.493)

In my research, close re-reading of transcribed interview responses and written data from open questionnaire questions and examination feedback sheets during the 'open coding' phase of the analysis led to the different categories for sources of difficulty in A-level mathematics. This 'open coding' was applied after each phase of data collection and different categories emerged after each phase, however during analysis of the final set of data, the written responses regarding the Core 4 examination, no new categories emerged, fitting the description of 'saturation' by Robson (2002). These categories included novelty, lack of instruction, number of steps required, position in the examination paper, use of mathematics from a previous module and mathematical concepts. The axial coding involved finding relationships between the different categories of why mathematics was considered to be difficult and trying to establish when particular reasons were given. Finally, during the selective coding the central category was identified as 'novelty'. If a mathematics question was different to those the students and teachers had seen before, then it was most commonly described as difficult. The subsequent reasons identified in the open coding phase were then used by students and teachers for why the question was considered difficult.

I did not use 'classical content analysis' as described by Ryan & Bernard (2003) as I did not decide on the coding categories prior to collecting data. 'Schema analysis' was also not used as I did not attempt to infer meaning from students' and teachers' written or verbal responses.

4.5 Summary

Robson (1983) and Silverman (2000) both advise that during qualitative research, analysis of some sort should start as soon as the first data is collected. If consideration is not only given to the research question but also to the potential emerging patterns in the results throughout the data collection, then analysis can begin during this phase of research. I followed this advice and when interviews were transcribed, my first reflections on the data and potential themes were also recorded. Themes that emerged from early reading of the data were novel questions, decisions about which method to apply and the number of steps in the solution were sources of difficulty for students. Data gathered from the end of Year 11 questionnaire, described at the start of Chapter 6, set up the expectations of what learning AS mathematics would be like. These formed a basis for which to explore students' experiences of AS mathematics and how it compared to the expectations as discussed in Chapter 6. Data collected in the final year of the longitudinal study is analysed in Chapter 7 where students' experiences of learning A2 mathematics and the Core 3 and Core 4 examinations are explored. Data was triangulated throughout by comparison of the data collected from case study students, teachers and mathematics cohorts. The use of the model to classify the demand of A-level mathematics questions was used as a framework to explore students' and teachers' descriptions of difficulty of the Core examinations. By adopting this multi-method case study approach I achieved a full and detailed picture of Grammar School students' perceptions of the difficulties of A-level mathematics.

CHAPTER 5: DEVELOPMENT OF MODEL TO CLASSIFY EXAMINATION QUESTIONS

5.1 Introduction

In this chapter I present the development of a model that classifies levels of demand in A-level mathematics examination questions. Encouraged by Torrance's (2007) paper 'Assessment as learning' and the current context of modular A-level examinations I decided to focus my research on the level of demand within the A-level papers to gain an insight into the challenges offered in learning A-level mathematics. Torrance (2007) found that examinations played a large part in learning, and suggested that the nature of the understanding required in answering A-level examination questions was the level of understanding that was demanded throughout the course. In Chapter 2 I explored several models from research on understanding in mathematics and I based my model on one of these in particular. Teachers and students were then interviewed to determine their perceptions of these examination questions and I used this data to amend my model. Analysis of this data along with analysis of the comments from the Examiners' report for these examination papers (OCR, 2007d) led to a refinement in my model. Finally I present an analysis of 24 A-level pure mathematics papers set over the years of this research (2008 to 2010) which I use later in my research for comparison with data collected from teachers and students over the same time period. In Chapters 6 and 7 I describe how this model was used to explore links between what students and teachers found difficult and the level of demand in a question determined by the model.

5.2 Defining a Model

The research reported in Chapter 2 revealed many researchers who set out to classify, organise and level aspects of mathematical understanding. The model I have based my own research on is adapted from the Sierpinska (1994) model. Sierpinska uses four levels of understanding, similar to that of Hoyles (1986, 1987) claiming:

> *"It came to me that understanding a concept could be measured by the number and quality of epistemological obstacles related to it that one has to overcome."*
> (Sierpinska, 1990, p.25)

Similarly, I decided to measure 'obstacles' as evidenced by the demand in an examination question that A-level mathematics students had to overcome. Duffin & Simpson (2000) stated that whilst teachers could not see their students' understanding, they could look at 'external manifestations' for evidence of understanding. I created my own model to gauge the level of obstacle and analysed examination questions to evidence the nature of the demand of understanding required of A-level mathematics students. I take the definitions of Sierpinska's four levels of understanding (identification, discrimination, generalisation and synthesis) and modify them to determine a level of demand offered in examination questions. The big shift from the Sierpinska (1994) model was that I analysed the content of the questions and not the actions of the student. As part of this shift, I defined the 'object of understanding' as the mathematics required to answer the question and I explore this in each of the examples presented.

I am not researching understanding, but the manifestation of understanding required to answer an examination question. In no way is this model indicative of 'how much' understanding a student has, nor a description of how difficult the mathematics is. Through my model I determine a level of demand in the examination questions.

To create model to classify the level of demand in examination questions I used three of the June 2007 A-level papers (OCR 2007a,b,c) the latest set of examinations available at the time I designed my model prior to the data collection. As described in Chapter 1, I considered the pure mathematics examinations as these were common to all students taking A-level mathematics. In these three examination papers there were a total of 62 parts of questions that were analysed. What follows is a description of each of the four categories of examination questions using the same labels as Sierpinska, with a discussion of examples from the June 2007A-level papers.

5.2.1 Identification

Sierpinska defined identification as when the object of understanding was singled out and recognised as an item to be understood. In this original definition, the object is described as the particular article or idea that requires understanding.

> *"I have identified the object of my understanding, I mean at first, that I have 'discovered' or 'unveiled' it, that is, isolated, singled out from the 'background of my field of consciousness' in which it was, so to say, hidden, and, second, that I have recognized it as something that I intend to understand."* (Sierpinska, 1994, p.56)

I defined Identification questions as ones that explicitly told the students which part of mathematics to apply. Thus the 'object of understanding' which I had taken to be the mathematics required, was identified by the wording of the question. Therefore questions that fitted the Identification classification were those where the mathematical technique had been singled out and included specific instructions which stated the method that was required. Thus there was no need for students to decide what mathematics to do; they were required to carry out a specific technique. Below are examples of questions classified as Identification from the June 2007 pure mathematics examination papers.

8 **(i)** Express $x^2 + 8x + 15$ in the form $(x + a)^2 - b$. [3]

The above question from the first AS pure mathematics paper Core 1(OCR, 2007a) is classed as Identification because the provision of the required form of the answer singled out the technique of writing a quadratic expression in completed square format. Hence students did not need to decide which technique to use; they had to follow this instruction. The object, in this question, writing a quadratic expression in completed square form, was identified. This classification was not an indication of the difficulty level of the mathematics involved; it was an indication of the demands in the process of answering the question. Similarly, the Core 1 (OCR, 2007a) question below gives the specific instruction for students to sketch curves. Thus the objects, sketching the reciprocal and quartic curves are identified.

2 **(a)** On separate diagrams, sketch the graphs of

(i) $y = \frac{1}{x}$, [2]

(ii) $y = x^4$. [1]

Of the Core 2 (OCR, 2007b) questions that were defined as Identification, where the instruction of what to do was explicit, the following is an example where students had to ‘write down’ specified values. Whilst the students were not told *how* to do this, it was clearly stated *what* they had to do. The object in this question was the use of the definition of a geometric progression to write down the values of three terms.

1 A geometric progression $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 15 \qquad \text{and} \qquad u_{n+1} = 0.8u_n \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [2]

As the students had been provided with the type of sequence, the first term, the relationship between one term and the next along with the instruction to write down the next three terms, this fitted the definition of Identification. In the second Core 2 (OCR, 2007b) example of an Identification question, a clear instruction of what was to be done was again provided. In this case, the method to be used to carry out this task was also given; the object here was solving an equation by taking logarithms and was identified in the question.

3 Use logarithms to solve the equation $3^{2x+1} = 5^{200}$, giving the value of x correct to 3 significant figures. [5]

The direction to use logarithms along with the instruction to solve the equation meant that students did not need to decide which mathematics to apply to the question. The first step in their solution was not decision making about what to do, but carrying out the specific task. Again, this is not a reflection on how difficult this mathematics was, but an indication of the processes involved in answering the question. Whilst students did not need to decide what mathematics to do, there were decisions about how to do the mathematics, in this case, the choice of base of the logarithms used.

The next question is from the final A2 pure mathematics examination paper, Core 4 (OCR, 2007c).

1 The equation of a curve is $y = f(x)$, where $f(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $f(x)$ in partial fractions. [2]

This was also an Identification question as it provided the explicit instruction to express an algebraic fraction in terms of partial fractions and hence this object was identified. Thus the question tested technique as students were required to follow this specific procedure rather than decide which mathematical method to apply. Similarly the following Core 4 (OCR, 2007c) question also fitted within the Identification category as it included the instruction to find $\frac{dy}{dx}$ in terms of t which identified the object; finding the gradient function.

5 A curve C has parametric equations

$$x = \cos t, \quad y = 3 + 2\cos 2t, \quad \text{where } 0 \leqslant t \leqslant \pi.$$

(i) Express $\dfrac{dy}{dx}$ in terms of t and hence show that the gradient at any point on C cannot exceed 8. [4]

Whilst students were not told how to find the gradient function of this curve that was defined parametrically, it was clear that this was what they had to do. For all of these questions the mathematical technique had been given and students needed to carry out the instructions or follow the specified process or method. All examination questions were classified as Identification when they singled out the method to be used in the answer. There was no decision making about *what* to do when answering Identification questions. There were possibly decisions about *how* to do the mathematics and if students were unable to do the specified process then they could not have answered the question. This illustrated that whilst the students did not need to decide the starting point for their answer, they did need to know the mathematics. The application of this model is based on the assumption that students had been taught and had learned the methods required to answer the examination questions. This paralleled the approach adopted by Pollitt & Ahmed (1999) as 'learning the subject' was step zero of their model for answering examination questions. The implications of whether the Identification type of questions could have been answered by memorising rote learned methods are explored in Chapters 6 and 7.

5.2.2 Discrimination

This was defined by Sierpinska as recognising the difference between the object to be understood and others.

> *"Discrimination between two objects is an identification of two objects as different objects"* (Sierpinska, 1994, p.57)

Examination questions that I defined to be of the Discrimination type were ones that required students to apply some mathematics that was not specifically mentioned. Having interpreted the 'object of understanding' as the mathematical technique required to answer the question, Discrimination questions initially required students to identify that there was no 'object' given before deciding on which technique to apply. Sierpinska (1990) described discrimination as 'finding relations between ideas' and this informed my decision to categorise questions as Discrimination when they required students to find the mathematical method required to answer them.

5 **(i)** Show that the equation

$$3\cos^2\theta = \sin\theta + 1$$

can be expressed in the form

$$3\sin^2\theta + \sin\theta - 2 = 0. \qquad [2]$$

The question above from the Core 2 (OCR, 2007b) paper was classified as Discrimination as it required students to recognise that $\cos^2\theta$ could be written as $1 - \sin^2\theta$. The use of this trigonometric identity was not given in the question and students had to decide that this was the correct mathematical technique to apply before re-arranging the resulting quadratic equation in $\sin\theta$ to the required form.

A Core 4 (OCR, 2007c) question of the Discrimination type is shown below.

6 The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point (2, 3) on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

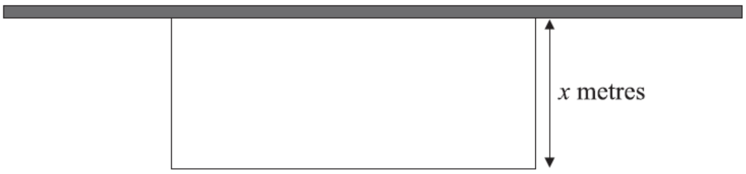
This question required students to decide that implicit differentiation of the equation of the curve was required in order to determine the gradient at the given point. This method, along with a subsequent method to form the equation of a straight line, was not given in the question and therefore the students were required to decide to apply these mathematical techniques.

The variety of demand within each of the four categories of the original model was stated in the following quote.

> *"There can be several degrees of discrimination as there can be several degrees of identification. One is mere perception that two objects are two and not one…another degree is that when two objects are compared with one another with respect to certain sensible circumstances, contingent to the objects themselves. A still higher degree is when two general ideas are compared from the point of view of abstract relations."*
> (Sierpinska, 1994, p.58)

Similarly with my use of the model to classify examination questions, there was a range in the demands within each category. In addition to the different mathematics that was the subject of the two Discrimination questions described above, there were different levels of decision making required. In the Core 2 question, there was only one decision required, to apply a trigonometric identity before re-arranging the resulting equation. Whereas the Core 4 question discussed above required students to recognise the equation as of the implicit type before selecting the appropriate method to find the gradient of the curve. Further decisions were also needed about how to find the equation of the normal. In contrast with the numerous decisions required in this Core 4 question, below is an example of a Discrimination question from Core 1(OCR, 2007a) that only required the application of the area of a rectangle.

5



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

(i) Show that the enclosed area, A m^2, is given by

$$A = 20x - 2x^2. \qquad [2]$$

Whilst all of these questions possessed the property that students were required to identify the mathematical method for themselves as the first step in their answer, the variation in demands in Discrimination questions came both from the number of the decisions required and the nature of these decisions. Students needed to decide on the mathematics they were going to apply by recognising properties in the question that meant that it was a question on a particular topic. Thus to answer a Discrimination question, decision making about which mathematics was required before the student could do the mathematics.

5.2.3 Generalisation

In the Sierpinska model Generalisation was described as when the object was seen as a specific case of another situation or class of objects.

> *"a given situation (which is the object of understanding) is thought of as a particular case of another situation. The term 'situation' is used here in a broad sense, from a class of objects (material or mental) to a class of events (phenomena) to problems, theorems or statements and theories."* (Sierpinska, 1994, p.58)

This act could include the awareness of the potential wider applications of the object or that one of the assumptions about the object was not essential (Sierpinska, 1990). When adapting the model to classify examination questions, I defined the 'Generalisation' category as those where the mathematical technique required was seen as a particular case of another situation. For example, if a question required students to find the equation of a tangent to a curve that was parallel to the x-axis, it was of the Generalisation type. The general situation was the conditions for a line to be a tangent to a curve and the conditions for that line to be parallel to the x-axis. Thus a general mathematical concept was initially identified, before it was then applied to the specific case in the question. It could have been argued that 'specialisation' may have been a more appropriate label, as students were required to identify the question as a particular example of a general concept, rather than generalising from a series of specific cases. However, I retained the original labels used by Sierpinska (1994) to maintain the links to the original four acts of understanding. The final part of the June 2007 Core 1 question below was classified as fitting the Generalisation category.

10 **(i)** Solve the equation $3x^2 - 14x - 5 = 0$. [3]

A curve has equation $y = 3x^2 - 14x - 5$.

(ii) Sketch the curve, indicating the coordinates of all intercepts with the axes. [3]

(iii) Find the value of c for which the line $y = 4x + c$ is a tangent to the curve. [6]

This final part of the question required the mathematical knowledge of what conditions were required for a line to be a tangent to a curve. This knowledge of the concept of a tangent, having the same gradient at a common point was required before students could decide which mathematical techniques to apply in order to meet these conditions. Thus Generalisation questions required decisions about which mathematical concepts were involved, prior to decisions about which mathematical techniques to apply to satisfy the specific conditions of the concept given in the question.

A Core 4 (OCR, 2007c) example of a Generalisation question required students to recognise that the given differential equation was of the 'separable variable' type.

8 The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

Whilst this question asked that students showed the requested result by integration, this was not an indication of the specific technique involved. Students were required to recognise the type of differential equation and then re-arrange it into the appropriate format $\int \frac{1}{6-h}\,dh = \int 20\,dt$ before carrying out the integration, subsequent substitution of initial conditions and re-arrangement to the given form. It is this further layer of decision making, in the recognition of a specific question on a general mathematical concept, which distinguishes a Generalisation question from a Discrimination question.

5.2.4 Synthesis

The act of Synthesis was described by Sierpinska (1994) as the quest for common themes, finding similarities in generalisations and forming a complete picture from previously separate concepts.

> *"'Synthesis' means for us here: the search for a common link, a unifying principle, a similitude between several generalizations and their grasp as a whole (a certain system) on this basis."* (Sierpinska, 1994, p.60)

In terms of examination questions, I defined Synthesis questions as those that required students to consider multiple representations of the same mathematical object and either combine them or consider the links between them in order to solve a problem. There were no questions on either of the June 2007 AS papers, Core 1 and Core 2 that fitted this description. Perhaps more surprisingly, I did not find an example of this type of question on the final pure mathematics paper in this examination series, June 2007 Core 4. However, many examination questions were presented in several parts, with the first parts often used to help answer later ones. Below is a question from Core 2 (OCR, 2007b) in the format it was given in the examination.

9 The polynomial $f(x)$ is given by

$$f(x) = x^3 + 6x^2 + x - 4.$$

(i) (a) Show that $(x+1)$ is a factor of $f(x)$. [1]

(b) Hence find the exact roots of the equation $f(x) = 0$. [6]

(ii) (a) Show that the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

can be written in the form $f(x) = 0$. [5]

(b) Explain why the equation

$$2\log_2(x+3) + \log_2 x - \log_2(4x+2) = 1$$

has only one real root and state the exact value of this root. [2]

In this format, the first two parts are Identification as they gave specific instructions about what to do and the third and fourth parts are Generalisation. It is the final part of the question that I consider in more detail. Whilst students had to recognise that the final part of this question was equivalent to solving f(x) = 0, I decided that this did not fit the description of Synthesis as the student only needed to spot the link between this request and earlier parts of the question rather than making this link for themselves.

However, had this question only have consisted of what was the final part of the original question then it would have been classified as Synthesis.

> *"Explain why the equation* $2log_2(x+3) + \ log_2 x - \ log_2(4x+2) = 1$ *has only one real root and state the exact value of this root."*

In this edited form students would have had to consider the different representations of the given equation and make links between the given logarithmic form and polynomial form in order to then determine the number of real roots.

Before fully deciding on the four categories of questions within my model, I next asked students and teachers about the examination papers in order to test my ideas. The data collected from students and teachers is considered in the following sections.

5.3 Testing the Model against Perceptions of Easy and Difficult

Once I had developed the model for classification of examination questions based of the four acts of understanding described by Sierpinska (1994), I tested the model against students' and teachers' perceptions of difficulty. I made the crucial decision not to share the classifications with students or teachers. I did not want to 'teach' them how to use my model but instead use my model as a framework to analyse their responses against. Instead I asked whether they found individual parts of examination questions 'easy' or 'difficult' and to give a reason for their choice. The use of these two categories was influenced both by literature on classifying difficulty of examination questions and my previous research experience. Ahmed and Pollitt

(1999) described ‘Sources of Difficulty’ and ‘Sources of Easiness’ in their analysis of GCSE questions. Similarly Crisp et al. (2008) asked Year 11 students about whether they found GCSE science questions easy or difficult. During my own practitioner researcher experiences when completing a Masters degree, I found that students were not used to talking about what they found difficult about learning mathematics. So I wanted to keep the written data collection forms simple and to also provoke a definite choice. As discussed in Chapter 4 there were also issues when designing a data collection format that students and teachers would be willing to complete five times over the research period.

In the initial phase that considered the June 2007 papers the data from the student volunteers and teachers was used to inform the classifications, and subsequently the definition of Identification questions was adjusted as described in section 5.3.4. In further exploration of the actual papers that the case study students sat, the final model was used as a framework to explore the sources of difficulty as perceived by the students and their teachers.

5.3.1 Initial Model

During the autumn term of 2008, I gave the June 2007 Core 1 and Core 2 to twenty Year 13 A-level mathematics students who volunteered to participate with my research. As my model for classifying examination questions was based on the challenge level of the questions within the context of the AS and A2 course, it assumed that students had learned the mathematical material that was examined. I asked Year 13 students as they had been taught the Core 1 and 2 courses in their previous year. Each student was asked to complete a written data collection sheet which required them to rate each question part as either ‘easy’ or ‘difficult’ and to give a reason for their choice. [3]

I asked the 20 students to consider each part of each question as presented in the examinations. The ‘difficult’ ratings were tallied for each question and given as a percentage of the total possible ratings. So for example, the 20 students considered the 15 Identification questions and made a total of 300 decisions about whether they were easy or difficult. Of these only 51 across the range of Identification questions were describes as difficult, giving a percentage of $\frac{51}{300} \times 100\% = 17\%$. The results of the analysis of students’ difficulty rating of questions are displayed in table 5.1.

Table 5.1 The percentage of students who classified Core 1 questions as ‘Difficult’

Classification	Identification	Discrimination	Generalisation
Core 1 questions	17%	22%	75%

[3] Further issues with using volunteers as discussed in chapter 4, were the increased target grade profile of the students who participated in comparison to the whole year group. The twenty volunteers were all targeted grades A to C with 65% targeted a grade A, these were higher than the whole Year 13 mathematics target grades of 84% A to C grades with 53% A grades. So their higher grade profile may have influenced their perceptions of the difficulty of the AS mathematics questions.

The Core 1 paper consisted of 22 question parts, of which 15 were of the Identification type, five the Discrimination type and one part was Generalisation. The above table showed that of the 15 Identification questions, the students described only 17% of these as difficult. This was as expected and indicated that most high achieving A-level students found questions which stated the mathematical method required easy. This was confirmed by the students' written responses that explained their reasons for describing these questions as easy. These comments often referred to the simple wording of the questions and the amount of information given. The following comments were typical.

> *"The question starts it off for you."*
> *"It gives you all the info you need."*

Of the five Discrimination questions, these were described as difficult in only 22% of the cases and this was a lower difficulty rating than I had anticipated. As this type of question required students to decide which mathematical method to apply before carrying out the selected technique, I had expected that students would have found this additional demand significantly more difficult than the Identification questions. Yet many students described finding these questions easy, with reasons based on having seen similar questions and many stated that these were 'basic' or 'simple' questions on the particular topics such as surds or transformations. The rating of one of the surds questions was particularly surprising, with only 10% of students describing it as difficult and this provoked me to re-consider its classification. The need for this change was substantiated by both the data from teachers and the comments from the Examiners' report (OCR 2007d) and is discussed in section 5.3.3. The single Generalisation question was most frequently described as difficult by the students. Whilst the percentage of students who rated this question difficult was significantly higher than the Identification and Discrimination questions, this was in line with what I had anticipated. This question required the knowledge of the conditions required for a line to be a tangent to a curve and this general concept in addition to decision making about the mathematical techniques applied offered a greater demand to students than questions which gave the required method. Comments written by students about the difficulty of this question referred to needing ‘more thought’ or ‘apply knowledge’ and the following quote suggested that this type of question was unfamiliar.

“You have to think about what you are doing as it is different.”

I repeated the exercise with the Core 2 examination and gave the 20 student volunteers a copy of the paper with a data collection sheet to fill in as they had done for Core 1. The table below shows the percentage of each type of question that was considered difficult by the students.

Table 5.2 The percentage of students who classified Core 2 questions as ‘Difficult’

Classification	Identification	Discrimination	Generalisation
Core 2 questions	23%	37%	80%

With the Core 2 examination paper, there were 20 parts of questions, 14 of which were Identification, 4 Discrimination and 2 Generalisation. A comparison between the perceptions of difficulty between the questions on Core 1 and Core 2 is discussed in the following section; here I consider the responses to Core 2. Identification questions were described as difficult least often and the Discrimination questions only slightly more so. Again, as to be expected with the increased demands of recognition of a specific mathematical concept, the two Generalisation questions were described as difficult by the majority of students. The reasons given by students for questions being described as difficult were very similar to those given for the Core 1 paper. These reasons included not knowing what mathematics to do and the number of steps involved in answering the question. It was interesting that one comment described the second Generalisation question as difficult because it required ‘understanding’.

“Explanation and understanding needed.”

As table 5.3 shows, the percentage of questions rated as difficult followed a similar pattern for both the Core 1 and Core 2 examinations. In both cases, the Identification questions were least often considered difficult and the Generalisation questions significantly more so. The overall increase in the percentage of difficult ratings might have been expected for the Core 2 questions as this was the second pure mathematics module and the syllabus built on the material covered in Core 1. Whilst the data from the students fitted with what I expected from my model, I was surprised that the Discrimination questions were not described as difficult more frequently and I investigated this through the data collected from teachers.

Table 5.3 The percentage of students who classified Core 1 and Core 2 questions as ‘Difficult’

Classification	Identification	Discrimination	Generalisation
Core 1 questions	17%	22%	75%
Core 2 questions	23%	37%	80%

As there were no students in school that had been taught all of the A2 core material at the time I was testing my model for classifying examination questions, I decided to ask mathematics teachers about their perceptions of difficulty of the Core 4 paper. As described in Chapter 4, I had to balance the amount of data that I wanted to collect with the amount of time I asked of teachers within the school year to participate in my research. As a result of this I decided to collect information from teachers about the Core 4 and Core 1 examinations. I issued the six mathematics teachers in my department with a copy of the June 2007 Core 1 and Core 4 papers and a written data collection sheet. However, rather than asking whether the teachers themselves found each part question easy or difficult, I asked them to consider whether they thought their students would find the questions easy or difficult and why. This provoked issues about whether all students would find the same questions difficult, yet teachers accounted for these differences in their comments when they used their own words to describe groups of students. Descriptions included ‘the majority’, ‘stronger’ or ‘weaker’ students.

The numbers of question parts that were considered to be difficult were compiled and the percentages out of the total number of responses are shown in table 5.4. The responses to the Core 1 paper are considered first.

Table 5.4 The percentage of teachers who classified Core 1 questions as ‘Difficult’

Classification	Identification	Discrimination	Generalisation
Core 1 Questions	26%	46%	100%

In the following section comparisons are made between the teachers’ and students’ perceptions, but first the teachers’ responses are considered in more detail. The reasons given for the 15 Core 1 Identification questions being described as easy often referred to their routine or standard nature. Similarly, when Discrimination questions were described as easy reasons centered on the typical nature of the request. Interestingly one comment indicated that these standard questions could have been answered by rote learning.

“Just learning is needed.”

When teachers rated the Discrimination questions as difficult, they gave reasons such as the numerous different steps required in the answer and that ‘most students’ would find it difficult to know where to start. It was interesting to again note that the requirement to ‘think’, in this case of a method to demonstrate the area of a rectangle in terms of algebra, was considered a source of difficulty by one teacher.

"'Show that' problems require reading and thinking."

Significantly, all six of the teachers considered that students would find the single Core 1 Generalisation question difficult. The most common reason for this difficulty was the number of stages required that were not given which meant that students had to decide how to approach the question. The unusual type of question was also given by two teachers and the quote below showed that the difference came from the additional variable included in the question.

"They seem to find problems with questions where some of the numbers are replaced with algebra."

The June 2007 Core 4 paper consisted of 20 question parts, 14 of which were Identification , 5 Discrimination and 1 of the Generalisation type. Table 5.5 shows the percentage of each type of question that were described by the teachers as difficult for students.

Table 5.5 The percentage of teachers who classified Core 4 questions as 'Difficult'

Classification	Identification	Discrimination	Generalisation
Core 4 Questions	32%	44%	60%

Whilst the percentages of questions considered difficult followed the previous pattern of difficulty increasing from Identification through Discrimination to Generalisation questions, these were not as high as I might have expected. Core 4 is often spoken about by students and teachers as significantly more difficult than the other pure mathematics papers and therefore I had expected the highest percentage ratings in all three types of questions when compared to Core 1. Yet the reasons given for the numerous questions that were rated as easy were very similar to those for the Core 1 paper. Many questions of both the Identification and Discrimination type were described as ‘routine’, ‘standard’ or ‘straight-forward’. The only difference in the reasons given by teachers for Core 4 questions being difficult was the reference to a specific mathematical topic. In two questions, one on differential equations and one on vectors, three of the teachers gave a reason related to the topic involved. These were the only examples of where the mathematics was identified as a source of difficulty. This may have been a reflection of the teachers’ own view of these topics, but may offer an insight into why students find these topics difficult and this is explored in Chapter 7.

5.3.2 Comparison of Students’ and Teachers’ Perceptions of Difficulty

I compared the perceptions of difficulty between students and teachers. As the students had only sat the Core 1 and 2 examinations at the time of this stage in my research and I had wanted to collect information regarding perceptions of Core 4, yet not overburden teachers, the only common data collected was regarding Core 1. Generally teachers thought more questions were difficult than the students did. It may have been that teachers had considered the full range of students rather than the higher achieving sample of students who volunteered to participate in the research. The differences between the data collected from students and teachers are now considered. Table 5.6 shows the comparison between the percentage difficult ratings of students and teachers.

Table 5.6 The percentage of students and teachers who classified Core 1 questions as ‘Difficult’

Classification	Identification	Discrimination	Generalisation
Core 1 questions Students	17%	22%	75%
Core 1 questions Teachers	26%	46%	100%

I had expected that there would be an increase in the percentage of questions from Identification, Discrimination to Generalisation questions. Whilst not predicting the exact percentages, this expected increase was displayed in both students' and teachers' responses. There were several differences in the perceptions of difficulty of the same examination questions. The proportion of questions that were rated as 'difficult' was higher for teachers rather than students and initially I was surprised by this. Overall, the teachers rated 34% of the Core 1 questions as difficult compared to 21% for the students. This data does reflect that the students were higher achievers than the whole cohort of Year 13 mathematics students and there may have been several additional explanations for this discrepancy. There are complex reasons why individuals perceive things as easy or difficult, and there are many possible conclusions that could be drawn. The students commented on their own perception whereas the teachers did not. This may have meant that the teachers were more honest in their answers as they did not have to admit they found something difficult. Both teachers and students were not required actually to do the questions before they rated them, teachers may have been more able to identify the mathematics involved and therefore have based their ratings on more accurate information. The teachers' comments often included references to 'weaker' students, and identified parts of questions that would cause these students difficulties. This did not fit the stronger target grade profile for the twenty students who voluntarily agreed to take part in the exercise. Several of these factors were investigated during interviews with the case study students and teachers following written data collection that rated the difficulty of AS and A2 pure mathematics examination questions. The data from these interviews and written data collection sheets is explored further in Chapters 6 and 7.

Overall the teachers gave more detailed reasons than the students and mathematics was mentioned more explicitly, with individual parts of questions highlighted as either easy or difficult. However the reasons were similar to those given by the students as were the types of questions more often considered difficult. Identification questions were least often described as difficult by either students or teachers and the majority of these questions were described as easy because they were standard or routine or the method required was given. These descriptions fitted with my category of Identification questions when the mathematical technique had been singled out.

Having considered the responses of students and teachers, I decided to analyse the 'Report on the Units' (OCR, 2007d) published by the OCR examination board that considered the responses of candidates to each of the June 2007 papers. Once I had analysed this report and the data collected from students and teachers, I revised one of the four categories of my initial model to create the final model to classify examination questions.

5.3.3 The Examiners' Reports

I next analysed the Examiners' report (OCR 2007d) for the Core 1, 2 and 4 papers (see Appendix 3, A3-1-9) in regard to my model. I analysed their comments and to see whether they described the performance of candidates on the different classifications of examination questions consistently. As with the data from students and teachers, in this section I will show that there was an agreement that Identification questions were considered difficult least often compared to the Generalisation questions which caused difficulty for the majority of candidates.

Within the report on the Core 1 paper, many of the Identification questions were described as 'done very well by the majority of candidates'. The additional challenge brought by the need for decision making in the Discrimination questions was also reflected by the Examiners' report comments with one question described as 'more challenging' and the quote below refers to the question being answered well only by the 'strongest' students.

> *"Only the strongest candidates scored both marks"* (OCR, 2007d, p.2)

Similarly the increased level of challenge of the single Generalisation question was reflected by the longest comment about an individual question in the Examiners' report on the Core 1 paper. Whilst this comment discussed the variety of approaches taken and the successes of students with one particular method it also referred to the number of students who were not able to recognise the general concept of the repeated solution of the simultaneous equations of a line and a curve if the line was to be a tangent. It was this component of the question which fitted the Generalisation category that was described as the source of difficulty.

> *"Those candidates who equated the 2 equations frequently did not realise that the discriminant of the resulting quadratic expression needed to be zero and so made little progress."* (OCR, 2007d, p.5)

The Core 2 paper was described in the Examiners' report as offering a combination of straightforward and routine questions and parts of questions that offered more challenge. Again this fitted with the application of my model to this examination paper. As 14 of the 20 question parts were of the Identification type that provided students with explicit instructions, this was consistent with the number of questions that the examiners referred to as 'straightforward'.

Both part questions of the Generalisation type were in the final question on the Core 2 paper that were discussed in section 5.2.4. The first part required knowledge of the general laws of logarithms, $log_k k = 1$ and specifically that $log_2 2 = 1$. The final part required recognition that solving the given equation was equivalent to solving the equation $f(x) = 0$ obtained in the previous part. The data collected from the Year 13 students had confirmed that there were additional challenges with Generalisation questions which fitted with my definition of the need to recognise a specific case of a general mathematical concept. Similarly this increased demand in the Generalisation type of question was also reflected by the comments in the Examiners' report.

> *"candidates struggled with manipulating logarithms and very few fully correct solutions were seen….It was disappointing to see a number of candidates attempting to 'expand' the logarithm, thus demonstrating a lack of understanding of the topic."* (OCR, 2007d, p.9)

However, the above comment raised the question whether it was actually the topic of manipulating logarithms that caused difficulties for students or whether it was the type of question on this topic. The type of question appeared to be significant as the Examiners' comment below on the manipulation of logarithms question, which I classed as Identification, described causing difficulty for fewer students than the Generalisation questions on logarithms.

> *"It was pleasing to see how many candidates gained full marks on this question. Whilst candidates struggle to manipulate logarithms, they are becoming increasingly adept at using them to solve equations."* (OCR, 2007d, p.7)

Although the Examiners' report compared students' performance when manipulating logarithms and using logarithms, to solve questions, it did not address the different style of question on these two topics. It seemed that the lower demand of an Identification question allowed students to demonstrate more proficiency in the area of using logarithms to solve equations, rather than the Generalisation question on manipulating logarithms.

The Examiners' report stated that there was a wide variety in the scores on June 2007 Core 4 paper. As in the report on the Core 1 and 2 papers, questions described as done well or as successful for the majority of students were all of the Identification type. Again this re-enforced the connection with decision making and difficulty as conversely students performed well when they had to follow explicit instructions. When describing the responses to one of the Discrimination questions, the Examiners' report implied that difficulties arose with the students' decision making and planning of their answer.

> *"Part (ii) was relatively easy but candidates did not sit back and plan an attack and so their efforts were often very convoluted and badly explained."* (OCR, 2007d, p. 15)

In the summary on pure mathematics, which comprised of the four core papers and the three further pure mathematics papers, the Chief Examiner's report on the June 2007 examinations commented on the high level of performance of many candidates.

> *"Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by these papers."* (OCR, 2007d, p.1)

However, the variety in the challenges provided were at odds with the findings from the application of my initial model which found that the majority of the questions were of the Identification type. Comments from the Examiners' report along with the data from the students and teachers were then used to make decisions about my model. These are discussed in the following section.

There were general patterns in the data from both students and teachers, with Identification questions described difficult less often than Discrimination questions. Generalisation questions were described as difficult by a significant majority of students and teachers and this increased difficulty was also reflected in comments from the Examiners' report. However there were a minority of Discrimination questions that did not fit with this general pattern. These Discrimination questions had a significantly lower percentage of both students and teachers describing them as difficult which were more typical of the Identification questions. These lower percentages combined with the consideration of written comments from students, teachers and the Examiners' report led to a change in my original model. There was a change to the definition of the category of Identification questions; a description of the refined definition with a reason for the change is described in the following section.

5.3.4 Refinement of the Definition of Identification Questions

Initially when developing the model I placed all questions which required the application of some mathematics not explicitly mentioned in the Discrimination category. However, after analysing the written data from students and teachers on the perceived difficulty against the classification of questions, I adapted my initial model. This change is explored through a discussion of the Core 1 question that prompted the shift in the model.

Initially I had classified the following Core 1 question as Discrimination because it required students to apply rules for manipulating surds, specifically $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ without the method being specified.

3 Simplify the following, expressing each answer in the form $a\sqrt{5}$.

(i) $3\sqrt{10} \times \sqrt{2}$ [2]

(ii) $\sqrt{500} + \sqrt{125}$ [3]

However, the data from the students was surprising. Whilst there appeared to be a correlation between the different classification of questions and the percentage of students who rated the question difficult, this was not the case with this question. This relationship was described in in section 5.3 of this chapter. This question was rated as difficult by only two students which, at 10% was more in line with the difficulty rating of the Identification questions. As a result of this difference I then explored the written reasons that students had provided for describing the question as easy. A typical reason given for the easy classification was "Once you learn the rules, it's easy to apply them". Also, many students commented that "The answer form points you in the right direction." These responses fitted with the comment from the Examiners' report which described the question as being well answered due to candidates' familiarity with the rules for surds.

> *"well answered by the vast majority of candidates, although a small number did not attempt it at all, seeming unfamiliar with the rules for manipulation of surds."* (OCR, 2007d, p.2)

This implied that examiners saw this question as rules for manipulation, as did the students. The Discrimination classification I had used had implied that students initially identified that there was no mathematical method given before deciding on which technique to apply. It appeared that the standard nature of the question meant that although the rules for the manipulation of surds were not explicitly included in the question, they were obvious to the students. Thus there was no need for discrimination, as the typical question format meant that the method required was implicitly identified. The classification that I had initially used had implied a greater level of demand, yet having talked to students and teachers it was clear that the technique required was signposted given the previous practice of similar questions. As I collected data from the teachers who taught the volunteer students, they were able to comment on the amount of practice and the standard nature of questions that the students had encountered in their learning of A-level mathematics.

Following the analysis of responses from students, teachers and Examiners to this question, I adjusted the definition of Identification question. In addition to questions which gave explicit instructions about what method to apply, I also included those questions that were of such a typical nature that whilst the mathematical technique required was not given, the format was so standard that the routine nature of the request meant that the method was implicitly identified for students.

This second type of Identification question brought a potential problem when using this method to classify the demand of mathematics examination questions. In order to apply these classifications consistently, knowledge of the course content and particularly questions given to students during the learning and revision stages was now a vital requirement. In order for questions to be described as standard so that the mathematical technique required to answer them was identified, detailed knowledge of the type and range of questions within the given syllabus were needed. It may be argued that the consideration of what was considered standard would not be the same for all students. However, Sierpinska (1994) described the original model for understanding as not having a rigid hierarchy and that the four acts were personal to the person trying to understand. Similarly the categorisation of examination questions would not be universally agreed. Yet, with knowledge of the AS and A2 course, departmental schemes of work, prior core mathematics examination questions and those given in the textbooks used to teach this material, these typical questions could be consistently classified as Identification.

Examples from each of the June 2007 papers with an explanation of why these were classified as Identification are provided below.

7 **(a)** In an arithmetic progression, the first term is 12 and the sum of the first 70 terms is 12 915. Find the common difference. [4]

This initially fitted the definition of a Discrimination question because it required students to recognise that there was no method provided, before deciding which method to apply. However, it was a very standard question of this part of the Core 2 syllabus. Thus students would have repeatedly practised answering this type of request and the typical format of the question meant that the method had been implicitly provided for students. Again, there were implications here of the potential role that rote learning methods could play in answering these questions and these are explored in Chapters 6 and 7.

Similarly, whilst the method was not explicit in the Core 4 integration question below, having talked to teachers, it was a very standard question that students would have met prior to the examination.

2 Find the exact value of $\int_0^1 x^2 e^x \, dx$. [6]

The integration techniques on the Core 4 syllabus consist of integration of trigonometric functions, of algebraic fractions, integration by parts and by substitution. However, as stated in the Core 4 syllabus the substitution would always be given and so the method of integration by parts could be identified by elimination of the other possibilities. As there was no substitution given, no trigonometric functions or algebraic fractions this could only have been a question on integration by parts. Thus the method of integration by parts was identified through the routine nature of the task that required students to integrate a product that could not be simplified algebraically. Hence this was also classified as an Identification question. The decision to classify this question as Identification was supported by the comment in the Examiners' report that stated most students recognised the method to be used.

"Almost everyone realised this was testing integration by parts" (OCR, 2007d, p.15)

Whilst the adaption of the definition of Identification questions was the only change to my original model, there was another factor that could have affected the classification of examination questions. The way that questions were presented in numerous, smaller parts rather than fewer, longer questions had an impact on the level of challenge within the question. The issue of staged questions is explored in the following section.

5.3.5 Staged Questions

As in the exploration of the Synthesis category of section 5.2.4, there were several examples on each of the June 2007 examination papers where the classification had been affected by the staging of the question. Many of the AS and A2 examination questions were split into stages that required the student to recognise how the earlier part of the question related to the latter parts. There were few opportunities when students were given an entire problem to solve independently, this was a feature of the description of A-level examinations as 'sat-nav' mathematics by Bassett et al. (2009) discussed in Chapter 3.Typically questions were broken down into stages that the student had to work through and then link. Often this link was made explicit by the words "Hence find". An example of this is a question from the June 2007 Core 4 paper.

7 **(i)** Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part **(ii)** to find the exact value of $\displaystyle\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}\, dx$. [5]

In this given format, each part of the question is of the Identification type as students were given the mathematical technique to use. In part one this was the quotient and remainder theorem implicitly given by the routine nature of the question, the second part specified both the form of the answer and 'hence' indicated to students that they should use their previous answer. Finally, the third part of the question instructed students to use their answer from part two of the question in order to re-arrange the algebraic fraction to a form where it could have been integrated. The help that these staged questions was acknowledged in the Examiners' report on this particular question.

> *"part (ii), which was designed to provide a bridge for candidates to the final part"* (OCR, 2007d, p.16)

However, had the Core 4 question above been edited to only the latter part, this revised question would have been of the Generalisation type. In the revised form, the question required students to recognise that this was a particular case of the integration of an improper algebraic fraction.

> *"Find the exact value of* $\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2+4}\, dx$*"*

If questions were comprised only of what was originally presented as a final part, then many would fit the Generalisation and Discrimination categories rather than just providing more

Identification questions. Students would then be required to think about what mathematics to apply to be able to solve a problem instead of being led through a series of steps, a process that Bassett et al. (2009) described as reading a map rather than following a sat-nav.

5.3.6 Unresolved Issues with the Model

Whilst the staging of examinations could have affected the classification of the questions, this was not an issue for my research as I used the model I developed to classify the examination papers sat by the case study students. So the classifications were applied to the format of the examinations as provided by the OCR examining body. However there were several issues that were relevant to the use of this model and these are described next.

When one teacher returned their data collection sheet they commented that they did not want to look at questions being either easy or difficult. Instead they had wanted to consider the difficulty level involved in 'parts of parts of questions'. This raised a potential issue with my model; I had only broken down questions into the parts stated in the examination, whereas I could have applied the model to the individual mathematical processes required in each solution. However, as I aimed to explore the links between these classifications and students' and teachers' perceptions of the difficulty of the questions, I needed a system that would work for both types of data collection. I did not feel it would be feasible to gather written and interview data on a more detailed level than parts of examination questions, so I defined the model in terms of the parts of examination questions so that it matched the format of the data collected from the nested case study students and teachers. My decision mirrored the approach of Fisher-Hoch et al (1997, p.2) who analysed 'the smallest unit of a question for which a mark can be awarded' when researching the difficulty of GCSE examination questions.

I applied these classifications from the starting point of a student whom had been taught the mathematical material that provided the subject of these questions, yet was meeting these examination questions for the first time. As described by Sierpinska (1994) and discussed in Chapter 2, the four acts of understanding were not static as new ideas became building blocks and what was once discrimination became identification. For example, at GCSE students worked to learn how to factorise a quadratic equation, but during their AS level this became a tool that could be applied automatically. Similarly, a question I defined as Discrimination because it required students to recognise that there was no method given and therefore decide on which technique to use, may have been described as Identification by those more familiar with the particular questions. These could have been teachers or Year 13 students if they looked back at a Core 1 question once they had studied the Core 3 and 4 modules. Yet following Sierpinska's original model, these classifications were defined from the point of view of a student who was sitting the particular examination for the first time. As discussed previously, this was not intended to be a universal model valid for all students, it was my interpretation of the level of demand provided to answer pure mathematics questions. I liked that the model was indicative and not accurate and I arrived at definitions that were robust. I wanted a model that I could use reliably to describe the nature of the demand involved in answering examination questions in order to make comparisons with these categories and the descriptions of difficulty given by students and teachers.

5.3.7 Final Model

This section is a final summary of each type of examination question, illustrated by questions that I constructed to fit each type within the Core 1 topic of co-ordinate geometry. Again this emphasises that this model does not categorise the difficulty of the mathematics as all four of these questions are about the gradients of straight lines.[4]

Identification Questions

Questions classed as Identification were those which did not require any decision making by the student about what mathematics to do. Instead, students had to follow instructions given either explicitly or implied by the standard nature of the question. These questions could have been answered by the application of a rote learned method or process. The question that I constructed on co-ordinate geometry to be of the Identification type was:

> *"Find the gradient of the line joining the points (2,-7) and (4,3)."*

This was a typical request which would have been familiar to students. It could have been answered using a rote learned method such as the substitution of the given co-ordinates into the gradient formula: $\frac{y_2-y_1}{x_2-x_1}$. Whilst this formula was not given explicitly in the question it was one that students were expected to memorise (as it was not listed in the formula booklet provided in mathematics examinations).

---

[4] These questions were then used in interviews with the case study students and the outcomes of these interviews are discussed in Chapter 6.

Discrimination Questions

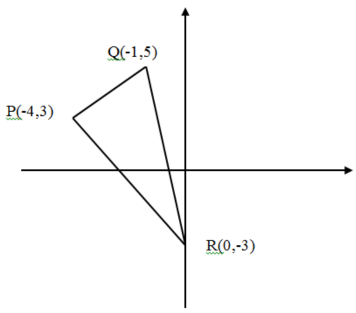
Questions categorised as Discrimination were those that required students to decide which part of mathematics to apply in order to answer them. Whilst students were given what to do, they needed to decide how to do it. Thus they needed to identify the object, the mathematics, for themselves. The co-ordinate geometry question written to be of the Discrimination category was:

"Find the equation of the line parallel to y - 2x = 6 passing through the point (-1, 8)."

This required students to find the gradient of the given line, then recognise that parallel lines have the same gradient and then find the equation of the required line.

Generalisation Questions

Questions classified as Generalisation were those which required students to recognise that the question was a particular case of a mathematical concept. Students had to work out both what mathematics to do and which technique to apply in order to do it. The following co-ordinate geometry question was constructed to fit the Generalisation category.



"Show that the triangle PQR is right-angled.

(Diagram not drawn to scale)."

This question required students to recognise the conditions for a triangle to be right angled (either PQ and PR are perpendicular or that $PQ^2 + PR^2 = QR^2$) and then to work out which mathematical techniques to apply in order to show this.

Synthesis Questions

Questions in the Synthesis category were those where students were required to consider multiple representations of the same mathematical object and either combine them or use the links between them in order to solve a problem. Students had to work out the different possibilities of what could be done, decide how the possibilities linked together, decide on the mathematical techniques to apply and then apply them. The consideration of multiple possibilities was illustrated by the co-ordinate geometry question that I created to be of the Synthesis type.

> *"For the given points A (-1,-1) and B (2, 3) find the three possibilities for the co-ordinates of C and D so that ABCD is a square."*

This question required students to find the different possibilities for the positions of the co-ordinates C and D in order to create a square. Students then had to work out how they could determine the position of those co-ordinates before finally using the techniques of calculating gradients, finding perpendicular gradients and the distance between two points.

This model was used as a framework within which to describe the structure and demands of different type of examination questions. In the following chapters the four categories of questions were used as a framework to explore what students and teachers perceived as difficulties both in mathematics examinations and their learning of mathematics. In the next section the composition of the initial set of papers used to develop the model and all of the pure mathematics papers between January 2008 and June 2010 are shown.

5.4 Use of Model to Classify Pure Mathematics Examination Questions

This section discusses the results from the application of my model. First the composition of the June 2007 papers that were used to develop the model and then all of the pure mathematics papers from 2008 to 2010. On each A-level examination paper, each part question is allocated marks. Once I had classified each question part I assigned all of the marks to the appropriate Identification, Discrimination, Generalisation or Synthesis category. The total number of marks for each classification were then tallied and considered as percentages of the total number of marks for each paper. Each AS and A2 mathematics paper has a total of 72 marks. This mark allocation strategy was used in all the analysis of examination papers. I also consider the grade boundaries for each of the examination papers that I analysed.

5.4.1 June 2007 Papers

Once I classified each question part I tallied the number of marks available for each category as described above. Table 5.7 shows the distribution of marks.

Table 5.7 Mark allocation for the June 2007 pure mathematics papers

Paper	Identification		Discrimination		Generalisation		Synthesis	
Core 1	54/72	75%	12/72	16%	6/72	8%	0/72	0%
Core 2	49/72	68%	16/72	22%	7/72	10%	0/72	0%
Core 4	50/72	70%	14/72	19%	8/72	11%	0/72	0%

As noted in section 5.2.4 there were no Synthesis questions on any of these three examinations. Whilst there were variations between the three papers, each fitted a pattern where the majority of the examinations were comprised of Identification questions, with fewer Discrimination questions. The marks available for Generalisation questions were significantly lower than the other categories. There were no significant differences between the composition of the two AS level papers, Core 1 and Core 2 and the final A2 paper, Core 4. I found this surprising as I had expected to see a progression in the nature of the demand provided, from mostly Identification questions on the Core 1 paper to more Generalisation and Synthesis questions on Core 4. This expectation was based on my experiences of teaching A-level and the commonly expressed perception amongst teachers and students that Core 4 was the most difficult mathematics module. I was surprised to find that the majority of the questions on each of the three 2007 Core mathematics papers that I developed the model with were of the Identification type.

In order to consider the level of demand expected of students, I wanted to compare the marks awarded for each type of classification and compare these with the grade boundaries for each paper. I was interested to see what grade could be achieved by correctly answering each classification of question, particularly what could be gained by only mastering the Identification questions. The grade boundaries for the June 2007 papers are shown in table 5.8.

Table 5.8 Grade Boundaries for the June 2007 pure mathematics papers

Paper	Grade A	Grade B	Grade C	Grade D	Grade E
Core 1	60/72 83%	52/72 72%	44/72 61%	36/72 50%	29/72 40%
Core 2	56/72 78%	48/72 67%	40/72 56%	33/72 46%	26/72 36%
Core 4	61/72 85%	54/72 75%	47/72 65%	40/72 56%	33/72 46%

Comparison of the values in table 5.8 with those in table 5.7 showed that on the Core 1 and Core 2 papers, correctly answering all the Identification questions would gain a grade B. However, on the A2 module, Core 4 the Identification questions would only result in a grade C. In order to make these comparisons easier to see, I combined the information of the classifications and grade boundaries to present this data in a bar graph. As the four types of questions presented increasing level of challenge from Identification through to Synthesis, I decided that a ‘stacked column’ graph was most appropriate. The information from the tables above is combined in the bar graph shown in figure 5.1.

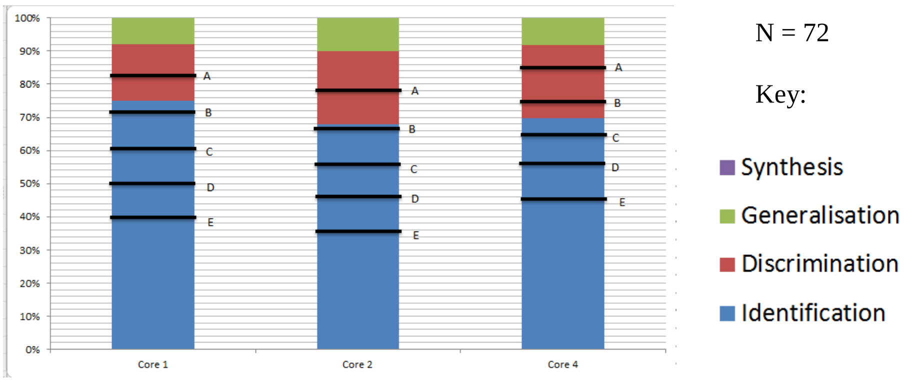


Figure 5.1: Classification and Grade boundaries for June 2007 papers

The graph clearly shows both the composition of each of the examination papers and the grade boundaries for each examination. Although the marks gained by students could come from any questions, with marks gained on Generalisation questions when lost on Identification questions, this method of data presentation shows the minimum demand within each examination to gain each grade. For example, the graph shows that to gain a Grade A none of the Generalisation questions needed to be answered on any of these three papers.

However, having only analysed three examination papers I did not know whether these were typical or indicative of the level of demand. As I subsequently used this model to analyse the examinations that the case study students sat as part of their two year A-level course, I wanted to discover whether their four examination papers were typical of those set throughout this time. So in a similar fashion I then analysed all of the papers from January 2008 until June 2010. I did this in six stages following each round of examinations over the course of three years. At each stage of the analysis, I classified all of the questions on each of the four pure mathematics papers. Over the three year period, this resulted in 24 examination papers being analysed with over 480 parts of questions considered and classified as Identification, Discrimination, Generalisation or Synthesis. The results of this analysis are considered in the following section.

5.4.2 Analysis of Pure Mathematics Examinations from 2008 - 2010

To consider whether there were any general patterns in the proportion of questions of each of the four classifications and the grade boundaries, 24 examinations were investigated. The results of this analysis are displayed in the following graphs, with each year of papers illustrated separately.

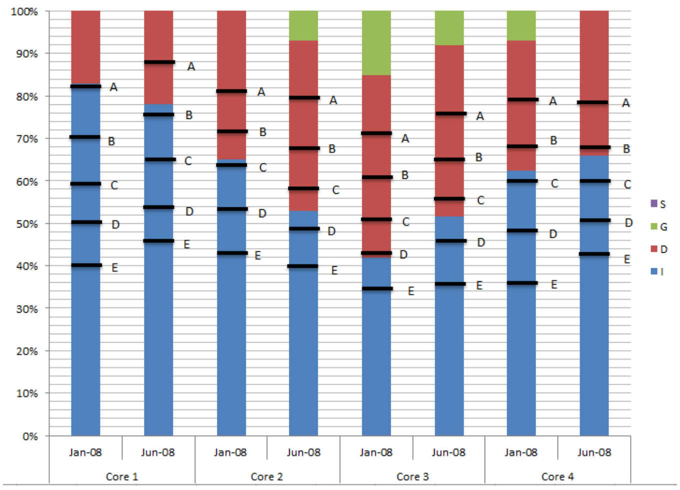


Figure 5.2: Classification and Grade boundaries for the 2008 papers

Figure 5.2 shows that as with the 2007 papers, the majority of questions on each A-level examination paper in 2008 were of the Identification type. There were no Synthesis questions on any of these papers and very few Generalisation questions. Only half of the 2008 papers had any Generalisation questions, and it is interesting to note that the grade boundaries on the June Core 2 and Core 3 and the January Core 3 and Core 4 were lower than the papers without any Generalisation questions. This suggested that students scored lower in examinations when there were more questions that required them to recognise that the question was a particular case of a mathematical concept. Whilst there was some variation in the marks required to gain each grade on each paper, there was one notable similarity. As with the June 2007 papers, no marks from the Generalisation questions were required for students to gain a grade A. Therefore students could obtain the highest A-level grade without having to apply their knowledge of general mathematical concepts. However, they were required to be accurate when carrying out specified methods to answer the Identification questions and when deciding which mathematical technique to apply on the Discrimination questions.

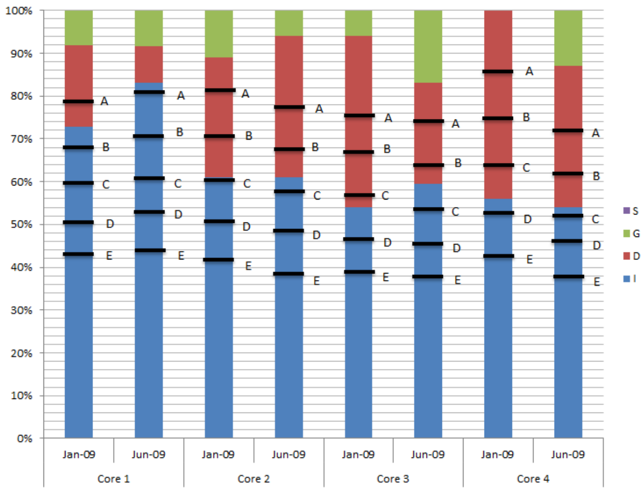


Figure 5.3: Classification and Grade boundaries for the 2009 papers

Figure 5.3 shows that, unlike the 2008 papers, the majority of examinations in 2009 had some Generalisation questions. The exception was the January Core 4 paper and the grade boundaries were significantly higher for this examination. Similarly, the June 2009 Core 1 paper had the greatest proportion of Identification questions and also higher grade boundaries. Conversely, the January 2009 Core 3 and June 2009 Core 4 papers had the least amount of Identification questions and also lower grade boundaries than the other examinations in this year. This again re-enforced the link between type of question and the number of students answering it correctly. As with the two previous years of pure mathematics examinations, there were no questions of the Synthesis type on any of the eight 2009 papers.

The pattern of no Synthesis questions continued for each of the eight pure mathematics papers in 2010. The composition and grade boundaries for this final year of examinations that were analysed are displayed in figure 5.4.

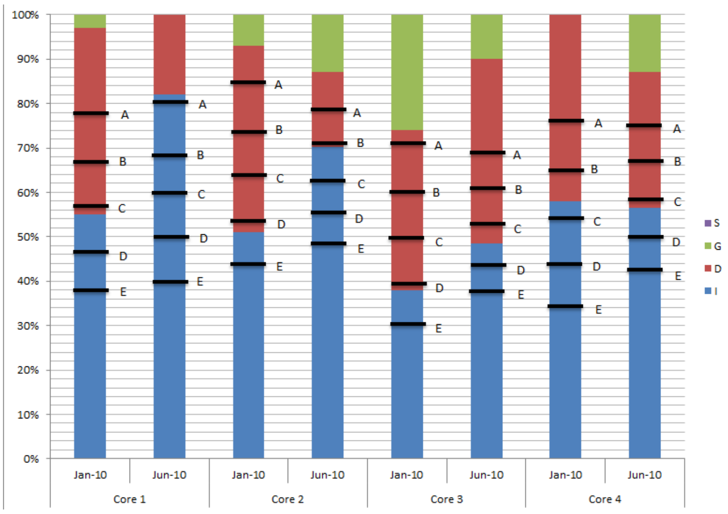


Figure 5.4: Classification and Grade boundaries for the 2010 papers

There was more variation in the composition of the 2010 pure mathematics examinations than in the previous years of papers analysed. Two of these papers, interestingly a Core 1 and a Core 4 examination contained no Generalisation questions. One paper, January 2010 Core 3 contained significantly more Generalisation questions than any of the other papers analysed over the three year period. This paper also contained the lowest proportion of Identification questions. Whilst the grade boundaries were lower than for the majority of examinations, the percentage of marks required to obtain a grade E was significantly lower than on any other paper. This was the Core 3 paper that the case study students sat and their responses along with those of their teachers to this examination are discussed in Chapter 7.

5.4.3 Summary

The composition of these examination papers was fairly consistent, and showed that with the exception of the unusual Core 3 examination, the papers that the case study students sat were typical of the pure mathematics examinations during this time period. Whilst the level of consistency was unexpected, it did not make interesting reading as the majority of the papers were so similar. I had anticipated that the composition of the A2 papers would have been significantly different to the AS papers, but this was not the case. The consistency of a majority of Identification questions, some Discrimination questions and a few, if any Generalisation questions was displayed throughout all the examinations from Core 1 to Core 4. A further surprise was that there were no Synthesis questions in any of the 27 pure mathematics papers that I analysed. Although this type of question had not appeared on the first of the three papers which I had used to develop the classification model, it was remarkable that it did not appear on any A-level examination over this four year period. It was striking that there were very few Generalisation questions on the examinations and that the grade boundaries were such that these questions did not need to be answered even to gain a grade A on any of these papers. It was noticeable that when an examination contained more Generalisation questions then the grade boundaries were lower than for papers with a higher proportion of Identification questions.

It is against this general picture of the composition of AS and A2 pure mathematics examinations that I consider the four papers that the case study students sat. The AS papers, Core 1 and Core 2 are analysed in Chapter 6 and the A2 papers, Core 3 and Core 4 in Chapter 7. In these chapters, the model for the classification of the demands of examination questions is used as a framework to analyse the data collected from students and teachers about the perceived difficulty of these examinations.

5.5 Conclusion

Existing research explores the goals of teaching and learning mathematics and values a depth of understanding. A variety models consider the different levels of understanding mathematics. As Torrance (2007) described the large part that examinations played in learning, I wanted to explore the understanding expected in AS and A2 mathematics examinations through the development of a model from Sierpinska (1994). Whilst Sierpinska considered that understanding could be measured by the type of epistemological obstacles required to be overcome, I considered the nature of the type of demand in examination questions. With this model, I defined four types of demand in examination questions. Analysis of 24 pure mathematics papers showed that the examinations mainly consisted of Identification questions, which do not require the students to make any decisions about what mathematics to do. This type of question presented students with limited demands as they provided instructions of what to do, either explicitly or by the standard nature of the request. Yet many students and teachers described these mathematics examinations as difficult. There is a tension between my analysis of examinations as mainly Identification questions and the perceptions of students and teachers of mainly difficult.

In Chapters 6 and 7 I explore students' and teachers' responses to the level of demand provided by the AS and A2 examination questions. Duffin & Simpson (2000) described that whilst teachers could not see their students' understanding of mathematics, they could interpret the 'external manifestations' of understanding. The level of demand in examinations questions provided these external manifestations that offered insight into the nature of understanding required of A-level mathematics students.

From Torrance's (2007) description of assessment as learning, the level of demand provided by the AS and A2 examinations is now what is taught throughout the A-level course. I explored the level of demand involved in learning A-level mathematics through interviews with students during each module. The data from the eight case study students was placed in context with questionnaire responses from the cohort of Year 12 and Year 13 mathematics students. In line with the findings of Torrance (2007), students' experiences of learning mathematics were very similar to their experiences of being examined in mathematics.

In addition to their responses to learning and being examined in mathematics, I also investigated students' expectations of A-level mathematics. As students' expectations of A-level were based on their previous experiences of learning and being examined in mathematics, as found by the study by Brown et al. (2008), I first explored their experiences of GCSE mathematics. These experiences of GCSE mathematics are discussed at the start of the following chapter.

CHAPTER 6: THE AS COURSE AND EXAMINATIONS

6.1 Introduction

In this chapter I present the findings from analysis of a range of data sources collected during the students' AS mathematics course which took place from September 2008 until July 2009. In order to put the AS experiences into context, I first discuss findings from a questionnaire issued to Year 11 students in July 2008 which gathered data on experiences of learning GCSE mathematics and their expectations for learning AS mathematics. I had identified that there were two aspects to students' experiences of AS mathematics; learning and being examined. I collected and analysed data about both aspects of the AS course and drew comparisons between them, finding striking similarities between the two components. Following the natural sequence of the students' experience I first considered the learning of the Core 1 course then the Core 1 examination, followed by the learning and finally the examination of the Core 2 module. The data from the aspects of Core 1 and Core 2 were combined and the learning of both pure mathematics AS modules is reported first, followed by the analysis of the AS examinations.

As reported in the research design (Chapter 4) the data sources consisted of three sets of interviews from the eight case study students in October 2008, February and July 2009 and interviews with four A-level mathematics teachers in February and July 2009. This data was set into context with questionnaire responses from whole Year 12 AS mathematics cohort in July 2009. In this questionnaire more than two-thirds of the Year 12 students felt that AS mathematics was more difficult than any of their other AS subjects. Reasons for this striking perception of increased difficulty of mathematics were explored. After each round of interviews, the data was analysed using the 'constant comparative' method (Thomas, 2011) for the recurrence of particular reasons for difficulties. Several themes emerged and these were used as headings to report the findings in this and the following chapter. These include workload, pace and intensity, memory, and decision making. However there was little insight into students' understanding as most difficulties were rooted in the external contexts of the questions.

In addition to the data gathered from the interviews and questionnaire, there was written data collected regarding the second AS pure mathematics examination, Core 2. The model to classify the level of demand in examination questions (Chapter 5) was used to analyse the questions on each paper. The composition of the examinations was calculated in terms of the number of marks allocated to the Identification, Discrimination, Generalisation and Synthesis questions (see section 6.3.2). Students and teachers were asked to rate each part examination question as either easy or difficult and to give a reason for their choice. These responses were used against the classification of questions to explore the links between the level of demand determined by the model and the perceived difficulties of students and teachers. Again data was analysed for recurring sources of difficulty and these themes; novelty, decision making and mathematics are used as section headings to report the findings under. Students and teachers also had strong views about what they expected from each examination, these included predicted levels of difficulty and anticipated structures such as order and mark allocation of questions.

When considering student responses to learning and being examined in AS mathematics I found it useful to consider the grade that each achieved in these modules. Table 6.1 shows the module scores for each AS module along with the overall AS grade. The uniform mark score (UMS) is out of 100, converted by the examination board from a mark out of 72 for each module examination. The grade boundaries for each module and the overall AS and A-levels are 80% for an A, 70% for a B, 60% for a C, 50% D and 40% E. Where two scores are listed for a module, the second indicates that the mark gained in a re-sit of that examination. The highest score automatically goes forward towards the overall AS grade.

Table 6.1 Case Study students' GCSE and AS module grades

'Name'	GCSE Grade	C1	C2	D1/S1	AS Grade and UMS
Alice	A*	87	86	81	A 254/300
Bella	A*	83 91	66 84	74	B 249/300
Craig	A*	99	94	83	A 276/300
Dan	A*	91	84	96	A 271/300
Emma	A*	97	99	86	A 282/300
Frankie	A	94	64 85	97	A 276/300
George	A	79	51	79	C 209/300
Henry	A	74	39	57	D 170/300

6.1.1 Experiences of learning GCSE mathematics and Expectations of AS mathematics

Data was collected from a questionnaire issued to the whole cohort of Year 11 students at the end of their GCSE course. Responses were analysed using the categories of Brown et al. (2008) and the results were found to be in line with this larger study (see Chapter 3). Of the 88 Year 11 students, 47 (53%) had chosen to study mathematics at AS-level, and 41 (46%) had not chosen to take their study of the subject further. My findings matched those of Brown et al. (2008), with difficulty of the subject and lack of enjoyment or application of mathematics being the reasons why students had chosen not to take it at AS-level. These findings may be surprising because of the high achieving nature of the Grammar School cohort with 100% A*-C grades in GCSE examinations compared to the mean of 65% for the schools in the Brown et al. (2008) study.

So that I could be sure of their AS choices, interviews with the eight case study students took place at the end of the first half term of Year 12. Five of the students described that GCSE mathematics had been more difficult than their other GCSE subjects and only Craig reported finding mathematics easier than all his other GCSE subjects. This was particularly surprising given their achievement of 5 A* and 3 A grades. Typical reasons for the difficulty referred to the amount of material that needed to be memorised and the following quote is representative.

> *"It was quite hard in comparison to the others, you could only just revise all of it because it was one of the longest ones. You've got to remember everything, you can't just make it up, if you forget it, you just get it wrong"* Frankie (A at GCSE)

The difficulty of mathematics was also reflected in the questionnaire responses. Despite 88% of the Grammar School students achieving A* or A grades at GCSE mathematics, many described it as their most difficult GCSE subject. Virtually all students thought there would be a big increase in difficulty from GCSE to AS level and this prevented many students with high target GCSE grades from continuing with mathematics. Students expressed their concern about being able to cope with the expected increased difficulty of AS mathematics. Surprisingly, one student felt that effort could not improve success in mathematics.

> *"No matter how hard you work if you can't really understand the principles, you're never going to get much better. With other subjects you can learn how to do it but for maths you have to know how to do it almost."* Dan (A* at GCSE)

Students were given a free response to what they thought learning mathematics would be like at AS-level, and the responses provide a starting point from which to explore the experiences of the case study students. The responses were then coded into the following categories: interesting (51%), challenging (48%), enjoyable (21%), and useful (15%). Some students offered mixed responses hence the total percentage exceeds 100. How their expectations compared with their experiences of AS mathematics is explored next.

6.2 Experience of Learning AS Mathematics

6.2.1 Introduction

Given that from the questionnaire responses at the end of Year 11 many students had listed finding mathematics 'fun' or 'enjoyable' as their reason for choosing to study the subject at AS, it was striking that so few described it as such at the end of Year 12. Only five students (5/65 = 8%) who completed the end of Year 12 questionnaire gave a response that included 'fun' or 'enjoyable' to an open question that asked students to describe what learning AS mathematics was like. However, this lack of a positive experience did not appear to be a cause for concern for the students as only four would not recommend the course to Year 11 students. Whilst only two students, one who gained a grade A and one a grade U, would recommend AS mathematics without reservation, many (20/65 = 31%) would recommend the course to students who were willing to work hard. Six students would recommend it to students who were 'good' at mathematics and five to students who enjoyed the subject. The following comment was typical of the reservations that came with students' recommendations.

> *"Only take it if you're really good at it and can understand it easy and pick things up quick."* Grade U student.

The comment also highlighted the pace of the AS mathematics course. This was a key feature in many students' descriptions of learning mathematics and will be explored further throughout this chapter.

Data from the end of Year 12 questionnaire showed the perceived difficulty of AS mathematics in comparison with other subjects. Two-thirds of students (44/65 =68%) thought mathematics was their most difficult subject and a quarter (16/65 =25%) thought it was at the same level as their other subjects. Only four students thought that mathematics was easier than one of their other subjects (all science AS levels). Surprisingly there was only one student out of sixty-five who felt that mathematics was his easiest subject. This was even more remarkable when the students' achievements at AS were considered, as almost a quarter (15/65 =23%) achieved an A grade.

Whilst there was agreement in the level of difficulty at the end of the AS mathematics course, this was not necessarily the case initially. The interview data from the case study students was revealing, after half a term of AS mathematics six felt the difficulty level had not noticeably increased and only two found it significantly more difficult. The two who thought AS mathematics was much more difficult than GCSE described not being able to understand explanations or answer questions, but they felt other students had similar experiences and they found this reassuring.

> *"Every lesson and every homework, without the textbook and the answers in the back, I have no idea how to even get to the answer. It happens a lot so you get disheartened, but then you talk to other people and they have the exact same problems so that makes me feel a bit better."* Bella, October 2008

Whilst the majority of the case students had not experienced a significant increase in difficulty from GCSE during the first few weeks of AS mathematics, all thought that the subject would get more difficult throughout the remainder of the course. This prediction of increased difficulty by the case study students in the first part of their AS course, was confirmed by the questionnaire responses of the whole cohort at the end of Year 12. When asked a free response question about how the AS mathematics course compared to their expectations, most students (45/65 =70%) described it was 'harder' or 'a lot harder'. Whilst these responses were evenly spread across the different grades, proportionally and to be expected, more students with E and U grades gave responses about the increased difficulty of the course.

Table 6.2 Difficulty of AS mathematics compared to Year 12 Students' expectations, by achieved AS grade

AS Grade	Number of Students	Easier than Expected	Same as Expected	More difficult than Expected
A	15	2	7	6
B	8	0	2	6
C	11	0	2	9
D	16	1	4	11
E	7	0	2	5
U	8	0	0	8
Totals	**65**	**3**	**17**	**45**

More striking is the breakdown of the difficulty of the AS mathematics course by the student's achieved GCSE results as shown in table 6.3. Given the school's entry requirement of only students with a grade B or higher at GCSE permitted to do AS mathematics, still almost 70% of students found the AS course more difficult than they had expected. Surprisingly, even the majority of those students with an A* at GCSE had found AS mathematics more difficult than they had anticipated.

Table 6.3 Difficulty of AS mathematics compared to Year 12 Students' expectations, by achieved GCSE grade.

GCSE Grade	Number of Students	Easier than Expected	Same as Expected	More difficult than expected
A*	29	2	11	16 (55%)
A	21	0	3	18 (86%)
B	15	1	3	11 (73%)
Totals	**65**	**3**	**17**	**45 (69%)**

The sources of the difficulties of learning AS mathematics are now explored under headings that emerged from the data collected from students and teachers.

6.2.2 Workload, Pace and Intensity

During the end of year questionnaire, students were asked to rate the amount of work done in mathematics lessons compared to their other subjects. The responses showed that the majority of students found AS mathematics required more work than their other subjects both inside the classroom (45/65 =70%) and particularly with homework (52/65 =80%). Significantly, no student felt that mathematics had been the least amount of work either in lessons or in terms of homework. Similarly the issue of increased workload arose during the interviews with the eight case study students part-way through the spring term of Year 12.

> *"There seems to be more stuff that you have to learn. C1 built up over topics, but C2 is hard from the start"* Emma February 2009

The reduction in teaching time for the Core 2 module was also given as a reason for the students increased workload. Four of the eight case study students thought they were receiving more homework on Core 2 than for Core 1 and two felt the work was more difficult.

> *"because we are having to rush through things, we'll pick up the pace. We've not been able to go into as much detail into the topics, so you have to do it at home."* Henry February 2009

The eight case study students were interviewed three times during the course of Year 12 and the issues of pace and intensity were recurring themes at each of these points. The following comments from October, February and July interviews show that for one student the pace of the AS mathematics course remained an issue throughout the year.

> *"He* [private mathematics tutor] *doesn't go really fast so I have time to ask every little question that I have, whereas in class you can't really do that because, everyone else, well lots of people have got it and the class just has to move on"* Bella in October of Year 12

> *"We have a lot less time* [for Core 2] *like we have just done trig and we didn't seem to have the amount of time to grasp what we needed."* Bella in February of Year 12

These descriptions closely match those of the 'top set' students (Boaler, 1997; Ireson, Hallam & Hurley, 2005; Boaler, Wiliam & Brown, 2000). Following study leave for the AS examinations, students return to school at the end of June to start their A2 courses. Individual subjects approach this transition from AS to A2 level in different ways. The comment below shows that for one student, the difference in the workload between mathematics and her other subjects was significant.

> *"Maths is so much harder than the others, they hardly did anything. But with maths they hit you hard with so much A-level work."* Bella in July of Year 12.

By June of Year 12, two of the eight case study students had decided not to continue with A2 mathematics. I interviewed these two students who both described the fast pace that they were required to work was the most difficult aspect of learning AS mathematics. Yet both of these students had begun the AS course having achieved a grade A at GCSE mathematics. Similarly, the issue of pace was raised by all eight of the case study students in interviews during their AS course. When interviewed in February of Year 12, about initial impressions of the Core 2 course, two students raised the issue of lack of time as a source of difficulty.

> *"It's harder in the actual lessons because we've got to cram more in as we are only having two lessons a week and not four and that doesn't help."* Henry February 2009

The lack of time was also apparent when students described how Core 2 lessons differed from Core 1 lessons. Whilst three students felt that there was no difference between lessons on the different pure mathematics modules, five described an increase in pace and covering more material in each lesson, the following quote is typical.

> *"classes are quite rushed and because there are so many people asking questions the teacher is like 'Oh no we have to hurry up now and get on with the rest'. Sometimes the teacher is like 'you can come and see me in your own time so you are not holding up the rest of the group' and I understand that. But it is quite frustrating because I have to sit in the rest of the lesson thinking oh, I don't understand."* Alice February 2009

This illustrated that the need for the teacher to cover a certain amount of material in a given lesson was shared with students, and that questions raised by students were seen as a barrier to achieving that aim. It indicated that questions that may have led to understanding were expected to take place outside of lesson time. Whilst frustrated by this, the student seemed to accept the situation that understanding mathematics was to be done in her own time rather than within lessons. The descriptions of Core 1 and Core 2 lessons contrast with the recommendation from the Cockcroft Report (DES, 1982) that mathematics lessons should provide a variety of activities. Instead my data reflects the findings of Boaler, Wiliam & Brown (2000) when describing transmission pedagogies tailored to the quickest students within the environment of a top attainment set.

6.2.3 The Role of Memory

A recurrent theme through students' descriptions of learning AS mathematics, both from the interviews and questionnaire was the need to memorise a lot of material. Students talked about the number of rules, formulae and methods that they were required to learn. Often the reliance on a memorised rule or formula was given as a reason that mathematics was easy. The following was an example from a student who described a specific question as 'easy' because he could remember what to do.

> *"I think that's quite easy, because it's quite a simple thing to remember, there's just a formula that's easy to apply, there's not that much working out to do."* Henry October 2008

However, the dependence on memory was more commonly described as a source of difficulty for many students. During the same round of interviews, when discussing four co-ordinate geometry questions (see section 5.3.7), two students described not being able to begin to form their answers because they could not remember what to do. It appeared that once a memorising approach to learning mathematics was adopted, if that memory failed then the student was left with no alternative strategies.

> *"I don't know how to do it (laughs) I know I should be able to do it, I just can't remember."* Bella October 2008

A reliance on memory left no alternative strategies for students who could not remember the specific technique that they were being asked to carry out. Similarly, the following comment highlights the difference in being able to answer a question whilst the topic was being taught, and at some time later, when more material had been covered.

> *"once I get started I can get into it. But if I comeback to these in 3 months time it could be that my mind goes blank, having forgotten about the gradient, I'll be staring at it for a while and I won't know how to get started, then I won't know how to do anything."* George October 2008

A further difficulty with memorising was that students felt unable to cope with questions that were in a different format to those seen before. When asked about what happened when she encountered something different, the following response demonstrated both the difficulty in seeking an alternative strategy and again a reference to time pressure and the need to work quickly.

> *"I probably skip it and then go back to it and try and find a way. I could do it if I had a lot of time, but if I've just got a couple of minutes to do it, like in an exam, I'd probably panic."* Bella October 2008

6.2.4 Decision Making

During the October 2008 interviews with the case study students, the difficulty of four co-ordinate geometry questions was discussed. Most of the case study students agreed that the amount of decision-making in a question was a factor in the level of difficulty. Interestingly no student discussed the mathematical subject of a question as a difficulty factor. Whilst the model for classifying examination questions was not introduced to students, there was a strong correlation between their descriptions of the types of questions and the classifications. The student word for Identification appeared to be 'standard' and these were the questions that were most often described as easy.

> *"Easy is when it's short, just one step and its obvious what you have to do."* Emma October 2008

Commonly, students also described the Discrimination question as straightforward. Whilst this question required some level of decision making, it appeared that it was of a standard nature and so students were familiar with the techniques involved. Whilst there was a lack of instruction in this type of question, often their familiarity meant that students were confident when deciding which mathematics to apply. However, the most difficulties arose when students had to make decisions about the application of mathematics to contexts that they were unfamiliar with. These were the questions of the Generalisation and Synthesis type. The latter was the most uncommon for students as it was not a typical examination style question of the type used in lessons or homework tasks and required students to consider multiple solutions. A further layer of decision making was described by one student as the need to select which information given in the question was relevant.

> *"if it involves loads of inputs, when you've got to discard things, like when you are given pointless information. You have to ask why are they giving me that, you have to think that's something I should actually ignore."* Craig October 2008

Again, this was a situation that the students were unfamiliar with, as the typical questions that students worked on in lessons did not provide opportunities for decisions regarding selection of information. The need for decision making prior to applying mathematical techniques was generally considered to be a source of difficulty and was summarised by the following quote.

> *"It's harder when you have to think what you have to do and then apply your knowledge. Because it's not telling you what to do, you have to think of that as well as the maths."* Emma October 2008

In addition to the amount of decision making required, students often described the number of steps required in a solution as a source of difficulty. This reason again resulted in the co-ordinate geometry questions of the Generalisation and Synthesis type being described as most difficult by seven of the eight case study students (one student felt he could not order the difficulty level of the questions as he could answer all of them). However, there was no agreement between which of these two questions was the most difficult, and two students rated them as having the same difficulty level as each other. There was an issue when interpreting the students' responses that arose from the format of the Generalisation question. This was the only question that included a diagram and it was unclear how this had influenced students. This issue was addressed by a different method of data collection in Year 13. However the reasons given for these two questions being difficult were very similar. The following quote was typical of those that referred to the number of steps involved that had not been stated explicitly in the question.

> *"Probably because there's a lot more working out to do, a lot more processes to get to the answer and I don't really know what those processes are".* Henry October 2008

The three possibilities for an answer to the Synthesis question were also a source of difficulty mentioned by three students.

> *"This is the hardest because there are three possibilities, that's always going to be a bit more puzzling. Also it says that it's a square, you've got to have a look at it in your mind and see where the different points are where it could be a square."* Craig October 2008

6.2.5 Mathematics

It was striking that throughout the different modes of data collection that mathematics was spoken of so infrequently. The descriptions and discussions were about the difficulty of the wording and structure of the questions rather than understanding of mathematical topics that they were concerned with. Whilst I had set up the data collection to discuss mathematics questions rather than mathematical topics, I was still surprised that the process of answering questions rather than the mathematics was the prominent feature in students' responses. This issue is considered further in Chapter 7. The descriptions of difficulty typically centered on question styles, lengths and familiarity rather than understanding of mathematical concepts.

One source of difficulty identified by both students and teachers was the use of mathematics from a previous module. Most of the instances of this reason for difficulty appears in Chapter 7, when the A2 modules depend and build upon the skills from AS. However, even as the students approached the second AS module, the use of mathematics from Core 1 was seen as a source of difficulty.

> *"C1 is just like a warm up almost for C2. You've got to have a knowledge of C1 not just the new bits of C2."* George February 2009

One teacher identified the compartmentalisation of mathematical knowledge as a problem that had resulted from the modular A-level. Their response fitted with the literature (Chapter 3) which explored the effects of the modularisation of the A-level course, described by Smith (2004, p.93) as 'splintering the unity and connectedness of mathematics'.

> *"In an ideal world we wouldn't have everything split down into tiny modules because that does de-compartmentalise their learning 'circles are C1', they sort of learn it and dump it."* Teacher 4 February 2009

Over the course of the data collected during Year 12, there were four mathematical topics that were raised as a source of difficulty. Two topics, differentiation and transformations of curves were from the Core 1 module and trigonometry and logarithms were from Core 2. There were again limited insights into understanding as students frequently made distinctions between difficult concepts and difficult questions. One student described that whilst the mathematical topic, differentiation, was difficult, the questions that he was expected to do were straight forward.

> *"We've just done differentiation. I didn't understand a word of that, I didn't know what it meant. But then in the second lesson we did some questions on it, and it actually turned out to be lot easier than it was almost trying to make out to be."* George October 2008

This quote illustrated the disparity between learning mathematics and learning how to answer AS mathematics questions. This student clearly struggled to understand the concept of differentiation, but could answer questions that required the use of the rule: if $y = ax^n$ then $\frac{dy}{dx} = anx^{n-1}$. The second Core 1 mathematical topic that was described as a source of difficulty for students was the transformation of curves. Yet all the difficulties described featured memorising a large number of rules rather than with understanding the mathematics.

When asked about their initial experiences of the Core 2 module, four of the eight case study students felt it was more difficult than Core 1. One thought it was a 'step up' from Core 1, but three students described trigonometry, the first topic that they had studied, as difficult.

> *"I think it's a bit harder, but I don't normally understand it fully until I actually revise it at the end of term. But generally I found the trigonometry quite hard, trying to remember the different points that each of the graphs go through and then last lesson we were doing the CAST diagram but I don't know what this means.*" Frankie February 2009

It was surprising that this student did not expect to understand the material as she progressed through the course, and only achieved understanding through her own revision after the module had been completed in lessons. However, this did fit with Alice's account (section 6.2.2) where she described not asking questions in lessons and left her understanding to outside of lessons. When asked about the Core 2 course during interviews in July, four of the eight case study students described logarithms as a difficult topic. Again all of the descriptions referred to the number of rules that were involved.

> *"Logarithms are always really difficult, there are so many rules to remember and which to apply, I find it quite daunting"* Alice July 2009

The students' descriptions of difficult topics at AS level showed memory rather than understanding of mathematical concepts were the sources of difficulty. This issue is explored further in Chapter 7, where the students' experiences of the A2 mathematical topics are considered.

6.3 Experience of AS Mathematics Examinations

6.3.1 Introduction

Most of what emerged from students' descriptions of their experiences of mathematics examinations was similar to their experiences of learning mathematics. Given that the students' descriptions of lessons emphasised the use of examination style questions it was perhaps unsurprising that the experiences of examinations were comparable to those of learning AS mathematics. Students talked about difficulties in deciding which mathematics to apply and this was described as particularly difficult when the examination questions were worded in an unfamiliar way.

In parallel with the data collected from students, I analysed the Core 1 and Core 2 papers with the use of the model to classify the level of demand of examination questions. This model is discussed in Chapter 5 along with the results from analyses of all of the pure mathematics papers from January 2008 to June 2010. The analysis showed that the construction of all of the AS papers was similar with comparable percentages of Identification, Discrimination and Generalisation questions. The two papers that the students sat, January 2009 Core 1 and June 2009 Core 2, were typical of the examinations set during this time.

The eight case study students were interviewed in the half term following the sitting of the Core 1 paper, their first AS examination. They talked through the questions on the examination describing whether they found them easy or difficult. How they tackled the questions and whether they used the order of the questions or mark allocation to signify difficulty was also discussed. After changes in data collection methods were considered (see section 4.4.3) in addition to discussing the questions in individual interviews, written data was also collected following the Core 2 examination. A further change was that both written and interview data was collected from the four teachers following the Core 2 examination.

The data from students and teachers was analysed against the classifications of demand of the examination questions. The difficulties described both by the students and teachers generally fitted with the classifications. Commonly, the questions classified as Identification were described as easy. Conversely, the Generalisation questions were considered to be the most difficult, seven out of the eight students and all of the teachers described these questions as difficult. Generally the views of the students were in line with those of the teachers, significant differences are explored later in this chapter. Data from students and teachers were analysed using a 'constant comparative' method (Thomas, 2011). Themes emerged from this analysis of the data on sources of difficulty and these are used as headings to report findings under. Common sources of difficulty were novel questions and the need for decision making about the method to be used in answers.

There were several aspects of the structure of mathematics examinations that influenced perceptions of difficulty; these included the high mark allocation of individual questions and the position in the paper. Students expected early questions to be easy and the last questions to be the most difficult. Surprisingly, the teachers also shared this view of the structure of the examination papers. Occasionally a specific mathematical topic was given by students as a reason for a question being difficult, this fitted with the findings from learning AS mathematics, with transformation of curves being seen as the most problematic material on Core 1 and logarithms on Core 2. Teachers did not give mathematical topics as a reason for difficulty on any question; they were more likely to qualify this as ‘a difficult question on’.

The students’ descriptions of questions being difficult were particularly interesting in light of their performance on these examinations (table 6.1). On the Core 1 paper, six students gained a grade A and two a grade B. For the Core 2 paper, four students gained a grade A, two a grade C, and the two students who chose not to continue with mathematics at A2 gained a grade D and U.

6.3.2 Classification of AS Questions and Perceptions of Difficulty

The questions in the Core 1 and Core 2 papers were categorised with use of the model (see Chapter 5). Each of the 25 question parts on the Core 1 paper, and 22 parts on Core 2 were categorised as Identification, Discrimination, Generalisation or Synthesis.

Table 6.4: Mark allocation for the AS papers

Classification	Identification	Discrimination	Generalisation	Synthesis
C1: Number of Marks	52/72 (72%)	14/72 (19%)	6/72 (8%)	0/72 (0%)
C2: Number of Marks	44/72 (61%)	21/72 (29%)	7/72 (10%)	0/72 (0%)

As in Chapter 5, the grade boundaries for each examination paper were considered in order to quantify what was expected of students. The grade boundaries are first presented in table 6.5 and then combined with the percentage of marks available for each category in figure 6.1.

Table 6.5 Grade Boundaries for AS papers

Grade	A	B	C	D	E
C1: Marks Required	57/72 (79%)	50/72 (69%)	43/72 (60%)	37/72 (51%)	31/72 (43%)
C2: Marks Required	56/72 (78%)	49/72 (68%)	42/72 (58%)	35/72 (49%)	28/72 (39%)

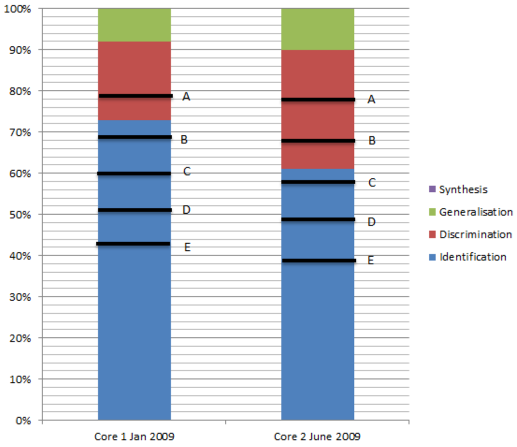


Figure 6.1: Classification and Grade boundaries for AS Core papers

As shown in Chapter 5, the two papers that the case study students sat were typical of the AS Core examinations set throughout the 2008 - 2010 period. The examinations were typical both in terms of the number of marks allocated to each classification of question and the percentage of marks required to achieve each grade. The majority of both examinations consisted of Identification questions, where the method was explicitly stated or implied by the standard nature of the request. As with all of the examinations analysed over the two year research period, there were no questions of the Synthesis type. There was only one part question of the Generalisation type on each of the papers. With the given grade boundaries students could achieve a grade A without having to master questions of the Generalisation or Synthesis types. A grade C could be achieved on these papers by answering only the Identification questions correctly. These questions did not require any applications of techniques other than those stated.

To explore links between the demands of examination and perceptions of difficulty I asked students and teachers to rate each part question as easy or difficult and to give a reason for their choice. These responses were then grouped according to the classification of the question, and the number and reasons for difficulty were analysed within the three categories of Identification, Discrimination and Generalisation. As in Chapter 5, the difficulty responses were tallied for each question and given as a percentage of the total possible ratings. For example, on the Core 1 paper there were 20 question parts that were classified as Identification and these were categorised by the eight case study students for a total of 160 ratings. Of these 160 ratings, 18 were labeled as difficult, which gave the Core 1 Identification questions a difficulty rating of 18/160 =11%. All percentages were rounded to the nearest whole number. Table 6.6 shows that the correlation between the classification as defined by the model and the perception of difficulty is strong for both the Core 1 and Core 2 examination papers.

Table 6.6: The percentage of Students and Teachers who classified AS questions as 'difficult'

Classification	Identification	Discrimination	Generalisation	Synthesis
C1: Student 'Difficult' Rating	11%	44%	88%	------
C2: Student 'Difficult' Rating	33%	44%	88%	------
C2: Teacher 'Difficult' Rating	27%	83%	100%	------

For both Core papers, as might be expected Identification questions were least often rated as difficult by both students and teachers. There was an increase in the percentage of Discrimination questions described as difficult. These questions, which required students to apply some mathematics that was not specifically mentioned, were described less frequently as easy by students and particularly teachers. The Generalisation questions were described as difficult by the majority of students and significantly, by all of the teachers.

Students rated fewer questions as difficult than their teachers did. Whilst there were many similarities in the responses given by students and teachers as to why they found particular questions difficult, there was one significant difference. It was noticeable that the teachers' comments contained more mathematical references than those of the students, particularly in their written responses. Below are first a student's and then a teacher's comment about the second part of the following question that they both described as difficult.

5 Solve each of the following equations for $0° \leqslant x \leqslant 180°$.

(i) $\sin 2x = 0.5$ **[3]**

(ii) $2\sin^2 x = 2 - \sqrt{3}\cos x$ **[5]**

"Found hard to get in the same form." George July 2009

"2x usually throws students. Many find one value for 2x, then halve and generate further solutions incorrectly. Having substituted for $\sin^2 x$, factorisation is just common factor and 'special angles result." Teacher 3 July 2009

Reasons for difficulty given by students and teachers were explored and are reported under headings that emerged from the data.

6.3.3 Expectations

Students and teachers held strong ideas of what they expected of AS mathematics examination papers. These expectations were based on their experiences and familiarity with previous examination papers and included the order of questions, the mark allocation of individual questions, and the time allocated to the questions. There was also a perceived level of difficulty of each examination paper, with Core 2 expected to be more difficult than Core 1. The students' expectations of Core 1 and Core 2 and the teachers' expectations of Core 2 are explored in this section.

When asked how the actual Core 1 paper compared to their expectations, five students felt it was about the same as they thought it would be. Most described similarities to the previous papers they had done as part of their revision and the following quote is typical.

> *"it was pretty much as I had expected it to be. On the past papers you can see a lot of recurring themes and a lot of the questions were pretty typical"* George February 2009

Only one student felt that the paper was more difficult than he expected, but he could not explain why. Two students thought the paper was easier than expected, and one expressed his disappointment about this.

> *"You get all this hype, oh it's so much harder than GCSE, but when we actually got to it, other than differentiation this is all stuff we did at GCSE anyway. Actually it's slightly easier than GCSE because GCSE was the summary of 2 years of work whereas this has only been half a year. I feel slightly cheated (laughs) that I was told it would be so hard."* Craig February 2009

However, whilst Core 1 had been in line with students' expectations, the Core 2 paper was generally considered to be more difficult than expected. Six students described it as harder than they thought it would have been, based on their previous experiences of Core 2 papers. The reasons given focused on the atypical nature of the questions and the following comment is representative.

> *"There wasn't the usual really easy questions at the beginning there were lots where you had to do several bits of working out answer."* Dan July 2009

However one student was positive about the different nature of the questions.

> *"the trapezium rule, we have only ever practised that with actual numbers but here they used constants. So you had to apply your knowledge of what you know and then work it out in a different way, But I had never seen one like that before, I didn't even know that you could do that until the exam. I guess it was nice to see something different"* Emma July 2009

One student described the paper as expected and the only student who described the paper as 'quite easy' was the student who had previously been disappointed by the lack of difficulty of the Core 1 paper.

> *"it was all very straight forward, you could see quite clearly what the topic was, and some of them will even say 'use logarithms to solve the equation'. I'm just getting the outputs from being given the inputs, a simple procedure."* Craig July 2009

However, all four teachers described the Core 2 examination as a difficult paper giving reasons based of the different wording of the questions. The following is a typical description of the different appearance of the examination questions.

> *"the way things had been written compared to the way they had in the past. Because they looked different, I think that panics particularly weaker students. So I think this was quite a daunting paper"* Teacher 3 July 2009

Each of the six mathematics module examinations are 90 minutes long and contain 72 marks. To gain full marks, students would be required on average to gain a mark every 75 seconds, which allows very little thinking time. During the interviews, students were asked about the time allocated both to the overall examination and how they used the time to answer individual questions. One student described how he used the mark allocation of a question as a guide to how long he should spend answering it.

> *"You start thinking hang on a minute, its only four marks and I'm taking five, six minutes here, that's far too long."* Craig July 2009

Whilst describing how they had attempted the Core 1 examination, seven of the eight students described 'skipping' questions that they couldn't do. This was due to concerns about running out of time. The following quote was a typical description.

> *"If I can't do it within a minute then I will skip it and then carry on to the next question and keep going until the end until I've done all the ones that I could do quickly. Then go back to the slightly harder ones."* Bella February 2009

When asked how she could work out what was required and think how to do it in such a short amount of time, her reply showed that her primary concern was running out of time.

> *"I always worry about running out of time so I think I might as well get the marks that I can get. Then I can work on the ones I find hard. Rather than trying to work on things and worry about not having time to do the rest."* Bella February 2009

Two students said that they had been told to adopt this approach of 'skipping' questions they found difficult and one went further saying it was an instruction on the examination paper. This advice does not in fact appear on the front of any of the AS or A2 examination papers. However, on the GCSE paper (Edexcel, 2008) that these students sat, there was the advice 'not to spend too long on one question'. This had been taken to heart by students with the vast majority, continuing to 'skip' questions that they could not answer immediately. This technique was also used by the same seven students in the Core 2 examination, with only Craig completing all the questions on the first attempt. As several students had described 'skipping' questions as a result of concerns about running out of time, it was surprising to discover that only one felt he actually needed more time. The remaining students felt that they had enough time to finish all the questions and check through their work. Two students felt that they were given too much time, and described that they spent a lot of time checking over their work having initially completed the paper in less than forty minutes.

During the interviews that followed Core 1, students were asked whether they thought the order that questions appeared on the examination paper was significant. Five students felt that the questions got more difficult from the beginning to the end of the paper. All but Henry, who said he couldn't see any pattern in the order, felt that the last three questions on Core 1 papers were 'wordy', 'tough', or 'long'. The position of a question in an examination paper could affect students' perception of its difficulty. This was highlighted by one student's comments about the first question on the Core 1 examination.

> *"Question 1 was the one I found harder than I was expecting. It probably wasn't hard if it had been further down. I just think it's not where it should be."* Bella February 2009

All of the students said that they answered the questions in the order that was given on the paper. This seemed to be an automatic approach as none of the students remembered being told to do this or could justify why they did so. The expectation that questions increased in difficulty throughout the paper was highlighted by the following comment from one teacher as they described the most difficult question on the paper.

> *"I think I'd have to go with nine as you kind of expect anyway because there isn't a ten is there?"* Teacher 4 February 2009

In addition to the position of a question in an examination, the number of marks it was allocated could also affect the perception of difficulty. The number of marks was not commonly used by students as the main reason given for their classification of a question as either easy or difficult, but it was mentioned in 25% of the students' written responses. Typically a question was described as easy as it was only worth one mark and conversely, questions were described as difficult because they were worth lots of marks. This was explored further during the interviews following each examination.

Generally, in the interviews following the Core 1 paper, the eight case study students did not appear to link the amount of marks directly with the level of difficulty with a question. Instead, they used the marks as an indication of how many steps their answer should contain. This was summarised by the description given by Henry which was representative of the other responses.

> *"I look at the marks as well to see roughly how much time to spend. If it's a two mark question and you thought you had to do quite a lot, then maybe not, so it sort of helps you pace your work. For the seven-mark question, I knew there was more to it than I first thought otherwise it wouldn't be worth seven"* Henry February 2009

However, five of the eight students described feeling uneasy if they thought their answer did not justify a higher mark allocation.

> *"If I do a question really easily and I look at the marks and see there is more than I expect, I assume I've done something wrong to have got it that easily so I go back and check."* Dan February 2009

In addition to the amount of work required, the mark allocation could also influence the perception of the difficulty of a question as the comments about the question below showed. Five students expressed concern about why the question was worth seven marks.

9 The curve $y = x^3 + px^2 + 2$ has a stationary point when $x = 4$. Find the value of the constant p and determine whether the stationary point is a maximum or minimum point. **[7]**

The mark allocation made some students question whether they had given a complete answer, as they didn't appear to value their attempts as worthy of the 7 marks. This was a high mark question, typically the AS pure mathematics examinations consisted of a majority of two and three mark questions. The question above was the only one allocated seven marks on the Core 1 paper, and similarly there was only one high mark question on Core 2 which was allocated eight marks. One student seemed to have been so influenced by the high mark allocation that she found it the most difficult question on the paper.

> *"I don't see what else I would do, I just thought it was a bit odd to have so many marks". Alice February 2009*

However, the remaining students appeared satisfied with their answers, and Bella described how the high marks for the above question could have been split if the question had been broken down into stages.

> *"I think it could have been split into 2, 2, 2 and the last part would be state whether it's maximum or minimum. You just have to know yourself what to do each step."* Bella February 2009

The different interpretations of the high mark allocation of individual questions continued in students' discussion of the Core 2 paper.

> *"8 marks in one step-makes it looks complex and lots to do."* Emma July 2009

Perhaps surprisingly, teachers also referred to high mark allocation as a reason for why they described questions as 'difficult'.

> *"Some will find that hard because it's seven marks not broken down"* Teacher 3 July 2009

6.3.4 Novelty

Unlike students' descriptions of learning AS mathematics when they frequently worked on typical questions within each topic, they faced novel questions in both the Core 1 and 2 examinations. These non-standard questions, unfamiliar to students, were often described as difficult. The most common reason that students described finding a question difficult of the on the Core 1 was that they had not seen a question like it before. This was regardless of the Identification, Discrimination or Generalisation type.

> *"I don't think I've done any practice papers that had similar questions. It was different to what I've been learning and practising."* George February 2009

This remained the case with students' descriptions of difficult questions on the Core 2 paper.

> *"Never done anything like it in class"* Frankie July 2009

Comments such as these showed that students expected to have covered all the types of questions that they could be asked in an examination in their lessons. They appeared surprised by and un-prepared for questions that were different from those they had seen before. This was in contrast with the recommendation of the Cockcroft Report (DES, 1982) which advocated that students should be able to apply their mathematical knowledge to unfamiliar contexts. However, the view that all question types rather than mathematical topics were covered was also suggested by the following comment by a teacher. They described an examination question as 'nasty' as it presented students with something that they had not covered in their lessons.

> *"Leaving the k is nasty- numerical methods, I only cover numerical examples in my teaching."* Teacher 3 July 2009

This fitted with the description of 'assessment as learning' of Torrance (2007) where only what is examined is taught, thus novel examinations are a surprise to teachers and cause difficulty for students. The particular Core 2 question that was the focus of the above quote is now discussed in more detail as it was named by all of the teachers and seven out of the eight students as the most difficult on the paper.

9 **(i)** Sketch the graph of $y = 4k^x$, where k is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. **[2]**

(ii) The point P on the curve $y = 4k^x$ has its y-coordinate equal to $20k^2$. Show that the x-coordinate of P may be written as $2 + \log_k 5$. **[4]**

(iii) **(a)** Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of

$$\int_0^1 4k^x \, dx.$$ **[3]**

(b) Given that this approximate value is equal to 16, find the value of k. **[3]**

The difficulty with this question most frequently raised by the students was the different style of question on the topic of the trapezium rule.

> *"I know the trapezium rule really well but you had to do something to be able to get to the trapezium rule and I didn't know how to do the first bit which stumped me for the whole question. I'd never seen one like that before so I didn't know what to do, it was new to me."* George July 2009

Similarly, all the reasons given by teachers about this question being difficult related to the atypical style of the question, rather than the topics covered.

> *"you've got an inequality sign and a k in there for the trapezium rule which is less usual. I think it's the question more than the topic, I don't think that the trapezium rule is a particularly difficult topic, I think it's more the k to the x is there rather than apply the rule, it requires more understanding."* Teacher 1 July 2009

One student expressed his frustration at the different style of question on the familiar topic of the trapezium rule.

> *"I knew these questions – so much revision, but what was with the 'to the power of x'? I can do the trapezium rule but this to the x made me cry."* George July 2009

The role of memory had been a source of difficulty for many students when learning AS mathematics (section 6.2.3). However, the need to memorise rules or methods was only rarely mentioned by students when discussing difficulties in AS mathematics examinations. Not remembering a method or a set of rules was seldom given by students as a reason for finding a question difficult. Below is one of five similar comments about the difficulty of remembering the rules associated with transformations of curves.

> *"I find sketching curves hard, I'm not sure why, because there are so many different ones that you have to remember they can get mixed up."* Alice February 2009

As novelty was seen as the most frequent source of difficulty, it was expected that remembering a method or set of rules was not often raised. Memory would not be helpful to students when faced with questions different to those they had met before. The high achieving Grammar School students did not appear to find memorising standard methods to answer typical questions difficult.

6.3.5 Decision Making

For both the Core 1 and Core 2 papers, after novelty the second most common reason that questions were described as difficult by students was that the required method was not given. Thus students were required to decide how to attempt the solution and which mathematical techniques to apply. These questions fitted the descriptions of the Discrimination and Generalisation types and were most often described as difficult by both students and teachers. The following quote was a student's response to why he described a Core 1 question, classified as Discrimination, which asked for a line to be verified as the diameter of a circle.

> *"I wasn't sure what I was meant to put for verify, like how am I meant to show that?"* Dan February 2009

An example of a Discrimination question from the Core 2 examination is shown below. This question required students to realise that the cosine rule was needed.

1 The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. **[3]**

(ii) Find the area of the triangle. **[2]**

However, the familiarity of the question was more key to students rather than the decision making. The lack of instruction was not a source of difficulty in questions that were regarded as ‘standard’ or ‘typical’ by students. The students who were familiar with this question found it easy as they had remembered that the largest angle in a triangle is opposite the longest side. However, they also acknowledged that some students had found the question difficult.

> *“I knew the largest angle would be opposite the longest side so I only had to work out one angle. I know a lot of people thought it was really hard, they didn’t know about the longest side, so they didn’t know where to start.”* Emma July 2009

There were significant similarities between the students’ and teachers’ responses. The most common responses by teachers about the difficulty of Discrimination questions on Core 2 described a lack of instruction about which method to use.

> *“Have to apply trig identities with no clues.”* Teacher 1 July 2009

However, it was only when questions were different than those that had been met before, that the lack of a specified method was given as a source of difficulty. The combination of novelty and a lack of instruction informed the reasons for which question was considered the most difficult on the Core 1 examination. Overall, five out of the eight students found the last part of question ten the hardest and significantly, this was the only Generalisation question on the paper. Whilst only the final part was described as the most difficult, to provide the context, the whole question is given below.

10 A curve has equation $y = x^2 + x$.

(i) Find the gradient of the curve at the point for which $x = 2$. **[2]**

(ii) Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

(iii) Find the values of k for which the line $y = kx - 4$ is a tangent to the curve. **[6]**

The following quote illustrates the struggle that a student had when deciding what mathematical method to apply when answering the final part of this question. The description of difficulty within a novel context is typical of the students' responses.

> *"I had no idea what to do because it had a k in it and because it was just x squared plus x, it didn't have any numbers. I'm not really sure, I just couldn't work out how to do it*." Bella February 2009

Another description of the final part of question ten showed that the difficulty lies with deciding what mathematics to apply rather than carrying out the mathematical techniques.

> *"I wasn't sure how to go about finding the answer. I spent absolutely ages trying to figure it out and I just couldn't do it at all."* Alice February 2009

The novelty along with the lack of instruction required may have accounted for the disparity between the perceptions of difficulty between students and teachers. Students rated fewer questions difficult than their teachers, there were many factors which may have accounted for this difference (see Chapter 5). The responses from students about the AS examinations showed that they were able to identify those questions that lacked instructions but were familiar to them. This familiarity meant that the need for decision making could be replaced with the application of a memorised technique. Thus many of the questions described as difficult by teachers because of a lack of instructions, may have been familiar to the high achieving case study students who gained 5A, 1B, 1C and 1D grades at AS level mathematics.

6.3.6 Mathematics

Unexpectedly, mathematics was very rarely seen as a source of difficulty in examination questions by either students or teachers. Whilst this mirrored the data collected about students' experiences of learning AS mathematics, I was still surprised that specific mathematical topics were not more commonly described as sources of difficulty in the AS examinations. In the rare instances of mathematical topics given as the reason that questions were difficult, as expected these were the topics previously raised in the learning of mathematics.

The one mathematical topic given by students as a source of difficulty in the Core 1 had been previously raised in the context of learning AS mathematics (see section 6.2.5). It was therefore unsurprising that 'transformations of curves' was the sole mathematical topic named as a source of difficulty. Four of the eight case study students described a Core 1 question as difficult because of this topic.

> *"I struggle to get my head around why it all meant what it did."* George February 2009

Whilst the above comment illustrates the struggle that a student had to understand the meaning of the different transformations, three students described that remembering all the different rules was the source of difficulty. Interestingly, the four students who did not describe finding this topic difficult gave the reason that they could remember the rules.

Mathematical topics were also rarely given as a source of difficulty in the Core 2 examination. From the written data collection sheets, of the 67 cases of students describing question parts as difficult only 11 (16%) of these responses cited the mathematical topic as the source of difficulty. However, there was no consensus on what this topic was, whilst logarithms was given most often by five students, graph sketching and trigonometry equations were both listed twice and roots and radians both once. The comments were typically brief yet often very powerful.

> *"Hate logs!"* Emma July 2009.

During the interviews that followed the written data collection, students expanded on their reasons for mathematical topics being the source of difficulty. The two Core 2 questions most frequently described as difficult because of the topic both involved the use of logarithms. The following quote was typical of those which described that whilst the general topic of logarithms had been identified, the required method was not clear to students.

> *"I don't know when to log and not to log. I know that I've got to use logs but I couldn't make it work out. I don't know why. Question nine, I was really confused by having to write it as log k 5, I couldn't work out or remember how to do it"* Dan July 2009

None of the four teachers gave the mathematical topic explicitly as the source of difficulty for any of the questions on the Core 1 or 2 examinations. Instead they gave qualified reasons that referred to the unusual style of a question on a topic.

> *"question eight looks difficult, it's not standard, they might not have seen a geometric progression with shapes. There wasn't many practice and past paper questions like it, so that's more difficult."* Teacher 1 February 2009

Rather than an individual mathematical topic being given as a source of difficulty, there was the possibility that mathematics from a previous specification was required. In the case of Core 1 this would be material covered at GCSE, and at Core 2 this would include Core 1 and GCSE mathematics. There were only two comments from teachers and none from students that gave the requirement to apply mathematics from the first module a source of difficulty in the second pure examination.

> *"I think a lot of this paper relies on C1 knowledge. They are so geared to the exam that they are sitting and don't think of assumed knowledge. That* [discriminant of a quadratic] *is on C1 and they should have remembered it, but that would have made that question quite difficult"* Teacher 3 February 2009

There were more examples of the need to apply mathematics from a previous module in the A2 examinations, where knowledge of Core 1 and Core 2 was assumed. To be expected the need to apply previous knowledge was given more frequently as a source of difficulty in the Core 3 and Core 4 examinations and is discussed in Chapter 7.

6.3.7 Summary of Experiences of AS Examinations

From comments made by both teachers and students, the difficulty in the Core 1 and Core 2 examination papers most often lay in the wording of individual questions and not in particular mathematical topics. If the questions were of a typical or standard nature and were similar to those that students had practised before, they were considered not to be difficult. These were most frequently the questions I classed as Identification. In addition, if questions had clear instructions that indicated which methods were to be used in solving them, they were also considered easier than those where students had to decide for themselves which mathematics to do. Consistently, both students and teachers described questions that were different as difficult. Whilst the majority of Identification questions were of a standard nature, it was the Discrimination and particularly the Generalisation questions that were most frequently described as different to those students had met previously. If questions were asked in novel ways or linked different parts of the syllabus together then they were considered difficult seemingly regardless of the topics that they covered.

It was particularly interesting that the students who were making these comments were such high achieving ones. Overall, the mean mark for the case study students at AS mathematics was 81% which was an A grade, with an average of 87% for the students who chose to continue to A2. Applying their knowledge to new or different contexts appeared to cause them great difficulties despite their overall success in the AS mathematics examination system.

6.4 Summary of AS Course and Examinations

The findings of the questionnaire of Year 11 students discussed at the start of this chapter showed that students hoped AS mathematics would be interesting, enjoyable and challenging. The student data collected during Year 12 reveals that the reality lies firmly in the third category. The majority of students describe learning AS mathematics as more difficult and requiring more work than their other AS level subjects. This is particularly surprising given their high levels of achievement with 23% gaining a grade A. Even given the expected increase in difficulty (section 6.1.1) 70% of Year 12 students describe AS mathematics as more difficult than they had expected. These finding fit the description of mathematics (section 3.2.2) as more difficult than other subjects (Dearing, 1996; Smith, 2004; Campaign for Science and Engineering in the UK, CASE, 2008; Ofsted, 2008). Descriptions of lessons which move at a pace which many students consider too fast reflect those of the 'top set' students (section 3.3.1) in the studies of Boaler et al. (2000) and Wiliam & Bartholomew (2004).

In addition to the pace at which students were required to learn, difficulties were described as the amount of material they had to memorise and making decisions about which mathematics to apply when answering questions. Surprisingly, specific mathematical topics were rarely mentioned as a source of difficulty and in the case of differentiation of polynomial functions, the mathematics was described as difficult, yet the questions on this topic described as easy. Comparisons between the data collected on experiences of learning and experiences of AS examinations show that they are overwhelmingly similar. The predominance of examination questions used in the learning of AS mathematics fit with the descriptions of the practice of 'teaching to the test' (Sierpinska, 1994; Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008; Torrance 2007) which focus on optimising results rather than furthering understanding (section 3.3.2).

The analysis of the AS examinations sat by the students show that they consist of a majority of Identification questions, where the method is provided. This fits with the descriptions of current A-level mathematics examination questions (section 3.3.2) as being more prescriptive than in previous specifications (ACME, 2009 and Basset et al., 2009)). There was only one question of the Generalisation type on each paper and significantly these questions were most often described as difficult by both students and teachers. Students and teachers held strong expectations of the format of AS mathematics examinations based on their experiences of previous papers. Questions were expected to increase in difficulty throughout the paper, and questions with high mark allocation were expected to require more work and in some cases were also considered as difficult. The reasons for examination questions being described as difficult in order of frequency are: novelty, decision making required and mathematical topic. Novelty was the most common reason given; often students commented that they had not seen a question of a particular type before. This typifies the practice of 'assessment as learning'

Torrance (2007) where students were learning mathematics questions rather than mathematics.

In Chapter 7 the students’ experiences of learning and being examined in Core 3 and Core 4 are explored and the similarities with the findings of this chapter are revealing.

CHAPTER 7: THE A2 COURSE AND EXAMINATIONS

7.1 Introduction

In this chapter I present the findings from analysis of a range of data collected during the students' A2 mathematics course from September 2009 until July 2010. The data collection methods and timings followed those of the AS year described in Chapter 6. In the first section of this chapter, the transition to A2 mathematics is considered. From a questionnaire issued to students at the end of their AS courses and interviews with the eight case study students, data was gathered about students' intentions to continue with mathematics and their expectations of learning the subject at A2 level. Two of the eight case study students decided not to continue with A2 mathematics and for continuity I decided not to replace them with two different Year 13 students.

The data from the A2 year was overwhelmingly similar to that gathered during the AS year. I had expected changes in what students perceived as difficult as they progressed through their two year A-level mathematics course, yet this was not reflected in the data. In Chapter 6 there were five subsections which were the reasons why AS mathematics was considered to be difficult: workload, pace and intensity; memory; novelty; decision making and mathematics. Here, the themes of difficulty were so similar that it was appropriate to report them under the same headings. Whilst this is a totally new data set, it shows many similarities with the Year 12 data.

Students and teachers felt that A2 mathematics was harder than AS because the Core 3 and Core 4 examinations were more difficult with a more challenging question style. However the model for classifying the demand of examination questions did not show this. The Core 3 and 4 examinations contained different content, but not different question style (section 7.4.2). As in Chapter 6, novelty is the over-riding factor in whether a mathematics question is considered difficult. However, unlike in Year 12, some of the case study students acknowledged that they were unprepared for novel questions. As with Chapter 6, I found it useful to consider the grade that a student got in their modules when considering their responses to learning and being examined in that module. Table 7.1 shows the module scores for each module along with the overall AS and A2 grades. At A2 if a student had an A grade overall and scored an average of 90% or higher in Core 3 and Core 4, then they were awarded an A* grade. Where two scores are listed for a module, the second indicates that the mark was gained in a re-sit of that examination. The highest score automatically goes forward towards the overall A2 grade. George and Henry decided not to continue with A2 mathematics, so their scores stop with their overall AS grade.

Table 7.1 Case Study students' AS and A2 module grades

'Name'	GCSE Grade	C1	C2	D1/S1	AS Grade and UMS	C3	C4	D2/D1	A2 Grade and UMS
Alice	A*	87	86	81	A 254/300	75	78	65	B 472/600
Bella	A*	83 91	66 84	74	B 249/300	82	71	70	B 472/600
Craig	A*	99	94	83	A 276/300	91	55	80	A 502/600
Dan	A*	91	84	96	A 271/300	80	68	80	A 499/600
Emma	A*	97	99	86	A 282/300	93	92	78	A* 545/600
Frankie	A	94	64 85	97	A 276/300	78	63	71	A 488/600
George	A	79	51	79	C 209/300	---	---	---	---
Henry	A	74	39	57	D 170/300	---	---	---	---

7.1.1 Transition to A2 Mathematics

In the final week of the summer term of Year 12 a questionnaire was issued to students who had taken mathematics at AS level. This took place during morning registration time, and although all students who had taken mathematics during the year were invited, attendance was relatively poor. Only 65 of the potential 104 students (63%) completed the questionnaire and due to the disruptions to the final week it was not possible to follow up with the remaining students. The questionnaire was issued after students had returned to begin their A2 courses for three weeks before the summer holiday, yet many were waiting until the AS results were published in August before they finalised their A2 choices.

Data for the whole cohort was provided by the school and showed that of the 111 students who started AS mathematics at the beginning of Year 12, 67 continued to A2. Of those who did not continue, seven left the school during the course of the year. Retention to A2 mathematics was not considered to be an issue for the school, and 64% of those who sat their AS examination chose to continue mathematics to A2. Overall, there were few differences between the retention rates of male (36/55 =65%) and female (31/49 = 63%) students. Table 7.2 shows that rather than gender, the grade achieved at AS was the key factor in whether students continued to study the subject at A2.

Table 7.2: Retention rates from AS to A2 by AS grade

AS Grade	Number of Students	Number of Students continuing to A2	% of Students continuing
A	29	29	100
B	11	10	91
C	17	11	56
D	21	14	65
E	10	3	30
U	16	0[5]	0
Totals	104	67	64

Most Year 12 students at the school studied 4 AS levels and the majority dropped one subject to continue with 3 subjects at A2 level in Year 13. From my role as Deputy Head of Sixth Form I had observed that many students dropped the subject in which they achieved the lowest AS grade. Thus it was unsurprising that all of the students who gained an A grade and the majority of those with a B grade at AS continued with mathematics at A2 level. Whilst more students with D grades continued compared to C grades, it may have been that the D

[5] It was school policy that students who gained a U grade at AS-level could not continue with a subject at A2 level. Thus twenty-one (21/88 =24%) students who gained a passing grade at AS level chose not to continue with their study of mathematics to A2 level.

grade students had achieved a lower grade in one of their other AS subjects. Similarly the three students who continued with a grade E had failed one of their other AS courses and so had to continue with mathematics in order to remain at the school.

Despite their experiences of the Core 3 material during the '3 week plan' sessions, most of the case study students believed that this module would be 'harder' or 'a step up' from Core 2 based on comments from the students in the year above. The following quote is typical of those describing that teachers and older students both talked about the increase in difficulty of A2 mathematics.

> *"we heard from some Year 13's that it is definitely a big step up 'if you think this is hard, wait until you do C3' so I'm guessing it's going to be quite hard. The teachers have said that if you have found AS hard then maybe consider not doing it next year."* Emma July 2009

The messages of increased difficulty that students received from their teachers were confirmed by the teachers' descriptions of Core 3. All four teachers were definite that Core 3 was significantly harder than Core 1 and Core 2 and the following comment is representative.

> *"It's harder, more content. It's all hierarchical, so the content relies on what you know from C1and C2 if you have just crammed for your exam, then everything is shaky."* Teacher 2 July 2009

How students responded to this commonly accepted and widely publicised increased level of difficulty was a significant factor, along with their AS grades in whether students chose to continue to study mathematics at A2 level. This was explained by the following teacher as either a motivating or deterring factor.

> *"they do say year after year that maths is the most challenging of their A-levels. It can go two ways, it can either really motivate them, or they can be completely de-motivated."* Teacher 3 February 2009

Whilst the majority of Year 12 mathematics students intended to continue with the subject in Year 13, they did so with the expectation that A2 mathematics would be more difficult than AS mathematics.

7.2 Experience of Learning A2 Mathematics

7.2.1 Introduction

The descriptions of learning A2 mathematics given by students were not significantly different to those that they had made about learning AS mathematics. Many comments about sources of difficultly were similar to those quoted in Chapter 6. I had thought that the students' descriptions may have developed over time, but this did not happen. Their responses did not become more detailed and surprisingly, other than a growing emphasis on factors that related to examinations, they did not alter significantly over the course of the two years. The comments from one student demonstrated how similar her descriptions of why she found a question difficult remained over a period of time from the second half term of Year 12 to the final term of Year 13.

> *"I spent absolutely ages trying to figure it out and I just couldn't do it at all."* Frankie February 2009

> *"It took me ages to figure out what I was actually supposed to do. That's what I always find hard, finding which method to use."* Frankie May 2010

I had considered the effect that the six interviews and three written reflections may have had on the students' responses, yet these did not influence the students as their descriptions of difficulty remained similar throughout the course of this study. One similarity in the descriptions of why students found particular aspects of learning mathematics difficult was the absence of detail. This lack of detail of why students found questions difficult was particularly surprising given their high academic achievement and that it was their final year of schooling. Participation in the data collection process did not develop the descriptions of difficulty or provide examples of understanding.

There had been an expectation that students believed that Core 3 and Core 4 would be more difficult than Core 1 and Core 2 (section 7.1.1). Yet even given this expectation of increased difficulty, significant numbers of students still felt that A2 mathematics was more difficult than they had anticipated. In the questionnaire responses from the cohort of A2 mathematics students at the end of Year 13 the majority (28/40 = 70%) felt that it was 'harder' or 'a lot harder' than they had expected. Only two students felt that the A2 pure mathematics modules were easier than they had expected.

These expectations of an increase in difficulty from AS to A2 mathematics were shared by teachers who described that the A2 core modules 'built on' the work covered in the AS modules. The teachers' perceived increase in difficulty had been shared with students in at least one case.

> *"*[Teacher 2] *said to me once at a parent's eve C4 would be where I start finding it hard. So that may control my thoughts on it (laughs) I've been told it's going to be hard so it is."* Craig February 2010

Data from the end of Year 13 questionnaire showed that 12 students (12/40=30%) felt that A2 mathematics was more difficult than all their other subjects, and fifteen (15/40=38%) felt that mathematics was more difficult than one of their other A2 subjects. These figures represented a significant change from the AS data when 68% of students described mathematics as their most difficult AS subject. This decrease supported the notion that mathematics is a subject for the 'clever core' as described by Matthews & Pepper (2007) and that the weaker students did not continue after AS. Thus the students who continued on to A2 Mathematics were those who found it less difficult than those who did not continue. However as with the AS questionnaire data, only one student (who went on to gain an A in A-level mathematics) described finding A2 mathematics less difficult than both his other subjects, Chemistry and Physics.

The difficulty of A2 mathematics still remained an issue for many students. When asked to describe what learning Core 3 and Core 4 had been like, many students (22/40=55%) included the word 'hard' or 'difficult' in their descriptions. An additional seven students (7/40=18%) included that it was harder or a lot harder than mathematics in Year 12. The case study students were interviewed during the second term of Year 13 when they had just completed their January module examinations and begun the Core 4 module. Whilst this final pure mathematics module was widely talked about by both teachers and students of the school as being the most difficult, this was not found to be true initially by most of the case study students.

> *"I thought C4 was going to be the biggest jump, but I think AS to A2 was probably the biggest, that's why C3 was so difficult. But C4 seems ok at the moment."* Alice February 2010

Instead of the expected increase in difficulty, four students described finding the work they had done on Core 4 no more difficult than that of Core 3. Despite not finding Core 4 difficult after one month of studying the module, one student was clear that he expected the work to get significantly harder. He also described how he expected the modules to increase in difficulty from Core 1 through to Core 4.

> *"I'd say it's alright actually, but we are only doing the foundations of it. I'm expecting it to get a lot harder as we go along. Its C4, it's got to be harder."* Craig February 2010

However, two students found Core 4 difficult from the start of the course; one described 'feeling a bit lost' and had continued working with a private mathematics tutor to help. The other student described feeling 'stuck all the time'.

> *"I find it really hard, a lot harder. The trigonometry integration and differentiation. It's hard enough to understand it in the first place let alone do more of it. At Core 2 I could follow it in class and then maybe get stuck on the homework. But now I feel I'm stuck all the time."* Emma February 2010

Whilst the questionnaire responses gave an indication of the perceived difficulty of A2 mathematics, both in comparison to other A2 subjects and to AS mathematics, the reasons for these perceptions were provided from data from the six case study students and four teachers. These are now explored under the headings used in Chapter 6.

7.2.2 Workload, Pace and Intensity

The structure of the school's timetable meant that all A2 subjects had an extra hour a week than at AS. This fifth lesson in Year 13 was used to teach Core 3 and Core 4 material. The 25% increase in teaching time, meant that there was less pressure on teachers to finish the pure mathematics syllabus. Whilst the time increased for the teaching of pure mathematics modules at A2 level, there were no descriptions that indicated how this affected the students' experiences of learning, other than to spend more time on a particular topic. Unlike in section 6.3.2 when the pace of work in lessons and amount of homework was repeatedly seen as a source of difficulty by students, only one student made a reference to the fast pace of lessons during Year 13.

> *"I think it's because my class is really clever, they automatically understand and I have to work a lot harder to understand it. In class I just have to keep up*." Bella February 2010

This lone reference to the pace of lessons was a significant change from Year 12 when all of the case study students and teachers described the speed that was required to complete the AS pure modules, particularly Core 2. In the only other reference to pace throughout all of the A2 data collection, one teacher explained how having more time for Core 4 made it easier. The reduction in workload was also evident in the end of Year 13 questionnaire responses where it was only mentioned by six (6/40=15%) students in their descriptions of learning A2 mathematics. Additionally, the descriptions of the workload in studying the A2 pure mathematics modules showed that this had lessened from the end of Year 12 questionnaire. Unlike at AS when a striking 70% of students felt that mathematics gave the most amount of classwork, only 30% felt that this was the case at A2. The homework picture was similar; there was a reduction from 80% at AS to 40% of students at A2 who described that mathematics had more homework than all of their other subjects.

7.2.3 The Role of Memory

As the A2 mathematics modules built on the AS modules (see Appendix 1, A1-3-15), there was more memorisation required. In the examination board specification (OCR, 2010a) of each module there was a list of formulae that were not provided in the examinations. Thus there was an increasing list of formulae that students were required to remember. These formed the minimum amount of memorisation as many students chose to learn additional formulae so they did not have to rely on the formula booklet and could quickly recall common formulae rather than construct them from the ones provided. Thus as the course continued through Core 3 and Core 4, the need to remember specific rules and techniques was more commonly described as a source of difficulty than it was at AS level. The following quote is typical of those which described the increase in the amount of memorisation required.

> *"Because there's a lot more of it, a lot more to remember. It's not harder than C2, there's just more of it."* Emma November 2009

Trigonometry was most commonly named by both students and teachers as the topic that required most memorisation. Whilst this was often described as a source of difficulty, there were several instances when being able to apply a memorised formula was the reason for a question being considered easy. From the description below, this teacher believed that remembering the definitions of a minor trigonometric function cotθ, and a multiple angle formula, cos2θ would not be a source of difficulty for their students.

> *"As long as they know that cotθ is one over tanθ then that shouldn't be hard to do. There isn't much calculation to do. It's the same with cos2θ, they just need to know the identity and it's not hard to work it out with a calculator."* Teacher 1 November 2009

Similarly, some students described that they found questions easy when they could just apply a technique that they had memorised. One student described a trigonometry question as easy because he 'knew' the mathematical technique required.

> *"It's not so much working out maths, its knowing maths. It's something that you could revise and just get anyway."* Craig November 2009

This student went further and described all of Core 3 trigonometry as an easy topic, and made the distinction between understanding mathematics and doing mathematics.

> *"It's something that you can just learn, that you can just see. You are given lots in the formula book, so you can just look it up. I don't think it's about understanding maths, it's just doing the work like a computer"* Craig November 2009

However, this was the same student who described finding all of Core 1 and 2 easy and scored 99, 94 and 91 in the Core 1, 2 and 3 examinations respectively. More commonly, the amount of formulae and methods that were required to be memorised was described as a source of difficulty. The other five case study students described the amount of formulae that they had to remember as the source of difficulty in trigonometry. The following comment was typical.

> *"There is lots to remember, I always forget that you are not given the double angle ones. It's too difficult to remember all of them."* Emma November 2009

One student described the difficulty with long term memory and the difference between remembering how to do questions within a lesson and then at some time later during the revision process.

> *"I forget it, so once I've learnt it, revised it, it should be alright. Going along you do loads of questions on the same thing and you get used to doing it, then two months later when you are doing exam questions, you've forgotten how to do it." Frankie November 2009*

7.2.4 Novelty

As it had been at AS mathematics, novelty was still the over-riding factor at A2 level in whether students considered a question to be difficult. If a question was in a form that they had seen before, regardless of the topic that it concerned, it was commonly classed as easy by the six case study students. There were strong parallels with the students' descriptions of the questions that they worked on in A2 mathematics lessons to those given in Chapter 6. The style of the lessons did not change over the duration of the A-level course and at A2 students spent the majority of their lessons as they had at AS, working on questions from the examination board textbook or past papers. Consequently, students rarely came across novel or different questions or tasks in their learning of A2 mathematics. It was striking that given the achievement (1A*, 3A and 2B's at A-level mathematics) of the case study students (table 7.1), that novel questions caused so much difficulty.

When asked what made a Core 4 question difficult the following response indicated the unfamiliar as the source of difficulty.

> *"I think when it's something that you are not used to seeing, not a standard question, or if it's really wordy and you can't work out what its saying."* Frankie May 2010

One student felt that he could learn standard methods and practice questions because he felt there were a limited number of questions that could be asked.

> *"You can learn all maths, you just have to apply it to the problem, because there can only be so many questions, just in different forms."* Dan May 2010

However this was an isolated viewpoint and the range of questions with unfamiliar wording was given as a source of difficulty for the other five case study students. This was also illustrated in the comments from the questionnaire at the end of Year 13. The following response is typical and described the need to apply mathematics to a wide variety of unfamiliar contexts as the reason why it was a difficult subject to learn.

> *"A lot more dependent on your own thinking in Y13. Teachers teach you everything you need to know but what to apply and how is never the same."*
> Year 13 student

Whilst the students and teachers were given an open choice about what type of questions they brought to discuss at the trigonometry and vectors interviews, all brought standard examination style questions and many, particularly teachers brought only examination questions. In the first year of data collection, I asked students to describe the difficulty level of four co-ordinate geometry questions. I had written the questions to fit with the model for classifying the demand of examination questions and the four questions were not typical of Core 1. For these interviews I had only asked for students and teachers to bring 'an easy and difficult question on trigonometry' and 'two difficult vectors questions' and did not specify examination questions in either case. Although previous interview questions and written data collection sheets may have influenced their choice of questions I still found it interesting that only examination questions were brought. A further surprise was that given a free choice of questions on trigonometry to bring to discuss in the interviews, two teachers brought the same question, OCR Core 3 June 2007 question 9 as their example of a difficult question. This question is shown below.

9 **(i)** Prove the identity

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) \equiv \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}. \qquad [4]$$

(ii) Solve, for $0^\circ < \theta < 180^\circ$, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = 4\sec^2\theta - 3,$$

giving your answers correct to the nearest 0.1°. [5]

(iii) Show that, for all values of the constant k, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = k^2$$

has two roots in the interval $0^\circ < \theta < 180^\circ$. [3]

The final part, which is an example of a Generalisation question, requires students to recognise this as a particular case of a quadratic equation having two real roots, was seen as the most difficult part of the question. Again the language of examinations dominated the discussion of what made a trigonometry question difficult.

> *"they don't usually have to do things like that, they think they are going wrong because it's so different. Part three is hard because it's not a normal trig question. They won't have seen it in past papers and they need a lot of understanding to solve it."* Teacher 1 November 2009

A second description of the same part of this question also emphasised the different nature of the mathematics required. It was interesting that the teacher felt that the question was difficult because it required 'actually thinking' in an examination situation.

> *"the last part it's not probably something that they have met before and they hate seeing the k in there. Actually thinking why k squared is greater than zero, I think they would have to keep quite a level head to get themselves through that."* Teacher 3 November 2009

Remarkably, this question was given as the reason why another question, was described as easy. Both teachers and students expected the examination questions to increase in difficulty as they went through the paper (section 6.4.3). So it was surprising that the last question on any examination would be considered as an example of an easy question. Yet one teacher brought the following question, 9 on the January 2008 Core 3 paper as their example of an easy trigonometry question.

9 **(i)** Use the identity for $\cos(A+B)$ to prove that

$$4\cos(\theta+60^\circ)\cos(\theta+30^\circ) \equiv \sqrt{3} - 2\sin 2\theta. \qquad [4]$$

(ii) Hence find the exact value of $4\cos 82.5^\circ \cos 52.5^\circ$. [2]

(iii) Solve, for $0^\circ < \theta < 90^\circ$, the equation $4\cos(\theta+60^\circ)\cos(\theta+30^\circ) = 1$. [3]

(iv) Given that there are no values of θ which satisfy the equation

$$4\cos(\theta+60^\circ)\cos(\theta+30^\circ) = k,$$

determine the set of values of the constant k. [3]

There are noticeable similarities between this and the previous question which had been described as difficult by two of the teachers. Particularly the final part of the question which was again of the Generalisation type. In this case, students had to recognise the particular case of conditions where an equation involving the trigonometric function $sin2\theta$ has no solutions. Yet it was this similarity to a previous examination question that was given for this question to be described as easy. It was also noticeable that this teachers' description of the question is dominated by the language of examinations.

> *"the reason I think it's easy is if you look at June 2007 and look at the question there it's so similar. So if you have done your revision, done your exam practice you would be able to do this. Which I think for a question nine is an easy question there is no novelty to it, it's the same as been asked before"* Teacher 2 November 2009

These different descriptions of two similar questions illustrated that it was the novelty rather than the mathematics that was seen as a source of difficulty. What had been considered a difficult question on trigonometry on first appearance on an examination was considered easy in a subsequent examination. This illustrated that the teachers did not expect students to experience difficulties with questions that had occurred before.

Conversely, lack of novelty was most frequently given for the reason why a question was considered easy, both by students and teachers.

> *"The harmonic form question, it's a very standard question. It's exactly the sort of thing that you would have done in class and they will know how to find R and α if they have been paying attention."* Teacher 4 November 2009

When describing what they considered as examples of easy questions on the Core 4 topic of vectors, one teacher explained that they looked for standard questions. The following quote from this teacher highlights that even though students may not have understood the mathematics of vectors, they could answer standard questions on it by memorising a method.

> *"I looked for things that were standard, things like find the equation, use the scalar product. They might not actually understand what is going on there but it is a method that they learn and anything like that they can apply their formula."* Teacher 3 May 2010

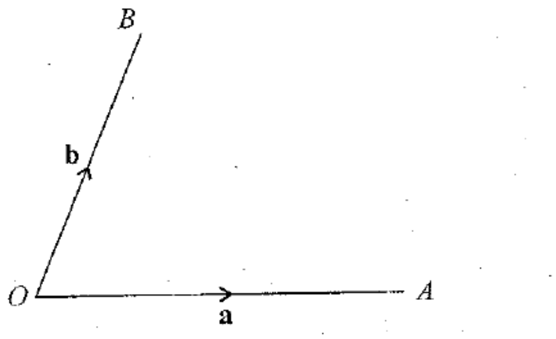
As described in Chapter 6, both students and teachers described questions that were different as difficult. Questions that were worded differently or required an unusual combination of skills were considered difficult regardless of the mathematical topic they involved. The most frequent reason that these novel questions were considered difficult was that they could not be answered by rote-learned standard methods.

> *"it's because it's just a different application, something that you haven't seen much of. It's like two topics mixed together and because they are taught as separate in class, it's too hard to merge them together in the same question."* Bella November 2009

When asked why she found questions that required the proof of a trigonometry identity difficult, one student described the variety of questions on this topic.

> *"You can't follow a set rule, you can't just learn 'do this, then this, then this' it's different for every single one."* Emma November 2009

Even when an A2 examination question was described by a teacher as having the mathematical content appropriate to a GCSE paper, surprisingly, the novelty of the wording still resulted in it being described as difficult. Below is the question and the teachers' description of difficulty follows.



"As shown in the diagram the points A and B have position vectors a and b with respect to the origin O.

(i) *Make a sketch of the diagram, and mark the points C, D and E such that $\overrightarrow{OC} = 2a$, $\overrightarrow{OD} = 2a + b$ and $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$.* *[3]*

(ii) *By expressing suitable vectors in terms of a and b, prove that E lies on the line joining A and B* *[4]"*

> *"Really it could have been in a GCSE paper but I think that was half the problem. It was so unexpected. There isn't a method, there isn't a concrete thing they can feel comfortable with and rely on."* Teacher 3 May 2010

This description that GCSE level mathematics was considered difficult when part of an A2 examination question, illustrated how powerful the novelty factor of a question was. The teacher felt students would find this question difficult because it was unexpected, even though they identified the content of the question to be suitable for GCSE, an examination that the students would have taken two years previously. This demonstrated that novelty was the dominant factor in whether a mathematics questions was considered to be difficult.

7.2.5 Decision Making

Just as the number of formulae and methods that students had to memorise increased from the AS to the A2 Core modules, so did the amount of decision making. As students learned more techniques, there were more possibilities for them to choose from when attempting unfamiliar questions that did not provide a specific method. Thus the need for decision making was more commonly featured as a source of difficulty at A2 than at AS.

Once a question was considered to be novel, the lack of full instructions was often given as the reason that students considered it to be difficult. The requirement to 'think for yourself' was repeatedly described as a source of difficultly and the following quote is typical.

> *"The harder ones are ones where you have to think through what to do for yourself, it doesn't tell you what to use or how to go about it."* Craig May 2010

Teachers also described that working out what a question was asking for was a difficult skill for students to master.

> *"It's just the skill of being able to spot it. I don't think the actual maths is too hard, it's just a difficult skill to acquire as opposed to something they have to learn."* Teacher 1 November 2009

There was strong agreement between students and teachers that the need to decide which mathematical technique was required was a major source of difficulty.

> *"I think they were hard because I couldn't work out what to do. It wasn't hard when you knew what to do, but knowing what to do makes them hard."* Bella May 2010

The lack of instructions fitted with the model classifications of Discrimination and Generalisation questions. These types of questions were those which required students to apply mathematics that was not specifically mentioned. The interview process prompted one teacher's realisation that interpretation of novel questions was a source of difficulty.

> *"But what they really struggle with and I've just realised, it's the way that these questions are worded. Understanding what the question is asking about is difficult. So if you can decipher it and picture it then the maths isn't too bad, but it's actually taking apart the question."* Teacher 2 May 2010

As described in Chapter 6, with the lack of instructions that occurred in novel questions, the number of stages in the solutions was also described as a source of difficulty. This was particularly true of the students' responses and the following quote is typical of the difficulties associated with answering a Discrimination question on trigonometry.

> *"Because of the different stages, they can get confusing and when you expand the brackets. I never know how to make it into a tan, then the next bit, I found that really difficult, it took me a while to work it out and you had to use the CAST diagram"* Alice November 2009

Just the increased amount of work required was considered to be difficult by one student.

> *"So it was harder because you need to do more steps, you had to do a proof. That was quite tough."* Dan November 2009

Most frequently, students described that the higher the number of steps in a solutions, the more chances there were for a mistake to be made.

> *"I think that's hard because you have to change it into those steps into tan, then factorise it and then solve it as well. There is a lot more things to do. With trig, if you go wrong a bit at the beginning, then it all just goes completely wrong."* Frankie November 2009

However, there were only two cases when a teacher gave the number of stages as a reason for difficulty. One was by Teacher 4 when describing a Core 4 vectors question and the following quote was from a different teacher describing why a Core 3 trigonometry identity was difficult.

> *"Because it requires a lot of steps to get there. That's a lot of algebra to go from the first part to the second part."* Teacher 2 November 2009

7.2.6 Mathematics

As to be expected, as the Core modules progressed there was a greater need to use mathematics from previous modules and this was clearly stated in the Core 3 and 4 syllabuses.

> *"Knowledge of the specification content of modules C1, C2 and C3 is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit C4"* (OCR, 2010a, p.42)

However, the need to use mathematics from a previous module was often given by students and teachers as a source of difficulty. With more modules to draw upon, it was to be expected this reason for difficulty was more common at A2 level that it had been at AS level. The following quote is representative of those given by teachers when describing how Core 3 and 4 depend on the knowledge of Core 1 and 2.

> *"In terms of the material they have to cover again with all the calculus you are doing if you don't really feel comfortable with what you have learned in C1 and C2 then you are going to struggle. It builds."* Teacher 4 November 2009

This issued was raised by one student who described differentiating trigonometry as difficult because she had found the previous Core 2 work on these topics difficult.

> *“The trigonometry integration and differentiation and the different ways you can do it, it’s hard enough to understand it I the first place let alone do more of it. ”* Emma February 2010

On occasions, specific parts of a previous module were required in a solution to questions from the current module being studied. This was described as difficult by one teacher when they explained how a Core 3 trigonometry question required the use of the discriminant of a quadratic equation which was part of the Core 1 syllabus.

> *“It’s difficult recalling discriminant, it’s awhile since they have done it, they don’t always relate a previous module into a current module.”* Teacher 3 November 2009

When describing a trigonometry question, one student was very clear that techniques from a previous module were the reason that she could not answer it.

> *“find the area, I didn’t know how to do that, it was from C2*.” Bella May 2010

It was interesting that this student was able to identify the mathematics as part of the Core 2 module, yet she was unable to remember the actual technique. This fitted with the description by Smith (2004) of the ‘splintering’ of mathematics by the modular A-level system (Chapter 3). In addition to individual formulae from previous modules, a significant proportion of the A2 pure mathematics modules built on the content of the AS modules. Three of the four teachers highlighted that the difficulty in Core 3 trigonometry was a result of students’ lack of understanding of previous work on this topic.

However, in some cases building on a topic was seen as an advantage, as the gap between meeting topics was seen as a source of difficulty. In the case of the Core 4 topic of vectors, students had not met this material since their GCSE course, two years earlier. One student identified the gap between learning vectors at GCSE and in the final A2 module as an issue.

> *"I think it's because it's something that we haven't done before really. The other stuff like trigonometry is building on earlier knowledge, but vectors we've not done since GCSE so I have forgotten it all."* Frankie May 2010

Similarly, one teacher described that the need to revise the GCSE material before continuing with the A2 material. The issue of which applied module students had studied was raised for the first and only time in the discussions of vectors. As vectors were used in the Mechanics 1 AS module, it was considered by teachers to be an advantage for students to have taken this option rather than the Statistics or Decision modules. None of the students in the case study had taken Mechanics as their applied option and this was not raised by any student in the interviews. However the following description given by a teacher again illustrated that the two year gap between students meeting the topic at GCSE to re-starting it at Core 4 was a source of difficulty.

> *"It's like a lack of familiarity, the statisticians will not have met them since GCSE's. It's been two long years with no contact with vectors at all."* Teacher 3 May 2010

As in Chapter 6, specific mathematical topics were rarely given as a reason for a particular question to be difficult. When asked how to describe the difficulty level of Core 3 only three topics were raised by students. Perhaps to be expected, given that students had been asked to bring trigonometry questions to discuss, three students named this as the most difficult part of the module. One student described the amount of formulae that she needed to remember for this topic, and one referenced the amount of work involved in proving trigonometry identities. Only one gave specific mathematical concepts, the minor trigonometry functions as a source of difficulty.

> *"But the cosec and cot questions are harder than the cos and sin ones because I don't understand them."* Frankie November 2009

Three topics received one individual mention. These were the modulus function, with the reason that it had been covered in 'the three week plan' at the end of Year 12, natural logarithms because of the different ways that they could have been rearranged and finally, integration. The quote below shows that the student had difficulty with remembering the rules of integration as he does not refer to understanding the processes involved.

> *"The chain rule in terms of integration. I struggled with knowing whether it's just $\frac{1}{n}$ or $\frac{1}{n+1}$ or when you start bringing logs or e into it"* Craig November 2009

This student talked about the need to remember rules without knowing where they came from in further interviews. The difficulty was described as having to remember rules without knowing the reasons why they worked, and the danger of forgetting a rule was also highlighted.

> *"It's just a rule because you are told it's a rule, but you are not told why. I'd say that was my only problem with C4, it means I have to just learn the rules. Sometimes in the exam I have forgotten a rule I'd be able to work it through. In terms of this C4 you can't work it through or draw a diagram for yourself, you just have to learn it off by heart."* Craig May 2010

In the end of Year 13 questionnaire, there were a wide variety of responses to what had been the most difficult part of learning Core 4. The most common response was an individual mathematical topic and there were four topics mentioned. From most to least frequent these were: integration (13/40=33%); vectors (6/40=15%); integrating trigonometric functions (4/40=10%) and differential equations (3/40=8%). However the reasons given were typically lacking in mathematical detail and included "I've always been bad at trig", "It's very complicated" (differential equations) and "It's the hardest topic" (vectors). The most common reason that each of the four topics was described as difficult was deciding which method to use to solve a problem.

Similarly, in the last interview during the final weeks of their A2 course, the case study students were asked what they had found the most difficult part of Core 4. Whilst the students had discussed specific vectors questions during these interviews, only two students listed this as one of the most difficult parts of the module and both students also mentioned differentiating and integrating trigonometric functions.

> *"Some of the topics aren't too bad, but differentiating and integrating trigonometry I just find really hard. I find vectors difficult, I don't get them. I don't know why, they just don't make any sense."* Frankie February 2010

The topic of differentiating and integrating trigonometry was also given by three other students, with reasons that there were many rules that needed to be memorised and that the method was not commonly given. The following quote was typical.

> *"Integrating trig because you have to know things and work things out. Like cos4x, you have to know how to manipulate it, what formula to use to work it out. You have to see it rather than just doing it." Dan May 2010*

The only student who did not mention this topic struggled to describe the mathematics that she had found most difficult. Whilst she was unable to remember the name of the topic, her description was of solving differential equations with separable variables.

> *"I've forgot what it's called. The ones when you have got x and y variables not just x. You have to integrate as well. I always get confused with that, I can't see what you have to re-arrange, it's really hard."* Alice May 2010

Even when discussing the difficulties within a specific mathematical topic, the focus still remained on deciding what to do or recalling a method or formula. Thus the language of difficulty did not provide the examples of 'not understanding' that I had anticipated.

7.3 Experience of A2 Examinations

7.3.1 Introduction

The data collection methods used were the same as those in Chapter 6. I analysed the Core 3 and Core 4 papers with the use of the model described in Chapter 5. This analysis showed that the June 2010 Core 4 paper was typical of the examinations set between 2007 and 2010 in terms of the number of marks allocated to each category: Identification, Discrimination and Generalisation with no Synthesis questions. The Core 3 paper had more Generalisation and fewer Identification questions than any other examination throughout this time period (see section 7.3.2). In parallel with the analysis of the examination questions, I collected data from the case study students and teachers about their perceptions of these papers.

After the Core 3 examination, each of the six case study students and the four teachers completed a written sheet on whether they had found each question part 'easy' or 'difficult' with a reason for their choice. Each was then interviewed individually about their thoughts on the examination, how it compared to their expectations and what they had considered to be the most difficult question. There were practical difficulties with the data collection for the Core 4 examination. Weeks prior to the final A2 examination period, the Year 13 students left school. Although agreeing to continue with providing data prior to leaving, once their examinations had finished it became difficult to contact the students and to gain further co-operation. As a result, it was not feasible to do a final round of interviews after the Core 4 examination and the written data sheets were issued and collected by post. However, not all students replied and written data only was collected from four students and three teachers.

Whilst the initial findings appeared similar to those of Chapter 6 there are some key differences and several of the reasons for examination questions being difficult are expanded on. Whilst raised in respect to the AS examinations, time allocation was not discussed by any students or teachers in regard to the Core 3 or Core 4 examinations.

Students held very strong expectations that A2 examination questions would be similar to those on past papers. Their learning and revision were tailored to this expectation and students found questions difficult that were different to those they had seen before. The extent that teachers held this belief was less clear, but they also described novel questions as difficult. The students did not have ideas for how they could prepare for unexpected questions.

7.3.2 Classification of A2 Questions and Perceptions of Difficulty

The questions on the Core 3 and Core 4 papers were categorised with the use of the model, each of the 22 question parts on Core 3 and 15 parts on Core 4 were categorised as Identification, Discrimination, Generalisation or Synthesis. Table 7.3 shows the number of marks awarded to each category of question.

Table 7.3 Mark allocation for the A2 papers

Classification	Identification	Discrimination	Generalisation	Synthesis
C3: Number of Marks	27/72 (38%)	26/72 (36%)	19/72 (26%)	0/72 (0%)
C4: Number of Marks	41/72 (57%)	22/72 (31%)	9/72 (12%)	0/72 (0%)

The amount of marks awarded for each category of question for the Core 3 paper was not typical of those throughout the period 2007-2010 (see Chapter 5). The atypical nature of the paper was reflected in the comments of students and teachers who described it as unlike the past papers and more difficult than expected. These comments are explored in the following section.

I had initially anticipated that there would be fewer Identification questions with more Generalisation and Synthesis questions on Core 4, given that it was the final pure mathematics examination. This was not the case with any of the Core 4 papers analysed throughout the research period (Chapter 5). As with all previous pure mathematics papers, there were no questions on either paper that fitted the Synthesis classification. A comparison of the four pure mathematics papers that the case study students sat is discussed in the concluding chapter.

Table 7.4 shows the grade boundaries for these papers. The 2010 A2 papers were the first set of examinations where an A* grade could be obtained. This was awarded to students who averaged at least 90% on Core 3 and Core 4 whilst achieving a grade A overall (at least 480/600 marks). Hence there was no grade boundary set for the new, highest grade on these individual papers.

Table 7.4 Grade boundaries for A2 papers

Grade	A	B	C	D	E
C3: Marks Required	51/72 (71%)	43/72 (60%)	36/72 (50%)	29/72 (40%)	22/72 (31%)
C4: Marks Required	54/72 (75%)	48/72 (67%)	42/72 (58%)	36/72 (50%)	31/72 (43%)

As previously, the grade boundaries were compared with the marks allocated to each classification of question. The results are displayed in figure 7.1.

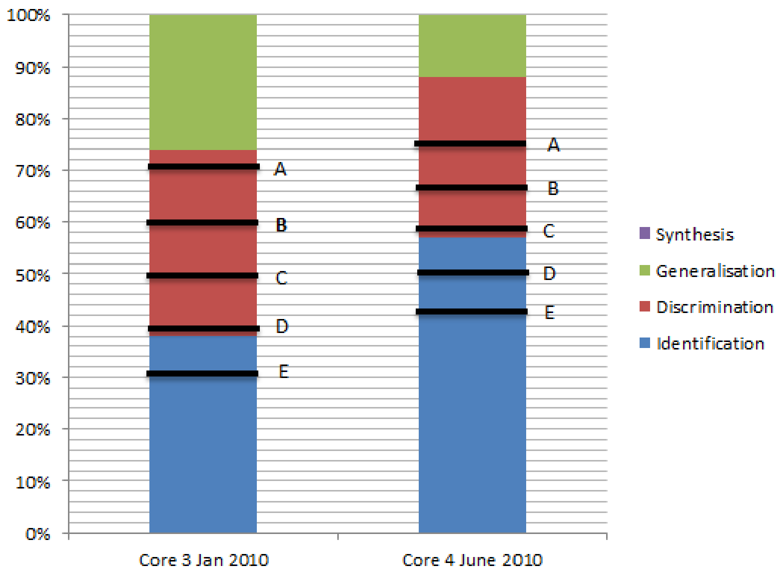


Figure 7.1 Classification and Grade boundaries for A2 Core papers

The atypical structure of the January 2010 Core 3 paper was reflected in the difference to the standard grade boundaries. It was noticeable that the grade boundaries were lower for this examination, particularly the E grade boundary which at 31% was 10% lower than on average throughout the research period. As reported in Chapter 5, to obtain a grade A on either of the Core 3 or Core 4 papers, students would not need to answer any of the Generalisation questions correctly.

Table 7.5 shows that the correlation between the classifications as defined by the model and the perception of difficulty remained as strong for these Core 3 and Core 4 papers as for the Core 1 and Core 2 examinations. Those questions classed as Identification, which explicitly told the students which part of mathematics to apply, or were of a very standard nature were least often described as difficult both by students and teachers.

Table 7.5 The percentage of students and teachers who classified A2 questions as 'difficult'

Classification	Identification	Discrimination	Generalisation	Synthesis
C3: Student 'Difficult' Rating	(13%)	(43%)	(83%)	------
C3: Teacher 'Difficult' Rating	(10%)	(71%)	(100%)	------
C4: Student 'Difficult' Rating	(31%)	(80%)	(75%)	------
C4: Teacher 'Difficult' Rating	(15%)	(53%)	(100%)	------

There were many similarities between the difficulty ratings of the A2 papers compared with those of the AS papers discussed in Chapter 6, notably that all of the teachers described all of the Generalisation questions as difficult. However, the percentages of questions rated as difficult by students were considerably higher for all categories of questions on the Core 4 paper. Slight caution is required when interpreting students' results regarding the Core 4 questions, as this data was collected just after students had taken this final examination and not triangulated with interview responses. Core 4 was widely considered to be the pinnacle of the A-level mathematics course and this may have accounted for the higher percentage of difficulty. It was also the only data collected about the difficulty of an examination when students were not studying a further mathematics module. However, reasons for this change are explored throughout the remainder of this chapter.

7.3.3 Expectations

There was agreement between student and teachers that the Core 3 and particularly Core 4 examinations were expected to be more difficult than the AS pure mathematics examinations. One teacher was clear that the increased difficulty was a result of a different question style.

> *"They put in these nasty bits of questions that they won't of seen before. That really brings out the difference between those with understanding and those who are applying a rote method."* Teacher 3 November 2009

Yet even with this expected increase in difficulty from AS level, the particular examinations that the students sat were considered to be more difficult than most students and teachers had anticipated. Whilst all the teachers responded to the request to write general comments about the overall difficulty of the Core 3 paper on the data collection sheet, only three of the six students did. However all of the comments indicated that this was considered to be a difficult Core 3 paper. Tellingly, each of the reasons given referred to the different style of questions which reflected the increase of the Generalisation type. The following is representative of the reasons given by students for the increased difficulty.

> *"I thought the paper was harder than lots of practice/past C3 papers. The questions seemed to be asked in a different way, which I probably wasn't expecting."* Emma February 2010

Comments given by teachers were more detailed and all made reference to the latter questions on the examination being proportionally more difficult than on previous papers. Whilst each of the student comments were made in reference to their own experience of the paper, all of the teachers made reference to 'weaker' and 'all but the brightest' students. Although the experience of 'weaker' students had been mentioned in teachers' responses to the difficulty of the AS examinations, the consideration of the majority of students was raised for the first time in the teachers responses to Core 3. The following response is typical.

> *"Whilst I understand the need to stretch the top end of students, questions eight and nine offered very little in terms of how the vast majority of students could demonstrate their understanding of C3 maths."* Teacher 2 February 2010

Students' expectations of similar questions on examination papers came from their experiences of the Core 1 and Core 2 papers and their revision. All students described that working on past papers was a major element of their revision. One student acknowledged that it would be better to understand the topics rather than just practice previous examination questions.

> *"I guess in Core 1 and Core 2 the questions were pretty much standard so you feel that for your revision, all you need to do is past papers. Maybe it would be better to go through each topic and make sure you understand because they can ask you anything."* Emma February 2010

However, students described being successful with the practising previous question approach to revision.

> *"because you do all the past papers and you get pretty used to the type of questions. Like 7b. I've done that question a hundred times, just with different numbers so it was easy to say that's that and do it."* Dan February 2010

The consequence of expecting examination questions to mirror those that have occurred before was highlighted by the following written comment which showed this student was unprepared to answer novel questions.

> *"Had no idea where to start with all three! Never seen these types of questions in previous papers so did not prepare for them."* Alice February 2010

Yet despite recognising that there were limitations to the approach of revision by repeated practice of similar questions, none of the six students were able to offer any alternative strategies to prepare for different style of questions.

> *"I don't know, because there are so many topics you can't really prepare for everything because it would just take forever (laughs). I revise by doing all the past papers over and over again until I'm used to the style of questions. And maybe I'm used to the style of questions and not really how to work them out."* Bella February 2010

The response of the students was put into context by one of the teacher's descriptions of the revision advice given to students which was to do 'lots of past paper questions'.

When asked how he felt when faced with a question that was different in an examination, one student described not knowing what to do. It was interesting that he took comfort in the fact that he thought other students found the unfamiliar question difficult.

> *"Well the only positive thing is that everyone found it really difficult, at least you are not the only one. Its relatively within the syllabus, it's just that we never done it before so I don't know how to do it.*" Dan February 2010

The statement that he did not know how to answer the question because he had not 'done it before' showed that he expected to have been taught each of the questions on the examination. The fact that he felt it was on the syllabus suggested that he recognised that the mathematics was part of the course, but the question had not been. It was unclear from the students' responses whether they thought that teachers expected the examinations to be similar. The following quotes represent the two differing views of students on this issue.

> *"Yes I'd say to a certain extent because that's the form, so they teach to what they know is going to come up on the exam."* Dan February 2010

> *"No I don't think so, they teach it from going through the textbook, and they say what the likely questions are so that you need to know how to do it."* Emma February 2010

The reasons for these mixed responses became clear when the teachers' own responses were considered. Three of the four teachers felt that there was a lot of similarity in previous A2 examinations. One teacher said that he “absolutely” expected papers to be similar and expressed how he felt the different questions on the Core 3 paper were unfair to students. The expected similarity of examination questions was also raised when teachers discussed why they described vectors questions as difficult. Their responses indicated how they tailored their teaching to what had occurred in previous examinations.

> *“I recall when these questions first came up they caught us out, the staff who had been teaching it and the students as well because they had been so used to practising the ones where it is all given in numerical form. So the ability to understand the geometry of what’s going on caught people out.”* Teacher 4 May 2010

This suggested that only what was examined was taught and that teachers as well as students were ‘caught out’ by novel questions. This was an example of the practice of ‘assessment as learning’ described by Torrance (2007) where examinations dominate the teaching and learning environment. The following comment was particularly surprising as ‘unit vectors’ are mentioned specifically in the Core 4 syllabus (OCR, 2010a). However, as they had not formed part of previous examination papers, it appeared that they had not been taught by this teacher.

> *“they didn’t know what a unit vector was. It was unexpected, it was like nothing that had come up before. It really depended on whether the teacher had picked up on unit vectors, had they even mentioned it in their teaching. I went through all the C4 questions and I could find no other questions which had that mentioned.”* Teacher 3 May 2010

Within this context, it was unsurprising that the Core 3 and Core 4 questions described as easy by students and teachers were, without exception, all of a standard and familiar nature. This may be an indication that the high achieving students that were the focus of my research were confident when applying techniques they had learned to questions similar to those they had worked on before. This was supported by the comment given by Teacher 4 which suggested that standard questions do not cause difficulties for students who have learned the material covered in mathematics lessons.

> *"what they are being asked to do it's exactly the sort of thing that they would have been expected to do in lessons."* Teacher 4 November 2009

However, as in Chapter 6, there was agreement between the students and teachers that the questions in the A2 examinations increased in difficulty throughout the paper. This view was repeatedly expressed by students and teachers in both written and interview forms. The following quote was typical of a student view.

> *"Because it's at the end of the paper and they tend to get harder as they go through. It doesn't matter about the topic, the questions at the beginning are easier questions, even if I find the topic hard. It's not a hard application of the topic."* Bella November 2009

One teacher took this progression of difficulty further and described a 'relative difficulty' where they rated the difficulty of a question in terms of its position in an examination paper. This was based on their belief that the examination questions start relatively straight-forwardly and then increase in level of difficulty as the examination progresses. The following quote described a question as difficult only because it was the first question on the Core 3 examination.

> *"Hard for a question 1. Not a settler because of necessary algebraic manipulation."* Teacher 2 February 2010

In addition to the expectation of increasing difficulty throughout the A2 examination papers, there were also expectations regarding the mark allocation of questions. There was a correlation between the amount of instructions given in a question and the number of marks it was allocated. Shorter questions with explicit instructions were often worth one or two marks, questions with fewer instructions had higher mark allocation. This pattern was evident in all the pure mathematics papers that I analysed. Three of the four teachers gave the number of marks as a reason for a question being difficult.

> *"A question that you can write in two lines yet is worth six marks, as soon as you get a big chunk like that its requiring them to break it down and structure their answer then it is harder."* Teacher 3 May 2010

One teacher described how a six mark question could be made easier by breaking it down into smaller, one mark questions with clear instructions. These descriptions explain why in Chapter 6, 'number of marks' was the only reason for a question being labelled difficult. A high mark allocation was an indication of need for students to apply mathematics that was not explicit. Thus it was the decision making, rather than the number of marks that was the source of difficulty and this is explored in section 7.3.6.

7.3.4 Novelty

Given the expectation of similarity and the revision strategy of practising past paper questions, it was unsurprising that novelty was the main reason given for examination questions being described as difficult. The students' descriptions often highlighted their helpless response (Dweck, 2000) when faced with novel questions.

> *"Question 8 and 9 I had never seen anything like it before so l didn't really know where to start."* Frankie February 2010

There was also recognition by students that understanding of mathematical topics was required to be able to successfully answer unfamiliar questions. This reflects the 'deeply unsatisfying' experience of mathematics described by Nardi & Steward (2002a) when students realise that they do not have the understanding to be successful in mathematics.

> *"I'd never seen them that way round, you had to really understand it in order to be able to apply it."* Emma February 2010

This need for understanding was also discussed by teachers. It was interesting that understanding was only seen as requirement to answer atypical questions, particularly when a high proportion of all the pure mathematics papers were considered to be standard.

> *"I've not seen a question worded like that before. There's a lot left for students to think about what to do. You had to be quite strong on your understanding to see how things work to be able to spot a suitable solution."* Teacher 4 February 2010

There was a common theme throughout all of the students' responses, that the different wording of the questions was the source of difficulty. This was also mirrored all four teachers' descriptions of the Core 3 paper, and the quote below is typical.

> *"from question six onwards it started to get fairly nasty. It is unlike what cropped up before so no amount of past paper practice would have prepared you for this. And it was the number that were different, yes you would expect it with maybe 8 and 9 but it was just the fact that by question six they were already thrown."* Teacher 3 February 2010

All of the teachers referred to the easier questions at the start of the paper, yet this distinction did not feature in any of the students' descriptions. The students' perception of the difficulty of an examination paper was more influenced by the questions that they couldn't do. They appeared to have discounted the questions that they were successful on. This mirrored the findings of Dweck (2000) where students who demonstrated a 'helpless approach' to challenges, once they experienced difficulty were quick to doubt their ability and ignore their previous successes.

The prevalence of the descriptions of difficulty arising from the different wording of the examination questions by both teachers and students was overwhelming. There were numerous references made to the type of question, rather than the mathematical topic as the source of difficulty. The following quote from a student was typical in the description of novel questions on a topic that they had not previously considered difficult.

> *"Had done sequences before but never in the way the two questions were worded. Hadn't been done in any exam before so didn't have practice."* Alice February 2010

This again demonstrated the extent that students and teachers expected examinations to be similar to the previous papers. One student gave an emotional response to the difficulty of the paper given that she had felt that she had prepared thoroughly.

> *"Overall I found this paper very, very difficult. I thought it was much harder than any of the practice papers I had tried. I was really annoyed how I couldn't do the quotient and remainder question, as these were topics I had found easy during revision, the questions in the paper seemed much harder."* Frankie July 2010

Similarly one teacher expressed anger at the difficulty of the final two questions of the Core 3 paper. Whilst they described these two questions as 'inaccessible' it was unsurprising to note that they were not typical questions and were not similar to any previous Core 3 questions on these topics.

> *"I understand that they want to test for the A*, they really want to stretch pupils, but questions 8 and 9 are of a very high level of difficulty and they don't offer much of a way in for most of our students. It's not going to be a fair reflection of the people who do this. The grades won't reflect the abilities of the pupils because 2/9 of the paper is unapproachable for them and they can't even access it."* Teacher 2 February 2010

When asked what they considered to be the most difficult question on the Core 3 examination, there was agreement between the majority of students and teachers that it was the following question:

8 **(i)** The curve $y = \sqrt{x}$ can be transformed to the curve $y = \sqrt{2x+3}$ by means of a stretch parallel to the y-axis followed by a translation. State the scale factor of the stretch and give details of the translation. **[3]**

(ii) It is given that N is a positive integer. By sketching on a single diagram the graphs of $y = \sqrt{2x+3}$ and $y = \frac{N}{x^3}$, show that the equation

$$\sqrt{2x+3} = \frac{N}{x^3}$$

has exactly one real root. **[3]**

(iii) A sequence $x_1, x_2, x_3, \ldots$ has the property that

$$x_{n+1} = N^{\frac{1}{3}}(2x_n + 3)^{-\frac{1}{6}}.$$

For certain values of x_1 and N, it is given that the sequence converges to the root of the equation $\sqrt{2x+3} = \frac{N}{x^3}$.

(a) Find the value of the integer N for which the sequence converges to the value 1.9037 (correct to 4 decimal places). **[2]**

(b) Find the value of the integer N for which, correct to 4 decimal places, $x_3 = 2.6022$ and $x_4 = 2.6282$. **[3]**

Again, the reasons given by students and teachers for this question being the hardest on the paper was to do with the different way that it had been written.

> *"Unlike any previous iteration exam question, only those who can really think on their feet will see what to do."* Teacher 4 February 2010

Strikingly, one teacher described that this question was difficult as it required understanding of the topic, rather than just a rote-learned approach.

> *"Really have to understand iteration rather than just applying a formula blindly."* Teacher 2 February 2010

The following reason given by a teacher also referred to the requirement of understanding as the source of difficulty. Yet this teacher offered a rare example of support for the novel style of the question.

> *"Eight part three, mainly because of the way it was presented, not because of what they actually had to do. It was presented in a way that they would be unfamiliar with which was good because it really tests the students' understanding"* Teacher 1 February 2010

The issue of some students being unable to begin to answer the question was expressed strongly by one of the case study students who couldn't distinguish between the difficulty levels of the last two questions.

> *"I couldn't do any of question 8 or any of question 9. I've just never seen anything like that before. The way they were written, they had never been like that in the textbook or the past papers. I didn't know how to start or what to do. I just couldn't do anything."* Bella February 2010

It was interesting to note that contrary to being a 'weaker' student as described by the teacher, this student scored 82 out of 100 on this module (equivalent to an A) and gained a B grade overall in A2 Mathematics. Yet despite her high achievement, she still felt unable to begin to solve a novel question under examination conditions.

7.3.5 Decision Making

When a question was novel, students had to decide how to approach it and the need to decide which method to apply was seen as a source of difficulty by many students. The following quote is a typical response to describing a question difficult when the method required had not been provided.

> *"No idea! No specific rule to follow."* Emma February 2010

Many of the teachers' comments also acknowledged that students would find the decision making required in unfamiliar contexts, difficult.

> *"Students may struggle to realise what is needed."* Teacher 1 February 2010

Whilst question eight had been described as the most difficult on the Core 3 examination by the majority of students and teachers, the final question on the paper was considered to be most difficult by the remaining students and teacher. The question is given next.

9 The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of

(i) $\tan 55^\circ$, **[3]**

(ii) $\tan 5^\circ$, **[4]**

(iii) $\tan\theta$, where θ satisfies the equation $3\sin(\theta + 10^\circ) = 7\cos(\theta - 10^\circ)$. **[5]**

The teachers' responses all contained a large amount of mathematical details, yet the reasons for difficulty were very similar to those of the students. The unfamiliar question style and lack of 'scaffolding' were seen as the sources of difficulty, rather than the topic of trigonometry. Two teachers described that the question was inaccessible and that many students would have been unable to attempt it due to the lack of instruction.

> *"some of them didn't even spot what they were trying to do. They didn't know where to start. You would be looking at your real top end students who would have been able to answer that."* Teacher 3 February 2010

The comment from the Examiners' report also identified that the decision making required about the method required to answer the question was a source of difficulty.

> *"This was a more demanding question requiring some thought by candidates on the choice of appropriate methods in the three parts."* (OCR, 2010b, p.14)

What follows is a quote by one student that is surprising because of the length and level of detail provided. Craig offers a perceptive insight into why he considered this question to be the most difficult on the Core 3 paper. His description is atypical of students' description of difficulty with the level of detail and references to specific parts of mathematics, the trigonometry double angle formulae.

> *"Well nine i was a requirement to know what tan 45 is. As soon as you knew that then it was easy. It was the other two parts that I found a lot harder. The second one, I could see that it was double angle formula, I just couldn't see how to work it through, in the end I just gave up. In all the other questions you are either told which rule to use or it is quite clear which one you should. But because there are like ten different rules for the angles, you have no idea what way to go down. You are looking at 9 part 3 and wondering do I need to get it in terms of sin or cos first, which way do I need to put them round? There are so many different factors there that you don't know which to get rid of first."* Craig February 2010

Whether providing the level of detail given by Craig or the more usual brief reference to a lack of method to follow, the absence of scaffolding was seen as the source of difficulty by both students and teachers. However it is significant that the lack of instructions occurred within the context of an unfamiliar question. The descriptions of students not knowing what to do would not have occurred if a similar question had been on a previous examination. Thus whilst decision making is seen as a source of difficulty, it is only within unfamiliar questions. The over-riding factor of difficulty remains difference.

7.3.6 Mathematics

To be expected, as the number of modules studied previously increased, the need to use mathematics from a previous module was more frequently given as a source of difficulty at A2 than at AS level. It may have been that because the majority of questions in a given examination featured mathematics from that module, students were surprised by the inclusion of mathematics from previous modules. The following was the reason given why a Core 4 question was described as difficult by one student.

> *"volumes of revolution isn't on C4 spec"* Bella July 2010

There appeared to be a further aspect of classifying difficulty, this was where difficulty arose from the dependence on a previous answer. The requirement to use mathematics from a previous part of a question had not occurred during the AS examinations, yet was mentioned several times in the description of why Core 4 questions were considered difficult. This reason was given by one of the students, Frankie, numerous times.

> *"I knew how to solve this but because I couldn't answer part (i) I couldn't solve it."* Frankie July 2010

Teachers similarly gave the requirement of answering a previous part of a question correctly as a source of difficulty.

As in the previous chapter, the actual mathematical content of an examination question was only rarely given as the reason it was considered difficult. There were only six comments of this type and interestingly they each referred to a different part of mathematics: modulus, stationary points, half angle formula, substitution, parametric equations and vectors. As with previous student responses, these comments were brief and did not provide any insight into why these topics were considered to be difficult. The following quote was typical.

> *"Don't get modulus"* Bella February 2010

As with the AS examination questions described in Chapter 6, none of the reasons given by teachers indicated a specific topic being the source of difficulty. Yet many teachers described questions requiring an atypical application of a topic as difficult. The following quote is a typical example of a teacher describing a question on integration by substitution as 'tricky' rather than the mathematical topic as difficult.

> *"Quite a tricky substitution question and have to sub in a negative so easy to make mistakes"* Teacher 1 July 2010

7.4 Summary

As with the findings from the AS examinations discussed in Chapter 6, novelty was the main source of difficulty at A2 level. Both students and teachers regularly described unusually worded questions as difficult. This contrasts with the description of understanding of mathematics (section 2.2) offered by Cockcroft (DES, 1982) which includes the ability to use and apply mathematics in unfamiliar contexts. Students prepared for A2 examinations as they had done so at AS level, by repeated practice of previous papers. As with the experiences described in the previous chapter, the students' descriptions of their experiences of A2 mathematics fit with the literature on 'teaching to the test' (Sierpinska, 1994; Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008; Torrance, 2007) a practice described as widespread in English mathematics classrooms (section 3.3.2). Data collected from the students show that they were 'learning to the test'. Their focus was on optimising their results and they did this by practising answering examination questions, mirroring the 'practising the finished product for the test' Prestage & Perks (2006, p.65).

There was a strong sense of expectation of similarity in examinations and students found the majority of standard questions as easy, indicating an instrumental understanding (Skemp, 1976). Similarly, being unprepared for and surprised by unfamiliar questions suggests a lack of relational understanding as students found difficulties with adapting to new tasks (section 2.3.1). Often students displayed emotional responses when describing how they had 'no idea' how to answer novel questions. Their responses fitted the 'helpless response' (Dweck, 2000) to difficulty (section 3.4.2) when faced with questions they had not met before. Whilst the students in my study fitted with the high achieving students described by Ames & Archer (1988), they did not display the awareness of effective learning strategies. This corresponds with the research that suggests that an emphasis on performance goals affects the approach to learning (Ames, 1992; Dweck, 2000; Turner et al., 2002) leading to a surface rather than a deep approach (Lublin, 2003).

Unlike the findings from the AS year, data collected during their A2 course showed that several students were aware of the limitations of a rote learning approach to mathematics. Students also acknowledged that they were unprepared to answer unusual questions. However, there were no examples of students suggesting alternative approaches and perhaps given their achievement at A-level mathematics (2B, 3A and 1A*) they did not see a need to alter their way of working. Similarly, three of the four teachers expected a large amount of similarity in examination papers and identified novel questions as requiring ‘really thinking’ or ‘full understanding’. The need for thorough understanding in order to answer unfamiliar questions was also acknowledged in the Core 3 Examiners’ report.

> *“This paper proved to be quite challenging for many candidates and the final three questions did, to an extent assess candidates’ understanding and their ability to apply their knowledge to the solution of more searching questions. At this level, candidates should expect an examination to do more than just ask them to reproduce well-rehearsed responses to requests of a routine nature. Candidates need to have a thorough understanding of all topics and techniques so that they can readily choose the appropriate response when faced with a situation which may be slightly unfamiliar.”* (OCR, 2010b, p.11)

Whilst the above quote specified that students should expect more than ‘requests of a routine nature’ it appeared that this was not the case for the students in my study. Whilst students described non-routine questions as difficult, they were still able to achieve highly on these A-level examinations. This belief was substantiated by the Examiners’ report comment on the June 2010 Core 4 paper which stated that the examination contained a high number of ‘straight-forward’ questions.

> *"This paper contained many straight-forward questions, provided candidates just sat and thought for a moment before attempting any solution."* (OCR, 2010c, p.15)

The students and teacher experiences of the Core 3 and Core 4 examinations fitted the practice of "Assessment as Learning" as described by Torrance (2007). For many students the mathematics examination questions had become the mathematics syllabus. For example, rather than learning numerical methods, students were learning how to answer numerical methods questions. Hence the different style of question on this topic question 8 part 3 on the Core 3 examination (section 7.3.4), caused many students difficulties as they felt they had not learned how to answer it. Rather than understanding a mathematical topic, students became competent at answering a range of questions provided by their teachers and sourced from the examination board textbook and previous past papers. Yet, whilst they acknowledged their difficulties when faced with novel questions, the students' approach resulted in high levels of achievement at A-level mathematics (table 7.1). The implications for students' experiences of learning and being examined in A-level mathematics are explored in the concluding chapter.

CHAPTER 8: CONCLUSIONS

8.1 Introduction

This thesis presents an exploration of high achieving students' experiences of A-level mathematics through the language of difficulty. The structure of the modular A-level course with students at most 12 learning weeks away (Chapter 1) from their next examination influenced my decision to research students' experiences of both learning and being examined in A-level mathematics. In Chapter 2 the curriculum reports suggested that understanding was central to learning mathematics and several models for describing understanding were explored as a way forward for my own research. Literature discussed in Chapter 3 suggest that external factors such as the frequency of modular examinations and high stakes testing had significant effects on students' learning experiences. Torrance (2007) describes a situation 'assessment as learning' where the examination questions had become the syllabus and only what was examined was taught. Internal factors that impact on student experiences are motivation for study and performance or learning goals (Ames, 1992; Ames & Archer, 1988; Dweck, 2000; Pintrick, 2000; Kaplan & Middleton, 2002) are considered to have an effect on students' response to difficulty. An environment dominated by performance goals, such as the current modular A-level system was found to foster a 'helpless' response to challenge (Dweck, 2000) where students had no strategies to overcome difficulties and questioned their previous successes. Through a two year mixed method case study approach, I explored students' perceptions of difficulties in learning and being examined in A-level mathematics and whilst I expected findings that gave insight into understanding, I instead gained insight into the external factors described in Chapter 3. In this chapter I review the work of the study, I take each of the research questions (section 1.1.1) in turn and summarise the findings from the data.

8.2 Findings

8.2.1 Do high achieving grammar school students find learning and being examined in A-level mathematics difficult?

Many students choose to do A-level mathematics even though they find GCSE mathematics difficult and expect the subject to become more difficult at AS-level (section 6.1.1). Five of the eight case study students described mathematics as their most difficult GCSE subject, and only one thought it was his easiest. Similarly, 48% of the questionnaire responses from Year 11 students who chose to continue with AS mathematics thought it would be challenging. Although students started the AS course with an expectation of increased difficulty, strikingly by the end of Year 12, 70% of students found that mathematics was more difficult than they had anticipated (section 6.2.1). Even though 23% of the students who completed the questionnaire achieved a grade A at mathematics, 68% found that it was their most difficult AS-level subject. These findings fit the description of mathematics (section 3.2.2) as more difficult than other subjects (Dearing, 1996; Smith, 2004; CASE, 2008; Ofsted, 2008).

Similarly, students begin A2 mathematics again expecting an increase in difficulty from AS level. The widely held perception that there is a 'step up' from GCSE to AS (section 6.1.1) and from AS to A2 (section 7.1.1) represent the 'sudden break points' (Brown et al., 2008) in learning mathematics (section 3.5). All six of the case study students who continued to A2 were expecting mathematics to be more difficult based on messages from older students and from teachers (section 7.2.1). Even with this anticipated increase, 70% of students in a questionnaire at the end of Year 13 described A2 mathematics as 'harder' or 'a lot harder' than they thought it would be. My findings do not fit with the description of A-level mathematics by Smith (2004) as insufficiently challenging for the high achieving students and overstretching for the low achievers. My data showed that many students felt overstretched with consistent descriptions of difficulty throughout their two year experience of A-level mathematics. I did not find evidence of the insufficient challenge for more able students other than from Craig, the only student to describe mathematics as his easiest subject throughout GCSE to A2.

There was one significant difference in the descriptions of difficulty of A2 compared with those of AS mathematics. Whilst a majority of Year 12 found mathematics their most difficult AS subject, only 30% of Year 13 students thought the same of their A2 subjects. This corresponds with the ‘clever core’ described by Matthews & Pepper (2006 & 2007), where the average GCSE point score of students studying AS mathematics is higher than for other subjects (section 3.2.2). This difference rises significantly between AS and A2 level, suggesting that the highest achieving students continue with A2 mathematics. My data is consistent with the finding that AS mathematics ‘weeds out’ the lower attaining students. Finding: **High achieving Grammar School students do find A-level Mathematics difficult.**

8.2.2 What are the nature of these difficulties?

The analysis of the data collected throughout the duration of the AS and A2 mathematics course showed that examinations dominated the talk of both students and teachers. During interviews where students were asked to describe what they found easy and difficult about learning a particular mathematical topic (trigonometry in section 6.3.4), their conversation often included the language of examinations. For example, one student felt that the question she had chosen was difficult because of it was ‘question eight in the exam’. The data collected over the course of the A2 year showed more of this overlap than was apparent in the AS data. The following quote showed the contradiction between learning a mathematical topic and learning how to answer specific questions.

> *"I had no idea how to approach it at all. Definitely it's the question, because the topic I had learned it. It's quite a specific type of trigonometry question that I'd never seen before, we had never been taught how to do questions like that."* Dan February 2010

This student expected to be taught how to answer questions on trigonometry. It was interesting that this expectation went alongside his statement that he had learned the topic. Thus he had interpreted that learning trigonometry was learning how to answer trigonometry examination questions. This fits with the findings of Mansell (2007) who found that 'hyper-accountability' (section 3.2.4) was a consequence of the high-stakes testing culture which results in learning being based on examination questions because only what is tested is valued. Similarly, Torrance (2007) describes a practice as 'assessment as learning' (section 3.3.2) where once examination questions become clearer and more standard, understanding of principles can be replaced with repeated practise of typical requests. There were many examples from the case study students that indicated that they were learning how to answer questions using the previous examinations as their syllabus. This fits with the descriptions of 'teaching to the test' a practice described as widespread (Sierpinska, 1994; Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008).

There are five reasons given by students why learning AS mathematics was considered to be difficult: workload, pace and intensity; memory; novelty; decision making and mathematics (section 6.2). As described in Chapter 7 the data collected from Year 13 students showed that these difficulties remained the same for learning A2 mathematics and no new categories emerged. With the significant overlap in the similarities of learning and being examined it was unsurprising that the reasons given that examination questions were difficult were similar to those given in learning mathematics. There were some differences that occurred in descriptions of AS examination questions, two additional sources of difficulty were identified; position in an examination paper and use of mathematics from previous modules. Both of these reasons were also used to describe difficulties in both learning and being examined at A2 level. At A2, there was one further reason given for examination questions being described as difficult; dependence on the previous answer. Each of these sources of difficulty is now considered concluding with the overwhelming source of difficulty- novelty.

The questionnaire responses of Year 12 students showed the majority of students found AS mathematics required more work than their other subjects both in lessons (45/65 =70%) and particularly with homework (52/65 =80%). Significantly, no student felt that mathematics had been the least amount of work either in lessons or with homework. The issue of the fast pace of learning was raised by all eight of the case study students, their descriptions matched those of 'top set' students (Boaler, 1997; Ireson, Hallam & Hurley, 2005; Boaler, Wiliam & Brown, 2000) described in section 3.3.1.

Throughout the data collection in both Year 12 and Year 13 students talked about the number of rules, formulae and methods that they were required to learn. Whilst the reliance on a memorised rule or formula was sometimes given as a reason that mathematics was easy, more often dependence on memory was described as a source of difficulty by many students. As the A2 course built on the content of the AS course (section 7.2.3) the need to remember specific rules and techniques was more commonly described as a source of difficulty in Year 13 when students were expected to retain the knowledge of the AS course. Whilst recall of key facts was seen as the ‘building blocks’ of learning mathematics Cockcroft (DES, 1982), the difficulties in distinguishing between factual recall and understanding were considered by many researchers (Dewey,1933; Byers & Herscovics, 1977; Sierpinska, 1994; Duffin & Simpson, 2000), explored in in section 2.4.3. The data collected from the students in my study offers more examples of students talking about their factual recall than their understanding.

When learning AS and A2 mathematics, students often worked on typical examination or textbook style questions (section 7.2.4) and so there were few opportunities for students to decide which mathematical techniques to apply in unfamiliar contexts. This fits with the literature of teaching to the test (section 3.3.2), particularly ‘practising the finished product for the test’ (Prestage & Perks, 2006, p.65). As Year 13 students learned more techniques, there were more possibilities for them to choose from when attempting questions that did not provide a method. Thus the need for decision making was more commonly featured as a source of difficulty in Year 13 than in Year 12.

It was striking that throughout the two years of mixed data collection methods that mathematics was spoken of so infrequently by students. The descriptions and discussions were about the difficulty of the wording and structure of the questions rather than understanding of mathematical topics that they were concerned with. Whilst I had set up the examination questions as the research instrument (Chapter 1) I was still surprised that the structure of the questions rather than the mathematics was the prominent feature in students’ responses. Students often made distinctions between difficult concepts and difficult questions, but there were few examples of the mathematics given as the source of difficulty.

All that is offered by the data collected from students is a list of mathematical topics that were raised as a source of difficulty. In learning AS mathematics these were: differentiation, transformations of curves, trigonometry and logarithms. In AS examinations; transformations of curves, logarithms, graph sketching, trigonometry equations and radians. In A2 mathematics difficult topics were considered to be: integration, modulus functions, natural logarithms, vectors, integrating trigonometric functions and differential equations. Descriptions of why these topics were difficult were brief and offered limited insights into understanding. The reasons given lacked mathematical detail and were often emotional 'I've always been bad at trig' or 'I hate logs'. Whilst I had considered that students would be more fluent in the language of difficulty rather than the language of understanding (Chapter 1) I was surprised that their descriptions regarding mathematical topics were so brief and did not provide any insight into why these topics were considered to be difficult.

There were two aspects of mathematics that were given as reasons why examination questions were considered to be difficult. One aspect only occurred in written responses from the Core 4 examination. This was where difficulty arose from the dependence on a previous answer. The requirement to use mathematics from a previous part of a question had not occurred during the AS examinations, yet was mentioned several times in the description of why Core 4 questions were considered difficult. The need to use mathematics from a previous module was often seen by students as a source of difficulty. With more modules to draw upon, it was to be expected this reason for difficulty was more common at A2 level that it had been at AS level. Students were surprised by mathematics from previous modules when learning or being examined in their current module. This fits the descriptions of the consequences of the modular structure of the mathematics A-level course (section 3.2.3), ‘splintering the unity and connectedness of mathematics’ (Smith, 2004, p.93). Unlike the warning from Kounine et al. (2008) that students cannot forget one module as they move on to the next, my data showed that whilst successful in the original modules, students saw previous content as difficult when it appeared in subsequent examinations.

The most frequent reason given for difficulties in A-level mathematics was novelty. When questions were different to those that students had seen before, they were considered to be difficult and the reasons given above were then used to describe the difficulty. Whilst students had little opportunity to experience non-standard questions in learning mathematics (section 7.2.4) they were faced with novel questions in examinations. The most common reason students gave for finding AS examinations questions difficult was they had not seen a question like it before (section 6.3.4). This was regardless of whether it was the Identification, Discrimination or Generalisation type. The structure of previous examination papers with the majority of questions being of the Identification type meant that the atypical questions were most often Discrimination or Generalisation category (section 5.4.2). This remained true for the A2 papers when novelty was again the main reason given for examination questions being described as difficult (section 7.3.4). Whilst the case study students achieved highly at A-level mathematics, they found applying their mathematical knowledge to unfamiliar contexts difficult. This was in contrast to the description of understanding offered by Cockcroft (DES,1982) that includes the capacity to use mathematics in a variety of settings; both familiar and unfamiliar (section 2.2).

As described with the overlap of learning and being examined, students expected the examination paper that they sat to be very similar to the past papers (section 6.3.3 & 7.3.3). Consequently their revision strategies focused on practising previous examination questions and it was unsurprising that students found novel questions difficult. The following quote was typical of responses which describe that students could not revise for unfamiliar questions.

> *"I didn't know what to do, I didn't know because I hadn't seen it before or had much practice. I didn't revise it really because I didn't see it in any of the papers, I didn't expect it to happen"* Alice February 2010

This description given by a student who achieved a grade B at A-level mathematics fits the helpless response (Dweck, 2000) when faced with novel questions. Dweck (2000) suggests that students with performance goals do not persist to overcome obstacles and instead offer a helpless response to difficulties (section 3.4.2). Finding: **The nature of students' difficulties in learning and being examined in mathematics include workload, pace and intensity, memory, decision making, with the overwhelming source of difficulty being novelty.**

8.2.3 What do A-level mathematics teachers consider to be students' difficulties?

The reasons given by teachers for what they considered to be students' difficulties closely matched the reasons offered by the students themselves. The teachers' responses showed the overlap between difficulties in learning and being examined in mathematics and teachers' descriptions of difficulty remained constant throughout the AS and A2 course. Thus the teachers' reasons for difficulty fitted the same categories as stated in the previous section: workload, pace and intensity; memory; novelty; decision making, the need to use mathematics from a previous module and occasionally, mathematics. As with data collected from students, the most frequent reason given by teachers for difficulties in A-level mathematics was novelty.

Again, mathematics was spoken of only rarely by teachers, with the descriptions of difficulty focusing on the wording of questions. Teacher descriptions were typically more detailed and included more mathematical references than those offered by students (section 6.3.2). A further difference was teachers did not give a mathematical topic as a reason why they considered a question to be difficult, instead they qualified their response as ‘a difficult question on’ a specific topic (section 6.3.6) usually with a reference to the ‘unusual’ wording of the question.

As with students’ descriptions of difficulty collected over the two-year A-level course, the language of examinations was widespread in the teachers’ descriptions. The teachers’ descriptions of difficulty included features from the structure of the examination papers such as a high mark allocation and position in the paper. Teachers described that easy questions were those that were standard or typical (section 7.2.4) and conversely that difficult questions were those which were novel or unusual (section 7.2.4). There were several cases where teachers expressed strong opinions about these novel questions, describing them as ‘nasty’ (section 6.3.4) or that they ‘caught us out’ (section 7.3.3). Frequently, teachers described that novel questions require ‘a lot of understanding’ or ‘actually thinking’ (section 7.2.4). Finding: **A-level mathematics teachers consider the nature of students’ difficulties in learning and being examined in mathematics to be those offered by the students themselves, with the overwhelming source of difficulty being novelty.**

8.2.4 What do the descriptions of difficulty reveal about the students' understanding?

Rather than gaining insight into understanding the data revealed insight into assessment, which was the dominant feature of students' experiences whether they were describing learning mathematics or being examined in the subject. I had planned to get at students' understanding through their descriptions of difficulty, but this did not happen because their descriptions remained focused on the external signals. When describing learning mathematics, students described difficulties that were centered on high workload, fast pace and the amount of content to be memorised. When describing difficulties with examinations, the question structure 'it's question nine', 'it's worth seven marks' rather than the mathematics featured in both student and teacher responses. I set up the data collection to provoke descriptions of difficulty of mathematics using the research instrument of mathematics questions. I had not anticipated that I would not get talk about mathematics and did not expect the data to be so focused on the external factors. Finding: **Descriptions of difficulty revealed little insight into students' understanding of A-level mathematics.**

8.2.5 What is the level of demand of A-level pure mathematics questions?

In Chapter 5 I describe how I took the definitions from a model based on four acts of understanding; identification, discrimination, generalisation and synthesis from Sierpinska (1994) and modified them to determine a level of demand offered in examination questions. This approach had similarities with that of Fischer-Hoch et al. (1997) in that it considered the parts of questions as structured in the examinations. My assumption for applying a model, that students had been taught and had learned the methods required to answer the examination questions, was similar to the approach adopted by Pollitt & Ahmed (1999) where 'learning the subject' was the first step in their model for answering examination questions. However my model to classify A-level pure mathematics questions with the analysis of students' and teachers' responses adds to the methods for collecting data on what makes an examination question difficult. Unlike the 17 'sources of difficulty' for GCSE mathematics questions identified by Fischer-Hoch et al. (1997) the majority of which focus on the wording and context of the questions, my model focuses on the processes involved in answering the questions.

The four categories of questions offer four different levels of demand. Questions of the Identification type do not require any decision making by the student about what mathematics to do. Instead, students have to follow instructions given either explicitly or implied by the standard nature of the question. Discrimination questions require students to decide which part of mathematics to apply in order to answer them. Whilst students are given what to do, they need to decide how to do it. Questions of the Generalisation type are those which require students to recognise that the question is a particular case of a mathematical concept. Students have to work out both what mathematics to do and which technique to apply in order to do it. Questions of the Synthesis type require students to consider multiple representations of the same mathematical object and either combine them or use the links between them in order to solve a problem. Students have to work out the different possibilities of what could be done, decide how the possibilities link together, decide on the mathematical techniques to apply and then apply them.

The model was applied to each of the pure mathematics papers from 2008 to 2010 and the composition of each of these 24 examinations was surprisingly consistent (section 5.4.2).

Table 8.1 shows that whilst there are some differences in the composition of the four pure mathematics examinations that the case study students sat, there were many similarities which contrast with the widely held perception that Core 4 is the most difficult paper.

Table 8.1 Comparison of the four Core Examination Papers

Paper	Identification	Discrimination	Generalisation	Synthesis
Core 1 January 2009	52/72 (72%)	14/72 (19%)	6/72 (8%)	0/72 (0%)
Core 2 June 2009	44/72 (61%)	21/72 (29%)	7/72 (10%)	0/72 (0%)
Core 3 January 2010	27/72 (38%)	26/72 (36%)	19/72 (26%)	0/72 (0%)
Core 4 June 2010	41/72 (57%)	22/72 (30%)	9/72 (13%)	0/72 (0%)

The majority of questions on each A-level examination in the period from January 2008 through to June 2010 were of the Identification type (section 5.4.2) and Discrimination questions were the second most common. There were very few Generalisation questions on the majority of the papers and the January 2010 Core 3 examination was exceptional in this respect. It contained significantly more Generalisation questions than any of the other papers analysed over the research period. The grade boundaries for this paper were lower than for other examinations (section 5.4.2) suggesting that candidates had not scored highly on these questions. As with all the papers analysed, there were no Synthesis questions on any of the papers that the students sat.

Whilst there was some variation in the marks required to gain each grade on each paper, there was one notable similarity; no marks from the Generalisation questions were required for students to gain a grade A. Therefore students could obtain a grade A without having to apply their knowledge of general mathematical concepts. However, they were required to be accurate when carrying out specified methods to answer Identification questions and when deciding which mathematical technique to apply to Discrimination questions.

Duffin & Simpson (2000) describe that whilst teachers could not see their students' understanding of mathematics, they could interpret the 'external manifestations' of understanding (section 5.1). I used my model to measure 'obstacles' as evidenced by the demand in an examination question. The nature of these demands provided insight into the nature of understanding required of A-level mathematics students. As the majority of the questions were of the Identification type, they presented students with limited demands as they provided instructions of what to do, whether stated explicitly or implied by the standard nature of the question. The Core mathematics examinations are structured in such a way that meant students could achieve highly without having to tackle and overcome the more demanding obstacles provided by Generalisation and Synthesis questions.

The findings from analysis of the demands of Core examination papers fit with the descriptions of ACME (2009) that modular A-level mathematics examinations are focused on techniques rather than problem solving (section 3.3.2). Similarly, the predominance of Identification questions with explicit instructions correspond to the numerous short questions with clear sign-posting of the method required to solve the problem, described as 'sat-nav' mathematics by Basset et al. (2009). Finding: **The demands of A-level pure mathematics questions do not increase from Core 1 to Core 4, the majority of questions are of the Identification type.**

8.2.6 What connections are there between the level of demand of the examination questions and the descriptions of difficulty given by students and teachers?

Whilst the application of my model showed that the demands of A-level pure mathematics questions do not increase from Core 1 to Core 4, to answer the final research question I combined the analysis of the examination papers with analysis of students' and teachers' perceptions of the difficulty of the questions. This approach follows the methods of several studies (Entwistle & Entwistle1991; Cooper & Dunne, 1998; Crisp et al., 2008) which use data collected from students to overcome the main disadvantage of document analysis when applied to examination questions or students' answers. Analysis of the documents does not provide insight into the reasons for the answers, yet reasons can be gained from interviewing students about their responses to the examination questions. There was a strong correlation between the classification of questions and whether they were considered to be difficult by either students or teachers (tables 6.6 and 7.4). For all four Core examinations, as might be expected Identification questions were least often rated as difficult by both students and teachers. The Discrimination questions were described more frequently as difficult by students and particularly teachers. On all four examinations, the Generalisation questions were described as difficult by the majority of students and significantly, by all of the teachers. With the exception of the OCR June 2010 Core 4 paper, students rated fewer questions as difficult than their teachers did. Although I was surprised initially, this also occurred in the data collected from students and teachers whilst I was developing the model. Reasons for this (section 5.3.3) centre on the students commenting on their own perception of difficulty, whilst the teachers did not. Teachers often commented on questions that would cause 'weaker' students difficulties which does not fit the higher achievement of the case study students.

The surprising similarity in the structure of the examinations from Core 1 to Core 4 was also reflected in the data I collected from students. I was expecting that students would describe different difficulties occurring within the different classifications as they progressed from AS to A2. I had considered that students may have experienced difficulties with Discrimination questions at AS which became difficulties with Generalisation and Synthesis questions at A2. But this did not happen; students got the same examination experience four times throughout the two years of their A-level course. Descriptions of difficulty remained with external factors; novelty; lack of instruction; number of steps required in the solution; requirement to use memorised formulae or methods; position in an examination paper; use of mathematics from previous modules and mathematical concepts (section 7.2.6). Their descriptions of difficulty did not develop; they found the same thing difficult throughout: 'different is difficult'. This is particularly surprising given the high achieving nature of a Grammar School cohort (the case study students achieved one A*, 3 A and 2 B grades at A-Level mathematics). Finding: **On each of the four pure mathematics papers, Identification questions were least often rated as difficult by students and teachers. Discrimination questions were more frequently described as difficult by students and particularly teachers. Generalisation questions were described as difficult by the majority of students and by all teachers.**

8.3 Limitations to the Research Design

In this section I present four limitations to my research design and then consider each one in turn. There is no doubt that as a practitioner researcher access to the range of data is limited. As a consequence of being a practitioner researcher, I collected and analysed data from my own school. This places my research within the case study field, and as such a major limitation is how far the findings from my particular data set can be generalised. The second limitation is a result of collecting data under the current modular examination structure as it restricted the breadth of data I collected. As I researched students' experiences of learning and being examined in A-level mathematics, I decided to follow the natural sequence of the students' experiences. Thus my data collection of experiences of learning and being examined was split into four components: Core 1, Core 2, Core 3, Core 4. A third limitation to my research design was my use of examination questions as the research instrument to provoke descriptions of difficulty in order to gain insight into understanding. Whilst the difficulty in observing understanding is well documented (Section 2.4.3) a way forward for my data collection was from the work of Duffin & Simpson (2000) who considered that teachers could recognise the 'external manifestations' (p.419) of their students' understanding. However, as the examination questions consisted mainly of Identification and Discrimination questions, these limited demands provided little variety in the 'external manifestations' offered to students. The fourth limitation to the research design was in the data collection methods I decided to use. Semi-structured interviews allowed students and teachers to develop their responses, but meant that there were inconsistencies between the interviews. Timings of interviews were designed to fit with students' and teachers' timetables and I needed to balance the richness of data I collected with the need to maintain the co-operation of participants over the two-year research period.

8.3.1 Practitioner-Researcher Issues

In section 4.2.1 I detail the pros and cons of being a practitioner-researcher- the advantages of being an 'insider' (Robson, 2003) and the tensions created by managing the relationships with those from whom I collected data (Allwright, 2005). The research design (section 4.4) details decisions about the tensions. As a consequence of the time restrictions of being a full-time teacher and part-time researcher, I took the decision to research the experiences of A-level students within my own school. Thus my research took the form of a case study, or ‘singularity’ (Bassey, 1995), and whilst this provided the opportunity to study the issues in depth, the most significant limitation of the research is how far, if at all, the findings can be generalised. I followed the advice to offer clear descriptions of the case (Robson, 1993; Burns, 2000; Stake, 2003) that would allow readers to identify commonalities and distinctions between this research, previous studies and their own situations. The commonality of my study with other research are the GCSE students who made choices about whether to continue on to mathematics post-sixteen, and the A-level mathematics students tracked throughout Year 12 and Year 13. The difference is that these are Grammar School students, selected for their high attainment and who go on to achieve highly in A-level mathematics. Consequently my findings relate to a limited and particular data set.

8.3.2 Modular Examination Structure

The structure of my research design mirrored the structure of the modular A-level examination system as I collected data about students’ experiences of learning and being examined in each of the four pure mathematics modules. As a consequence of this decision, the limitations of my research design are the limitations of the modular examination system. As discussed in Chapter 3, the current structure of the A-level mathematics specification has had an impact on the subject, described as ‘narrow achievement in six separate mini-courses’ Bassett et al. (2009, p.18). Similarly, the timings and foci for my data collection took place within this fragmented modular environment and having talked to students about their experiences of learning Core 1 mathematics and then the Core 1examinations, it may have moved the focus onto the modular examination structure and away from learning mathematics. The description of the effect of modular examinations ‘splintering the unity and connectedness of mathematics’ Smith (2004, p.93) may also be applied to the effect that the modular structure had on splintering the data. I could have chosen to take a thread of calculus and look at how students’ understanding and perception of difficulty developed over the course of the two years of their A-level study. However, there is rarely a point in any term where students are just learning mathematics (Chapter 1) so there was very limited time in which to explore mathematics without an examination focus. The structure of the A2 course is the same as that of the AS course and it was therefore unsurprising that data collected whilst students were in Year13 showed many similarities with that collected during Year 12. Yet I decided to explore students’ experiences within the existing structure of the current A-level course.

8.3.3 Understanding

Having analysed the nature of demand of A-level mathematics papers through the use of a model based on understanding, ‘assessment of learning’ (Torrance 2007) suggests that this might provide insight into the understanding required when learning A-level mathematics. It was significant to my findings that the majority of examination questions were of the Identification type and none were of the Synthesis type. So using the examination questions as a research instrument limited the evidence of student understanding.

Student and teacher descriptions of difficulties in learning and being examined in A-level mathematics did not offer much insight into understanding. Instead the difficulties described features of assessment such as question structure, position in the examination and mark allocation. As the curriculum reports suggest that understanding is fundamental to learning mathematics I had thought that assessment of the curriculum would reflect some understanding in mathematics. As I researched high achieving students who were successful in their examinations, their experiences do demonstrate understanding but only in relation to successfully completing examination questions.

8.3.4 Data Collection Methods.

As I required the participation of the case study students and teachers over the two-year research period I depended upon volunteers. This presented a threat to the ‘external validity’ (Burns, 2000, p.18) of my research however the students in my study were not a random sample, they were selected from a high achieving cohort of Grammar School students (section 4.2.5). Maintaining the willingness of students and teachers to participate affected my decisions regarding data collection.

Limitations to the data collected from the questionnaire include the combination of questions which asked students to select up to responses from a list and free response questions (section 4.3.2). The items listed may have affected students’ responses to the open questions, and as with all questionnaires, the truth of responses cannot be guaranteed. The practicality of gaining access to a large number of students at one time within the school week meant I administered questionnaires during assembly time. This may have affected the quality of students’ responses in terms of the amount of time they had to consider and write their responses. In order to address these issues I triangulated the questionnaire responses with data collected from interviews.

Having considered the literature on different interviews I decided to adopt a semi-structured approach (section 4.3.3). Whilst these interviews consisted of a series of questions for all interviewees, the flexible nature of the semi-structured format meant individual interviews developed differently. Over the two-year data collection period the interviews became less conversational and more consistent as my interviewing skills developed. Maintaining participation of students over the research period meant that essentially students had to enjoy the interview process and not find it too burdensome on their time. Having designed the research in the AS year for students to talk about four co-ordinate geometry questions, I observed that it seemed awkward for them to do so in an interview situation. As a result, in the A2 year I asked students to bring their own questions to discuss. Whilst this provoked more fluent talk about the nature of difficulties, students still talked about the wording and structure of the questions rather than mathematics. Similarly, the frequency of the interviews was affected by how much time I felt it was appropriate for students and teachers to give up and at what point in the academic year it was reasonable to call on their time.

8.4 Implications of the Study

In this section I consider the implications of the study for students, teachers and examiners. For students, these include perceptions of difficulty and the dominance of assessment on their learning experiences. For teachers implications include teaching to the test with widespread use of examination questions and expectations of similarity in examination papers. Implications for examiners include the difference in their ambitions for the challenges posed by examinations and the grade boundaries which result in the most challenging questions not being required to achieve the highest grades.

There is a general agreement that A-level mathematics is easier now than in the initial Curriculum 2000 specification and previous linear versions, yet there has been no shift in the perception of the difficulty of the qualification by students (Smith, 2004 and Matthews & Pepper, 2007 in section 3.2.2). Similarly the increase in pass-rate and high percentages of A grades has not led to a shift in perception. This was reflected in my data where over two-thirds of students describe mathematics as their most difficult AS level when over a quarter went on to gain a grade A in the subject. There was a reduction in this percentage to 30% of students who felt mathematics was their most difficult A2 subject. This supports the notion that mathematics is a subject for the 'clever core' (Matthews & Pepper, 2007) and that weaker students did not continue after AS. Yet despite mathematics having the highest proportion of A and A* grades at the school it is still spoken about as a difficult subject. If the achievement of students is not the source of the perceived difficulty, the implication is that it is the experiences of the students which contribute to the belief that mathematics is a difficult A-level. Whilst the notion that 'different is difficult' prevails, students will continue to describe novel questions difficult and until this is addressed mathematics is likely to retain its reputation as a difficult subject.

This study found that the assessment structure had significant impact on students' learning experiences, with many students learning mathematics by practising past examination questions. This is particularly significant given the predominance of Identification and Discrimination questions in the pure mathematics examinations at both AS and A2 level. As there were no Synthesis questions on any of the 24 examination papers analysed during the research period, questions of this type did not form part of students' experience of learning A-level mathematics. Thus students who went on to achieve highly at A-level mathematics did not engage with questions that required them to consider multiple representations of the same mathematical object and either combine them or use the links between them to solve a problem (Section 5.2.4). It may surprise many who are not familiar with the current specifications that high achieving A-level mathematics students rarely engage with challenging problems. The mathematicians and engineers of the future begin their undergraduate courses with high grades in A-level mathematics yet many are not confident in problem-solving. Those who run university courses need to be aware that students have had limited opportunities to apply their mathematical knowledge in novel contexts during their experiences of learning and being examined in A-level mathematics.

Data was collected from teachers regarding what they considered difficulties for students in learning and being examined in A-level mathematics, and this study supports the findings of Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008; Torrance 2007 that ‘teaching to the test’ (section 3.3.2) is widespread. Teachers’ talk was dominated by the language of examinations whilst talking about students’ experiences of learning mathematics. Reasons for questions being difficult for students included position in the examination paper and the number of marks allocated. When asked to bring questions on trigonometry and vectors to discuss during interviews, all four teachers brought only previous examination questions (section 7.2.4).

In addition to the frequent references to examinations when describing students’ experiences of learning A-level mathematics, teachers had strong expectations of the format of the examinations. Teachers expected examinations to be structured with questions increasing in difficulty through the paper (section 6.3.3). Core 3 and particularly Core 4 were considered to be difficult examinations (section 7.3.3). There was an expectation that the majority of questions would be similar to those on previous papers, and that novel questions were ‘nasty’ (section 6.3.2). Teachers talked about themselves as well as their students being ‘caught out’ by unusual questions (section 7.3.3). Whilst teachers described a minority of questions on any examination as unusual, it was striking that these novel questions were consistently described as difficult. The reasons offered for why novel questions were difficult included that they required ‘a lot of understanding’ or ‘actually thinking’ (section 7.2.4). In interview the four teachers talked about the time pressure of teaching the A-level mathematics course within the allocated lesson time. Within the context of performance related pay, individual teachers and departments judged on students’ examination grades in comparison to nationally set targets there is significant pressure on teachers to deliver results (Mansell, 2007 in section 3.2.4). The system of ‘hyper-accountability’ (Mansell, 2007) has led to teachers making decisions to base their teaching on examination questions. Data collected from the four teachers showed that they were aware that novel questions caused difficulty for students. Yet because of the structure of the examination papers they knew that students could still achieve highly because of the standard nature of most of the questions. The implications are that teachers want their students to get the best A-level grades rather than the best learning experiences of mathematics. Within the context of ‘teaching to the test’ (Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008; Torrance 2007) if students’ experiences of learning A-level mathematics are to change, the change depends on the next generation of A-level mathematics examinations. This study shows that the modular examination structure dominates students’ experiences of learning A-level mathematics, with students being at most twelve learning weeks away from their next examination (Chapter 1). Changes to the A-level courses being considered at the time of writing of this concluding chapter include a move away from modular, back to a linear format. However, it is the structure and style of questions of future examinations which are of particular importance as they play a significant part in students' experiences of learning mathematics. In line with the Cockcroft Report (DES, 1982 in section 3.3.2) the data from both teachers show the powerful influence that examination specifications have on mathematics teaching. Unless these new examinations value 'thinking, contemplation and understanding for their own sake' (Sierpinska, 1994 in section 2.3.3) then it is unlikely that these will feature in the learning experiences of future A-level mathematics students.

The final implications of this study are for the external examining bodies who write the mathematics syllabi and A-level examinations. Comments from the Examiners' reports describe that papers consist of questions 'that challenged even the most able candidates' (OCR, 2007d, p.6 in Chapter 5) and 'many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by these papers' (OCR, 2007d, p.1 in Chapter 5). However, external examiners might reflect more carefully when questions identified as posing greater challenge are so few in number. When describing the January 2010 Core 3 paper as 'quite challenging' (section 7.4), the Examiners' report (OCR, 2010, p.11) states that students are expected to be able to apply their knowledge in unfamiliar contexts. However, it is significant that on all the examination papers analysed during the research period, none of the Generalisation questions would need to be answered for students to gain a grade A (Section 5.4.3). Whilst the examiners' comments suggest they are writing papers which require students to have a 'thorough understanding' and apply their knowledge to novel contexts, the structure of these examinations and corresponding grade boundaries indicates this is not the case. As long as students can omit the most challenging questions and still achieve an A grade, the ‘well-rehearsed responses to requests of a routine nature’ (OCR, 2010b, p.11) will form the majority of their experiences of learning and being examined in A-level mathematics.

8.5 Areas for Further Research

A next stage for this research could be to increase the data set to include more views of teachers. An exploration of aspects of pedagogy with a group of A-level teachers could be a possible way forward to gain further insight into the experiences of students. Sharing the model for classifying the demand offered by A-level mathematics questions could allow an investigation into the types of questions used within the teaching of A-level.

Work on the motivations for study and the promotion of learning goals (Ames, 1992; Ames & Archer, 1988; Dweck, 2000; Pintrick, 2000; Kaplan & Middleton, 2002) could offer the way forward for action research with a group of A-level students. Work on developing mastery-oriented responses to difficulties may offer an insight into approaches to novel questions.

A further possibility would be to apply the model of classification to the applied and the further pure mathematics modules. Analysis from other areas of mathematics may add to the information gained from the application to the pure mathematics modules. This model could also be extended to other A-level subjects, whilst this may most closely fit with the sciences, there may be possibilities for adaptations that would allow its application to other subjects.

8.6 Conclusion

I was motivated to begin this research after noticing that the descriptions of mathematics as a difficult subject were shared not only by the GCSE re-sit students that were the focus of my Masters research, but also the high achieving A-level mathematics students whom I taught. Although the attainment of both groups of students was significantly different I was struck by their similar descriptions of learning mathematics. Whilst I set out to use A-level students' language of difficulty in order to explore their understanding, I instead achieved a 'thick description' (Robson, 1993) of difficulty that focused on external factors influenced by the language of examinations. The findings of this study fit within the 'assessment as learning' (Torrance, 2007) where the mathematics examination questions have become the mathematics for many A-level students.

The development and use of a model from Sierpinska (1994) to evidence the nature of demand offered by the pure mathematics examinations found that the majority of questions were of the Identification type. Thus students' experiences of being examined were dominated by standard questions which required no decision making. The grade boundaries are such that students are not required to answer Generalisation questions to achieve an A grade. Because of the high stakes testing environment (Mansell, 2007) and the practice of 'teaching to the test' (Sierpinska, 1994; Smith, 2004; Golding, 2007; Ofsted, 2006 & 2008) as greater obstacles, evidenced by Synthesis questions, are not present in examinations they do not form part of the teaching and learning of A-level mathematics. Thus students do not need to overcome significant obstacles in order to achieve highly at A-level mathematics. As the majority of the experiences of learning and being examined in A-level mathematics consist of standard questions, students are surprised by and unprepared for novel questions. Despite the high achievement of the six case study students, they experienced difficulties when faced with novel questions and these difficulties remained consistent throughout their two year A-level course, a situation summarised by ‘different is difficult’.

LIST OF REFERENCES

Advisory Committee on Mathematics Education (ACME) (2009) **ACME Response to the QCA consultation on Level 3 Mathematics** [online]. Available from:www.acme-uk.org/media/1593/acme%20response%20to%20qca%20consultation%20on%20level%203%20mathematics%20(july%202009)%20final.pdf [Accessed 20 September 2012]

Ajdukiewicz, K. (1975) **Problems and Theories of Philosophy** Translated by Skolimowski, H. & Quinton A. Cambridge: Cambridge University Press

Ahmed, A. (1987) **Better Mathematics: A Curriculum Development Study based on The Low Attainers in Mathematics Project** London: Her Majesty's Stationary Office

Alexander, R. & Armstrong, A. (2010) **Children, their World, their Education Final Report and Recommendations of the Cambridge Primary Review** London: Routeledge

Allwright, D. (2005) Developing Principles for the Practitioner Research: The case of Exploratory Practice. **The Modern Language Journal** 89(3): 353-366

Ames, C. (1992) Classrooms: Goals, Structures and Student Motivation. **Journal of Educational Psychology** 83 (3): 261-271

Ames, C. & Archer, J. (1998) Achievement Goals in the Classroom: Students' Learning Strategies and Motivation Processes. **Journal of Educational Psychology** 80 (3): 260-267

Askew, M., Brown, M., Rhodes, V., Johnson, D., Wiliam, D. (1997) **Effective Teachers of Numeracy Final Report. Report of a study carried out for the Teacher Training Agency 1995-96 by the School of Education King's College London.** London: King's College

Backhouse, J.K. (1978) Understanding School Mathematics- A Comment. **Mathematics Teaching** 82: 39-41.

Bassett, D., Cawston, T., Thraves, L., Truss, E. (2009) **A new level** [online]. Available from: http://www.reform.co.uk/client_files/www.reform.co.uk/files/a_new_level.pdf [Accessed 20 September 2012]

Bassey, M. (1995) **Creating Education Through Research: a Global Perspective of Educational Research for the 21st Century**. Newark: Kirklington Moor Press

Bell, J.F & Emery, J. (2006) The curious case of the disappearing mathematicians In Crisp, V. & Johnson, M. (eds.) **Research Matters: A Cambridge Assessment Publication** 2, 11-13 pp.21-23

Bell, D. & Nutt, C. (2002) Divided Loyalties divided Expectations: Research ethics, professional and Occupational Responsibilities In Mauthner, M et al. (eds.) **Ethics in Qualitative Research** pp70-90

Biggs, J. and Collis, K. (1982) **Evaluating the quality of learning The SOLO Taxonomy (Structure of the observed Learning Outcome)** New York: Academic Press
Biggs, J. and Tang C. (2007) **Teaching for Quality Learning at University What the student does** Third Edition New York: Open University Press

Blackwell, L.S., Dweck, C.S. & Trzesniewski, K.H. (2007) Implicit Theories of Intelligence Predict Achievement Across an Adolescent Transition: A Longitudinal Study and an Intervention. **Child Development** 78 (1): 246-263.

Boaler, J. (1997a) **Experiencing school mathematics-teaching styles, sex and setting.** Buckingham: Open University press

Boaler, J. (1997b) Reclaiming school mathematics: The girls fight back. **Gender and Education** 9 (3): 285-305

Boaler, J., William, D & Brown, M. (2000) Students' Experiences of Ability Grouping – disaffection, polarisation and the construction of failure. **British Educational Research Journal** 26 (5): 631-648

Boehm, V. R. (1980) Research in the "real world" – A conceptual model **Personnel Psychology** 33: 495-503

Brown, M., Brown, P. & Bibby, T. (2007) 'I would rather die': attitudes of 16-year-olds towards their future participation in mathematics In Kuchemann, D. (Ed.) **Proceedings of the British society for Research into Learning Mathematics** 27 (1): 18-23 March 2007

Brown, M., Brown, P. & Bibby, T. (2008) "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics **Research in Mathematics Education** 10 (1): 3-18

Burns, R.B. (2000) **Introduction to Research Methods**. London: SAGE

Burton, L. (1994) **Children Learning Mathematics Patterns and Relationships** London: Simon & Schuster Education

Buxton, L (1981) **Do you panic about maths? Coping with maths anxiety** London: Heinemann

Byers, V. & Herscovics, N. (1977) Understanding School Mathematics **Mathematics Teaching** 81: 24-27.

Campaign for Science and Engineering in the UK (CASE) (2008) **Higher UCAS Points for Science and Mathematics** [online]. Available from http://www.sciencecampaign.org.uk/documents/2008/UCASpoints.pdf [Accessed 20 September 2012]

Carr, W. & Kemmis, S. (1986) **Becoming Critical: Education, Knowledge and Action Research** London: Falmer Press

Charmaz, K. (2003) Grounded Theory Objectivist and Constructivist Methods In Denzin, N.K. & Lincoln, Y.S. (eds.) **Strategies of Qualitative Inquiry** Second Edition California: SAGE

Cole, N.S. (1990) Conceptions of Educational Achievement. **Educational Researcher** 19 (4) 2-7

Cooper, B. & Dunne, M. (1998) Anyone for tennis? Social class differences in children's responses to national curriculum mathematics testing. **Sociological Review** 46 (1):115-148.

Corbin, J. and Strauss, A. (2008) **Basics of Qualitative Research.** Third Edition California: SAGE

Crisp V., Sweiry, E., Ahmed, A. & Pollitt, A. (2008) Tales of the expected: the influence of students' expectations on question validity and implications for writing exam questions **Educational Research** 50 (1): 95 - 115

Dearing, R. (1994) **The National Curriculum and its Assessment. Final Report** London: School Curriculum and Assessment Authority [online] Available from: http://www.voced.edu.au/content/ngv13228 [Accessed 14 September 2012]

Dearing, R. (1996) **Review of qualifications for 16-19 year olds: full report** London: School Curriculum and Assessment Authority

Denscome, M. (1998) **The Good Research Guide: for small-scale social research projects** Buckingham: Open University Press

Department for Education and Skills (DES) (1982) **Mathematics counts. Report of the Committee of Inquiry into the Teaching of Mathematics in primary and secondary Schools in England and Wales.** London: Her Majesty's Stationery Office

Department for Education and Skills (DfES) (2005) 14-19 Education and Skills White Paper**: CM6476 London: The Stationery Office**

Department for Education (DFE) Statistical First Release (2011) **GCE/Applied GCE A/AS and equivalent examination results in England 2009/10 (Revised)** [online] Available from www.education.gov.uk/researchandstatistics/a00196667/ [Accessed 24 August 2012]

Dewey, J. (1960) **How We Think**. Massachusetts: Heath

Dick, T.P. & Rallis, S.F. (1991) Factors and influences on high school students' career choices. **Journal for Research in Mathematics Education** 22 (4): 281-292

Donaldson, M. (1978) **Children's Minds.** Glasgow: Collins

Duffin, M. & Simpson, A. (2000) A Search for Understanding. **Journal of Mathematical Behaviour** 18 (4): 415-427.

Dweck, C.S. (2000) **Self-Theories: Their role in Motivation, Personality, and Development.** Philadelphia: Psychology Press

Dweck, C.S. & Leggett E.L. (1998) A Social-Cognitive Approach to Motivation and Personality. **Psychological Review** 95 (2): 256-273.

Eder, D & Fingerson, L. (2002) Interviewing Children and Adolescents In Gubrium, J.F. & Holstein, J.A. (eds.) **Handbook of Interview Research: Context and Method** London: SAGE

Edexcel (2008) **GCSE Mathematics A (Linear) Paper 4 (Calculator) Higher Tier** [online]. Available from: http://www.edexcel.com/migrationdocuments/QP%20Current%20GCSE/June%202008/[ARCHIVED]%205540H_4H_que_20080602.pdf [Accessed 20 September 2012]

Entwistle, N. (1981) **Styles of Learning and Teaching** Chichester: John Wiley & Sons Ltd.

Entwistle, N. (1987) **Understanding classroom learning** London: Hodder and Stoughton

Entwistle, N.J. & Entwistle, A (1991) Contrasting forms of understanding for degree examinations: the student experience and its implication. **Higher Education** 22 (3): 205-227

Fisher-Hoch, H., Hughes, S. & Bramley, T. (1997) "What makes GCSE examination questions difficult? Outcomes of manipulating difficulty of GCSE questions" In **British Educational Research Association Annual Conference, University of York,** September 11-14 1997

Flyvbjerg, B. (2004) Five misunderstandings about case-study research In Seale, C. et al. (eds.) **Qualitative Research Practice** London: SAGE

Fontana, A & Frey, J.H. (1994) Interviewing: The Art of Science In Denzin, N.K. & Lincoln, Y.S. (eds.) **Handbook of Qualitative Research** London: SAGE

Fontana, A & Frey, J.H. (2003) The Interview: From Structured Questions to Negotiated Text In Denzin, N.K. &. Lincoln, Y.S (eds.) **Collecting and Interpreting Qualitative Materials** Second Edition California: SAGE

French, D. (2007) Simplicity and Surprise in School Mathematics. **The Mathematical Gazette** 91 (522): 387-405

French, D. (2002) Examining some changes in mathematics post-16 In Haggarty, L. (Ed.) **Aspects of Teaching Secondary Mathematics Perspectives on practice** London: RoutledgeFalmer pp.33-51

Gagne, R.M (1974) **Essentials of learning instruction** Illinois: Dryden Press

Gillard, D. (2011) Education in England: a brief history [online] Available from: www.educationengland.org.uk/history [Accessed 14 September 2012]

Gillham, B. (2000) **Case Study Research Methods.** London: Continuum

Glesne, C. & Peshkin, A. (1992) **Becoming Qualitative Researchers: An introduction** New York: Longman

Golding, J. (2007) Testing and Assessment: A response from the Mathematical Association. **Mathematical Association News** 134: 5-7

Gorard, S. (2001) **Quantitative Research Methods in Educational Research** London: Continuum

Hernandez-Martinez, P., Black, L., Williams, J., Davis, P., Pampaka M., Wake, G. (2008) Mathematics students' aspirations for higher education: Class, ethnicity, gender and interpretive repertoire styles. **Research Papers in Education** 23 (2): 153-65

Hernandez-Martinez, P., Williams, J., Black, L., Davis, P., Pampaka M., Wake G. (2011) Mathematics coursework as a facilitator of formative assessment, student-centred activity and understanding. **Research in Mathematics Education** 13 (2): 197-212

Hiebert, J. (1984) Children's Mathematics Learning: The Struggle to Link Form and Understanding. **The Elementary School Journal** 84 (5): 496-513.

HMI (1979) **Mathematics 5-11 HMI Series: Matters for Discussion No. 9** London: Her Majesty's Stationary Office [online] Available from: www.educationengland.org.uk/documents/hmi-discussion/maths-primary.html [Accessed 14 September 2012]

HMI (1982) **Mathematics in the Sixth Form HMI Series: Matters for Discussion No. 14** London: Her Majesty's Stationary Office [online] Available from: www.educationengland.org.uk/documents/hmi-discussion/maths-sixth.html [Accessed 14 September 2012]

HMI (1985) **The Curriculum from 5 to 16 HMI Series: Curriculum** London: Her Majesty's Stationary Office [online] Available from: www.educationengland.org.uk/documents/hmi-curricmatters/curriculum.html [Accessed 14 September 2012]

HMSO (1989) **Education Reform Act 1988** London: Her Majesty's Stationary Office [online] Available from: http://www.legislation.gov.uk/ukpga/1988/40/pdfs/ukpga_19880040_en.pdf [Accessed 14 September 2012]

Hodder, I. (2000) The Interpretation of Documents and Material Culture In Denzin, N.K. & Lincoln, Y.S. (eds.) **Handbook of Qualitative Research Second Edition** London: SAGE

Hodgson, A. & Spours, K. (1997) (eds.) **Dearing and Beyond: 14-19 Qualifications, Frameworks and Systems** London: Kogan Page Limited

Holliday, A. (2002) **Doing and Writing Qualitative Research** London: SAGE
Holloway, W. & Jefferson, A. (2000) **Doing Qualitative Research Differently: Free association, narrative and the interview method** London: SAGE

Hoyles, C. (1986) Scaling a mountain: a study of the use, discrimination and generalisation of some mathematical concepts in LOGO environment. **European Journal of Psychology of Education** 1 (2): 111-126

Hoyles, C. (1987) Tools for Learning: Insights for the mathematics Educator from a logo programming environment **For the Learning of Mathematics** 7 (2): 32-37

Hughes, S., Pollitt, A. & Ahmed, A. (1998) The development of a tool for gauging the demands of GCSE and A level exam questions In **BERA 1998 August 27-30 The Queen's university of Belfast**

Ipsos MORI (2006) **GCSEs and A level: the experiences of teachers, students, parents and the general public Research Study conducted for the Qualifications and Curriculum authority (QCA)** [online] Available from: http://www.ipsos-mori.com/Assets/Docs/Archive/Polls/qca.pdf [Accessed 20 September 2112]

Ireson, J. & Hallam, S. (1999) Raising standards: is ability grouping the answer? **Oxford Review of Education** 25 (3): 343-358

Ireson, J., Hallam, S. & Hurley, C. (2005) What are the effects of ability grouping on GCSE attainment? **British Educational Research Journal** 31 (4): 443-458

Janesick V.J. (2003) The Choreography of Qualitative Research Design: Minuets, Improvisations, and Crystalization Text In Denzin, N.K. & Lincoln, Y.S. (eds.) **Strategies of Qualitative Inquiry** Second Edition (California, SAGE)

Kaplan, A. & Middleton, M.J. (2002) Should Childhood be a Journey or a Race? Response to Harackiewicz et al. (2002) **Journal of Educational Psychology** 94 (3): 646-648.

Kounine, L., Marks, J. & Truss, E. (2008) **The value of mathematics** London: Reform

Kutnick, P., Sebba, J., Blatchford, P., Galton, M., Thorp, J. (2005) **The Effects of Pupil Grouping: Literature Review** Research Report No. 688 Department for Education and Skills: University of Brighton

Lawton, D., Gordon, P., Ing, M., Gibby, B., Pring, R., Moore, T. (1981) **Theory and Practice of Curriculum Studies.** London: Routledge & Kegan Paul)

Lublin, J. (2003) **Deep Surface and Strategic approaches to learning** Centre for teaching and learning [online] Available from www.warwick.ac.uk/services/ldc/development/pga/introtandl/resources/2a_deep_surfacestrategic_approaches_to_learning.pdf [Accessed 1 September 2012]

Mansell, W. (2007) **Education by Numbers The Tyranny of Testing.** London: Politico's

Marton, F. and Säljö, R. (1976) On qualitative differences in learning – 1: outcome and process **British Journal of Educational Psychology** (46):4-11

Matthews, A. & Pepper, D. (2006) **Evaluation of participation in A level mathematics***:* **Interim report**, London: Qualifications and Curriculum Agency

Matthews, A. & Pepper, D. (2007) **Evaluation of participation in A level mathematics**: **Final report,** London: Qualifications and Curriculum Agency

McLeod, J. (1999) **Practitioner Research in Counselling** London: Sage Publications

Menard, S. (2002) **Longitudinal Research** Second Edition**.** Sage University Papers Series on Quantitative Applications in the Social Sciences: 07-76 California: SAGE

Mendick, H. (2005) Mathematical stories: why do more boys than girls choose to study mathematics at AS-level in England? **British Journal of Sociology of Education** 26 (2): 235-251

Mendick, H. (2006) **Masculinities in mathematics.** Maidenhead: Open University Press

Mendick, H. (2008) Subtracting difference: troubling transitions from GCSE to AS-level mathematics. **British Educational Research Journal** 34 (6): 711-732

Midgley, C., Kaplan, A. & Middleton, M.J. (2001) Performance-approach goals; Good for what, for whom, under what circumstances, and at what cost? **Journal of Educational Psychology** 93 (1): 77-86

Minards, B. (2006) **An exploration of GCSE re-sit students' responses to the teaching and learning of mathematics.** MPhil thesis, University of Birmingham

Minards, B. & Prestage, S. (2008) What constitutes a 'hard' question in GCSE Mathematics: A bit of thought is required In Joubert, M. (ed.) **Proceedings of the British Society for the Research and Learning of Mathematics** 28 (3) November 2008:78-83

Moore, N. (2000) **How to Do Research The complete guide to designing and managing research projects.** Third Edition London: Facet Publishing

Mousley, J. (2005) What does Mathematics Understanding Look like*?* In P. Clarkson et al (eds.) **Mathematics Education Research Group of Australia 28 Building Connections: Research, Theory and Practice Proceedings of the Annual Conference** pp.553-560

Nardi, S. & Steward, S. (2002) I could be the best mathematician in the world…if I actually enjoyed it. **Mathematics Teaching** 179: 41-42

Nardi, S. & Steward, S. (2002) I could be the best mathematician in the world…if I actually enjoyed it: Part 2. **Mathematics Teaching** 180: 4-9

Nardi E. & Steward, S. (2003) Is Mathematics T.I.R.E.D? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom. **British Educational Research Journal** 29 (3): 345-367

Nuffield Review (2006) **Higher Education Focus Groups Preliminary Report of 14-19 Edcuation and Training** [online] Available from: www.docstore.mch-net.info/nuffield.pdf [Accessed 14 September 2012]

OCR (2007a) **Core Mathematics 1 June 2007** [online] Available from: http://pdf.ocr.org.uk/download/pp_07_jun/ocr_18145_pp_07_jun_l_gce.pdf [Accessed 20 September 2012]

OCR (2007b) **Core Mathematics 2 June 2007** [online] Available from: http://pdf.ocr.org.uk/download/pp_07_jun/ocr_18146_pp_07_jun_l_gce.pdf [Accessed 20 September 2012]

OCR (2007c) **Core Mathematics 4 June 2007** [online] Available from: http://pdf.ocr.org.uk/download/pp_07_jun/ocr_18148_pp_07_jun_l_gce.pdf [Accessed 20 September 2012]

OCR (2007d) **Reports on the Units June 2007** [online] Available from: http://pdf.ocr.org.uk/download/rep_07/ocr_19201_rep_07_l_gce_jun.pdf [Accessed 20 September 2012]

OCR (2009a) **Core Mathematics 1 January 2009** [online] Available from: http://pdf.ocr.org.uk/download/pp_09_jan/ocr_34765_pp_09_jan_l_gce_4721_01.pdf [Accessed 20 September 2012]

OCR (2009b) **Core Mathematics 2 June 2009** [online] Available from: http://pdf.ocr.org.uk/download/pp_09_jun/ocr_41550_pp_09_jun_gce_4722_01.pdf [Accessed 20 September 2012]

OCR (2010a) **AS/A2 GCE Mathematics Specification** [online] Available from: http://pdf.ocr.org.uk/download/kd/ocr_10096_kd_gce_spec.pdf [Accessed 20 September 2012]

OCR (2010b) **Reports on the Units January 2010** [online] Available from: http://pdf.ocr.org.uk/download/rep_10/ocr_40928_rep_10_l_gce_jan.pdf [Accessed 20 September 2012]

OCR (2010c) **Reports on the Units June 2010** [online] Available from: http://pdf.ocr.org.uk/download/rep_10/ocr_48466_rep_10_gce_jun.pdf [Accessed 20 September 2012]

OCR (2010d) **Core Mathematics 3 January 2010** [online] Available from: http://pdf.ocr.org.uk/download/pp_10_jan/ocr_52277_pp_10_jan_gce_4723.pdf [Accessed 20 September 2012]
OCR (2010d) **Core Mathematics 4 June 2010** [online] Available from: http://pdf.ocr.org.uk/download/pp_10_jun/ocr_57450_pp_10_jun_gce_4724.pdf [Accessed 20 September 2012]

OCR (2010e) **AS/A2 GCE Mathematics All documents** [online] Available from: http://www.ocr.org.uk/qualifications/type/gce/maths/maths/documents/[Accessed 20 September 2012]

Ofqual (2009) **Review of Standards in GCE mathematics in 2004 and 2007** London: Office of the qualifications and Examinations Regulator

Ofsted (2006) **Evaluating mathematics provision for 14-19 year olds** London: Ofsted

Ofsted (2008) **Mathematics: understanding the score** London: Ofsted

Pendlington, S. (2006) The Balance of Power in the Mathematics Classroom In Hewitt, D. (ed.) **Proceedings of the British Society for Research into Learning Mathematics** 26 (1):.59-64

Patton, M, Q. (2002) **Qualitative Research & Evaluation Methods** Third Edition California: SAGE

Pearsall, J. (Ed.) (1998) **The New Oxford Dictionary of English** New York: Oxford University Press

Pintrich, P.R. (2000) Multiple goals, multiple pathways: The role of goal orientation in Learning and Achievement. **Journal of Educational Psychology** 92 (3): 544-555

Pirie, S. & Kieren, T. (1994) Growth in mathematical understanding: How can we characterise it and how can we represent it? **Educational Studies in Mathematics** 26: 165-190.

Pollitt, A & Ahmed, A. (1999) A New Model of the Question Answering Process In **International Association for Educational Assessment, Bled, Slovenia, May 1999**

Pollitt, A & Ahmed, A. (2000) Comprehension Failures in Educational Assessment In **European Conference on Educational Research, Edinburgh, September 2000**

Pollitt, A & Ahmed, A. (2008) Outcome Space Control and Assessment In **9th Annual Conference of the Association for Educational Assessment – Europe Hissar, November 2008**

Prestage, S & Perks, P. (2006) Doing Maths or Practicing the Finished Product In Hewitt, D. (ed.) **Proceedings of the British Society for Research into Learning Mathematics** 26 (1): 65-70

Pring, R. (2000) **Philosophy of Educational Research** London: Continuum

Rittle-Johnson & Alibali M.W. (1999) Conceptual and Procedural Knowledge of Mathematics: Does One Lead to the Other? **Journal of Educational Psychology** 91 (1): 175-198.

Rieber R. W. (Ed.) (1987) **The Collected Works of L. S. Vygotsky Volume 6 Scientific Legacy** New York: Kluwer Academic/ Plenum Publishers

Roberts, G. (2002) **SET for success: The supply of people with science, technology, engineering and mathematics skills** London: The Stationery Office

Robson, C (1993) **Real World Research A Resource for social Scientists and Practitioner-Researchers** Oxford: Blackwell

Robson, C (2002) **Real World Research A Resource for social Scientists and Practitioner-Researchers** Second Edition Oxford: Blackwell

The Royal Society (2008) A 'state of the nation' report on the participation and attainment of 14-19 year olds in science and mathematics in the UK, 1996-2007 **[online]. Available from: http://royalsociety.org/uploadedFiles/Royal_Society_Content/education/policy/state-of-nation/SNR2-full-report.pdf [Accessed 20 September 2012]**

Ruspini, E. (2002) **Introduction to Longitudinal Research**. London: Routledge

Ryan, G.W & Bernard, H.R (2003) Data Management and Analysis Methods In Denzin, N.K. & Lincoln, Y.S. (eds.) **Collecting and Interpreting Qualitative Materials** Second Edition California: SAGE

Saldana, J. (2003) **Longitudinal Qualitative Research: Analyzing Change through Time** California: Altamira Press

Sierpinska, A. (1990) Some Remarks on Understanding in Mathematics**. For the Learning of Mathematics** 10 (3): 24-41

Sierpinska, A. (1994) **Understanding in Mathematics.** London: Falmer

Skemp, R. R. (1971) **The Psychology of Learning Mathematics** New York: Routledge

Skemp, R.R. (1976) Relational Understanding and Instrumental Understanding. **Mathematics Teaching** 77: 20-26.

Skemp, R.R. (1982) Symbolic Understanding. **Mathematics Teaching** 99: 59-61.

Slavin, R. E. (1990) Achievement Effects of Ability grouping in Secondary Schools: A Best-Evidence Synthesis. **Review of Educational Research** 60 (3): 471-499

Smith, A. (2004) **Making Mathematics Count The Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education** London: The Stationary Office Limited

Spens (1938) **The Spens Report Secondary Education with Special Reference to Grammar Schools and Technical High Schools** London: Her Majesty's Stationary Office [online] Available from: www.educationengland.org.uk/documents/spens/ [Accessed 14 September 2012]

Stake R.E (2003) Case Studies In Denzin NK, & Lincoln YS (eds.) **Strategies of Qualitative Inquiry** Second Edition California: SAGE

Swan, M. (2000) GCSE Mathematics in Further Education: Challenging beliefs and practices. **The Curriculum Journal** 11 (2):199-223.

Tall, D (1992) Success and failure in arithmetic and algebra [online] Available from: http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1992d-edinburgh-plenary.pdf [Accessed 14 September 2012]

Taverner, S. & Wright, M (1997) Why go modular? A review of modular A-level Mathematics. **Educational Research** 39 (1): 104-112

Tebbutt, M.J. (1993) Sixth Formers' Perceptions of a level and Degree Courses in Physics and Mathematics. **Research in Science and Technological Education** 11 (1): 27-37

TGAT (1988) **National Curriculum Task group on Assessment and Testing A Report** London: Department of Education and Science [online]. Available from: www.educationengland.org.uk/documents/pdfs/1988-TGAT-report.pdf [Accessed 14 September 2012]

Thomas, G. (2011) **How to do Your Case Study A guide for students and researchers** London: SAGE

Tomlinson, M. (2004) **14-19 Curriculum and Qualifications Reform: Final Report of the Working group on 14-19 Reform** London: The Stationary Office Limited

Torrance, H (2007) Assessment as Learning? How the use of explicit learning objectives, assessment criteria and feedback in post-secondary education and training can come to dominate learning. **Assessment in Education: Principles, Policy & Practice** 14 (3): 281-294

Turner, J.C., Midgley, C., Meyer, D.K. et al. (2002) The Classroom Environment and Students' Reports of Avoidance Strategies in Mathematics: A Multimethod Study. **Journal of Educational Psychology** 94 (1): 88-106.

Tymms, P. (1995) **Comparing Examination Boards and Syllabuses at A-Level: Students Grades, Attitudes and Perceptions of Classroom Processes** Reference 85678 CEM Centre, University of Durham

Tymms, P., Coe, R., Merrell, C. (2005) **Standards in English schools: changes since 1997 and the impact of government policies and initiatives** [online]. Available from: http://www.civitas.org.uk/pdf/Tymms_SundayTimes_v03.pdf [Accessed 20 September 2012]

Van Hiele, P. (1958) **The Child's Thought and Geometry** New York: City University

Vidal Rodeiro, C. (2007) **A-level subject choice in England: Patterns of uptake and factors affecting subject preferences**. Cambridge: Cambridge Assessment

Von Glasersfeld, E. (1983) Learning as Constructive Activity In Bergeron, J.C. & Herscovics, N. (Eds.) Pro**ceedings of the 5th Annual Meeting of the North American Group of Psychology in Mathematics Education Vol.1 Montreal: PME-NA** pp.41-101

Von Glasersfeld, E. (1989) Learning as Constructive Activity In Murphy, P. and Moon, B (Eds.) **Developments in learning and assessment** Great Britain: Hodder and Stoughton Educational pp.5-18

Wake, G. (2011) Introduction to the Special Issue: deepening engagement in mathematics in pre-university education. **Research in Mathematics Education** 13 (2): 109-118

Wiliam, D. & Bartholomew, H. (2004) It's not which school but which set you're in that matters: the influence of ability grouping practices on student progress in mathematics **British Educational Research Journal** 30 (2): 279-294

Yin, R. K. (2009) **Case Study Research Design and Methods** Fourth Edition California: SAGE

AS Teaching Timetable September 2008-July 2009

Term	'Pure Teacher'	'Applied Teacher'
Autumn Term Year 12	GCSE revision	GCSE revision
7 Weeks	GCSE revision	GCSE revision
'Initial Test' in 2nd week	C1	C1
	C1	C1
	C1	C1
	C1	C1
	C1	C1
HALF TERM	HALF TERM	HALF TERM
7 Weeks	C1	C1
	C1	C1
	C1	C1
	C1	C1
	C1 Revision	C1 Revision
Mocks Thurs & Fri	C1 Revision	C1 Revision
Last Week	Go over Mock (C1 Revision)	(C1 Revision)
CHRISTMAS	CHRISTMAS	CHRISTMAS
Spring Term: 6 Weeks	C2	S1 or D1
'Flexi-time' for 2 weeks	C2	S1 or D1
whilst AS Exams on	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
HALF TERM	HALF TERM	HALF TERM
6 Weeks	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
	C2	S1 or D1
EASTER	EASTER	EASTER
Summer Term: 3 Weeks, 2days	C2/Mock	S1 or D1/Mock
Mocks Thurs Fri 1st week	Finish C2 & go over mock	Finish S1/D1, go over mock
	C2 Revision	S1 or D1 Revision
EXAM LEAVE	EXAM LEAVE	EXAM LEAVE
3 Weeks	'The three week plan' C3	'The three week plan' C3
	'The three week plan' C3	'The three week plan' C3
	'The three week plan' C3	'The three week plan' C3
Last Week	Last Week	Last Week

A2 Teaching Timetable September 2008-July 2009

Term	'Pure Teacher'	'Applied Teacher'	'Fifth Lesson'
Autumn Term Year 13	Revision of '3 week plan'	Revision of '3 week plan'	Not on in first week
7 Weeks	C3	C3	C3
Test of '3 week plan'	C3	C3	C3
Material in 2nd week	C3	C3	C3
	C3	C3	C3
	C3	C3	C3
	C3	C3	C3
HALF TERM	HALF TERM	HALF TERM	HALF TERM
7 Weeks	C3	C3	C3
	C3	C3	C3 Revision
	C3	C3	C3 Revision
	C3	C3	C3 Revision
	C3 Revision	C3 Revision	C3 Revision
Mocks Thurs & Fri	C3 Revision	C3 Revision	C3 Revision
Last Week	Go over Mock (C3 Revision)	(C3 Revision)	C3 Revision
CHRISTMAS	CHRISTMAS	CHRISTMAS	CHRISTMAS
Spring Term: 6 Weeks	C4	S2 or D2	Not on during
'Flexi-time' for 2 weeks	C4	S2 or D2	'flexi-time'
whilst AS Exams on	C4	S2 or D2	C4
	C4	S2 or D2	C4
	C4	S2 or D2	C4
	C4	S2 or D2	C4
HALF TERM	HALF TERM	HALF TERM	HALF TERM
6 Weeks	C4	S2 or D2	C4
	C4	S2 or D2	C4
	C4	S2 or D2	C4
	C4	S2 or D2	C4
	C4	S2 or D2	C4 Revision
Mocks Thurs Fri 1st week	C4 Mock	S2 or D2 Mock	Not on during mocks
EASTER	EASTER	EASTER	EASTER
Summer Term: 3 Weeks	Go over mock & Revision	Go over mock & Revision	C4 Revision
	C4 Revision	S2 or D2 Revision	C4 Revision
	C4 Revision	S2 or D2 Revision	C4 Revision
EXAM LEAVE	EXAM LEAVE	EXAM LEAVE	

Name Form Target GCSE Maths Grade

Please answer the following questions honestly and in as much detail as possible. Your responses will be kept confidential and will not be seen by any other teachers.

1. **List the AS-level subjects you have chosen to do in order of preference. Write a brief reason why you chose each subject.**

Choice	Subject	Reason
1		
2		
3		
4		
(5)		

2. **If you did not choose maths, tell me why. Then go to question 4.**

3. **What do you hope learning A-level maths will be like?**

4. **If you were talking to a Year 9 Student, how would you describe what learning maths is like in Key Stage 4?**

5. **What do you think makes a person good at maths?**

P.T.O.

6. Please complete the percentages:

Being good at maths = ____% effort + ____% ability

7. **What do you think are the most important qualities for a person to be good at A-Level maths?** Please rate the items below **1 as most important** to **5 as least important** in each box.

- ❑ An A* at GCSE
- ❑ Willing to work on hard questions
- ❑ Likes to be challenged
- ❑ Asks questions when they don't understand
- ❑ Found GCSE easy

7a. Please tick only one choice: **Will you revise for your GCSE maths exams for**

- ❑ More time than other subjects?
- ❑ About the same time as other subjects?
- ❑ Less than other subjects?

7b. **Why?**

Thank-you for your time.

Would you be willing to answer some follow-up questions in an interview?
This would take less than 15 minutes during one lunchtime next term.

❑ Yes ❑ No

Name ____ Form ________

Please answer the following questions honestly and in as much detail as possible. Your responses will be kept confidential and will not be seen by any other teachers.

1. **What maths course have you done this year?** Circle one of the options.
   AS with Decision AS w. Statistics AS w. Mechanics Further

2. **Why did you choose to study maths in Year 12?**

3. **What other subjects are you doing? Please give a reason why you chose each subject.**

4. **How did your maths course compare to what you thought it would be like?**

5. **If you were talking to a Year 11 Student, how would you describe what learning maths is like in Year 12, would you recommend it?**

6a. **Are you continuing with maths next year?** Tick one of the options below.

❑ Definitely Continuing ❑ Might Continue ❑ Definitely Not Continuing

6b. **Please give a reason for your decision for whether to continue with maths next year.**

P.T.O.

6. **What do you think makes a person good at maths?**

8. **How did the amount of work done in lessons compare to your other subjects?** Tick a box

- ❑ More than all
- ❑ About the same
- ❑ Less than all

More than some (please name subject(s))
Less than some (please name subject(s))

8. **How did the amount of homework set compare to your other subjects?** Tick a box

- ❑ More than all
- ❑ About the same
- ❑ Less than all

More than some (please name subject(s))
Less than some (please name subject(s))

8. **How did the difficulty of the work compare to your other subjects?** Tick a box

- ❑ More difficult than all
- ❑ About the same
- ❑ Less difficult than all

More difficult than some (please name subject(s))
Less difficult than some (please name subject(s))

11. **What do you think are the most important qualities for a person to be good at A-Level maths?** Please rate the items below **1 as most important** to **5 as least important** in each box.

- ❑ An A* at GCSE
- ❑ Likes to be challenged
- ❑ Found GCSE easy
- ❑ Willing to work on hard questions
- ❑ Asks questions when they don't understand

12a. Please tick only one choice: **Did you revise for your AS maths exams for**

- ❑ More time than other subjects?
- ❑ About the same time as other subjects?
- ❑ Less than other subjects?

12b. **Why?**

13. Please complete the percentages:

Being good at maths = ____% effort + ____% ability

Thank-you for your time.

Would you be willing to answer some follow-up questions in an interview?
This would take less than 15 minutes during one lunchtime next term.

- ☐ Yes
- ☐ No

Name ______________________________ Form ________

Please answer the following questions honestly and in as much detail as possible.
Your responses will be kept confidential and will not be seen by any other teachers.

1. **Which applied module have you done this year?** Circle one of the options.
D2 S2 M2 D1 S1 M1

2. Why did you choose to continue with maths in Year 13?

3. **What other subjects are you doing? Give a reason why you chose each subject.**

Subject	**Reason**

3. How did Core 3 and Core 4 compare to what you thought they would be like?

5.a. If you were talking to a Year 12 Student, how would you describe what learning maths is like in Year 13?

5.b. Would you recommend A2 Maths to an AS student? Please give a reason for your answer.

6a. What are you planning to do next year?

6b. Does your planned course/training/job involve learning any more maths?
Please circle one of the following.

Mainly **Partly** **None** **Don't know**

P.T.O.

7.a. What has been the most difficult part of learning Core 4?

7.b. Why?

8. What do you think makes a person good at maths?

9. **How did the amount of work done in C3 & 4 lessons compare to your other subjects?**
   - ❑ More than all
   - ❑ More than some (please name subject(s))________________________
   - ❑ About the same
   - ❑ Less than all

10. **How did the amount of C3 & 4 homework set compare to your other subjects?**
    - ❑ More than all
    - ❑ More than some (please name subject(s))________________________
    - ❑ About the same
    - ❑ Less than all

11. **How did the difficulty of the work in C3 & 4 compare to your other subjects?**
    - ❑ More difficult than all
    - ❑ More difficult than some (please name subject(s))____________________
    - ❑ About the same
    - ❑ Less difficult than all

12a. What do you think are the most important qualities for a person to be good at A-Level maths? Please rate the items below **1 as most important** to **5 as least important** in each box.

❑ An A* at GCSE

❑ Likes to be challenged

❑ Found GCSE easy

❑ Willing to work on hard questions

❑ Asks questions when they don't understand

12b. What other qualities are important for a person to be good at A-Level maths?

13a. Please tick only one choice: **Are you planning to revise for your Core 4 exam for**

- ☐ More time than other subjects?
- ☐ About the same time as other subjects?
- ☐ Less than other subjects?

13b. **Why?**

14. Please complete the percentages:

Being good at maths = ____% effort + ____% ability

Thank-you for your time.

October 2008: Student Interviews

Why Choose AS Mathematics and Co-ordinate Geometry Questions

1. Why do you want to take maths at AS level?
2. Did you choose your AS levels in any order?
    a. If so, which order?
3. Are you planning to do A2 maths or stop at AS?
4. How did the difficulty of GCSE maths compare to that of your other GCSEs?
5. When were you challenged when learning GCSE mathematics?
6. How does the maths you are doing now compare to GCSE maths?
7. What is your perception of what learning AS maths will be like?
8. When do you enjoy maths?
9. What do you do when you find maths difficult?
10. What makes a person good at maths?

    Introduce four co-ordinate geometry questions and have mini whiteboard and pen available.

11. Can you describe whether any of these questions are easy or difficult and why?
    a. What makes it easy?
    b. What makes it difficult?
    c. If you had to order them from least to most difficult, what would the order be?
    d. What sort of questions are you getting in class and for HW?

Thank you for your time.

February 2009 Interview: Students
Core 1 Examination and initial thoughts on Learning Core 2

1. Please talk me through the C1 paper.
Which questions did you find easy, which did you find difficult?
    a. Why?

2. How do you use the marks allocated for each parts of the question?

3. Do you think the order of the questions is significant?
    b. Do you answer the questions in the same order as the question paper?

4. How did the actual paper compare with your expectations?

5. How do you feel about the time limit of 90 mins?
    a. Did you have enough time?
    b. What would you do differently if there was no time limit?

6. What do you think C2 will be like compared to C1?

7. How are C2 lessons compared to C1?

8. Are there any differences in C2 HW compared to C1?

Thank-you for your time

February 2009 Interview: Teachers
Core 1 Examination and initial thoughts on Learning Core 2

1. Please talk me through the C1 paper.

Which questions do you think students found easy, which do you think they found difficult?

   a. Why?

2. Do you think the marks allocated for each parts of the question are significant?

3. Do you think the order of the questions is significant?

4. How did the actual paper compare with your expectations?

5. How do you feel about the time limit of 90 mins?
   a. Do you think students have enough time?
   b. What would you get students to do differently if there was no time limit?

6. How do you think C2 compares to C1?

Thank-you for your time

July 2009: Core 2 Examination and Learning Core 3

Students

1. What do you think about the overall difficulty of the C3 paper?
2. What do you think is the easiest question and why?
3. What do you think is the most difficult question on the paper and why?
4. What have 3 week sessions been like in maths?
5. How does the amount of work compare to that in other subjects?
6. How does the difficulty of the work compare?
7. Do you think there is any difference in the difficulty level of C3 compared to C2?
8. Have you received any messages about the difficulty of C3?
    a. From whom?

Teachers

1. What do you think about the overall difficulty of the C3 paper?
2. What do you think is the easiest question and why?
3. What do you think is the most difficult question on the paper and why?
4. What have 3 week sessions been like in maths?
5. Do you think there is any difference in the difficulty level of C3 compared to C2?
6. How do you describe A2 maths to Year 12 students who are considering whether to continue to Year 13?

November 2009 Interviews about learning Core 3 topic: Trigonometry

Students

1. Describe why you chose this as your 'Easy' question?
    a. What makes Core 3 questions easy?
2. Describe why you chose this as your 'Difficult' question?
    a. What makes Core 3 questions difficult?
3. Where did you find the questions?
4. What do you think are the difficult parts of trig to learn?
5. What do you think about the difficulty of C3 you have learned so far?

Teachers

1. Describe why you chose this as your 'Easy' question?
    a. What makes Core 3 questions easy?
2. Describe why you chose this as your 'Difficult' question?
    a. What makes Core 3 questions difficult?
3. Where did you find the questions?
4. What do you think are the difficult parts of trig for students to learn?
5. What do you think about the difficulty of C3 material?

February 2010 Core 3 Examination

Students

1. What do you think about the overall difficulty of the C3 paper?
2. What do you think was the easiest question on the paper and why?
3. What do you think was the most difficult question on the paper and why?
4. To what extent do you expect the exam to be similar to past papers?
5. To what extent do you think the teachers expect the papers to be similar?
6. What effect do you think the new A * has had on the exams?
7. How difficult have you found learning the C4 material so far?
8. Have you heard any messages about the difficulty of C4?
    a. From whom?

Teachers

1. What do you think about the overall difficulty of the C3 paper?
2. What do you think was the easiest question on the paper and why?
3. What do you think was the most difficult question on the paper and why?
4. To what extent do you expect the exam to be similar to past papers?
5. What effect do you think the new A * has had on the exams?
6. How difficult do you think students have found learning the C4 material so far?
7. How do you describe the difficulty of C4 compared to the other core modules?

Core 4 Vectors Interviews: May 2010

Students

1. How did you decide these questions were 'difficult'?
2. What about the questions made them difficult?
3. Where did you get the questions from?
4. Where else did you look?
5. How did you find learning vectors as a topic?
6. How have you found the difficulty of Core 4 compared to Core 3?
7. What has been the most difficult topic to learn and why?

Teachers

1. How did you decide these questions were 'difficult'?
2. What about the questions made them difficult?
3. Where did you get the questions from?
4. Where else did you look?
5. How do you think students find learning vectors as a topic?
6. How do you compare the difficulty of Core 4 compared to Core 3?
7. What is the most difficult Core 4 topic for students to learn and why?

Sample of Interview Discussions.

Questions in bold indicate those which were asked in all interviews.
I is used to denote interviewer question or prompt.
S is used to denote student response.

Interview with 'Bella': October 2008
Reasons for Choosing AS Mathematics & Co-ordinate Geometry Questions

1. **Why do you want to take maths at AS level?**

S Well I enjoy it and it goes well with the other subjects I do, business studies, economics and German.

2. **Did you choose your AS levels in any order? If so, which order?**

S I actually took 5 to start with but then I dropped Psychology. I think they are all the same, they all go together.

Do you know what you want to do in future?
S Yes Business or Economics. Universities for some courses, they need you to have maths. It's a good A-level to have really.

3. Are you planning to do A2 maths or stop at AS?
S Well at the moment, I'm kind of thinking that I might have to stop, it's really hard. I don't know if I can cope with A2. But then again I should probably drop business studies or economics because of the university thing. But then I don't know whether it's better to get good grades that I can get in them, or have the subject like maths. I'll see how the January exams go.

4. How did the difficulty of GCSE maths compare to that of your other GCSEs?
S Well I can't really remember, I guess because I always work hard no matter what it is. Everyone says A-levels are particularly hard, but maybe because I've always worked so hard it doesn't really seem that bad.

I Can you compare it to your other GCSE subjects?
S It was more difficult. It was one of my best grades so it was Ok. But I didn't expect the grade I got. I got an A*, I didn't expect to get it at all, it was a shock.

I What did you expect?
S I was expecting, well hoping for an A.

I So how did you feel when you got that A*?
S I don't know, I was really happy because maths was a subject that I really wanted to do well in. I don't know why, I Just did.

I Can you remember any particular bits of GCSE that were difficult?
S The graphs. I never understood the graphs, like the f(x) thing, I've never been very good at shape.

I What about graphs makes them difficult?

S I dunno, I just see them and panic. I just can't see how a number can relate to, can be put onto a graph, I just can't see it. It takes me ages to grasp it.

I Is that the same with the shapes?
S Yes I think so, if they describe a shape I can't visualise it or put it into the context of the question.

5. When were you challenged when learning GCSE mathematics?
S Yes (laughs) I dunno really, every lesson it took me quite a long time to grasp and most other people were able to get it. Whereas I had to write down every single step otherwise I couldn't do it. It took me a lot longer than everyone else in the class. So maybe sometimes in the lessons they would be going really fast and I couldn't keep up

6. How does the maths you are doing now compare to GCSE maths?
S Erm, well certain aspects are just like carrying on, like indices. You kind of use some of the things you've learned at GCSE in a different way. It seems quite different what you do.

7. What is your perception of what learning AS maths will be like?
S I don't know, like differentiation and stuff. We did a problem today that was a GCSE coursework that took 2 weeks, but we did it in 25 minutes just by using differentiation. Every lesson and every homework, without the textbook and the answers in the back, I have no idea how to even get to the answer.

I Did that happen at GCSE?
S No, at GCSE I always did the questions and then look out the answer, whereas at A-level sometimes I don't have any idea how to answer. So I look at the answer and try to work out how to get to it. But that never really happened at GCSE.

I How do you feel when that happens?
S The main thing is that it happens a lot so you get disheartened, but then you take to other people and they have the exact same problems so that makes me feel a bit better and then I go and ask for help.

9. What do you do when you find maths difficult?
S I speak to or ask some people, or well… I've just started private tutoring outside of school.

I What's that like?
S It really helps me because he really makes me understand it rather than just showing me a method which is why I'm getting it. Also he doesn't go really fast, so I have time to ask what he's doing.

I How does your tutor make you understand?
S Erm, because I can ask every little question that I have, whereas in class you can't really do that because everyone else, well lots of people, have got it.

I How do you know everyone else has got it?
S I don't know (laughs) I ask questions but sometimes you and the class just has to move on.

10. What makes a person good at maths?
S They have to be logical and be prepared to keep going at something. I think you have to have a natural ability, I think it's also to do with attitude. But yes, I have to work a lot harder than someone who is more natural. But I guess they can't really do much about it, they don't have to do as much work.

I Do you think you have a natural ability for maths?
S I don't think so.

I Do you think you are good at maths?
S No, not particularly (laughs) I'm ok at it, I guess I'm comparing myself to people who are really good. Like in my class virtually everyone apart from a couple of us were in the top set last year and so are really good at maths.

I What about the people who have just joined the school?
S They all seem to be really good as well, there's a few that seem to be the same level as me, but no-one seems particularly bad.

I Do you always look to people who you think are better than you?
S I dunuo, I don't really think about it.

I what made you get a tutor?
S Well I told my Mum that I'm finding maths really hard and she said that she had a tutor when she did maths A- level, so that's why.

Co-ordinate Geometry Questions

11. Can you describe whether any of these questions are easy or difficult and why?
S That one's (I) hard, I don't know why, I just can't picture it on the graph. I can't see how they could be connected. I could probably draw it though and work it out from that. I think that one's alright (G) because it's just Pythagoras and if you did the dotted *lines (describes making 3 right angled triangles with hypotenuse on each side of given triangle whilst sketching on the diagram).* Then I'd find the length of all the sides and check.

a. Why is that (G) easier than the other one (I)?
S I dunno, I can just see it, the diagram definitely helps and we've done it recently as well.

I How about the other 2?
S I think that one's alright (S) and that one's hard (D).

b. Why do you think that's difficult?
S Because I don't know how to do it (laughs) I know I should be able to do it, I just can't remember.

I What do you do if you can't remember?
S Erm, dunno I'd try to figure it out by seeing what it looks like.

I How about this one (S)?

S I'd have to draw it to see if I could do it.

I Once you've drawn it, how do you feel the difficulty level?
S Erm because like finding the equation I'm not so good at that, I don't know. But when you're just finding the co-ordinates it's not really algebraic, it's just numbers.

c. If you had to order them from least to most difficult, what would the order be?
S That would be the least difficult (G) then (S), then (D) then (I).

I Why do you think (I) is the most difficult one?
S Because I can't see how you do it, I can't remember.

I Then how would you start it?
S Erm, I don't know, if that just came up in the exam I'd leave it and come back to it at the end if I've got time left over and then feel out some way to do it. I could probably do it eventually it would just take me awhile.

I Is remembering a really important part of you doing maths?
S Yes, I try to remember what I've been taught.

I So what happens when you come across something different?
S Er, well I probably skip it and then go back to it and try and find a way. I could do it if I had a lot of lime, but if I've just got a of couple of minutes to do it, like in an exam, I'd probably panic.

Interview Feb 2010 with 'Craig':
Core 3 Examination

1. What do you think about the overall difficulty of the C3 paper?
I found it a lot more difficult than the mock had been and quite possibly any paper I had done before. But I think that's more down to the last question, it was more of a shock I could see how to do the rest of the questions it was just inputting numbers into formulas or applying methods, but question 4 required actual thinking (laughs). The mock I got a hundred percent in so I just settled in with an air of confidence I guess and it was more difficult, but I think everyone found that. Seven a told you the rate of the area was increasing and asked you about the radius so you had to work one way and then work backwards. Whereas what we have done before it was only working forwards and quite a few of these questions were like that. Eight iii, if you knew how to do that, you just input the numbers, but working that through in your head you had to see what they had done to get those equations. It was a bit more logical.

2. What do you think was the easiest question on the paper and why?
Question 4 i, just input in number twice. In fact the whole of four really. The last part is the one rule it doesn't matter what graph you get, you always put the same answer for that, I can't see how you would drop a mark for that if you had just listened in those five minutes you were told the rule for that.

3. What do you think was the most difficult question on the paper and why?
Nine. Well nine i was a requirement to know what tan 45 is. As soon as you knew that then it was easy, to be fair really you did have a calculator so you could have found that. It was the other two parts that I found a lot harder. I came out with an answer for part two but not so much for part three where I had an answer but I knew it was wrong, I'm not expecting many marks for that. I could see where I had just made up some maths halfway through the question. The second one, I could see that it was double angle formula, I just couldn't see how to work it through, in the end I just gave up. I think that the topic is quite hard because there are so many rules and you don't necessarily know which way to go. You are given all the rules so you don't need to learn them, it's just knowing how to apply them. In all the other questions you are either told which rule to use or it is quite clear which one you should. But because there are like ten different rules for the angles, you have no idea what way to go down. You are looking at 9 part 3 and wondering do I need to get it in terms of sin or cos first, which way do I need to put them round, do I need to get rid of the 7? There are so many different factors there that you don't know which to get rid of first.

7. How difficult have you found learning the C4 material so far?
I'd say it's alright actually, but we are only doing the foundations of it. I'm expecting it to get a lot harder as we go along. A lot of it just seems to be the stuff we have already done before. Its C4, it's got to be a third harder than C3 (laughs). It has got harder, I remember C1 essentially being the top end of GCSE and C2 just seemed to be we are just taking that into some different areas. C3 was the only area where we started learning properly new maths, new ideas.

8. Have you heard any messages about the difficulty of C4?
Not really. Mr. A said to me once at a parents eve C4 would be where I start finding it hard. So that may control my thoughts on it (laughs) I've been told its going to be hard so it is. Nothing from other students, no.

Interview 5 May 2010: 'Teacher 4' Core 4 Vectors

1. **How did you decide these questions were 'difficult'?**

From my recollection of how students coped with them when they did them and examiner's reports for the feeding back generally. The two that I've picked are from the old P3 module that has the same vectors content as C4 now.

2. **What about the questions made them difficult?**

[OCR, P3, June 2003]
It's question 7of 8 so its towards the end so you are going to expect that to be a slightly harder question anyway. The thing that makes the first part of this tricky is that in one of the vectors 2 of the values are not given, they are given as letters and then you are given some facts that they are perpendicular and intersecting. The first part is worth six marks and you have to pull together quite a lot of different parts of your understanding about vectors and mostly the kind of question that most would have practiced would be ones where they know all the values in the vectors as well. So that is quite tricky as you have to synthesise quite a few different things and make connections between what you have learned. If you don't get that done you may well be struggling with the rest of the question as well. Part two is a hence, so if you haven't got a and b there's no chance of hence. The last part of the question is a slightly

unusual one in the sense that we are on a particular line and we are looking at 2 points either side of the point a certain distance away. Sometimes with vectors questions you could almost have done the same question yourself in your revision and this isn't one of those at all.

[OCR, P3, June 2005]
It's one of those rather than giving you nice column vectors in the question, you have a diagram. So you have to show geometrical understanding. I recall when these questions first came up they caught us out, the staff who had been teaching it and the students as well because they had been so used to practicing the ones where it is all given in numerical form. So the ability to understand the geometry of what's going on caught people out. The examiners' report suggested that people got in a muddle with signs as well. You are looking to find an equation of a line which is not on the diagram as well which they don't like. Then the last part of the question is asking you to find the angle between two direction vectors which do not intersect. In terms of students being able to look at that and grasp that intellectually, I think they find that tougher than ones they can actually see where there's an angle between them. Again it is question 7 out of 9 or 10 so its going to be a tougher one.

What were the student's responses?
It was just not knowing how to get started. The good students will be fine with it because they won't be phased by the information they expect to be there isn't there, But weaker and middle ability students had to make-up the equations from the information given failed to see the connections. There were several stages of processing needed, rather than the easier questions might be find the magnitude or find the equation, standard questions that are worked through.

How standard do you think vectors questions are?
The Core 4 questions, I think that really a decent student, not a high flyer, should be able to be quite successful on the problems they would be set. You just have to know the equation of a line-get to the line, move up and down the line. I need to know about parallel and perpendicular vectors, solving simultaneous equations. I think conceptually students should be able to cope with them. It's not just vectors, any question where you have to pull together several bits of information and use that to solve the problems you are given is going to be more difficult than one that says do this, do that.

3. **How do you think students find learning vectors as a topic?**

Fine. I do find the Autograph stuff with the 3-D vectors is really nice and you can visualise stuff with that. I think if you try and get away from some of the nasty formula and just get students to think: get to the line, move along the line and make relationships between what they are doing in vector equations and what they do in linear equations. Students often panic a bit when they do a vector question, look at the answer in the back of the book and it's not their answer, but they could be right it could look very different.

Why do you think students find learning vectors difficult?
They are meeting it for the first time really since GCSE so there has been a big gap, so they can't remember it at all and that makes it tough. It's the time it takes to get back into the way of thinking. They are very different mathematical objects to the rest what they are used to.