A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

ESSAYS ON MULTIVARIATE VOLATILITY MODELS

AN APPLICATION TO EMERGING FINANCIAL MARKETS

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For Mum and Dad

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1. Introduction

1.1. Motivation of this thesis

The global financial market has experienced large fluctuations over the past decade. At the beginning of the 2000s, the birth of the Euro indicated a significant trend of integration of the European financial markets. From 2007 to 2009, the occurrence of the Global financial crisis, caused by the sub-prime crisis in the US, had a negative impact on the global financial system. The ongoing problems of the Euro currency put a large pressure on financial markets in the Euro Zone. The recent important negative events of the Global financial market indicated that it has internationalized so that an event, occurring at one financial market or economic zone, has a significant effect on the other financial markets. There are many studies, which try to depict how financial markets are connected to each other. However, it is always necessary to have an empirical research, which uses most advanced econometric methods, to deliver important empirical results for answering a question how the Global financial market has changed recently. This motivated me to perform this study, which applied the latest techniques in financial econometrics to give a precise report on the change of the global financial market from the point of view of emerging financial markets.

Recent studies on this topic mainly focus on the developed financial markets while there are a limited number of studies on the emerging financial markets. This is because of a belief that emerging financial markets are inefficient and not normal. Therefore, it limits the performance of models, which are designed to work under a Normal distribution assumption. Besides, there is a lack of study, which applies advanced methodology in financial econometrics to answer how emerging financial markets are connected to developed financial markets, especially during times of financial crisis. Specifically, the consistent positive performance of some emerging countries, especially the BRICs (Brazil, Russia, India and China) has raised an idea that the emerging financial markets possibly have an important role in the stabilization of the global financial market. On the other hand, the emergence of these financial markets has offered investors a chance to diversify their portfolios. Hence, a comprehensive research on how emerging financial markets are linked to developed financial markets is highly important for both investors, who manage to optimize the value of portfolios at risk and financial authorities, who are concerned by the instability of financial markets during times of financial crisis.

Structural changes in the global financial market have made the study of the interdependence of financial markets an important focus of financial econometrics and financial economics. In early stage of this research, Vector Autoregressive Models (VARs) were used to study the cross-market linkages. However, the main drawback of VAR method is that it cannot capture the heteroskedastic property of financial time series, which is now shown to be in presence in all financial time series. To complement the VAR method, the multivariate volatility models, which can account for the heteroskedasticity of financial time series, are used to examine the linkages in volatility rather than in return. The study of financial interdependence, via linkages in volatility, has benefited from the development and the extensions of multivariate volatility models. Thus, the multivariate models are now widely used both by practitioners; such as Riskmetrics filters, and by researchers; for example the multivariate GARCH models (Generalized Autoregressive Conditionally Heteroskedasticity). There is a rich literature on volatility models, which are applied to the more integrated and developed financial markets. However, one research question is how does a large number of volatility models perform when applied to the emerging markets. It helps financial authorities and investors to choose the most appropriate model to have the best practical views of the effects of the structural changes in the global financial market. Specifically, the study of the interdependence between emerging financial markets and developed financial markets will provide empirical evidence on the connections between these two types of financial market.

One important application of multivariate volatility model is that it can generate a dynamic conditional correlation between financial markets. It means that a multivariate volatility model can detect how financial markets are connected to each other in the short-run, especially in crisis period. Hence, another research question is whether, and how, the recent financial crises occurred in the US market have contagion effects on the emerging financial markets. The bad effect of the recent Global financial crisis on the World economy has raised a question whether, and how a financial crisis in one market could be outspread to other financial markets. This phenomenon is known as a financial contagion, which is defined as a significant increase in the correlation between financial markets due to the effect of a major shock. There is a rich literature in using a multivariate volatility model to test for financial contagion. However, almost studies in the literature choose a specific volatility model without explaining why, and whether a chosen model is the best model to test for a financial contagion. This may lead to conclude a market interdependence, which is defined as a temporary effect of a financial crisis, as a contagion. Therefore, a good choice of multivariate volatility model will efficiently detect the relationship between emerging markets and developed markets. A study of financial contagion with the use of an appropriate multivariate volatility model will give investors and financial authorities a better view of how emerging financial markets are affected by a financial crisis from developed markets. It is also a guidance that helps investors to have a better asset allocation and allows them to have a better risk management of portfolios containing stocks from emerging markets.

The success of the multivariate GARCH models, when applied to both developed and emerging financial markets, can be noticed by a large number of its extensions and popular applications in finance. However, recent practical changes in financial markets have shown that there exists a non-linearity in the dependence between financial markets, which the standard multivariate GARCH models cannot efficiently capture. This requires a development of a new class of models or the modification of a standard model. The simple use of copula is an efficient way that improves the performance of the multivariate

GARCH models. This is a reason why there are now an increasing number of studies, which apply a copula model in finance. Therefore, a research question is how the copula-based multivariate GARCH model performs relatively to the standard model when applied to the emerging financial markets. Hence, a copula-based model that is fitted well to the emerging data will be of great help to the financial authorities who are concerned by the interdependence among financial markets and to the international portfolio managers, who are in need of a precise estimator for the Value at Risk of their portfolios containing stocks from emerging financial markets.

1.2. The scope and contribution of this thesis

This thesis is an empirical study of how multivariate volatility models can be applied to analyse the dependence between emerging financial markets and the US financial market. This thesis comprises 3 complete papers which employ this data set in a number of ways.

1. The first paper, presented in chapter 2, is a comparative research on the estimation and evaluation of 54 individual volatility models, which belong to 10 different classes of models namely the Riskmetrics models, the Constant Conditional Correlation model (CCC), the Orthogonal-GARCH model (O-GARCH), the Dynamic Conditional Correlation model (DCC), the Asymmetric DCC model (ADCC), the Consistent DCC model (CDCC) and the Student's t-DCC model (TDCC). All of these models were estimated and then ranked by using both in-sample and out-of-sample performance. This research is to emphasize the importance of model selection in modelling the volatility of financial time series from emerging financial markets. In existing literature, our paper is the first paper to evaluate the performance of a large number of volatility models using the emerging data. Our paper also suggested that the calibrations to select the appropriate values for the risk aversion and to specify the reasonable range for the evaluation sample are the key to achieve proper statistical results. The Student's $t(6)$-distribution assumption is more relevant than the Gaussian distribution assumption for the volatility models

to fit for the emerging markets. This also indicates the difference of the emerging data from the developed data, which Pesaran *et al* (2009) used 7 degrees of freedom for the Student's t-distribution assumption. The use of CDCC model in this study, which is a consistent in large-scaled portfolio, indicated that the DCC-type models are consistent estimator using a portfolio of 19 emerging and the US stock indices. For the emerging data, the DCC-type models continued to be in the top models while the TDCC model outperforms other models in in-sample evaluations and managed to be in the top 3 models in the out-of-sample evaluations. The filters of Riskmetrics, widely used by practitioners, were not suggested by our study. However, the empirical result also showed that that there is no best model at all times. Therefore, the choice of an appropriate model for each data set is important. Moreover, the result also showed that the emerging data is more volatile than the developed data while the risk aversion coefficient, used in this research, is higher.

2. The second paper, presented in chapter 3, uses the TDCC model which performed relatively well among the 54 volatility models to analyse the volatilities and correlations of the emerging markets. Specifically, the pair-wise conditional correlations between each of the emerging markets and the US market, generated by the TDCC model, were used to perform empirical tests for contagion in the 3 recent financial crises, i.e. the Dotcom crisis in 2000, the Sub-prime in 2007-2008 and the Global financial crisis in 2008-2009. The use of the TDCC model, which assumes a Student's t-distribution, is more appropriate for the empirical tests for contagion as it deals with the fat-tailed distribution of the financial data. Using the methods of Forbes and Rigobon (2002) and Chiang *et al* (2007) to test for contagion, this is the first paper which used the TDCC model to generate the dynamic conditional correlations between each of the 19 emerging financial markets and the US financial market, which were used for the tests. The devolatization technique, initially introduced by Pesaran *et al* (2007) for the estimation of the TDCC model, was used firstly to remove the heteroskedasticity in the conditional correlation series. The technique is more relevant than techniques, used in the previous studies, while it

showed that contagion in 2 recent financial crises which are the Sub-Prime crisis in 2007 and the Global financial crisis in 2008 was not as widespread as concluded in the previous studies in the literature. Moreover, the timing of the outbreak of a financial crisis and the choice of a financial crisis period are the key factors to achieve a good result while there is no robust method to test for a contagion.

3. The third paper, presented in chapter 4, is the application of multivariate copula, which provides a connection between the univariate distributions and the multivariate distribution inside the DCC model, to analyse the emerging data. The flexibility of the copula model, separating the multivariate distribution assumption from those for univariate series, allows us to have an efficient examination of the dependence structure of emerging financial markets. Following the success of the copula model in recent studies, our research, which is the first to use the copula model to analyse high-dimensional data, confirmed a significant improvement of the copula from the standard DCC model. The results indicated that the t-copula DCC model outperforms the Gaussian copula DCC model in both in-sample and out-of-sample evaluations. Especially, the t-copula DCC model passed the VaR-based diagnostic test under the active risk management manner in the evaluation period of 12 years (from 1998 to 2010), which showed that the use of copula significantly improves the performance of the DCC model. Another contribution of this paper to the existing literature is the flexibility of copula helped us to show that the choice of relevant marginal model is highly important to the performance of the copulas. The our application of copula to the DCC framework, which works with high-dimensional data, offers investors a chance to apply copula to estimate the Value at Risk of their portfolios, which contain a large number of stocks.

The last chapter summarizes all the findings and contributions of the three papers and suggests an outlook for future research on the topics of volatility models and copulas.

1.3. Data Analysis

The data set, used in this thesis, contains the market indices of 19 emerging financial markets and the US market. The 19 emerging markets under consideration are eight markets in Asia (China, India, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand), five markets in Latin America (Brazil, Chile, Colombia, Mexico, and Peru) and six markets in Europe (Czech, Hungary, Israel, Poland, Russia and Turkey). The criterion for a country to be selected into the portfolio as an emerging market is suggested by Morgan Stanley Capital International (MSCI). MSCI is also a leading global index provider so we used indices of the 19 emerging countries and the US, which are provided by MSCI, for the analysis in this thesis. The MSCI indices, used in this thesis, are free float-adjusted market capitalization weighted indices that are designed to measure the equity market performance.

The MSCI indices are constructed using Free-float Methodology, which refers to an index construction methodology that takes into consideration only the free-float market capitalization of a company for the purpose of index calculation and assigning weight to stocks in the index. Free-float market capitalization takes into consideration only those shares issued by the company that are readily available for trading in the market. It generally excludes holding of promoters, government holding, strategic holding and other locked-in shares that will not come to the market for trading in the normal course. In other words, the market capitalization of each company in a Free-float index is reduced to the extent of its readily available shares in the market

1.3.1. Definition of Free-float

Shareholdings of investors that would not, in the normal course, come into the open market for trading are treated as 'Controlling/Strategic Holdings' and hence not included in free-float. Specifically, the following categories of holding are generally excluded from the definition of Free-float:

- Shares held by founders/directors/acquirers which has control element

- Shares held by persons/bodies with 'Controlling Interest'
- Shares held by Government as promoter/acquirer
- Holdings through the FDI Route
- Strategic stakes by private corporate bodies/ individuals
- Equity held by associate/group companies (cross-holdings)
- Equity held by Employee Welfare Trusts
- Locked-in shares and shares which would not be sold in the open market in normal course.

1.3.2. Major Advantages of Free-float Methodology

- A Free-float index reflects the market trends more rationally as it takes into consideration only those shares that are available for trading in the market.
- Free-float Methodology makes the index more broad-based by reducing the concentration of top few companies in Index.
- A Free-float index aids both active and passive investing styles. It aids active managers by enabling them to benchmark their fund returns vis-à-vis an investible index. This enables an apple-to-apple comparison thereby facilitating better evaluation of performance of active managers. Being a perfectly replicable portfolio of stocks, a Free-float adjusted index is best suited for the passive managers as it enables them to track the index with the least tracking error.
- Free-float Methodology improves index flexibility in terms of including any stock from the universe of listed stocks. This improves market coverage and sector coverage of the index. For example, under a full-market capitalization methodology, companies with large market capitalization and low free-float cannot generally be included in the Index because they tend to distort the index by having an undue influence on the index movement. However, under the free-float Methodology, since only the free-float market capitalization of each company is considered for index

calculation, it becomes possible to include such closely held companies in the index while at the same time preventing their undue influence on the index movement.

- Globally, the free-float Methodology of index construction is considered to be an industry best practice and all major index providers like MSCI, FTSE, S&P and STOXX have adopted the same.

For all of the above advantages of the free-float Methodology, the MSCI indices are relevant for the study in this thesis, which use a various type of multivariate volatility models to analyse the dynamics of emerging financial markets. Moreover, this type of the market index is helpful for the evaluations of the volatility models, which are based on the trading performance of the volatility models.

1.3.3. Some preliminary data analysis

The whole sample covers over the period from 15/05/1995 to 07/05/2010, with a total of 3910 observations. Hence, price indices of the emerging markets are obtained from Datastream following source of MSCI and quoted in US dollar. The market index for each country is computed by incorporating all listed and investible securities within the country. Then daily returns are calculated from the market indices as follows

$$r_{k,t} = \frac{100 \times (P_{k,t} - P_{k,t-1})}{P_{k,t-1}} \text{ with } k\text{=1, 2, ..., 20 and } P_{k,t} \text{ is the index of } k^{th}\text{market}$$

Figure 1.1, Figure 1.2 and Figure 1.3 show the plots of daily returns of the 20 market indices. From the first graphical inspection, the daily returns show substantial volatility clustering, which can be noticed for most emerging markets. This gives motivation for the case of GARCH models to explain the volatility behaviour of each individual market, and the multivariate parameterization allows us to estimate the conditional correlations among emerging markets as well as between an individual emerging market and the US. At the availability of various multivariate volatility models, we need to answer a research question of how to select the best model that fit well to the emerging data. This will be discussed in more details in the next chapter, which explains the model choice for the case of emerging financial markets.

Moreover, Figure 1.4, Figure 1.5 and Figure 1.6 display the plots of both prices and returns of the 20 financial markets at quarterly frequency. This gives a clearer graphical view of how financial markets, expressed in the price and return levels, behave at the occurrence of negative shocks. At the lower frequency, we can observe a clearer response of financial markets to a major negative shock that cause a price index to have a sharp fall. At Dotcom crisis in 2000, the US financial market experienced a large decrease in the price level and a large negative return. However, only a few other financial markets, such as Russia, Israel, had a fall in price levels and negative returns. Until a major shock, caused by the Global financial crisis from 2007-2009 in the US market, all of the 19 emerging financial markets experienced a sharp decrease in the price levels, which were associated with large negative returns. So this also raise the question whether there is a financial contagion from the US and how emerging financial markets are affected by a large negative shocks, which occurred in the US market during a crisis period. This will be discussed in chapter 3, where the best volatility model for the emerging data, which is suggested in chapter 2, will be used to test for a financial contagion of the three recent financial crises in the US financial market.

Table 1.1 gives a primary descriptive statistics of the return series with the unconditional means, the standard deviations, the skewness, the kurtosis and the estimates of the univariate t-GARCH(1,1) for each return series which is individually assumed to follow univariate Student's t-distribution with $\hat{\nu}$ degrees of freedom. All observations are used and the results indicate that all countries except Philippines have a positive mean. The European and Latin American countries generally have higher mean in returns than the countries from Asia. The significant degrees of skewness of some markets such as China, Korea, Malaysia is positive and significantly different from zero and all markets show a clear excessive kurtosis from the lowest level at 5.497 for Taiwan to the highest being 59.259 for Malaysia and the mean of the kurtosis centres around 15. This suggests

Table 1.1.: Descriptive Statistics and Univariate GARCH under Student's t-distribution Assumption (3910 observations)

Countries	**Mean**	**St.Dev.**	**Skewness**	**Kurtosis**	***t*-GARCH(1,1)**					
	(× 100%)				$\hat{\lambda}_{2i}$	**S.E.**	$\hat{\lambda}_{1i}$	**S.E.**	$\hat{\nu}_i$	**S.E.**
Asia										
CHINA	1.809	2.126	0.270	08.374	0.095	0.0125	0.894	0.0108	5.900	0.4744
INDIA	5.143	1.823	0.186	11.076	0.108	0.0160	0.868	0.0123	6.221	0.4557
INDONESIA	5.077	2.922	-0.003	22.400	0.101	0.0130	0.890	0.0116	4.085	0.2193
KOREA	4.989	2.609	0.809	18.417	0.061	0.0079	0.934	0.0072	5.892	0.4883
MALAYSIA	1.786	1.890	0.892	59.259	0.094	0.0123	0.901	0.0115	4.452	0.1964
PHILIPPINES	-0.390	1.812	0.960	18.993	0.164	0.0248	0.784	0.0173	5.012	0.3180
TAIWAN	1.381	1.757	0.045	05.497	0.052	0.0098	0.939	0.0077	5.630	0.3322
THAILAND	0.310	2.245	0.894	13.987	0.091	0.0151	0.893	0.0118	4.680	0.2511
Mean	*2.513*	*2.148*	*0.507*	*19.751*	*0.096*		*0.888*		*5.234*	
Latin America										
BRAZIL	6.967	2.413	0.144	10.600	0.094	0.0150	0.893	0.0126	7.045	0.7294
CHILE	2.492	1.324	0.135	16.820	0.111	0.0170	0.861	0.0123	8.530	1.0621
COLOMBIA	6.139	1.670	0.170	14.088	0.236	0.0270	0.714	0.0220	5.050	0.5649
MEXICO	6.417	1.915	0.134	12.540	0.103	0.0156	0.873	0.0116	5.851	0.4874
PERU	6.048	1.792	0.026	10.347	0.080	0.0129	0.909	0.0107	4.833	0.2641
Mean	*5.613*	*1.823*	*0.122*	*12.879*	*0.125*		*0.850*		*6.262*	
Europe										
CZECH	6.194	1.786	0.181	15.209	0.100	0.0134	0.884	0.0108	6.634	0.6201
HUNGARY	4.126	1.449	-0.223	07.767	0.043	0.0088	0.948	0.0069	4.230	0.2145
ISRAEL	7.468	2.259	0.088	12.674	0.109	0.0177	0.868	0.0138	5.727	0.4796
POLAND	3.613	2.086	-0.024	06.687	0.075	0.0122	0.906	0.0091	7.201	0.4330
RUSSIA	11.278	3.243	0.189	12.382	0.126	0.0137	0.861	0.0117	4.965	0.3077
TURKEY	7.914	3.162	0.250	09.230	0.110	0.0229	0.855	0.0155	5.412	0.3880
UNITED STATES	2.733	1.267	-0.022	11.414	0.068	0.0084	0.926	0.0076	7.533	0.8012
Mean	*6.190*	*2.179*	*0.063*	*10.766*	*0.090*		*0.893*		*5.957*	
Mean(all countries)	*4.575*	*2.078*	*0.255*	*14.888*	*0.101*		*0.880*		*5.744*	

that the use of the volatility models with the normality assumption is likely to be too restrictive.

The maximum likelihood estimates of the univariate t-GARCH model also give evidence of high volatility persistence and the parameters show similar behaviour of the conditional volatility across emerging markets. The similarity can also be found in the estimates of the degrees of freedom of the Student's t-distribution, which range from the highest of 8.530 for Chile to the lowest of 4.085 for Indonesia with the mean for all markets of 5.744. This result suggests again that the normality assumption should be replaced by that of Student's t-distribution. However, as presented by Pesaran *et al* (2009), the estimation of the multivariate volatility models has a significant technical difficulty when assuming Student's t-distributions due to the fact the Quasi Maximum Likelihood Estimation (QMLE) gives the estimates of the multivariate volatility models using normally distributed errors. To overcome this difficulty, the authors used the assumption of a Student's t-distribution with generic degrees of freedom, which was fixed at 7 to estimate all 52 volatility models. Thus, in Pesaran *et al* (2009), based on the descriptive analysis of data from developed markets, the mean of estimated degrees of freedom ($\hat{\nu}$), given by the univariate t-GARCH model for 18 developed markets, is 6.7. Therefore, those authors chose the 7 degrees of freedom for the Student's t-distribution assumption for the volatility models, which are designed to work under the Normal distribution. Following our descriptive statistics of return series of 20 financial markets, where the mean of $\hat{\nu}$ for 20 markets is 5.7, we are going to estimate the 53 volatility models assuming that the errors follow a Student's t-distribution with 6 degrees of freedom and the TDCC model was estimated using the t-distributed errors with the endogenous degrees of freedom, ν_t which was estimated at every sub-sample.

The difference in $\hat{\nu}$ between emerging data and developed data showed that the distribution of emerging data, which has the smaller $\hat{\nu}$, has fatter tails than those of the distribution of developed data. This indicates that the emerging financial markets experience larger positive and negative returns than the developed markets. It means that the emerging markets are more volatile than the developed markets. Therefore, the data analysis helps to choose the appropriate degrees of freedom for the Student's t-distribution assumption used in the model estimations and evaluations in the next 3 chapters to give empirical results relevant to the emerging data.

Recent development in the research of the volatility of financial time series showed that the DCC-type models, though efficient in modelling the volatility, have reach its limitation when it can not fully capture the non-linearity in the dependence structure in multivariate time series. There are a number of new research, which try to solve this problem. One of the new research, which is the copula model, will be discussed in the chapter 3. In this chapter, we will integrate a copula function, which provides a mapping connection between the univariate and multivariate distributions, into the DCC framework. The copula-based DCC model will then be applied to estimate the Value at Risk of a portfolio of the 20 market indices, which are mentioned above. Interestingly, the evaluation period for the copula-based DCC model will cover the whole period of the recent financial crisis in 2007-2009. Hence, this chapter will give us a better view of how good the recent development in volatility research could be.

Figure 1.1.: Daily Returns of Eight Asian Countries

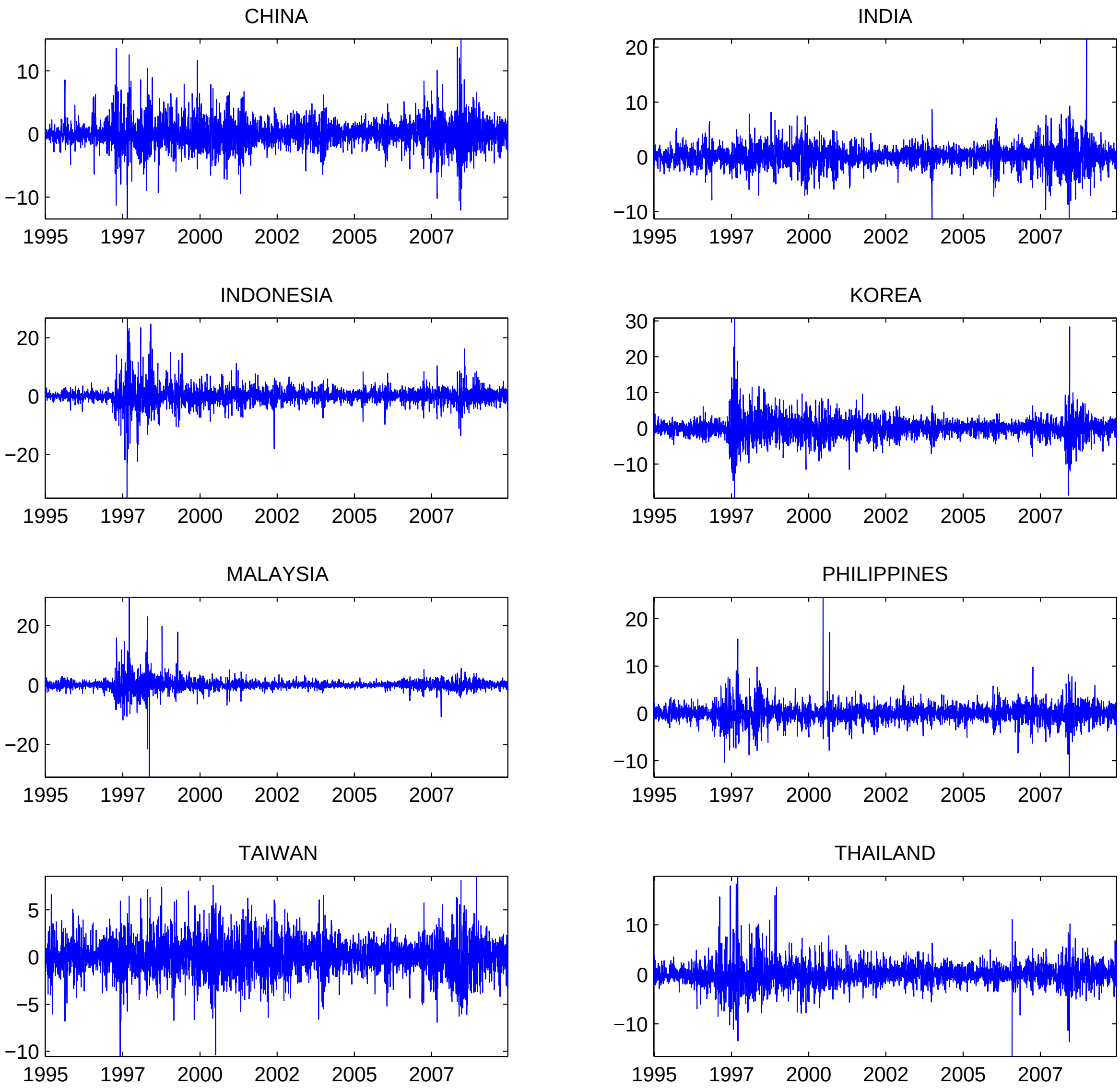


Figure 1.2.: Daily Returns of Five Latin American Countries

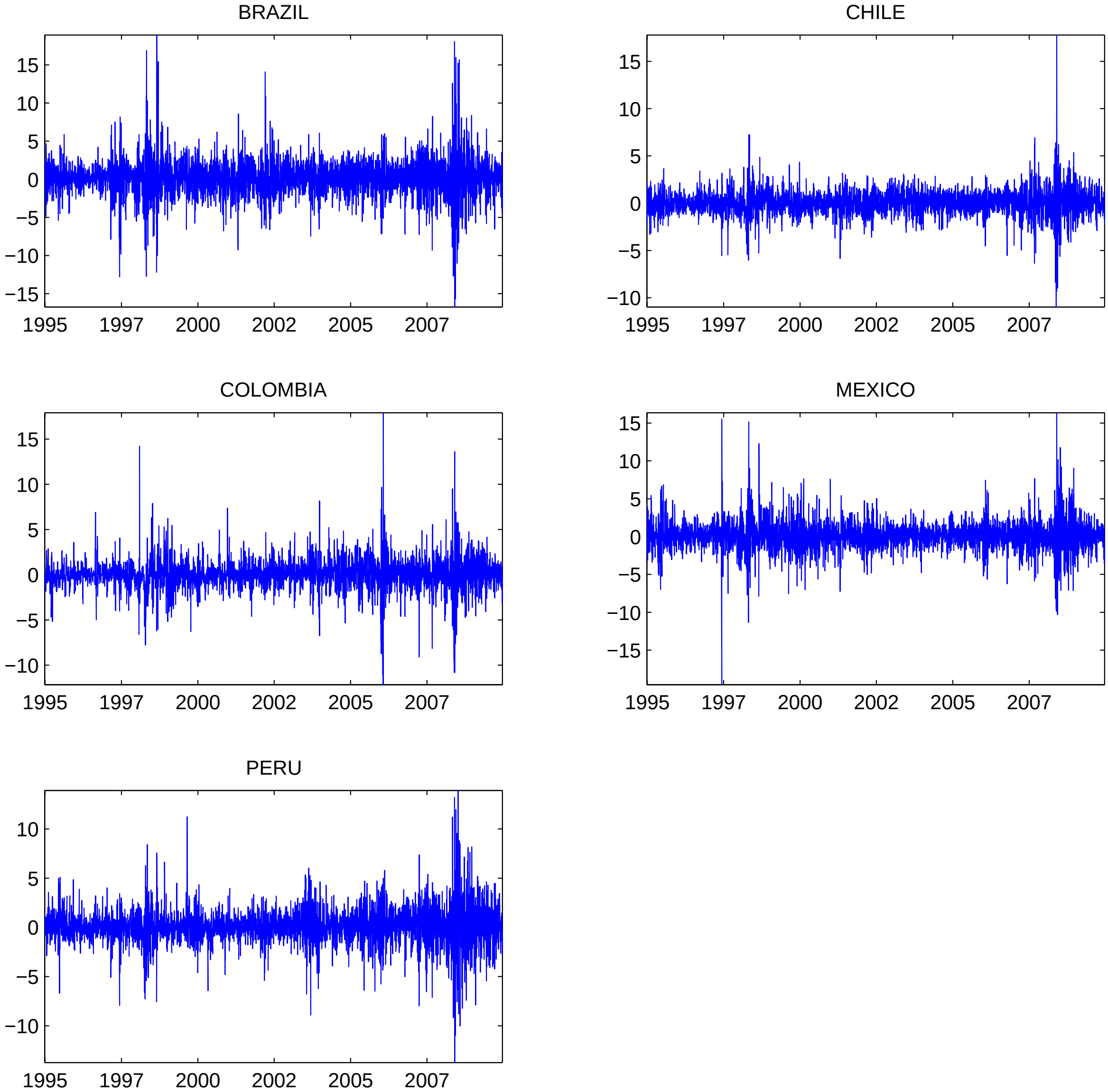


Figure 1.3.: Daily Returns of Six European Countries and the US

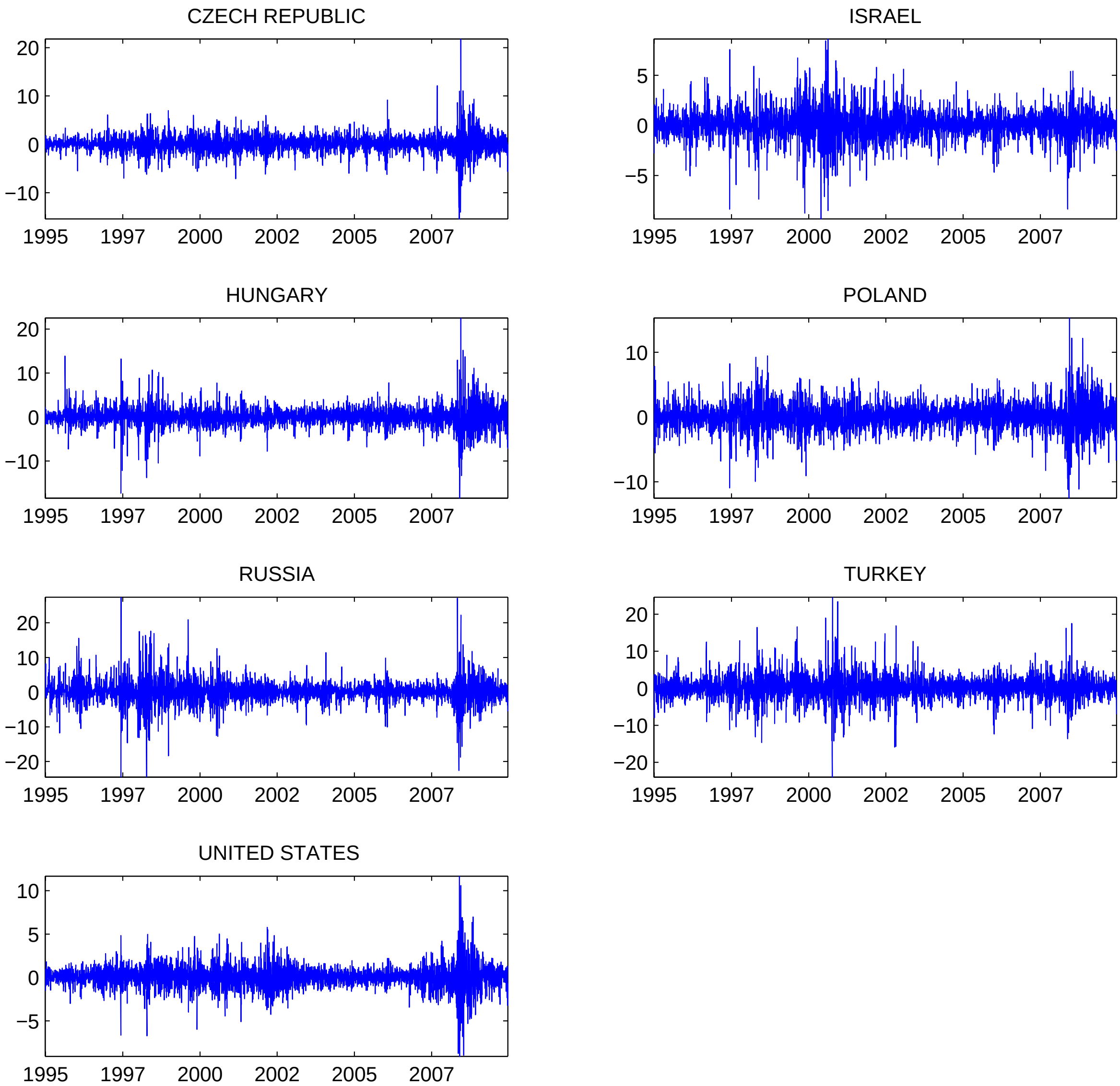


Figure 1.4.: Quarterly Prices and Returns of Eight Asian Countries

Figure 1.5.: Quarterly Prices and Returns of Five Latin American Countries

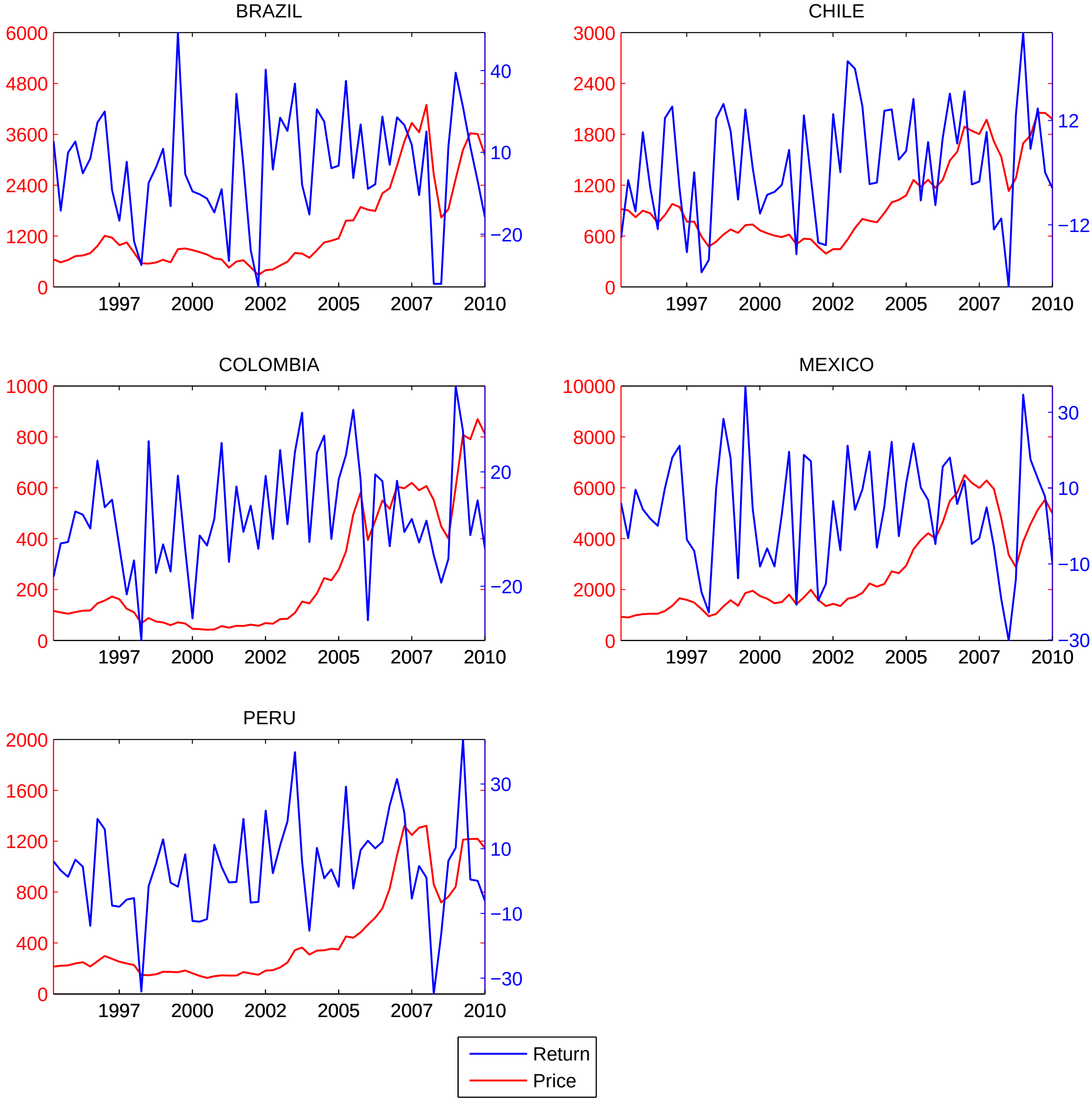
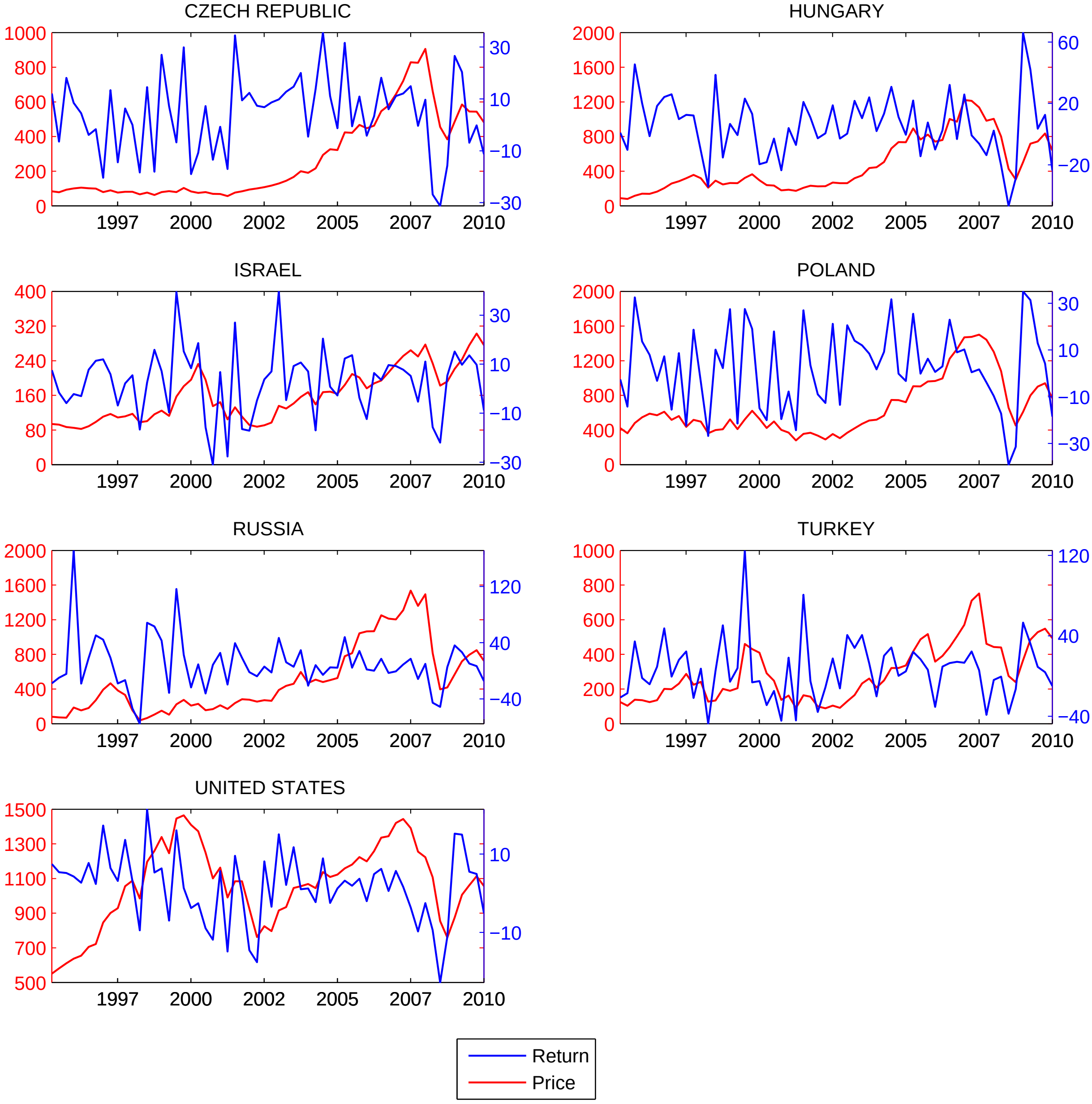


Figure 1.6.: Quarterly Prices and Returns of six European Countries and the US



2. RANKING MULTIVARIATE VOLATILITY MODELS: AN APPLICATION TO EMERGING FINANCIAL MARKETS

Abstract

The increasing number of extensions of the Dynamic Conditional Correlation (DCC) model and computational capability offer various measures for risk management, portfolio selection and the analysis of financial market interdependence or contagion. The success of the TDCC model proposed by Pesaran and Pesaran (2007) in the study of Pesaran *et al* (2009) in analysing the performance of the volatility models (both by finance practitioners and academics) motivates us to check how the TDCC model compares to previously developed multivariate models in the context of the 19 emerging financial markets and the US market. Thus, this study empirically compares the TDCC model with the different specifications of the multivariate GARCH model such as the Riskmetrics model, the Constant Conditional Correlation model (CCC), the Orthogonal-GARCH model, the DCC model, the Asymmetric DCC model (ADCC) and the Consistent DCC model (CDCC). In total, 54 models, categorized into 10 classes, were evaluated for the in-sample performance using the maximized values of the log-likelihood function, the AIC and the SBIC. The out-of-sample evaluation procedure, based on the one-step-ahead fore-

cast of the covariance estimators, utilizes Value-at-Risk analysis to produce the VaR-based diagnostic tests for the models following both the active and the passive risk management manners. Besides, the Kolmogorov-Smirnov (KS) and Kuiper (Ku) tests were also used to give evaluations from different points of view which focuses on the tail distributions. The empirical results using data from the emerging markets give a broader view on the trading strategies and the volatility model selection in the risk management manners. Similar to the study of Pesaran *et al* (2009), the TDCC model dominated the other models in in-sample performance, the DCC-type models were in the top models for both types of evaluations. However, the TDCC and the CDCC models were rejected by the Ku and the KS tests at the 1% significance level while the Riskmetrics filters were reasonably suggested by these two tests. For the data from the emerging markets, the calibrations to select the appropriate values for the risk aversion (δ) and to specify the reasonable range for the evaluation sample are the key to achieve proper statistical results. The Student's $t(6)$-distribution assumption is more relevant than the Gaussian distribution assumption for the volatility models to fit for the emerging markets.

2.1. Introduction

Over the past two decades, the empirical applications of theoretical models in finance have benefited greatly from developments in financial econometrics. The interrelationship between the theoretical models and the statistical methods in finance which has become dominant trend was a prediction of Pagan (1996[83]). Thus, financial markets have suffered from a lot of structural changes, the behaviours of investors, etc. which cause the market anomalies. Consequently, the theoretical models need to be modified so as to adapt flexibly to the practical changes. The modern financial econometrics, which develops sufficient tools to deal with the anomalies in the financial markets such as non-stationarity, non-normal distributions, heteroskedasticity, etc., is empirically helpful to realize the modified theoretical models in finance.

The volatility of financial returns, which has been the central focus of financial economics and is known as the unobservable second moment of financial data, shows that the financial returns are not as homoskedastic as assumed in many theoretical models in finance. The introduction of the ARCH model (Autoregressive Conditionally heteroskedasticity) of Engle (1982) puts a rigorous framework to measure the volatility of financial data. Since the introduction of the ARCH model, there have been an incredibly burgeoning number of extensions of it. Bollerslev (2008) gives a glossary of over 100 extensions of the ARCH models, although some extensions are still missing from the list. For reviews of the literature on the univariate ARCH/GARCH models, see Bera and Higgins (1993); Bollerslev, Engle and Nelson (1986); Bollerslev, Chou and Kroner (1992); Diebold and Lopez (1995); Pagan (1996); Palm (1996); Shephard (1996). The most recent review on the univariate GARCH models can be found in Poon and Granger (2003) who performed a broad survey on 93 papers and compared different methods for modelling univariate volatility.

Initially, the GARCH models, which are the generalised version of the ARCH and was proposed by Bollerslev (1986), E-GARCH [Nelson (1991)], GJR [Glosten, Jagannathan and Runkle (1993)], etc., are the univariate extensions of the ARCH model which deal with the movement of a single financial series. However, the applications of the univariate GARCH models are somewhat limited in finance. A multivariate parameterization of the GARCH model, therefore, was expected to have wider applications in financial studies. For example, in the portfolio selection, following the theory of Markowitz (1952, 1959), the weights of financial assets in a portfolio can be optimally chosen with respect to the estimates of conditional volatility and correlations, which can be estimated by a multivariate GARCH model. Specifically, following the coming introduction of the Basel Accord III, as a response to the Global financial crisis, banks are required to calculate the minimum amount of bank capital based on their traded financial assets. In other words, in risk management, the necessary capital amount that a bank is required to maintain is computed based on the Value-at-Risk of the portfolio of financial assets being traded by the bank. A multivariate GARCH model, which delivers a precise forecast of conditional volatilities and correlations of assets, will be a useful tool to obtain correct estimates of the

Value-at-Risk of financial assets held by banks. Besides, the pricing method of derivatives can also benefit from the development of the multivariate GARCH models which can be used to estimate the dynamic model of volatility of the underlying asset return, the option price and the contracts traded on volatilities.[1] Last but not least, a multivariate GARCH model can be used to examine the interdependence as well as contagion between financial markets by modelling the conditional covariances and the conditional correlations.

The study of the multivariate GARCH model was initiated in late 1980s by the introduction of the VECH model of Bollerslev, Engle and Wooldridge (1988) and early 1990s by the introduction of the Constant Conditional Correlation (CCC) model of Bollerslev (1990). The two models were proposed by different approaches in the construction of the covariance matrix. The former was constructed based on the direct estimation of the covariance matrix which is regressed on the lagged covariances and past squared errors while the latter was constructed by the recombination of the estimated diagonal matrix of the standard deviations obtained by the univariate GARCH and a time-invariant matrix of the conditional correlations. The BEKK representation proposed by Engle and Kroner (1995) is a modification of the VECH model which ensures the positivity definiteness of the variance matrix. However, both VECH and BEKK parameterizations face the problem that the number of parameters to be estimated rises exponentially with the number of return series, which is known as the 'curse of dimensionality'.[2] Those types of models, therefore, are then difficult to use in practice for the issue of the number of parameters will cause the model to be over-parameterized in even a modestly-sized portfolio of financial assets. The CCC specification was proposed to avoid the curse of dimensionality. In this approach the number of parameters rises linearly with the number of return series.[3] However, conditional correlations are often not constant over time due to different degrees of financial integrations or financial crises which cause the correlations of financial assets to vary over time. Therefore, Engle (2002) introduces a new class of the multivariate

[1] Nowadays, a volatility contract is designed to be exchangeable or tradeable in a similar way to a futures contract. It relies on the measurements of realized volatility of the underlying instrument. For details, see Krause (2000): *Volatility Contracts - A new alternative.*

[2] The VECH and the full BEKK models involve $\mathcal{O}(k^4)$ parameters, the diagonal VECH and the standard BEKK model involve $\mathcal{O}(k^2)$ parameters.

[3] The CCC model involves $\mathcal{O}(k)$ parameters

GARCH model, namely the Dynamic Conditional Correlation model (DCC) which relaxes the assumption of constant conditional correlations to allow for time-varying correlations. Since then, the DCC model has achieved a great success for its popular applications in finance, such as in risk management, asset allocation, derivative pricing and the analysis of interdependence of financial markets. There is also an increasing number of extensions of the DCC model such as the AG-DCC (Asymmetric Generalised DCC) model of Capiello, Engle and Sheppard (2006) which is able to capture the asymmetric properties of the volatilities and correlations, the introduction of the TDCC (Student t-DCC) model of Pesaran and Pesaran (2007) which assumes a multivariate Student's t-distribution for the return series or the CDCC (Consistent DCC) model of Aielli (2011) which is consistent in modelling the portfolios containing a large number of financial assets.

There are several survey papers on the multivariate GARCH models. Laurent, Bauwens and Rombouts (2006) is a comprehensive review of the multivariate GARCH models in terms of model selection, model estimation and the diagnostic checking for model specification. The purpose of their paper is to give a comprehensive background that acts as an indication for appropriate applications of the multivariate GARCH models in financial economics. In this literature, there is a key note that there is a co-existence of both types of the multivariate GARCH model, which are the BEKK model and the DCC model. Hence, there was a need to compare the differences and the similarities between the BEKK model and the DCC model. On this topic, an important review paper is given by Silvennoinen and Teräsvirta (2009) who focused on the comparison between the BEKK model and the CCC model or its generalisations. Moreover, Caporin and McAleer (2011) also gave a clear in-depth discussion of where the DCC models are preferred to use in practical applications and BEKK models are mainly mentioned in the theoretical aspects due to their dimensionality curse which makes the model estimation unrealisable in even a portfolio containing a modest number of financial assets. As the DCC models are easier to estimate than the BEKK model, they are mainly used by researchers. Practitioners tend to apply simple models to estimate the covariances and the correlations of financial returns such as the Riskmetrics specifications proposed by J.P.Morgan (1996). However, there has been no research to explain the difference between researchers and practitioners in using the multivariate parameterizations to model the volatility and correlations until Pesaran, Schleicher and Zaffaroni (2008) proposed the average modelling technique which also included the evaluations on the performances of a large number of the multivariate volatility models. The models included in their study range over 9 different classes of models in which the Riskmetrics specifications, such as the EQMA, the EWMA, the MMA, and the GEWMA, used by practitioners are compared with the models used by researchers, such as the CCC model of Bollerslev (1990), the Orthogonal GARCH model of Alexander (2001), the DCC model of Engle (2002), the AG-DCC model of Cappiello *et al* (2006) and the TDCC model of Pesaran and Pesaran (2007). The data set used in this paper comes from developed futures markets, including equity markets, currency markets, bond markets and commodity markets. Hence, the paper is limited to the analysis of the developed markets. In our study, we use data of 19 emerging markets and the US market to re-evaluate the performances of the set of 53 specific models used in the paper of Pesaran *et al* (2009) with the addition of the Consistent DCC model (CDCC) of Aielli (2011) which is the consistent estimator in large-scaled portfolios.

The rest of this paper is organized as follows: Section 2 gives an overview of 10 classes of models to be evaluated in this paper. Section 3 provides the empirical results as well as model evaluations and rankings. The last section provides some concluding remarks on the implications of the empirical results.

2.2. Specifications for the covariance and correlation models

This section provides an introduction to the specifications of the multivariate GARCH models which are examined in this paper. When the number of financial time series is larger than 5, only a few models are feasible to estimate. Therefore, there are 10 estimation-feasible classes of the multivariate GARCH model selected for the analysis in this paper. Nine of them are used in the study of Pesaran, Schleicher and Zaffaroni (2009):

- The Riskmetrics filters [see J.P. Morgan(1996)], including the Equally-Weighted Moving Average (EQMA), the Exponentially-Weighted Moving Average (EWMA), the Mixed Moving Average (MMA), the Generalised Exponentially-Weighted Moving Average (GWEMA).
- The Orthogonal GARCH (O-GARCH) by Alexander (2001).
- The Constant Conditional Correlation (CCC) by Bollerslev (1990).
- The Dynamic Conditional Correlation (DCC) by Engle(2002).
- The Asymmetric Dynamic Conditional Correlation (ADCC) by Capiello *et al* (2006).
- The Student's t-Dynamic Conditional Correlation (TDCC) by Pesaran and Pesaran (2007).

We included one more model class into the selection of the considered models, the Consistent Dynamic Conditional Correlation (CDCC) proposed by Aielli (2011). This extension of the DCC model is to solve the main problem of the DCC model which has estimated parameters being biased when the dimension of the portfolio is larger. Each class of the models may have more than one representation depending on how many past lags are included. Hence, there are totally 54 different specific volatility models being estimated and evaluated in this study.

Our research is focused on how the various multivariate GARCH models perform both in-sample and out-of-sample using the data from the 19 emerging markets and the US market. So let r_t be $m \times 1$ vector of return series at time t. Without a precise assumption about their distribution, the conditional returns of m financial series, r_t at time t are denoted as $E(r_t|\digamma_{t-1}) = \mu_t$. We specifically assume that the return series follow a first-order autoregressive process, AR(1) characterized as follows

$$r_t = c_0 + c_1 r_{t-1} + \epsilon_t \tag{2.1}$$

Hence, the $E(r_t|\digamma_{t-1}) = \mu_t = c_0 + c_1 r_{t-1}$. Therefore, we have $\epsilon_t = r_t - \mu_t$ with $\epsilon_t \sim (0,\ H_t)\quad where\quad H_t = Var(\epsilon_t|\digamma_{t-1})$ is the conditional covariance matrix at time

t of the innovations, ϵ_t. Consequently, the innovation series, ϵ_t can be standardized to satisfy the requirement of the multivariate volatility model estimations and evaluations by using the conditional covariance matrix, H_t as $z_t = \frac{\epsilon_t}{\sqrt{H_t}}$. We denote $\mathcal{H}_M(r_t|\digamma_{t-1})$ as the joint probability distribution of ϵ_t under some model M, which can be specified by the choice of H_t and the specification of the distribution of the standardized returns, z_t. Here, we shall consider both cases of a multivariate Normal distribution and a multivariate Student's t-distribution with ν degrees of freedom. There are many specifications that use parametric methods to estimate the conditional covariance matrix, H_t. Bollerslev, Engle and Wooldridge (1988) put forward the multivariate generalised autoregressive heteroskedasticity model of order (1,1) (which is also known as the VECH specification of the MGARCH(1,1)) as follows

$$VECH(H_t) = C + A_0 VECH(H_{t-1}) + B_0 VECH(\epsilon_{t-1}\epsilon'_{t-1}) \tag{2.2}$$

where VECH(.) denotes the column stacking operator applied to the upper portion of the symmetric matrix, C is $\frac{m(m+1)}{2} \times 1$ parameter vector, $A_0,\ B_0$ are $\frac{m(m+1)}{2} \times \frac{m(m+1)}{2}$ matrices of unknown parameters. However, the drawback of the MGARCH is that it requires a very large number of parameters as the size of matrices $A_0,\ B_0$ increases quadratically in the number of assets, m, in the portfolio. Hence, the model expressed in Equation 2.2 is rarely used in practice.

The introduction of the CCC model, the DCC model and its extensions is to realise the estimation of the conditional covariance matrix, H_t. Besides, the simple specifications, introduced by Riskmetrics for practical use in finance, are easy to use in estimating the conditional covariance matrix, H_t. The study of Pesaran, Schleicher and Zaffaroni (2009), aims to compare the performance of the model classes used by researchers with the ones used by practitioners. In their paper, there are 53 specific multivariate volatility models which are used to estimate the conditional covariance matrix, H_t. The 53 models are categorized into 9 different groups which are the CCC group of Bollerslev (1990), the DCC group of Engle (2002), the ADCC group of Cappiello *et al* (2006), the orthogonal GARCH group of Alexander (2001) and the TDCC model of Pesaran *et al* (2007), the group of the equally-weighted moving average models (EQMA), the group of the exponentially-weighted moving average models (EWMA), the group of the mixed moving average models (MMA) and the group of the generalised exponentially-weighted moving average models (GEWMA). Our paper differs from theirs in that we add the newly developed model of Aielli (2011), namely the CDCC and evaluate the performance of the 54 models on a data set from the US market and 19 emerging markets, which are more challenging for both the theoretical models and the practical models initially designed for developed markets.

The procedure for the estimation of each of the 54 specific models is based on the framework suggested by Engle (2002) for the decomposition of the conditional covariance matrix, H_t as in Equation 2.3 below

$$H_t = D_t R_t D_t \tag{2.3}$$

where $D_t = diag\left\{\sqrt{\sigma_{kk,t}}\right\}$ is the $m \times m$ diagonal matrix of time-varying standard deviations from the univariate GARCH models with $\sqrt{\sigma_{kk,t}}$ being the k^{th} position on the diagonal; $R_t = \{\rho_{kj,t}\}$ is the $m \times m$ one-step-ahead conditional correlation matrix. This decomposition allows for the feasible estimation of the multivariate volatility models as well as cross-asset correlations regardless of the number of assets in the portfolio. So each of the 54 models is used to estimate and forecast $\sqrt{\sigma_{kk,t}}$ and $\rho_{kj,t}$ by using the $m \times 1$ vector of residuals, ϵ_t obtained from the OLS regressions of the first-order autoregression of return series, r_t.

2.2.1. Equally-weighted Moving Average [EQMA(d_0)]

In this specification, the conditional covariance matrix, $H_{t,EQMA}$ can be simply computed by using the rolling moment estimates based on the last d_0 observations as follows

$$H_{t,EQMA} = \frac{1}{d_0}\sum_{s=1}^{d_0} r_{t-s}r'_{t-s} \tag{2.4}$$

To ensure that H_t is positive definite, the last d_0 observations must be larger than the dimension of returns vector, m. However, setting d_0 too high makes the conditional variance matrix too similar to the unconditional variance. In this application, following the common practice, suggested by the Riskmetrics, four variants of $H_{t,EQMA}$ are considered by setting d_0 = 50, 75, 125 and 250. For this class of model, the the conditional covariance matrix, $H_{t,EQMA}$ will behave like the unconditional covariance matrix if the choice of d_0 is too large. The choice of d_0, ranginf from 50 to 250, gives us a chance to select a model that best fit to our data set.

2.2.2. Exponentially-Weighted Moving Average [EWMA(d_0, λ_0, ν_0)]

The EWMA model is essentially the simple extension of the historical average volatility measure which allows the more recent observations to have a stronger impact on the forecast of the volatility than the older observations. This approach gives the EWMA model a practical application where recent events, in practice, are more influential on volatility. The one-parameter EWMA (setting $\lambda_0 = \nu_0$) officially used by Riskmetrics can be expressed in the following recursion:

$$H_{t,EWMA} = \lambda_0 H_{t-1} + \frac{1-\lambda_0}{1-\lambda_0^{d_0}}\epsilon_{t-1}\epsilon'_{t-1} - \frac{1-\lambda_0}{1-\lambda_0^{d_0}}\lambda_0^{d_0-1}\epsilon_{t-d_0-1}\epsilon'_{t-d_0-1} \tag{2.5}$$

where $0 < \lambda_0 < 1$ a constant parameter, d_0 is the window size. The kj^{th} element in the variance-covariance matrix $H_{t,EWMA}$ can be obtained from

$$\sigma_{kj,t} = \frac{1-\lambda_0}{1-\lambda_0^{d_0}} \sum_{s=1}^{d_0} \lambda_0^{s-1} \epsilon_{k,t-s} \epsilon_{j,t-s} \tag{2.6}$$

One of the drawbacks of the EWMA model is that when the infinite sum is replaced with a finite sum of observable data as in Equation 2.6, the weights from the given expression will sum up to less than one. So the parameter $\dfrac{1-\lambda_0}{1-\lambda_0^{d_0}}$ is added to make the sum of all parameters λs equal to 1. Moreover, the EWMA does not have the property of being mean-reverting. That is, the forecast of the conditional volatility of a series does not converge towards the unconditional variance like for other volatility models. The parameter d_0 is usually fixed as *a priori*. In J.P. Morgan (1996), it is suggested that the decaying factor is $\lambda_0 = 0.94$ (this can be estimated in practice). However, due to the asymptotic properties and the uncertainty over the parameters, we follow the suggestions of Pesaran *et al* (2009) that set the parameters $\lambda_0 = 0.95,\ 0.97$ and $d_0 = 250,\ 125,\ 75\ and\ 50$.

It is documented in both practice and academic research that there are different rates applied to decaying process of transmission of shocks to the conditional volatilities and the conditional correlations. Therefore, it can be assumed that there is some parameter, ν_0 different from λ_0, to construct the conditional covariance dynamic in Equation 2.6 as follows (normally we have $\nu_0 < \lambda_0$ and those two parameters are set as *a priori*)

$$\sigma_{kj,t} = \frac{1-\nu_0}{1-\nu_0^{d_0}} \sum_{s=1}^{d_0} \nu_0^{s-1} \epsilon_{k,t-s} \epsilon_{j,t-s} \tag{2.7}$$

Hence, the kj^{th} element of the correlation matrix, R_t in Equation 2.3 can be computed by using entries from the variance-covariance matrix, $H_{t,EWMA}$ as follows

$$\rho_{kj} = \frac{\sigma_{kj,t}}{\sqrt{\sigma_{kk,t}\sigma_{jj,t}}} \tag{2.8}$$

2.2.3. Mixed Moving Average [MMA(d_0, ν_0)]

This specification is actually a generalised version of the equally-weighted moving average model. The k^{th} entry in the standard deviation matrix, D_t is obtained similarly to what we can see in Equation 2.6 of the EWMA model. The conditional covariance is estimated by using Riskmetrics filter as $\sigma_{kj,t} = \frac{1-\nu_0}{1-\nu_0^{d_0}} \sum_{s=1}^{d_0} \nu_0^{s-1} \epsilon_{k,t-s}\epsilon_{j,t-s}$. The conditional correlation matrix, R_t, is constructed by using the structure in Equation 2.8. Consequently, the conditional variance matrix, $H_{t,MMA}$ is obtained by combining D_t, R_t as in Equation 2.3.

2.2.4. Generalised Exponentially-Weighted Moving Average [GEWMA(d_0, p, q, ν_0)]

This specification is a generalisation of the equally-weighted moving average model where there are two different decaying parameters used for the conditional volatilities and correlations. Firstly, we use the univariate GARCH(p,q) models to estimate for each return series in vector of return series, r_t by using the quasi-maximum likelihood to obtain conditional variances to form matrix D_t. The conditional correlation matrix is then computed by using the conditional covariances obtained by applying the Riskmetrics filter in Equation 2.5 with the conditional correlations computed following Equation 2.8. Once matrices D_t and R_t are available we can construct the variance-covariance matrix, $H_{t,GWEMA}$ following the structure in Equation 2.3. The total number of parameters being estimated in the GEWMA model is $m(1+p+q)$ where p, q are the order of the univariate GARCH model.

2.2.5. Constant Conditional Correlation [CCC(p,q)]

The Constant Conditional Correlation model, proposed by Bollerslev (1990), parameterizes the variance-covariance matrix, H_t, in Equation 2.3 by assuming that the conditional correlations in R_t are constant over time. The kk^{th} element, which is the conditional standard deviation in the diagonal matrix D_t, is estimated by using a univariate GARCH(p,q) of Bollerslev (1986)

$$\sigma_{k,t} = c_{0k} + \sum_{i=1}^{q} \lambda_{1,ki}\epsilon_{k,t-i}^{2} + \sum_{j=1}^{p} \lambda_{2,kj}\sigma_{k,t-j} \tag{2.9}$$

where c_{0k}, $\lambda_{1,ki}$, $\lambda_{2,kj}$ are positive parameters which are sufficient to ensure the positivity of $\sigma_{k,t}$. The conditional correlation matrix, R_t, is comprised of $\dfrac{m(m-1)}{2}$ constant parameters, ρ_{kj} which are computed by using the innovations, $\epsilon_{k,t}$ with $k = 1, 2, ..., m$ as follows

$$\rho_{kj} = \frac{\sigma_{kj}}{\sqrt{\sigma_{kk}\sigma_{jj}}} \tag{2.10}$$

The entries in the correlation matrix, in fact, are the unconditional correlations estimated by using the quasi-maximum likelihood to give the total estimated parameters which are comprised of $m(p + q + 1)$ parameters from the univariate GARCH estimations and $\dfrac{m(m-1)}{2}$ parameters from the constant correlation matrix. The conditional variance-covariance matrix $H_{t,CCC}$ is then constructed by the computed matrices D_t and R_t following Equation 2.3.

2.2.6. Orthogonal GARCH [O-GARCH(p,q)]

Alexander (2001) proposed the Orthogonal GARCH model by using a static principle component decomposition of residuals standardized as follows

$$\epsilon^{\star}_{k,t} = \frac{\epsilon_{k,t} - \bar{\epsilon}_k}{\bar{\sigma}_k} \text{ with } t{=}1,\ 2,\ ...,\ m\text{and } t{=}1,\ 2,\ ...,\ T.$$

where $\bar{\epsilon}_k$ is the sample mean of the return of asset k and $\bar{\sigma}_k$ is the sample standard deviation of the returns of asset k. The standardized returns are used to construct the sample covariance matrix which are, in turn, used to give the eigenvectors, a_k ($k = 1,\ ...,\ m$) and eigenvalues. The sample covariance matrix can be expressed as

$$\bar{S} = \frac{\sum_{t=1}^{T} \epsilon^{\star}_t \epsilon^{\star\prime}_t}{T}$$

In this specification, the time-varying conditional variance-covariance matrix, $H_{t,O-GARCH}$ is defined in the following formula

$$H_{t,O-GARCH} = AD_tA' \tag{2.11}$$

where $A = U \times a$ is a $m \times m$ matrix of standard deviations normalized by the weighting matrix $a = (a_1,\ a_2,\ ...,\ a_m)$ containing the eigenvectors a_{k_s} with respect to the first m largest eigenvalues of the sample covariance matrix $\bar{S}$ and U is diagonal matrix containing sample standard deviations of the innovations $\epsilon_{k,t}$; D_t is the orthogonal matrix of the conditional variances with $\sigma_{k,t},\ the\ kk^{th}$ entry on the diagonal being estimated by the univariate GARCH(p,q) with $k = 1,\ 2,\ ...,\ m$ as follows

$$\sigma_{k,t} = c_{0k} + \sum_{i=1}^{q} \lambda_{1,ki} s^2_{k,t-i} + \sum_{j=1}^{p} \lambda_{2,kj} \sigma_{k,t-j}$$

where $s_{k,t} = (\epsilon_1, \epsilon_2, ..., \epsilon_T) \times a_k$ is the orthogonalised return series of asset k with $k = 1$, 2, ..., m. The number of total parameters in the specification of O-GARCH is $m \times (p+q+1)$.

2.2.7. Dynamic Conditional Correlations [DCC(p,q,M,N)]

This model was proposed by Engle in 2002 based on the modification of the CCC model of Bollerslev (1990) and allows the elements in the conditional correlation matrix, R_t to vary overtime. The matrix of the standard deviations, D_t, is constructed similarly to what is done in the CCC method where the k^{th} element on the diagonal is estimated by a univariate GARCH(p,q) so that

$$\sigma_{k,t} = \omega_{0k} + \sum_{i=1}^{p} \lambda_{1,ki}\epsilon_{k,t-i}^2 + \sum_{j=1}^{q} \lambda_{2,kj}\sigma_{k,t-j}^2 \tag{2.12}$$

The restrictions imposed on the parameters in Equation 2.12 to ensure the non-negativity and the stationarity of the conditional variances are $\omega_{0k} > 0,\ \lambda_{1,ki} > 0,\ \lambda_{2,kj} > 0$ for all i, j, k and $\sum_{i=1}^{p} \lambda_{1,ki} + \sum_{j=1}^{q} \lambda_{2,kj} < 1$. The time-varying dynamic conditional covariance is presented as follows

$$Q_t = (1 - \sum_{i=1}^{M}\alpha_i - \sum_{j=1}^{N}\beta_j)\bar{Q} + \sum_{i=1}^{M}\alpha_i(\epsilon_{t-i}^{\star}\epsilon_{t-i}^{\star\prime}) + \sum_{j=1}^{N}\beta_j Q_{t-j} \tag{2.13}$$

where $\bar{Q} = E(\epsilon_t^{\star}\epsilon_t^{\star\prime})$ is the unconditional covariance of the standardized residuals, $\epsilon_t^{\star} = (\epsilon_1^{\star}, \epsilon_2^{\star}, ..., \epsilon_m)'$ which are standardized by using the conditional variances estimated in Equation 2.12 as follows

$$\epsilon_{k,t}^{\star} = \frac{\epsilon_{k,t}}{\sqrt{\sigma_{k,t}}} \tag{2.14}$$

Consequently, the time-varying conditional correlation matrix, R_t is formed by

$$R_t = Q_t^{\star-1} Q_t Q_t^{\star-1} \tag{2.15}$$

where

$$Q_t^{\star} = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{q_{22}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \sqrt{q_{mm}} \end{bmatrix}$$

the $\sqrt{q_{kk}}$ of $Q_t^{\star}$ is the k^{th} diagonal of Q_t. So the kj^{th} entry of R_t is defined as $\rho_{kj} = \dfrac{q_{kj}}{\sqrt{q_{kk}q_{jj}}}$ giving the correlation matrix positive semi-definite with elements on the diagonal.

In this study, we will use the DCC(p,q,1,1) meaning that there is only one lag of the covariance term and of the standardized residual. Hence, the general structure, as in Equation 2.13, is reduced as follows

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha(\epsilon_{t-1}^{\star}\epsilon_{t-1}^{\star\prime}) + \beta Q_{t-1} \tag{2.16}$$

The covariance matrix, $H_{t,DCC}$ is obtained by recombining D_t and R_t following Equation 2.3. The DCC is estimated by two-stage quasi-maximum likelihood. The first stage is carried out by estimating the univariate GARCH(p,q) for individual series to compute D_t. In the second stage, the log-likelihood function is set up, using R_t, D_t as in Equation 2.17, to give estimated parameters $\hat{\alpha}$, $\hat{\beta}$ in the dynamic correlation structure in Equation 2.16.

$$LLF = -\frac{1}{2}\sum_{t=1}^{T}(klog(2\pi) + 2log(|D_t|) + log(|R_t|) + \epsilon_t^{\star\prime}R_t^{-1}\epsilon_t^{\star}) \tag{2.17}$$

The number of total parameters in the DCC(p,q,1,1) specification is equal to $m(1 + p + q) + \dfrac{m(m+1)}{2} + 2$.

2.2.8. Asymmetric Dynamic Conditional Correlation [ADCC(p,q,q,M,O,N)]

In the standard framework of the DCC, Engle also mentioned the possibility of extending the DCC model by adding an asymmetric term that allows the DCC model to capture the asymmetric behaviour of financial assets. Cappiello, Engle and Sheppard (2006) proposed a modified specification of the DCC that allows for the estimation of asymmetric properties of financial time series. In the asymmetric DCC framework, the asymmetric term enters both stages of the model estimation. In the first stage of the conditional variance estimation for every single series, the standard univariate GARCH of Bollerslev (1986) is replaced by the asymmetric GARCH such as the GJR-GARCH. Cappiello *et al* (2006) suggested a choice of the 9 asymmetric univariate GARCH models. In this paper, we used the GJR-GARCH model proposed by Glosten, Jagannathan and Runkle (1993) for the first stage of estimation as in the following structure

$$\sigma_{k,t} = \omega_{0k} + \sum_{j=1}^{q} \lambda_{1,kj}\epsilon_{k,t-j}^2 + \sum_{j=1}^{q} \gamma_{kj} d(\epsilon_{k,t-j} < 0)\epsilon_{k,t-j}^2 + \sum_{i=1}^{p} \lambda_{2,ki}\sigma_{k,t-i} \tag{2.18}$$

where d(ζ) is an indicator function that takes the value of unity if $\zeta < 0$ and zero, otherwise. All parameters in Equation 2.18 must be positive and $\sum_{j=1}^{q} \lambda_{1,kj} + \sum_{j=1}^{q} \gamma_{kj} + \sum_{i=1}^{q} \lambda_{2,ki} < 1$ to satisfy the positivity and stationary conditions of the conditional variances that will be used to form the matrix of conditional standard deviations, D_t and to obtain the normalized residuals $\epsilon_t^\star$ given by $\epsilon_{k,t}^\star = \frac{\epsilon_{k,t}}{\sqrt{\sigma_{k,t}}}$.

In the second stage, the standardized residuals are used to compute the time-varying correlation dynamic as presented below

$$q_t = (1-\alpha'\bar{\rho}\alpha - \beta'\bar{\rho}\beta - \delta'\bar{\eta}\delta) + \sum_{j=1}^{N} \alpha_j (\epsilon_{t-j}^\star \epsilon_{t-j}^{\star\prime}) + \sum_{j=1}^{O} \delta_j d(\epsilon_{t-j}^\star < 0)(\epsilon_{t-j}^\star \epsilon_{t-j}^{\star\prime}) + \sum_{i=1}^{M} \beta_i q_{t-i} \tag{2.19}$$

where d(ζ) is an indicator function that takes the value of unity if $\zeta < 0$ and zero, otherwise; $\bar{\rho}$, $\bar{\eta}$ are the unconditional covariance matrices given by

$$\bar{\rho} = \frac{1}{T}\sum_{t=1}^{T} \epsilon_t^{\star}\epsilon_t^{\star\prime}$$

$$\bar{\eta} = \frac{1}{T}\sum_{t=1}^{T} d(\epsilon_t^{\star} < 0)\epsilon_t^{\star}\epsilon_t^{\star\prime}$$

The condition necessary to hold such that the q_t in Equation 2.19 is positive and stationary is $\sum_{j=1}^{N}\alpha_j^2 + \sum_{j=1}^{O}\delta_j^2 + \sum_{i=1}^{M}\beta_i^2 < 1$, the intercept $(1 - \alpha'\bar{\rho}\alpha - \beta'\bar{\rho}\beta - \delta'\bar{\eta}\delta)$ is positive semi-definite with the initial value of the covariance matrix, Q_0 is positive definite. These conditions are sufficient to make all realisations of ADCC possible.

It can be noticed that the DCC representation is a special case of the ADCC model. In the scalar version of the ADCC, the correlation dynamic structure can be expressed as

$$q_t = (1 - \alpha^2\bar{\rho} - \beta^2\bar{\rho} - \delta^2\bar{\eta}) + \alpha^2(\epsilon_{t-1}^{\star}\epsilon_{t-1}^{\star\prime}) + \delta^2 d(\epsilon_{t-1}^{\star} < 0)(\epsilon_{t-1}^{\star}\epsilon_{t-1}^{\star\prime}) + \beta^2 q_{t-1} \quad (2.20)$$

with the GJR-GARCH(1,1,1) as below

$$\sigma_{k,t} = \omega_{0k} + \lambda_{1,k}\epsilon_{k,t-1}^2 + \gamma_k d(\epsilon_{k,t-1} < 0)\epsilon_{k,t-1}^2 + \lambda_{2,k}\sigma_{k,t-1} \quad (2.21)$$

The sufficient condition to secure the positivity of q_t is that the intercept in Equation 2.20 must be positive semi-definite. Hence, it is necessary and sufficient to derive the condition that makes the model estimations feasible in any realisations is that $\alpha^2 + \beta^2 + \kappa\delta^2 < 1$ where κ is the maximum eigenvalue of matrix $\left[\bar{\rho}^{-1/2}\bar{\eta}\bar{\rho}^{-1/2}\right]$. This nonlinear constraint will be imposed on the maximization process of the log-likelihood function which can be written in a similar form to that of the DCC specification in Equation 2.17. The conditional correlation matrix R_t is then computed by

$$R_t = q_t^{\star -1} q_t q_t^{\star -1} \tag{2.22}$$

where $q_t^\star = diag\left(\sqrt{q_{kk,t}}\right)$ with $q_{kk,t}$ is the kk^{th} element of matrix q_t meaning that it is on the k^{th} diagonal position of q_t.

The variance-covariance matrix $H_{t,ADCC}$ is recombined as in Equation 2.3 by using D_t which contains the conditional variances from Equation 2.18 and R_t given by Equation 2.22. Quasi-maximum likelihood is also used for the estimation of the ADCC model. The total number of parameters in the specification of the ADCC(p,p,q,1,1,1) is $m(1 + 2p + q) + \dfrac{m(m+1)}{2} + 3$ which gives $m \times p + 1$ parameters more than those of the DCC(p,q,1,1) due to the inclusion of the asymmetric terms. Thus, the p asymmetric terms are included in the m univariate-GARCH processes and 1 asymmetric term added in correlation dynamic structure as in Equation 2.20.

2.2.9. Consistent Dynamic Conditional Correlation [CDCC(1,1,1,1)]

The DCC-type estimators have been shown in Aielli (2011) to be biased and inconsistent in large systems of financial assets. Hence, the consistent DCC model (CDCC) has been proposed by Aielli (2011) to solve the problem of inconsistency of the DCC models. The main modification of the CDCC model is the correction in the dynamic correlation structure in Equation 2.16 as follows

$$Q_t = (1 - \alpha - \beta)\tilde{Q} + \alpha(\mathcal{E}_{t-1}\mathcal{E}'_{t-1}) + \beta Q_{t-1} \tag{2.23}$$

where

$$\mathcal{E}_{t-1} = (\mathcal{E}_{1,t-1},\ \mathcal{E}_{2,t-1},\ ...,\ \mathcal{E}_{m,t-1}),\ where\ \mathcal{E}_{i,t-1} = \epsilon^{\star}_{i,t-1}\sqrt{q_{i,t-1}}$$

$\tilde{Q} = E(\mathcal{E}_t\mathcal{E}_t')$ is the sample correlation matrix of $\mathcal{E}_t$

For the small and medium portfolios, the DCC and the CDCC models have similar performances. However, the CDCC model performs better with large portfolios. Hence, it allows for a wider range of applications in practice. The number of parameters in the CDCC parameterization is similar to those of the DCC.

2.2.10. *t* - Dynamic Conditional Correlation [TDCC(1,1,1,1)]

All previous DCC-type frameworks are based on the use of residuals normalized by using the conditional variances estimated in the first stage while the second stage is to estimate the dynamic correlation process. The 2-stage estimation procedure of the DCC-type specifications is realised by assuming that the distribution of innovations is multivariate Gaussian. However, financial time series show fat-tailed behaviour which can be better approximated by the assumption of the Student's t-distribution with ν different degrees of freedom. Moreover, the 2-stage estimation procedure of the DCC-type framework is proven to be inefficient even though it is still consistent. The TDCC model of Pesaran and Pesaran (2007) is fitted to returns series which are assumed to have a multivariate Student's t-distribution with ν degrees of freedom.

To improve the performance of the DCC-type models, Pesaran *et al* (2007) introduced a crucial modification to the standard framework of the DCC model of Engle (2002). In the TDCC specification, the method used to standardize the residuals, ϵ_t, is replaced by the devolatization method that uses a realised variance to get the series of residuals devolatized as in the following computations

$$\sigma_{k,t}^{2,realized}(p) = \frac{\sum_{j=0}^{p-1} \epsilon_{k,t-s}^{2}}{p} \quad with \;\; k = 1, 2, ..., m; \;\; p \text{ is lag order} \tag{2.24}$$

The lag order p, indicating p latest observations being included to compute realised volatility, needs to be chosen so as to give the most appropriate variances to render almost Gaussian series of innovation as computed in the following formula

$$\tilde{\epsilon}_{k,t} = \frac{\epsilon_{k,t}}{\sigma_{k,t}^{realised}} \tag{2.25}$$

Pesaran *et al* (2007) indicated that p should be calibratedly equal to 20 to make the devolatized series of residuals nearly Gaussian. The noted difference of the devolatization process from the standardization technique used in the DCC of Engle (2001) is that the devolatization technique includes the contemporaneous residuals while the technique of the DCC does not. This feature works better in the case of intradaily data and also reduces the data-driven effects on estimated parameters for daily data or data of higher frequencies. It also helps when dealing with jumps in data that cause the financial data to have a non-Gaussian distribution that can be observed more often in emerging markets where shocks usually occur.

The devolatized returns that have approximate Gaussian distribution are utilized for the construction of the time-varying conditional correlations, in similar fashion to the previous structures of the DCC family. However, the TDCC is offered with an option of two types of the dynamic conditional correlation which are non-mean reverting and mean reverting.

The non-mean reverting structure is expressed in Equation 2.26

$$q_{kj,t} = \phi q_{kj,t-1} + (1-\phi)\tilde{\epsilon}_{k,t-1}\tilde{\epsilon}_{j,t-1} \tag{2.26}$$

The mean reverting structure is given by

$$q_{kj,t} = \bar{\rho}_{kj}(1 - \phi_1 - \phi_2) + \phi_1 q_{kj,t-1} + \phi_2 \tilde{\epsilon}_{k,t-1}\tilde{\epsilon}_{j,t-1} \tag{2.27}$$

where the $\bar{\rho}_{kj}$ is the sample correlation of residuals $\tilde{\epsilon}_{k,t}$ and $\tilde{\epsilon}_{j,t}$. For the mean-reverting case, the condition for all possible realisations of the TDCC model is $\phi_1 + \phi_2 < 1$. It can be seen that the non-reverting case is a special case of the mean-reverting case when $\phi_1 + \phi_2 = 1$. The conditional correlation $\tilde{\rho}_{kj,t}$ of residuals $\tilde{\epsilon}_{k,t}$ and $\tilde{\epsilon}_{j,t}$ is computed following Engle (2002) as

$$\tilde{\rho}_{kj,t} = \frac{q_{kj,t}}{\sqrt{q_{kk,t}q_{jj,t}}} \tag{2.28}$$

And $\tilde{\rho}_{kj,t}$ is also the kj^{th} entry of conditional correlation matrix $\tilde{R}_t$. Hence, the conditional variance-covariance matrix, $H_{t,TDCC}$ is obtained by recombining D_t and $\tilde{R}_t$ based on Equation 2.3. It is noted that $\sigma_{k,t}$, the kk^{th} diagonal element of the diagonal matrix, D_t is given by the univariate GARCH(1,1) model as follows

$$\sigma_{k,t} = \bar{\sigma}_k(1 - \lambda_{1k} - \lambda_{2k}) + \lambda_{1k}\sigma_{k,t-1} + \lambda_{2k}\epsilon^2_{k,t-1} \tag{2.29}$$

where $\bar{\sigma}_k$ is the unconditional variance of k^{th} return series.

The estimation procedure of the TDCC is also a modified version of those of the DCC when all parameters are estimated in one stage that helps to improve the efficiency of the TDCC and the return series are assumed to follow Student's t-distribution with ν degrees of freedom. The structure of the log-likelihood function is given in the following equation

$$\begin{cases} LLF_\tau(\theta) = & -\frac{m}{2}ln(\pi) - \frac{1}{2}ln \mid \tilde{R}_{\tau-1}(\theta) \mid -ln \mid D_{\tau-1}\left(\lambda_1, \lambda_2\right) \mid +ln \left[\frac{\Gamma\left(\frac{m+\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \right] \\ & -\frac{m}{2}ln(\nu-2) - \left(\frac{m+\nu}{2}\right) ln \left[1 + \frac{e_\tau' D_{\tau-1}^{-1}(\lambda_1,\lambda_2)\tilde{R}_{\tau-1}^{-1}(\theta)D_{\tau-1}^{-1}(\lambda_1,\lambda_2)e_\tau}{\nu-2} \right] \\ & with\ \ e_\tau = r_\tau - \mu_{\tau-1} \end{cases} \tag{2.30}$$

where $\theta = (\lambda_1,\ \lambda_2,\ \phi_1,\ \phi_2,\ \upsilon)'$ is the vector of parameters with $\lambda_1 = (\lambda_{11}, \lambda_{12}, ..., \lambda_{1m}),\ \lambda_2 = (\lambda_{21}, \lambda_{22}, ..., \lambda_{2m})$ denoted as parameters obtained from the univariate GARCH(1,1) model; $\phi_1,\ \phi_2$ are parameters that drive the dynamic correlation process and ν is the degrees of freedom of the multivariate Student's t-distribution. The total of parameters in θ for TDCC(1,1,1,1) model is $2m$+3 including $2m$ parameters from the univariate GARCH, 2 parameters from the correlation dynamic process and the degrees of freedom of the Student's t-distribution.

2.3. Empirical results and discussion

We used the data set described in Chapter 1 to obtain the empirical results in this chapter. To estimate the 54 volatility models, which belong to 10 different model types, we followed the method introduced by Pesaran *et al* (2009). Firstly, we used the AR(1) model defined in Equation 2.1 to generate one-day-ahead forecast of the conditional mean, $\hat{\mu}_{k,t+1}$ which is denoted as follows

$$\hat{\mu}_{k,t+1} = E(r_{k,t+1}|F_t) = \hat{c}_{0,k} + \hat{c}_{1,k} r_{k,t} \text{ with } k\text{=1, 2, ..., 20} \tag{2.31}$$

The AR(1) model presented in Equation 2.1 was fitted to individual return series using a window of 800 observations which was rolling forward by each day when the AR(1) model was re-estimated, then the forecasted error of the conditional mean, $\hat{\epsilon}_{k,t+1} = r_{k,t+1} - \hat{\mu}_{k,t+1}$

was generated recursively. So our 54 models were estimated using the one-day-ahead forecast errors, $\hat{\epsilon}_{k,t+1}$ and a rolling window of 800 observations. All models were re-estimated at the frequency of 25 days indicating that the risk updates for the parameters of the volatility models is monthly, which is considered as reasonable in terms of risk management. A daily risk update was considered, which requires the 54 models to be estimated every day. This means that the total times of model estimations is 167,949 (=54 x (3910-800)). With the current ability of computation, it is imposible to realise this type of risk update. Therefore, monthly risk update is more reasonable, where each of the 54 models was estimated 125 times over the whole estimation sample and the total times of model estimations is 6750. For the data from 19 emerging financial markets and the US market, no model failed to converge.

2.3.1. Model Ranking

2.3.1.1. In-sample evaluations

2.3.1.1.1. The methodology of in-sample evaluations

This section will give a brief discussion of how the 54 volatility models can be evaluated in this study. In this study, we used the two popular methods in financial econometrics for the evaluations of model performance, which are the in-sample and out-of-sample evaluations.

The method of in-sample evaluation is to evaluate how well a model fits to a data set. In this study, all multivariate volatility models were estimated by the method of maximum likelihood so the in-sample evaluation was performed by using the maximized value of the log-likelihood (LL) function. The best-fitting model will have a highest LL value. Moreover, one can argue that models with larger number of parameters are more likely to have high LL values. Hence, AIC and SBIC are introduced to give robust in-sample evaluations, which use a penalty for the number of parameters used by a volatility model.

Based on the estimation results of the 54 volatility models, we use the maximized log-likelihood values to compute the AIC and the SBIC as follows

$$AIC_{i,t} = LL_{i,t} - \kappa_i \; and \; SBIC_{i,t} = LL_{i,t} - \frac{\kappa_i}{2} ln(W) \tag{2.32}$$

where $LL_{i,t}$ is the maximized log-likelihood value of model i at time t; κ_i is the total number of parameters used by model i; W is the size of estimation window which is set to 800 observations. The information criteria were computed based on the estimates of the volatility models using the Gaussian errors and the Student's t-distributed errors with 6 degrees of freedom.

2.3.1.1.2. Result and discussion

Table 2.1 and Table 2.2 display the maximized log-likelihood values of the estimated volatility models using the Gaussian and the Student's t-distribution with 6 degrees of freedom, respectively. These tables deliver the values of $LL_{k,t}$ for the first sub-sample starting from 15/05/1995 to 16/06/98, for the last sub-sample starting from 03/04/2007 to 07/05/2010 and for the average of 125 sub-samples for each individual volatility model. For the Gaussian assumption, the log-likelihood values are between the lowest of -38,386 for MMA(50,0.95) model to the highest of -28,299 for the TDCC model. Also, these values vary among the different model types while they are similar among the models in the same model type. For the family of the Riskmetrics filters, none of the averages of the LL values is above -30,000. The best model of this type is the GEWMA(2,2,0.97) model with the LL value of -30,566. The O-GARCH models which the LL values centering around -29,390 performed better than the Riskmetrics specifications. However, the DCC-type models showed the best performance with the average values of the LL centering around -28,600 for the DCC models, around -28,620 for the CCC models, at -29,717 for the CDCC model and the highest value of -28,299 for the TDCC model. The ADCC models except the ADCC(1,1) provided the poorest performance in this model type with the average values of the LL being between -35,886 for the ADCC(2,2) and -30,111 for the ADCC(2,1). The ADCC(1,1), in contrast, was the second best model after the TDCC model. The reason for the differences in the performance of all estimated models is that the Gaussian assumption is not relevant for the data of emerging markets which show the fat-tailed behaviour. That is, the TDCC model, which was estimated using t-distributed innovations with the degrees of freedom, ν_t which was estimated at every sub-sample, showed the best in-sample performance while the poor-performing ADCC models, which allows for the asymmetric shocks causing the fat-tailed behaviour of the returns series, did not fit the data well under the Gaussian assumption. However, by assuming the Student's t-distribution with 6 degrees of freedom for the returns series, all models, except the TDCC, showed a significant increase in the maximized LL values. The TDCC model, which was fitted to the t-distributed innovations with the endogenous degrees of freedom, was still the best model with the highest LL value of -28,299 while all models of the ADCC type showed the biggest increase in the LL values, for example the LL value for the ADCC(2,2) increased by -7,256 from -35,886 to -28,630. The DCC-type models also fitted best under the assumption of the Student's t innovations with the LL value of the DCC models, ranging from -28,418 for the DCC(1,1) to -28,403 for the DCC(2,2); the LL value of the CCC model being between -28,435 for the CCC(1,1) and -28,418 for CCC(2,2). The Riskmetrics filters under the Student assumption were still the worst-performing models with the LL values ranging from -32,519 for to the MMA(50,0.95) to -29,584 for the EQMA(250) while the O-GARCH models continued to be ranked in the middle between the Riskmetrics and the DCC-type models.

Table 2.3 and Table 2.4 display the AIC values for the 54 models under the Gaussian assumption and the Student's t-distribution with 6 degrees of freedom, respectively while Table 2.5 and Table 2.6 deliver the SBIC values for the 54 models under the Gaussian assumption and the Student's t-distribution with 6 degrees of freedom, respectively. In these tables, models were ranked by using the information criteria and the ranking results were reported in parentheses, where 1 means the best model and 54 means the worst model, for the first sub-sample (15/05/1995 to 16/06/98), the last sub-sample (03/04/2007 to 07/05/2010), and for the average values of 125 sub-samples for each of 54 models. As

Table 2.1.: Maximized Values of Log-Likelihood for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods				Sample periods		
	16-Jun-98	07-May-10	Average		16-Jun-98	07-May-10	Average
	(1)	(2)	(3)		(4)	(5)	(6)
EQMA				(1,1,0.97)	-31216	-31413	-30586
(n_0)				(1,2,0.97)	-31134	-31400	-30572
(250)	-32562	-32680	-30700	(2,1,0.97)	-31210	-31484	-30592
(125)	-31599	-32033	-30870	(2,2,0.97)	-31124	-31460	-30566
(75)	-32288	-32479	-31813				
(50)	-34622	-34427	-34186	OGARCH			
				(p,q)			
EWMA				(1,1)	-29573	-29235	-29397
(n_0,λ_0,ν_0)				(1,2)	-29555	-29220	-29382
(250,0.95,0.95)	-35014	-34056	-33694	(2,1)	-29573	-29235	-29408
(250,0.97,0.95)	-35685	-34604	-33977	(2,2)	-29552	-29221	-29386
(250,0.95,0.97)	-32111	-31673	-31197				
(250,0.97,0.97)	-32496	-32019	-31337	CCC			
(125,0.95,0.95)	-34410	-34220	-33659	(p,q)			
(125,0.97,0.95)	-35015	-34790	-33966	(1,1)	-28006	-28693	-28636
(125,0.95,0.97)	-31740	-31899	-31331	(1,2)	-27973	-28692	-28626
(125,0.97,0.97)	-32081	-32228	-31485	(2,1)	-28001	-28699	-28628
(75,0.95,0.95)	-34850	-34786	-34342	(2,2)	-27961	-28684	-28610
(75,0.97,0.95)	-35461	-35397	-34700				
(75,0.95,0.97)	-32534	-32689	-32317	DCC			
(75,0.97,0.97)	-32897	-33030	-32508	(p,q)			
(50,0.95,0.95)	-37580	-37124	-36980	(1,1)	-27995	-28651	-28614
(50,0.97,0.95)	-38285	-37699	-37398	(1,2)	-27962	-28649	-28604
(50,0.95,0.97)	-35447	-35211	-35136	(2,1)	-27990	-28661	-28607
(50,0.97,0.97)	-35917	-35557	-35385	(2,2)	-27950	-28646	-28589
MMA				ADCC			
(n_0,ν_0)				(p,q)			
(250,0.95)	-42570	-40983	-36871	(1,1)	-27924	-28513	-28516
(250,0.97)	-36990	-36687	-33343	(1,2)	-27900	-28516	-34136
(125,0.95)	-38208	-38485	-35847	(2,1)	-27904	-28540	-30111
(125,0.97)	-34183	-34735	-32756	(2,2)	-27860	-28502	-35886
(75,0.95)	-37409	-37598	-36001				
(75,0.97)	-34217	-34495	-33388	TDCC			
(50,0.95)	-39849	-39081	-38386	(p,q)			
(50,0.97)	-37026	-36496	-36057	(1,1)	-27733	-28252	-28299
GEWMA				CDCC			
(p,q,ν_0)				(p,q)			
(1,1,0.95)	-33391	-33366	-32593	(1,1)	-28495	-35528	-29717
(1,2,0.95)	-33286	-33350	-32582				
(2,1,0.95)	-33381	-33442	-32602				
(2,2,0.95)	-33267	-33416	-32580				

Table 2.2.: Maximized Values of Log-Likelihood for 54 Multivariate Volatility Models under Student's *t*-distribution Assumption

Model type	Sample periods				Sample periods		
	16-Jun-98	07-May-10	Average		16-Jun-98	07-May-10	Average
	(1)	(2)	(3)		(4)	(5)	(6)
EQMA				(1,1,0.97)	-30195	-30574	-29897
(n_0)				(1,2,0.97)	-30164	-30573	-29892
(250)	-30436	-30264	-29584	(2,1,0.97)	-30190	-30593	-29894
(125)	-29980	-30201	-29768	(2,2,0.97)	-30156	-30595	-29885
(75)	-30271	-30559	-30322				
(50)	-31064	-31457	-31382	OGARCH			
				(p, q)			
EWMA				(1,1)	-29182	-28771	-29052
(n_0, λ_0, ν_0)				(1,2)	-29168	-28770	-29044
(250,0.95,0.95)	-31568	-31569	-31160	(2,1)	-29181	-28770	-29061
(250,0.97,0.95)	-31767	-31568	-31221	(2,2)	-29168	-28771	-29051
(250,0.95,0.97)	-30482	-30558	-30075				
(250,0.97,0.97)	-30632	-30543	-30104	CCC			
(125,0.95,0.95)	-31091	-31490	-31102	(p, q)			
(125,0.97,0.95)	-31270	-31515	-31173	(1,1)	-27791	-28374	-28435
(125,0.95,0.97)	-30075	-30559	-30112	(1,2)	-27773	-28376	-28428
(125,0.97,0.97)	-30205	-30560	-30151	(2,1)	-27790	-28376	-28428
(75,0.95,0.95)	-31108	-31574	-31352	(2,2)	-27769	-28374	-28418
(75,0.97,0.95)	-31296	-31626	-31436				
(75,0.95,0.97)	-30305	-30849	-30582	DCC			
(75,0.97,0.97)	-30447	-30872	-30634	(p, q)			
(50,0.95,0.95)	-31885	-32303	-32236	(1,1)	-27778	-28336	-28418
(50,0.97,0.95)	-32048	-32363	-32320	(1,2)	-27761	-28337	-28412
(50,0.95,0.97)	-31301	-31798	-31698	(2,1)	-27776	-28350	-28413
(50,0.97,0.97)	-31428	-31834	-31755	(2,2)	-27757	-28349	-28403
MMA				ADCC			
(n_0, ν_0)				(p, q)			
(250,0.95)	-32817	-31952	-31744	(1,1)	-27740	-28253	-28353
(250,0.97)	-31601	-31006	-30604	(1,2)	-27726	-28261	-28626
(125,0.95)	-31943	-31784	-31543	(2,1)	-27711	-28272	-28486
(125,0.97)	-30805	-30811	-30482	(2,2)	-27703	-28261	-28630
(75,0.95)	-31811	-31842	-31719				
(75,0.97)	-30896	-31049	-30873	TDCC			
(50,0.95)	-32396	-32514	-32519	(p, q)			
(50,0.97)	-31725	-31951	-31915	(1,1)	-27733	-28252	-28299
GEWMA				CDCC			
(p, q, ν_0)				(p, q)			
(1,1,0.95)	-31186	-31518	-30915	(1,1)	-28200	-30476	-28932
(1,2,0.95)	-31158	-31519	-30912				
(2,1,0.95)	-31180	-31539	-30911				
(2,2,0.95)	-31146	-31540	-30905				

the AIC and the SBIC are computed based on the LL values, the ranking of the 54 models is also similar to the one above obtained using the LL values. Following the average of the AIC and the SBIC of 125 sub-samples, the DCC-type models were ranked in the top 10 in which the TDCC model was the best model. The reason why the TDCC was the best model based on either the AIC or the SBIC is that the degrees of freedom were estimated for every sub-sample and the devolatization technique made the innovations approximately Gaussian rather than standardization technique used in the DCC model. There is not much difference in the ranking between the AIC and the SBIC under the same distribution assumption. However, a significant difference can be noted between the different distribution assumptions of the same information criterion. Specifically, the ADCC models were ranked in the top 10 models based on the SBIC and in the top 15 models based on the AIC under the assumption of Student's t-distribution while only one of the ADCC models (the ADCC(1,1)) was in the top 10 models based on both the information criteria under the Gaussian distribution. Moreover, under the assumption of the Gaussian distribution, 2 out of 4 ADCC models (the ADCC(1,2) and the ADCC(2,2)) were ranked in the bottom 10 models. The DCC models were consistently in the top 10 models regardless of the distribution assumptions while the CDCC model, which was newly developed by Aielli (2011), was ranging from a rank of 10 to 15 following the different information criteria under different the distributional assumptions. This result indicates that using the medium-scaled data containing 20 returns series the DCC model still performs consistently as the CDCC model is suggested by Aielli to show the consistency for large-scaled data set, i.e. data set containing over 100 return series. The next best model after the DCC model was the CCC ranking from 7 to 18. For the CCC models, it is suggested that these were fitted better using the assumption of normally distributed emerging data rather than the t-distributed emerging data. For example, the CCC models were ranked from 7 to 10 for the SBIC under the Gaussian assumption while they were ranked by the same information criterion from 12 to 18 under the Student's t-distribution assumption. The models following the CCC models in the ranking table were the O-GARCH models with the ranks ranging 11 to 18. The family of the Riskmetrics models was ranked in the bottom of all models, which indicates that the Riskmetrics filters are not relevant to fit to the data from emerging markets. Thus, following Pesaran *et al* (2009), one member of the Riskmetrics filters (the EQMA(250)) was ranked in the top 10 model for the data from developed financial markets. However, in our study, no Riskmetrics models were ranked even in the top 15 models. Among the Riskmetrics model, the simplest specification, the EQMA(250), with the ranks ranging from 17 to 21, and the most advanced filter, the GEWMA(1,1,0.97), with the ranks being between 17 and 21, performed considerably better than the other filters in the family.

For in-sample evaluation, the TDCC model continued to be the best model when being fitted to the emerging data. The noticeable difference is that the group of the ADCC models which used to be the second best group in the study of Pesaran *et al* (2009) has now 3 out of 4 models having the worst performance under the Gaussian assumption. For the medium-scaled data of 20 return series, the rank of the CDCC(1,1) model was lower than those of the DCC models, which is also consistent with the conclusion proposed by Aielli (2011) that the CDCC model will only be more consistent when used with large-scaled data. The DCC models consistently performed better than the ADCC models. However, the ADCC(1,1) shows exceptional performance being ranked in the top 3 by any in-sample criteria. This suggests that it is necessary to consider the asymmetric property of the financial emerging data. The poor performance of the Riskmetrics models indicates that these specifications maybe are designed to fit to the data from more integrated markets. The dominance of the DCC-type models in the in-sample performance when they are fitted to both developed and developing data shows that the DCC-type models are more reliable in general and can be applied to various types of financial markets and assets.

2.3.1.2. Out-of-sample evaluations

The in-sample evaluation can only tell us the goodness of fit of a model while it cannot tell how well a model is in forecast. That is why we also need to have out-of-sample evaluation, which is based on the forecast of a model to evaluate its performance. In econometrics, the popular method to evaluate the out-of-sample performance of a volatility model is

Table 2.3.: AIC Values for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods 16-Jun-98 (1)		07-May-10 (2)		Average (3)	
EQMA						
(n_0)						
(250)	-32562	(30)	-32680	(28)	-30700	(21)
(125)	-31599	(23)	-32033	(25)	-30870	(22)
(75)	-32288	(27)	-32479	(27)	-31813	(27)
(50)	-34622	(39)	-34427	(37)	-34186	(41)
EWMA						
(n_0, λ_0, ν_0)						
(250,0.95,0.95)	-35014	(41)	-34056	(35)	-33694	(38)
(250,0.97,0.95)	-35685	(45)	-34604	(39)	-33977	(40)
(250,0.95,0.97)	-32111	(26)	-31673	(22)	-31197	(23)
(250,0.97,0.97)	-32496	(28)	-32019	(24)	-31337	(25)
(125,0.95,0.95)	-34410	(38)	-34220	(36)	-33659	(37)
(125,0.97,0.95)	-35015	(42)	-34790	(42)	-33966	(39)
(125,0.95,0.97)	-31740	(24)	-31899	(23)	-31331	(24)
(125,0.97,0.97)	-32081	(25)	-32228	(26)	-31485	(26)
(75,0.95,0.95)	-34850	(40)	-34786	(41)	-34342	(43)
(75,0.97,0.95)	-35461	(44)	-35397	(44)	-34700	(44)
(75,0.95,0.97)	-32534	(29)	-32689	(29)	-32317	(28)
(75,0.97,0.97)	-32897	(31)	-33030	(30)	-32508	(29)
(50,0.95,0.95)	-37580	(50)	-37124	(49)	-36980	(52)
(50,0.97,0.95)	-38285	(52)	-37699	(51)	-37398	(53)
(50,0.95,0.97)	-35447	(43)	-35211	(43)	-35136	(45)
(50,0.97,0.97)	-35917	(46)	-35557	(45)	-35385	(46)
MMA						
(n_0, ν_0)						
(250,0.95)	-42570	(54)	-40983	(54)	-36871	(51)
(250,0.97)	-36990	(47)	-36687	(48)	-33343	(35)
(125,0.95)	-38208	(51)	-38485	(52)	-35847	(47)
(125,0.97)	-34183	(36)	-34735	(40)	-32756	(34)
(75,0.95)	-37409	(49)	-37598	(50)	-36001	(48)
(75,0.97)	-34217	(37)	-34495	(38)	-33388	(36)
(50,0.95)	-39849	(53)	-39081	(53)	-38386	(54)
(50,0.97)	-37026	(48)	-36496	(47)	-36057	(50)
GEWMA						
(p, q, ν_0)						
(1,1,0.95)	-33451	(34)	-33426	(31)	-32653	(30)
(1,2,0.95)	-33366	(32)	-33430	(32)	-32662	(31)
(2,1,0.95)	-33461	(35)	-33522	(34)	-32682	(33)
(2,2,0.95)	-33367	(33)	-33516	(33)	-32680	(32)

Model type	Sample periods 16-Jun-98 (4)		07-May-10 (5)		Average (6)	
(1,1,0.97)	-31276	(21)	-31473	(18)	-30646	(17)
(1,2,0.97)	-31214	(19)	-31480	(19)	-30652	(18)
(2,1,0.97)	-31290	(22)	-31564	(21)	-30672	(20)
(2,2,0.97)	-31224	(20)	-31560	(20)	-30666	(19)
OGARCH						
(p, q)						
(1,1)	-29633	(15)	-29295	(14)	-29457	(11)
(1,2)	-29635	(16)	-29300	(15)	-29462	(12)
(2,1)	-29653	(18)	-29315	(16)	-29488	(14)
(2,2)	-29652	(17)	-29321	(17)	-29486	(13)
CCC						
(p, q)						
(1,1)	-28256	(12)	-28943	(10)	-28886	(7)
(1,2)	-28243	(10)	-28962	(11)	-28896	(8)
(2,1)	-28271	(13)	-28969	(12)	-28898	(9)
(2,2)	-28251	(11)	-28974	(13)	-28900	(10)
DCC						
(p, q)						
(1,1)	-28057	(8)	-28713	(6)	-28676	(3)
(1,2)	-28044	(6)	-28731	(7)	-28686	(4)
(2,1)	-28072	(9)	-28743	(8)	-28689	(5)
(2,2)	-28052	(7)	-28748	(9)	-28691	(6)
ADCC						
(p, q)						
(1,1)	-28007	(4)	-28596	(2)	-28599	(2)
(1,2)	-28003	(3)	-28619	(3)	-34239	(42)
(2,1)	-28027	(5)	-28663	(5)	-30234	(16)
(2,2)	-28003	(2)	-28645	(4)	-36029	(49)
TDCC						
(p, q)						
(1,1)	-27776	(1)	-28295	(1)	-28342	(1)
CDCC						
(p, q)						
(1,1)	-28557	(14)	-35590	(46)	-29779	(15)

Table 2.4.: AIC Values for 54 Multivariate Volatility Models under Student's *t*-distribution Assumption

Model type	Sample periods							Sample periods					
	16-Jun-98		07-May-10		Average			16-Jun-98		07-May-10		Average	
	(1)		(2)		(3)			(4)		(5)		(6)	
EQMA							(1,1,0.97)	-30255	(23)	-30634	(26)	-29957	(21)
(n_0)							(1,2,0.97)	-30244	(22)	-30653	(27)	-29972	(22)
(250)	-30436	(28)	-30264	(19)	-29584	(19)	(2,1,0.97)	-30270	(25)	-30673	(28)	-29974	(23)
(125)	-29980	(19)	-30201	(18)	-29768	(20)	(2,2,0.97)	-30256	(24)	-30695	(29)	-29985	(24)
(75)	-30271	(26)	-30559	(23)	-30322	(29)							
(50)	-31064	(34)	-31457	(35)	-31382	(44)	OGARCH						
							(p, q)						
EWMA							(1,1)	-29242	(15)	-28831	(14)	-29112	(15)
(n_0, λ_0, ν_0)							(1,2)	-29248	(16)	-28850	(16)	-29124	(16)
(250,0.95,0.95)	-31568	(45)	-31569	(39)	-31160	(40)	(2,1)	-29261	(17)	-28850	(15)	-29141	(17)
(250,0.97,0.95)	-31767	(48)	-31568	(38)	-31221	(42)	(2,2)	-29268	(18)	-28871	(17)	-29151	(18)
(250,0.95,0.97)	-30482	(30)	-30558	(22)	-30075	(25)							
(250,0.97,0.97)	-30632	(31)	-30543	(21)	-30104	(26)	CCC						
(125,0.95,0.95)	-31091	(35)	-31490	(36)	-31102	(39)	(p, q)						
(125,0.97,0.95)	-31270	(41)	-31515	(37)	-31173	(41)	(1,1)	-28041	(10)	-28624	(10)	-28685	(8)
(125,0.95,0.97)	-30075	(20)	-30559	(24)	-30112	(27)	(1,2)	-28043	(11)	-28646	(12)	-28698	(9)
(125,0.97,0.97)	-30205	(21)	-30560	(25)	-30151	(28)	(2,1)	-28060	(13)	-28646	(11)	-28698	(10)
(75,0.95,0.95)	-31108	(36)	-31574	(40)	-31352	(43)	(2,2)	-28059	(12)	-28664	(13)	-28708	(11)
(75,0.97,0.95)	-31296	(42)	-31626	(44)	-31436	(45)							
(75,0.95,0.97)	-30305	(27)	-30849	(31)	-30582	(31)	DCC						
(75,0.97,0.97)	-30447	(29)	-30872	(32)	-30634	(33)	(p, q)						
(50,0.95,0.95)	-31885	(50)	-32303	(52)	-32236	(52)	(1,1)	-27840	(5)	-28398	(5)	-28480	(3)
(50,0.97,0.95)	-32048	(52)	-32363	(53)	-32320	(53)	(1,2)	-27843	(6)	-28419	(7)	-28494	(4)
(50,0.95,0.97)	-31301	(43)	-31798	(47)	-31698	(47)	(2,1)	-27858	(8)	-28432	(8)	-28495	(5)
(50,0.97,0.97)	-31428	(44)	-31834	(48)	-31755	(50)	(2,2)	-27859	(9)	-28451	(9)	-28505	(6)
MMA							ADCC						
(n_0, ν_0)							(p, q)						
(250,0.95)	-32817	(54)	-31952	(51)	-31744	(49)	(1,1)	-27823	(2)	-28336	(2)	-28436	(2)
(250,0.97)	-31601	(46)	-31006	(33)	-30604	(32)	(1,2)	-27829	(3)	-28364	(3)	-28729	(12)
(125,0.95)	-31943	(51)	-31784	(46)	-31543	(46)	(2,1)	-27834	(4)	-28395	(4)	-28609	(7)
(125,0.97)	-30805	(32)	-30811	(30)	-30482	(30)	(2,2)	-27846	(7)	-28404	(6)	-28773	(13)
(75,0.95)	-31811	(49)	-31842	(49)	-31719	(48)							
(75,0.97)	-30896	(33)	-31049	(34)	-30873	(34)	TDCC						
(50,0.95)	-32396	(53)	-32514	(54)	-32519	(54)	(p, q)						
(50,0.97)	-31725	(47)	-31951	(50)	-31915	(51)	(20)	-27776	(1)	-28295	(1)	-28342	(1)
GEWMA							CDCC						
(p, q, ν_0)							(p, q)						
(1,1,0.95)	-31246	(38)	-31578	(41)	-30975	(35)	(1,1)	-28262	(14)	-30538	(1,1)	-28994	(14)
(1,2,0.95)	-31238	(37)	-31599	(42)	-30992	(37)							
(2,1,0.95)	-31260	(40)	-31619	(43)	-30991	(36)							
(2,2,0.95)	-31246	(39)	-31640	(45)	-31005	(38)							

Table 2.5.: SBIC Values for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods 16-Jun-98 (1)		07-May-10 (2)		Average (3)			Sample periods 16-Jun-98 (4)		07-May-10 (5)		Average (6)	
EQMA							(1,1,0.97)	-31417	(20)	-31613	(18)	-30787	(18)
(n_0)							(1,2,0.97)	-31402	(19)	-31668	(19)	-30839	(19)
(250)	-32562	(30)	-32680	(28)	-30700	(17)	(2,1,0.97)	-31478	(22)	-31751	(21)	-30859	(20)
(125)	-31599	(23)	-32033	(25)	-30870	(21)	(2,2,0.97)	-31458	(21)	-31794	(22)	-30901	(22)
(75)	-32288	(27)	-32479	(27)	-31813	(27)							
(50)	-34622	(39)	-34427	(37)	-34186	(41)	OGARCH						
							(p,q)						
EWMA							(1,1)	-29774	(15)	-29436	(10)	-29598	(11)
(n_0,λ_0,ν_0)							(1,2)	-29823	(16)	-29487	(11)	-29649	(12)
(250,0.95,0.95)	-35014	(41)	-34056	(35)	-33694	(38)	(2,1)	-29840	(17)	-29503	(12)	-29676	(13)
(250,0.97,0.95)	-35685	(45)	-34604	(39)	-33977	(40)	(2,2)	-29886	(18)	-29555	(14)	-29721	(14)
(250,0.95,0.97)	-32111	(26)	-31673	(20)	-31197	(23)							
(250,0.97,0.97)	-32496	(28)	-32019	(24)	-31337	(25)	CCC						
(125,0.95,0.95)	-34410	(38)	-34220	(36)	-33659	(37)	(p,q)						
(125,0.97,0.95)	-35015	(42)	-34790	(42)	-33966	(39)	(1,1)	-28841	(11)	-29528	(13)	-29471	(7)
(125,0.95,0.97)	-31740	(24)	-31899	(23)	-31331	(24)	(1,2)	-28875	(12)	-29594	(15)	-29528	(8)
(125,0.97,0.97)	-32081	(25)	-32228	(26)	-31485	(26)	(2,1)	-28903	(13)	-29602	(16)	-29530	(9)
(75,0.95,0.95)	-34850	(40)	-34786	(41)	-34342	(42)	(2,2)	-28930	(14)	-29653	(17)	-29580	(10)
(75,0.97,0.95)	-35461	(44)	-35397	(44)	-34700	(44)							
(75,0.95,0.97)	-32534	(29)	-32689	(29)	-32317	(28)	DCC						
(75,0.97,0.97)	-32897	(31)	-33030	(30)	-32508	(29)	(p,q)						
(50,0.95,0.95)	-37580	(50)	-37124	(49)	-36980	(52)	(1,1)	-28202	(3)	-28858	(3)	-28821	(3)
(50,0.97,0.95)	-38285	(52)	-37699	(51)	-37398	(53)	(1,2)	-28237	(4)	-28923	(5)	-28878	(4)
(50,0.95,0.97)	-35447	(43)	-35211	(43)	-35136	(45)	(2,1)	-28264	(6)	-28935	(6)	-28881	(5)
(50,0.97,0.97)	-35917	(46)	-35557	(45)	-35385	(46)	(2,2)	-28291	(7)	-28987	(9)	-28930	(6)
MMA							ADCC						
(n_0,ν_0)							(p,q)						
(250,0.95)	-42570	(54)	-40983	(54)	-36871	(51)	(1,1)	-28201	(2)	-28791	(2)	-28794	(2)
(250,0.97)	-36990	(47)	-36687	(48)	-33343	(35)	(1,2)	-28244	(5)	-28860	(4)	-34480	(43)
(125,0.95)	-38208	(51)	-38485	(52)	-35847	(47)	(2,1)	-28315	(8)	-28951	(7)	-30522	(16)
(125,0.97)	-34183	(36)	-34735	(40)	-32756	(30)	(2,2)	-28338	(9)	-28980	(8)	-36364	(50)
(75,0.95)	-37409	(49)	-37598	(50)	-36001	(48)							
(75,0.97)	-34217	(37)	-34495	(38)	-33388	(36)	TDCC						
(50,0.95)	-39849	(53)	-39081	(53)	-38386	(54)	(p,q)						
(50,0.97)	-37026	(48)	-36496	(47)	-36057	(49)	(1,1)	-27876	(1)	-28396	(1)	-28443	(1)
GEWMA							CDCC						
(p,q,ν_0)							(p,q)						
(1,1,0.95)	-33592	(33)	-33567	(31)	-32794	(31)	(1,1)	-28702	(10)	-35735	(46)	-29924	(15)
(1,2,0.95)	-33553	(32)	-33618	(32)	-32849	(32)							
(2,1,0.95)	-33648	(35)	-33710	(33)	-32869	(33)							
(2,2,0.95)	-33601	(34)	-33750	(34)	-32914	(34)							

Table 2.6.: SBIC Values for Multivariate Volatility Models under Student's empht-distribution Assumption

Model type	Sample periods							Sample periods					
	16-Jun-98 (1)		07-May-10 (2)		Average (3)			16-Jun-98 (4)		07-May-10 (5)		Average (6)	
EQMA							(1,1,0.97)	-30395	(24)	-30775	(26)	-30098	(22)
(n_0)							(1,2,0.97)	-30432	(25)	-30840	(28)	-30160	(26)
(250)	-30436	(26)	-30264	(19)	-29584	(19)	(2,1,0.97)	-30457	(28)	-30860	(30)	-30161	(27)
(125)	-29980	(19)	-30201	(18)	-29768	(20)	(2,2,0.97)	-30490	(30)	-30929	(32)	-30219	(28)
(75)	-30271	(22)	-30559	(22)	-30322	(29)							
(50)	-31064	(34)	-31457	(35)	-31382	(44)	OGARCH						
							(p,q)						
EWMA							(1,1)	-29382	(15)	-28971	(10)	-29252	(11)
(n_0,λ_0,ν_0)							(1,2)	-29435	(16)	-29037	(12)	-29312	(13)
(250,0.95,0.95)	-31568	(45)	-31569	(39)	-31160	(37)	(2,1)	-29449	(17)	-29037	(11)	-29329	(14)
(250,0.97,0.95)	-31767	(48)	-31568	(38)	-31221	(41)	(2,2)	-29502	(18)	-29105	(13)	-29385	(17)
(250,0.95,0.97)	-30482	(29)	-30558	(21)	-30075	(21)							
(250,0.97,0.97)	-30632	(31)	-30543	(20)	-30104	(23)	CCC						
(125,0.95,0.95)	-31091	(35)	-31490	(36)	-31102	(35)	(p,q)						
(125,0.97,0.95)	-31270	(37)	-31515	(37)	-31173	(38)	(1,1)	-28627	(11)	-29209	(14)	-29270	(12)
(125,0.95,0.97)	-30075	(20)	-30559	(23)	-30112	(24)	(1,2)	-28675	(12)	-29278	(16)	-29331	(15)
(125,0.97,0.97)	-30205	(21)	-30560	(24)	-30151	(25)	(2,1)	-28692	(13)	-29278	(15)	-29331	(16)
(75,0.95,0.95)	-31108	(36)	-31574	(40)	-31352	(43)	(2,2)	-28739	(14)	-29343	(17)	-29387	(18)
(75,0.97,0.95)	-31296	(38)	-31626	(41)	-31436	(45)							
(75,0.95,0.97)	-30305	(23)	-30849	(29)	-30582	(31)	DCC						
(75,0.97,0.97)	-30447	(27)	-30872	(31)	-30634	(33)	(p,q)						
(50,0.95,0.95)	-31885	(50)	-32303	(52)	-32236	(52)	(1,1)	-27985	(2)	-28543	(3)	-28625	(2)
(50,0.97,0.95)	-32048	(52)	-32363	(53)	-32320	(53)	(1,2)	-28035	(4)	-28611	(5)	-28686	(4)
(50,0.95,0.97)	-31301	(39)	-31798	(45)	-31698	(47)	(2,1)	-28050	(5)	-28624	(6)	-28687	(5)
(50,0.97,0.97)	-31428	(42)	-31834	(47)	-31755	(50)	(2,2)	-28098	(7)	-28690	(8)	-28744	(6)
MMA							ADCC						
(n_0,ν_0)							(p,q)						
(250,0.95)	-32817	(54)	-31952	(51)	-31744	(49)	(1,1)	-28018	(3)	-28531	(2)	-28630	(3)
(250,0.97)	-31601	(46)	-31006	(33)	-30604	(32)	(1,2)	-28070	(6)	-28605	(4)	-28970	(8)
(125,0.95)	-31943	(51)	-31784	(43)	-31543	(46)	(2,1)	-28122	(8)	-28683	(7)	-28897	(7)
(125,0.97)	-30805	(32)	-30811	(27)	-30482	(30)	(2,2)	-28181	(9)	-28739	(9)	-29108	(9)
(75,0.95)	-31811	(49)	-31842	(48)	-31719	(48)							
(75,0.97)	-30896	(33)	-31049	(34)	-30873	(34)	TDCC						
(50,0.95)	-32396	(53)	-32514	(54)	-32519	(54)	(p,q)						
(50,0.97)	-31725	(47)	-31951	(50)	-31915	(51)	(20)	-27876	(1)	-28396	(1)	-28443	(1)
GEWMA							CDCC						
(p,q,ν_0)							(p,q)						
(1,1,0.95)	-31386	(40)	-31718	(42)	-31116	(36)	(1,1)	-28407	(10)	-30683	(25)	-29139	(10)
(1,2,0.95)	-31426	(41)	-31786	(44)	-31180	(40)							
(2,1,0.95)	-31448	(43)	-31806	(46)	-31179	(39)							
(2,2,0.95)	-31480	(44)	-31875	(49)	-31239	(42)							

to use some standard statistics such as MSE (Mean Squared Error), Mean Absolute Error (MAE), etc. However, it is difficult to apply this traditional method to evaluate a large number of multivariate volatility models from different families. Moreover, a major drawback of MSE method is that it relies on the fourth moment (squares of squares) of realised return. We can see in the following formula

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\bar{\sigma}_t - \hat{\sigma}_t\right)^2 \tag{2.33}$$

where $\bar{\sigma}_i$ is the realised covariance of a portfolio return; $\hat{\sigma}_i$ is the forecast of covariance of a portfolio. Both of them are considered as the second moment of return. If the distribution of return has a fat-tailed behaviour, the MSE criterion will be heavily biased due to effect of large shocks, thereby is less reliable in measuring a portfolio performance in finance.

Hence, another method, recently suggested in the literature, is the use of VaR-based diagnostic tests to check the performance of volatility models. This method differs from the traditional one by evaluating a volatility model based on decisions on how it performs in trading and risk management. The core of this method is the application of the Value at Risk theory in financial econometrics.

2.3.1.2.1. Value at Risk Theory in finance

Following its introduction in October, 1994 by JP Morgan, VaR is widely accepted by portfolio managers as a reliable method of quantifying market risk and by financial regulators as a milestone in the revolution of risk management. The role of risk measures, using VaR, in risk management are the interest of both academia and practitioners. VaR is defined as the maximum loss, which can occur with possibility of $X\%$ over a holding period of t days. It means that VaR gives an estimate of downside risk of a portfolio.

Therefore, the VaR of a portfolio at time t can be defined in the following formula

$$Pr\left[p_t < VaR_t(\alpha)\right] = \alpha \tag{2.34}$$

This means that there is a possibility of $\alpha\%$ for a portfolio return, p_t, at time t to fall below $VaR_t(\alpha)$. The advantage of VaR is that it can summarize risk in a single number, which is a loss of a portfolio over a period from t-1 to t. For example, if a daily VaR is stated as 1% to a 95% level of confidence, this means that during the day there is a only a 5% chance that the portfolio return (the loss) will fall below 1%. The VaR measures a potential loss in market value of a portfolio using the estimated volatility and correlations. Therefore, the Value at Risk theory is now popular in risk management in finance. Thus, it is applied in the risk management of financial assets such as fixed-income instruments, options and stocks. Moreover, the VaR theory can be applied in credit risk management.[4]

To estimate VaR, there are three methods: Historical Method, Variance-Covariance Method and Monte-Carlo Simulation Method. The Historical Method simply rearranges actual portfolio returns, putting them into a histogram. It then assumes that history will repeat itself, from a risk perspective. The Variance-Covariance method assumes that portfolio return has Normal distribution. We then need to estimate the expected portfolio return and its standard deviation, which allow for plotting a Normal distribution curve along the actual return. The normal curve will help to locate where the worst 5% or 1% of actual portfolio return is. The Monte-Carlo Simulation approach uses a model to predict the future portfolio return and randomly runs hypothetical trials for this model. The predicted returns, generated by performing a number of trials, are now re-organised from the worst to the best. Looking at the 5% or 1% from the left tail, we can tell the maximum loss of a portfolio, which is the VaR of portfolio.

In financial econometrics, a multivariate volatility model can be used to estimate the conditional volatilities and correlations of a number of financial assets. So it can estimate

[4] For the details of application of VaR in finance, see Introduction to Value at Risk, 4th ed., Choudhry (2006).

the VaR of a portfolio constructed from those financial assets. The Variance-Covariance approach is appropriate to estimate the VaR of portfolio in this case. However, this approach assumes a multivariate Normal distribution for the return of a portfolio and linearity in the dependence structure between financial assets in portfolio. In fact, the distribution of financial return is more likely to have fatter tails than Normal distribution due to the presence of extremes in financial data. Moreover, the asymmetric property of financial return may cause a non-linearity in correlation between financial asset. Hence, a multivariate volatility model may not give a precise estimate VaR of a portfolio due to those reasons. It, therefore, initiate an idea that an estimate of the VaR of a portfolio can be use to evaluate the performance of a multivariate volatility model. A well-performing model will have to deliver an adequate VaR of a portfolio.

Using VaR method, there are two manners of model evaluation: one is from the point of view of financial authorities who monitor the trading behaviour of investors; the other is from the point of view of investors themselves. The manner for financial authorities is known as passive risk management, which assumes equal weights for assets in a portfolio, as they do not know the structure of portfolio. For investors, who know the weights of assets in a portfolio, the manner is called active risk management. Therefore, the passive risk management uses pre-determined weights for assets in a portfolio and the active risk management employs flexible weights for assets in a portfolio, which are optimally computed by using the popular approach of mean-variance analysis.

2.3.1.2.2. VaR-based diagnostic tests

The Value-at-Risk theory is used to focus on the estimate of a portfolio return based on the risk represented by the covariance matrix, H_t estimated by a volatility model. In our study, this technique is also appropriate for the out-of-sample evaluations of the 54 volatility models. The first step of the diagnostic test is to construct a portfolio based on $m \times 1$ vector of returns, $r_t \sim (\mu_t,\ H_t|\Omega_{t-1})$.

Let $\rho_{i,t}$ be the return on a portfolio comprised of m assets with weights, $w_{i,t-1}$ which can be pre-determined weights, $w^p_{i,t-1}$ for the passive risk management or the optimal weights, $w^a_{i,t-1}$ of model i for the active risk management. In this study, for the passive risk management, the weights for assets in the portfolio, $w^p_{i,t-1}$ are equally set to $\frac{1}{20}$ to compute the portfolio return, $\hat{\rho}^p_{i,t}$. For portfolio return, $\hat{\rho}^a_{i,t}$ in the active risk management, the optimal portfolio weights, $w^a_{i,t-1}$ were computed using the forecast of the conditional mean, $\mu_{i,t}$ given by Equation 2.31 and the one-step-ahead recursive forecast of the $H_{i,t}$ of each of the 54 volatility models under the assumption of the Gaussian distribution or the Student's t-distribution with 6 degrees of freedom. So the portfolio return can be generally expressed as

$$\rho_{i,t} = w'_{i,t-1} r_t \tag{2.35}$$

In risk management, managers always need a benchmark for the loss of their portfolios. For example, $\rho^\star_{i,t-1}$ is considered as the maximum daily loss so managers would expect a probability, α for their portfolio return, $\rho_{i,t}$ at time t to fall below the benchmark, $\rho^\star_{i,t-1}$ conditionally specified by all available information up to time t-1. Hence, α is the risk tolerance of managers which could be set at 1% or 5%. This constraint in risk management can be expressed as follows

$$Pr(\rho_{i,t} < -\rho^\star_{i,t-1} | \Omega_{t-1}) \leq \alpha \tag{2.36}$$

The main idea of this diagnostic test to check for the validity of a volatility model is to count the number of times this VaR constraint is violated using a count function, I_t as follows

$$I_t(\rho_{i,t} + \rho^\star_{i,t-1}) \begin{cases} = 1 & \text{if } \rho_{i,t} + \rho^\star_{i,t-1} < 0 \text{ or VaR constraint in Equation 2.36 is violated} \\ = 0 & \text{otherwise} \end{cases}$$

(2.37)

For the evaluation for each model, the whole sample is divided into two sub-samples $T_{est}(t = 1 : T)$ *and* $T_{eval}(t = T + 1 : T + N)$ where T_{est} is for model estimation and T_{eval} is for model evaluation. The VaR indicator, I_t is recursively computed by using N observations in the evaluation period. We can count the number of days when the VaR constraint is violated by using the VaR indicators for model i as follows

$$\hat{\pi}_i = \frac{1}{N} \sum_{t=T+1}^{T+N} \hat{I}_t \tag{2.38}$$

Hence, under the specification of a volatility model, $\hat{\pi}_i$ will have mean α and variance $\dfrac{\alpha(1-\alpha)}{N}$. Moreover, a standardized test statistic can be obtained based on the result from Equation 2.38 as follows

$$z_{\hat{\pi}_i} = \frac{\sqrt{N}(\hat{\pi}_i - \alpha)}{\sqrt{\alpha(1-\alpha)}} \tag{2.39}$$

For a sufficiently large sample of evaluation, the above test statistic is asymptotically normally distributed with zero mean and unit variance. The standardized test statistic is used to test the null hypothesis under which volatility model i is correctly specified

$$H_0 : \quad H_t = H_t\left(\hat{\theta}_{S_{est}}\right) \; or \; \hat{\pi}_i = \alpha \tag{2.40}$$

2.3.1.2.3. Passive risk management

In passive risk management, the VaR constraint in Equation 2.36 becomes

$$Pr(\rho_t < -\bar{\rho}_{i,t-1}|\Omega_{t-1}) \leq \alpha \tag{2.41}$$

where ρ_t is constructed with no need of a volatility model i and weights are equally set to $\frac{1}{20}$. A volatility model i is only used to compute the benchmark for the maximum daily loss, $\bar{\rho}_{i,t-1}$.[5]

The first 800 observations were used for model estimation and the last 3104 observations were used for the recursive computations of the above statistics of model performance with the update frequency is 25 days or monthly update. By setting the risk tolerance probability $\alpha = 1\%$ and $\alpha = 5\%$ and based on the equally-weighted portfolio returns, we can compute the VaR exceedance ratio ($\hat{\pi}_i$) and the standardized test statistic ($z_{\hat{\pi}_i}$) for each model.

Table 2.7 and Table 2.8 display the results for VaR-based diagnostic tests following the passive risk management with $\alpha = 1\% \; and \; \alpha = 5\%$, respectively. These two tables give the estimated VaR exceedance ratio ($\hat{\pi}_i$, in percent) defined as the percentage of the days in the evaluation sample when the VaR constraint in Equation 2.36 was violated and the standardized test statistic ($z_{\hat{\pi}_i}$) described in Equation 2.39 following the Gaussian distribution and the Student's t-distribution with 6 degrees of freedom.

In Table 2.7, the risk tolerance, the violation rates for all models under the Gaussian distribution assumption, which range from a low of 1.90% for 6 out of the 8 GEWMA models to a high of 2.58% for the EQMA(250) and the EQMA(125) models, are consistently higher than those of all models under the Student's t-distribution assumption, which are between a low of 1.16% for the GEWMA(1,1,0.97) and a high of 2.03 for the EQMA(250) and the MMA(125,0.97) models. Therefore, the volatility models in this study performed better under the Student's t-distribution assumption. Under the Student's $t(6)$-distribution

[5] For the derivation of $\bar{\rho}_{i,t-1}$, see Appendix in section A.3

Table 2.7.: VaR-based Diagnostic Tests under Passive Risk Management Using 54 Multivariate Volatility Models ($\alpha = 1\%$)

Model type	normal		$t6$		Model type	normal		$t6$	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$		$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
EQMA					(1,1,0.97)	1.90	5.05	1.16	0.90
(n_0)					(1,2,0.97)	1.90	5.05	1.19	1.08
(250)	2.26	7.03	1.87	4.87	(2,1,0.97)	1.93	5.23	1.19	1.08
(125)	2.58	8.84	2.03	5.77	(2,2,0.97)	1.90	5.05	1.22	1.26
(75)	2.58	8.84	2.00	5.59					
(50)	2.48	8.29	1.93	5.23	OGARCH				
					(p, q)				
EWMA					(1,1)	1.97	5.41	1.22	1.26
(n_0, λ_0, ν_0)					(1,2)	1.97	5.41	1.29	1.62
(250,0.95,0.95)	2.26	7.03	1.58	3.24	(2,1)	1.97	5.41	1.26	1.44
(250,0.97,0.95)	2.16	6.49	1.58	3.24	(2,2)	1.97	5.41	1.35	1.98
(250,0.95,0.97)	2.16	6.49	1.61	3.42					
(250,0.97,0.97)	2.16	6.49	1.68	3.78	CCC				
(125,0.95,0.95)	2.26	7.03	1.58	3.24	(p, q)				
(125,0.97,0.95)	2.19	6.67	1.58	3.24	(1,1)	2.19	6.67	1.64	3.60
(125,0.95,0.97)	2.19	6.67	1.61	3.42	(1,2)	2.26	7.03	1.58	3.24
(125,0.97,0.97)	2.16	6.49	1.71	3.96	(2,1)	2.13	6.31	1.61	3.42
(75,0.95,0.95)	2.26	7.03	1.64	3.60	(2,2)	2.19	6.67	1.61	3.42
(75,0.97,0.95)	2.19	6.67	1.61	3.42					
(75,0.95,0.97)	2.19	6.67	1.68	3.78	DCC				
(75,0.97,0.97)	2.22	6.85	1.68	3.78	(p, q)				
(50,0.95,0.95)	2.29	7.21	1.64	3.60	(1,1)	2.09	6.13	1.55	3.06
(50,0.97,0.95)	2.45	8.11	1.74	4.14	(1,2)	2.13	6.31	1.55	3.06
(50,0.95,0.97)	2.35	7.57	1.64	3.60	(2,1)	2.06	5.95	1.58	3.24
(50,0.97,0.97)	2.32	7.39	1.84	4.69	(2,2)	2.09	6.13	1.55	3.06
MMA					ADCC				
(n_0, ν_0)					(p, q)				
(250,0.95)	2.16	6.49	1.68	3.78	(1,1)	2.16	6.49	1.48	2.70
(250,0.97)	2.22	6.85	1.71	3.96	(1,2)	2.26	7.03	1.55	3.06
(125,0.95)	2.35	7.57	1.97	5.41	(2,1)	2.29	7.21	1.48	2.70
(125,0.97)	2.48	8.29	2.03	5.77	(2,2)	2.26	7.03	1.68	3.78
(75,0.95)	2.38	7.75	1.93	5.23					
(75,0.97)	2.51	8.47	1.93	5.23	TDCC				
(50,0.95)	2.48	8.29	1.80	4.51	(p, q)				
(50,0.97)	2.58	8.84	1.84	4.69	(1,1)	-	-	1.80	4.51
GEWMA					CDCC				
(p, q, ν_0)					(p, q)				
(1,1,0.95)	1.90	5.05	1.26	1.44	(1,1)	2.22	6.85	1.77	4.32
(1,2,0.95)	1.90	5.05	1.29	1.62					
(2,1,0.95)	1.97	5.41	1.29	1.62					
(2,2,0.95)	1.90	5.05	1.32	1.80					

Table 2.8.: VaR-based Diagnostic Tests under Passive Risk Management Using 54 Multivariate Volatility Models ($\alpha = 5\%$)

Model type	normal		t6		Model type	normal		t6	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$		$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
EQMA					(1,1,0.97)	5.22	0.56	5.77	1.96
(n_0)					(1,2,0.97)	5.19	0.48	5.80	2.05
(250)	5.38	0.98	5.77	1.96	(2,1,0.97)	5.22	0.56	5.77	1.96
(125)	5.93	2.38	6.45	3.69	(2,2,0.97)	5.16	0.40	5.80	2.05
(75)	6.09	2.79	6.48	3.78					
(50)	6.12	2.87	6.48	3.78	OGARCH				
					(p, q)				
EWMA					(1,1)	5.00	-0.01	5.64	1.64
(n_0, λ_0, ν_0)					(1,2)	5.16	0.40	5.67	1.72
(250,0.95,0.95)	5.77	1.96	6.12	2.87	(2,1)	5.03	0.07	5.67	1.72
(250,0.97,0.95)	5.48	1.22	6.16	2.95	(2,2)	5.06	0.15	5.67	1.72
(250,0.95,0.97)	5.74	1.88	6.28	3.28					
(250,0.97,0.97)	5.45	1.14	6.09	2.79	CCC				
(125,0.95,0.95)	5.74	1.88	6.12	2.87	(p, q)				
(125,0.97,0.95)	5.58	1.47	6.19	3.04	(1,1)	5.93	2.38	6.57	4.02
(125,0.95,0.97)	5.74	1.88	6.28	3.28	(1,2)	5.96	2.46	6.54	3.94
(125,0.97,0.97)	5.45	1.14	6.22	3.12	(2,1)	5.90	2.29	6.51	3.86
(75,0.95,0.95)	5.70	1.80	6.35	3.45	(2,2)	5.96	2.46	6.48	3.78
(75,0.97,0.95)	5.70	1.80	6.25	3.20					
(75,0.95,0.97)	5.64	1.64	6.38	3.53	DCC				
(75,0.97,0.97)	5.61	1.55	6.38	3.53	(p, q)				
(50,0.95,0.95)	5.83	2.13	6.54	3.94	(1,1)	5.70	1.80	6.48	3.78
(50,0.97,0.95)	5.77	1.96	6.64	4.19	(1,2)	5.74	1.88	6.54	3.94
(50,0.95,0.97)	5.87	2.21	6.51	3.86	(2,1)	5.80	2.05	6.48	3.78
(50,0.97,0.97)	5.96	2.46	6.61	4.11	(2,2)	5.70	1.80	6.54	3.94
MMA					ADCC				
(n_0, ν_0)					(p, q)				
(250,0.95)	5.38	0.98	5.83	2.13	(1,1)	5.83	2.13	6.48	3.78
(250,0.97)	5.45	1.14	5.83	2.13	(1,2)	6.09	2.79	6.67	4.27
(125,0.95)	5.87	2.21	6.51	3.86	(2,1)	5.96	2.46	6.41	3.61
(125,0.97)	6.03	2.62	6.51	3.86	(2,2)	5.99	2.54	6.51	3.86
(75,0.95)	5.74	1.88	6.32	3.36					
(75,0.97)	5.90	2.29	6.48	3.78	TDCC				
(50,0.95)	5.93	2.38	6.54	3.94	(p, q)				
(50,0.97)	5.87	2.21	6.57	4.02	(1,1)	-	-	5.61	1.55
GEWMA					CDCC				
(p, q, ν_0)					(p, q)				
(1,1,0.95)	5.22	0.56	5.87	2.21	(1,1)	6.51	3.86	7.12	5.42
(1,2,0.95)	5.22	0.56	5.83	2.13					
(2,1,0.95)	5.25	0.65	5.90	2.29					
(2,2,0.95)	5.22	0.56	5.90	2.29					

assumption, the rate of VaR constraint exceedance, $\hat{\pi}_i$ centres around 1.5% and does not vary markedly across the 54 models. The VaR exceedance rate of three Riskmetrics filters of the GEWMA(1,1,0.97), the GEWMA(1,2,0.97) and the GEWMA(2,1,0.97) are 1.16%, 1.19% and 1.19%, respectively. These are the lowest rates among the 54 model and marginally close to 1% with the test statistic, $z_{\hat{\pi}_i}$ of the GEWMA(1,1,0.97), the GEWMA(1,2,0.97), the GEWMA(2,1,0.97) and the GEWMA(2,2,0.97) models are 0.90, 1.08, 1.08, and 1.26, respectively. This indicates that the 3 models are correctly specified as the null hypothesis in Equation 2.40 cannot be rejected at 99% significance levels. The GEWMA(2,2,0.97) and O-GARCH(1,1) models also passed this test with the value of $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being 1.22% and 1.26, respectively. The group of the DCC-type models performed well in this test with the low estimates of $\hat{\pi}_i$ ranging from 1.48% for the ADCC(1,1) model to 1.80% for the TDCC model under the Student's t-distribution assumption. However, no models in this group passed the test.

Under the Normal distribution assumption, the VaR violation rates of all model classes, excepts the GEWMA, OGARCH classes, are well above 2% and the values of test statistic for all model are significantly larger than 5. This shows that all models are clearly rejected by this test under the Gaussian assumption. So the Gaussian assumption used for innovations is not relevant as it cannot capture the fat-tailed behaviour of the financial returns.

The results presented in Table 2.8 consider larger risk tolerance probability, $\alpha = 5\%$. The majority of the volatility models had the estimated VaR violation rates around 5.5% following the Gaussian distribution assumption and around 6.5% following the Student's t-distribution assumption. According to this result, the Gaussian distribution assumption is more relevant than the Student's t-distribution when all GEWMA models with $z_{\hat{\pi}_i}$ ranging from 0.40 for the GEWMA(2,2,0.97) to 0.65 for GEWMA(2,1,0.95) were significant at the 1% level under the Gaussian distribution assumption. Similarly, all the O-GARCH models passed the test under the Gaussian assumption with $\hat{\pi}_i$ ranging from 5% to 5.16% and $z_{\hat{\pi}_i}$ being between -0.01 and 0.4. The TDCC model also passed this test at the 5% significance level while all other members of the DCC family failed to pass this test regardless to any distribution assumption. Interestingly, the TDCC model is the only model that managed to pass the test under the Student's t-distribution with the lowest value of $\hat{\pi}_i$ being at 5.61 and the lowest value of $z_{\hat{\pi}_i}$ being at 1.55.

Under the Student's $t(6)$-distribution, the VaR exceedance rates of all models, being above 6%, are clearly larger than the hypothesized level of 5%. This suggests that the use of the Student's t-distribution assumption is not necessary when the risk tolerance of managers is getting higher. In the passive risk management manner, the use of the distribution assumption depends on the choice of the risk tolerance parameter to choose the best volatility model to fit to the emerging data.

2.3.1.2.4. Active risk management

The use of VaR constrain, $\bar{\rho}_{i,t-1}$ in Equation 2.41, which is given by a model i, is relatively efficient in controlling the violation rate with respect to the risk tolerance. However, the limit of this test is that the portfolio return is computed with no need for a multivariate volatility model. So we are not sure if the portfolio is optimally computed with respect to the variance estimated by a multivariate volatility model. The limit of the VaR-based diagnostic test for equal weights is indicated by Pesaran *et al* (2009) in which the power of the test is dependent on the weights, $w_{i,t-1}$ and the correlations between the assets in portfolio are likely to be time-varying due to shocks during the time of the recent financial crises and the financial integration of the emerging markets. Therefore, the VaR-based diagnostic test is more likely to be biased in practice. Hence, in the active risk management, the portfolio return is directly constructed by the complete use of a volatility model to compute optimal portfolio weights, $w^a_{i,t-1}$ as follows

$$\begin{cases} w^a_{i,t-1} = \dfrac{1}{\delta}\hat{H}^{-1}_{i,t-1}\hat{\mu}_{i,t-1} & \text{if the VaR constraint in Equation 2.44 does not bind} \\ w^a_{i,t-1} = \dfrac{1}{\delta^\star_{i,t-1}}\hat{H}^{-1}_{i,t-1}\hat{\mu}_{i,t-1} & \text{if the VaR constraint in Equation 2.44 binds} \end{cases} \tag{2.42}$$

where δ is the risk aversion coefficient representing the attitude of managers to risk; $\delta^{\star}_{i,t-1}$[6] is the risk aversion at time t-1 which is chosen when the VaR binds for the optimal weights; $\hat{\mu}_{i,t-1}$ is the conditional mean of portfolio given by model i; $\hat{H}_{i,t-1}$ is the one-step-ahead forecast of the conditional variance-covariance matrix given by model i.

Once the portfolio return is obtained based on the use of a volatility model, the maximum daily loss can be pre-specified based on the preference of risk managers. So the VaR constraint in Equation 2.36 now becomes

$$Pr(\rho_{i,t} < -L_{t-1}|\Omega_{t-1}) \leq \alpha \tag{2.43}$$

Table 2.9 provides the results of the VaR-based diagnostic test for the 54 models following the active risk management manner mentioned above. Using the same risk aversion, $\delta = 75$ as in Pesaran *et al* (2009), we obtained the VaR binding rate of the models for the optimal weights with the data from the emerging markets, which is well above 80%, is significantly higher than those in the original research, which focused on more integrated and developed financial markets. This is because the risk aversion was set too small relative to the evaluation sample that covers the whole period of the global financial crisis with large unexpected shortfalls in the emerging markets. To obtain a reasonable VaR binding rate for the optimal weights, the risk aversion coefficient is changed to 105. Following the results in these two table, the VaR constraint bound more often in the case of the $t(6)$-distributed returns and for the Riskmetrics specifications. This indicated that the VaR constraint is more reasonable if a t-distribution is used. Under the Student's t-distribution assumption, the TDCC model has the lowest VaR binding rate which is 35% while the DCC(1,2) has the lowest VaR binding rate of 22% under the Gaussian assumption. Besides, a volatility model can be simply evaluated using the Information Ratio (IR) obtained by dividing the optimal portfolio return by it standard deviation. A model that performs well is expected to have a positive and high value of IR.

[6] For the derivation of $\delta^{\star}_{i,t-1}$, see Appendix in section A.2.

For the evaluations on the trading performance by using the Information Ratio (IR), all values of the IR are positive and between the lowest of 0.45 for the ADCC(2,2) model under the Student's $t(6)$-distribution assumption to the highest of 4.71 for the DCC(2,1) model under both of the assumptions. The DCC models perform best with the IR ranging from 4.60 to 4.71. The TDCC model with the IR of 4.41 and the ADCC(1,1) model are also in the top models. The CCC models are the second best models after the DCC models. Amongst the Riskmetrics filters, the GEWMA models have the highest IRs which are in the range from 3.29 for the GEWMA(1,2,0.95) to 3.98 for the GEWMA(2,1,0.97).

With the risk tolerance probability $\alpha = 1\%$ and the maximum daily loss $L_{t-1} = 1\%$, the estimated rate of VaR exceedance for each model under the Normal distribution is higher than that under the Student's t-distribution. Under the Gaussian assumption, those rates for all Riskmetrics filters are ranging from the lowest of 4.25% for the GEWMA(1,2,0.97) model to the highest of 13.21% for the MMA(50,0.95) model while under the Student's t-distribution those rates are between the lowest of 3.22% for the GEWMA(1,1,0.97) model and the highest of 11.25% for the EWMA(50,0.97,0.95) model. The O-GARCH models perform better than the Riskmetrics filters with the rates centering around 3% for Gaussian innovations and around 2.5% for $t(6)$-distributed innovations. The DCC-type models are ranked in the top models with the violation rates close to the hypothesized level of 1%. Specifically, under the Student's t-distribution assumption, the DCC models are the best models with the value of $\hat{\pi}_i$ being from 1% for the DCC(1,2) model to 1.13% for the DCC(2,2) model and with the values of $z_{\hat{\pi}_i}$ being from -0.01 for the DCC(1,2) model to 0.72 the DCC(2,2) model. This result indicates that the standard DCC models are correctly specified by the test. Following this test, under the t-distribution assumption: the ADCC(1,1) model is significant at the 1% level with the estimated rate of VaR violation being 1.22%; the CCC(1,1) model is also significant at the 5% level with the estimated rate of VaR violation being 1.26%; the CDCC model is not significant with the violation rate of 2% and the value of test statistic 5.59. However, for the Gaussian assumption under which the models were actually estimated, there is only the DCC(1,1) model which managed to pass the test with $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being 1.22% and 1.26, respectively. The TDCC model, which is the only model with the degrees of freedom being endogenously estimated, has the values of $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being at 1.47% and 2.70, respectively. In term of practical estimations of the 54 models, the TDCC model, therefore, is considered as the second best model in this test while the DCC(1,1) is the best model.

In the two approaches for the VaR-based test, the DCC-type models showed that it is more relevant in modelling volatilities and correlations of the emerging financial markets while this model type consistently remained in the top models in both of the tests. The Riskmetrics filters failed the VaR-based test in the active risk management where the complete knowledge of a multivariate volatility model is utilized.

2.3.1.2.5. The Kolmogorov-Smirnov and the Kuiper tests

Another diagnostic test based on Berkowitz (2001), as proposed in Pesaran and Pesaran (2007), is the probability integral transforms (PITs) as follows

$$\hat{U}_{i,t} = F_\nu\left(\frac{w'_{t-1}r_t - w'_{t-1}\hat{\mu}_{i,t-1}}{\sqrt{\dfrac{\nu-2}{\nu}w'_{t-1}\hat{H}_{i,t-1}w_{t-1}}}\right),\ t \in T_1 = \{T+1,\ T+2,\ ...,\ T+N\} \tag{2.44}$$

where w_{t-1} is the $m \times 1$ matrix of portfolio weights which are equally set to $\dfrac{1}{20}$; $\hat{\mu}_{i,t-1}$ is the conditional mean of the portfolio given by model i; $\hat{H}_{i,t-1}$ is the one-step-ahead forecast of the conditional variance-covariance matrix given by model i; ν is the degrees of freedom of the portfolio, which takes the endogenous value if model i is the TDCC or is equal to 6 degrees of freedom, otherwise.

Under the null hypothesis that the model i is correctly specified, the estimates of $\hat{U}_{i,t}$ are uniformly distributed within the range (0,1). To test if $\hat{U}_{i,t}$ is uniformly distributed over time t ranging over the evaluation period, the Kolmogorov-Smirnov Test and the Kuiper Test are suggested with KS and Ku statistics, respectively. These two test statistics are

Table 2.9.: Information Ratios and VaR-based Diagnostic Tests under Active Risk Management using 54 Multivariate Volatility Models ($\alpha = 1\%$)

Model type		Normal					Student-t(6)				
		mean return	IR	$\hat{\pi}$	$z_{\hat{\pi}}$	% VaR binds	mean return	IR	$\hat{\pi}$	$z_{\hat{\pi}}$	% VaR binds
EQMA	(n_0)										
	(250)	44.16	2.83	4.38	18.94	36	40.62	2.95	3.64	14.79	56
	(125)	48.63	2.92	5.51	25.25	42	43.88	3.04	4.35	18.76	64
	(75)	56.52	3.17	6.90	33.01	51	50.04	3.26	5.48	25.07	71
	(50)	69.87	3.11	10.70	54.30	60	61.02	3.16	9.02	44.92	78
EWMA	(n_0, λ_0, ν_0)										
	(250,0.95,0.95)	64.48	3.14	8.48	41.85	52	57.67	3.15	7.22	34.82	73
	(250,0.97,0.95)	63.77	2.64	8.25	40.59	50	57.44	2.65	7.12	34.27	70
	(250,0.95,0.97)	54.30	3.83	5.45	24.89	45	48.07	3.90	4.54	19.84	67
	(250,0.97,0.97)	52.77	3.36	5.35	24.35	44	47.30	3.46	4.51	19.66	65
	(125,0.95,0.95)	64.72	3.13	8.54	42.21	53	57.86	3.15	7.28	35.18	73
	(125,0.97,0.95)	64.09	2.65	8.41	41.49	51	57.62	2.66	7.25	35.00	71
	(125,0.95,0.97)	55.31	3.76	6.06	28.32	47	48.86	3.83	4.74	20.92	69
	(125,0.97,0.97)	53.95	3.32	5.77	26.70	45	48.31	3.40	4.80	21.28	66
	(75,0.95,0.95)	68.01	3.09	9.35	46.72	55	60.53	3.11	7.96	38.97	75
	(75,0.97,0.95)	68.42	2.73	9.15	45.64	54	60.95	2.73	8.06	39.51	74
	(75,0.95,0.97)	60.24	3.61	7.12	34.27	51	53.00	3.68	5.99	27.96	73
	(75,0.97,0.97)	59.77	3.30	7.44	36.08	51	53.19	3.36	6.16	28.86	71
	(50,0.95,0.95)	81.24	2.83	12.28	63.14	62	71.48	2.81	10.89	55.38	80
	(50,0.97,0.95)	83.87	2.61	12.79	66.03	61	73.86	2.57	11.25	57.37	80
	(50,0.95,0.97)	73.10	3.18	11.31	57.73	61	64.06	3.23	9.47	47.45	79
	(50,0.97,0.97)	74.59	2.99	11.41	58.27	60	65.65	2.99	9.76	49.07	78
MMA	(n_0, ν_0)										
	(250,0.95)	73.04	1.42	8.96	44.56	45	67.32	1.35	7.80	38.06	64
	(250,0.97)	55.05	1.72	6.25	29.40	39	51.40	1.71	5.48	25.07	59
	(125,0.95)	68.30	1.57	9.28	46.36	48	62.36	1.49	8.28	40.77	67
	(125,0.97)	54.50	2.00	6.83	32.65	43	50.00	1.96	5.61	25.80	63
	(75,0.95)	71.48	2.03	10.02	50.51	53	63.36	1.92	8.73	43.30	72
	(75,0.97)	60.30	2.49	7.86	38.42	50	54.20	2.48	6.41	30.31	70
	(50,0.95)	89.28	2.24	13.21	68.37	60	78.36	2.11	11.70	59.89	77
	(50,0.97)	77.66	2.59	11.70	59.89	59	68.59	2.55	9.93	49.97	77
GEWMA	(p, q, ν_0)										
	(1,1,0.95)	56.48	3.35	7.03	33.73	45	50.80	3.33	5.80	26.88	68
	(1,2,0.95)	55.19	3.31	6.86	32.83	45	49.68	3.29	5.54	25.43	67
	(2,1,0.95)	60.63	3.39	6.96	33.37	46	54.36	3.36	5.74	26.52	69
	(2,2,0.95)	58.96	3.41	6.90	33.01	46	52.86	3.40	5.83	27.06	69
	(1,1,0.97)	46.80	3.89	4.42	19.12	37	42.24	3.96	3.22	12.44	62
	(1,2,0.97)	45.54	3.82	4.25	18.22	36	41.21	3.89	3.35	13.17	61
	(2,1,0.97)	49.70	3.92	4.45	19.30	38	44.81	3.98	3.38	13.35	63
	(2,2,0.97)	48.36	3.91	4.61	20.20	38	43.66	3.96	3.51	14.07	62
OGARCH	(p, q)										
	(1,1)	43.40	3.61	3.09	11.72	32	39.80	3.76	2.51	8.47	51
	(1,2)	42.73	3.60	2.96	11.00	31	39.22	3.74	2.42	7.93	52
	(2,1)	42.55	3.39	3.13	11.90	32	39.11	3.57	2.55	8.65	52
	(2,2)	41.86	3.38	3.06	11.54	32	38.58	3.58	2.48	8.29	52
CCC	(p, q)										
	(1,1)	40.41	4.56	1.51	2.88	26	37.12	4.57	1.26	1.44	49
	(1,2)	39.38	4.50	1.58	3.24	26	36.25	4.51	1.35	1.98	48
	(2,1)	42.08	4.62	1.55	3.06	28	38.53	4.63	1.32	1.80	50
	(2,2)	41.36	4.58	1.77	4.32	27	37.92	4.59	1.48	2.70	50
DCC	(p, q)										
	(1,1)	38.39	4.66	1.22	1.26	23	35.53	4.66	1.03	0.18	45
	(1,2)	37.38	4.61	1.29	1.62	22	34.68	4.60	1.00	-0.01	44
	(2,1)	40.07	4.71	1.39	2.16	25	36.92	4.71	1.06	0.36	47
	(2,2)	39.39	4.67	1.45	2.52	24	36.37	4.67	1.13	0.72	46
ADCC	(p, q)										
	(1,1)	41.21	4.60	1.71	3.96	27	37.75	4.60	1.22	1.26	49
	(1,2)	87.09	0.50	2.32	7.39	26	82.80	0.48	1.97	5.41	48
	(2,1)	62.92	0.59	2.38	7.75	29	58.84	0.55	1.97	5.41	51
	(2,2)	138.10	0.46	2.58	8.84	28	134.08	0.45	2.26	7.03	50
TDCC	(p, q)										
	(1,1)	-	-	-	-	-	37.39	4.41	1.48	2.70	35
CDCC	(p, q)										
	(1,1)	42.74	1.82	2.22	6.85	24	40.18	1.83	2.00	5.59	45

defined by

$$\mathrm{KS}_N = \max_{T+1 \leq j \leq T+N} \left| \frac{j}{N} - \hat{U}_j^\star \right| \tag{2.45}$$

$$\mathrm{Ku} = \max_{T+1 \leq j \leq T+N} \left(\frac{j}{N} - \hat{U}_j^\star \right) + \max_{T+1 \leq j \leq T+N} \left(\hat{U}_j^\star - \frac{j}{N} \right) \tag{2.46}$$

where $\hat{U}_1^\star \leq \hat{U}_2^\star \leq ... \leq \hat{U}_N^\star$ are ordered values of $\hat{U}_t(x)$ for t ranging in the evaluation period, $S_{eval} = \{T\mathit{+1},\ T\mathit{+2},\ ...,\ T\mathit{+N}\}$.

In this study, the choice of the evaluation range significantly affects the statistical results of the two tests due to the fact that the Global financial crisis in 2007-2008 included in the evaluation period, which caused all models to fail to be well specified because of the large unexpected jumps during the time of the financial turmoil. Table 2.10 delivers the p-values of these tests for the 54 models with the evaluation period expanding from 17/06/1998 to 07/05/2010. Consequently, all 54 models were rejected by the two tests at the 1% significance level with all p-values being equal to zero. However, the test results are clearly different after the evaluation was changed so as to rule out the period of the financial crisis. Thus, these tests focus on the tail behaviour of the distribution. During the crisis period, we can observe a fatter left tail of the distribution, which causes the models to fail the tests. These results displayed in Table 2.11 indicates that under the Gaussian distribution assumption all models were rejected by both tests at the 1% significance level. However, when assuming the Student's $t(6)$-distributed innovations, there are a considerable number of models which cannot be rejected by the tests at the 1% or even at the 5% significance level. Thus, all of the ADCC, the CCC and the O-GARCH models were only rejected by the Ku test at the 10% significance level and by the KS test at the 5% level. The DCC models were rejected by both of the tests at the 5% significance level. However, the CDCC and TDCC models were both rejected by the two tests at the 1% significance level. It is because the average of the estimates of the degrees of freedom of the TDCC model for 125 sub-samples is 13.62 with the highest value of 9.6 for the 122th sub-sample, the lowest value of 20.20 for the 85th sub-sample and 12.88 for the first sub-sample while the other models were assumed to fit to the t-distributed innovations with generic 6 degrees of freedom. The nature of the Ku and KS tests is to emphasize on the tail behaviours of the distribution so by assuming the 6 degrees of freedom for the Student's t-distribution which is clearly higher than the estimated degrees of freedom of the TDCC, it is more likely for the other models to pass the Ku and the KS tests.

Amongst the Riskmetrics filters, the EQMA and the GEWMA models were only rejected by the two tests at the 5% significance level. For the other members in this family, the EWMA(250,0.97,0.97) and the MMA(125,0.97) are the only two models rejected by the Ku test at the 10% significance level and the MMA(125,0.95) model is the only model rejected by the KS test at the 10% significance level. Eight of the 16 EWMA models were rejected by the KS test at the 1% significant while the rest of the EWMA models was rejected by the same test at the 5% level. Also the Ku and the KS tests rejected all the MMA models at the 5% significance level except the MMA(50,0.95) model which was rejected by Ku test at the 1% level. The best model in this class is the MMA(125,0.95) which was not rejected by the Ku test with p-value being 0.112 and the second best model in this class is the MMA(125,0.97) which was only rejected by the Ku test at the 10% significance level. By this results, the Student's $t(6)$-distribution suggested by the descriptive statistic in Table 1.1 is appropriate for the Riskmetrics specifications to fit to the emerging data in the scope of the Ku and KS tests. The DCC-type models continue to perform better than the Riskmetrics filters except the two recent extensions which are the TDCC and the CDCC models that performed well in the previous tests were rejected by the Ku and KS tests at the 1% significance level.

Table 2.10.: Kuiper and Kolmogorov-Smirnov Tests of the Validity of 54 Multivariate Models: Evaluation sample from 17-June-1998 to 07-May-2010

Model type	normal		$t6$		Model type	normal		$t6$	
	Ku	KS	Ku	KS		Ku	KS	Ku	KS
EQMA					(1,1,0.97)	0.000	0.000	0.000	0.002
(n_0)					(1,2,0.97)	0.000	0.000	0.000	0.002
(250)	0.000	0.000	0.000	0.000	(2,1,0.97)	0.000	0.000	0.000	0.002
(125)	0.000	0.000	0.000	0.004	(2,2,0.97)	0.000	0.000	0.000	0.002
(75)	0.000	0.000	0.000	0.002					
(50)	0.000	0.000	0.000	0.001	OGARCH				
					(p,q)				
EWMA					(1,1)	0.000	0.000	0.000	0.000
(n_0,λ_0,ν_0)					(1,2)	0.000	0.000	0.000	0.000
(250,0.95,0.95)	0.000	0.000	0.000	0.000	(2,1)	0.000	0.000	0.000	0.000
(250,0.97,0.95)	0.000	0.000	0.000	0.002	(2,2)	0.000	0.000	0.000	0.000
(250,0.95,0.97)	0.000	0.000	0.000	0.001					
(250,0.97,0.97)	0.000	0.000	0.000	0.002	CCC				
(125,0.95,0.95)	0.000	0.000	0.000	0.000	(p,q)				
(125,0.97,0.95)	0.000	0.000	0.000	0.001	(1,1)	0.000	0.000	0.000	0.000
(125,0.95,0.97)	0.000	0.000	0.000	0.001	(1,2)	0.000	0.000	0.000	0.000
(125,0.97,0.97)	0.000	0.000	0.000	0.002	(2,1)	0.000	0.000	0.000	0.000
(75,0.95,0.95)	0.000	0.000	0.000	0.000	(2,2)	0.000	0.000	0.000	0.000
(75,0.97,0.95)	0.000	0.000	0.000	0.000					
(75,0.95,0.97)	0.000	0.000	0.000	0.000	DCC				
(75,0.97,0.97)	0.000	0.000	0.000	0.001	(p,q)				
(50,0.95,0.95)	0.000	0.000	0.000	0.000	(1,1)	0.000	0.000	0.000	0.000
(50,0.97,0.95)	0.000	0.000	0.000	0.000	(1,2)	0.000	0.000	0.000	0.000
(50,0.95,0.97)	0.000	0.000	0.000	0.000	(2,1)	0.000	0.000	0.000	0.000
(50,0.97,0.97)	0.000	0.000	0.000	0.000	(2,2)	0.000	0.000	0.000	0.000
MMA					ADCC				
(n_0,ν_0)					(p,q)				
(250,0.95)	0.000	0.000	0.000	0.002	(1,1)	0.000	0.000	0.000	0.000
(250,0.97)	0.000	0.000	0.000	0.001	(1,2)	0.000	0.000	0.000	0.000
(125,0.95)	0.000	0.000	0.000	0.003	(2,1)	0.000	0.000	0.000	0.000
(125,0.97)	0.000	0.000	0.000	0.003	(2,2)	0.000	0.000	0.000	0.000
(75,0.95)	0.000	0.000	0.000	0.001					
(75,0.97)	0.000	0.000	0.000	0.002	TDCC				
(50,0.95)	0.000	0.000	0.000	0.000	(p,q)				
(50,0.97)	0.000	0.000	0.000	0.000	(1,1)	0.000	0.000	0.000	0.000
GEWMA					CDCC				
(p,q,ν_0)					(p,q)				
(1,1,0.95)	0.000	0.000	0.000	0.001	(1,1)	0.000	0.000	0.000	0.000
(1,2,0.95)	0.000	0.000	0.000	0.001					
(2,1,0.95)	0.000	0.000	0.000	0.001					
(2,2,0.95)	0.000	0.000	0.000	0.001					

Table 2.11.: Kuiper and Kolmogorov-Smirnov Tests of the Validity of 54 Multivariate Models: Evaluation sample from 17-June-1998 to 30-Aug-2004

Model type	normal		$t6$		Model type	normal		$t6$	
	Ku	KS	Ku	KS		Ku	KS	Ku	KS
EQMA					(1,1,0.97)	0.000	0.000	0.028	0.042
(n_0)					(1,2,0.97)	0.000	0.000	0.024	0.042
(250)	0.000	0.000	0.037	0.020	(2,1,0.97)	0.000	0.000	0.024	0.036
(125)	0.000	0.000	0.050	0.036	(2,2,0.97)	0.000	0.000	0.028	0.042
(75)	0.000	0.000	0.043	0.023					
(50)	0.001	0.001	0.017	0.013	OGARCH				
					(p, q)				
EWMA					(1,1)	0.000	0.000	0.050	0.036
(n_0, λ_0, ν_0)					(1,2)	0.000	0.000	0.058	0.042
(250,0.95,0.95)	0.002	0.003	0.007	0.006	(2,1)	0.000	0.000	0.050	0.036
(250,0.97,0.95)	0.000	0.001	0.028	0.013	(2,2)	0.000	0.000	0.058	0.042
(250,0.95,0.97)	0.001	0.002	0.032	0.015					
(250,0.97,0.97)	0.000	0.001	0.066	0.031	CCC				
(125,0.95,0.95)	0.002	0.003	0.009	0.007	(p, q)				
(125,0.97,0.95)	0.001	0.002	0.037	0.015	(1,1)	0.000	0.000	0.066	0.048
(125,0.95,0.97)	0.001	0.002	0.032	0.017	(1,2)	0.000	0.000	0.050	0.042
(125,0.97,0.97)	0.000	0.001	0.066	0.031	(2,1)	0.000	0.000	0.058	0.048
(75,0.95,0.95)	0.003	0.003	0.003	0.005	(2,2)	0.000	0.000	0.058	0.048
(75,0.97,0.95)	0.002	0.002	0.020	0.009					
(75,0.95,0.97)	0.002	0.002	0.020	0.013	DCC				
(75,0.97,0.97)	0.001	0.001	0.037	0.015	(p, q)				
(50,0.95,0.95)	0.003	0.003	0.001	0.005	(1,1)	0.000	0.000	0.043	0.042
(50,0.97,0.95)	0.002	0.004	0.002	0.008	(1,2)	0.000	0.000	0.037	0.042
(50,0.95,0.97)	0.003	0.005	0.002	0.005	(2,1)	0.000	0.000	0.043	0.042
(50,0.97,0.97)	0.003	0.004	0.004	0.007	(2,2)	0.000	0.000	0.032	0.036
MMA					ADCC				
(n_0, ν_0)					(p, q)				
(250,0.95)	0.000	0.000	0.024	0.027	(1,1)	0.000	0.000	0.066	0.036
(250,0.97)	0.000	0.000	0.012	0.017	(1,2)	0.000	0.000	0.066	0.036
(125,0.95)	0.000	0.000	0.112	0.055	(2,1)	0.000	0.000	0.076	0.036
(125,0.97)	0.000	0.000	0.076	0.048	(2,2)	0.000	0.000	0.087	0.042
(75,0.95)	0.000	0.000	0.024	0.011					
(75,0.97)	0.000	0.000	0.032	0.015	TDCC				
(50,0.95)	0.002	0.003	0.009	0.011	(p, q)				
(50,0.97)	0.001	0.002	0.015	0.011	(1,1)	0.000	0.000	0.000	0.001
GEWMA					CDCC				
(p, q, ν_0)					(p, q)				
(1,1,0.95)	0.000	0.000	0.028	0.042	(1,1)	0.001	0.003	0.000	0.002
(1,2,0.95)	0.000	0.000	0.028	0.036					
(2,1,0.95)	0.000	0.000	0.032	0.042					
(2,2,0.95)	0.000	0.000	0.032	0.042					

2.4. Concluding remarks

The introduction of the DCC-GARCH model of Engle (2002) initiated a large number of studies that extend the DCC model or apply the DCC specifications in risk management, portfolio selection or in the analysis of market interdependence. However, the increasing extensions of the DCC model, as well as the convenience of the Riskmetrics filters, raise the question of the uncertainty of the multivariate volatility models, specifically when they are fitted to the data of emerging financial markets. This paper focuses on the evaluations of the 54 volatility models categorized into 10 classes to provide the detail analysis of model selection in the context of the emerging markets. The methods of evaluation comprise in-sample evaluations, which used the maximized log-likelihood values, AICs and SBICs and out-of-sample evaluations, and used the VaR-based diagnostic test with both the pre-determined weights and the optimal weights in portfolio selection. Moreover, the Kolmogorov-Smirnov and the Kuiper tests are also suggested to test the out-of-sample fit of the 54 models.

In summary, the in-sample and out-of-sample evaluations using the different statistics provide an in-depth view of the applications of the volatility models to the data including 19 emerging financial markets and the US market. The DCC-type models generally performed better than the Riskmetrics specifications and O-GARCH models in both types of the evaluations. Specifically, the TDCC model was selected to be the best model by both of the AIC and SBIC in the in-sample evaluations.

In active risk management, the optimally-computed portfolio return allows for computing the Information Ratio which evaluates how volatility models perform in trading. The IR values were all positive, which indicates that all volatility models perform well in trading. However, the DCC-typed models continued to outperform the Riskmetrics filters in this evaluation by producing significantly higher values of IR. Models under the Student's $t(6)$-distribution had higher IR values than the values of those under Gaussian distribution.

Based on VaR analysis using optimally-weighted portfolio returns, the DCC(1,1) model was ranked as the best model while the TDCC was ranked as the second best model in the VaR-based test. The ADCC(1,1) and the CCC(1,1) models also passed the VaR-based diagnostic test at the 1% and the 5% significance levels, respectively, if assumed to be under the Student's $t(6)$-distribution. However, all Riskmetrics filters were rejected at the 1% significance level by the test under either of the assumptions. The limit of this test is that it is hard to realise in practice when it needs to compute the optimal portfolio weights which are conditionally suggested by a volatility model. However, transaction costs are more likely to prevent risk managers from obtaining the expected optimal portfolio weights.

An alternative approach suggested to overcome the difficulty is the passive risk management by which the weights of portfolio are set equally. This test was performed using two values of risk tolerance probability, $\alpha = 1\%$ and $\alpha = 5\%$. With $\alpha = 1\%$, models using $t(6)$-distributed innovations outperformed those using Gaussian innovations. However, with larger risk tolerance of 5%, models under Student's t-distribution assumption were rejected in favour of those under Gaussian assumption. This contradictory result indicated that the VaR-based diagnostic test in passive risk management is not consistent when the risk tolerance changes. This is due to the fact that this test only used a volatility model to compute VaR constraint rather than to directly compute the portfolio return. The test is more likely to be inconsistent and biased when financial returns experience unexpected shortfalls. Following this test, the choice of the risk tolerance probability is decisive in selecting the best model. However, it is interesting that the DCC(1,1) and the TDCC models consistently managed to be in the top 2 models regardless to the choice of risk tolerance probability.

Using the Kuiper and the Kolmogorov-Smirnov tests with the exclusion the period of the Global financial crisis from evaluation sample, the DCC models were chosen to be the best models. The Riskmetrics filters were also reasonably suggested by the tests. However, the TDCC and the CDCC models were rejected at 1% levels. This is explained by the fact that the TDCC model used the endogenous degrees of freedom for Student's t-distribution which is lower than the generic 6 degrees of freedom for the other models and the CDCC model was designed to fit to the large-scaled portfolios rather than such the medium-scaled portfolio as in this study. The different approach of the Ku and the KS tests in evaluating the models by mainly focusing on the tail behaviour of the innovations to deliver the different test results by which the attractive TDCC and CDCC models were rejected contribute to highlight the fact that there is no best model at all times. This is importantly helpful for investors and risk managers in choosing the most appropriate volatility model.

The Student's $t(6)$-distribution assumption is more relevant than the Gaussian distribution assumption for all of the volatility models to fit to the data from the emerging financial markets. The TDCC model outperformed the other models in in-sample evaluation and performed well in the VaR-based analysis but failed to pass the Ku and KS tests. This is because of the difference between the estimates of the endogenous degrees of freedom of the multivariate Student's t-distribution (centering around 13.62) of the TDCC model and the estimates of the same parameter of the univariate Student's t-distribution given by the univariate t-GARCH model (centering around 5.744). This could be a suggestion for the future research.

The VaR-based diagnostic tests as well as the Ku and KS tests provide more practical ways to evaluate volatility models than the usual in-sample evaluation. It helps both financial authorities, who use passive risk management, and investors, who use active risk management, in choosing the best model for their own purposes. That there is no best model at all times highlights the fact that it is unlikely to find a model that works well in both calm and noisy periods of financial markets. A model with t-distribution assumption is more relevant for crisis period but too conservative in normal period. Hence, designing a model that can deal with both calm and turmoil period is still a real challenge. Therefore, the results in this chapter could not be generalised. However, it helps us to find the best practical model that work relatively well during the time of the Global financial crisis. It is the TDCC model, which is suggested to be applied in the next chapter for the test of financial contagion.

3. TDCC GARCH MODELLING OF VOLATILITIES AND CORRELATIONS OF EMERGING FINANCIAL MARKETS

Abstract

Our results from the previous paper (Barassi *et al*, 2010) showed that the TDCC model outperformed the DCC models and the Riskmetrics filters in the in-sample evaluations using AIC and SBIC and also managed to stay in the top 2 models in the out-of-sample evaluations using the VaR-based diagnostic tests. Moreover, the TDCC model is the only model which had the degrees of freedom of the Student's t-distribution assumption practically estimated while the other models used generic degrees of freedom. This motivated us to use the TDCC model to model the conditional volatility and dynamic correlations of emerging stock markets under the condition of financial crises. So this study investigates the dynamic of the volatilities and the conditional correlations between the US stock market and 19 emerging stock markets from Asia, Latin America and Europe. Using the TDCC GARCH model proposed by Pesaran and Pesaran (2007), which is based on the DCC GARCH model by Engle (2002) with the additional assumption of t-distributed returns and the use of devolatized returns computed by realized volatilities instead of using conditional volatility estimates of the GARCH model, we modelled the dynamics of con-

ditional volatilities and correlations between those emerging markets and the US market. The pair-wise correlations of emerging markets in Asia, Latin America and Europe with the US market generated by the TDCC model allow for our re-evaluations of the empirical tests for contagion proposed by Forbes and Rigobon (2002) using t-tests and by Chiang *et al* (2007) using the AR model with dummies. By using the TDCC-estimated correlations, we review and re-examine the empirical contagion tests performed in 10 previous studies using other parameterizations of the DCC model. With the devolatization in the correlation series to adjust for the heteroskedasticity, our results show that contagion in 2 recent financial crises which are the Sub-Prime crisis in 2007 and the Global financial crisis in 2008 was not as widespread as concluded in the previous studies in the literature.

3.1. Introduction

In the complicated modern world, the behaviour of financial markets and financial assets has recently become the main interest of researchers and the increasing concerns of financial authorities while it has been the central focus of financial practitioners. Thus, examining the nature of financial markets and financial assets becomes the first priority when the Subprime crisis was triggered in 2007 in the US financial markets and spread to other developed financial markets that consequently caused the great recession of the world economy. However, there are debates on how the recent financial crises in the US spread to emerging markets based on financial contagions using the multivariate GARCH models. This paper is going to re-examine how the financial crises spread to emerging markets by using the TDCC model introduced by Pesaran and Pesaran (2007).

In the last three decades, financial econometrics has witnessed significant development, which provides efficient tools to measure and explain the nature of the risk in financial markets. The key point that initiates volatility research is that the volatility of stock markets shows high time-dependence. The volatility dependency of the returns of financial assets also appears in clusters of different sizes at different periods, that gives strong proof that the risk in stock markets cannot be measured by the unconditional volatility, which is given by sample variance and is constant over time. The initial measurement of time-varying volatility is the method that measures historical volatility by calculating the standard deviation of the return of stock price for a short period, e.g. for a month. However, the drawback of historical volatility is that this measurement is 'noisy' because it accumulates a limited number of observations. Besides, the long-run volatility computed by using the whole sample range does not respond instantaneously to the news that is currently updated in the market. Thanks to the introduction of the ARCH model (Autoregressive Conditionally Heteroskedasticity) by Engle (1982) and the GARCH model (Generalized Autoregressive Conditionally heteroskedasticity) by Bollerslev (1986), the time-varying volatilities of financial assets and financial markets can be explained rigorously in a formal framework. This is the breakthrough in financial econometrics as the ARCH/GARCH model provides rigorous background in modelling the volatility of financial markets. Thus, the ARCH/GARCH model gives a parametric measurement of how the effects of past errors and past variances can be transmitted to the variance at present which is known as the conditional variance or the conditional volatility. The ARCH/GARCH model has become popular because of its ease of estimation and its good forecastability which shows the high efficiency of the model. Hence, the conditional volatility given by the ARCH/GARCH model is widely used by financial practitioners and researchers around the world to capture the risk of financial markets.

The univariate ARCH/GARCH model is applied to generate the conditional volatility of a single series because it was initially invented to model inflation in the UK. But the wide applications of the GARCH model in finance lead to a necessity of developing a new volatility framework that deals with multivariate financial time series. The multivariate volatility model is especially helpful when the internationalization of financial markets offers investors chances to trade in financial markets of different countries. Moreover, portfolio diversification suggests investors to allocate their resources to a portfolio of many stocks rather than a single stock. So the question is how different financial markets with different conditional volatilities are related to each other. In more detail, investors may want to know how financial markets dominated in different conditional volatilities are correlated to each other. Thus, the knowledge of risk-updated correlation, known as conditional correlation or time-varying correlation between different markets, helps investors minimize the risk of a portfolio containing stocks from different financial markets. There are several approaches to capture the conditional correlation of financial time series. The multivariate parameterizations of the GARCH model are the prevailing method which can efficiently estimate the conditional correlation between financial markets. Early studies on volatility transmissions and correlations among financial markets can be found in Engle, Ito and Lin (1990), Bollerslev (1990) who introduced the Constant Conditional Correlation model (CCC) which assumes all conditional correlations to be constant to produce a more parsimonious procedure. However, it is, in fact, likely that the conditional correlations vary over time. For example, financial markets can be affected by financial crises or financial integrations. So the cross-market correlations cannot remain constant in the long-run. For this, the introduction of the multivariate GARCH model with the BEKK representations (Engle and Kroner, 1995) delivers different results from that of the CCC model though being applied to the same data sets. However, the BEKK representations include a large amount of unknown parameters which rise exponentially with the number of series in the portfolio. So it is impossible to apply the BEKK to even a small portfolio containing a modest number of assets.

To solve the problem of dimensionality of the BEKK, as well as the problem of constant correlation of the CCC model, Engle (2002) introduced the Dynamic Conditional Correlation model (DCC-GARCH) that significantly improves the CCC model by relaxing the constant correlation assumption to allow for the time-varying correlation and limiting the number of unknown parameters in the DCC model to rise linearly with the number of assets, thereby solving the curse of dimensionality. The DCC framework put forward by Engle (2002) provides the background for risk management of large portfolios that contain a large number of financial assets. Besides, it was proven that conditional volatility may also show asymmetric behaviour. Thus, volatility tends to rise more when negative shocks occur than when positive shocks occur. During a crisis period, the conditional volatility of financial markets is substantially high. To examine the asymmetric property of multivariate time series, the Asymmetric Generalized DCC model (AG-DCC) was proposed by Cappiello, Engle and Sheppard (2006). The AG-DCC is an important extension of the DCC model with an asymmetric term to capture leverage effects to conditional volatilities and correlations. However, it is well documented that the financial data, particularly those recorded at daily or higher frequency, show a fat-tailed distribution which cannot be fully modelled by using the Normal distribution, which is the primary and key assumption of many econometric frameworks in finance. The multivariate volatility models mentioned above face the same problem, so their performance can be improved by assuming a multivariate Student's t-distribution that characterizes the fat-tailed behaviour of financial time series well. But the 2-step procedure of the standard DCC parameterization does not allow for a change to the Student's t-distribution where the degree of freedom is needed as an unknown parameter. This happened because the innovations are standardised to be Gaussian by using the estimates of the conditional volatility obtained in the first step of the DCC procedure. To this extent, another important modification to the DCC model was made by Pesaran and Pesaran (2007) who introduced a devolatization procedure for the innovations and combined the 2 steps in the DCC into one-step estimation using maximum likelihood.

In the study of Pesaran *et al* (2009), the TDCC model is empirically more effective than the standard DCC with applications to integrated markets. Moreover, our study in the previous chapter of this thesis also showed that the TDCC model outperformed the other volatility models. Therefore, the Student's t-distribution assumption of the TDCC suggests that this model is likely to offer a great chance to analyse the volatility of emerging financial markets under the condition of crises where most of models have shown poor performance due to its Gaussian assumption. These are the reasons why the TDCC model was applied to examine the volatilities and conditional correlations among emerging financial markets in this paper. There are 19 emerging markets in Asia, Europe and South America and the US market being considered.

There are a number of applications of DCC-type models in finance, such as portfolio risk management or the analysis of financial market interdependence. Thus, the latter application of the DCC-type models has recently become an important focus of financial econometrics as the threads of the widespread contagion of financial crises which are likely to occur at any time due to the possible collapse of many indebted countries in Europe, such as Greece, Ireland, Portugal. Moreover, the great concern of traders in stock markets, CEOs of financial corporations and financial authorities is how the emerging financial markets are correlated with the developed markets in the short run, especially during the time of financial turmoil. It is, therefore, crucial to evaluate how emerging markets can be affected by the financial crises from developed markets. In short, is there contagion between developed and emerging markets? This study contributes to the literature by investigating how emerging markets are linked to the US by using the attractive TDCC model. It is a guidance that gives investors more choices in allocating their funds and allows them to have a better risk management of portfolios containing stocks from emerging markets.

Specifically, this study is to explain the short-run linkages of the emerging financial markets with the US market based on the pattern of the analysis of the contagion effects of the Dotcom crisis in 2000, the Subprime from 2007-2008 and then the Global financial crises from 2008 to 2009. Because these financial crises have been considered to have a global impact on emerging financial markets. The Asian financial crisis is also considered for comparison with previous studies on this topic. We also provide a review of 10 previous studies in the literature of testing for the contagion using the DCC models. We then apply the TDCC model to estimate the conditional correlations between the emerging markets and the US market which are then devolatized following the method of Pesaran *et al* (2007) before being used in the empirical tests for the financial contagion due to Forbes and Rigobon (2002) and Chiang, Jeon and Li (2007). Our results are significantly different from the previous research by indicating that the two recent financial crises in 2007-2009 are not severely contagious to emerging markets while the Dotcom crisis hit, through contagion, only 2 out of 19 emerging markets.

The structure of the paper is as follows: This section is followed by a literature survey on studies of the stock market volatilities and correlations as well as the empirical tests for the financial contagion. Section 3 presents the estimation procedure of the TDCC model. Section 4 gives the empirical results and discussions. The last section is to conclude and offer some possibilities of applications of the model in future research.

3.2. Literature review

3.2.1. Review of the univariate ARCH/GARCH models with application to Emerging Markets

It is documented by Andersen *et al* (2010) that volatility modelling and forecasting can be both parametric and non-parametric. Among the two methodologies, the introduction of Engle's (1982) ARCH model gives a breakthrough in parametric modelling and forecasting of volatility. Thus, the use of the GARCH volatility modelling can be found in applications in medicine (Ewing, Piette and Payne, 2003), political science (Gronke and Brehm, 2002), electricity market (Battle and Barquin, 2004), agricultural economics (Ramirez and Fadiga, 2003), environmental economics (Chemarin, Heinen and Strobl, 2008), monetary economics (Ruge-Murcia, 2004). However, the estimation and forecast of the GARCH model, which works well with high frequency data, allow for a wide application of parametric volatility modelling in finance.

Mentioned by Campbell, Lo and MacKinlay (1997) as the central point of financial economics, the research of volatility of financial time series has been the mainstream in financial econometrics for over the last 30 years. Hence, it can be seen that there is a rich literature on this topic. Andersen, Bollerslev and Diebold (2006) classify research on univariate volatility into 3 basic frameworks, which are the GARCH method, stochastic volatility and realized volatility, while Poon and Granger (2003) state that volatility can be forecasted following 4 methods, which are historical volatility, the GARCH model, implied volatility and stochastic volatility. Among the methods of forecasting volatility, the ARCH model, introduced by Engle in 1982, was initially proposed to reparameterize the conditional heteroskedasticity in the wage-price equation for the United Kingdom.

However, the family of ARCH models were soon to be successful in financial econometrics as it is actually in form of the ARMA models. Besides, the ARCH-type models are easy to estimate and it can be generalized to different forms flexibly to describe different characteristics of volatility of financial assets. The drawback of the ARCH model is that it requires more lags of the squared errors to model efficiently the conditional variance. Then Bollerslev (1986) proposed a generalized version of the ARCH model, known as the GARCH model, by adding lagged conditional variance that summarizes the effect of past shocks. Hence, the GARCH model is parsimonious while it is equivalent to the ARCH model of order going to infinity.

Since the introduction of the ARCH/GARCH model, there have been a burgeoning number of extensions of the GARCH model. Bollerslev, Chou and Kroner (1992) list an early survey on extensions of the ARCH model and its applications in finance. A large volume of surveys of ARCH/GARCH literature exist, see *inter alia* Bera and Higgins (1993), Bollerslev, Engle and Nelson (1994), Diebold and Lopez (1995), Pagan (1996), Palm (1996), Shephard (1996), Andersen and Bollerslev (1998), Engle and Patton (2001), Andersen, Bollerslev, Christoffersen and Diebold (2006), Bauwens, Laurent and Rombouts (2006). Among them, Engle and Patton (2001) give a clear explanation of forecastability of the GARCH-type models. A comparative review of the univariate GARCH family is given by Poon and Granger (2003) who did a survey 93 papers that compare different methods of modelling and forecasting volatility. Among 93 studies, included in Poon and Granger's paper, seventeen papers compared the alternative versions of the GARCH model. As it is stated in their results that the exponential GARCH of Nelson (1991) and the GJR-GARCH of Glosten, Jagannathan and Runkle (1993) generally outperform the standard GARCH because they are able to capture the asymmetric behaviour of financial returns. Other extensions such as the FI-GARCH (Fractionally Integrated GARCH) of Baillie, Bollerslev and Mikkensen (1996), which allows for high persistent of volatility, and the RS-GARCH (Regime Switching GARCH) of Hamilton and Susmel (1994), which can model volatility among different regimes of low, moderate and high volatility are proven to perform better in some cases. Besides, there are a growing number of extensions of the GARCH model, for example the Absolute-Value GARCH [Taylor (1986)], the GARCH-in-Mean [Engle, Lilien and Robins (1987)], the Non-linear ARCH [Higgins and Bera (1992)], the Threshold GARCH [Zaikonan (1991)], the Asymmetric-Power PARCH [Ding, Engle and Granger (1993)], the Asymmetric GARCH [Engle (1990)], the Non-Linear-Asymmetric AGARCH [Engle and Ng (1993)]. For a comprehensive overview of research development of the ARCH/GARCH model, see Bollerslev (2008) who gives an alphabetical list of over 100 extensions of the ARCH/GARCH model.

Similar to many other models, the ARCH/GARCH was initially designed for research of developed markets where a rich data source is available and market conditions closely reach the assumptions of the theoretical models to accommodate for various empirical tests to produce excellent results. Hence, the ARCH/GARCH model is widely applied to various research in finance and economics in developed countries. However, there is a significant difference between developed markets and emerging markets. There are, for example, the inefficiency of emerging market, the abundance of structural changes caused by inconsistent government policies towards markets, the impacts of high inflation and currency devaluation, liquidity and contagion problems. These reasons provide an immense challenge for standard theoretical models. Early analyses of volatility in emerging markets were performed by Bekaert and Harvey (1997), De Santis and Imrohoroglu (1997), Aggarwal *et al* (1999), Kim and Singal (2000), Jayasuriya (2005), Cunado *et al* (2006), Daal, Naka and Yu (2006), Brooks (2007), and generally indicate the following properties of emerging markets volatility:

- First, there is also strong evidence of time-varying volatility. Quantitatively, they found that volatility in emerging markets changes over time and, as in the developed markets, emerging market volatility tends to cluster implying that high or low volatility can persist for a long time. Volatility shows a high persistence and is predictable.
- Second, volatility in emerging markets is considerably higher than in developed markets at both conditional and unconditional levels.
- Third, there is strong evidence for a fat-tailed conditional distribution of returns,

which indicates that large changes in speculative stock prices are expected to occur frequently.

- Fourth, there exists asymmetric behaviour of volatility of emerging markets. However, the degree of asymmetry is different among emerging markets and is lower than that of developed markets.

- Fifth, an important characteristic of emerging markets is that the financial liberalization process has been brought about at different levels and in different ways. Bekaert and Harvey (1997) studied the time-series property of volatility as well as the cross-sectional effect of volatility, and specifically the effect of equity market liberalization on volatility. On variance estimation from world factor model, with data from 1976-1992 of 17 emerging markets, they showed that stock market liberalization (SML) significantly reduces volatility. Subsequently, Bekaert, Harvey and Lundblad (2000) pooling time series for a cross-section of 20 emerging markets under regulatory changes, but cannot find any significant change in volatility. Kim and Singal (2000) study 14 emerging markets under the effect of removal of restrictions on capital inflows, and find that volatility did not respond to SML after 2 year, but decreased in the fourth and fifth year. By contrast, Levin and Zervos (1998) use the model of Schwert (1989) to test for a structural break and a possible significant change in volatility at the time of liberalizing restrictions on international capital flows and on the repatriation of dividends. The result indicated that volatility generally tends to be higher in the period after liberalization. Koot and Padmanabhan (1993) find that volatility is significantly higher in the period after SML. However, De Santis and Imrohoroglu (1997) test the effect of the degree of market openness, defined as share capital issuance, on volatility. They found no evidence of changes in volatility. Kwan and Reyes (1997) use the GARCH model to show that SML in Taiwan significantly reduces volatility.

- Sixth, an important implication of all asset pricing models is the positive relationship between the conditional return and the conditional variance of invested assets. What is the risk-return relationship in emerging markets? Many researchers found that the link between the return and risk premium is not constant overtime. De Santis and Imrohoroglu (1997), using the GARCH (1,1) of Bollerslev (1986) and the International CAPM, find no evidence on the risk-return relation at the Latin-American markets and the Asian markets. The reason for that result is that the CAPM model and the GARCH (1,1) are not clearly always adequate to capture the variations of developed markets let alone emerging markets. Haque *et al* (2001), using the GARCH (1,1)-in-mean to detect the risk-return relationship in the Latin-American markets, find that five out of seven markets and the Latin American Index have a positive coefficient of risk premium at the 10 percent level. However, Hassan *et al* (2006), using the GARCH (1,1)-in-mean to analyse 7 emerging markets in Europe, document that five of seven markets do not show positive and significant time-varying risk premia.

The early research above utilized the univariate GARCH frameworks. However, it is not irrelevant to use the univariate GARCH to deal with multivariate data set, related to many different emerging markets. Hence, the development of the multivariate GARCH models offers an appropriate tool to analyse the conditional volatility as well as the conditional correlation between emerging markets.

3.2.2. Review of the multivariate GARCH models and the empirical tests for financial contagions

3.2.2.1. Review of the multivariate GARCH models

Researchers have used different methods to model linkages between different financial markets or different financial assets. In early studies, the Vector Autoregressive model (VAR) used by Eun and Shim (1989), Oertmann (1995), Cha and Cheung (1998,[24]) and Janakiramanan and Lamba (1998) has been proven to be effective in modelling the linkages among the first moment of financial markets. Koch and Koch (1991) investigated the linkages of national stock indexes by using a dynamic simultaneous equation system. Hamao, Masulis and Ng (1990) used the univariate GARCH-in-mean to document volatility spillovers among the US, the UK and Japanese stock markets. Engle, Ito and Lin (1990) use a volatility type of VAR to detect the cross-market impact on volatility. However, the multivariate GARCH model with the VECH specification, introduced by Bollerslev, Engle and Wooldridge (1988), proposed a rigorous method to analyse the linkages of the second moment of financial markets and assets. Later, Hafner and Herwartz (1998) introduced volatility impulse response functions, based on the multivariate GARCH model with the VECH representation, to model the impact of independent shocks on the conditional covariances of bivariate exchange rate series. Nevertheless, the initial parameterization of the multivariate GARCH has certain disadvantages, namely, the impossibility to guarantee a positive semi-definite covariance matrix, and the dimensionality problem. To overcome the difficulties of the VECH, the BEKK representation proposed by Engle and Kroner (1995) ensures the positivity of the covariance matrix.

The specification of the multivariate GARCH models starts from the assumption that the vector of the return series $\{r_t\}$ with $k \times 1$ dimension of k assets can be characterized as an autoregressive processes of the first order

$$r_{i,t} = \omega_i + a_i r_{i,t-1} + \varepsilon_{i,t} \text{ with } i\text{=1, 2, ..., } k \tag{3.1}$$

where $\varepsilon_t \sim N(0, H_t)$ is heteroskedastic with $H_t = \{\sigma_{ij,t}\}$ denoting as the covariance matrix with $k \times k$ dimension of the innovation series, ε_t conditional on all information at time t-1, Ω_{t-1}. One would use the VAR as an alternative method to construct the matrix of heteroskedastic error terms. However, VAR-MGARCH, in practice, also delivers results similar to those of AR-MGARCH.

In general, there are several ways to model the conditional multivariate covariance matrix. The first way is documented by Kroner and Ng (1998), summarizing several General Dynamic Covariance models, which include the VECH, BEKK. The BEKK specification of H_t is as follows

$$H_t = C_0' C_0 + \sum_{k=1}^{K} \sum_{i=1}^{q} A_{ki}' H_{t-1} A_{ki} + \sum_{k=1}^{K} \sum_{j=1}^{p} B_{kj}' \Xi_{t-1} \Xi_{t-1}' B_{kj} \tag{3.2}$$

in which the condition to ensure the positivity of H_t is that at least one of the matrices C_0 or B_{ki} has full rank and the matrices $H_{t-1},\ H_{t-2},\ ...,\ H_{t-q}$ are positive definite. The BEKK representation shows how the conditional variance of one variable depends on not only its own lagged conditional variances and squared errors but also those of the other variables. Causality in variance can thus be analysed using the BEKK model. However, the standard multivariate GARCH model has the drawback that the dimension of the model increases exponentially when we increase the number of variables. Although the diagonal-VEC and BEKK versions of the MGARCH models make a large improvement by being more parsimonious, they are still difficult to apply in practice. The extensions of this multivariate GARCH model are to solve the computational burdens constituted by the large number of parameters. In larger models, the over-parameterizations will lead to flat likelihood functions and make statistical inference quite troublesome.

An alternative approach to cope with the dimensionality curse of the VECH and the BEKK approaches is the use of the conditional variance matrix and the conditional correlation matrix to construct the covariance matrix. The Constant Conditional Correlation (CCC) model of Bollerslev (1990) is the initial method that parameterizes the covariance matrix, H_t, directly from a diagonal matrix composed of the conditional variances derived from a univariate GARCH process and a matrix of constant correlations. Moreover, the introduction of the CCC model and the Dynamic Conditional Correlation (DCC) model by Engle (2002) give a formal background for modelling correlation between financial series where the correlation is autoregressively estimated conditional on the estimations of the conditional variance given by the univariate GARCH models and the estimation of the conditional covariance as follows

$$H_t = D_t R D_t \tag{3.3}$$

where D_t is the $n \times n$ diagonal matrix of the conditional standard deviations derived from the univariate GARCH(1,1) process as follows

$$q_{i,t} = \bar{q}_i(1 - \lambda_1 - \lambda_2) + \lambda_1 q_{i,t-1} + \lambda_2 \varepsilon^2_{i,t-1} \tag{3.4}$$

R is the $n \times n$ time invariant matrix of correlation elements. So the condition for H_t to be positive definite at all t is that all conditional variances are well defined and all elements of R matrix are positive.

The CCC model is a parsimonious procedure as the number of the estimated parameters is much lower than those in the BEKK model. However, assuming constant correlations may be fairly restrictive as there may be periods, such as crisis periods, during which correlations may be conditionally time-varying. The DCC model modifies the construction of the covariance matrix H_t in the CCC by allowing the elements of the correlation matrix to vary overtime as follows

$$H_t = D_t R_t D_t \tag{3.5}$$

There has been a growing literature on the family of multivariate GARCH models since the introduction of the DCC model. Bauwens, Laurent and Rombouts (2006) give an overview assessment of the development of multivariate GARCH models. The paper is a survey of the multivariate GARCH models, initially introduced from the VECH and the BEKK representations to the parameterization of the CCC and the DCC models, in which leverage effects and copulas are also considered. The survey also presents a clear review of those multivariate GARCH models in terms of different methods of model estimation, diagnostic checking for model specification with various diagnostic tests. Silvennoinen and Terasvirta (2008) present another important survey on the multivariate GARCH development that includes more recent generalizations of the DCC GARCH model.

The dynamic correlation matrix, R_t in Equation 3.5 is presented as

$$R_t = Q_t^{\star-1} Q_t Q_t^{\star-1} \tag{3.6}$$

The $k \times k$ matrix Q_t can be obtained from the process below

$$Q_t = \bar{Q}(1-\alpha-\beta) + \alpha(\epsilon_{t-1}\epsilon'_{t-1}) + \beta Q_{t-1} \; with \; \epsilon_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{q_{i,t}}} \tag{3.7}$$

where $\bar{Q}$ is the unconditional covariance of the standardised residuals, ϵ_t. The conditions for the existence of the DCC are that all parameters in Equation 3.7 are positive ($\alpha, \ \beta \geq 0$) and the sum of the two is less than one: $\alpha + \beta < 1$. $Q^\star$ is the diagonal matrix with diagonal elements extracted from diagonal elements of Q_t

$$Q_t^\star = \begin{bmatrix} \sqrt{q_{1,t}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{q_{2,t}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \sqrt{q_{k,t}} \end{bmatrix} \tag{3.8}$$

Hence, $\rho_{ij,t}$ which is ij^{th} the element of dynamic conditional correlation matrix, R_t is formally constructed as follows

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{i,t}q_{j,t}}} \tag{3.9}$$

With this computation, we have unit values on the diagonal of R_t and a positive definite matrix, H_t. The correlation coefficients at time t are conditionally estimated using information at time *t-1*. One can extend the model using other parameterizations than the univariate GARCH model such as TGARCH, EGARCH, to capture the asymmetric responses of the univariate conditional variances from negative shocks in returns.

To improve the performance of the DCC model, Cappiello, Engle and Sheppard (2006) proposed the Asymmetric Generalized Dynamic Condition Correlation model (AG-DCC) to obtain smoother regressions and to examine conditional asymmetries in correlation dynamics. The univariate GARCH in Equation 2.12 becomes an asymmetric GARCH, such as the GJR model, and follows

$$q_{i,t} = \omega_{0i} + \lambda_{1,i}\varepsilon_{i,t-1}^2 + \kappa_i d(\epsilon_{i,t-1} < 0)\epsilon_{i,t-1}^2 + \lambda_{2,i}q_{i,t-1} \tag{3.10}$$

In the AG-DCC model, the dynamic structure in Equation 3.7 can be rewritten as

$$Q_t = \bar{Q}(1 - \alpha - \beta) - \bar{N}\gamma + \alpha(\epsilon_{t-1}\epsilon_{t-1}') + \beta Q_{t-1} + \gamma\eta_{t-1}\eta_{t-1}' \tag{3.11}$$

The difference is that the asymmetric term $\eta_{t-1} = I[\epsilon_{t-1} < 0] \circ \epsilon_{t-1}$ represents the effects of negative shocks in the previous period, in which $I[.]$ is the $k \times 1$ indicator function taking on value 1 if the argument is true and 0 otherwise, $\circ$ represents for the hamadard product. $\bar{N} = E[\eta_{t-1}'\eta_{t-1}]$ is the unconditional covariance matrix of negative innovations. So the DCC model is a special case of the AG-DCC when $\gamma = 0$. The conditions for the existence of the ADCC model are $\alpha,\ \beta,\ \gamma > 0$ and $\alpha+\beta+\delta\gamma < 1$ with δ as the maximum eigenvalue of matrix $\left[\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}\right]$ to ensure that the matrix $\bar{Q}(1-\alpha-\beta)-\bar{N}\gamma$ and the initial value of Q_t are positive semi-definite. The latter condition makes the AG-DCC model more restrictive and sometimes harder to converge when it is fitted to emerging data.

However, this major modification of the DCC model helps explaining how the conditional correlations respond to the joint bad news of different markets or different equities. The model showed its improvement over the standard DCC when applied to analyse the conditional correlations of 21 national equity indices in the EU, Americas and Asia as well as that of 5-year average maturity government bond indices of 13 countries (Cappiello *et al*, 2006). They reported stronger co-movements in equity markets during periods of financial turmoil such as the 1987 stock market crash, the beginning of the Gulf war and the Asian financial crisis in 1997. Hyde *et al* (2007) also discovered significant increases in correlations among Asian-Pacific, EU and the US markets during the Asian financial crisis in 1997 by using the AG-DCC model to analyse data from 1991 to 2006. The advantage of the DCC-type models is that the number of parameters only rises linearly with the number of series included into the model. So it can be applied to some cases with a large number of assets. However, the main criticism to the DCC-type models is that they assume the the return series have a multivariate Normal distribution while performing estimations by the 2-step procedure (for the case of standard DCC) or the 3-step procedure (for the case of AG-DCC). However, the distribution of asset returns has fat tails and therefore Gaussian assumptions in the 2-step procedure can be violated. To solve the problem, Pesaran and Pesaran (2007) suggest a procedure namely the TDCC GARCH in which the DCC model is utilized with an assumption that asset returns follow a multivariate Student's t-distribution and the 2-step estimation procedure is replaced by a new one with only one step of estimation. As the t-distribution is more appropriate to describe the fat-tailed features of financial time series, the TDCC model is more capable of capturing the characteristics of the data even though this is a symmetric extension of the DCC-type models. The estimation procedure in the TDCC model also does not follow the old method in the DCC. The introduction of the use of devolatized innovations which are approximately Gaussian instead of the standardised innovations is the another important modification. The devolatized innovations $\tilde{\epsilon}_{it}$, presented in Equation 3.13, are computed by letting returns be normalized by the realized volatilities rather than by the conditional volatilities in the GARCH-type models.

The use of devolatized innovations has more advantages than standardised innovations used by the DCC. Firstly, it is more data intensive as it requires intradaily observations. Thus, realized volatility is proven to be more efficient with high frequency data.[1] Secondly, standardised innovations in the Engle's DCC model do not effectively deal with jumps as we can see in Equation 3.7 that the effect of a jump at time t can only affect the standardised innovations $\epsilon_{i,t}$ at time t via $\varepsilon_{i,t}$ while the conditional variances in the denominator are computed by using all information at up to time $t-1$ as in Equation 2.12. While jumps are supposed to cause some non-normal behaviour, devolatized innovations, computed by using realized volatilities as in Equation 3.13, are more updated by the effect of jumps at time t that enter to both the innovations $\varepsilon_{i,t}$ and the realized volatilities $\tilde{\sigma}^2_{i,t}$.

The application of the TDCC model has been successful recently. Pesaran, Schleicher and Zaffaroni (2009[91]) show that the TDCC outperformed all other multivariate GARCH models, such as the CCC, the DCC, the AG-DCC, in modelling conditional correlations and volatilities. Our study in the previous chapter of this thesis also indicates that the TDCC model performed quite well when it is fitted to the emerging data. Based on these empirical performances of the TDCC, we propose an use of the TDCC model to examine the conditional volatilities and correlations between emerging markets and the US market.

3.2.2.2. Review of application of the multivariate GARCH models in modelling the interdependence and contagion between financial markets

Over the past decades, the regional financial crises, as well as the crises at global level, have significantly changed the structure of international equity markets by increasing the

[1] See Andersen, Bollerslev and Diebold (2005): Practical Volatility and Correlation Modelling for Financial Market Management

interdependence of financial markets all over the world. There are some changes of the market interdependence which are documented as financial contagions. Hence, the important focus of financial econometrics is now not only to model the risk but also to model financial interdependence, which is defined by Forbes and Rigobon (2002) as the market co-movement. Therefore, the term 'financial contagion' is used in contrast to the term 'financial interdependence'. The current studies on interdependence and contagion originate from studies by Sharpe (1964), Grubel and Fadner (1971) and Engle, Lin and Ito (1990). There is a reasonably large literature of empirical studies of the interdependence and the contagion. Early surveys of those studies can be found at Dornbusch, Park and Claessens (2000), Pericoli and Sbracia (2003). Moreover, the most recent and comprehensive overview of methodologies and research of financial contagion is given by the textbook by Kolb (2010). However, the most important literature of financial contagion is the Dungey, Fry, Hermosillos and Martin (2005) who make an empirical comparison of four main approaches to test for contagion. These are the correlation analysis approach of Forbes and Rigobon (2002), the VAR approach of Favero and Giavazzi (2002), the probability method of Eichengreen, Rose and Wyplosz (1996,[35]) and the co-exceedance proposal of Bae, Karolyi and Stulz (2003). Among those approaches, the method of Forbes and Rigobon is the most popular as it relies directly on the measurement of cross-market correlations. Furthermore, the in-depth discussion of contagion tests using correlation analysis is presented by Corsetti, Pericoli and Sbracia (2010).

Forbes and Rigobon (2002) indicated that during times of high volatility, such as in times of financial crises, the correlation between markets is higher because it is positively correlated with the conditional volatility, which implies that the contagion test using the conditional correlation is a biased test due to the heteroskedasticity in the conditional correlation. A general increase in the correlation between financial markets does not definitely imply that there is financial contagion. It is just a short-run change in the interdependence of financial markets. Financial contagion is, therefore, defined by those authors as a significant increase in the market co-movement due to a shock that occurs in one market. Following the approach of Forbes and Rigobon, a significant shift in the mean of the conditional correlation between financial markets can be interpreted as financial contagion. However, the VAR-based strategy, used by the authors to estimate the correlation, easily fails to detect contagion. The drawbacks of this method are indicated in the study of Chiang, Jeon and Li (2007) as omitted variable bias is easily caused by a VAR model. Moreover, the VAR method is unable to capture the non-linearity in the conditional correlations. The multivariate GARCH is the most advanced technique which can give a clear picture of the interdependence of financial markets and hence the conditional correlations estimated by MGARCH is more relevant for the tests of contagion.

Both specifications of the BEKK and the DCC can formally give estimations of the conditional covariances and correlations. However, the DCC model is more practical than the BEKK model as it can feasibly work on portfolios of a large number of financial returns. This is the reason why there is a large number of extensions and realizations of the DCC, which provides a generous environment for contagion tests. Table 3.1 presents a list of recent studies with empirical tests for financial contagion and volatility spillover by using the DCC GARCH or the BEKK parameterizations.

The evolution of financial markets in developing countries also raises the question of whether those markets are correlated with the developed markets. The listed research in Table 3.1 show that the effect of financial crisis on emerging markets is one of the main focuses. However, no research has used the TDCC model to test for financial contagions so far. The TDCC model assumes that the financial returns follow a Student's t-distribution which well characterizes the fat-tailed natures of financial return series. Pesaran *et al* (2010) indicated that the TDCC model passed the serial correlation LM test and Kolmogorov-Smirnov test. However, it failed the VaR-based diagnostic test when applied to a portfolio of 17 financial returns at weekly frequency. However, it is a real challenge to propose a model that performs well in both tranquil and turmoil periods. While there are a number of possible extensions to improve the current realizations of the DCC model, it is worth applying the TDCC model to model the dynamic correlation between any of the emerging markets and the US market and hence re-perform the contagion test to re-evaluate if and how the effect of financial crises is transmitted between

Table 3.1.: Researches on empirical tests for financial contagion or volatility spillover using Multivariate GARCH models

Author(s)	Year	Model	Topic
Chiang, Jeon and Li	2007	DCC 8 East Asian FMs	Financial crisis in Asia in '97, the effect of credit-rating agencies on the structure of correlation dynamic.
Cheung, Fung and Tam	2008	DCC, Spillover Index 11 EMEAP and US	Interdependence of financial markets, Contagion risk in EMEAP region.
Cho and Parhizgari	2008	DCC 8 East Asian FMs	Financial crisis in Asia in 1997
Kuper and Lestano	2008	DCC 6 East Asian FMs	Effect of financial crisis on the interdependence of financial and FX markets
Pereira, Martin and Nakamura	2009	DCC, GJR-DCC 7 FMs: Asia, L. America and US	Financial crisis in Asia in 1997, financial crises in Brazil and in Argentina
Beirne, Caporale, Ghattas, Spagnolo	2009	MGARCH-in-mean 41 FMs: Asia, L. America, Middle East	Global and Regional volatility spillover
Munoz, Marquez and Chulia	2010	TSFA,[a] DCC 19 FMs: N. America, Europe, E. Asia	Asian financial crisis, Dot-com crisis, Globa l financial crisis
Yiu, Ho and Choi	2010	PCA,[b] ADCC 11 EMEAP and US	Asian financial crisis, Global financial crisis
Naoui, Khemiri and Liouane	2010	DCC 10 FMs: Asia, Latin America and US	Sub-prime crisis 2007
Kenourgios, Samitas and Paltalidis	2011	copula-DCC, AG-DCC BRIC, UK, US	Five recent financial crises

[a] Time Series Factor Analysis
[b] Principle Component Analysis

the US market and the emerging markets. In this paper, we use the TDCC to obtain the dynamic conditional correlations and follow the methods of Forbes and Rigobon (2002) and Chiang *et al* (2007) to test for financial contagions. Following the method of Forbes and Rigobon, the correlation series are divided into 2 periods which are pre-crisis period and post-crisis period and t-tests are used to investigate if there are a significant increases in the conditional correlations after a financial crisis. Following Chiang *et al* (2007), we use dummy variables, which are 1 in the crisis periods and zero otherwise, to test for financial contagion.

3.3. Econometric Methodology of the Student's *t* Dynamic Conditional Correlation Model

3.3.1. The framework of the Student's *t* Dynamic Conditional Correlation Model (the TDCC)

The TDCC model is formally presented as follows: We start with an assumption that the vector of asset returns, r_t at the end of day t has a conditional multivariate Student's t-distribution with degrees of freedom $\nu_t > 2$, the conditional mean μ_t and the non-singular variance-covariance matrix H_t. To make the computations more simple we can assume that the conditional mean, μ_t, can be predicted hence it is taken as a given. The variance matrix H_t can be expressed as in Equation 3.5.

From the decomposition of the conditional covariance matrix H_t in Equation 3.5, we can obtain the specifications for the conditional volatilities, which are expressed by diagonal elements of matrix $Q^\star$ by utilizing the popular univariate GARCH(1,1) process as follows

$$Var(\varepsilon_{i,t}|\Omega_{t-1}) = \sigma_{i,t}^2 = \bar{\sigma}_i^2(1-\lambda_{1i}-\lambda_{2i}) + \lambda_{1i}\sigma_{i,t-1}^2 + \lambda_{2i}\varepsilon_{i,t-1}^2 \tag{3.12}$$

Pesaran and Pesaran (2007) use devolatized residuals, $\tilde{\epsilon}_{i,t}$ as in Equation 3.13 to estimate the conditional correlation matrix as in Equation 3.15 rather than standardised residuals used by the standard DCC. Thus, in the DCC model the standardised residuals are computed based on the dividing ε_t by the conditional standard deviations obtained from the univariate GARCH process from the previous stage. However, the devolatized residuals are computed by dividing ε_t by the realized volatilities obtained by taking the square root of a d-day moving average of the squared residuals which includes also the current residuals at time t. Hence the devolatized residuals, $\tilde{\epsilon}_t$ can capture the most updated information of the data.

$$\tilde{\epsilon}_{i,t} = \frac{\varepsilon_{i,t}}{\tilde{\sigma}_{i,t}} \qquad \tilde{\sigma}^2_{i,t}(d) = \frac{\sum_{s=0}^{d-1} \varepsilon^2_{i,t-s}}{d} \tag{3.13}$$

Following the calibration of Pesaran *et al* (2007), the choice of d is suggested to be 20 so that $\tilde{\epsilon}_{i,t}$ is approximately Gaussian. The conditional correlation matrix, $\tilde{R}_t$ will have the ij^{th} element as follows

$$\tilde{\rho}_{ij,t} = \frac{\tilde{q}_{ij,t}}{\sqrt{\tilde{q}_{i,t}\tilde{q}_{j,t}}} \tag{3.14}$$

It is then clear that elements on the diagonal of $\tilde{R}_t$ are equal to 1 for $i = j$. The covariance $q_{ij,t}$ is the ij^{th} element of the matrix $\tilde{Q}_t$ which is defined by the structure as follows

$$\tilde{Q}_t = \bar{Q}(1 - \alpha - \beta) + \alpha(\tilde{\epsilon}_{t-1}\tilde{\epsilon}'_{t-1}) + \beta\tilde{Q}_{t-1} \tag{3.15}$$

At Equation 3.15, we can find that the condition for the existence of unconditional correlation, $\bar{Q}$ is that $1-\alpha-\beta > 0$. If this condition holds, we obtain mean-reverting conditional covariances. In the special case that $1 - \alpha - \beta = 0$ *or* $\alpha + \beta = 1$, we have a non-mean reverting version of the TDCC model where conditional covariances in Equation 3.15 can be as follows

$$\tilde{Q}_t = \alpha(\tilde{\epsilon}_{t-1}\tilde{\epsilon}'_{t-1}) + (1-\alpha)\tilde{Q}_{t-1} \tag{3.16}$$

Finally, the conditional variance matrix H_t can be obtained by recombining the matrix $Q^{\star}$ and $\tilde{R}_t$ as in Equation 3.5.

3.3.2. Estimation Strategy

The estimation procedure below follows the methodology of Pesaran and Pesaran (2007). Consider a $k \times 1$ vector of returns, r_t with observations ranging as follows $t = 1,\ 2,\ ...,\ T,\ T+1,\ ...,\ T+N$. The first T observations are used to compute the unconditional variance $\bar{\sigma}_i^2$ in Equation 3.12 and the unconditional correlation matrix, $\bar{Q}$ in Equation 3.15 as follows

$$\bar{\sigma}_i^2 = \frac{\sum_{\kappa=1}^{T} \epsilon_{i,\kappa}^2}{T} \tag{3.17}$$

$$\bar{q}_{ij} = \frac{\sum_{\kappa=1}^{T} \epsilon_{i,\kappa}^2 \epsilon_{j,\kappa}^2}{\sqrt{\sum_{\kappa=1}^{T} \epsilon_{i,\kappa}^2 \sum_{\kappa=1}^{T} \epsilon_{j,\kappa}^2}} \text{ with } \bar{q}_{ij} \text{ is the } ij^{th} \text{ element of } \bar{Q}\ matrix \tag{3.18}$$

The first d observations are then used to compute the realized standard deviation, $\tilde{\sigma}_{i,t}$ in Equation 3.13. The choice of d cannot be too large so that realized volatility can behave like the unconditional variance. It is suggested by Pesaran *et al* (2007) that d should be 20 so that the devolatized innovations, $\tilde{\epsilon}_t$ in Equation 3.13 are approximately Gaussian.

N observations from $T+1$ to $T+N$ are saved for model evaluations. In summary, the whole sample can be divided into 2 parts which are $S_{est},\ S_{evl}$ which are described as follows

$$S_{est} = \{r_t : \ t = 1, \ 2, \ ..., \ T\} \tag{3.19}$$

$$S_{evl} = \{r_t : \ t = T+1, \ T+2, \ ..., T+N\} \tag{3.20}$$

In the TDCC(1,1) model, there are $2k+3$ parameters to be estimated which include *2k* coefficients from vectors $\lambda_1 = (\lambda_{11}, \ \lambda_{12}, \ ..., \lambda_{1k})$ and $\lambda_2 = (\lambda_{21}, \ \lambda_{22}, \ ..., \lambda_{2k})$ that enter the univariate GARCH (1,1) model for individual asset returns, the coefficient α, β in the dynamic correlation structure as in Equation 3.15 and finally the degrees of freedom of the multivariate Student's t-distribution, ν. Those *2k+3* parameters are estimated by the technique maximum likelihood.[2]

The TDCC model is rather parsimonious as the number of unknown parameters in the model rises linearly with the number of return series, k, as compared with the general multivariate GARCH model such as the BEKK where it rises quadratically with k. However, the interesting assumption of the distribution used by the TDCC may lead to an issue that different univariate GARCH(1,1) models give different degrees of freedom whereas the multivariate Student's t-distribution needs to have the same ν across all returns.

3.3.3. Diagnostic checking of the TDCC model

The TDCC model were evaluated in the same way as in the previous chapter in this thesis by using Value at Risk analysis, Kuiper and Kolmogorov-Smirnov tests. For the VaR-based test, we firstly construct a portfolio based on a $k \times 1$ vector of returns, $r_t \sim (\mu_t, \ H_t|\Omega_{t-1})$. Let ρ_t be the portfolio return with $k \times 1$ vector of weights, w_{t-1}. So the

[2] For the MLE of TDCC, see Appendix in section A.1

portfolio return is given by

$$\rho_t = w'_{t-1} r_t \tag{3.21}$$

At a probability of α, a risk manager would expect the portfolio return, ρ_t at time t to fall below a certain loss $\rho^{\star}_{t-1}$. By this setting, VaR constraint can be formed as

$$Pr(\rho_t < -\rho^{\star}_{t-1} | \Omega_{t-1}) \leq \alpha \tag{3.22}$$

If this risk manager prefers an active manner of risk management, the portfolio return will be computed by using optimal portfolio weights as presented in Equation 2.42. The maximum daily loss is pre-determined as L_{t-1}. Therefore, the VaR constraint will be

$$Pr(\rho_t < -L_{t-1} | \Omega_{t-1}) \leq \alpha \tag{3.23}$$

If the risk manager prefers to use passive risk management, the portfolio will be computed by using equal weights. The daily loss becomes $\bar{\rho}_{t-1}$ which is the function of α, $\hat{\mu}_{t-1}$, $\hat{H}_{t-1}$.[3] The VaR constraint becomes

$$Pr(\rho_t < -\bar{\rho}_{t-1} | \Omega_{t-1}) \leq \alpha \tag{3.24}$$

The main idea of the diagnostic test is to use a VaR indicator, I_t, to check for the validity

[3] For the computation of $\bar{\rho}$, see Appendix in section A.3

of the TDCC model as follows

$$\begin{cases} I_t = I(\rho_t + L_{t-1}) & \text{for active risk management} \\ I_t = I(\rho_t + \bar{\rho}_{t-1}) & \text{for passive risk management} \end{cases} \tag{3.25}$$

in which the indicator function, $I(A)$ takes value of unity if $A < 0$ and is equal to 0 otherwise. So $I(A)$ is the function that counts the number of days when $\rho_t < -L_{t-1}(or\ -\bar{\rho}_{t-1})$, indicating that VaR constraint is violated or the portfolio return falls below the maximum daily loss. The VaR indicator, I_t is recursively computed by using N observations in the evaluation period, S_{eval} as in Equation 3.20. By using the VaR indicators, We can compute the rate of VaR exceedance as follows

$$\hat{\pi}_N = \frac{1}{N}\sum_{t=T+1}^{T+N} \hat{I}_t \tag{3.26}$$

Hence, under the specification of the TDCC model, $\hat{\pi}_N$ will have mean α and variance $\dfrac{\alpha(1-\alpha)}{N}$. Moreover, the standardised test statistic can be obtained based on the result from Equation 3.26 as follows

$$z_{\hat{\pi}_N} = \frac{\sqrt{N}(\hat{\pi}_N - \alpha)}{\sqrt{\alpha(1-\alpha)}} \tag{3.27}$$

Then the above test statistic is asymptotically normally distributed with zero mean and unit variance. The standardised test statistic is used to test the null hypothesis under which the TDCC model is correctly specified

$$H_0: \quad H_t = H_t\left(\hat{\theta}_{S_{est}}\right) \tag{3.28}$$

Another diagnostic test based on Berkowitz (2001) as proposed in Pesaran and Pesaran (2007) is the probability integral transforms (PITs) as follows

$$\hat{U}_t = F_\nu \left(\frac{w'_{t-1} r_t - w'_{t-1} \hat{\mu}_{t-1}}{\sqrt{\frac{\nu - 2}{\nu} w'_{t-1} \hat{H}_{t-1} w_{t-1}}} \right), \ t = T+1,\ T+2,\ ...,\ T+N \tag{3.29}$$

Under the null hypothesis that the TDCC model is correctly specified, the estimates of $\hat{U}_t$ are serially uncorrelated and uniformly distributed within the range (0,1). The Lagrange Multiplier test can be used to detect for the serial correlations in $\hat{U}_t$ where $\hat{U}_t$ is regressed on its own lags. To test if $\hat{U}_t$ is uniformly distributed over time t, a Kolmogorov - Smirnov test is suggested with the KS statistic defined by $\mathrm{KS}_N = \max_{T+1 \leq j \leq T+N} \left| \frac{j}{N} - \hat{U}^\star_j \right|$ where $\hat{U}^\star_{T+1} \leq \hat{U}^\star_{T+2} \leq ... \leq \hat{U}^\star_{T+N}$ are ordered values of $\hat{U}_t(x)$ for t ranging in the evaluation period, S_{eval} from T+1, T+2, ..., T+N.

3.3.4. Methodology of contagion test

In this study, we used two recent methods to test for a contagion, which are of Forbes and Rigobon (2002) and Chiang *et al* (2007). Both methods, used in our study, rely on the time-vary conditional correlation, generated by the TDCC model, to test for a contagion between an emerging financial market and the US financial market. The method of Forbes and Rigobon, introduced in 2002, uses the t-tests to test whether there is a significant change in the dynamic conditional correlation after an effect of a major shock in one market. The method of Chiang *et al* (2007), also using the dynamic conditional correlation series, was introduced in a different approach, which uses dummies to test for a contagion.

3.3.4.1. Timeline of events

In both those methods of contagion test, we need to specify the time of the outbreak of a financial crisis and the length of the turmoil period from the outbreak of a crisis.

To decide the start date of a financial crisis, we chose the date, which are consistently suggested in the literature as a single factor leading to the crisis. In this chapter, we consider financial crises, which are the Asian financial crisis in 1997, the Dotcom crisis in 2000, the Subprime crisis in 2007 and the Global financial crisis in 2008.

For the Asian financial crisis, it is difficult to choose the breakpoint as the related financial markets in Asia experienced a sharp fall in different dates. However, Forbes and Rigobon suggested that the outbreak of the Asian financial crisis is 17/10/1997 when the Hong Kong market crashed. From this date, events in Asia, related to this crisis, became the headlines of news in the US and UK.

The Dotcom crisis is considered to occur in the US financial market on 10/03/2000 when the NASDAQ composite index, which is based on the stock price of dotcom companies in the US, closed at 5048.62 after peaking 5132.52 in an intraday trading. From that date, the NASDAQ lost 78% of its market value in the next 31 months when it reached a trough of 1114.11 on 09/10/ 2002.

The Subprime crisis, which is known as the early stage of the Global financial crisis, is popularly recognised by researchers to start on 15/08/2007 when the stock price of Countrywide Financial Corporation, the largest mortgage lender in the US, fell by 13%. This event was followed by the avoid of a bankruptcy of this corporation by taking out an emergency loan of $11 billion from a group of banks on 16/08/2007. An attempt of the Federal Reserves was to stabilise the financial market by cutting in the discount rate by 0.5% and leaving the fed funds rate unchanged on 17/08/2007.

The Global financial crisis is recognised to start on 15/09/2008 when the Lehman Brothers filed for bankruptcy protection to become the first major bank to collapse since the start of the Subprime crisis in 2007. Meanwhile, the Merrill Lynch, another large investment bank and also affected by the credit crunch, was sold to Bank of America.

Figure 3.4 displays the timeline of the three financial crises that occurred in the US. The figure used the MSCI index for the US market, which is constructed by using all listed investible stocks in the US market. The three recent crises in the US financial markets are the Dotcom crisis, the Subprime crisis and the Global financial crisis. We can see that at the breakpoint of each crisis, the MSCI index had a sharp fall from a peak.

3.3.4.2. Methods of Forbes and Rigobon and Chiang *et al* to test for a contagion

We have already mentioned how the dynamic conditional correlations, $\rho_{ij,t}$, between each emerging market and the US market are estimated by following Equation 3.14 and Equation 3.15. Using the method of Forbes and Rigobon, let $\rho,\ \rho^S\ and\ \rho^T$ be the mean of the conditional correlation series generated by the TDCC during the full period, stable period and turmoil period, respectively. Therefore, the hypotheses of no contagion and contagion, being tested by t-tests, can be stated as below

$$H_0:\ \rho \geq \rho^T \text{ (No Contagion) vs } H_1:\ \rho < \rho^T \text{ (Contagion)} \tag{3.30}$$

The alternative hypothesis implies a significant increase in the mean of the conditional correlation during a crisis period. The conditional correlation, estimated by using the VAR-based method of Forbes and Rigobon, is likely to be biased due to the omitted variable problem. Although the omitted variable problem is solved by using the TDCC model to obtain the correlation series, there are still a number of limitations in this strategy. Firstly, the correlation series still faces the heteroskedasticity problem due to the fact that the TDCC cannot perfectly capture the non-linearity of the dependence between financial markets. Secondly, there is still the bias in the selection of the window length to compute $\rho,\ \rho^S$ and ρ^T. So there is possibly no contagion even when the null hypothesis in Equation 3.30 is rejected. Thus, that the correlation is conditionally high during the crisis due to high volatility resulting in higher mean of the conditional correlation during crisis, ρ^T. This just implies a higher interdependence but not contagion. To overcome this problem, Chiang *et al* (2007) suggest to use an AR model with an intercept and dummy variables for the conditional series as follows

$$\rho_{ij,t} = \gamma_0 + \sum_{m=1}^{p} \gamma_m \rho_{ij,t-m} + \sum_{n=1}^{q} \delta_n dummy_n + \upsilon_t \tag{3.31}$$

in which the dummy variables for the financial crises take value of 1 during turmoil periods and zero, otherwise. The index q is the number of most recent crises. The advantage of this method is that financial contagion is comprehensively tested in the presence of all recent crises. In event study, one difficulty is to decide how long a crisis period is from the breakpoint of this crisis. A long crisis period could be irrelevant for the test of contagion. Because it may include the effects of many other events, which are not related to the shock that happens at the start of the crisis. Hence, a long window for a crisis could lead to a result of a shift in the mean of the conditional correlation, which is caused by a series of many different shocks, not by the initial shock at the breakpoint of a crisis. Consequently, the result may include the effects of many shocks rather than the initial shock. Therefore, this increase in the mean of the conditional correlation cannot be interpreted as a contagion. Forbes and Rigobon (2002) suggested that the crisis period should one month from the outbreak of a crisis. At daily frequency for the data, used in this study, the one-month window for the post-period of the recent financial crises in the US, which could have global effects, is reasonable. Therefore, we used this method to specify the post-crisis period for the dummies in Equation 3.31.

In Equation 3.31, index j is for the US market, where a shock occurs, while i is the index for one of the 19 emerging financial markets. Using the timeline of events in the previous section, we can specify the four dummy variables as follows

1. $dummy_1$: for the Asian financial crisis with the breakpoint at 17/10/1997, post-crisis period: 17/10/1997 to 16/11/1997.
2. $dummy_2$: for the Dotcom financial crisis with the breakpoint at 10/03/2000, post-crisis period: 10/03/2000 to 09/04/2000.
3. $dummy_3$: for the Subprime crisis with the breakpoint at 15/08/2007, post-crisis

period: 15/08/2007 to 14/09/2007.

4. $dummy_4$: for the Asian financial crisis with the breakpoint at 15/09/2008, post-crisis period: 15/09/2008 to 14/10/2008.

During a post-crisis period, a dummy takes value of 1 and zero, otherwise. So a positive and significant estimate of δ_n will cause a shift in the mean of the correlation (γ_0) in Equation 3.31 between an emerging market i and the US, which is interpreted as a financial contagion.

3.4. Empirical results and discussions

Table 3.3 reports the unconditional correlations among the emerging markets and the US markets while Table 3.2 provides information on the unconditional correlations as average pairwise correlations for markets within and across regions. The results indicates that markets in the same region are highly positively correlated while low correlations of markets across regions are also reported except the case of Latin America where 4 countries except Colombia are highly correlated with the US market. It is also noticeable that 8 Asian emerging markets have only very low correlation with the US market with correlation coefficients ranging from the lowest of 0.044 for the case of Philippines to the highest of 0.159 for the case of India.

3.4.1. Estimation of the TDCC model

The primary data analysis suggested the use of the TDCC model under the assumption that the return series follow the multivariate Student's t-distribution. The whole sample from 15/05/1995 to 07/05/2010 with 3910 observations will be divided into 2 sub-samples for model estimation and model evaluation. Following the general procedure of the TDCC estimation, the first step is to get the residual series for each return series following the AR(1) model specified in Equation 2.1. And one-step-ahead forecast of mean, denoted as $\hat{\mu}_{i,t+1}$ with i=1, 2, ..., 20, were generated to compute forecast errors, $\hat{\varepsilon}_{i,t+1} = r_{i,t+1} - \hat{\mu}_{i,t+1}$,

Table 3.2.: Average Pairwise Correlations of Returns Within and Across Continent Classes

Countries	Asia	America	Europe	US
CHINA	0.437	0.226	0.272	0.126
INDIA	0.343	0.227	0.251	0.159
INDONESIA	0.402	0.154	0.179	0.049
KOREA	0.396	0.198	0.234	0.128
MALAYSIA	0.375	0.130	0.172	0.033
PHILIPPINES	0.392	0.162	0.183	0.044
TAIWAN	0.381	0.152	0.206	0.079
THAILAND	0.421	0.202	0.223	0.110
BRAZIL	0.184	0.607	0.326	0.525
CHILE	0.203	0.574	0.330	0.446
COLOMBIA	0.158	0.443	0.247	0.191
MEXICO	0.177	0.588	0.318	0.614
PERU	0.185	0.543	0.306	0.374
CZECH	0.232	0.333	0.507	0.213
HUNGARY	0.168	0.252	0.398	0.334
ISRAEL	0.233	0.343	0.538	0.265
TURKEY	0.179	0.268	0.436	0.179
POLAND	0.265	0.332	0.536	0.238
RUSSIA	0.213	0.304	0.477	0.237
UNITED STATES	0.091	0.430	0.244	1.000

Table 3.3.: Unconditional Cross-Market Correlation Matrix

Countries	CHN	IND	INA	KOR	MAL	PHI	TWN	THA	BRZ	CHI	COL	MEX	PER	CZE	HUN	ISR	POL	RUS	TUR	US
CHINA	1.000	0.348	0.347	0.388	0.322	0.353	0.369	0.371	0.242	0.252	0.185	0.228	0.225	0.293	0.225	0.299	0.328	0.261	0.225	0.126
INDIA	0.348	1.000	0.223	0.277	0.184	0.212	0.249	0.255	0.222	0.240	0.204	0.226	0.245	0.277	0.193	0.278	0.297	0.242	0.219	0.159
INDONESIA	0.347	0.223	1.000	0.257	0.352	0.379	0.264	0.395	0.137	0.178	0.162	0.129	0.163	0.205	0.124	0.208	0.240	0.188	0.110	0.049
KOREA	0.388	0.277	0.257	1.000	0.253	0.272	0.381	0.338	0.220	0.208	0.161	0.222	0.177	0.240	0.183	0.247	0.293	0.234	0.209	0.128
MALAYSIA	0.322	0.184	0.352	0.253	1.000	0.289	0.237	0.365	0.121	0.142	0.103	0.136	0.146	0.177	0.127	0.189	0.210	0.186	0.145	0.033
PHILIPPINS	0.353	0.212	0.379	0.272	0.289	1.000	0.270	0.364	0.157	0.179	0.151	0.145	0.180	0.209	0.146	0.216	0.245	0.156	0.128	0.044
TAIWAN	0.369	0.249	0.264	0.381	0.237	0.270	1.000	0.277	0.165	0.181	0.145	0.129	0.139	0.221	0.184	0.196	0.240	0.192	0.203	0.079
THAILAND	0.371	0.255	0.395	0.338	0.365	0.364	0.277	1.000	0.205	0.244	0.156	0.198	0.206	0.235	0.163	0.234	0.270	0.246	0.191	0.110
BRAZIL	0.242	0.222	0.137	0.220	0.121	0.157	0.165	0.205	1.000	0.569	0.305	0.660	0.502	0.327	0.287	0.349	0.346	0.346	0.301	0.525
CHILE	0.252	0.240	0.178	0.208	0.142	0.179	0.181	0.244	0.569	1.000	0.326	0.541	0.432	0.358	0.286	0.372	0.368	0.306	0.289	0.446
COLOMBIA	0.185	0.204	0.162	0.161	0.103	0.151	0.145	0.156	0.305	0.326	1.000	0.270	0.312	0.304	0.159	0.288	0.276	0.246	0.211	0.191
MEXICO	0.228	0.226	0.129	0.222	0.136	0.145	0.129	0.198	0.660	0.541	0.270	1.000	0.471	0.317	0.328	0.349	0.335	0.314	0.263	0.614
PERU	0.225	0.245	0.163	0.177	0.146	0.180	0.139	0.206	0.502	0.432	0.312	0.471	1.000	0.358	0.200	0.358	0.338	0.307	0.275	0.374
CZECH	0.293	0.277	0.205	0.240	0.177	0.209	0.221	0.235	0.327	0.358	0.304	0.317	0.358	1.000	0.242	0.534	0.535	0.403	0.329	0.213
HUNGARY	0.225	0.193	0.124	0.183	0.127	0.146	0.184	0.163	0.287	0.286	0.159	0.328	0.200	0.242	1.000	0.320	0.305	0.293	0.231	0.334
ISRAEL	0.299	0.278	0.208	0.247	0.189	0.216	0.196	0.234	0.349	0.372	0.288	0.349	0.358	0.534	0.320	1.000	0.585	0.424	0.364	0.265
POLAND	0.328	0.297	0.240	0.293	0.210	0.245	0.240	0.270	0.346	0.368	0.276	0.335	0.338	0.535	0.305	0.585	1.000	0.419	0.369	0.238
RUSSIA	0.261	0.242	0.188	0.234	0.186	0.156	0.192	0.246	0.346	0.306	0.246	0.314	0.307	0.403	0.293	0.424	0.419	1.000	0.324	0.237
TURKEY	0.225	0.219	0.110	0.209	0.145	0.128	0.203	0.191	0.301	0.289	0.211	0.263	0.275	0.329	0.231	0.364	0.369	0.324	1.000	0.179
UNITED STATES	0.126	0.159	0.049	0.128	0.033	0.044	0.079	0.110	0.525	0.446	0.191	0.614	0.374	0.213	0.334	0.265	0.238	0.237	0.179	1.000

which will be used in the estimation.

However, the forecasted errors need to be devolatized following the specifications in Equation 3.13. The lag d is calibratedly suggested by Pesaran *et al* (2007) to be 20 so that the devolatized series are rendered to be approximately Gaussian. Moreover, all series can be estimated together or they can be grouped together for the same region. For the primary purpose of calculating dynamic correlations, a fixed estimation window was used with window size ranging from 15/05/1995 to 07/05/2009 totalling 3910 observations and all series were estimated at the same time. Based on the primary analysis on the estimated parameters of the univariate t-GARCH in Table 1.1, the mean-reverting version of the TDCC is selected, where the conditional covariance structure is described in Equation 3.15 with parameter α representing the ARCH term and parameter β representing the GARCH term. Therefore, the total number of estimated parameters is 43 which includes 40 country-specific volatility parameters, $\hat{\lambda}_1 = \left(\hat{\lambda}_{1,1},\ \hat{\lambda}_{1,2},\ ...,\hat{\lambda}_{1,20}\right)$ (GARCH parameters) $\hat{\lambda}_2 = \left(\hat{\lambda}_{2,1},\ \hat{\lambda}_{2,2},\ ...,\hat{\lambda}_{2,20}\right)$ (ARCH parameters); 2 conditional correlation parameters, $\hat{\alpha}$ and $\hat{\beta}$ and the degrees of freedom parameter, $\hat{\nu}$. With the data from emerging markets, the TDCC model converged well in the whole sample estimation as well as in recursive estimation strategy which is used for diagnostic checking in subsubsection 3.4.1.2.

Table 3.4 reports the maximum likelihood estimates of 43 parameters of the TDCC model in the whole sample. Interestingly, all estimated parameters are highly significant at the 1% level. All estimates of country-specific volatility parameters, $\hat{\lambda}_1$, $\hat{\lambda}_2$ and two correlation parameter, $\hat{\alpha}$, $\hat{\beta}$ satisfied the condition that the sum of GARCH and ARCH parameters needs to be less than unity to ensure the stationarity condition of the conditional correlation. This result suggests a statistically significant mean-reverting process of all conditional volatilities and correlations of emerging markets and the US market. The average estimate of GARCH parameters, $\hat{\lambda}_1$ is 0.9034, while Riskmetrics, by using their exponential smoothing model for volatility parameter, suggest a bandwidth for the parameter ranging from 0.95 to 0.97. The estimated degrees of freedom, $\hat{\nu}$, which is also consistent with previous research on equity markets is 10.226 and is significant at the 1%

level. The estimate of degrees of freedom of the Student's t-distribution, $\hat{\nu}$, is well below 30, supporting our assumption that returns follow a multivariate Student's t-distribution.

Table 3.5 provides the information on the in-sample performance of the DCC(1,1) models by reporting maximized log-likelihood values, AIC and SBIC computed by following different distributions such as Normal distribution, t-distribution. The choice of the assumption that return series follow a Student's t-distribution is now supported by the results in Table 3.5. The DCC model under Gaussian assumption is rejected by the version under t-distribution assumption as the maximized log-likelihood values of the DCC with Student's t-distribution at any level of degrees of freedom is far larger than that of the DCC with Normal distribution. Hence, the computed values of AICs and SBICs based on the maximized log-likelihood values also suggest the DCC model with a t-distribution. The TDCC models with endogenous degrees of freedom, $\hat{\nu} = 10.226$ which was simultaneously estimated with the other parameters is also selected by the maximized log-likelihood, the AIC and the SBIC which are far better than those of the TDCC of exogenous degrees of freedom, say $\nu = 6$.

3.4.1.1. VaR-based diagnostic test for the TDCC model

We used the result from our previous chapter in this thesis when the TDCC model is suggested to be one of the best models among 54 models. A window of 800 observations rolling at the frequency of 25 observations (monthly update) were used for model estimation. The estimation output was then used to compute recursively one-step-ahead forecast of the conditional mean, $\hat{\mu}_{i,t}(i = 1, 2, ..., 20)$ and the one-step-ahead forecast conditional variance-covariance matrix, $\hat{H}_t$. The evaluation period starts from the 801^{st} observation and extends to the last observation of 3910. Over the evaluation period, the VaR exceedance rate was computed by using the pre-determined weights $\omega_{t-1} = \dfrac{1}{20}$ for passive risk management or optimal weights for active risk management,[4] risk tolerance probability $\alpha = 1\%$, risk aversion coefficient $\delta = 105$, the maximum daily loss $L_{t-1} = 1\%$ (if using active risk management). The estimated VaR exceedance rate, $\hat{\pi}_N$ (in percent) and the

[4] For the details of optimal weights, see Appendix in section A.2.

Table 3.4.: Estimates of the TDCC(1,1) Model for 20 Countries

(Mean-Reverting Case; Sample from 15/05/1995 to 07/05/2010: 3910 obs)

Countries	**Maximum Likelihood Estimates**				
	$\hat{\lambda}_1$	t-Statistic	$\hat{\lambda}_2$	t-Statistic	$1-\hat{\lambda}_1-\hat{\lambda}_2$
Asia					
CHINA	0.915	118.91	0.072	11.57	0.013
INDIA	0.887	119.71	0.088	16.21	0.025
INDONESIA	0.912	120.83	0.080	11.83	0.008
KOREA	0.938	156.07	0.056	10.59	0.005
MALAYSIA	0.922	213.93	0.074	18.20	0.004
PHILIPPINES	0.854	141.04	0.106	23.40	0.040
TAIWAN	0.945	153.36	0.044	09.59	0.011
THAILAND	0.903	112.78	0.078	12.23	0.019
Latin America					
BRAZIL	0.918	144.82	0.065	13.43	0.018
CHILE	0.918	091.21	0.060	08.72	0.021
COLOMBIA	0.761	154.88	0.186	38.45	0.054
MEXICO	0.926	131.07	0.058	11.33	0.016
PERU	0.923	108.73	0.063	09.32	0.015
Europe and US					
CZECH	0.890	088.04	0.080	12.46	0.030
ISRAEL	0.941	137.04	0.045	09.00	0.014
HUNGARY	0.890	187.27	0.084	21.28	0.026
POLAND	0.936	145.04	0.045	10.18	0.019
RUSSIA	0.888	154.76	0.099	20.00	0.014
TURKEY	0.895	272.47	0.080	31.34	0.025
UNITED STATES	0.937	133.01	0.053	09.11	0.009

	Conditional Correlation and Degrees of Freedom		
Log-likelihood	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\nu}$
-135,221	0.003	0.995	10.226
	(18.348)	(3295.9)	(3051.8)

	VaR-based Diagnostic Test ($\alpha = 1\%$)	
	$\hat{\pi}_N$	$z_{\hat{\pi}_N}$
Optimally-Weighted Portfolio	1.48%	2.70
Equally-Weighted Portfolio	1.80%	4.45

Table 3.5.: Maximized Log-Likelihoods and Information Criteria of TDCC Model

(For the whole sample under different Distribution Assumptions)

Distribution	Log-likelihood	AIC	BIC
Normal	-137248	137425	137291
Student, $\nu = 5$	-135673	135850	135716
Student, $\nu = 6$	-135453	135631	135496
Student, $\nu = 7$	-135334	135511	135377
Student, $\hat{\nu} = 10.226$	**-135221**	**135399**	**135264**

Notes: *bold number denotes the best information criterion*

corresponding standardised statistic, $z_{\hat{\pi}_N}$ are also presented in Table 3.4. Based on our analysis in the previous chapter in this thesis, this is the best result which is obtained by estimations, although the result shows that the TDCC model is still rejected. In active risk management, the violation rate is 1.48% while the allowed rate is 1%. The estimated rate in passive risk management is 1.8%.

3.4.1.2. Analysis of the TDCC model during the Subprime and the Global crises from 2007-2009

Following the method of Pesaran *et al* (2010) and as mentioned in subsection 3.3.3, another way to evaluate the TDCC model is to use the LM test for serial correlation in $\hat{U}_t$ and the Kolmogorov - Smirnov test for uniformity of $\hat{U}_t$ defined by Equation 3.29. This is to check how the TDCC model performed during the period of financial turmoil. Hence, the estimation sample is set from 03/01/2000 to 28/12/2007, and the parameters were recursively estimated every 25 days by a window with size of 800. To evaluate how the TDCC model performed during the global financial crisis, we use the period from 02/01/2008 to 30/10/2009 for model evaluation where the estimated parameters were fixed. The computation of $\hat{U}_t$ is based on equally-weighted portfolio with the weight, w_{t-1} in Equation 3.29 being set to $\frac{1}{20}$. The parameter of risk tolerance, α is assumed to be 1%.

Table 3.6 reports the Lagrange Multiplier test results for the serial correlation in $\hat{U}_t$. It can be seen that the null hypothesis of no serial correlation in $\hat{U}_t$ cannot be rejected at any significance level and at any lag up to 12 lags. This result provides strong support for the claim that the TDCC model is correctly specified. Table 3.7 reports the Kolmogorov - Smirnov test statistics and p-values for the uniformity test of the probability integral transforms, $\hat{U}_t$. Under any assumption of the distribution of returns, the result suggested that the null hypothesis cannot be rejected at any significance level. This indicates that the $\hat{U}_t$'s are uniformly distributed and hence the TDCC model is well specified during the turmoil period.

Both the LM and KS tests performed on $\hat{U}_t$'s indicate that the TDCC model is capable in modelling the volatilities and correlations of the 19 emerging markets and the US market during the period of financial crisis. This result allows us to apply the TDCC model to re-examine the contagion effects of the Asian financial crisis in 1997, the Dotcom crisis in 2000, the Subprime crisis in 2007 and the Global financial crisis in 2008 on the 19 emerging equity markets around the world.

Table 3.6.: LM Test for Serial Correlation in Probability Integral Transforms $\hat{U}_t$

(H_0: $\hat{U}_t$'s are serially uncorrelated where the TDCC is well specified)

Lags	Under Assumption of Distribution					
	Normal		Student with $\nu = 6$		Endogenous	
	LM-Statistic	p-value	LM-Statistic	p-value	LM-Statistic	p-value
q = 1	0.3325	0.5642	0.3770	0.5392	0.3627	0.5470
q = 2	0.4795	0.7868	0.5294	0.7674	0.5143	0.7733
q = 3	1.3455	0.7183	1.5620	0.6680	1.4942	0.6836
q = 4	1.6170	0.8057	1.8337	0.7663	1.7747	0.7771
q = 5	1.4783	0.9156	1.6377	0.8967	1.5880	0.9027
q = 6	1.9568	0.9236	2.1885	0.9016	2.1143	0.9089
q = 7	3.4196	0.8437	3.5852	0.8261	3.5307	0.8320
q = 8	3.5116	0.8983	3.6370	0.8883	3.5933	0.8918
q = 9	4.2047	0.8974	4.3025	0.8904	4.2805	0.8920
q = 10	4.8661	0.8999	4.8309	0.9022	4.8619	0.9002
q = 11	5.7471	0.8897	5.7232	0.8912	5.7475	0.8897
q = 12	6.6275	0.8812	6.8594	0.8668	6.7666	0.8726

Table 3.7.: Kolmogorov - Smirnov Test for Uniformity of Probability Integral Transforms $\hat{U}_t$

(H_0: $\hat{U}_t$'s are uniformly distributed in (0,1) the TDCC is well specified)

Under Assumption of Distribution					
Normal		**Student with ν=6**		**Endogenous**	
KS-Statistic	p-value	KS-Statistic	p-value	KS-Statistic	p-value
0.9494	0.3282	0.7121	0.6911	0.8307	0.4950

3.4.2. The conditional correlation analysis and the empirical test for financial contagions

3.4.2.1. Primary correlation analysis

The estimated parameters presented in Table 3.4 allow for the computations of the conditional correlations between any two of the 20 countries in the consideration of this chapter following the dynamic structure described by Equation 3.14 and Equation 3.15. The initial idea of this paper is to examine how emerging markets conditionally correlate with the US market. We are now going to look at the correlation between the US market and each individual emerging market.

Figure 3.1, Figure 3.2 and Figure 3.3 plot both the conditional volatilities and the correlations of each emerging market with the US market. All emerging markets showed a positive correlation at different levels except some Asian countries such as Korea, Taiwan, in very short periods experience negative correlation with the US. The Figure 3.1 shows that all Asian markets generally have a very low correlation with the US. The markets that have negative correlations with the US market during the period of financial crisis in 1997 are Taiwan, Thailand. In recent years, China, India, Korea and Thailand show a clearly increasing trend in correlation which centres around 0.2. The correlation relationships between the US and emerging countries in South America indicated by Figure 3.2 are highest among all emerging markets. In the last 8 years, all markets in this region showed an increasing trend in the correlation with the US. Among these countries, Mexico is the country with the highest correlation with the US when the correlation sometimes went beyond 0.7. For emerging markets in Europe, the increasing trend in correlation with the US can also be noticed in Figure 3.3. Over the last 5 years, Russia, Hungary, Poland, Turkey have all shown a consistently increasing trend in correlation with the US while Israel has shown a stable correlation with the US.

It can be seen that the conditional volatility of all markets in Asia, Latin America and Europe significantly increased when the financial crisis occurred in 2007-2009. Moreover, it also indicates that the dynamic correlation of almost all Latin American and some major markets in Asia and Europe such as China, Thailand, Russia, Israel with the US market experienced sharp falls in 2008 when the Global financial turmoil was at its peak.

3.4.2.2. Empirical tests for the contagions of 4 recent financial crises

In this section, we re-examine the contagion effects of 4 recent financial crises which are the Asian financial crisis in 1997, the Dotcom crisis in 2000, the Subprime crisis in 2007-2008 and the Global financial crisis in 2008-2009. Thus, the empirical tests for contagion were previously performed by a large number of studies based on the popular method of Forbes and Rigobon (2002), using t-tests, and the recent method of Chiang *et al* (2007), using the AR model with dummy variables. Both methods rely on the computation of the conditional correlations. Forbes and Rigobon (2002) use the VAR method to compute the cross-market conditional correlations with the adjustment for heteroskedasticity by using the change in the unconditional volatility caused by the crisis while Chiang *et al* (2007) utilized the DCC model to obtain the cross-market correlation. In this chapter, we re-evaluate the empirical test for the contagion of the recent financial crises by using both of those methods with the application of the TDCC model, which statistically and empirically outperforms the standard DCC parameterizations to generate the conditional series between each of the 19 emerging markets and the US market. The heteroskedasticity in the correlation series is also treated by using the devolatization method, suggested by Pesaran and Pesaran (2007) as in Equation 3.13.

Table 3.8, Table 3.9 and Table 3.10 report the results of the method of Forbes and Rigobon (2002). These tables provide $\hat{\rho}^S$, $\hat{\rho}^T$ which are the means of the estimated conditional correlations during the stable and turmoil period, respectively, and the test statistic for the null hypothesis under which $\rho \geq \rho^T$ (no contagion) against the right-sided alternative one that $\rho \leq \rho^T$(contagion). The estimated standard deviations, $\hat{\sigma}^S$, $\hat{\sigma}^T$, of the conditional correlations during the stable and turmoil periods are also given in these tables. Table 3.11 summarizes the contagion effects of 4 recent financial crises detected by the method of Chiang *et al* (2007). This table displays the estimates of parameters in Equation 3.31, with the constant, $\hat{\gamma}_0$, the AR(1) parameter, $\hat{\gamma}_1$ and four dummies namely **Asian**, **Dotcom**, **Subprime** and **Global**.

In Table 3.8, the standard deviation of the conditional correlations during the stable and turmoil periods are not equal to each other for each country so the test statistic is given by the right-sided t-test with unequal variances. The bold statistics indicate that the null hypothesis of no contagion is rejected. It is clear that the Dotcom crisis did not have widespread contagion effects on the emerging markets. There are a number of paradigms to explain the channels of the transmission of the contagions, such as the information channel, financial linkages, trade ties. A possible evaluation of the effect of the Dotcom crisis is that countries in Europe, Asia and Latin America which are related to high-tech industries are likely to be sensitive to the crisis caused by the collapse of many Dotcom-related corporations in the US. In Asia, four countries which were affected by the Dotcom crisis are China, India, Korea and Taiwan with the increase in the mean of the conditional correlations ranging from 3.574% for China to 45.007% for India. It is noticeable that the four affected countries are closely related to high-tech industries in the US. Among those, China was slightly affected with around 3% increase in mean correlation while the other 3 countries were severely hit with around 30% increase in mean correlation. Therefore, the result supports the argument that the Dotcom crisis hit only countries related to high-tech industries in the US. The remaining emerging markets in Asia which were not hit by the crisis are from manufacturing-export countries such as Thailand, Indonesia, etc. Europe is the continent that was worst hit by the Dotcom crisis, while all emerging markets except Turkey got infected. For the case of Turkey, the mean of the conditional correlation dropped by 6.273% after the crisis, this can be explained that Turkey is the emerging country in Europe which is less related to the high-tech industry. The remaining emerging markets in Europe are affected by the crisis with the increase in the mean of the conditional correlations being from 10.187% for Poland to 49.069% for Israel. The result also indicates that Israel being the high-tech-dependent country was infected severely. In contrast, Latin America is less affected by the crisis, though it is closely linked to the US in term of trade ties. There are only 2 affected countries which are Chile with an increase of 4.936% in the mean of the conditional correlation and Mexico with this number being at 7.753%. The analysis shows that the channel of volatility spillover of the Dotcom crisis cannot be the trade linkage. Only countries that rely on the high-tech industries such as internet, computer, semiconductor export, were affected by the crisis.

The Subprime crisis and consequently the Global financial crisis are different from the Dotcom crisis in the way it originated from the collapse of the housing markets in the US and some large investments banks in the US. Hence, the two recent crises are generally supposed to have the effects on financial markets all over the world. We are now going to re-evaluate the effect of the two financial crises using the empirical results in Table 3.9 and Table 3.10. The difference in the reported standard deviations for each individual market in Table 3.9 and Table 3.10 supports the right-sided t-test with unequal variances. The test statistics given by the t-test in these two tables demonstrate that for 17 out of 19 countries the t-test rejects the null hypothesis of no contagion of the Subprime crisis at the 1% significance level and for 18 out of 19 countries the t-test rejects the null hypothesis of no contagion of the Global financial crisis also at the 1% significance level. For the Subprime crisis, the two emerging markets which were not affected are China and Taiwan with an decrease in the mean of the conditional correlation being at -20.131% for China and at -18.916% for Taiwan. The remaining emerging markets were severely hit by the crisis with the significant increase in the mean of the conditional correlations ranging from the lowest of 8.552% for Czech Republic to the highest of 82.5% for Indonesia. Recall that a large and significant increase in the mean of the conditional correlation implies a contagion in the method of Forbes and Rigobon (2002). For the Global financial crisis, all the emerging markets, except Israel which has a decrease in the mean of the conditional correlation by -8.163%, were affected with the increase in the mean of the conditional correlations being between 4.651% for Poland and 89.44% for Taiwan. The increase in the mean of the conditional correlations displayed in Table 3.10 also indicated that the emerging financial markets in Asia got the worst hit by the Global financial crisis with 5 out of 8 markets having the increase in the mean of the conditional correlations larger than 40%.

Following the results given by the method of Forbes and Rigobon, the channel of the contagion effect of the Subprime crisis is likely to be the financial linkage. Thus, the collapse of some large investment banks in the US and consequently in the UK, such as Lehman Brothers or Northern Rock Plc., caused banks and financial institutions in emerging markets to be in trouble due to their connections dominated in loans or investments in equities. Hence, the bad news brought to the emerging markets then induced a sharp fall in the emerging markets resulting in a rise in the conditional volatility and correlations. The scenario for the contagion of the Global financial crisis is slightly different. Thus, during the time of the crisis, the US economy and the world economy were in deep recession so the the spread of contagion can be explained by such channels as trade or investment linkages. Therefore, the Asian markets, such as China, Taiwan, India, which have close economic and financial ties with the US economy were severely infected. For Latin America, the effect of the two crises was minor with the average increase in the mean of the conditional correlations of the Latin American markets being less than 10% for the Subprime crisis and being less than 20% for Global financial crisis. The reason is that the Latin American countries being the neighbours of the US mainly have trade links with the US exporting materials and agricultural produces to the US and the financial markets in those countries are highly integrated with the US financial market. Hence, the Subprime crisis which affected the other markets via news channel had less effect than those of the Global financial crisis which hit the other markets via trade or financial linkages. In Europe, Israel is the special case that the financial market of this country was only slightly hit by the Subprime crisis due to the spread of bad news and has not got infected by the Global financial crisis because of the fact that the Israeli economy is based on the high-technology industries.

Table 3.8.: Forbes and Rigobon test for the contagion of the Dotcom crisis using the correlation generated by the TDCC model

(*Dotcom crisis period from 22/11/1997 to 31/10/2002. Turmoil breakpoint at 10/03/2000*)

Country	**Stable**[1]		**Turmoil**[2]		**Change in**	**Test**[3]	***p*-value**
	$\hat{\rho}^S$	$\hat{\sigma}^S$	$\hat{\rho}^T$	$\hat{\sigma}^T$	**Mean(%)**	**Statistic**	
Asia							
CHINA	0.090	0.020	0.093	0.032	3.574	**2.166****	0.015
INDIA	0.078	0.024	0.113	0.042	45.007	**18.689*****	0.000
INDONESIA	0.048	0.022	0.027	0.018	-43.772	-18.068	1.000
KOREA	0.111	0.025	0.136	0.032	22.107	**15.592*****	0.000
MALAYSIA	0.066	0.022	0.026	0.016	-60.691	-36.190	1.000
PHILIPPINES	0.112	0.030	0.070	0.024	-37.314	-27.552	1.000
TAIWAN	0.053	0.031	0.067	0.026	26.091	**8.639*****	0.000
THAILAND	0.100	0.033	0.092	0.020	-8.330	-5.360	1.000
Latin America							
BRAZIL	0.487	0.016	0.482	0.039	-1.068	-3.197	0.999
CHILE	0.392	0.021	0.411	0.033	4.936	**12.877*****	0.000
COLOMBIA	0.143	0.018	0.111	0.023	-22.393	-27.879	1.000
MEXICO	0.545	0.020	0.588	0.024	7.753	**34.567*****	0.000
PERU	0.311	0.024	0.228	0.054	-26.434	-36.225	1.000
Europe							
CZECH	0.121	0.031	0.163	0.032	34.413	**23.790*****	0.000
ISRAEL	0.287	0.028	0.428	0.052	49.069	**61.458*****	0.000
HUNGARY	0.208	0.026	0.240	0.033	15.163	**19.128*****	0.000
POLAND	0.193	0.025	0.213	0.036	10.187	**11.549*****	0.000
RUSSIA	0.196	0.020	0.251	0.026	28.258	**42.766*****	0.000
TURKEY	0.122	0.047	0.114	0.048	-6.273	-2.904	0.998

[1] Stable preriod including 601 observations is from 22/11/1997 to 09/03/2002. $\hat{\rho}^S$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the stable period.

[2] Turmoil preriod including 689 observations is from 10/03/2000 to 31/10/2002. $\hat{\rho}^T$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the turmoil period.

[3] ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively. The test statistic is given by right-sided t-test with unequal variances. Hence, bold values indicate a significant increase in the mean of the correlation implying that there is the effect of contagion caused by the Dotcom crisis outbreaking in 10/03/2000.

Table 3.9.: Forbes and Rigobon test for the contagion of the Subprime crisis using the correlation generated by the TDCC model

(*Subprime crisis period from 30/12/2005 to 14/09/2008. Turmoil breakpoint at 15/08/2007*)

Country	Stable[1]		Turmoil[2]		Change in	Test[3]	p-value
	$\hat{\rho}^S$	$\hat{\sigma}^S$	$\hat{\rho}^T$	$\hat{\sigma}^T$	Mean(%)	Statistic	
Asia							
CHINA	0.152	0.019	0.122	0.013	-20.131	-24.872	1.000
INDIA	0.131	0.023	0.160	0.008	22.372	**24.400*** **	0.000
INDONESIA	0.060	0.024	0.109	0.012	82.500	**36.360*** **	0.000
KOREA	0.129	0.016	0.149	0.012	15.484	**18.350*** **	0.000
MALAYSIA	0.037	0.033	0.060	0.016	59.538	**11.881*** **	0.000
PHILIPPINES	0.049	0.018	0.068	0.012	37.950	**16.412*** **	0.000
TAIWAN	0.089	0.014	0.072	0.016	-18.916	-13.961	1.000
THAILAND	0.104	0.027	0.123	0.022	17.811	**10.142*** **	0.000
Latin America							
BRAZIL	0.543	0.046	0.605	0.027	11.562	**22.712*** **	0.000
CHILE	0.403	0.063	0.465	0.009	15.335	**19.808*** **	0.000
COLOMBIA	0.235	0.052	0.282	0.020	20.259	**17.082*** **	0.000
MEXICO	0.609	0.036	0.680	0.007	11.566	**39.450*** **	0.000
PERU	0.348	0.021	0.410	0.025	17.728	**34.732*** **	0.000
Europe							
CZECH	0.190	0.031	0.206	0.036	8.552	**6.157*** **	0.000
ISRAEL	0.303	0.024	0.332	0.025	9.563	**15.151*** **	0.000
HUNGARY	0.194	0.037	0.275	0.016	41.651	**39.909*** **	0.000
POLAND	0.230	0.035	0.301	0.024	30.935	**32.145*** **	0.000
RUSSIA	0.199	0.027	0.297	0.021	49.139	**54.280*** **	0.000
TURKEY	0.193	0.038	0.295	0.013	52.707	**50.946*** **	0.000

[1] Stable preriod including 424 observations is from 30-Dec-2005 to 15-Aug-2007. $\hat{\boldsymbol{\rho}}^{\boldsymbol{S}}$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the stable period.

[2] Turmoil preriod including 283 observations is from 16/08/2000 to 14/09/2008. $\hat{\boldsymbol{\rho}}^{\boldsymbol{T}}$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the turmoil period.

[3] ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively. The test statistic is given by right-sided t-test with unequal variances. Hence, bold values indicate a significant increase in the mean of the correlation implying that there is the effect of contagion caused by the Subprime crisis outbreaking in 15/08/2007.

Table 3.10.: Forbes and Rigobon test for the contagion of the Global financial crisis using the correlation generated by the TDCC model

(*Global financial crisis period from 15/09/2007 to 31/12/2009. Turmoil breakpoint at 15-Sep-2008*)

Country	**Stable[1]**		**Turmoil[2]**		**Change in**	**Test[3]**	***p*-value**
	$\hat{\rho}^S$	$\hat{\sigma}^S$	$\hat{\rho}^T$	$\hat{\sigma}^T$	**Mean(%)**	**Statistic**	
Asia							
CHINA	0.122	0.012	0.175	0.029	42.714	**30.519*****	0.000
INDIA	0.161	0.007	0.241	0.035	49.303	**42.012*****	0.000
INDONESIA	0.112	0.011	0.136	0.012	22.130	**26.315*****	0.000
KOREA	0.150	0.012	0.163	0.016	8.372	**10.944*****	0.000
MALAYSIA	0.061	0.016	0.108	0.040	75.598	**19.634*****	0.000
PHILIPPINES	0.068	0.013	0.080	0.011	17.428	**11.532*****	0.000
TAIWAN	0.071	0.015	0.134	0.019	89.440	**45.459*****	0.000
THAILAND	0.125	0.019	0.180	0.036	43.897	**24.329*****	0.000
Latin America							
BRAZIL	0.608	0.025	0.628	0.028	3.342	**9.346*****	0.000
CHILE	0.465	0.009	0.511	0.032	9.861	**25.512*****	0.000
COLOMBIA	0.285	0.017	0.309	0.026	8.580	**13.856*****	0.000
MEXICO	0.680	0.008	0.710	0.010	4.352	**42.123*****	0.000
PERU	0.416	0.021	0.474	0.052	14.154	**19.396*****	0.000
Europe							
CZECH	0.208	0.030	0.237	0.046	14.115	**9.408*****	0.000
ISRAEL	0.337	0.022	0.309	0.023	-8.163	-14.794	1.000
HUNGARY	0.278	0.015	0.339	0.036	21.688	**28.678*****	0.000
POLAND	0.305	0.021	0.319	0.039	4.651	**5.772*****	0.000
RUSSIA	0.301	0.018	0.339	0.045	12.298	**14.082*****	0.000
TURKEY	0.298	0.012	0.341	0.028	14.557	**25.850*****	0.000

[1] Stable preriod including 424 observations is from 15/09/2007 to 15/09/2008. $\hat{\rho}^S$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the stable period.

[2] Turmoil preriod including 283 observations is from 16/09/2008 to 31/12/2009. $\hat{\rho}^T$ is the mean of the conditional correlation between the emerging markets and the US market estimated by the TDCC model during the turmoil period.

[3] ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively. The test statistic is given by right-sided *t*-test with unequal variances. Hence, bold values indicate a significant increase in the mean of the correlation implying that there is the effect of contagion caused by the Global financial crisis outbreaking in 15/09/2008.

However, the main criticism of the Forbes and Rigobon's methodology is that the choice of window for stable and turmoil periods is likely to cause the test to be a noisy instrument that detects for the financial contagion despite of using the TDCC. Moreover, the adjustment for heteroskedasticity in the correlation series introduced by Forbes and Rigobon (2002) also makes the method become a passive test that is likely to reject all possibilities of contagion. Following the method of Chiang *et al* (2007), 4 dummies which were used to test for the financial contagion are ***Asian*** for the Asian financial crisis starting from 17/10/1997, ***Dotcom*** for the Dotcom crisis starting from 10/03/2000, ***Subprime*** for the Subprime crisis starting from 15/08/2007 and ***Global*** for the Global financial crisis staring from 15/09/2008. The turmoil periods in which the dummies take value of 1 are defined as 1 month from the starting point of the crises. Following the result in Table 3.11, the devolatized correlation series fit well in AR(1) model with all autoregressive parameters being significant at the 1% level. The estimates of the AR(1) parameter, $\hat{\gamma}_1$ for each of 19 countries range from 0.879 for Indonesia to 0.973 for Taiwan with the mean centering around 0.92 which is below unity to ensure the stationarity condition of the AR model. Interestingly, almost all estimates of the mean term, $\hat{\gamma}_0$ are positive and statistically significant at the 1% level. The devolatization to remove the heteroskedasticity in the correlations gives clearly different result from those given by the method of Forbes and Rigobon. To conclude a contagion, the estimated parameter of a dummy needs to be positive and significant indicating that the shocks in the US market during the time of crisis shift the mean of the conditional correlation and consequently. So the context to conclude a contagion in this methodology is essentially similar to that of the Forbes and Rigobon's methodology, which is also based on a shift in the mean of the conditional correlation to conclude a contagion.

In Table 3.11, the estimates of parameter for the Asian dummy is reported to ensure that there are no contagion effects of the Asian financial crisis in 1997 on the US market. Thus, the crisis in 1997 did not cause an increase in the correlation between the US and Thailand with the estimate of the Asian dummy for Thailand is not significant and negative. The Dotcom crisis had contagion effects only on Malaysia and Brazil with

Table 3.11.: Test for the contagion of the financial crises using the correlation generated by the TDCC model and dummy variables

(*4 Crises in consideration: the Asian financial crisis, the Dotcom crisis, the Subprime crisis and the Global financial crisis*)[1]

Country	$\hat{\gamma}_0$	*t*-Stat	$\hat{\gamma}_1$	*t*-Stat	Asian[2]	*t*-Stat	Dotcom[3]	*t*-Stat	Subprime[4]	*t*-Stat	Global[5]	*t*-Stat
Asia												
CHINA	0.095***	6.00	0.905***	57.74	-0.0144	-0.87	-0.0081**	-1.86	-0.0009	-1.04	**0.0137*****	3.03
INDIA	0.120***	3.61	0.879***	25.77	**0.0291****	1.91	0.0017	0.11	**0.0009*****	2.42	0.0176	1.27
INDONESIA	0.078***	2.57	0.921***	30.76	-0.0141***	-3.14	0.0033	1.22	0.0036	0.73	**0.0065****	1.76
KOREA	0.079*	1.41	0.921***	22.51	0.0054	0.00	0.0058	0.83	-0.0018**	-1.80	0.0002	0.02
MALAYSIA	0.028***	2.69	0.964***	100.74	**0.0084*****	2.71	**0.0197*****	3.76	**0.0102*****	4.26	**0.0236*****	5.78
PHILIPPINES	0.102***	4.33	0.897***	38.81	-0.0138***	-6.51	-0.0025	-0.71	-0.0047***	-4.86	**0.0112*****	2.43
TAIWAN	0.026**	1.98	0.973***	71.57	-0.0435	-0.62	-0.0258	-0.09	**0.0034***	1.33	-0.0077	-1.02
THAILAND	0.098***	5.45	0.903***	51.86	-0.0441	-0.86	0.0016	0.01	**0.0035***	1.59	**0.0231*****	5.49
Latin America												
BRAZIL	0.072***	92.96	0.928***	1190.12	**0.0048*****	15.97	**0.0004****	2.23	-0.0003	-0.19	**0.0020*****	21.16
CHILE	0.068***	71.13	0.932***	954.06	**0.0061*****	4.87	-0.0009***	-5.50	**0.0007*****	3.39	**0.0012*****	6.87
COLOMBIA	0.081***	19.31	0.919***	222.17	**0.0095*****	3.88	-0.0068***	-27.47	-0.0000	-0.00	**0.0027*****	4.98
MEXICO	0.065***	316.14	0.935***	4331.96	**0.0041*****	10.02	-0.0006***	-7.93	**0.0003*****	5.87	**0.0010*****	8.85
PERU	0.083**	2.02	0.917***	22.41	**0.0073*****	7.27	-0.0032	-0.88	**0.0014****	2.30	**0.0060*****	2.34
Europe												
CZECH	0.068***	4.43	0.932***	60.10	**0.0097*****	4.39	0.0002	0.00	-0.0005	-0.21	**0.0070*****	8.96
ISRAEL	0.087***	6.78	0.913***	71.70	**0.0061****	2.28	-0.0015	-1.22	**0.0011****	1.69	**0.0012****	1.79
HUNGARY	0.072***	6.45	0.928***	216.33	0.0028	0.00	-0.0031***	-9.87	0.0009	1.09	0.0074	0.01
POLAND	0.090***	42.63	0.910***	422.56	**0.0078****	1.76	0.0039	0.91	**0.0011*****	19.09	**0.0046*****	17.02
RUSSIA	0.081***	3.27	0.919***	22.07	0.0032	0.00	-0.0020	-0.51	**0.0012****	2.23	0.0003	0.00
TURKEY	0.085***	14.30	0.914***	153.59	0.0155	0.12	0.0101	0.03	**0.0021*****	3.03	**0.0042*****	4.73

[1] ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively. The bold numbers indicate that there is the effect of contagion.

[2] **Asian** is the dummy variable that takes value of 1 during crisis period which is one month statring from 17/10/1997 to 16/11/997.

[3] **Dotcom** is the dummy variable that takes value of 1 during crisis period which is one month statring from 10/03/2000 to 09/04/2000.

[4] **Subprime** is the dummy variable that takes value of 1 during crisis period which is one month statring from 15/08/2007 to 14/09/2007.

[5] **Global** is the dummy variable that takes value of 1 during crisis period which is one month statring from 15/09/2008 to 14/10/2008.

the estimates of parameter of the Dotcom dummy, being 0.0197 for Malaysia and being 0.0004 for Brazil. The estimated parameter for Brazil compared with the mean of 0.072 is too small to report a significant shift in the mean indicating a contagion. So it can be concluded that following this method, the Dotcom crisis did not have contagion effects on 18 out of the 19 emerging markets. Consequently, the result of the contagion of the Dotcom crisis, concluded by the former technique, can be interpreted as the changes in the market interdependence. Furthermore, according to the results given by the latter technique, the effects of the Subprime and the Global crises, in the contagion point of view, is less severe those of the former. In Asia, 4 markets, including China, Indonesia, Korea and Philippines, were immune from the Subprime crisis while India, Malaysia, Taiwan and Thailand were hit by the crisis. However, only Malaysia with the estimate of the dummy parameter being 0.0102 is large enough to be considered as contagion, the 3 remaining markets with the estimates of the Subprime dummy parameter ranging from 0.0009 for India to 0.0035 for Thailand received a small effect from the Subprime crisis. The cases of Latin America and Europe are the not different from the case of Asia. The 3 affected markets in Latin America, Chile, Mexico and Peru, have estimates of the Subprime dummy parameter ranging from 0.0003 for Mexico to 0.0014 for Peru, indicating little contagion effects. The results for 4 affected markets in Europe, Israel, Poland, Russia and Turkey, are similar to those of Latin American markets with the estimates of the Subprime dummy parameter ranging from 0.0011 for Israel and Poland to 0.0021 for Turkey. Therefore, these estimates are considerably small to report a severe contagion. For the Global crisis, five Asian markets that suffered from the contagion are China, Indonesia, Philippines and Thailand. Among those markets, only Indonesia saw a moderate effect of the crisis, with the estimate of the Global dummy parameter being 0.0065 while the other 4 Asian markets experienced a significant contagion effect, with the estimates of the Global dummy parameter ranging from 0.0112 for Philippines to 0.0236 for Malaysia. In Latin America, all markets got slight contagion effects of the Global financial crisis with the estimates of the Global dummy parameter being between 0.0010 for Mexico and 0.0060 for Peru. In Europe, only Hungary and Russia were not affected while the rest of the European markets was in mild effect of contagion with the estimates of the Global dummy parameter ranging from 0.0012 for Israel to 0.0070 for Czech Republic.

In summary, the method of Chiang *et al* (2007), being combined with the devolatization in the conditional correlations to remove its heteroskedasticity, gives a comprehensive picture of contagion of recent financial crises, which is different from that of the Forbes and Rigobon's method. Following the Chiang *et al* (2007), the Dotcom crisis did not have a contagion effect on emerging markets except Malaysia, indicating that the context of the contagion of the Dotcom crisis given by the Forbes and Rigobon's method can be interpreted as market interdependence. The Subprime crisis in 2007 has only significant effect on Malaysia and a little effect on several emerging markets. It is consistent to the the context that the Subprime crisis, being the early stage of the Global financial crisis had a contagion effect on the other developed markets rather than the emerging market. The Global financial crisis is described to have a significant effect on only Asian markets while other markets in Latin America and Europe only experienced from mild to moderate effects of the crisis, indicating a scenario different from those of Forbes and Rigobon's method that reports a significant effect of the Global crisis on majority of emerging markets in Asia, Latin America and Europe. Among the 3 recent financial crises, only the latest one which caused the world economy to be in the deep recession has some significant contagion effects on Asian emerging markets. Therefore, it appears that the financial crises in the US financial market tend to have contagion effects on the other developed markets which are financially integrated with the US financial market, leaving the emerging financial markets unaffected or slightly affected somehow.

3.5. Concluding remarks

Emerging markets normally give a real challenge to standard econometric models which are initially designed to work with data from more integrated or developed markets. However, the TDCC model, in this chapter, is proven to work well with data from emerging stock markets. This is supported by the summary statistics in Table 1.1 which showed that the degrees of freedom parameters estimated separately by the univariate GARCH model are not much different among those countries. Once there is a large difference in this type of parameter, may the accuracy of the estimates be doubted. The log-likelihood function converged in the whole sample estimation as well as in recursive estimations. Interestingly, the TDCC model passes almost the diagnostic checking. Thus, the AIC, SBIC and maximized log-likelihood values indicated that in-sample performance of the TDCC was far better than standard DCC framework. In the evaluation for out-of-sample performance, that the model passed the LM tests and the Kolmogorov-Smirnov tests allows for important implications of time series analysis under the effects of financial crisis. It is interesting that the TDCC model passed the diagnostic tests performed on the probability integral transforms (PITs) with the evaluation period set from 2008 to 2009 when the global financial crisis took place. This is an important result when almost previously developed volatility models fail to explain the volatility and the dynamic correlation of stock markets during the time of financial turmoils. This suggests future research into how good volatility models can perform in estimating the volatility of emerging markets with the presence of financial crashes.

The good performances of the TDCC model allow us to perform the contagion test using the conditional correlations estimated by this successful model. Hence, the two popular methods to test for contagion, which are the method of Forbes and Rigobon (2002) using t-tests and the method of Chiang, Jeon and Li (2007) using AR model with dummies, are applied to the estimated correlations which are adjusted for the heteroskedasticity by using the devolatization method suggested by Pesaran *et al* (2007). The results showed that the Asian financial crisis triggered in Thailand did not have a contagion effect on the

US market. This is consistent with previous studies suggesting that the financial crisis in Asia in 1997 was a the regional crisis. Therefore, it affected the markets in Asia rather than the US market. The results obtained by the test of Forbes and Rigobon suggested that the Dotcom crisis had effects on the high-tech related markets in Europe and in Asia rather than the markets from manufacturing-export countries. The t-test of Forbes and Rigobon (2002) indicated that the Subprime and the Global financial crises badly hit almost all emerging markets while the test of Chiang *et al* (2007) showed that the effects of the two recent crises were not as severe: it indicated that there are 8 out of 19 markets affected by both of the crises; 2 markets immune to both of the crises and the rest are affected by either of the crises. The degree of the contagion effects is ranging from mild to moderate levels.

The results are very important as guidance to investors who consider emerging stock markets with the idea of minimizing the volatility of their portfolio's returns. However, there are still questions for further studies such as the TDCC model with an asymmetric term could perform better during the time of crisis. The Student's t-distribution assumption may not be a relevant assumption during the calm periods of financial markets so the use of a mixture of different models may be more effective. Moreover, different financial returns may follow different distributions. A copula model, therefore, could be worth considering. Further research is also needed to improve the empirical tests for contagion. Heteroskedasticity in the correlations could be removed by other methods, such as a switching-regime model or a copula model.

3.6. Chapter 3 - Figures

Figure 3.1.: Conditional Correlation between US and Emerging Countries in Asia

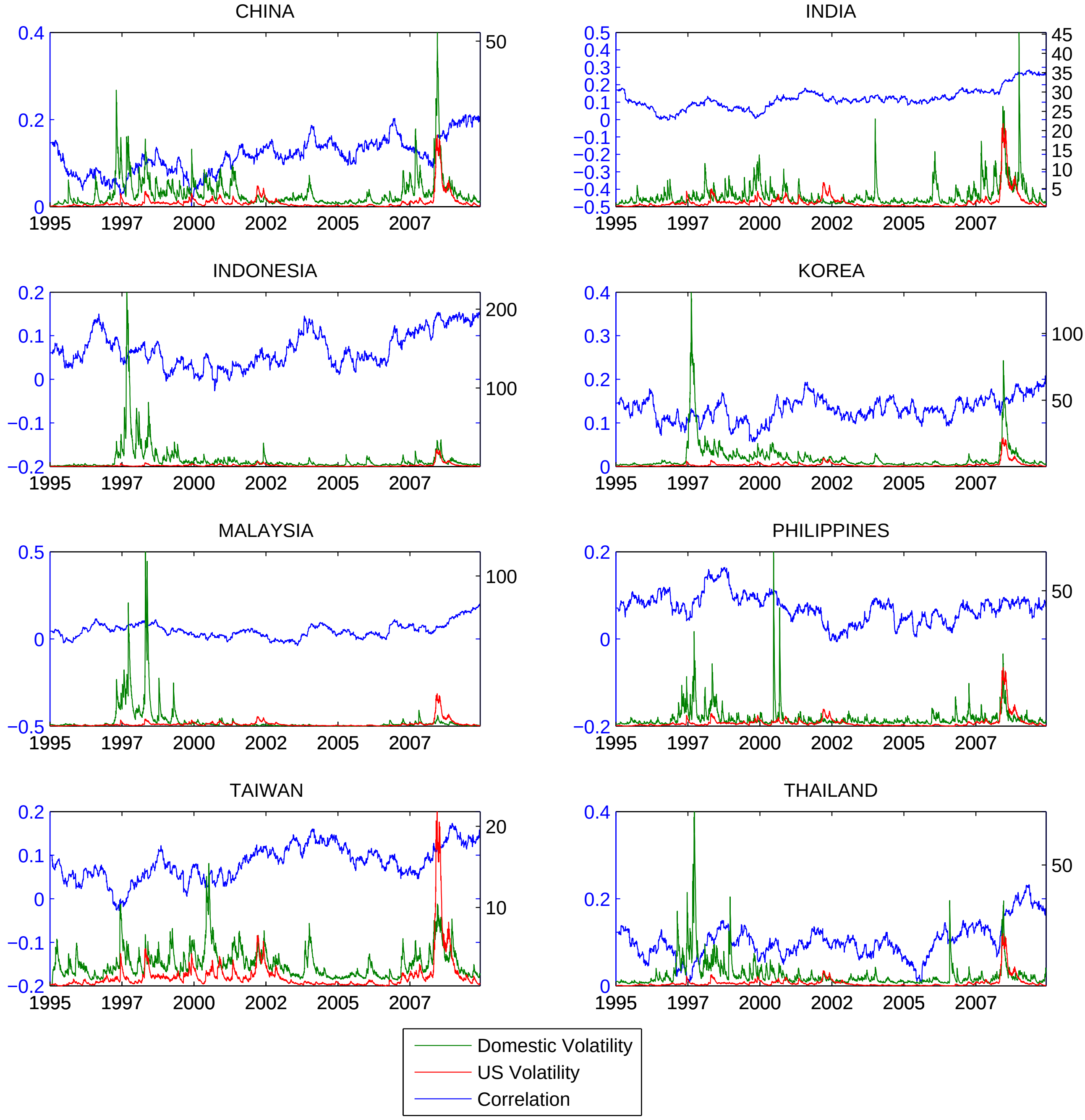


Figure 3.2.: Conditional Correlation between US and Emerging Countries in Latin America

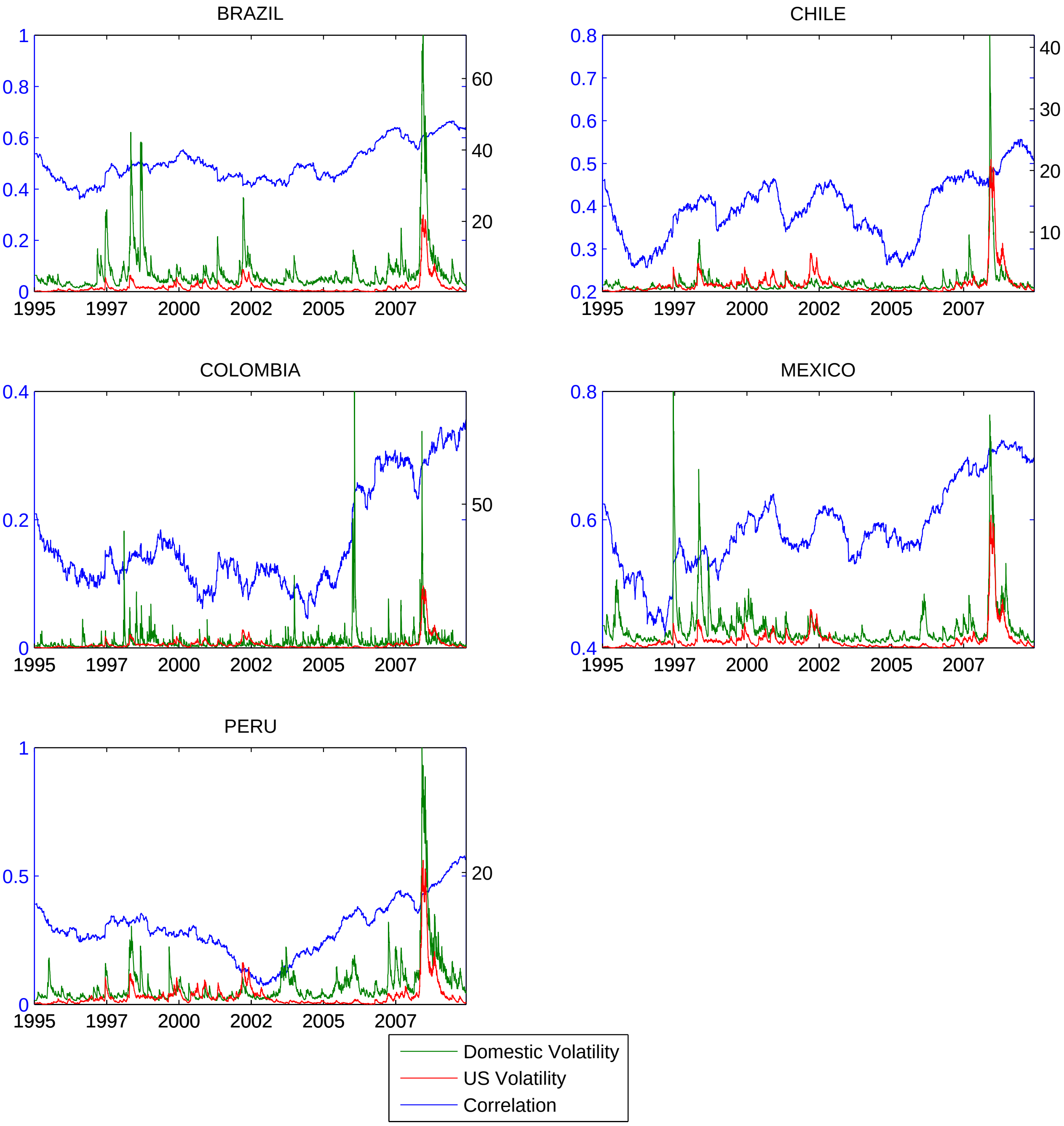


Figure 3.3.: Conditional Correlation between US and Emerging Countries in Europe

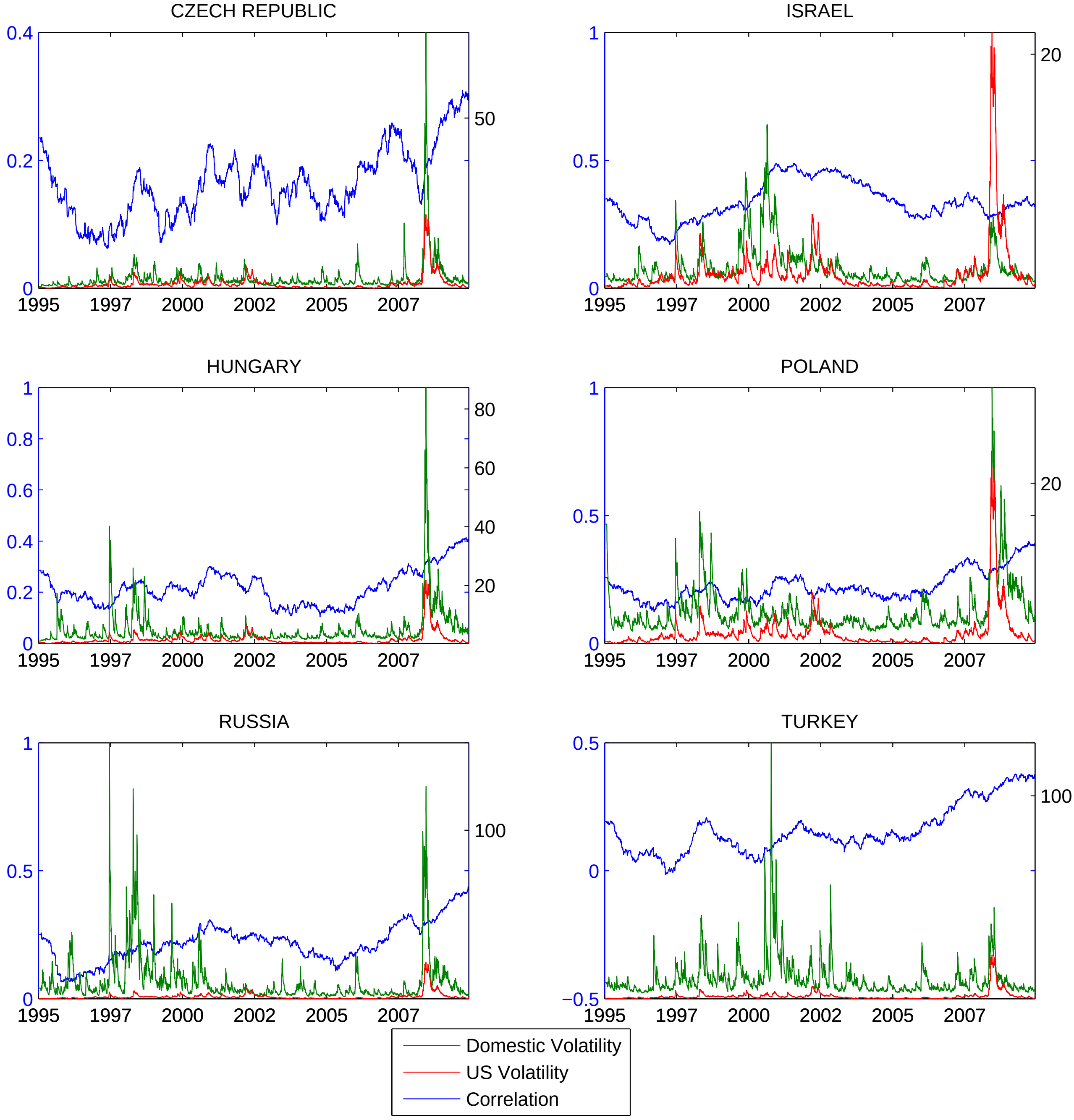
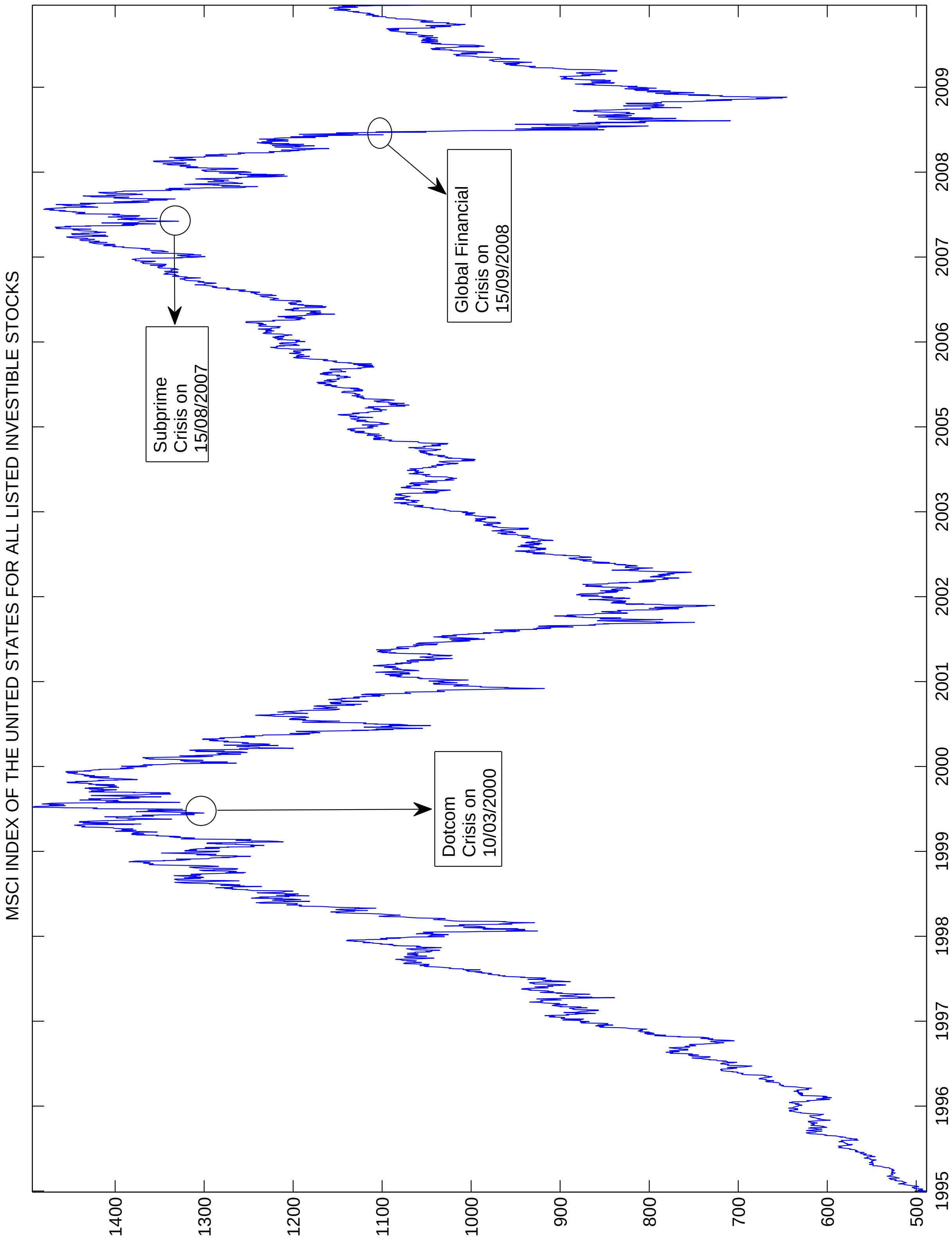


Figure 3.4.: Timeline of three recent financial crises in the US: Dotcom crisis, Subprime crisis, Global financial crisis



4. MULTIVARIATE COPULA: AN APPLICATION TO EMERGING FINANCIAL MARKETS

Abstract

In this study, we examine the dependence structure of 20 financial markets, including 19 emerging financial markets and the US financial market, using the multivariate conditional copula of two types: the Gaussian copula and the Student's t-copula. To adapt the multivariate conditional copulas to the high-dimensional portfolio containing 20 series, we replace the bivariate BEKK representation, used by the copula of Patton (2006) with the DCC specification of Engle (2002). Utilizing the flexibility of a copula function, we construct 12 copula types, each of which is formed by a choice of the GARCH(1,1) or the GJR(1,1,1) for the marginal model assumed by Gaussian or Student's t or Hansen's skewed Student's t-distribution to be coupled with the Gaussian or the Student's t-copula. These 12 copulas are estimated by the 'Inference Functions for Margins' method (IFM) which is performed via 2-step maximum likelihood estimation. The evaluation of copulas is carried out by using the AIC, the SBIC for in-sample fit and diagnostic test statistics based on the Value-at-Risk theory are used for the evaluation of out-of-sample fit. The results, indicating that the Student's t-copula with the GARCH-t margin passes the VaR-based test and is ranked in top place in all evaluations, show that the Student's t-copula is an appropriate method to model financial dependence. Besides, the relevant choice of

a univariate GARCH model for the margin has a significant impact on the performance of the copulas. The result of our study is important for financial authorities who are concerned by financial contagions and for international portfolio managers who need a precise estimator for the Value at Risk of their portfolios.

4.1. Introduction

Modelling the dependence between financial markets has become a rising concern of practitioners and authorities since the occurrence of the two recent financial crises which are the Subprime crisis in 2007-2008 and the Global financial crisis in 2008-2009. The study of the dependence between financial markets is important for financial authorities who are concerned about financial contagion and for international portfolio managers who realise that the non-normal dependence structure of international stocks is more likely to incur big losses for their internationalized portfolios. The emergence of the BRICs (Brazil, Russia, India and China) suggests the rising role of emerging financial markets in maintaining the stability of the global financial market. Therefore, the research on the linkages between developed financial markets and emerging financial markets has become one of the main focuses in financial economics.

Early investigations of cross-market linkages can be found in some studies in the early 1990s which used a VAR (Vector Autoregressive) model to analyse cross-market relationships via the first moments (the returns of financial series). Specifically, Eun and Shim (1989), Oertmann *et al* (1996), Cha and Cheung (1998) and Janakiramanan *et al* (1998) used a VAR to model the linkages between financial markets in mean. Koch and Koch (1991) investigated the linkages of national stock market indexes by using a dynamic simultaneous equation system. However, this method is limited by the heteroskedasticity in the financial return series. Therefore, the multivariate GARCH, introduced by Bollerslev (1990) in the specification of the CCC model (Constant Conditional Correlation), by Engle and Kroner (1995) in the form of the BEKK representation and by Engle(2002) in the version of the DCC model (Dynamic Conditional Correlation), allows for modelling the dependence structure of financial assets and markets via the second (and unobservable) moments which are known as the conditional correlations between financial returns.

The success of the multivariate GARCH model can be noticed from the growing number of extensions of the DCC model. The two important extensions are the AG-DCC model proposed by Cappiello, Engle and Sheppard (2006), which allows for a component to capture the asymmetry in the dependence structure, and the TDCC model, introduced by Pesaran and Pesaran (2007), which adopts an one-step procedure of estimation instead of the two-step procedure used in the original DCC model and assumes a multivariate Student's t-distribution for the residuals. These two models have shown some improvements in modelling the dependence structure between financial series, which can be found in the comparative research of Pesaran, Schleicher and Zaffaroni (2009) which investigated the performance of 53 volatility models for the 18 financial returns from developed financial markets and in the study of Barassi, Dickinson and Le (2010) that performed a similar exercise on 54 volatility models for 19 emerging financial markets and the US financial market. These studies indicated that the main drawback of the standard DCC-type models, which outperform the Riskmetrics filters and other volatility models, is that the innovations used in the second step to model the financial dependence are assumed to be multivariate normal while they are, in fact, non-normally distributed. Thus, the standardization process used by the DCC model or the devolatization process used by the TDCC model is to ensure that the distribution of innovations is Gaussian.

The copula approach which has recently been used some researchers, such as Patton (2006), Jondeau and Rockinger (2006), Rodriguez (2007) or Peng and Ng 2011), has initially shown some early success in filling this gap in the literature. By the approach based on the Sklar's theorem (1959) of a copula function which is defined as a function providing a mapping between the marginal distribution of each univariate series and the joint distribution of all series in consideration. It implies is that there is no need to ensure that the marginal distribution and the multivariate distribution must be of a same type as in the DCC model. Patton (2006) provides the theoretical framework for the multivariate conditional copula and allows for the application of the copula to analyse the dependence structure using high-dimensional data. In the current family of copulas, two copula types that can work with high-dimensional data are the Gaussian copula and the Student's t-copula.

Previous studies focus mainly on bivariate analysis. In this study, we use these two types of the multivariate conditional copula to analyse the dependence structure between 19 emerging markets and the US markets. In these two multivariate copulas, the correlation between the financial markets is conditionally specified in the popular DCC representation of Engle (2002). Our research is to check if and how the copulas improve the DCC model's performance. The flexible feature of the copula function allows us to form 12 different copulas which are combinations of the GARCH or the GJR models with the choice of distribution assumption being Gaussian or Student's t or Hansen's skewed Student's t and the Gaussian copula or the Student's t-copula chosen for the dependence structure. These 12 copulas are estimated by a two-step ML estimation by which the marginal model and the copula are estimated separately. The estimates obtained by this method are shown by Patton (2006) to be asymptotically efficient and normal.

We use the maximized values of log-likelihood function to compute the AIC and SBIC that evaluate the in-sample performance of the copulas. The Value-at-Risk theory is also used to evaluate the out-of-sample performance of the copulas. The Value-at-Risk methodology, to construct the diagnostic tests for the copulas, is applied in both passive and active manners of risk management. For the passive risk management, an equally-weighted portfolio is used for VaR analysis while an optimally-weighted portfolio is used for VaR analysis in the active risk management.

The remaining parts of this paper is organized as follows: the literature review of the development and the application of copula in finance and in financial econometrics are presented in section 2. Section 3 provides the methodology of the copula constructed with the DCC model. The empirical results are presented in section 4. The last section gives concluding remarks and suggests some future work.

4.2. Literature review

4.2.1. Review of the development of copula in financial econometrics

Based on Sklar's theorem, introduced in 1959, the studies and discussions on the applications of copula in financial econometrics and financial economics can now be widely found in books such as Cherubini *et al* (2004) who were the first authors to provide a detailed explanation of the theoretical framework of the copula applied to mathematical finance and derivatives pricing. Joe (1997) and Nelsen (2010) provided a rigorous introduction of the copula from the statistical and mathematical perspective. Trivedi and Zimmer (2007) also wrote a book introducing some additional applications of the copula in finance. An important review of the applications of the copula in finance is the study of Patton (2009) which focuses on the reviews of recent key developments in the applications of the copula in finance. It is a useful review as it gives a comprehensive discussion of applications of the copula in both univariate and multivariate time series analysis. It is also a presentation of profound background of the copula-based models relied on the break-through research of Patton (2006) that defines the 'conditional copula' which acts so as to connect all possibly-correlated univariate series to specify the joint distribution of these series. The introduction and the application of copulas to finance and financial econometrics has filled a gap in the methodology.

Until recently, the success of the DCC-GARCH model, proposed by Engle (2002), and its various extensions in modelling the multivariate dependence structure in finance rested on that the DCC-type models can capture the time-varying conditional correlations between financial assets - for examples of the existence of the time-varying conditional correlations see Andersen *et al* (2006) and Laurent *et al* (2006). However, the main drawback of the DCC-type models is the 2-step estimation procedure which cannot efficiently resolve the significant difference between the distributions of each univariate series and the multivariate distribution assumption used to specify the likelihood function of the multivariate dependent structure. Specifically, Pesaran and Pesaran (2007) indicate that the standardized residuals from the first step are not likely to be Gaussian to satisfy the multivariate Normal distribution assumption in the second step. Therefore, the TDCC model proposed by Pesaran and Pesaran (2007) uses the devolatization process to replace the standardization process used by the DCC-type models and also suggests the 1-step estimation procedure. However, the success of the TDCC model is also limited due to the fact that the multivariate Student's t-distribution, used by the TDCC model, does not imply the univariate Student's t-distribution for each individual return series with the same degrees of freedom. The method of using the time-varying conditional copula, initially introduced by Patton (2006) to provide an informative link between the marginal distribution and the multivariate distribution, is expected to solve this problem.

The conditional copula used by Patton (2006) can allow for the time-varying behaviour of the conditional correlation in the multivariate structure of dependence which utilized the bivariate BEKK specification of Engle and Kroner (1995). However, the BEKK model has the drawback that the dimension of the model increases exponentially when we increase the number of variables. Therefore, the estimation of the conditional copula using the BEKK (even the diagonal BEKK) becomes infeasible for a portfolio containing even a modest number of assets. Jondeau and Rockinger (2006,[65]) used the same method to derive the conditional copula for the TVC model (Time-Varying Correlation) of Tse and Tsui (2002) and the Markov-Switching model. Rodriguez (2007) applied the conditional copula of different types such as the Student's t-copula, Frank copula, Clayton copula or Gumbel copula for the regime-switching model of Hamilton (1994), assuming a constant correlation in the dependence structure. Chollote *et al* (2009) considered a similar methodology with the use of a switching regime model for the combination of the Gaussian copula for calm period and a choice of the Gaussian copula or the Student's t-copula or the canonical vine copula for noisy periods. While all above authors used the bivariate copulas, Garcia and Tsafack (2011) applied tetra-variate copulas, a mixture of two bivariate copulas to model the dependence of the stock and bond markets between a selection of two countries. The dependence structure assumed in these two latter studies is also the constant correlation which is represented by the sample correlation. This is because of the fact that is explained in the survey by Patton (2009) that there is now lack of a flexible way to specify a conditional copula that can characterize well the high-dimensional portfolio returns being fitted to the structure of dependence specified by the DCC-type models. The high persistence in the conditional correlation generated by the DCC-type models due to the existence of frequent unexpected small breaks in return series is a real challenge for a feasible realisation of the copula-based models for high-dimensional portfolios. One suggestion to overcome the dimensional issue is the 2-step Maximum Likelihood method of copula estimation which has been introduced by Shih and Louis (1995) and Joe (2005). In this method, the parameters of the marginal models are estimated in the first step and the copula parameters are estimated in the second step conditional on the estimates of the marginal parameters from the previous step. The 2-step procedure is shown by Joe (2005) and Patton (2006) to deliver asymptotically efficient and normal estimates of the copula parameters though there is some loss of efficiency. However, this is now the only feasible method to estimate the copula using the DCC-type specifications for a medium or large-scaled portfolio while there are on-going efforts to give a new way of construction of a copula of high dimensionality.

4.2.2. Review of applications of the copula in finance and economics

There are some early applications of copula in finance; Embrechts *et al* (1999) were among the first authors who mentioned the copula in risk management. For the review of the advent and the spectacular growth of applications of the copula in finance, Genest *et al* (2009) give a broad survey on 353 papers which use the term of 'copula' in connection to finance and are categorized into four main areas, which are management and measurement of risk, portfolio management, pricing of derivatives and the studies on interdependence between financial markets.

In risk and portfolio managements or risk measurement, the central focus of managers is the use of VaR (Value at Risk) to measure the riskiness of an asset or a portfolio. For a portfolio containing a large number of assets, the computation of VaR for the portfolio becomes more complicated as VaR basically depends on the correlation between assets in the portfolio. A constant correlation is likely to give a biased estimate of VaR while there is strong evidence of time-varying correlation. However, the fat-tailed behaviour of financial returns causes the non-normal dependence between financial returns which is shown by the fact that the conditional correlation generated by the multivariate GARCH models is heteroskedastic. Consequently, the VaR of portfolio computed using the conditional covariance estimated by the multivariate GARCH is biased. The reason is that the multivariate distribution assumption for the dependence structure is easily violated due to the fact that each individual asset may follow a distribution of a type different from the type of the multivariate distribution. Hence, the conditional copula, which allows for the time-varying dependence structure, is considered to give a better estimate of the VaR.

For the literature of this area, the textbook of Cherubini *et al* (2004) introduced the use of copula in VaR study while Alexander (2009) wrote a textbook which provides detailed instructions on the use of the copula in risk management using VaR. In early empirical research, Embrechts *et al* (2003) and Mendes *et al* (2004) used a bivariate copula to compute the VaR of a portfolio of two stocks which are assumed to have a linear and constant correlation. Embrechts and Höing (2006) extended the use of a copula for the VaR scenarios to the case of higher dimensions. Ozun and Cifter (2007) used a Joe-Clayton copula to compute VaR of a bivariate portfolio and the result showed that it deals with extremes better that the Riskmetrics filter namely EWMA (Exponentially Weighted Moving Average). Similarly, Hotta *et al* (2008) performed comparative research on the performances of the Gumbel copula and the bivariate BEKK or the DCC model in computing a VaR of a portfolio in which the copula outperformed the multivariate GARCH models. In the research of Fantazzini (2008), Gaussian and Student's t-copulas were applied to model the dependence structure of a bivariate portfolio in order to obtain the forecasts of VaR of portfolio. Huang *et al* (2009) conducted an interesting empirical study on the performance of 8 types of copulas which are Gaussian, Student's t, Clayton, Rotated-Clayton, Plackett, Frank, Gumbel and Rotated-Gumbel copulas to compute the VaR of portfolio containing 2 assets which are assumed to have a constant correlation. Their empirical results suggest that the best model is the Student's t-copula with the Gaussian-GARCH for the marginal model. One important intuition of this research is that the correlation of assets in the portfolio is the key factor to deliver an appropriate VaR amount. In one of the most recent piece of research, Wang *et al* (2010) considered 3 multivariate conditional copulas for a portfolio of 4 currencies to compute VaR and CVaR (Conditional VaR) with the assumption of a constant correlation between the currencies. It is clear that the recent research generally has applied the bivariate copulas, with the except of Wang *et al* (2010). So there is a need to perform an empirical research on copulas with higher dimensions. Another issue is that the VaR in this research was computed by using equal weights of assets while the correlation between assets in the dependence structure was assumed to be constant over the whole sample. If this assumption is violated by the fact that the correlation is more likely to be time-varying, the VaR of an equally-weighted portfolio is no longer a correct risk indicator. Therefore, a VaR of portfolio with dynamic weights, which are optimally computed by using the conditional covariance between assets, is an appropriate measurement for the riskiness of a portfolio.

Research on the interdependence and contagion of financial markets also receives a significant support from the use of copula. Forbes and Rigobon (2002) defined financial contagion as a significant shift in mean of Pearson's cross-market correlation after an initial shock. This broad research topic has increasingly become of interest to many researchers in finance since the recent financial crisis in 2007-2009. The early survey of those studies can be found in Dornbusch *et al* (2000), Pericoli and Sbracia (2003), or for a more recent and comprehensive overview of methodologies and research on financial contagion, in Kolb (2011). Also, Dungey *et al* (2004) who made the empirical comparison of four main approaches to test for contagion which are the correlation analysis approach of Forbes and Rigobon (2002), the VAR approach of Favero and Giavazzi (2002), the probability method of Eichengreen *et al* (1996) and the co-exceedance proposal of Bae *et al* (2001). Furthermore, an in-depth discussion of the contagion test using correlation analysis is presented by Corsetti (2005). Thanks to the DCC-type models, the research on contagion can somehow examine the non-linearity of cross-market correlation which is generated by various types of the DCC model. For a review of recent studies of contagion using the DCC models, see Barassi *et al* (2011). However, the conditional correlation generated by the DCC model is still heteroskedastic due to the non-normal feature of the dependence structure caused by the extreme values in stock markets. Even though the TDCC model of Pesaran and Pesaran (2007) is used to test for contagion in the study of Barassi *et al* (2011), the conditional correlations still show the property of heteroskedasticity. The use of a copula is then introduced to the research of this area; the first study of contagion test using copula is by Rodriguez (2007) who used a bivariate copula, with the switching-regime model for the dependence structure. Kenourgios (2011) tested for contagion using the regime-switching copula to compare with the AG-DCC model of Cappiello (2006). The copula type adopted in this research is the Gaussian copula which is limited by being unable to capture asymmetric dependence and to deal with fat-tailed properties. The most recent research of contagion using a copula is possibly that of Peng and Ng (2011) who used mixtures of different dynamic copulas to model the dependence between developed financial market such as the US, the UK and the Japanese financial markets where the volatility indices are also available to fit to the models beside the return series.

4.3. Methodology of the Copula-DCC-GARCH

4.3.1. The DCC-GARCH

Consider a vector of asset returns, r_t at the end of day t which contains k asset returns with marginal univariate distribution function $F_{i,t}$, conditional mean $\mu_{i,t}$ and conditional variance $\sigma^2_{i,t}$ for $i = 1, 2, ..., k$. To make the computations simpler, we can assume that the conditional mean, $\mu_{i,t}$ can be predicted and hence it is taken as a given. In this study, we consider 3 different t-distribution types which are Gaussian, Student's t and Hansen's skewed Student's t for the marginal models. To deal with the heteroskedasticity of the return series, we use the standard univariate GARCH(1,1) model of Bollerslev (1986) for each individual asset as follows

$$\begin{cases} r_{i,t} = \omega_i + a_i r_{i,t-1} + \varepsilon_{i,t} \\ \text{where } \varepsilon_{i,t} = \sigma_{i,t} z_{i,t} \text{ for } i\text{= 1, 2, ..., } k \\ \text{(1) } z_{i,t} \text{ follows one of the below distributions:} \\ \quad \text{(a) Gaussian: } z_{i,t} \sim N(0,1) \\ \quad \text{(b) Student's } t\text{: } z_{i,t} \sim t_{\nu_i} \qquad \nu_i\text{: degrees of freedom} \\ \quad \text{(c) Hansen's skewed } t\text{: } z_{i,t} \sim SkT(\nu_i, \delta_i) \qquad \delta_i\text{: skewness parameter} \\ \text{(2) } \sigma_{i,t}^2 = \bar{\sigma}_i (1 - \lambda_{i,1} - \lambda_{i,2}) + \lambda_{i,1} \varepsilon_{i,t-1}^2 + \lambda_{i,2} \sigma_{i,t-1}^2 \qquad \bar{\sigma}_i\text{: unconditional variance} \\ \text{where } \lambda_{i,1} + \lambda_{i,2} < 1,\ \lambda_{i,1} > 0,\ \lambda_{i,2} > 0 \end{cases} \tag{4.1}$$

To capture the asymmetric behaviour in the financial returns, we can use the GJR(1,1) model of Glosten *et al* (1993) for each individual asset which is defined as

$$\begin{cases} r_{i,t} = \omega_i + a_i r_{i,t-1} + \varepsilon_{i,t} \\ \text{where } \varepsilon_{i,t} = \sigma_{i,t} z_{i,t} \text{ for } i\text{= 1, 2, ..., } k \\ \text{(1) } z_{i,t} \text{ follows one of the below distributions:} \\ \quad \text{(a) Gaussian: } z_{i,t} \sim N(0,1) \\ \quad \text{(b) Student's } t\text{: } z_{i,t} \sim t_{\nu_i} \qquad \nu_i\text{: degrees of freedom} \\ \quad \text{(c) Hansen's skewed } t\text{: } z_{i,t} \sim SkT(\nu_i, \delta_i) \qquad \delta_i\text{: skewness parameter} \\ \text{(2) } \sigma_{i,t}^2 = \lambda_{i,0} + \lambda_{i,1} \varepsilon_{i,t-1}^2 + \lambda_{i,2} \sigma_{i,t-1}^2 + \gamma_i I_t \varepsilon_{i,t-1}^2 \quad \text{for } I_t = \begin{cases} 1 & \text{if } \varepsilon_{i,t} < 0 \\ 0 & \text{otherwise} \end{cases} \\ \text{where } \lambda_{i,0}, \lambda_{i,1}, \lambda_{i,2} > 0,\ \lambda_{i,1} + \lambda_{i,2} + \frac{1}{2}\gamma_i < 1 \end{cases} \tag{4.2}$$

Using the residuals from the mean equation and the conditional variance of the GARCH or GJR model, we can obtain the standardized innovation as follows

$$\epsilon_{i,t} = \frac{\varepsilon_{i,t}}{\sigma_{i,t}} \tag{4.3}$$

The vector of the standardized innovations, $\epsilon_t = \begin{bmatrix} \epsilon_{1,t} & \epsilon_{2,t} & \dots & \epsilon_{k,t} \end{bmatrix}'$, which is assumed to have multivariate Normal distribution and the covariance matrix, H_t, is constructed as follows

$$H_t = E(\epsilon_t \epsilon_t') = \begin{bmatrix} E(\epsilon_{1,t}\epsilon_{1,t}) & E(\epsilon_{1,t}\epsilon_{2,t}) & \cdots & E(\epsilon_{1,t}\epsilon_{k,t}) \\ E(\epsilon_{2,t}\epsilon_{1,t}) & \ddots & & \\ \vdots & & \ddots & \vdots \\ E(\epsilon_{k,t}\epsilon_{1,t}) & & \cdots & E(\epsilon_{k,t}\epsilon_{k,t}) \end{bmatrix} \tag{4.4}$$

or

$$H_t = \begin{bmatrix} \sigma_{11,t}^2 & \sigma_{12,t} & \dots & \sigma_{1k,t} \\ \sigma_{21,t} & \ddots & & \\ \dots & & \ddots & \vdots \\ \sigma_{k1,t} & & \dots & \sigma_{kk,t}^2 \end{bmatrix} \tag{4.5}$$

Due to the existence of the dependence between assets, the covariance between the assets, $\sigma_{mn,t}$ is not zero. Moreover, the dependence between assets is likely to vary overtime because of the fact that financial crisis or financial integration may cause the correlation between financial assets to be time-varying. Therefore, the DCC model of Engle (2002) is used to capture the dynamic conditional correlation between financial assets. The representation of the DCC(1,1) model can be expressed using the following structure

$$H_t = h_t R_t h_t \tag{4.6}$$

where $h_t = diag(\sigma_{i,t})_{k \times k}$ $with$ $\sigma_{i,t}$ is the conditional standard deviation generated by Equation 4.1 or Equation 4.2; $R_t = (\rho_{mn})_{k \times k}$ is the dynamic conditional correlation matrix. The mn^{th} element of the R_t matrix is defined as

$$\rho_{mn,t} = \frac{\sigma_{mn,t}}{\sigma_{mm,t}\sigma_{nn,t}} \tag{4.7}$$

The dynamic correlation matrix, R_t in Equation 4.6 is presented as follows

$$R_t = Q_t^{\star -1} Q_t Q_t^{\star -1} \tag{4.8}$$

The $k \times k$ matrix Q_t can be estimated by the process below

$$Q_t = \bar{Q}(1 - \alpha - \beta) + \alpha(\epsilon_{t-1}\epsilon'_{t-1}) + \beta Q_{t-1} \; with \; \alpha + \beta < 1, \; \alpha, \, \beta \in (0, 1) \tag{4.9}$$

where $\bar{Q}$ is the unconditional covariance of the standardized residuals, ϵ_t and $Q^\star$ is the diagonal matrix with diagonal elements extracted from diagonal elements of Q_t

$$Q_t^\star = \begin{bmatrix} \sqrt{q_{1,t}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{q_{2,t}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \sqrt{q_{k,t}} \end{bmatrix} \tag{4.10}$$

Since the introduction of the DCC-GARCH model model, there have been an increasing number of extensions of the DCC model. There are some major contributions that improve the performance of the DCC model such as the Asymmetric DCC model by Cappiello

(2006) that allows for an asymmetric component in the dynamic covariance structure in Equation 4.9 or the TDCC model by Pesaran and Pesaran (2007) that assumes the Student's t-distribution of return series and the devolatization process for the innovations. However, the main criticism of the DCC-type models is that the multivariate distribution assumption is likely to be inappropriate as each individual univariate return series, in fact, may follow an empirical distribution which is not similar to what is assumed for the multivariate distribution function. Hence, the copula function introduced by Sklar (1959) which provides a mapping connection between the multivariate and the univariate distribution assumptions can solve the problem of the DCC models.

4.3.2. The copula for the DCC model

The theorem presented by Sklar (1959) shows that there exists a copula defined as a function that merges all marginal distributions into a multivariate distribution function. The Sklar's theorem states the following:

Sklar's theorem:

Consider a vector of residuals, $\varepsilon_t = \{\varepsilon_{1,t}, \varepsilon_{2,t}, ..., \varepsilon_{k,t}\}$ with a joint k-dimensional distribution function, $\mathbb{D}$ and marginal distribution functions $D_1,\ D_2,\ ...,\ D_k$. Based on the univariate distributions of the residuals, $\varepsilon_{i,t}$ which can be 'Gaussian', 'Student's t' or 'Skew T' to define the log-likelihood function of the marginal models, to apply the copula theory to the DCC model the residuals ε_t need to be standardized following Equation 4.3 to give ϵ_t and the standardized innovations are transformed to the uniform distribution by the probability integral transform method (PIT) using the corresponding distribution of residuals, ε_t as follows

$$u_{i,t} = D(\epsilon_{i,t})\ with\ u_{i,t} \sim U[0,1] \tag{4.11}$$

Therefore, there exists a k-variate copula $\mathbb{C}$ that provides a connection between the marginal distribution functions and the joint distribution function for all $\epsilon_{i,t} \in \mathbb{R}$ as follows

$$\mathbb{D}\left(\epsilon_{1,t},\, \epsilon_{2,t},\, ...,\, \epsilon_{k,t}\right) = \mathbb{C}\left(D_1(\epsilon_{1,t}),\, D_2(\epsilon_{2,t}),\, ...,\, D_k(\epsilon_{k,t})\right) \tag{4.12}$$

From Equation 4.12, the copula can be obtained according to the following equation

$$\mathbb{C}(u_{1,t},\, u_{2,t},\, ...,\, u_{k,t}) = \mathbb{D}\left(D_1^{-1}(\epsilon_{1,t}),\, D_2^{-1}(\epsilon_{2,t}),\, ...,\, D_k^{-1}(\epsilon_{k,t})\right) \tag{4.13}$$

The useful feature of the above structure is that the marginal distributions are not required to be in the same distribution class. For any given continuous marginal distributions, there always exists an unique copula, $\mathbb{C}$ that satisfies Equation 4.12.

Corollary

If the multivariate distribution function, $\mathbb{D}$ is k-times differentiable, the Sklar's theorem shows that the joint density of $\mathbb{D}$ can be derived as below

$$\boldsymbol{d}\left(\epsilon_{1,t},\, \epsilon_{2,t},\, ...,\, \epsilon_{k,t}\right) = \frac{\partial^k \mathbb{D}\left(\epsilon_{1,t},\, \epsilon_{2,t},\, ...,\, \epsilon_{k,t}\right)}{\partial\epsilon_{1,t},\, \partial\epsilon_{2,t},\, ...,\, \partial\epsilon_{k,t}} \tag{4.14}$$

$$\boldsymbol{d}\left(\epsilon_{1,t},\, \epsilon_{2,t},\, ...,\, \epsilon_{k,t}\right) = \frac{\partial^k \mathbb{C}\left(D_1(\epsilon_{1,t}),\, D_2(\epsilon_{2,t}),\, ...,\, D_k(\epsilon_{k,t})\right)}{\partial u_{1,t},\, \partial u_{2,t},\, ...,\, \partial u_{k,t}} \times \prod_{i=1}^{k} d_i(\epsilon_{i,t}) \tag{4.15}$$

$$\boldsymbol{d}\left(\epsilon_{1,t},\, \epsilon_{2,t},\, ...,\, \epsilon_{k,t}\right) = \boldsymbol{c}(u_{1,t},\, u_{2,t},\, ...,\, u_{2,t}) \times \prod_{i=1}^{k} d_i(\epsilon_{i,t}) \tag{4.16}$$

where $\boldsymbol{d}$ and $\boldsymbol{c}$ are the joint densities of the distribution function, $\mathbb{D}$ and the copula, $\mathbb{C}$, respectively.

From Equation 4.16, one can solve for the corresponding copula density as follows

$$\boldsymbol{c}(u_{1,t},\, u_{2,t},\, ...,\, u_{2,t}) = \frac{\boldsymbol{d}\left(D_1^{-1}(u_{1,t}),\, D_2^{-1}(u_{2,t}),\, ...,\, D_k^{-1}(u_{k,t})\right)}{\prod_{i=1}^{k} d_i(D_i^{-1}(u_{i,t}))} \tag{4.17}$$

Taking logs of both sides of Equation 4.16 to give the log-likelihood form of the joint density, $\boldsymbol{d}$

$$\mathcal{L}(\phi;\, \theta;\, \varepsilon_t) = \sum_{t=1}^{T}\left(\sum_{i=1}^{k} log\left(d_i(\phi_i;\, \varepsilon_{i,t})\right) + log\left(\boldsymbol{c}\left(D_1(\epsilon_{1,t}),\, ...,\, D_k(\epsilon_{k,t})\right);\, \theta\right)\right) \tag{4.18}$$

where ϕ is the vector containing the parameters of the marginal univariate models, $\phi = (\phi_1,\, ...,\, \phi_k)$; θ is the vector containing the copula parameters for the multivariate model. Due to the limited computability of the log-likelihood function for a medium or large size of the dimension of the return series, Equation 4.18 is divided into two parts which are the marginal log-likelihood $d\mathcal{L}$ and the copula log-likelihood $c\mathcal{L}$ as follows

$$d\mathcal{L}(\phi;\, \varepsilon_t) = \sum_{t=1}^{T}\sum_{i=1}^{k} log\left(d_i(\phi_i;\, \varepsilon_{i,t})\right) \tag{4.19}$$

$$c\mathcal{L}(\theta;\, \epsilon_t) = log\left(\boldsymbol{c}\left(D_1(\epsilon_{1,t}),\, ...,\, D_k(\epsilon_{k,t});\, \theta\right)\right) \tag{4.20}$$

The decomposition of the log-likelihood function allows for a feasible estimation process for a medium or large-scaled vector of return series that can be realised in a 2-step procedure using the MLE. The marginal models are estimated in the first step and the copula model is estimated, conditional on the first step, in the second step.

4.3.2.1. The marginal models

The marginal models used in this study are the GARCH and the GJR models. Based on Equation 4.19, the log-likelihood for the marginal model is specified by a marginal distribution as follows

For the Gaussian distribution:

$$d\mathcal{L}_i(\phi_i,\, \varepsilon_{i,t}) = -\frac{1}{2}(log(2\pi) - log(\sigma_{i,t}^2) - \varepsilon_{i,t}^2) \tag{4.21}$$

For the Student's t-distribution:

$$d\mathcal{L}_i(\phi_i,\, \varepsilon_{i,t}) = log\left(\frac{\Gamma\left(\frac{\nu_i+1}{2}\right)}{\Gamma\left(\frac{\nu_i}{2}\right)}\right) - \frac{1}{2}\left(log(\pi(\nu_i-2)) + log(\sigma_{i,t}^2) + (\nu_i+1)log\left(1+\frac{\varepsilon_{i,t}^2}{\nu_i-2}\right)\right) \tag{4.22}$$

where $\Gamma(.)$ is the gamma function

For the Hansen's skewed Student's t-distribution:

$$d\mathcal{L}_i(\phi_i,\, \varepsilon_{i,t}) = -\frac{1}{2}log(\sigma_{i,t}^2) + p_{i,t}(\nu_i,\, \delta_i,\, \varepsilon_{i,t}) \tag{4.23}$$

where $p_{i,t}$ is the natural logarithm of the density function:

$$p_{i,t}(\nu_i,\, \delta_i,\, \varepsilon_{i,t}) = \begin{cases} bc\left(1+\frac{1}{\nu_i-2}\left(\frac{b\varepsilon_{i,t}+a}{1-\delta_i}\right)^2\right)^{-\frac{\nu_i+1}{2}} & for\ \varepsilon_{i,t} < -\frac{a}{b} \\ bc\left(1+\frac{1}{\nu_i-2}\left(\frac{b\varepsilon_{i,t}+a}{1+\delta_i}\right)^2\right)^{-\frac{\nu+1}{2}} & for\ \varepsilon_{i,t} \geq -\frac{a}{b} \end{cases} \tag{4.24}$$

with $a = 4\delta_i c \dfrac{\nu_i - 2}{\nu_i - 1}$, $b^2 = 1 + 3\delta_i^2 - a^2$ and $c = \dfrac{\Gamma\left(\dfrac{\nu_i + 1}{2}\right)}{\Gamma\left(\dfrac{\nu_i}{2}\right)\sqrt{\pi(\nu_i - 2)}}$

The vector of unknown marginal parameters, $\phi_i = (w_i,\, a_i,\, \lambda_{1,i},\, \lambda_{2,i},\, \gamma_i,\, \nu_i,\, \delta_i)$ is obtained from the first-step estimation and used to construct log-likelihood function of the copula for the DCC(1,1) model. In this study, there are two types of copula which are Gaussian copula and Student's t-copula.

4.3.2.2. The Gaussian copula for the DCC(1,1) model

Based on standard representation of the copula in Equation 4.13 and Equation 4.20, the Gaussian copula for the DCC(1,1) model can be constructed as follows

$$\mathbb{C}_{Gaussian}(u_{1,t},\, u_{2,t},\, ...,\, u_{k,t},\, R_t) = \mathbb{N}\left(\Phi^{-1}(\epsilon_{1,t}),\, \Phi^{-1}(\epsilon_{2,t}),\, ...,\, \Phi^{-1}(\epsilon_{k,t})\right) \tag{4.25}$$

where $\mathbb{N}$ is a k-dimensional multivariate Normal distribution; $u_{i,t} = \Phi(\epsilon_{i,t})$ with Φ is the univariate Normal distribution. If the innovations follow the Hansen's skewed Student's t-distribution, the univariate Normal distribution, Φ in Equation 4.25 is replaced by SkT_{ν_i,δ_i}.

The log-likelihood function of the Gaussian copula for the DCC(1,1) is:

$$c\mathcal{L}_{Gaussian}(\theta) = \sum_{t=1}^{T} \boldsymbol{c}_{t,Gaussian}(\theta) \tag{4.26}$$

$$For\ \boldsymbol{c}_{t,Gaussian}(\theta) = -\frac{1}{2}log(|R_t(\theta)|) - \frac{1}{2}\tilde{u}_t \times (R_t^{-1}(\theta) - I) \times \tilde{u}_t' \tag{4.27}$$

where $\tilde{u}_t = \Phi^{-1}(u_t)$, with Φ^{-1} the inverse univariate Normal distribution; $\theta = (\alpha,\, \beta)$

is the vector of copula parameters for the DCC models; R_t is the dynamic conditional correlation matrix generated by the DCC(1,1) model.

4.3.2.3. The Student's *t*-copula for the DCC(1,1) model

The Student's t-copula is increasingly popular in applications to analyse financial dependence because the choice of the degrees of freedom can effectively describe the interdependence of financial assets. Similar to the Gaussian copula, the Student's t-copula for the DCC(1,1) model can be derived using the standard copula construction in Equation 4.13 and Equation 4.20 as

$$\mathbb{C}_{\boldsymbol{t}}(u_{1,t},\, u_{2,t},\, ...,\, u_{k,t},\, R_t) = \mathbb{T}_{\boldsymbol{\nu}}\left(\boldsymbol{t}_{\nu_1}^{-1}(\epsilon_{1,t}),\, \boldsymbol{t}_{\nu_2}^{-1}(\epsilon_{2,t}),\, ...,\, \boldsymbol{t}_{\nu_k}^{-1}(\epsilon_{k,t})\right) \tag{4.28}$$

where $\mathbb{T}_{\boldsymbol{\nu}}$ is the multivariate Student's t-distribution with $\boldsymbol{\nu}$ degrees of freedom; $u_{i,t} = \boldsymbol{t}_{\nu_i}(\epsilon_{i,t})$ with $\boldsymbol{t}_{\nu_i}$ is the univariate Student's t-distribution function for the i^{th} innovation. In the case that the innovations follow the Hansen's skewed Student's t-distribution, the univariate Student's t-distribution, $\boldsymbol{t}_{\nu_i}$ in Equation 4.28 is replaced by SkT_{ν_i,δ_i}.

$$\begin{aligned} c\boldsymbol{\mathcal{L}}_{Student}(\theta) = \;& -Tlog\frac{\Gamma\left(\dfrac{\boldsymbol{\nu}+k}{2}\right)}{\Gamma\left(\dfrac{k}{2}\right)} - kTlog\frac{\Gamma\left(\dfrac{\boldsymbol{\nu}+1}{2}\right)}{\Gamma\left(\dfrac{k}{2}\right)} - \frac{\boldsymbol{\nu}+k}{2}\sum_{t=1}^{T} log\left(1+\frac{\tilde{u}_t' R_t^{-1}\tilde{u}_t}{\boldsymbol{\nu}}\right) \\ & - \sum_{t=1}^{T} log\,|R_t| + \frac{\boldsymbol{\nu}+1}{2}\sum_{t=1}^{T}\sum_{i=1}^{k} log\left(1+\frac{\tilde{u}_{i,t}}{\boldsymbol{\nu}}\right) \end{aligned} \tag{4.29}$$

where $\tilde{u}_{i,t} = \boldsymbol{t}_{\nu_i}^{-1}(u_t)$, with $\boldsymbol{t}^{-1}$ is the inverse Student's t-distribution; $\theta = (\boldsymbol{\nu},\, \alpha,\, \beta)$ is the vector of copula parameters for the DCC models; R_t is the dynamic conditional correlation matrix generated by the DCC(1,1) model.

4.4. Empirical application of the copula - DCC models

Figure 1.1, Figure 1.2 and Figure 1.3 show the plots of daily returns of the 20 market indices. From graphical inspection, the daily returns showed that volatility clustering can be noticed from most emerging markets. This gives motivation for the use of the GARCH models to explain the volatility behaviour of each individual market. The multivariate parameterization allows us to estimate the conditional correlations among emerging markets as well as between an individual emerging market and the US market.

In this study, we used the same data set from our previous chapter (see Barassi et al) to perform a comparative analysis on the performance of the copulas based on the DCC(1,1) model. Table 1.1 gives a primary descriptive statistics of the return series with unconditional means, standard deviations, skewness, kurtosis and the estimates of the univariate t-GARCH(1,1) for each return series which is individually assumed to follow univariate Student's t-distribution with $\hat{\nu}$ degrees of freedom. All observations are used and the results indicate that all countries except Philippines have positive mean. The European and Latin American countries generally have higher mean in returns than the countries from Asia. The degree of skewness of some markets such as China, Korea, Malaysia, etc. is positively different from zero and all markets show a clear excess of kurtosis from the lowest level at 5.497 for Taiwan to the highest being 59.259 for Malaysia and the mean of kurtosis centers around 15. This evidence suggests that the use of Gaussian DCC models is likely to be mispecified and instead suggests the use of the univariate GARCH with the skewed Student's t assumption specified in Equation 4.1 for the symmetric model and in Equation 4.2 that is specified for the asymmetric model.

The ML estimates of the univariate t-GARCH models also give evidence of high volatility persistence and the parameters show a clear similarity of the conditional volatility across emerging markets. The similarity can also be found in the estimates of the degrees of freedom of the Student's t-distribution which range from the highest of 8.530 for Chile to the lowest of 4.085 for Indonesia with the mean for all markets of 5.744. This result our choice to replace the normality assumption should by that of Student's t-distribution.

4.4.1. The choice of copula models

The flexible feature of a copula allows for various constructions of econometric models of multivariate time series to model the financial interdependence. In this study, we construct 12 different specifications for the copula-DCC models. For the marginal model, it is either GARCH(1,1) or GJR(1,1,1) models with the choice of distribution assumption being Gaussian or Student's t or Hansen's skewed Student's t-distribution. For the copula-DCC model, it is the choice between the Gaussian copula with the DCC specification or the Student's t-copula with the DCC specification. So the 12 model names are GARCH-n(1,1) with Gaussian copula, GARCH-t(1,1) with Gaussian copula, GARCH-skt(1,1) with Gaussian copula, GARCH-n(1,1) with Student's t-copula, GARCH-t(1,1) with Student's t-copula, GARCH-skt(1,1) with Student's t-copula, GJR-n(1,1,1) with Gaussian copula, GJR-t(1,1,1) with Gaussian copula, GJR-skt(1,1,1) with Gaussian copula, GJR-n(1,1,1) with Student's t-copula, GJR-t(1,1,1) with Student's t-copula, GJR-skt(1,1,1) with Student's t-copula.

4.4.1.1. In-sample evaluations

In this study, a 2-step method of estimation is adopted to obtain the estimates of the copula model for the high-dimensional data set containing 20 series. In the first step, the marginal model as represented by a specific univariate model in Equation 4.1 or in Equation 4.2 is estimated by maximizing the log-likelihood function defined in Equation 4.21 under the Gaussian assumption or in Equation 4.22 under the Student's t or in Equation 4.23 under the Hansen's skewed Student's t. In the second step, the copula is estimated conditional on the estimates of the first step. Specifically, the residuals standardized by the conditional variances generated by the univariate GARCH or the GJR model are now transformed into the uniform distribution from 0 to 1 by using the corresponding distribution assumed for the residuals to allow for method of 'IFM'.[1] The log-likelihood function of the copula is then constructed by using the dynamic conditional correlation matrix,

[1] Inference Function for Margins: see Joe (1997), Chapter 10.

R_t, obtained by the DCC(1,1) model and the transformed standardized residuals as in Equation 4.27 for the Gaussian type or in Equation 4.29 for the Student's t type.

To evaluate the performance of the 12 copula models, we use the estimation strategy to estimate each of these 12 models using a window of size fixed to 800 observations and rolling along the whole sample of 3910 observations with a frequency of 25 days (implying a monthly risk update) to make 125 sub-samples. Hence, the total estimations of these 12 models are 1500. The estimation results are used to evaluate the in-sample performances of the copulas using the maximized log-likelihood values of the copula defined in Equation 4.27 for the Gaussian copula and in Equation 4.29 for the Student's t-copula. The AIC and the SBIC, the two information criteria, are also computed by using the maximized value of the log-likelihood function of the copula as follows:

$$AIC_{i,t} = -c\mathcal{L}_{i,t} + \kappa_i \; and \; SBIC_{i,t} = -c\mathcal{L}_{i,t} + \frac{\kappa_i}{2} ln(W) \tag{4.30}$$

where $c\mathcal{L}_{i,t}$ is the maximized log-likelihood value of a copula i at time t; κ_i is the total number of parameters used by the copula i; W is the window size. The best-performing model is selected by minimizing the information criterion. Table 4.1 reports the maximized values of the 12 copulas while Table 4.2 displays the values of the information criteria which are AIC and SBIC. In these tables, the values for the in-sample evaluation criteria are reported for the first sub-sample (from 15/05/1995 to 16/06/98), the last sub-sample (from 03/04/2007 to 07/05/2010), and for the average values of 125 sub-samples for each of the 12 multivariate conditional copulas. In Table 4.1, the maximized log-likelihood values of the copulas are presented in two groups which are the copulas with symmetric marginal models and the copulas with asymmetric marginal models. In the former group, the log-likelihood value ranges from the highest value, indicating the best in-sample model, of 2522 for the Student's t-copula with the GARCH-t margin to the lowest value, indicating the worst in-sample model, of the Gaussian copula with the GARCH-*skt* margin. In the latter group, the Student's t-copula with the GJR-t margin has the highest log-likelihood value of 2446 while the Gaussian copula with the GJR-*skt* margin has the lowest log-likelihood value of 2182. Hence, the Student's t-copula with the GARCH-t margin has the highest log-likelihood value among the 12 copulas in consideration. Clearly, Figure 4.1 plots the maximized LL values for all of the 12 copulas. We can see that the best model of the Student's t-copula with the GARCH-t margin has the LL values consistently higher than the other copula in almost 125 sub-samples. The second best model is the Student's t-copula with the GJR-t margin.

Table 4.1.: Maximized Log-Likelihood Values for the 12 Multivariate Conditional Copulas

Margin	Copula	Sample periods		
Type	Type	08/06/1998 (1)	07/05/2010 (2)	Average (3)
Symmetric				
GARCH-n	Gaussian	869	4959	2253
GARCH-t	Gaussian	842	4968	2243
GARCH-*skt*	Gaussian	826	4953	2235
GARCH-n	Student's t	912	5094	2301
GARCH-t	Student's t	1229	5334	2522
GARCH-*skt*	Student's t	857	5118	2292
Asymmetric				
GJR-n	Gaussian	843	4866	2190
GJR-t	Gaussian	827	4879	2188
GJR-*skt*	Gaussian	810	4865	2182
GJR-n	Student's t	880	4988	2234
GJR-t	Student's t	1207	5187	2446
GJR-*skt*	Student's t	839	5017	2238

There are 2 unknown parameters in the Gaussian copula which are the α and β in the dynamic structure of the DCC model as presented in Equation 4.9 whereas the Student's t-copula used 3 unknown parameters which are α, β for the dynamic correlation and the degrees of freedom, $\boldsymbol{\nu}$ of the multivariate Student's t-distribution. Hence, the reported values of the information criteria in Table 4.2, computed by using the maximized log-likelihood and the number of copula parameters, also suggest that the best model for in-sample evaluation is the Student's t-copula with the GARCH-t margin. The Student's t-copula with the GJR-t margin, which was expected to outperform the symmetric model by using the asymmetric term, is selected as the second best model. The result for the

Table 4.2.: AIC, SBIC Values for the 12 Multivariate Conditonal Copulas

Margin Type	Copula Type	Sample periods 08/06/1998 (1)		07/05/2010 (2)		Average (3)	
AIC							
GARCH-*n*	Gaussian	-863	(5)	-4952	(8)	-2246	(5)
GARCH-*t*	Gaussian	-836	(8)	-4962	(7)	-2236	(6)
GARCH-*skt*	Gaussian	-819	(11)	-4947	(9)	-2228	(7)
GARCH-*n*	Student's *t*	-902	(3)	-5084	(4)	-2291	(3)
GARCH-*t*	Student's *t*	-1219	(1)	-5324	(1)	-2512	(1)
GARCH-*skt*	Student's *t*	-847	(6)	-5108	(3)	-2282	(4)
GJR-*n*	Gaussian	-836	(7)	-4859	(11)	-2183	(10)
GJR-*t*	Gaussian	-820	(10)	-4872	(10)	-2181	(11)
GJR-*skt*	Gaussian	-804	(12)	-4859	(12)	-2176	(12)
GJR-*n*	Student's *t*	-870	(4)	-4978	(6)	-2224	(9)
GJR-*t*	Student's *t*	-1197	(2)	-5177	(2)	-2436	(2)
GJR-*skt*	Student's *t*	-829	(9)	-5007	(5)	-2228	(8)
SBIC							
GARCH-*n*	Gaussian	-867	(5)	-4957	(8)	-2251	(5)
GARCH-*t*	Gaussian	-840	(8)	-4966	(7)	-2241	(6)
GARCH-*skt*	Gaussian	-824	(11)	-4951	(9)	-2233	(8)
GARCH-*n*	Student's *t*	-909	(3)	-5091	(4)	-2298	(3)
GARCH-*t*	Student's *t*	-1226	(1)	-5331	(1)	-2519	(1)
GARCH-*skt*	Student's *t*	-854	(6)	-5115	(3)	-2289	(4)
GJR-*n*	Gaussian	-841	(7)	-4864	(11)	-2188	(10)
GJR-*t*	Gaussian	-825	(10)	-4877	(10)	-2186	(11)
GJR-*skt*	Gaussian	-808	(12)	-4863	(12)	-2180	(12)
GJR-*n*	Student's *t*	-877	(4)	-4985	(6)	-2231	(9)
GJR-*t*	Student's *t*	-1204	(2)	-5184	(2)	-2443	(2)
GJR-*skt*	Student's *t*	-836	(9)	-5014	(5)	-2235	(7)

in-sample fit of the copulas indicated that:

Firstly, the choice of marginal model is highly important for the performance of a copula. Thus, the marginal model with the skewed t-distribution appears not to be a good choice for the copulas as the copulas coupled with the skewed-t marginal model are always ranked in the bottom half of the table. Moreover, the copulas with the simple GARCH margin, being ranked from the first place for the Student's t-copula with the GARCH-t to the 7^{th} place for the Gaussian copula with the GARCH-skt, have in-sample performance better than that of the copulas with the GJR margin which are usually ranked from the 7^{th} place to the 12^{th} place by either of the information criteria.

Secondly, the Student's t-copula generally outperforms the Gaussian copula as the top 3 copulas selected by either the AIC or the SBIC are all Student's t-copulas regardless of the choice of marginal model and the bottom 3 copulas suggested by either the AIC or SBIC are all Gaussian copulas. The findings also showed that the dependence structure cannot be adequately characterized by a multivariate Normal distribution. A Student's t-distribution is more relevant in describing the dependence between financial returns as it can fit the fat tails by using the different degrees of freedom. This also explains the poor performance of the DCC-type models that need to ensure Gaussianity of the standardized innovations used in the second step of estimation for the dependence structure. The results of the well-performing t-copula is also supported by the previous studies such as that of Huang *et al* (2009) which show that the Student's t-copula outperforms 7 other copulas in a bivariate analysis with the use of a constant correlation structure.

4.4.1.2. Out-of-sample evaluations

We adopt the method used in the subsubsection 2.3.1.2 to evaluate the performance of the 12 copulas. Therefore, both passive risk management and active risk management will be employed in this section. We shall consider a portfolio comprising 20 individual return series with $r_t \sim (\mu_t,\ H_t|\Omega_{t-1})$ denoted as the vector containing these 20 individual return series. Let ρ_t be the return on this portfolio with weights w_{t-1} which can be predetermined weights w^p_{t-1} for the passive risk management or the optimal weights $w^a_{i,t-1}$ of a copula i

for the active risk management. So the portfolio return can be expressed as

$$\rho_t = w'_{t-1} r_t \tag{4.31}$$

In risk management, investors focus on the possibility of the return of portfolio to decrease in a certain period of time, say from *t-1* to *t*. Therefore, a benchmark $\rho^{\star}_{i,t-1}$, which is known as the maximum daily loss, is needed such that the possibility of the portfolio return, ρ_t to fall below the benchmark is $\alpha \in (0, 1)$. This constraint in risk management can be expressed as follows

$$Pr(\rho_t < -\rho^{\star}_{i,t-1} | \Omega_{t-1}) \leq \alpha \tag{4.32}$$

In portfolio theory, the computation of the VaR constraint relies on the assumption of constant correlation between assets in portfolio and this assumption is also used for the computation of the portfolio return. However, the VaR constraint presented in Equation 4.32 is likely to be inappropriate when the dependence structure of the assets in portfolio, in fact, is non-normal. The multivariate conditional copula contributes to the literature of risk management in such a way that the covariance between assets in portfolio is parametrically or semi-parametrically estimated using the non-normal distribution assumption such as the Student's *t*-distribution. The VaR constraint is, therefore, estimated in the way that it is updated by the dynamic change in the dependence structure of the portfolio. Moreover, the optimal portfolio return also benefits from the precise estimation of the conditional covariance which is used to compute the optimal portfolio weights in the strategy of active risk management.

In the reverse way, the VaR theory can be applied to evaluate the performance of the multivariate copulas which deliver the forecast of the conditional covariance of assets in portfolio and the VaR constraint is based on the forecast of the conditional covariance.

So a count function I_t is set up to check for the validity of a copula as follows

$$I_t(\rho_t + \rho_t^{\star}) \begin{cases} = 1 & \text{if } \rho_t + \rho_t^{\star} < 0 \text{ or VAR constraint in Equation 4.32 is violated} \\ = 0 & \text{otherwise} \end{cases} \tag{4.33}$$

For the evaluation for each model, the whole sample is divided into two sub-samples $T_{est}(t = 1 : T)$ and $T_{eval}(t = T + 1 : T + N)$ where T_{est} is for model estimation and T_{eval} is for model evaluation. The VaR indicator, I_t is recursively computed by using N observations in the evaluation period. We can count the number of days when the VaR constraint is violated by using the VaR indicators as follows

$$\hat{\pi}_i = \frac{1}{N} \sum_{t=T+1}^{T+N} \hat{I}_t \tag{4.34}$$

Hence, under the specification of a copula, $\hat{\pi}_i$ will have mean α and variance $\dfrac{\alpha(1-\alpha)}{N}$. Moreover, the standardized test statistic can now be obtained based on the result from Equation 4.34 as follows

$$z_{\hat{\pi}_i} = \frac{\sqrt{N}(\hat{\pi}_i - \alpha)}{\sqrt{\alpha(1-\alpha)}} \tag{4.35}$$

If N is sufficiently large, the test statistic, $z_{\hat{\pi}_i}$ is asymptotically Normally distributed with zero mean and unit variance. This standardized test statistic is used to test the null hypothesis under which a copula i is correctly specified

$$H_0: \quad H_t = H_t\left(\hat{\theta}_{S_{est}}\right) \; or \; \hat{\pi}_i = \alpha \tag{4.36}$$

4.4.1.2.1. Out-of-sample evaluations using passive risk management technique

In using the passive risk management technique to evaluate the performance of a copula, the weights for assets in the portfolio, w_{t-1}^{p} equally set to $\frac{1}{20}$ was used to compute the portfolio return, $\hat{\rho}_{i,t}^{p}$. In this technique, the VaR constraint in Equation 4.32 becomes

$$Pr(\rho_t < -\bar{\rho}_{i,t-1}|\Omega_{t-1}) \leq \alpha \tag{4.37}$$

where ρ_t is constructed with no need of a copula i. A copula i is only used to compute the benchmark for the maximum daily loss which is denoted as $\bar{\rho}_{i,t-1}$.[2]

Based on the computed portfolio returns and by setting the risk tolerance probabilities: $\alpha = 1\%$ and $\alpha = 5\%$, we can compute the statistics of the model performance which is the VaR indicator I_t, π_i and z_{π_i}. The first 800 observations were used for model estimation and the last 3110 observations were used for the recursive computations of the above statistics of model performance with the update frequency is 25 days or monthly risk update.

Table 4.3 presents the results for VaR-based diagnostic tests for the 12 conditional copulas which follows the passive risk management manner at two levels of risk tolerance $\alpha = 1\%$ *and* $\alpha = 5\%$. At each level of risk tolerance, the test results are obtained by using the endogenous distribution assumption which means the Normal distribution for the Gaussian copula and Student's t-distribution with the degrees of freedom which is recursively estimated at every update frequency and by using the Student's t-distribution assumption with a generic degrees of freedom $\boldsymbol{\nu} = 6$ for all copulas. The generic degrees of freedom is set equal to 6 based on the suggestion of the mean of the degrees of freedom of individual return series estimated by the univariate t-GARCH displayed in Table 1.1. For the risk tolerance 1% and 5%, the violation rate or the VaR exceedance rate, π_i is assumed to be 1% and 5%, respectively. For the copula i under the null hypothesis, the hypothesized rate of violations indicates that the copula i is correctly specified. Hence, the

[2] For the derivation of $\bar{\rho}_{i,t-1}$, see Appendix in section A.3

estimates of the violation rate of the 12 copulas are expected to be equal to the assumed violation rate under the null hypothesis which is tested by the standardized statistic, $z_{\hat{\pi}_i}$ following the standard Normal distribution with zero mean and unit variance.

Table 4.3.: VaR Diagnostic Tests for the 12 Multivariate Conditional Copulas using Equally-Weighted Portfolio (α=1%, α=5%)

Margin Type	Copula Type	α=1%				α=5%			
		Endogenous		Student-t6		Endogenous		Student-t6	
		$\hat{\pi}_i$	$z_{\hat{\pi}_i}$	$\hat{\pi}_i$	$z_{\hat{\pi}_i}$	$\hat{\pi}_i$	$z_{\hat{\pi}_i}$	$\hat{\pi}_i$	$z_{\hat{\pi}_i}$
Symmetric									
GARCH-n	Gaussian	2.19	6.65	1.61	3.41	5.92	2.35	6.59	4.08
GARCH-t	Gaussian	2.09	6.11	1.51	2.87	5.53	1.36	6.18	3.01
GARCH-skt	Gaussian	2.09	6.11	1.54	3.05	5.63	1.61	6.27	3.25
GARCH-n	Student's t	2.12	6.29	1.61	3.41	5.95	2.43	6.63	4.16
GARCH-t	Student's t	1.90	5.03	1.58	3.23	5.76	1.94	6.24	3.17
GARCH-skt	Student's t	2.03	5.75	1.58	3.23	5.63	1.61	6.24	3.17
Asymmetric									
GJR-n	Gaussian	2.70	9.54	2.03	5.75	7.46	6.30	8.04	7.78
GJR-t	Gaussian	2.61	9.00	1.96	5.39	7.17	5.56	7.75	7.04
GJR-skt	Gaussian	2.61	9.00	1.96	5.39	7.14	5.48	7.91	7.45
GJR-n	Student's t	2.70	9.54	2.03	5.75	7.46	6.30	8.04	7.78
GJR-t	Student's t	2.28	7.19	1.96	5.39	7.37	6.05	7.78	7.12
GJR-skt	Student's t	2.54	8.64	1.96	5.39	7.20	5.64	7.78	7.12

In Table 4.3, at the risk tolerance level of 5%, we can see that all the estimates of the violation rate are above the hypothesized level of 5% and the standardized statistics, $z_{\hat{\pi}_i}$ are larger than the 95% critical value of 1.96, which indicates that all the copulas are rejected by this diagnostic test; except the Gaussian copula with GARCH-t margin under the endogenous assumption with the estimated violation rate of 5.53% and the test statistic of 1.36 (the null hypothesis cannot be rejected at the 95% significant level). This is also the best model with the rate of violation closest to the benchmark of 5%. In comparison between the symmetric and asymmetric groups, the symmetric margins continue to show its adequacy to match with copulas with the estimated violation rates ranging from the lowest of 5.53 to the highest of 5.95 under endogenous assumption and from 6.18 to 6.63 under the Student's $t(6)$ assumption, while those rates of the asymmetric group are clearly higher with the estimates of violation rates ranging from the lowest of

7.17 to the highest of 7.46 for endogenous assumption and from the lowest of 7.04 to the highest of 7.78 under the Student's $t(6)$ assumption. These rates of the asymmetric group are dramatically higher than the benchmark of 5% for this test.

At the risk tolerance level of 1%, all the copulas are rejected by this test at the 95% significant level with the lowest rate of violation being at 1.51% for the Gaussian copula and the GARCH-t margin under Student's $t(6)$ distribution. The symmetric group marginally performs better than the asymmetric group under both distribution assumptions. However, the estimated rates of VaR exceedance of the copulas under the endogenous distribution assumption, centering around 2%, is much higher than the hypothesized level of 1%. The copulas under the Student's $t(6)$ distribution assumption have these rates ranging from the lowest level of 1.51% to the highest level of 2.03%. At both risk tolerance levels, the Student's t-copula with the GARCH-t margin, which is ranked at the first place in the in-sample evaluation, continues to stay in the top 3 models with the VaR exceedance rates approaching the benchmark levels and with the lowest values of the standardized test statistic, though higher than the critical value.

Figure 4.2 and Figure 4.3 plot the VaR exceedance rate, $\hat{\pi}_i$ and the VaR constraint, $\bar{\rho}_{i,t-1}$ for the best copula at both of the risk tolerance probabilities $\alpha = 1\%$ and $\alpha = 5\%$. For $\alpha = 1\%$, the expected violations are 31 while the actual violations are 59 ($\hat{\pi}_i = 1.90\%$). For $\alpha = 5\%$, the actual violations are 179 while the expected violations are 155. Based on the graphical view, the violations mainly occurred in the periods when the Asian financial crisis, the Dotcom crisis and the Global financial crisis happened. Especially at $\alpha = 5\%$, the best copula clearly failed in the period of the Global financial crisis during 2007-2009 with a large number of VaR violations.

4.4.1.2.2. Out-of-sample evaluations using active risk management technique

That few models explicitly passed the passive VaR-based diagnostic test can be explained by the fact that the equal weights being set to $\frac{1}{20}$ are likely to generate a portfolio return, ρ_t which is not optimal at the estimated VaR constraint, $\bar{\rho}_{t-1}$. Hence, the information obtained from Equation 4.32 to be sent to the count function in the equation is more likely to be incorrect, especially at the presence of the shocks in financial markets. Consequently, the estimates of the VaR exceedance rate as well as its standardized test statistics will be biased. So the VaR-based test performed in the manner of passive risk management may deliver wrong decisions on the validity of the copulas. It, therefore, needs to be performed in the style of active risk management by which the portfolio weights are optimally computed as follows

$$w^a_{i,t-1} = \frac{1}{\delta} H^{-1}_{i,t-1} \mu_{i,t-1} \text{ if the VaR constraint in (34) does not bind} \tag{4.38}$$

$$w^a_{i,t-1} = \frac{1}{\delta^\star_{i,t-1}} H^{-1}_{i,t-1} \mu_{i,t-1} \text{ if the VaR constraint in (34) binds} \tag{4.39}$$

where δ is the risk aversion parameter which is set so as to get an acceptable rate of VaR bindings; $\delta^\star_{i,t-1}$ is the risk aversion at time *t-1* which is chosen when the VaR binds for the optimal weights, for the derivation of $\delta^\star_{i,t-1}$ see Appendix in section A.2; $H_{i,t-1}$ is the covariance matrix estimated by the copula i; $\mu_{i,t-1}$ is the conditional mean of portfolio. This means that the portfolio return is computed by using the full knowledge of copula i so risk managers can simply set the maximum daily loss, L_{t-1} based on their attitude to risk and the VaR constraint in Equation 4.32 now becomes

$$Pr(\rho_t < -L_{t-1} | \Omega_{t-1}) \leq \alpha \tag{4.40}$$

Using the same risk aversion $\delta_{t-1} = 75$ as in Pesaran *et al* (2009), we got the VaR binding rates of the copulas for the optimal weights with the data from the emerging markets well above 80% which is significantly higher those in the original research which focused on the integrated financial markets only. This is because of the fact that the risk aversion was set too small relative to the evaluation sample that covers the whole period of the Global financial crisis with large unexpected shortfalls in the emerging markets. To obtain reasonable VaR binding rates for the optimal weights, the risk aversion coefficient is changed to 97. The maximum daily loss, L_{t-1} is fixed to 1%.

Table 4.4 provides the results of the VaR-based diagnostic test for the 12 copulas following the active risk management manner discussed above. The risk tolerance rate for this test is set to 1%. This table reports the annualized mean of portfolio return, the Information Ratio (IR), which is computed by dividing the mean of portfolio return by its own standard deviation, the percentage of VaR bindings for optimal portfolio weights. In the same condition, the VaR constraint under Student's $t(6)$ assumption tends to bind more often than that under the endogenous assumption. The Information Ratio, showing the trading performance of the copulas, reported in positive values and indicates that the copulas perform well in trading. The trading performance of the copulas is relatively stable across the different copula types. However, there is a noticeable difference in the IR between the symmetric group and the asymmetric group. Specifically, the IR for the symmetric group is between 4.54 for the Gaussian copula with the GARCH-t and 4.75 for the Student's t-copula with the GARCH-n under the endogenous assumption. Under the Student's $t(6)$ assumption, the IR ranges from 4.54 for the Gaussian copula with the GARCH-t to 4.74 for Student's t-copula with the GARCH-n. The IR for the asymmetric group is between 3.16 for the Gaussian copula with the GJR-t and 3.59 for the Gaussian copula with the GJR-n under the endogenous assumption and the IR of this group, under Student's $t(6)$ assumption, ranges from 3.03 for the Gaussian copula with the GJR-t to 3.47 for the Gaussian copula with the GJR-n or with the GJR-skt. The IRs of the portfolio obtained under the two assumption for the innovations are relatively similar to each other. This indicates that the trading performance does not obviously rely on the distribution of innovations. Following this criterion, the Student's t-copula continues to outperform the Gaussian copula in trading and the choice of margin is decisive to obtain higher IRs. However, this criteria does not explicitly show a strong statistical penalization when the IRs remain stable across copula type in the same group.

Table 4.4.: Information Ratios and VaR Diagnostic Tests for the 12 Multivariate Conditional Copulas using Optimally-Weighted Portfolio ($\alpha = 1\%$)

Margin	Copula	Endogenous							Student's t ($\nu = 6$)						
Type	**Type**	**Mean Return**	**IR**	$\hat{\pi}_i$	$pval_{\hat{\pi}_i}$	$z_{\hat{\pi}_i}$	$pval_{z_{\hat{\pi}_i}}$	**% VaR Binds**	**Mean Return**	**IR**	$\hat{\pi}_i$	$pval_{\hat{\pi}_i}$	$z_{\hat{\pi}_i}$	$pval_{z_{\hat{\pi}_i}}$	**% VaR Binds**
Symmetric															
GARCH-n	Gaussian	40.71	4.73	1.42	0.01	2.33	0.01	37	37.02	4.73	1.06	0.32	0.34	0.37	60
GARCH-t	Gaussian	36.76	4.54	1.25	0.07	1.43	0.08	32	33.74	4.54	1.03	0.39	0.16	0.43	55
GARCH-skt	Gaussian	39.07	4.64	1.42	0.01	2.33	0.01	36	35.59	4.63	1.06	0.32	0.34	0.37	59
GARCH-n	Student's t	40.37	4.75	1.35	0.02	1.97	0.02	40	37.00	4.74	1.00	0.46	-0.02	0.51	60
GARCH-t	Student's t	36.77	4.66	1.19	0.13	1.07	0.14	47	35.38	4.66	1.03	0.39	0.16	0.43	58
GARCH-skt	Student's t	38.46	4.65	1.38	0.02	2.15	0.02	41	35.60	4.64	1.06	0.32	0.34	0.37	59
Asymmetric															
GJR-n	Gaussian	49.08	3.59	3.02	0.00	11.34	0.00	51	43.19	3.47	2.19	0.00	6.65	0.00	70
GJR-t	Gaussian	43.62	3.16	2.80	0.00	10.08	0.00	46	39.07	3.03	1.99	0.00	5.57	0.00	67
GJR-skt	Gaussian	47.10	3.39	2.89	0.00	10.62	0.00	49	41.78	3.24	2.09	0.00	6.11	0.00	69
GJR-n	Student's t	48.44	3.58	3.02	0.00	11.34	0.00	53	43.10	3.47	2.22	0.00	6.83	0.00	70
GJR-t	Student's t	43.52	3.37	2.48	0.00	8.28	0.00	59	41.31	3.31	2.03	0.00	5.75	0.00	68
GJR-skt	Student's t	46.20	3.37	2.77	0.00	9.90	0.00	54	41.88	3.25	2.09	0.00	6.11	0.00	69

We are now looking at the VaR diagnostics computed using the risk tolerance probability of 1% and the optimal portfolio weights. It is clear that with unbiased diagnostics, the VaR violation rate differs markedly between the 2 groups of copula. Under the endogenous assumption, the copulas with the GARCH margins clearly show lower rates of violation varying from the lowest of 1.19% for the Student's t-copula with the GARCH-t to the highest of 1.42 for the Gaussian copula with either the GARCH-n or the GARCH-skt. For this group, the standardized test statistics shows that only the Student's t-copula with the GARCH-t margin ($z_{\hat{\pi}_i} = 1.07$ with p-value $= 0.14$) is not rejected at the 95% significance level while all other types in this groups are rejected at the 95% level. The VaR violation rates of the asymmetric group are significantly higher than those of the symmetric group, ranging from the lowest of 3.16% for the Gaussian copula with the GJR-t to the highest of 3.59% for the Gaussian copula with the GJR-n. This indicates the significant difference from the hypothesized level of 1%. Besides, the high values of the test statistics show the rejection of all copula types in this group. Under the Student's $t(6)$ assumption, the symmetric group shows an excellent performance when all copulas in this group pass this test with the test statistics being between -0.02 and 0.34, indicating that the null hypothesis in Equation 4.36 for the copula validity cannot be rejected, and the estimated VaR violation rates are almost 1% for all the copulas in this group. For the asymmetric group, the estimates of the VaR violation rate are lower under the Student's $t(6)$ assumption but still significantly higher than 1% and all the copulas in this group are rejected by the high values of the test statistics being between 5.57 and 6.83.

In combination with the previous test results, the Student's t-copula with the GARCH-t margin is the best-performing model in both in-sample and out-of-sample evaluations by being ranked in the top place and passing the VaR-based diagnostic tests. The Student's t-copula with the GJR-t which is ranked in the second place for in-sample performance does not pass any VaR-based diagnostic tests. Figure 4.4 plots the VaR exceedance rates of the best model in this test which is the Student's t-copula with the GARCH-t margin. With the maximum daily loss being fixed at 1%, the portfolio return is expected to fall below this benchmark no more than 31 days (equivalent to 1%) in the whole evaluation sample of 3110 days. The reported VaR exceedance rate, which is 1.19%, equivalent to 37 days of violation, is from the endogenous assumption under which the copula is actually estimated with the maximized value of the log-likelihood function. The VaR exceedance occurred mainly during the time of financial crises. The two periods when the portfolio return dropped far below the maximum daily loss are the period of Dotcom crisis and the period of the Global financial crisis.

4.5. Concluding remarks

The non-normal and non-linear properties of financial dependence structure has been identified as the main reason that limits the efficiency of all current multivariate models assuming the multivariate Normal distribution for the dependence structure. Typically, the dynamic conditional correlation estimated by the popular DCC model of Engle (2002) shows a heteroskedastic behaviour when it is applied in the test for financial contagion. This indicates that the standardization technique used in the DCC to normalize the innovations used to estimate the conditional correlation is not relevant to characterize efficiently the non-normal behaviour of financial dependence which are caused by the frequent breaks and unexpected shortfalls in stock markets. The use of multivariate conditional copula based on the theorem of Sklar (1959) to provide a mapping connections between the marginal and the multivariate distributions allows for an appropriate assumption for the dependence structure.

In this study, using multivariate conditional copula of the Gaussian and Student's t types, we find that the Student's t-copula with GARCH-t margin outperforms the Gaussian copula in all evaluations for performance. It means that using the copula function to explicitly allows the Student's t-distribution to characterise financial dependence structure is an efficient way to overcome the difficulty in modelling the financial dependence. In applying a copula to financial time series, our results also suggested that an appropriate choice of marginal model is crucially important to the performance of the copulas. The GJR model which is expected to describe the asymmetric behaviour of financial series has shown an inappropriate match with the copulas when it is outperformed by the symmetric GARCH model in coupling with the copulas. The Hansen's skewed Student's t-distribution is also rejected by the use of the Student's t or even the Gaussian distribution for the margin.

In using a copula to estimate the Value at Risk of portfolio, we also find that the optimal portfolio weights calculated by using the conditional covariance estimated by Student's t-copula can deliver the portfolio return at the minimized risk. Moreover, the accurate estimates of Value at Risk of portfolio given by the Student's t-copula are helpful for portfolio and risk management. However, further studies are required for the multivariate copula when there is a loss in efficiency in the 2-step ML estimation. It indicated that the Student's t-copula with Student's $t(6)$ has better performance that the Student's t-copula with the endogenously estimated degrees of freedom being larger than 6. The one-step estimation procedure with no loss in efficiency is only feasible for a bivariate system while portfolios normally include a large number of assets.

4.6. Chapter 4 - Figures

Figure 4.1.: Maximized Log-likelihood values of the 12 multivariate conditional copulas for 125 sub-samples

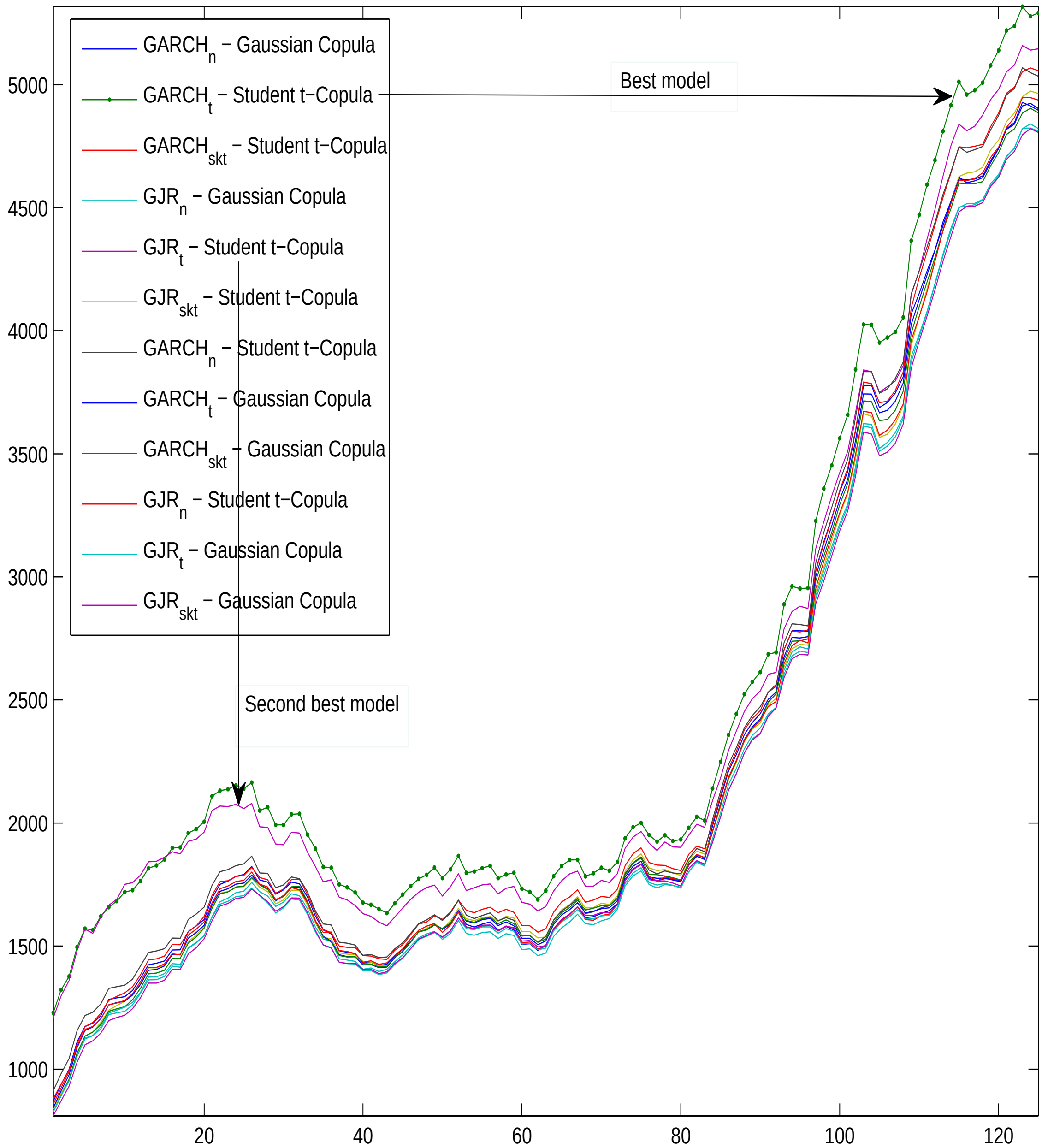


Figure 4.2.: VaR exceedance of the best copula (Student's t-copula with GARCH-t) in passive risk management with $\alpha = 1\%$

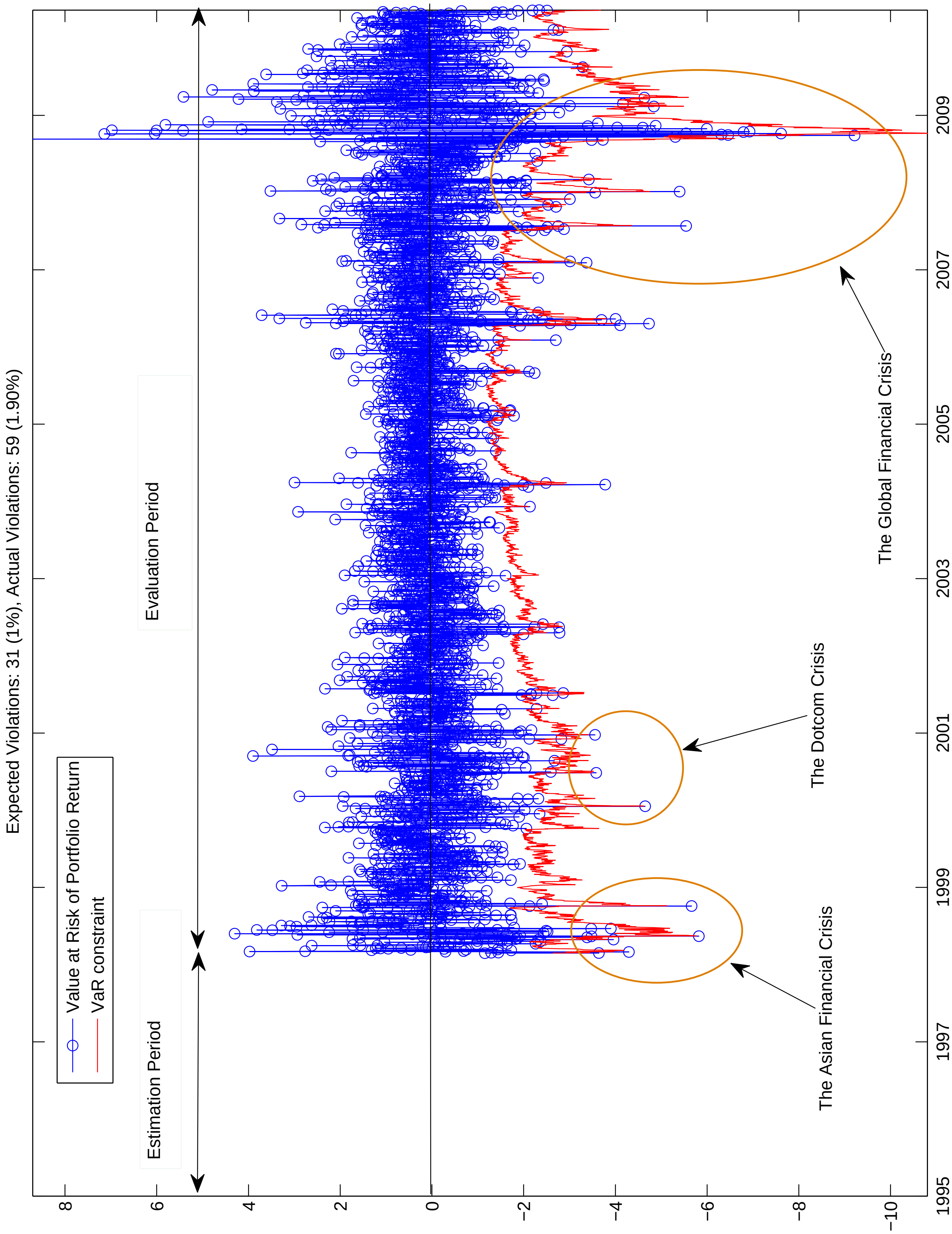


Figure 4.3.: VaR exceedance of the best copula (Student's t-copula with GARCH-t) in passive risk management with $\alpha = 5\%$

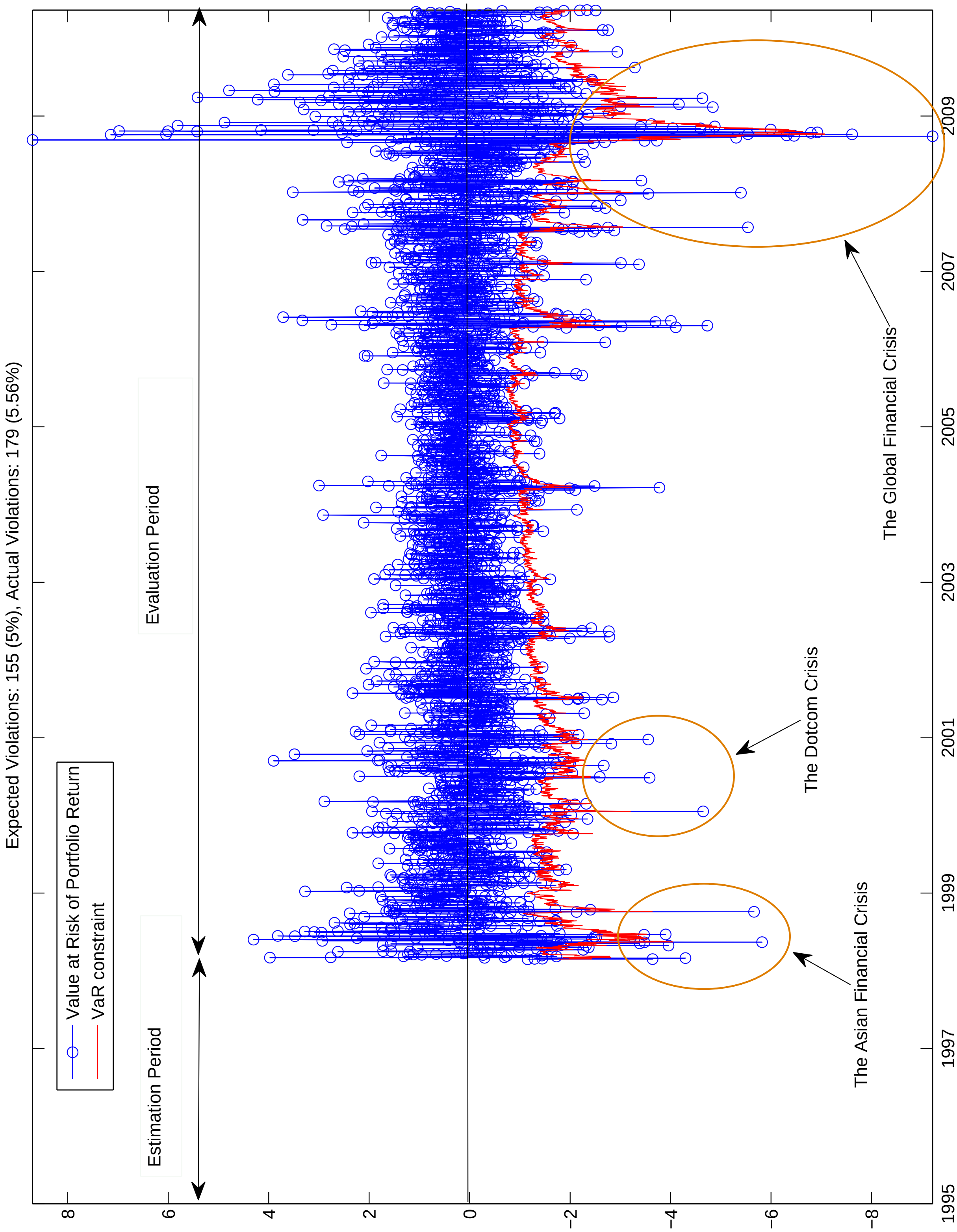
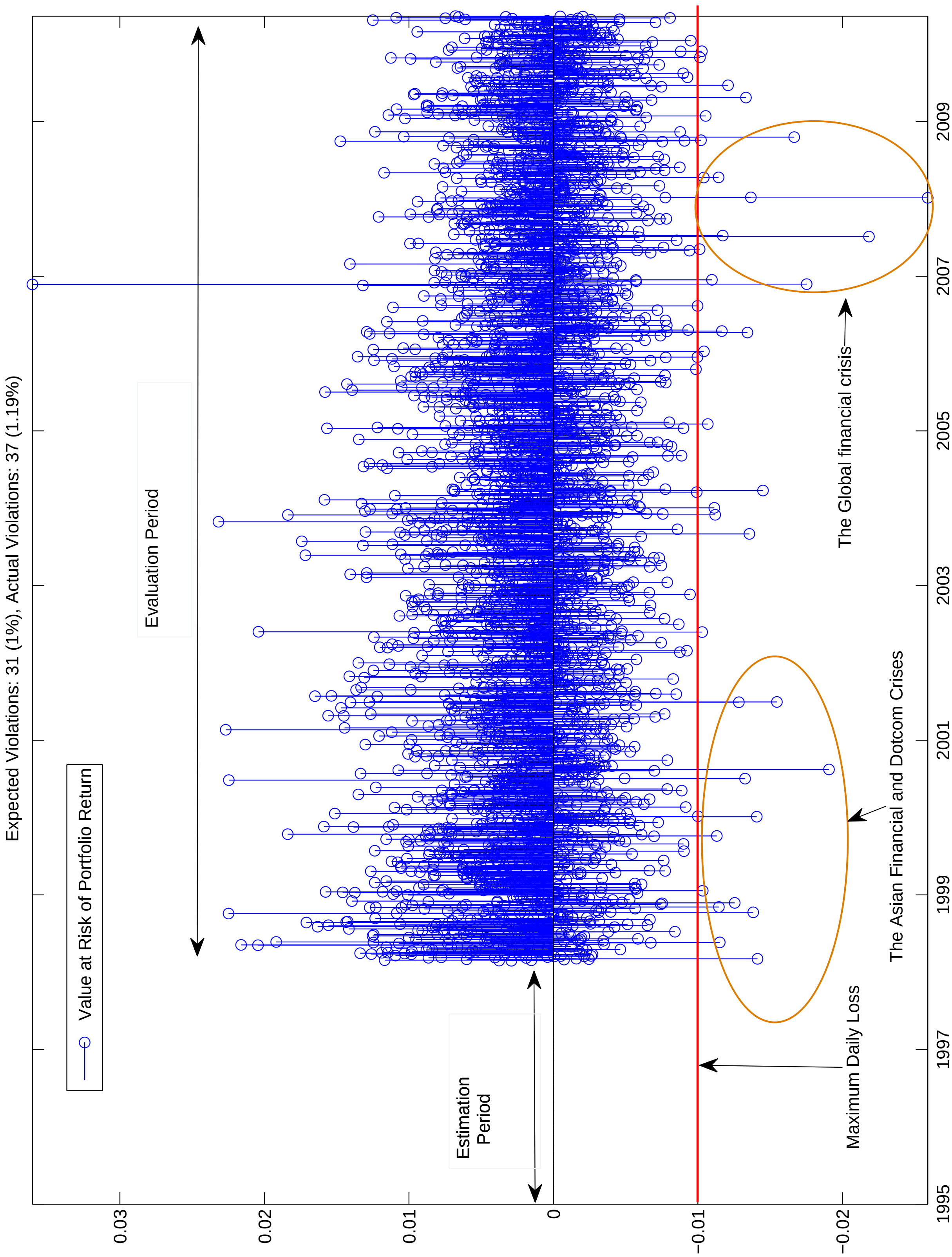


Figure 4.4.: VaR exceedance of the best copula (Student's t-copula with GARCH-t) in active risk management with $\alpha = 1\%$



5. CONCLUSIONS AND OUTLOOK FOR FUTURE RESEARCH

5.1. Conclusions

This thesis evaluates and applies several multivariate volatility models to data from emerging financial markets. All of the models, considered in this thesis, were initially built to fit data from developed markets. Therefore, the results in this study, obtained by using data from 19 emerging financial markets, provide a different point of view on the use of the volatility models

Firstly, the second chapter is a comprehensive review on the performance of a large number of volatility models. Since the introduction of the Riskmetrics filters, practitioners have been provided an efficient tool to quantify the risk in finance. However, recent development of volatility models, proposed by researchers, has offered them more choice of models used in risk management. Pesaran *et al* (2009) showed that the volatility models, used in academic research such as the CCC or the DCC models, outperform the Riskmetrics filters in the application to developed financial markets. In our study, 54 volatility models of 10 classes, which are used by both practitioners and researchers, were evaluated using the data set of 19 emerging financial markets and the US market.

It is interesting that the evaluation period covers the time of the financial crisis from 2007-2009. The results on the ranking of 54 volatility models show that there is no best model for all time. Thus, the TDCC model of Pesaran and Pesaran (2007,[89]) performed best in in-sample evaluation and was ranked on the group of top models by VaR-based tests but rejected by Ku and KS test. The standard DCC models consistently remain among the top models for both in-sample and out-of-sample performances, showing that it is important to consider this model class for modelling emerging markets data. The second best performance of the ADCC(1,1) model in many tests suggests the usefulness of asymmetric component for a volatility model in analyzing emerging data. The poor performance of the Riskmetrics filters, which ignore the time-varying properties of cross-market conditional correlations shows that these model types are not relevant to analyze the emerging data at market level where the structural changes are more likely to cause the conditional correlations to vary over time. Generally, the choice of an appropriate volatility model is crucial to obtain the best result because there is unlikely to have a model that can perform well both in calm and in noisy periods.

For example, the TDCC with the Student's t-distribution assumption or the ADCC model with an asymmetric component is efficient in capturing the dependence during the noisy period but is outperformed by the standard Gaussian DCC model during calm period. A model with t-distribution assumption is more relevant for crisis period but too conservative in normal period. Hence, designing a model that can deal with both calm and turmoil period is still a real challenge. Therefore, the results in this chapter could not be generalised. However, it helps us to find the best practical model that work relatively well during the time of the Global financial crisis. Moreover, the result in this chapter showed that all standard volatility models, which work under the Normal distribution assumption, have very poor performance when applied to emerging data. This finding confirms the fact that emerging financial markets are less efficient and more volatile than developed financial markets due to more frequent presence of unexpected sharp falls. However, the use of a Student's t-distribution, though with a generic degree of freedom, to replace the Normal distribution showed that standard volatility models could fit to the emerging data well. This is shown by the performance of the TDCC model, which are designed to have a Student's t-distribution assumption, at both developed data as in Pesaran *et al* (2009) and emerging data as in our study in chapter 1.

However, a main reason that limits popular application of the DCC models, especially the TDCC model, in risk management is that these model classes require a high computability for the estimation and the evaluation of the models. In risk management, it may require us to have daily risk update. However, for a medium-scaled portfolio, i.e. a portfolio of 20 stocks, it is only feasible to use a monthly risk update to evaluate the performance of models. This is the reason why the Riskmetrics models are still widely used in practical risk management although it is outperformed by the models suggested by researchers in both developed and emerging financial markets.

In the evaluation of volatility models for emerging financial data, the risk aversion coefficient, which is used in the active risk management to choose the optimal weights for stocks in a portfolio, should be larger than that of Pesaran *et al* (2009) for the data from developed financial market. The risk aversion coefficient, which represents the attitude of investors towards risk, is normally a constant number. Hence, a larger risk aversion coefficient for the emerging data means that investors are more risk averse to emerging financial markets or the emerging financial markets are considered as being riskier than the developed financial markets. Therefore, a future research question could be whether the risk aversion coefficient is time varying. Because, after a financial crisis or an effect of a financial contagion, the attitude of investors to risk may be changed. They may become more or less risk averse towards a specific financial asset or market. Therefore, a time-varying risk aversion coefficient could be more relevant to be used in the evaluation of a volatility model.

The outperformance of the TDCC model in chapter 2 is the motivation of the third chapter, which uses the TDCC model in testing for financial contagion. The results, obtained in this study, are different from those of the existing literature using the other volatility models or methods to specify cross-market correlations. Based on the evaluations of the TDCC model in the second chapter, the TDCC model, which deals with the fat-tailed behaviour of the financial returns better than the other model types, is suggested as an efficient tool to test for financial contagion. The log-likelihood function converged in the whole sample estimation as well as in recursive estimations. Interestingly, the TDCC model passes almost the diagnostic checking. Thus, the AIC, SBIC and maximized log-likelihood values indicated that in-sample performance of the TDCC was far better than standard DCC framework. In the evaluation for out-of-sample performance, that the model passed the LM tests and the Kolmogorov-Smirnov tests allows for important implications of time series analysis under the effects of financial crisis. It is interesting that the TDCC model passed the diagnostic tests performed on the probability integral transforms (PITs) with the evaluation period set from 2008 to 2009 when the global financial crisis took place. This is an important result when almost previously developed volatility models fail to explain the volatility and the dynamic correlation of stock markets during the time of financial turmoils. This suggests future research into how good volatility models can perform in estimating the volatility of emerging markets with the presence of financial crashes.

In this study, both methods of the active risk management, which opts to choose the optimal portfolio weights of stocks in a portfolio, and the passive risk management, which employs fixed weights for stock in a portfolio, are applied for a comparison. The active risk management method showed that it is more relevant than the passive risk management in evaluating a volatility model during a time of financial crisis. This can be explained that the perception of investors to risk may change due to the effect of a major shock. Hence, the weights of stocks in a portfolio must be changed. So a method with fixed weights is not relevant to evaluate how a volatility model performs during a time of financial crisis.

The two popular methods to test for contagion in our study, which are the method of Forbes and Rigobon (2002) using t-tests and the method of Chiang, Jeon and Li (2007) using AR model with dummies, are applied to the estimated correlations, which are adjusted for the heteroskedasticity by using the devolatization method suggested by Pesaran *et al* (2007). The results of the empirical tests for contagion show that the effects of the recent financial crises triggered in the US on the emerging markets is not as severe as reported in the previous studies. The emerging financial markets, which rely on the stability of emerging economies, are somewhat not closely connected to the developed markets. The short-run increases in the conditional correlations between the emerging markets and the

US market during the crisis time are only interpreted as the 'interdependence' of financial markets. This finding is important for investors who want to diversify their portfolios by using emerging stocks. However, in the tests for financial contagion, two key factors, which affect the empirical result, are the point of outbreak of a financial crisis and the length of a financial crisis. In this literature, there is no robust method for the two methods of Forbes *et al* (2002) and Chiang *et al* (2007). Therefore, different choices of the two factors may deliver different results. Hence, a reasonable justification for the choice of the two factors is a key to achieve an acceptable result for the test of financial contagion.

However, there are still questions for further studies such as the TDCC model with an asymmetric term could perform better during the time of crisis. The Student's t-distribution assumption may not be a relevant assumption during the calm periods of financial markets so the use of a mixture of different models may be more effective. Moreover, different financial returns may follow different distributions. This is the reason why the estimates of the degrees of freedom of t-distribution are different between the univariate and the multivariate t-distributions. Beside, the heteroskedasticity in the dynamic conditional correlation, which is reported in this chapter, is an evidence of the non-linear dependence structure between financial markets, which cannot be fully captured by the TDCC model. A copula model, therefore, could be worth considering. Further research is also needed to improve the empirical tests for contagion. Heteroskedasticity in the correlations could be removed by other methods, such as a switching-regime model or a copula model.

Finally, the application of multivariate conditional copula using the DCC model to specify the dependence structure is the focus of the fourth chapter in this thesis. The result in my fourth chapter indicated that the dependence among financial markets is more likely to be non-linear. This property can be noticed by the fact that the conditional correlation generated by either the DCC or the TDCC models is heteroskedastic. Hence, the use of copula function, based on Sklar's theorem (1959,[100]), is to improve the performance of the DCC-type models. Our results in this chapter show a significant improvement of the copula-based DCC model from the DCC-type model. Specifically, the Student's

t-copula outperforms the Gaussian copula, which shows that the multivariate Student's t-distribution used in the t-copula is more relevant in featurising the non-linear characteristics of financial dependence. These results are also robust to our results in the second chapter where the volatility models performed better under the Student's t-distribution with a generic degree of freedom. Our contribution in this chapter is the use of copula with the DCC specification to analyze the medium-scale portfolio containing 20 financial series while all previous research used copula for bivariate analysis.

In an application to estimate the Value at Risk of a portfolio, the DCC-based t-copula showed that it can deliver a better estimate of the VaR of a portfolio than what could be done by the DCC-type models. The mean-variance approach to obtain optimal weights for stocks in a portfolio is useful for the estimation of Value at Risk of a portfolio. The result in our study confirmed that the copula function, used to provide a link between the univariate and the multivariate distributions in the DCC framework, is helpful to explain the non-linearity of the dependence structure in multivariate time series. It improved the performance of the standard DCC model. With the use of copula function, we found that the choice of a univariate model is highly important in the DCC framework.

Our study in this chapter is the first to apply a multivariate copula model to a data set containing more than 4 series. Therefore, it is helpful for investors who want to have a precise estimator of Value-at-Risk of their medium-scale portfolios. However, the computability limits the realisation of the estimation of a copula-based DCC model in the one-step Maximum Likelihood procedure. The result was obtained by the two-step Maximum Likelihood procedure with some loss of efficiency. Therefore, further research is required for the application of copula in this literature. This would help investors, who manage to find a better estimator of the Value at Risk for their large-scaled portfolios, which may contains hundreds of stocks. Moreover, in active risk management, transaction cost of changing portfolio weights, in fact, may not be zero. Therefore, it would be interesting to include transaction costs in this research in the future.

5.2. Outlook for future research

Based on my research methodology in these 3 main chapters, there are some topics that could be continued in future works

The success of the copula model encourages us to construct copula models based on the different classes of volatility models. We can apply a similar methodology used in the second chapter for the different classes of copula-based models. This will be a large project, as it requires a large amount of works to fit 10 classes of models to the copula structure.

It is likely that it could be more successful if the TDCC model includes an asymmetric component to deal with the asymmetric behaviour of financial dependence. However, the one-step estimation procedure of the TDCC model is fairly restrictive to adapt an asymmetric component as the likelihood function easily fails to converge or it is impossible to converge due to the limit of computationality. Further research is required on this topic. Or another way is the use of ADCC model of Cappiello *et al* (2006,[23]) for the dependence structure in the Student's t-copula. However, the sensitivity of convergence of the ADCC needs to be considered when it is integrated into the Student's t-copula.

A combination of two model classes could be performed by the use of the Switching-regime model where the DCC model is used for the calm period and the TDCC or the t-copula is used for the noisy period. However, a proper prediction of the change in a volatility regime that allows these combined models to work efficiently could be a real challenge.

For risk managers who want to apply a volatility model to estimate the value at risk of large-scale portfolios, the CDCC model could be suggested for its consistency in large-scale portfolios. However, it is more likely to have non-linearity in the dependence structure of large-scale portfolios. Hence, the use of the Student's t-copula is recommended to replace the Gaussian assumption used in the CDCC model. Further works will be required to match the CDCC model with the copula function.

A. Appendix

A.1. Maximum Likelihood Estimation of the TDCC model

In the TDCC (1,1) model, there are $2k+3$ parameters to be estimated which include $2k$ coefficients from vectors $\lambda_1 = (\lambda_{11},\ \lambda_{12},\ ...,\lambda_{1k})$ and $\lambda_2 = (\lambda_{21},\ \lambda_{22},\ ...,\lambda_{2k})$ that enter the univariate GARCH (1,1) model for individual asset returns, the coefficient α, β in the dynamic correlation structure (Equation 3.15) and finally the degrees of freedom of the multivariate Student's t-distribution, ν.

Let θ be a vector of unknown coefficients as follows

$$\theta = (\lambda_1,\ \lambda_2,\ \alpha,\ \beta,\ \nu)'$$

The log-likelihood function available at time t which is built based on the variance decomposition and can be fitted by the observations in the estimation sample, S_{est} noted in Equation 3.19

$$l_t(\theta) = \sum_{\tau=d+1}^{t} f_\tau(\theta)$$

in which density function $f_\tau(\theta)$ of multivariate Student's t-distribution with ν degrees of freedom is expressed as follows

$$f_\tau(\theta) = -\frac{k}{2}ln(\pi) - \frac{1}{2}ln \mid \tilde{R}_{\tau-1}(\theta) \mid - ln \mid D_{\tau-1}(\lambda_1, \lambda_2) \mid + ln \left[\Gamma(\frac{k+\nu}{2})/\Gamma(\frac{\nu}{2}) \right]$$

$$-\frac{k}{2}ln(\nu-2) - (\frac{k+\nu}{2})ln \left[1 + \frac{e'_\tau D^{-1}_{\tau-1}(\lambda_1, \lambda_2)\tilde{R}^{-1}_{\tau-1}(\theta)D^{-1}_{\tau-1}(\lambda_1, \lambda_2)e_\tau}{\nu-2} \right]$$

in which

$$e_\tau = r_\tau - \mu_{\tau-1}$$

$$\text{and} \quad ln \mid D_{\tau-1}(\lambda_1, \lambda_2) \mid = \sum_{i=1}^{k} ln \left[\sigma_{i,\tau-1}(\lambda_{1i}, \lambda_{2i}) \right]$$

In Engle's specification, R_{t-1} depends not only on λ_1 and λ_2 but on α and β also. However, under a symmetric TDCC, R_{t-1} only depends on α, β and d, the lag order used in the devolatization process.

Under a Gaussian distribution of the devolatized returns, it gives rise to the log-likelihood function, $l_t(\theta)$. So the Maximum likelihood estimate of θ which can be denoted as $\hat{\theta}$ and is based on the sample observation S_{est} in Equation 3.20. $\hat{\theta}$ can now be obtained by maximizing $l_t(\theta)$ with respect to θ. The vector of estimated parameters, $\hat{\theta}$ can be expressed more specifically as

$$\hat{\theta}_t = \text{Arg}\,\underset{\theta}{\text{Max}} \{l_t(\theta)\} \text{ for } t \sim S_{est} = \{1, \ 2, \ldots, \ T\}$$

A.2. Mean-Variance Approach for the Optimal Portfolio Weights

Following this approach, under a volatility model i, the portfolio weights are optimally derived by using an objective function is as follows

$$\mathcal{O}(w_{i,t-1}|\Omega_{t-1}) = w'_{i,t-1}E(r_t|\Omega_{t-1}) - \frac{\delta}{2}w'_{i,t-1}V_i(r_t|\Omega_{t-1})w_{i,t-1} \tag{A.1}$$

where δ is the risk aversion, V_i is the covariance matrix estimated by model i under the distribution assumption $\mathcal{F}$. For example, if the distribution of r_t is assumed to be multivariate Student's t-distribution with ν_{t-1} degrees of freedom, the portfolio return will also have Student's t-distribution with the same degrees of freedom. So the portfolio return can be expressed as follows

$$\frac{w'_{i,t-1}r_t - w'_{i,t-1}\mu_{i,t-1}}{\sqrt{\dfrac{\nu_{t-1}-2}{\nu_{t-1}}w'_{i,t-1}H_{i,t-1}w_{i,t-1}}} \sim t_{\nu_{t-1}} \tag{A.2}$$

Hence, the VaR constraint becomes

$$\frac{-L_{t-1} - w'_{i,t-1}\mu_{i,t-1}}{\sqrt{w'_{i,t-1}H_{i,t-1}w_{i,t-1}}} \leq -\bar{c}^{\alpha}_{v_{t-1}} \tag{A.3}$$

where $\bar{c}^{\alpha}_{v_{t-1}} = \dfrac{\nu_{t-1}-2}{\nu_{t-1}}c^{\alpha}_{v_{t-1}}$with $c^{\alpha}_{v_{t-1}}$ is the α percent critical value of the Student's t-distribution with ν_{t-1} degrees of freedom. Under this constraint, the optimal portfolio weights, $w^{a}_{i,t-1}$ are obtained by maximizing the objective function presented in

Equation A.1 as follows

$$w^a_{i,t-1} = \begin{cases} \frac{1}{\delta} H^{-1}_{i,t-1}\mu_{i,t-1} & \text{if } \delta > \delta^\star_{t-1} \\ \frac{1}{\delta^\star_{i,t-1}} H^{-1}_{i,t-1}\mu_{i,t-1} & \text{otherwise} \end{cases} \tag{A.4}$$

The risk aversion, $\delta^\star_{i,t-1}$ is derived in case the VaR constraint in Equation A.3 binds by using the ex ante daily Sharpe ratio of the portfolio, $s_{i,t} = \mu'_{i,t-1}H_{i,t-1}\mu_{t,t-1}$

$$\delta^\star_{i,t-1} = \frac{\sqrt{s_{i,t}}(\bar{c}^\alpha_{\nu_{t-1}} - \sqrt{s_{i,t}})}{L_{t-1}} \tag{A.5}$$

The Sharpe ratio is conditioned so as it does not exceed the α percent critical value, $\bar{c}^\alpha_{\nu_{t-1}}$ to ensure the positivity of the risk aversion, $\delta^\star_{i,t-1}$ in Equation A.5.

A.3. Capital Value at Risk of portfolio, $\bar{\rho}_t$

Capital Value at Risk of portfolio, $VaR(w_{t-1}, \alpha) = \bar{\rho}_t(w_{t-1}, \alpha)$ is a function of α. Under the condition that we have all information at $t-1$ and the model i is valid, the portfolio return, $\rho_{i,t}$ has conditional mean $\mu_{i,\rho t} = w'_{t-1}\mu_{i,t}$ and conditional variance $\sigma^2_{\rho t} = w'_{t-1}H_{i,t}w_{t-1}$. Therefore, we can compute the standardized returns as follows

$$z_t = \sqrt{\frac{\nu_{t-1}}{\nu_{t-1}-2}} \frac{w'_{t-1}(r_t - \mu_{i,t})}{\sigma_{\rho,t}} | \Omega_{t-1} \sim \left(F_\nu,\ 0,\ \frac{\nu_{t-1}}{\nu_{t-1}-2} \right)$$

$$or\ z_t = \sqrt{\frac{\nu_{t-1}}{\nu_{t-1}-2}} \frac{w'_{t-1}r_t - w'_{t-1}\mu_{i,t}}{\sigma_{\rho,t}}$$

$$or\ z_t = \sqrt{\frac{\nu_{t-1}}{\nu_{t-1}-2}}\frac{\rho_{i,t}-\mu_{i,\rho t}}{\sigma_{i,\rho,t}}$$

The model is fully specified when the standardized returns, z_t have a joint cumulative distribution function F_t so that $c'z_t$ also has a distribution function similar to any type of fixed k-dimensional vector c. In this case, we assume that z_t follows the multivariate Normal distribution or the multivariate Student's t-distribution with ν degrees of freedom.

The cumulative distribution function is denoted as $F_\nu(z)$. Value-at-Risk of the portfolio, $\bar{\rho}_t$ is the solution to

$$F_\nu\left[\sqrt{\frac{\nu_{t-1}}{\nu_{t-1}-2}}(-\bar{\rho}_{i,t}-\mu_{i,\rho t})\sigma^{-1}_{i,\rho,t}\right] \leq \alpha$$

where $\bar{\rho}_{i,t}(w'_{t-1},\alpha) = max\,\{\rho_{i,t}\}$. As $F_\nu(.)$ is a continuous and monotonically non-decreasing function we have

$$\sqrt{\frac{\nu_{t-1}}{\nu_{t-1}-2}}(-\bar{\rho}_{i,t}(w_{t-1},\alpha)-w'_{t-1}\mu_{i,t})\sigma^{-1}_{i,\rho,t} = F^{-1}_\nu(\alpha) = -c_\alpha$$

$$\bar{\rho}_{i,t}(w_{t-1},\alpha) = c_\alpha\sigma_{i,\rho,t}\sqrt{\frac{\nu_{t-1}-2}{\nu_{t-1}}} - w'_{t-1}\mu_{i,t}$$

with c_α is the α percent critical value of the distribution of $z_{\rho t}$ conditional at Ω_{t-1}.

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