Sharp Boundaries and Supervaluationism

by

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Abstract

It is claimed to be a crucial advantage of supervaluationism over other theories of vagueness that it avoids any commitment to sharp boundaries. This thesis will challenge that claim and argue that almost all forms of supervaluationism are committed to infinitely sharp boundaries and that some of these boundaries are interesting enough to be problematic. I shall argue that only iterated supervaluationism can avoid any commitment to sharp boundaries, but on the other hand that is the model that Terrance Horgan has recently argued is a form of transvaluationism and thus logically incoherent.

I shall first argue that infinitely higher-order vagueness gives rise to an infinite number of boundaries. I will then argue that an infinite number of these boundaries are, in the case of the vague term 'tall', located over a finite range of heights. I will argue that because of this, these boundaries must be infinitely sharp. I shall argue that on every plausible non-iterated supervaluationist model, some such boundary will mark a sharp boundary between heights that would make someone 'more tall than not tall' and heights that would not. Finally I shall argue that this is the sort of interesting sharp boundary supervaluationism must not admit.

For Martina, even though she hates vagueness.

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Table of Contents

1 Introduction

In this thesis I will argue that interesting and infinitely sharp boundaries arise out of the most popular supervaluationist models of vagueness. I shall argue that the only model of supervaluationism that might avoid sharp boundaries entirely is iterated supervaluationism and conclude that this is the only form of supervaluationism worth considering as a theory of vagueness. I'll argue that supervaluationist models that admit sharp boundaries share the only major problem of epistemicism and epistemicism's greater simplicity means it should be favoured over models of supervaluationism that admits sharp boundaries.

The thesis can be split into two parts: -

Sections 1 through 5 form an extended introduction to vagueness and supervaluationism. Section 2 introduces the phenomena of vagueness and high-order vagueness, the Sorites paradoxes they give rise to and the threat these paradoxes make against the meaningfulness of everyday terms such a 'tall' and 'human'. Section 3 introduces supervaluationism and its solution to the Sorites paradox. Section 4 briefly introduces three other theories of vagueness, namely epistemicism, degree theory and transvaluationism. Section 5 gives details of the supervaluationist account of higher-order vagueness.

Sections 6 through 10 present arguments against various supervaluationist models on the basis that they give rise to sharp boundaries. Section 6 will give initial case for sharp boundaries in a very simple non-iterated model of supervaluationism with logic for the *D* operator that obeys S4. Section 7 will detail Williamson's argument for sharp boundaries in another non-iterated model of supervaluationism with logic for the D operator that is weaker than S4, and his two suggested replies the ‘near impossible standard’ reply and iterated supervaluationism. It will also draw attention to Horgan’s most recent argument that iterated supervaluationism is a form of transvaluationism. Section 8 will assume the ‘near impossible standard’ reply has been adopted and argue that the supervaluationist model still gives rise to infinitely many infinitely sharp boundaries. Section 9 will argue that whatever model the supervaluationist takes, one of the infinitely many infinitely sharp boundaries will be problematic as it will mark the exact point of greatest indefiniteness. Section 10 will present my conclusion that sharp boundaries make all non-iterated supervaluationist models inferior to epistemicism and conclude that the only supervaluationist model worth considering further is iterated supervaluationism.

2 Introduction to Vagueness and the Sorites Paradox

Vagueness is the linguistic phenomenon where certain terms have borderline cases to which it is uncertain whether or not the term applies. Some paradigm examples of vague terms are 'blue', 'tall' and 'heap'. Take for instance 'blue', for some shade on the borderline between those that are blue and those that are green, some bluey-green colour, you will find it impossible to decide whether that shade is blue or not. It is a borderline case of 'blue', not clearly blue and yet also not clearly not blue. Such borderline cases exist because the extension of the term 'blue' is vague. That means that we at least don't know its exact extension and intuitively most believe is has no exact extension. Similarly 'tall' has a vague extension, so if Francis was 180cm tall then he would be a borderline case of 'tall' and we would be unable to decide if he was tall or not. If pushed we might claim that there is no fact of the matter as to whether Francis is tall or not.

2.1 The Sorites paradox

We might be tempted to just shrug and accept that some terms are simply vague and refuse to give any further analysis. However, vagueness gives rise to logical problems that make such naive analyses of vagueness extremely unattractive. The main problem which vagueness gives rise to is the Sorites Paradox. The Sorites Paradox is a two premise argument which entails an absurd conclusion, the exact formulation varies but an example of a Sorites paradox would be: -

(1) A human 10m in height is tall.

(2) All humans less than 0.1mm shorter than a tall human are tall.

(3) Therefore, a man 1mm in height is tall.

The paradox can be rewritten in various ways but importantly the 2nd premise can be replaced by many premises for each step, in this case for each 0.1mm shorter height considered. For instance: -

(4) A human 10m in height is tall.

(5-1) If a human 10000mm in height is tall then a human 9999.9mm in height is tall.

...

(5-1,000,000) If a human 1.1mm in height is tall then a human 1mm in height is tall.

(6) Therefore, a human 1mm in height is tall.

2.2 The threat of global nihilism

We might however still be tempted to try and shrug this off as a problem that need only concern mathematicians, logicians and those philosophers who are seeking construct perfect proofs.

Vague terms do seem to be effective in communication after all so we might be tempted not to be too concerned that they give rise to paradoxes. Why after all should we be worried that the logic of the terms of our language isn't perfect when we don't generally in our everyday lives away from our desks dealing with non-philosophers bother to create logically perfect arguments. If we aren't aiming for logical perfection then why should we be worried if the

Sorites paradox demonstrates that we would be bound to fall short of such logical perfection? We don't in our ordinary discourse aim for logical perfection and yet our flawed everyday discourse does often serve to bring us closer to holding true beliefs. This is surely because although what we say and hear may not be perfectly logical, we fill in the gaps as it were in the reasoning presented to us. Similarly perhaps vague terms such as 'tall' might not be completely logically coherent, but almost every English speaker would know what you meant if you told them “Francis is a tall man”.

Unfortunately for anyone wishing to adopt such a dismissive line towards the problem presented by the Sorites paradox, the problem runs so deep that it challenges the idea that vague terms are meaningful at all. The problem is that without any solution to the Sorites paradoxes we must accept them as valid arguments with true premises and we must thus accept their conclusions. If we accept that any argument is valid and the premises are true then we must accept the conclusion or else we're just not playing the game of rationality anymore and can resist any argument on a whim.

If we wish to hold as true that (1) 'A human 10m in height is tall' and (2) 'All humans less than 0.1mm shorter than a tall human are tall' and accept that the argument is valid we must also accept (3) ''Therefore, a human 1mm in height is tall', so it seems we will be forced to accept that all humans are tall and 'tall' applies to all humans. What is worse however is that for each Sorites argument there is a complementary argument that is equally strong and that points us to the opposite conclusion. Consider for instance: -

(7) A human 1mm in height is not tall.

(8) All humans less than 0.1mm taller than a human who is not tall are not tall.

(9) Therefore, a human 10m in height is not tall.

If we can't find fault with this argument then we will be forced to accept that all humans are not tall and that 'tall' applies to no humans. I find myself very similarly confident in the truth of (7) and (8) as I am of the truth of (1) and (2) and the logical form is identical so it would be hard to reject either of the two contradictory conclusions. Combine this with the fact that each argument leads to a conclusion that also contradicts our ordinary beliefs about the meaning of tall and it seems that the best conclusion one can come to is that in fact 'tall' is simply meaningless.

That conclusion itself is bad enough. The conclusion that any term in our language as common as 'tall' should turn out to be meaningless is something we should baulk at since surely we would have noticed that such a common term was actually meaningless and not conveying anything. The real problem however, is that the same argument applies not just to 'tall' but to a vast number of terms. I gestured earlier to other paradigmatic vague terms such as 'heap' and 'blue', the problem is not even limited to terms such as these. 'Human' for instance might initially appear not to be susceptible to such an argument, but upon closer inspection the series is simply hard to formulate because of the difficulty in defining ‘human’. If we did try to define what it is to be human then we might well end up referring to common genetic code, but in that case it might seem like a single gene wouldn't make the difference between being a human and not being a human and so we might accept both: -

(10) Any organism that has all but one gene in common with a human is a human.

(11) Any organism that has all but one gene in common with a non-human is non-human.

However, if we accept that these two premises are true then we could in theory construct a Sorites series of organisms, each with all but one gene in common with their neighbours. One end of such a series could start with a genuine human such as you or I, and have at the other end an amoeba. This is of course exactly why evolution leads us to believe that there was no particular first human and no answer to which came first, the chicken or the egg.

The point is that as Priest points out: -

> Using Sorites arguments we can prove almost anything. For example, we can prove that you are a scrambled egg as follows. Let b_0 be you, and suppose that there are n molecules in your body; let $b_0, b_1, \ldots, b_n$ be the sequence of objects each of which is obtained from its predecessor by replacing one molecule of you with a molecule of scrambled egg, so that b_n is all scrambled egg. Let β_i be the statement that you are b_i. Then clearly β_0, and for any i, $\beta_i \rightarrow \beta_{i+1}$. Hence by n applications of modus ponens β_n: you are a scrambled egg. (Priest, 2003, 9)

And we can hardly object to this line of reasoning for as he goes on to say: -

> Thus, even if you were changed by replacing one molecule of your body with a molecule of scrambled egg, you would still be as you as you could be. You change more than that every morning after breakfast. (Priest, 2003, 12)

Using Sorites arguments we can prove quite absurd things. We can prove that pretty much anything falls within the extension of a vague term and we can also switch the argument round to prove that nothing does. We could trivially construct a counter argument to Priest to prove that we are not scrambled eggs and even things we think are scrambled eggs are not in fact scrambled eggs. These arguments can, if taken as sound, prove that vague terms have in their extension both everything and nothing. They reduce vague terms to meaninglessness, and since almost all terms are vague this threatens the meaningfulness of all of language.

This threat of what Williamson (1994, 166) termed global nihilism regarding meaning that springs from the Sorites paradox, elevates the paradox to something more than a bare intellectual puzzle. If we are intellectually honest and rational then we must accept the best available solutions to the puzzles we encounter. Therefore, if nihilism is the only working solution we have to the Sorites paradox then we must accept it as the best. However, nihilism regarding meaning strikes against the very foundations of our everyday beliefs since it renders the expressions of those beliefs formalised into language quite literally meaningless and those are the building blocks of our very thoughts. We can't accept nihilism and so if we are to maintain our intellectual honesty and rationality we must find an alternative. We must find another working solution to the Sorites paradox.

2.3 Introduction to higher-order vagueness

The phenomenon described above can be referred to as 1^{st} order vagueness and a related phenomenon can be identified as 2^{nd} order vagueness. 2^{nd} order vagueness is vagueness about whether something is or isn't a borderline case of whether the vague term applies. So going back to our example, there are shades between the shades you can pick out as being on the borderline between blue and green and the shades you can pick out as definitely being 'blue'.

These shades are such that you will find it impossible to decide whether they are borderline cases of 'blue' and might claim that there is no fact of the matter as to whether or not they are borderline cases of 'blue'. Instead such a shade could be described as 'borderline borderline blue'. That there are such borderline borderline shades of blue constitutes the term 'blue' having 2nd order vagueness. The situation is very similar in the case of ‘tall’. We can pick out values such as 180cm that are borderline cases of 'tall' and we can pick out definite cases of 'tall' such as people 190cm. Then there is a vague borderline between these two sets of cases where there are heights, say 184cm, that we would be unable to classify as borderline cases of 'tall' or not. Such heights are borderline borderline cases of tall and thus 'tall' has 2nd order vagueness.

Similarly for some other heights between those which are borderline borderline tall and those which are simply borderline tall you'd find it impossible to decide whether they are borderline borderline tall or not, so those heights are borderline borderline borderline tall. The feature of a term having such borderline borderline borderline cases is called 3rd order vagueness. This process of picking out further borderline cases of borderline cases could seemingly go on indefinitely since it seems all the borderlines will admit borderline cases. So it seems that for all natural numbers *n* there will be heights such that the predicate formed by *n* repetitions of 'borderline' followed by 'tall' will apply to them. This is because it seems at least that for all *n* 'tall' has *n*th order vagueness. All these infinitely many orders of vagueness over 1st order vagueness are collectively referred to as higher-order vagueness.

Versions of the Sorites paradox can be constructed concerning the higher-order vagueness of terms as easily as regarding the 1st order vagueness. For instance: -

(12) A man 184cm in height is borderline borderline tall.

(13) A man 0.001mm shorter than a man who is borderline borderline tall is borderline borderline tall.

(14) Therefore, a man 1mm in height is borderline borderline tall.

Since 1st order vagueness is the subject of the original Sorites paradox and in some sense the most basic form of vagueness, most accounts are built to explain 1st order vagueness and/or Sorites paradoxes relating to 1st order vagueness. This top down approach to the construction of theories of vagueness means most theories have a very firm and technically solid account of 1st order vagueness that isn't easy to criticise. It seems then that a good approach for a critic of any theory of vagueness is to try and find fault with the theory's account of higher-order vagueness. The theory's account of higher-order vagueness is likely to be some sort of extension of the theory's account of 1st order vagueness and since higher-order vagueness throws up some unique problems of its own this extension may not be entirely fitting. The critic wishing to probe a theory of vagueness for weak points does well therefore to start with a close examination of a theory's account of higher-order vagueness since it is there if anywhere that problems are likely to be found.

Perhaps it might be thought that higher-order vagueness doesn't present as important a problem as 1st order vagueness. After all, we do not commonly in everyday life consider whether someone is borderline borderline borderline tall or not, these things simply don't matter to us and some of our intuitions regarding the nature of higher-orders of vagueness weaken the higher the order of the vagueness considered. This is true, but it misses the threat that a lack of explanation of higher-order vagueness poses to the meanings of first order vague terms. Without some adequate explanation of higher-order vagueness global nihilism once again threatens. The threat from higher-order vagueness stems from the Sorites paradoxes it gives rise to, just as it did with 1st order vagueness. As noted, Sorites paradoxes can easily be constructed in higher-order vagueness. Now, just as a pair of Sorites paradoxes for 'tall' threatens to force us to accept nihilism regarding the meaning of 'tall', a similar pair for 'borderline tall' immediately threatens to force us to accept nihilism regarding the meaning of 'borderline tall'. If the results of the opposing Sorites paradoxes for 'borderline tall' are accepted, then everyone is accepted to be 'borderline tall' and ‘not borderline tall’ and therefore ‘borderline tall’ is meaningless. However, intuitively (although not uncontroversially), it can be claimed that 'borderline tall' means nothing more or less than 'not tall and not not tall'. If it is accepted that everyone is ‘borderline tall’, then everyone is 'not tall and not not tall'. That would mean that everyone would be both tall and not tall and we would be forced to accept the nihilistic result that 'tall' is meaningless. Furthermore, since no property of 'tall' other than its Sorites susceptibility was used in this argument, global nihilism quickly follows. This is the problem with higher-order vagueness: if no solution is given to Sorites paradoxes based on higher-order vagueness, then the nihilism regarding the meaning of terms like 'borderline borderline... borderline tall' quickly spreads down to nihilism regarding the meaning of the simple term, in this case 'tall', and so once again there is a threat of global nihilism that needs to be answered.

In order to answer the threat of global nihilism we need to therefore present some analysis of vagueness and a solution to the Sorites paradox that doesn’t lead to the sort of meaninglessness that threatens when the absurd conclusions of the paradoxes are simply accepted. There are at the moment several competing theories of vagueness but this paper shall concentrate on just one, that of supervaluationism, which will be introduced in the next section.

3 Introduction to Supervaluationism

One of the most popular theories of vagueness is that of supervaluationism. The focus of this paper will be an examination of possible sharp boundaries arising in the formal model endorsed by this theory. In this section I'll introduce the basic structure of supervaluationism and outline its solution to the Sorites paradox. Supervaluationism, like most such terms in philosophy refers not to a single theory but rather, as noted by Varzi (2007, 633) a mixed bag of theories of vagueness that all share certain features. In the case of supervaluationism they all give a similar analysis of 1st order vagueness based on the idea of supervaluating over a set of delineations. I'll explain exactly what that means in a moment, but in a single sentence you could say that all supervaluationist theories hold that vagueness should be analysed by appeal to the multitude of ways in which our ordinary vague language could be made precise.

3.1 Delineations

A delineation of a vague language can be thought of as a way in which a vague language could be made precise completely. Leaving aside supervaluationism for a moment, if we decided to try and do away with the vagueness of English we could try to systematically invent a new precise language and call it ‘Precise English’ or ‘Penglish’. We could attempt to go about this task by replacing every vague term with a precise one that aside from its lack of vagueness meant roughly the same thing[†]. So for instance 'tall' might be replaced by 'ptall', which for a Caucasian human male might simply mean over 182cm in height[‡]. What exact meaning we choose for 'ptall' is simply a matter of choice; we could equally well choose to

[†]Such a task is almost certainly impossible, but we can imagine what such an exercise would be aiming to create.
[‡]Or rather 183 pcentimetres.

define 'ptall' as being over 183cm or over 182.89cm or over 185cm. The way this ties back to the idea of a delineation in the supervaluationist model is that the possible precise definitions we could choose between as definitions of 'ptall' are known as delineations of 'tall', they are the ways in which 'tall' could be made precise. Any possible complete precise language, such as ‘Penglish’, that contains in place of the vague terms of English delineations of those terms is a delineation of English.

Some delineations of 'tall' are closer to being precise analogues of 'tall' than others: over 182.8963824cm; or over 180cm; or over 183cm etc. are all quite close to the intuitive meaning of tall since most English speakers would agree that the vague borderline between 'tall' and 'not tall' is somewhere around 6 foot. However, on some models not all delineations need be so confined by natural language. Williams (2008, 195) for instance endorses a model that admits extreme delineations that stretch the meaning of the term. An example of such an extreme delineation of 'tall' would be one such that a human is tall if and only if they are over 3m tall. Another even more extreme delineation of 'tall' that bares almost no relation to the actual vague meaning of 'tall' would be the delineation where a human is tall if and only if they are *less than* 400m tall. Such extreme delineations are unimportant to the arguments given in this paper but Williams provides a good argument for the inclusion of such extreme delineations in the supervaluationist model so I will follow him in allowing for them in the model of supervaluationism I will give here.

The delineations of individual terms within a delineated version of a natural language must be consistent. So if 'tall' in Penglish means over 183cm then it must do so on all instances of the use of that term within Penglish. So if we have the sentence in Penglish “Francis is tall and

Louis is not tall", then whatever tall means it must mean the same in both instances[§]. If the precise meaning of the first instance of 'tall' is over 183cm then it must mean over 183cm in the second instance as well rather than say 184cm.

3.2 Sharpenings

On the supervaluationist account there's nothing privileged or special about any particular delineation. However there is a subset of delineations that are more privileged, more legitimate and following Williams' terminology (2008, 195) I'll refer to the members of this subset as the sharpenings. Most supervaluationists treat vagueness as semantic indecision (Weatherson, 2002, 30). They treat cases of vagueness as cases where we have simply failed to decide on a single precise delineation. However, according to such a model the set of precise delineations between which we have failed to decide is far smaller than the total set of delineations. Looking at our 'tall' example again, if we consider various delineations there are some we can rule out, some we have decided that that is not the meaning of the term 'tall'. For instance we have implicitly decided that 'tall' does not mean ‘less than 20cm in height’ or ‘exactly 183cm in height’. There are other delineations, which if we treat the vagueness as semantic indecision, we can say we haven't decided whether the term means that or not, for instance it could be claimed that we haven't decided whether 'tall' means ‘over 183cm in height’ or not. It would be consistent with our previous usage of the term to decide that it does mean that, but nothing in our previous usage has committed us to the term having that exact meaning.

[§]At least assuming certain contextual consistency, if for instance Francis is a man and Louis is a building then perhaps 'tall' could have different precise meanings if we treat 'tall' as applied to men and 'tall' as applied to buildings as two different terms.

Another simple way to understand the concept of a sharpening is by analogy. If we imagine the process of creating a knife from a thin bar of metal we could do this by using a file to remove metal from one or both sides until the long edge of the metal was sharp.

Figure 1

Shown in figure 1 are representations of the cross section of the original unsharpened strip of metal, two possible results of sharpening the metal and one sharp edge that can't be formed by sharpening the original. Clearly (a) is the unsharpened original and (b) and (d) could be obtained simply by removing metal from (a). (c) on the other hand would require some form of bending or the addition of new metal onto the side, it cannot be obtained from (a) merely by sharpening. There are many different ways the metal can be sharpened by taking different amounts of metal from each side, but each of them leaves a sharp edge that was somewhere along the board edge of the original unsharpened metal. Supervaluationist sharpenings are similar, there are many ways a vague term could be replaced with a precise one, but sharpenings place the precise boundary somewhere in the region of the vague boundary of the original vague term. A sharpening is a delineation that is the possible result of filing the vagueness off the edges of language so to speak. Completing the analogy: (a) is equivalent to a vague term and (b), (c) and (d) are all delineations of (a), but only (b) and (d) are sharpenings of (a).

Sharpenings of languages will also obey certain rules regarding the interconnectedness of vague terms in language. The colour terms 'blue' and 'green' for instance are taken to be exhaustive and exclusive over a certain range of the spectrum colours made up of single wavelengths of light and all sharpenings will obey this rule. There are delineations of green that place the boundary between the two at 500nm and 510nm and there are delineations of blue that place the boundary at those points as well. There are also sharpenings that place the green-blue boundary at 500nm and 510nm because these wavelengths are on the vague boundary between blue and green (Hardin, 1988, Plate 1), but although there are delineations there are no sharpenings that place the boundary of green at 500nm and the boundary of blue at 510nm, because then the terms would not be exhaustive in the way required by their use in everyday language. Similarly there are no sharpenings that place the boundary of blue at 500nm and the boundary of green at 510nm because then the terms wouldn't be mutually exclusive in the correct way. Kit Fine (1999, 124) termed such logical relations between vague terms ‘penumbral connections’ and that all sharpenings respect such connections is a key feature of supervaluationism that gives rise to many of its advantages over other theories.

3.3 Supertruth, superfalsity and supervaluationism

The core principle of supervaluationism is that the multitude of different sharpenings may be used to analyse the truth value of the original vague terms. This is done by assessing the truth value of a vague statement at each of its sharpenings. If a statement is true according to every sharpening then it is said to be supertrue. Similarly if a statement is false according to every sharpening it is said to be superfalse. If a statement is neither supertrue nor superfalse, so true on some sharpenings but false on others, then it is indeterminate.

So if Francis is over 300cm tall, then since all the sharpenings of the vague term 'tall' will place the boundary between 'tall' and 'not tall' below 300cm the statement <Francis is tall> will be supertrue because Francis will be tall according to every sharpening. If Francis were shorter, say 183cm, then since 183cm is a borderline case of tall there will be different sharpenings that place the boundary above and below that height. According to the sharpenings where the boundary is below that height, <Francis is tall> will be true and according to the others that place the boundary above that height it will be false. This disagreement means that <Francis is tall> is neither supertrue nor superfalse and is thus indeterminate. Now looking at a more complex example, such as <Francis is tall and not tall and wearing blue jeans>, whatever Francis' height this statement will always be superfalse. That is because even if Francis is a borderline case of tall at each sharpening of the whole language, one of the conjuncts of the statement will be false and so the whole statement will be false at all sharpenings and thus superfalse.

The penumbral connections between vague terms lead to the correct analysis of certain other complex statements. Take for instance some patch of colour S on the vague borderline between blue and green. A supervaluational analysis of <S is green> will come out as indeterminate because the patch is green according on some sharpenings but not on others. Similarly <S is blue> will also turn out to be indeterminate. However <S is green or blue> will come out as supertrue since the penumbral connections between the two terms guarantee that on every sharpening every colour on the borderline between green and blue will be either green or blue. <S is green and blue> on the other hand comes out as superfalse since the penumbral connections between the terms blue and green guarantee that on all sharpenings no colour will be both blue and green. That seems quite correct and that supervaluationism reaches plausible truth values for such sentences involving interrelated vague terms in one advantage of supervaluationism.

Most supervaluationists hold that statements involving vague terms are only true if they are supertrue (Weatherson, 2002, 31). So if Francis is 300cm tall then the supervaluationist claims that <Francis is tall> is true because <Francis is tall> is supertrue if Francis is that height.

3.4 Supervaluationism and the Sorites paradox

The supervaluationist solution to the Sorites paradox is that the second premise, the inductive premise, is always superfalse since on every sharpening some step in the Sorites series *does* make the difference; the second premise is false on every sharpening and thus superfalse. The sharpenings disagree about which step will make the difference but not that there will be one.

Looking back at the original example of a Sorites I presented in section 2.1: -

(1) A man 10m in height is tall.

(2) A man 0.1mm shorter than a tall man is tall.

(3) Therefore, a man 1mm in height is tall.

In this form premise (2) is superfalse, because according to every sharpening of 'tall' premise (2) is false since a man 0.05mm taller than when the sharpening places the boundary will be tall and someone 0.1mm shorter than that will not be tall. To put it another way the sharpenings disagree regarding the location of the counter-example, but according to every sharpening there is a counter-example so the premise is superfalse and thus false. So the argument doesn't have true premises and thus there's nothing paradoxical about the false conclusion.

Now what of the second formulation given in section 2.1 where the second premise is replaced with a large set of premises: -

> (4) A man 10m in height is tall.
>
> (5-1) If a man 10000mm in height it tall then a man 9999.9mm in height is tall.
>
> ...
>
> (5-1,000,000) If a man 1.1mm in height it tall then a man 1mm in height is tall.
>
> (6) Therefore, a man 1mm in height is tall.

Here none of the premises are superfalse because they are each true on all but a small subset of sharpenings, but according to every sharpening there will always be a sharp boundary somewhere between 10m and 1mm, so on every sharpening one of the premises (5-1) - (5-1,000,000) will be false. So according to all the sharpenings of 'tall', one of the premises of the argument is false and so some of the premises of the argument aren't supertrue. If truth is supertruth as most supervaluationists claim, then this means that not all the premises are true as many of them are indeterminate since they are false on some sharpenings and true on others, and so there is nothing paradoxical about the false conclusion.

4.0 Alternative Theories of Vagueness

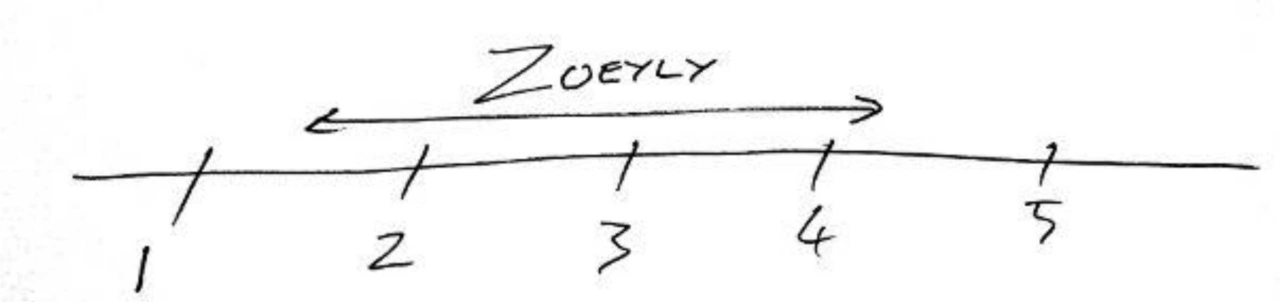
The thrust of the criticism of supervaluationism in this paper will be to question what theoretical advantages it offers over other competing theories by trying to pin on supervaluationism a commitment to sharp boundaries. Therefore for the purposes of comparison I will in this section very briefly set out three other theories of vagueness with an emphasis on their disadvantages and the criticisms that have been levelled against them. None of these should be taken as critiques of the positions themselves since space constraints will not allow any sort of rigorous analysis of any position and each of these positions are worthy of book length discussion.

4.1 Epistemicism

The technically simplest solution to the Sorites paradox is to bite the bullet and accept that for some particular number *x*, <A man *x* mm in height is tall> is true and <A man *x* – 0.1 mm in height is tall> is false and this is what epistemicism claims. Epistemicism claims that for any vague term such as 'tall' there is a sharp line and the extension of the term stops there. Technically this is flawless since it amount to nothing more than a straight rejection of premise (2) with none of the revisions or extensions to logic and semantics that cause problems for other theories. The problem with the theory is equally simple: the solution is intuitively wrong because there simply isn't any such sharp line. The whole reason the Sorites paradox is problematic is that premise (2) is intuitively true. It beggars belief that something as fuzzy as the word 'tall' should underneath it all harbour a perfectly sharp meaning, never mind multidimensional vague terms such as 'intelligent'.

The hard challenge for the epistemicist then is not to ensure their solution doesn't hide any nasty technical hitches or counter-intuitive implications, but to provide some account that will make their central massively counter-intuitive claim plausible. The approach they take to this is to explain why we believe that there isn't a sharp boundary when, according to them, there is. The explanation is that there is an epistemic barrier that prevents us from discovering the precise meaning of vague terms and thus leads us to mistakenly believe that there is none. Williamson explains this by virtue of margin for error principles (1994, 226). We can't discern the difference in height between a man 1751.3mm and one 1751.4mm tall, so if we don't know that 1751.3mm is tall we can't know that 1751.4mm is tall. We can only know that someone of a certain height is tall if we also know that everyone within a certain margin for error of that height is also tall, otherwise we may simply be right by accident. This seems a reasonable explanation of why terms have the appearance of vagueness since basically our certainty that we're outside of the margin for error, and thus our warrant, trails off as we get closer and closer to the hidden sharp borderline. It doesn't however tackle the underlying misgiving that a lot of people have, which is to question how any such precise boundary could mysteriously arise by default out of a collection of uses of the vague term. Take the term 'zoeyly' which I believe I just coined, which is a term that picks out a certain vague section of the real number line. You can see roughly what numbers are zoeyly by looking at figure 2 below.

Figure 2



Now that I’ve introduced you to this term we can have a discussion of whether a number is zoeyly or not. We could discuss what numbers are zoeyly and agree that 4 and 2.3874 are zoeyly, but 1, 5.6 and 200 are not. The epistemicist holds that there are now hidden sharp boundaries to the extension of zoeyly and given its definition and uses to date the suggestion that its extension has sharp boundaries seems completely implausible. Most do not find it plausible that there are sharp boundaries hidden under all vague terms and that is the major problem with epistemicism.

4.2 Degree theory

Similar in some respects to epistemicism is the degree theory of vagueness. This approach revises logic to admit degrees of truth, but maintains a commitment to hidden sharp boundaries between these many truth values. The simplest model introduces a third truth value, 'indeterminate', so that every truth evaluable statement can be true, false or indeterminate. This has the advantage over epistemicism that when applied to vagueness it respects our gut instinct that there is no fact of the matter as to whether some shades are blue or whether <Francis is tall> if Francis is 180cm in height. Unfortunately this was not the most problematic feature of epistemicism, the most problematic feature of epistemicism was its commitment to sharp boundary and the three valued logic approach merely replaces one sharp boundary with two. That doesn't look like any sort of improvement.

More promising is infinitely many-valued logic which replaces true and false with infinitely many values of truth. These are generally represented as real numbers between 0 and 1 with 1 assigned to absolute truth and 0 assigned to absolute falsity. This still commits the degree theorist to sharp boundaries, but they can at least attempt to argue that none of their sharp boundaries represent significant sudden changes of truth value like those of epistemicism. There are sharp lines, but there are infinitely many sharp lines so that moving across them is more akin to moving along a continuum. That perhaps isn't so counter-intuitive. After all, as we consider shorter heights that Francis could be, he would be less tall, so it seems reasonable that that constant change in height should be reflected in a slow constant change in the truth value of <Francis is tall.>

That doesn't seem so unreasonable until you consider that there are some truth-values which are far more interesting and significant than others. Keefe (2000, 92) points out that there will be some point at which the truth value of vague statements will go from 1 to less than 1. There will be a sharp boundary that marks the edge of the cloud of vagueness. There are replies the degree theorist could give to that, such as claiming nothing or almost nothing is absolutely true, but it seems at least the degree theorist will be committed to there being one particular point at which a vague statement is 0.5 true. Some height must be the precise point above which Francis would be 'more tall than not tall' and below which he would be 'more not tall than tall'. Similarly there will be a pair of real numbers of which it is exactly 0.5 true that they are zoeyly. These interesting sharp boundaries between slightly different degrees of truth arising mysteriously out of our usage of the terms are implausible for the same reason and to my mind to the same degree as the epistemicist's sharp boundaries between truth and falsity. The thought is as Keefe puts it: -

> For a start, what could determine which is the correct function, setting that my coat is red to degree 0.322 rather than 0.321? This, again, can be treated as parallel to the

> question asked of the epistemic view about what determines the valuations fixing sharp boundaries to the extensions of predicates. (Keefe, 2000, 114)

Aside from the shared problem of sharp boundaries the degree theory also suffers from a great deal of additional complexity and technically questionable logical revisionism that must also be counted against it. This makes degree theory relatively unattractive compared to epistemicism to anyone who lacks independent reasons for believing there are degrees of truth.

4.3 Transvaluationism

Finally let me gesture in the direction of transvaluationism. Transvaluationism is the theory that vague language is in fact incoherent in the way Williamson argued lead to nihilism regarding meaning, but that it is also a genuine phenomenon and still viable. Horgan (1995, 97) claims that the incoherence of vague discourse, far from robbing vague language of its meaning, is an essential feature of working language. This is clearly quite a radical position and has theoretical costs as far as its revision of semantics is concerned, but one that should be kept in mind as a possible alternative to the other theories of vagueness.

Horgan has repeatedly argued (1998, 23 and 2010, 81) that all theories of vagueness that don't admit sharp boundaries are actually implementations of transvaluationism. He has explicitly argued that a particular sort of supervaluationism, iterated supervaluationism, is a form of transvaluationism. I'll say more regarding that form of supervaluationism and Horgan's most recent argument that it is a form of transvaluationism in section 7.2.

5 The Supervaluationist Account of Higher-Order Vagueness

The above simple account of supervaluationism says nothing about higher-order vagueness, but allows for supervaluationists to give several subtly different accounts of high-order vagueness with the addition of a couple of new features to the framework.

5.1 The access relation

The first thing required is an access relationship between different delineations. The access relation will be something like a similarity or resemblance relation that may hold between delineations. So if two delineations access each other they are similar in some important respect. Weatherson comments that "To help get clear on the picture, think that one [delineation] is accessible from another if the second resembles the first insome (sic) salient respects."(2002, 42) The role of the access relation in supervaluationism's formal model is exactly analogous to that of the possibility relation that may hold between possible worlds in possible world semantics. It allows some delineations to access each other in a formally similar way to that in which worlds can be possible at other worlds. The exact details of the access relation will vary considerably between different supervaluationist models. Almost all fine details such as whether the relation is symmetric (Mahtani, 2008, 506) are debatable.

5.2 The D operator

In ordinary language we talk of things being 'definitely true', people being 'definitely tall' and cricket balls being 'definitely red'. The concept of 'definitely' plays an important role in the supervaluationist account of vagueness and higher-order vagueness. It is represented in the supervaluationist model by a logical operator *D*.

To borrow from Williamson: “As a first approximation, for the supervaluationist, definiteness is truth is truth under all sharpenings of the language consistent with what speakers have already fixed about its semantics” (1999, 128) This means that if *p* is true at all sharpenings then it is definitely true, or to put it another way *Dp* is true. So *Dp* is true if and only if *p* is supertrue. The truth condition for *Dp* is therefore that *p* be true at all sharpenings.

The supervaluationist account of 1st order vagueness in terms of the *D* operator can be expressed as follows: if *p* is a 1st order vague sentence, then $\neg Dp \;\&\; \neg D\neg p$ is true at some sharpenings. That simply means there are sharpenings at which *p* is neither 'definitely true' nor 'definitely not true' so at these sharpenings it is indefinite, it is vague, whether *p* is true or not. This is equivalent to saying that there are borderline cases of *p* and is thus equivalent to 1st order vagueness.

The *D* operator can also be iterated to give expressions such as *DDp* and *DDDDp*. These should be read as “definitely definitely *p*” and “definitely definitely definitely definitely *p*” respectively. Later in this thesis I will be considering extreme long series of D operators so for notational purposes let D^n represent a string of *n D* operators where *n* is any natural number, so D^4p is shorthand for *DDDDp*. Let $D_{set}{}^{n}p$ represent the set of delineations on which the truth of D^np depends. D^np is true if and only if $D^{n-1}p$ is true at all delineations accessed by $D_{set}{}^{n-1}p$. Then we can say that across all the common supervaluational models the truth condition for D^np is that $D^{n-1}p$ must be true at every member of $D_{set}{}^{n-1}p$ and every delineation accessed by

$D_{set}^{n-1}p$. Thus the set $D_{set}^{n}p$ is the union of $D_{set}^{n-1}p$ and the set of delineations accessed by $D_{set}^{n-1}p$. So for instance $D_{set}p$ is simply the set of sharpening of *p* and thus *DDp* is true just in case that *Dp* is true at all delineations accessed by the sharpenings of *p*.

The logic of the *D* operator depends on the nature of the access relation between the delineations. One of the key splits among supervaluationists is regarding the logic of the *D* operator and centres around whether or not $Dp \vDash DDp$ holds for all *p*.

5.3 The S4 rule

The dividing line between these two camps is the following rule: -

$$Dp \vDash DDp$$

I'll follow the convention of referring to this rule as S4 since the logic of supervaluationism is formally very similar to that of modal logic and in modal logic the analogous rule $\Box p \vDash \Box\Box p$ is known as S4. Whether S4 holds or not will depend on the behaviour of the access relation between delineations. I'll now look at each of these two opposed camps in turn, starting with those who hold that the logic for the *D* operator is S4 or stronger.

5.3.1 S4 or stronger logic for D

Simply put, if for all sentences *p*, the members of $D_{set}^{1}p$ only access members of $D_{set}^{1}p$ then

S4 will hold. To illustrate this let us coin a new vague[**] term, ‘littleheap’. Now if you have 6 grains of corn then you have a littleheap of corn and if you only have 1 grain of corn then you don't have a littleheap of corn, but if you have 3 or 4 grains of corn then you might have a littleheap of corn. In other words the sharpenings of ‘littleheap’ put the boundary at either having 3 or more grains or 4 or more grains. I'll call these by names of the form 'the sharpening at *x'* by which I mean 'the sharpening that gives the boundary for having a littleheap as having *x* or more grains'. Now let’s say that the sharpening that places the boundary at 4 accesses itself and the sharpening at 3, and the sharpening at 3 only accesses the sharpening at 4 and itself. Let *h* be the sentence <You have a littleheap>. If you have 4 or more grains of corn, then Dh is true since it is true according to every member of $D_{set}{}^{1}h$, because $D_{set}{}^{1}h$ comprises the delineations at 3 and 4. Similarly if you have 4 or more grains of corn, then DDh is true since it is true according to every delineation in $D_{set}{}^{2}h$. This is because $D_{set}{}^{2}h$ also only comprises the delineations at 3 and 4, since $D_{set}{}^{1}h$ only accesses itself and so $D_{set}{}^{2}h = D_{set}{}^{1}h$. Similarly $D_{set}{}^{2}h$ only accesses itself, as it is the same set as $D_{set}{}^{1}h$, and so $D_{set}{}^{3}h$ is the same set once again and so on. Therefore by induction we can see that for any natural numbers i and j, $D_{set}{}^{i}h = D_{set}{}^{j}h$. So for any natural number n, the truth of $D^{n}h$ relies on the truth of h at the same set of delineations (the delineations at 3 and 4), and so $D^{n}h$ is true if and only if Dh is true and thus the logic will obey S4 in the case of ‘littleheap’. More generally the rule is that whenever $D_{set}{}^{1}p$ only accesses itself the S4 rule will hold for that sentence. Thus if for all p, $D_{set}{}^{1}p$ only accesses itself the S4 rule will hold for all p.

[**] It is contentious to claim that Littleheap is vague (Keefe, 2000, 31) since as we will see it is vague in a very simple and limited sense, but for the sake of creating a simple example I hope any unsympathetic readers will allow me to speak of it as vague.

5.3.2 Weaker than S4 logic for D

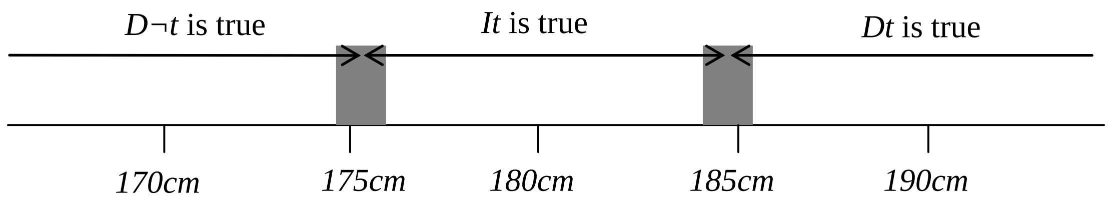
If $D_{set}^{1}p$ accesses delineations other then those contained in $D_{set}^{1}p$, then $D_{set}^{2}p$ may contain more extreme delineations than $D_{set}^{1}p$. It could then be the case that the statement *p* will be true according to all the delineations contained in $D_{set}^{1}p$ but false according to the more extreme delineations contained in $D_{set}^{2}p$ but not $D_{set}^{1}p$. In that case *Dp* will be true but *DDp* indeterminate and therefore $Dp \nvDash DDp$.

Going back to the example of 'littleheap', if we now assume the sharpening at 4 also accessed the delineation at 5, then $D_{set}^{1}h$ would comprise the delineations at 3 and 4 but $D_{set}^{2}h$ would be comprised of the delineations at 3, 4 and 5. That being the case, if you had 4 grains of corn then *Dh* would be true but *DDh* false and thus $Dp \nvDash DDp$.

5.4 The I operator

Fine (1999, 140) defined the indefinitely operator *I,* read as 'it is indefinite that', as $Ip = \neg Dp \;\&\; \neg D\neg p$. *Ip* is thus true at the borderline cases of *p* and if *Ip* is true at any delineations then *p* has 1st order vagueness. Figure 3 below which roughly illustrates the boundaries of *It* where *t* is the statement <Francis is tall> and Francis' height increases from left to right, so at 170cm $D\neg t$ is true, at 180cm *It* is true and at 190cm *Dt* is true.

Figure 3



The I operator is no more than a shorthand notation $\neg Dp$ & $\neg D\neg p$ and so frequently ignored in the literature. However, the meaning of it is quite simple to understand and since some of the examples later in this thesis will involve series of I operators that would otherwise have to be written as long conjunctions of expression involving long strings of D operators, it will be a useful notational shorthand to bear in mind.

5.5 The supervaluationist account of higher-order vagueness

Now that the whole framework is in place the standard supervaluationist account of higher-order vagueness is as follows. 2nd order vagueness of a statement p is equivalent to the 1st order vagueness of Ip. So 2nd order vagueness of p is equivalent to there being delineations at which $\neg DIp$ & $\neg D\neg Ip$ is true and that is in turn the equivalent to there being delineations at which IIp is true. Similarly 3rd order vagueness of p is equivalent to 1st order vagueness of IIp which is equivalent to there being delineations at which $IIIp$ is true. Let I^n represent a string of n I operators, so for example I^4p is shorthand for $IIIIp$. Generally if p exhibits nth order vagueness then there are delineations at which I^np is true.

6 The Case Against S4 or Stronger Logic for D

The main problem with accounts where S4 is obeyed by *D* is that they appear to give rise to a hidden sharp boundary between cases where *Dp* is true and those where *Dp* is indeterminate or false. Such a sharp boundary would be a serious problem for any supervaluationist model. The first problem is that any such boundary is contrary to our expectations of how something as fluid as natural language should be structured. The second is that violating this intuition is the only significant flaw with the epistemic theory of vagueness to which supervaluationism must show itself to be superior. This section will outline where and why the apparent sharp boundary arises from the Stronger than S4 model.

As mentioned above the supervaluationist analysis of higher-order vagueness is given in terms of repeated indefinitely operators. Different orders of vagueness analysed by appeal to different numbers of repetitions of *I* operators. The boundaries to set of delineations where *IIp* is true are those of 2[nd] order vagueness etc. Now consider the expansion of *IIp*: -

$$\neg DD\neg p \;\&\; \neg DDp \;\&\; (\neg D\neg D\neg p \vee \neg D\neg Dp)$$

Now if S4 holds then *DDp* is equivalent to *Dp*. So we can simplify the explanation to: -

$$\neg D\neg p \;\&\; \neg Dp \;\&\; (\neg D\neg D\neg p \vee \neg D\neg Dp)$$

There are two major problems with this. The first stems from the fact that the first two conjuncts $\neg D\neg p \;\&\; \neg Dp$ are the definition of *Ip*. That means that if *IIp* is true at any delineation then *Ip* is true at that delineation. So if something is a borderline borderline case then it is a borderline case and that doesn't seem right. If a shade is borderline borderline blue then that does not mean it is borderline case of blue. Borderline borderline cases are just that, not borderline and not not borderline. An account of higher-order vagueness that takes borderline borderline as implying borderline seems to be simply wrong.

The second problem is that the boundary between IIp and Dp is the same as the boundary between Ip and Dp. Similarly the boundary between being $II\neg p$ and $D\neg p$ is the same as the boundary between Ip and Dp. That means that two of the boundaries given rise to by 2nd order vagueness are equivalent to the two boundaries of 1st order vagueness. More generally since we can replace p in the above expansions with $I^{n-2}p$ and observe that

$$\neg DD\neg I^{n-2}p \;\&\; \neg DDI^{n-2}p \;\&\; (\neg D\neg D\neg I^{n-2}p \vee \neg D\neg DI^{n-2}p)$$

is equivalent to

$$\neg D\neg I^{n-2}p \;\&\; \neg DI^{n-2}p \;\&\; (\neg D\neg D\neg I^{n-2}p \vee \neg D\neg DI^{n-2}p)$$

and that

$$\neg D\neg I^{n-2}p \;\&\; \neg DI^{n-2}p$$

is equivalent to

$$I^{n-1}p$$

From this we can deduce that half the boundaries given rise to by *n*th order vagueness will be equivalent to the boundaries given rise to by *n-1* order vagueness. This means that none of the boundaries are actually analysed, they simply show up again in the next higher order of vagueness.

All plausible models of supervaluationism that endorse the S4 rule also endorse the equivalent to modal logics S5 rule, that $\neg Dp \vDash D\neg Dp$. The expansion of IIp

$$\neg D\neg p \;\&\; \neg Dp \;\&\; (\neg D\neg D\neg p \vee \neg D\neg Dp)$$

can therefore be expressed as

$$D\neg D\neg p \;\&\; D\neg Dp \;\&\; (\neg D\neg D\neg p \vee \neg D\neg Dp)$$

That is unsatisfiable since if the first two conjuncts in the expansion are true, then the third is false. There are thus no borderline borderline cases on such a model. This amounts to either a denial of all orders of vagueness above the 1st or an admission that higher-orders of vagueness must be analysed by some method other than that of the normal supervaluational framework.

Going back to the example of 'tall', let *t* again be <Francis is tall>. 'Tall' appears to be a term that has higher-order vagueness. There doesn't appear to be any sharp boundary between those heights that would make DDt true and those that make $\neg DDt$ true. It is to my mind no more plausible to suggest that there is a sharp boundary between those heights that make DDt true and $\neg DDt$ true than it is to suggest that there is a sharp boundary between those heights that make Dt true and those that make $\neg Dt$ true. It is implausible that there should be any sharp boundaries in the model of *t* because *t* is not just vague but higher-order vague, the question is how that higher-order vagueness is to be accounted for. The problem is that the supervaluationist who holds that the logic for *D* is S4 or stronger can't provide a satisfactory analysis of this higher-order vagueness within their supervaluational framework. The framework will need expanding in some way to take into account higher-order vagueness. Either that or S4 or stronger logic for *D* should be rejected.

6.1 Hybrid theories of vagueness

One possible way for the framework to be expanded would be to adopt a hybrid theory of vagueness. Most obviously a theory could be adopted where 1st order vagueness is analysed by appeal to supervaluationism, but where higher-order vagueness is to be analysed in accordance with some different theory. The most obvious candidate for such a hybrid theory is one where there is a sharp boundary between *Dp* and ¬*Dp* and higher-order vagueness is merely epistemic. The problem for such a theory is that for many terms it seems no more plausible that there are hidden sharp boundaries for *Dp* than there are for *p*. It is no more plausible that higher-order vagueness is purely epistemic than it is that vagueness is epistemic. This means that such a hybrid supervaluational/epistemic theory of vagueness would inherit epistemicism's major flaw. This might not be a huge problem if epistemicism had other major problems that the hybrid theory avoided, but this doesn't appear to be the case. Rather epistemicism is a theory that, aside from its commitment to sharp boundaries, is a fairly attractive theory in its simplicity and lack of major technical problems. Epistemicism is a relatively simple theory that is technically very solid and if you're willing to bite the bullet and admit sharp boundaries there seems little reason to reject it in favour of a more complex theory involving supervaluationism.

The general problem that dogs all hybrid theories is that they tend to inherit most of the problems and complexity of both. Unless it can be shown that there is a hybrid theory that has significantly fewer problems than either of its parts then it will inevitably fall victim to an argument from simplicity that one of its parts is simpler and has the same or fewer problems. I've argued this explicitly in the case of an epistemic hybrid of supervaluationism and it seems unlikely that any other hybrid would fare any better when the argument against is so simple and generic. If there were a theory of vagueness that could explain higher-order vagueness effectively but struggled to give a good account of 1st order vagueness, then attempting to construct a hybrid theory would seem a promising project but there is no such theory that I am aware of.

In section 7.2 we will look at a way in which supervaluationists who endorse S4 or stronger logic for *D* can extend the supervaluational framework to get around this problem and give a supervaluational analysis of higher-order vagueness. For the moment however, let us put S4 and stronger logic aside and look at a problem that affects supervaluationist accounts where the logic of *D* is weaker than S4.

7 Williamson on Potential Sharp Boundaries

It seems then that we should probably reject supervaluationism with stronger than S4 logic for *D* and consider instead a supervaluationism with a logic for the *D* operator that's weaker than S4 so as to avoid the problematic sharp boundaries that arise out of the S4 and S5 rules. However, Williamson in considering potential sharp boundaries in supervaluationism observed that even with logic for *D* that is weaker than S4 the supervaluationist model may still turn out to contain sharp boundaries and thus might still turn out to be inferior to epistemicism for this reason.

The problem revolves around a new operator defined in terms of *D* that I shall refer to as *D*.* Williamson (1994, 160) defined D^* such that *D*p* is the infinite conjunction: *p* & *Dp* & *DDp* & *DDDp & ...* . Williamson points out that the logic of this will be S4. This is the case because if we were to expand out D^*D^*p into the infinite conjunction then every term in the expansion would also appear in the infinite conjunction represented by D^*p.

To see why *D*D*p* expands out to become equivalent to *D*p,* let's quickly run through the first few steps of such an expansion exercise.

*D*D*p*

substituting the first *D* w*ith its expansion gives

*D*p & D*Dp & D*DDp & …*

which if the first *D** is in turn substituted by its expansions gives

p & *Dp* & *DDp* & *DDDp & …* & *D*Dp & D*DDp & …*

and substituting out the next *D** gives

$p \;\&\; Dp \;\&\; DDp \;\&\; DDDp \;\&\; \ldots\; Dp \;\&\; DDp \;\&\; DDDp \;\&\; \ldots \;\&\; D^*Dp \;\&\; D^*DDp \;\&\; \ldots$

which once the repeated terms are dropped is

$p \;\&\; Dp \;\&\; DDp \;\&\; DDDp \;\&\; \ldots \;\&\; D^*DDp \;\&\; \ldots$

and so on until all the infinitely many D^* terms are expanded to give

$p \;\&\; Dp \;\&\; DDp \;\&\; DDDp \;\&\; \ldots$

which is the definition of D^*p. Thus the two expressions are equivalent and since trivially $D^*p \vDash D^*p$, if we substitute D^*D^*p for D^*p on the right hand side we get the result $D^*p \vDash D^*D^*p$ which is an S4 rule for D^*.

Furthermore, since the longest chain of operators in D^*p is infinite in length, the only difference between DD^*p and D^*p is that the expansion of DD^*p does not contain the term p. However, it does contain the term Dp and trivially $Dp \vDash p$ because common sense dictates that if p is definitely true then it must at least be true. This means that the only conjunct in the expansion of DD^*p but not in the expansion of D^*p is implied by one of the conjuncts of DD^*p, so we have the result that $D^*p \vDash DD^*p$. This means that any vagueness of the boundary between D^*p and $\neg D^*p$ can't be analysed by adding another D operator. This means that using the established framework supervaluationism is unable to give any analysis of vagueness in D^*p and so it would seem that the supervaluationist must admit that D^*p is precise, or at least that their model is inadequate for analysing its vagueness. Williamson does however also suggest two possible replies that a supervaluationist could give to his example without admitting sharp boundaries.

7.1 Williamson's first reply – near impossible standard

The first reply the supervaluationist could give is that D^*p is never or hardly ever true if p is vague, that D^*p is an almost impossible standard that will only be met if there is some limit to the medium over which the vague term has its extension. So going back to the example of height, if Francis is x meters in height and t is again <Francis is tall>, then there are no positive finite values of x for which either D^*t true or $D^*\neg t$ are true. In some cases this fits quite closely with our intuitions, for instance in the case of the term 'bald'. One might be intuitively attracted to the idea that one is only D^* bald if one has no hair at all. Dorr (2010, 555) takes this line even further and argues that D^*p can never be true for any p under any circumstances, so that even a man with no hair at all isn't D^* bald. A discussion of Dorr's argument is beyond the scope of this paper but whatever its merits, that D^* is a near impossible standard is a position that attracts some philosophers. Leaving Dorr's argument aside then the more moderate position suggested by Williamson (1994, 160-161), that D^*p is hardly ever true strikes me as more intuitive and it is a strong enough position to get the full argumentative force of the reply so that is the position I'll discuss here. If Dorr's argument is correct and the supervaluationist must admit that count-intuitively even men with no hair at all aren't D^* bald, then that is so much the worse for supervaluationism since it turns out the theory must do even greater violence to our intuitions.

Looking more closely at what it means for D^*t to be a near impossible standard we see that for any finitely large value of x there will be a finite number m such that $D^m t$ is false. So in other words if you put enough D operators in front of t, then no matter what Francis' height is the statement will be false. I find this exceedingly counter-intuitive, my intuition is that if say Francis grew to be 30 foot in height, then no matter how many D operators were put in front of statement <Francis is tall> is would always be true because at 30 foot in height Francis is so far above the vague borderline of 'tall' that no amount of *D* operators could generate a statement that was anything less than definitely true. It is no vague matter that a 30 foot man is absolutely, completely and utterly tall! Imagine you have in your care a small child and you meet a 30 foot tall man in the street, the child asks “That man was tall, wasn’t he?” and you would of course agree. Now imagine the child asks “That man was definitely tall, wasn’t he?” to which you truthfully agree and then “That man was definitely definitely tall, wasn’t he?” and so on until hours later the child is repeating “definitely” hundreds of times, each time being met with your exasperated assent. Surely the child’s interrogation is pointless since no matter how many times it repeats “definitely”, if you answer truthfully your answer will always be the same. The supervaluationist has to deny this, they have to hold that if you are to answer the child perfectly truthfully at some point you need to say “Oh no, he wasn’t definitely… definitely tall.” That seems to be a mistake, a 30 foot man is tall and part of what makes the child’s line of questioning so annoying is that the answer will always be “yes” and the child cannot expect to learn anything more from pestering you so. What's more, if these intuitions hold for a human who is a mere 30 foot in height then surely it is the case for a human who is many light years in height, and yet these are things that the supervaluationist must deny if they wish to use this reply.

The near impossible standard reply also entails that at the other end of the scale for any finitely small value of x there will be a finite number m such that $D^m\neg t$ is false. So if Francis were a human being less than 1mm in height, then then there would be some finite number of *D* operators m such that if D is repeated m times in front of $\neg t$ then it will be false. This is again extremely counter-intuitive; intuitively any human less than 1mm in height is absolutely not tall and that this is simply not a vague matter.

The problem is perhaps even more acute when you consider the case of the colour term blue. Let *b* be the statement <This shade is not blue>. It seems plausible that for any shade of green between what we might pick out as a paradigmatic green and the blue-green border D^*b isn't true, but it is much less plausible that D^*b isn't true of a shade between paradigmatic green and the green-yellow borderline. It is less plausible still that D^*b isn't true for any shade of yellow and less plausible still that is isn't true of any shade of red. However the problem doesn't even end there, because the term 'visible light' is also vague and lacks a sharp extension. There isn't any sharp boundary at the edge of what wavelengths of electromagnetic radiation are 'visible light', even for individuals there is no sharp cut off, merely a drop off in the probability of detection of photons as their wavelength increases (Hardin, 1988, 26). There simply isn't any natural sharp boundary at which D^*b can switch to true without introducing just the kind of arbitrary hidden sharp boundary that this response is meant to avoid. That means D^*b must surely still be false and remain false through the spectra of infra-red, microwaves and into that of radio waves. So it seems that if this reply is to be adopted then the supervaluationist must hold that D^*b is false for radio waves. According to this account it seems that radio waves aren't D^* not blue and that seems to not only be counter-intuitive but seems to be almost on a par with a category mistake since it assigns something invisible the property of being not absolutely not blue. It seems little better to assign colour to sounds than to radio waves since neither can ever be seen by anything that speaks English. I can see two possible replies to this line of reasoning. First the supervaluationist could point out that there isn't anything akin to a category mistake, because if the extension of 'visible light' is also vague then radio waves aren't D^* not visible. That seems correct within the model but still rather odd considering no human being could ever see them. The second would be to reply that there is a sharp boundary at which D^* not visible and D^*b both become true, perhaps at the longest wavelength of light that has in fact ever been seen? Either way the results are still very counter-intuitive.

7.2 Williamson's second reply – iterated supervaluationism

The other solution Williamson proposes is to claim that D^* is still vague and only appears precise because it can't be used to measure its own vagueness. He draws the analogy to a cloud being said to have an exact length because it is exactly as long as itself (Williamson, 1994, 160). To understand exactly what this analogy means we need to consider what would happen if we were asked to give the length of a certain cloud in a language which only had a single length term, the cloudymetre, which was defined as being the length of the very same cloud. All we could say in such a language was that the cloud was exactly one cloudymetre long, so in the absence of any other description or knowledge of the definition of a cloudymetre the person who asked us the question might reasonably assume that the cloud had exact length. This would be quite incorrect since in fact the appearance of precision is merely an illusion created by the vagueness of the term exactly matching the vagueness of the object of the description. The case is analogous for D operators since for any finite string of n D operators it is trivially true that $D^n p \vDash D^n p$, and so if we are asked to give the extension of $D^n p$ in a language with only n D operators it might appear precise because in the terms available to describe its extension it is best described as being exactly $D^n p$. So if we don't analyse $D^n p$ using more than n D operators we might get the illusion of precision and so we can say that the vagueness of $D^n p$ can't be captured using less than $n+1$ D operators.

So perhaps D^* is also vague and similarly needs a new vague operator, call it D_2, to capture its vagueness. D and D^* are both part of the metalanguage used to discuss natural language, the central claim of this reply is that the metalanguage is also vague and any analysis of its vagueness needs to take place in a metametalanguage of which D_2 is a term. D_2 can't be any part of the infinite series of definitely operators that are used in the definition of D^*, because firstly it wouldn't be able to capture the vagueness of D^* since it couldn't capture its own vagueness, and secondly the D operators and D^* are in the metalanguage and D_2 isn't. D_2 appears in the metametalanguage but not the metalanguages, D_2 is a completely new D operator that is able to capture the vagueness of the metalanguage because it is outside of the metalanguage. However, like all D operators it is vague and unable to capture its own vagueness which is the vagueness of the metametalanguage, and so to capture the vagueness of D_2 a new vague operator D_3 in the metametametalanguage is needed and so on without end. That means that there is an infinite hierarchy of metalanguages above the object language and every one of them is vague and needs to one above for its analysis.

Keefe (2000, 210) points out that actually the same response can be given to the earlier criticism of supervaluationism with stronger than S4 logic for D. Recall that supervaluationism with stronger than S4 logic for D was alleged to have admitted sharp boundaries and failed to give an account of higher-order vagueness. However, if D is taken to be vague because the metalanguage is vague, then higher-orders of vagueness within the supervaluationist model with stronger than S4 logic for D can be analysed as the vagueness of the hierarchy of metalanguages. So 1st order vagueness is vagueness in the object language and is captured by analysis in the metalanguage using the D operator, and 2nd order vagueness is vagueness in the metalanguage and is captured by analysis in the metametalanguage using the D_2 operator and so on so that nth order vagueness is vagueness in the meta^{n-1}language and is captured using the D_n operator. This has the advantage that the model of supervaluationism with S4 or stronger logic for *D* is simpler than for weaker than S4 logic, so if the move to an infinite hierarchy of vague metalanguages has to be made it should be made in response to the problem of sharp boundaries in the S4 model rather than in response to the problem of sharp boundaries associated with Williamson's *D** operator. This is the type of supervaluationism Keefe endorses.

Supervaluationist models that employ this move to an infinite hierarchy of vague metalanguages are the type of supervaluationism that Horgan refers to as 'iterative supervaluationism' and has repeatedly argued that is it a form of transvaluationism (1998, 23 and 2010, 83). Horgan's most recent argument starts by questioning why it is that an epistemically ideal agent couldn't assign final semantic statues to all statements and thus discover sharp lines within the iterated supervaluationist model. He claims that the only plausible explanation for why this is the case is because it obeys principles that commit it being a form of transvaluationism. If the argument is correct then the iterative supervaluationists must accept either sharp boundaries or transvaluationist style logical incoherence. Horgan's argument is by inference to the only plausible explanation and thus open to reply from a supervaluationist who can come up with an alternative plausible explanation. Yet in spite of that weakness it does seem hard to see what reply a supervaluationist might come up with and so it does look quite strong. I find it very hard to see what principled response the supervaluationist could that wouldn't commit them to either problematic sharp boundaries or transvaluationism. Given how unattractive both the logical incoherence of transvaluationism and sharp boundaries to vague terms are for most philosophers this must count as a reason to be wary of iterated supervaluationism.

Taking Williamson's second reply leaves the supervaluationist with the hard task of coming up with a reply to Horgan's challenge. If no such reply can be found then they will be forced to accept the logical incoherence of transvaluationism, which unless they are convinced by Horgan's arguments that this incoherence is benign looks like a high price to pay. If supervaluationism is to avoid the force of Horgan's argument and the threat of the logical incoherence of transvaluationism that it entails, then it seems they should adopt Williamson's first reply that D^* is a near impossible standard. This reply as I said seems extremely counter-intuitive to me but not everyone shares this intuition and such people will presumably be able to accept this reply with ease. Therefore what the critic of supervaluationism needs is to find a non-extreme sharp boundary, a boundary that the supervaluationist can't simply dismiss by shoving it off to infinity. How we might search for such a non-extreme sharp boundary will be the focus of the next section.

8 The Search for a Non-Extreme Sharp Boundary

The critic of supervaluationism should in response to the 'near impossible standard' reply look for sharp boundaries somewhere in the middle of the range of vagueness, since such boundaries cannot be explained away as only occurring at infinity or be so easily dismissed by claiming they never arise at all. The remainder of this section will concentrate on arguing that there are infinitely many such sharp boundaries.

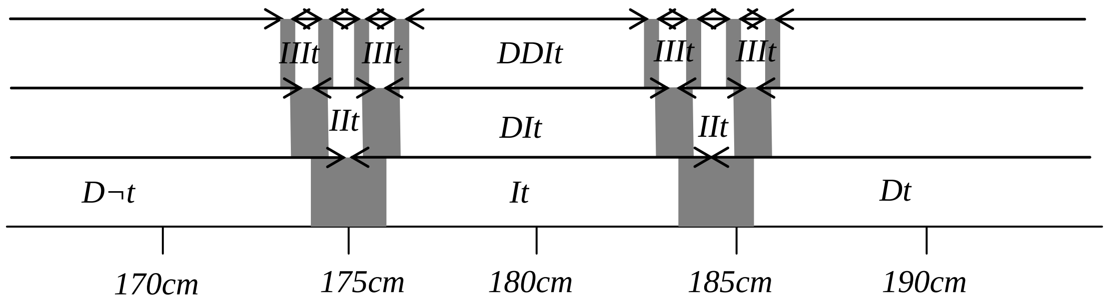
8.1 Infinitely many boundaries

One feature of $D^n p$ is that although it can have any number of boundaries it is also the case that in most cases $D^n p$ will have the same number of boundaries as $D^{n-1} p$, and so for many vague statements in natural language $D^n p$ has only one or two boundaries. $D^n p$ has one boundary in cases like that of $D^n t$ where it is true in all cases where Francis is over a certain height. $D^n p$ has two boundaries in cases such as those involving colour predicates blue, so in the previous example where b is <This shade is not blue>, $D^n b$ is false for some vague set of delineations roughly around light of wavelength 465nm[††]. $I^n p$ on the other hand doesn't share this feature, rather for many statements in natural language Ip has two or four boundaries, but for those same statements $I^2 p$ typically has four or eight boundaries, $I^3 p$ typically has eight or sixteen boundaries and so on. Generally $I^n p$ typically has twice as many boundaries as $I^{n-1} p$. If we let x be the number of boundaries to the sets of delineations at which Ip is true, then $I^n p$ will typically have $2^{n-1} x$ boundaries. To see why this is the case it is probably easiest to first

[††]This is true for small values of n at least, for higher values the supervaluationist should probably claim that $D^n b$ is false for all wavelengths of light.

examine figure 4 below that gives a graphical representation of the boundaries of *It, IIt* and *IIIt.*

Figure 4



Because *t* is higher-order vague *It* has vague boundaries, so *IIt* is true at both the boundaries of *It.* Now for each distinct extension where *IIt* is true, it has both an upper and a lower boundary so it has twice as many boundaries as *It. S*imilarly *IIIt* is true at each of the boundaries of *IIt* and has upper and lower boundaries for each of these distinct extensions and so has twice as many boundaries as *IIt* and four times the number of *It.* Following this reasoning we can then see that in general $I^{n+1}t$ will have twice as many boundaries as $I^n t$. Combine the facts that *It* has two boundaries and $I^{n+1}t$ has twice as many boundaries as $I^n t$ and we get the result that $I^n t$ will have 2^n boundaries. Looking at figure 4 once more a quick count shows that this does indeed hold for the first three orders of vagueness depicted.

However, as we've noted *Ip* could have any number of boundaries[‡‡], so to arrive at a more general formula for the number of boundaries for $I^n p$ from the result that the formula for *It* is 2^n, we need to divide that formula by 2 because that is the number of boundaries *It* happens to

[‡‡] For an extreme example where *Ip* has infinitely many boundaries consider the vague proposition <Cos z is high>, this will be true when z is 0, 2π, 4π etc and indefinitely true at two sets of points around each of those infinitely many values.

have, and then multiply by *x*, the number of boundaries there are to *Ip*. This is still far from a completely general formula since on some models there may be cases where $I^n p$ has greater or fewer than twice as many boundaries as $I^{n-1} p$, but this formula does hold for all plausible models of natural language terms like 'tall' and 'blue'. Now for all vague statements *x* will be greater than or equal to 1 because *x* corresponds to the number of boundaries there are to the 1st order vague boundaries, so if there are any vague boundaries then there must be at least one boundary to those boundaries. Thus since *x* is non-zero and positive, as *n* tends to infinity so does $2^{n-1}x$. This means that for most vague statements $I^\infty p$ will have an infinite number of boundaries.

It may be claimed that some of these strings will eventually not be true at any delineations. However it still seems that the number of boundaries that this supervaluational model gives rise to will be infinite, or as many as the underlying medium over which the vague term has its extension allows. So for example in the case of the vague term ‘flock’ as applied to birds, it would seem likely that there will be at least as many boundaries as there might be birds. The alternative is that at some order of vagueness the number of boundaries stops increasing at all. Since each vague boundary requires at least two boundaries in higher-order vagueness to be analysed, that would imply that higher-order vagueness has cut out and we have reached a state of precision. The supervaluationist should not admit that vagueness cuts out at any order, because if higher-order vagueness cuts out then we have precision in the boundaries at that point and thus sharp boundaries which is ultimately what the critic of supervaluationism is aiming to prove.

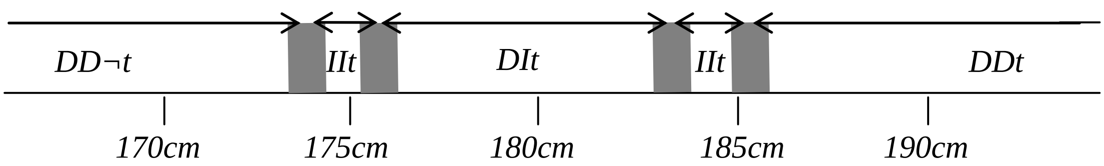
8.2 The distribution of the boundaries

The supervaluationist can also point out that if $I^n p$ is expressed purely in terms of D operators, then one of the terms in the formula is $\neg D^n p$ and so as n tends to infinity that will become $\neg D^\infty p$. This will give reason for supervaluationists like Dorr who claim that $D^* t$ is an impossible standard to claim that the infinite number of boundaries to $I^\infty t$ will be spread over the entire range of heights from 0cm to infinity. If the range over which the infinite number of boundaries is spread is also infinitely, then the vague extension of each may in fact be finite. This would be an effective reply if the distribution of the boundaries was unknown. However, although the range of all the boundaries generated by $I^\infty t$ may be infinite, we have a pretty good idea as to the location of some subsets of these boundaries. Looking back at figure 4 you can see that the boundaries, if they exist, of any string of the form *DD...DIIt* will be around roughly 175cm for the lower case of *IIt* or 185cm for the higher case of *IIt.* This is because they can't move outside the vague range of *IIt* as *D* operators are added, since $DIIt \vDash IIt$. Similarly we know that all the boundaries to sets of heights at which strings of the form *...DIt* are true will fall between these two values since $DIt \vDash \neg IIt$, and as figure 4 illustrated the range where *DIt* is true is between the two ranges over which *IIt* is true. It appears therefore that an infinite number of boundaries, those to areas where strings of the form …*DIt* are true, fall inside a roughly 10cm range.

8.3 Infinitely sharp boundaries

Furthermore, all these boundaries that arise from the supervaluational analysis of the vagueness of *t* are ordered. Take for example the case of boundaries at the level of 2nd order vagueness. As you can see in figure 5 below, the boundary between *DD¬t* and *IIt* is lower than that between *IIt* and *Dit,* and this has to be the case. Consider for a moment the alternative, if say the vague boundary between *DD¬t* and *IIt* was higher than *IIt* and *DIt.* Then there would be delineations at which *DIt* would be true, but *IIt* and *DD¬t* would both be indeterminate as it would be on the vague boundary between those two. That is impossible because *DIt* implies *¬IIt* and *¬DD¬t,* so anything that is *DIt* cannot be on the *DD¬t* | *IIt* borderline and so the vague boundary between *DD¬t* and *IIt* can't extend higher than the vague boundary between *IIt* and *DIt.* Similar reasoning will apply in all other cases, so a similar ordering will hold no matter how long the strings become. The boundaries may of course overlap since they're vague, but they must also maintain their order.

Figure 5



According to the supervaluationist all of these boundaries are vague and for the sake of argument let us suppose that that is indeed the case. There is a problem with having an infinite number of vague boundaries between the two regions where *IIt* is true. They can't be identically located because they have to maintain their order, but they also all have to fit into an approximately 10cm range. So either (1) the vague boundaries are finitely wide but the differences in the vague locations of the boundaries must be infinitely small, or (2) the boundaries themselves must be infinitely narrow.

Now let's look at (1) in more detail. Let *u*, *v* and *w* be three extremely long, 10^{100} say, chains of ¬, *D* and *I* operators such that *ut*, *vt* and *wt* are true at adjacent regions so they define two boundaries *ut* | *vt* and is *vt* | *wt.* Suppose these two boundaries are vague and have some small finite extension, but they differ in location by some far smaller extension so that they overlap almost entirely. In that case, since *Ivt & D¬ut* is only true at the region that is on the *vt* | *wt* boundary but not the *ut* | *vt* boundary, the set of delineations at which it is true that *Ivt & D¬ut* will have a far smaller extension than either boundary. Now if the difference between the vague locations of *ut* | *vt* and *vt* | *wt* were infinitely small, then the extension of the region at which *Ivt & D¬ut* is true must be infinitely small and *Ivt & D¬ut* | ¬(*Ivt & D¬ut*) will be an infinitely thin borderline. Therefore (1) implies (2), so the supervaluationist must accept that there are infinitely many infinitely sharp vague boundaries.

An infinitely sharp vague boundary in some quantifiably medium such as height is very similar to an irrational number such as π. Just like π it can't be given a precise finite decimal explanation, but at every stage it has a location on the number line precise enough that there is a determinate next digit in its decimal expansion. So for π, if a mathematician with a computer were asked for the first 6 figures of π she could calculate that the answer is 3.14159, and if she were asked for the next digit then she could calculate that the answer is 2, and if asked for the eighth digit she could calculate that it is 6 and so on. What she couldn't do is to at any point give the precise decimal value of π, but at every stage she could give the next number in the expansion. An infinitely sharp vague boundary is similar. If the supervaluationist account is correct and we could somehow get past the epistemic constraints, then if someone were to ask the location of one particular boundary of $I^{\infty}t$ in cm then we could tell them to as many figures as they required. The only thing we couldn't do is ever give a precise finite decimal location for it.

If this is the case then it puts the supervaluationist in real trouble. It doesn't give the precise boundaries that epistemicism admits, but the intuitive difference between a precise boundary and one that it infinitely sharp seems to be almost non-existent. The intuitive problem with precise boundaries is that to suggest that the addition of one atom to the top of Francis' head could make any difference to the semantic value of anything related to the analysis of the statement <Francis is tall>. Such an addition is completely imperceptible, so it seems wrong to suggest that such an addition could make the difference between Francis being taller or shorter than $Ivt \,\&\, D\neg ut$. In this crucial respect an infinitely sharp boundary is as bad as a precise one since either way one tiny step down the Sorites series will make the difference.

8.4 One infinitely sharp boundary

One reply that the supervaluationist could try to make to the above is that past a certain order of vagueness there are simply no more definite cases. Everything is indefinite except for those things that are true or false at all delineations. For instance, if we assume D^*p is taken to be an impossible standard then $\neg D^*p$ is true at all delineations. The argument could go that past a certain point everything is indefinitely the case and ultimately $I^{\infty}p$ is true at every delineation, so at the level of infinite-order vagueness there no boundaries because everything is perfectly indefinite. This might seem fairly plausible at first glance, but the problem is that $I^{\infty}p$ is simply shorthand for its expansion in terms of D operators. Recall that Ip is simply shorthand for $\neg Dp \,\&\, \neg D\neg p$ and so IIp is just shorthand for $\neg DDp \,\&\, \neg DD\neg p \,\&\, (\neg D \neg Dp \vee \neg D \neg D\neg p)$ etc. So when each I operator in $I^{\infty}p$ is replaced by the equivalent in terms of D operators and the result is expanded out it becomes an infinite disjunction and we can still ask about the boundaries between the different ways of fulfilling the requirement of the disjunction. There will still be infinitely many sharp boundaries between the different sets of delineations at which different combinations of disjuncts are true.

It will perhaps be clearer if we look at one particular infinitely small set of delineations that must in turn have infinitely sharp boundaries. This set, let's call it *L*, is the set of all delineations at which the following infinite conjunction is true: $\neg Dt\ \&\ \neg D\neg Dt\ \&\ \neg D\neg D\neg Dt\ \&\ \neg D\neg D\neg D\neg Dt$ ... Now $\neg Dt$ will be true if Francis is below the height that would make him definitely tall, so it's true if Francis is below roughly 184cm. $\neg D\neg Dt$ on the other hand will be true if Francis is above the height that would make $D\neg Dt$ definitely true, so let us say for the sake of argument, if Francis is above roughly 182cm. $\neg D\neg D\neg Dt$ is true as long as Francis is below the height that would make $D\neg D\neg Dt$ true. That's a lower height than is necessary to make Dt true, so the requirement that $\neg Dt$ is true for all members of *L* doesn't actually restrict the set at all. Similarly ¬D¬D¬D¬Dt is a more restrictive standard than ¬D¬Dt and so on. If we look at each pair of terms in the conjunction we can also define further sets based just on the conjunction of two successive pairs in the series, so let L^1 be the set of heights that would make $\neg Dt\ \&\ \neg D\neg Dt$ true and L^2 be the set of heights that would make $\neg D\neg Dt\ \&\ \neg D\neg D\neg Dt$ true, etc. Now at each stage the set of heights that would make each L^n true is smaller than the set of heights that would make L^{n-1} true since they are defined on the basis of greater standards of indefiniteness at each stage.

Now clearly no delineation can definitely be a member of any of these sets since no delineation can definitely adhere to any of the conditions of the conjunction else it would be violation of one of the other conditions and this means the set must be vague[§§]. However, as

[§§] I owe this point to a conversation with Cian Dorr.

vague as the set is we can still ask about its size and since at each step along the conjunction it becomes smaller and the conjunction is infinite in length, either the size of that *L* is infinitely small or it has a finite size, but the differences in sizes of successive members of the L^n series will tend to be infinitely small as *n* tends to infinity. Now if the set is infinitely small, then its boundaries will be infinitely sharp and so supervaluationist has to admit to infinitely sharp boundaries. Alternatively if *L* is finite, then as *n* tends to infinity the extension of the region of heights at which L^{n-1} is true but L^n false will tend to be infinitely small, and so *L* will again have infinitely sharp boundaries.

The supervaluationist could perhaps reply that in fact the set is empty and that once again we have defined an impossible standard that nothing meets. However, even if that happens to be true in this particular case, it can't be a general reply since the conjunction is nothing more than one element of the expansion of $I^{\infty}t$ and to claim that $I^{\infty}t$ is an impossible standard would be nothing more than to deny infinite higher-order vagueness. Denying infinite higher-order vagueness would be a problem for the supervaluationist because in that case sharp boundaries will be admitted at the order at which higher-order vagueness cuts out.

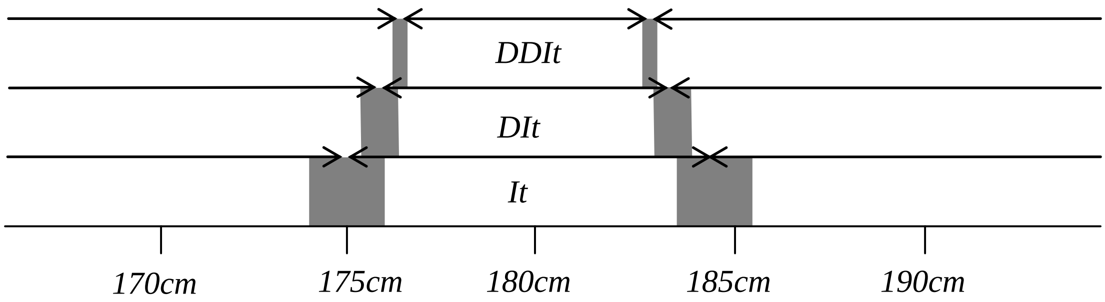
9 Problematic Non-Extreme Sharp Boundaries

Supervaluationism is of course built on the idea that vague terms are to be analysed with regard to their sharp delineations, so perhaps the fact that each of the vague boundaries are, at the level of infinite-order vagueness, made up of an infinite number of sharp boundaries is nothing problematic. After all these aren't the boundaries we're interested in when we wonder whether Francis is tall or some patch of colour is blue. These are boundaries in infinite-order vagueness and perhaps it is just unproblematic for boundaries in infinite-order vagueness to be infinitely sharp. Perhaps that's just to be expected and isn't problematic in the same way as sharp boundaries in 1st or 2nd order vagueness.

Such a line of argument seems fairly plausible when considering such abstract boundaries as those of *L* or as arbitrary as $Ivt \,\&\, D\neg ut \mid \neg(Ivt \,\&\, D\neg ut)$. However, there are other boundaries that arise out of the model that seem far more clearly problematic. This section will concentrate on picking out problematic sharp boundaries in the weaker than S4 supervaluationist models of the vagueness of *t* and arguing that whatever the exact details of the model it must give rise to one such problematic sharp boundary. If it can be argued that weaker than S4 models of supervaluationism admit a boundary as problematic as that admitted by epistemicism, then supervaluationism is in real trouble since it looks like on either model it shares the major weakness of epistemicism.

Let us start by looking at figure 6 below which illustrates the central portions of the ‘tall’ boundary structure at each of the first three orders of vagueness where *It, DIt* and *DDIt* are true. With each additional *D* operator that area is shown as decreasing in width and indeed that will be the case. That decreasing structure would continue if the diagram was extended to cover higher-orders of vagueness. Depending on the details of the model $D^{\infty}It$ will either: be true at an infinitely small region of points; won't be true anywhere; or it will be true at a finite region of points. In the next three subsections I'll run through each of these scenarios in turn and argue that they each give rise to at least one interesting sharp boundary.

Figure 6



<u>*9.1 $D^{\infty}It$ is true at an infinitely small region of heights*</u>

The first case is when $D^{\infty}It$ is true at an infinitely small region of heights. This fits in well with the position that D^* is a near impossible standard since D^*It would only be true at this infinitely small region of heights as well. The problem is that on this scenario, because $D^{\infty}It$ is true at an infinitely small region, then that region will constitute an infinitely sharp boundary at the most indefinitely tall height. An infinitely sharp boundary such that if Francis is 0.1mm above that height he can be said to be 'more tall than not tall', and if he is 0.1mm below that height then he can be said to be 'more not tall than tall'. This isn't identical to the type of sharp boundary epistemicism admits that divides simply 'tall' from 'not tall', but the difference between that type of sharp boundary and this one doesn't seem that great, certainly not great enough for it not to be a problem for supervaluationism.

Firstly both boundaries mark a sudden change in truth value for something which intuitively shouldn't change suddenly. For epistemicism one extra micron in height might make Francis tall whereas for supervaluationism that one extra micron in height might make Francis more tall than not tall. 'More tall than not tall' is a more complex notion than simply 'tall' and not one we are so prone to consider, but it is hardly an alien concept and it still seems as wrong that it should have an infinitely sharp cut-off as it is for 'tall'.

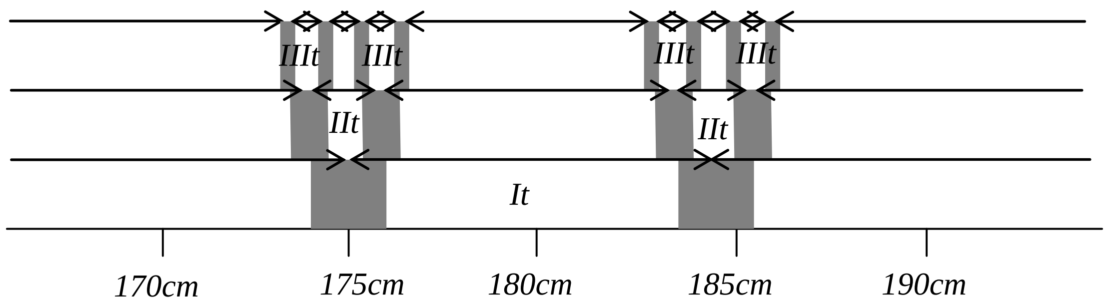
Secondly the problem with sharp boundaries isn't so much that they admit sharp change in truth value from true to false, but that they introduce any sharp features at all into something that seems completely vague. Recall that the degree theory of vagueness also eliminated the stark change from true to false for simple vague statements like t, but it did so at the expense of introducing a possibly infinite number of sharp boundaries that marked infinitely small changes in truth value. This was still judged to be problematic on the basis that it was still introducing interesting sharp lines, such as the line where t would be 0.5 true, where there are no such sharp lines found in natural language.

The degree theorist's sharp boundary above which t is more than 0.5 true marks the dividing line at which Francis would be 'more tall than not tall' and below which he would be 'more not tall than tall'. This is the type of interesting infinitely sharp boundary that supervaluationism was meant to avoid. However on this model of supervaluationism, where the logic for D is weaker than S4 and $D^{\infty}It$ is true at an infinitely small region, this is exactly the type of sharp boundary that is introduced. That degree theory's 0.5 line is precise and supervaluationism's line is merely infinitely sharp hardly seems like a significant advantage.

9.2 $D^{\infty}It$ isn't true at any height

In the second case $D^{\infty}It$ isn't true at any height. This would be the case if we accept that nothing can be D^* true since $D^{\infty}It \vDash D^*It$. Consider figure 7 and notice how depicted in this way the boundaries that arise at successive orders of vagueness form two almost tree-like structures. If $D^{\infty}It$ isn't true at any height then the gap between these two trees will disappear and the branches will at some order of vagueness meet.

Figure 7



Crucially, if and when the two trees do meet, they will at the infinite order of vagueness have an infinitely sharp boundary between them and this is the boundary that I wish to draw your attention to as problematic for this model. Consider the three operator case again and let's assume for the sake of argument that *DDIt* is an impossible standard, then the boundary pattern would look like this:-

DDD¬t| IIIt | DIIt | IIIt | IIIt | DIIt | IIIt |DDDt

There is a clear central boundary between the two extensions of *IIIt* around which the regions and boundaries are symmetrical[***]. Now as we move in from both ends, clearly *t* is more indefinite in the regions where *DIIt* is true than in the end sections where *IIIt* is true, and similarly *t* is more indefinite in the central regions where *IIIt* is true than in either of the regions where *DIIt* is true. The indefiniteness of *t* increases as we move in from the ends towards the centre and each boundary that is crossed as we move towards the centre marks a move towards greater indefiniteness of *t*. The most indefinite point is the boundary between the two central regions since the pattern of increasing and decreasing indefiniteness pivots around this point.

The supervaluationist may reply to this by simply disagreeing with my assertion that the central boundary marks the point of greatest indefiniteness and claim that since both regions of *IIIt* are in some sense as indefinite as each other the boundary between them is nothing special. However, this is a moot point when considering the important case in which the regions are defined not by chains of only three operators but with infinitely long chains of operators. This is the case because as the number of regions tends to infinity they must also tend to be infinitely thin and therefore the two regions at which *t* is most indefinite when taken together will still have infinitely small extension, and so they will still define an infinitely sharp point of greatest indefiniteness.

[***] It would be a mistake to think that because *IIIt* is true at both of these areas they can blend seamlessly together, because if the *I* operators are replaced with the equivalent statements in terms of *D* operators then *IIIt* becomes: -

¬*DDDt* & ¬*DDD*¬*t* & (¬*DD*¬*Dt* v ¬*DD*¬*D*¬*t*) & ((¬*D*¬*D*¬*Dt* & ¬*D*¬*D*¬*D*¬*t*) v (¬*D*¬*DDt* v ¬*D*¬*DD*¬*t*))

The difference between these areas is that at one area the conjunct (¬*DD*¬*Dt* v ¬*DD*¬*D*¬*t*) is satisfied by ¬*DD*¬*Dt* being true and at the other by ¬*DD*¬*D*¬*t* being true.

9.2.1 Picking out the most indefinite point

However, at the infinite order of vagueness some strings of operators won't be true at any delineation, indeed by assumption $D^{\infty}It$ won't be true at any delineation. Given that we can't guarantee that any particular string of operators will be satisfied at any delineation and so we can't say which infinite string of operators will be the most indefinite that is true at any delineation, and thus we can't give any sort of direct characterisation of the nature of the infinitely sharp point of greatest indefiniteness. If we can't provide such a characterisation then perhaps the supervaluationist could claim that since we can't pick it out with a characterisation in terms of *D* operators then there is no such most indefinitely tall height. This however is not the case, because although we can't pick out exactly what will be true at the most indefinite point we can be given an algorithm that, were it not for epistemic boundaries, could be used to pick out the most indefinitely tall height. This algorithm is sufficient to show that there will be such a height within this supervaluationist model and thus present the problem for this model. I will now detail the algorithm that can be used to pick out this point.

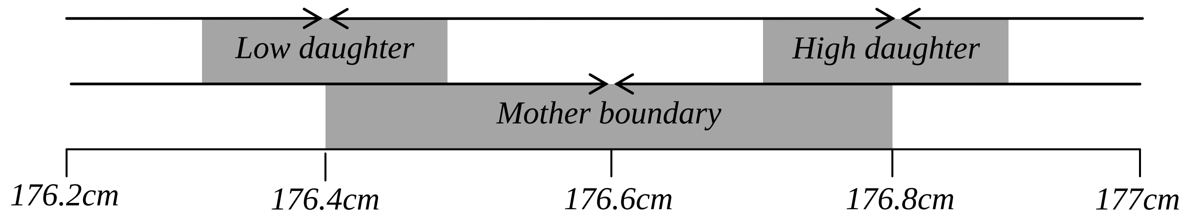
9.2.2 The algorithm

The first step is to systematically assign a finite real number to every vague boundary in higher-order vagueness. We can start this process in a fairly arbitrary fashion by specifying that the two 2nd order vague boundaries, those between *It* and $\neg It$, are to both be numbered $1/2^2$ [†††]. Now as illustrated in Figure 7 above, each boundary in *n*th order vagueness will

[†††] $1/2^2$ is of course simply 1/4, but for the sake of understanding how the algorithm works it is simpler to think of it as $1/2^2$.

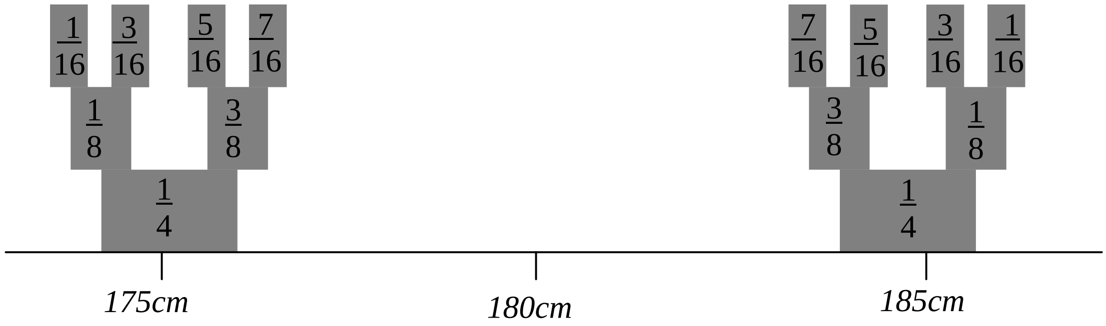
typically give rise to two boundaries in *n+1* order vagueness. What is more, in the case of 'tall' one will have a vague location that is higher than the other. I'll refer to these as the ‘mother boundary’, the ‘high daughter boundary’ and the ‘low daughter boundary’. So if the mother is a boundary in *n*th order vagueness, then the high and low daughters are boundaries in *n+1* order vagueness, and if the mother is in the vague area of 176.4cm – 176.8cm then the high daughter will be at about 176.8cm and the low daughter at about 176.4cm as illustrated in figure 8.

Figure 8



Now let *m* be the number assigned to the mother boundary and *n* be the order of vagueness of *t* that gives rise to the daughter boundaries. Now for boundaries that are given rise to by the higher-order vagueness of the *It* |*Dt* boundary, let the number of the low daughter be $m + 1/2^n$ and the number of the high daughter be $m - 1/2^n$. Alternatively for boundaries that are given rise to by the higher-order vagueness of the *D¬t* |*It* boundary, let the number of the low daughter be $m - 1/2^n$ and the number of the high daughter be $m + 1/2^n$. This might seem very complex, but the end result is quite simple as illustrated in figure 9. It simply assigns every boundary a rational number between 0 and 1/2 such that the boundaries that are closer to the centre of the vague extension of *It* are assigned higher numbers and those further from the centre of the vague extension of *It* are assigned lower numbers.

Figure 9



Now assuming as we are that there is no finite gap between those two “trees” of boundaries, then there will be an infinite number of boundaries at the level of infinite order vagueness between every two points between 185cm and 175cm. This means that every point between 185cm and 175cm will be on at least one infinitely thin boundary in infinitely higher-order vagueness. The final step in the algorithm is to assign to every point between 185cm and 175cm the lowest of the numbers of the boundaries in infinitely higher-order vagueness that the point corresponds to. With this assignment of numbers in place the most indefinitely 'tall' height that Francis could be is the height that has the highest number assigned to it, whatever that number may be.

9.2.3 Defence of the choice of algorithm

However, if the trees overlap the supervaluationist could reasonably ask why the point with the highest number assigned to it is the most indefinite. After all the assignments of the numbers were not only arbitrary, but systematically different for the boundaries that arise from the higher-order vagueness of the $D\neg t$ $|It$ and the It $|Dt$ boundaries. The general flavour of the whole algorithm might come across as unappealingly ad hoc. Under another assignment, if say, we instead for the boundaries that are given rise to by the higher-order vagueness of the $D\neg t\ |It$ assigned the number to the high daughter according to the formula $m + 1/2^{2n}$, then the point with the highest number that the critic claims is the point of greatest indefiniteness would be quite different. The key difference between the assignment that was actually used and other assignments that would give different results is that according to the assignment chosen the numbers are assigned symmetrically to reflect the symmetry of the boundaries. So for instance both the boundaries that are assigned 7/16 are of the form $DDIp|IIIp$, the difference between them is merely that p is in one case t and in the other $\neg t$. Since these are higher-order boundaries of the same boundary as in 1st order vagueness there seems no reason to expect that the two boundaries of the form $DDIp|IIIp$ would occur at different levels of indefiniteness and so they should be assigned the same number. Any system of assigning finite numbers to the boundaries that respects this symmetry and gives higher numbers to more indefinite boundaries will assign the highest number to the same point. So in spite of the fact that other number assignments could easily have been chosen, any assignment which respect these minimal principles will return the same point as the point of greatest indefiniteness.

Thus the weaker than S4 supervaluationism once again has the same problem as degree theory. This model where $D^{\infty}It$ isn't true at any delineation also gives rise to a single infinitely sharp point of greatest infiniteness above which Francis could be said to be ‘more tall than not tall’ and below which he could be said to be ‘more not tall than tall’. Once again this model of supervaluationism gives rise to the sort of problematic infinitely sharp boundary that supervaluationism is meant to avoid and it seems doubtful that the benefits of supervaluationism over epistemicism will be worth the complexity.

9.3 D^∞It is true at a finite region of heights

The final model we need to consider is that at which $D^\infty It$ is true at a finite region of heights. This model, unlike the previous two, doesn't give rise to an infinitely sharp point above which Francis would be 'more tall than not tall' and below which Francis would be 'more not tall than tall' since there is no infinitely sharp boundary at the centre of the range of vagueness of *t*. Rather there is a central region where $D^\infty It$ is true and contains no other boundaries. This might initially seem like a promising model to adopt to bypass the criticisms of the previous two models, but unfortunately it has problematic features that are at least as bad.

The first and most immediate criticism is that it merely replaces one problematic sharp boundary with two. Both the boundaries to $D^\infty It$ will be precise sharp boundaries: above the higher one Francis would be 'more tall than not tall' and below it 'not more tall than not tall'; below the lower boundary Francis would be 'more not tall than tall' and above it 'not more not tall and tall'. These seem like two quite problematic boundaries. It is analogous to a degree theory where there is an extended region where *t* has truth value 0.5, which is a model a degree theorist could also adopt if there were anything intuitively desirable about it. However it seems to me to be intrinsically less attractive than the previous two models since it seems mysterious that there should be such a calm eye to the storm of vagueness that surrounds it.

The second problem is that it undermines the reply from the near impossible standard to Williamson's initial argument. It seems that if we are to accept that *D*t* isn't true even if Francis were tall enough to bump his head on the moon, then it seems implausible that *D*It* will be true of any height and since $D^{\infty}It \vDash D^{*}It$ that is exactly what the supervaluationist must claim. There doesn't seem to be any principled reason to make such a claim apart from to get out of a commitment to a single central infinitely sharp boundary, and so if such a model were accepted then the asymmetry in the model in its treatment of the standard required for something to be *D** true would look more than a little ad hoc.

9.4 The case against supervaluationism with weaker than S4 logic for D

We've worked through the three possible scenarios and the conclusion is that weaker than S4 supervaluationism is tied to there at least being an infinitely sharp boundary between heights that are 'more tall than not tall' and heights that are 'not more tall than not tall'. There are also significant disadvantages to denying there is an infinitely sharp boundary between heights that are 'more tall than not tall' and heights that are 'more not tall than tall'. What is more, I put this debate in terms of 'tall', but the argument could easily be adapted for any vague term once a supervaluational analysis of that term has been constructed. I could for instance have just as easily have put it in terms of whether numbers are zoeyly or not. I could then have shown that there are two infinitely sharp boundaries between those numbers that are 'more zoeyly than not zoeyly' and those that are 'not more zoeyly than not zoeyly'. Supervaluationism with a weaker than S4 logic for *D* thus fails to maintain any significant advantage over supervaluationism with S4 or stronger logic for *D.* The extra complexity of the theory has failed to provide a solution to the problem of sharp boundaries in supervaluationism and this should be rejected in favour of the simpler supervaluationist model with S4 or stronger logic for *D.* I conclude this section by agreeing with Keefe (2000, 210) that the correct response for a supervaluationist to the problem of sharp boundaries is to reject weaker than S4 logic for *D* as a needless complexity and adopt the iterated supervaluationist model.

10 Conclusion

This is an extremely damaging result for supervaluationism since one of the most fundamental advantages of the theory was that it supposedly got away from the commitment to interesting sharp boundaries arising mysteriously from our use of terms. What I have demonstrated is that the only way for supervaluationism to rid itself of the spectre of sharp boundaries is to accept that the metalanguage must also be vague and that supervaluationism must be iterated. In light of Horgan's most recent argument that iterated supervaluationism is a form of transvaluationism this does not look like a good result for supervaluationism.

Supervaluationalists are faced with a stark choice between iterated supervaluationism and sharp boundaries. If they choose iterated supervaluationism, then they face Horgan's argument that their theory is weakly logically incoherent and a form of transvaluationism. Alternatively if they choose sharp boundaries they must accept that their theory shares the main flaw of epistemicism and degree theories.

Supervaluationism that admits sharp boundaries looks extremely unpalatable on the basis that it will be almost impossible to justify the complexity of supervaluationism's formal structure once it is admitted that it shares epistemicism's major weakness.

Iterated supervaluationism on the other hand faces a serious challenge if it wishes to avoid being a form of transvaluationism. Failure to meet this challenge would be bad news for supervaluationism and good news for epistemic theories. It would, as Williamson noted, "be good news for epistemicism if transvaluationism turned out to be the only alternative" (2002, 279). Good news because most will find the logical incoherence of transvaluationism extremely unpalatable and would rather bite the bullet of sharp boundaries than the bullet of incoherence. However, my arguments above show that iterated supervaluationism is the only way a supervaluationist might be able to avoid sharp boundaries. For supervaluationism having the risk of being shown to be a form of transvaluationism makes the theory less attractive, but if supervaluationism is shown to be committed to sharp boundaries then for simplicities sake it should be forgotten in favour of simple epistemicism. I thus conclude that because of the problem of infinitely sharp boundaries the only good form of supervaluationism is iterated supervaluationism. It is unfortunate for supervaluationism that if Horgan is correct then the only "good" form of supervaluationism is a form of transvaluationism and committed to logical incoherence.

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