The design of aggregate gradings to minimum and controlled porosity - a study of packing characteristics and void characteristics in aggregates

by

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CONTENTS

SYNOPSIS

A study of the factors affecting the packing and porosity of particles has been made and a general theory for the combining of aggregates of varying shapes and sizes in order to achieve minimum porosity is proposed.

Arising out of this theory experiments have been carried out from the results of which graphs have been prepared to enable the determination of the optimum percentage of fine material for maximum density in two component systems. A method for extending the application of these results into the province of the design of multicomponent systems of both the continuous grading and intermittent grading types has been devised, and in the latter type the inherent gaps in the grading have been related to measured void characteristics. From another graph the value of porosity appropriate to any such mixture can be estimated.

The theory and relevant graphs are believed to be of general application in the fields of mixtures comprising aggregates of any shape or combination of shapes whether these be dry aggregates, wet aggregates, bitumen or tar coated aggregates or concrete aggregates, and to cover all possible environmental conditions of the particulate mass such as the applied compactive effort and boundary effects.

PART I. EXAMINATION OF FUNDAMENTAL CONCEPTS.

CHAPTER 1.

SCOPE OF THE PRESENT STUDY

This study of the various factors controlling the mass aggregate porosity of a bed of particles (i.e. excluding the internal porosity of the particles) has had as its main aim the development of an improved method for combining aggregates of mixed size and mixed shape so as to produce mixtures of minimum or controlled void content.

An initial assumption has been made in respect of this study that negligible particle deformation and particle crushing take place, i.e. that the particles remain substantially the same size and the same shape, during whatever treatment they receive, and that therefore no reduction in porosity is to be sought from either the flow of particles, or the fragmentation of particles, into adjacent voids. These two actions do, of course, frequently take place to a greater or less extent, but it has been decided here to deal only with cases in which such events are considered to be of minor importance.

From a theoretical and practical appraisal of the factors which affect the packing and porosity of a mass of particles, an uncomplicated experimental prodedure has been developed for testing aggregates in the laboratory, from the results of which may be determined the optimum percentage of fine material to obtain maximum density in a two component mix. It has been subsequently verified that this approach can be extended into the field of multi component mixes, enabling maximum density gradings of both the intermittent and continuous type to be derived. These gradings, in contrast to those of a more empirical nature such as those due to (i) Fuller and Thompson (1907) - the "Fuller" curve, (ii) the Asphalt Institute (1962) (iii) Talbot and Richart (1923), (iv) Andreasen and Andersen (1929) and (v) the Road Research Laboratory's Road Note 4 (1950), take into account any change in packing properties which may occur at any size level.

As will be shown in the following pages these packing properties are affected by changes in shape and surface texture of the particles, by surface static effects, by degree and type of compactive effort, by effect of the container wall (including its size and shape, relative to the size and shape of particles) and of any other boundary such as an open surface, reinforcement bars, etc., and by the lubricating or cohesive effects of any liquid film or coating. The complexity of the interactions of these factors, which may all be varied independently of each other, makes it practically impossible to derive a porosity value for any given case, by way of calculations based upon correction factors for the variables concerned. Accordingly, and although reference is made in Chapter 2 to corrections which have been attempted for the "wall effect" referred to above, it has been decided in this work to base all relationships, compositions, porosities, particle void characteristics, etc. upon experimentally determined conditions. Provided the initial laboratory (or field) tests on which the method depends are appropriately designed so as to provide a faithful imitation of the full scale case as regards aggregate (including coatings), boundary effects and compactive effort the method here proposed constitutes a general solution to the problem, taking all the above mentioned intrinsic and environmental factors into account.

It thus becomes possible to design minimum or controlled void content mixtures for concrete, for bituminous materials and for dry or water-bound stone construction work, for any given section and any given effort in accordance with scientific principles, limited only by the extent to which the method of laboratory compaction, the materials employed and the container reproduce the field conditions which it is desired to simulate.

No overall attempt has been made at this stage to evaluate the strength properties of the mixes designed according to the method, but a number of asphaltic mixes so designed have been compared for Stability by the Marshall Test, with mixes prepared according to current British Standard (1961) and American Asphalt Institute Specifications (1961) and have shown up favourably in this comparison (Appendix I).

CHAPTER 2.

FACTORS AFFECTING MASS AGGREGATE POROSITY - PREVIOUS & CURRENT RESEARCH

2.1 An outline discussion of the factors.

Previous research, which has principally been carried out during the past 40 years, has concerned several aspects of the influence on the porosity of a mass of particles,of the mode of their packing, their size-distribution, their shape and shape distribution. Other factors have been mentioned in the literature but all exert their influence ultimately via one of these four main factors.

A brief account of these factors is now given, to be followed by a more extensive treatment.

Mode of packing may vary between theoretical loosest and theoretical closest and between practical loosest and practical closest. The study of packing, through the years, has accordingly taken account not only of theory (Graton and Fraser, 1935) but of such practical considerations as the conditions of deposition (natural or artificial) (Kolbuszewski, 1948) and of any post depositional effects such as tamping, rodding, rolling and vibration (Shergold, 1953; Kolbuszewski and Alyanak, 1964). Further, several authors have directed their attention to the question of the influence on packing, of the size and shape of the container relative to the size and shape of the particles (Dunagan, 1940; Heywood, 1946; Verman and Banerjee, 1946; Brown and Hawksley 1945/6; Hughes, 1956; Hosking, 1961).

Size Distribution * (Grading). Since the earliest days in man's history it is apparent that he has applied the lesson of his observation that small particles can be used to fill the voids between large particles

* It will have been noted that size, that is to say, absolute size, has not been listed among the main factors influencing porosity, in the opening paragraph of this chapter. In fact it has been shown theoretically (Graton and Fraser, 1935) and confirmed experimentally (Westman and Hugil, 1930) that absolute size has no direct effect on porosity. It does however frequently have a considerable indirect effect by way of its effect on the mode of packing. This indirect effect will be considered later.

From this early realisation has developed a whole group of theories, some supported by practical experiment, on the subject of the proportioning of aggregates to achieve minimum porosity. This present work also devotes considerable attention to this subject and to the related but generally neglected one, of proportioning aggregates to achieve a controlled porosity, which is not necessarily a minimum. Further reference to this aspect of grading is deferred to Part II.

Particle Shape. The shape of particles varies from the completely spherical to an infinite range of non-spherical forms, differing in their degree of departure from the sphere in terms of elongation ratio, flatness ratio, sphericity, roundness, angularity and surface texture. Definitions of these shape parameters and descriptions of the means of measuring them are to be found in the literature (Wadell, 1933;Zingg, 1935; Krumbein,1941; Wright, 1955; Aschenbrenner, 1956; Lees, 1964 (a) & (c)). Summaries of this extensive literature exist (Lees, 1964 (b)) and no attempt will be made here to repeat the summary. The influence that particle shape has on porosity will however be one of the main subjects under discussion in the following pages, and reference will be made to another generally neglected parameter - namely shape-distribution.

The influence of shape on porosity is felt not only directly but also indirectly via its effect on mode of packing. That is to say that not only may there be a difference in the porosity of a high angularity aggregate at its closest packing compared with that of a low angularity aggregate at its closest packing, but that also the difference in shape may affect how close either of these aggregates approaches its closest packing under a given compactive effort.

It has been said above that the indirect effect of size on porosity may be considerable. It should be added here when considering shape, that while its effect may be large at sizes in the coarse sand range and upwards, the effect of shape of porosity becomes swamped by the size effect as size reduces.

The number and complexity of the permutations and combinations of the variables of packing, packing distribution, size distribution, shape and shape distribution are reflected in the volume of literature on the subject of the porosity of aggregates (see list of references, page 137).

It is this complexity which justifies the present study in spite of this already extensive literature and will undoubtedly lead to many future studies on the same subject.

In the further examination of the listed variables which follows, some of the more important results and conclusions which have emerged out of the earlier work and which are relevant to the present study will be examined and the interrelations between them considered.

2.2. Mode of Packing

(a) the theoretical and practical packing of single-size spheres.

The classical work of Graton and Fraser (1935) described systems of theoretical, i.e. geometrically perfect packings, of single size spherical particles. Six different "cases" of packing were figured and described (Fig.1) but these represent only four fundamentally different spatial arrangements, namely case 1) cubic packing, cases 2) and 4) orthorhombic packing, case 5) tetragonal-sphenoidal packing and cases 3) and 6) rhombohedral packing. Cases 2) and 4) differ only in orientation relative to a datum plane; likewise with cases 3) and 6).

In all cases it is fundamental that the attendant porosity is independent of the size of the grains.

Of the four regular packings listed above, the two which have provoked most interest have been the cubic, giving the maximum value of porosity, 47.64%, and the rhombohedral, giving the minimum porosity value of porosity, 25.95%.

Excluding, for the moment, the effect of the container wall, i.e. imagining an infinitely large container or a container so large relative to the size of the particles that its effect may be ignored, the chances of producing an approximation to either of these porosities in practice, even with perfect spheres, rest upon the degree and type of effort employed during deposition and/or subsequent compaction. For example if under type of effort is included the degree of care taken in packing the units, then careful hand-placing is able to produce both maximum and minimum porosity packings. If we exclude hand placing then other methods of placing affect the results as follows:-

The packing becomes closer, and hence porosities fall, as the velocity of deposition rises, as the intensity of deposition falls, as

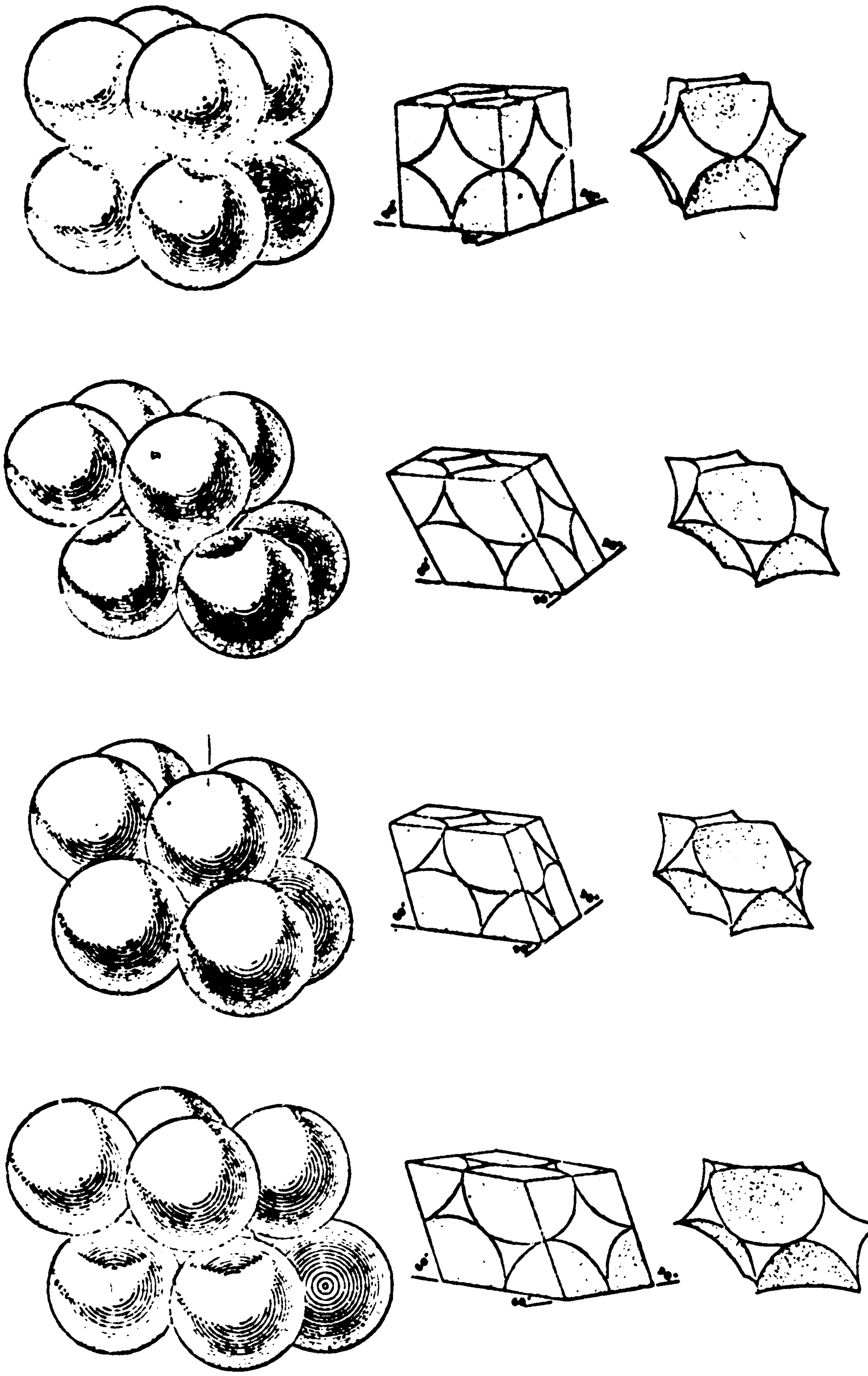


Fig. 1. SYSTEMATIC ARRANGEMENTS OF EQUAL SPHERES
(After Graton and Fraser 1935)

surface adhesive effects reduce and as the efficiency of any contemporary or post-depositional tamping, rolling or vibrational effort increases.

In this connection Kolbuszewski's observation (1948) that "porosity is a fundamentally valid means of describing a state of packing only in so far as it can be shown that any state of packing has a unique porosity" is a statement of fundamental importance, but whose full importance is only realised when considered together with his other observation that "it has been shown that it is possible to have different states of packing with the same porosity."

In dealing with examples of the effects mentioned above (namely velocity, intensity, adhesive effects and compaction), recourse will necessarily have to be made to experimental evidence obtained on particles which are not strictly single size and often deviate more or less from the spherical form. Nevertheless the conclusions drawn are believed to be the same as would have followed, had the original condition of uniform spherical grains been maintained. This assumption made, the discussion can at this stage be widened to include,

(b) the practical packing of 'near' single size grains of spherical and non-spherical form.

(i) Velocity and intensity. Kolbuszewski (1948) has dealt with these two depositional effects. He has shown, in addition to the reduction in porosity with rise in velocity and fall in intensity, that the effect of high velocity is only apparent at the lower orders of intensity, i.e. that at high intensity the extra energy possessed by the fast descending grain over that of the slow descending grain cannot benefit it (in the sense of encouraging it to find its most stable resting place) because of its crowded environment.

(ii) Surface adhesive effects contribute the major reason for the influence of grain size on porosity. The forces involved will include electrostatic and London / van der Waal's forces as well as the binding effect of adsorbed films and of low viscosity and high viscosity liquids. Under high pressure compactive efforts "solid bridging" or welding may occur (Orr 1966, p.403).

It can be readily shown theoretically that for identical packings, porosity is independent of size. However as grain size reduces and the ratio area/volume increases, surface adhesive effects increase, leading to important changes in packing because of the locking of the grains into metastable arched or domed structures. The smaller the size, the greater is this effect on packing and, other factors being constant, the higher the porosity.

A theoretical maximum porosity, for spheres in cubical packing, of 47.64% has been quoted above. For the reasons just given porosities well in excess of that amount can be achieved in practice even with spherical particles, if their size is small and especially if the intensity of deposition is high.

In an early statement of the view that porosity rises as size falls, Fraser (1935) quoted results obtained by Ellis and Lee (1919) on thirty-six samples of 'single-size' particles ranging from coarse sand to silt:-

Coarse sand	39 - 41%
Medium sand	41 - 48%
Fine sand	44 - 49%
Silt	50 - 54%

These results appear to show an increase in porosity with decreasing size, but no attempt at shape analyses was made and some effect due to variations in shape may also have contributed to the observed trend.

Terzaghi (1925) too, recorded porosities ranging from 50% for particles larger than 0.02 m.m. to about 95% for particles smaller than 0.002 m.m.

Heywood (1946) also emphasized the arching and cohesion effects in fine powders stating that "these effects usually more than nullify the effect of particle size variation so that fine powders may have very high porosities". Heywood's results are shown in Table 1, and refer

Percentage passing 200 mesh B.S.sieve	Porosity (%)		
	Loosely Poured	Vibrated	Compressed (10000 lb/sq.in)
100	83.0	68.8	29.6
93.7	71.0	47.6	23.6
84.5	68.7	48.8	22.8
78.7	66.7	45.9	22.8
76.6	63.8	43.6	21.8
62.5	58.8	39.2	21.5
57.4	56.6	35.4	20.3
46.5	51.7	33.1	20.2

TABLE 1.

to measured porosities of ground limestone powder.

Although there is again no certainty that shape was constant in the various sizes, or that the size ranges of the samples tested were constant, it is probable that adhesive effects were mainly responsible for the increase in porosity with decreasing size and that even vibration could not entirely remove its effect. The values of porosity in the final column are much closer to one another and it is at least possible that these differences are attributable to grading or shape differences as mentioned above. Certainly the effect of compression seems to have been to remove very largely the arching caused by surface bonding effects.

One of the most careful attempts to separate the effect of absolute size from the effects of size distribution and particle shape has been that due to Frederick (1962). Some of Frederick's results are repeated here as Figs. 2,3,4 and 5. Figure 2 shows an attempt made to quantify shape by use of the parameter "Roundness". Frederick recognised the inadequacy of using but a single parameter for shape, but none the less the figure shows a rising porosity (1) with decreasing roundness and, more relevant to this section, a rising porosity with decreasing mean diameter (volume). Figure 3 shows that the same increase in porosity with decreasing size is also observed with Ballotini (glass spheres) in which the size distribution is varied. Frederick used σ_g (the standard deviation of the distribution of mean volume diameters - Hatch and Choate (1929))as a measure of particle size distribution and Figure 3 shows further how porosity falls as σ_g increases (i.e. as range of size increases). This influence is only mentioned in passing in this Chapter. A full account of the influence of grading on porosity follows in Part II.

Frederick carried out a similar study with respect to the effect of absolute size on minimum porosity, but for sand samples the variation in minimum porosity with size was noticeably less than in the maximum porosity test, (Fig.4) and for ballotini was nil (Fig.5). Frederick considered a number of possible explanations, but perhaps the most likely, and one that he did not consider, is that arching is more likely, for a given size, if conditions of deposition are designed to be 'quiet', as in maximum porosity test, than if conditions are 'violent' as in a

(1) the porosity determined was the maximum porosity after the method of Kolbuszewski (1948).

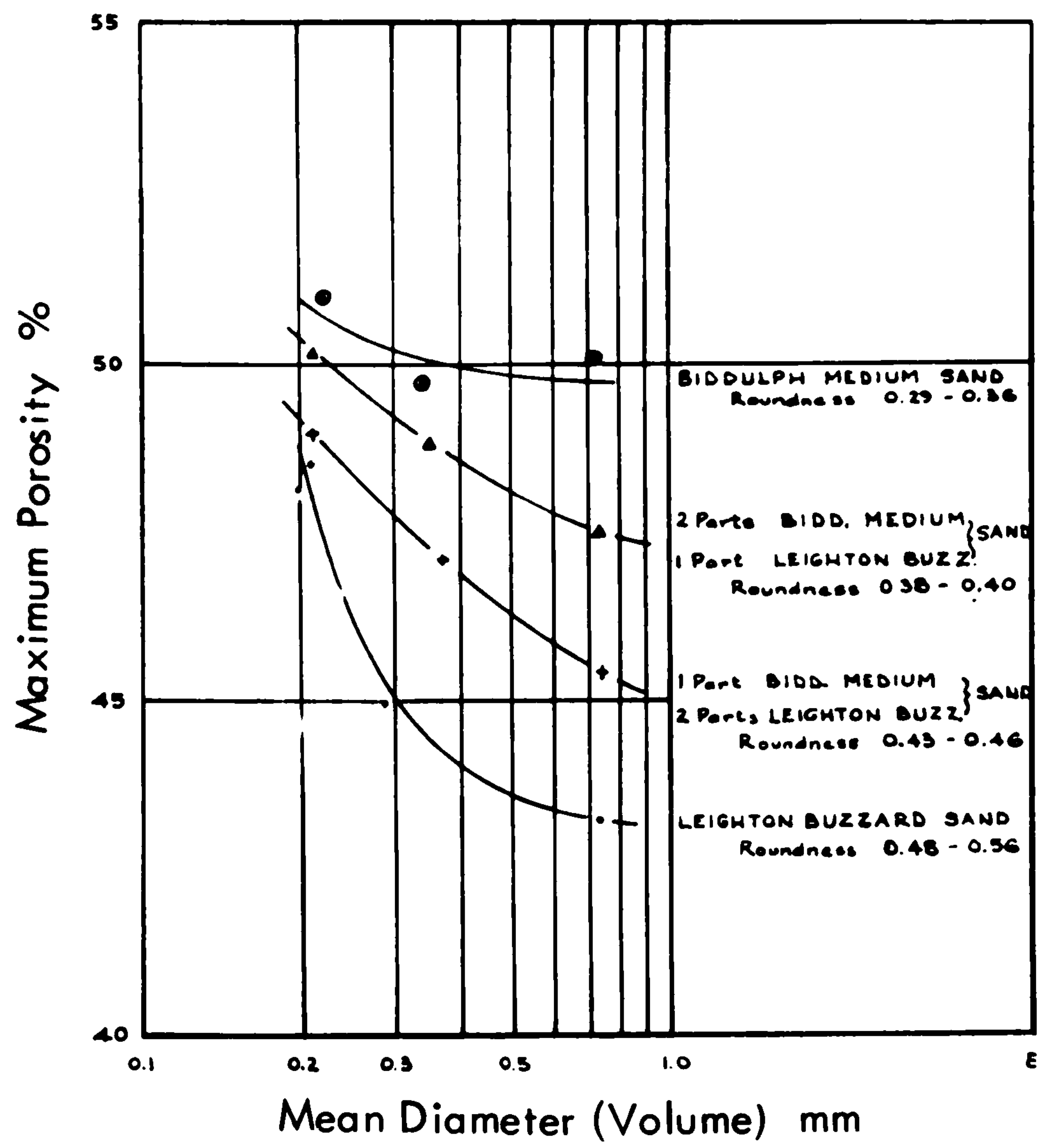


Fig. 2. INFLUENCE OF GRADING AND ABSOLUTE SIZE ON MAXIMUM POROSITY OF SAND. (after Frederick 1962)

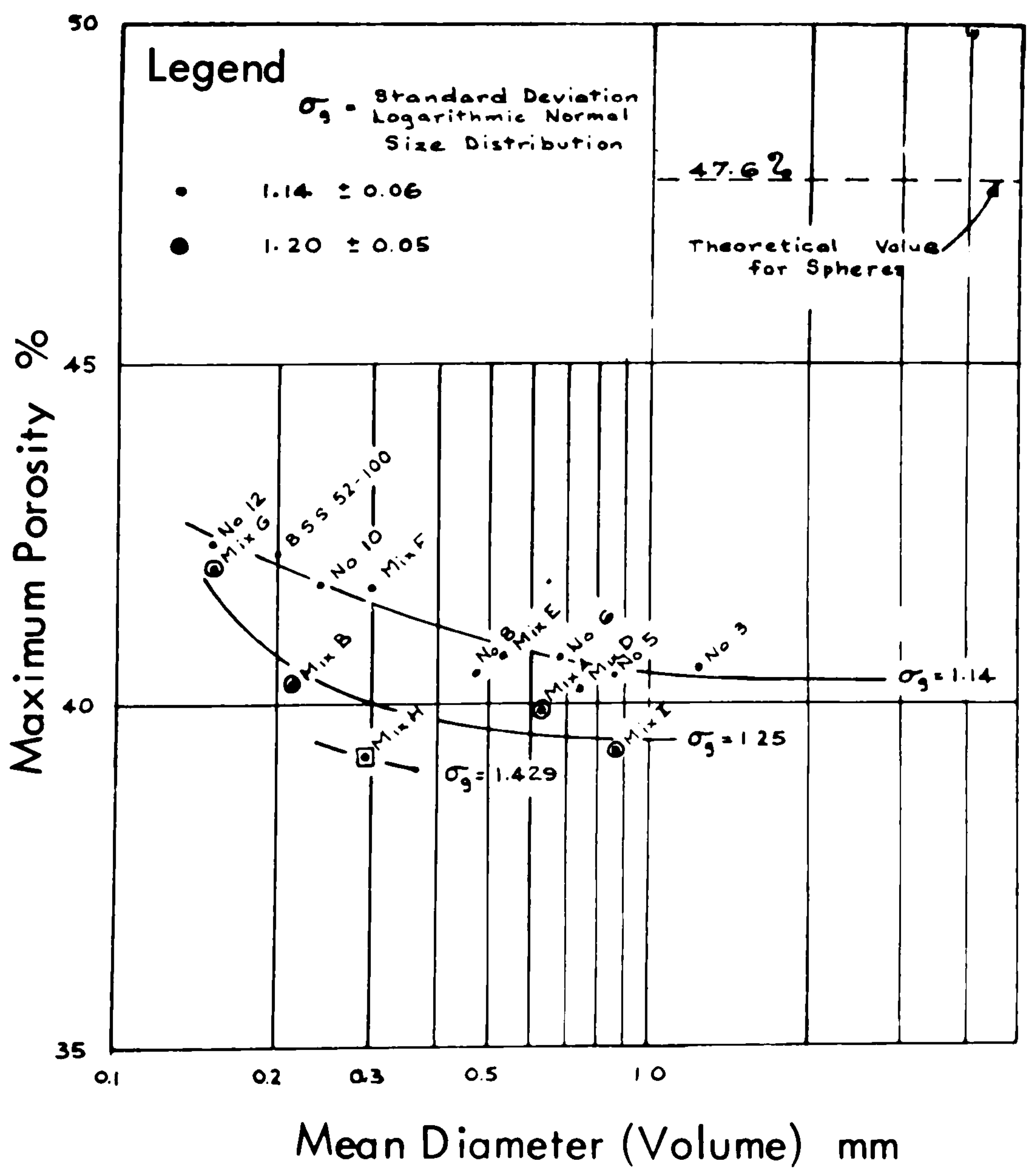


Fig. 3. INFLUENCE OF ROUNDNESS AND ABSOLUTE SIZE ON MAXIMUM POROSITY OF BALLOTINI. (after Frederick, 1962)

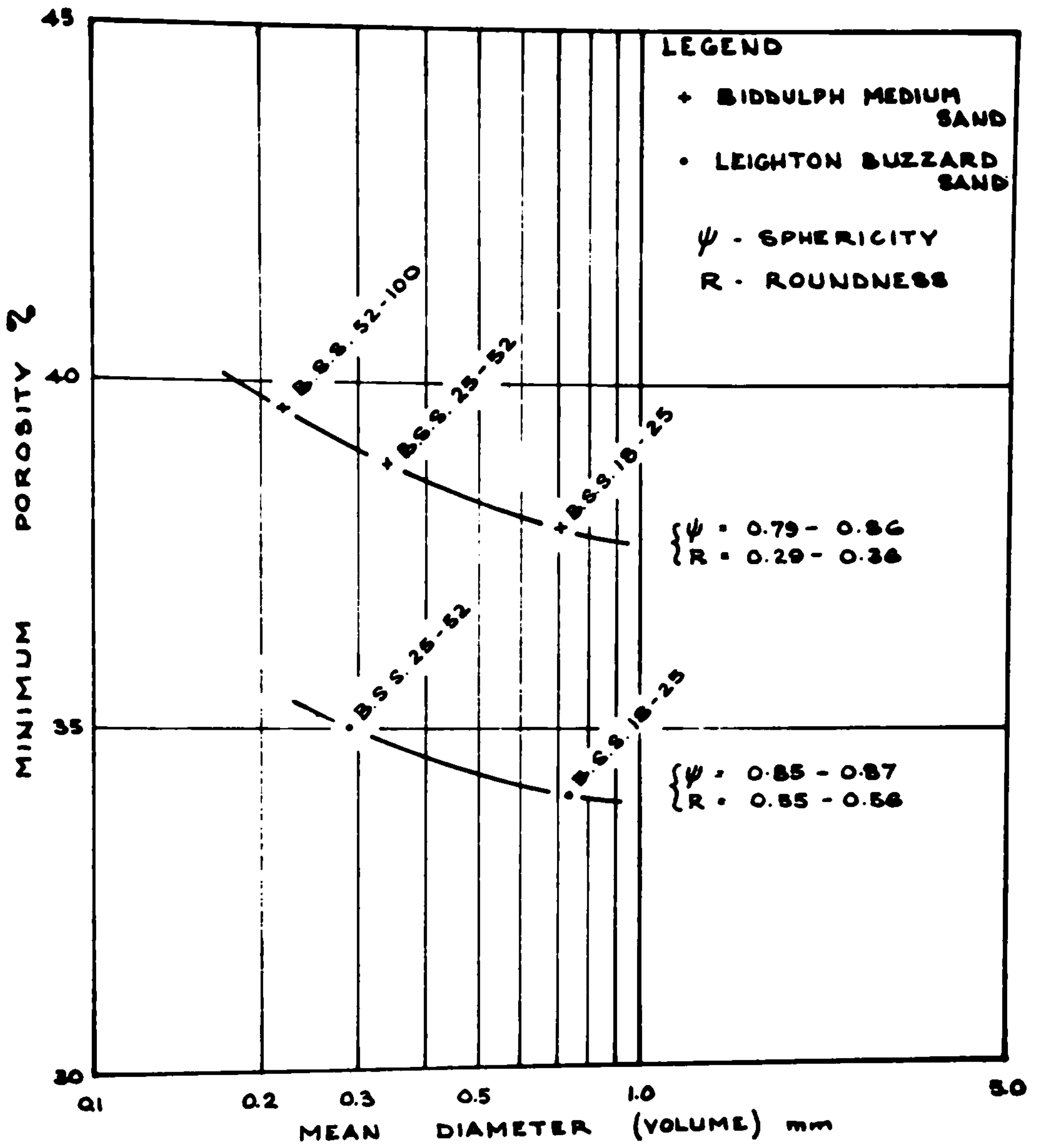


Fig. 4. INFLUENCE OF ROUNDNESS, SPHERICITY AND ABSOLUTE SIZE ON MINIMUM POROSITY OF SANDS. (after Frederick, 1962).

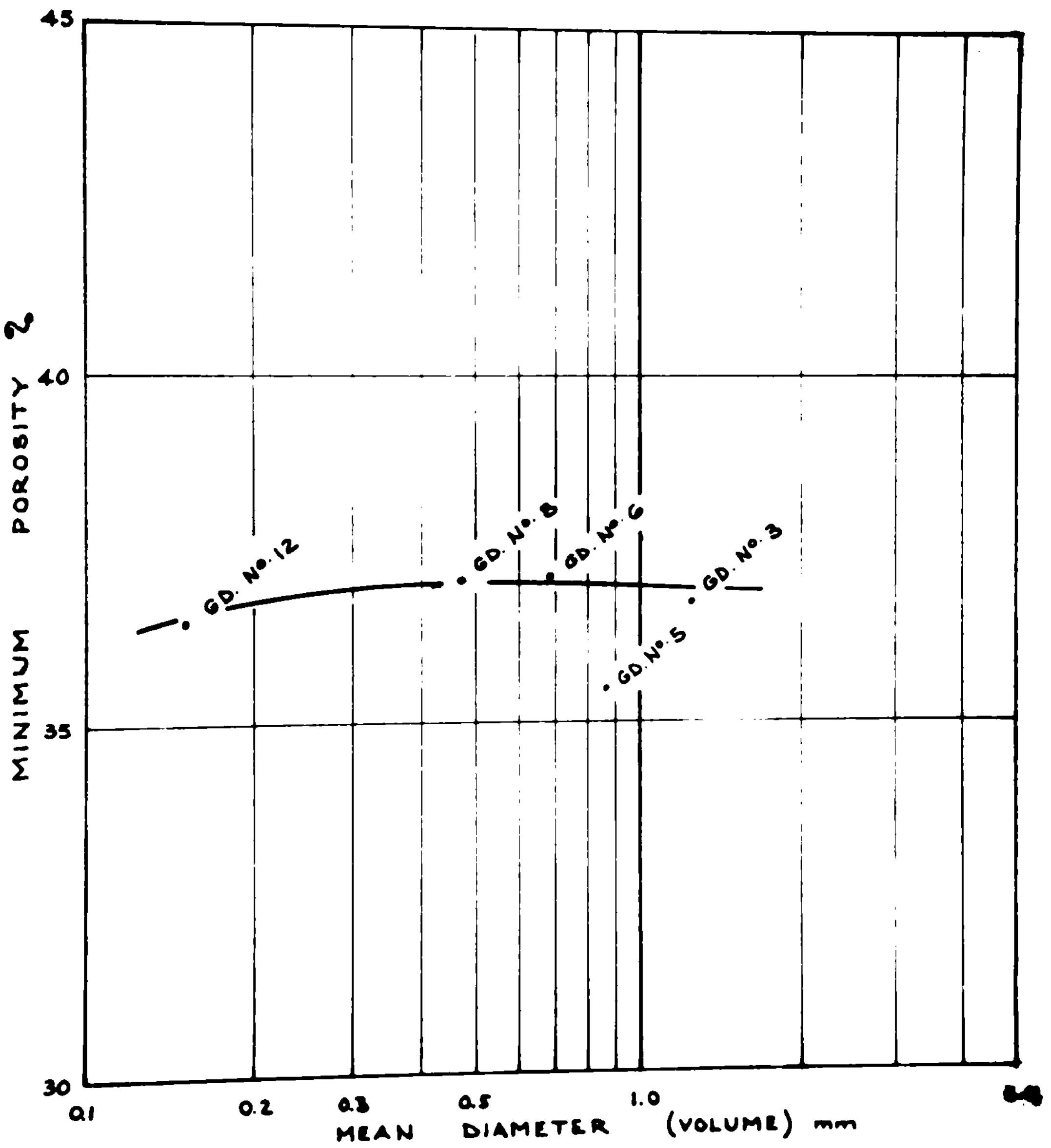


Fig. 5. INFLUENCE OF ABSOLUTE SIZE ON MINIMUM POROSITY OF BALLOTINI. (after Frederick 1962)

minimum porosity test. It would thus seem natural that the indirect effect of size on porosity would be much more clearly seen in the maximum porosity test than in the minimum porosity test. Indeed in the extreme case of the ballotini sample in which, apparently, the spherical shape and low friction value of the surface make it impossible for any significant number of metastable structures to persist in the presence of the higher compactive effort applied,the graph can be looked upon as further confirmation of the statement that grain size has no direct effect on porosity.

(iii) Efficiency of the compactive effort. Some examples of the effect of compaction on porosity have been given in the previous section where the porosities of ground limestone powder loosely poured, vibrated and under a compressive load were compared, and in the comparison of maximum and minimum porosities for sands and ballotini of various gradings and various shapes.

Amongst the extensive literature on this subject, which concerns variously, rolling, rodding, tamping, kneading and vibrated procedures, Fraser (1935) was among the first to show the effect of compaction upon porosity of single-size spherical particles in reporting the following results:-

	Porosity %	
	loose	compacted
Lead shot	40.06	37.18
	41.72 (ave.)	37.26 (ave.)
Sulphur shot	43.38	37.35

(Assuming that Fraser was attempting in his experiment to use methods of deposition and compaction which would give him values as near as possible to the practical maximum and minimum porosity, it appears from a comparison of his results with the theoretical values of 47.64 and 25.95, that although he could not reach either extreme, it was possible to come considerably closer to the theoretical maximum than to the theoretical minimum). The influence of the container wall was a real but unmeasured quantity in these results. Further mention of this influence will be made later.

Both magnitude (intensity) and duration are concerned when considering the efficiency of the compactive effort. The type of effort applied also affects to an important degree, details of the packing structure such as degree of preferred orientation of particles and the degree of particle breakdown. Studies of these effects have been carried out and Table 2 (Salehi, 1966) illustrates the variation in one of these parameters

TABLE 2. Average Degradation Factor. (% increase in the surface area of the coarse aggregate).

Method of Compaction					
Static		Impact		Field Roller	
Flaky	Non Flaky	Flaky	Non Flaky	Flaky	Non Flaky
Average Degradation Factor:-					
87.2	63.1	54.1	23.6	23.4	15.5

for 3 contrasting types of compactive effort.

Greater magnitude of effort of a given type may hence reduce porosity not only by the breaking down of arches and other metastable structures, but also by the fragmentation or deformation of particles. This may lead to a continuous fall in porosity with magnitude. Although it is never possible completely to ignore fragmentation and deformation the present discussion of previous work and the experimental work described hereafter will concern only those compactive efforts and those aggregate types in which these effects are small and can be neglected.

As regards duration of application of effort at constant magnitude, there is generally an early rapid fall in porosity, but the tendency is generally for the porosity to reach or approach some constant value with time (Kolbuszewski and Alyanak, 1964).

With regard to the intensity of effort in vibration, these authors have also shown that it does not follow that the density of sand is invariably increased by increasing the energy of compaction. Their results, are illustrated in Fig. 6 of which they say "the porosity of a sand decreases with increasing intensity of vibration down to a minimum . Further increase in the intensity causes the sand to shear at such a rate that the grains do not get a chance to occupy favourable places for close packing."

Clearly then with vibration the greatest densification is achieved at an optimum acceleration.

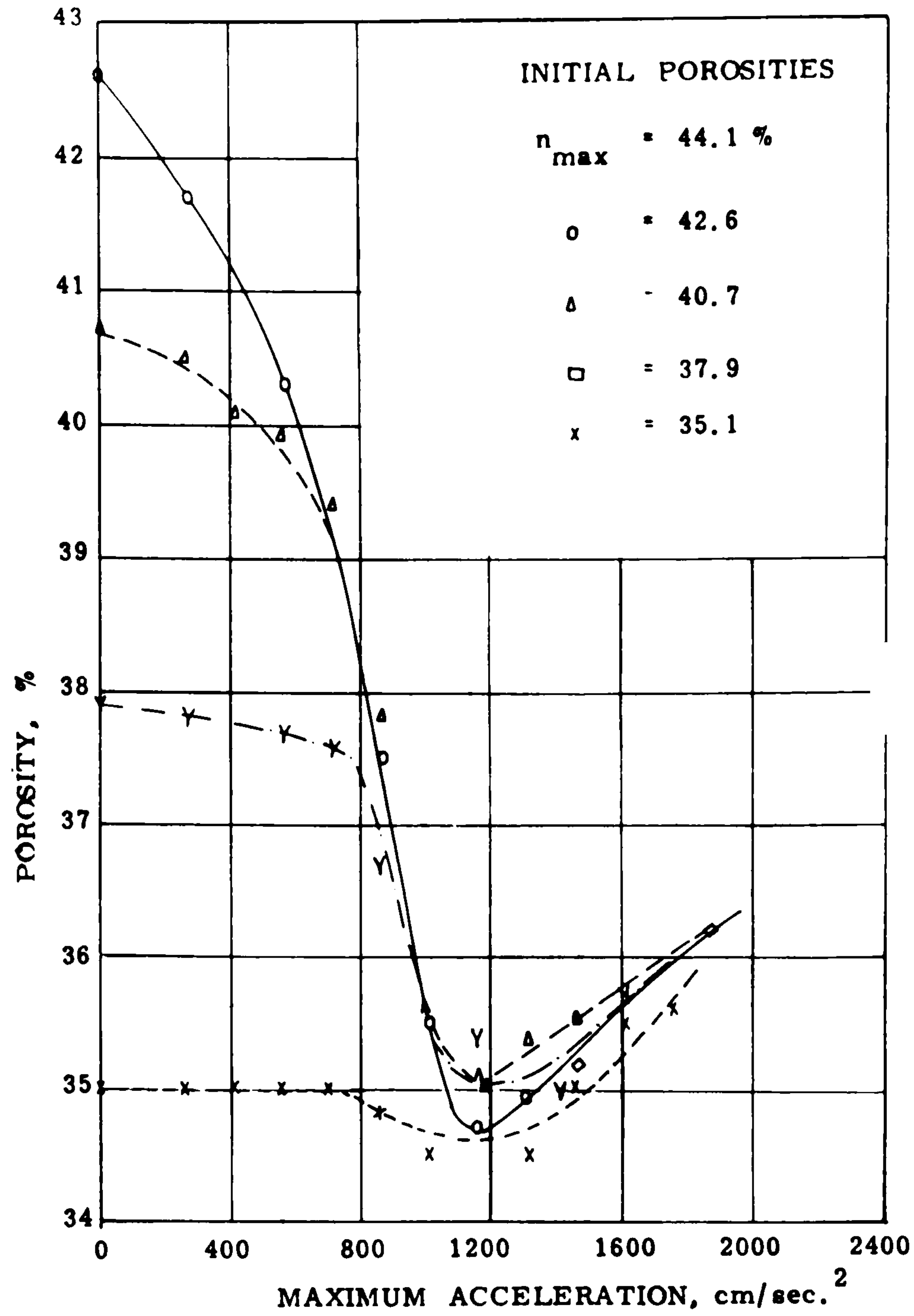


Fig. 6. INFLUENCE OF MAXIMUM ACCELERATION ON MINIMUM POROSITY (after Kolbuszewski and Alyanak 1964).

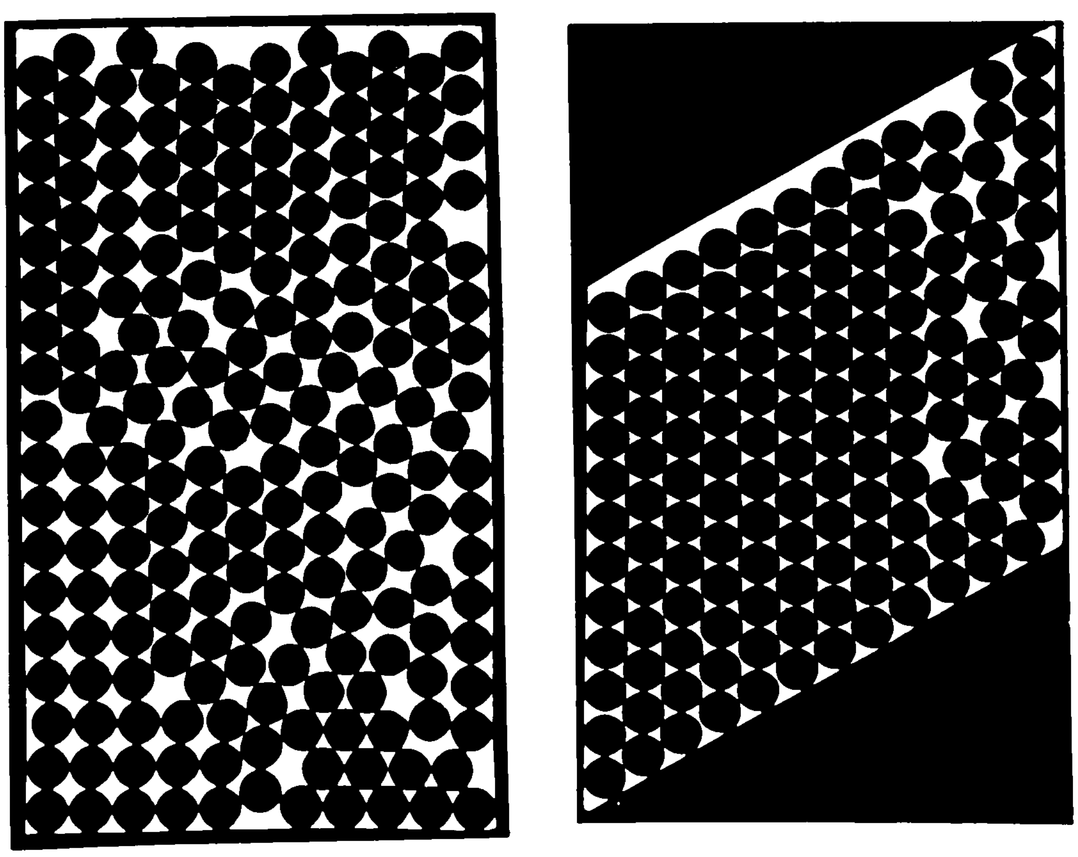


Fig. 7(a) & (b). INFLUENCE OF THE CONTAINER WALL ON PACKING AND POROSITY OF SINGLE SIZE SPHERES. (after Fraser 1935)

(iv) Influence of the container wall on the packing and porosity of single size spheres and particles of other shapes. The discussion of the last few pages has concerned some of the factors affecting packing and porosity but making an assumption that the effect of the container wall could be ignored.

Farouki and Winterkorn (1964) referred to this effect in the following passage. "The cubical packing has the greatest potential energy, whereas the rhombohedral has the least. Because bodies try to attain the position of least potential energy, these systems tend to form the rhombohedral state, especially if a mechanical disturbance is applied to the system. However the side walls of the container act against this tendency by preventing any lateral spreading of the system" and, a few lines later in remarking that:- "The angle which the side wall makes with the bottom influences the packing formation. A 90° angle will favour Cases 1,2,4 and 5; a 60° or 120° angle favours Cases 2 and 3, whereas a 70° 32' or 109° 28' angle favours Case 6. The packing is also influenced by the angles which the side walls make with each other. A 90° angle favours formation of the square pattern and hence Cases 1,2 and 3. Intersection of the side walls at 60° with themselves and at 90° with the bottom favours Cases 4 and 6. Intersection of the side walls at 60° with themselves and at 70° 32' with the bottom favours Case 6 packing".

Fraser (1935) had produced photographs of several two-dimensional models, two of which are reproduced here as Figs. 7 (a) and (b). These illustrate the broad principles of Farouki and Winterkorn's case and show clearly the influence of the inter wall angle on mode of packing. They also demonstrate that local increase in porosity at boundary walls may occur due to two causes, namely 1) the presence of incomplete cells of a given pattern at the boundary, without any change occurring in the nature of the packing, (top of Fig. 7 (b)), and 2) the modification of packing structure adjacent to and due to the presence of the boundary, such that the pattern of structure in the interior of the mass differs from that at the exterior (right of Fig. 7(b)), (right of Fig. 7(a)).

Failure to appreciate the importance of the container wall effect has led to some confusion regarding the possibility of certain types of packing occurring in the mechanical placing of spheres. McGeary (1961) for example, viewing his packing structures through transparent containers with the aid of a low power stereoscopic microscope reached the conclusion that "All single size packings were found to consist almost entirely of the orthorhombic arrangement" (Fig. 8(a)), the remainder being of the tetragonal-sphenoidal pattern. He continues, "close packed arrangements" (i.e. rhombohedral) "cannot occur in mechanical packings because the vertical force would cause the two elevated spheres" (text Fig. 8(c)) "to move downward to a layer position identical to the double nested" (i.e. tetragonal sphenoidal) "case" (Fig. 8(b)).

This statement is incorrect. As drawn by McGeary it seems quite logical to say that the two elevated spheres would move downward, but even so there is no reason why a rhombohedral packing should not build upwards from this base since there is no reason why the upper of the two spheres drawn by him adjacent to the container wall should not also move inward away from the wall and slip into the hopper formed by the spheres of the lower layer. Depending on the configuration of this lower layer either Case 3 or Case 6 packings (both rhombohedral) could be built up provided there was no interference from the container walls. McGeary neglects that the structures seen by him through the tube wall are undoubtedly modified by the presence of that wall and not likely to be representative of the structures existing in the interior of the mass.

Chief interest in the container wall indeed arises out of this local effect upon packing and porosity.

Many experimental workers (e.g. Hosking 1961 (Fig.9) and Hughes (1962)) had observed that the measured porosity of a given sand or gravel increased as the size of the container in which the determination was made was reduced. In addition it was clear that a similar effect was produced by increasing the size of the aggregate in a given container.

It was clear from studies such as these that the presence of the wall had a disturbing i.e. loosening effect on the packing of the particles and that the total (or average) porosity was determined by the relative magnitude of the outer loosely packed zone and the internal more densely packed zone.

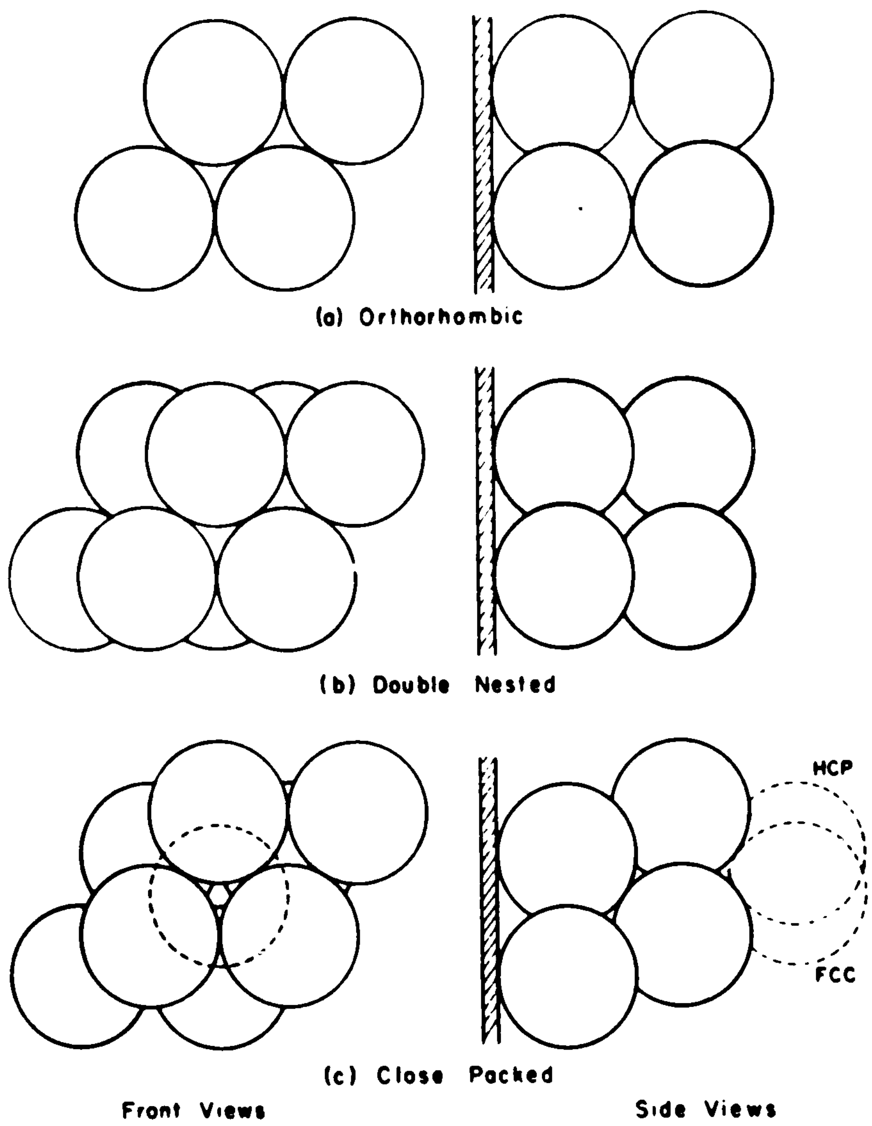


Fig. 8. PACKINGS OF SINGLE SIZE SPHERES AS SEEN THROUGH TRANSPARENT WALL OF CYLINDRICAL CONTAINER. (after McGeary 1961)

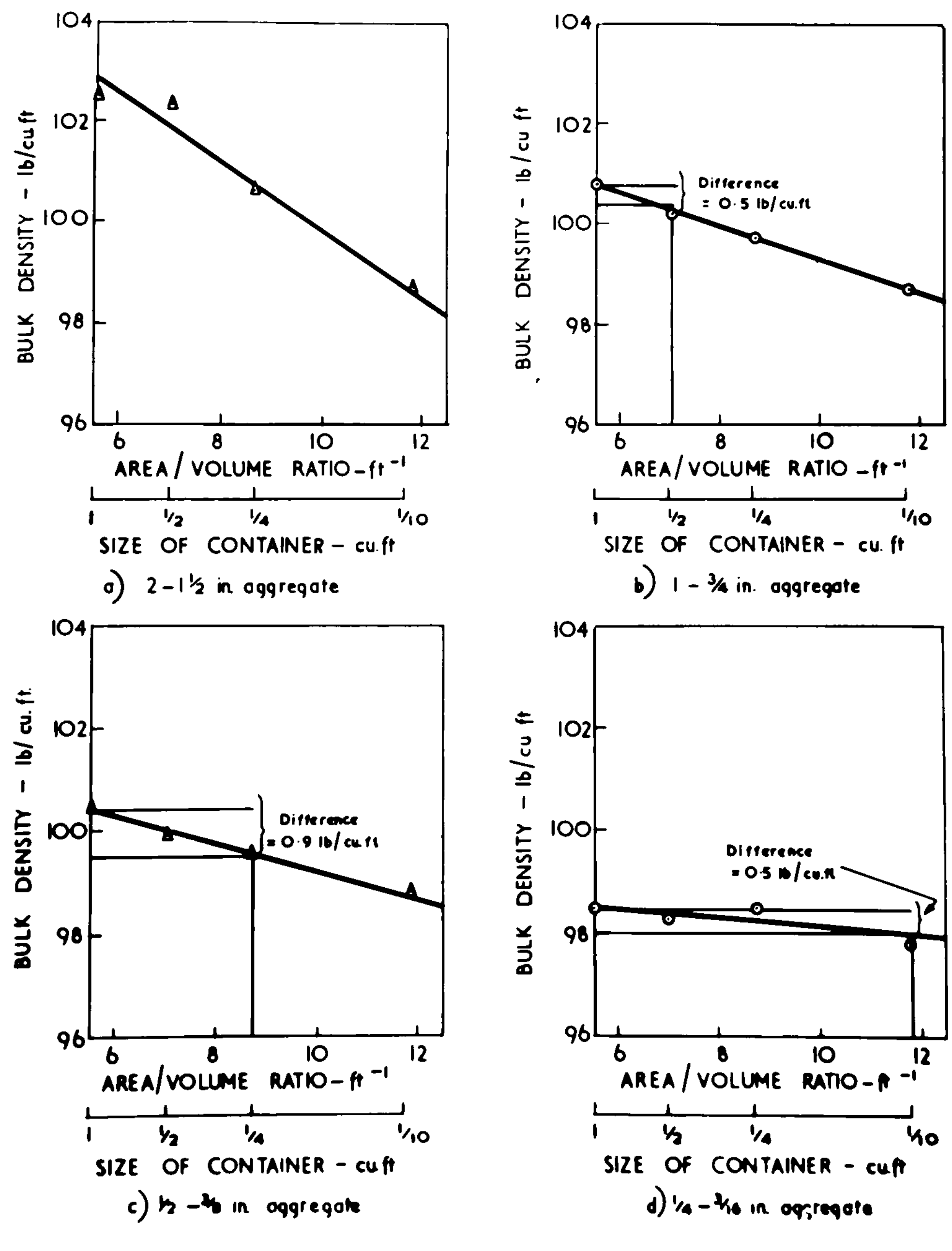


Fig. 9. INFLUENCE OF RELATIVE SIZE OF AGGREGATE AND CONTAINER ON 'RODDED' POROSITY. (after Hosking 1961)

Attempts to derive a correction for container wall effect have followed the two lines indicated earlier, namely by making allowance for

1) Increase in boundary porosity without alteration of packing geometry
2) Increase in boundary porosity due to local modification of packing adjacent to that boundary.

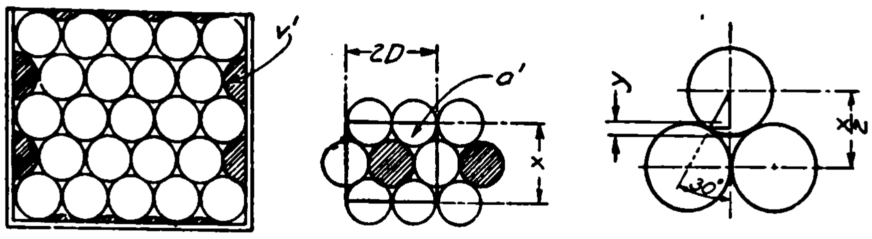
With regard to the first of these, it has already been seen in Fraser's figures (e.g. top, base and left side of Fig.7(b)) that even where the container wall does not modify the character of the packing, its presence as a boundary may still affect the porosity in those cases where the boundaries of the unit cell of packing structure do not conform geometrically with the wall boundaries. It was this aspect of the wall effect which was first to be studied.

It will be apparent for example that in the case of a cubical packing of spheres in a cubical container OF SIDE LENGTH EQUAL TO A SIMPLE MULTIPLE OF THE SPHERE DIAMETER , the porosities of those unit cells adjacent to the boundary are no different from those of interior cells. Once this condition of conformity of shape and size of the container with form of packing and size of the spheres is departed from, the presence of the boundary wall necessarily leads to the formation of incomplete cells, even if the form of packing is unchanged, and hence to local zones of higher porosity.

A special form of this condition of non-conformity was described by Dunagan (1940). The container he considered was cubical and the mode of packing of the spherical particles contained therein, rhombohedral. With these conditions there was an obvious misfit between unit cell boundaries and container wall.

Dunagan employed an entirely mathematical method to derive a correction factor for the container wall effect. This correction factor K, calculated as indicated in Fig.10, was to be added to the experimental value d_o (the volumetric solid content).

Both size of container and size of particle are considered in the correction factor since this will rise as either D (diameter of particle) or $\frac{A}{V}$ (area/volume ratio of the container) rises. Area/volume ratio of the container rises of course as the container gets smaller. It may be asked why diameter is used to express the size of the particle and



SECTION — SIDE VIEW

Ideal arrangement of particles considered as spheres

DERIVATION OF VALUE FOR x

$$\frac{x}{2} = D - y, \quad x = 2D - 2y$$

$$\frac{y}{2} = \frac{D}{2} - 0.433D, \quad y = 0.134D$$

$$x = 2D - 0.268D$$
$$x = 1.732D$$

Shaded portions are unfilled void spaces at container boundaries.

Each area that includes one unfilled void space equals $(2D)(x) \quad a' = (2D)(1.732D) = 3.462D^2$.

The volume of each particle v' per area a', equals $v' = \frac{\pi D^3}{12}$

For the entire container the correction will be $K = \left(\frac{A}{3.464D^2}\right)\left(\frac{\pi D^3}{12}\right) / V = \frac{AD}{13.231V}$

where A = the inside area of the container, including top.
D = the average diameter of the size group tested.
V = the volume of the container.

Fig. 10. SUGGESTED METHOD FOR CALCULATION OF A CORRECTION FACTOR FOR BOUNDARY VOIDS. (after Dunagan 1940)

$\frac{A}{V}$ ratio to express the size of the container. The use of $\frac{A}{V}$ ratio for the container allows one to make due allowance for the variations in shape of the container which will produce varying degrees of boundary effect. Although one could also use $\frac{A}{V}$ ratio to describe the particle, measurements of surface area are much more complicated than with the containers. Hence the simpler measurement of D (or, if not spherical, an Equivalent Spherical Diameter) is made and is probably sufficient for most purposes.

However the fallacy underlying Dunagan's study lies in the unjustified assumptions that are made that

(a) the packing would necessarily be of the form described

and (b) the character of the packing would be unmodified at the container wall.

A more general case was studied by Verman and Bannerjee (1946). In their first analysis and making the same assumption (b) as Dunagan, they showed that any system of packed spheres (or other particles) may be sectioned by a series of imaginary planes, representing the walls of a container, which will cut through a certain number of particles at each surface. Treating these in the same way as Dunagan had done, particles which had been so cut were considered as removed from the container. If there are n particles in a row across a cubical container then on the assumption that an average of one particle per row will be removed because of intersection with the wall, the average density will be lower than the density in the interior portion of the system in the ratio $\left|\frac{n-1}{n}\right|^3$. That is, if the mean density is d_n and density in the undisturbed interior packing is d_o

then
$$d_n = \left|\frac{n-1}{n}\right|^3 d_o \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Verman and Bannerjee stated that equation 1 would apply equally well to a cylindrical container, the diameter and height of which were equal to 2 nr and that the equation could be further generalised to apply to right cylinders of unequal dimensions and to rectangular containers as follows:-

$$d_n = \left|\frac{n-1}{n_1}\right|\left|\frac{n_2-1}{n_2}\right|\left|\frac{n_3-1}{n_3}\right| d_o \ldots\ldots\ldots (2)$$

where n_1, n_2 and n_3 represent the dimensions of the container, in 3 orthogonal directions, in terms of particle diameter.

In all cases the assumption was retained that there was no change in packing geometry at the boundary.

It was suggested above however, that another, possibly sometimes even more important, effect on porosity than the 'incomplete cells' just described, arises out of the restriction on freedom of particle movements imposed by the physical presence of the boundary. This restriction, by encouraging the formation of arches in the grain structure adjacent to the boundary, leads to entirely different forms of packing from those which may be predominant in the interior of the mass.

Verman and Banerjee in their second analysis, recognised that a disturbing effect of the container wall would probably exist in addition to the 'particle removal' effect allowed for in equations (1) and (2).

In the absence of experimental evidence they proposed an amendment to their initial assumption, namely that the influence of the walls on the spatial distribution of the particles be considered to be confined to the first layer of particles immediately adjacent to the boundary.

Equation (1) thus becomes modified as follows:-

$$d_n = \left|\frac{n-1}{n}\right|^3 d_o + \left[1 - \left|\frac{n-2}{n}\right|^3\right] \Delta d \quad \ldots\ldots (3)$$

where Δ d is the change in density due to the disturbing effect only i.e. not including the missing particle or cutting effect.

Brown and Hawksley (1946) in the discussion on Verman and Banerjee's paper, produced experimental evidence to suggest that for regular particles (regular discs and spheres) the influence extended further inwards than the first layer of particles mentioned by the authors and showed that for these particles the equation,

$$d_n = \left|\frac{n-1}{n}\right|^3 d_o + \left[1 - \left|\frac{n-3}{n}\right|^3\right] \Delta d \quad \ldots\ldots\ldots (4)$$

would hold, implying a disturbing effect to a depth of 3 particles.

However, these workers produced additional evidence to show that for irregular particles (crushed coal) the container wall influence extended inwards less deeply than suggested by equation (4) but proposed no new equation to cover this, or other, cases.

It appears that none of these formulae can claim to have solved entirely the problem of correction for boundary effect, since it seems that different constants are required in the formulae for aggregates of different shape (and probably for different size distributions, different compactive efforts, and different container shapes also).

Hughes (1962) carried out a thorough investigation in which aggregates varying from crushed igneous rock to rounded river gravels were used, in containers of varied shape and with contrasting methods of deposition/compaction. From his graph (Fig.11), the varying slopes confirm the truth of the statement made above that different correction factors would be needed for aggregates of different shape and size distribution, and containers of different shape.

The majority of Hughes' experiments were carried out in plane sided containers of square cross section. For such containers Hughes considered that a linear relationship could be established between specific bulk volume (and hence porosity) and a factor which he called the "Packing Number". This is in effect a number derived by multiplying the area volume ratio of the container by the diameter of the particles (since the particles were not single size an effective mean diameter was used in the calculation). This has the effect of removing the effects of absolute size of aggregate and container on porosity values. This is a useful device, in considering the influence of the container wall since it recognises that what really matters is the size of the container relative to the size of the aggregate, i.e. the container wall effect is nil if either the container is infinitely large (when $\frac{A}{V} = 0$), or the particles infinitely small (& d = 0) but increases as either $\frac{A}{V}$ rises or as d rises.

The effect of this can be seen by comparison of Fig.11 with Fig.12. In Fig.12 Bulk Density (an opposite expression of porosity, but affected by differences in Specific Gravity between aggregates) is plotted against the measure of container size i.e. $\frac{A}{V}$ ratio - called here r). It is seen that this results in different slopes for aggregates of similar shape but different size (cf. slopes 1 and 2, where the coarser the aggregate the steeper the slope). When the same results are plotted as Specific Bulk Volume $(1 - \frac{\text{Porosity}}{100})$ against $\frac{A}{V} \times d$ (i.e. "Packing Number") as in Fig.11, these results now plot at the same slope. In this figure it follows that any differences in Specific Bulk Volume, when $\frac{A}{V} \times d = 0$ (and container wall effects are eliminated),

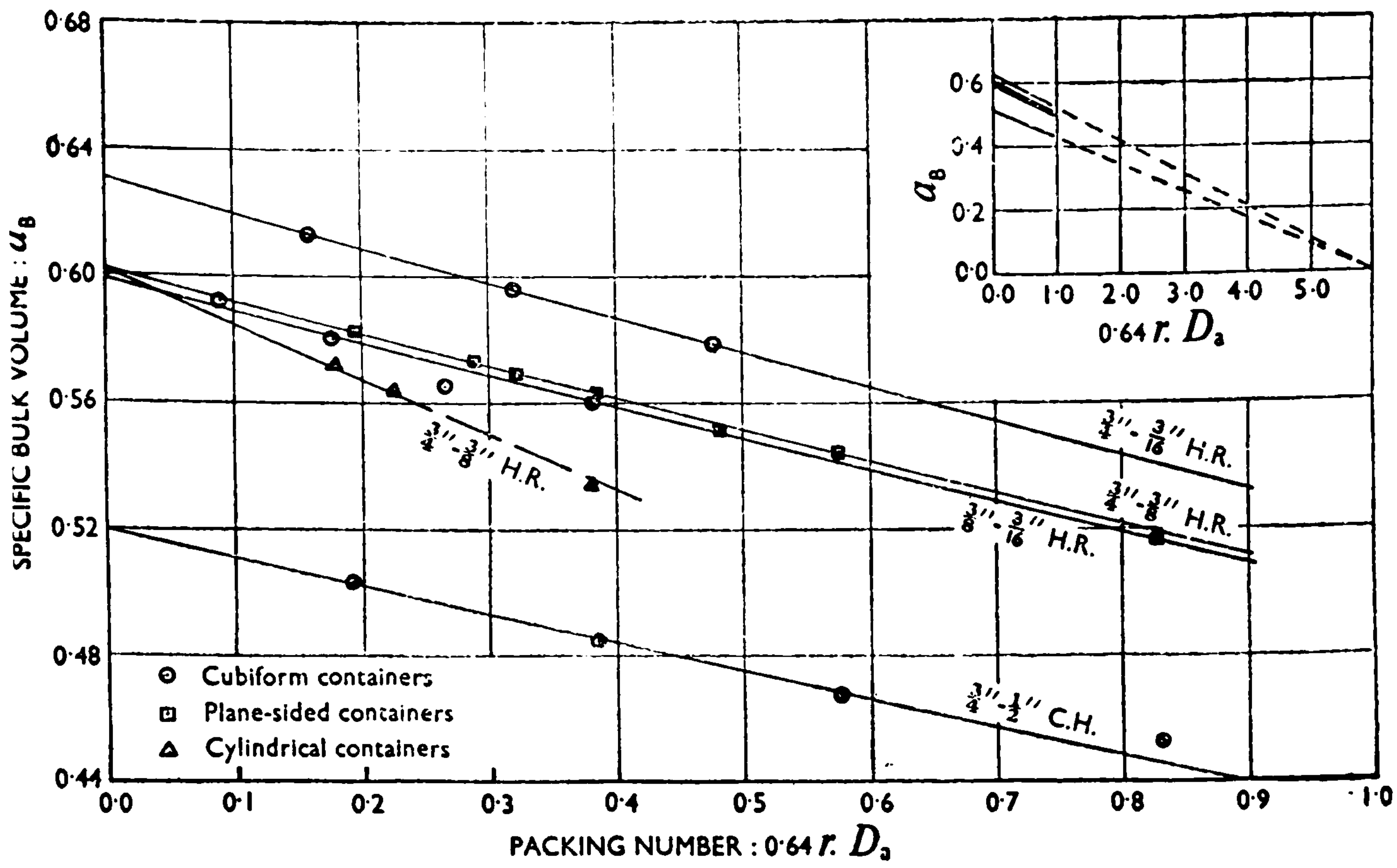


Fig. 11. BOUNDARY EFFECTS ON SPECIFIC BULK VOLUME FOR A VARIETY OF AGGREGATES AND CONTAINERS. (after Hughes 1962)

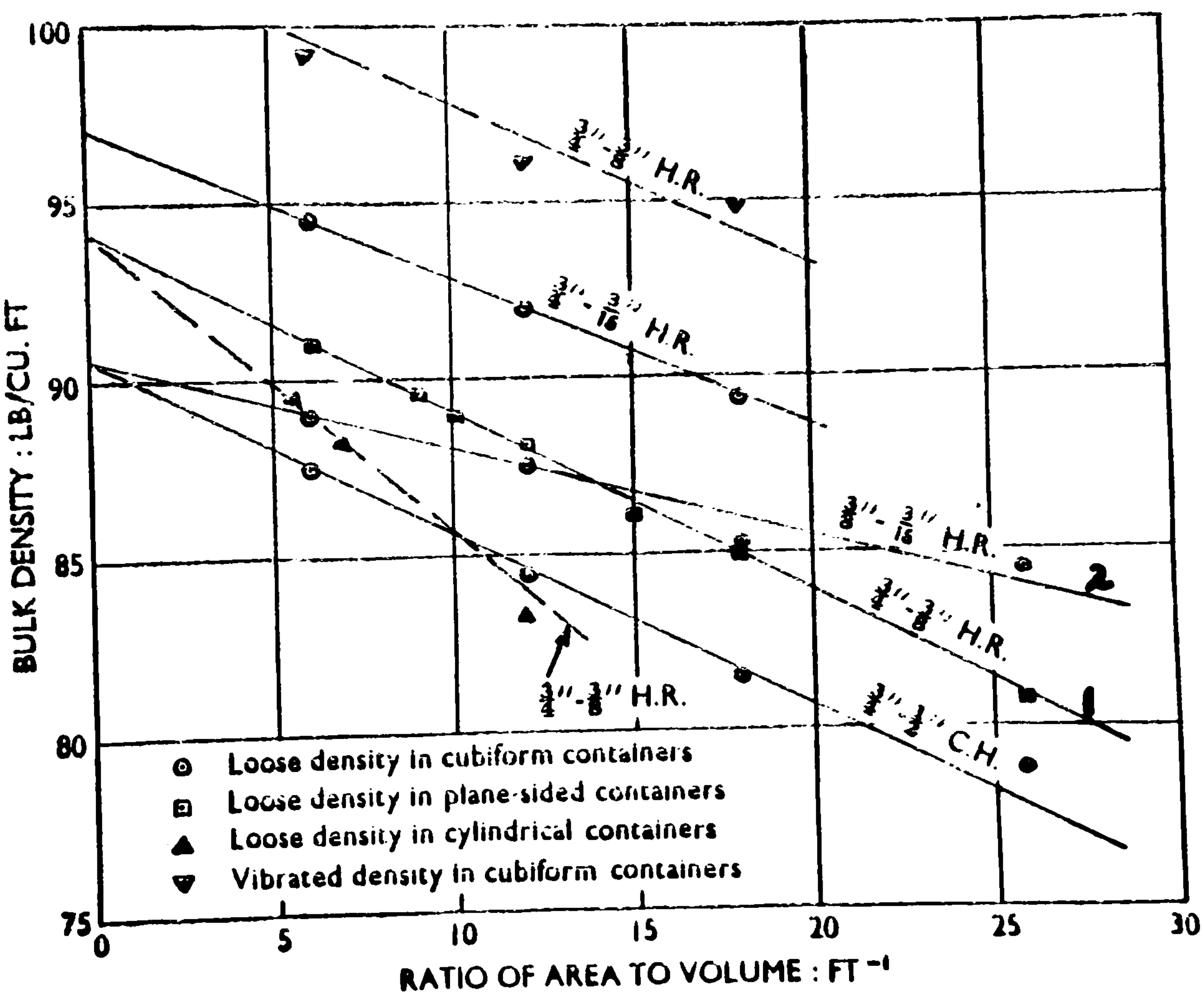


Fig. 12. BOUNDARY EFFECTS ON BULK DENSITY FOR AGGREGATES AND CONTAINERS AS IN Fig. 11. (after Hughes 1962)

are due to differences in one of the factors which influence the fundamental packing of the aggregate, namely compactive effort, particle shape, surface adhesion (influenced in its turn by absolute size and presence or absence of any lubricant or adhesive coatings, e.g. water, clay, bitumen, tar, etc.), and particle size distribution.

In the small insert graph in Fig.11 Hughes showed that he considered that the lines established for plane sided containers in the range of Packing Number values from 0 to 1, could be extrapolated to meet at a common point for zero Specific Bulk Volume (i.e. 100% porosity) at Packing Number equal to 6. This assumption arises out of the statement made by Hughes "The bulk density of an aggregate of minimum particle size d_o in a cubiform container of side $d_o - \delta d_o$ was undoubtedly zero. Alternatively the bulk density was zero for $r = \frac{6}{d_o}$ ", whence Hughes derives the conclusion that bulk density is zero when $r \times d_o$ (his Packing Number) = 6. It will be shown later that this assumption is probably not justified. However in the "practical range of values" the figure offered by Hughes as "Correction to Specific Bulk Volume" (Fig.13) based upon Fig.11, would change little or not at all by the amendment to the assumption which will be proposed. The difficulty presented by containers with curved sides (noted in Fig.11) remains however. Hughes recognised this problem in stating "Although the actual value of r for the compacting factor cylinder was $\frac{5}{6}$ in $^{-1}$, it was convenient to relate it to a cubiform container giving the same bulk density. This "effective" area/volume ratio, r_e , was found to be $1\frac{1}{4}$ in. $^{-1}$. A single bulk density determination was now sufficient for the bulk density of that aggregate in any container of known r_e value to be deduced".

This statement is in fact of no aid in the problem since the requirement that the container should be "of known r_e value" necessitates a preliminary bulk density determination in that container. The value being sought has thus had to be found for the determination of r_e . It might be argued that the r_e value sould be a constant unaffected by changes in aggregate grading and shape or in compactive effort but neither Hughes nor any other worker has confirmed this.

Notwithstanding these comments the corrections shown in Fig.13 may be relied upon provided they are (a) used only for plane sided sections and without such complications as reinforcement and (b) not extrapolated beyond the experimental limit of Packing Number = 1.0.

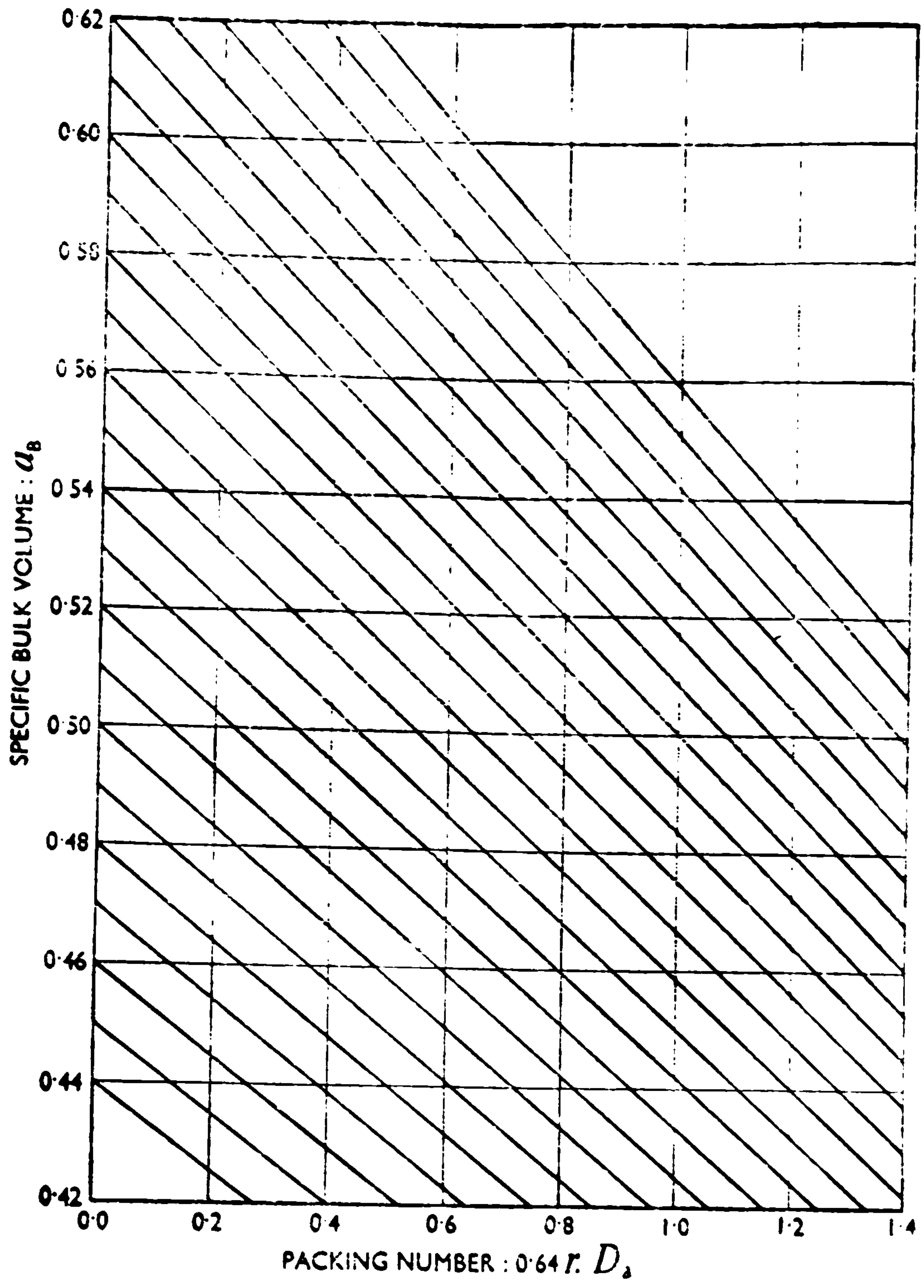


Fig. 13. CORRECTION TO SPECIFIC BULK VOLUME DUE TO BOUNDARY EFFECTS. (after Hughes 1962).

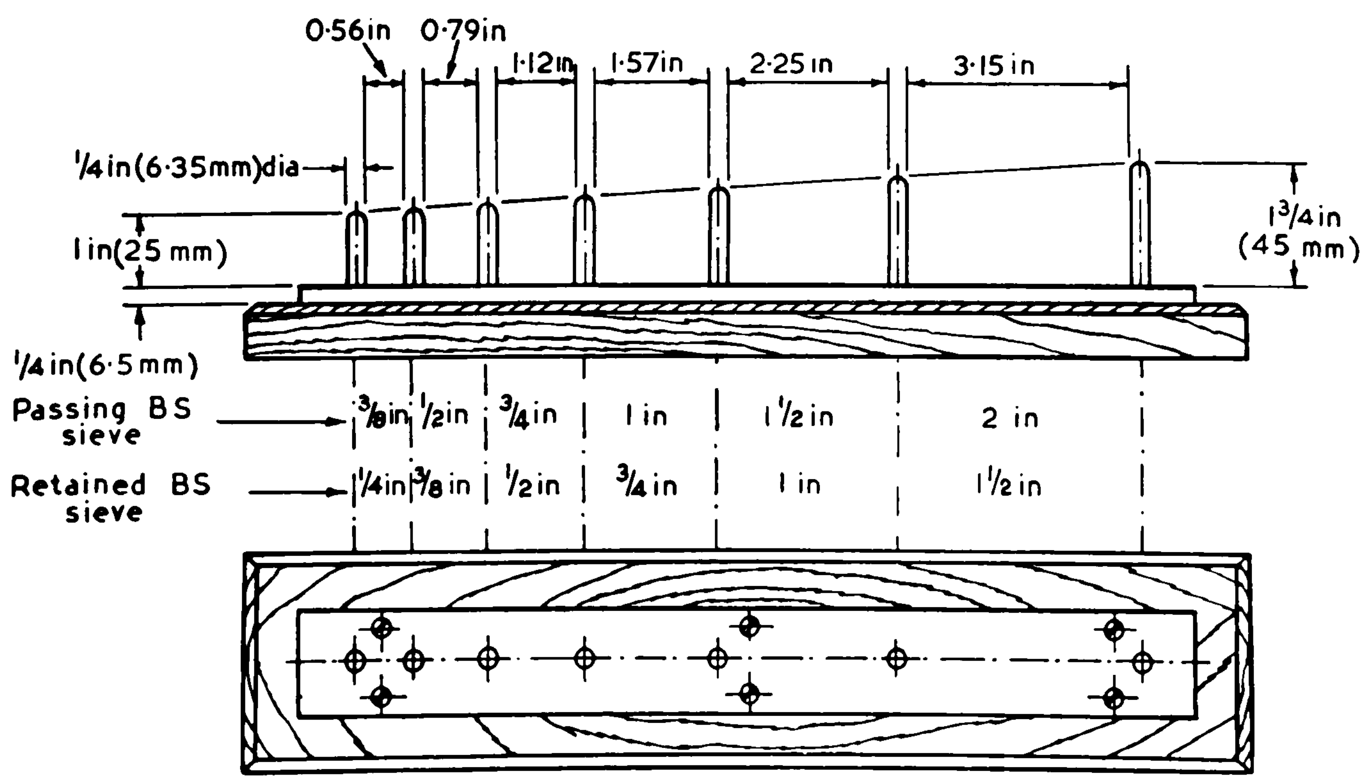


Fig. 14. ELONGATION GAUGE BRITISH STANDARD 812:1961.

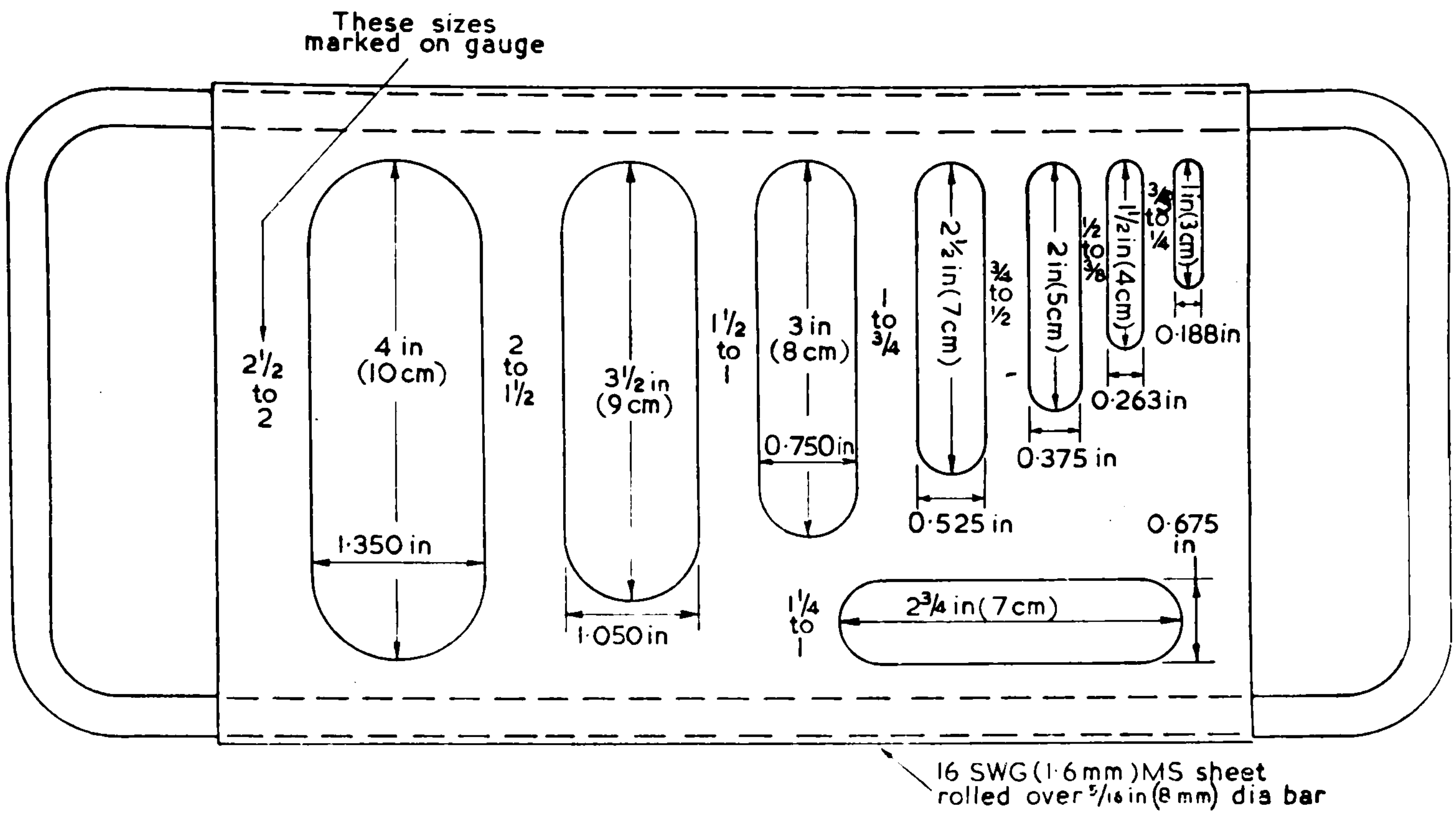


Fig. 14 (b) FLAKINESS GAUGE - BRITISH STANDARD 812:1961

In order to investigate still further this important subject of container wall effect and particularly the effect of particle shape on porosity in containers of various size, a further comprehensive experimental study was undertaken and the results of this and experimental studies by other workers have been interpreted in theoretical light.

Aggregates studied were shape sorted, after sieve separation, into categories based upon elongation ratio and flatness ratio. The separation was effected by means of the British Standard 812 elongation and flakiness gauges (Fig.14). This part of the work was carried out at an early stage, before the deficiencies of these gauges had been realised and investigated (Lees 1964 (c)). However, this was of no great importance in this case since the prime object had been to extend the range of aggregate shapes studied by producing some extreme types.

Shape categories studied were designated as follows:- Crushed Basalt (Rowley Regis, Nr. Birmingham) sub-divisions: Equidimensional, Discs, Rods and Blades, and Rounded Gravel (Weeford Pit, Staffs) sub-divisions: Equidimensional, Discs, Rods and Blades.

In order to examine the relationships between porosity and $\frac{A}{V}$ ratio over a wider range than that examined by previous workers, containers of quite small size relative to the size of the particles were employed for some of the tests. All containers used in this experiment were cylinders, but of varying height diameter ratio. It follows that their different $\frac{A}{V}$ ratios were not only a consequence of their varying size, but also to some extent of their varying shape. Figures 15 to 18 show on graphs of porosity against $\frac{A}{V}$ ratio, plotted points which are the mean of six determinations *. Gently curvi-linear trends are discernible (the origin of the curvature and of the frequently large deviations of points from the trend line at high $\frac{A}{V}$ ratios are discussed below).

General observation can be made firstly of the tendency towards increase in slope of the trend lines with increase in size of the particles and secondly that the extrapolations to the vertical ordinate, at which $\frac{A}{V} = 0$, do not meet at a point, but cross over one another. This is in apparent contradiction of the theory that for particles of the same shape, porosity is independent of size, to satisfy which theory, all sizes should show the same porosity in an infinitely large container.

* The porosity determinations were of the loose porosity obtained by dropping the particles freely and in mass into the container from a height of a few inches above the container rim. Particles projecting above the rim were carefully removed without disturbing the structure beneath.

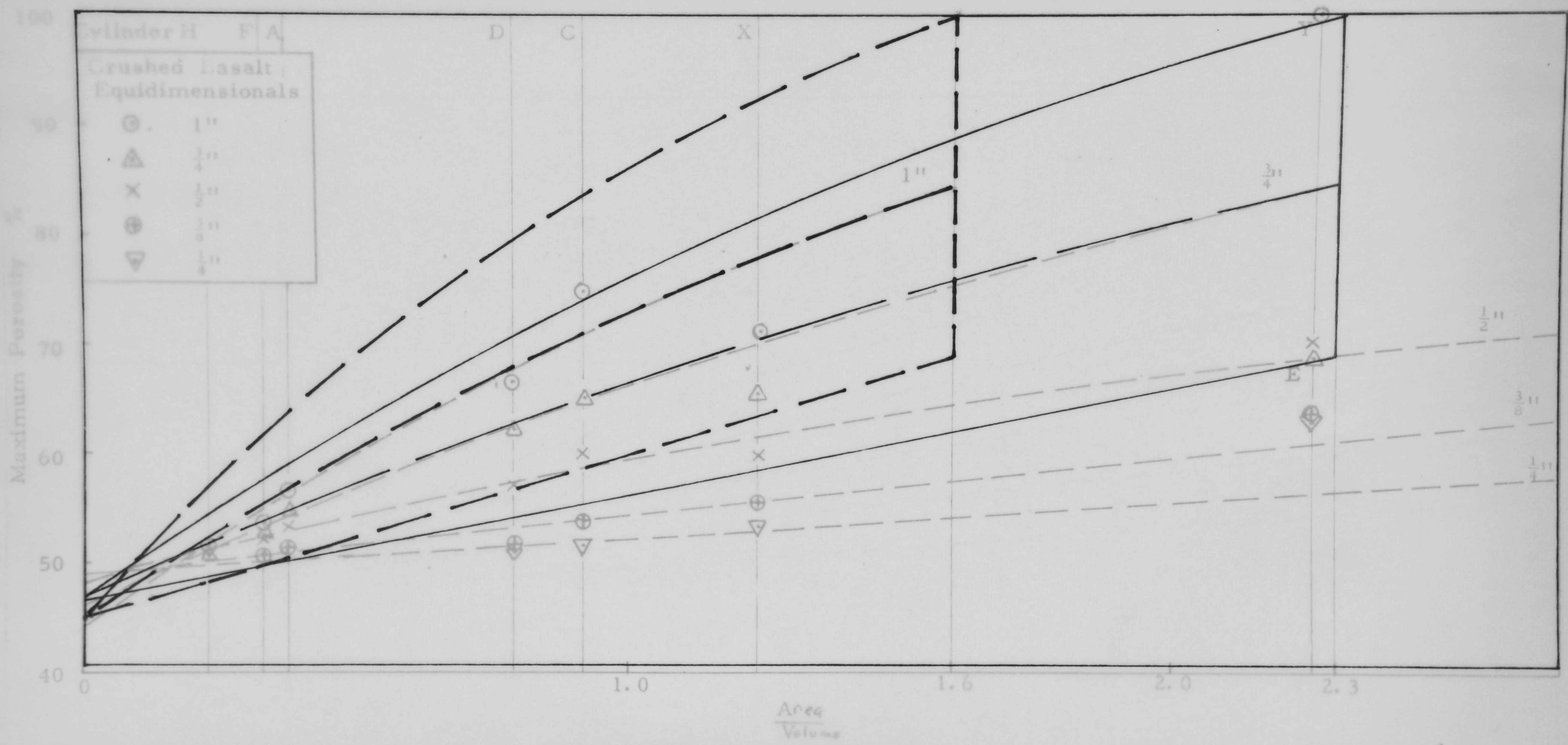
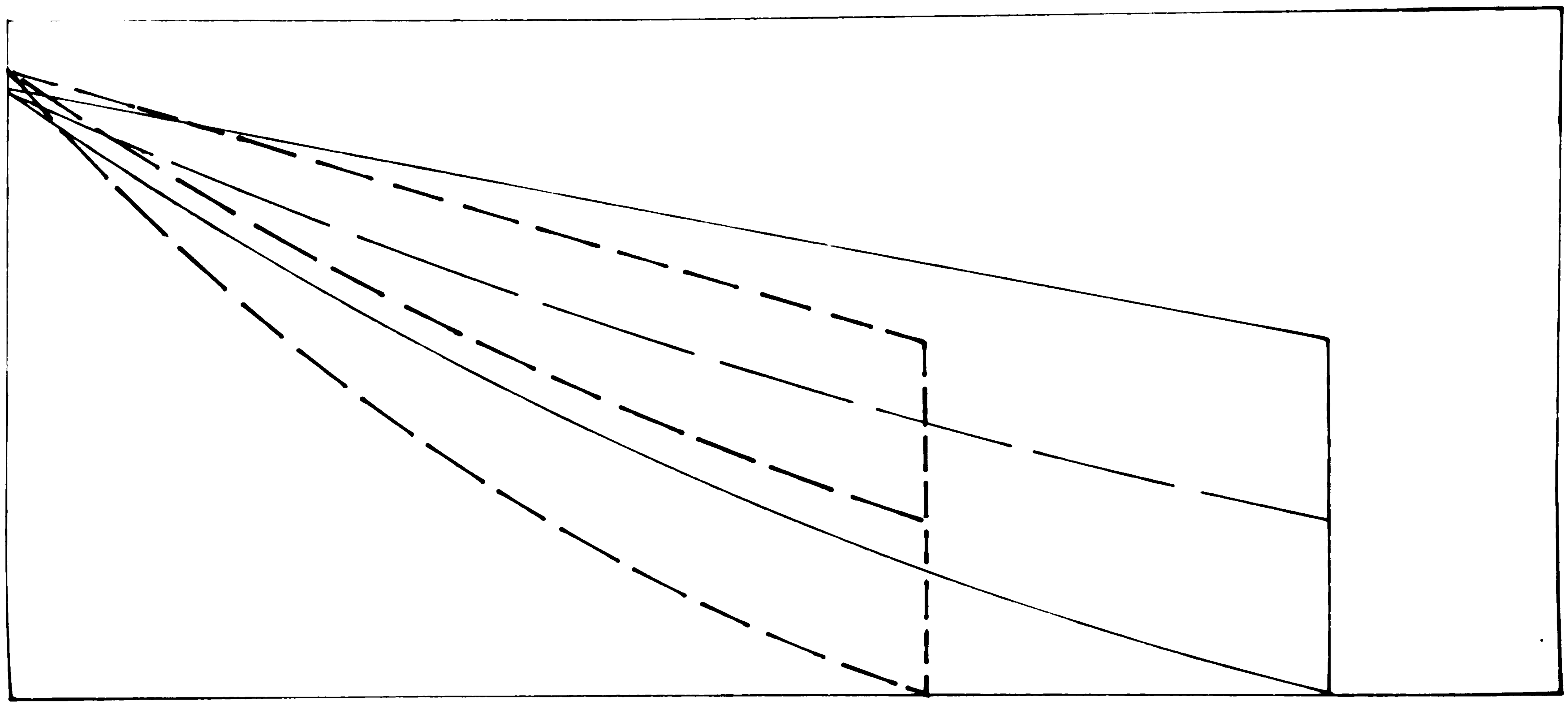


Figure 15. MAXIMUM POROSITY VERSUS $\frac{A}{V}$ RATIO FOR CONTAINER.



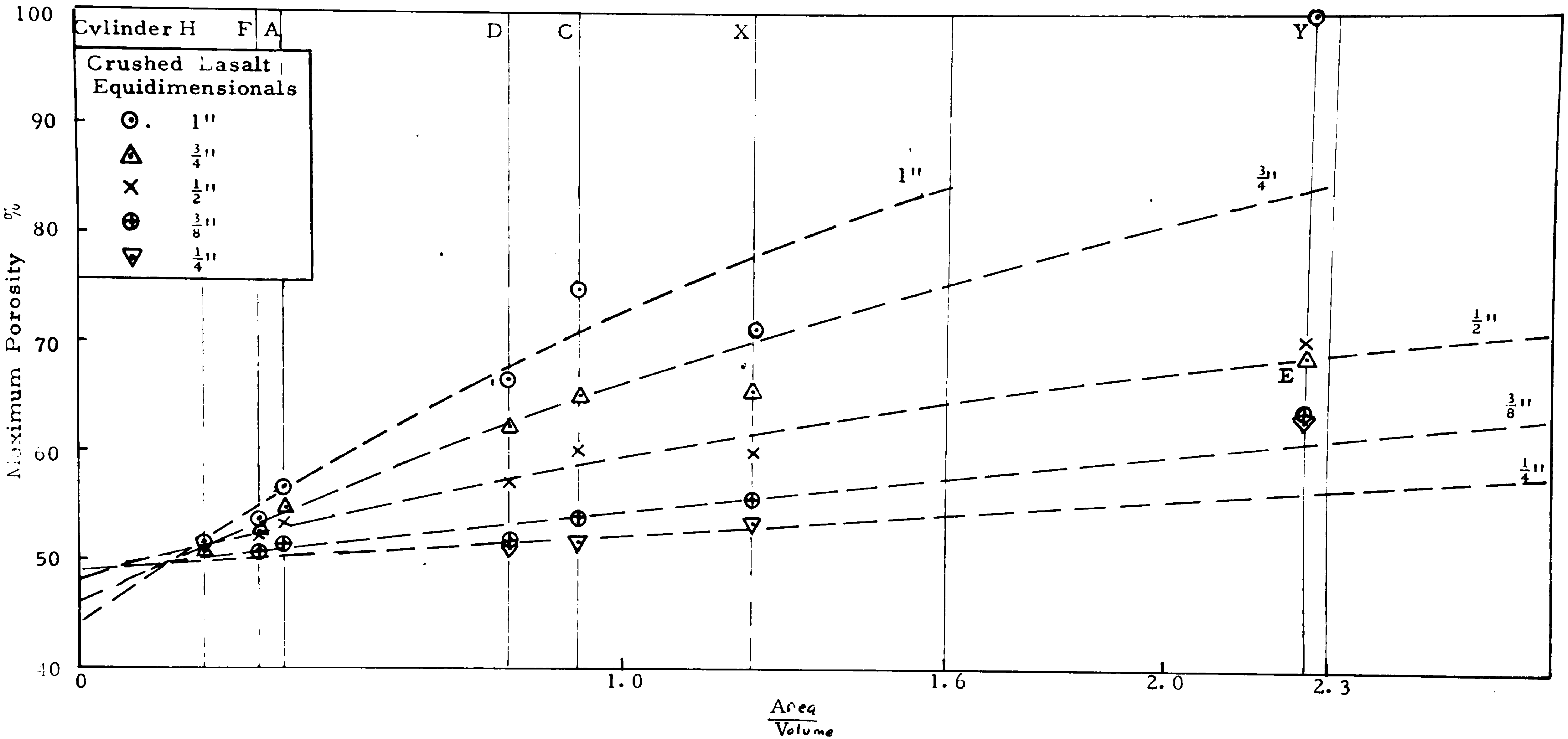


Figure 15. MAXIMUM POROSITY VERSUS $\frac{A}{V}$ RATIO FOR CONTAINER.

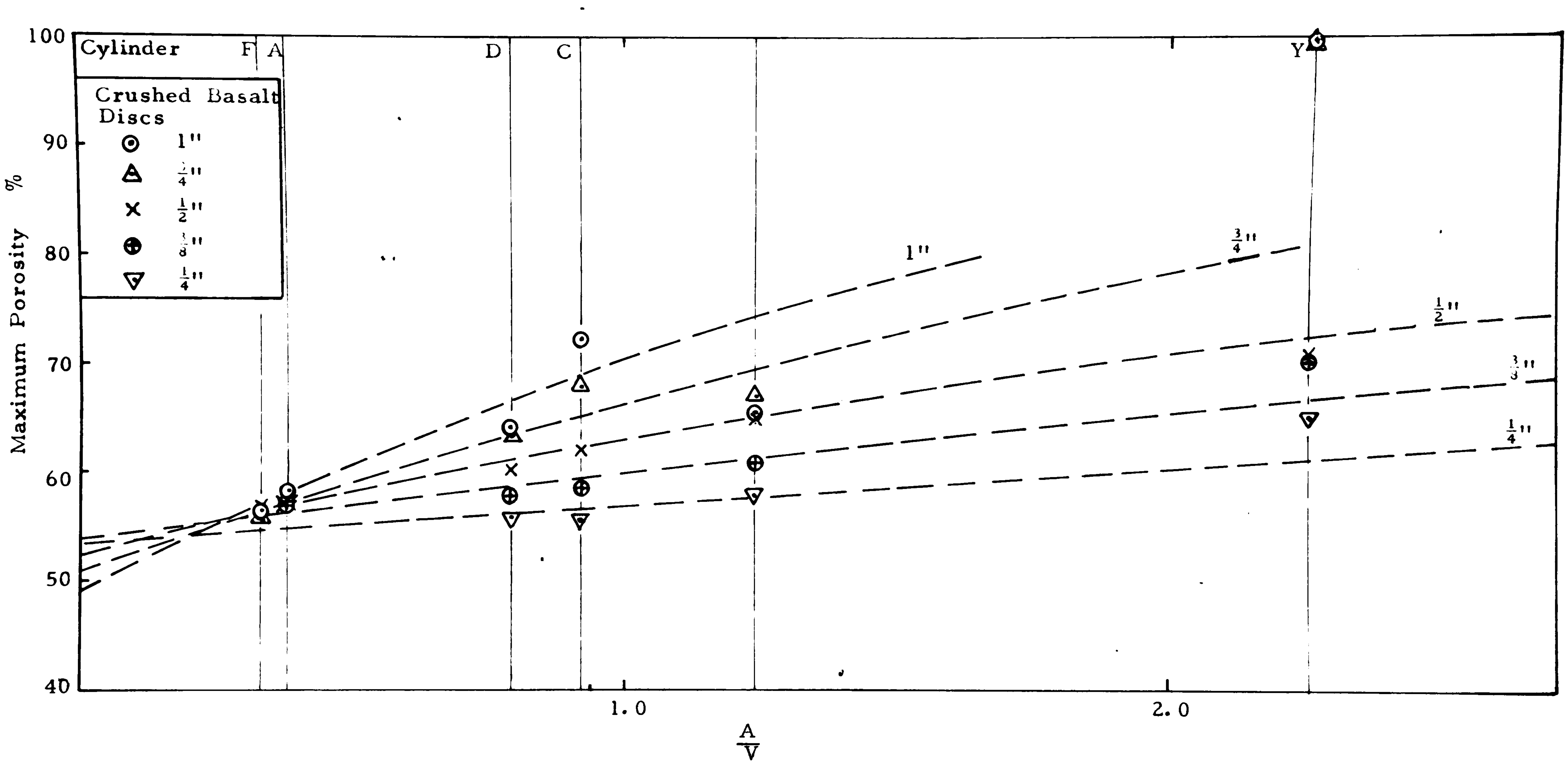


Figure 16. MAXIMUM POROSITY VERSUS $\frac{A}{V}$ RATIO FOR CONTAINER.

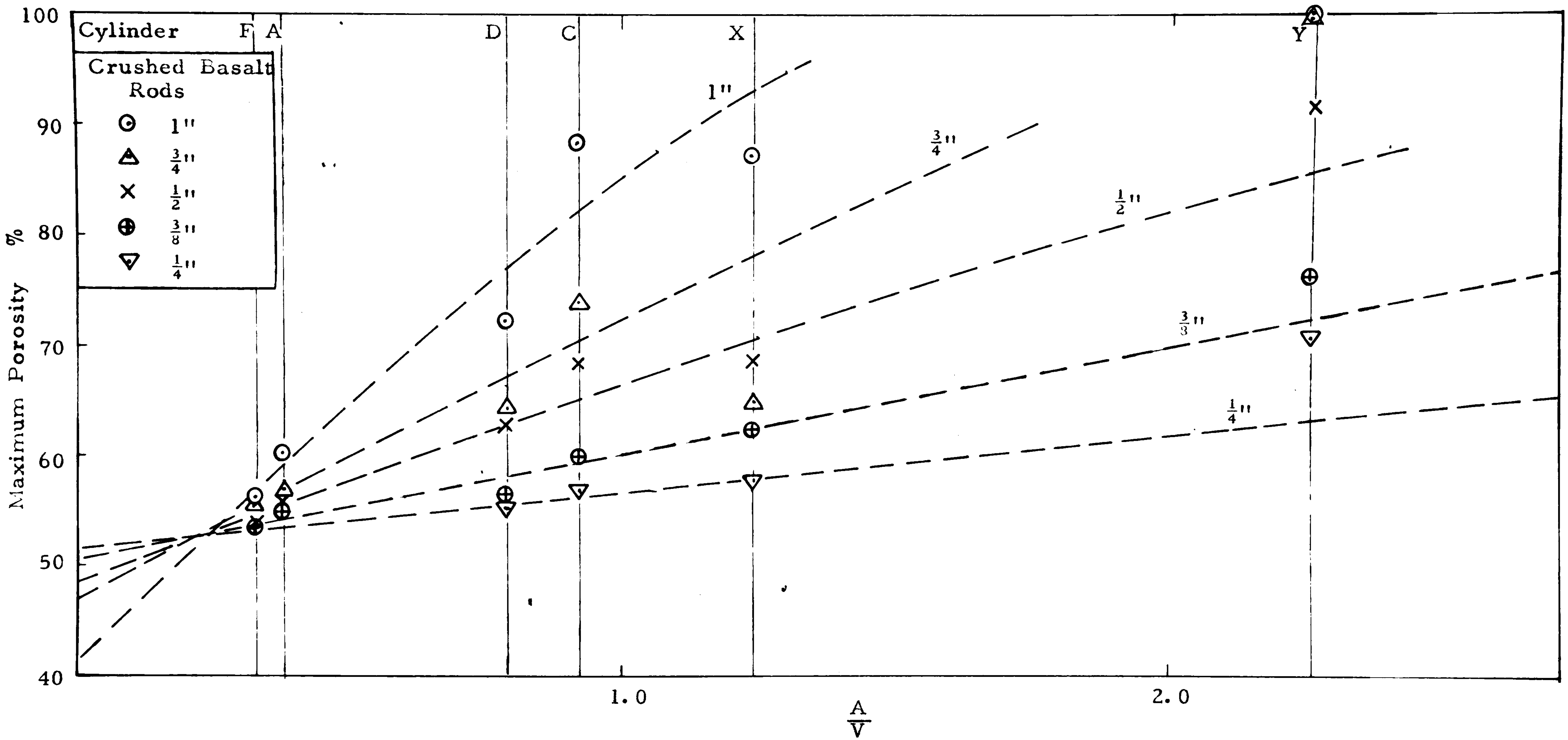


Figure 17. MAXIMUM POROSITY VERSUS $\frac{A}{V}$ RATIO FOR CONTAINER

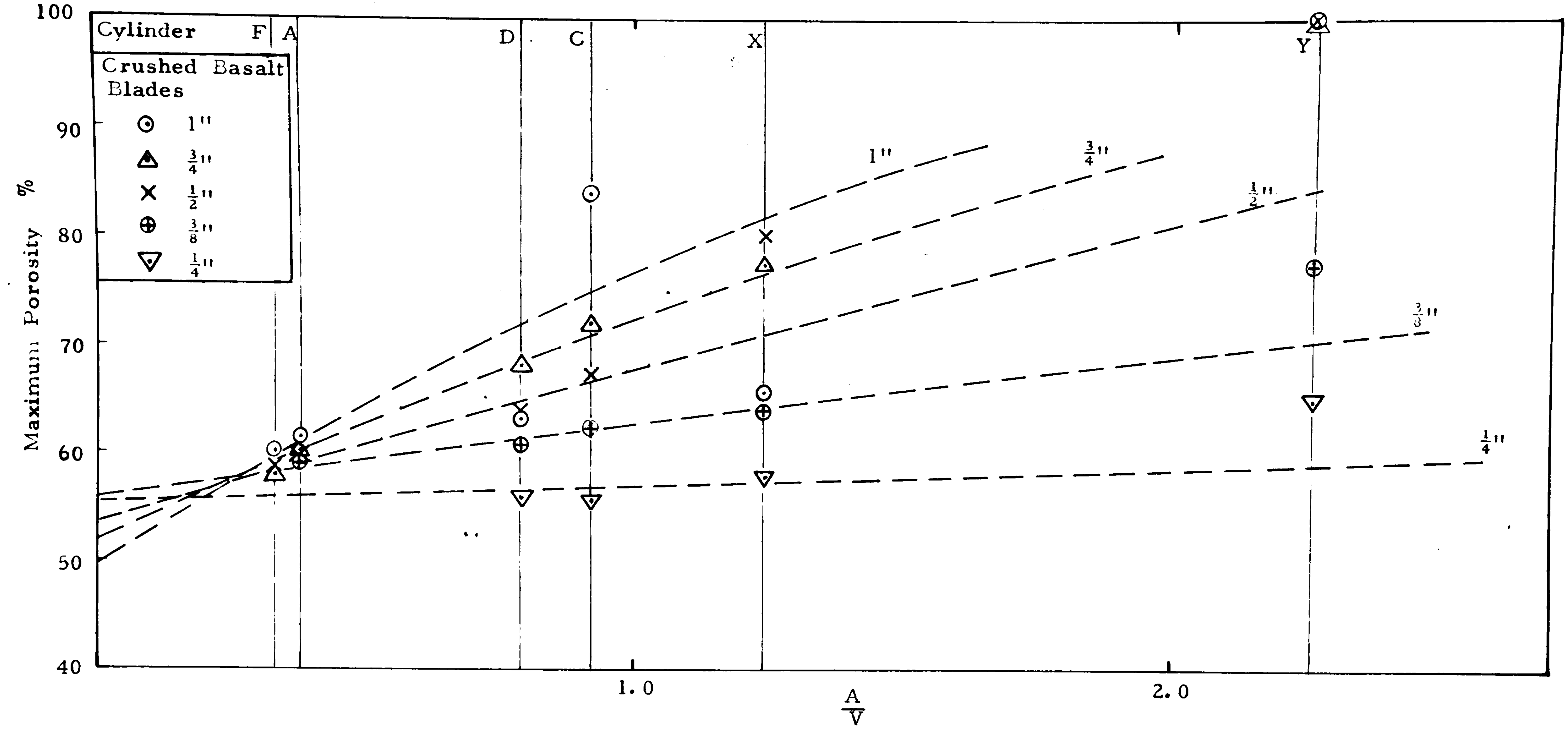


Figure 18. MAXIMUM POROSITY VERSUS $\frac{A}{V}$ RATIO FOR CONTAINER

It will be recalled however that the shape categories employed were based upon separation according to elongation ratio and flatness ratio only. Measurement of the Degree of Angularity by the method according to Lees (1964 (a)) revealed a general progressive increase in angularity with decreasing size within any given shape category (Table 3).* It

TABLE 3.

Degree of Angularity and Maximum Porosity for a series of shape-sorted aggregates of various sizes.

Sieve Size		Equidimensionals		Discs		Rods		Blades	
Passing	Retained	a	b	a	b	a	b	a	b
⅜"	¼"	1984	49.0	2412	53.5	2198	51.7	2445	55.5
½"	⅜"	1942	49.0	2492	54.0	2188	50.7	2405	56.0
¾"	½"	1858	48.0	2129	52.5	2113	48.5	2262	53.5
1"	¾"	1804	46.0	1998	51.0	2072	47.0	2150	52.0
1½"	1"	1645	44.5	2000	49.0	1850	41.0	1984	49.5

a = Angularity b = Maximum Porosity %

was apparent therefore that in spite of superficial similarity in shape according to axial ratios, shape was not a constant factor in these tests and that the differences in porosity at $\frac{A}{V} = 0$ arose out of the variation in angularity. Fig.19 shows the relationship between porosity at $\frac{A}{V} = 0$ and Degree of Angularity for all samples. All points fall within a fairly narrow band indicating a range of porosity of about 7% for any given angularity.

Another tendency is for the results to show increasing erratic behaviour i.e. to scatter widely from the simple curvilinear trend as the particles increased in size or as the container is reduced. (This effect is seen even more clearly in Figs. 20 to 23 where the results are replotted as porosity against $\frac{A}{V} \times d$).

The reproducibility of results decreases as the size of particle increases relative to the size of the container. This alone would cause

* The reason for the increase in angularity with decreasing size is that the smaller sizes were freshly crushed in the laboratory while the larger sizes, having been crushed at the quarry had suffered some abrasion of sharp corners during transportation and handling. It will be noted that the proportion of freshly crushed material increases towards the smaller sizes but that the ⅜" and ¼" sizes are of equal order of angularity. Some other consequences of the change in angularity with size (namely the effect on passage of grains through sieves and the consequent effect on the dimensions of the particle average intermediate diameter) are discussed in Lees 1964 (b) and (c).

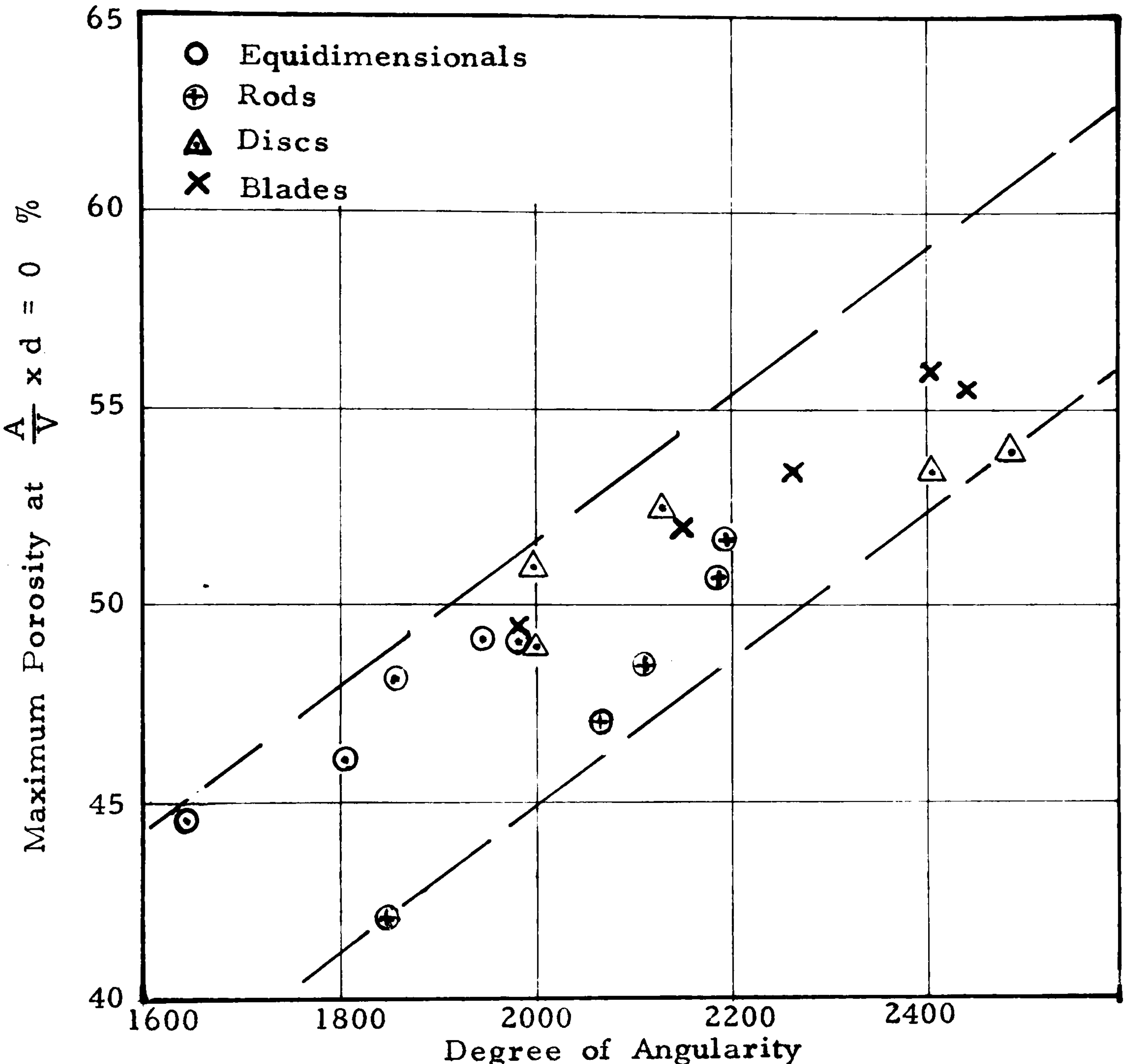


Fig. 19. INFLUENCE OF PARTICLE SHAPE AND ANGULARITY ON MAXIMUM POROSITY.

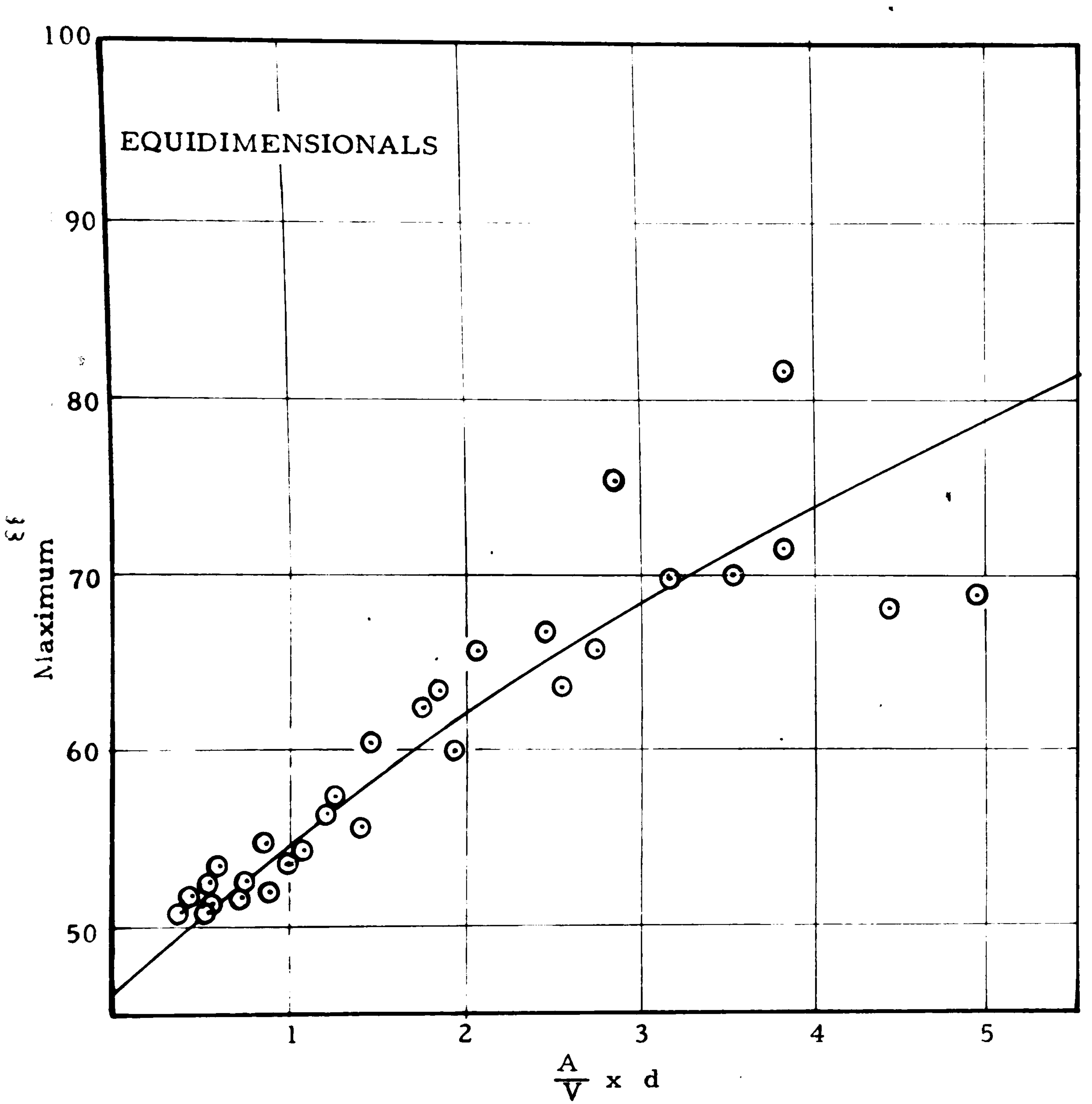


Fig. 20. CONTAINER WALL EFFECT ON MAXIMUM POROSITY CRUSHED BASALT - EQUIDIMENSIONALS - ALL SIZES

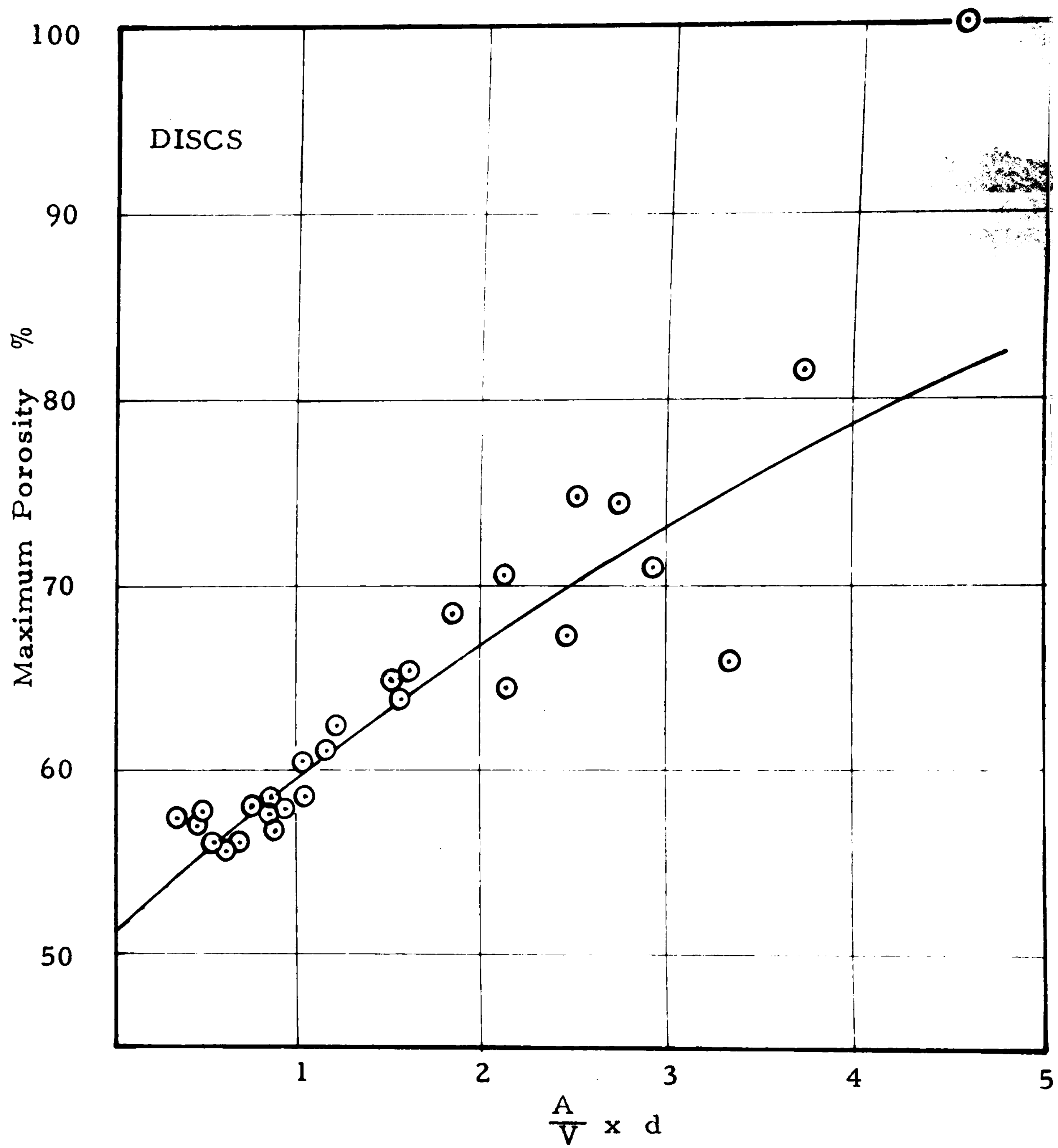


Fig. 21. CONTAINER WALL EFFECT ON MAXIMUM POROSITY. CRUSHED BASALT - DISCS - ALL SIZES

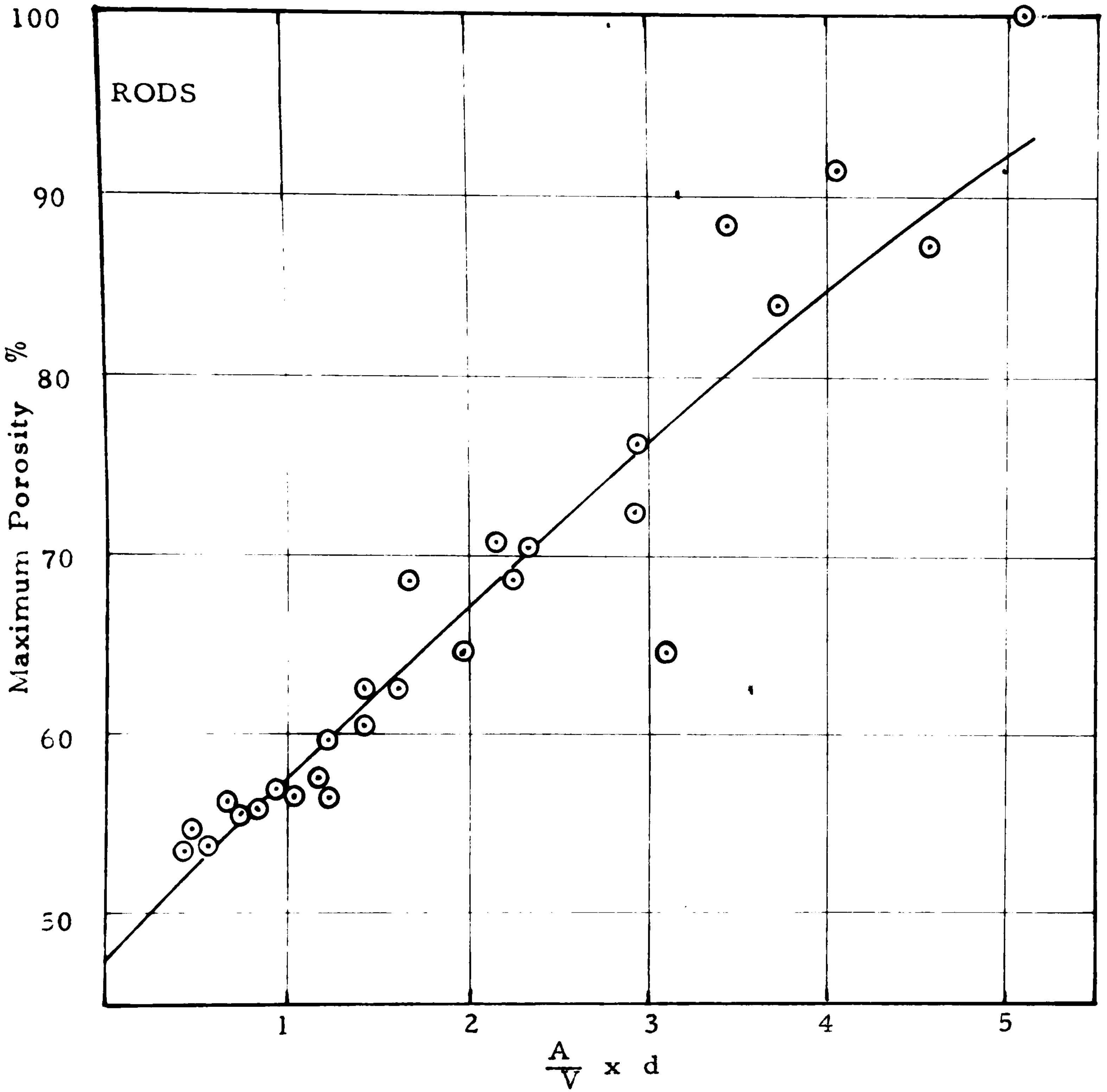


Fig. 22. CONTAINER WALL EFFECT ON MAXIMUM POROSITY CRUSHED BASALT - RODS - ALL SIZES

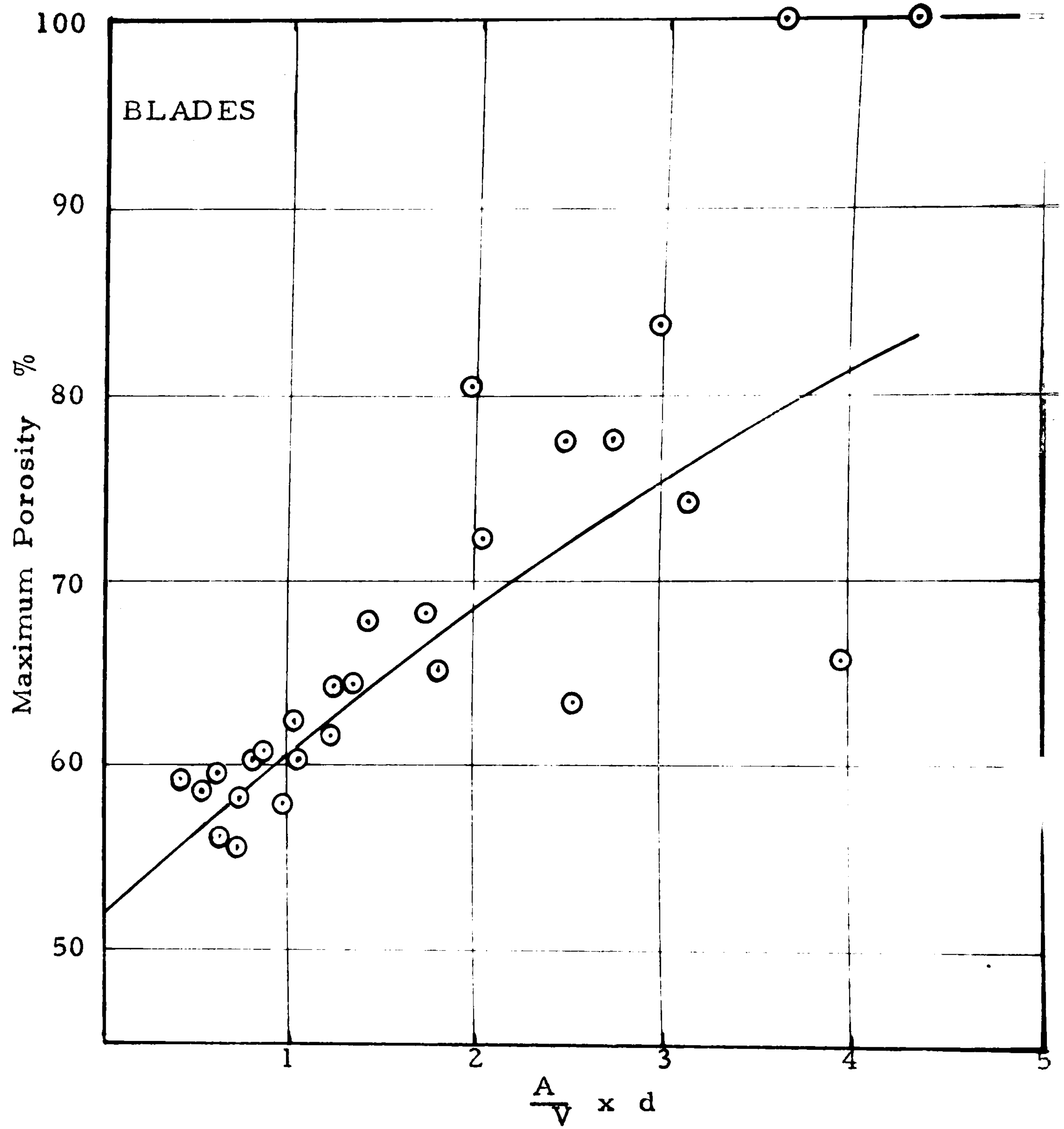


Fig. 23. CONTAINER WALL EFFECT ON MAXIMUM POROSITY. CRUSHED BASALT - BLADES - ALL SIZES

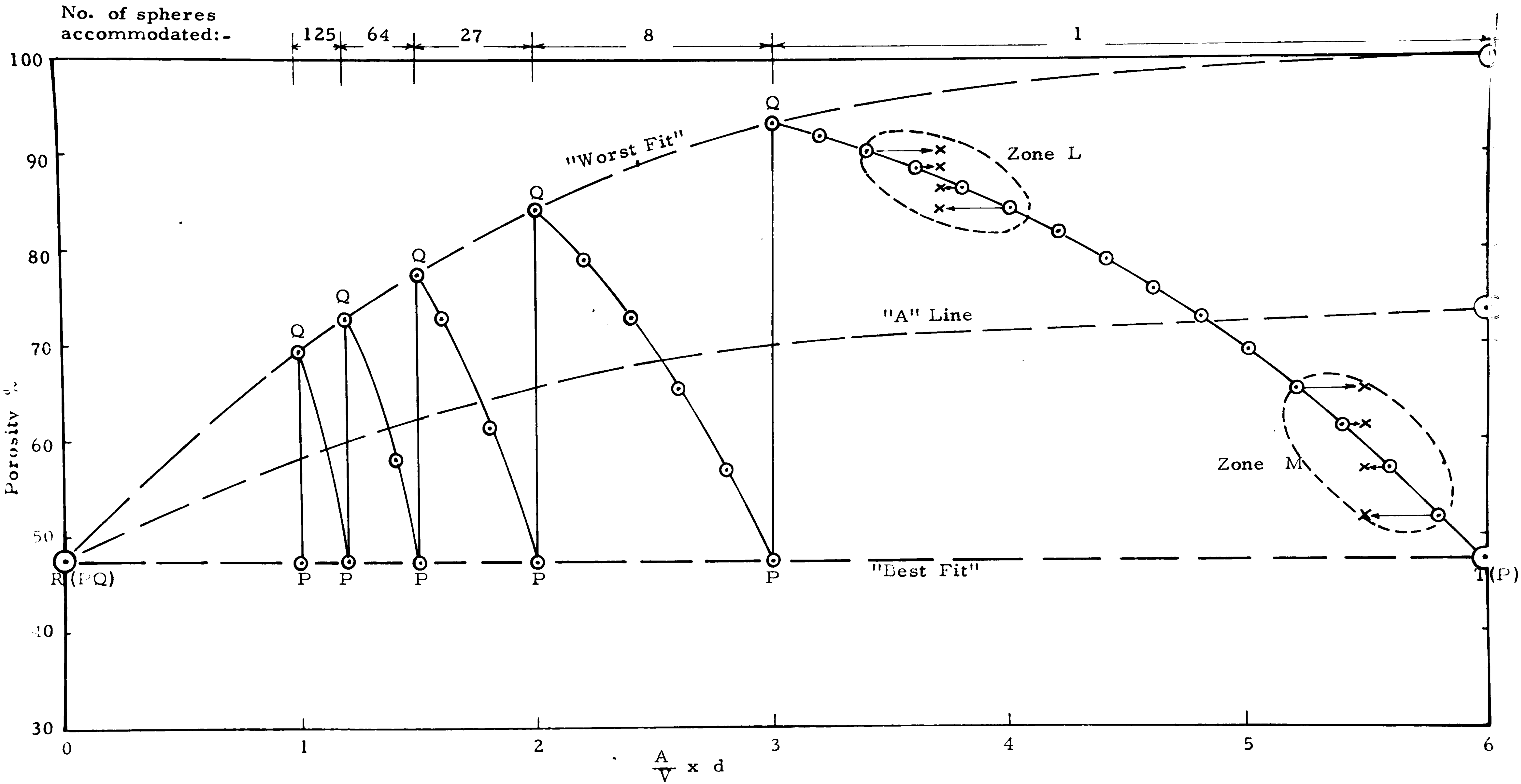


Fig. 24. THEORETICAL POROSITY RELATIONSHIPS FOR CUBICAL PACKING OF SINGLE SIZE SPHERES IN CUBICAL CONTAINERS OF VARYING SIZE.

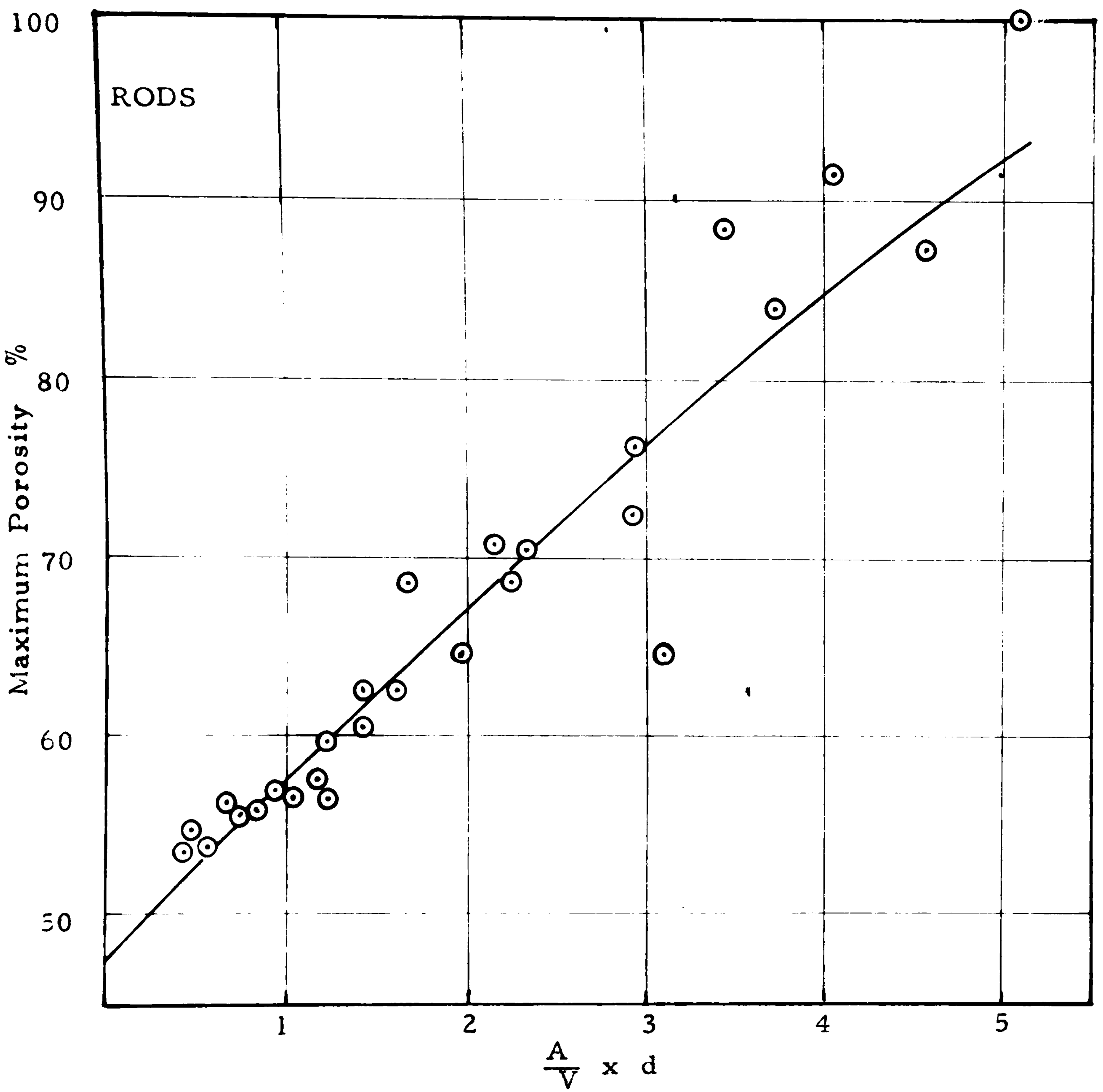


Fig. 22. CONTAINER WALL EFFECT ON MAXIMUM POROSITY CRUSHED BASALT - RODS - ALL SIZES

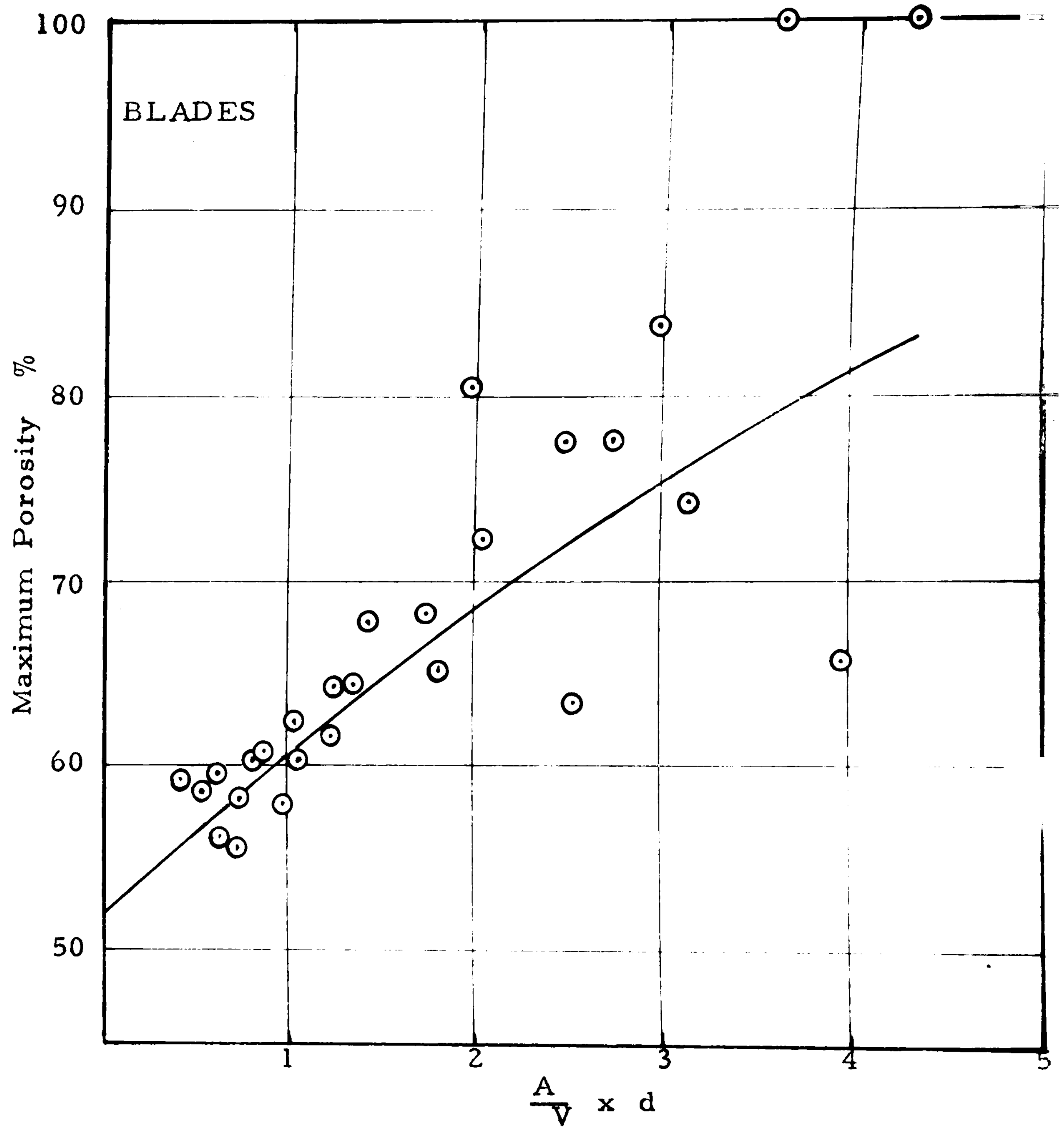


Fig. 23. CONTAINER WALL EFFECT ON MAXIMUM POROSITY. CRUSHED BASALT - BLADES - ALL SIZES

an increase in the scatter of points. It is suggested however, on theoretical grounds, that it is not just the lower reproducibility which causes the erratic distribution of points, but that the true relationship is neither a straight line nor a simple curve, but follows a line of complex ascending and descending portions.

It has been said above that the presence of a boundary wall may contribute to increased porosity in single size aggregates in two ways, one by causing a change in packing adjacent to the wall, and the other by resulting in the formation of incomplete cells because fractional particles are not possible.

The following theoretical discussion begins by dealing with the latter case and shows firstly how the kind of erratic behaviour described above may arise and secondly how the original concept of other authors of a linear relationship between porosity (or specific bulk volume) and $\frac{A}{V}$ x d may be false.

The discussion, while framed around an assessment of the behaviour of uniform spherical particles in a cubical container, applies equally to aggregates of other shapes and in containers of other shapes. Uniform spherical grains and the cubical container are chosen merely because they constitute the simplest case to understand.

It has been stated previously in this work that to obtain the theoretical loosest porosity for single size spheres of 47.64%, granted the appropriate perfect cubic packing, one would need either to have an infinitely large container, for which the $\frac{A}{V}$ ratio equals 0 (or infinitely small particles) or to have a container of the appropriate shape (in this case, a cube) and of a size which just fitted exactly a whole number of the stacked spheres. (Theoretically the same applies to other systems of packing as well, but the containers would be of complex form).

The points marked P on Figure 24 denote these conditions of exact correspondence of packing and container for cubical packing in cubical containers. It will be seen that these points occur only at intervals as the $\frac{A}{V}$ x d value falls. At $\frac{A}{V}$ x d = just greater than 6, the container is just too small to accommodate a single sphere and porosity = 100%. At $\frac{A}{V}$ x d = just less than 6, one sphere just fits inside the container and the porosity = 47.64%. If the container is now enlarged for a constant size sphere $\frac{A}{V}$ x d falls and so long as only

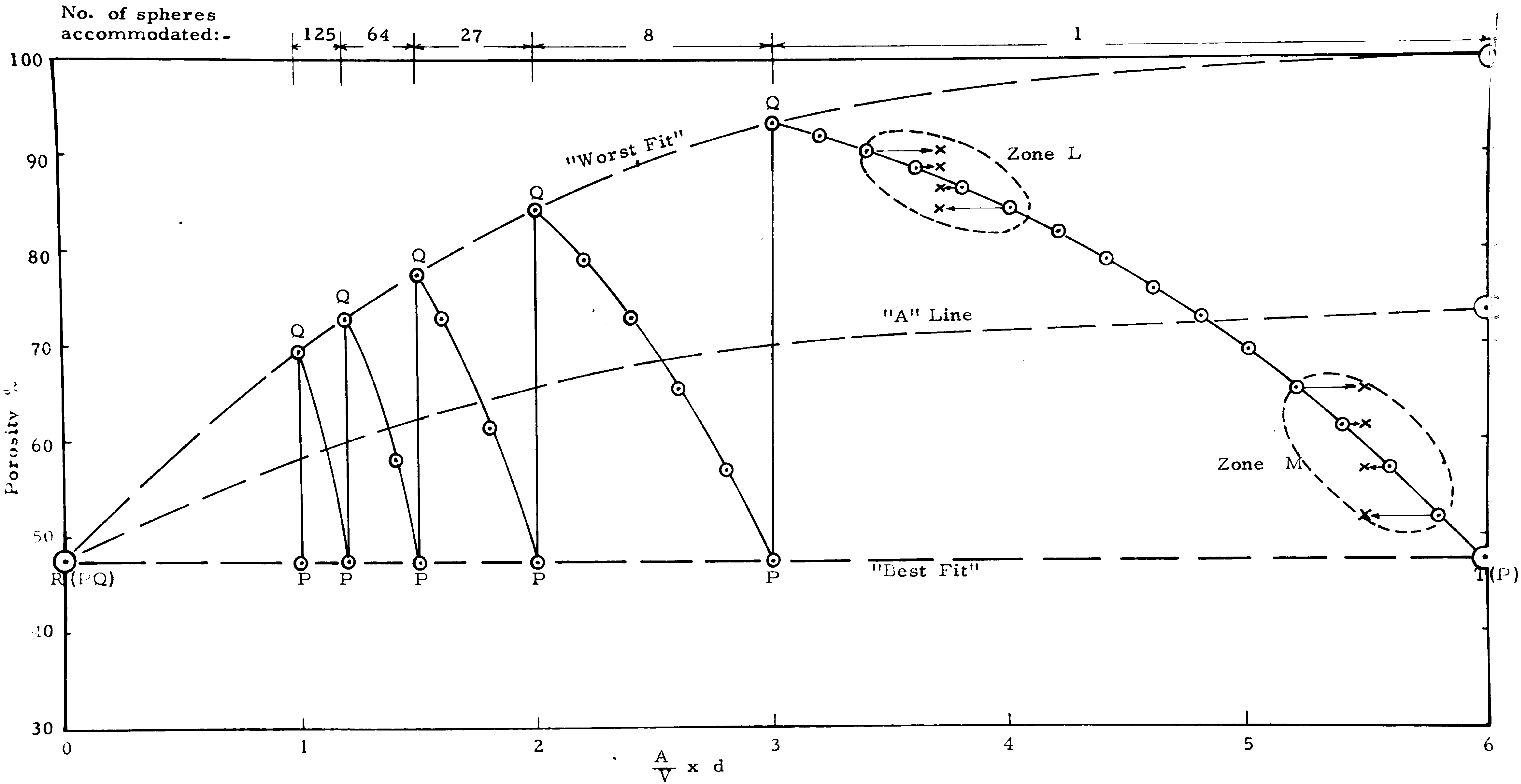


Fig. 24. THEORETICAL POROSITY RELATIONSHIPS FOR CUBICAL PACKING OF SINGLE SIZE SPHERES IN CUBICAL CONTAINERS OF VARYING SIZE.

one sphere can occupy the space in the container the porosity will rise. Maintaining the condition of cubic packing $\frac{A}{V}$ x d would fall to 3 before another 7 spheres could be accommodated. At this point the porosity would again fall to 47.64%. This would again happen at $\frac{A}{V}$ x d = 2 (with the admission of a total of 27 spheres), and with increasing frequency as $\frac{A}{V}$ x d $\rightarrow$ 0. These recurring cases of identical porosity of 47.64% may be regarded as a series of "best fit" conditions which show that in theory, and provided the mode of packing is unchanged, infinitely large containers or infinitely small particles are not essential to the production of the lowest porosity for that type of packing.

Between these "best fit" cases, occurring at certain definite values of $\frac{A}{V}$ x d, the porosity rises along a curved line, as shown, with falling $\frac{A}{V}$ x d value. Fractionally before the next " best fit " condition the porosity values are at a maximum (Q) and these positions therefore represent a corresponding series of "worst fit" conditions. In other words the problem is seen as one of recurring "best fit" and "worst fit" conditions as $\frac{A}{V}$ x d falls. However while the "best fit" porosities remain constant at 47.64% during this change, the value of the "worst fit" porosities reduces as shown, from 100% at $\frac{A}{V}$ x d = 6 to 47.64% at $\frac{A}{V}$ x d = 0, i.e. there is no distinction between "best fit" and "worst fit" conditions at this point.

The form of the "worst fit" trend line is curved while that of the "best fit" is a straight line. It follows that an average trend line, i.e. averaging the "best fit" and "worst fit" cases would also be curved and would terminate on the ordinate of $\frac{A}{V}$ x d = 6 at the porosity value of 73.82% i.e. the average of 47.64 and 100%. It must be emphasised that the "best fit" and "worst fit" trend lines are not real lines. The average line ('A' line) too, is a "virtual" line since it does not represent the average of an even scatter of points above and below it. The line connecting all possible points is one of complex rising and falling sections contained within the envelope RST, for which the 'A' line represents only a general trend. It will be noted that at low values of $\frac{A}{V}$ x d the spread of porosity values about the 'A' line is not great, but becomes considerable at high values of this ratio. This greater spread is a consequence of the stepped behaviour of the relationship and since the points plotted are theoretical, is clearly not associated with any reduction in the reproducibility of experimental values. Consequently when experimental results are, later, considered the probability must be considered that the greater scatter of results at high values of $\frac{A}{V}$ x d, results also from a similar stepped relationship upon which may be superimposed a further scatter due to lower reproducibility.

In the case just described the variations in porosity were a consequence of incomplete cells with no variation in packing. In the case of actual particles it is certain that disturbance of packing also occurs.

A closer simulation to this case may be made by modifying the previous example as follows.

In that example cubical packing in cubical containers was considered. Rhombohedral packing in complex rhombohedral containers would show a similar behaviour but with the "best fit" line occurring at the lower constant level of 25.95% porosity. In both of these cases there would be conformity of shape between packing "cells" and container. However if the alternative case of rhombohedral packing in cubical containers is considered it is clear that as size of particle approaches size of sphere there would be increasing disturbance of the packing, superimposed upon the incomplete cell effect which has already been described. It follows that at the one extreme case of the infinite container when $\frac{A}{V} \times d = 0$, the porosity would be 25.95% but that at the other extreme of $\frac{A}{V} \times d =$ just less than 6, the porosity would again be 47.64% since this is again the case of the single sphere in a cubical container. This would lead to the "best fit" trend line sloping upwards as in Fig.25, indicating a progressive increase in the proportion of loose cubical packing relative to dense rhombohedral packing as the wall effect increases.

The porosities of intermediate $\frac{A}{V} \times d$ values, for which the packing arrangements would be quite complex, have not been calculated and undoubtedly this is an oversimplified view, however it is suggested by the short-dashed lines in Fig. 25 that the behaviour should be basically similar to the case previously described and illustrated in Fig. 24.

When actual particles are considered similarities appear between their behaviour and that hypothesised in Fig. 25.

In Fig. 15 for example, the $\frac{A}{V}$ ratio at which just one average particle of 1" size could be accommodated in a cylindrical container

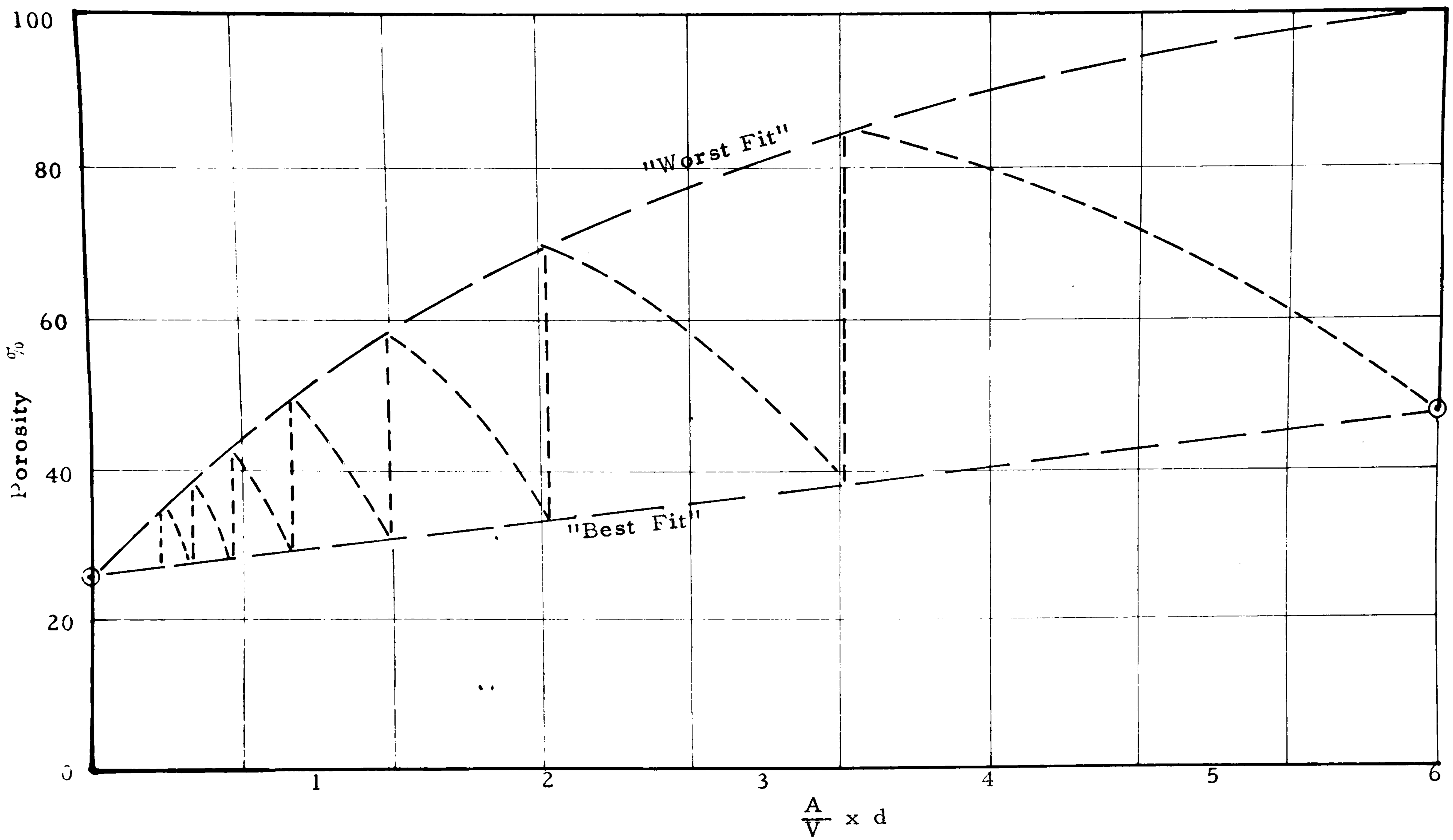


Fig. 25. HYPOTHETICAL POROSITY RELATIONSHIPS FOR RHOMBOHEDRAL PACKING OF SPHERES IN, AND MODIFIED BY BOUNDARIES OF, CUBICAL CONTAINERS.

of equal height and diameter has been calculated as 1.6. Similarly for one average particle of $\frac{3}{4}$" size the $\frac{A}{V}$ ratio of the container just to fit has been calculated as 2.3. Assumihg then that these values represent the limits for their particular cases, the fields of existence for the experimental points may be hypothesised as shown in the overlay to Fig. 15 which show a close similarity to the theoretical Fig. 25. Particular attention may be drawn to the fact that cylinder Y is very close to the value of 2.3 for $\frac{3}{4}$" size aggregate and that the point E on the Y ordinate represents just one particle in that container. In other words this represents the "best fit" porosity at the extreme limit just before the container must become empty and it is seen that the "A" line trends towards the mid position between E & 100% porosity. The maximum "best fit" porosity for the 1" aggregate has been supposed to be of the same order as this value E, since for the close geometrical similarity which exists between these two sizes it is logical to assume a similar porosity for the condition of one particle just within the container. Again the Average line comes sensibly towards the mid point between this porosity value and the maximum "worst fit" value of 100%.

Actual particles are of course never so single-sized as the theoretical sphere and so will not behave exactly in the manner described for idealised particles and packing. The plotted points will scatter more widely and less regularly but nevertheless it appears that their general trend can still be represented by the "A" line. The curvature of the Average lines in Figs.15 to 18 and 20 to 23 referring to actual particles, can thus be explained in terms of the theories which have been advanced to describe the behaviour of simpler ideal systems. The concept of the straight line relationship between porosity (or Specific Bulk Volume) and $\frac{A}{V}$ x d has arisen apparently because previous workers have only experimented in the range of $\frac{A}{V}$ x d values up to 1, where the curvature would not be easily apparent, and because of the error in thinking that the Average line would terminate at a porosity value of 100% for $\frac{A}{V}$ x d = 6, instead of at the average between 100% and the actual porosity value when 1 particle is just fitted into the smallest container of a given shape which will accommodate it. Further the greater scatter of points which occurs at higher values of $\frac{A}{V}$ x d now appears mainly as a consequence of the stepped behaviour of the relationship, similar to the case illustrated in Fig.24 where the mode of a cluster of points in Zone L would lie considerably above the "A" line and equally the mode of a cluster of points in Zone M would lie well below the "A" line. (In Zones L and M in Fig.24 the theoretical points have also been plotted on an assumed common ordinate for $\frac{A}{V}$ x d, analagous to the case of actual particles retained between two sieves, which although varying in their individual equivalent spherical diameter would be plotted as for the average equivalent spherical diameter for that size fraction, in a cylinder of constant size. This would cause them to be plotted at a constant value of $\frac{A}{V}$ x d in spite of slight differences in particle size, just as in Fig.24 the points in Zones L and M have been plotted at a constant value of $\frac{A}{V}$ x d in spite of slight differences in container size.)

In conclusion it may be thought that in practice $\frac{A}{V}$ x d values never reach these high values at which the curvature and the wide scatter become important. However these conditions do indeed exist where concrete or bitumen bound aggregates are placed in extremely narrow or thin sections and where concrete is placed in positions of congested reinforcement. Serious errors in aggregate mix design would arise from attempts to predict the porosity in such situations by extrapolation from the known porosity in larger containers.

(v) Internal boundary effect. The previous discussion has emphasised the effect of container walls on the packing and porosity of adjacent particles. Figures 26 to 28 after Fraser (1935) illustrate another very important type of boundary, which though internal, will have a similar effect to the external container walls referred to previously. Fraser showed with these figures, that the wall-effect exists also, for smaller particles in the vicinity of larger particles placed among them - especially, as will be noted, on the underside of such particles.

Figs. 26 and 28 show diagrammatically what may occur to the packing of sand grains in the presence of rounded gravel particles and similarly Fig. 27 shows what may occur to the packing of sand grains in the presence of angular crushed rock particles.

The importance of this internal boundary effect in the theory of packing will be referred to later, but it is interesting to remark on two "side" effects in passing, which refer to aggregates in concrete:-

(1) that during failure of concrete these looser-than-average zones beneath the coarse aggregate particles frequently provide the sites for initial crack formation, from which general fracture proceeds. This may be seen for example in Fig. 29

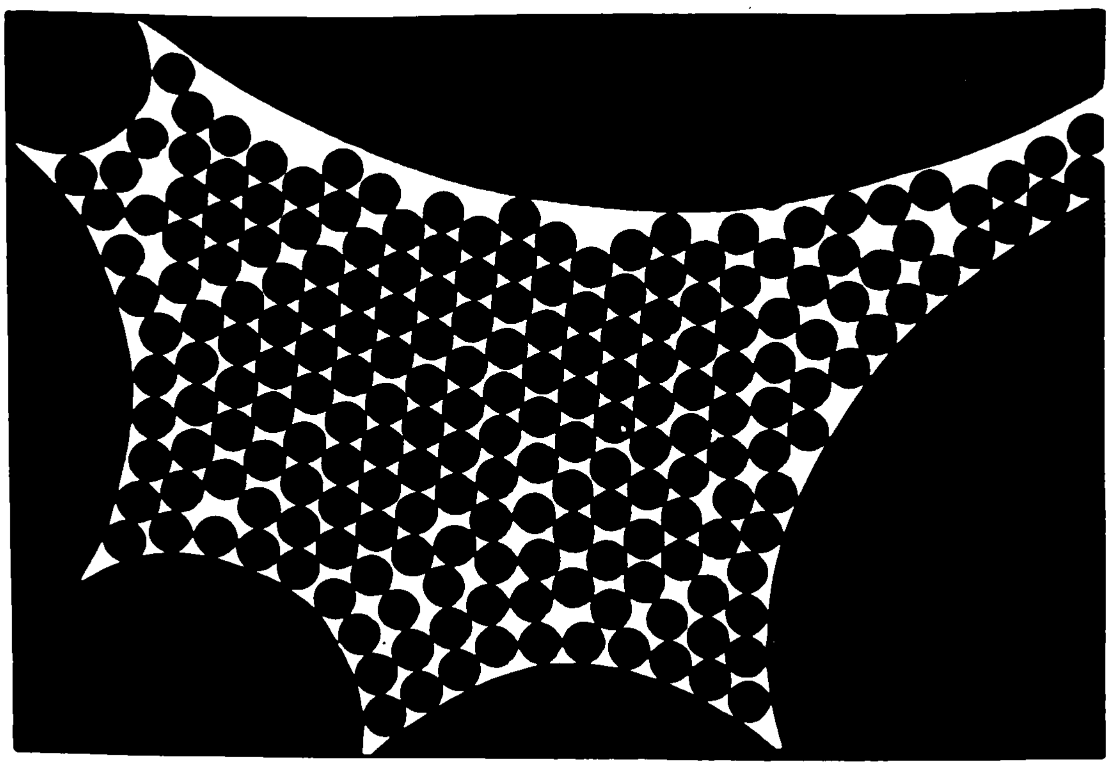


Fig. 26. DIAGRAMMATIC REPRESENTATION OF THE INFLUENCE OF ROUNDED GRAVEL PARTICLES ON PACKING AND POROSITY OF SPHERICAL 'SAND' PARTICLES IN COARSE AGGREGATE VOIDS. (after Fraser 1935).

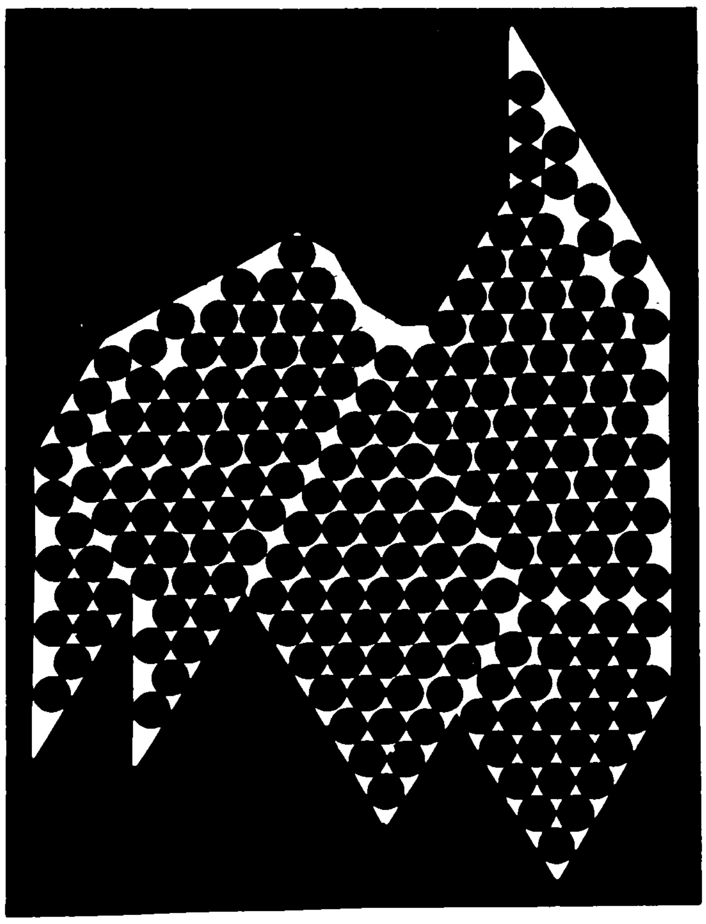


Fig. 27. AS Fig. 26, BUT WITH ANGULAR COARSE AGGREGATE. (after Fraser 1935)

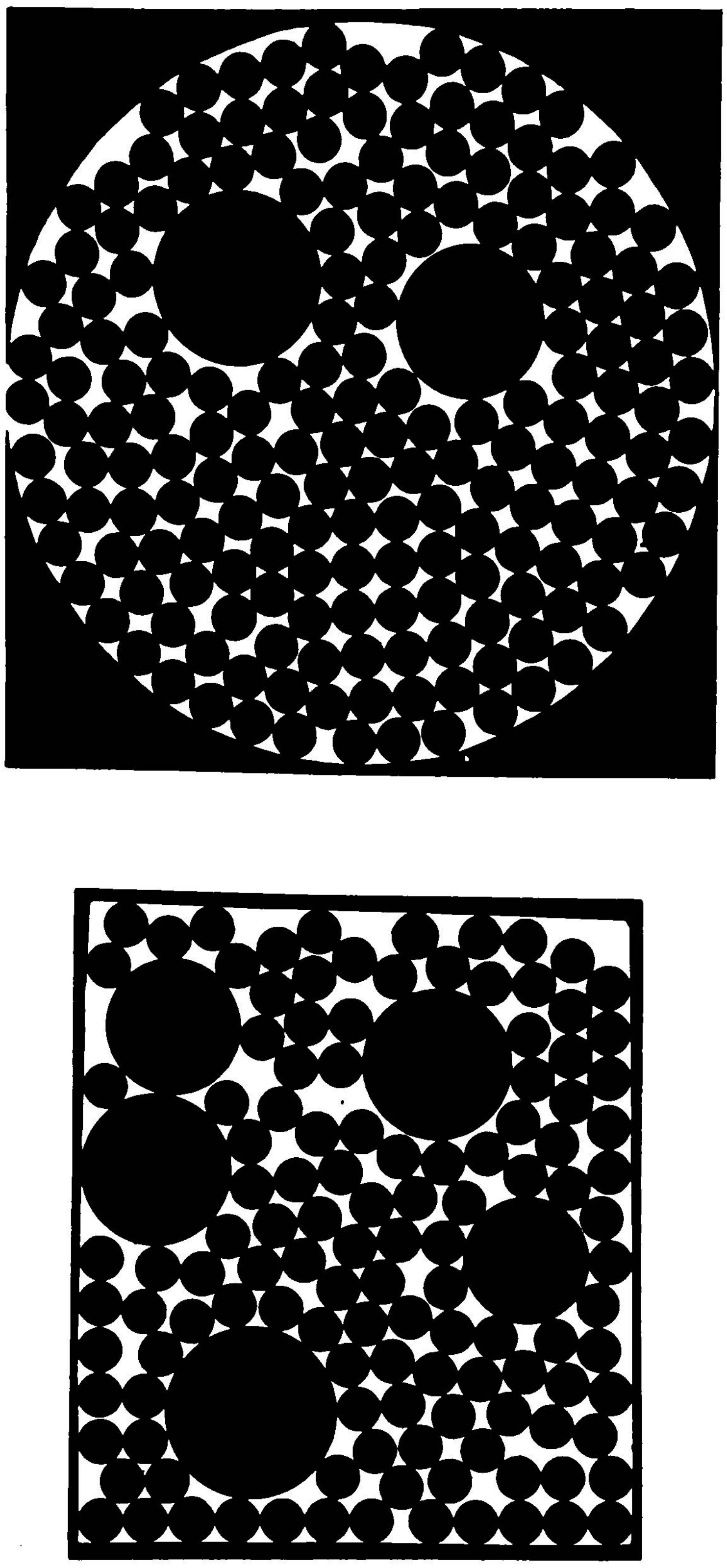


Fig. 28. DIAGRAMMATIC REPRESENTATION OF INFLUENCE OF INCLUDED GRAVEL AND/OR REINFORCEMENT BARS ON PACKING AND POROSITY OF 'SAND' PARTICLES. (after Fraser 1935).

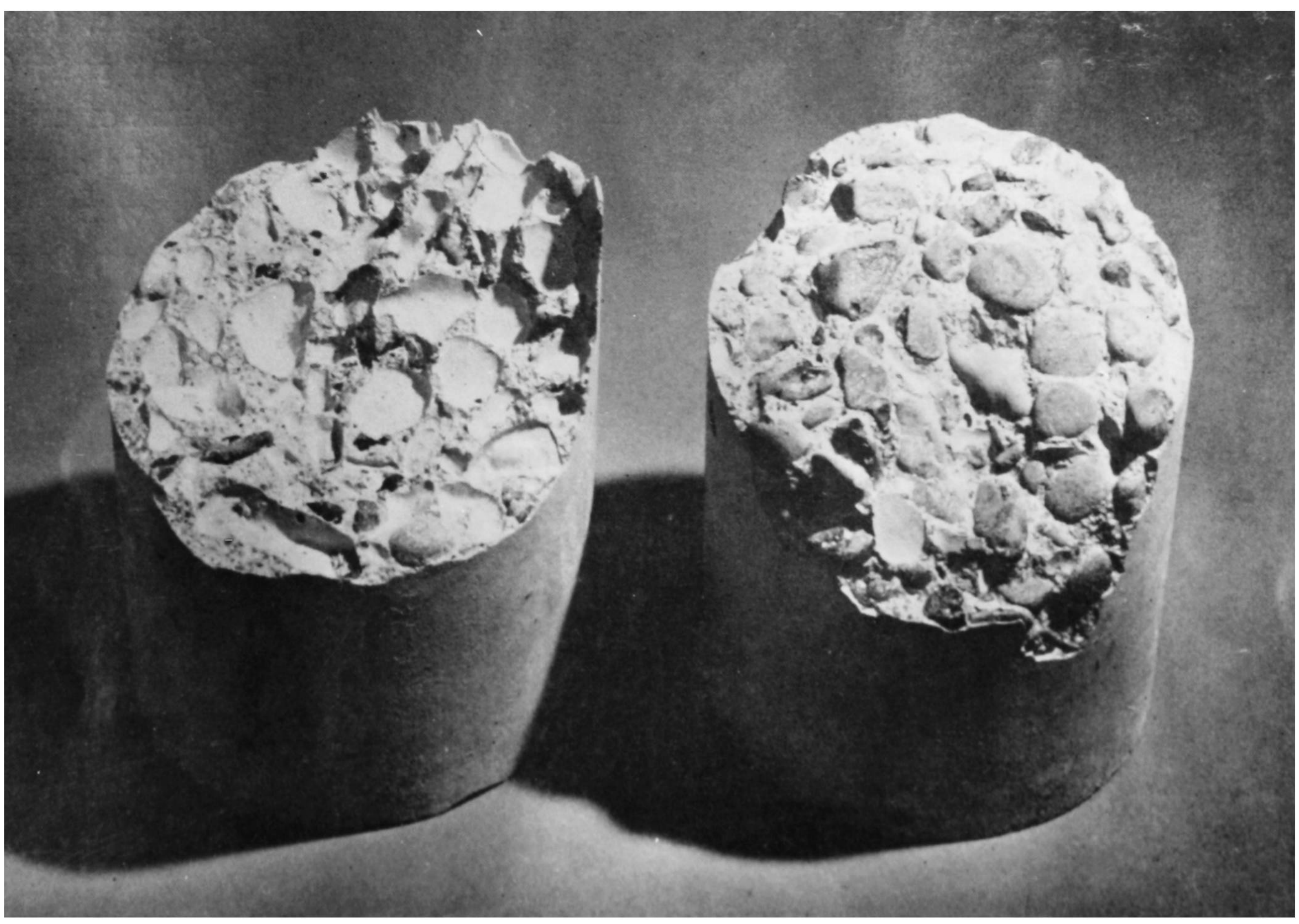


Fig. 29. ILLUSTRATION OF CONCRETE FAILURE INITIATING ON UNDER SIDE OF COARSE PARTICLES. (after Bache & Nepper-Christensen 1965)

after Bache and Nepper-Christensen (1965).

(2) that reinforcement in concrete also acts as an internal wall, in that its presence similarly disturbs the local packing of fine aggregates. The large circles in Fig. 28 could, for example, represent the cross sections through reinforcing bars.

The degree of disturbance will vary according to the attitude, i.e. vertical, horizontal or inclined, of the reinforcing bars, as well as to their size and spacing, but it is important to note that even if the reinforcement bars were to approach infinitely small cross section, their very presence as restrictive boundaries would still cause local increases in porosity.

2.3. Particle shape and shape distribution.

(a) Particle shape.

Several aspects of the influence of particle shape on porosity have already been mentioned during the course of discussion on the influence of absolute size and of container wall effect. These results are summarised in Figs. 3 & 4 (Frederick, 1962) and in Fig. 19 and Table 3 (present author). From these results, from the work of Kolbuszewski and Alyanak (1964) from which Fig. 30 is taken as an example, from the studies of Shergold (1953) and Hughes and Bahramian (1966), there is no doubt of the importance of the influence of particle shape on porosity. The major difficulty in the past appears to have been that of adequately defining and separating the influences of the several different aspects of particle shape. For example Shergold employed the rodded porosity of aggregate in a cylindrical container as the basis of his Angularity Number test (which now forms part of the current British Standard 812, 1961) without recognising that the measured value of voids was a function of the flatness ratio and elongation ratio as well as of the angularity of surface projections.

Kolbuszewski, Frederick and Alyanak used the parameters Roundness and Sphericity defined by Wadell (1932/3). These parameters could be considered adequate for those sections of their work which dealt with uncrushed quartz sands but were inappropriate and insufficient respectively for the description of crushed aggregates. Roundness, for example, being

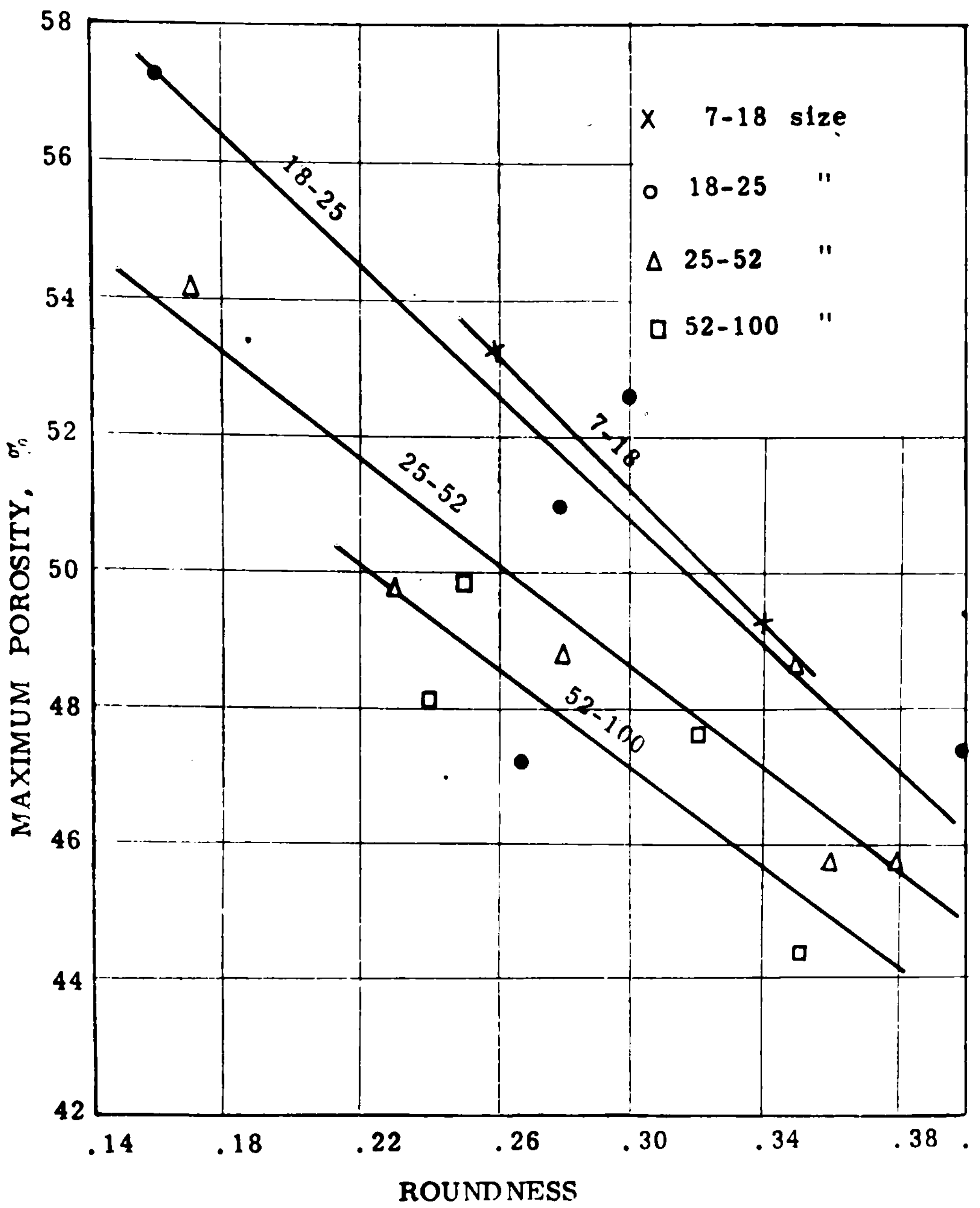


Fig. 30. INFLUENCE OF PARTICLE SHAPE (ROUNDNESS) ON MAXIMUM POROSITY. (after Kolbuszewski and Alyanak 1964)

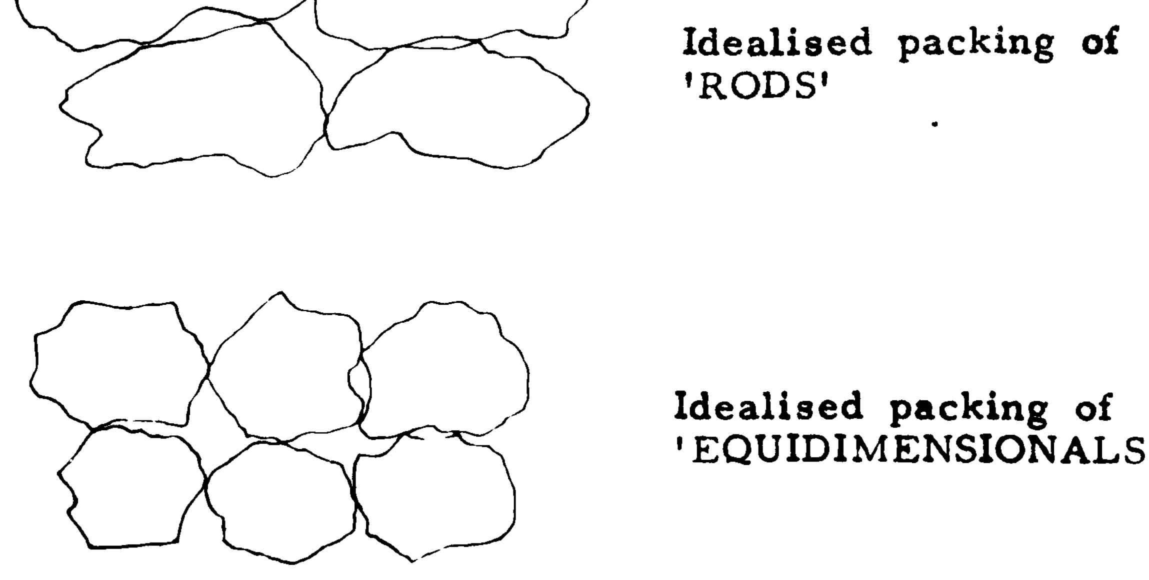


Fig. 31. DIAGRAM TO ILLUSTRATE THAT THE POROSITY OF 'ROD-LIKE' PARTICLES MAY BE EXPECTED TO BE LESS THAN THAT OF EQUI-DIMENSIONAL PARTICLES OF EQUAL ANGULARITY (IN ABSENCE OF DIFFERENTIAL CONTAINER WALL EFFECTS).

based on measurements of radii of curvature of surface projections is unsuitable for description of truly angular corners, while Sphericity on its own is an insufficient quantity for the description of the 3 dimensional shape of crushed aggregates, since with the greater range in flatness ratio than is present with natural quartz sands it is possible for particles of widely different shape to have the same sphericity. Use of Zingg's (1935) Shape Factor in addition to Sphericity would get over this problem, but a simpler solution would appear to be to use the Elongation Ratio and Flatness Ratio.

These points are further discussed by Lees 1964 (a) and (b) and Fig. 19 based on the shape classification proposed by the author in the latter paper shows how, when maximum porosity is plotted against degree of angularity the points for different shape categories follow separate trends within the band. It is interesting to note for example, that for a given angularity the porosity for the Rod-like particles is less than that of the Equidimensionals. In consideration of the fact that these two shape categories differ only in Elongation Ratio (while having similar high values of Flatness Ratio), the lower porosity of the Rod particles can be explained as due to their greater length for a similar cross section. Adjacent particles, able to pack occasionally with their long axes parallel, have a smaller number of the large "end-voids" in a given volume than would occur between the Equidimensional particles, as shown diagrammatically in Fig. 31 *.

Fraser (1935) invoked a similar argument in explanation of the lower values of porosity shown by a "Standard sand" of marine origin over that shown by spherical particles of shot (Table 4). In regard to the standard sand Fraser remarked "the grains are pretty uniformly disk-shaped and consequently possess many advantages of spheres in packing, but, at the same time, may be packed more tightly in the third dimension (with the flat sides together)", and continues regarding the other results:- "More angular materials show in marked degree the effect

* It will be noted that the lower porosity of the Rods over that of the Equidimensionals applies to the condition of equal angularity and at $\frac{A}{V} \times d = 0$, i.e. when container effects are removed. It does not however apply to the condition of particles of these shapes which are of the same sieve size. Here the order is reversed, the reason being that Rods and Equidimensionals of the same sieve size are not of the same true size (Lees 1964 (b)) and hence the container wall effect is greater for the Rods, which are in fact larger particles.

TABLE 4.

Influence of Grain Shape on Porosity.

Material	Specific Gravity	Type of Packing			
		Dry		Wet	
		Loose	Compacted	Loose	Compacted
Lead shot	11.21	40.06%	37.18%	42.40%	38.89%
Sulphur shot	2.024	43.38	37.35	44.14	38.24
Standard sand(marine)	2.681	38.52	34.78	42.96	35.04
Beach sand	2.658	41.17	36.55	46.55	38.46
Dune sand	2.681	41.17	37.60	44.93	39.34
Crushed calcite	2.665	50.50	40.76	54.50	42.74
Crushed quartz	2.650	48.13	41.20	53.88	43.96
Crushed halite	2.180	52.05	43.51	-	-
Crushed mica	2.837	93.53	86.62	92.38	87.28

after Fraser (1935)

of their angularity on packing, which is to increase the porosity".

(b) Shape Distribution - materials of constant size. Little attention has been given to this aspect of particle variation yet it is clearly just as possible to vary the proportions of particles of various shapes in a constant size material as to vary the proportions of particles of various sizes while keeping shape constant.

In an attempt to discover some of the effects of mixing shapes, in aggregate of the same sieve size, quantities of $\frac{3}{4}$" to $\frac{1}{2}$" Crushed Basalt were mixed with $\frac{3}{4}$" to $\frac{1}{2}$" Rounded Gravel in the Compacting Factor cylinder, in varying proportions and the maximum and minimum porosities determined.

The results shown in Fig. 32 demonstrate that there are no apparent minima in these curves, when Size Ratio = 1, as there is in general when mixtures of Size Ratio = < 1 are considered, but the form of the curve suggests that the addition of materials of the contrasting shape is only effective at proportions greater than 10%.

The difference between maximum and minimum porosity is of the order of 10% approximating to the value of 9% found by Kolbuszewski and Alyanak (1964). It may be observed that due to the steeper slope of

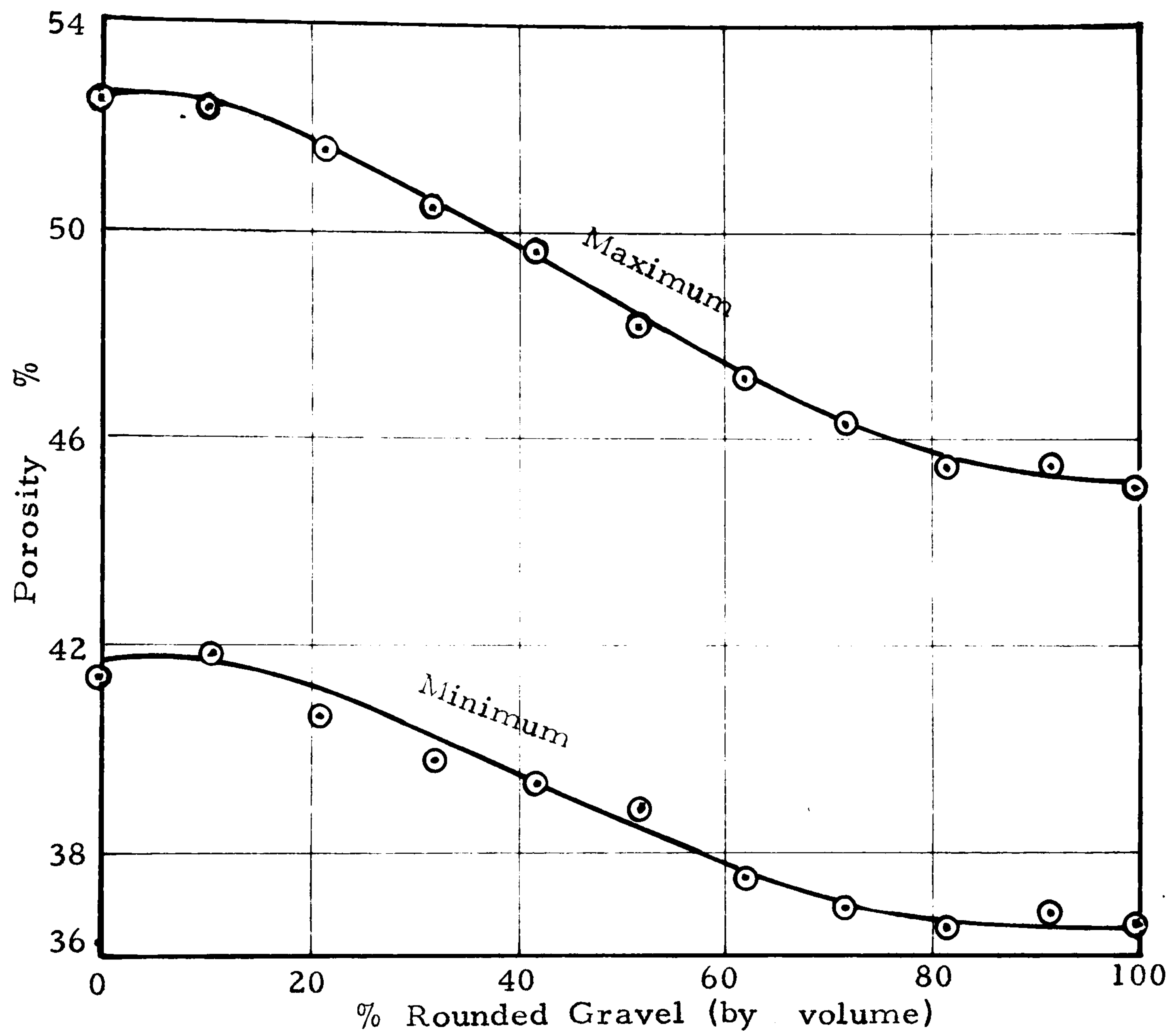


Fig. 32. POROSITY AGAINST % GRAVEL IN A MIXTURE OF ½" ROUNDED GRAVEL AND ½" CRUSHED BASALT.

the maximum porosity curve the range in fact increases from about 9% for 100% Rounded Gravel to about 11% for 100% Crushed Basalt, confirming the general view that the range between maximum and minimum porosity increases with reduced sphericity and increasing angularity.

(c) Shape Distribution - materials of varying sieve size. From the point of view of aggregate grading design a more important aspect of the influence of shape distribution occurs in the frequent cases where shape varies with size. This may happen even where aggregate from a single source is concerned, as for example with crushed rocks which are subjected to change in crusher type and/or reduction ratio and/or choke-feeding condition at various stages of crushing, and as also with water deposited sands which decrease in roundness with decreasing size due to the greater cushioning effect of water on the smaller particles.

Even more extreme contrasts in shape at different size levels occur as a result of deliberate blending of materials from different sources, as for example in asphalt gradings to B.S. 594, Table 7 which specifies crushed rock coarse aggregates to be combined with sand fine aggregates or for example in concreting sands which, in order to satisfy some specified grading requirement, may incorporate a quantity of crushed gravel to supply the wanting sizes.

The importance of these shape contrasts with size is recognised. Allowance for these effects and for the effect of absolute size on mode of packing, for the effect of compactive effort and of the container wall form the basic philosophy of the method proposed in Part II for the design of aggregate gradings.

CHAPTER 3.

STUDIES OF VOID CHARACTERISTICS

Aggregate grading design, having as its object the filling of space with particles has in its development made frequent reference to the study of particles. Less attention has been paid to the character of the inter particle spaces, yet from the nature of the problem this would seem to be no less important a subject for study.

The comparative rarity of studies on interparticle voids undoubtedly arises out of their continuous and intangible nature. However means have been developed for overcoming some of the difficulties so arising.

The approach adopted has been to use one of the newer plastic materials (Welvic L11/17 P.V.C.) as a void filling medium, which because of its pliable and sectile nature can be removed from the particles which served as its mould, for the purpose of visual appraisal and measurement. In this way the voids can be rendered both tangible and arbitrarily discontinuous. The arbitrary division has to be performed with great patience, but careful section of the plastic with knife and scissors along the planes of minimum interparticle distance between 2 particles and across the "throat openings" between 3 or more particles enables isolated void units to be extracted. Figures 33 to 40 illustrate some of the void units obtained from packings of a variety of particle shapes. The measurements which can be made upon these void units and the uses which can be made of these measurements will be referred to below.

3.1. Voids in theoretical packings of uniform spherical particles.

The first steps in the description and measurement of the characteristics of inter-particle voids were taken by Fraser (1935).

Apart from permeability characteristics (which will not be dealt with here), Fraser described the void system by reference to the largest spherical particles which could (a) occupy the voids, supposing that they had reached that position during deposition and (b) pass through the throat openings from one void cell to an adjacent cell.

Fig. 33. INTER AGGREGATE VOIDS

Fig. 34. INTER AGGREGATE VOIDS

Fig. 35. INTER AGGREGATE VOIDS.

Fig. 36. INTER AGGREGATE VOIDS.

Fig. 37. INTER AGGREGATE VOIDS.

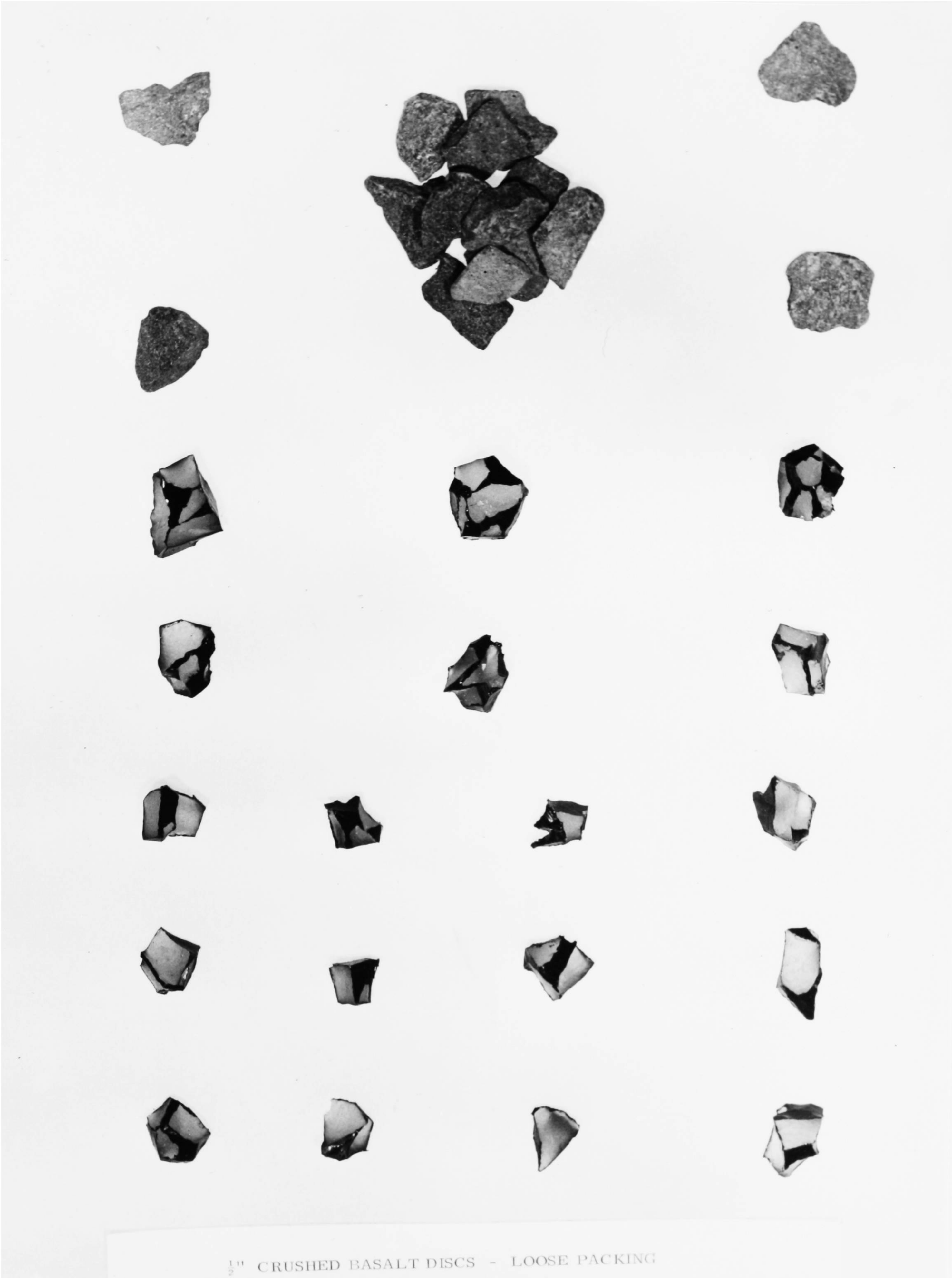


Fig. 38. INTER AGGREGATE VOIDS.

Fig. 39. INTER AGGREGATE VOIDS.

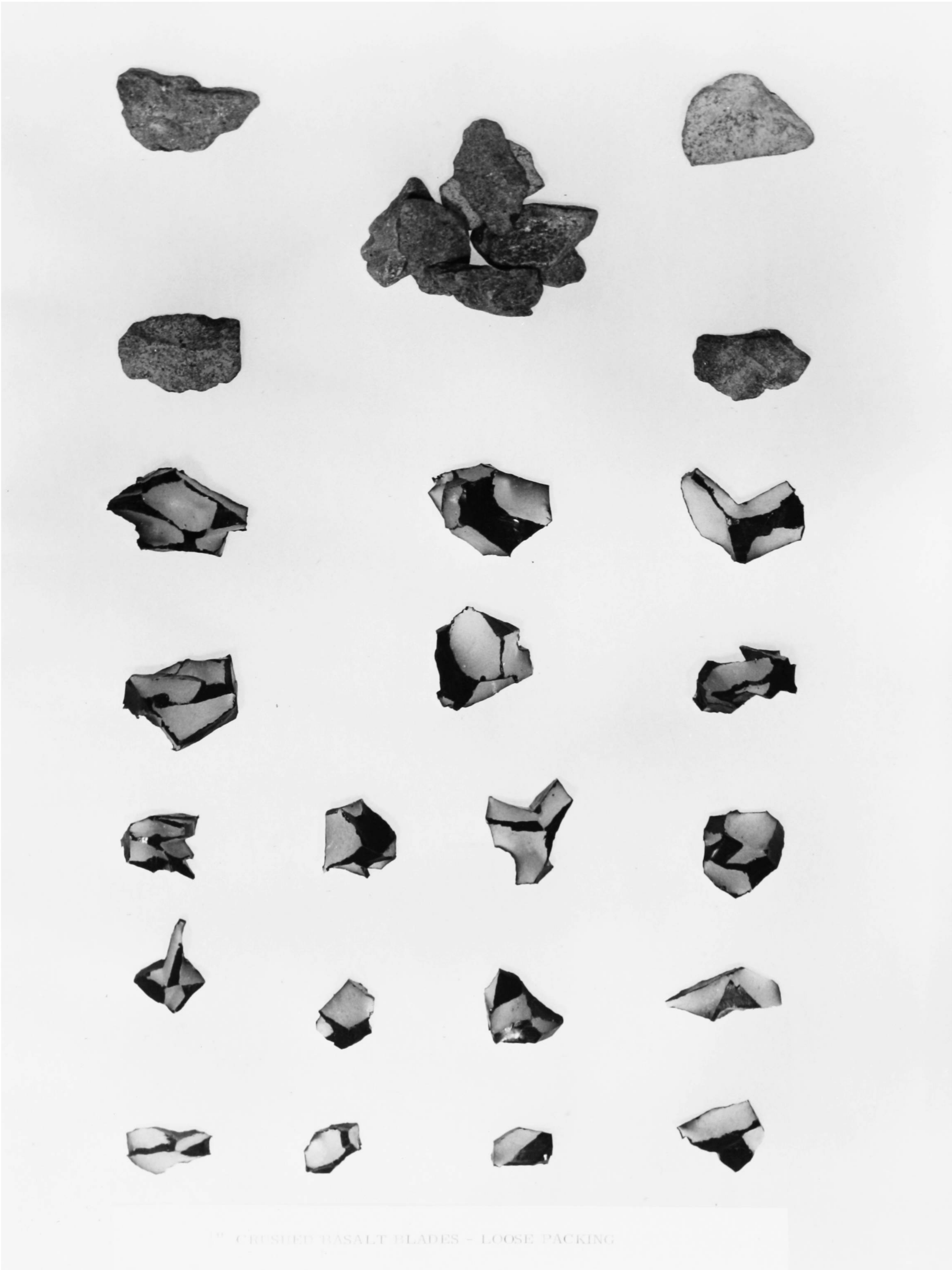


Fig. 40. INTER AGGREGATE VOIDS.

The ratio of the diameter of the occupation sphere to the diameter of the major sphere was termed by Fraser the critical ratio of occupation. The ratio of the diameter of the sphere that could pass through one of the "throats", to the diameter of the major sphere was termed the critical ratio of entrance.

Fig.41 shows examples of void units cut by the technique described, from the cubical (loosest) and from the rhombohedral (closest) packing of uniform spheres.

It will be noted that the void of a unit of rhombohedral packing contains in fact 3 cells, one a concave cube and the other two concave tetrahedra. The critical ratio of entrance is the same for both cell types but the critical ratio of occupation differs between the concave cube and the concave tetrahedra. The void of a unit of cubical packing contains but one cell - a concave octahedron, with larger values of both critical ratios than are present in the rhombohedral packing.

The values of the critical ratios shown in the figure were calculated from geometric considerations by Fraser, but may be checked by measurements on the voids cells dissected from the mass. The critical ratio of entrance would be calculated from the diameter of the largest circle which could be fitted to the throat section - (painted black in Fig.41), and the critical ratio of occupation calculated from a measurement of the diameter of the largest circle on a plane cut across the centre of the void cell.

This was in fact the technique adopted in the subsequent examination of voids dissected from packings of actual particles.

The use of a spherical diameter to express the critical ratios cannot be regarded as in all respects ideal. Even when packings of major spherical particles are considered, it does not follow that only particles of smaller size which are of spherical form could (a) occupy or (b) enter the voids in the system. When the major particles are non spherical further errors are introduced by the use, in the calculation of critical ratios, of an equivalent spherical diameter as a measure of the size of the major particles. However it appears that at least for the present, the inaccuracies that undoubtedly ensue from the unjustified assumptions made, must be tolerated in view of the extreme difficulties involved in trying to deal with the problem in any other way which would aim at giving more accurate expression to the actual shape of particles used.

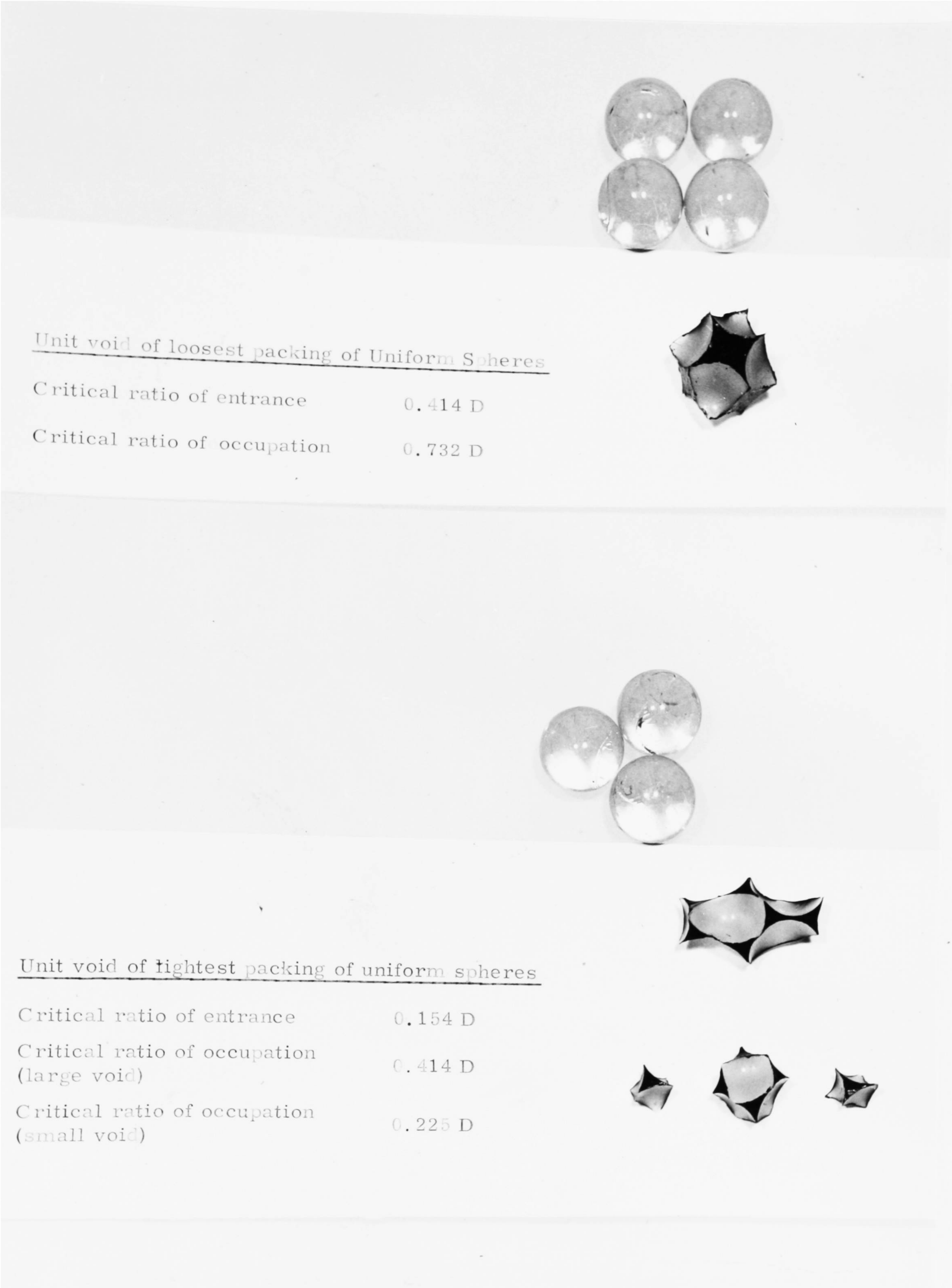


Fig. 41. CRITICAL RATIOS OF ENTRANCE AND OCCUPATION FOR PACKINGS OF UNIFORM SPHERICAL PARTICLES.

3.2 Voids in practical packings of gravel and crushed rock particles.

The void impregnation and dissection technique described enables the direct measurement of critical ratios of entrance and occupation for void cells cut from packings of actual particles. Figs. 33 to 40 illustrate some characteristic void cells from loose packings of particles of the 8 previously listed shape categories. (The larger units towards the top of each figure represent several void cells before final dissection). Each void entrance was painted black and measured by means of the Shadomaster measuring projector (Fig.42). This instrument enables the image of the object under investigation to be projected on a screen to which may be applied an accessory plate marked with numbered concentric circles (Fig.43). The diameter of a spherical particle which could pass through this entrance is thus readily determined and the critical ratio of entrance calculated as the ratio of this diameter to the equivalent spherical diameter of the particles which formed the packing. The critical ratio of occupation is similarly determined from measurement on a section out along any diametral plane of the maximum inscribed sphere of the void cell under consideration.

The entrance and occupation diameters cover a range of values whose distribution, as with particles, can be represented on a curve of summation percentage against size. From each of 18 packings investigated, between 60 and 350 entrance diameters and between 18 and 51 occupation diameters were measured, care being taken that the void units so measured were from zones at least 2 particles depth inward from any boundary. These results were then plotted as indicated on graphs of summation % (by number) against size.

Figs. 44 to 47 show the size distribution of the entrance and occupation diameters for loosest packings of $\frac{3}{4}$" to $\frac{1}{2}$" Equidimensional Rounded Gravel and of $\frac{3}{4}$" to $\frac{1}{2}$" Equidimensional Crushed Basalt and have been chosen as typical examples of the 36 such graphs constructed from the measured data. From these graphs the D_{50} size was obtained, i.e. the diameter at which 50% of the diameters were this size or less. For distributions which have, as in this case, a similar form, the D_{50} size is a quantity which may be used to compare different void systems. Table 5 gives the D_{50} size for all cases examined. It will first be noted that the entrance and occupation diameters for the Disc particles are in general of a lower order than those of the other particles. The explanation lies in the actual smaller size of the Disc particles compared with the other particles, (the origin of this difference in size, even among particles retained between the same sieves is discussed

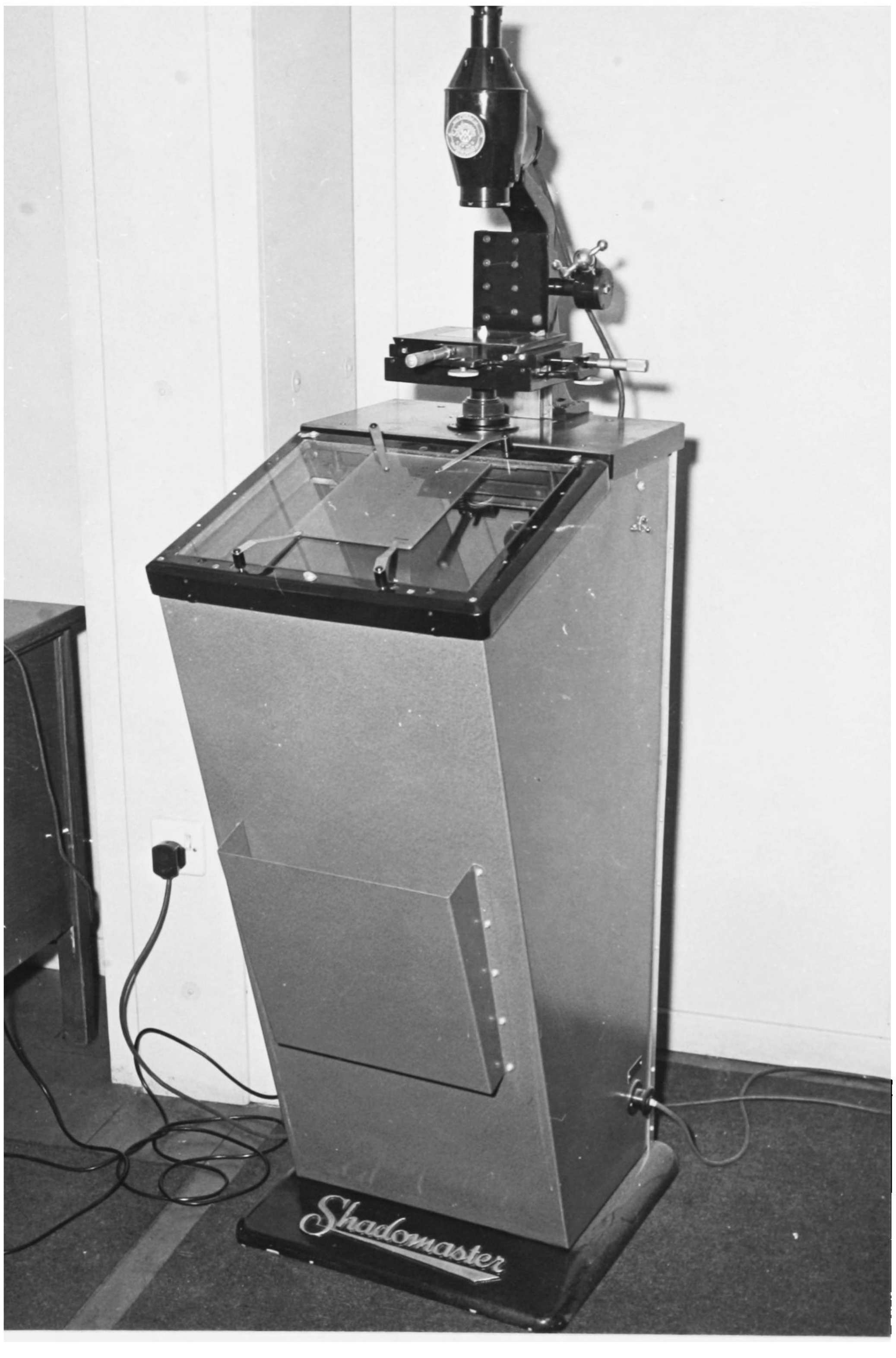


Fig. 42. SHADOMASTER MEASURING PROJECTOR

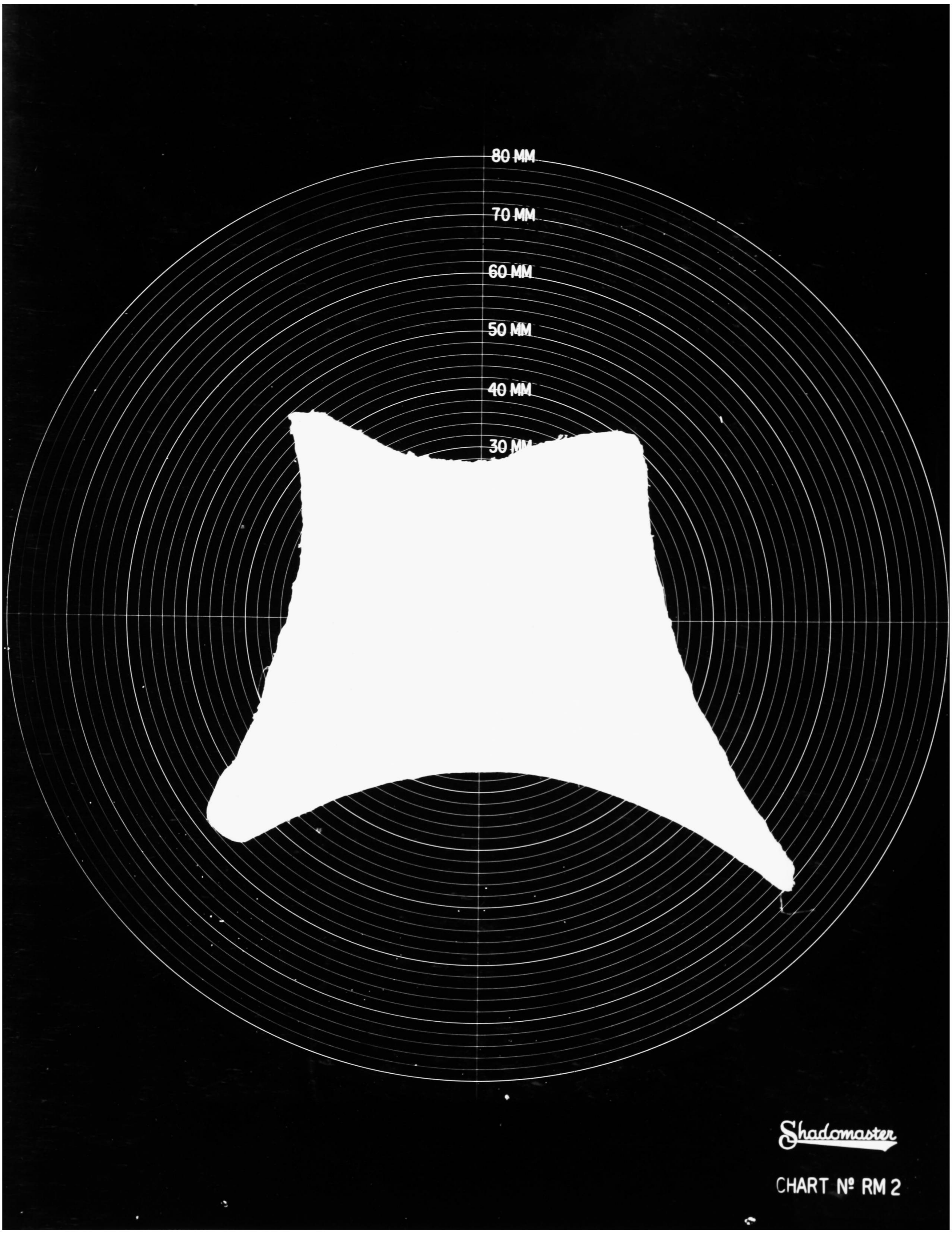


Fig. 43. ILLUSTRATION OF METHOD OF DETERMINATION OF ENTRANCE AND OCCUPATION DIAMETERS BY USE OF SHADOMASTER ACCESSORY CHART.

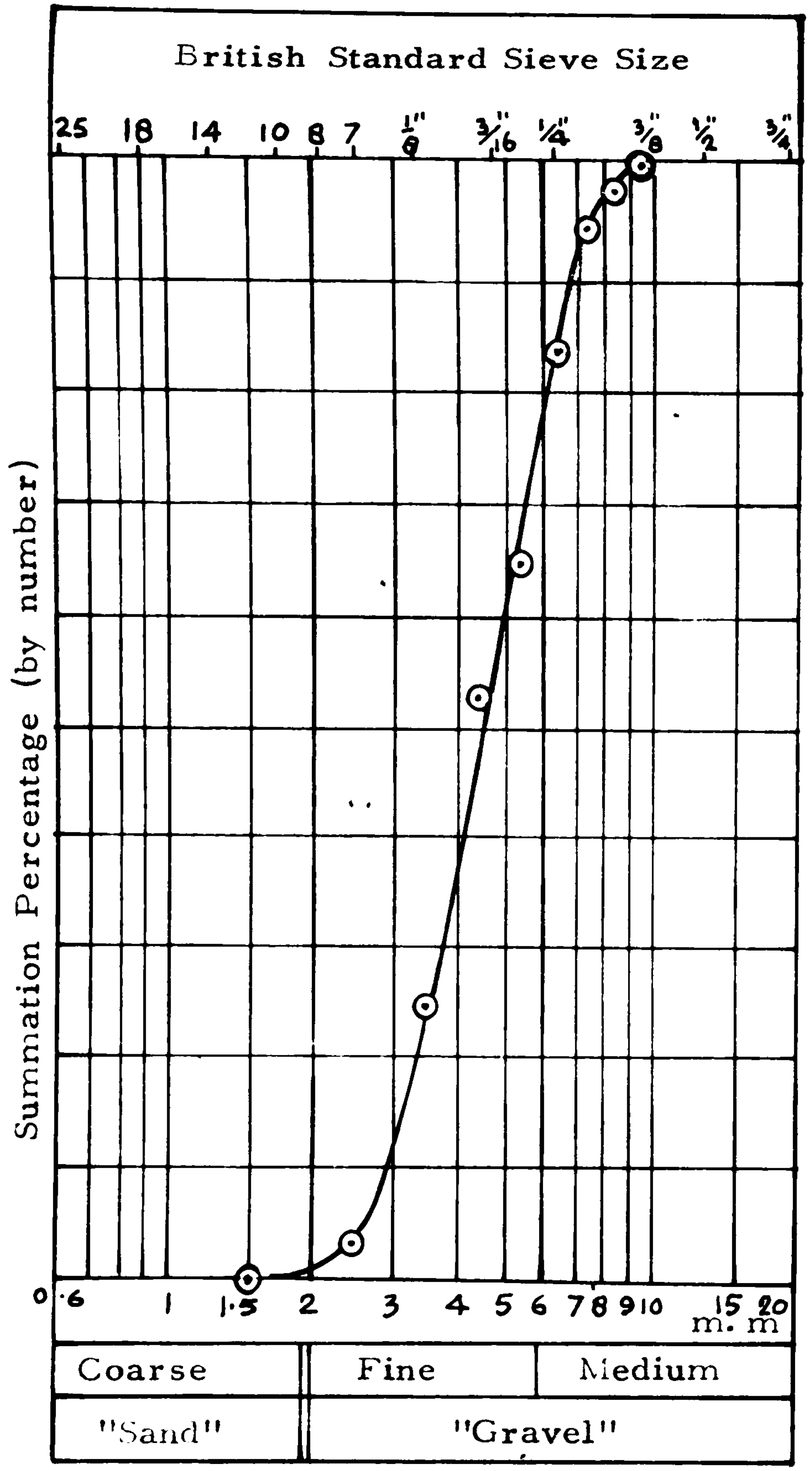


Fig. 44. ENTRANCE DIAMETER - ½"
EQUIDIMENSIONAL ROUNDED GRAVEL
LOOSEST PACKING

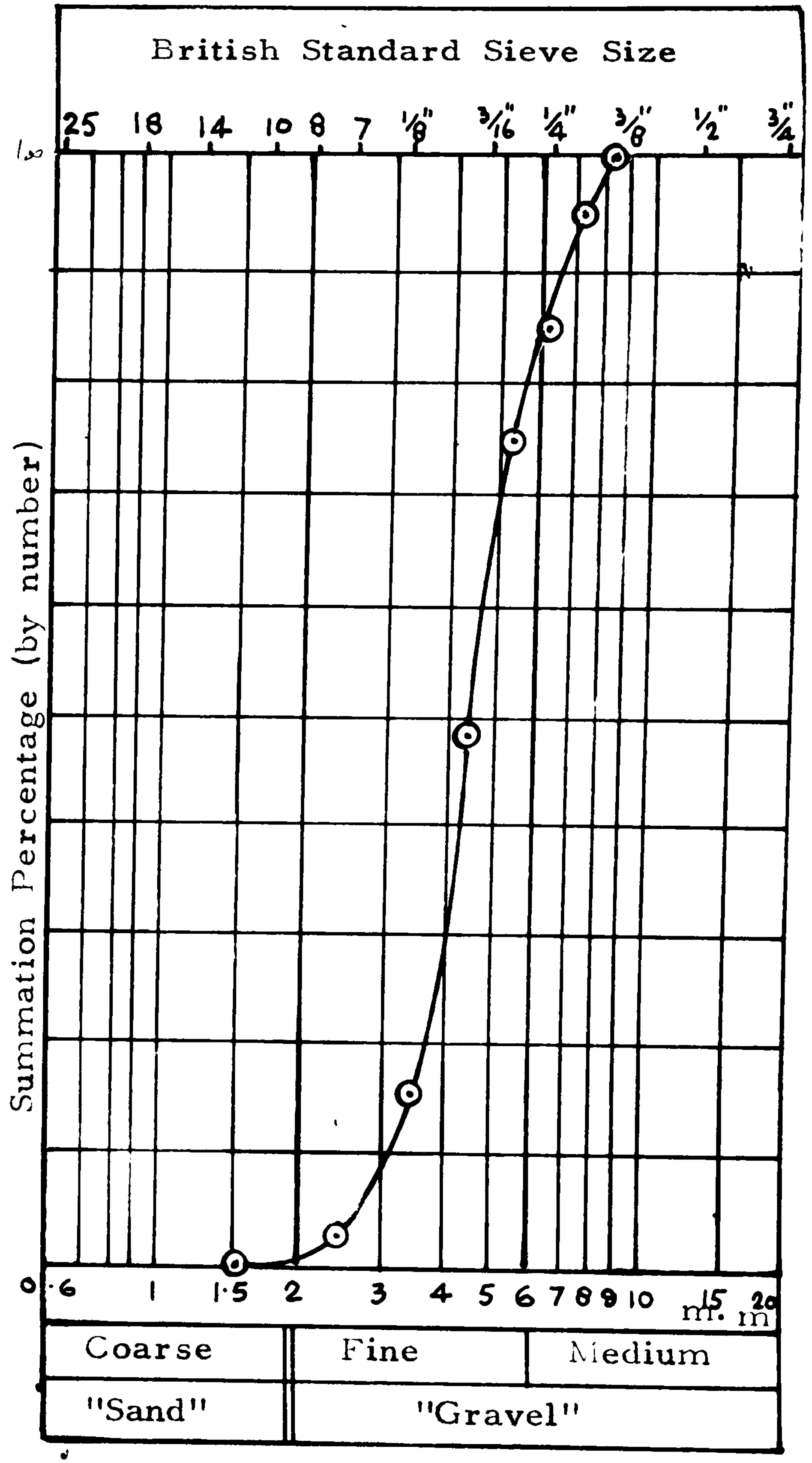


Fig. 45. ENTRANCE DIAMETER - ½"
EQUIDIMENSIONAL CRUSHED BASALT
- LOOSEST PACKING

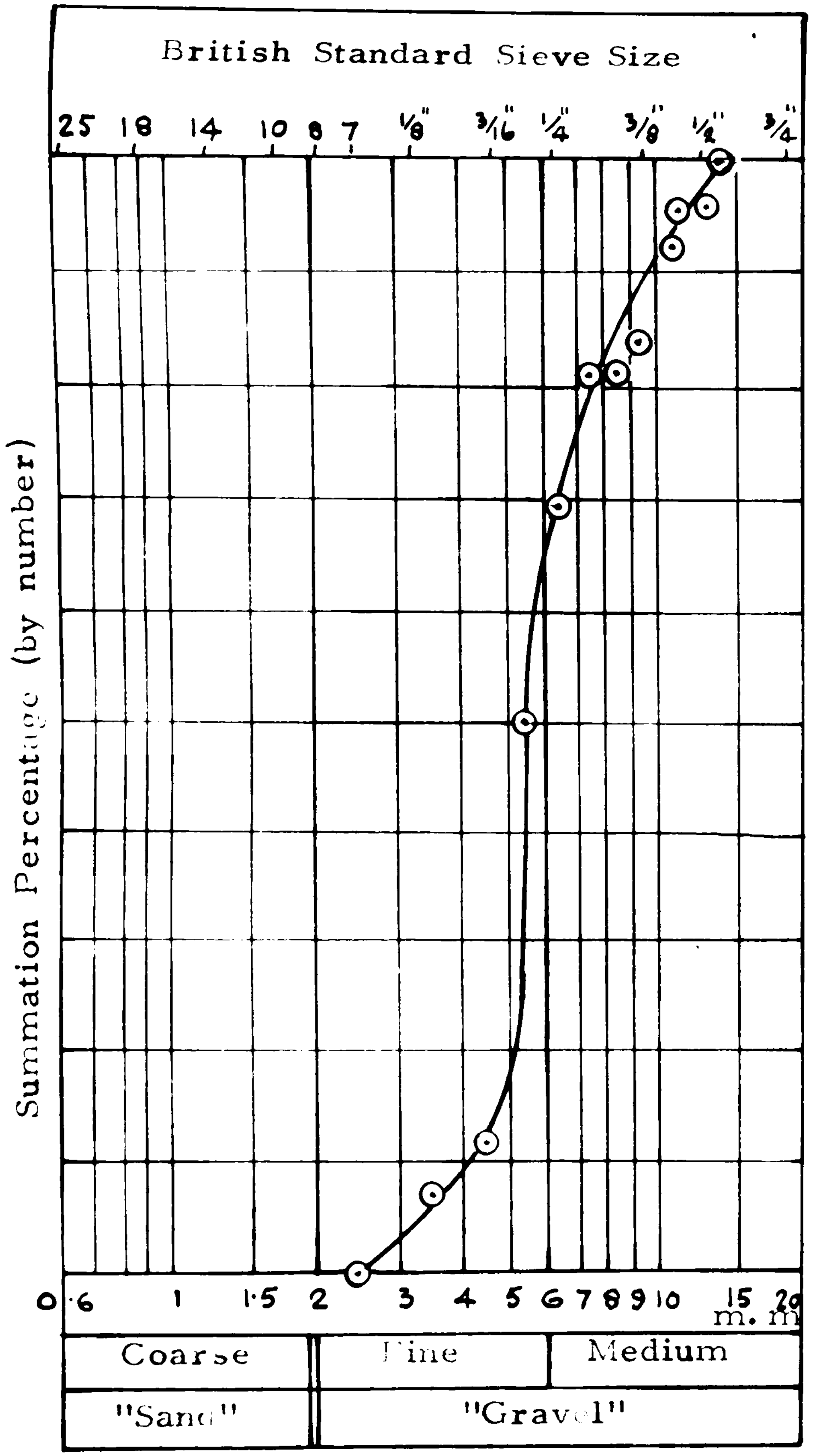


Fig. 46. OCCUPATION DIAMETER - ½" EQUIDIMENSIONAL ROUNDED GRAVEL - LOOSEST PACKING

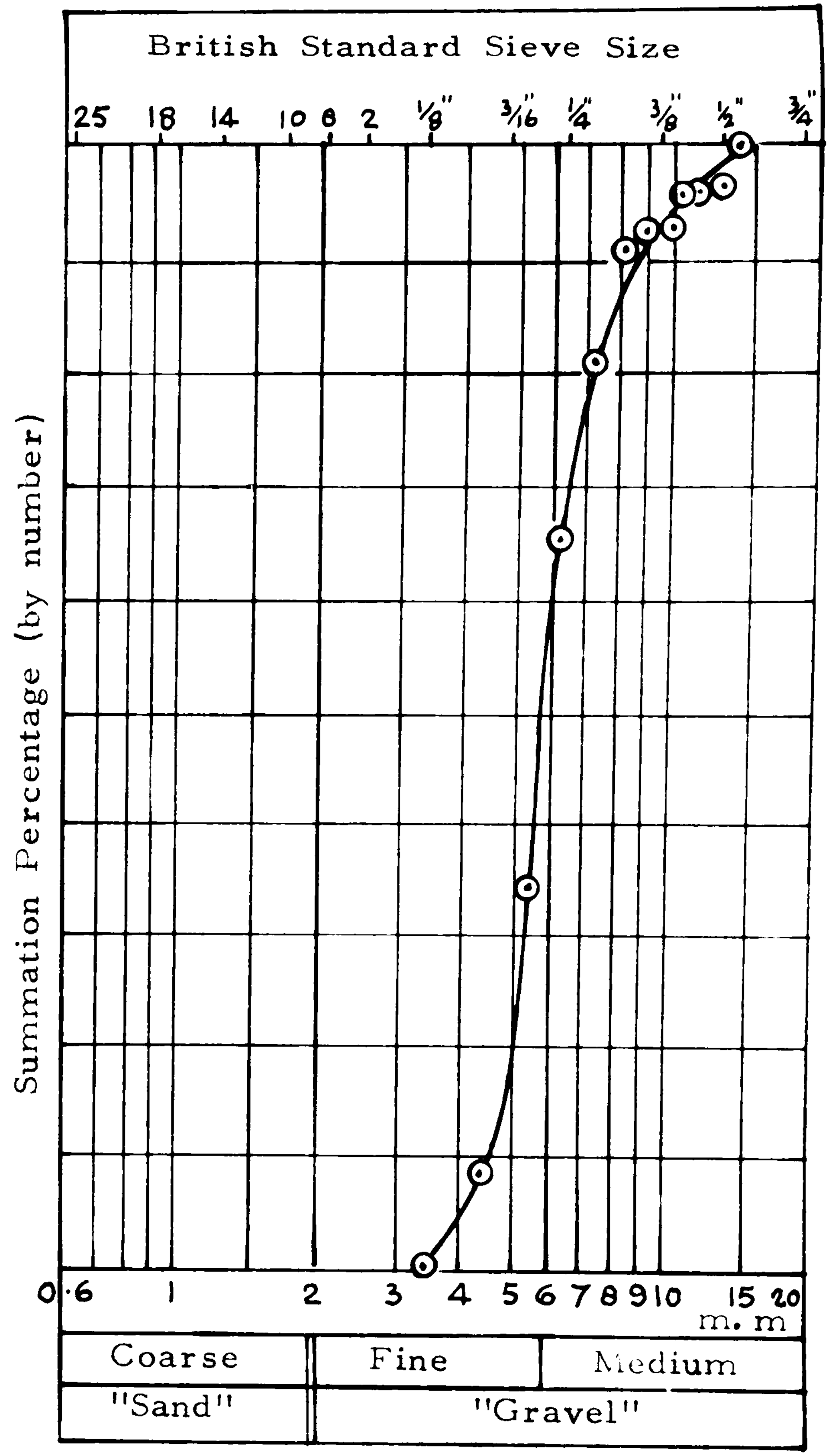


Fig. 47. OCCUPATION DIAMETER ½" EQUIDIMENSIONAL CRUSHED BASALT - LOOSEST PACKING

TABLE 5. CRITICAL RATIOS OF ENTRANCE (Cr E) & CRITICAL RATIOS OF OCCUPATION (Cr O) FOR LOOSE PACKINGS OF AGGREGATES OF VARIOUS SHAPE CATEGORIES.

½" Rounded Gravel	E.S.D. cm.	Entrance D_{50} cm	Entrance $\frac{D_{50}}{E.S.D.}$ (=Cr E)	Occupation D_{50} cm	Occupation $\frac{D_{50}}{E.S.D.}$ (= Cr O)
EQUIDIMENSIONALS	1.65	0.45	0.27	0.55	0.33
DISCS	1.47	0.32	0.22	0.52	0.35
RODS	1.98	0.47	0.24	0.77	0.39
BLADES	1.71	0.45	0.26	0.69	0.40
AVERAGE			0.25		0.37
½" Crushed Basalt					
EQUIDIMENSIONALS	1.56	0.45	0.29	0.59	0.38
DISCS	1.34	0.37	0.28	0.58	0.43
RODS	1.83	0.47	0.26	0.72	0.39
BLADES	1.60	0.47	0.29	0.70	0.44
AVERAGE			0.28		0.41

in the author's paper 1964 (b)). Being of significantly smaller size it follows that their voids will also be smaller - and this is so - in spite of the Discs showing for example a greater porosity than the Equidimensionals. C_r entrance and C_r occupation are, however, not significantly different.

The second observation, that the entrance and occupation diameters of the Crushed Basalt are not appreciably different from those of the Rounded Gravel is perhaps not so readily understood. Since the crushed particles have invariably a higher porosity than the rounded gravel particles of the same shape category, while the particle size is not significantly different, it might have been expected that the entrance and occupation diameters for the former would be greater than those for the latter.

However it has already been suggested that the use of a spherical form for the entrance and occupation particles may lead to some misconceptions if not carefully treated and this could be so in this case. That is to say that the greater porosity of the crushed particles is apparently due to the fundamentally different shape of their voids, as shown in Figs. 33 to 40, rather than that the particles are in any sense further apart, or the voids between them of greater diameter at either the 'throats' or in the expanded sections. Thus while the use of the entrance diameter gives useful information on the size of near spherical particles which could filter through a fixed bed of the material, and the occupation diameter indicates the maximum size that can be accommodated singly in a single void without causing dilation of the coarse aggregate structure, neither of these diameters bears any relation to the porosity values, since they give no information on the space remaining when the void cell is occupied by its largest sphere, or of how many additional particles, and of what sizes, would be needed to fill this space.

The values given in Table 5 referred to loosest packings of the aggregates concerned.

Some measurements were also made of critical ratios of entrance of somewhat denser * packings of the same aggregate categories, in order to observe the effect of packing on this ratio. These results given

* The packings in question were not considered to be the densest possible since the practical problems associated with filling the voids necessitated using a rather small container. This would of course have the effect of opening up the structure.

TABLE 6. CRITICAL RATIOS OF ENTRANCE (Cr E) FOR MEDIUM PACKINGS OF AGGREGATES OF VARIOUS SHAPE CATEGORIES

½" Rounded Gravel	E.S.D. cm	Entrance D_{50} cm	$\frac{\text{Entrance } D_{50}}{\text{E.S.D.}}$ = Cr E
EQUIDIMENSIONALS	1.65	0.35	0.21
DISCS	1.47	0.33	0.22
RODS	1.98	0.38	0.19
BLADES	1.71	0.34	0.20
AVERAGE			0.21
½" Crushed Basalt			
EQUIDIMENSIONALS	1.56	0.36	0.23
DISCS	1.34	0.30	0.23
RODS	1.83	0.45	0.25
BLADES	1.60	0.38	0.24
AVERAGE			0.24

in Table 6 confirm a tendency for the Cr E to fall with the denser packing but further measurements on these cells were discontinued (a) because of the uncertainty in the nature of the packing produced, discussed in the footnote and (b) because, for reasons which will be given in Part II, now to follow, the author considered that the critical ratios for loosest packings were more relevant to the problem of the design of aggregate gradings than the critical ratios for densest packings.

PART II. AGGREGATE GRADING DESIGN.

The most commonly studied aspect of the design of aggregate gradings is that which aims at maximum solid volume density (i.e. minimum porosity), although it will also be apparent that if the factors which influence particle packing are understood well enough to achieve or approach this aim, then the design of gradings to a controlled void content - not necessarily the minimum - will also be possible. The quest for minimum voids gradings is long established, that for controlled porosity gradings is relatively new, having arisen with the relatively recent technology of bituminous mixtures. It is frequently claimed that the requirement of flexibility in such materials demands sufficient voids to accommodate a minimum quantity of the bituminous binder, without undue separation of the aggregate particles by the binder and consequent loss of frictional resistance.

The first essential, before studies of multi-component systems may be undertaken, is to understand the factors involved in the relationship between aggregate proportions and porosity in 2 component systems.

CHAPTER 4.

2 - COMPONENT SYSTEMS - PROPORTIONS FOR MINIMUM VOIDS

The first major step in the study of the proportions for minimum voids in 2-component systems was due to Furnas (1928).

On the basis of an unspecified number of experiments Furnas produced the generalised and idealised curves (Figs. 48 to 50) showing the relationship between "normal voids" * of the separate components, size ratio, proportion by volume of coarse to fine aggregate, and the voids in the mix at any given proportion. Each set of curves is for one value of "normal voids" and adopts the same value for both coarse and fine components.

Each curve has a clear minimum, and for any given proportion of fine to coarse aggregate, including that at the minimum position, it may be seen that the voids in the mix tends to decrease with decreasing size ratio. The lowest minimum point is that for the size ratio = 0 curve, and since this value of size ratio implies an infinitely small fine aggregate (or infinitely large coarse aggregate) which of course is impossible to obtain, it may be enquired how this line was plotted.

* "Normal voids" is defined by Furnas as "the amount of voids in a system of broken solids when the particles composing the bed have been allowed to fall into place individually from a short distance above the bed. If the pieces are allowed to fall into place one by one, the opportunity for bridging is eliminated, and the pieces tend to distribute themselves in a random manner which appears to be definite and easily duplicated. Experimentally it is not necessary to drop the particles on the bed one by one, but it is necessary to pour the stream of particles quite slowly and thus to avoid bridging."

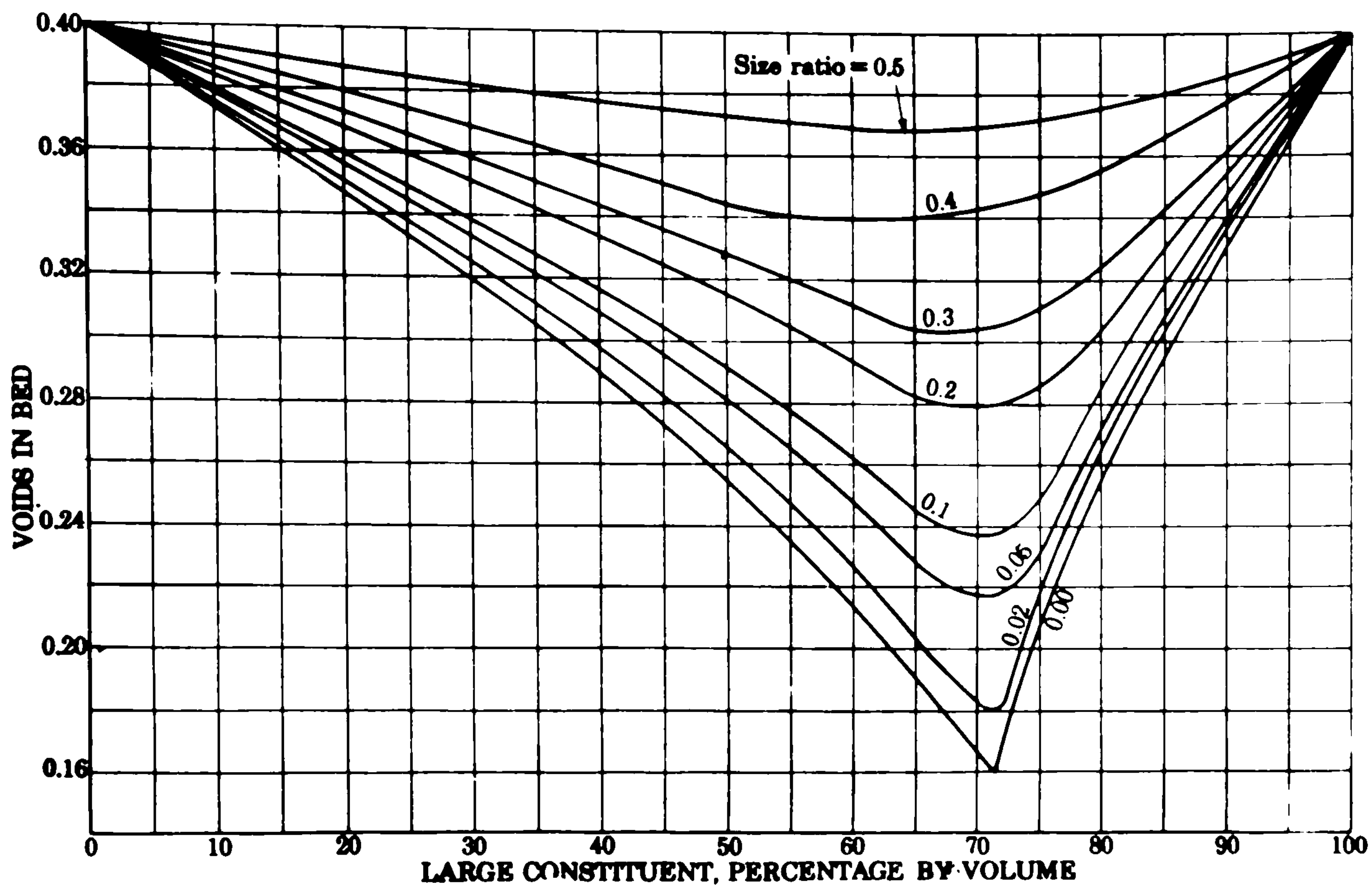


Fig. 48. VOIDS IN MIX/MIX PROPORTIONS (NORMAL VOIDS = .40) (after Furnas 1928)

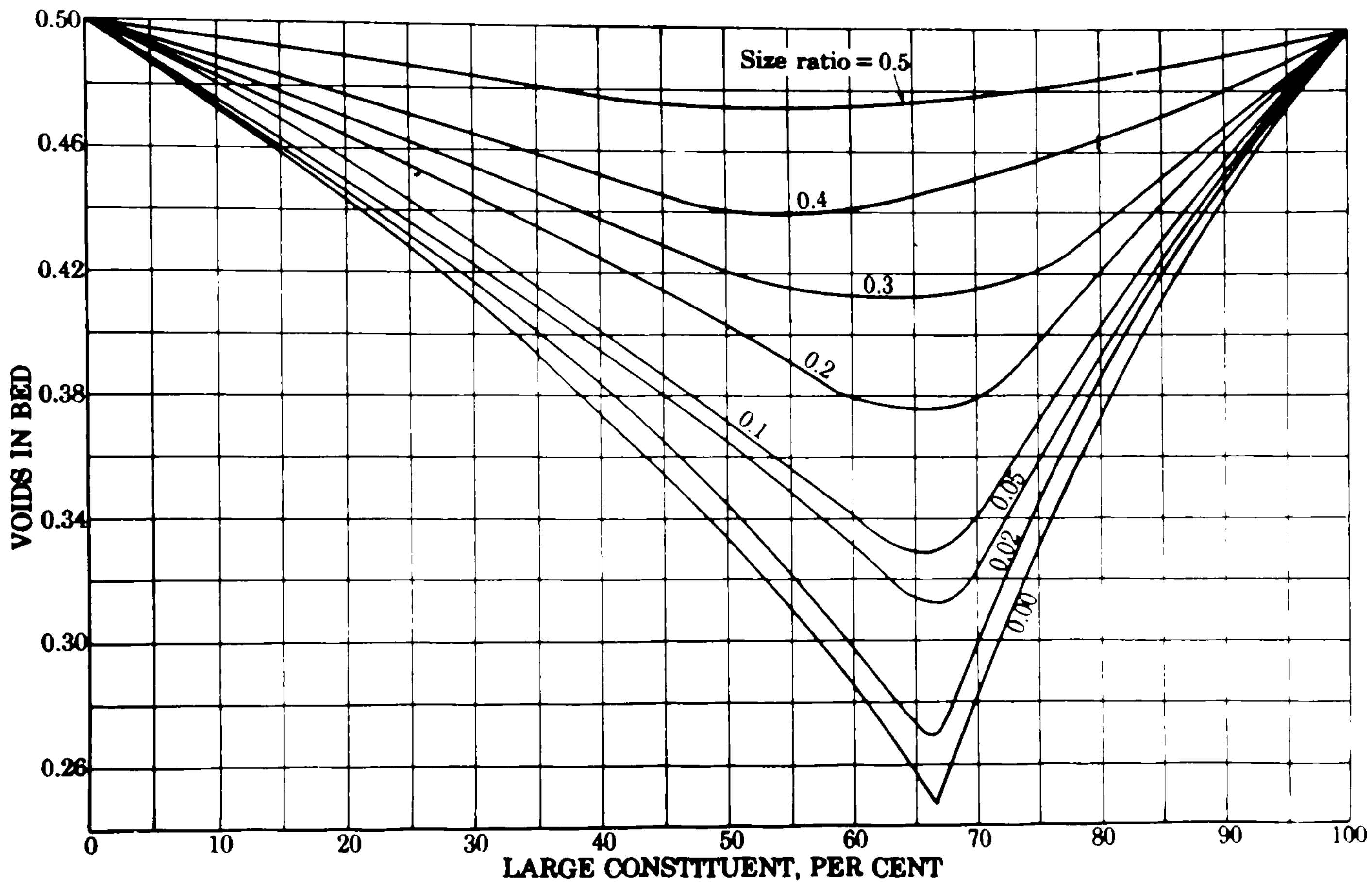


Fig. 49 VOIDS IN MIX/MIX PROPORTIONS (NORMAL VOIDS = .50) (after Furnas 1928)

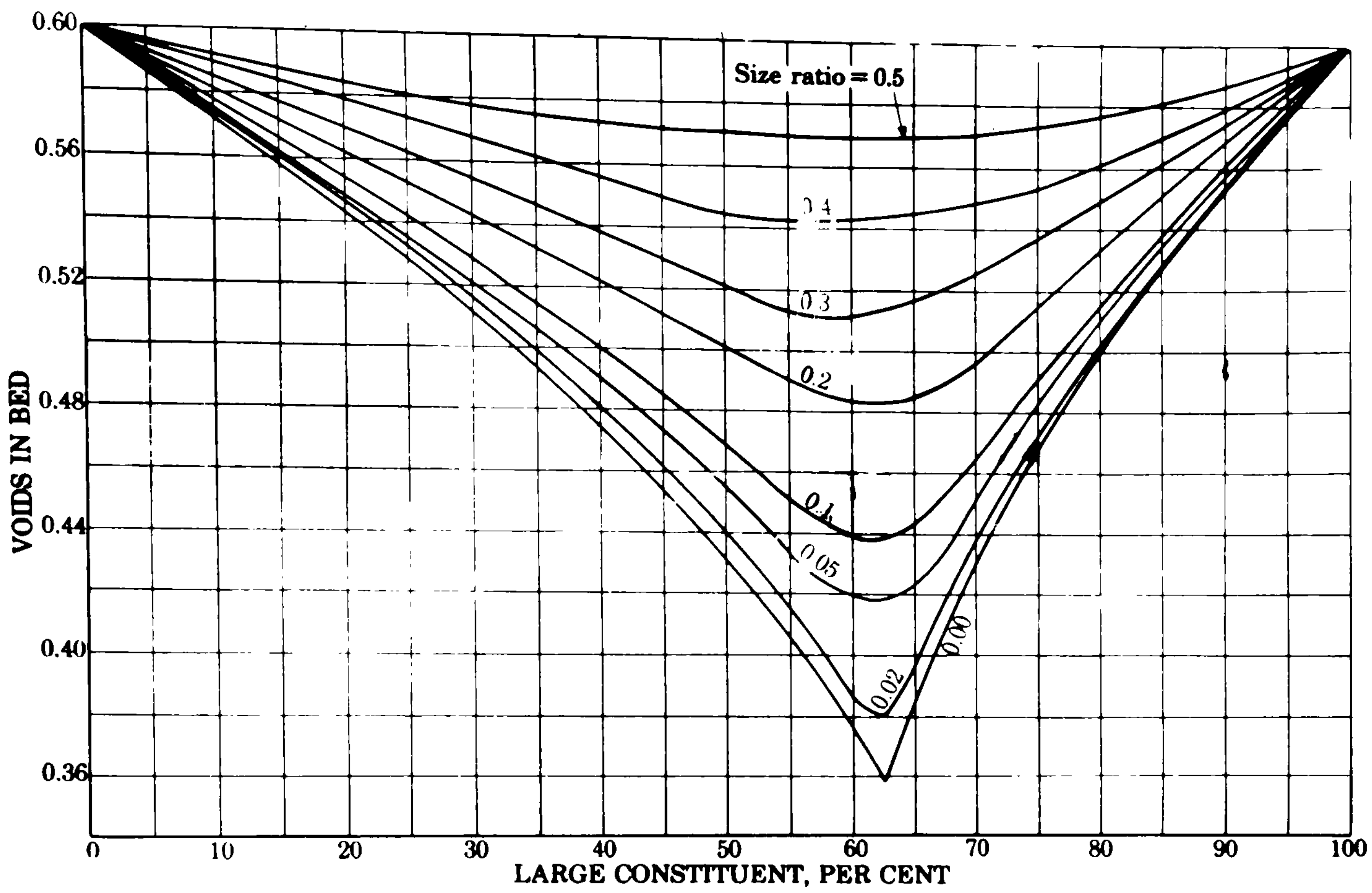


Fig. 50. VOIDS IN MIX/MIX PROPORTIONS (NORMAL VOIDS = .60) (after Furnas 1928)

This is done by plotting the normal voids values, recalculated as volume occupied per unit weight, on a Specific Volume graph and constructing lines to theoretical minimum points on the opposite vertical ordinates.

$$\text{Specific Volume} = \frac{1}{\text{True Specific Gravity} \times (1 - e)} \qquad (5)$$

where e = voids in the bed.

4.1. Specific volume and specific void content relationships for the theoretical case of size ratio = 0.

Considering the example illustrated in Fig.51 in which the specific volumes for 3 different 2-component systems were calculated from the above equation, Furnas showed for example that the triangle ABC represented the total field of possible existence of the mixtures of Nos. 000 and 12 shot. The line A-B-Origin connects the specific volume of the coarse aggregate alone with the value of the partial specific volume of the fine aggregate in any mix of the two. (Partial specific volume equals the contribution to the total bulk volume provided by the ingredient concerned). Now, if a coarse aggregate skeletal framework exists, occupying a certain bulk volume then the introduction of fine (theoretically - infinitely fine) material into this structure without disturbing it, adds weight to the system without increasing the volume. This, on the above definition of the term, represents a partial specific volume of 0. A line joining A to the origin therefore represents the lower limit of possible existence of the system (since the partial specific volume cannot be negative) - except that at B another mechanism begins to operate. Here the quantity of fine material would have become great enough to fill the voids of the coarse aggregate skeleton so that it is no longer possible to add weight without also adding volume. What determines the location of B ?

Considering now the line CBD, C represents the specific volume of the 100% fine fraction, while D represents the partial specific volume of the coarse aggregate combined with the fine in any mix. It will be noted that this time D is not at zero but at the value equal to

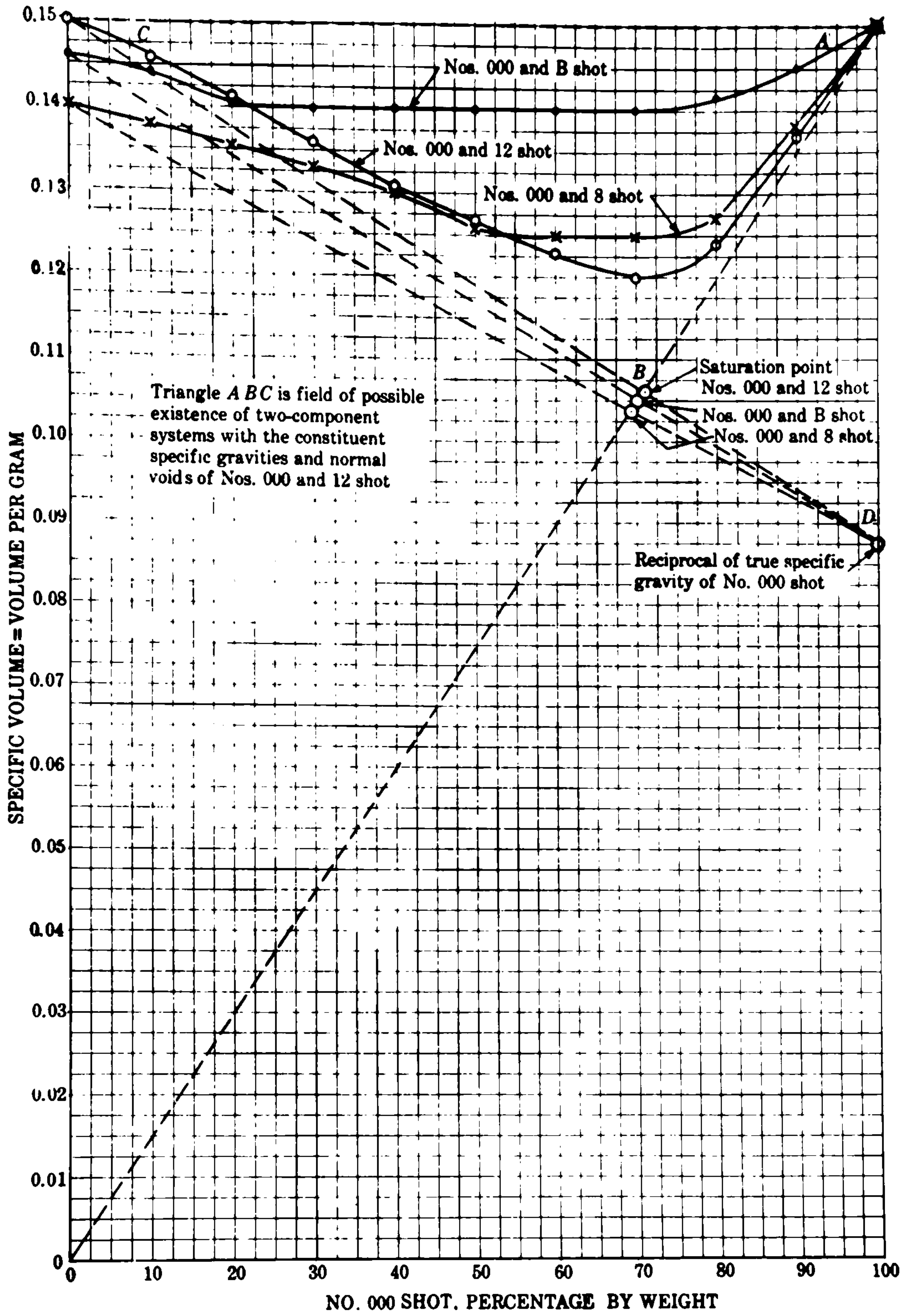


Fig. 51. SPECIFIC VOLUME/MIX COMPOSITION (after Furnas 1928)

the reciprocal of the true Sp. G of the coarse ingredient. This is because in this case it is not possible to add weight without at the same time increasing the bulk volume. Theoretically the most effective way in which the coarse aggregate could be added would be if it were introduced in such a way that it added its own weight and NO MORE THAN ITS OWN VOLUME (i.e it did not dilate the fine aggregate into which it was placed). Therefore the lowest possible value of partial specific volume for the coarse aggregate is the reciprocal of the true specific gravity and the line C D is a lower limit boundary. C D however crosses A D at B, the point to the right of which the quantity of fines becomes too small to occupy fully the voids in the coarse aggregate skeleton. Hence beyond B the lower boundary is already determined as the line to A. The triangle ABC therefore represents the total field of possible existence of mixtures of the two considered components.

The theoretical boundary line ABC, representing the most favourable possible conditions for minimum voids, i.e. when size ratio = 0, can be easily converted back from Specific Volume into "Voids in mix", for inclusion on the graphs shown as Figs. 48 to 50. AB and BC now become curves.

Powers (1964) later showed how a similar approach could be adopted when the results were plotted on a Specific Void Content graph instead of Specific Volume graph (Fig.52) and this method has the advantage that for purposes of comparison, differences in specific gravity are eliminated.

Specific voids (u) is calculated as

$$u = \frac{e}{1 - e} \qquad \text{.....................(6)}$$

where e = unit voids. This can also be expressed as

$$= \frac{V}{a\,V_a + b\,V_b} - 1 \qquad \text{.................. (7)}$$

where
V = over-all volume of aggregate
a = weight of fine aggregate
b = weight of coarse aggregate
V_a = specific solid volume of fine aggregate
V_b = specific solid volume of coarse aggregate

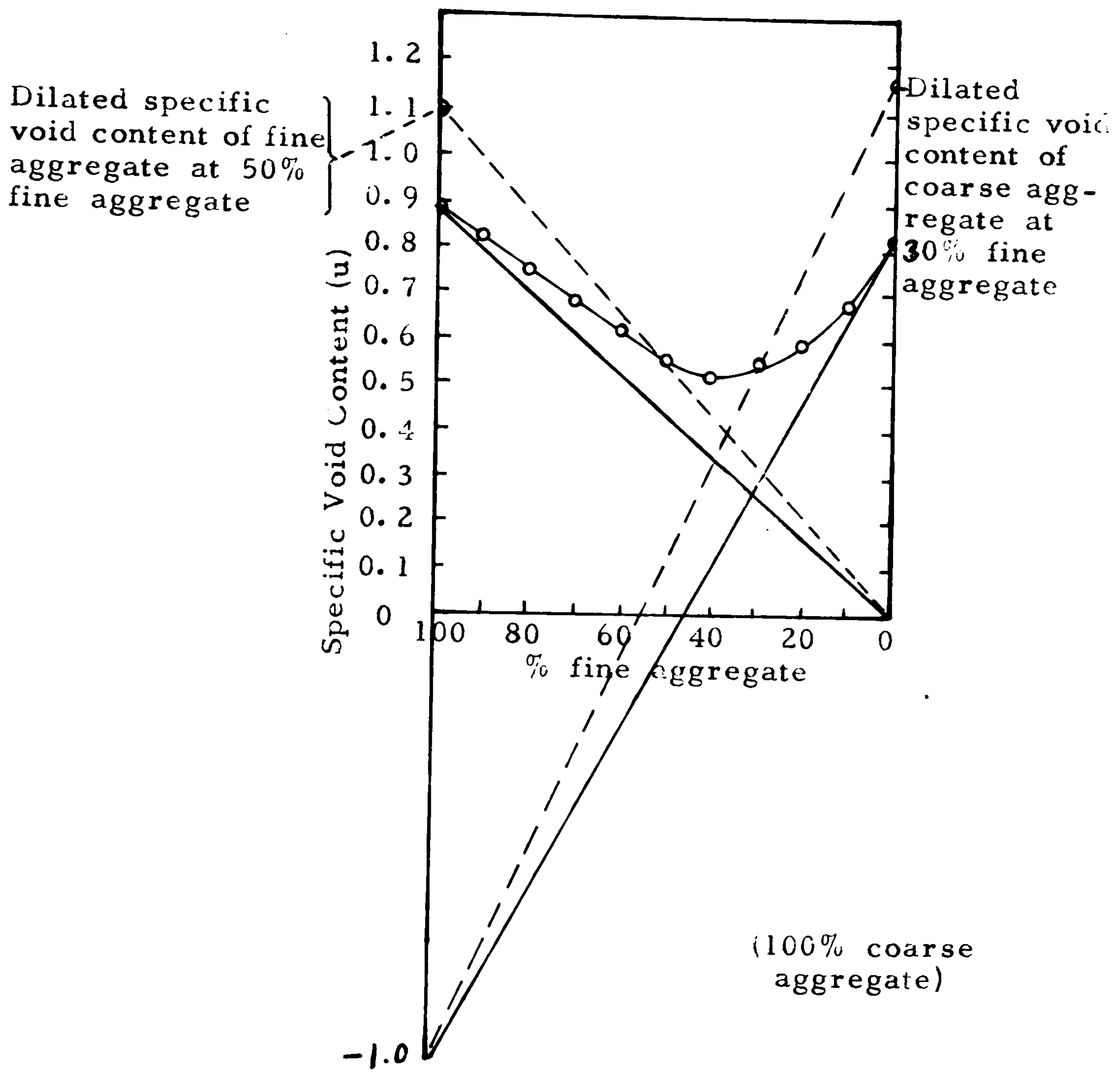


Fig. 52. SPECIFIC VOID CONTENT DIAGRAM - 2 COMPONENT SYSTEM - SIZE RATIO = 0.07. (after Powers, 1964). The projection of the dashed construction lines from the lower theoretical end points, through experimental points onto the vertical ordinates gives the dilated specific voids (hence porosity) of the respective component in the presence of the given quantity of the other.

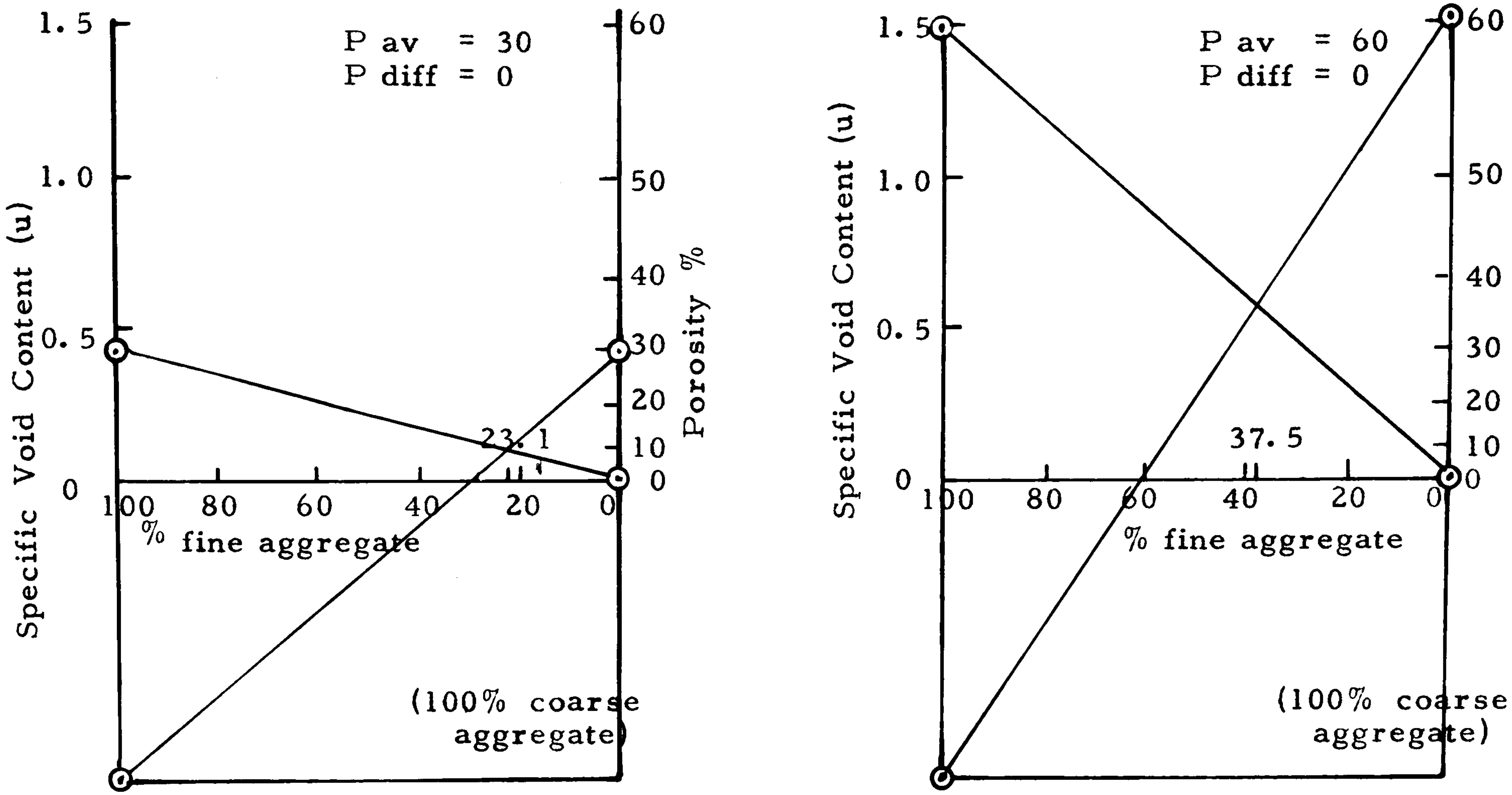
The theoretical boundary lines for the size ratio = 0 condition, in this case connect (i) the specific void content of the coarse aggregate with a specific void value of -1 (arising from equation 3) on the fine aggregate ordinate and (ii) the specific void content of the fine aggregate with a specific void value of 0 on the coarse aggregate ordinate. These lower points (-1, 0) correspond with the terminal points on the vertical ordinates for the Specific Volume graph whose derivation has already been described.

In both types of graph the position of the intersection of the boundary lines locates and identifies the minimum theoretical porosity and the mix proportions to produce that minimum.

Employing the Specific Void Content graph, which is, for most purposes the most convenient to use, Figs. 53 to 56 show several hypothetical cases for which the theoretical boundary lines have been drawn. It will be seen from Fig.53 that the actual value of the porosity of the separate component aggregates (which are in this Fig. assumed equal) has a marked influence on the position of the point of intersection, i.e. on the mix proportions for minimum voids. Fig. 54 shows further that even though the average porosity of the coarse and fine aggregates remains the same, increasing differences between the values produce large changes in the proportion for minimum voids. In the present study these differences have been called positive when the porosity of the coarse aggregate exceeds that of the fine aggregate, and negative when the fine aggregate has the greater porosity.

In considering the causes of the variations in porosity, probably the first that comes to mind is the influence of aggregate shape, and for example it will be expected from consideration of the graphs that a system of angular coarse aggregate and angular fine aggregate (as Fig. 53 (b)) will require a higher percentage of fines for minimum voids than a system of rounded gravel plus sand (as Fig. 53 (a)).

Comparison between Figs. 53 (b) and 54 (b), in terms of shape effects, suggests that a still higher percentage of fines will be required for a given angular coarse aggregate, if the fines are rounded (and hence pack to a low voids content) than if they too are angular. In other words a high value of positive $P_{difference}$ implies need for a high proportion of fines. Conversely and as shown in Fig. 54 (c) a high negative $P_{difference}$, such as would arise from, say the combination of a rounded gravel coarse aggregate with an angular fine aggregate indicates the need for a much lower proportion of fine material.



(a) Generally rounded aggregates compacting to low porosity

(b) Generally angular aggregates compacting to high porosity.

Fig. 53. SPECIFIC VOID CONTENT DIAGRAM - INFLUENCE OF PARTICLE SHAPE ON MIX PROPORTIONS FOR MINIMUM VOIDS (Theoretical case - size ratio = 0)

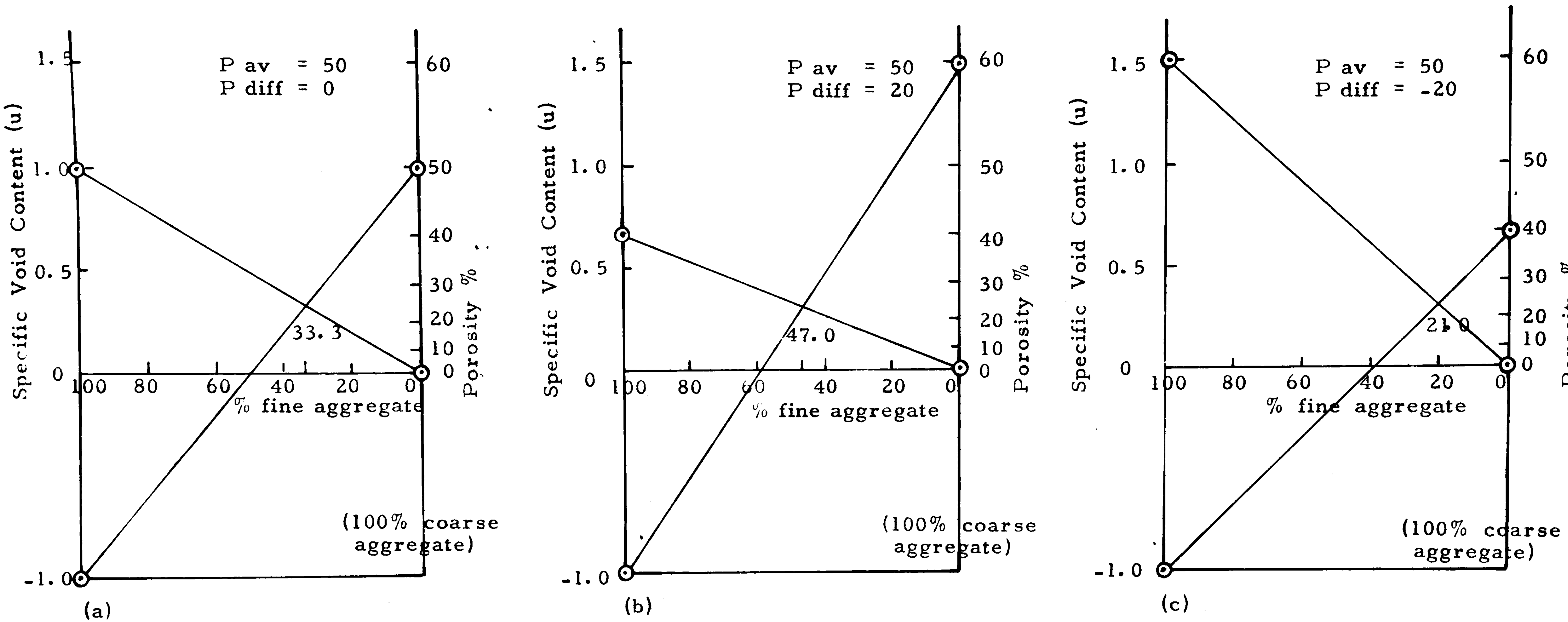


Fig. 54. SPECIFIC VOID CONTENT DIAGRAM - INFLUENCE OF POSITIVE AND NEGATIVE VALUES OF P diff ON MIX PROPORTIONS FOR MINIMUM VOIDS. (Theoretical case - size ratio = 0)

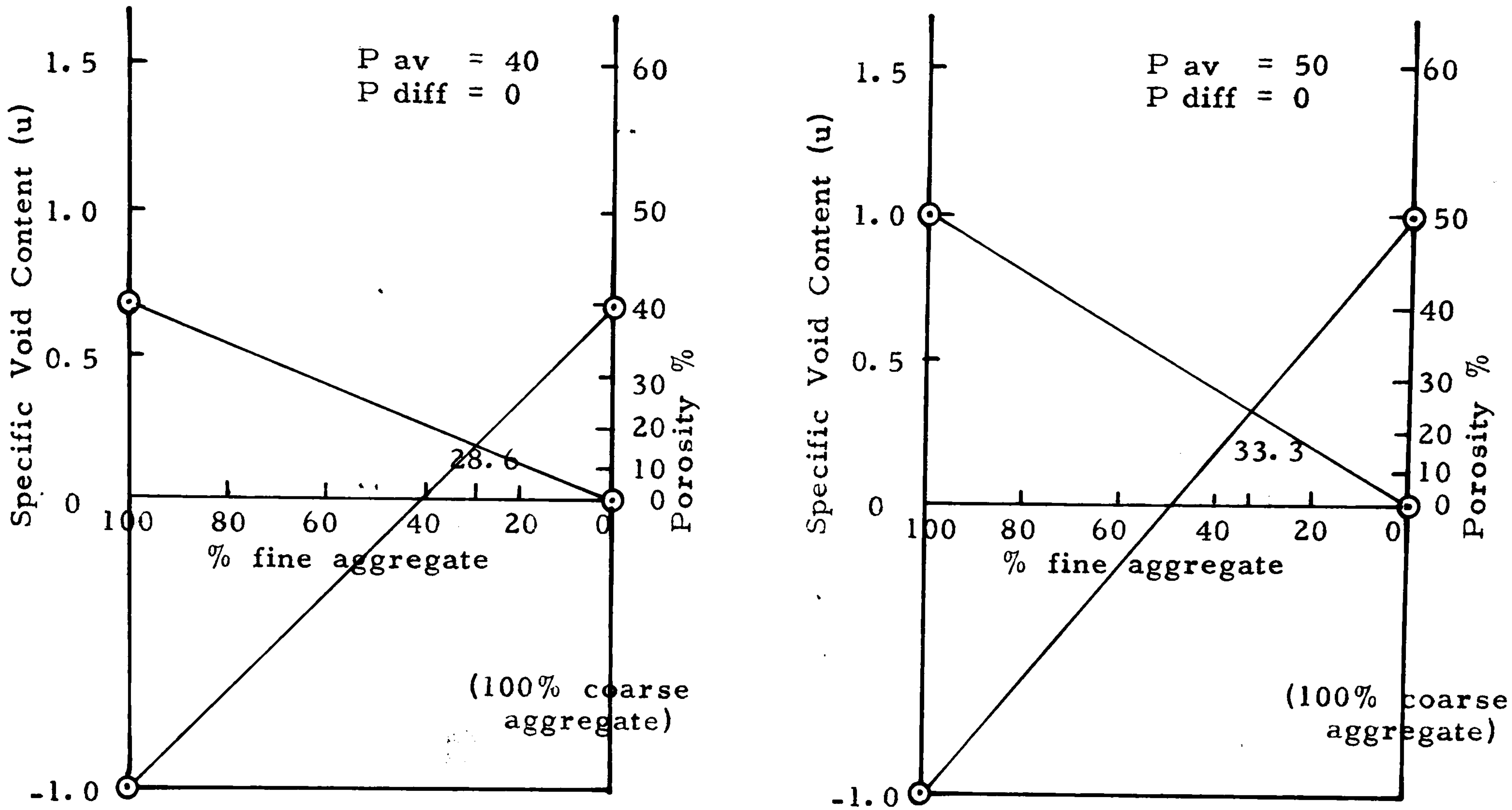
Particle shape, it has been pointed out, while playing an important part in determining the porosity of a single size aggregate is not the only controlling factor. The mode of packing as influenced by compactive effort, boundary effects (container wall and internal boundaries) and surface adhesion effects also play a large part in affecting porosity and hence also in affecting the proportions for maximum density.

For example it does not require stating that the porosity of a given aggregate is higher at its loosest state than at its densest. It does require, however, to be said that the proportions in which to mix two components for maximum density can change appreciably with the compactive effort applied, WITHOUT ANY ALTERATION IN THE TYPE OF AGGREGATE.

Fig. 55 illustrates the principle with reference to a hypothetical pair of aggregates of similar packing properties, say a rounded gravel and rounded sand, which possess a porosity of say 40% when densely packed and 50% when at their loosest state. It is apparent that the optimum proportions for minimum porosity of the mix will be different in the two cases. In fact they will differ in exactly the same way and by the same amount as if the differences in porosity had been caused by different shape at constant effort instead of by constant shape at varying effort.

It begins to be clear that what determines the mix proportions for maximum density is the porosity of the separate ingredients, irrespective of how that porosity was produced.

Further it has been shown previously how porosity is dependent on the size of the container. As before then, a given aggregate will show a higher porosity when confined in a smaller container. However for the two aggregates, coarse and fine, which have just been postulated and which had, for the sake of argument, equal porosity in a given container and for a given compactive effort, a reduction in the size of the container will cause a greater increase in the porosity of the coarse aggregate than of the fine aggregate. This follows from the general discussion on container wall effect in Chapter 2 and is demonstrated by Figs. 15 to 18. The resulting positive value of $P_{diff.}$ causes a shift in the position of the optimum to the left, i.e. in the direction of a higher requirement of fines (Fig. 56). This trend continues progressively as the size of the container further reduces. Its effect may even in certain circumstances outweigh that of shape



(a) Densely packed

(b) Loosely packed

Fig. 55. SPECIFIC VOID CONTENT DIAGRAM - INFLUENCE OF COMPACTIVE EFFORT ON A GIVEN AGGREGATE, ON MIX PROPORTIONS FOR MINIMUM VOIDS. (Theoretical case - size ratio = 0)

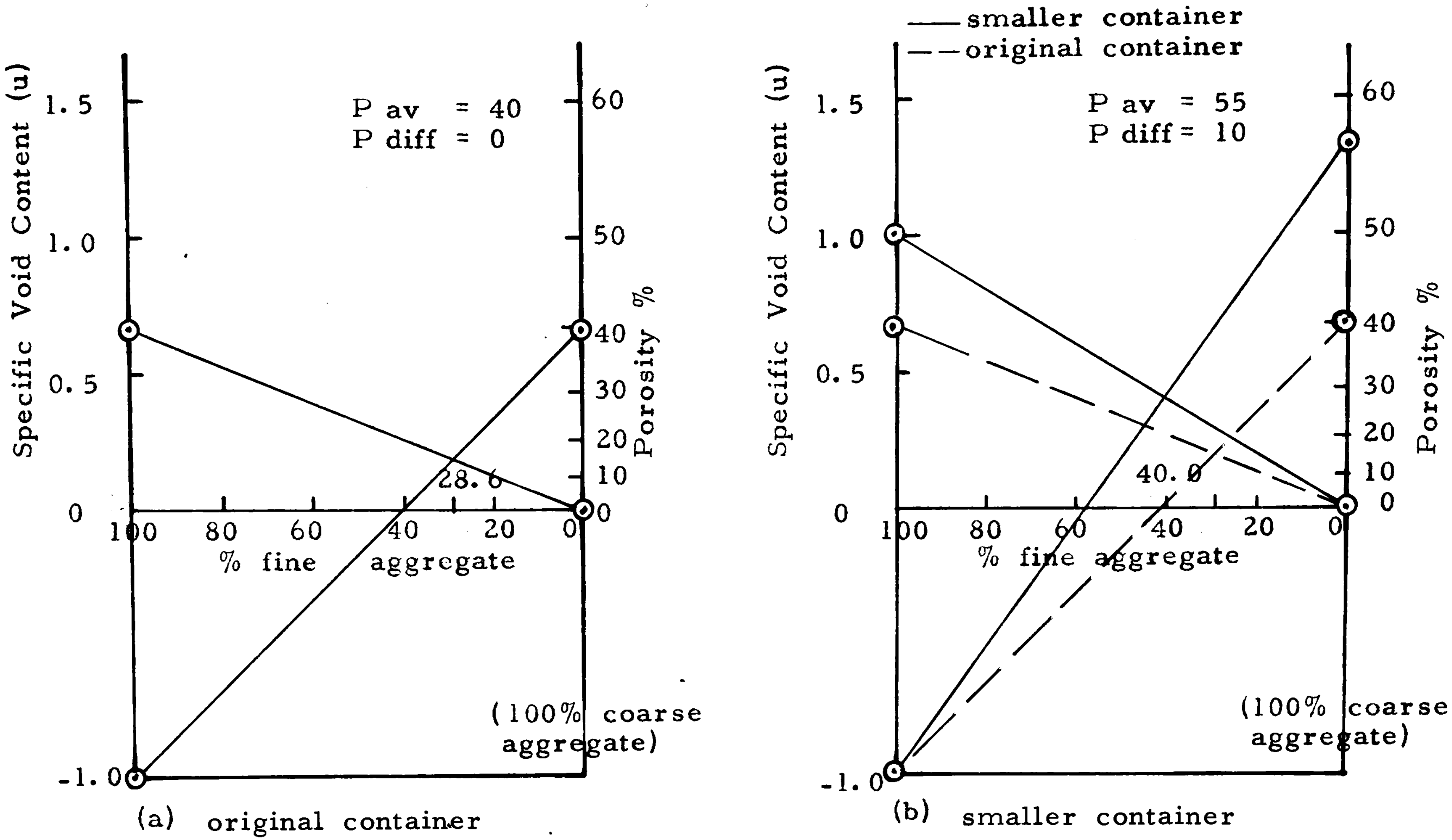


Fig. 56. SPECIFIC VOID CONTENT DIAGRAM - INFLUENCE OF CONTAINER WALL AND ITS DIFFERENTIAL INFLUENCE ON COARSE AND FINE COMPONENTS, ON MIX PROPORTIONS FOR MINIMUM VOIDS. (Theoretical case - size ratio = 0)

i.e. with a small container it is possible for a positive P_{diff} and a high fines requirement to obtain with a rounded coarse aggregate combined with angular fines - a combination of shapes which would normally produce a negative P_{diff} and a low fines requirement.

Herein lies the origin of the need for those several empirical corrections, which appear in other works and specifications, for adjustment of the proportions of fines required in, for example, a concrete mix to be placed in narrow sections, in positions of congested reinforcement or requiring to be piped. Of similar empirical nature are the adjustments in grading built into British Standard Specifications for asphalt where these adjustments are related to the placing of surfacing courses of varying thickness.

The proper appreciation of the nature of boundary effects on packing and proportioning, provide the means for lifting this problem out of the realm of empiricism and onto a rational basis.

An interesting reversal of the above trend is seen if instead of a variation in the size of the container the influence of increasing surface adhesive effects with decreasing particle size is considered. As discussed earlier (Chapter 2) an extremely fine grained fine aggregate may on account of its high surface charge pack to a very loose state and attain a much higher value of porosity than a given coarse aggregate with which it is to be combined. This condition of high negative P_{diff} (cf. Fig. 54 (c)) will be associated with a reduction in the percentage of fine material at the optimum mix proportion, the greater the effect the further the drop in fines required. Again this effect could be, in certain cases, great enough to outweigh the effect of shape leading to a low % of fines required, even with an angular coarse material and well-rounded fines (for example slightly damp sand) - a combination of shapes that would normally demand a high proportion of fine to coarse material.

In the light of the foregoing analysis the approach in the present study has been to consider that a common law might exist for the proportioning of particles in a two component system based upon the measured porosities of each of the components but irrespective of the means by which the porosities were produced - In other words upon the hypothesis that so far as proportioning for maximum solid volume density is concerned, the optimum proportions remain unchanged whether a given porosity results from say a light compaction to a well rounded aggregate or a heavy compaction to a more angular aggregate, or again to a rounded aggregate in a relatively small container or an angular aggregate in a relatively large container, etc. etc..

Consequently it has been considered essential that in any attempted prediction of the optimum proportion for any given site and placement conditions, the associated laboratory (or field) test upon whose results the prediction is to be made must imitate as faithfully as possible the dimensions of the section in which the mix is to be placed (especially its least dimension or dimensions), the intensity, duration and type of the compactive effort, and of course must employ as representative a sample as can be obtained of the aggregate in question. This latter item will properly include, where appropriate the thickness and viscosity of any lubricating or adhesive coatings of water, bitumen, tar, etc. This aspect will be referred to in greater detail in a later section, the present discussion being restricted to dry uncoated aggregates.

4.2. Porosity/Mix composition relationships for practical cases of finite size ratio.

Up to this point the discussion of 2 component mixes has been framed round the theoretical boundary conditions for a size ratio of 0. Furnas (1928,1931) while recognising that this condition is never realised, formed the opinion from his experimental results that for small size ratios (less than about .2) the proportions for minimum voids were not appreciably different from the optimum proportions for the theoretical case.

Furnas had therefore employed two relationships derivable from the theoretical case as a basis for his studies on gradings. These are:-

$$Z = \frac{1}{1 + V} \qquad \text{.........................(8)}$$

where Z = unit volume of coarse aggregate at minimum voids

V = normal voids (identical for fine and coarse aggregate) if the voids are not identical then

$$Z = \frac{(1 - V_1)}{(1 - V_1) + V_1 (1 - V_2)} \quad \ldots\ldots\ldots\ldots \quad (9)$$

where V_1 = normal voids in coarse aggregate

V_2 = normal voids in fine aggregate

For higher size ratios, however, the optimum proportions depart from the theoretical value, as seen in Figs. 48 to 50, the trace of the minimum point curving off towards a higher value of % fines with increasing size ratio. The Furnas formulae would therefore be inapplicable to these cases.

Interest in the general problem at all size ratios therefore led to the carrying out by the author of a full scale series of experiments with a variety of aggregate combinations over a wide range of size ratios.

Generally it had been concluded from the previous considerations that the relationship between % fines for minimum voids and all other relevant factors could be expressed as follows:

$$\%\text{ fines (by volume)} = f\left\{\underbrace{\begin{matrix}\text{particle shape \& surface texture,}\\ \text{surface charge, lubricating and}\\ \text{adhesive coatings}\end{matrix}}\quad \underbrace{\begin{matrix}\text{container wall and internal}\\ \text{boundary effects}\end{matrix}}\quad \underbrace{\text{compactive effort}}\quad \underbrace{\text{size ratio}}\right\}$$

$$= f\left\{\underbrace{\text{particle 'friction'} \quad \text{boundary effects} \quad \text{effort}}_{\text{(measured porosities of fine and coarse components expressed as:-)}}\quad \underbrace{\text{size ratio}}\right\}$$

$$= f\left\{ P_{av.}, \quad P_{diff.}, \quad \text{size ratio} \right\} \quad \ldots\ldots \quad (10)$$

These 3 variables alone then, govern the mix proportions for minimum voids for any given conditions of environment and effort. Accordingly the experimental programme designed involved the measurement of the porosities of a variety of fine and coarse materials separately and in combination at proportions of approximately 10% intervals, and of the size ratio between the components. *

* The size ratio taken was that of the Equivalent Spherical Diameters computed from weight per particle determinations. The container used was the Compacting Factor Cylinder. Efforts appropriate to (a) maximum porosity and (b) minimum porosity conditions were employed. Maximum porosities were obtained as previously described (p.9) and minimum porosities by vibration as described in Appendix 2.

In the initial experiments, 55 such two-component systems were investigated over the full range of mix proportions, 6 porosity determinations being made at each mix-composition. The results were plotted graphically in order to determine the % fines at minimum voids for each mix. For the purposes of analysis these results were combined with 21 due to Furnas (1928) and 4 due to Worthington (1953), making a total of 80.

A multiple linear regression analysis by KDF 9 computer, on these 80 results gave

$$y = 3.889\,x_1 + .373\,x_2 + 38.36\,e^{x_3^2} - 22.79 \quad \ldots\ldots \quad (10a)$$

where y = % fine material

x_1 = P diff

x_2 = P av

x_3 = Size ratio.

A check on accuracy, by comparison of calculated and measured values gave as follows:

Calculated values of % fines were within ± 2% of measured values 48.8%times

" " " " " " " ± 4% " " " 68.4% "

" " " " " " " ± 6% " " " 79.7% "

" " " " " " " ± 8% " " " 86.1% "

This accuracy was considered reasonable, but notwithstanding this, it was felt that there was some reason to doubt whether the linear relationships would apply outside the experimental range of P av and P diff values. Fig. 57* supplies the reason for this doubt. This figure, being that relating % fines at the minimum to P av and P diff for Size Ratio = 0, is completely theoretical. Within the range of P av values from 40 to 60 and within the range of P diff values from -5 to +10, i.e. within the experimental ranges of these values, the relationships do appear reasonably linear, but curve off to varying degrees at the extremities.

For this reason graphs Figs. 58 to 68* were constructed for finite values of size ratio in which the slopes of linear portions were calculated on the basis of the regression equation, but the extreme portions of which were produced along curved lines to calculated theoretical end points. (That is to say that not only is the size ratio = 0 case theoretical, but also are the cases for 0% fines and 100% fines,

* graphs Figs. 57 to 68 inclusive, are contained in wallet inside rear cover.

where the size ratio is finite).

The final forms of these graphs were produced with the aid of a number of additional experiments in which extreme values of P av and P diff were deliberately produced.

The accuracy of the graphs was then tested against the original 80 results as follows:

Graphical	values	of	%	fines	were	within	± 2%	of	measured	values	43.4%	times
"	"	"	"	"	"	"	± 4%	of	"	"	59.2%	"
"	"	"	"	"	"	"	± 6%	"	"	"	76.3%	"
"	"	"	"	"	"	"	± 8%	"	"	"	84.2%	"

This accuracy is of the same order as for the regression equation since the results against which it was tested fall within the linear portion of the graphs. However the graphs would show a greater accuracy for mixes with extreme values of P av and/or P diff.

In either case the accuracies are considered adequate in view of (i) the shallow forms of the curves of porosity against mix composition in mixes of high size ratio, which makes determination of the minimum points open to some error, and (ii) the comparison with the accuracies worked out for the same mixes using the Furnas formula given as (9) above.

For this formula the accuracies were as follows:

Predicted	values	of	%	fines	were	within	± 2%	of	actual	values	15.8%	times
"	"	"	"	"	"	"	± 4%	"	"	"	23.7%	"
"	"	"	"	"	"	"	± 6%	"	"	"	27.6%	"Z
"	"	"	"	"	"	"	± 8%	"	"	"	35.5%	"

The poor accuracy of the latter results is due solely to the neglect of the size ratio influence in the Furnas formula.

The 80 experimental results referred to, give of course not only the mix compositions for maximum solid volume density but also the value of voids at those compositions. As shown earlier by Furnas the lower the size ratio the lower the porosity of the mixture. For the benefit of more general application to all mixes this phenomenon is more usefully expressed in terms of the reduction in bulk volume on mixing the 2 components at their optimum proportions compared with the sum of their separate bulk volumes.

The ratio of this reduction to the theoretical maximum possible reduction at a size ratio = 0 is termed the relative contraction.

$$\text{Thus relative contraction} = \frac{a - c}{a - b} = \frac{\text{actual reduction}}{\text{possible reduction}} \quad (11)$$

where

a = Porosity at minimum point (P_{min}) at size ratio = 1

b = " " " " " " " " = 0

c = " " " " " " " " = x

x = actual size ratio of components

It follows that for two components of size ratio = 1, i.e. of the same size, the relative contraction = 0, and that for the theoretical case when size ratio = 0, the relative contraction = 1. By definition the quantity relative contraction is independent of P av.

Analysis of the experimental results showed further that the relative contraction was dependent upon the size ratio and on the value of P diff (but irrespective of its sign).

The graph Fig. 69* was constructed according to the observed relationships between these three quantities. From known values of size ratio and P diff, the porosity at the minimum point can thus be calculated, from the value of relative contraction read from the graph, according to the formula

$$c = a - \left[(a - b) \times \text{relative contraction}\right] \quad \ldots\ldots\ldots\ldots (12)$$

(derived from equation 11)

a is taken as the lower of the 2 porosities measured for the separate fine and coarse aggregates (since this would necessarily be the minimum porosity when the size ratio = 1).

b is calculated from

$$b = \frac{P_{coarse} \times P_{fine}}{100} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

the derivation of this formula is given in Appendix 3.

* graph Fig. 69 is contained with Figs. 57 to 68 in wallet inside rear cover.

Combining these equations the resultant porosity (c) is calculated as

$$c = a - \text{relative contraction} \left[a - \left(\frac{P_{coarse} \times P_{fine}}{100} \right) \right] \quad ..(14)$$

The accuracy of the graphically predicted voids against the measured values is as follows:

Calculated values of porosity were within ± .5% of the measured values 42.1% times

Calculated values of porosity were within ± 1.0% of the measured values 64.5% times

Calculated values of porosity were within ± 1.5% of the measured values 76.3% times

Calculated values of porosity were within ± 2.0% of the measured values 86.8% times

Calculated values of porosity were within ± 2.5% of the measured values 96.1% times

Calculated values of porosity were within ± 3.0% of the measured values 98.7% times

The graph (Fig.69) of size ratio / relative contraction, and the graphs (Figs. 57 to 68) of optimum % fines thus enable the prediction with reasonable accuracy, of both the porosity value at the minimum point and the mix composition to produce that minimum, for any pre-mixed 2 component system of aggregates in any given conditions of environment and compactive effort.

It is emphasised that the above statement has been restricted to pre mixed 2-component systems.

There is reason to believe that the graphs do not apply (without some modification) to the mixing technique which has been applied by some authors, notably Hopkins (1962) in which the coarse aggregate is placed first and subsequently held in the container while the fine aggregate, necessarily of a low size ratio, is filtered into the coarse aggregate voids.

Hopkins himself noted that the minimum point by this technique gave a few % lower porosity (i.e. higher bulk density) than that obtained by the pre-mix technique on the same pair of aggregates (Fig.70).

It appears also probable that the mix proportion for maximum density was different in the two cases, for although there are insufficient

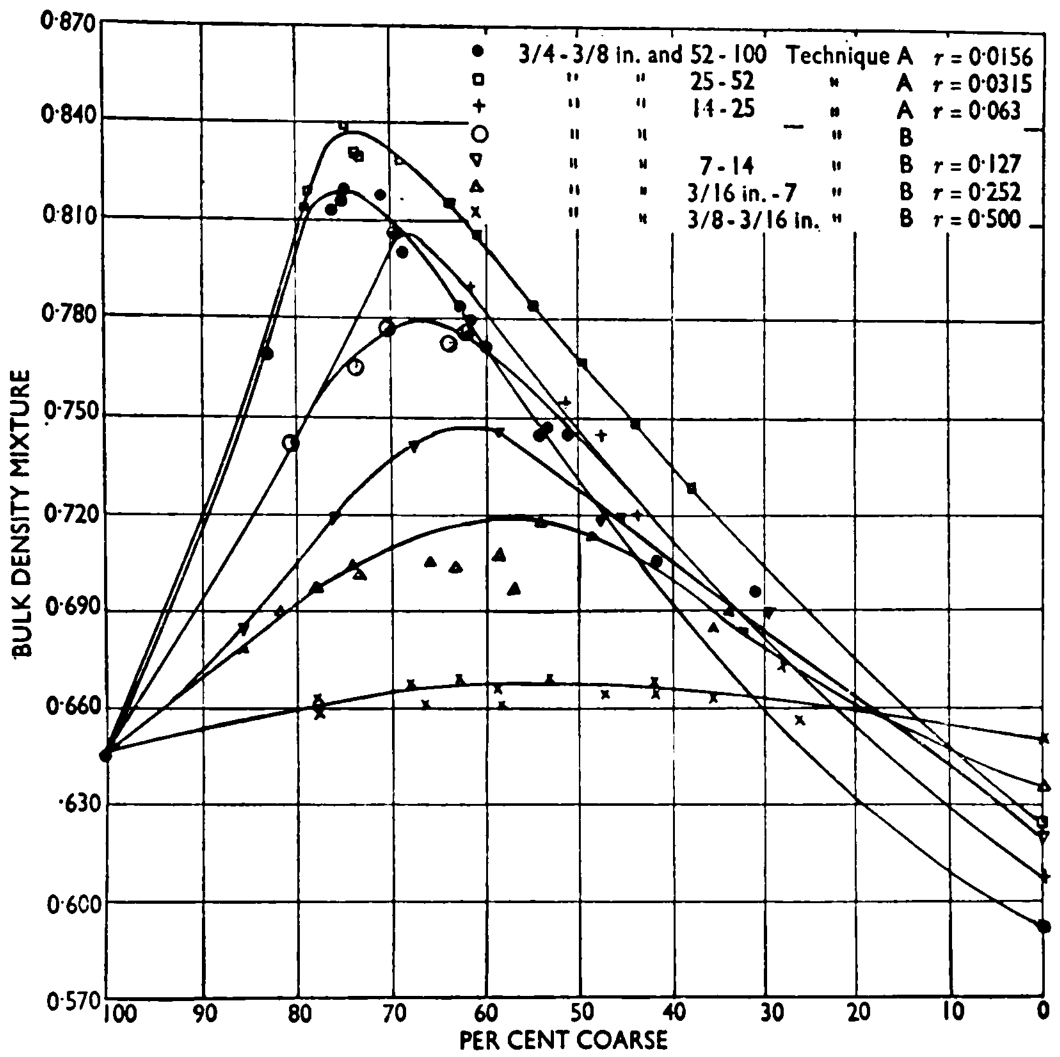


Fig. 70. BULK DENSITY OF 2 COMPONENT MIXTURES, COMBINED BY (a) Filtration
(b) Pre-mixing techniques

(after Hopkins 1962)

points plotted, in the case of the one sand measured by the two techniques (namely the 14-25 sand), for a direct comparison to be made, there does appear to be a sudden discontinuity in the trend of location of the points marking the position of maximum density corresponding with the change over in technique. The discontinuity is such as to suggest a decrease of from 5 to 10% in the proportion of fine material at the optimum.

It would seem highly probable that a different law operates with the filtering technique as compared with the pre-mixing technique, since in the former case the normal dilation of the coarse aggregate due to the presence of the fine aggregate is restricted or wholly prevented by reason of its pre-placement while the dilation of the fine aggregate in the presence of the coarse is as great (if not greater, should the filtration be incomplete due to occasional void entrance blockage) as when the two materials are mixed together before placing.

This would invalidate a determinitive procedure based upon conditions of mixing in which each component has been allowed freely to modify the packing of the other. However it would appear that the optimum mix proportions could still be derived (for the case where the coarse aggregates were completely constrained) by constructing on the specific void content graph the theoretical boundary line for the coarse aggregate and noting its point of intersection with the construction line joining the value of dilated specific voids for the fine aggregate. The latter would be obtained from a specific void content graph based upon the voids and % fines values obtained from Figs. 57 to 69.

This procedure not only involves the assumption that the coarse aggregate remains undilated (though this can be assured) but also that the dilation of the fine aggregate is the same with both techniques. Some errors though probably not serious, may arise due to this assumption, However it may be thought that there is less necessity for predicting the optimum % fines, with the filtration technique since the value is so readily determined by direct experiment.

Practical applications of the filtration technique are the use of "blinding" fines, coated grit etc. for infilling of "pitching" and coarse macadam layers. The American "Prepakt" concrete employs the same principle, and Bahrner (1951) speaking of the use of this material in Sweden reports "By placing stones of a suitable grading beforehand in the moulds and packing them well, the voids between them become less than for ordinary concrete and the quantity of mortar required to fill these voids becomes correspondingly smaller. It is therefore possible to obtain the desired strength with a quantity of cement which according to information received is up to 30% less per cubic metre of concrete".

Certain site conditions may permit this potentially useful technique, but these will tend to be rare compared with those demanding the conventional pre-mix concrete.

4.3. 2 component systems in which one component is a liquid.

The case just described in which the coarse aggregate was placed first and the fine aggregate subsequently filtered in, is in some respects similar to the case now to be considered where one component is a liquid. In the filtration case it was stated that dilation of the coarse aggregate could, if it was held in place, be prevented while the fine aggregate was filtered into place, but that the latter would be dilated as normal or to an even greater extent. When a liquid is substituted for the fine aggregate in a mix it becomes possible, in appropriate circumstances, to achieve in practice what was previously only a theoretical possibility, namely for the effective size ratio to equal zero. That is to say that it is possible for the 'coarse' aggregate solid volume density to be unchanged, in the presence of the liquid, from what it was dry and for the liquid to contain no air pockets.

On the specific volume graph the theoretical boundary lines would be drawn as normally, utilising the reciprocal of specific gravity of the liquid as the "anchor point" on the left hand ordinate, and would show a clear minimum as in the case for two aggregates. With the specific void content graph the boundary line for the liquid becomes horizontal along the $u = 0$ line (Fig.71), hence the optimum proportion giving the maximum possible content of solid matter, with voids completely filled with liquid, is given by the intersection of the aggregate line (QR) with this horizontal line.

The volume % of liquid indicated by the point of intersection is of course equal to the measured porosity of the aggregate on its own. Mix compositions to the left of this point would still be air-voidless but the particles would become increasingly separated and "floated off"

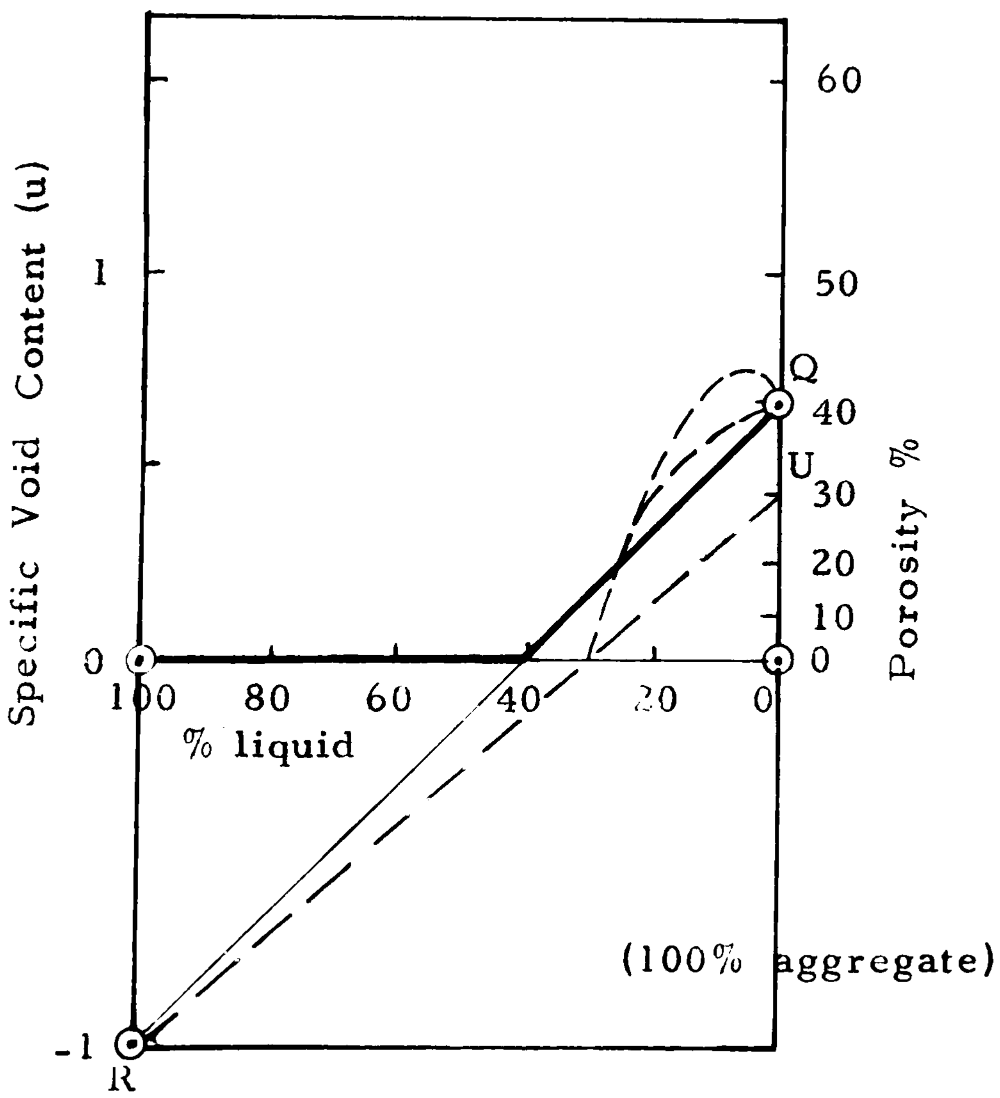


Fig. 71. SPECIFIC VOID CONTENT DIAGRAM - AGGREGATE/ LIQUID COMBINATION

from one another by the liquid. This is indeed the state so feared by asphalt technologists in their recognition of the "unstabilising" of asphalt mixtures in which the voids become "overfilled" with bitumen.

Although the liquid/aggregate system appears at first sight simpler than the case of two aggregates and could in fact be as described, there are possibilities, in three directions, of a more complicated system existing.

In the first instance it is always possible for air bubbles to be trapped within the liquid body or at liquid solid contacts which dannot be expelled by the expedient of adding more liquid. Again this is the case in asphaltic mixes in which the early addition of bitumen produces rapid fall in voids, but the last remaining 2 or 3 % of air cannot be expelled before the "unstabilising" condition just described with reference to the specific void content graph, is reached.

If it is possible to make an assumption of the amount of air which cannot be readily dispelled without dispersal of the aggregate (in the case of asphaltic mixes we might for example be tempted to accept the strong empirical evidence which suggests the content of this "obstinate" air to be in the region of 3% (Hveem, 1940)), then the simple operation of subtracting this % from the initial porosity determination and assigning the remaining space to the liquid, provides directly the mix proportions for maximum aggregate density. The value however undoubtedly alters with both type of binder (increasing with higher viscosity) and with shape, surface texture and grading of aggregate.

Corrections of an opposite kind require to be made for absorbent aggregates. The initial porosity may here be increased by a value equal to the % of binder absorbed in order to arrive at the optimum mix proportions.

Lower and higher binder contents respectively will be indicated in the two cases above.

However even were the quantities of entrapped air and absorbed binder readily determinable, the solution would not be so simple as just described because of the further effects now to be described.

The third complication which may enter is that a liquid may exert an influence on the other, particulate, component which is not normally seen when both components are particulate aggregates. In the latter case dilation of each aggregate in the presence of the other is the normal case, (as shown in Fig.52)- the relative degrees of dilation of coarse and fine components depending on the mix proportions. In the case of a liquid component, bulking i.e. dilation of the aggregate is common (perhaps even ever-present) at low percentages of the liquid, as noted in the case of damp sand and low binder content bituminous mixtures, but at higher contents of the liquid the phenomenon of lubrication enters, the effect of which may be to cancel in whole or in part any early tendency towards dilation and in some cases even so far as to condense the aggregate structure to ~~produce~~ a state of packing denser than that which could be achieved with the same effort with the aggregate tested alone.

The major difference now apparent between this and previous cases discussed is that whereas previously the lower boundary of the field of possible existence of systems corresponded with the theoretical lines for size ratio = 0, constructed as described on specific volume or specific void content graphs, this is no longer true where lubrication is possible.

Fig.71 illustrates on the specific void content graph, trends which have commonly been described from both bitumen/aggregate systems and soil/water systems. Initial dilation (bulking) in the presence of small proportions of the liquid may occur as described for 2 component particulate mixtures, i.e. with a net reduction in air voids (lower dashed line), but is sometimes so extreme as to result in a real increase in porosity (upper dashed line). Further increases in the liquid content and hence in film thickness are in general accompanied by reduction in viscosity, more complete burial of surface texture irregularities and reduction of surface tension forces at liquid air interfades. An accompanying rapid increase in the lubrication effect is common, leading in some cases to the aggregate packing even more densely than when compacted alone, as suggested by the fall of the practical line below the level of the theoretical line QR.

Inevitably this leads, as observed, to saturation being reached at a lower liquid content than would have been predicted from the intersection of QR with the horizontal base line.

Should it be accepted however that there is for any aggregate an ultimate state of dense packing, beyond which the porosity cannot be further reduced (without particle breakage), then the more complete the initial compactive effort on the "dry" sample, the more accurate may be the prediction of the optimum liquid content which will give the minimum air voids consistent with maximum aggregate density, for the aggregate in its final fully lubricated condition.

Being a static case this will be a true optimum, irrespective of any propensity towards dilation at liquid contents less than the optimum.

The practical difficulties which so far have prevented the application of this seemingly simple solution are concerned with the attainment in the laboratory of the ultimate state of density as represented say by several years of traffic compaction, without producing at the same time an unrealistic degree of particle degradation and an unrepresentative structure in terms of particle arrangement and orientation, compared with the field conditions.

These problems are at present under further investigation by the author and fellow research workers (Salehi, 1966) with the object of perfecting a method of laboratory compaction which in terms of both degradation and structure will more nearly represent the field condition.

A further difficulty faced by the asphalt technologist is that even were he able accurately to predict the ultimate state of aggregate density, there is always a greater or less time lag between laying and the achievement of this ultimate state. That is to say that the structure is, because of the viscous nature of the binder, more open in the immediate post laying period than it is after several years of traffic compaction (McLeod 1955). According to the principles already stated, the optimum % of fines or liquid is related to the state of packing at any time.

It follows that the binder content which will give maximum aggregate density with least possible air voids at an early stage of life shortly after rolling when the structure is still somewhat open cannot also be the binder content to achieve the same condition at a later stage when traffic compaction has completed the process of densification.

This then is the dilemma of the asphalt technologist, towards the resolution of which he must make further estimates or measurements of the range of packing densities through which the structure will pass during its life, and make the most satisfactory engineering compromise based upon the relative risks of his pavement being permeable in its early life, if he chooses the lower binder content which would satisfy the condition of minimum voids consistent with maximum aggregate density after several years of traffic compaction, or of becoming unstable during its later life if he chooses the higher binder content which would satisfy the same condition under the less complete compactive effort applied at the time of laying.

The smaller the difference between post-rolling density and ultimate density, the less is the risk of failure due to one of the causes mentioned above - hence the trend towards heavier and more efficient (e.g. pneumatic tyred) rollers in recent years.

In the discussion in this section on 2-component systems involving a liquid component, the aggregate has been considered as the other component irrespective of whether it were a single size or graded material.

A graded aggregate might be considered multi-component but for the purpose of this section the nature of the grading has not been considered.

The question of design of multi-component aggregate systems is now considered further.

CHAPTER 5.

MULTI-COMPONENT SYSTEMS

The problem of maximum density (minimum porosity) mixes, devolves into one of ascertaining the grading which, when defined in terms of both sizes and proportions of particles, will produce a mixture filling the maximum possible space with solid material. The author, in applying the principles learned from a study of particles, of voids and of 2-component systems, towards the aim of gradings giving maximum solid volume density makes no claim that these gradings are necessarily those which will also give maximum crushing, flexural or tensile strength, maximum fatigue life, maximum workability (for concrete) or maximum durability (for bituminous mixtures). In fact it is certain that no one mix-composition of any source, would satisfy all of these requirements or even satisfy the condition of one of them, say maximum crushing strength, for all environments (i.e. confining pressures).

Nevertheless the general relationship of strength and impermeability with density (solid volume density) and the need for a more complete understanding of the factors governing the packing structure of aggregates justifies the present study and may form a basis on which further studies of the relationships between density and engineering properties may follow.

Further as previously stated the method proposed will also allow the design of gradings to a controlled void content, should the engineering property which it is desired to achieve, indicate the need for a porosity greater than the minimum which it is possible to produce.

The main conclusion to be drawn from the studies described in previous sections is that there can be no such thing as a unique ideal maximum density grading curve. It has been emphasised that the correct proportions for minimum voids must inevitably be affected by changes of shape from size to size, by the degree of compactive effort applied, by the presence of lubricating or adhesive coatings, and by the size and shape of the section in which the material is to be used. Yet the literature contains many references which claim to have found "the ideal grading curve" - some according to a mystical acceptance of some supposed desirable constant mathematical relationship between adjacent sizes of the form

$$p = 100\left|\frac{d}{D}\right|^{n} \qquad \text{.........} \quad (15)$$

where p = per cent by weight finer than the sieve
d = the sieve in question
D = maximum size of aggregate

as quoted by Yoder (1959), (Yoder states further:- "Maximum density generally occurs when the exponent n equals 0.5"), some according to laboratory experimentation, (Fuller and Thompson 1907, Campen 1940, Worthington 1953) and some as a result of survey and analysis of mixtures which have performed satisfactorily in the field (e.g. Hveem 1940 - though Hveem, in quoting recommended gradings from a variety of sources some of which claim maximum density and from which he derived his ideal grading envelope, was careful to state that 'ideal' in his definition did not necessarily imply giving minimum voids).

Bulking large in the literature on maximum density gradings is the discussion of the relative merits of the gap (or "skip" or "intermittent") type of grading compared with the continuous type of grading. As the names imply, the former consists of a system of several single sizes or restricted-range sizes in combination, between each of which a certain size or range of sizes is deliberately omitted. In the continuous grading some quantity of every size in the total range from largest to smallest is present.

Before entering into the description of gap grading and continuous grading design procedures, it may be instructive to enquire into the controversy which surrounds these two alternative forms of grading. Each has itssown proponents, those who favour the gap grading claiming that it is the only grading capable of rationalisation since it can theoretically be designed on the 'telescope' principle. That is to say that each lower size added can be chosen so as to 'fit' into its appropriate place in the framework provided by the sizes above, and that this must therefore make it possible to design mixes to the lowest possible porosity.

The proponents of the continuous type of grading claim that such mixes are in general free from the segregation (of particles of different size) to which gap graded mixes are sometimes subject and which would lead directly to a higher void content, and extend, probably without justification, the observation that the porosity of a 2-component mix can be reduced by the insertion of a quantity of an intermediate size into such statements as "one can hardly expect greater density from systems consisting of a few sizes fitted together, than from systems in which all sizes are represented in appropriate amount", (Andersen and Andreasen, 1929).

The present author's main purpose at this point is to show that there need be no conflict in either the matter of the porosities which can be achieved or in the matter of susceptibility to segregation.

Dealing firstly with porosity reference will be made to Fig.72 after Powers (1964). This figure shows, on the specific void content plot, the boundary lines (for the theoretical case of S.R. = 0) for a number of different combinations of aggregate sizes. For the purposes of this experiment a continuous grading (from 1½" to passing 200 mesh) has been arbitrarily divided at the no. 4 size of sand, and the 3 sieve sizes above this sieve have been designated the coarse fraction, while the 6 sizes below this sieve have been designated the fine fraction. These separate composite coarse and fine fractions have each been subjected to porosity tests (effort and size of cylinder unspecified), in which they achieved porosities corresponding to specific void contents of 0.51 ($C_n = 3$) and 0.43 ($A_n = 6$) respectively. The boundary lines from these two values to the theoretical partial specific void contents for S.R. = 0 intersect at B_c . This then represents the minimum possible void value, but is far from attainable since the condition of continuous grading (albeit arbitrarily divided) having been set, the size ratio between the two components is also fixed. Powers calculates this as 0.056 (the ratio of the mean volume diameters). The experimental curve of results confirms that the actual mihimum voids does not even begin to approach the theoretical value at B_c . The relatively high size ratio is the cause.

Gaps in the grading can be introduced in various ways, of which the simplest would be to omit some or all of the middle sizes. This can be done by omitting the smaller sizes of the coarse aggregate and the larger sizes of the fine aggregate. For example $A_{n=5}$ refers to

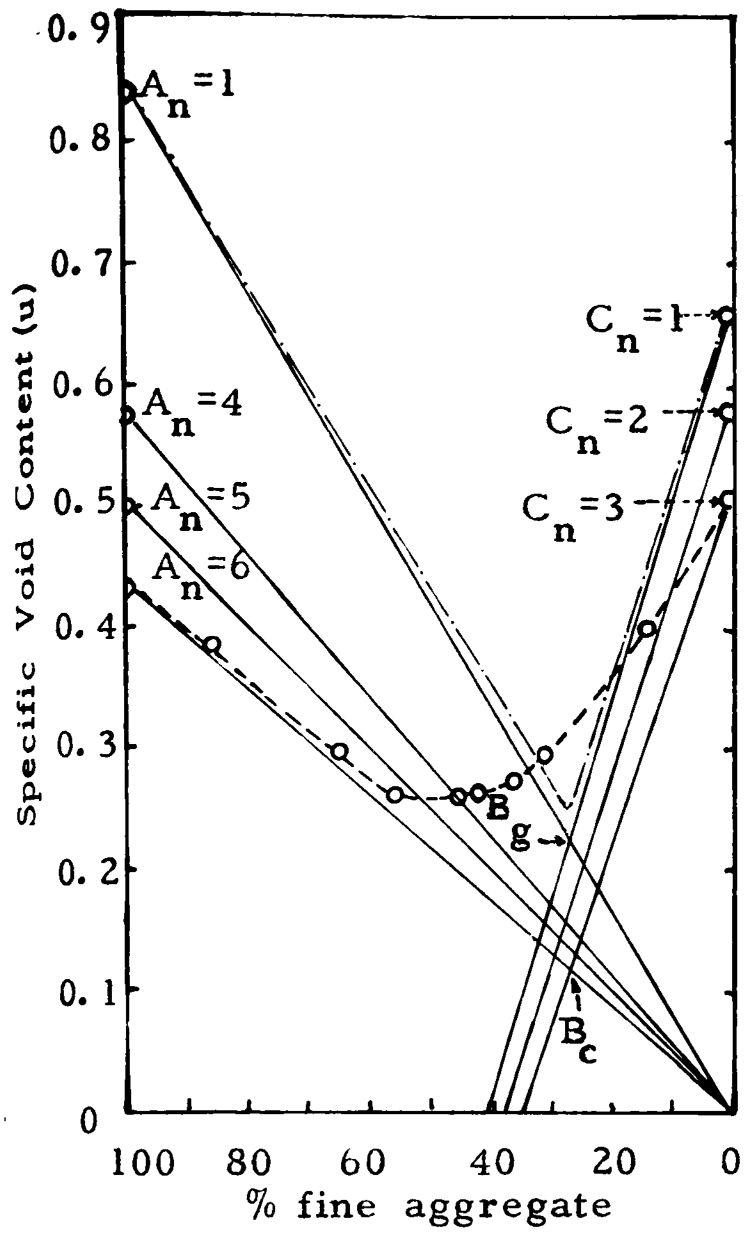


Fig. 72. SPECIFIC VOID CONTENT DIAGRAM ILLUSTRATING THEORETICAL AND PRACTICAL RELATIONSHIPS FOR VARIOUS CONTINUOUS GRADED AND GAP GRADED MIXTURES. (Modified, after Powers 1964)

the fine aggregate from which the coarsest size is omitted, $A_{n=4}$ omits 2 sizes and so on until at $A_{n=1}$ only the finest fraction is present for which the specific void content = 0.84. Similarly for $C_{n=1}$ only the coarsest fraction of the coarse aggregate is present. In both cases there is, as would be expected, a rise in the specific void content of the single size material compared with the corresponding fine or coarse component which covered a wider range of sizes. In consequence the point of intersection of the boundary lines for S.R.=0 for this gap graded mix (B_g) is higher than the point B_c previously mentioned, which applies to the continuous grading. It might appear to have been proved therefore that the continuous grading will have the lower porosity, but it will be remembered that the actual experimental curve for the continuous grading could not approach B_c because of the relatively high size ratio of 0.056. On the other hand the size ratio for the mixture of $A_n = 1$ and $C_n = 1$ is only about 0.004, and although no experimental curve for this mix was given by Powers, it would seem reasonable to suppose that with such a low size ratio the minimum on such a curve would approach B_g much more closely than the minimum on the other curve for the continuous grading could approach B_c. A check upon this hypothesis was made by consulting the author's graph of relative contraction against size ratio for the calculated value of P diff. The value of relative contraction so determined was 0.985, from which a porosity of 20.3% for the minimum point was calculated.

A hypothetical curve descending to the corresponding specific void content value of 0.255 is drawn (chain-dotted) on Fig.72 for comparison with the other experimental curve, from which it can be confirmed that the difference in void content between the continuous and the gap grading is likely to be small.

There are of course many other ways in which (a) the continuous grading could have been subdivided and its component parts composed and (b) the gaps could have been chosen, as regards size and number, in the gap grading. Nevertheless the illustration provided does suggest that it would be wrong to assume that the combination giving the lowest point of intersection of the boundary lines for S.R. = 0 necessarily provides the lowest voids when real mixes and finite size ratios are considered.

5.1. Design of gap gradings.

Gap graded aggregates are encountered in three engineering environments namely in waterbound macadam, in rolled asphalts to B.S. 594 and in concrete mixture. The principles on which these have been designed (or evolved) have varied from the purely empirical to the theoretical, but defects, more or less serious, exist in all methods so far proposed.

In general the problem involved in the design of a gap grading is a two-fold one namely

1) to assess the magnitude and number of the gaps that will be introduced and hence to determine the sizes that will be represented in the grading

2) to assess the optimum mix proportions for minimum porosity for the aggregate sizes chosen in 1).

In some of the engineering examples quoted above the design of the mix has arisen purely by chance and remains on an ill-defined and empirical basis. Such a case is the range of mixes for rolled asphalt, contained in B.S.594, mixes in general use on all major roads in Britain today. These evolved from a sand sheet mixture to which quantities of a coarser stone were later added in order to increase the stability. Because the stone so chosen was considerably coarser than the sand base a gap was introduced into the mix grading almost by accident rather than by design. In addition to there being no rational basis for the sizes chosen, there is also no strict control over the proportions in which the coarse and fine components are to be mixed. For example mixes are permissible which contain from 0% to 55% coarse aggregate. That these mixes work in practice in spite of this lack of attention to basic principles of aggregate grading is due in the main to the quantity of stiff binder which is employed to fill the voids. However even if it is agreed that in general such mixes do work, though there have been notable cases where they have failed, there can be no denying that, because of this lack of design, they have in general a higher cost than would have mixes designed to a lower voids content.

Other methods of gap-grading design, some of which have not advanced out of the theoretical stage, have been based on one of two principles. These are 1) that the size of the fine aggregate to be combined with a given coarse aggregate at any stage in the grading should approximate to the size of the voids present in the coarse aggregate skeleton, i.e. the size ratio between coarse and fine should approximate to the critical ratio of occupation and 2) that the fine aggregate size should be such that it may enter easily into the voids of the coarse aggregate skeleton during emplacement. In other words that the fine aggregate size should be less than the critical ratio of entrance of the coarse aggregate.

In consideration of the first of these, a common attitude has been to consider the packing of spheres and to assume that because the closest packing of uniform spheres gives 25.95% porosity (compared with 47.64 % for the loosest packing) then the densest grading of aggregate can be made up with this basic arrangement of major spheres to which the second, third and fourth etc. orders of successively smaller spheres are added, in sizes and proportions appropriate to them.

Such an approach to the problem has been made by Horsfield (1934) who calculated with great ingenuity the sizes of these secondary, tertiary, quaternary and quinternary spheres. Horsfield's table reproduced in abbreviated form below (Table 7) shows the calculated size ratios of these spheres and the calculated reduction in porosity that would result at each successive stage.

TABLE 7.

	Primary	Secondary	Tertiary	Quaternary	Quinternary	Filler
Radius of sphere	a	0.414 a	0.225 a	0.175 a	0.017 a	very small
Relative no. of spheres	1	1	2	8	8	
% Voids in mix	25.95	20.7	19.0	15.8	14.9	3.9
% Vol.of spheres in mix	77.1	5.5	1.7	3.3	1.0	11.4
Total Surface Area of spheres in mix	$12.57a^2$	$14.73a^2$	$16.00a^2$	$19.08a^2$	$20.46a^2$	

The concept that the achievement of a lowest void content for the combined mix is necessarily associated with the densest packing of its coarsest component has been a popular one, but a misleading one. As has been shown, an experimentally determined densest packing of the coarse aggregate is impossible to reproduce in a multi-component mix because of the interference of the smaller particles on the packing of the larger.

Further if, in addition and as Horsfield attempts, the proportion of fines required is calculated on the basis of a theoretical closest packing of perfect spheres, additional errors are introduced since, as has been shown earlier, no method of depositing and compacting spherical particles, other than hand placing, has ever been able to achieve or even approach the low porosity of the theoretical closest packing.

Both proportion of fines required and size would be affected, since dilation alters not only the total porosity, but also the size of the individual voids. It follows that an experimentally determined closest packing (minimum porosity) is a very poor choice on which to base an estimate for a maximum density grading, but a theoretical closest packing is an even worse choice.

✓ While Horsfield's approach, intended for application to asphalt gradings, never advanced beyond the theoretical stage; Vallette (1954, 1963) derived practical gap gradings for concrete which were based on a similar principle of a discontinuous series of aggregates chosen so that each particle size would be able to fit into the voids of the next larger size.

The different fractions in this discontinuous series are specified by Vallette as follows:- (a four component mix is used in this example).

If D_1 and d_1 represent the upper and lower sieve limits of the coarsest 'single' size present and D_2 and d_2, D_3 and d_3, D_4 and d_4 represent similar limits for 'single' size fractions of successively smaller size than the following relationships should hold

$$\frac{d_1}{D_1} = .67 \; ; \quad \frac{d_2}{D_2} = .67 \text{ to } .50 \; ; \quad \frac{d_3}{D_3} = .50 \; ; \quad \frac{d_4}{D_4} = .50 \quad \ldots\ldots (16)$$

These ratios merely ensure that the 4 components approach single size condition. The gap between these components is fixed, at ratios given as follows, which are chosen according to Vallette to ensure that each particle size can fit into the voids of the next larger size

$$\frac{D_2}{d_1} = .25 \text{ to } .20 \; ; \quad \frac{D_3}{d_2} = .33 \text{ to } .25 \; ; \quad \frac{D_4}{d_3} = .33 \text{ to } .25 \ldots (17)$$

The further stage of selecting the quantities to employ, of the chosen sizes, is performed by means of porosity tests on each size.

Vallette's approach has much to commend it and some of his concepts will be compared later with the author's own conclusions. Bahrner (1951) however referring to earlier work of Vallette commented "It is however rather doubtful whether the estimation of the mixing proportions of the concrete can be carried out as easily as described without specially trained staff. Further it may be difficult and very costly to obtain these sharply bounded single sizes of aggregate".

Compared with Vallette's method, the relationships established by the author and available in graphical form would tend to minimise the first objection (although the author would suggest that it would be no disservice to the construction industry if it were to aim for a higher content of specially trained staff, to deal with materials which though overtly commonplace are in fact highly complex engineering materials).

In regard to the second comment the author's method is available for all aggregate components whether single size or more widely graded.

The second of the existing methods of gap grading design is that in which the gap is fixed relative to the critical ratio of entrance and accordingly in which it is required that the finer particles of any adjacent pair in the series be of this ratio or less. The logic of this requirement is easily seen when considered in relation to such materials as penetration macadam and 'prepakt' concrete where fine material has to be vibrated or grouted into the voids of a coarse aggregate already laid, but the argument for its adoption in the design of a pre-mixed gap graded aggregate is much less sound. This has nevertheless been the criterion for gap size adopted by Bate and Stewart (1955) in their approach to concrete mix design.

In making this choice Bate and Stewart reasoned that honeycombed zones in concrete were sometimes due to the inability of a too coarse fine aggregate to filter into the voids in local pockets of high coarse aggregate concentration. In their view the choice of a fine aggregate smaller than the C_r entrance ensures that those voids do not remain empty but become filled by migration of the fine particles through the inter void "throats".

These two authors although using the other critical ratio from that used by Horsfield, nevertheless initially made the same basic mistake of choosing their value of critical ratio from the ideal rhombohedral packing of spherical particles described by Graton and Fraser, i.e. they considered 0.154 D as the maximum permissible size for the fine aggregate.

This packing and its associated critical ratio of entrance have no direct relevance when actual particles of a variety of possible shapes are considered. Stewart (1952) recognised the need to work with real particles in his statement "The maximum size of the fine aggregate is therefore the admittance size. In practice this size will be found to be 0.125 D. One method of determihing whether a sand is suitable is to sieve it through four or five layers of tightly packed coarse aggregate held between two nesting $\frac{3}{16}$ in. sieves. The sand should always be as coarse as possible while still complying with the conditions stated above." (Surprisingly no mention of this approach is mentioned in Bate and Stewart, 1955). Even with this method the criticism raised previously against Horsfield's approach that a dense packing even of the real aggregate particles cannot be reproduced in a premixed material where other sizes are present, still applies. Also Bate and Stewart's design method while attempting a solution of both parts of the problem, namely choice of size ratio and proportioning, failed to recognise the interdependence of these two, since the degree of dilation and hence the critical ratio of entrance depend on the proportion of the given fine aggregate added.

The present author's position may be summarised as follows:

1) Design on the basis of a critical ratio of entrance is sound for a mix of the 'prepakt' category,

2) For pre-mixed aggregates the critical ratio of entrance determined at closest packing is smaller by several units than the true critical ratio of entrance for the aggregate in its final compacted, but dilated state. Furthermore, in consideration of the fact that a mix during compaction is a collapsing not a static structure, there is a period of time during the earlier stages of compaction when the void entrances are still larger.

These considerations suggest that the ratio suggested by Bate and Stewart is considerably smaller than need be, even to satisfy their own criterion of filtration. The phenomenon of segregation which has been suggested as one of the failings of gap graded mixes arises largely from this fact. Indeed the basic cause of the fault is the criterion accepted for design, for what can filter in can, by definition, also filter out and produce the undesirable segregated mix. It must however be stressed that segregation is not therefore an intrinsic hazard associated with all gap graded mixes but only on those based upon the critical ratio of entrance. It is the author's view

1) that a design of gap gradings based upon a ratio somewhat greater than the critical ratio of entrance (close packing), as will be described, is no more subject to segregation than a continuously graded mix,

2) that particles considerably larger than those equal to this critical ratio of entrance can be accommodated within the framework of coarse particles without causing these to lose contact. Hence lower porosities can be achieved for such mixes, within a given range of maximum and minimum mixes, than when designed on the basis of critical ratio of entrance (close packing) - for there can be in general one or two more components within the range in the former case,

3) that contact between the particles forming the coarse aggregate skeleton clearly begins to be lost when the size of the finer aggregate exceeds the critical ratio of occupation (loose packing) PROVIDED THERE ARE SUFFICIENT OF THESE PARTICLES TO OCCUPY ALL VOIDS. As the previous sentence implies however, coarse aggregate contact is a function not only of size of the finer aggregate but also of its proportion. Hence it is recognised that contact can be lost even with particles smaller than occupation size, if the proportion of such particles is high.

5.2 Proposed method for the design of gap gradings.

In consideration of the defects of present design methods for gap grading based upon an assessment of the critical ratio of entrance (close packing) it has been decided in this work that some value in between this ratio and the critical ratio of occupation (loose packing) would form a sounder basis for design, but that in adopting such criterion for the gap, it is necessary to avoid the error of trying to calculate sizes and size ratios from theoretical packings of idealised particles.

It has already been said that measurements of critical ratios of occupation and entrance for loosest packings of aggregates of a variety of shapes have been made by use of a plastic, sectile, void filling medium. These measurements were therefore available for use in this part of the study.

To summarise the arguments for giving consideration to the criterion for size ratio as stated above it appears that

1) The critical ratio of occupation at closest packing, however desirable theoretically, has no practical significance because in practice dilation precludes this form of packing.

2) The critical ratio of entrance (close packing), or any smaller ratio, tends to provide particles so small as to

(a) encourage segregation

(b) fill the voids of the coarser aggregate with a large number of particles and hence with a large number of their attendant voids, some of which particles and voids could have been replaced by larger particles, hence giving a lower total porosity.

3) Size ratios greater than the critical ratio of occupation imply that the coarser aggregate particles of the considered pair are so far separated as to have largely lost contact even when there is only one occupation size particle per void, and to be still further displaced where there are more than one of the occupation size particles per void. (In the extreme case of still higher size ratios the number of components becomes large and the grading continuous. Even here too, the aim must be not to allow the coarsest particles to be too far separated by the smaller sizes or the objective of a dense structure will be lost).

For a gap graded mix it would appear that the ideal structure would be one in which there is a step-like reduction in size of the components, such that each successive lower size is (a) not so small that voids in the framework of coarser particles are occupied by bodies of small particles plus their attendant voids, which could have been replaced by larger particles, and (b) not so large as to cause the coarser particles to which they are added to be unduly separated, leaving in between void-zones requiring to be filled by a third smaller component at the next stage.

In recognising the latter requirement it is not intended to suggest that exactly the right number of lower size particles to fill each coarse aggregate void once and once only, will be used at the optimum proportions.

Counts made on 2-component mixtures of low P diff, at minimum porosity proportions indicate that when the size ratio is in the region of, or greater than the critical ratio of occupation, there are from 4 to 10 times as many particles of the smaller size present as of the larger size. (For smaller sizes the number of small particles of course increases manifold). This suggests that, for statistical reasons, a mix in which there were equal numbers of large particles and occupation particles, which for theoretically ideal conditions of placing should give the minimum voids, would in practice not do so, presumably because of a mal-distribution of the occupation particles. That is to say that presumably while some voids of the coarse component remain empty, others may be filled, in fact "over-filled", with 2 or more of the occupation size. Apparently it requires at least 4 times as many particles of the smaller size to provide enough chances for 'all' voids to receive a minimum of one occupant particle, and thereby to produce the minimum porosity.

The foregoing considerations lead to the conclusion that if more than one occupation particle (size based on loosest packing) is generally present in each void, at the optimum proportions, then the coarse aggregate will, under these conditions, be on average dilated to a state of porosity above that of its loosest packing.

The approach was therefore considered of investigating, by means of the specific void content graph, the size ratio for which the fine aggregate would dilate the coarse aggregate to the state of its loosest packing, when both components were combined at their optimum proportions at the compactive effort designed to produce their densest state of packing.

Figure 73 illustrates this new concept namely that there is a critical size ratio (referred to hereafter as the critical ratio of dilation, by analogy with the critical ratios of entrance and occupation), above which the coarse aggregate is dilated to a porosity greater than that of its loosest packing and below which the coarse aggregate is dilated to a porosity intermediate between that of its closest and that of its loosest packings.

Re-examination of the experimental porosity versus mix composition results previously referred to, on specific void content diagrams, showed that the critical ratio of dilation lay in the region of 0.23. Further investigations are necessary to define its precise range in respect of all aggregate types, but this value was considered a reasonable one upon which to base further experiments when considered in relation to the limiting values of 0.154 (theoretical critical ratio of entrance (close packing)) (and 0.37 (mean experimental value of critical ratio of occupation (loose packing) which for the reasons given above, it would not be desirable to approach. It is of interest further to observe that the value of 0.23 is within the range suggested by Vallette, and the present author would tend to agree with the implications of Vallette's figures that the precise

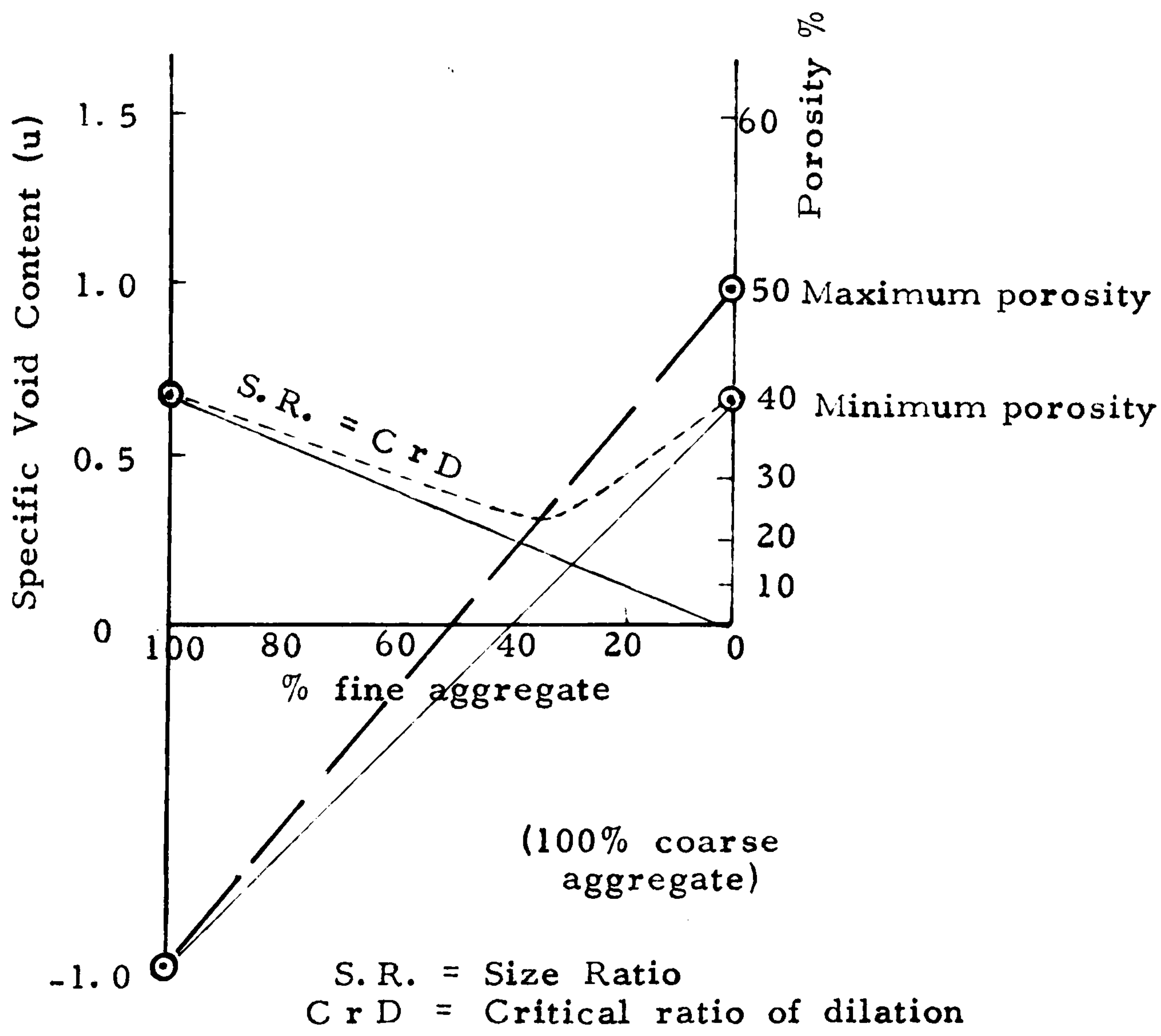


Fig. 73. SPECIFIC VOID CONTENT DIAGRAM TO ILLUSTRATE CONCEPT OF CRITICAL RATIO OF DILATION.

value is not critical but that it is preferable to err on the side of slightly larger sizes rather than to accept significantly lower ratios.

This average value of critical ratio of dilation was therefore chosen as the most reasonable and practical size ratio to employ in fixing the gaps in a gap grading since this is the only way in which to avoid dealing with the two parts of the gap grading design problem in isolation. It would appear illogical to choose the size of particles to employ, separately from the choice of proportion or vice versa. The use of the critical ratio of dilation enables both parts of the problem to be tackled at the same time since the size so chosen to produce a certain deliberate effect on the coarser aggregate structure is, by definition, to be combined with it at the optimum proportion.

This proportion will be decided in the manner already described for any 2 component mix, i.e by reference to the parameters P av, P diff and size ratio, and the procedure repeated as many times as is relevant to the total range of sizes represented. The wider the total range the more frequent will be the gaps and the number of individual sizes.

The first experiments carried out in this connection were aimed at testing the applicability of the method of determining the optimum % fines for 2 component systems with reference to multi-component systems in general. 3 component mixes were initially selected, at size ratios ranging between 0.14 and 0.40 for adjacent pairs of components , in order to test (a) the accuracy of the prediction of porosity by the method for 2 component mixes adapted as described below, and (b) the value of porosity as compared with mixes of the same materials prepared according to the Furnas (1931) equation:-

total absolute volume of solids =

$$\frac{d_1}{\dfrac{1 - V_1}{(1 - V_1) + V_1(1 - V_2)}} + \frac{d_2}{1 - \dfrac{1 - V_1}{(1 - V_1) + V_1(1 - V_2)}}$$

$$+ \frac{d_3}{\left[1 - \dfrac{(1 - V_1)}{(1 - V_1) + V_1(1 - V_2)}\right]\left[\dfrac{1 - \dfrac{1 - V_2}{(1 - V_2) + V_2(1 - V_3)}}{\dfrac{1 - V_2}{(1 - V_2) + V_2(1 - V_3)}}\right]}$$

$$+ \ldots\ldots\ldots\ldots \qquad \ldots\ldots (18)$$

where d_1 , d_2, d_3, represent the proportions by absolute volume of the first (i.e. coarsest), second and third sizes.

This equation was based upon equation (9) previously quoted (p.87) and makes the same assumptions as there discussed.

For the proposed method it was hypothesised that the following techniques could be applied for the extension of the 2 component system analysis into the field of 3 or more component systems:-

The 2 largest components are considered as combined in the proportions determined as previously described in Chapter 4. The resultant porosity, determined also as described in Chapter 4, by reference to the graph of relative contraction against size ratio, is then considered as the porosity of a new "2 part" coarse aggregate, then to be combined with a third and finer component. New values of P av and P diff are calculated from the known separate porosities (1 calculated, 1 experimental) and a new size ratio calculated between the third component and the composite coarse component.

The calculation of the size ratio in itself poses another problem, namely how to calculate a mean equivalent spherical diameter for the 2 sizes comprising the coarse component.

There are as Hughes (1966) has shown 4 ways in which a mean diameter between particles of different size groups can be calculated. These are as follows (quote):-

" (a) The diameter given by the simple arithmetical mean , and referred to as the "equivalent mean diameter", D.
(b) The mean diameter, E, as calculated by Butcher and Hopkins (1956).
(c) The mean diameter, F, as given by a sphere having the average volume of the particles.
(d) The mean diameter, 6/G, as given by a sphere having the average surface area per unit volume of the particles, when the particles are assumed to be spherical; G is the "grading modulus".

The mathematical expressions from which these mean diameters are calculated are:

$$\text{(a)} \quad D = \sum_{i=1}^{n} p_i d_i$$

$$\text{(b)} \quad E^{-3} = \sum_{i=1}^{n} p_i d_i^{-3}$$

$$\text{(c)} \quad F^3 = \sum_{i=1}^{n} p_i d_i^{\ 3}$$

$$\text{(d)} \quad G/6 = \sum_{i=1}^{n} p_i d_i^{\ -1}$$

where p_1, p_2, ... p_n are the proportions by volume of particles of diameter d_1, d_2, ... d_n respectively."

Although Hughes points out that one of these may emphasise one characteristic, while another emphasises a different characteristic, it has been on the basis of purely empirical reasoning, namely following comparison of measured and predicted values of porosity, that the mean diameter $\frac{6}{G}$ has been chosen for application to the method here proposed for multi-component systems.

From the size ratio thus calculated and from the computed values of P av and P diff, the new value of % fines for combination of the third component is determined from the graphs. Successive fine components are added to the system in like manner.

It must be recognised in this connection with gap-grading design that the size ratio is being used for 2 purposes. In the method as proposed, one use of the size ratio is as the basis on which are chosen the sizes that will be used in the gap grading and for this purpose a size ratio of the order of the critical ratio of dilation between ADJACENT pairs has been proposed. The second usage of the size ratio is as one of the three parameters from which the optimum % fines is determined. As just discussed, this size ratio is best determined on a CUMULATIVE basis, i.e. as a ratio between the size of the finest fraction and the mean size of all coarser fractions considered together.

Once the two different usages are appreciated it may be recognised that there is no inconsistency in choosing the sizes to be used in the gap grading by reference to an ADJACENT size ratio, while calculating the optimum % fines at each stage in the design procedure by reference to a CUMULATIVE size ratio.

To summarise the procedure diagrammatically, Fig.74 shows how the design method for a 2 component system may be extended to cope with additional components. In the diagram the successively finer aggregates hypothesised have all the same porosity. This of course is not

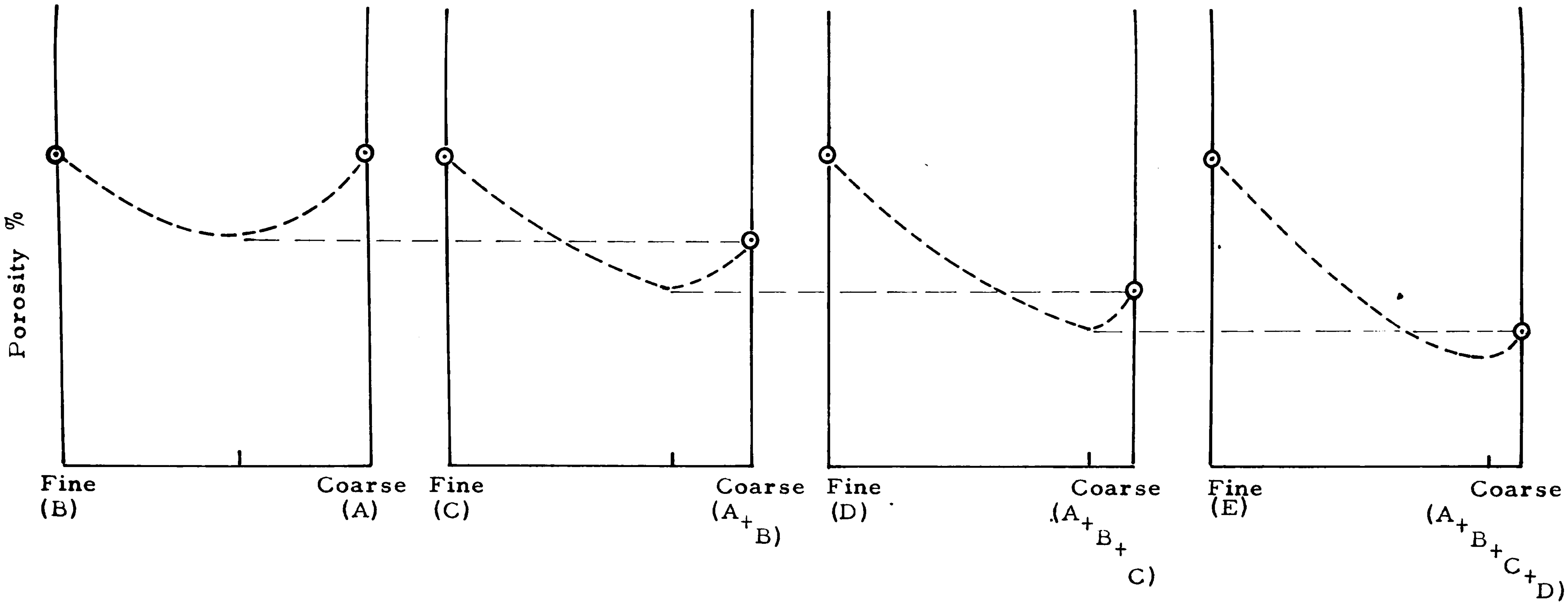


Fig. 74. DIAGRAM TO ILLUSTRATE THE CONCEPT OF THE EXTENSION OF TWO COMPONENT MIX DESIGN INTO THE REALM OF MULTI-COMPONENT SYSTEMS.

necessarily true but illustrates for their case, a progressive downwards trend in porosity at the minimum points with addition of successive fine components and a progressive movement to the right of the optimum mix proportion. This takes it to the coarser side, but against that it will be noted that the coarse component itself becomes progressively finer as more components are added to it.

Experimental results provided the following data (Table 8), from which the validity of the extension of the 2 component analysis into the realm of 3 components is confirmed. Predicted porosities for the proposed method at the 2 component stage and the final 3 component stage are generally close to the measured value with a similar order of accuracy (the maximum difference in the results given in Table 8, plus a further 7 results not recorded in the table, is 2.8%). In all cases the mixes designed by the new method gave porosities several per cent less than those achieved or predicted by use of the Furnas equation.

In the mixes referred to in Table 8 size ratios as extreme as 0.14 and 0.40 had been used in order to demonstrate that the method can be applied to all size ratios which may be chosen, or dictated by circumstance. Indeed although a preference has been stated for a size ratio of the order of the critical ratio of dilation, it will be rare (even with laboratory facilities) that aggregates of the precise size required will be available.

As a final stage in this study of gap gradings consideration was given to their major use in one of the most important forms of road construction material, namely rolled asphalt to B.S. 594 (1961). The mixes specified in this standard are comprised, in the main, of 3 aggregate components (coarse aggregate, sand fines and filler) and a liquid component (a bituminous binder). A specimen mix given below in Table 9, shows size ratios between adjacent pairs of the 3 aggregate components to be very low - of the order of 0.04 and 0.12. Consequently only 3 components can be fitted into the limits fixed by maximum and minimum size. On the other hand if larger size ratios of the order of 0.23 (or as near as is possible with available aggregates) are employed, 5 components may be fitted into the required range.

The lower section of Table 9 gives the design calculations and final mix proportions for such a 5 component mix, for comparison with the 3 component mix which would comply with B.S. 594.

Table 8. Comparison of Composition and predicted porosities for 3 component mixes prepared according to 2 methods.

Mix		FURNAS METHOD			PROPOSED METHOD		
		Recommended Proportions % (vol.)	Approximate Predicted Porosity %	Measured Porosity %	Recommended Proportions % (vol.)	Predicted Porosity %	Measured Porosity %
G 1	¾" - ½" Rounded Gravel	66.3			60.3		
	¼" - 3/16" Crushed Gravel	24.6			28.4	2compt. 31.0	30.5
	14-25 L.Buzzard Sand	9.1	3compt. 27.5		11.3	3compt. 19.8	21.9
B 1	1½"- 1" Eq.,Basalt	60.1			34.0		
	5/16"-¼" Crushed Gravel	29.2			46.9	2compt. 30.0	32.1
	7 - 14 L. Buzzard Sand	10.7	3compt. 27.5	27.3	19.2	3compt. 23.0	24.5
B 2	1½" -1" Discs Basalt	61.3			35.9		
	¼" - 3/16" Crushed Gravel	27.7			46.7	2compt. 30.3	32.2
	14 - 25 L. Buzzard Sand	11.0	3compt. 24.0	28.3	17.5	3compt. 20.7	23.5
B 3	1½"- 1" Blades Basalt	58.2			17.7		
	5/16" - ¼" Crushed Gravel	30.4			49.9	2compt. 32.0	34.8
	7 - 14 L. Buzzard Sand	11.4	3compt. 27.5	30.3	32.3	3compt. 24.5	24.4
B 4	1½"- 1" Rods Basalt	63.2			31.3		
	5/16" - ¼" Crushed Gravel	25.5			51.8	2compt. 30.0	31.8
	7 - 14 L. Buzzard Sand	11.3	3compt. 26.0	31.0	17.0	3compt. 24.0	24.6
B 5	½" - ⅜" Eq. Basalt	63.7			45.3		
	3/16" - 5 Crushed Basalt	26.0			30.3	2compt. 34.9	33.8
	14 - 25 L. Buzzard Sand	10.3	3compt. 29.7		24.4	3compt. 22.8	23.9
B 6	½" - ⅜" Rods, Basalt	62.6			42.5		
	3/16" - 5 Crushed Gravel	26.8			36.2	2compt. 34.5	33.1
	14 - 25 L. Buzzard Sand	10.6	3compt. 28.5		21.4	3compt. 21.9	24.7
B 7	1" - ¾" Eq., Basalt	63.7			57.5		
	3/16" - 5 Crushed Gravel	26.0			30.9	2compt. 29.2	30.6
	14 - 25 L. Buzzard Sand	10.3	3compt. 25.5		11.6	3compt. 21.7	23.9
B 8	1" - ¾" Discs, Basalt	61.4			37.8		
	3/16" - 5 Crushed Gravel	27.6			41.0	2compt. 32.1	33.0
	14 - 25 L. Buzzard Sand	10.9	3compt. 26.5		21.3	3compt. 22.8	23.5
B 9	1" - ¾" Blades, Basalt	60.6			31.4		
	3/16" - 5 Crushed Gravel	28.3			44.4	2compt. 32.1	33.7
	14 - 25 L. Buzzard Sand	11.9	3compt. 25.8		24.3	3compt. 22.8	24.6
B 10	1" - ¾" Rods, Basalt	62.2			35.6		
	¼" - 3/16" Crushed Gravel	27.4			44.3	2compt. 31.0	32.8
	14 - 25 L. Buzzard Sand	10.4	3compt. 24.9		20.1	3compt. 20.8	22.7

B.S. 594 does not give any preferred or recommended proportions of fine to coarse aggregate. The proportions of the 3 component mix were therefore also determined by the method here proposed.

The predicted porosities in both mixes were close to the measured porosities, confirming again the validity of the method, but the final porosity of the 5 component mix was slightly lower than of the 3 component mix.

It cannot be assumed without further evidence that the minimum possible voids content in the aggregate is necessarily the most desirable property to produce, in a mix of an asphaltic composition. It may be, as discussed previously, that a void content above the minimum which can be obtained, is necessary in order to accommodate a minimum quantity of bituminous binder necessary to provide adequate durability (weathering resistance) and fatigue flexural strength. Nevertheless it has been shown that conventional 3 component mixes to B.S. 594, and the newly suggested mixes with a slightly larger number of components can both be designed to give low voids if needed, or alternatively, because the porosities are predictable with reasonable accuracy at all stages, be designed by omission of certain components, to give some void value which is not the minimum obtainable, should this be desired.

5.3 Design of continuous gradings.

Although at first consideration of the larger number of component sizes involved, the problem of the design of continuous gradings might seem more complex than that of gap gradings this is in fact not so. On the contrary since all sizes will be represented, one of the problems of gap grading design is eliminated - namely that of choosing the sizes to be used. The problem remains of deciding the position of arbitrary divisions of the continuous range and of selecting the proportions that will be used of these arbitrarily divided size groups.

Historically the best known of all systems of continuous grading and still frequently referred to, is that known as the Fuller curve and is due to Fuller and Thompson (1907). These authors made up gradings of a wide variety of types, and derived a "maximum density curve" from consideration of their results. As observed by Hveem (1940) this curve, on an arithmetic plot of summation % against grain size has the form of an ellipse on the finer portion of the curve and has been projected either as a straight line or (more usually) as a parabola from the vertical axis of the ellipse to the upper right hand corner of the chart (Fig.75).

On Fig.76 the curve is shown on the more familiar semi-logarithmiz plot.

The two portions of the curve, elliptical then parabolic, follow the equations,

$$p = 100 \times \left| \frac{d}{D} \right|^{0.2} \qquad \text{.........} \ (19)$$

and

$$p = 100 \times \left| \frac{d}{D} \right|^{0.5} \qquad \text{.........} \ (20)$$

Many authors (e.g. Road Research Laboratory, 1952) however, seem to have neglected the elliptical section of the Fuller curve and assumed equation (20) as synonymous with the Fuller grading. This assumption tends to produce mixes with a lower fines content (e.g. 6% passing No.200 sieve with maximum size $\frac{3}{4}$") compared with the elliptical/parabolic curve (which would produce approximately 15% of the stated size at the same maximum size).

Equations (19) and (20) above are special forms of equation (15)

$$p = 100 \times \left| \frac{d}{D} \right|^{n}$$, previously given.

Other authors have investigated continuously graded systems of this type with varying values of the exponent n. Talbot and Richart (1923) have, for example, investigated a large number of such systems with n varying from 0.24 to 1.20. The systems studied by them, varied in their maximum size up to 2" and were investigated both as 'dry' aggregate systems and as concrete mixes with cement and water added (at approximately constant cement/aggregate ratio and constant workability).

From their results Talbot and Richart noted "that for the dry aggregate the point of minimum voids in any group of aggregates, is away from the coarse gradations and much nearer the finest gradations. With the addition of cement and water the point of minimum space outside of the aggregate, which is also the point of minimum voids in the concrete corresponds to a much coarser gradation".

Talbot and Richart offer no explanation for their observation, but it would seem that 2 explanations are possible:-

(1) that since the cement also constitutes a fine aggregate (but is not included in the grading analysis), a smaller quantity

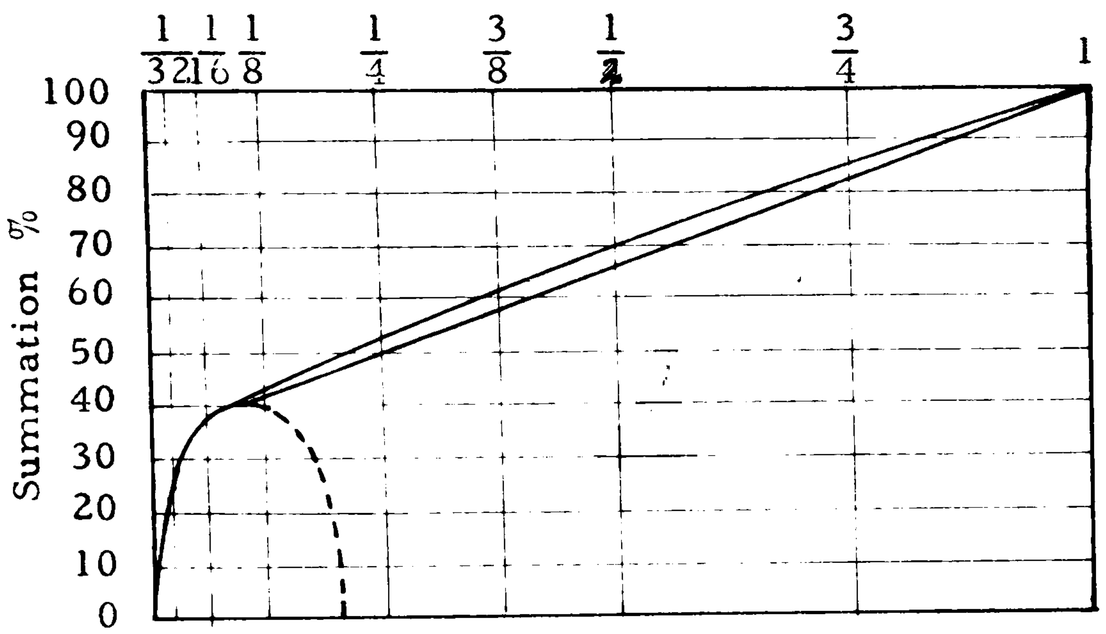


Fig. 75. THE "FULLER" CURVE
(abscissa-arithmetic scale)

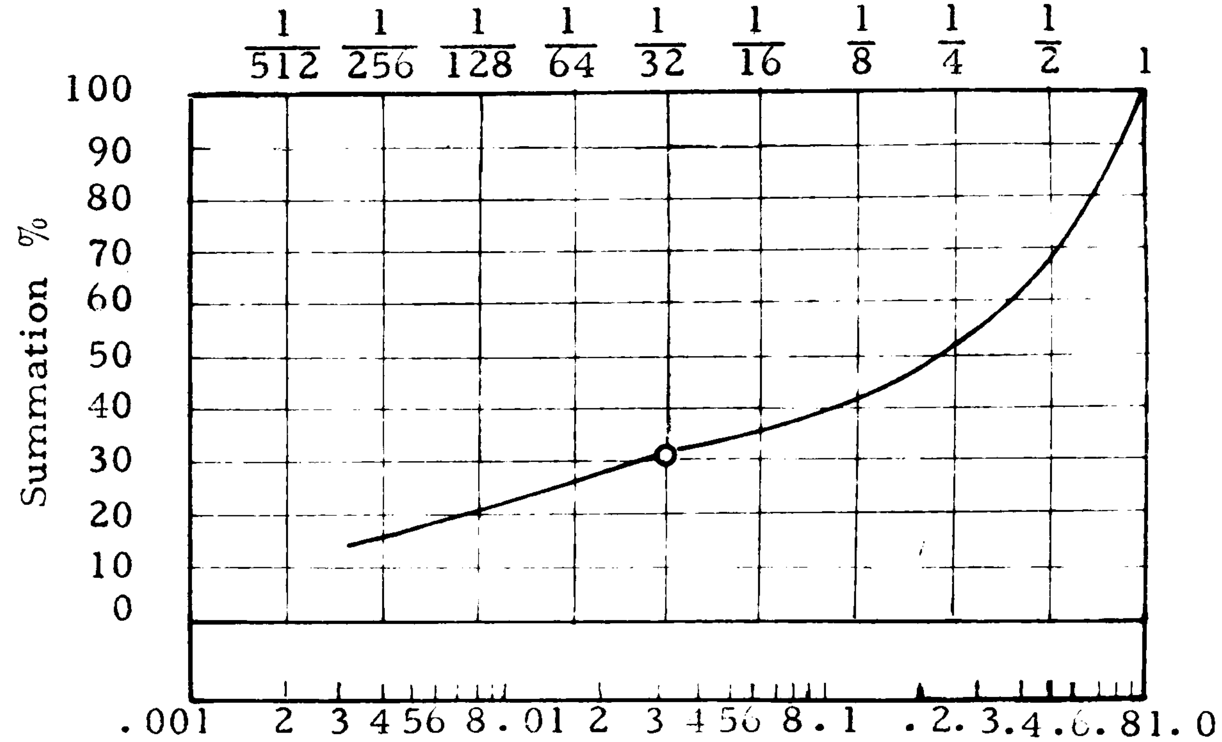


Fig. 76. THE "FULLER" CURVE
(abscissa-logarathmic scale)

of the sand fine aggregate is required for conditions of maximum solid volume density when cement is also present,

alternatively, or in addition,

(2) that bulking of the finer particles takes place in the presence of water, as discussed in Chapter 4, implying that a smaller quantity of this expanded fine material can be accommodated within the voids of the coarse aggregate skeleton than when this same material is dry.

These explanations, if accepted, confirm the view previously expressed of the importance of taking all components and all adhesive and lubricating coatings into account.

All approaches of this nature however whether they follow the Fuller tradition,one of Talbot and Richart's curves or arbitrary curves or envelopes such as those of the Asphalt Institute, Road Note 4, etc., etc., suffer from the unjustified assumption that some one grading curve is the best to use regardless of the packing properties of the aggregates or variations in these packing properties from size to size, and regardless of the effort to be applied, the boundary restraints, and usually regardless of the cement content in concretes and of the filler and binder contents in asphalts.

Some attempts by other authors have been made to improve the situation by relating the proportions of the various sizes to be used to certain of these factors but have usually been incomplete in one way or another.

Weymouth (1933), for example, referring to the workability of concrete mixtures took as his criterion the arrangement of the aggregate particles when in place in the finished concrete, suggesting that in this situation, "no particle interference should exist", that is to say that the most desirable relationship between the proportions of the various size groups used, should be one in which the inter particle spacing is such that each particle has just sufficient room to be able to move into the space between particles of the next larger size.

Dunagan (1940) employed Weymouth's formula

$$t = \left[(do / da)^{1/3} - 1 \right] D$$

where t = ave. distance between particles of ave.diameter D

do = density of a size group (solids per unit volume)

da = ratio of absolute volume of a size group to the space available to that size in the concrete, to compute aggregate gradings which were then compared with corresponding Fuller gradings , e.g.

Sizes	Percent by weight of each size	
	Weymouth	Fuller
$1\frac{1}{2}$" - $\frac{3}{4}$"	40.4	57
$\frac{3}{4}$" - $\frac{3}{8}$"	32.8	29
$\frac{3}{8}$" - No.4 sieve	26.9	14

As Dunagan points out the first point of contrast is that the Weymouth formula distributes the material with relatively more of the intermediate sizes and second that Weymouth's formula, in contrast to the Fuller curve allows variations in the grading according to type of aggregate (since 'do' varies with type of aggregate).

This is certainly the approach which it is desirable to follow, but suffers from the following three disadvantages which would apply to both the criterion of workability (as applied by Weymouth) and the criterion of density (should an application in this respect be sought), notwithstanding the fact, as discussed by Hopkins (1962), that it cannot be assumed that the same gradings will necessarily give the best results in respect of both qualities:-

1) the formula depends upon an assumption that there exists "a most favourable spacing" between particles. Tests carried out by Dunagan suggested this to be equal to 0.5 D, but it cannot be regarded as proved that this value would apply to all aggregates and all environmental conditions.
2) the computation of mix proportions rests upon a single value of 'do' (obtained from one 'single-size' fraction of the coarse aggregate) and as Dunagan says "depends upon the assumption that 'do' will be the same for all size groups of a given material ."

The method thus takes no account of such variations in packing properties as may occur in the different size groups.

3) the method takes no account of the packing characteristics of the fine aggregate. Dunagan's paper contains the statement "consider the remainder of the mix to be fine aggregate" when referring to the calculation of mix proportions - a statement which clearly implies that the fine aggregate content is to be found by difference and that no alteration in proportions is intended for different fine aggregate types or gradings.

This might also be considered as one of the limitations of Hughes'(1960, 1966) approach so far as a general theory of packing is concerned, he also neglected the packing properties of the fine aggregate. It must be recalled however that Hughes' aggregate proportioning method was also based upon the aim of achieving maximum workability rather than maximum density. Hughes reports that his experimental results showed that the packing properties of the fine aggregate did not require to be taken into consideration when computing the optimum coarse aggregate content for maximum workability. The author, not having investigated the workability aspect of grading, cannot comment on this view other than to suggest that the workability tests may be rather insensitive to this parameter *. It has already been shown that porosity, on the other hand, is sensitive to the fine aggregate packing properties as well as to those of the coarse aggregate.

5.4. Proposed method for the design of continuous gradings.

The method here proposed for the design of continuous gradings follows directly from the procedures already outlined for design of 2 component and multi-component gap-graded mixes. The method is in fact identical with that described for gap grading design, once the intermittent component sizes of that grading had been chosen.

The details of the design procedure will therefore not be repeated here. Rather, attention will be drawn to examples of the design of specific mixes in which 13 to 15 arbitrarily divided size components of contrasting shape characteristics were combined. In these examples (Tables 10 & 11) the predicted porosities were checked at selected stages

* Footnote. Hughes (personal communication) explains this insensitivity in relation to normal concrete mixes, as due to the comparatively small proportion of the fine aggregate in relation to the other components in such mixes. In lean mixes, where the voids in the coarse aggregate contain a higher ratio of sand to cement/water paste, he feels that the influence of fine aggregate packing properties would begin to be felt in workability, as here noted in respect of density.

a Particle	b P_{dense}	c ESD.(cm)	d P_{av}	e P_{diff}	f S.R.	g % fine	h Unit Volume	i Total Unit Vol.	j Mean Diameter	k Relative Contraction	l P_{min} Graph Predicted	m P_{min} Actual	n Unit Wt.	o % Wt.	p Summation %	q Col.o .8500 gm
¾"–½" C.B. Eq.	42.1	1.563					36.70 A	36.70					36.70	8.30	100.00	705
½"–⅜" C.B. R.	43.7	1.313	42.9	– 1.6	.840	63.30	63.30 B	100.00	1.395	.011	41.8	41.3	63.30	14.32	91.70	1217
⅜"–¼" C.B. U	48.5	0.7213	45.1	– 6.7	.517	14.30	16.69 C	116.69	1.231	.018	41.4	40.2	16.69	3.77	77.38	320
¼"–3/16" C.B. U.	49.1	0.4933	45.3	– 7.7	.401	12.50	16.67 D	133.36	1.037	.029	40.8	38.3	16.67	3.77	73.61	320
3/16" –7 C.G.	44.7	0.3066	42.8	– 3.9	.296	22.8	39.39 E	172.75	0.6720	.132	37.8	36.3	39.39	8.91	69.84	757
7 – 14 S.	35.3	0.1730	36.5	+ 2.5	.257	33.9	88.59 F	261.34	0.3397	.248	29.8	29.4	82.11	18.57	60.93	1578
14 – 25 S.	35.0	0.08395	32.4	– 5.2	.247	13.6	41.13 G	302.47	0.2402	.185	26.2		38.12	8.62	42.36	733
25 – 36 S.	35.7	0.0579	30.9	– 9.5	.241	9.0	29.91 H	332.38	0.1872	.840	24.8		27.72	6.27	33.74	533
36 – 52 S.	36.7	0.03651	30.7	–11.9	.195	8.6	31.27 I	363.65	0.1382	.109	23.1		28.98	6.55	27.47	557
52 – 72 S.	37.9	0.02600	30.5	–14.8	.188	7.0	27.38 J	391.03	0.1061	.077	22.0		25.38	5.74	20.92	488
72 –100 S.	38.7	0.01814	30.3	–16.7	.171	6.4	26.74 K	417.77	0.0809	.072	21.0	21.2	24.78	5.61	15.18	477
100–150 S.	41.3	0.01294	31.1	–20.3	.160	5.4	23.85 L	441.62	0.0630	.059	20.3	19.6	22.10	5.00	9.57	425
150–200 S.	39.0	0.01117	29.6	–18.7	.177	4.7	21.78 M	463.40		.049	19.7	19.1	20.19	4.57	4.57	388

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P = porosity
S.R. = size ratio
C.B. = crushed basalt
U. = unsorted by shape
av. = average
E.S.D. = equivalent spherical diameter
Eq. = equidimensional
C.G. = crushed gravel
diff. = difference
min. = minimum i.e. at optimum proportions
R = rods
S. = sand

Table 10. MIX-DESIGN FOR MINIMUM POROSITY FOR 13 COMPONENT CONTINUOUS GRADING OF CRUSHED BASALT (ROWLEY REGIS), CRUSHED GRAVEL (WEEFORD, STAFFS.) & NATURAL SAND (LEIGHTON BUZZARD) (curve L, Fig.77)

a Particle	b P_{dense}	c E.S.D.(cm)	d P_{av}	e P_{diff}	f S.R.	g % fine	h Unit Volume	i Total Unit Vol.	j Mean Diameter	k Relative Contraction	l Pmin Graph (Predicted)	m P_{min} Actual	n % wt.	o Summation %	p Col.n X 8500 gm
3/4" – 1/2" G.	37.6	1.615					43.14 A	43.14					3.02	100.01	257
1/2" – 3/8" G.	37.1	1.193	37.3	+0.5	.739	56.86	56.86 B	100.00	1.345	.036	36.3		3.97	96.99	337
3/8" – 5/16" G.	37.1	0.947	36.7	–0.8	.704	48.00	52.00 C	152.00	1.119	.040	35.4		3.64	93.02	309
5/16" – 1/4" G.	36.6	0.794	36.0	–1.2	.710	47.2	135.90 D	287.90	0.9378	.032	34.7		9.50	89.38	807
1/4" – 3/16" G.	37.8	0.600	36.3	–3.1	.640	34.6	152.31 E	440.21	0.7849	.011	34.5		10.65	79.88	905
3/16" – 5 C.G.	39.6	0.450	37.1	–5.1	.573	22.6	128.54 F	568.75	0.6897	.011	34.3		8.99	69.23	764
5 – 7 S.	48.4	0.273	41.4	–14.1	.396	6.7	40.85 G	609.60	0.6257	.001	34.3	34.95	2.86	60.24	243
7 – 14 S.	35.3	0.173	34.8	–1.0	.276	25.7	210.90 H	820.50	0.3741	.279	28.1		14.74	57.38	1253
14 – 25 S.	35.0	0.08395	31.5	–6.9	.224	12.4	116.10 I	936.60	0.2618	.170	25.0		8.12	42.64	690
25 – 36 S.	35.6	0.0579	30.3	–10.6	.221	9.1	93.76 J	1030.36	0.1982	.092	23.5	24.75	6.56	34.52	558
36 – 52 S.	36.7	0.03651	30.1	–13.2	.184	8.7	98.18 K	1128.54	0.1431	.106	21.9	22.96	6.86	27.96	583
52 – 72 S.	37.9	0.0260	29.9	–16.0	.182	6.5	78.46 L	1207.00	0.1107	.065	21.0	22.41	5.49	21.10	467
72 –100 S.	38.7	0.01814	29.8	–17.7	.164	6.3	81.15 M	1288.15	0.08377	.070	20.1	20.94	5.67	15.61	482
100–150 S.	41.3	0.01294	30.7	–21.2	.154	5.2	70.65 N	1358.80	0.06521	.061	19.4	20.15	4.94	9.94	420
150–200 S.	39.0	0.01117	29.2	–19.6	.171	5.0	71.52 O	1430.32		.042	18.9	18.66	5.00	5.00	425
															8500

P = porosity
av. = average
diff. = difference

S.R. = size ratio
E.S.D. = equivalent spherical diameter
min. = minimum i.e. at optimum proportions

G = natural gravel
C.G. = crushed gravel
S. = sand

Table 11. MIX-DESIGN FOR MINIMUM POROSITY FOR 15 COMPONENT CONTINUOUS GRADING OF NATURAL GRAVELS, CRUSHED GRAVELS, (WEEFORD, STAFFS.) AND NATURAL SANDS (LEIGHTON BUZZARD) (curve L_2 : Fig.78)

against the measured porosities and the final porosity compared with that obtained with other gradings arrived at by reference to the methods of other authors or arbitrarily chosen in order to investigate the effect of deviations from the designed composition.

As in previous cases the reasonably high accuracy of the prediction of porosity values by the method is confirmed (columns l and m of Tables 10 & 11) as also is the realisation of a lower porosity for the mixes as designed than could be achieved with any other of the gradings used for comparison (Figs. 77 and 78).

Gradings lying close to the designed mix composition did not of course differ from it very much in porosity; so far as could be determined the porosity rose in gradings that lay either to the finer or the coarser side of the design composition.

The main point of contrast between the author's grading curves (L1 & L2) and the curve $p = 100 \times \left(\frac{d}{D}\right)^{0.5}$, often quoted as the maximum density curve, is that the former do not follow the smooth form of the latter. However it is in this irregularity that their main merit is believed to lie, being a reflection of the irregular packing characteristics of the aggregates at their various size fractions.

It should be noted in conclusion that although the curves as designed do represent, in the author's view, the ideal gradings for minimum porosity for the aggregates chosen, they satisfy this condition only in relation to

(a) the particular compaction procedure adopted

and (b) the container employed, namely the compacting factor cylinder.

Should either be significantly changed then the minimum porosity grading curve would also be changed.

The argument may be offered that the need to design each mix individually in respect of changing aggregate type, changing compaction and changing environment increases the complexity of the design.

This is indeed true, as compared with a design chosen according to a standard grading curve or grading envelope. However in the author's view the complexity of the method offered, originates in and reflects the inherent complexity of the granular system, the neglect of which can only lead to less satisfactory results.

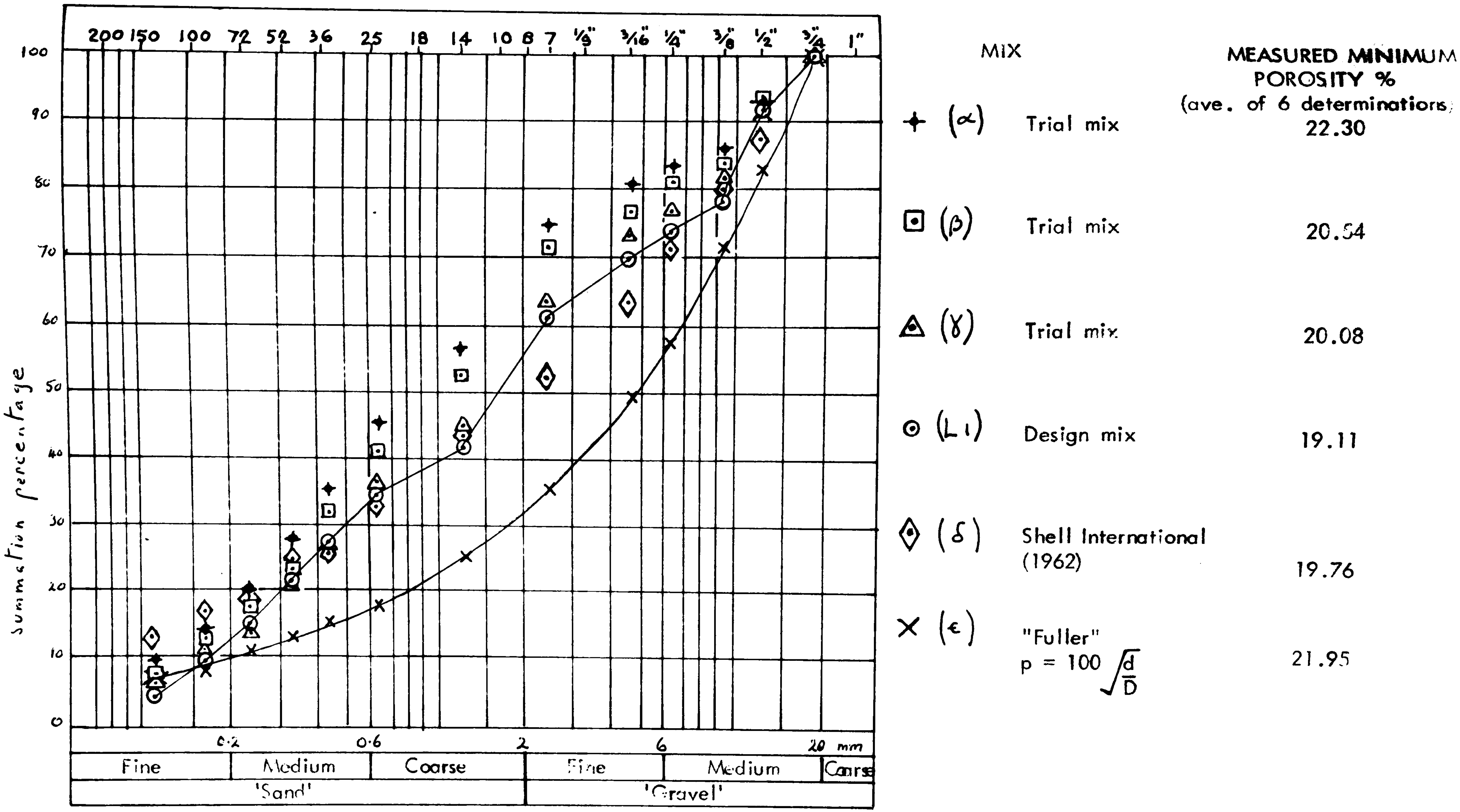
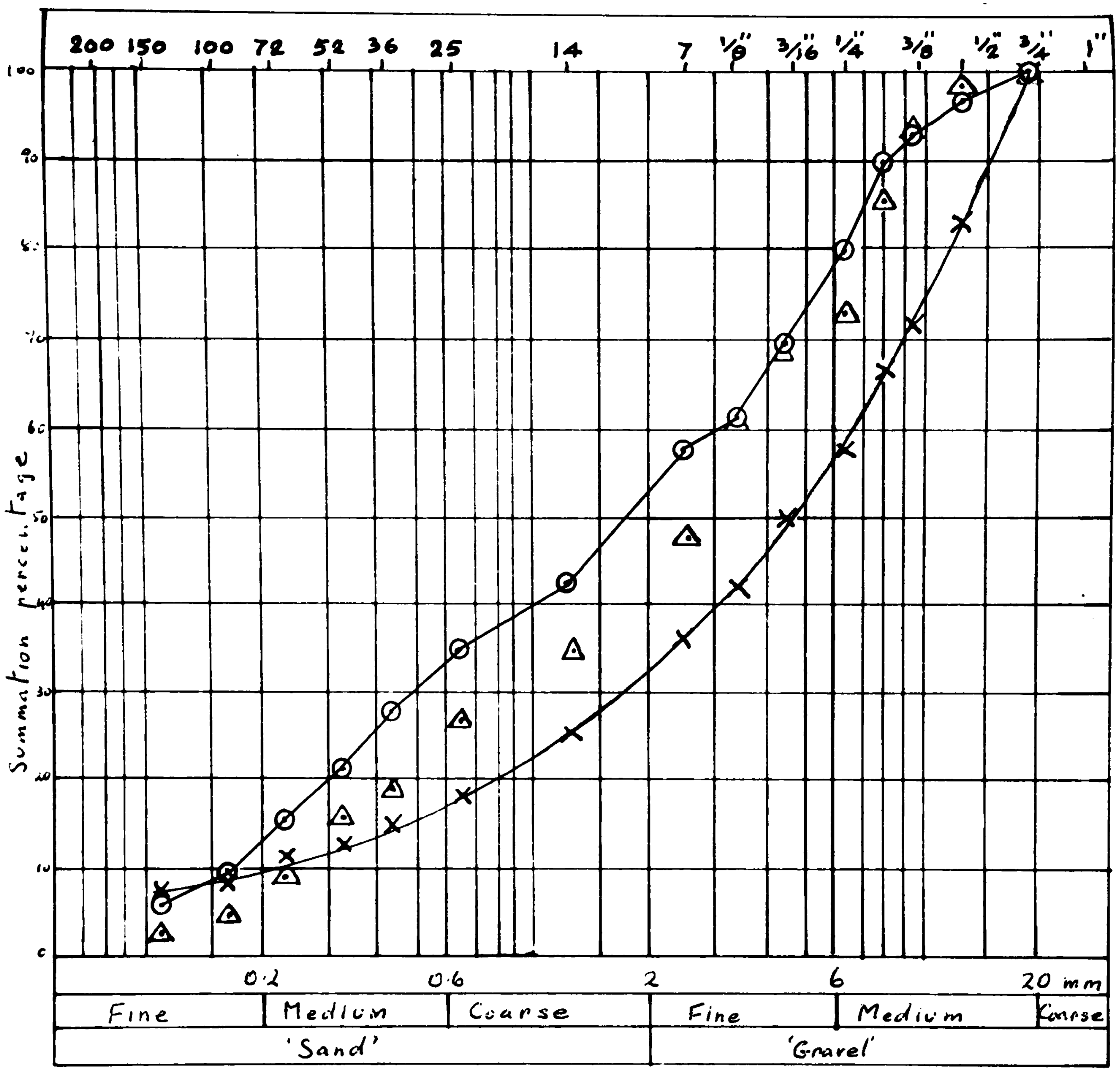


Fig. 77 GRADING CURVES AND MEASURED POROSITIES FOR MIXTURES OF CRUSHED BASALT, CRUSHED GRAVEL AND NATURAL SAND (AS TABLE 10)



MIX		MEASURED MINIMUM POROSITIES % (ave. of 6 determinations)
⊙ (L 2)	Design Mix	18.66
△ (a)	Trial Mix	19.58
× (b)	"Fuller" $p = 100 \sqrt{\frac{d}{D}}$	20.14

Fig. 78. GRADING CURVES AND MEASURED POROSITIES FOR MIXTURES OF NATURAL AND CRUSHED GRAVEL AND NATURAL SAND (AS TABLE 11)

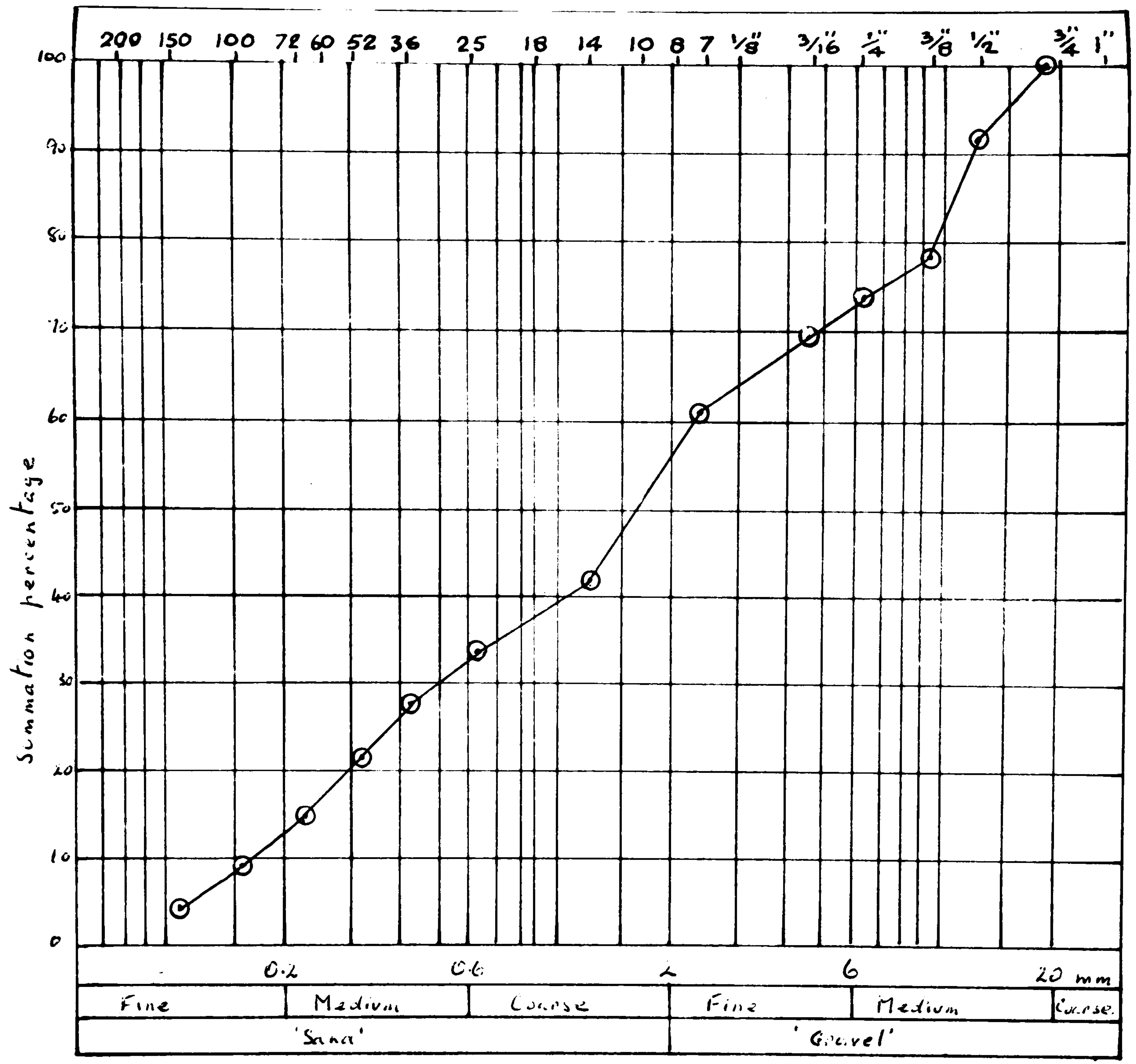


Fig. 79. DESIGN GRADING CURVE - BASALT/GRAVEL/SAND MIX (AS TABLE 10)

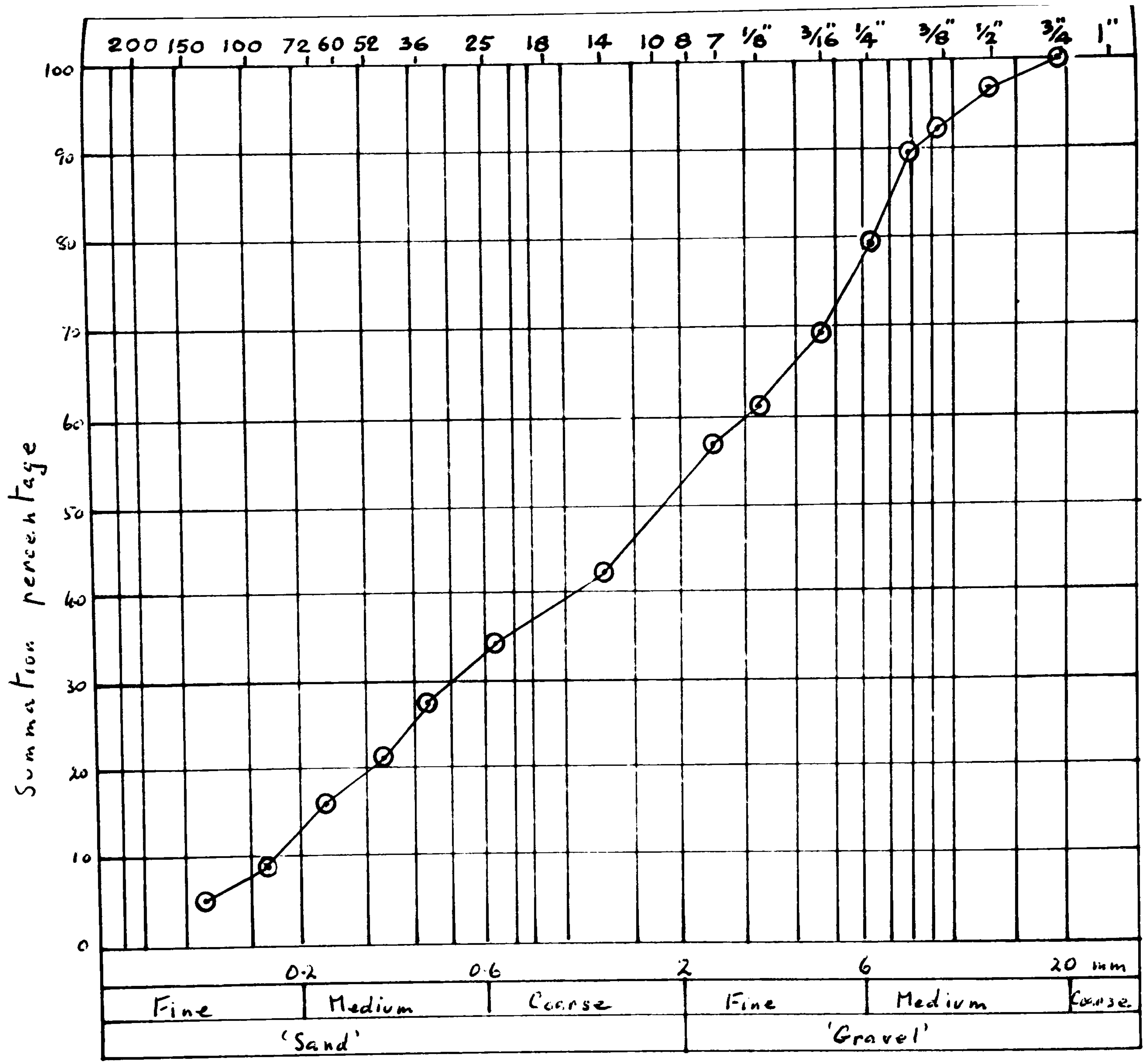


Fig. 80. DESIGN GRADING CURVE - GRAVEL/SAND MIX (AS TABLE 11)

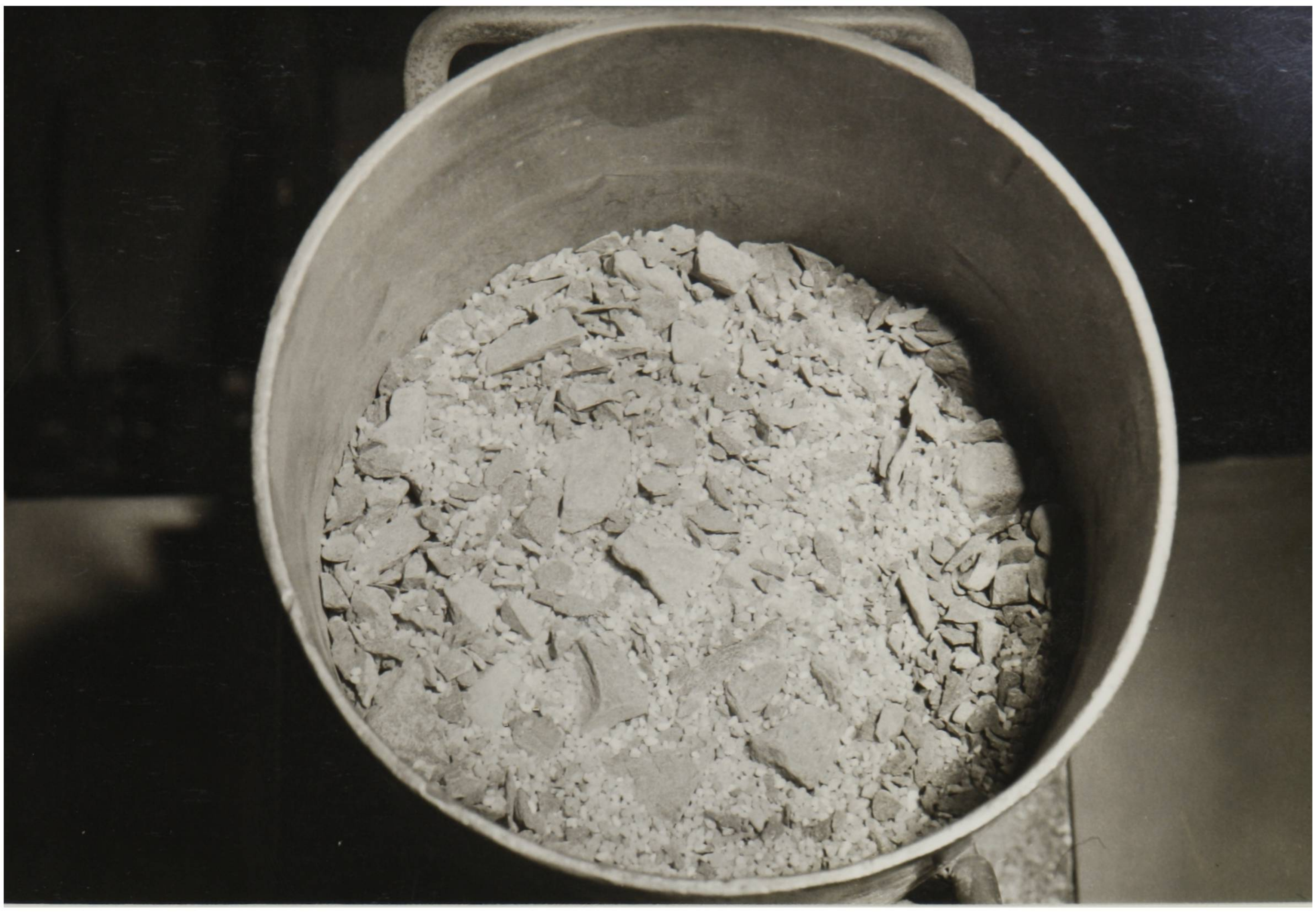


Fig. 81. CONTINUOUS GRADED MIX OF COMPOSITION GIVEN IN TABLE 10, AT 6 COMPONENT STAGE.

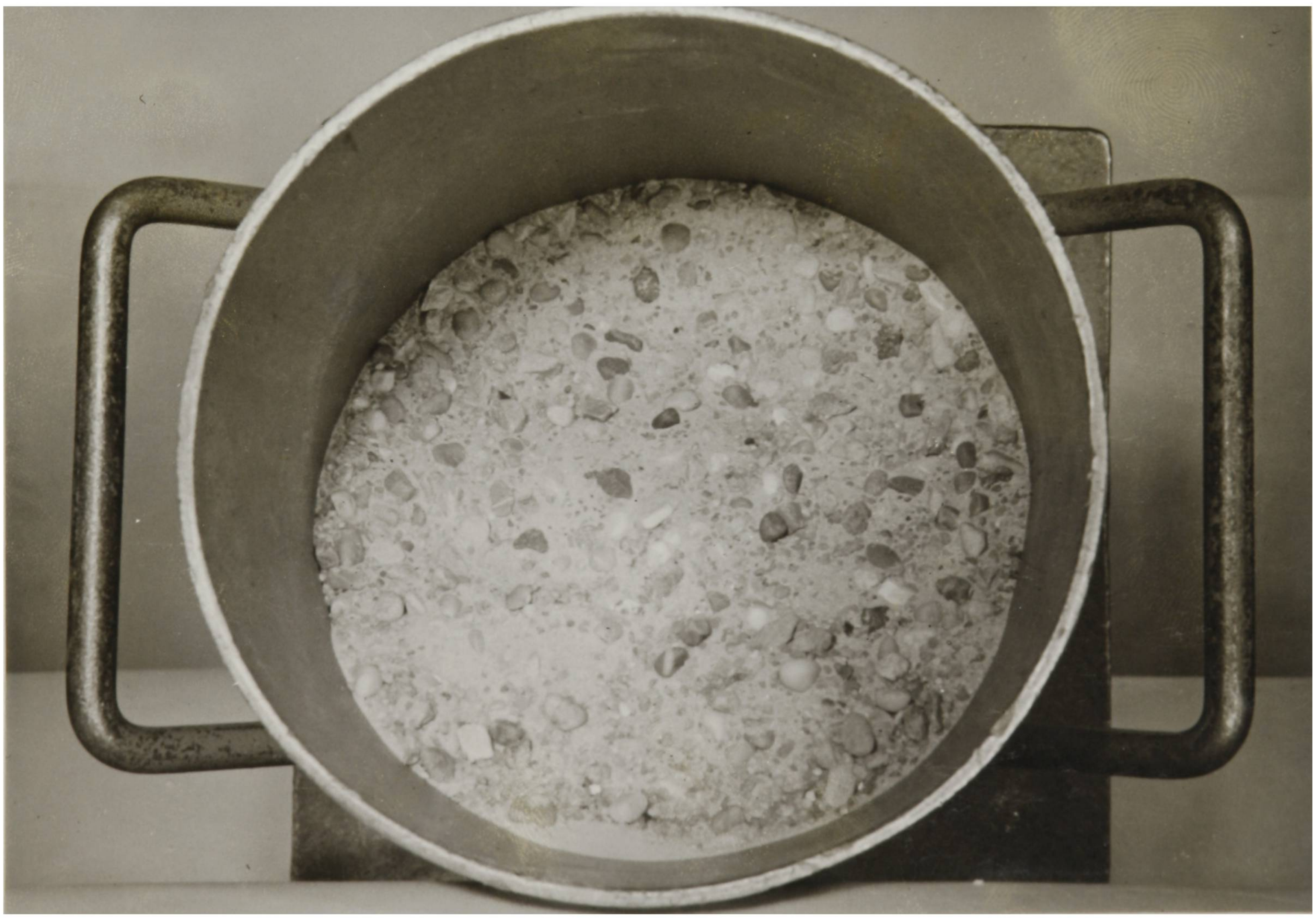


Fig. 82. CONTINUOUS GRADED MIX OF COMPOSITION GIVEN IN TABLE 11, AT FINAL 15 COMPONENT STAGE.

Notwithstanding the apparent complexity of the design method, the experiments and calculations involved do not take up more than a few hours once the appropriate compactive procedure and a container of appropriate dimensions have been chosen.

Should the container or 'section' to be employed in the field case be larger in one or two dimensions than can reasonably be duplicated in the laboratory then the aim should be to reproduce the minimum dimension or 2 dimensions in the laboratory, making the remaining dimension(s) as large as can be conveniently managed.

Alternatively either (a) corrections according to Hughes' graph (Fig.13) or (b) field tests for porosity employing the same compaction plant, as will be employed for the final job, may be attempted.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH.

In this study a review has been made of the factors which govern mass aggregate porosity, with particular reference to the problem of combining aggregates of different size and shape under different environmental conditions with the aim of achieving a minimum voids content.

Attempts have been made to measure certain characteristics of the inter particle voids and where appropriate to relate these measurements to the above mentioned problem.

The suggested methods for design of 2 component and multi-component gap- or continuous graded systems, which have arisen out of this study, would appear to have application in several fields connected with engineering construction materials. Nevertheless many problems, some of which have been mentioned in passing, in the previous pages, remain.

It has for example frequently been said with reference to both bituminous materials and concrete that an aggregate system giving minimum voids is not suitable for one reason or another, as when say stability, durability and workability respectively, are concerned. These statements may be true (in which event it has been suggested that aggregate systems to a controlled void content could be designed by reference to the proposed method), but the author feels that in other cases the statements so made, have arisen out of the assessment of the behaviour of so called maximum density systems which have not been designed in accordance with the fundamental principles as here discussed.

The most common failing seems to have been to assume that a grading curve established as giving a minimum voids for 'dry' aggregate, is necessarily the same as would give maximum solid volume density in the presence of binding materials such as bitumen, tar or cement paste.

Although the author too has worked chiefly with dry aggregates, the importance of taking into account the presence of all constituents including lubricating or adhesive coatings has been stressed and it is in the direction of confirming or amending the method as it applies to these more complicated materials that further research is required.

Finally further detailed studies are required of void characteristics, including the criticle ratios of entrance and occupation and the new criticle ratio of dilation, in aggregate systems of other compositions and otherppacking arrangements than those which have been studied so far. These studies together with studies of particle shape and orientation and their relation to such engineering properties as permeability, workability and the various strength parameters, offer scope for research in all quarters for many years.

REFERENCES

AGGREGATE POROSITY AND GRADINGS

1. ALYANAK, I. 1961. Vibration of sands with special reference to the minimum porosity test for sands. Ph.D. Thesis. University of Birmingham.

2. ANDEREGG, F.O. 1931. Grading Aggregates. The application of mathematical formulas to mortars. Ind. and Eng.Chem. 23, 1058 - 64.

3. ANDREASEN, A.H.M. and ANDERSEN, J. 1929. Über die Beziehung zwisohen Kornabstufung und Zwischenraum in Produkten aus losen Körnern. Kolloid Zeitschrift. 49(3), 217 - 28.

4. Asphalt Institute. 1964. Specifications and construction methods for asphalt concrete(SS-1) 3rd ed., Maryland.

5. BACHE, H.H. and NEPPER-CHRISTENSEN, P. 1965. Observations on strength and fracture in lightweight and ordinary concrete. International conference on the structure of concrete, London.

6. BAHRNER, V. 1951. Gap graded concrete. Cement och betong 26 (2). C.A.C.A. Library translation No.42, 1 - 19.

7. BATE, E.E.H. and STEWART, D.A. 1955. A survey of modern concrete téchnique. Proc. Inst. Civ. Engrs. 4 (3), 589 - 661.

7a. British Standards Institution. B.S. 1881 : 1952. Methods of testing concrete.

8. British Standards Institution. B.S. 812 : 1960. Mineral aggregates, sands and fillers.

9. British Standards Institution. B.S. 594 : 1961 Specification for Rolled Asphalt (Hot process)

10. BROWN, R.L. and HAWKSLEY, P.C.W. 1945. Packing of Regular (spherical) and Irregular Particles. Nature, Lond. 156, 421 - 22.

11. BROWN, R.L. and HAWKSLEY, P.G.W. 1946. Discussion on paper by Verman and Banerjee (q.v.)

12. BUTCHER, B.J. and HOPKINS, H.J. 1956. Particle interference in concrete mixes. Proc.Am.Conc. Inst. 53, 545 - 56.

13. CAMPEN, W.H. 1940. The development of a maximum density curve and its application to the grading of aggregates for bituminous mixtures. Proc. Ass. Asph. Pvg. Tech.11, 302-14.

14. DUNAGAN, W.M. 1940. The application of some of the newer concepts to the design of concrete mixes. Jour. Am. Conc. Inst. 36, 649 - 84.

15. ELLIS, A.J. and LEE, C.H. 1919. Geology and ground waters of the western part of San Diego county, California. U.S. Geol. Surv. Water Supply Paper. 446, 121 - 3.

16. FAROUKI, O.T. and WINTERKORN, H.F. 1964. Mechanical properties of granular systems. Highway Research Record 52, 10 - 58.

17. FRASER, H.J. 1935. Experimental study of the porosity and permeability of Clastic sediments. Jour.Geol. 43, 910 - 1010.

18. FREDERICK, M.R. 1962. The significance of particle shape in sand behaviour. Ph.D. Thesis. University of Birmingham.

19. FRITSCH, J. 1951. Vibrated mass concrete. Civil Engineering and Public Works Review (January/February) 3 - 8.

20. FULLER, W.B. and THOMPSON, S.E. 1907. The laws of proportioning concrete. Trans. Am. Soc. Civ. Engrs. 59, 67 - 172

21. FURNAS, C.C. 1928. Relations between specific volume, voids and size composition in systems of broken solids of mixed sizes. U.S. Bur. of Mines. Report of Investigations. 2894, 1 - 10.

22. ———— 1931. Grading aggregates. Mathematical relations for Beds of broken solids of maximum density. Ind. and Eng. Chem. 23, 1052 - 1058.

23. GRATON, L.C. and FRASER, H.J. 1935. Systematic packing of spheres with particular reference to porosity and permeability. Jour. Geol. 43, 785 - 909.

24. HATCH, T. and CHOATE, S.P. 1929. Statistical description of the size properties of non uniform particulate substances. Jour. Franklin Inst. 207, 369 - 87.

25. HEYWOOD, H. 1933. Calculation of the specific surface of a powder. Proc. Inst. Mech. Engrs. 383 - 459.

26. ________ 1946. The packing of spheres. Jour. Imperial Coll. Chem. Eng. Soc. 2, 3 - 20.

27. HOPKINS, H.J. 1962. Discussion on paper by Hughes 1960 (q.v.)

28. HORSFIELD, H.T. 1934. The strength of asphalt mixes. Jour. Soc. Chem. Ind., 107T - 115T.

29. HOSKING, J.R. 1961. An investigation into some factors affecting the results of bulk density tests for aggregates. Cement, Lime and Gravel. 36 (11), 319 - 26.

30. HUDSON, D.R. 1949. Density and packing in an Aggregate of mixed spheres. Jour. Appl. Phys. 20, 154 - 62.

31. HUGHES, B.P. 1956. Concrete mix design. Ph.D. Thesis. University of Birmingham.

32. ________ 1960. Rational concrete mix design. Proc. Inst. Civ. Eng. 17, 315 - 32.

33. ________ 1962. Discussion on above paper. Proc. Inst. Civ. Eng. 21, 927 - 52.

34. ________ 1966. The optimum coarse aggregate content of concrete. Mag. Conc. Res. 18 (54), 3 - 8.

35. HUGHES, B.P. and BAHRAMIAN, B. 1966. A laboratory test for determining the angularity of aggregate. Mag. Conc. Res. (18) 56, 147 - 52.

36. HVEEM, F.N. 1940. Gradation of mineral aggregate for dense graded bituminous mixtures. Proc. Ass. Asph. Pvg. Tech. (11) 315 - 39.

36a. JACKSON, G.P. and BRIEN, D. 1962. Asphaltic concrete. Shell International Petroleum Co. Ltd., Lond.

37. KAHN, J.S. 1956. The analysis and distribution of the properties of packing in sand size sediments. Jour. Geol. 64, 385 - 95 and 578 - 606.

38. KOLBUSZEWSKI, J. 1948. An experimental study of the maximum and minimum porosities of sands. Proc. 2nd Internat. Conf. on Soil Mech. and Found. Eng., Rotterdam, 1, 158 - 65.

39. ___________ and ALYANAK, I. 1964. Effect of Vibrations on the Shear Strength and porosity of sands. Surveyor (May 30) 3756, 23 - 7; (June 6) 3757, 31 - 4.

40. ___________ and FREDERICK, M.R. 1963. The significance of particle shape and size on the mechanical behaviour of granular materials. Proc. European Conf. Soil Mech. and Found. Eng. Wiesbaden 1 (4), 253 - 63.

41. LEES, G. 1964.(a) A new method for determination of the angularity of particles. Sedimentology 3 (1), 2 - 21.

42. ___________ 1964 (b) The measurement of particle shape and its influence in engineering materials. Jour. B.G.W.F. 4 (2), 17 - 38.

43. ___________ 1964 (c) The measurement of particle elongation and flakiness - a critical discussion of British Standard and other methods. Mag. Conc. Res., 16 (49), 225 - 30.

44. ___________ and SALEHI, M. 1967. Particle orientation with reference to bituminous materials. Research Journal 1966 pt. 2. Materials and Construction Management. Dept. of Transportation and E.P. Univ. of B'ham, 58 - 63.

45. McGEARY, R.K. 1961. Mechanical packing of spherical particles. Jour. Am. Ceram. Soc. 44 (10), 513 - 22.

46. McLEOD, N.W. 1955. Discussion on "Laboratory and field densities of bituminous mixtures" by Dillberg, Proc. Ass. Asph. Pvg. Tech. 24.

47. MORRIS, H.C. 1959. Effect of particle shape and texture on the strength of non cohesive aggregates. ASTM Spec. Publ.254.

48. ORR, C. 1966. Particulate technology. New York (Macmillan Co.)

49. POWERS, T.C. 1964 Geometric properties of particles and aggregates. Jour. Portland Cement Assoc. 6 (1), 2 - 15.

50. Road Research Laboratory. 1950. Road Note 4. Design of concrete mixes. H.M.S.O., Lond.

51. Road Research Laboratory. 1952. Soil mechanics for road engineers. H.M.S.O. Lond.

52. SALEHI, M. 1966. Characteristics of bituminous paving mixtures. M.Sc. Qualifying Thesis. Dept of Transportation and Environmental Planning, Univ. of Birmingham.

53. SHERGOLD, F.A. 1953. The percentage voids in compacted gravel as a measure of its angularity. Mag. Conc. Res.,5(13), 3-10.

54. STEWART, D.A. 1937 Fundamental research on the application of vibration to the pre-casting of concrete. Proc. Inst. Civ. Engrs., 318 - 48.

55. ____________ 1949. Vibrated concrete. Engineering (August 12, 19 and 26), 1 - 8.

56. ____________ 1952. Some recent developments in concrete technology. Engineering (Sept.5), 313 - 15; (Sept.12) 354 - 6.

57. TALBOT, A.N. and RICHART, F.E. 1923. The strength of concrete. Its relation to the cement, aggregate and water. Univ. of Illinois, Eng. Expt. Station Bull. 137, 1 - 118.

58. TERZAGHI, C. 1925. Engineering News Record, 914.

59. VALLETTE, R. 1954. Composition des bétons. Revue des materiaux de construction et de travaux publics. 463, 87-98 and 464, 127 - 36.

60. VALLETTE, R. 1963. Manuel de composition des bétons. Eyrolles. Paris.

61. VERMAN, L.C. and BANERJEE, S. 1946. Nature Lond. 157, 584 - 5y. Effect of container walls on packing density of particles.

62. WADELL, H. 1933. Sphericity and roundness of rock particles. Jour. Geol. 310 - 31.

63. ____________ 1935. Volume, shape and roundness of quartz particles. Journ. Geol. 43 (3), 250 - 80.

64. WESTMAN, A.E.R. and HUGIL, H.R. 1930. The packing of particles. Jour. Am. Ceram. Soc. 13, 767 - 79.

65. WEYMOUTH, C.A.G. 1933. Effects of particle interference in mortars and concretes. Rock Products, 26. - 30.

66. ____________ 1938. A study of finea aggregate in freshly mixed mortars and concretes. Proc. A.S.T.M. 38, Pt.II,354-72.

67. WISE, M.E. 1952. Dense random packing of unequal spheres. Phillips Res. Rept. 7, 321 - 43.

68. WORTHINGTON, P.M. 1953. An experiment on the theory of voids in granular materials. Mag. Conc. Res. 12, 121 - 6.

69. YODER, E.J. 1959. Principles of pavement design. John Wiley & Sons Inc.

70. ZINGG, Th. 1935. Beitrag zur Schotteranalyse. Schweiz Mineral. Petrog. Mitt., 15, 39 - 140.

APPENDIX I

Density, voids and stability analysis of 3 & 5 component gap-graded asphalt mixes with aggregate gradings computed according to the proposed design method (page 2 refers).

Table 12. HOTMIX DESIGN TEST REPORT FORM

b	f	g	h	i	j	k	l	m	n	o	p
% Binder by wt. of mix	Density		Volume - % total			Voids %		Unit Weight lbs/ft³	Stability		Flow
	Bulk $\frac{c}{e}$	Max. Theor. 'x'	Binder $\frac{b \times f}{G_4}$	Aggregate $\frac{(100-b) \times f}{G_2}$	Voids 100-h-i	Aggregate 100 - i	Filled with Binder $100 \times \frac{h}{k}$	62.4 x f	Measured lbs.	Adjusted lbs.	$\frac{1}{100}$ "
3 Component - MIX 1 (Proportions to B.S. 594 (1961) Table 7 Schedule 1:55% coarse aggregate content)											
5.65	2.42 2.44 2.46 2.43 Av.2.44	2.53	13.8	83.0	3.2	17.0	81.2	152.1	4900 4600 4750 4900	5440 5240 5290 5440 Av.5353	10 10 5 7 Av.8
3 Component - MIX 2 (Proportions calculated by proposed method: sizes and bitumen content as for MIX 1)											
5.65	2.52 2.49 2.50 2.50 Av.2.50	2.53	14.1	84.9	1.0	15.1	93.4	156.0	4900 4750 4825 4900	5440 5270 5350 5440 Av.5360	12 15 10 11 Av.12
5 Component - MIX 3 (Proportions and sizes calculated according to proposed method)											
6.5	2.33 2.36 2.34 Av.2.34	2.46	15.2	80.6	4.1	19.4	79.0	146.2	2800 2750 2850	3110 3050 3020 Av.3060	30 27 29 Av.29
6.0	2.39 2.39 2.39 2.39 Av.2.39	2.48	14.3	82.9	2.8	17.1	83.9	149.1	4650 5100 4900 5125	4930 5660 5440 5690 Av.5430	23 25 21 20 Av.22
5.5	2.44 2.41 2.43 2.42 Av.2.43	2.49	13.3	84.3	2.4	15.7	85.0	151.6	5800 5700 5900 6025	6440 6040 6250 6690 Av.6355	13 14 12 9 Av.12
5.0	2.43 2.45 2.46 2.46 Av.2.45	2.50	12.3	85.7	1.9	14.3	85.4	152.8	5050 5250 5200 5350	5750 5660 5440 5690 Av.5635	7 8 8 9 Av.8
4.5	2.42 2.41 2.37 2.40 Av.2.40	2.53	10.8	84.4	4.8	15.6	69.1	149.8	5325 5935 5950 6100	6270 6760 6130 6460 Av.6405	5 5 6 6 Av.6

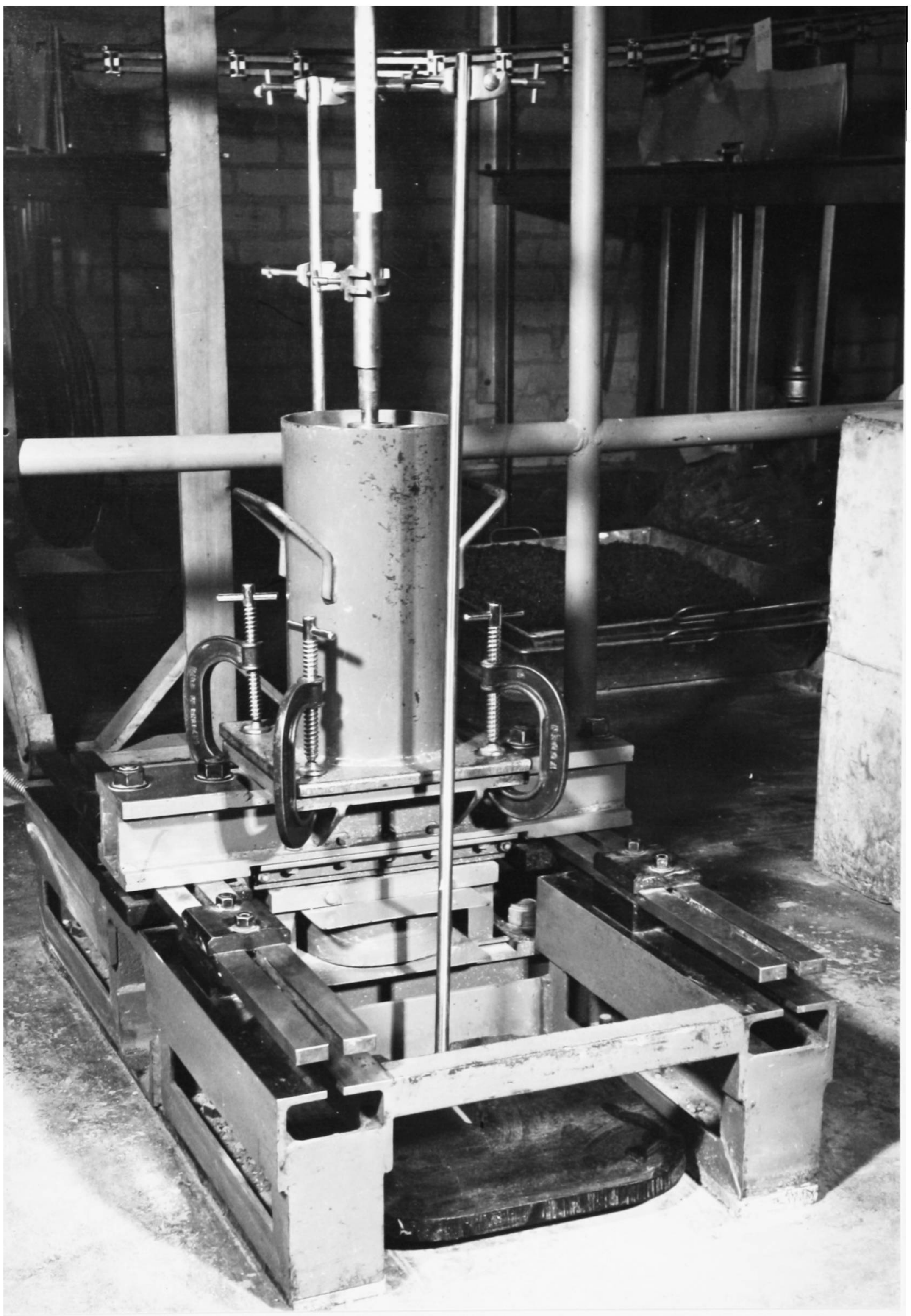


Fig. 83. WESTOOL-STEWART RESEARCH VIBRATING TABLE WITH COMPACTING FACTOR CYLINDER.

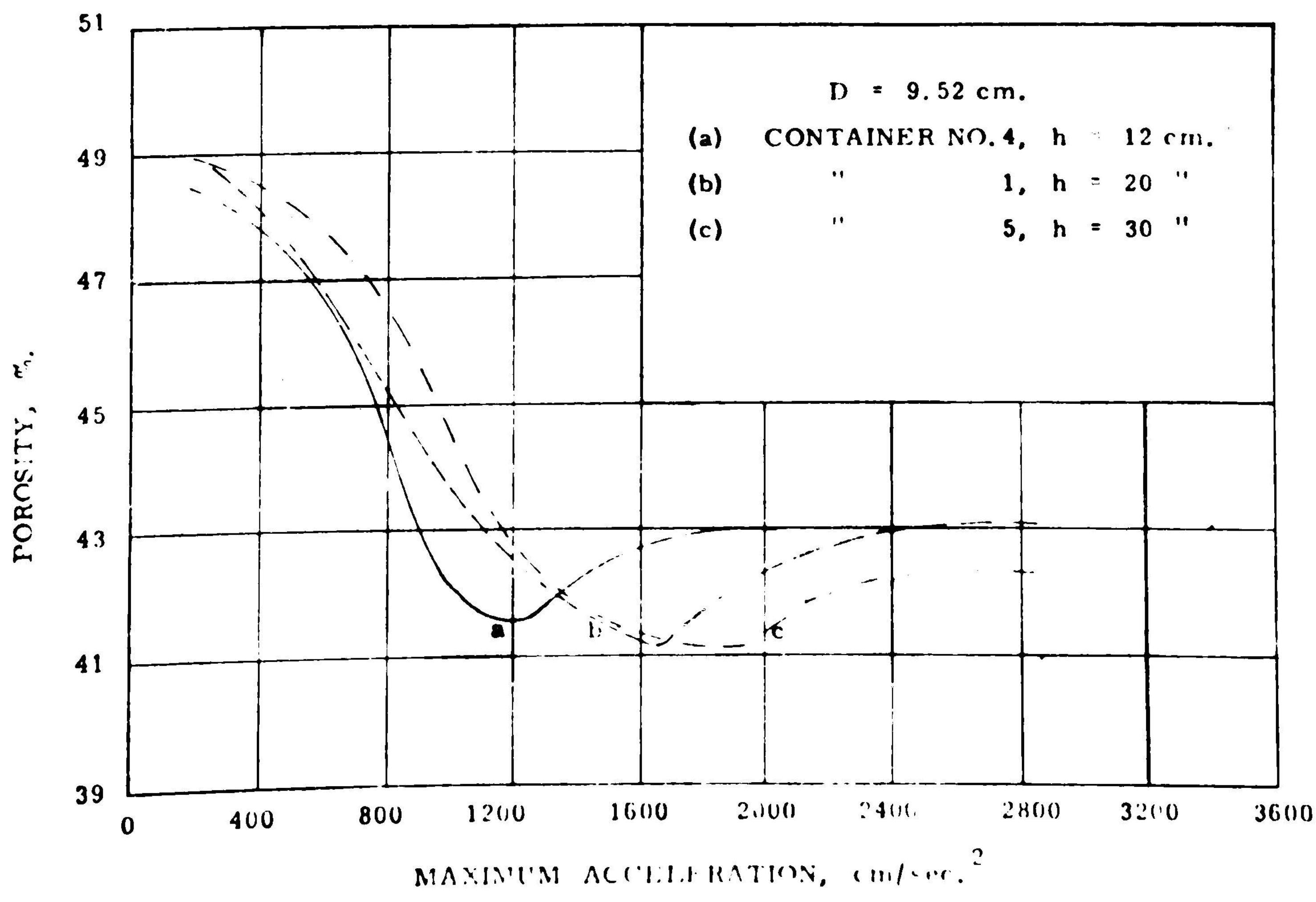


Fig. 84. INFLUENCE OF CONTAINER HEIGHT (HENCE SURCHARGE) ON OPTIMUM ACCELERATION FOR MINIMUM POROSITY (after Kolbuszewski & Alyanak 1964)

observed on the scale placed on the plunger resting, in the cylinder, on the surface of the aggregate (Fig. 85). This generally took place in under two minutes. Progressively lower accelerationswwere applied under similar conditions until the compaction process was complete.

It should be noted that these conditions are not conditions of equal energy input into the system, but are designed to produce minimum porosity for any given aggregate notwithstanding that it may require different compactive efforts to achieve this condition.

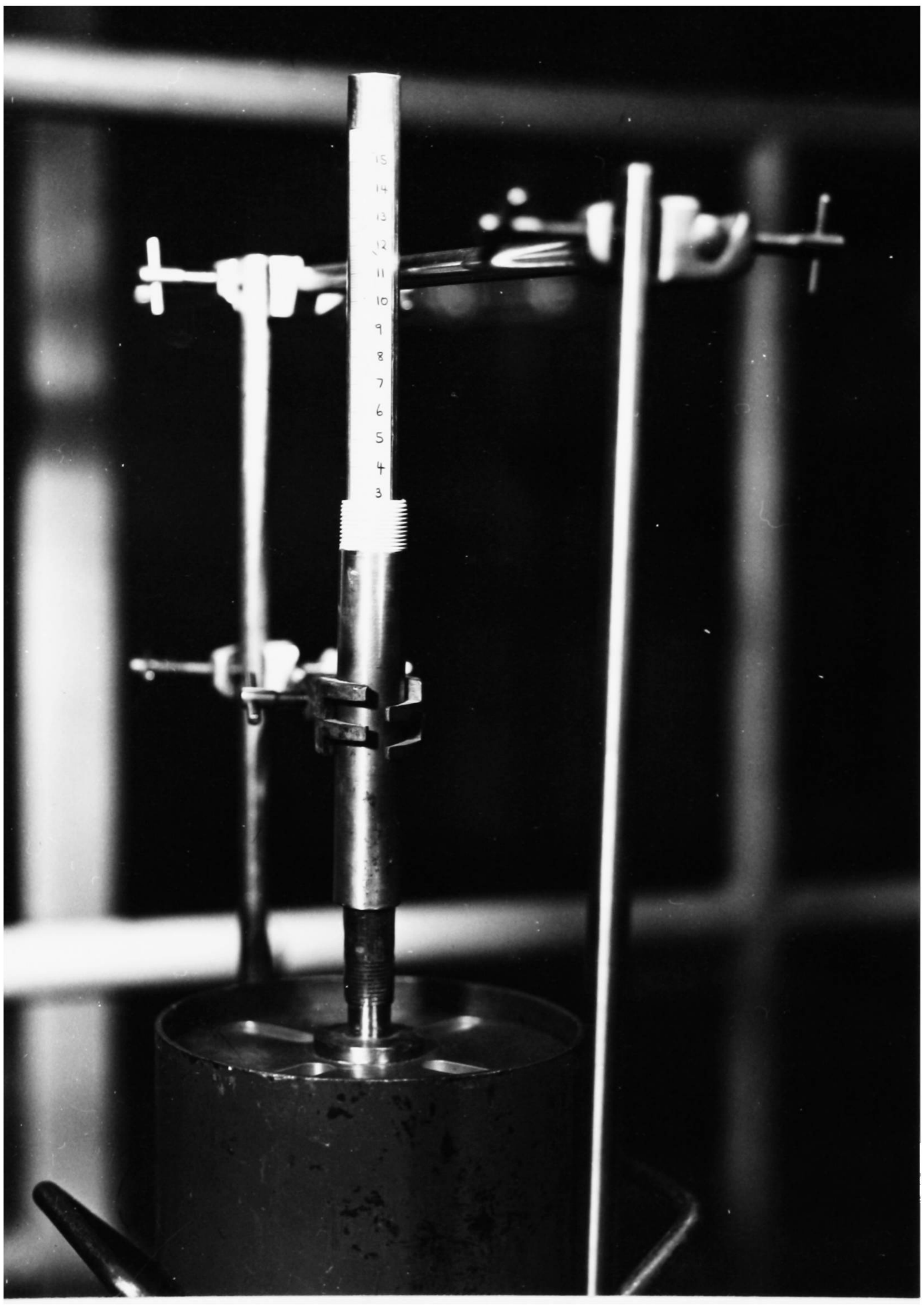
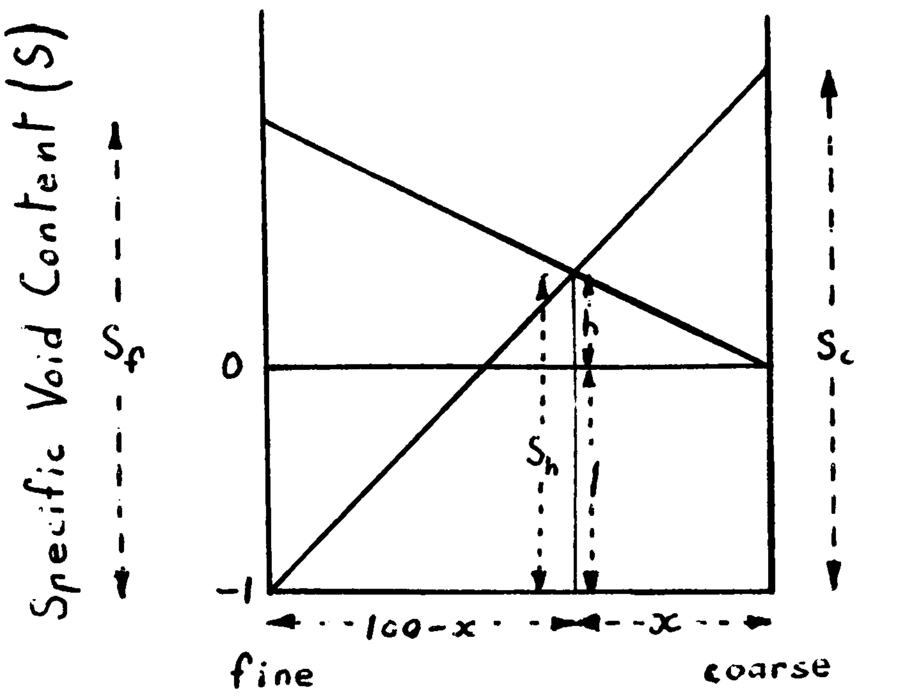


Fig. 85. PLUNGER & SCALE FOR DETERMINATION OF VOLUME CHANGES IN COMPACTING FACTOR CYLINDER.

APPENDIX III.

Derivation of formula (13)

$$b = \frac{P_{coarse} \times P_{fine}}{100} \qquad \text{(p.90 refers)}$$



$$\frac{S_f - 1}{100} = \frac{h}{x} \; ; \quad \therefore \; x = \frac{100\,h}{S_f - 1} \quad \ldots\ldots\ (i)$$

$$\frac{S_c}{100} = \frac{h + 1}{100 - x} \; ; \quad \therefore \; 100 - x = \frac{100\,(h + 1)}{S_c} \quad \ldots\ldots\ (ii)$$

from (i) and (ii) $100 - \frac{100h}{S_f - 1} = \frac{100(h + 1)}{S_c}$

$$1 - \frac{h}{S_f - 1} = \frac{h + 1}{S_c}$$

$$\therefore \; 1 - \frac{h}{S_f - 1} = \frac{h + 1}{S_c}$$

$$\therefore \; \frac{S_f - 1 - h}{S_f - 1} = \frac{h + 1}{S_c}$$

$$\therefore \; (h + 1)(S_f - 1) = S_c\,(S_f - 1 - h)$$

$$\therefore \; h\,S_f + S_f - h - 1 = S_c\,S_f - S_c - h\,S_c$$

$$\therefore \; h\,(S_f + S_c - 1) = S_c\,S_f - S_c - S_f + 1$$

$$\therefore \quad h = \frac{S_c\,S_f - S_c - S_f + 1}{S_f + S_c - 1}$$

$$S_h = 1 + h$$

$$= 1 + \frac{S_c\,S_f - S_c - S_f + 1}{S_f + S_c - 1}$$

$$= \frac{S_f + S_c - 1 + S_c\,S_f - S_f + S_c + 1}{S_f + S_c - 1}$$

$$= \frac{S_c\;S_f}{S_f + S_c - 1} \qquad \text{......... (iii)}$$

$$S = \frac{100}{100 - P} \qquad \text{where } P = \text{porosity}$$

$$\therefore \quad S_h = \frac{100}{100 - P_h} \qquad (\text{ \& similarly for } S_c + S_f)$$

$$\therefore \quad S_h = \frac{\left(\frac{100}{100 - P_c}\right)\left(\frac{100}{100 - P_f}\right)}{\left(\frac{100}{100 - P_f}\right) + \left(\frac{100}{100 - P_c}\right) - 1}$$

$$= \frac{\dfrac{100^2}{(100 - P_c)\,(100 - P_f)}}{\dfrac{100(100-P_c)+100(100-P_f)-[(100-P_f)(100-P_c)]}{(100 - P_c)(100 - P_f)}}$$

$$= \frac{100^2}{100^2-100\,P_c+100^2-100P_f-100^2+100P_c+100P_f-P_cP_f}$$

$$= \frac{100^2}{100^2 - P_c\,P_f}$$

$$= \frac{100}{100 - \frac{P_c\,P_f}{100}}$$

$$\text{but} \quad S_h = \frac{100}{100 - P_h}$$

$$\therefore \quad \frac{100}{100-P_h} = \frac{100}{100 - \frac{P_c\,P_f}{100}}$$

$$\therefore \quad P_h = \frac{P_c \, P_f}{100}$$

let $P_h = b$ (in notation of text),

$$\therefore \quad b = \frac{P_{coarse} \times P_{fine}}{100} \qquad \text{Q.E.D.}$$