MEDIUM AND MESSAGE: THE USE AND DEVELOPMENT OF AN ENGLISH MATHEMATICS REGISTER IN TWO MALTESE PRIMARY CLASSROOMS

by

Marie Therese Farrugia

A Thesis submitted to
The University of Birmingham
For the degree of
Ph.D

School of Education
The University of Birmingham
September 2006

ABSTRACT

The National Minimum Curriculum (NMC) of Malta recommends the use of the country’s second language, English, for the teaching of mathematics. The aim of my study was to enhance the local medium-of-instruction debate by focusing on the use and development of a mathematics register, and distinguishing between issues relevant to second-language classrooms and ones more generally applicable Assuming a social perspective of learning, I used a grounded methodology, thus generally allowing my reflections to develop out of the data I collected. The research design consisted of lesson observations in two primary classrooms and interviews with the teachers and pupils. I concluded that the use of English in class created tensions with other NMC principles; I also noted variations in the way some mathematical words were used when compared to what I might expect as part of an English mathematics register. On the other hand, the frequency of pupils’ use of mathematical vocabulary during lessons seemed to depend on the teacher’s pedagogic approach. Also applicable to general mathematics classrooms appeared to be three conditions I identified as important for word meanings to be effectively shared with pupils: frequency of use, clarity, and significance, that is, how crucial a word appeared to be when used.

DEDICATION

To Walter and Daniel

ACKNOWLEDGMENTS

Special thanks go to all the people who assisted me throughout this project.

I am especially grateful to my supervisor, Dr. Dave Hewitt, for his guidance and constant encouragement. His insights and constructive feedback have helped me greatly in developing my ideas, and our discussions have had a significant impact on my development as a researcher.

I am indebted to the Head of School who accepted me in her school, as I am to the teachers and pupils who welcomed me into their classrooms. The project would not have been possible without their keen participation.

I would like to thank my colleague Antoinette Camilleri-Grima who, as a linguist, helped me to appreciate issues relating to bilingualism. I also acknowledge as helpful, the feedback I received from various researchers when I presented my ideas at BSRLM and BERA conferences. Their reactions helped me to establish a sense of direction along the way.

Finally, I would like to mention the friends and colleagues who kindly read various parts of my work and whose constructive comments guided me towards the final version: Leonard Bezzina, Leone Burton, Antoinette Camilleri-Grima, Dylan Jones, Yosanne Vella and Alice Wakefield. Special thanks go to Christine Hockings who gave so generously of her time by reading all the chapters.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

C H A P T E R   O N E

Introduction

1.1 Background to the study

In 1999, the Ministry of Education in Malta published a new National Minimum Curriculum (NMC) for all Maltese primary and secondary schools. The aim of the document was to encourage schools to reflect on issues such as assessment, inclusion, creativity and technology and to come up with school-based policies on such matters (Ministry of Education, 1999). Apart from stressing areas that were deemed important by the writers of the NMC, this document was a step in the direction of school autonomy in a system of education that has been centralised for decades.

One of the points mentioned in the document was that of a policy on bilingualism. Although the island has its own language – Maltese - English is also widely used as a result of 165 years of British colonial rule that ended when Malta gained independence in 1964. Indeed, Maltese and English are the country's two official languages. As Camilleri-Grima (2003) stated: "the average Maltese person lives daily with two languages, moving from one to the other as the context demands" (p.56). As a result of this, code-switching between Maltese and English has become common practice, resulting in a language pattern that Borg (1980) referred to as Mixed Maltese English.

In local mathematics classrooms at both primary and secondary levels, English is the language of written texts, while for spoken language, 'mathematical' words are usually retained in English even when the rest of the speech is in Maltese. The NMC writers appeared to disapprove of such language patterns and favoured consistent use of one language; they also expressed a wish to strengthen both the Maltese and English languages. To this end, they strongly recommended that English should be used as the medium of instruction for mathematics: the apparent assumption was that this approach would help to enhance students' command of English. It was this suggestion that first sparked off my interest in conducting research on language in local mathematics classrooms.

My initial reaction had been to agree with the NMC recommendation, being influenced by my own experience of using English frequently in personal and academic situations. However, conversations with linguists and early reading around the subject encouraged me to reflect beyond my own particular experiences and to recognise that the issue was a complex one. This awareness coincided with the time that I, a mathematics educator involved in primary-teacher training, started to think about conducting doctoral research. Consequently, I decided that I would base my project on language use in local mathematics classrooms.

1.2 The aim of the study

The local debate regarding the choice of language of instruction generally centres on two main arguments. Those in favour of using English tend to support the approach as a way to enhance pupils' knowledge of the language, while those who favour code-switching argue that our first concern should be the understanding of mathematics. I feel that the debate generally does not go beyond this dichotomy, and the aim of this study is to enhance the discussion by introducing reflections relating to spoken *mathematical* language and to the sharing of meaning for mathematical words.

A challenge that I faced was to find a way to discuss and link the English / Maltese dilemma with the use and understanding of mathematical vocabulary. In this regard, I found Halliday's (1978) notion of register helpful. According to Halliday (*ibid*), a register is:

> "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p.195).

Working with the idea of a mathematics register, I could consider the extent and cohesion of talk (hence focusing on issues relating to the use of English), examine the inclusion of mathematical words in speech and also explore the apparent meanings expressed for these same words.

My study was based in two classrooms: Grade 3 (7 to 8-year-olds) and Grade 6 (10 to 11-year-olds). I observed a series of mathematics lessons conducted in English, focusing on interaction that 'surrounded' mathematical words and also conducted interviews with the two teachers and some children. I asked them their opinions regarding the use of English and also asked the children to offer meanings for a selection of topic-related words. The idea was to draw comparisons between apparent meanings expressed during the lesson and those expressed by the children. I hoped to be able to identify features of the classroom interaction that seemed to be helpful for successful sharing of meaning of mathematical words. In my analysis, I planned to reflect on the extent to which the points at hand appeared to be particular to classrooms wherein a second language was used, and to what extent they were general mathematical language issues. I hoped that these reflections would then help to both deepen and broaden the local discussion: deepen, by reflecting in more detail on learning through a second-language and broaden, by reflecting on other language matters.

1.3 Overview of thesis chapters

I start this project by explaining the local medium-of-instruction debate, contextualising it historically and with respect to contemporary international situations (Chapter 2). I then discuss what might be considered as 'mathematical language', and the development of a register (Chapter 3). Drawing on Vygotstky (e.g. 1962), I assume a social view of teaching and learning, and consider the use of a register as a community practice to be shared by the teacher with the pupils. I then explain my methodology and develop a semiotic model in order to assist me in my discussion of 'meaning' (Chapter 4). Details about the process of data collection follow (Chapter 5).

In order to help me structure my analysis, I draw on Pimm's (1987) view that language is both a medium and a message. Hence the title of my thesis: '*Medium and message: the use and development of an English mathematics register in two Maltese primary classrooms*'. In my discussions, I first present points relating to language as a medium: reflections on the choice of the medium of instruction, the extent and cohesion of English talk, the frequency of use of mathematical words for the various topics observed and the apparent influence of Maltese on how mathematical words are used in English (Chapters 6 & 7).

In the second part of the analysis, I consider language as a message by focusing my attention on meanings for topic-related words. Across the topics considered – 'Multiplication and Division' and 'Length' in Grade 3, and 'Graphs' and 'Length' in Grade 6 – I attempt to identify features of the classroom interaction that appeared to be helpful for successful sharing of meaning (Chapters 8 –11). I end the study by bringing together the main conclusions and suggesting a way forward (Chapter 12).

CHAPTER TWO

Bilingualism in Malta

2.1 Introduction

In order to provide a backdrop for my reflections, I begin by presenting the socio-historical context within which my study is set. I then outline the local debate regarding the medium of instruction for mathematics, locating it within the wider international context. Finally I present the research questions that this study aims to address.

2.2 The use of Maltese and English in Malta

The Republic of Malta lies in the centre of the Mediterranean Sea and consists of two main islands and a number of smaller uninhabited ones. The total area is 316 square kilometres and the population is 400 000. Malta's geographic position has attracted a succession of powers over the centuries, the main ones among them being the Phoenicians, Romans, Arabs, Normans, Knights of Saint John and the French. The last colonisers were the British, who ruled Malta for 165 years until Independence was achieved in 1964.

The island has its own language, Maltese. Camilleri-Grima (2003) described the Maltese language as having a Semitic foundation originating during the Arabic domination (870 – 1249 A.D.). The Arabic elements form the basis of the sound systems of the language (that is, the phonology), word formation (morphology) and sentence structure (syntax). Over a period stretching from 1249 to the present, Maltese vocabulary was extended through Romance elements. The Romance influence is also evident in some phonological and syntactic aspects of the language. Finally, Maltese has been influenced by English on the lexical level, that is, by the addition of new vocabulary. Today, the language continues to absorb English words as a result of the prominent role English enjoys as a global language. Some words have been wholly assimilated into written Maltese since they are now given a Maltese spelling, as in the case of **friġġ** (fridge) and **kowt** (coat) (Azzopardi, 2003). Although Maltese is the national language of the country, both Maltese and English are recognised as official languages by the Constitution.

Camilleri-Grima (2003) explained that while Maltese is widely spoken as a means of daily communication and is the official language of parliament and the courts, English is essential for international communication, the tourism industry and in local education. This type of bilingualism is known as societal bilingualism, since it is common to a group of people (Baker, 2001). Of course, many differences exist in individual competences with regard to the receptive skills (reading and listening) and productive skills (speaking and writing) of the language. Furthermore, Camilleri (1995) pointed out that a small portion of the population use English as a first language at home, and popular perceptions persist in associating English with higher levels of education and social standing. While varying attitudes exist regarding the use of English, a person who uses English exclusively tends to be regarded as a 'snob' by others who do not.

Generally, a Maltese person may use the two languages to different extents depending on their backgrounds, preferences, and the context in which they find themselves. As an example, and following Camilleri-Grima's (2003) personal illustration, let me consider my own language biography. I use either language in family and social circles, depending on whether the person I am speaking to prefers to use English or Maltese. Since TV stations broadcast programmes in both languages I may listen to the news in Maltese and, just afterwards, watch a film in English. At the University of Malta where I work, I discuss with colleagues during meetings in Maltese, but will take notes in English. I converse with my students in Maltese, but generally lecture in English, unless my students request I do otherwise. I am aware that these options are open to me since I have a working knowledge of both languages, having first been exposed to English as a child at home and at school, and later as an adult needing to be fluent in Maltese as my life experiences widened.

As Camilleri (1995) pointed out, it is not possible to talk about language compartamentalisation in Malta, since Maltese and English frequently overlap. Indeed, code-switching between Maltese and English has become common practice locally, resulting in a language pattern that Borg (1980) referred to as Mixed Maltese English. Baker (2001) defined code-switching as the practice of deliberately alternating between two or more languages. When the switch within a sentence is for only one or a few words, this may be referred to as 'code-mixing', while longer stretches may be considered as 'switches'. As Baker (*ibid*) pointed out, the distinction in practice between mixing and switching is not clear-cut and I use the expression 'code-switching' to refer to both. The extent to which code-switching occurs will depend on the speaker and the context, but it is certainly a feature of classrooms, as will be discussed shortly.

2.3 A brief history of the recent struggle for language supremacy in Malta

The language of a nation is often a symbol of its identity and allegiance, and the medium of instruction used in schools is a powerful means of maintaining or revitalizing a culture (Tollefson and Tsui, 2004). The choices of official languages and media of instruction are shaped by political, social and economic forces and every country in which there has been a struggle for language supremacy has its own complex story to tell (Tollefson and Tsui, *ibid*, offer a comprehensive compilation of such stories). Common factors often include the imposition of a language by a dominant group, and/or a perception of the dominant language as being a gateway to economic or social success. Malta is no exception, and here I offer but a brief and simplified account of the events of recent times.

In the mid-nineteenth and early twentieth century, Italian was the language used and promoted by the local professional elite and the nobility. Indeed, Italian was a marker of social standing, knowledge and class (Sultana, 1997). The powerful Catholic Church also used Italian and, fearing proselytism, resisted interference in education by the Protestant colonial power. At the same time however, English was becoming increasingly relevant to those closely connected with the British naval activity on the islands, and was a compulsory requirement for the civil service. Thus, when education first started being organized on a national level in the mid-nineteenth century, the three languages – Italian, English and Maltese - were considered as a medium of instruction (Camilleri, 1995). The problem with Maltese was that little existed in the way of books and the form of the alphabet had not yet been firmly established. Given these circumstances, and the fact that most teachers spoke and wrote Italian rather than English, the Maltese Director of Education based most of instruction on Italian (Blouet, 1981).

In 1879, the British government commissioned an inquiry into the state of Maltese education. Among other things, the report (Keenan, 1879) suggested that Maltese should be the medium of instruction, English should be taught through Maltese while Italian should be offered as an optional language outside normal school hours. These and other recommendations were implemented by the Maltese Director of the time, although English as a subject was introduced in the second year of schooling and Italian in the third and within school hours (Xerri, 1994). The change brought about a furious reaction from the pro-Italian lobby and according to Xerri (*ibid*), the ‘language question’ was to hamper progress in local education for many years. In 1897, more changes were made, with Maltese being taught in the first two years and used as a medium of instruction and a choice between English and Italian as subjects offered in the third year of schooling. Most parents chose English, presumably because of its practical relevance. Primary schooling became compulsory in 1924 and over the years some slight amendments were made to the language arrangement, with the most significant change coming in 1931, when Italian was moved to the secondary school.

According to Marshall (1971), one of the shackles of education in Malta had long been the lack of an established spelling system or orthography, but a group of distinguished writers working on it in the 1920s was strong enough to make themselves heard, and a standard orthography was confirmed on 1st January 1934. In the same year, the Maltese language was raised to the status of national and official language. Interestingly, Frendo (1975) suggested that this move served, in part, as a solution to the rivalry between pro-English and pro-Italian political factions of the time. The policy now was for Maltese to be used as the medium for all subjects including for the teaching of English; at this time, History and Geography books were also produced using the new Maltese orthography.

However, in 1948, a committee set up to assess the problems being experienced by the educational system suggested that *English* be used as a medium of instruction (Zammit Mangion, 2000) and Camilleri (1995) stated that during the 20th century the preference of school authorities did tend to be English. This was mainly due to the language's importance as the colonial language and to its vast and respected literature. Another reason may be have the fact that between 1944 and the late 1970s, teacher training was run by British Catholic religious orders in residential courses and this may have influenced the teachers' mode of teaching. The preference for English was perhaps more evident in private schools, who tended to cater for the well-to-do. Today, some private schools still promote English as the medium of instruction and reasons sometimes cited are the usefulness of English and 'tradition'.

Although what exactly went on in classrooms over the years is not documented, I can say from my personal experience and anecdotal evidence that today, many teachers, especially those in State-run schools, prefer to use Maltese, or rather code-switching, for mathematics instruction. One reason for this may be what Camilleri (1995) considered to be the ever-increasing respect for the language over the decades; other reasons may be the moving of teacher training to the University of Malta in the late 1970s, and as in the case of other professions, the increased accessibility of teacher training to a wider section of the population.

2.4 Code-switching in classrooms and the local language debate

Brincat (2000) stated that code-switching between Maltese and English occurs when the two languages are used alongside each other, as in the case of schooling. In our classrooms, a situation exists where written Maltese is used only for Social Studies, Religion (Roman Catholic) and, of course, Maltese, that is subjects closely tied to the local traditional culture. All other subjects continue to utilise English for written texts including books, handouts, whiteboard work, copybook notes, computer software and exams. Hence, it becomes necessary for teachers and students to move from one language to another. As an illustration of how code-switching might occur in a mathematics classroom, I present a piece of interaction between a teacher and an 8-year-old child. This is taken from my pilot study, and was observed during a lesson on 'Money'. (In the transcription, the original Maltese speech and its translation are indicated in a **bold** print):

Teacher:	**Qed nistsaqsikom liema** coin**? Liema hi l-kelma bil-Malti**? Which coin has the smallest value? [**I'm asking you which** coin? **What's the word in Maltese?** Which coin has the smallest value?]
Pupil:	**Kemm tiswa** … one cent. [**Its value** … one cent]

Here the interaction was a preparation for the subsequent activity where questions such as 'Which coin has the smallest / biggest value?' were written on the board to be answered in writing.

In her study of various secondary school classrooms, Camilleri (1995) found that linking with written texts was the most common reason for code-switching. Furthermore, code-switching occurred when subject specific words were used: for the various subjects she observed, including mathematics, Maltese equivalents of some technical words did not exist and when they did, the Maltese versions were more commonly used in 'every day' life, rather than as part of the 'academic' classroom talk. Finally, Camilleri (*ibid*) found that the use of code-switching allowed a flexible and comfortable mode of communication. Thus, she concluded that code-switching served as a useful pedagogical and communicative resource. This concurs with Setati and Adler's (2000) view that in the South African mathematics classrooms they observed, code-switching appeared to be something positive. They stated that:

> "Code-switching is a practice that enables learners to harness their main language as a learning resource" (*ibid*, p. 244).

Various international researchers writing about situations in ex-colonies have noted code-switching in classrooms for similar reasons as those observed by Camilleri (1995) in Malta. I summarise their observations in Table 2.1.

Author	Country	Medium of Instruction (MoI)	Consequences Noted
Arthur (1996)	Botswana	English used instead of Setswana or Ikalanga beyond Grade 4. Policy derived from colonial conventions.	Grade 6 teachers switched to first language to translate ideas and to encourage participation. Pupils not 'allowed' to switch.
Gfeller & Robinson (1998)	Cameroon	A Cameroonian language to be used for first 3 years of primary, French /English taught as subject, then used as MoI from Grade 4 onwards except for culture, history and geography.	(Details not given)
Lin (1996)	Hong Kong	English the language of power, education and socio-economic advancement [note: article written prior to handing over of HK back to China in 1997]. Used as MoI instead of Cantonese.	Teachers switched to Cantonese to establish a friendly atmosphere, English used for academic terms and to facilitate a task. Switching also enabled learners to relate the English 'distant' topics with student's familiar experiences.
Merritt et al (1992)	Kenya	English used as MoI instead of Swahili from Grade 4 onwards. Terminal exams in English. Reasons: colonial language, international language, 'neutral' language since first language of only few people, used for scientific terms, more resources available.	English used for formal interaction, Swahili for informal. Code-switching occurred for whole sentences, in translation or discourse markers. Code-switching patterns depended on: school policy, cognitive and classroom management concerns, attitude re languages in society at large.
Ndayipfuk-amiye (1996)	Burundi	French taught as subject and then used as MoI after Grade 4. Policy derived from need for international language and because Kirundi not suitable for science and technology.	Code-switching occurred in observed Grade 5, and main reason appeared to be an attempt to bridge the gap between the world of books and the pupils' knowledge.
Setati (1998)	South Africa	English instead of first language/s.	English used mainly to reformulate pupils' responses and to familiarize the pupils with formal assessment. Setswana used for translation, to facilitate understanding of concepts and to encourage learner participation.
Setati (2003)	South Africa	English instead of first language/s.	In observed Grade 4, English used for 'procedure' aspects of mathematics, Setswana for conceptual knowledge.
Setati & Adler (2000)	South Africa	English instead of first language/s.	Teacher and learners used English for the 'public' domain. Teacher used main language for reformulation in whole class teaching and also interaction with individuals / small groups. Learners' shared language Setwsana, interspersed with mathematical English.

Table 2.1. Medium of instruction policies in some ex-colonial countries.

Exactly when and why a teacher should code-switch is not a straight forward choice, and Adler (2001) referred to it as a dilemma that involves finding the balance between using the first language to aid understanding, and using English to provide access to a language deemed useful not only for mathematics, but for other contexts too. However, code-switching in classrooms appeared not be approved of by the writers of the NMC. This attitude of disapproval seems to reflect an apparent assumption that code-switching constitutes language deficiency (Baker, 2001), a view which according to Lin (1996) reflects a normative-based perception of what counts as standard or legitimate language. This apparent belief coupled with a wish to improve Maltese students' standard of spoken and written English, prompted the writers of the NMC to suggest that mathematics, science and technology be taught through English at both primary and secondary levels.

The Maltese/English dilemma for mathematics is one of which local educators are well aware. During an informal discussion for which I was present between teachers, linguists and education officials, various opinions were aired. The main points mentioned in favour of using English were that such an approach supports children's learning of the language and their 'overall' education. Furthermore, it may be beneficial to high achieving students who go on to follow academic paths. Other opinions expressed were that English is useful when non-Maltese children are present in the classroom and that since technical words are in English, then it makes sense to conduct all the conversation in English. Those present who had reservations regarding the use of English expressed concerns about pupil participation, disadvantaging lower-achievers and the teachers' own confidence in using English. Another opinion was that it was not practical to expect teachers to stick to English, especially at points where they felt that their students did not understand the mathematical concepts. Some argued that since Maltese is a living language, then it should be allowed to develop according to the use to which it is put, and this may necessitate code-switching with English. Finally, it was suggested that if English competence is seen to be a problem, then it should be tackled in its own right rather than through other subjects.

It may seem unusual that there is a drive by Maltese people to promote the ex-colonial language, after all the historic struggles we have experienced. However, this is not a unique situation, and Bunyi (1997) noted that societies continue to use the language of their colonisers for a variety of reasons long after the colonisers have left. For example, I note a similarity with the situation in Singapore, where according to Pakir (2004), one of the reasons why English has been established as the medium of instruction for schools is that people have a strong perception of English as a language that empowers its users. (Another reason is that it is seen as politically ‘neutral’ for the three main ethnic groups, a situation that does not arise in Malta). I believe that some Maltese people share the same perception of the usefulness of English, including the writers of the NMC. Of course, I cannot exclude the fact that they may have a personal perception of English as being in some way ‘superior’ to Maltese, or as Baker (2001) suggested, that the recommendation is motivated by an ulterior agenda to offer an advantage to certain social groups. However, being acquainted with some of the people involved in the writing of the NMC, I believe that the aim may have been a genuine search for finding ways to improve students’ competence in English.

Students in many parts of the world find themselves learning mathematics through a second language for a variety of reasons, including ex-colonial ones like ours, or migration. Various researchers have commented that such an experience is generally problematic for the learner. For example, Brodie (1989) commented on the difficulties non-English speaking South African students faced when they pursued education in English when this was their third or even fourth language. These included difficulties in understanding the teacher and the material, and with expressing themselves in English during the lessons. Arthur (1996) commented that the teachers she observed in Botswana tended to ask closed questions, and that the use of English as a medium of instruction inhibited attempts by the teacher and pupils to pursue more challenging and culturally congruent learning; MacGregor (1993) described the difficulty immigrant students in Australia had in following what the teacher was saying because she talked too fast and because sentence structure may have varied from that familiar to them in their own language.

Sometimes it was word problems or ‘story sums’ that were found to be a source of difficulty. Non-English speaking children may not understand them, as reported by MacGregor (1993) in the Australian context; Jones (1982) and Clarkson (1991) found in their studies that Papua New Guinean children lagged behind in word problems involving the expressions *more* and *less.* Adetula (1990) found that Nigerian children fared better in word problems involving these same words when these were set in the children’s native language (Yoruba or Hausa). Olivares (1996) drew attention to the fact that synonyms or words of similar meaning such as *add*, *plus*, *combine*, *and*, *join* may be a source of difficulty to students with limited English proficiency since they may not know their equivalents in their own language.

In the contexts mentioned above, there may not have been the option of code-switching, either because of political reasons, or because the teacher and pupils were not familiar enough with each others' language. This contrasts with the local situation where both teachers and children understand both Maltese and English, albeit to different degrees, and code-switching practices are common. In situations were children are *necessarily* immersed in a second language, researchers (e.g. Capps and Pickreign, 1993; Earp and Tanner, 1980) strongly recommend systematic support for the learning of mathematics, with particular attention being given to the mathematical vocabulary and expression. This point will be taken up in the next chapter.

2.5 Learning a subject through a second language

Baker (2001) stated that:

> "a repertoire of languages gives wider access to social, cultural, political economic and educational information" (*ibid*, p.112).

The method of targeting a second language by teaching all, or part of the curriculum through this language is known as the immersion approach (Baker, 2001). In western countries, this approach originated in Canada in the 1960s, the success of which Baker (*ibid*) attributed to teacher enthusiasm and competence in both languages (English and French) together with parental motivation and support. There are different forms of immersion: Baker (*ibid*) lists early total immersion (i.e. at a very young age), early partial immersion and late immersion as three types, and he discusses research findings regarding the effectiveness of the various programmes. For the sake of this thesis, I will be using the general expression 'immersion approach', taking it to mean teaching and learning through the second language.

A comparable idea that has picked up in popularity in the EU in recent years is Content and Language Integrated Learning or CLIL[1], whereby a subject is taught through a targeted language. The main reasons for promoting language competence within the EU are: professional and personal mobility, cross-cultural contacts and mutual understanding (European Communities, 2004a). This has led to a language policy referred to as 'Mother Tongue Plus Two' (European Communities, 2004b). 'Two' refers to two foreign languages, one to be introduced at an early age, and the other in secondary school. The CLIL approach emerged as a "pragmatic solution" (Marsh, 2002, p.11) to the pressure that extra language lessons put on school curricula,

[1] This approach is also often referred to by its French acronym EMILE (Enseignement d'une Matière par l'Intégration d'une Langue)

In the comprehensive report that Marsh (*ibid*) compiled with other language experts, details were given about ongoing projects being carried out in various EU countries where the approach was being trialled. For each project, a list of objectives was stated based on five dimensions: cultural, environmental, learning, language and content (i.e. subject). The content objective/s were generally cited as the accessing of subject-specific terminology and/or the preparation for future studies or work.

The compilers' overall impression was that CLIL can prove to be an inclusive method suitable for different ages, abilities and leaner competencies. They believed that CLIL can be implemented in appropriate context-specific ways in widely differing situations if the situational variables are understood, and taken into consideration. The consultancy group stressed that teachers need to be competent in both the target language and the subject and Kelly *et al* (2004) offered details of how different countries are addressing the development of dual competency as part of teacher training programmes. The writers concluded that high exposure was not necessarily correlated with high competence. Rather, it was the form, intensity and timing of exposure that were the important factors. For example, the report stated that low exposure (5 – 15% of teaching time) over a longer period appeared to be advantageous. The writers also stated that although the preferred subjects for CLIL have traditionally been those regarded as 'less academic', opinions were changing to the view that whatever non-language subject matter is adopted, it must be relevant in terms of the dual-plane learning. That is, subjects or themes should link into the true contexts of the world in terms of both language and non-language topics. The writers quoted an example at a higher level of education where a module on European law may be appropriate for CLIL, but not so a module on national law. The report noted that interest was shifting to theme-designed, modular approaches rather than subjects as such, with the core issue being relevance.

In Malta, the situation is slightly different since, for us, English is a second language rather than a foreign one in the way Spanish or French are. English has now long been taught alongside Maltese as a 'subject' from the early primary years, with another one or two languages taken up at secondary school level, generally Italian, French, German or Spanish. However, although the NMC recommendation was suggested five years before Malta's entry into the EU in May 2004, it may be the case that projects such as CLIL reinforce the idea that a language and a subject can be taught together. I feel that Malta is in an interesting historical situation: we are a new member state, yet only recently left our colonial past behind by gaining independence from another member state, whose language is now dominating the international scene. We now have to merge feelings of sovereignty with feeling associated with European ideals; we also have to continue to strive to find a balance between valuing the Maltese language while strengthening our second one.

2.6 Conclusion

One question that was asked in the introductory section of the consultative report on EU CLIL projects (Marsh, 2002, p.25) was: 'What is the impact on subject teaching and learning?', by which they generally meant achievement. The report recommended reflection on exposure time, relevance, methods and context. This recommendation supports an idea stated by Brodie (1989) for bilingual education in general:

> "Every situation where the need for bilingual education arises is unique, and will require its own analysis, research and solutions" (*ibid*, p.52).

My project offered one way of reflecting on the use of a second language as a medium of instruction. My first research interest was to consider the NMC language recommendation in the light of other NMC principles; the second focused more specifically on the pupils' use and development of mathematical language; finally I was interested in exploring what helped or hindered the sharing of meanings for mathematical vocabulary. Hence, I posed the following research questions:

> *(1) How does the NMC recommendation regarding the use of English for mathematics fit in with other educational principles promoted in the same document?*
>
> *(2)(a) How much, and with what ease, do pupils talk in immersion classrooms?*
> *(b) How 'mathematical' is their talk, in terms of the inclusion of mathematical vocabulary?*
>
> *(3) What conditions appear to be helpful for a teacher to 'share' the meaning of (a selection of) mathematical words with the pupils?*

While the first question was directly related to the immersion situation, I planned to reflect on whether other observations were particular to immersion classrooms, or whether they might be more generally applicable.

In this chapter I have offered some background with respect to medium-of-instruction issues. In preparation for discussions related to pupil talk and mathematical vocabulary, I now consider literature related to mathematical language.

CHAPTER THREE

Mathematical Language

3.1 Introduction

> "Mathematics is, among other things, a social activity, deeply concerned with communication" (Pimm, 1987, p.xvii).

The relationship between language and mathematics has been a source of interest for several researchers and educators over the years. The relationship can be viewed in a variety of ways. For example, Aiken (1972) and Austin and Howson (1979) suggested that attention be given to children's reading ability, teacher-pupil interactions, culture, multilingualism and to mathematical written symbols as a language. More recently, Usiskin (1996) considered diagrams as part of language, while Borba (2005) discussed technology as a communicative means. My own interest lies in verbal classroom communication, in particular in a 'whole-class' teacher-directed style of teaching and learning, which is a very prevalent mode in Malta.

In the U.K., the importance of increasing pupils' contribution to classroom talk was brought to the fore through the document that came to be known as 'The Cockcroft Report' (DES, 1982). This stated that students should be encouraged to discuss and explain the mathematics they were learning. This recommendation was offered alongside the promotion of practical work and investigations, in what Pimm (1987) considered an attempt to move teaching away from traditional exposition teaching methods.

I am interested in examining how much children talk, with what ease, and how mathematical their talk is. Hence, in this chapter, I consider how different styles of classroom interaction encourage different degrees of pupil verbal participation. Since verbal communication is often complemented by the use of gestures, I also include a short section about gestures. Assuming the use and development of mathematical language to be an integral part of learning the subject, I offer examples of how teachers can help pupils move from 'everyday' to 'mathematical' language, reflecting on what actually renders language 'mathematical'. Furthermore, I consider how mathematical vocabulary is added to a language to constitute a 'register' (Halliday, 1978), and describe English and Mixed Maltese English mathematics registers. Finally, I explain how the notion of register was helpful to me in structuring my discussions within this study.

3.2 Communication in the mathematics classroom

Verbal communication can be useful in the mathematics classroom since through talk, understandings can be clarified and misconceptions addressed (Griffiths and Clyne, 1994). Brissenden (1988) considered pupil talk as a means for a teacher to assess understanding, while Pimm (1987) recognised the value of pupil talk as a means for talking things through and organising one's thoughts. Similarly, Marks and Mousley (1990) believed that through talk, pupils make their mathematical meanings explicit and "talk themselves into understanding" (p.132).

In a teacher-directed pedagogy, it is usually the teacher who initiates talk, and very often contributions are invited through questioning. The questions tend to serve a very different purpose from those used in everyday conversations (Wood, 1994). In the latter situation, a person usually asks a question to find out something he or she does not know. On the other hand, teachers usually already know the answer to their own question and according to Seeger (1998), the role of the pupils is to fill in the 'slots' left by the teacher's questions with the right answer. Hence, pupils' answers are often short, after which the teacher reacts by perhaps confirming or otherwise the appropriateness of the answer. This type of three-part interaction is known as the Initiation - Response - Feedback (IRF) pattern of interaction (Sinclair and Coulthard, 1975). An example of this type of interaction, taken from the data I collected for this study, is the following:

> (*The class is working on converting units. The teacher has just written 2m34cm on the whiteboard*).
>
> | | |
> |---|---|
> | Teacher: | Now we said two metres are how many centimetres? [INITIATION] |
> | Pupils: | Two hundred! [RESPONSE] |
> | Teacher: | So I have two hundred and …? [FEEDBACK – INITIATION] |
> | Pupils: | Thirty-four. [RESPONSE] |
> | Teacher: | Thirty-four. (*Address a particular pupil*). How many centimetres do I have? [FEEDBACK – INITIATION] |
> | Pupil: | Two hundred and thirty-four. [RESPONSE] |
> | Teacher: | (Writes = *234cm* on the board). [FEEDBACK] |

This question-and-answer type of interaction can be used in order to 'funnel' an incorrect answer towards a correct one. In defining funneling, Bauersfeld (1998, p.170) offered the following example, where the teacher has just asked a young boy for the answer to 9+7:

Pupil: 14.
Teacher: OK. 7 plus 7 equals 14. 8 plus 7 is just adding one more to 14, which makes …?
Pupil: 15.
Teacher: And 9 is one more than 8. So 15 plus one more is …?
Pupil: 16.

Although the teacher here is guiding the pupil towards a correct answer, it is still the case that the pupil does not get to speak much, nor is it necessary for him to think in detail about number relationships. Generally, pupil talk tends to be limited in teacher-led interaction, as noted by English (1981), Torbe and Shaurd (1982) and Kerslake (1982). Bauersfeld (1998) contrasted the funneling type of interaction with what he calls 'focusing', where the teacher creates opportunities for the pupils to explain and give reasons for their ideas, and for listeners to ask questions. The teacher's questions in this situation serve to encourage the pupils to focus on their own reasoning and this type of interaction has the potential to increase pupil talk. Some comments and questions that can promote pupil participation are 'Tell John how it works', 'Go on …', 'Where did the eight come from?' 'Show me!' (Clemson and Clemson, 1994). Similarly, Kamii (1994) suggested questions such as: 'What are you trying to find?' 'Does everyone agree?' These researchers suggested that open-ended questions encourage children to articulate their own thoughts, offer various methods, reflect on their answers and on those of others, and to participate in an exchange based on possible disagreement.

As part of communication, I conjecture that both teachers and pupils might use gestures to complement verbal contributions and indeed, McNeill (1992) argued that gestures and speech used together form a single unified system and are co-expressive. Roth (2000, 2001) classified gestures into four types: first there are 'beats' such as the flicking of a hand, or tapping motions that are used to emphasise certain utterances. These gestures may serve the role of coordinating speaking turns, acknowledging understanding or requesting a response. Second, there are symbolic gestures like the 'O.K.' finger gesture that function independently and gain their meaning through convention. Third, there are deictic, or pointing gestures: these draw attention to some aspect of a context and therefore make it salient against everything else, which is rendered background. Deictic gestures are often used with deictic words such as *here*, *there*, *this*, *that* etc. Fourth, iconic gestures can provide an image since the movements bear a perceptual resemblance to concrete things or events, for example a chopping action, or a curve 'drawn' in the air. In a similar vein to iconic gestures, Goldin-Meadow *et al* (1999, p.721) suggested that gestures can render concepts 'concrete'. They gave an example of holding two pointing hands together and then ‘drawing’ a horizontal line by pulling them apart while saying ‘that would be a line measurement’.

Researchers have reflected on the usefulness of gestures in the classroom. Roth and Welzel (2001) noted from their observations in science classrooms that almost all incidences of gestures occurred when the teachers they observed attempted to produce descriptions and explanations. Indeed, Roth (2001) suggested that students might be able to use the teacher’s gestures as additional resources for making sense of the teacher-talk. Pupils, too, may use gestures in communication and this might help the teacher better assess the student’s current understanding (Roth, 2001). Wagner Alibali *et al* (1997) commented that children’s gestures are particularly revealing when they convey information that is not expressed in speech, or when the speech is inarticulate or vague, as is often the case when children are working out a new idea.

Hadar and Butterworth (1997) stated that body movements generally tend to increase when hesitation in speech occurs and hence Roth and Welzel (2001) suggested that the points when students use gestures to express themselves can be used as opportunities for the teacher to supply appropriate terms or expressions related to the subject at hand. They stated this in relation to scientific words but I suggest that the same argument can be made for mathematics.

3.3 The importance of mathematical expression

The talk used in a mathematics classroom is likely to consist of a mix of ‘everyday’ and ‘mathematical’ vocabulary such as *shape*, *angle*, *graph*, *axis*, *twenty*, *multiplication*, *addition*, *length*, *metre* and so on. According to Harvey (1982), ‘technical’ language is not always essential and pupils may very well use informal language to express themselves. For example, a child might call an *angle* a *corner*, or refer to the *perimeter* of a shape as the *outside line*. However, Harvey (*ibid*) also stated that more technical language is convenient, since standard words or expressions increase the potential of more effective communication with others in, and beyond, the immediate classroom, and also reduce the chances of ambiguities. For instance, Kerslake (1982) gave the example of a teacher’s idiosyncratic expression ‘park the 1 in the garage’ to signify regrouping in addition. Although helpful in the context it was used, Kerslake suggested that the expression may not be easily transferred when children move up a Grade. Hence it is useful that teachers help pupils to use more conventional language (Miller, 1993) which, according to Pimm (1995), allows us to talk about things and to ‘point’ with words. This argument also holds for Maltese children who learn through a mixed code with mathematical words retained in English.

According to Pimm (1987), learning to speak mathematically implies learning to *mean* mathematically. Hence, I consider that the teaching and learning of mathematical vocabulary to be an integral part of teachers' and pupils' participation in classroom mathematics. In Lave and Wenger's (1991) terms, I can consider mathematical language as part of the experience of the 'community of (mathematical) practice'. According to Lave and Wenger (*ibid*), opportunities for new practices to develop occur when a person participates with others in an activity. For example, when a child is out shopping, he or she may become aware of the practices of comparing prices, counting change or using certain expressions. In this situation, learning is considered to be an 'apprenticeship' where a novice learns a particular practice through participation with more 'expert' people. Lave and Wenger's (*ibid*) original theory was developed by the authors from research projects carried out with for example, midwives, tailors and butchers. In her later writing, Lave (1997) acknowledged that it is not generally possible to map all the features of a traditional craft apprenticeship onto other examples of situated learning and wrote about 'apprenticeship forms of learning' instead (Lave, 1997, p.19). Indeed, Adler (1998) commented that applying the idea of apprenticeship to classroom practice is particularly challenging since it is not so easy to identify the practice that is in focus. Adler (*ibid*) asked whether the practice is *teaching* (what the 'expert' is doing), *learning* (what the children are engaged in) or *mathematics* (the discipline at hand). She concluded that school mathematics is a hybrid practice shaped by both the discipline of mathematics and its applications to the curriculum.

Indeed, in some ways, the apprenticeship model is not completely applicable to my interest in learning and using mathematical language. A traditional apprenticeship implies starting and end points, in between which the novice progresses from a 'green' position to being as skilled as their master or mistress (albeit, lacking his or her extensive experience). With respect to knowledge of mathematical words, I suggest that beginning and endpoints are not identifiable, since mathematical words carry layers of meaning (Roberts, 1998). For example, a young child may perceptually associate the word *triangle* with a particular shape; an older pupil may be aware of the properties of the shape or the relationship of a triangle with other shapes; later still, *triangle* may be understood in terms of Pythagoras' theorem and so on. This developing knowledge may be accompanied by the use of increasingly more complex language regarding triangles.

Furthermore, a traditional apprenticeship implies a certain degree of modeling but this idea is not so easily applied to mathematical language. Language in general serves to create a temporarily shared social reality that Wertsch (1985, p.59) referred to as ‘intersubjectivity’. Meaning is what is agreed upon - or at least accepted - as a working basis (Bruner, 1986) and therefore language is flexible. This implies that there exists the potential for the ‘same’ idea to be expressed differently, that is, in different semantic terms (Chapman, 2003). Hence the idea of modeling implied in an apprenticeship needs to be widened to include appropriate variations of language and an acknowledgment that meanings may vary.

On the other hand, as Adler (1998) noted, the classroom practice of mathematics involves novices gaining control over the resources present in the practice, and mathematical language can be considered a resource that can enable fuller participation in the practice. This may be in the sense of overt communication and also in the sense of coming to understand the language of the discipline. Another aspect of the apprenticeship analogy that I find useful is that it highlights the expert/novice relationship between the teacher and the pupils. It is generally assumed (by employers, colleagues, parents, children and indeed, myself) that a teacher possesses more subject knowledge than the pupils and thus is expected to guide the children through their learning experiences. The teacher can guide and challenge ‘novices’ towards increased involvement that, according to Rogoff (1995) does not necessarily imply action or spoken interaction, but can also include observation and listening.

3.4 Helping pupils to learn mathematical vocabulary

Rothman and Cohen (1989), Earp and Tanner (1980) and Zaskis (2000) suggested that mathematical vocabulary needs to be taught explicitly. Indeed, Capps and Pickreign (1993) recommended that teachers present mathematical words in the four language forms - listening, speaking, writing and reading - and suggested that teachers should emphasise pronunciation and word meaning. Capps and Pickreign (*ibid*) suggested that this is particularly useful for mathematical vocabulary that may not be used outside the classroom.

Hatch and Brown (1995) suggested five essential steps in vocabulary learning in general: encountering new words, getting the (aural and oral) word form, getting the word meaning, consolidating form and meaning, and finally *using* the word. Hatch and Brown (*ibid*) admitted that the final step is not necessary if all that is desired is a receptive knowledge of the word. However, they argued that if a teacher’s goal is to help learners move as far along the continuum of word knowledge as they can, then word use is essential. Hatch and Brown (*ibid*, p.390) suggested that this provides “a mild guarantee” that words and their meanings will not fade from memory.

It is not always easy for a teacher to decide when to insist on language that is more mathematical, and Pimm (1987) even warned of the danger of over-emphasising form (sentence structure etc.) over free expression of meaning. Adler (2001) called the related dilemma one of mediation: On one hand a teacher would wish students to express themselves comfortably and freely, while on the other hand, they may wish to intervene to guide the students to more effective communication. Furthermore, Adler (*ibid*), drawing on Lave and Wenger (1991), talked about a dilemma of ‘transparency’ that can arise when teaching mathematical expression. On the one hand, a teacher may wish to focus explicitly on language thus making the mathematical language ‘visible’, while on the other hand the teacher needs to ensure that the language is available enough to the students to allow them to talk about ideas. In the latter situation, the language would be ‘invisible’ in the sense that students ‘see mathematics through it’ as one would look through a window. Adler’s interpretation was based on multilingual classrooms in South Africa, but she recommended awareness of the in/visibility balance for monolingual classrooms too.

One of the difficulties that might arise for pupils when moving from everyday to mathematical language is the problem of ambiguity of meaning. One instance of this is when a word can have two completely different meanings in the different contexts. For example, some mathematical words may have ‘ordinary English’ meanings that children may already be familiar with from their everyday lives, an interpretation they may then give to the mathematical word. These are words like *odd*, *table*, *volume*, *net*, *record*, *left*, *product* and *relation* (Clemson and Clemson, 1995; Kerslake, 1982; Orton, 1992). Confusing meanings is not restricted to young children. Monaghan (1991) reported how A-level students studying limits in their calculus course found difficulty with interpreting the expressions *tends to*, *approaches*, *converges* and *limit*. The author stated that while these terms may be used interchangeably in mathematics, the students’ previous knowledge of the words influenced their mathematical interpretation. I conjecture that ambiguity for Maltese pupils might arise only if the pupils already know the everyday meaning. If the child does *not* know the everyday meaning, then *net* and *odd* may be as ‘mathematical’ as *quadrilateral* or *graph.*

Other types of lexical ambiguity are also possible. Durkin and Shire (1991) stated that some words have *related* meanings in everyday and mathematical contexts. This is known as polysemy and is common among spatial terms. For example, we may say “the numbers are going up” or “eight is higher than five” where the meaning is not in fact spatial, but concerned with number relations. Hence, when asked to write a ‘big’ number on the whiteboard, a young child may write in a very large script or when asked to write a ‘higher’ number, he or she may write in a higher position on the whiteboard. Durkin and Shire (1991) recommended that teachers be aware of ambiguous words and confront this ambiguity explicitly. Orton (1992) suggested that one way of doing this is to write down the words by category: unique to mathematics, not unique and subtly different in meaning, and not unique and very different in meaning.

Another possible difficulty is when two words sound the same but have different meaning, for example, *sum/some* or *two/too/to*. This aspect of language is called homonymy. A shift in application can also cause difficulty. This is when a word can be considered from different perspectives, such as when a number is considered in its nominal, ordinal or cardinal meaning (Durkin and Shire, 1991). Yet another possible source of ambiguity is when different words can be used to describe the same mathematical symbol. For example the symbol ‘=’ can be expressed as *equals*, *means*, *makes*, *leaves*, *the same as*, *gives*, and *results in*. Furthermore, Rothman and Cohen (1989) and Hanley (1978) drew attention to synonyms, or words with similar meanings. In particular, Hanley (*ibid*) stated that for example, *sum* and *total* are both used to refer to the same additive operation. Finally, Anghileri (1991) stated that differences in meaning of expressions used in word problems may be quite subtle, as in making a distinction between *shared by*, *shared among*, *shared between* and *shared out*.

Learning how to ‘speak mathematically’ may pose particular problems for children learning through a second language. For example, the lexical difficulties outlined above may be even more problematic for these learners. Olivares (1996) commented that synonyms such as *add*, *plus*, *combine*, and, *join* may be a source of difficulty to students with limited English proficiency in the U.S., Garbe (1985) reported on the difficulties that Native American Navajo students had with mathematical terms having everyday meanings, and comparison terms such as *greater* / *less than*. Furthermore, these students also mixed up mathematical words with similar sounding everyday ones, such as *sum*/*sun*. Garbe (*ibid*) suggested that their teachers may not have used the vocabulary often enough or enunciated them well enough for the students to detect slight differences in pronunciation. Similarly, writing within the Australian context, Miller

(1993) recommended clear pronunciation so that Aboriginal students do not mix words such as *size* and *sides*, *ankle* and *angle*. The points related to possible ambiguities may very well be applicable to Maltese children in an immersion setting. Indeed, they may even be applicable in a setting where Mixed Maltese English is used, since even here, pupils tend to experience mathematical vocabulary in English.

Brodie (1989) and Cuevas (1991) recommended that second language learners experience systematic support, in the sense that the teacher should plan specifically to focus on mathematical language. In practice, it is not possible for a teacher to concentrate on language at all times, but Gibbons (1998) suggested that this might be done occasionally and with purpose. Writing about science education, Gibbons (*ibid*) suggested that a teacher's intention of focusing on language should be made explicit to the students by stating 'we're going to talk like scientists' and 'your language has to be precise' or by evaluating children's responses in terms of the language used ('you explained very well'). Students should "focus not only on what they wish to say, but on how they are saying it" (Gibbons, *ibid*, p.103-4). Such a recommendation is equally appropriate within a mathematics classroom.

Some teachers and researchers have described specific classroom practices. For example, Appleby (2003) described her method of introducing new vocabulary to her Grade 3 ethnic minority pupils, whereby she encouraged them to identify such words in problems and then practice the words by writing out their own simple problems. Other useful classroom tasks suggested by MacGregor (1993) included card games, fill-in-the-gap exercises, text reconstruction and picture dictation (that is, describing a picture). Campbell (1986) described how a teacher in the Philippines helped her Grade 6 pupils talk about mathematical ideas in English, when English was not the first language for either the teacher or the pupils. Campbell noted that the teacher gave a lot of attention to the key mathematical terms, verb phrases and sentence structures which eventually came together in longer utterances. Similarly, Moschovich (1999) reported how a teacher helped his Hispanic students in the U.S as they offered informal or incomplete mathematical language. This teacher focused on the mathematical content of the students' contributions, asking them for clarifications, and accepting and building on them by re-voicing statements. Finally, van Eerde and Hajer (2005) designed activities for immigrant 12 to14-year-olds in Holland with the specific aim of addressing both mathematical ideas and language concurrently.

The U.K. Department for Education and Skills (DfES, 2002) provided guidelines for classroom teachers regarding how they can help second language students to understand and use mathematical expression, and also how to support reading and writing in mathematics. If I contrast this document to our NMC document, I note that the NMC's attention when recommending the use of English for mathematics is on the English language, which the writers seem to assume, will be learnt over time. No attention is given to either mathematical language itself nor indeed, to the relationship between language and mathematical practices. Although these aspects are, perhaps understandably, outside the scope of a national curriculum document, I think it is useful for us, as local educators, to turn our attention in this direction.

3.5 Further considerations of what renders language 'mathematical'

Up to now I have considered mathematical language in terms of vocabulary. However, two qualifications are necessary. First, it is not possible to say that a stretch of speech either is, or is not, mathematical. According to Chapman (2003), the idea of language being 'more' mathematical is related to the *gradual* move from informal to formal language and Chapman (2003) offered a way of considering this shift by considering two overlapping continua. Drawing on Hodge and Kress' (1988) notion of modality or degree of certainty, Chapman (*ibid*) suggested that the higher the modality, the more mathematical a statement sounds. For example, she considered the following utterance to have high modality: "What angle does the minute hand of the clock pass through?" (p.122). In this question there is implied that the angle exists, the hand passes through it, it always happens, it is a fact. Similarly, "So eighteen divided by three is six" (*ibid*, p.143) implies certainty. A statement of *low* modality is "Does it make sense to say three is a multiple of one?" (p.160) since it indicates uncertainty.

Then, drawing on Walkerdine (1988), Chapman also considered a continuum that transforms from metaphor to metonymy. Walkerdine (1988) had shown how mathematical meanings are developed through a series of shifts from focusing on particular everyday objects to more general relationships. Chapman (2003) suggested that the *less* metaphoric and the *more* metonymic elements present in a statement, the more mathematical that statement is. For example, the question "Thirty cents is what fraction of a dollar?" (p.113) contains the metaphoric element of money which provides a context for the fractional relation, while "Find the value of two to the power of five" (p.113) operates in a purely metonymic way. Chapman (*ibid*) overlapped the two continua as shown in Figure 3.1.

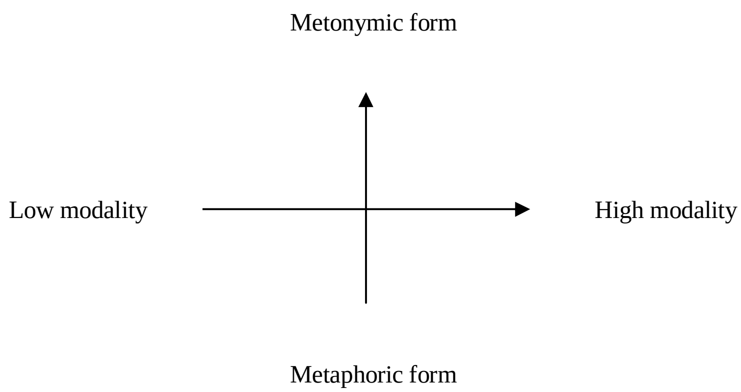


Figure 3.1 Language shifts in mathematics (Chapman, 2003, p.128)

According to Chapman (*ibid*), considering mathematical language involves considering shifts along these related continua. However, I am aware that at primary school level, much of the work carried out is metaphoric in nature. So, for example, multiplication is represented through repeated sets of buns, or fractions are considered through pizza slices and so on. Therefore, I conjecture that although perhaps various degrees of modality may occur in a primary classroom, the majority of situations may be close to the metaphoric end of the continuum. Hence, I may not be able to use Chapman's model in an effective way to discuss how mathematical the talk is, and I prefer to gauge this aspect by considering the inclusion of mathematical vocabulary in a stretch of interaction.

However, even here, a qualification is necessary regarding how to recognise particular vocabulary as being 'mathematical'. My experience with the subject allows me to distinguish between say, *rectangle* as a mathematical word and *pencil* as an everyday one. However, I also note that sometimes the distinction between an everyday and mathematical word may not be so clear-cut. I realised this in a discussion I had with a teacher, in fact, during the pilot study I carried out prior to this study. At one point in a conversation, I suggested that *coin* was a mathematical word. The teacher disagreed, arguing that, by virtue of its everyday familiarity, it was not mathematical. This prompted me to think about what it was that rendered a word mathematical, since like *coin* I could mention *long*, *minute* and others that are often used in everyday life. I realised that I had considered *coin* to be mathematical because of the context that the teacher and I were considering, that is, a weeks' lessons focusing on the topic Money, during which key points to be addressed were coin recognition and value. Thus perhaps I can say that rather than a word being mathematical *per se*, it might or might not be defined as such depending on the function that it plays in a given context. The everyday words *coin*, *long* and *minute* may be considered ‘everyday’ ones in many contexts, but in a mathematics lesson that focuses on money, length or time, the words may be rendered key mathematical words because of their significance for the topic at hand.

It is interesting to note that for some ‘clearly recognisable’ English mathematical words or expressions such as *graph*, *square root* and *sine*, no Maltese translations appear in the most comprehensive Maltese dictionary available (Aquilina, 1999). In cases where translations do exist, such as in the case of *multiplication*, *axis* and *perpendicular*, it is still the English one that tends to be used in class. Furthermore, although translations exist for English words that are often used in everyday situations such as *coin*, *long* and *minute*, again these are likely to be used in English in the classroom. Thus, in classrooms in which code-switching is used, I might tentatively say that how ‘mathematical’ the language is, might be gauged by the presence of the English words. I conjecture that the practice of retaining the English version may actually draw attention to the mathematical word by virtue of it being in different language. Thus, code-switching may serve a further role to those identified by Camilleri (1995) and discussed in Section 2.4, namely that of highlighting mathematical vocabulary.

3.6 Mathematical vocabulary as part of a ‘register’

Halliday (1978) referred to the way of talking within, or about, a particular context as a ‘register’, which he defined as:

> “A set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings” (p.195).

Halliday (1978) explained how vocabulary is added to a language to express mathematical ideas. I now summarise his key ideas in relation to English, and also comment on the spoken Mixed Maltese English mathematics register in order to elaborate further on what the NMC writers may have found unsatisfactory. I base my comments on personal experiences since no large scale studies of the mixed mathematics register have been carried out and therefore it is not possible for me to talk in terms of established patterns of speech.

<u>*Newly-created words.*</u> Some mathematical terms such as *quadrilateral* and *parallelogram* are made up out of Latin and Greek elements, even if the actual word did not exist in the original languages. Halliday (1978) noted that this method of creating words is the most typical procedure

in contemporary European languages for the creation of new technical terms, and that these terms are not normally used in everyday situations. In Maltese classrooms, the English version of these terms is always used, although Maltese equivalents might be found in a good dictionary (e.g. Aquilina, 1999).

<u>Borrowing of terms</u>. Sometimes terms are borrowed from ordinary English, but they are given a different meaning when applied in a mathematical context. Examples are *volume* and *net*. Words may also be borrowed from another language, as in the case of *infinite* and *probable*. In Malta, we borrow the same English terms. Indeed, it is rather common practice (even outside the classroom) to say numbers themselves in English, even though Maltese equivalents exist and are commonly known. However, as Camilleri (1995) pointed out, words borrowed from English tend to be partially assimilated since the pronunciation is usually a bit different, shaped by Maltese phonology. One such example is the word *graph* which a Maltese speaker would pronounce as *gruff*. For some words, whether the English or Maltese version is used depends on the context. For example, *longer* and *shorter* might be used in class, while **itwal** (*longer*) and **iqsar** (*shorter*) used in other situations.

<u>Calquing</u>. Another feature noted by Halliday (1978) is that of calquing or the creating of new words in imitation of another language. Although rare in modern English, Halliday (*ibid*) suggested that this process is a regular feature of many languages. A key area of vocabulary where this has happened in Malta is verbs, which have been adapted to Maltese morphological (word formation) structures. For example, *to label* is **illejbilja** (pronounced *il-label-ya*). This word and many like it feature in dictionaries and have become a generally acceptable feature of spoken and written Maltese. Camilleri (1995) referred to such words as loan shifts and found evidence of them in classrooms. In the mathematical context, I can mention as examples *to plot* which becomes **ipplottja** (pronounced *ip-plot-ya*) and *to construct* [a triangle] which becomes **ikkonstraktja** (*ic-construct-ya*). An interesting example that illustrates a move from ordinary Maltese to mathematical English is the verb *to share*. The Maltese word for *share* is **qasam** which is very commonly used in everyday life. However, in classrooms, it is possible to find both teachers and children using the loan shift **ixxerja** (pronounced *ish-share-ya*) instead. This word serves the apparent purpose of linking with written English.

<u>Locutions</u>. According to Halliday (1978), the language of mathematics also contains specialised locutions or whole expressions that function as semantic units on their own, such as *square root*

and *right angle*. Shuard and Rothery (1984) gave further examples such as *simple interest*, *pie chart*, *square root*, *closed figure* and pointed out that the meaning of the expression is not simply the sum of the parts. In the Mixed Maltese English register, such expressions are generally retained in English, so that a teacher / child might say: "**Għalija din tidher li hija** right angle" ("**This looks like a** right angle **to me**"). However, Pimm (1992) stated that the aim of mathematical language should be to help construct, express and communicate mathematical meanings, and this certainly cannot be done using isolate words or expressions. Indeed, Halliday (1978) himself stated new styles of meaning and the combining of existing elements into new combinations also play a part, as in the examples 'teacher-to-pupil ratio' and 'the sum of the series to *n* terms' and "*if and only if*" and "*if … then*" as forms of argument. In Malta, these expressions are generally retained in this same form.

Pimm (1987) also identified more subtle points regarding mathematical expression. For example, there may be a subtle change in meaning from everyday ones, as in the case of the word *any*, which in a mathematical context means *every;* a word's grammatical category may change, as in the case of number names *one*, *two* …etc., which may serve as nouns in a mathematical context while in ordinary English they serve as adjectives (e.g. 'two birds'). Pimm (*ibid*) gave an example of a student who interpreted the term 'diagonal' as an adjective in the sense of a sloping line rather than understanding it in its mathematical nominal sense - *a* diagonal. Mathematical language may also include differences in the use of prepositions and grammatical connectives, as when referring to 'the area *of* a triangle' rather than '*in/inside* a triangle'. These subtle changes in meaning will also be experienced in our local classrooms when we borrow the English expressions.

It is important to state that a mathematics register can also be written. Indeed, written mathematical English in general needs attention in its own right, even for first language speakers, since it varies from ordinary written English. Kane (1968) noted that letter, word and syntactical redundancies are different and the grammar and syntax of mathematical English are less flexible, while Morgan (1998) listed characteristics of a written register as the presence of symbols, specialist vocabulary and conciseness. Morgan (*ibid*) also mentioned certain grammatical structures that appear to render language more mathematical: the use of imperative ('draw a diagram'), nominalization (i.e. changing a process like *to rotate* into a noun – *rotation* - thus creating a mathematical 'object'), the use of the passive structure ('a line is drawn') or the use of the more formal 'we' rather than the personal 'I' ('we draw a line' rather than 'I draw a line').

Styles of writing may also indicate deductive reasoning by including words like ‘hence’, ‘therefore’, ‘by Theorem 1’ and so on. Furthermore, Morgan argued that the presence of elements such as diagrams, tables, labels and even neatness contribute to giving an impression to a reader of a written text being more mathematical.

Halliday (1988) observed that as a new register evolves, it gains value by virtue of its being functional. However, the requirements of expressing mathematical meanings can place strains on a language and, according to Pimm (1991), it is this strain that results in new ways of expression. This was evidenced in the Welsh, Māori and Aboriginal registers, which have only recently been developed as part of a wave of language revival. In Table 3.1 overleaf, I outline the development of these registers. As can be seen from the outline given, deliberate register development is not without difficulty, especially when familiar words begin to be used for mathematical purposes (Halliday, 1978).

	Background to Register Development	Some General Consequences	Classroom observations
Welsh (Jones 1997, 1998, 2000; Jones and Martin-Jones, 2004)	Bilingual education started in the late 1930s as part of the rise of nationalism in Wales and opposition to the dominance of English. Welsh medium mathematics eventually became available and children are today offered the opportunity study mathematics through either Welsh or English (including exams). In order to achieve this aim, intensive work was started in the late 1970's on dictionaries of mathematical terms and textbooks in Welsh.	Some difficulties in assessment procedures: (a) sometimes the English word is more familiar than the Welsh one, even to Welsh speakers; (b) a Welsh term may convey the meaning of the notion more explicitly than the English counterpart, favouring students who opt for the Welsh version of the exam (e.g. *pedrochr* (quadrilateral) literally translates to 'four sides'); (c) a Welsh word may fail to capture the necessary scientific sense (e.g. *cyflymder* which means speed rather than velocity); (d) a situation presented in Welsh may not be 'realistic', since such a situation would always be experienced through English in real-life.	In five secondary classrooms observed by Jones (2000) code-switching was common, since students were not grouped by language choice. Welsh main language for classroom management, English used for calculations / numbers; explanations relating to concepts often offered in both languages, reiterated in English if they were initially expressed in Welsh and vice versa; less code-switching used for teacher-student individual interaction; worksheets either prepared in two versions back-to-back or if sheet contained only a little text, two versions of a statement where given one beneath the other.
Māori (Barton *et al* 1998)	The process of developing a Māori mathematics register started in the mid-1980's and formed part of a drive to revive the language that had started in the previous decade. Register developed systematically by specially appointed persons. By 1996 two mathematics dictionaries had been produced.	Expression moved towards English modes and conventions in order to articulate Western mathematical concepts (which may differ from traditional ones) and changes have occurred in the traditional grammatical nature of some words. Meanings themselves may have change due to words having been borrowed from English. (e.g. *whakaruau* - multiplication - traditionally meant to multiply in the sense of pro-create). On the other hand, some words carry the mathematical meaning more effectively than their English version, as in the case of *tapamaha rite* (regular polygon) literally meaning 'many equal.	In some classrooms, both English and Māori registers are used. Code-switching and forced re-phrasing appears to be beneficial to students since two modes of thought are available.
Aboriginal (Roberts, 1998)	School mathematics is entrenched in Western ideas, but mathematical ideas are talked about in Aboriginal languages outside school. The various Aboriginal mathematics registers are still under development and choices will have to be made regarding (a) how to blend world views, (b) changes in language.	Various difficulties that need to be resolved: (a) differences in world views, e.g. the Aboriginal tendency to view objects in terms of their relationship to this object, rather than in its own right; (b) the rare use of logical connectives ('if… then'); (c) use of verbs for processes rather than nouns as is case in English; (d) varying use of metaphor across languages.	Teachers are teaching mathematics and discussing it with their students in their own language and in English. They feel that children understand better in their own language. On the other hand, some ideas are difficult to express in their language, so teacher uses an English expression itself. This results in code-switching.

Table 3.1. The recent development of the Welsh, Māori and Aboriginal mathematics registers

In her reflections on Aboriginal mathematics registers, Roberts (1998) stated that these are still under development (there are about 90 distinct Aboriginal language in Australia), but she believed that there will come a time when community decisions will determine what 'sounds right'. Perhaps the dilemma expressed in the NMC is a reflection of a sense of uncertainty about how appropriate the language form used for mathematics is; perhaps we too are at a stage when we need to collectively reflect on what 'sounds right'.

One point I would like to highlight is that both the English and Mixed registers that I have considered in this chapter are *western* registers, in the sense that the words included in the register denote western ideas. Bradford and Brown (2005) noted the difficulties Bradford herself encountered when using an English register with some teachers in Uganda. A problem cropped up with the use of the word *circle* not because of the nomenclature as such, but because the teachers did not share Bradford's concept of a circle, being culturally oriented to thinking in terms of *circular* instead. Differences in concepts or views of the world were also noted for the following peoples by various researchers: Aborigines (Australia) by Meaney (2005), Māori (New Zealand) by Barton *et al* (1998), Navajo (U.S.) by Mellin-Olsen (1987) and the Yoruba (Nigeria) by (Morris, 1974, cited in Orton, 1992). With regard to the Maltese situation, I think I can safely assume that we share 'the same' western concepts implied in the mathematics words expressed in English.

Furthermore, while I have argued that teachers should try to encourage pupils to use mathematical language, it should be acknowledged that sometimes, participating in the use of a register can be problematic. For example, describing a classroom context in Catalonia, Spain, Gorgorió and Planas (2001) noted how the home culture of minority students interfered with desired patterns of participation. The authors gave as an example a student whose values suggested that it was impolite to tell her teacher that she could not understand, or to talk to friends while her teacher spoke; another girl would not communicate with the boys in the class since this would land her in trouble with her family. Hence in these cases, the students' values impinged on the use of talk in general. Again, such situations may not *generally* occur in Malta, although of course I cannot exclude the occasional possibility.

In this section, I have focused on mathematical vocabulary, but Halliday's (1978) notion of register goes beyond the consideration of the inclusion of mathematical word, and in the following section I look at more detail at his definition in relation to my study.

3.7 The three components of register

Halliday's (1973, 1978, 1985) view of language was a functional one, whereby he believed that people put language to use according to the situation in which they find themselves. Thus, context is a crucial notion in the consideration of register. Researchers have defined context in different ways, for example, Rogoff (1984), considered that a context for an activity is constituted by the activity's physical structure, the purpose of the activity and the social milieu within which it is embedded; Lave (1997) suggested that activity is its own context, implying that context is flexible and changing. Halliday's (1978, 1985) own definition of a 'context of situation' includes three elements. He referred to these as:

- the *field of discourse,* or the activity that the participants are engaged in;
- the *tenor of discourse* or the set of relationships between the participants; and
- the *mode of discourse* or the role language plays, that is, whether is it spoken or written, spontaneous or rehearsed, whether its purpose is to persuade or to explain and so on.

These three contextual elements then determine what Halliday (1985) referred to as a 'text' which is:

> "any instance of living language that is playing some part in a context of situation" (*ibid*, p.10).

Within a text, Halliday (1978) identified three functional meanings which are the respective realisations of the context's field, tenor and mode. These are:

- *ideational meanings*, which express categories of experience;
- *interpersonal meanings*, which express social and personal relationships; and
- *textual meanings* which make the language "operationally relevant" (Halliday, 1973, p.42)[3].

[3] Strictly speaking, in his 1978 publication, Halliday subdivides the ideational element into two: the *experiential* component which is the content function of the language and the logical component that expresses *logical* relationships such as 'and', 'if - then' and 'or'. However, in his 1985 publication, the author lists the four components separately as follows: experiental, interpersonal and textual (as realizations of the field, tenor and mode respectively) and the logical component which is built into natural languages and serves to link the experiential and interpersonal elements.

As an example to illustrate the idea of context and its resulting text, Halliday (1985) considered a radio talk given by an Archbishop. He identified the field as the maintenance of an institutionalised system of beliefs and the members' attitude towards it. Hence, ideational meanings in the spoken text were carried by words that functioned as names (e.g. 'Christian'), metaphorical expressions ('to defend'), and also processes that were being talked about ('to take seriously', 'to question'). The tenor related to the institutionalised relationship of authority to an unseen and unknown audience, and thus the interpersonal meanings were reflected in expressions such as 'I ask you …', 'let us consider'. Furthermore, Halliday suggested that the use of imperatives ('the Christian should') and declaratives ('three motives have impelled men') also contributed to interpersonal meanings by setting a 'mood' of authority and specialist. Finally, in Halliday's example, the mode was that of a written text to be spoken aloud, carefully prepared as a rational argument. The nature of the language - spoken but not spontaneous - and the rhetorical function, were reflected in the resulting textual meanings characterised in the text by simple grammatical structures and conjunctives such as 'therefore', 'in turn', 'first/next' which created a kind of cohesion in the talk.

The functional meanings are also identifiable in *written* texts as illustrated by Morgan (1998) in her study of secondary school students' written examination texts. As part of the ideational aspect, Morgan (*ibid*) considered words that expressed different processes, and suggested that these indicated what students thought mathematics was about. For example, a student's use of statements such as: "If you add … then multiply … you get …" (Morgan, *ibid*, p.80), reflected a view of mathematics as coming into existence through human involvement. In Halliday's (1976) terms, this indicates an action process, with a participant clearly present. On the other hand, Morgan suggested that the nominalisation of processes (e.g. *rotation* and *permutation*) and the use of symbolisation, obscured human presence and hence portrayed a picture of mathematics as an autonomous system. In this case, the processes implied were relational ones (Halliday, *ibid*). With regard to interpersonal meanings, Morgan suggested that, for example, an expression such as "We shall show that …" inferred the authority of a community, while "For my extension I am going to …" provided a more personal development (Morgan, *ibid*, p.84). Finally, Morgan interpreted the textual function in terms of the writing styles used by students such as narrative, description, explanation and so on. For example, she considered that a phrase such as "there is one definite pattern …" (*ibid*, p.113) formed part of a descriptive report, while "… so if I multiply …" (p.113) formed part of an argumentative style. Morgan (1998) admitted that it is difficult to define a mathematics register precisely, and indeed there exist more than one mathematics register. For example, the language used in a primary classroom is different to that used in an A-level class; the text found in an infants' workbook is far removed from an academic research paper. And yet, all four examples could classify as making use of a mathematics register.

In my own study I will not be reflecting in detail on the elements of a context or the functional meanings of a text, but I am aware of their relevance to my interests to the extent as follows:

- *I ask how much pupils talk* (cf. question 2a, p.15). The relationship between the teacher and the pupils (tenor) may determine a particular style of interaction, and hence I might consider the actual length of pupils' contributions as fulfilling an interpersonal function.

- *I ask with what ease (i.e. how coherently) pupils talk in English* (cf. question 2a). This textual component may be considered to be the realisation of the purpose to which language is put (mode). For example, whether language is used to offer brief answers, explanations, investigative strategies etc.

- *I ask how frequently mathematical words are used by pupils* (cf. question 2b). Mathematical vocabulary expresses ideational meanings; word use is determined by what the talk is about (field).

The notion of field can also support my third research interest, which is to reflect on sharing of meaning of mathematical words with pupils (cf. question 3, p.15). For reasons I will explain in the following chapter, I planned to interview some pupils after their lessons in order to ask them for the meanings of words. I also planned to compare their responses with what had been expressed in class. I realize that, in an interview, the register used by the pupils is likely to be different to that which they use in class. First, interpersonal meanings expressed are different since the relationship between the interviewer (myself) and the pupil is different to that between the teacher and pupil. Furthermore, textual meanings vary, since the role the pupil's language plays is also different (explicitly and systematically explaining meanings for words). However, the *field* of talk at certain points in the lessons and during the interview can be considered 'common', since in both situations the talk is about, say, perimeter. Hence, ideational meanings expressed in the two contexts can be compared. I will continue to discuss sharing of meaning in more detail in Chapter 4.

3.8 Conclusion: language as medium and message

My research questions included three elements: immersion issues, the use of mathematical vocabulary within stretches of talk, and sharing of meaning. Initially, I was concerned that dealing with all these aspects within one project might prove to be impractical or even inappropriate. Yet, I was reluctant to narrow the focus: the immersion issue was very topical and, as stated by Torbe and Shuard (1982), the role of language for communicating is inextricably bound up with the role it plays in conveying meaning. Furthermore, it seemed important to me to address the various aspects since, as explained in Chapter 2, I hoped that my study would serve as a seminal one on language in Malta that might encourage interest in language among colleagues beyond the English/Maltese dichotomy. (The only local studies that I am aware of that focus on technical vocabulary are the quantative ones investigating English *scientific* vocabulary carried out by Ventura (1991) and Farrell and Ventura (1998). For the reader's interest, these studies are summarised in Appendix A).

The notion of register itself provided the means for me to bind the three research interests together coherently. By viewing register as Pimm (1987) suggested, that is, as being simultaneously a 'medium' and a 'message', I was able to present my discussions within one encompassing perspective. As shown in Table 3.2 overleaf, this approach helped me to shift my attention smoothly from the use of English to focusing on mathematical language. It also allowed me, through the mathematical words themselves, to shift from medium to message: while the inclusion of the words was part of the overt medium, the apparent meaning ascribed to them as they were used constituted a particular message.

<table>
<tr><th colspan="2">The use and development of an English mathematics register in Maltese primary classrooms</th></tr>
<tr><td>Language
as a
medium</td><td>(1) Reflections of immersion recommendation in the light of other NMC principles.

(2) Reflections on the English register:
(a) General language: extent and cohesion of talk
(b) Mathematical aspect: frequency of mathematical words.</td></tr>
<tr><td colspan="2">Shifting the focus from medium to message through mathematical words:
frequency of use → meanings expressed</td></tr>
<tr><td>Language
as a
message.</td><td>3) Sharing of meaning: conditions that appear to be helpful for sharing of meaning (across various topics).</td></tr>
</table>

Table 3.2. Binding my research interests together

In the following chapter, I explain my methodology, offer a model to help me address ‘meaning’ and show how my assumptions and beliefs guided me towards a suitable research design.

CHAPTER FOUR

Methodology and an Analytic Tool for 'Meaning'

4.1 Introduction: The roots of my interest in language

Having established my research interests, I needed to consider a suitable approach to addressing them. As suggested by Burton (2002), it is important that research methods are justified by the researcher's values, beliefs and attitudes. In this chapter I explain how my view of learning as a flexible, social activity guided me to conduct a qualitative case study. I start by going back to the root of my interest in mathematical language, since it is here that I recognise the first pointers for the approach I took.

My interest in language finds its beginning in a project I carried out some years ago as part of a Master's degree (Farrugia, 1995). In this project, I had explored children's understanding of fractions and had used as a theoretical base, the theory of Radical Constructivism as espoused by von Glasersfeld (e.g.1990, 1995). This neo-Piagetian position places the individual at the centre of learning and utilises a key notion of mental 'schemes' or structures which are modified or elaborated as the individual interacts with the environment. I had set some fraction tasks for nine children to do individually, and used their responses to describe a hierarchy of schemes. Following von Glasersfeld's view of reality, I had rejected the Platonic view that the external world exists as a given reality. Von Glasersfeld's ontological view of the world was that reality was not something external to be received passively by the individual. Rather, he stressed that knowledge was an individual construction and that the function of cognition is adaptive, and serves to organise experiences. Von Glasersfeld's (1996) view of the social realm was that the world and 'others' are for each individual what he or she perceives and conceives them to be.

In the course of the interviews I had carried out, I had noted that one pupil gave different interpretations for the English word *half* and its Maltese translation **nofs**. At the time it surprised me that a child might come to hold two different meanings for what seemed to me to be the same word. I supposed that the explanation lay in examining pupils' experiences in, and outside, the classroom but reflections in this regard lay beyond the scope of the study. The point remained forgotten for some years, until the publication of the draft NMC. The language recommendation included in this document brought the incident back to me and this sparked off the general interest in language that led to the present project.

4.2 A personal shift from a Constructivist to a Socio-cultural view of knowledge

For my Master's project, I had believed the individual to be at the centre of knowledge and considered language only in as much as it offered a "symptom of thought" (Sierpinska, 1998, p.32). I did not consider the experiences through which the interviewed children may have learnt what they expressed.

However, the Radical Constuctivist view has been criticized for its lack of adequate explanation of social and cultural practices (Cobb and Bauersfeld, 1995). Lerman (1994, p.43) also criticised the perspective on the grounds that within this perspective, language is "undervalued, or at least under-elaborated". Although 'social' Constructivist writers have attempted to give a more elaborate account of the role of social interaction (see, for example, Steffe and Thompson, 2000; Steffe and Tzur, 1994), still, Lerman (1994) suggested that by virtue of the centrality of the individual, the Constructivist view does not take into account how the 'social' impinges on individuals *without* their choice. Furthermore, he stated that Constuctivism does not tackle important issues such as what teacher-actions will actually prod the 'right' mental schemes, and ultimately the theory is unable to explain a great deal of human behaviour.

As I looked back at my Master's project and forward to the present one, I came to share Mercer's (2000a) belief that Constructivism does not handle well the complex social circumstances involving several people, as is the case of classroom situations. Yet my new research interests in mathematical language foregrounded the social element of learning. I now considered the classroom as a context where learning of mathematical vocabulary took place and was interested in examining the process of learning in detail; I considered the language used in the classroom as an integral part of learning how to participate in the community of (classroom) mathematical practice; Viewing mathematical language as a register, I valued the context-dependent nature of language use. This shift in view-point implied that although I continued to believe that reality is not an ontological 'given' and that knowledge is individual, my perspective regarding the way knowledge was actually constructed changed, resulting in an epistemological move away from Radical Constructivism. I felt that I needed a perspective that fore-grounded social aspects of learning, in particular, one that admitted the centrality of language. Hence, I turned to Vygotsky's writing.

Vygotsky's (1981a) fundamental tenet is expressed in the following quotation:

> "Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition" (Vygotsky, *ibid*, p.163).

As an illustration, Vygotsky (1978) offered the example of a young child who uses a hand movement to grasp unsuccessfully at an object. The child's mother may interpret the action as an indicator and she may intervene by say, picking up the object. The combination of the child's behaviour and the adult's response transforms a noncommunicative behaviour into a sign on an interpersonal plane and the child begins to understand the movement as 'pointing'. Thus he or she begins to gain voluntary control on the intrapsychological plane over what had formerly existed only in social interaction. The emergence of this voluntary control over what Vygotsky considered to be external signs forms is the general process involved in internalization or the 'turning inward' of communicative speech to become 'inner speech' (*ibid*, p.57). (This transformation is the result of a long series of developmental events; details of these stages can be found in Vygotsky 1962 and 1978). Wertsch and Stone (1985) explained that it is the mechanism of coming to recognise the significance of an external sign that makes possible the general acculturation and cognitive development of a child.

Generally speaking, a 'sign' is something that stands for something else in the sense of *X represents Y* (Tobin, 1990). Vygotsky (1981b) considered various things as signs including art, writing, diagrams, pictures, counting systems, algebraic symbols and even language itself. Vygotsky (*ibid*) considered these signs to be psychological tools, and drew an analogy between them and technical tools used at a work place. He argued that although the latter are externally oriented to cause change in the outside environment while the former are internally oriented and result in mental change, yet both types of tools mediate activity. Thus, using the idea of signs, Vygotsky attempted to link the external (the social plane) with the internal (individual plane).

Vygotsky (1962) considered concepts or word meanings that are learnt at school to be 'scientific' concepts. He distinguished these from 'spontaneous' concepts that he believed emerged from the child's reflection upon everyday experiences and which he suggested are not subject to conscious awareness. On the other hand, scientific concepts are mediated through other concepts with their internal, hierarchical system of interrelationships. Vygotsky argued that in school instruction, a word assumes a particular form of communication whereby word meanings are learnt as part of a system of knowledge. Thus these signs begin to function not only as a means of communication, but as the objects of the communication activity: through instruction, the child's attention is directed explicitly toward word meanings and their interrelationships.

This socio-cultural perspective of learning was particularly useful to me since it helped me to focus on the classroom interaction as a 'starting' place for learning mathematical words. It also allowed me to shift my focus from this social milieu to the individual, in order to be able to discuss 'sharing of meaning'. In the following section, I elaborate further on how the consideration of signs helped provide a theoretical foundation for my study.

4.3 Meaning through signs

Wertsch and Tulviste (1996) believed that Vygotsky's own interest in signs may have been derived from the writing by the contemporary linguist Saussure, who gave importance to the social aspect of word meanings. Saussure (1983) wrote about the links between concepts and sound patterns, as in the case of when the notion of a tree (the 'signified') is represented by the word *tree* (the 'signifier'). Saussure (*ibid*) considered such links as signs which, although arbitrary, come to be established within a community. However, according to Wells (1999), since Vygotsky's main interest was inner speech, his statements regarding language and culture were rather general. Wells (*ibid*) suggested that Vygotsky did not give details about the role that mediation plays in social encounters, both in terms of instantiating the culture and in modifying it.

For the purpose of my own study, I felt that I needed to address explicitly how signs relate to each other, since I anticipated that word meanings in the classroom would be conveyed through links with other words, diagrams, objects and notation. Hence, I turned to a theory of semiotics - that is, a consideration of sign systems - as espoused by Charles S. Peirce. I chose to work with Peirces' theory rather than Saussure's since the latter was mainly concerned with the relatively stable linguistic structures of language (Whitson, 1997). I wished to use a theory that took into consideration the "dynamic and productive activity of signs" (Whitson, 1997, p.99), and one that incorporated the idea of flexibility of individual meaning.

Peirce's sign relation was expressed as a triadic one and included the following elements: a sign or *representamen*, the *object* that the sign represents and the *interpretant* or interpretation[4]. Eco (1976, p.59) illustrated Peirce's epistemological view by means of the following diagram:

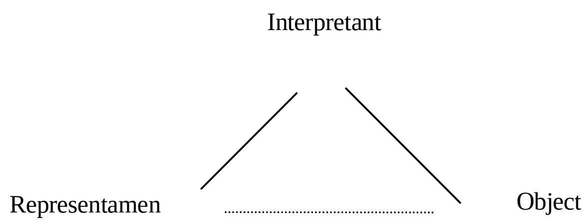


Figure 4.1. Peirce's semiotic model as shown by Eco (1976, p. 59)

In Hausman (1993), Peirce stated:

> "The essential nature of a sign is that it mediates between its Object, which is supposed to determine it and to be, in some sense, the cause of it, and its Meaning … the object and the interpretant being the two correlates of every sign … the object is the antecedent, the interpretant the consequence of the sign" (*ibid*, p.68).

The *representamen* corresponds to the common notion of a sign or symbol, or something used to represent something other than itself. Peirce identified three types, and these are explained in Greenlee (1973) and Rosensohn (1974): icons which resemble the objects (an example could be a picture of a dog to represent the real animal), indices which have the power to compel attention to an object although they do not resemble them (e.g. pointing or stating "there!") and symbols which bear no resemblance, but represent the object by virtue of interpreters' habits (e.g. the numeral *3* to represent three-ness).

With regards to the notion of an 'object', Hausman (1993) explained that Peirce did not use the word *object* in a realist sense. Rather, the object implies that there are external resistances or independent conditions that constrain interpretation. Hausman offered the example of using the utterance 'Abraham Lincoln' as a sign to refer to the historical person (the 'object'). In order to establish a relationship between the utterance and the person, the speaker must be acquainted with various aspects of the object such as the idea of presidents and that of referring to persons

[4] It should be noted that sometimes Peirce used the word *sign* interchangeably with representamen, while on other occasions he used it to refer to the whole triad. I use it in the former sense.

by proper names. These in themselves are not objective external realities, but can be considered as cultural units that, as Eco (1976) suggested, a human group decides to use and recognise as a vehicle for something else. Hence, Eco (*ibid*) considered the meaning of a term to be a cultural unit, which is "anything that is culturally defined and distinguished as an entity" (Eco, *ibid*, p.67) and I can include mathematical notions among such entities.

The third element of the triad is the interpretant that Greenlee (1973) likened to an 'interpretation'. According to Lidov (1999), the interpretant is the awareness of the object in the light of its representamen and thus the interpretant of a sign is what might be call 'meaning'. Eco (1976) provided the following conception of the interpretant:

> "The most fruitful hypothesis would seem to be that of conceiving the *interpretant as another representation which is referred to the same 'object'*. In other words, in order to establish what the interpretant of a sign is, it is necessary to name it by means of another sign which in turn has another interpretant to be named by another sign and so on. At this point here begins a process of unlimited semiosis, which, paradoxical as it may be, is the only guarantee for the foundation of a semiotic system capable of checking itself entirely by its own means… Thus the very definition of 'sign' implies a process of *unlimited semiosis"*(Eco, *ibid*, p.68-9, italics original).

Hence, although the model presented in Figure 4.1 appears static, Eco (1976) explained that a chain of interpretants may be viewed as continual shiftings which refer a sign (representamen) back to another sign.

I found Peirce's view of semiotics useful on three inter-related counts. First, the interpretant element allows for flexibility of meaning. Eco (1976) suggested that meaning is not fixed, but rather it is a possible interpretation by a possible interpreter and this offers a theoretical justification of why pupils sometimes express meanings differently to that offered by their teacher. As suggested by Sáenz-Ludlow (2004), if signs produced the same interpretant in all interpreters, communication in general and the teaching of mathematics in particular, would be a straightforward enterprise, offering no challenges to teachers, pupils and I might add, researchers.

Another strength of Peirce's theory was that it took into account the duality of objective/subjective knowledge and hence linked the social with the individual, an idea that was essential to me as I considered how meaning was shared in the classroom. Whitson (1997) stated that Peirce considered an interpreter as an external condition of the sign and not an essential internal constituent of the triad itself. Whitson (*ibid*, p.105) argued that for an individual, a representamen "appears within and against particular horizons and backgrounds". This accounts for the element of individual subjectivity within a social context and also accounts for the objective element that 'conditions' an interpretation. Vile (2003, p.43) suggested that since an interpretant becomes a 'thought sign' or a representamen in the mind of the interpreter, it is impossible to pin down the interpretant as being inside or outside one's head. Rather, it exists through an intertwining of the private and the public. Radford (2001) explained the double life of signs as follows:

> "On the one hand they [signs] function as tools allowing the individual to engage in cognitive praxis. On the other hand, they are part of those systems transcending the individual and through which a social reality is objectified" (*ibid*, p.241).

Third, the theory provided a way to engage in a discussion about 'meaning'. Sierpinska (1994, p.13) suggested that "few concepts have caused as much trouble in philosophy as the concept of meaning". Kilpatrick *et al* (2005) explained that the discussion of meaning has given rise to different philosophies of mathematics and the writers offered a collection of essays in which meaning is discussed in a variety of ways. In one of the essays, Otte (2005) offers Peirce's theory of meaning as a possible perspective, stating that for something to have meaning, it must be related to something else, that is, nothing can just mean itself. This idea was relevant to me since I anticipated that mathematical words used as part of the medium of the classroom, might be endowed with meaning through their association with other words, diagrams, objects and notation. In fact, using the triadic view, I developed an analytic tool that allowed me to discuss this association. The tool will be explained in Section 4.5.3.

4.4 Establishing a methodological approach

Just as I assumed that pupil knowledge was individual and was created through social contexts for pupils, so too, as a researcher I considered that any 'reality' I might observe or experience was a context-bound personal interpretation of events. Hence, I chose to carry out an 'interpretative' type of project. Mertens (1998) offered three broad categories of research, namely: Positivism/Postpositivism, Interpretive/Constructivist and Emancipatory. The categories are summarised in Table 4.1.

	Assumptions re nature of reality (Ontology)	Assumptions re nature of knowledge (Epistemology)	Approach to enquiry
Postivism / Postpositivism	One reality	Researcher manipulates and observes in a dispassionate, objective manner.	(Primarily) quantitative; interventionist; decontextualised.
Interpretive / Constructivist	Multiple, socially constructed realities	Interactive link between researcher and participants; values are made explicit; created findings.	(Primarily) qualitative; contextual factors are described; hermeneutical; dialectical.
Emancipatory	Multiple realities shaped by social, political, economic, ethnic, gender, disability (etc.) values.	Interactive link between researcher and participants; knowledge is socially and historically situated.	(Primarily) qualitative (dialogic); contextual and historical factors are described especially as they relate to oppression.

Table 4.1 Three main research approaches (adapted from Mertens, *ibid*, p.10)

Certainly, my interest in the complexity of socially situated learning and acknowledgement of flexibility of meanings implied that I did not assume a positivist position. I did not locate myself in an emancipatory methodology either, since although one of my aims was to problematise the use of English as a medium of instruction for mathematics, it was not my intention to highlight in detail teachers' and pupils' experiences as they lived them, in an attempt to empower them to contemplate or bring about change.

Being aware of my research questions and beliefs, I then reflected on an appropriate research design. I rejected a quantative approach, since this is associated with a positivist perspective and would not allow me to observe the teaching-learning process as it unfolded, nor discuss in detail with teachers and pupils. Rather, I was prompted to conduct a qualitative study, which Denzin and Lincoln (2000) defined as follows:

> "Qualitative research is a situated activity (…). It consists of a set of interpretive, material practices that make the world visible. These practices […] turn the world into a series of representtions, including field notes, interviews, conversations, photographs, recordings, and memos to the self" (*ibid*, p.3).

Qualitative studies may take various forms. Mertens (1998) listed seven main strategies which are indicated in Table 4.2.

Possible research strategies for qualitative research		
Ethnographic research		A method used to describe practices and beliefs of cultures and communities. The focus of such research is to understand a situation from an insider and outsider perspective. Researchers may start with a theory, which they then modify or abandon if it does not 'fit' in with the data.
Case study		An example of ethnographic research that involves intensive and detailed study of one individual or group.
Grounded theory		Theoretical propositions are not stated at the outset. Rather, generalisations (theory) emerge out of the data itself.
Participative inquiry		(1) *Cooperative inquiry*. Co-researchers identify a research problem they want to work on together; they implement research procedures in everyday life and work; they draw conclusions for change in practice or need for additional research. (2) *Participatory action research*. This emphasises the establishment of liberating dialogue between oppressed groups and the political production of knowledge. The role of the researcher is a change agent to provides opportunities for oppressed voices to be heard.
Phenomenological research		A study that emphasises an individual's subjective experience of the world and life around them.
Clinical research		The application of qualitative methods to bio-medical problems to investigate physical, behavioural, social, emotional and spiritual aspects in relation to a physical or psychological problem.
Focus groups		Group interviews that rely not a question-and-answer approach, but on the interaction within the group. This gives evidence of say, the struggle to understand others' interpretations and how differences are resolved.

Table 4.2. Types of qualitative studies (adapted from Mertens, 1998)

Since my interest was in classrooms, the interaction therein, and pupils' expression of meanings, I considered that the last four strategies explained above were not applicable to me. However, I felt that I could draw on the first three.

Ethnographies generally tend to focus on the 'culture' of a group of people, say, highlighting aspects of their life-styles or social relationships over time (LeCompte and Preissle, 1993). Although this was not exactly what I hoped to do, I did wish to focus on on-going settings in socio-cultural contexts where events occur as human interaction takes place (Shimahara, 1990), and my interest was in fact to deal with real-life contexts (Hammersley, 1994). Hence I considered methods that Hammersley (*ibid*) offered as suitable for an ethnography: observations and interviews, interpreting the meanings and functions of human actions, and expressing results through descriptions and explanations.

It also seemed appropriate to consider a case-study approach. According to Sturman (1994) 'case study' is a generic term for the investigation of an individual group or phenomenon. However, the expression 'case study' has been defined differently by various authors (see Bassey, 1999, for a detailed discussion), and therefore Bassey (1995) preferred the term 'singularity'. I find the term singularity useful in that it is set in opposition to the term *generalisation* so often invoked in quantitative studies based on large samples. Bassey (1995, 1999) defined a singularity as a study of a particular set of events bounded by space and time and stated that the focus of research is an issue rather than the case as such. I had no wish to find and present generalisations and I did not believe that every primary classroom would yield the same data or issues. Yet, Bassey (1995) stated that a singularity is chosen because it is expected in some way to be typical of something more general and as Wolcott (1995) aptly put it:

> "Each case study is unique, but not so unique that we cannot learn from it and apply its lessons more generally" (*ibid*, p.175).

Watson (2002) stated that such studies can help us to develop tentative characterisations and alert others to issues that might help them make sense of their work. I believed that carrying out a small scale study would allow me to recognise that certain situations can, and do, occur. Since the aim of my study was to offer reflections that could be used in discussion with other colleagues to enhance the immersion debate, I felt that a 'singularity' could serve as a springboard for such reflection. Indeed, because of the detail that a small-scale study would allow, I considered such an approach to be a strength, rather than a weakness, of a possible research design.

Hence, assuming a small number of participants, I planned to collect data through observations and interviews. In Table 4.3, I show why I considered these methods to be appropriate to help me address each of my research questions.

	Research Question	Data Required	Research Method
Language as medium	*(1) How does the NMC recommendation regarding the use of English for mathematics fit in with other educational principles promoted in the same document?*	Teacher and pupil opinions regarding the medium of instruction policy; Classroom observations to note teachers' and pupils' actual use of English/Maltese; Personal reflections on classroom interaction and opinions in the light of the NMC document.	Interviews with teachers and pupils Lesson observations Reading of NMC document
	(2)(a) How much, and with what ease, do pupils talk in immersion classrooms? *(b) How 'mathematical' is their talk, in terms of the inclusion of mathematical vocabulary?*	Stretches of classroom interaction to focus on length and ease of pupil contributions; Quantification of the use of mathematical words during lessons; Teacher opinion regarding the usefulness of mathematical vocabulary in order to aid my reflections on pupils' use of the words.	Lesson observations Lesson observations Interviews with teachers
Shifting from medium to message	- -	Frequency of word use in class; teachers' and pupils' opinions regarding which mathematical words were 'new'/ previously familiar to pupils; pupils' expression of meanings for new words. This in order for me to be able to relate frequency of use with (a) previous familiarity or otherwise; and (b) newly shared meaning.	Interviews with teachers and pupils Lesson observations
Language as message	*(3) What conditions appear to be helpful for a teacher to 'share' the meaning of (a selection of) mathematical words with the pupils?*	Pupil explanations of meanings for words to gauge whether meaning has been 'shared' or not. Classroom interaction within which mathematical words are used in order to attempt to explain conditions of word use that appear helpful or otherwise. Teachers' intentions for the week in order to help reflections on what was 'covered' during the week; teachers clarifications regarding selected parts of lessons to help my interpretation of lesson interaction.	Interviews with pupils Lesson observation Interviews with teachers

Table 4.3. General plan of research design

I can now outline how I organised the data collection (details are presented in Chapter 5). I observed primary mathematics lessons, focusing my attention on stretches on interaction that ‘surrounded’ mathematical words. The same excerpts allowed me to reflect on my various interests. For aspects relating to language as a medium, I considered the pupils’ extent of talk and use of English and mathematical English, while for aspects of language as a message, I re-considered the same excerpts in terms of meanings expressed (by the teacher and /or pupils).

I also conducted interviews with the teachers before a set of lessons. This was mainly to gauge their intentions for the lessons, and ask their opinions regarding the use of English as a medium of instruction. After the lessons I also interviewed the teachers to ask them about certain points that had occurred in the lessons. These interviews helped me to ‘see’ the unfolding of the lessons from the teacher’s point of view. For example, I asked them which mathematical words they had considered to be key, and this fed into my discussion regarding the inclusion of mathematical words; I asked them about what they had wished to ‘share’ with the pupils, and this helped me in my discussion of sharing meaning.

In order to address pupils’ expression of meaning, I interviewed some pupils after the lessons. I compared the pupils’ expression of meaning to that expressed during the lessons (generally by the teacher) and reflected on any similarity or dissimilarity of meaning. I then attempted to explain successful ‘sharing’ or otherwise by considering how clearly meanings had been expressed in class. Following Mercer (2000a), I assumed that teachers introduce technical words to pupils by using them in contexts that render their meanings clear. However, I wished to explore what, in fact, rendered meanings ‘clear’, and I was also open to note other features of word-use that helped or hindered sharing of meaning.

My disposition to ‘discover’ ideas from my data implied that my study was, to a certain extent, ‘grounded’ (Glaser and Strauss, 1967). According to Charmaz (2000), grounded theory offers flexibility because researchers can modify their emerging or established analyses as conditions change. Indeed, as my project unfolded, I developed what Mason (1998) called new sensitivities, and these in turn prompted me to consider new or different aspects as data. Strictly speaking, in a grounded study, data collection and analysis occur simultaneously until ‘theoretical saturation’ is reached (Dey, 2004, p.80). That is, decisions regarding data collection are taken in the course of the study. In my case, ‘new’ data was the main study data, as opposed to the pilot study, and *re-*

visiting the main study data as new sensitivities developed. As I came to 'see' things that I had not been attuned to notice before, new evidence came into being through renewed interpretation.

In Table 4.4, I outline how I restructured my attention over time (Mason, 1998) and indicate how main themes appeared. These ideas will be discussed in detail in Chapters 6 – 11, but three things are important to note in relation to reading the Table: (a) for reasons I explain in Chapter 5, the focus of my study changed from the pilot to the main-study phase from a focus on code-switching practices to issues of immersion; (b) my research questions themselves actually took shape through the course of the project; (c) the semiotic model mentioned in the Table will be explained in Section 4.5.

Starting assumptions		Open to 'discover'…		New sensitivities		New assumptions		Open to 'discover'…		Main points arising that formed the basis of this write-up
Mathematical language is an important aspect of the subject. Code-switching is a helpful practice for sharing meaning with pupils.		How is a Mixed Maltese English mathematics register constituted?	PILOT	Mathematical vocabulary appears to serve 'different' roles e.g. references, denote concepts.		Mathematical language is an important aspect of the subject. The immersion approach for mathematics may be problematic.	MAIN STUDY	In what ways is immersion problematic? What are teacher's and pupils' opinions about the use of English? What does the English mathematics register 'sound' like?		The English immersion recommendation is in tension with other NMC recommendations. English mathematical vocabulary may be used non-'conventionally'. Gestures play a part in communication. Pupil inclusion of mathematical vocabulary depends on various aspects.
Teachers introduce technical words by using them in dialogues that make their meanings clear (Mercer, 2000a).		How are the two languages used in order to render meaning 'clear'?		New English words are taught and then assumed familiar by teacher. Links between Maltese and English words, and notation and diagrams appear helpful.		Teachers introduce technical words by using them in dialogues that make their meanings clear. Clarity considered as links with diagrams, notation.		What renders meaning clear? (Focus on links between words, diagrams, notation; focus on different 'types' of words).		Clarity can be discussed for words that serve as references and denote properties and concepts. Two other conditions apart from clarity appear to be helpful for sharing meaning.
				The development of a semiotic model prior to the collection of main study data.						

Table 4.3. The development of sensitivities over time.

4.5 Towards an analytic semiotic model

After the pilot study (see Chapter 5 for details), and prior to collecting the main study data, two important developments occurred. First, I became aware of the different ‘roles’ that mathematical words played, for example as references, or to denote properties, relationships or verbs. Second, I came to feel the need to have an analytic tool that would enable me to talk about ‘clarity of meaning’. In this section I show how the two can come together in a semiotic model. I first present Vygotsky’s views regarding reference and meaning, then explain a triadic model developed by Heinz Steinbring (1997). Finally, I build on his model to create my own as a tool to help me focus on my discussion of meaning of mathematical words.

4.5.1 Different ‘types’ of mathematical words

In his discussion of how children learn new concepts, Vygotsky distinguished between ‘referent’ as opposed to ‘meaning’, and considered the former to be the original function of a word (Vygotsky, 1981c). In the early phases of a child’s development, a word uttered by an adult serves the purpose of an indicator or index and – perhaps accompanied by a gesture or verbal indicator like the word ‘that’ - it regulates the child’s attention to an object. Werstch (1985) drew a parallel between Vygotsky’s reference and Peirce’s index, since both serve a role of indicating objects. Both writers implied that for a reference, the sign and its object must be co-present and that the object is characterised in only a minimal way. Indeed, indexing does not say much about the object beyond drawing attention to it (Goudge 1965, cited in Wertsch 1985). In contrast, the symbolic function of speech involves the classification of objects and events in terms of generalised categories and eventually the formation of relationships among these categories. Vygotsky (1962) stated that in the most advanced form of generalisation, a concept involves a relationship to another concept. At this point, language can operate independently of the concrete context and meaning is mediated through words alone.

In my discussion of mathematical words up to now, I have not distinguished between different ‘types’ of words. I would now like to make a distinction between the various roles words may serve. A word serving as a reference serves to ‘point’ to something. For example, the word *square* can be used to refer to a particular shape: a teacher might attach a large coloured cardboard shape on the whiteboard, touch it and say ‘This is a square’. Or she might from a distance say ‘That coloured piece of cardboard is called a square’. Here, the word *square* acts as a label for the shape, while the gesture or word index ‘that’ draws attention to the tangible object.

Other words that I suggest can be used in a referential way are for example, *graph*, as a name for a particular diagram, and *plus sign* as the name for a symbol. Such words are used in close proximity (in the sense of perception) to the object to which they refer. I consider this nominative role of a word to be a first layer of meaning (Roberts, 1998), which can then be extended to further layers of understanding.

On the other hand, words can denote properties or relationships. Respective examples are *long* [line] or *odd* [number]) and *greater* or *ratio*. These words cannot be referred to indexically like a square or a graph, so that the meaning for these words belongs to what Vygotsky called a 'symbolic' plane.

Finally, I also note that a mathematical word may be a verb, that is, it denotes an action. I may be able to understand some verbs if they are used in a referential way. For example, if I am told that an observed person is swimming, I might associate the word *swimming* with the action taking place in water. However, I suggest that understanding the meaning of a verb includes being aware of the function the action serves, so that I fully appreciate the meaning of *swimming* if I am aware that the action can serve as exercise or to cool down. Furthermore, some verbs imply a more complex interpretation. For example, suppose I am told that an observed person is welding. If I do not know what the word *welding* means, then I am unsure on what to focus in order to interpret the word. Is it the creation of sparks that is welding? Is welding the action of holding a piece of metal? Is it the covering of one's face with a dark mask? What exactly is the person doing to the metal rod? And so on. In order to understand what the word *welding* means I need to appreciate the *function* of the action, that is, what it can achieve, when it is carried out, why, on what objects, the tools needed and so on. Thus understanding verbs generally goes beyond the referential. Examples of mathematical verbs are *share*, *plot* and *subtract*.

4.5.2 Steinbring's triadic model

Within mathematics education, there has been a recent surge of interest in semiotic issues (Vile, 1997). Vile (*ibid*) suggested that this is because semiotics offers a vocabulary for thereotical description of meaning-making that accounts for the socio-cultural nature of experience and it allows for empirical focusing on signs that can help researchers understand the meaning-making process in specific contexts. Semiotics is particularly relevant to mathematics since mathematical notions cannot be directly perceived; rather, access to mathematics must necessarily pass by way of semiotic representations (Duval, 2001). Such representations in the classroom are many and diverse, including number systems, figures, graphical representations, algebraic and formal notations and language (Duval, *ibid*). For the sake of brevity I do not give details of research that has been carried out, but for the reader's interest, I indicate some studies in Appendix B.

Of particular interest to me is Heinz Steinbring's work, and I built on his model for my own purposes in order to address not only 'meaning', but also the notion of 'clarity of meaning'. Steinbring (1997, 2005) developed a triadic model similar to the one I presented in Figure 4.1 but he used alternative terms: 'sign/symbol' for representamen, 'object/reference context' for object, and 'concept' for interpretant. (His model is actually an adaptation of the one offered by Ogden and Richards (1923) who use the terms *symbol*, *referent*, and *thought or reference* respectively).

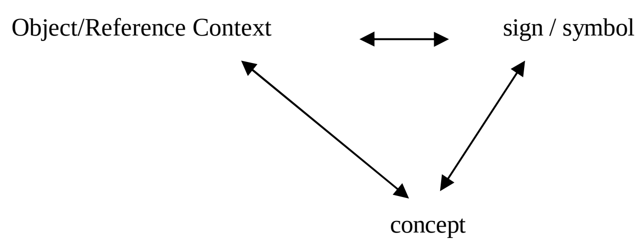


Figure 4.2. Steinbring's (1997, 2005) epistemological triangle

Steinbring focused his attention on mathematical notation and diagrams. As an example of a triad, Steinbring (2002) offered '3' as a sign/symbol, diagrams of three apples / balls as a reference context and 'elementary number concept' as the third component. In another example Steinbring gave the respective elements as: $\sqrt{2}$, a unit square with a diagonal marked in, and 'aspect of the concept of real numbers' (Steinbring, 1997). Steinbring (*ibid*) considered that the notation functions as a sign because it represents the object in some respect. For example, the symbol 3 refers to the numerosity of the set of balls and not to say, their colour or shape. According to Steinbring (*ibid*) children learn to mediate between signs/symbols and objects through examples, and hence meanings of mathematical concepts emerge in the interplay between sign/symbol and reference contexts/object domains.

Steinbring (*ibid*) suggested that the object should be known in at least some basic aspect. Certainly, a unit square with a diagonal is not some independently existing entity, but the perceiver must already have some conception of aspects of it through previous cultural experiences (e.g. lines, shapes as mathematical entities, orientations, and so on). Thus, a transfer of relations can occur from a relatively familiar reference context to a new, unknown sign system (Steinbring, 2005). I believe that Steinbring's use of the expression *reference context* instead of *object* in his model is a significant one since it seems to make explicit that an object is understood "primarily in terms of the context of involvement in which it is normally encountered" (Van Valin, 1980, p.218) and therefore the relationship set up between object and sign depends on the context in which they are associated.

According to Steinbring (2002, p.8), the epistemological model allows for a flexible switching between sign and reference context, by exchanging the roles of each: "sign systems and reference contexts are then temporarily equal … none precedes the other". For example an empty number line, with only the numbers 17 and 25 shown, can serve as a reference context, which is symbolized by '17+ __= 25'. In this case the arithmetic-symbolic structure is partly explained by the number line. On the other hand, the notation 17+__= 25, which may already be familiar to the children, can be symbolised by the empty number line. For the benefit of young children, the reference context is often a real life context or a picture, but Steinbring (1997, 2005) stated that the empirical character of knowledge can be increasingly replaced by diagrams or other sign systems in order that relational connections are set up.

In his recent publication, Steinbring (2005) presented consecutive epistemological triangles to illustrate successive interpretations by the learner in the construction of mathematical knowledge. Steinbring (*ibid*) developed the triangles alongside a parallel discussion regarding the classroom communication that accompanied the signs and reference contexts. For brevity's sake, it is not possible for me to go into detail about this work, but the point that is necessary to make here is that my own model grew out of Steinbring's earlier writing dated 1997, 2002.

4.5.3 A semiotic analytic tool for mathematical meanings

As an analytic tool to aid me in my discussion of meaning, I chose to represent meaning through two different types of diagrams. I used the following representation to illustrate the meaning of a word in a referential role:

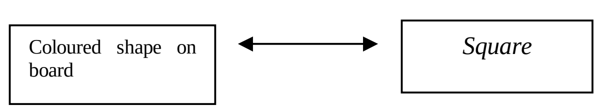


Figure 4.3. Diagrammatic representation of a word as a reference

On the other hand, I used a triadic model to represent meaning of words that go beyond the referential. Lemke (2003) recommended that it is only by cross-referencing and integrating verbal language with say, written expression, diagrams and chalkboard cues that meaning is effectively made and shared. In anticipation of the linking of these signs, I adapted Steinbring's model as follows:

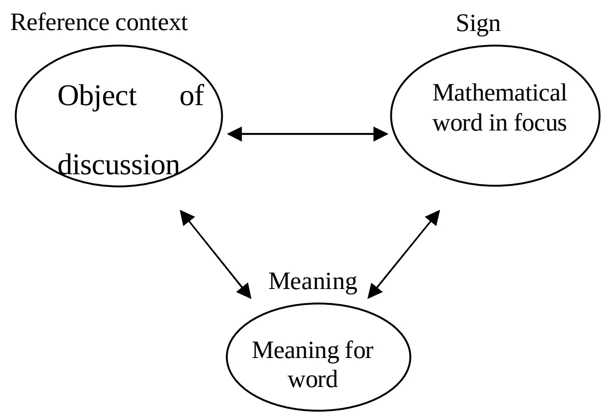


Figure 4.4. Epistemological triangle for a mathematical word

I considered that although a 'sign' might be – as originally suggested by Steinbring – notation or a diagram, it could also be a mathematical word. I replaced Steinbring's label *concept* with *meaning*. The reason I did this was because I anticipated that I would wish to discuss meanings for words that denoted properties (*odd*, *equilateral*) and verbs (*share*, *plot*), which I do not normally refer to as 'concepts', a word I tend to reserve for relationships such as *multiplication*.

As part of what I viewed as a reference context, I incorporated the element 'familiar words' along with the object of discussion. As stated by Mercer (2000b):

> "words gather meanings from 'the company they keep' - that is, from the influence of the meanings of other words which are used with them" (Mercer, *ibid*, p.67)

Mathematical words keep company with many other words and the teacher may wish to draw attention to *particular* ones to establish associations. This might be done either by stressing the relevant words or changing intonation or indicating their importance in some way or another. (For the sake of simplicity of presentation, I have not included gestures and vocal stressing in Figure 4.4).

Wertsch (1985) explained that any situation, event or object has many possible interpretations and the speech used serves to impose a particular interpretation. Hence, I suggest that the object serves its purpose in the development of mathematical ideas thanks to what is rendered salient through familiar language. So for example, when handling a 10 cents coin, a teacher might use language to direct attention to the number on a coin in order to lend meaning to the word *value.* The choice of this language can be contrasted to other alternatives that would draw attention to the images on the coin, its thickness, the material it is made of and so on. In cases where there are no perceivable objects or pictures available, language serves as its own context and meaning is conveyed through words alone.

As a new word becomes established, it may then be used to support further learning by assuming a new role as one of the 'familiar words' in a reference context, and hence a chain of signification can be set up. This chain can be represented as a sequence of triangles as shown in Figure 4.5:

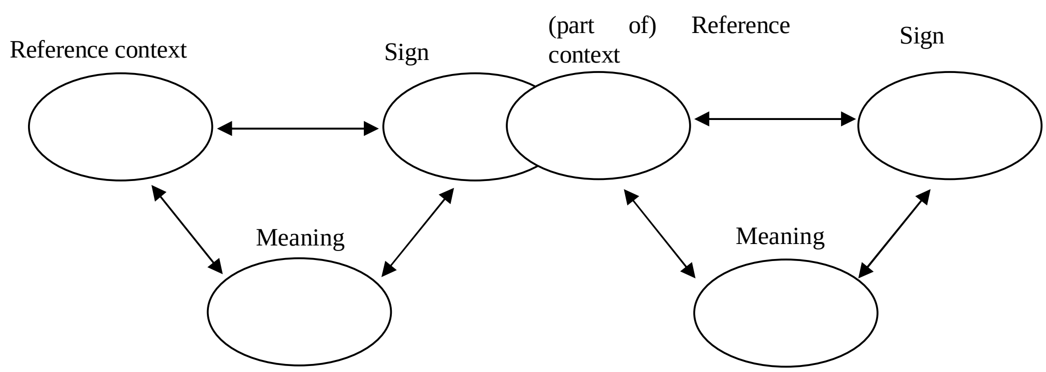


Figure 4.5. Chains of signification

In my study, I planned to use this model to illustrate chains of meaning that were apparently set up through classroom interaction in an attempt to share meaning. In a teacher-directed approach, it is likely to be the teacher who would set up the links, although the pupils' input may also contribute. I acknowledge that the model only takes into consideration things that I, as a researcher, can perceive: spoken language, whiteboard work, pictures, gestures etc. The model does not take into consideration other factors that may very well impinge on a pupil's attention during a lesson, factors that may therefore detract from meaning being shared. These might include affect for the subject (Evans, 1999), interest in the specific task at hand, mood or frame-of-mind, momentary distractions and so on. Another limitation of my model is the use of the blanket expression ‘familiar words’. I acknowledge the difficulty in establishing which words used by a teacher are familiar to the children in a class, and if indeed familiar, to what extent. I will return to this point in Chapter 8.

4.6 Reflections on ‘*discourse*’

As a final reflection, I would like to comment on a dilemma I had as I worked on this project This was whether to consider mathematical language in terms of a ‘discourse’ or not. The reason for my uncertainty was that although my study did have some features common to studies that claim to be analyses of discourse, it also had features that were quite different.

According to Sierpinska (2005), the meaning of the word *discourse* within educational research has changed over the years, but an important feature of the word’s use is that it brings into focus social aspects of a situation. Hodge and Kress (1988) used the word *discourse* to refer to the social process in which texts are embedded, while Halliday himself used it in his discussion of register. Phillips and Jorgensen (2002) explained that the concept has become vague through different applications, but that in many cases, the general idea is that language is structured according to different patterns that people’s utterances follow when they take part in different domains of social life. Discourse analysis is then the analysis of these patterns. Phillips and Jorgensen (*ibid*) reported that some researchers choose to analsyse people’s discourse in everyday social interaction (e.g. the significance of national identity for a work-place interaction) while others prefer more abstract mapping of the discourses that circulate in society (e.g. ‘medical discourse’). One particular form of discourse analysis is ‘discursive psychology’ developed by Potter and Wetherell (1987). In this approach, the rhetorical function of talk is explored. An example of such analysis is that carried out by Barwell (2003a) who studied pupils’ interaction within a group as they solved word problems together. Barwell (*ibid*) considered ‘attention’ as a discursive resource, identified patterns of attention in the pupils talk and considered how the areas of attention were constructed and deployed by the pupils as part of the sense-making process.

A research interest in *critical* discourse analysis implies reflecting on participants’ ‘positioning’ or their “socially significant identities” within a context (Gee, 1997, p.256). Often, in critical discourse analysis projects, power relations are studied and problematised. For example, Burton and Morgan (2000) discussed the identities that mathematicians present to the world and the ways in which they represent mathematical activity through their writing. Creese (2004) reflected on the power relation apparently at play between classroom teachers and EAL facilitators, a relation she believed was created and maintained through the respective use of ‘expert’ subject knowledge and the apparently less important language used for general support.

If I consider my own research interests, I note that I did plan to problematise the immersion method, and hence did consider the social situation in a critical way. Also, for my second focus, and drawing on Halliday (1978), I commented on the relationship between the teacher and the pupils, and the role of language when considering the extent and cohesion of pupils’ talk, and the inclusion of mathematical words. However, beyond that point, I did not examine the patterns of classroom talk nor see how these are related to action. For example, as I discussed the inclusion of mathematical words as part of the medium being used, and the meanings apparently expressed, I did not deal with the ‘type’ of mathematics that pupils may be learning through the talk (procedural, investigative etc.); I did not question the pupils’ roles in the creation of mathematics (receivers / creators of knowledge etc.) nor their resulting possible affect for the subject. My analysis of meaning centred on associations that teachers and pupils appeared to make between words, and how the words were used in conjunction with diagrams and notation. (More detail in this regard will be given in Section 5.7).

Furthermore, in my study, I did not tackle issues of power or identity. Rather, I *assumed* an asymmetry of power in environments where learning is likely to be teacher-directed. Following Pimm (1994), I took the view that this need not be a negative thing. Like Pimm (*ibid*), I believed that it is not inappropriate that teachers should use their positional power to teach what they hold to be important. Hence, I decided not to use the word *discourse* but used the expression *mathematical language* instead, since I felt that the latter would better encompass the various aspects of my study.

4.7 Conclusion

In this chapter, I explained my methodology and devised a semiotic tool to help me to deal with the notion of ‘meaning’. I also indicated the research design I believed appropriate for addressing my research questions. I will now explain in more detail all the choices and practical aspects of the data collection phase.

CHAPTER FIVE

The Data Collection Process

5.1 Introduction

In the previous chapter I indicated the research method I planned to undertake in order to address my research questions. As a general research design, I established that I would observe classrooms and conduct interviews with teachers and pupils. In this chapter, I explain all the choices I made and give details of the practical implementation of the research design. As a way of introduction, suffice it to say that I first carried out a pilot study that focussed on the teaching of two topics in one classroom, while for the main study, I considered two topics for each of two classes.

5.2 An unforeseen change in direction

One important development in my study was the unforeseen change in my sense of direction. My original research interest had been to study classrooms in which code-switching was used, in order to reflect on the structure of the Mixed Maltese English register and on the sharing of meaning of mathematical words through the two languages. The code-switching element was, in fact, present in the pilot study. However, in the main study I actually observed two *immersion* classrooms in one school and this significant change needs explaining.

On first approaching the pilot and main study teachers, I stressed that I needed classes in which code-switching occurred. Although both schools had informal policies to use English during mathematics as much as possible, all three teachers explained that they did in fact use code-switching. I had no reason to doubt this, since such a situation – that is, a theoretical stress on English, but a practical compromise with Maltese – is a common one in Malta. The pilot teacher did use code-switching and this suited my original aim. On the other hand, the two teachers involved in the main-study used only English. I became aware of this when I observed a first extra ‘trial’ lesson. I expressed concern with the teachers that I was going to lose out on the crucial code-switching aspect, but they assured me that code-switching would occur, especially when children did not understand. However, as I watched the second trial lesson, English continued to be used exclusively by the teachers and almost exclusively by the children during whole-class interaction, and I began to realise that the policy to use English was being adhered to. At this point I considered whether I should change schools, but decided to stay on for two reasons.

First, the Head of School had made an effort to accommodate me and the teachers had welcomed me willingly and with trust. We had already established the topics to be observed, made out a schedule for lesson observation and the interviews and sent a letter home to the pupils' parents. I had already sat in for, and recorded, two trial lessons. Hence I felt uncomfortable informing the Head of School and the teachers that the situation would now not match my research interests. Second, on the same day that I sat in for the second trial lesson, the school received a notification from the local Teacher's Union: as a result of an incident arising out of a totally unrelated research project in another school, the Union issued a directive to all schools not to allow any video-recording in classrooms. The Head of School and I clarified my own situation with the Union, explaining the approach I was using with regard to confidentiality (see Section 5.4), and that I had my University's support to conduct my research. I was 'allowed' to continue with my recordings and therefore decided to stay on at the school. It was highly unlikely that any other school would have accepted me at that point since locally, the Union directives are strictly followed. Indeed, had the Union insisted that I stop recording, the Head of School would have, albeit reluctantly, asked me to withdraw.

Staying on however, meant that I had to alter my foci by including a discussion on the immersion recommendation itself, considering an *English* mathematics register and reflecting on sharing of meaning through a second language. This change in direction resulted in the study I am presenting.

5.3 Issues of validity and reliability

Since I carried out a small scale study, I would like to begin by commenting about two features that have traditionally been considered crucial to a research project: validity and reliability. The concepts have traditionally been associated with quantitative tests. Validity refers to the degree to which a method or research tool actually does measure what it is supposed to measure (Wellington, 2000). However, it takes on a different significance in qualitative studies if, as Mason (1998) suggested, we accept to let go of the notion of certainty associated with quantitative approaches. Mason (*ibid*) stated that an alternate stance opens up the way to view assertions as invitations and conjectures that may be checked out with others who find such assertions helpful. In qualitative research, validity may be considered within the parameters of the particular setting, population and theoretical framework and therefore it is important to state these parameters clearly (Marshall and Rossman, 1989). In order that different readers share the same interpretation as much as possible, an interpretative study builds in 'safe-guards' (Wiliam, 1998). In my case, this safe-guard is my attempt to make explicit any assumptions and decisions and the exposition of the analytic tool with which I choose to interpret the data.

Reliability, on the other hand, is a judgment of the extent to which the method, test or tool gives consistent results across a range of settings, and if used by a range of researchers (Wellington, 2000). However, in a *qualitative* study, the assumption that the social world is always changing renders the very notion of replication problematic (Marshall and Rossman, 1989). Indeed, according to Le Compte and Preissle, (1993) the qualitative focus on unique situations implies that establishing reliability is actually an impossible task. The most I can hope for is that other researchers may borrow from the theoretical framework and design, and compare and contrast their own research interpretations.

5.4 The ethical dimension

Burgess (1989) emphasised the necessity for an ethical relationship between a researcher and the participants involved in a study, a relationship that implies:

> "a respect for the rights of the individual whose privacy is not invaded and who is not harmed, deceived, betrayed or exploited" (*ibid*, p.60).

In both the pilot and main study, I was careful to respect ethical considerations and proceeded in a similar manner in both cases. Hence, in this section I will focus only on the details of the main study.

The BERA[4] ethical guidelines (2004) recommend that one way of respecting participants is to offer them enough information so as to allow them to understand the process in which they are to be engaged, and to make them aware of how the study will be used and reported. I explained my interests to the teachers so that their consent would be an informed one (Zevenbergen, 1998). According to Wellington (2000), participants should know what they are letting themselves in for and so I estimated the amount of time I expected them to dedicate and made this explicit in our first meeting. I also offered to provide the interview questions beforehand which, however, none

[4] British Educational Research Association

of the teachers felt was necessary. I assured them that any recordings would not be viewed by anyone but myself and that I would use them purely for transcription purposes. Of course, the notion of informed consent is not watertight, since the initial questions often change, with possible effects on the role of the researcher and the participants (Sowder, 1998). In fact, my own foci did change and, although our respective roles were not altered, I informed the teachers that I had become aware of the immersion issue and that a change of direction in my study was going to be necessary.

My research plan included recording pupils during lessons and interviews. Tymchuk (1992) suggested that it is appropriate to obtain parental permission, since parents have a right to have a say in what happens to their children. Therefore, I first obtained the children's parent's permission for their children to participate (all guardians were parents). I did this by sending a letter home (see Appendix C). In this letter, I gave a brief explanation of my interests and offered a contact number in case parents wished for further details or clarification. One parent did in fact phone. In the letter, I explained that the recordings were for my own use, and that no-one else would be viewing them. The parents were offered the option of withholding permission, an option that four out of fifty-three families chose. I respected this by not interviewing the pupils in question and also keeping them out of camera-view during any class recordings. However, as Tymchuk (*ibid*) explained, it is also appropriate that the child him- or herself assents to participation. I admit an oversight in not confirming with the pupils themselves whether they wished to be caught on video, but did confirm assent for the interviews. Only one pupil whose parents had in fact granted permission for participation indicated discomfort and so I did not interview her; all the other pupils were keen. I explained that I was interested in observing the teaching and learning of some mathematics topics and that I wished to ask them some questions to clarify what they had understood during the lessons. I assured them that I was not testing them in any way.

As recommended by Cohen *et al* (2000), I was careful to show that I would be respecting confidentiality. During the data collection process, I did not repeat what was said by either teacher to the other or to the Head of School and did not quote individual children's comments to their teachers. Furthermore, I assured all participants that they would be given fictional names in my writing, as suggested by Bassey (1999). Such assurance is very important in a small country like ours, where schools and individuals may be much more easily identified than in a larger country.

Despite my attempt at openness, I acknowledge that the researcher enters the field in a position that is privileged, in the sense that he or she has power over what will be observed, what will be asked, the discourses that will frame the research and how the data will be used (Zevenbergen, 1998). For example, although I designed interview questions to help me see things from the teacher's and pupils' point of view, at the end of the day, I am the one who is interpreting and, as pointed out by Cameron *et al* (1992), the final reporting is mine.

5.5 Choices regarding participants and topics

For the purpose of the pilot study, I felt it would be sufficient to observe one teacher. I observed two topics and interviewed two pupils per topic. I believed that this would be sufficient to allow me to get a 'feel' of the approach and try out technical aspects such as recording. As part of the process, I tried out some transcription of classroom and interview data and concluded that the volume of work involved implied that it would be unreasonable to work with more than two classrooms for the main study. In fact in the main study, I observed two teachers teaching two topics each, and interviewed six pupils for each topic. These choices are explained in more detail in the following sections.

5.5.1 Schools, classes and teachers

In line with my original interest in code-switching classrooms, it was necessary to observe classes where this occurred. Thus the main consideration centered on choosing a school where children tended to use Maltese as a first language, since I assumed that this factor would imply that the teacher would need to code-switch for mathematics. I acknowledge that viewing the children as 'Maltese speakers' homogenises the group (Barwell, 2003b), and fails to make a distinction between different language backgrounds. After all, children may use code-switching in various degrees in different situations, and have varying degrees of familiarity with English. However, I have chosen to use the umbrella term 'Maltese speakers' as a working position since I believe that within the scope of my study, it is neither possible nor necessary to make such detailed distinctions between the children.

From my experience, I knew that the majority of Maltese primary schools offered the required situation, and I only had to eliminate a few private independent schools that I knew attracted a high proportion of English speaking families and used English as a medium of instruction throughout. Furthermore, my research questions did not dictate the actual age group I would study. I set myself a minimum age of Grade 3 (7 – 8 year olds), since I was concerned that it might be harder to interview children younger than that regarding their understanding of words. For the pilot study, I did not have any particular preference, although for the main study, I thought it might be useful to observe classes that were 'far apart' in the sense of age. I tentatively conjectured that this might bring to light differences in the number of mathematical words used and I was curious about any possible differences in the amount of talk going on in class. The furthest apart the classes could be were Grade 3 and Grade 6. Beyond these parameters, my choice of schools and teachers was what Wellington (2000) called opportunistic as I shall explain.

I broached the pilot study through the teacher herself and her class happened to be a Grade 3 class. The teacher was an ex-University student of mine whom I contacted to ask if she would help me out. Although I had never actually been her tutor on teaching-practice, I remembered her as a highly motivated student; furthermore she could express herself confidently in both Maltese and English, and so I hoped that she would feel comfortable participating in a study that focused on language. She accepted to participate and paved the way for me to approach the Head of her school.

The relationship between the teacher and myself was generally positive and I am grateful for her participation. However, I felt that working with an ex-student had a disadvantage: it was difficult to break down the student-lecturer relationship. Even though I tried to make it clear that I was not evaluating her as a teacher, I found that she often attempted – unnecessarily - to justify her actions. I found myself feeling a bit uncomfortable with the asymmetrical relationship she seemed to perceive and which I had not intended. I had hoped (naively perhaps) that she would perceive me in a different role to the one she had previously known me in. Occasionally, I even found that I compromised on my research needs in an attempt to present a more balanced distribution of 'power'. Therefore, I thought it best if for the main study, I worked with teachers who were not ex-students of mine, so that we could start up a new relationship. Of course, as stated by Cameron *et al* (1992), researchers bring their biographies with them, and I was not to know what perception any teacher would have of me and how this would influence the development of the study. However, I thought it best to eliminate the student-lecturer element.

For my main study, I opted for a Church school over a State school, since it was unlikely that I would have had access to Grade 6 in a State school. In this year group, pupils are prepared for a competitive exam for entry to State grammar schools and any forms of intrusion are kept to a minimum (including, for example, not placing trainee-teachers on practice in Grade 6). I gained access to the school I will call St. Helen's – a girls' school - through the Head of School who I knew personally. Thus in this case, it was the Head of School who acted as the 'gate-keeper' (Wellington, 2000).

I explained my research interests to the Head of School and left it up to her to identify volunteers. As pointed out by Zevenbergen (1998), where teachers volunteer to participate in a study, access to information is likely to be greater and richer, and indeed, I found that the teachers involved in my study were very forthcoming. I will call them Rose (Grade 3) and Gina (Grade 6). Both teachers were in their late forties and had several years of teaching behind them. I believe that this experience may have given them the confidence to feel at ease with me watching them. This is not to say that they did not feel self-conscious to begin with: Gina admitted that in all her years of experience she had never had another adult nor a camera in her classroom and was initially nervous about my presence; Rose said she had trouble sleeping the night before we were to begin recording lessons because she claimed that mathematics was not her 'top' subject. However, once things got under way, both teachers appeared to relax.

I am grateful for the interest Rose and Gina showed in my project. Whenever necessary, they adjusted their lesson times to accommodate me, photocopied things and offered practical advice regarding the use of the cameras. As suggested by Mertens (1998) a good rapport with participants is facilitated if the researcher accommodates herself to the routines of the participants. Therefore, I fitted in with their schedules and was sensitive to any stress on their part, caused by the forthcoming Open-Day and school concert. During the interviews I allowed them to express themselves freely, and was careful not to sound judgmental in any way.

5.5.2 Children chosen for interviews

The classes I observed at St Helen's were mixed ability classes with 27 and 26 girls in Grade 3 and Grade 6 respectively. The pupils were understandably conscious of my presence to begin with, and this was evident in their frequent side-glances in my direction and lack of interaction with the teacher. However, three trial lessons enabled them to relax, so that by the time I started recording the necessary lessons, they appeared to have become accustomed to my presence.

After each topic, I wished to interview some pupils per topic. In the pilot study, the teacher had offered her 'best' pupils, suggesting that they would be the most forthcoming and would catch up easily with any missed schoolwork. Although I accepted, I realised that I should be more pro-active in choosing the pupils. Therefore for the main study, I stated from the outset that I would like to select the children to be interviewed myself, a preference that Rose and Gina accepted. I considered that 'any' children would be appropriate for the interviews, but I believed it might add interest to have a spread of abilities in the sense of what the teachers themselves considered to be 'good', 'average' and 'weak' achievers for mathematics. When choosing the pupils, I eliminated the two per class for whom permission had been withheld, and any pupils who had missed a day or more of school while the topic was being taught.

The *number* of pupils to interview was not guided by my research question, and was based mainly on practical considerations of what would be manageable in terms of time and processing of data. Since I wished to interview children in pairs (see Section 5.6.4), it seemed reasonable to interview six pupils (that is, three interviews) for each topic, using as a rough guide two 'good', two 'average' and two 'weak' pupils per topic. The list of pupils can be viewed in Appendix D.

5.5.3 Topics and words focused on

The choice of topics to observe was partly influenced by the time of year that the project was carried out. I approached the pilot teacher in November 2001 and the main study Head of School in early December 2002 in order to allow time for the scholastic year to get under way. Both pilot teacher and the main study teachers expressed a wish to work on the project after the mid-yearly exams, and by that time a certain amount of work would already have been covered in agreement with other teachers of the same Grade.

For the pilot study, the teacher offered the topics Money, Block Graphs, Time (mainly reading the clock) and Roman Numerals. I opted for the first two: Money and Shopping because of the potential use of Maltese equivalents and Graphs because of *lack of* such equivalents. My pilot data heightened my awareness of the different roles that words play in terms of references and concepts and in denoting properties, relationships and verbs. Thus, for the main study, I wished to select topics that utilised different 'types' of words, again keeping in mind the idea of English/Maltese equivalents. I offered Rose and Gina several examples of paired topics for them to consider. Examples of paired topics were Money and Length for Grade 3 and Fractions and Graphs for Grade 6. They then considered the possibilities in the light of the syllabus yet to be covered after the mid-yearly exams. Rose suggested that any combination of the topics Graphs, Length and Multiplication/Division would be possible, while Gina offered Graphs, Length (including perimeter) and Area. I decided on Length and Multiplication/Division for Grade 3 and Length and Graphs for Grade 6. I believed that this would create a balance of various words, address three key primary mathematical areas (Number, Measurement and Data Handling) while providing the opportunity for comparison across Grades by virtue of the common topic Length.

Having decided on the topics, it was also necessary to select particular words to focus on during observations and interviews. For both pilot and main study, I did this by looking through textbook pages and notes that the teachers were going to use. However, I also discussed with the teachers what *they* believed to be the key mathematical words for the topic. If any other words came to my attention during the week of lessons, then I included these in the discussion with the pupils and teachers after the lessons. Examples of words related to the topics were: *multiply* and *sharing* for Grade 3 Multiplication & Division and *block graph* and *represent* for Grade 6 Graphs. For the topic Length, there was some overlap, with the words *width* and *metre* being included for both age groups, and *perimeter* and *metric* included for Grade 6. The number of words per topic ranged from 8 to 17 and the full list of words for each topic may be found in Appendix E. For simplicity's sake, I grouped some words together. For example, I assumed that the pupils' knowledge of English would be such that they would recognise say, *measure*, *measures*, *measured*, *measuring* as variations of the same verb and therefore considered these as 'one' word.

5.6 The collection of data

In this section I will explain the data collection procedures related to time-frames, classroom observations and interviews.

5.6.1 Time frames

One lesson I learnt from the pilot study was the importance of comfortable time-frames. During the pilot study, I had observed one topic immediately after the other, thus not allowing myself enough time for reflections and planning of pupil interviews. On the other hand, too much time passed between the end of the topics and the teacher interviews (although not originally planned this way) with the result that the teacher had not been able to recall certain situations. Regarding the actual duration of the interviews, the teacher had opted to have her interviews after school hours and the time allowed in this respect was generally sufficient. However, the time allocated for the children's interviews had been too tight. The main reason had been that the teacher preferred the pupils not to miss lessons, and therefore the time they could leave class was limited (e.g. 20 – 30 minutes). This had an adverse effect on the interview data I collected, as I tended to rush through the questions.

In order to schedule the data collection better in the main study, I prepared a detailed time schedule and discussed this with Rose and Gina beforehand. I planned to interview the teachers and the pupils as soon as possible after a topic so that things would still be fresh in their minds, but on the other hand, I had to be flexible because of school activities. Together we came up with a schedule convenient for all. The preliminary stages of the main study (that is, contact with the school, letter to parents, trial lessons and first interviews) were held between December 2002 and January 2003. Lesson observations and the main interviews were carried out from mid February through March 2003. A detailed record of the time-frame can be found in Appendix F.

The teachers and I also discussed the necessity of appropriate time-slots for all the interviews. I am grateful for the time Rose and Gina gave up during their breaks and 'free' periods. I also recognise that the adjustments they made in their daily activities allowed me ample time for the pupil interviews, approximately 45 minutes for each interview.

5.6.2 Observation and recording of classroom interaction

One of the main research approaches I planned to use was classroom observation. The advantage of observing is that it allows a researcher to study the process of education as it unfolds in the classroom (Anderson and Burns, 1989). My role in the observation of classroom lessons can be described as a non-participant one, since I did not take part in the event I was observing. However, as Swann (1994) pointed out, the distinction between participant and non-participant roles is not straightforward, since just by being in the classroom and observing, implies participation to some extent. Shipman (1997) stated that social researchers are part of the world they research and their activities provide clues for those they study. The participants' perceptions of me and my interests may have had some effect on their behaviour, although I have no way of knowing whether this was the case, and if so, in what way. As stated by Robson (1997) we cannot know what the behaviour would have been like if it had not been observed. The possibility of participants not acting 'naturally' was not of concern to me. I believed that a lesson could unfold in a multitude of ways and my interest was not to describe regular patterns of behaviour, but to use the observed lessons as a springboard for reflection. Thus I could consider 'any' lesson development to be of value.

In order to be able to reflect on the lessons and to transcribe excerpts at a later date, I needed to record the lessons. I opted for video-recording, since a camera acts as an extra pair of eyes that are able to observe and record in far greater detail than is otherwise possible (Zevenbergen, 1998). During the pilot study I had used only one camera. This had allowed me to capture *either* the teacher and whiteboard (thus losing out on pupil talk and actions) *or* the pupils (so that I had to copy down whiteboard work). Thus for the main study I used two cameras, identifying optimal positions through the trial recordings. I placed one camera at the back of the class and it picked up the whiteboard and the teacher, who spent much of the time at the front of the class. The other camera faced the pupils, picking up their speech and gestures. One limitation was that although I maximized the number of pupils in view and occasionally panned, it was not possible to view all pupils at all times. Therefore, although all pupils could be heard in the recordings, it was not always possible to identify them, or to note their gestures. Floor plans illustrating the classroom settings may be found in Appendices G1&2.

Since I did not wish to distract the participants' attention during the lessons, I sat by the front camera and only crossed the room if I needed to change a camera tape. On a couple of occasions in each class, I moved around the pupils as they worked in groups on an activity, in order to listen in to the language they used for a group activity. Before starting the recording of the required lessons, I sat in for three extra lessons so as to allow the teacher and pupils to get used to my presence, a process referred to by Robson (1997) as habituation. These lessons also enabled me to become accustomed to the pupils' names.

Generally speaking, I was successful in recording the data I required. However, one limitation of the recordings was that the cameras did not pick up interaction that occurred between say, two pupils at their desks, or between the teacher and a pupil as the teacher monitored written work (unless the pupil happened to be seated very close to either camera). This means that I may have lost occasions when mathematical words were used by either the teacher or a pupil.

To supplement my observations, the teachers provided me with copies of the hand-outs or textbook pages to which they had referred. With the pupils' permission, I also photocopied two sets of copybook notes per class. Although I had not asked permission from their parents for this, the teachers assured me that it would not be a problem and I believed this to be the case, since I only needed the copybook scripts as a reference when going through the recordings at a later stage. To ensure confidentiality, I did not make a note of the pupils' names.

I was prepared to record all the lessons related to each topic. As I had expected from experience, the Grade 3 lessons were about 1 hour long, but the Grade 6 lessons were substantially longer, often 2 – 2 ½ hours. In all, I recorded approximately 9 ½ hours of lessons in Grade 3 (10 lessons) and 24 ½ hours of lessons in Grade 6 (13 lessons). For each lesson I kept a record of the main steps of the lesson as shown in Figure 5.1, documenting the lesson time in running minutes and indicating where a transcript was available with a 'X'. This was useful to be able to refer back easily to parts of a lesson without having to revisit the recording itself. The illustration below is taken from the fourth lesson on Length held in Grade 3:

Time	***Development of Lesson***	***Transcript***	***Notes***
…	…	…	…
29-33	Next exercise. Now cm are given e.g. 271cm on whiteboard, and children are to change to m's and cm's. Focus on place value: 'how many tens?'	X	
34-38	Teacher asks children to look at book exercise; they are to work it out. After some minutes, teacher writes exercise on board.		
39-41	Correction of SW.		
42-54	Go to page 5 to revise 1km = 1000m. Examples of the type 2km 265cm = 2365m on the board. Teacher erases and they work it again 'together' since teacher states that kilometres are a bit hard for them.	X	At one point in the lesson (I believe earlier than here) teacher states that they will do more about km in Year 4.
…	…	…	…

Figure 5.1 Sample of overview of a lesson

A classroom situation is a complex one and it was necessary to focus my attention on what was relevant to my research interests. Mason (2002) explained that observing or noticing in any situation is based on making distinctions (light-dark, regions of colour or texture and so on) and the distinctions one makes are based on one's theory about what is worth attending to. That is, Mason stated that what we notice is *what we are prepared to notice.* Research itself also involves noticing and thus involves stressing some aspects and ignoring others. So for example, I focused on a point in a lesson where a teacher introduced a new mathematical word, but ignored – in the sense of not attributing significance – the teacher's summing up of a previous topic; I focused on a pupil's question related to a mathematical word, but ignored her outlining of a computation.

During the pilot study, using one camera meant that I had to copy down all the whiteboard work, leaving me with very little time to reflect on the lesson. On the other hand, the camera arrangement in the main study freed my time to observe the lesson and jot down reflections in a notebook. These notes dealt with the use of English or Maltese and things that struck me in the use of mathematical words. In order to be able to match the timing of my reflection with the classroom video, I made a note of the actual time (e.g. 8.25am) and the running minute of the lesson (e.g. 15). I organised the notes in the following format:

Time	Minute	Reflections

Figure 5.2. Format for keeping field notes

Two examples of reflections are the following:

> *Discussing what we need for a graph. Vocabulary doesn't seem to be new. Ask girls about axes. T said: "Daniela named the lines". Reference to standard [names]. [Grade 6 Graphs, Lesson 1 minute12]*
>
> *T gives procedure. If [she] doesn't allow speaking and children's own suggestions, they [children] have little opportunity to use language. [Grade 3 Multiplication and Division, Lesson 4 minute 50]*

Every evening I went through my notes and typed them out systematically, grouping points according to categories such as mathematical word use, the Maltese/English issue, possible questions to ask teacher/pupils after the lessons and so on. These reflections paved the way for future analysis of the data.

Another step in the processing of lesson data was tracking the use of the mathematical words. This was done by watching the recording of each lesson and keeping a handwritten record as in the following sample taken from the first lesson on Length in Grade 6:

Minute	Teacher Use	Pupil Use
…	…	…
34	Height	
	Height	
	Length	
	Length	
35	Height	
	Height	
36	Measure	
	Measure	
	Long	
37	Long	
	Measure	
	Long	
	Length	
	Long	
38	Height	
		Hand span
	Hand spans	
	Width	
	Hand spans	
		Height
		Height
39	…	…

Figure 5. 3. Sample of method for tracking word use

A key step in the processing of the lesson data was transcription of interaction. It was not necessary for me to transcribe all the lessons since, as stated by Furlong and Edwards (1993), underlying theories and assumptions determine not only how data is explained but what is to count as data in the first place. I transcribed parts of the lessons that I considered relevant to my research questions, namely parts that would help me develop a discussion on the use of English and parts where mathematical words were used. However, transcribing is a very time-consuming activity (Swan, 1994), and notwithstanding selection, transcribing lesson excerpts proved to be a lengthy and intensive exercise. First, since the selected words were interspersed throughout the lesson, identifying beginning and end-points of transcription excerpts was not always straightforward. Furthermore, producing a transcript complete with talk, gestures and whiteboard work, involved viewing both camera recordings.

5.6.3 Interviews with teachers

For qualitative research, the interview is a predominant means of data gathering (Sanger, 1996). Since I proceeded in a very similar manner with the teachers in both the pilot and the main study, I will only give details of the main study.

I started off by conducting an interview where we discussed the language policy and mathematical vocabulary in general. This discussion was held with both teachers together. I chose this arrangement so that the teachers would feel relaxed and also because I thought that a more dynamic discussion might be developed around the issue. The rest of the interviews were held individually. This was partly because the topics and/or words differed across the Grades and therefore could not be discussed together, and partly because when the teachers had been together, I noticed that one of them tended to initiate points of conversation, while the other followed. When interviewed alone, I felt that the latter could speak more autonomously.

I conducted two interviews per topic. The first discussion helped me to prepare myself for the topic at hand. I asked the teacher what her main intentions for the week were, what she considered to be the key words, which she considered 'new' to the pupils, and if she anticipated that Maltese might be helpful at any stage. In the second interview, held *after* the week's lessons, I asked her what she believed she had shared with the pupils over the week with respect to the selected mathematical words. I also discussed particular statements with her. For example, I asked Rose what she had meant when she told the girls '*division is repeated subtraction*'. I did this in order to add to my interpretation of what I had observed. Occasionally I included a question related to something the children may have said during their interviews (without quoting individual children directly). For example, I asked Rose for her view regarding the fact that the girls did not recall a particular word at all. Hence, the teacher's responses sometimes served to "supplement, clarify or validate the data gained from other sources" (Zevenbergen, 1998, p.23). I considered this to be a form of triangulation. Mertens (1998) defined triangulation as a means for checking information or establishing consistency of evidence.

According to Wellington (2000), the style and approach to interviewing depends on the purpose of the interview and since I wished to get the teacher's own opinions and interpretation of events, I avoided indicating agreement or otherwise, or giving my own version of things. I avoided asking leading questions, but used a 'tell me about it' approach instead. I consider the type of interviews I carried out to be semi-structured (Wellington, *ibid*). That is, although I had pre-

prepared questions that I wished to go through, I allowed for flexibility in the discussion. The list of interview questions may be found in Appendices H1 – H9, These provided a framework to be built on, although I may have added on other pertinent questions in the course of the interviews.

Prior to the interviews with Rose and Gina, I tried not to show a preference for any language (English or Maltese), in order to leave them free to use the language they preferred. I noted that they used Maltese for informal conversation, so I did the same. However, for the interviews, they opted for using mostly English, possibly because my questions were written and therefore read out in English, or because the activity itself was perceived as more formal. However, some occasional code-switching did occur.

During the pilot study I had been alerted to the issue of 'authenticity' (Cooper, 1993) that refers to participants' recollection of events. Cooper (*ibid*) suggested that statements are authentic when they develop from the interviewee's perceptions of how they actually think they behave when they are teaching or learning. During the pilot phase, the teacher had claimed that one aspect of her approach to teaching Graphs was based on a particular textbook scheme. I was familiar with the scheme and was aware that it did not in fact, advocate the said approach. I decided against pursuing the point, unsure of how to interpret the teacher's statement. On reflection however, I concluded that a teacher – or a child for that matter – can only express their awareness as it is at the time of the research project, and it is these beliefs that constitute the data with which I have to work. If they believe things to be so, then their statements in any regard should be considered as authentic.

The interviews were carried out in a comfortable lounge, which helped to create a relaxed atmosphere. I recorded the sessions using a video-camera and also a cassette-recorder, in order to have a backup. The video indicated gestures, but these tended to be of the 'beats' type as described in Section 3.2 and I did not consider them significant. I chose to record rather than to take notes, since this allowed me to concentrate on the discussion at hand and to maintain eye-contact with the interviewees (Wellington, 2000). Note-taking would have been a constant reminder of the recording aspect, and would have lengthened the interview time. The total running time of the interviews was 3½ hours. I transcribed the interviews verbatim, opting to leave selection for a later stage. The average transcription time was 6 times the interview length which is in line with Bassey's (1999) prediction of 5 – 10 times.

5.6.4 Interviews with pupils

After each topic, I interviewed the pupils regarding their opinions about use of English as a medium of instruction, and also asked them to explain the meanings of some mathematical words. The general approach I suggested was that the children explain to a new schoolmate what they had learnt during the week. I started off the discussion by asking them to relate what they had learnt during the week, in the hope that they might indicate meanings in an unguided manner. I then went through a list of mathematical words systematically. I also prepared some questions that paralleled some teacher-questions. For example, if I asked the teacher what she had meant by saying that division is repeated subtraction, I asked the children: "*I heard your teacher say 'division is repeated subtraction' – I wonder what she meant by that*". The pupils were requested to bring their copybook and textbook along, so that we could refer to them if necessary; I considered that these might serve as contexts around which mathematical talk might centre. I also provided paper and pencils in case the pupils wished to write or draw anything.

The interviews with the children were semi-structured. As with the teachers, I had a list of questions to go through, but also allowed for some flexibility. For example, I sometimes changed the order of the questions to follow on in the same direction the pupils may have taken in the course of the talk; or I allowed the pupils to digress a little before getting them back on track, in order not to pressurise them. The list of interview questions can be viewed in Appendices I1 – I4.

For the pupil interviews, I made three significant changes from the pilot study. First of all, during the pilot study, I had interviewed the pupils individually but I had felt that the interviews had had a formal 'question-and-answer' feel about them. In order to render interviews more informal, I considered interviewing the St. Helen's girls in small groups. Eder and Fingerson (2002) suggested that when interviewing children, a power differential exists not only by virtue of the researcher's role in posing questions, but also because of the age difference. Eder and Fingerson (*ibid*) believed that when interviewing takes place in group settings instead, children feel more relaxed. This is partly because they are in the company of their peers and also because they outnumber the adults. Cohen *et al* (2000) also noted that group interviews may be less intimidating than individual ones.

Since grouping had been missing in the pilot study, I needed to try it out. I did this with a few girls in relation to a previous topic covered in class. I first opted for threes, but as Dunne and Bennett (1990) warned, one girl got side-lined because of the creation of a dyad and an outsider.

I found a paired interview to be more satisfactory, and hence used pairs for my main interviews. I avoided placing two weak pupils together since Dunne and Bennett (*ibid*) suggested that in such a grouping, pupils' knowledge and understanding may be insufficient to provide each other with support. Rather, I mixed the abilities, but avoided placing a weak student with a very good one, in the fear that she may let the better pupil do the talking. Another possible difficulty for group interviews is domination and feeling uncomfortable with certain peers (Cohen *et al*, 2000). Hence, I avoided pairing up a very dominant character with a very shy, quiet one. I relied on the teachers' advice regarding the suitability of pairs. Although a little self-conscious to begin with, the pupils were very forthcoming and I am grateful for their enthusiasm and trust. In general I found it easier to interview the older pupils, who gave each other space to talk and supported each other in their explanations. The younger girls needed occasional prodding and some reminders to let each other talk.

A second change was that during the pilot, I had recorded the interviews using only a cassette-recorder, while for the main study, I also used a video-camera. This served not only as a back-up, but also enabled me to document gestures as part of expression (although I did not consider 'beats' as significant).

Third, during the pilot study, I had interviewed the pupils using Mixed Maltese English, for example "***Xi tfisser il-kelma*** *cost?" [****What does the word*** *cost* ***mean****?]."* This was done because the lesson interaction had been conducted in this fashion. The situation at St. Helen's was different since the medium of instruction was English. Therefore a new element in the interviews was to ask the children their opinion regarding the use of English. Another new element was to ask them for two explanations of the chosen mathematical words. They were asked to explain them to a 'new' Maltese schoolmate who we pretended was going to join their class, then to an *English* one. I believed that this would be useful since English was the language they were expressing themselves in during the lessons, while I conjectured that they might feel more at ease or be able to speak at more length in their first language. It is worth noting that in four out of the twelve interviews, I interviewed the girls in English *first*, then Maltese, so that what is listed as 'Question 2' in the interview sheet was actually the first question for these pupils, while 'Question 1' was tackled second. The reason why I did this was in anticipation of general differences in the length or appropriateness of pupils' explanations when they used either English or Maltese. I supposed that if I varied the order, any differences would be more confidently attributed to the language itself, rather than to the fact that the language was consistently used for a *first* or *second* explanation.

In reality, I found that pupils often gave the ‘same’ explanation in both languages, as in the following example where a Grade 6 pupil Monica is giving a meaning for *perimeter*. (The **bold** print represents words that were originally in Maltese, while the key to referencing can be found in Section 5.7):

> **You’ve got to do** fifty times two, **because you’ve got this and this** (*she touches the shorter sides of the coffee table in front of her*), **and** sixty times two (*she touches the longer sides of the table*). **Then you** plus **the** answers. [G6Length(C)Q1]
>
> Fifty times two and sixty times two. And then you add the answers and the answer you get is the perimeter. [G6Length(C)Q2]

However, on some occasions, the pupils offered a different aspect of meaning for a word, as in the following explanations for *kilometre* offered by a Grade 3 pupil, Sonia:

> **When we had** kilometre, **often the** teacher **would tell us for example ‘how much are** two kilometres?**’ That’s** two thousand [metres]. [G3Length(A)Q1]
>
> Kilometres are when you measure something big. [G3Length(A)Q2].

Thus, giving two versions allowed pupils to sometimes *add* to their explanations. I also noted that occasionally, pupils used Maltese during an ‘English’ explanation and vice versa. As for the remaining interview questions, where I asked children about their views about language use or about particular incidents during the week, they were left free to use any language that they preferred. Most pupils opted to use Mixed Maltese English, especially in the case of the younger ones, and I myself followed their lead in my use of language.

Fontana and Frey (1994) stated that an interviewing style has a bearing on the results of the study. Indeed, although my main motivation of pairing the children up was to help the children to relax, the fact that I paired the children had a significant effect on the data I collected. Wellington (2000) suggested that advantages of grouping include helping each other relax, ‘warming up’ and jogging each other’s memories and thoughts, and these were elements that I noted when interviewing the children in pairs. Indeed, they sometimes gave joint explanations as in the following excerpt that formed part of the pupils’ general overview of the week’s work. The speakers are myself, and Grade 3 pupils Petra and Charlotte. Interruptions are indicated by /:

(*The pupils are looking through their textbook and recalling what they learnt during the week. Petra is looking at a page that shows an estimation exercise. Pictures of various objects are shown*).

I:	So what did you have to do on this page?
Petra:	Em, we, the teacher find it, all these things and /
Charlotte:	/ gave /
Petra:	/ gave it to four groups. One group we have to measure/
Charlotte:	/ this things (*indicates the pictures on the page*).
Petra:	This thing. But first we have to guess what they are [i.e. estimate the length].

[G3Length(B)Q2]

Therefore, at times it was not possible to separate the statements to document individual expression of meaning. In such cases, I kept the conversation intact during transcription, viewing understanding as shared expression.

In all, I collected 8½ hours of pupil interview data which I documented as follows: first I produced a full transcription of the interview. For ease of reference, I then grouped the responses to the questions together, so that all opinions regarding the use of English were kept together, all expressions of meaning for the word *length* were grouped together and so on. In anticipation of excerpts being read by non-Maltese readers, I translated the 'Maltese' explanations of words into English as faithfully as possible. The organisation of this data is illustrated in Figure 5. 4.

Length			
	English explanation	**'Maltese' explanation translated**	**Original 'Maltese' explanation**
Petra & Charlotte	I: Length. I think the teacher mentioned that word… Ch: No! P: Yes, yes. A length is like measuring. I: (*I ask for a sentence with the word 'length'*). Ch: I have a length. I: Suppose 'Rosie' asks you 'what do you mean you have a length?' Ch: (*Silence*). I: Can you help her out Petra? P: Em, have a length means you have something to measure? I: Can you give me a sentence with the word length? P: My measure, my measuring tape has a length. (p4-5)	P: (*Thinks and looks up her copybook).* Ch: **Here in the** Notes **we had some …** (*opens notes and finds the Length-Measurement table*)**. Here, look, here's the word** 'length' (*points to title 'length' on page*). P: length **means** \\ Ch\\ (*interrupts*) kilometres. P: Measurement. **That's kilometre.** Ch: length **is** measurement. P: Measurement, that means **to measure**.	P: (*Thinks and looks up her copybook).* Ch: **Hawnhekk fin-** Notes **kellna xi …** (*opens notes and finds the Length-Measurement table*)**. Hawn ara, il-kelma** 'length' **hawn qegħda** (*points to title 'length' on page*). P: length **jiġifieri** \\ Ch\\ (*interrupts*) kilometres. P: Measurement. **Kilometru jiġi.** Ch: length **hija** measurement. P: Measurement, **jiġifieri tkejjel.**
Sonia & Jessica	…	…	…
Kim & Fiona	…	…	…

Figure 5.4. Sample of documentation of children's explanations for a mathematical word.

The transcription and translation of pupil interviews was a time-consuming process, taking an average of 11.5 times the original interview time to complete.

I am aware of some limitations of this phase of the data collection. The first regards the question I asked regarding whether the words had been new to the girls or not. It was important for me to ask this in order to be confident in concluding that shared meaning was a result of the teaching process. However, I recognise that I asked them for meanings of words in isolation (i.e. not in a sentence, or situation), and this might have influenced their judgment regarding whether they knew the word or not. Furthermore, even if they stated that a word had already been familiar, the term 'familiarity' does not tell me exactly what it was that they knew with respect to the word. I will revisit the idea of familiarity in the course of the forthcoming analysis.

Another limitation was that despite my efforts to encourage both pupils to speak, there were some instances when one of a pair did not contribute or when a response was not very clear. This may have happened when a pupil abandoned her explanation of a word because her friend took over the conversation. I also noted a few instances in the Grade 3 interviews when one pupil's contribution was 'I think the same' (as her friend), leaving me in doubt as to whether to consider this pupil's response or not. The consequence of such situations was that I might have collected data from only four or five children regarding a particular point. However, I did not consider this to be too problematic, since whether I considered four, five or six pupils, I could not take their expression of meaning to be representative of the class. As is typical of case studies, the data collected was considered valuable in its own right and I considered each available contribution as a possible interpretation by a possible interpreter (Eco, 1976).

5.7 Conclusion: General plan of analysis

In Table 5.1 I show how I organised the analysis chapters that follow, and indicate which of the above described data I utilised for the various discussions.

	Chapter	Focus of Discussion	Data utulised
Language as Medium	6	Tensions arising between the language recommendations and other NMC policies.	Mainly interview data (teacher and pupils); Some classroom transcriptions; personal reflections on classroom observations and the NMC document.
	7	Reflections on the extent and coherence of language used; inclusion of mathematical words by pupils; some initial reflections on a possible Mixed Maltese English register.	Mainly classroom transcriptions; some interview data (teacher and pupils).
Shifting from medium to message	8	Teachers' and pupils' opinions regarding how familiar the selected words were prior to lessons; relating this with the frequency of the word use in class.	Interview data (teacher and pupils); the tracking of word use in class.
Language as message	9	Reflections on what rendered meaning clear for words related to the Grade 3 topic 'Multiplication and Division'. Also searching for other features that enable sharing of meaning.	Classroom excerpts and pupil explanations of words (pupil interviews); some teacher interview data.
	10	As above for the Year 6 topic 'Graphs'.	As above.
	11	As above for the Year 3 and 6 topic 'Length'.	As above.

Table 5.1. Indication of which data was utilised for the various discussions

In my discussion on shared meaning, I started by comparing the expression of meaning offered by the pupils and by the teacher in the classroom. For example, I considered the following statements as 'similar' (see Section 5.7 for key to referencing):

> (*Teacher in class, with reference to a rectangular desk top*): "To find the perimeter, we [can] multiply the length by two, the breadth by two, then add the answers" [G6Length5minute6]
>
> (*Monica during interview, referring to the respective sides of a rectangle coffee table*]: "Fifty times two and sixty times two. And then you add the answers and the answer you get is the perimeter." [G6Length(C)Q2]

Both the teacher and the pupil linked *perimeter*, *length* and *breadth* through a similar relationship determined by multiplication and addition. The reference context may be slightly different in that the tables referred to were different ones, and the 'familiar words' used varied. However, as stated by Chapman (2003), similar meanings can be expressed through different semantic terms, so that I can consider the meanings expressed to be comparable. On the other hand, if the pupil had suggested that, say, *perimeter* was the name of a particular polygon, or that it referred to the height of an object, then I would have concluded that the meaning had not been shared. Another form of dissimilarity could have been if the pupil had no recollection of the word *perimeter* being used in class.

Having established similarity / dissimilarity, I then used my semiotic model to explore how clearly the word had been used during the lessons. Thus, I attempted to explain why the word meaning appeared to have been shared with the pupils or otherwise. Although I assumed that clarity was important, I remained open to other features of word use in the classroom that might have had some bearing on the pupils' ability to recollect and offer an appropriate explanation for the word.

It is important to note the boundaries of my study. It is not my intention to discuss all relevant words that might potentially be used for a given topic. Rather, I will be restricting my reflections to words – and hence aspects of the respective topics - that the teacher chose to work on. Furthermore, although teacher knowledge of mathematics is an important issue in itself, it is also beyond the scope of this study to reflect on it in detail. Occasionally, I noted that a meaning expressed by Rose and Gina did not coincide with that generally held by the wider mathematical community. For example, Gina's definition of regular shapes was that such a shape had all sides equal, without any consideration of the angles. I will be drawing attention to such instances in the course of my analysis, but whatever the teacher's own apparent meaning for a word, I will reflect on whether this meaning was shared or not.

In the following chapters, I will be presenting transcribed excerpts to support my discussions. In many cases, I could have presented more than one excerpt, but do not do so for the sake of being concise. I will reference the excerpts as indicated in Table 5.2:

Reference	**Point in the interview or lesson**
G3Length2minute45	The 2^{nd} lesson on Length in Grade 3, starting at the 45^{th} minute
RoseLength(2)Q4	Rose's 2^{nd} interview related to the topic Length, Question 4
G3Length(A)Q5	Grade 3 pupil interview A, Question 5

Table 5.2. Referencing of interview and lesson excerpts

In cases when Maltese speech was used, I have generally presented the translated version for the benefit of a non-Maltese reader. Original transcripts were retained when the use of the two languages was essential to the point being made. The conventions I will use for transcription are as follows:

Abbreviation / Feature	**What it represents**
I	Myself
T	Teacher
P	Unidentified pupil (e.g. out of camera view)
A, B, C etc.	Pupils by name (Angela, Barbara, Claire etc.)
…	Brief pause in speech
[words]	Words added in to render the text clearer
(…)	Some speech is omitted since it is not essential for my discussion
/	Interrupted speech
Bold	Maltese or translated speech
CAPITALS	Words stressed by speaker's tone of voice

Table 5.3. Transcript conventions

I now start my discussions by reflecting on the NMC recommendation for the use of English as a medium of instruction for mathematics.

CHAPTER SIX

The Immersion Recommendation and Other NMC Principles: Tensions Arising

6.1 Introduction

> *How does the NMC recommendation regarding the use of English for mathematics fit in with other educational principles promoted in the same document?*

My first research interest is the consideration of the National Minimum Curriculum recommendation for the use of English as a medium of instruction for mathematics. I can view the mathematics classroom as a community of practice (Lave and Wenger, 1991), one which involves using and understanding mathematical language. However, Lave and Wenger also suggested that any community of practice exists in relation to other overlapping practices. I can consider as potential overlapping practices, the realisation of other NMC principles. For example, the NMC recommends the four practices of inclusive education, collaborative learning and - more specifically related to language - 'consistency' of language and the strengthening of Maltese.

My interest in how well the immersion recommendation 'fits in' with other NMC principles can be viewed in terms of the existence of tensions between the use of English and the realisation of the four afore-mentioned various ideals. I became aware of these tensions as I reflected on the NMC document in the light of what I observed in the lessons or was told during the interviews. In this chapter I discuss these tensions, but first I give an overview of the adoption of the immersion policy at St. Helen's, and the participating teachers' opinion regarding the immersion approach.

6.2 The adoption of the immersion policy at St. Helen's

The Head of School explained to me that the learning of English had always been a priority for the school. However, the academic year 2002/2003 – the year in which I collected my data - was the first year that a policy was stipulated more 'officially' for both the primary and secondary levels of the school. Since the vast majority of the girls were Maltese speaking, parental opinion had been sounded out through a survey prior to deciding on the immersion approach. I do not know the exact format of the circular posted to parents, but the Head of School reported to me that a high number of the parents had been in favour of the use of English as a medium of instruction. Hence, all the academic subjects were to be taught through English except for those closely tied to the local culture, that is, Social Studies, the Catholic Religion and Maltese. Furthermore, students were expected to address administrative staff in English.

My two participating teachers were in favour of the immersion approach. Rose, the Grade 3 teacher, said that she was aware that 'some research' had shown Maltese children to be generally weak in English and so, as a school, they were making it a point to use a lot of English. Both teachers mentioned the fact that mathematics textbooks and exams were written in English and they believed that using English in the classroom helped the pupils to understand these texts. Rose suggested that although the main textbook they were using at the time (a U.K. publication, Merttens and Kirkby (1999)) contained little written language, this was likely to increase over the years. Hence she wished to prepare the pupils for this eventuality:

> "I don't work for today. I think about the coming years. I really wish for them that they will not find it difficult. I wish to train them" [RoseLength(2)Q8].

Similarly Gina, the Grade 6 teacher, believed that using English in the primary school was a good preparation for the secondary level:

> **"So** it happens that when they go up to the secondary classes, they are shy to speak in English, because once it's not drummed in[to] them in the primary … By the time they are in Form 4 and Form 5 they realise that they have to start speaking the language *fluently*. If for nothing else, they have to sit for an exam, and they have their oral… They find it very difficult because they are so conscious of speaking the language. And they're just stuck **then**". [Gina GeneralDiscussionQ2].

Both teachers acknowledged that there was still a long way to go, but they were generally optimistic about the approach:

> "We are still in the very early stages of this, and we're trying very, very hard … most of them don't speak English at home, so they still find it difficult to express themselves between themselves, even with me" [RoseLength(2)Q7]

> "There is a trend … that speaking in English amongst themselves is something silly…. If you speak in English, you're – you know – a snob. Nowadays we're trying to break that attitude. It's not easy, but it's coming. At least in class they don't feel that they are snobs speaking in English. [GinaLength(2)Q8].

The generally positive views held by the Head of School, teachers, parents (apparently) and, as I will shortly indicate, a number of pupils, appeared to be allowing the policy to be implemented: high motivation of the participants has been found to be a key factor in immersion programmes (Baker, 2001). My personal experience led me to conclude that the girls at St. Helen's were generally less inhibited in using English than I would expect children their age to be, although I am not in a position to comment on the effectiveness of the programme in terms of the teaching and learning of English.

Rose admitted to me that deep down she believed it would be 'easier' to teach the children mathematics using Maltese, and that she believed the children would be able to understand her better. She said that from her past experiences:

> "Maltese has always helped them [the pupils] more, especially in our school [where children come from Maltese speaking backgrounds]" [RoseLength(2)Q8].

However, Rose chose to suppress her personal views in favour of the new school policy, saying:

> **"The Curriculum tells us to use English more**" [RoseGeneralDiscussionQ2]

Indeed, the Head of School and both teachers considered the NMC document to be a major influence on the school's policy and they seemed to accept the document as an authority that 'must be right'. It is interesting to note that in the published document, the NMC is *personified* through expressions such as 'the NMC encourages' (Ministry of Education, 1999, p.79), 'the NMC accepts' (p.82), the NMC advocates' (p.79) and so on. I feel that this apparent detachment from human agency masks the fact that the document was written by a group of people - albeit after a period of consultation with various stakeholders - all with particular backgrounds and opinions. A first step in our local discussion is to recognise that these opinions can be questioned and challenged. Reflecting on possible tensions is one way of doing this.

6.3 The issue of inclusion

One educational aspect that is given prominence in the NMC is the idea of inclusion. The document states:

> "An inclusive education is based on a commitment (…) to fully acknowledge individual difference" (Ministry of Education, 1999, p.30) (…) The National Minimum Curriculum commits the State to ensure that all students are provided with the best possible educational experiences, irrespective of their social realities and abilities (*ibid*, p.36).

Educational inclusion requires an awareness of difference, and one difference that may be present in a Maltese classroom is pupils' different levels of English. Such variation may be a consequence of varying degrees of exposure to, and support for the language outside school. Indeed, I believe that the intention of the writers of the NMC is precisely to narrow the gap between the pupils' different levels. They appear to believe that this can be achieved through further exposure and use of English, a belief echoed in Rose's comment:

> "What they lack at home, we make up here" [RoseGeneralDiscussionQ3a].

It is interesting to contrast the general satisfaction expressed by Rose and Gina regarding the approach being used, with the opinions expressed by the pupils interviewed. I asked the pupils for their opinion regarding the use of English as a medium of instruction for mathematics and have summarised their opinions in Table 6.1 overleaf. I have grouped the pupils as 'high achievers', 'average pupils' and so on, as described by their respective teachers. As indicated in the Table, some pupils said that it made no difference which language was used, two of them even saying that they generally preferred English as a language over Maltese.

Grade 3		**Grade 6**	
'High' Achievers		*'Very Good' pupils*	
Petra	Feels comfortable with use of English.	Claudette	Lived in Scotland for four years, so finds it easier to express herself in English during the maths lesson.
Sonia	Understands mathematics through English. Asks questions in English.	Joanne	It makes no difference to her whether maths is taught through English or Maltese.
Maria	Understands both English and Maltese.	Clare	Thinks that using English helps them practice expressing themselves; sometimes she has to use Maltese because it is difficult to translate every word into English.
Sandra	Feels good about using English for mathematics.		
		'Good' pupils	
		Celia	Prefers the lessons in English, because generally prefers the English language to Maltese.
		Federica	It makes no difference to her whether mathematics is taught through English or Maltese
'Average' pupils		Rachel	Feels that since she is now used to using English for maths, she would not want to change to using Maltese.
Jessica	Likes English more than Maltese. If lesson is in English, she asks in English, if in Maltese, she asks in Maltese.	Dorianne	English is an important language for life. She has now got used to asking questions in English. As the teacher speaks in English, she translates into Maltese in her head. When she does not understand she asks the teacher to explain in Maltese and she understands better.
Kelly	Likes both languages; Has no particular preference regarding maths lessons.		
Kim	Does not always understand the lesson in English, so she asks questions. Sometimes she does *not* ask, assuming the teacher will explain shortly.	*'Fair' pupils*	
Charlotte	Likes English a lot, but does not always understand it. Sometimes does not know how to word a question, so has to think about it, or uses Maltese instead. Believes that using English for maths helps them get used to the language.	Charmaine	Using English helps her to learn more words. Now that they got used to using English, she would not want to change to Maltese.
Lara	(Discussion not held with her and her partner)	Stefania	Sometimes there are words she does not understand, so she asks and the teacher explains in Maltese.
		Monica	Using English helps them to learn English. Thinks that they would understand more if the lesson were done in Maltese.
'Weak' pupils		*'Weak' pupils*	
Fiona	Sometimes understands, sometimes does not. Feels that she does not know much English. Generally asks questions in Maltese because she is afraid to use English.	Josephine	When she needs to ask a question, she sometimes thinks she will 'mess up', but she tries.
Ramona	Sometimes understands English, but would prefer Maltese, because English is a bit 'hard'. Thinks she would understand more if lessons were done in Maltese.	Katrina	Would be pleased if lessons were to be given in Maltese, because she would understand better.
Melissa	(Discussion not held with her and her partner).		

Table 6.1: Pupils' opinion regarding the use of English as a medium of instruction for mathematics

The pupils' responses indicate that there was a tendency for those who sounded more confident to be the 'higher achievers', while the 'average' and 'weak' ones were more likely to express some reservations related to potential understanding. For example, Grade 3 pupil Fiona said:

> "**I don't know too much English… Sometimes I understand the lesson, sometimes I don't.**" [G3Length(C)Q7].

Similarly, when I asked the Grade 6 'weak' pupil Katrina how she would feel if mathematics were to be taught through Maltese rather than in English, she answered:

> "**I'd be pleased, because I'd understand it better**" [G6Length(B)Q7].

The tendency in favour of Maltese was more evident with the younger pupils. One reason for this may have been that by virtue of their age, the girls may generally have had less experience with English than the older girls. Another reason may have been that the Grade 3 teacher tended to adhere to the immersion policy more strictly than the Grade 6 teacher, offering the pupils less flexibility with language.

Some of the 'weaker' pupils indicated a degree of inhibition regarding the asking of questions. For example, Fiona (Grade 3) stated:

> "**I don't like asking (…) I ask in Maltese because I'm afraid to include something in English**" [G3Length(C)Q7].

I examined the level of participation of the pupils in order to see if this 'matched' what they said during the interviews. However, I found that it was difficult to establish a match or otherwise in Grade 3, since pupil participation was limited by the Initiation-Response-Feedback style of interaction (Sinclair and Coulthard, 1975). In this class, the talk was very teacher-directed and pupil responses tended to be very short, usually one or two words (e.g. "one hundred"). Furthermore, giving answers to an exercise was usually conducted in a turn-taking fashion, with the teacher asking pupils along the desk rows. Consequently, the Grade 3 pupils rarely asked questions spontaneously or commented in any length and therefore I had limited opportunity to check whether they actually used Maltese or English for questions or suggestions. So for example, although Sonia said during the interviews that she asked questions in English, in the lessons I observed, she did not actually ask any questions; similarly, Fiona, who said that she preferred to ask in Maltese, did not ask questions in either language.

Hence for Grade 3, I can only comment on the few occasions when longer statements were offered by the pupils which in fact were given in Maltese. For example, Jessica, who during the interview said that she would ask questions in English, once voiced a query regarding a homework task as follows:

> *(The teacher has just asked the pupils to draw three lines of any length for homework).*
> Jessica: **Tikteb kemm tkun għamilthom [twal]?**
> **[Do you write how [long] you've done them?]**
> [G3Length3minute63]

In Grade 6, the pupils were given more opportunities to express themselves spontaneously and therefore there was more opportunity for me to reflect on the 'match' between Grade 6 opinions and what actually went on the classroom. Although the girls used English more than their younger counterparts, still, there was a tendency for them to use Maltese more than they indicated in their interviews. So, for example, although Rachel said that she would not wish to change back to learning mathematics through Maltese since she was now used to English, yet she herself often asked questions in Maltese.

Another indicator of language preference came through the interviews themselves: the 'weaker' pupils appeared to have greater difficulty in expressing themselves during the English part of the interview, and switched back to Maltese at the first opportunity. Indeed, Fiona, Ramona (Grade 3) and Katrina (Grade 6) pulled faces or initially showed reluctance to speak when I suggested that part of the interview was to be conducted in English. Admittedly I cannot say to what extent individual pupils were being restricted by the medium being used. After all, whether Katrina, quoted previously, would understand more if taught through Maltese or not, neither she nor I can be sure. I can only report what Katrina believed.

Although in my study it was the 'weak' pupils who tended to voice reservations, this may not always be the case. I suggest that as part of the local debate it would be useful to consider the discomfort and possible lack of understanding that pupils may experience through the immersion approach. If a practice of inclusion is considered to be a desirable by the NMC writers, then I suggest that further reflection may be necessary on how language can be used in an inclusive way.

I felt that the teachers were using English in the belief that in time pupils would 'pick it up'. For example, Rose stated:

> "When we meet together – staff meetings or seminars – we are told to stick to English. They tell us 'don't worry, they'll [the pupils] get used to it as they go along'" [RoseLength(2)Q7]

However, if considered with respect to mathematical language, this attitude is not in line with researchers' recommendations (e.g. Rothman and Cohen, 1989, Zaskis, 2000) who suggested that mathematical language should be taught *explicitly*, especially when it is presented in a second language. Perhaps, given our local situation, a possible alternative to the immersion approach may be to teach through code-switching as suggested by Adler (2001) while offering *explicit attention* to mathematical English. I conjecture that this might contribute not only to the development of mathematical language and understanding, but also to the development of *general* English competence. Perhaps more explicit attention to language may be a more effective way to help 'close the gap' between pupil differences in English as the NMC so desires.

6.4 The issue of consistency

Another tension I recognised concerns the notion of consistency. The NMC recommends 'consistency' of language for the teaching of mathematics:

> "In classroom situations when teaching [Mathematics, Science and Technology] in English poses difficulties, code-switching can be used as a means of communication. These situations apart, the National Minimum Curriculum advocates consistency in the use of language during the teaching-learning process" (Ministry of Education, 1999, p.79).

By consistency, I understand the writers of the document to mean that the participants should, if possible, use only *English*. I assume that the recommendation cannot be referring to Maltese, since the ingrained practices of retaining subject-specific words in English, and of using English textbooks are unlikely to change in the foreseeable future. In this section, I explore to what extent it appeared possible to put the NMC principle of consistency into practice.

During the interviews, Rose said that she would use Maltese if she felt that the pupils were not understanding. However, in practice, her use of Maltese was negligible. If she needed to repeat an explanation, this was done in English. The interaction was very structured, leaving little opportunity for pupil questions or diversions. The pupils' answers were short, often consisting of mathematical words. Since in Malta these tend to be said in English, then the interaction between teacher and pupils was in fact carried out in English, thus fulfilling the NMC ideal for 'consistent' use of language. A typical stretch of interaction is illustrated in the excerpt below.

> (*Rose has introduced that relationship between metres and kilometres and they class is practising some examples*).
> Teacher: Now if I had two metres?
> Pupils: (*Hands go up*)
> Teacher: How much is that? Kim?
> Kim: One thousand.
> Teacher: A thousand? Jessica?
> Jessica: Two hundred.
> Teacher: Two hundred.
> (…)
> Teacher: If I have three metres? Yolande?
> Yolande: Three hundred.
> Teacher: I want to hear the whole of it.
> Yolande: Centimetres.
> [G3Length4minute2]

The pupils told me that while they generally spoke to the teacher in English during the mathematics lesson, they spoke to their *classmates* in Maltese. In fact, I did note that the classroom pupil-pupil talk took place in Maltese. This may have been 'social' talk as, for example, when a pupil asked to borrow a ruler, or when one pupil commented to a classmate about her broken arm in plaster. It could also have been talk related to the mathematical work at hand. For example, in the two occasions that group work was set (both during the topic 'Length') I heard statements such as the ones shown below as pupils addressed each other [G3Length3minute19]:

> (*The girls are working in groups of four, estimating and measuring various objects*).
> Pupil 1(group1): **Kemm taħseb li hi? … Naħseb** forty.
> [**How much do you think it is? … I think it's** forty**]**
> (…)
> Pupil 2(group2) **Ikteb x'taħseb, imbagħad kejjel.**
> [**Write down what you think, then measure**].
> (…)
> Pupil 3(group3) **L-ewwel tiegħi mbagħad ta' Kim.**
> **[First mine, then Kim's].**
>
> [G3Length3minute19]

Hence, while the whole-class interaction offered 'consistent' use of English, pupils' interaction situations did not. Interestingly, this is a similar situation to that observed by Setati and Adler

(2000) in South Africa. In the contexts observed by these researchers, pupils used their first language interspersed with mathematical English for group interaction, but switched to English for the 'public' domain, that is, the whole-class interaction.

In Grade 6, the whole-class interaction generally took place in English, with Gina being generally consistent in her use of English. Like Rose, Gina said that she would use Maltese if the need arose and indeed, she did code-switch 'consciously' twice during the topic 'Graphs' in order to help the girls tackle the task at hand.

> (*The pupils are expected to express a pass rate of 4 / 5 as a percentage. Some of the girls are having difficulty understanding what was required of them).*
>
> Pupil 1: **Miss, jien *din* ma nistax nifhem.**
> **[Miss, it's *this* I can't understand**]
>
> Teacher: What percentage passed you mean? What percentage of STUDENTS passed. Not the percentage of marks.
>
> Pupil 2: **Mhux kollha għaddew**.
> [**They didn't all pass**].
>
> Teacher: **Ara, ħa ngħidlek mil-Malti.**
> [**Look, let me tell you in Maltese**].
>
> Listen to me (sic). Let's pretend one, two, three, four, five (*indicates five pupils*) and I'm giving a lesson.
>
> (…) **Issa, minnkom il-ħamsa, jekk waħda m'għaddietx, x'persentaġġ**, what percentage, **x'persentaġġ għaddew**? [**Now, from you five, if one of you did not pass, what percentage**, what percentage, **what percentage passed?**]
>
> Pupil 3: Four all over five.
>
> Teacher: (*Addresses Pupil 1*). Can you understand that? **Issa** [**now**] four all over five is not a percentage, it's a number. You bring it to percentage, **jiġifieri** [**that is**] you bring it out of a hundred as if you were a hundred [girls].
>
> [G6Graphs2minute32]

Gina justified the use of Maltese as follows:

> It's pointless keeping to English when I am not reaching to certain girls … I won't leave anybody behind! **Now** if they get mixed up, if they are not sure of what they need to do and perhaps they feel much better if I speak in Maltese, and they can keep up the pace with me, I prefer to speak in Maltese and then translate in English." [GinaGraphs(2)Q7]

In a similar vein, Adler (2001) noted that teachers in the South African classrooms she observed were faced with a 'dilemma of code-switching' between English, the official classroom language, and the pupils' first language. The dilemma of code-switching was also evident in Gina's classroom, but only to a limited degree as explained above. Gina, like Rose, generally did not 'allow' herself to code-switch during whole-class interaction, but rather, made a conscious effort to use English in line with the school policy.

If Maltese was used, it tended to be used by the pupils. The Grade 6 pupils contributed to classroom talk more than the Grade 3 girls since Gina offered them more opportunities to talk by asking them open-ended questions. She also allowed the girls to pass spontaneous comments and encouraged them to ask questions. Sometimes, Gina would ask the pupil to repeat in English, as in the following excerpt:

> (*The teacher has started drawing a graph on the whiteboard. She has written two sets of scales in the course of her explanation*).
>
> Kirsty: **Miss, għaliex għandna tnejn [skali]?**
> [**Miss, why do we have two? [scales]**]
>
> Teacher: English!
> Kirsty: Miss, why do we have two, em, x-axis and y-axis?
> Teacher: Because first I explained, then I had to draw. Then I explained again! They're both the same.
> [G6Graphs2minute133]

The insistence on English appeared to be an attempt by Gina to promote 'consistency' on the pupils' part. However, in the course of the lessons there were many occasions when Gina did *not* insist on the switch to English, although she herself generally answered in English as in the following excerpt:

> (*The class is correcting a straight-line graph they had done for homework. The pupil in question is commenting that she could not find the answer to a question from her graph because her y-axis was not long enough*).
>
> Pupil: **Miss, ma stajtx insibha għax waqaft** one hundred and fifty.
> [**Miss, I couldn't find it, because I stopped at** one hundred and fifty].
>
> Teacher: Em, don't you have any more space [on the copybook]?.
>
> Pupil: **Għax imbagħad il-**line (**unclear**).
> [**Because then the** line (**unclear**)]

Teacher: OK, but you can work it out. (*They then go on to work out the answer by simple proportion*).
[Graphs3minute67]

Instances when more Maltese tended to be used was when Gina spoke to pupils on a one-to-one basis, as she did when for example, she walked around the classroom monitoring pupils' written work. In these situations, the pupils often used Maltese, with Gina sometimes answering in English and sometimes in Maltese (this was more evident with the 'weaker' pupils). An interesting pattern of talk was evident when Gina was discussing a graph with Josephine, a pupil she had described as 'weak' in mathematics. Gina used Maltese when addressing Josephine directly, but changed to English when she addressed the whole class. As she turned her attention back to Josephine, she shifted back into Maltese. The excerpt illustrates this 'private / public' distinction identified by Setati and Adler (2000).

(The teacher is walking around the class, checking the graph that the pupils had drawn. Josephine had found some difficulty marking kilograms on the x-axis).

Teacher: **Fejn huma l-**kilos? (…) **Dan hawn, dan hawn** …
Where are the kilos? (…) **This [goes] here, this [goes] here** …
(*The teacher writes in '1', '2' on the x-axis on Josephine's copybook. She then raises her head and addresses the whole class*). Mark your numbers girls!

(*Turns to Josephine again*). Where is your half?
Hawn il-half?
[**Is the** half **here**?](*Touches a point on the x-axis*).

Mela hawnhekk trid tiktbu l-half.
[So this is where you've got to write the half].

[G6Graphs3minute17]

Finally, as was the case in Grade 3, the pupil-to-pupil talk, both social and that related to the task at hand, was conducted in Maltese. In this classroom, more pupil-to-pupil talk occurred than in Grade 3, since the pupils were allowed to talk to each other quietly as they worked on an exercise. Hence, I noted more use of Maltese in Grade 6 than in Grade 3 and in this respect the language used was not 'consistent' in the way the NMC presumably intended. I concluded that ultimately, Gina was trying to find a practical balance between promoting English, and at the same time ensuring understanding of mathematical ideas. As she admitted:

> "One thing I always make sure is that I don't sacrifice a maths lesson for English" (GinaLength(2)Q6].

However, I think that if a teacher allows pupils to use Maltese, this creates a tension between the implementation of the immersion and 'consistency' recommendations, and undermines the whole objective of immersion. After all, the main problem perceived in Malta is not actually exposure to English, but production (speaking and writing) of the language. Thus a tension is created between the ideal of the policy and its practical implementation. Furthermore, I believe that consistency of language use may be generally easier for a teacher to achieve than for the pupil - assuming that a teacher feels confident using English. The writers of the NMC may have underestimated the difficulty encountered in insisting that pupils use their second language to discuss at length, or they may have inadvertently assumed that in a mathematics classroom it is the teacher who is to do most of the talking. While this may have been the case traditionally, I have argued in Chapter 3 that more recently, it is considered much more desirable for pupils to contribute more significantly to classroom talk. Furthermore, the NMC assumption actually works counter to another NMC principle, that of 'developing thinking through co-operation'.

6.5 The issue of developing thinking through co-operation

One NMC principle promotes a 'new' pedagogy of co-operation, expressed in the document as follows:

> "The pedagogy of co-operation, based on group work, should transform the hitherto competitive and individualistic tendencies typical of Maltese classrooms, into a hive of synergetic collective endeavour. It is through discussion, exchange of ideas and collaboration with others that we clarify our thoughts, learn how to ask questions, change and elaborate our concepts and gain exposure to different modes of thinking and action." (Ministry of Education, 1999, p.35).

As a mathematics educator, I welcome the NMC's promotion of the development of thinking through co-operative discussion. However, I think that learning mathematics through English may work counter to this ideal. Although perhaps the pupils' English may improve enough over time to allow 'discussion', 'exchange of ideas' and 'collaboration', I conjecture that at any one time, primary school pupils might communicate more effectively in their first language than in their second. Furthermore, the teacher herself may feel reluctant to set activities that require the use of a lot of talk because of this anticipated difficulty. Indeed, I observed the Grade 6 girls communicating with their teacher using longer stretches of English and they often appeared to experience some difficulty. I will discuss this point in the following chapter,.

It is also worth noting that while the teachers I observed were fluent in English and were able to use it throughout the lessons and beyond, it may be the case that some Maltese teachers lack this confidence and flexibility of language, so that they themselves may not be in an ideal position to lead creative and analytic discussions in a second language.

6.6 Strengthening the Maltese language

The NMC document recommends a strengthening of the Maltese language:

> "This document regards bilingualism as entailing the effective, precise and confident use of the country's two official languages (…) The process of strengthening Maltese, the language used by the majority of Maltese children in their home and community environment, contributes to their holistic development". (Ministry of Education, 1999, p.37)

I feel that the immersion approach suggested by the NMC writers actually works against this ideal since the method may actually pass an indirect message to young learners that Maltese is not a suitable language for mathematics, thus detracting from the value of the language. Furthermore, when English is used as a medium for mathematics, Maltese mathematical vocabulary is neither used nor developed explicitly. The ideal of strengthening the language is a far-reaching one that goes well beyond the mathematics classroom, but I will broach the discussion through one of the questions I set the girls during the interviews.

I asked the pupils for Maltese equivalents for the English mathematical words under consideration and I found that for a number of the words, the girls could not give equivalents. In such cases, the girls either stated that no Maltese equivalents existed, or if they did that they themselves did not know them since they used the English ones. At one point, Celia (Grade 6) said:

> "I think I know much more [mathematical] words in English than Maltese" [G6Length(B)Q4]

Now while it might be true that the English versions might be more accessible through the classroom register, and that for some mathematical words Maltese translations are not commonly used, it is also true to say that for many of the words I was focusing on, translations do actually exist. Furthermore, many of these translations are not simply 'dictionary entries', but words which in my experience can, and are, used. It is not easy to define in a clear-cut manner which words one might 'expect' pupils to be able to translate; as a very general rule, words that have common everyday usage such as *length*, *height* etc. are more likely to have commonly used Maltese equivalents than words that I might loosely call more 'technical' such as *axis*.

The girls' attempts at offering translations are summarised below although not all the words under consideration are shown. This is because for the Grade 6 topic 'Graphs', I did not ask for translations because some of them do not exist, while others are 'dictionary' entries that are rarely used. Hence, I assumed that the girls would not be familiar with them. Some Grade 3 words are not mentioned below since some English mathematical words that had been used by the teacher were not recalled at all by the pupils and hence they could not comment about possible translations. Furthermore, in the summary below, not all the six pupils interviewed are represented since occasionally, a pupil may have remained silent, not offering a translation, yet not stating explicitly that none existed.

Grade 3		Grade 6
<u>*Length*</u> One girl offered appropriate translations of **metru**, **ċentimetru** and **kilometru** for *metre*, *centimetre* and *kilometre* respectively. The other pupils stated that no translation existed for these three words. Two pupils said that no equivalent for *length* existed, two did not offer a translation and two gave the Maltese for **big** and **measure** as possible translations. Three pupils offered the appropriate verbs **kejjel** or **immexerja** for *to measure*, while **kejjel** was again offered by two of them as a possible translation for *measurement*. *Longer* and *shorter* were translated correctly by same girl who offered translations for the units (see above), while the other pupils gave Maltese equivalents of **long/short**, **longer/small**, **big/small**.		<u>*Length*</u> The girls offered appropriate translations for *metre*, *centimetre*, *kilometre* and *millimetre*. Appropriate translations were also given for *length, height, longer /shorter* and *to measure* (two versions). For *width*, one pupil offered the correct translation, one **thickness**, and the other four could not give a translation. No translations were known for *perimeter* and *measurement* (in my experience, Maltese **perimeter** in rarely used, but **qies** /**kejl** [measurement] are common words).
<u>*Multiplication and Division*</u> Four pupils offered the loan-shift **ixxerja** for *share*, although I noted that during the course of the interviews they also used the other translation **qasam**. Two pupils offered the appropriate **iggruppja** for *to group.* *Multiplication, multiply by, times, division, divide by* and *tables* were said by all pupils not to have translations. These words, except for (*multiplication) tables* do have translations, which however are not commonly used as part of the classroom register.		<u>*Graphs*</u> Discussion not held for this topic (see main text for reason).

Table 6.2 Translations offered by pupils for mathematical words

The Grade 6 girls seemed more able to offer Maltese equivalents for Length related words. I can only guess at why this was the case, since I did not specifically ask them if they could recall when and where they learnt them. Possibly their age and experience played a part. Another possibility is that the connections between the languages may have been made in earlier Grades, prior to the establishing of the school immersion policy.

Strictly speaking, the pupils' inability to suggest translations does not necessarily mean that they did not know them or that they did not use them in other contexts; all I can say is that they did not make the link when asked. Admittedly, I was asking for translations in isolation, rather than as part of a meaningful situation, and this might have accounted for their difficulty. However, their difficulty might suggest that the divide between school and home mathematics (Whitebread, 1995) is reinforced when the subject is taught through a second language. If this is the case, then the immersion approach also works against yet another, fifth, ideal expressed in the NMC, that of linking school with home:

> "Students consider the learning process to be relevant when they establish a link between school work and their personal experiences" (Ministry of Education, 1999, p.32).

I do not wish to suggest that pupils should learn Maltese mathematical words that are rarely used, since this would be an artificial use of language. However, I favour children being able to make the connections between the two languages for two reasons. First of all, if they have not yet encountered the word in Maltese through other experiences, becoming familiar with the word may enhance their knowledge of the language, thus fulfilling the NMC ideal of 'strengthening' Maltese. Secondly, if the pupils *do* know and use the Maltese word outside school, then this knowledge can be tapped into when teaching the English words in order to help convey meaning. I saw this strategy being used in my pilot study during the teaching of the topic 'Money'. The teacher had explicitly drawn on her pupils' knowledge of the Maltese words for *change* and *cost* in order to teach the new English mathematical words. However, linking with Maltese equivalents can only happen in a classroom where code-switching is 'allowed' while the immersion approach eliminates the possibility of translation being used as a pedagogic strategy by the teacher.

6.7 Conclusion

The overlap between various practices – a 'mathematics' classroom, an 'inclusive' classroom, a 'cooperative' classroom – render a teaching/learning situation a multi-faceted one. The insistence on English may complicate the situation by creating tensions between the ideals.

If the NMC immersion recommendation is adopted on a large scale in Malta, then strategies that teachers have developed over time to help them cope with the code-switching situation will be side-lined. Indeed, the NMC suggestion actually reduces opportunities or even the *need* to identify and share with others such strategies. On the other hand, accepting that code-switching can be used for mathematics implies viewing code-switching not as a problem, but as a resource (Setati and Adler, 2000). If we hold this latter view collectively, then we could focus our attention on finding effective ways to link the (mixed) spoken mathematics with the (English) written texts. Rather than eliminate the situation by using English throughout – something that my data suggests might not be so easy, nor advisable, to do – I suggest that we look for and share effective ways to tackle the link. I believe that this is a fundamental issue in our language debate for mathematics.

CHAPTER SEVEN

The Use and Development of an English Mathematics Register

7.1 Introduction

> *How much, and with what ease, do pupils talk in immersion classrooms? How 'mathematical' is their talk, in terms of the inclusion of mathematical vocabulary?*

In the second part of my consideration of language as a medium, I now reflect on the use and development of the English mathematics register. I am interested mainly in the language used by the pupils, although I consider this in relation to the teachers' contributions, since whole-class interaction is constituted by both.

I start this chapter by looking at the extent to which the pupils were generally encouraged to talk and also the ease with which they used English to express themselves; I then direct my attention to the insertion of mathematical words, reflecting on similarities to, and variations from, 'conventional' mathematical English. I end the chapter by digressing from the English mathematics register to reflect on a possible Mixed Maltese English one. This latter discussion had not been planned, but since I collected some Mixed data as part of the pupils' interviews, I found that I was able to offer some initial thoughts in this regard.

7.2 Extent and ease of pupil talk

The first aspect of the class talk that I would like to examine is how much opportunity the Grade 3 and Grade 6 pupils were given to talk in class. In both classes, a 'whole-class' teacher-directed approach was used. Rose and Gina tended to teach from the front of the class, guiding the pupils by means of a series of questions. Within each class, pupils covered the same work, at roughly the same pace and class corrections were commonly carried out. However, the styles of interaction were not identical, resulting in different levels of pupil involvement. In Grade 3, the style of teacher–pupil interaction was almost exclusively the I-R-F type (Sinclair and Coulthard, 1975). Pupils tended to give short responses which, when correct, were repeated or confirmed by the teacher. If incorrect, Rose generally turned to another pupil for an alternative response. Sometimes, Rose allowed chorus answers as in the following excerpt:

(*The class is working out a textbook exercise wherein given a number of metres and centimetres, the pupils are required to express the length in centimetres*).

Teacher: If I have two metres, thirty-four centimetres … (*writes '2m 34cm' on the whiteboard*). Now we said two metres are how many centimetres?

Pupils: Two hundred!

Teacher: So I have two hundred and …?

Pupils: Thirty-four!

[G3Length3minute10]

On other occasions, Rose selected particular pupils to answer a question:

(*The class is correcting a textbook exercise. Given a number of gloves, the pupils were required to find the number of fingers*).

Teacher: Kelly, how many gloves did we have?

Kelly: Two.

Teacher: Two. And how many fingers?

Kelly: Five.

Teacher: And that gives me?

Kelly: Ten.

Teacher: Ten.

[G3Mult&Div1minute37]

The Grade 3 pupils did occasionally utter statements of more than one word, but even these were rather short. For example:

(The class is recalling the difference between a horizontal and vertical line, an idea discussed in a previous week).

Melissa: Horizontal, it is … (*smiles and inclines her head to the side*) sleeping.

[G3Mult&Div3minute2]

One of the longest statements I heard was offered by Nadia:

(*The teacher has just indicated that a metre is approximately the length of her arm span. She has asked pupils if they can estimate a length of 50cm. Nadia stretches one arm out to her side. Teacher asks her to stand up, apparently inviting an explanation*).

Nadia: You put out your hand only [only one arm] and that's a fifty centimetres.

[G3Length3minute35]

For a great part of the time, the girls' were required to supply an answer that was already 'in the teacher's head', and their responses were of one or two words only. I cannot exclude the fact that Rose used this style because she was conscious of the potential difficulty the pupils might find in using English, but from my observations of the trial lessons and Rose's general relationship with the pupils, I believe that this was her preferred pedagogical style. I concluded that in the case of the Grade 3 pupils, opportunities to talk were de-limited by the style of interaction encouraged by the teacher. If I draw on Halliday's (1978) discussion of register, then I can consider the 'style of interaction' to be a result of both the interpersonal relationship and the role the pupils' language was expected to play. The fact that the interventions were short meant that the pupils contributed with ease to the generally predictable pattern of talk; in this class it seemed that it was possible for English to be used by the pupils as a medium of communication.

Although a similar style was often used in the Grade 6 class, Gina also encouraged her pupils to use more speech. Furthermore, she allowed spontaneous calling out of answers and questions and occasional light-hearted comments. Gina referred to her pupils as her 'young ladies' and outside lessons, Gina often listened to what her pupils had to say about a variety of things. This relationship was also reflected in the mathematics lessons by means of open-ended questions (e.g. why? / how?) or invitations to share ideas. For example:

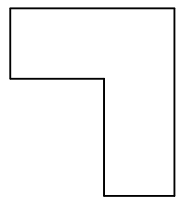
> (*A class discussion is taking place regarding methods primitive man may have utilised to measure*).
> Teacher: Can you use your body to measure?
> Monica: Yes!
> Teacher: How Monica? Tell us.
> [Gr6Length1minute6]

Following such questions, I would expect that the role played by pupils' language is that of offering opinions and spontaneous ideas. Indeed, 'why?' and 'how?' requests are generally intended to promote pupil talk (Clemson and Clemson, 1994). However, I noted that at these points in the lesson, the Grade 6 girls often showed some difficulty in expressing themselves, and resorted to using gestures. For example, the above quoted episode involving Monica developed as follows:

> Teacher: How Monica? Tell us.
> Monica: (*Shows up her right pointer finger*).
> Teacher: Your finger. Tell me how. Measure the desk with your finger.
> Monica: (*Lays her finger along the edge of the desk and moves it along like a worm. The other girls laugh*).
> Teacher: That's a bit difficult, but good!
> [G6Length1minute6]

Hence, although invited to 'tell', Monica *showed* instead. Of course, gestures are an integral part of any communication and indeed, both the teachers themselves used gestures as they taught. However, in the teachers' case, gestures tended to *complement* the speech rather than *replace* it. For example, in the following excerpt, Gina used a deictic gesture to direct pupils' attention:

> *(An L-shaped desk arrangement is drawn on the whiteboard. The teacher is discussing the dimensions with Celia).*



> Teacher: This is ninety-nine you said? (*She touches the longest vertical side*).
> Celia: Yes.
> Teacher: What else? (*She touches the lower adjacent side*).
> Celia: Forty centimetres.
> [G6Length5minute29]

At other times, the gesture may have been used to "make concrete the concept" (Goldin-Meadow *et al*, 1999, p.4) as follows:

> *(Rose is giving the pupils an indication of the length of a metre)*
> Teacher: (*Stretches arms out at her sides*). This is about one metre, about one hundred centimetres.
> [G3Length 1minute35]

Gestures did not seem to be used much in Grade 3, and I conjecture that this was because of the brevity of the pupils' responses. On the other hand, it seemed to me that the Grade 6 pupils sometimes used gestures because they found some difficulty in expressing themselves verbally. Often, when this happened, Gina would rephrase the sentence or 'fill in' the missing language, sometimes stating: 'So you're saying that' or 'she is saying that …'. Such a situation is illustrated in the following excerpt:

(*The pupils are working on a tiling problem. The teacher has just distinguished between area and perimeter. Dorianne looks up at the ceiling beam that runs across the width of the room.*).

Dorianne: That … (*points to the beam*).

Teacher: The beam.

Dorianne: (*Shrugs her shoulders as though to say 'whatever'*). Em, it's like perimeter, it's around. But if it's like a wall … It's the area (*moves hand up and down as one would do when painting a wall*).

Teacher: Ah, we can take the length of the beam, she says, but if we need to wall the space in between, we have to find the area.

Dorianne: (*Nods*).

[G6Length8minute64]

This is not to say that the Grade 6 pupils found difficulty with every contribution. There were many occasions when they offered responses of say, one sentence long and these were offered in full. For example:

(*The class is reading off information from a graph*).

Teacher: Why do the numbers in the vertical axis stop at hundred? Caroline?

Caroline: Because the total [highest] mark is hundred.

[G6Graphs4minute18]

Most of the Grade 6 pupils indicated that they could express themselves effectively in statements of this length, although the accuracy of expression (in the sense of correct use of English) varied from pupil to pupil. However, generally speaking, the longer the response expected, the greater the tendency for the girls to falter in their expression. For example in the following excerpt, Clare - who was considered by her teacher to be one of the 'brightest' pupils in the class, and from my observations was one of the pupils who participated most in class - tried to explain how prehistoric man may have measured objects:

"If I was living in those times, I would get something straight (*gestures a horizontal orientation with one hand*) and example, if I know that a leaf, em, a leaf is this long (*holds her hands about 15cm apart, palms facing each other*), I do straight. And for example I do like this (*hands about 50cm apart*). I get a small ruler like this (*hands 20cm apart*) and I know that it is like this (*hands a bit further apart*). (…) I do this (*makes an action with one hand resembling a chop*), em … I would do a sign (*repeats action*) so that if I want to measure a leaf, I see if it's smaller, if it's big or smaller". [G6Length1minute 3]

Although I got a general sense of what Clare meant to say (that is, to compare the length of a leaf with that of a given ruler), the details of her argument were not altogether clear to me. Whether Gina and the other pupils followed or not, I cannot tell; after Clare's explanation the teacher simply said:

> "Yes, yes, even in the very olden days, man did find a way to measure."[G6Length1minute3]

Adler (2001) noted what she referred to as a 'dilemma of mediation' in the South African classrooms she observed. This involved the teacher finding a balance between allowing the students to express themselves freely and on the other hand, intervening to guide them to more effective communication. In the case of Grade 6, 'free expression' sometimes meant that the girls expressed themselves somewhat vaguely or through gestures. This type of expression was accepted by Gina, just as she accepted Maltese on some occasions. Gina admitted that sometimes her pupils would be 'fighting for words' when trying to use English, and in relation to the possible use of Maltese in such a situation, she told me during the first interview:

> **"Then you get some Maltese and some English… [then I say] 'so,** you mean that …' and you [I] say it in English. **Then you [I] tell her, '**let me see if you've understood' … And she tries it in English. That would be a step forward, **no**?" [GinaGeneralDiscussionQ3d]

However, I noted that in practice, this second attempt at expressing themselves did not actually take place, whether it was Maltese, gestures or incorrect English that had been used. Gina had also stated that she was reluctant to discourage her pupils' efforts to use English and in fact it appeared to me that once the teacher understood the pupil's contribution, the lesson moved on. Hadar and Butterworth (1997) suggested that body movements increase when hesitation in speech occurs and this is what might have been happening as the Grade 6 girls tried to participate through English. Hence even though Gina generally tried to encourage her pupils to express themselves by asking open-ended questions, the English immersion approach might have been restricting the pupils' verbal contributions.

As a point of interest, I would like to note a difference between the pupils' ease of articulation during the lessons and the interviews I carried out. I noted that during the interviews, the Grade 3 girls expressed themselves at greater length than in the classroom while the Grade 6 girls were often able to express themselves more articulately (whether their ideas were correct or not). For example:

> (Melissa, Grade 3): Sharing means that we have … (*long pause*) … when we have three, three girls and we have three, three children. You have to share them with the three girls. [G3Mult&Div (B)Q2].

> (Clare, Grade 6): Regular is when you have a shape that has all the sides equal; and irregular, you have a shape and it doesn't have all the sides equal [G6Length(C)Q2].

The longer and clearer articulation may be a result of the fact that the English explanation may have even been the second attempt to explain the words, after an explanation in Maltese had already been given (although for some pupils it was the other way round). For example, with regard to the two explanations just quoted, both Melissa and Clare had explained the words *sharing* and *regular* once before, through Mixed Maltese English. However, I think that a fuller explanation lies in the elements of the social situation that defined the interview context. As pointed out by Halliday (1978), the elements of a context determine the register. In the case of the Grade 3 pupils, the role of their language was substantially different to that in the classroom. In the classroom, the role of their language was to 'fill in the blanks' left by the teacher who was in an obvious authoritative position; on the other hand, during the interview they were asked to offer fuller explanations to help me out, which they willingly attempted to do. With regard to Grade 6, if I compare opportunities to talk in the classroom with the interview context, then I can say that during the lessons, Gina moved from closed to open questions in a rather unpredictable manner, and the discussions were various. On the other hand, the interview took on a predicable pattern since much of it consisted of me reading out a word from a printed list and the pupils offering an explanation. I allowed all the pupils as much time as necessary to talk and even encouraged them to support each other whenever they felt it was helpful. Consequently, the interpersonal relationships during the interview and the role that the pupils' language got to play were different to those of the classroom, resulting in different types of contributions by the pupils.

Morgan (1998) suggested that there exists more than one mathematics register, in the sense that the language in a primary classroom is different to that in an A-level class in terms of say, vocabulary and level of argumentation. I can add that even for the same speakers, the register can vary given different situations and I acknowledge that the register used by the pupils in the classroom is not the only situation in which these pupils may communicate their mathematical ideas. However, ultimately I think that it is more relevant to reflect on how language was used in the classroom than in the interviews, since this is what constitutes teaching and learning of mathematics, unlike the one-off interview experience with a researcher.

The inhibition of extended talk in class should be a source of concern for us in Malta since we are in the process of phasing in a new Mathematics Scheme for primary schools (Merttens and Kirkby, 1999). The scheme includes investigative tasks, practical work and the sharing of mental strategies, activities that generally promote and indeed, *require* more use of language than traditional approaches. Most independent and Church schools (including St. Helen's) had already introduced the scheme for all Grades when I carried out my study, while the State sector adopted a phasing-in approach, with the aim of having all Grades use the Scheme by the academic year 2006 – 2007. I have had informal conversations with education officers who visit schools regularly and they report that although some teachers are skirting around the above-mentioned 'innovative' activities (I got the impression that both Gina and Rose themselves were two such teachers), many others wish to use the scheme to its full advantage. This being the case, I am curious about the dilemmas that these latter teachers might face if they choose to insist on English in line with the NMC recommendation.

7.3 Inclusion of mathematical words

One element that characterises a mathematics register is the presence of mathematical vocabulary. I was interested in exploring whether topic-related words were used by the teacher and pupils and if so, how frequently. I was also curious about which tasks appeared to encourage their use, and hence support the development of the register. Before presenting my results, two points should be noted. First, I feel that it is not possible to establish what can be considered 'many' or 'few' mathematical words in a stretch of talk, and so I will restrict myself to comparing the teacher's use of the words with the pupils' use. Second, my discussion only involves the presence of *topic-related* words (see Section 5.5.3) for how these words were selected) and not *all* mathematical words that were used.

In order to compare the use of the topic-related words by the teacher and pupils, I tracked the number of times the words were used in the lessons. (For simplicity's sake, I grouped variations of a word together as, for example, the verbs *measure*, *measures*, *measured*, and *measuring*). A limitation of this exercise was that I could only count words that were picked up by the cameras. Generally these were words used during the 'whole-class' interactions. However, I cannot exclude the fact that the words under consideration may have been used on occasions when they were not recorded by the video-camera, such as one-to-one teacher-pupil or pupil-pupil interaction while the girls worked on a task. However, I believe that the frequencies I present offer a good general picture of word use.

The frequency of use by teachers and pupils are shown in Tables 7.1a&b. The entries under 'Teacher' refer to either Rose (Grade 3) or Gina (Grade 6), while the entries under 'Pupils' refer to use by any one of the pupils in the class. Some pupils contributed to this section more than others: for example Grade 6 pupil Kirsty used the expression *x-axis* three times in the second lesson on 'Graphs', while several pupils did not use it at all during that same lesson. I have presented the words in order of frequency of use during the hours dedicated to the respective topics.

Grade 3						
'Length' (4.2 hours)				**'Multiplication and Division' (5.2 hours)**		
Word	*Teacher*	*Pupils*		*Word*	*Teacher*	*Pupils*
Centimetre/s	464	200		Multiply by	162	23
Metre /s	301	43		Division	112	27
Measure/s/ed/ing	99	0		Times	62	43
Long/er/est	67	1		Tables	61	0
Kilometre /s	52	1		Multiplication	54	4
Length/s	37	1		Divide by	39	3
Short/er/est	21	0		Grouping	19	0
Measurement/s	18	1		Sharing	13	1
Width	9	0				
Estimate	7	0				
Height	5	0				
Total	1080	247		**Total**	522	101

Table 7.1a. Frequency of topic-related mathematical words use in Grade 3

Grade 6						
'Length' (12.9 hours)				'Graphs' (11.6 hours)		
Word	*Teacher*	*Pupils*		*Word*	*Teacher*	*Pupils*
Centimetre /s	430	130		Graph	153	15
Metre /s	267	88		x-/y-axis /axes	118	47
Millimetre /s	231	64		Represent / ing	74	26
Measure /s/ed/ing	191	36		Plot /ing /ed	35	2
Length /s	154	36		Scale	30	8
Spans (hand/arm/foot/ wing)	65	7		Pie-graph/chart	23	10
Longer / er /est	58	4		(Drop a) perpendicular	16	0
Perimeter	47	17		(Straight)line graph	15	8
Width	45	17		Block graph	15	1
Kilometre /s	43	19		Data	12	2
Height	35	16		Bar graph	6	4
Measurement /s	32	2				
Breadth	28	15				
Short / er /est	21	1				
Irregular	12	1				
Regular	9	0				
Metric	4	0				
	1727	428			497	123

Table 7.1b. Frequency of topic-related mathematical words use in Grade 6

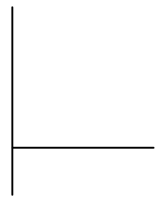
The Tables reveal very clearly that the teachers used the words much more than the pupils. This is perhaps not surprising considering that they talked much more than the pupils and that the exposition of ideas was very much their responsibility. However, the relative use brings to light an interesting point if compared across the Grades. Table 7.2 shows the ratio *teacher frequency / pupil frequency* for the topic-related words under consideration:

Word frequency (teacher) ÷ word frequency (pupils)						
Grade 3				Grade 6		
Length	**M&D**	**Both Topics**		**Length**	**Graphs**	**Both Topics**
4.37	5.17	4.60		4.04	4.04	4.04

Table 7.2. Relative use of topic-related words (teacher frequency ÷ pupil frequency).

Interpreted differently I can say that for every one mathematical word the teacher used, the Grade 3 and Grade 6 pupils used respectively 0.22 and 0.25 words. The results for Grade 3 and Grade 6 do not appear too different and I consider this impression to be one worth reflecting on since the Grade 6 pupils were in fact given much more opportunity to talk in the class than their Grade 3 counterparts. One reason may have been that although the younger girls' contribution was limited, it often included mathematical words (e.g. "twenty centimetres"). However, I was also struck by how easily the Grade 6 pupils 'got by' without using topic-specific words. For example in the following interaction, I might have expected Rachel to follow her teacher's modelling of the term *y-axis* as part of her query, but she did not:

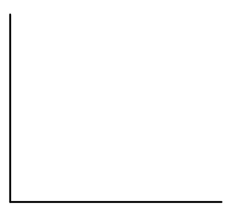
(*The class is about to start copying a graph presented in a textbook. The teacher has just drawn the x- and y-axes on the whiteboard as shown:)*



Rachel: Miss, why at the end of the graph, you drew, you drew another line? (*Gestures with one hand in the air, moving hand vertically downwards*).
Teacher: Where? (*Looks at the board*). Where?
Rachel: The graph/
Teacher: /The x-axis? The x? The y? (*Runs her hand along the y-axis*).
Rachel: **Yes.** Because under … you, you did another line (*repeats vertical gesture*).
Teacher: For the time being we're not using this (*touches 'extension' of y-axis*). Later on, yes.
[G6Graphs2minute41]

According to Pimm (1995), using mathematical words allows the speaker to 'point' and I suggest that one advantage of doing this is to use time more efficiently since not having a name for something can slow down communication. For example, in the following excerpt, time was 'wasted' as Gina established to what it was that Federica was referring:

(*The class is correcting an exercise where the pupils had to draw a straight line*



The teacher asks Federica for instructions:)
Teacher: How am I going to plot these (*runs hand along the x-axis*). Where is your 'birth'?
Federica: (*Points to blackboard*).
Teacher: **Where? Here? Here? Here?**

	(*Touches three points along the x-axis, the first point being the origin*).
Federica:	**There.** (*Points towards the origin*).
Teacher:	**Here?** (*Touches the origin*).
Federica:	(*Nods*). And then three kilogrammes/
Teacher:	Where? Down here? (*Touches origin*).
Federica:	No. There. (*Points to indicate further up the y-axis*).
Teacher:	(*Marks three short horizontal lines on the y-axis and labels the third one '3'*).One, two, three kilos.

[G6Graphs5minute51]

Federica was at the side of the classroom and could not touch the whiteboard as the teacher could. Through further observations and interview data, I have evidence that both Gina and Federica were aware of the expressions *x-axis* and *y-axis*, and Gina even used the word *origin* once during the week. However, in the above excerpt, these words and expressions were not used, leading to what I considered to be inefficient communication. Other potentially helpful expressions may have been 'two centimetres above the origin', 'along the x-axis', 'further up the y-axis' and so on. I do not know whether Federica was inhibited by the use of English at this stage. Possibly, had the interaction been taking place in Mixed Maltese English, she *might* have used expressions such as "two centimeters **iktar 'il fuq miz-**zero" [two centimetres **further up than the** zero], "**mal-**x-axis" [**along the** x-axis]. Of course, this can only be a conjecture on my part, since I have no way of knowing what Federica might have said in an alternative situation.

Admittedly, it is not practical to expect that a mathematical word will be inserted at every potential opportunity, but given that various researchers (e.g. Chapman, 2003, Harvey, 1982; Miller, 1993; Zaskis, 2000) have recommended a gradual move from informal language to more mathematical language, it would seem necessary to encourage pupils to use topic-related mathematical vocabulary. It seems that increasing the amount of talk on the part of pupils may not be sufficient to ensure this.

On the other hand, some words were in fact used substantially more than others by the pupils in both classes. These included *centimetres*, *metres*, *multiply by* and *division* for Grade 3 and *centimetres*, *metres*, *millimetres* and *x/y-axis* for Grade 6. In both classes, the measuring units were used frequently as part of conversion exercises (e.g. expressing 3m as 300cm):

(*The Grade 3 class is working out a conversion exercise together*).
Teacher: What if I have four hundred and nineteen centimetres? How many metres and centimetres is that?
(…)
Kelly: Four metres and nineteen centimetres.
[G3Length4minute32]

(*The Grade 6 class was correcting an exercise where they had to measure the length of some objects drawn on a handout*).
Teacher: How long is our pencil?
Pupil 1: Thirteen centimetres, five millimetres.
Teacher: Another way of saying it Charmaine?
Charmaine: Thirteen and a half.
Teacher: Sara?
Sara: One hundred and thirty-five millimetres.
[G6Length 2minute61]

For the Grade 3 topic ‘Multiplication and Division’, many of the tasks involved working out multiplication and division operations, with the teacher and the pupils often using the expression *multiply by* or *division by* as part of their expression (these were used in the sense of *multiplied by* and *divided by*):

(*The class is looking at a textbook illustration showing six monsters with three legs each. The object of the exercise is to find the total number of legs*).
Teacher: What are you going to do [Angela]?
Angela: Six multiply by three.
[G3Mult&Div5minute6]

In Grade 6, the expressions *x/ y-axis* were used when the girls offered their own suggestions for the scale of a graph:

(*The class is correcting a graph that had been set for homework, where they had to show test marks obtained by students on a bar graph. The pupils had been expected to choose a scale for both axes. The girls are stating the scales they have used).*
Dorianne: I did it, the x-axis, two boxes represents one name of a child **and** y-axis, two centimetres represents two marks.
(…)
Pupil: Miss, **can you do ‘**y-axis **em**… two centimetres represents the mark each child got’?
[G6Graphs2minute11-16]

Hence the words that tended to be used by the teacher and the pupils were those that were ‘needed’ for the exercises at hand. While this might seem like an obvious observation, put another way I can say that words that were not needed were not used. This leads me to conclude that if I wish a word to be used, then I may need to create a situation where it is needed.

Hewitt (1996) suggested that one way to encourage students to use mathematical vocabulary is to subordinate words to other activities thus 'forcing' the words to be practised. This does not mean that complete revamping of activities is necessary – an activity may only need a slight variation in order to fulfill this objective. For example, in one of the exercises Rose set for homework, she asked the pupils to measure some items, giving instructions as follows:

> "Measure your table at home, measure a chair, measure a copybook …"[G3L1minute46]

At home, the pupils wrote the name of the object and a measurement on their copybook (e.g. *book* - *15cm*); it was not necessary to write, say, 'the width of the table' etc. However, a slight variation to this task would have ensured the use of the words *width*, *length* and *height* that Rose had just introduced. The words would have been used not only in the written work, but also in the reporting back of the homework the following day.

Although during the interviews the teachers had stated that they did draw attention to key mathematical words, during the lessons I got the impression that the vocabulary was not a main objective for them. For example, for the above-mentioned activity, Rose explained to me that the aim of the task had been for the pupils to practise using the measuring tape, so in fact it was not important for them to use the words *width*, *length* and *height*. I think Rose's view 'it was not important for them to use the words' is significant and was also reflected in one of Gina's statements, where she said that it was not a source of concern for her that her pupils did not use mathematical words:

> "If I use it [the mathematical word] and use it and use it, it will fall in place, so to speak. (…) As they grow older they will hear it more, so the most important thing for me is that when I say the word they understand it and they know it, and they can express themselves. It's OK with me for the time being [that they do not use the mathematical words]. Later on things will fall into place" [GinaLength(2)Q4]

and that:

> "It's exposure to the language which is most important I think" [GinaLength(2)Q9].

Gina's opinion reflected an emphasis on the receptive aspect of language. However, as Hatch and Brown (1995) suggested, if a teacher is to move pupils as far as possible on the continuum of word knowledge, then word use is essential. I have reservations about adopting an attitude that things will 'later fall into place' since this may result in a situation where a teacher may not consciously give attention to language. That is, it would seem necessary that a teacher appreciate increased use of mathematical vocabulary both as a means of communication and as evidence that the pupils have developed a meaning for the word. This belief may then result in the teacher trying to find a balance between focusing on mathematical language explicitly and on the other hand ensuring that the language is available to students to allow them to talk about ideas. The attempt at this balance is referred to by Adler (2001) as a 'dilemma of transparency'. I suggest that this dilemma is a useful one, but one that was not apparent in the classrooms I observed.

I think that if a teacher is to go to the trouble of adapting her pedagogic approach, then an underlying belief in the usefulness of mathematical language is necessary. I cannot assume that a teacher will hold this belief. Indeed, in her work with primary trainee-teachers, Zaskis (2000) found that some trainees initially resisted a focus on more precise mathematical terminology, feeling that it was unnecessary. This potential resistance is something worth reflecting on for me as a mathematics educator for primary level mathematics.

I note that the issue regarding the inclusion of mathematical words seems to be a pedagogical issue, rather than a consequence of the English immersion approach. I conjecture that, even if the lessons were to be given through Mixed Maltese English, or any other language, the same situation might have arisen. Hence, this issue is one that appears to relate to general mathematical language or 'any' classroom, rather than specifically to an immersion one.

7. 4 Variations in the use of mathematical words

Many of the words were used by the teachers and the pupils in a similar way grammatically to that which, from my experience, I might expect. So for example, *centimetre* was used as a noun to denote the measuring unit, *plot* was used as a verb in relation to graphs, *long* was used as an adjective in relation to size and so on. The following examples illustrate this point:

Rose: How many centimetres do I have?
[G3Length4minute11]

Rose: To work multiplication, you need to know the tables.
[G3Mult&Div2minute0]

Gina: Are the measurements all the same?
[G6Length7minute80]

Gina: You can start plotting it [the graph] if you know how to, and I think that you do.
[G6Graphs1minute84]

Similarly for the pupils:

Melissa: They are not small, they are big.
[G3Length1minute28]

Petra: Four times three.
[G3Mult&Div5minute5]

Caroline: You measure the string.
[G6Length8minute49]

Pupil: [The cents are] on the y-axis.
[G6Graphs3minute4]

However, there were some words that were used in an unexpected way by the teacher and /or the pupils during the lessons or interviews. These are outlined in Table 7.3. Here I also include the word *plus*, since it is an interesting example, even though generally I have not considered this word in my analysis:

Word/expression	Users	Examples
Do plus used as an alternative to *add*	Pupils (both G3 and G6), occasionally Rose	Rose: We cannot do plus. [G3Mult&Div2minute30] Daniela: You do those two plus, and the answer times two. [G6Length5minute7]
Multiply as a noun *Do multiply* as an alternative to *multiply* *Multiply by* as an alternative to *multiplied by*	Grade 3 pupils, Rose	Rose: (*Touching the ÷ notation on the board*). We use this for division not the multiply, all right? [G3Mult&Div2minute17] Lara: When you do multiply, the sum, em, the … answer begins [becomes] smaller. [G3M&D(B)Q2] Pupil: Three multiply by ten. [G3Mult&Div1minute48]
Division as an alternative to *divide/divided by*	All pupils, Rose and Gina	Rose: Ten division by two, because you have two girls. [G3Mult&Div2minute19] Maria: You make … em .. three … three division by three. [G3Mult&Div(A)Q2] Pupil: Can you do the answer division by two? [G6Length5minute18]

Table 7.3. Examples of different uses of some mathematical words.

I think that more 'conventional' modes of mathematical expression could have been achieved with a heightened awareness of how the words were being used. I say this because in fact, Gina and Rose did at times use expressions such as 'you added', 'we multiply' and 'divided by' [for example, G6Graphs2minute26, G3M&D1minute8, G3M&D4minute17 respectively]. There was also evidence that some pupils were familiar with these expressions. For example, Grade 6 pupil Clare said 'you divided', Claudette stated 'you can't multiply' while Grade 3 Sandra used the expression 'we wouldn't add' [G6Length3minute41, G6Length5minute15, G3M&D(A)Q2]. Thus the vocabulary was in fact known to the teachers and to *some* of the pupils.

I discussed the pupils' examples of unusual style of expression with Gina, who acknowledged that they were not altogether correct. She said:

> "I have those kind of sentences when they are little bit excited… As a matter of fact, I don't correct their English. I focus more on what they are doing on their copybook. I do accept 'I did them plus'. I say 'OK, it doesn't matter' or 'OK, you are right' because eventually when they are SURE of what they're doing, they will realise the wrong sentence construction and they will improve on their English as well" [GinaGraphs(2)Q8].

I cannot say how widespread the examples quoted are locally; this can only be established through the observation of many classrooms. However, variations like the ones mentioned are worth reflecting on from both an epistemological and a linguistic point of view.

If I consider the word *multiply* as used in Grade 3, that is: ‘three multiply by four’, ‘do multiply’ and ‘the multiply’, it seems to me that one word *multiply* is sufficing for three expressions namely *multiplied by, multiply* and *multiplication.* Although related, each of these variations offers potentially different mathematical meaning. If *three is multiplied by four*, then the verb is in a passive form, implying that one number is acted on by another and thus obscuring agency (Anghileri, 1995). On the other hand, [*I*] *multiply* is an active verb, that implies a human involvement (see Morgan, 1998, for a detailed discussion of an author’s /speaker’s relation with mathematics). Hence, these variations suggest different ways of how mathematics is brought about and the role of human beings in its creation (Morgan, 2001). Furthermore, as stated by Morgan, using a nominalisation like *multiplication* has an effect on what can be said about this process object, for example it allows the process to act as the theme of a clause (for example, “Multiplication IS … commutative / the inverse of division / etc.”).

Admittedly, I did note that variation was offered by my participants through the use of *do* and *the*: ‘do multiply’ appeared to indicate an active verb, ‘the multiply’ appeared to imply a nominalisation. However, if we are to promote the use of English as the medium of instruction for mathematics, then we need to keep in mind that within the English register there are particular ways of saying things that have now become established. Hence, I think that when promoting *general* English we need to be careful that we do not overlook the importance of promoting appropriate *mathematical* English. Explicit attention to this is important even if a mixed verbal code is accepted, since written mathematics continues to be in English.

The variations outlined above may also be of interest from a linguistic point of view and it may be worth examining how Maltese may be influencing our use of English words. I say this because I noted that some of the variations listed above utilise the words *do* or *make* (e.g. ‘you do multiply’). The over-use of *do/make* by Maltese speakers using English goes beyond the discipline of mathematics. In his research on ‘negative transfer’ from Maltese to English in secondary school students’ written texts, George Camilleri (2004)[5] noted that one of the most

[5] The author cited here is not to be confused with Antoinette Camilleri (1995, or as Camilleri-Grima, 2003) previously quoted in this thesis.

common mistakes he found was the inappropriate use of the verbs *do /make*. The Maltese verb **għamel** translates into both *to do* and *to make*, but is often used in Maltese where *do / make* would not be used in English. Indeed, I observed the verb *to do* being used often by the pupils, for example:

> Ramona: You want [need] to do the tables.
> [G3M&D(C) Q2]

> Clare: I do straight.
> [G6Length1minute3]

It seems that the **do+** structure may be a result of translation, a point that I will continue to discuss in the following section. Indeed, at this stage, I would like to digress a while in order to touch on a possible Mixed Maltese English register. Although not part of my main research interests, I felt that the data I collected during the pupil interviews allowed me to offer some initial thoughts.

7.5 Reflections on a possible Mixed Maltese English register

The context-dependency of register implies that it is too simplistic to suggest that mathematics 'should' or can be expressed totally in English or totally in Maltese. As Camilleri (1995) noted, codeswitches usually serve a pedagogic reason. One reason may be to form links with written texts. For example, in the excerpt below, Kelly is referring to a textbook exercise where the girls had been expected to find the total number of legs of monsters.

> Kelly: **Hemm** two monsters (…) **Allura nagħmlu** two times three. **Kemm hemm** monsters **u saqajn.**
> **[There are** two monsters (…) **So we do** two times three. **How many** monsters **there are, and legs].**
> [G3Mult&Div(C)Q1]

While I have generally argued against immersion and I am in favour of mixing the languages according to need, this does not imply that it is not necessary to reflect on this mix. At present, I can only offer a starting point for this discussion, and the aspect that I will touch on is how well English mathematical words and expression 'fit' into otherwise Maltese speech. I noted that words that were grammatically nouns 'fitted' well into the Maltese speech. These were words like such as *metre, measurement, width, graph, x-axis, multiplication, tables*:

Rachel: **Imma l-**bar graphs **iktar sempliċi mill-**line graphs.
[But the bar graphs **are more simple than the** line graphs].
[G6Graphs(A)Q1]

Petra: **Irridu nużaw il-**kilometres.
[We need to use [the] kilometres].
[G3Length(a) Q1]

The same could be said about words that were adjectives (e.g. *long/longer/longest*, *shorter, tall, regular*/*irregular*).

Sonia: **L-għalliema**] **bdiet tgħajjat lit-tfal biex joħorġu (…) biex niftakru kemm huma twal u** short.
[**The teacher**] **stated to call out children (…) so that we'll remember how tall or** short **they are]**.
[G3Length(A)Q1]

Celia: *(Tpinġi shape fuq karta).* Irregular **dik.**
(*Drew a shape on a sheet of paper).* **That one is** irregular**.**
[G6Length(B)Q1]

Sometimes English insertions were longer than one word, as in the following example:

Joanne: Square **hija** regular shape **għax għandha** all sides equal.
[[A] Square **is a** regular shape **because it's got** all sides equal].
[G6Length(A)Q1]

From a linguistic point of view, the mixing illustrated above complies with Myers-Scotton's (1993) discussion of how two languages come together in code-switching patterns. Of particular interest to me from Myers-Scotton's work is her point that when two languages come together, it is the 'matrix language' (Maltese) rather than the 'embedded language' (English) that provides the morphosyntactic frame. That is, the grammatical procedures are matrix-language based. So for example, in the quotation above, Celia literally said "Irregular **that [one]**" which is in line with Maltese sentence structure. As a result of this phenomenon, I became aware of one potential difficulty with regard to the use of verbs. Maltese verbs are conjugated so that an English verb cannot simply be inserted as is. That is, one cannot simply say '**Jiena** plot **il-**graph' to mean 'I plot the graph' because the verb *plot* must be conjugated to tally with the first person pronoun according to Maltese grammar. In fact, some of the verbs used by the girls were conjugated translations or loanshifts, as in the case of **kejjel** and **immexerja,** respective alternatives for *to measure.* The excerpt below illustrates the use of both words by one pupil: (the reader should note that the transcripts include conjugations which change the appearance of a word – I have underlined the words to draw attention to them):

Jessica: **Meta <u>tkejjel</u> xi ħaġa, meta tkun qed <u>timmexerja</u>.**
[When <u>you measure</u> something, when <u>you're measuring</u>].
[G3Length(A)Q1]

Conjugation was possible if the mathematical Maltese verb was known, as was the case for *measure* in Grades 3 and 6, and *share* in Grade 3. On the other hand, if the Maltese verb was not known, the pupils used alternatives. In Grade 6, for example, the girls did not seem aware of the verb **ipplottja**, a loan shift derived from *plot* and instead they used the conjugations for '**do/make/draw'** [a graph]. The Grade 3 girls did not seem to have single-word Maltese verbs available to them for *multiply* and *divide*, and instead the pupils rendered these words nouns while preceding them by the verb **do [għamel]**. For example:

Lara: **Ma toqgħodx <u>tagħmel</u> il-ħin kollu** <u>plus</u>. **<u>Tagħmel</u>** <u>multiply</u> **mill-ewwel**.
[<u>You</u> don't keep <u>doing</u> <u>plus</u> **all the time**. **<u>You do</u>** <u>multiply</u> **straight away]**.
[G3Mult&Div(B)Q5]

Maria: (…) **<u>tagħmel</u>** <u>adding</u>, **il-**plus. (…) **U <u>tagħmilhom</u>** <u>division</u>.
[(…) **<u>you do</u>** <u>adding</u>, **the** plus. (…) **And <u>you do them</u>** <u>division</u>].
[G3Mult&Div(A)Q5&Q6]

Kelly: **Mbagħad meta <u>tagħmel</u>** <u>dividing</u> **[the answer] ħa jiġik iżgħar**.
[Then when you <u>do</u> <u>dividing</u> **[the answer] will come smaller].**
[G3Mult&Div(C)Q6]

The Maltese excerpts reproduced above may explain the use of the English expressions 'do plus', 'do multiply' and 'make division' by the pupils and even occasionally by the teacher. The expressions appear to be literal translations of what they would say in Maltese. Pimm (1991) stated that the necessity of expressing mathematical ideas can place strains on a language and it is this strain that may result in new ways of expression. Developments in a language may be viewed as *accommodation* or, on the other hand, it can be perceived as *corruption*.

Although talking about mathematics through Mixed Maltese English is quite widespread in Malta, the mixed talk has developed informally over the years unlike say, the Welsh and Māori (see Section 3.6) ones that were developed deliberately. Hence, I cannot say that there actually exists a recognised register in the way that an English mathematics register exists. However, as Roberts (1998) suggested, communities need to decide about what 'sounds' right in developing registers. My data suggests that the development of the Mixed Maltese English mathematics register begs discussion between mathematics educators and linguists alike since to date, the mixed mathematics register has not been studied. As part of our language discussion, it might be helpful to reflect on how mathematical verbs are to be used, since such verbs would seem to be an essential part of any mathematical register. Another point for discussion may be identifying mathematical words and expressions for which no translation exists at present, and reflecting on how the English versions are inserted into Maltese speech or suggesting appropriate translations.

A Parliamentary Act passed in July 2004 and entitled 'The Maltese Language Act' (Department of Information, Malta, 2004) saw the setting up in April 2005 of a National Council for the Maltese Language to further promote and develop the language. One of the Council's duties is to:

> "Establish the correct manner of writing words and phrases which enter the Maltese language from other tongues (…) develop, motivate and enhance the recognition and expression of the Maltese language (…) (*ibid*, p.A384)

Furthermore, the Council is to appoint Technical Committees in sectors of specialisation as follows:

> "Each committee shall be a consultative organ for the discussion of the linguistic policy to be adopted in specialised sectors, such as specific terminology (…) (*ibid*, p.A387)

Such a committee for mathematics may be instrumental in organising discussions on the local spoken register.

7.6 Conclusion

While some language issues discussed in this chapter are related closely to the immersion approach, other points are 'general' ones that appear to be more dependent on general pedagogic approaches or teacher beliefs. I think that it is important for us to tease out these differences in our continuing discussion on language use for mathematics in Malta.

I noted that the younger pupils were required to give only short answers and therefore did not find much difficulty in expressing themselves in English. On the other hand, the Grade 6 pupils were prompted to contribute more, but often they were not very articulate and / or used gestures in replacement of language. Hence, the immersion approach needs to be reflected on in terms of its impact on the use of an English mathematics register.

In this chapter I also reflected on variations in the use of English mathematical words by the teachers and pupils. Since some variations may imply epistemological differences in terms of how mathematics comes into being, and also alter the 'sound' of the established English register, I suggest heightened awareness of such variations. I also reflected briefly on the Mixed Maltese English register used by the pupils during the interviews. Even though I am generally in favour of a Mixed code being used, I am also aware that this brings with it aspects that also need to be discussed between linguistics and mathematics educators. In particular, I suggested that perhaps we need to give attention to how mathematical verbs are used in Maltese.

On the other hand, one issue that may be a general one is the use of mathematical vocabulary. Collectively, the Grade 3 and Grade 6 pupils used mathematical vocabulary in a similar relation to their teacher. This seems to imply that increased pupil talk may not be sufficient to ensure the use of mathematical words, that is, to promote the development of mathematical register. On the other hand, some activities did encourage the use of certain topic-specific words, and this prompts me to conclude that if particular words are to be used, then activities may need to be designed in such a way as to encourage their use. This aspect of register development appears to be a pedagogic one, that is, it is related to a teacher's approach to teaching mathematics rather than a consequence of the immersion approach. Furthermore, increased effort may depend on a teacher's own belief in the importance of promoting mathematical language.

As stated earlier in the study, I can consider language to be both a medium and a message. In the following chapter, I offer a shift from a focus on medium to one on message, by linking the frequency of word-use outlined in this chapter with the apparent familiarity of the words. I will then, in Chapters 9 -11, view language as a message by considering the meaning of the selected mathematical words in the sense of their relation with other words, notation and diagrams.

CHAPTER EIGHT

Shifting the Focus from Medium to Message: Reflections on Familiarity and Frequency

8.1 Introduction

In the second part of my analysis, I will be considering the sharing of meanings of mathematical words. In this regard, it was important for me to be aware of which words were already familiar to the pupils or not, since my interpretation of the success or otherwise of 'sharing meaning' could only be carried out in the light of whether the words were known beforehand or not. If they were known, then the girls may very well have given an appropriate explanation even before the lessons were held. This does not imply that I will not be considering topic-related words that had already been familiar: indeed, the varying explanations that the girls gave for these words problematise the simplistic blanketing notion of 'familiarity'. As Clare (Grade 6) stated with regard to the words *width*, *length* and *height*:

> We used them [in Grade 5], but they weren't that … (*trails off*). I didn't know them exactly like I know them this year. [G6Length(C)Q3]

In this chapter, I discuss overall familiarity. First, I will gauge the teachers' and pupils' opinion about which words were 'new' and check if their opinions matched. Second, I will reflect on whether the words that were considered new by the teacher or the pupils were used more than others in an attempt to share their meaning. I also explore the relationship between the frequency of word use and the pupils' ability to recall the words afterwards since I consider that recollection is a first indication of shared meaning.

8.2 Familiarity of selected mathematical words

An integral part of learning mathematics is the process of coming to know the meaning of words as part of the mathematics register. These words often denote concepts being taught intentionally at school, rendering them non-spontaneous scientific concepts (Vygotsky, 1962). Wells (1999) suggested that the construction of such concepts requires conscious awareness and deliberate application. It seems necessary to me that a teacher is consciously aware of which mathematical words are 'new' to the pupils, since I suggest that these words may need more 'deliberate application' than already-familiar words which may in fact serve a supporting role for the new ones, as indicated in the model I presented in Chapter 4.

I believe that ideally, there should be a close correspondence between what the class teacher and the pupils perceive to be already familiar or 'new' words. Thus, as part of my study, I attempted to see how closely matched the pupils' and teachers' opinions were with respect to the topic-specific words under consideration. The teachers' opinion regarding various words was gauged through interview data collected both before and after the week's work. Unfortunately, the words *metre*, *measurement* (Grade 3), *represent* and *data* (Grade 6) were not commented upon by the respective teachers. The pupils' opinions were explored during the interviews: I asked them which words were new to them this year, and which ones they had already known previously. Sometimes the opinions of the paired pupils varied and a common reason given by the pupils in such a case was that they had been taught by different teachers in the previous Grade.

As already indicated in Chapter 4, the notion of familiarity is not without its limitations. First, for any individual, perhaps more so for young pupils, it may be difficult to recall exactly when a word became familiar. Furthermore, I acknowledge that since the interviews were carried out in pairs, it is possible that children may have been influenced by their classmate's answer, that is, a pupil may have said that a word had been familiar because her classmate had just expressed this opinion. Another point is that the Grade 3 topic 'Multiplication and Division' had already been dealt with three months earlier, so that my question regarding whether the word was new to them this year was a potentially problematic one: it might have been interpreted by some pupils as 'new this year' and by others as 'new this week'. Another limitation is that when asking the Grade 6 pupils about the 'Graphs' vocabulary, I presented the list and asked the girls to indicate which words were new. I assumed that since the list was short, the girls would easily identify the new words. I did not go through the list word by word as I had done with the other topics and as a consequence, some words were not specifically commented on by the girls. Therefore, for the 'Graphs' words, some of my conclusions are based not on a clearly stated opinion, but rather on the fact that the words were not identified as new.

Despite the limitations attached to this exercise and indeed, to the very notion itself of familiarity, I think that the pupils' responses are useful because they indicate the general perception held by the pupils. The opinions expressed by the teachers and pupils regarding what they believed to be new words are summarised in Tables 8.1a & b. The words are listed in order of how frequently they were used in the classroom by the teacher, so as to retain the same order as that presented in Tables 7.1a & b. The teachers' opinions are classified as 'new [to the pupils], 'previously known [by pupils]' or 'fairly /maybe new'. I classified the pupils' opinions as 'new' and 'previously known' if *all* the six pupils interviewed agreed on the point, or as 'opinion varied' if they gave different opinions. (Words that were inadvertently not discussed are left out of the Tables and indicated by (-)).

Grade 3						
Teacher's Opinion				*Pupils' Opinion*		
New	**'Fairly' or 'maybe' new**	**Previously known**		**New**	**Opinion varied**	**Previously Known**
	Centimetre					Centimetre
-	-	-				Metre
To measure					To measure	
		Longer				Longer
Kilometre				Kilometre		
	Length			Length		
		Shorter				Shorter
-	-	-		Measurement		
Width				Width		
Estimate				Estimate		
Height				Height		
Multiply by				Multiply by		
	Division				Division	
		Times Tables				Times Tables
	Multiplication				Multiplication	
Divide by				Divide by		
		Grouping				Grouping
		Sharing				Sharing

Table 8.1a. Opinions regarding word familiarity (Grade 3)

Grade 6						
Teacher's Opinion				*Pupils' Opinion*		
New	**'Fairly' or 'maybe' new**	**Previously known**		**New**	**Opinion varied**	**Previously Known**
		Centimetre				Centimetre
		Metre				Metre
		Millimetre				Millimetre
		Measure				Measure
		Length				Length
		Spans			Spans	
		Longer				Longer
		Perimeter				Perimeter
		Width				Width
		Kilometre				Kilometre
		Height				Height
		Measurement				Measurement
		Breadth				Breadth
		Thickness				Thickness
		Shorter				Shorter
		Tall				Tall
		Thick				Thick
		Irregular			Irregular	
		Regular			Regular	
		Metric		Metric		
		Graph				Graph
x-/y-axis					x-/y-axis	
-	-	-		Represents		
Plot					Plot	
		Scale *			Scale	
	Pie chart				Pie chart	
Drop a perpendicular				Drop a perpendicular		
	Line graph				Line graph	
		Block graph			Block graph	
-	-	-			Data	
		Bar graph				Bar graph

*Familiarity based on the fact that she had mentioned the word briefly during the topic 'Length'

Table 8.1b. Opinions regarding word familiarity (Grade 6)

Examination of the data indicated that although for most words there was a 'match', that is, both teacher and pupils stated that the word was new or already familiar, there were some words for which different opinions were given. So for example, while Rose thought that *measure* would be new, five out of the six children interviewed said that they had used it before; while Gina thought that *x-/y-axis* and *plot* would be new, two of the pupils said that they had used the expressions in the class they had been in the previous year. The implication of such cases is that some pupils in the class may have been more familiar with a word than assumed by the teacher. A more potentially problematic situation may be when the pupils are *less* familiar with a word than assumed by the teacher. For example, while Rose thought that the pupils might already be familiar with the word *length*, the pupils all agreed that this was a new word. While Gina assumed that all the words for the topic 'Length' would be already familiar, the pupils stated that *metric* was new, while there were varying pupil opinions for the words *spans*, *regular* and *irregular*.

I would consider a close correspondence between teacher and pupil opinions to be a positive thing. This would provide a good starting point for the tackling of a new topic in terms of which words (and hence ideas) need particular attention and which words can be assumed familiar (and thus lend support to the meaning of others). Admittedly, school is not the only place that pupils might learn new words and their meanings: for example, Grade 3 pupil Kim said that her mother was a seamstress and she had learnt the word *metre* from home before learning it at school, while Charlotte reported that she had learnt the word *measure* from her older sister. However, as their classmate Petra pointed out:

> **It's mostly when you go to school that you start to learn these words**. [G3Length(B)Q5]

As part of the local language discussion, we may wish to reflect on how teachers of various Grades may work towards a coordinated progression of vocabulary. The new mathematics scheme recently introduced in local schools may be helpful in this regard, since it gives attention to 'key vocabulary' as part of the development of mathematical ideas.

8.3 Relating familiarity with frequency, frequency with recollection

If a topic-related word is assumed to be new by a teacher, then I might expect it to be given more importance during the week and possibly used more in an attempt to share its meaning. I was interested in checking if this was indeed the case for the lessons I observed, that is, Grade 3 'Length' (4.2 hours) and 'Multiplication and Division' (5.2 hours); Grade 6 'Length' (12.9 hours) and 'Graphs' (11.6 hours). I revisited Tables 7.1a & b where I had presented the frequency of word use, and compared them with Tables 8.1a & b. Hence I found the number of times the new words were used by the teacher and these are shown in Table 8.2. I have also included the number of times the pupils used them as a reminder of their relative use (teacher/pupils). Since some words were thought new by the teacher, and some thought new by the pupils, I have indicated these opinions by using different columns for the teacher / pupils. Some words were thought new by both the teacher and the pupils and these words are printed across the two columns (for simplicity's sake, I only considered the words that the teacher and/or *all* the pupils viewed as new):

<table>
<tr><th colspan="4">Grade 3</th><th></th><th colspan="4">Grade 6</th></tr>
<tr><th colspan="2">Thought new by:</th><th colspan="2">Frequency</th><th></th><th colspan="2">Thought new by:</th><th colspan="2">Frequency</th></tr>
<tr><th>Teacher</th><th>All pupils</th><th>Teacher</th><th>Pupils</th><th></th><th>Teacher</th><th>All pupils</th><th>Teacher</th><th>Pupils</th></tr>
<tr><td colspan="2">Multiply by</td><td>185</td><td>23</td><td></td><td>x/y-axis</td><td></td><td>165</td><td>47</td></tr>
<tr><td>measure</td><td></td><td>99</td><td>0</td><td></td><td></td><td>represent*</td><td>100</td><td>26</td></tr>
<tr><td colspan="2">kilometre</td><td>53</td><td>1</td><td></td><td>Plot</td><td></td><td>37</td><td>2</td></tr>
<tr><td colspan="2">divide by</td><td>42</td><td>3</td><td></td><td colspan="2">drop a perpendicular</td><td>16</td><td>0</td></tr>
<tr><td></td><td>Length</td><td>38</td><td>1</td><td></td><td></td><td>metric</td><td>4</td><td>0</td></tr>
<tr><td></td><td>measurement*</td><td>19</td><td>1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Width</td><td>9</td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Estimate</td><td>7</td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Height</td><td>6</td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

*Teacher did not give opinion for this word

Table 8.2 Frequency of use for words considered new by teacher and/or all the pupils

As can be seen from the Table above, some of the words that were recognised as new by both the teacher and/or pupils were not used very frequently. In the classrooms I observed, word frequency depended on the nature of the activities carried out and I noted that these activities did not necessarily encourage the use of the *new* words. As already argued in Section 7.3, a pre-requisite for the use and promotion of mathematical words is the teacher's belief in the value of doing this.

It is interesting to note that the words that were used the least were not recalled by the pupils afterwards. (I acknowledge that there may be a difference in Grade 3 and Grade 6 pupils' ability to recall due to their ages, but it is beyond the scope of this study to explore such a possibility). On the other hand, the Grade 3 pupils stated categorically that they had not heard the words *width, height, estimate;* only two recalled the word *measurement* and they were not confident about its meaning. In Grade 6, only Clare recalled the word *metric* – although not its meaning – while only two pupils recalled the expression *drop a perpendicular.*

On the other hand, I noted that words that were used more frequently in class were, in fact, recalled by the pupils. This seems to imply that if the teacher deems a new word to be important, then it may be a good thing to use it more often, and ideally, as argued in Chapter 3, encourage the pupils to use it themselves. I say 'deem it important' since from Rose's comments it transpired that 'new' did not necessarily imply 'important'. I commented to Rose that the children I interviewed did not recollect some key mathematical words that had been used during the week. To this she reacted:

> "It doesn't concern [worry] me (…) They will be repeating this topic in Grade 4, 5, and they will do it more elaborate[ly]" [RoseLength(2)Q4]

It is interesting to note that although Rose was not worried about the pupils not recalling some words, she was in fact *surprised*. Of particular interest to me is her comment:

> "Mind you, after a week's lessons, I expect them to understand what I said and what we did." [RoseLength(2)Q4)]

In the context of our conversation, I believe that by 'understand' Rose meant 'recall and understand'. Rose did not seem to be aware of how little she had actually used the words as can be seen in Table 8.3 below.

Word	**Lessons for the Topic 'Length'**				
	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**	**Lesson 5**
Measurement	4	3	9	0	2
Width	4	2	3	0	0
Estimate	2	0	5	0	0
Height	2	2	2	0	0

Table 8.3. Distribution of the least-used Length related words by the Grade 3 teacher

Admittedly, in practice it is not possible for teachers to be conscious of the number of times they use each mathematical word, but I believe that an increased awareness in this regard may result in their using the words more frequently and thus offering more opportunities for its meaning to be shared with the pupils.

8.4 Conclusion: medium to message

Hewitt (2001) stated that students need assistance with memorising words. It appeared to me that the teachers' frequent use of the words had some bearing on the pupils' ability to recall a word, and it was for these recalled words only that pupils could offer explanations of meaning. The apparent link between frequency (generally by the teacher) and recollection (by the pupils) can be viewed as a shift from the overt medium or social realm to the message or meaning held by the pupils, that is, the individual realm. I consider recollection of a word to be a first layer of meaning (Roberts, 1998), so that frequency of use appeared to be one condition necessary for sharing meaning. This observation, therefore, begins to address my third research question:

> *What conditions appear to be helpful for a teacher to 'share' the meaning of (a selection of) mathematical words with the pupils?*

Of course, memorising or recalling words is not an end in itself, but a means to support working on mathematical understanding (Hewitt, 2001). In the following chapters, I turn my attention to meanings expressed by pupils and consider them in the light of classroom interaction. I focus on what appeared to render meaning 'clear', while remaining open to any other helpful features of word use that might come to light. I have previously suggested that words can serve as indicators or references, they can name actions, or denote properties and concepts. For each topic, namely, Multiplication and Division (Grade 3), Graphs (Grade 6) and Length (both Grades), I will consider words in these categories.

One thing that became apparent to me in the course of my reflections was that my study did not in fact allow me to draw conclusions regarding to what extent the pupils' understanding was inhibited by the use of English. Although there were instances where I could reflect on the use of either Maltese or English, I felt that the pupils generally appeared to 'follow' the lessons and I could not identify points where the use of English was evidently detrimental. I found that any differences between meanings expressed by the teacher in the classroom and by the pupils during the interviews, could be explained by factors that went beyond the use of English. I found that I could base my interpretations on the general teaching approach used, that is, the way the teachers explained ideas. Although I cannot exclude the fact that the teachers' approach and/or use of language was itself influenced by their 'obligatory' use of English, I think many of the discussions I present in the following chapters may be applicable to 'any' classroom.

C H A P T E R 9

Multiplication and Division

9.1 Introduction

The first topic I consider is Multiplication and Division as taught in the Grade 3 class. The week's lessons I observed were not the pupils' first encounter with the topic. Rose reported:

> "They learn multiplication and division in Year 2, but the emphasis is on the TABLES (…) To study them, to know them by heart" [RoseM&D(2)Q9]

In the first term of Grade 3, the girls had 'revised' the tables previously learnt, and also listed the tables of 6, 7 8 and 9 in their copybooks. During the lessons I observed, the pupils responded with ease to Rose's requests for an answer to, say, 'three times five'. Indeed, the girls admitted to me that the tables were very familiar to them. Rose explained that her intention was now to provide opportunities for the tables to be applied to what she called 'situations'. Two examples of such situations were: finding the total number of legs for a given sets of three-legged monsters (multiplication) and establishing how many 5p coins are needed to buy stamps of 15p, 25p, 45p etc. (division). The tables utilised during the week were of 2, 3, 5 and 10.

In the first term, the pupils had also listed in their copybooks what appeared to be 'key' words and symbols as shown below:

Multiplication	Division
To multiply is a repeated addition	To divide is a repeated subtraction
$2 + 2 + 2 + 2 = 8 \qquad 2 \times 4 = 8$	divide
times	divided by
multiply	division
multiplication	share
×	grouping
	÷

Figure 9.1. Facing pages in pupils' copybooks.

Rose used these notes as a starting point for the week's lessons. She asked the pupils to look at the left-hand side page, and referred to the words listed there as:

> "Words and signs that tell us we are multiplying" [G3M&D1minute0]

Indeed, listing the words under a common heading suggests an association between the words, although the semantic relationship between them is not specified. During the interviews, this association was expressed by the pupils as follows:

> Melissa: [**We wrote them**] **to remember how many** words **we've got in the** multiply. [G3M&D(B)Q7]
>
> Sandra: The division is em … it is … em, divide by, em … share, grouping and many things. [M3M&D(A)Q7]
>
> Maria: [The words] they are the same thing. [G3M&D(A)Q7]

The words listed are, of course, closely related: *multiplication* involves *multiplying*, which can be expressed as a number *times* another number. Similarly, *division* involves *dividing*, and can be expressed as a number *divided by* another number (although in the case of the Grade 3 classroom, the expression *division by* tended to be used instead of *divided by* as discussed in Section 7.4). In this chapter I look in more detail at the apparent meanings the pupils had for the respective words. I relate the meanings expressed during the interviews to the process of sharing of meaning in the classroom in an attempt to identify what helped or hindered 'clarity' of meaning. I identified words that served as references, implied actions and denoted concepts. I start my discussion by examining the sharing of meaning of words that served as references, then move on to discussing the procedures (actions) of multiplying and dividing. As part of the latter discussion, I also reflect on three points: the maxim 'multiplying makes bigger and dividing makes smaller' that was often stated in the classroom, the property of commutativity, and the idea of division as the inverse of multiplication. Finally, I turn my attention to the sharing of meaning of concepts for multiplication and division and reflect on why division is sometimes considered to be a 'hard' operation to teach and learn.

9.2 Words as references

According to Vygotsky (1981c), a word serves a primary function when it acts as an indicator. I applied this idea to the words *times*, *multiply by*, *division by*, *multiplication* and *division* when they were used as names for notation. Their referential role is indicated in Figure 9.2, where *m*, *n*, *x* and *y* denote particular numbers used within the lessons:

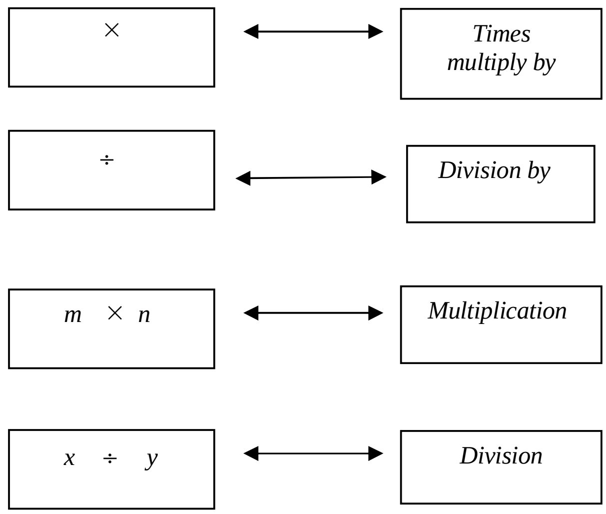


Figure 9.2. Words as names for notation

So for example, the notation 3 × 4 was read as "three times four", where the word *times* referred to the symbol ×. The expression *multiply by* came to serve as an alternative for *times*, since Rose encouraged the pupils to use it instead of *times*. In fact, Rose explained that this was one of her objectives for this topic:

> Rose: Instead of saying 'times' (…) I want them to say 'multiply'. [M&D(2) Q9].

Similarly, the expression *division by* was used as a name for the symbol ÷ as in the case of, say, 12 ÷ 3 ("twelve division by three"). For example:

> T: We have thirty division by five (*writes 30 ÷ 5 on the whiteboard as she speaks*). [G3M&DLesson2minute35]

Examples of Rose's use of the words *multiplication* and *division* as references to notation are:

> T: (*The teacher has written 8 ÷ 2 = 4 on the whiteboard*). Now he [author of textbook] wants us to draw fish to show that division [MD4minute13]
>
> T: (*The pupils are looking at their homework exercise and are about to correct the worked out examples of the type:* $\square$ × 3 = 15 *and* $\square$ ÷ 3 = 1 *etc.)* Yesterday we did some multiplications and divisions [MD5minute1]

During the lessons and the interviews, the pupils indicated that they could use the words in a similar way to their teacher:

> Sandra: Times means the sign (…) it's like a cross (*makes an X sign with both pointer fingers*). [G3M&D(A)Q1]
>
> Kelly: Multiplication **is** times (…) **And you can do, for example**, three multiply by three (…) [G3M&D(C)Q1]
>
> (*I have just shown a card on which is written 12 ÷ 3 to Melissa and Kelly and asked them to read it*).
> Melissa: Twelve division by three. [G3M&D(B)Q9]

All pupils agreed that the word *times* had already been familiar to them from the previous Grade, while they all agreed that *multiply by* was a new expression. Opinions regarding *divide by*, *division*, and *multiplication* were varied. Whatever the degree of previous familiarity, I concluded that this particular use of the words had been shared with the pupils. I attribute this success to two features of their use. First, as previously discussed in Chapter 8, the expressions were needed for the tasks at hand, they tended to be used frequently (although generally, more by the teacher than the pupils). Second, the words were used in close conjunction with the object to which they referred. That is, as suggested by Wertsch (1985), the index and the object to which it referred were temporally and spatially co-present.

However, as stated by Goudge (1965, cited in Wertsch 1985), indexing alone does not say much about the perceived object. As the words 'name' the notation, they also serve to express a particular relationship between the numbers that is different to, say, "four add three" or "twelve subtract three". In order to gain more insight into the meaning of the words, I need to look further into their use. The development of the topic over the week addressed the relationships at both a procedural and conceptual level. I believe that the procedural element was shared successfully with the pupils and in the next section I will illustrate how this was achieved.

9.3 Mathematics as doing: *multiplying* and *dividing*

During the week, Rose frequently asked the question 'what are we going to do?', thus emphasizing what Halliday (1976) referred to as action processes, where mathematics is seen as 'doing' something. For example:

> (*The class is working out a textbook exercise where the pupils have to find how much money was earned during a skittles game. Each skittle dropped earns the player 10p*).
> Teacher: What are we going to do to find how many for six skittles? Jessica?
> Jessica: Six times ten.
> [G2M&D1minute43]

In such situations, the pupils would suggest the relevant operation of the type "*m* times *n*", then go on to recite the relevant multiplication table. Tables were recited in one of two forms, so that for "three times five" the girls might offer either "five, ten, fifteen" or "one five is five, two fives are ten, three fives are fifteen". These alternatives were usually accompanied by opening up fingers one at a time, or touching the fingers of one hand with the fingers of other hand, a gesture that Anghileri (2000) suggested is a useful calculating aid. For example for 'three fives are fifteen' a pupil would open out the thumb and two fingers or touch the left hand thumb and two fingers with her right hand fingers. The practice was encouraged by Rose who often used it herself.

A similar approach was used to work out division. In this case, the expression that prompted the tables was, say, 'twelve division by three'. A pupil would then recite the multiples of three until she reached twelve: "one three is three, two threes are six, three threes are nine, four threes are twelve". The answer was recognised by the 'four' expressed along with the 'twelve' or from the number of fingers opened up. It was interesting to note that the gesture appeared to be an integral part of the recitation and seemed to be used to concretise the mathematical idea (Goldin-Meadow *et al*, 1999). In fact, Rose told the girls:

> "Use your fingers. The answer is on your fingers" [G3M&D2minute35]

Depending on whether the operation at hand was multiplication or division, the working out of the operation through the tables was referred to as *multiplying* or *dividing* respectively. For example:

> (*The class is working out a textbook exercise together while the teacher writes the notation on the whiteboard. The answer to $6 \div 3$ has been found by reciting the tables 'one three is three, two threes are six'*).
> Teacher: (*Touches the written notation $6 \div 3 = 2$*). Over here we are dividing.
> [MD4minute45]

As evidence of shared meaning for the words, I had to rely on the classroom data. The interview data was a bit limited since only Sandra and Maria used the words as verbs:

> Sandra: ‘Multiply by’ means that you multiply by three, or four or ten. You could multiply by eleven (…) [G3M&D(A) Q1].

As mentioned in Chapter 7, the pupils sometimes rendered verbs as nouns, resulting in, for example, Melissa’s explanation below:

> Melissa: **The** ‘divide by’ **is [when] the** number **becomes more** small. [G3M&D(B)Q1].

However, the girls did use the words *multiplying* and *dividing* in the classroom. As the class worked out textbook exercises, Rose would ask ‘What can I do here?’ to which a pupil might answer ‘We divide’; in response to ‘What am I doing here?’ a child might answer ‘We are multiplying' and these suggestions were then followed on quickly by the recitation of the tables.

Therefore, although both the pupils and Rose agreed that *multiply* and *divide* were new words, I concluded that meanings for *multiplying* and *dividing* as procedures were successfully shared with the pupils. As was the case for references, the words were used frequently. Furthermore, the words were used in close association with the notation that prompted the required action. I have argued previously that a meaning for a word that denotes an action involves an awareness of the *function* of the action. It appeared that the function of the actions ‘to multiply’ and ‘to divide’ was clear to the pupils: carrying out the tables procedure in order to find a solution to multiplication or division notation. In a sense, the procedure was the same one (reciting the table) except that the solution was recognised as the spoken number that was ‘missing’ in the notation e.g. “four threes are TWELVE” for 3×4, or “FOUR threes are twelve” for $12 \div 3$. A meaning for the word *multiply* as a procedure can be illustrated as in Figure 9.3.

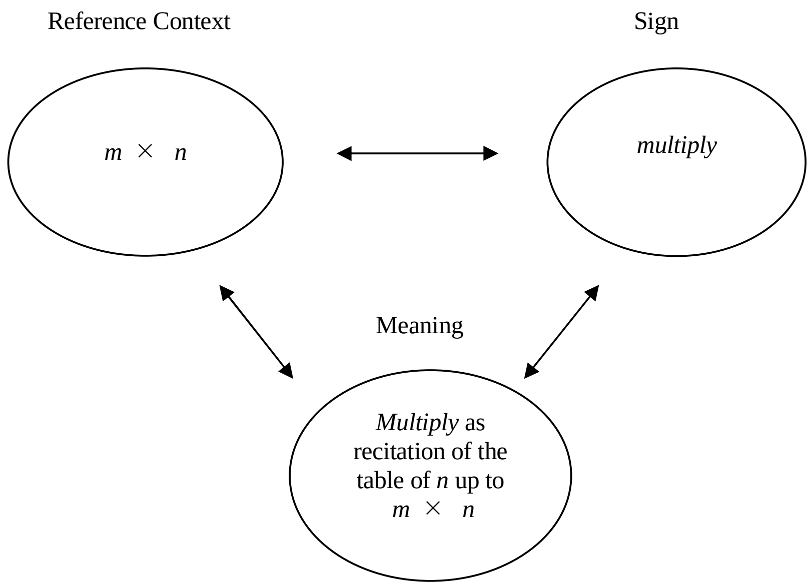


Figure 9.3. A meaning for *multiply*

In relation to multiplying and dividing, Rose tackled three ideas, namely, the maxim that 'multiplication makes bigger / division makes smaller', the property of commutativity; and the inverse relationship between multiplication and division. I will reflect on these in turn, since they contributed to meanings for the words *multiply* and *divide*.

9.3.1 Making 'bigger' or 'smaller'

On several occasions throughout the week, Rose told the pupils - and encouraged them to state – that 'multiplication makes bigger and division makes smaller'. The maxim was sometimes stated in a general sense:

> Rose: We are multiplying. We are making something LARGER, BIGGER. [G3M&D1minute9]

On other occasions, Rose indicated an increase through the word *more*:

(*The class is working out an exercise orally. The aim of the activity is to find the prize money for skittles knocked down, @ 10p a skittle).*)
Teacher: For one skittle you are going to get …?
Pupils: Ten pence.
Teacher: For six skittles, are you going to get MORE or less?
Pupils: More!
[G3M&D1minute43]

Similarly, in division situations, Rose would ask 'is the answer more or LESS?' to which the pupils answered 'less'. The statements 'makes bigger / smaller' were used frequently and in close conjunction with the procedures to which they related. I concluded that the maxims as such were shared successfully with the pupils, since the pupils also repeated them confidently during the interviews. However, when pressed for an explanation about what the answer was bigger *than*, or smaller *than*, the pupils could not give an appropriate response. For example, I pressed Ramona and Kelly for a more specific explanation regarding division:

Ramona: **You'll have** eleven, **and you do** divide b- b- divide by three equals … **and the** answer **comes** smaller.
I: Smaller **than the** eleven **or the** three?
Ramona: (*Looks unsure. Looks across to Kelly*).
Kelly: **Than the** three.
Ramona: **Than the** three.
[G3M&D(C)Q6]

The interview evidence suggested that the pupils had not reflected on what becomes bigger or smaller than what, and certainly this point was not attended to during the lessons. Thus, the meaning that the pupils appeared to express was limited to repeating the statements. This may be explained by the fact that Rose's classroom statements were incomplete, since *bigger* and *smaller* are comparative adjectives and are best followed by the word *than.* Similarly, *more* [*than*] and *less* [*than*] are generally used for comparison of quantities. In class, what the answer was bigger or smaller *than* was not clearly indicated. Hence, I believe that while the maxim itself had been shared thanks to frequent use, its actual meaning was not clear. This, therefore, was a case of divergence between frequency and clarity in the classroom, which was then reflected in the pupils' responses.

While the maxims obviously do not hold when the multiplier or divisor is a fraction, nor when a multiplier is a negative number, I suggest that even for examples involving numbers greater than 1 (the type of situations met with at this young age), the maxim needs some reflection: while a product is in fact greater than both its factors, the situation for division is not so straightforward.

In order to illustrate this point, I present three situations in Table 9.1. , and for each case offer a reflection.

Structure	Reflection on an Example
<u>Multiplication</u> Product greater than both of the factors.	*e.g. One monster has 3 legs. How many legs do 4 monsters have*? 4 monsters together have MORE legs than one monster. So the solution to the question is 12 legs which is MORE THAN the original 3 legs. The answer is also more than the number of monsters (4).
<u>Division as a sharing activity</u> This is the partitioning structure of division (van de Walle, 2004) or more informally, sharing. The number of groups to be formed is known, but number of items per group is not. In sharing situations, divisor is always a whole number (e.g. children, boxes), and quotient is always smaller than the dividend (Frobisher *et al*, 1999), but not necessarily smaller than the divisor.	*e.g. 12 stickers are shared out among 3 friends. How many stickers does each friend get*? 1 friend has LESS stickers than the original 12. The solution to the question is 4 stickers, which is LESS THAN the original 12 stickers. The solution gives the number of stickers, so the relationship LESS THAN compares quantities of the *same* item. In this case the quotient is *not* numerically smaller than the number of friends (although in other situations it may be).
<u>Division as the formation of equal groups.</u> This is the measurement structure of division (Van de Walle, 2004) or more informally 'grouping'. The number of items in a group is known, but the number of groups is not. The quotient is smaller than the dividend, but not necessarily less than the divisor. The quantities being compared are not of the same item.	*E.g. Ms Borg has 20 buns. She packs them into bags of 4. How many bags does she need*? The solution 5 is LESS THAN 20. However, the solution gives the number of bags not buns, so the relationship LESS THAN compares quantities of *different* items. In this case the quotient is *not* numerically smaller than the divisor (although in other situations it may be).

Table 9.1. 'Multiplication makes bigger' / 'Division makes smaller'

The examples presented highlight the difficulty involved in quoting the maxims without qualifying exactly what is more or less than what, particularly in the case of division. I suggest that clarity may be established through the continuation of the comparative statement smaller/bigger *than*, with clear reference to the two numbers and/or objects being compared. I also conjecture that focusing on the elements of the situations would also help pupils differentiate between sharing and grouping, a point that I will develop further in Section 9.5.

9.3.2 The property of commutativity

One aspect of understanding multiplication concerns the property of commutativity, which according to Anghileri (2000) is an abstract principle that can offer flexibility of strategies for working out problems. Schliemann *et al* (1998) suggested that since this property may not develop easily outside school, instruction plays an important role in encouraging its appreciation. However, they also stated that images of repeated sets are not helpful for the development of the idea. For example, four monsters with three legs each are not 'the same' as three monsters with four legs each, even though the total number of legs is the same. Suitable contexts to support the idea of commutativity are arrays (Anghileri, 2000). In the class I observed, stamps and pegboard arrays were used and an example is shown in Figure 9.4:

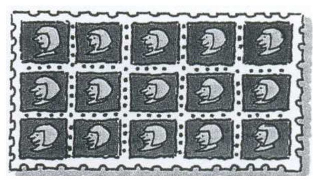


Figure 9.4. Textbook depiction of an array

Although the property of commutativity was not 'named', the third lesson was dedicated almost entirely to working with arrays. Rose guided the pupils to consider first what she referred to as 'stamps on the vertical line' (i.e. three in the above diagram) followed by the number of 'stamps on the horizontal line' (i.e. five), then to multiply the numbers instead of counting them. Early on in the exercise, Melissa offered the numbers in the opposite order (first 'horizontal', then 'vertical') and it was at this point that Rose brought out the idea of commutativity:

Teacher: Four times five, or Melissa said five times four. Do you think I'm going to have the same answer?
Pupils: Yes!
Teacher: Four fives are …?
Pupils: Twenty.
Teacher: And five fours are …?
Pupils: Twenty!
Twenty: Twenty. So over here there is another thing. That when we multiply four by five (*writes 4 × 5 on the board as shown*), and five by four (*writes 5 × 4 underneath*) we are going to have always the same answer.

4×5

5×4

The numbers, they just change position. (*Looks at Petra who has put her hand up*). Yes?
Petra: Miss, em, of the four times five and five times four … if you watch them, four equals like that (*makes diagonal gesture in the air towards the right*) and five equals like that (*diagonal gesture towards the left*).
Teacher: **Yes**, because we have five which is the same and four which is the same. We have the same numbers. (*On the board she joins the numbers with lines as shown*:

4×5

5×4

[G3M&D3minute8]

The following page in the textbook offered pegboard representations:

(*The class is looking at the example of a pegboards that shows an array of 2 by 3,and the notation 2 × 3 and 3 × 2. The teacher has just read the notation 2 × 3).*
Teacher: We can do it the other way round. We can do …?
Pupil: Three times two.
Teacher: Three times two, and we get the same answer. (…) It's important to remember that two times three and three times two are the same.
[G3M&D3minute22]

The point was addressed again a few times as the class went through the exercises orally, although for the written exercise, Rose suggested that the girls need only write down one interpretation.

Clarity in conveying the idea (although not a particular word as such) seemed to be assured since the notation was manipulated on the board, although I also felt that the pupils seemed to recognise immediately, from the tables that they knew fluently, that $a \times b = c$ and $b \times a = c$. Interestingly, I believe that something else that contributed to the sharing of the idea of commutativity was Rose's inconsistent use of notation whereby she changed the order of the numbers. While she tended to use $m \times n$ for m sets of n objects (the convention used by the textbook), she also sometimes used $n \times m$ instead (including, for example, as part of the 'notes' shown in Figure 9.1). Although this is not what is meant by commutativity, the different use of notation may have suggested that re-ordering of numbers is acceptable for multiplication.

During the interviews, all six pupils explained the idea confidently in terms of the notation:

> (*I show the two girls a card with* '6 $\times$ 5' *written on it, which they read as "six times five" and suggest it means "multiply". I then take out another card with 5 $\times$ 6 written on it).*
> I: **What can you tell me about this?**
> Lara: **We can turn it around**.
> Melissa: **But it's still the same** answer.
> [G3M&D(B)Q8]

I also presented Sandra and Maria with an array (a rectangular grid) and they explained that the answer could be found in two different ways: three times five, or five times three.

Unlike multiplication, division is not commutative, but this point was not addressed during the week. Rather, the pupils were guided by the teacher and / or the textbook with regard to which number to offer first for a division operation. So for example, in an exercise showing stamps with values printed on them (e.g. 30p), the authors offered an example, 30p ÷ 5p = 6, which the pupils then modelled. When left free to offer their own suggestions as to which number featured first in a division notation, the girls often gave the wrong order, for example:

(*The class is working out a story sum wherein a boy is packing 5 cans into boxes. He has 20 cans to pack. They have already established that 4 boxes are needed, since a pupil had counted in 5s up to 20. The teacher has guided them to write what she calls a 'statement' as follows*:

5 cans = 1 box
20 cans = ?

She would now like the pupils to suggest a 'division' in line with what they had written (45 ÷ 5) for a previous story sum.

Teacher: What do I write here (*touches the board beneath the 'statement')*.
Rita: Five division by twenty.
Teacher: You can't divide five by twenty! Can you turn it the other way round?
Rita: Twenty division by five.
Teacher: (*Writes 20 ÷ 5 on the board*).
[G3M&D2minute67]

Of course, five *can* be divided by twenty, although such an operation is not usually presented to young children. Data from the interviews indicated that the pupils applied the commutativity idea to division: during the interview with Melissa and Lara, I presented the girls with two cards with the following written on them: '12 ÷ 3' and '3 ÷ 12'. Apparently echoing their teacher's comments for multiplication, the girls suggested that the notations were 'the same', just 'the other way round'. Furthermore, during my discussion with Ramona I started by showing her the card '12 ÷ 3' for which she said:

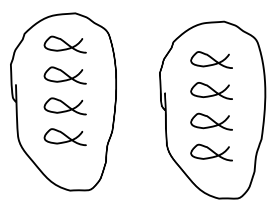
Ramona: **You do** twelve division by three, **you find the** number [answer]. **You can do** three division by twelve. **The** number [answer] **is the same**. [G3M&D(C)Q9]

I noted further inappropriate suggestions of numbers when Sandra and Ramona offered examples for division that could not easily be worked out, considering the pupils' age and level of mathematical knowledge. For example, Ramona offered examples such as 'three division by six', 'five division by ten' and 'five division by three' while Sandra offered 'six division by four'. Thus, although Ramona and Sandra defined division through examples, the examples they offered were not appropriate (considering their assumed level of knowledge). The other pupils did not offer examples and only Maria offered examples that could be worked out using the tables, for example: 'six division by three' and 'three division by three'.

I concluded that during the lessons, Rose had guided the pupils in establishing the order of the numbers, however, the opportunities for the pupils to reflect on the significance of the numbers involved in the operations were limited. This was more detrimental in the case of division than multiplication since the former is not commutative, and therefore the numbers' position in the notation is more significant. It seems that clarity for the idea of commutativity may need to include focusing explicitly on the notation in relation to situations and objects at hand (cans, etc.). In the absence of such reflection, the pupils applied phrases like 'the other way round' inappropriately to division.

9.3.3 Multiplication and division as inverse operations

Another aspect of 'knowing' multiplication and division is the awareness of their inverse relationship. This relationship was addressed by Rose during two of the four division activities. For example:

> (*The teacher has copied the textbook diagram on the board as shown):*
>
> 
>
> *The notation $8 \div 2 = 4$ and $2 \times 4 = 8$ is shown on the book. Rose writes $8 \div 2$ on the board. She suggests that they work with the table of 2. One of the pupils recites the table and the answer is established as 4*).
>
> Teacher: And if we turn it the other way round, I'm going to have the same answer again. Now I have four multiply by two give me eight (*writes $4 \times 2 = 8$ on the board).* If I take eight fish and divide them into two groups (*touches the two groups of fish drawn on board*), I have four fish in each group. If I take the four (*touches one group of fish on the board*) and multiply by two, I'm going to get an eight again.
>
> [G3M&D4minute17]

I noted once again Rose's use of the expression 'the other way round' which she had used for re-ordering of numbers within multiplication. I think that this was an interesting example of how informal language, although serving an immediate purpose, may not be mathematically specific.

The inverse relationship was revisited for all the examples, and two operations for each written on the board and copybooks. The idea was mentioned again during the following exercise that involved computer ‘blasts’ worth 3 points each. In the first part of the exercise, diagrams of blasts on a screen were shown and the pupils were expected to find the total number of points won, while the second part of the exercise involved mixed multiplications and divisions using 3 as a multiplier or divisor. The two depictions are shown in Figure 9.5:

Figure 9.5 a & b Textbook depiction of computer blasts

The classroom data indicated that the girls began to talk about the situations in the way desired by the teacher. For example:

> (*The open sentence at hand is* $\square \times 3 = 18$. *Jessica has recited the table of three till she got to 18 and offered ‘6’ as the solution*).
> Teacher: Very good. Can you think of another way of getting that answer? Six.
> Jessica: Eighteen division by three.
> (…)
>
> (*The open sentence at hand is* $\square \div 3 = 4$).
> Teacher: A number division by 3 gives me 4 that number. Cheryl?
> Cheryl: Twelve.
> Teacher: Twelve. How did you get the number 12?
> Jessica: Three, six, nine, twelve. Twelve.
> Teacher: Twelve. Can you find another way to get twelve?
> Cheryl: Four multiply by 3.
> [G3M&D4minute53 & 56]

The girls appeared to be successful in suggesting the inverse, although I am not sure whether this was because of the guided approach; as class-work, Rose set the same examples the class had worked out together on the board. Unfortunately, I do not have interview data in this regard and therefore cannot say how successful or otherwise the pupils would have been in expressing the inverse relationship on their own.

Up to now I have considered aspects of the topic that are related to the naming of notation and working out of the procedures for finding solutions for multiplication and division operations. Generally speaking, Rose was successful in sharing these layers of meaning (Roberts, 1998) with the pupils. Now I would like to go beyond procedures to concepts. In the sections that follow, I start by reflecting on how a concept for multiplication as repeated addition was successfully shared with the Grade 3 pupils. I then go on to contrast this with the less successful exposition of a concept for division. Finally, I present a theoretical discussion about various representations through which the two concepts might be presented.

9.4 A concept for multiplication

The word *multiplication* was considered new this year by some of the pupils and 'fairly new' by the teacher. However, since the topic had actually been touched on three months earlier, the topic-related words may not actually have been new for the week. An association between the words *multiply*, *multiplication* and *times* was suggested through the fact that the words were listed in the pupils' copybooks under one heading. Furthermore, the words were all used in relation to the same notation $m \times n$ and linked to the idea of repeated addition. During the interviews, I had evidence that all the pupils could give an appropriate explanation for the words, linking them with each other and/or the notation and repeated addition. For example:

> Sandra: **In** multiplication, **you don't keep doing** six plus six, plus six (*opens one finger at a time*). You just multi-, you just multi- … three tim- … multiply by six and you get the answer
> [G3M&D(A) Q2+Q5].
>
> Lara: **To do** multiply, **you don't keep doing** plus **all the time. You do** multiply **straight away**.
> Melissa: **It depends on how many you've got.**
> [G3M&D(B) Q5]
>
> Kelly: (*Opens her book on the 'monsters' page*). These are monsters. Now here is two monsters. And they have three legs [each]. Now you to find (sic) … to times … because there are two monsters, and then you count the legs (…) and when you count the legs you write them here (*touches the page where there is written in pencil* $2 \times 3 = 6$) and … and … then you write the answer.
> [G3M&D(C)Q2]

I suggest that the links presented in the lessons were clear, and through three excerpts I will identify how this clarity was achieved. In my discussion, I assume that the children were familiar with all the following: everyday words such as *we, fingers*, *for example*, *legs* and so on; the mathematical words *plus* and *addition, how many* and the symbol + . I also assume that they would recognise that the expressions *repeating the number* and *repeated addition* can both refer to the notation $n + n + n + \ldots$.

In the first excerpt, Rose related objects to multiplication notation and used the word *multiply* in conjunction with the situation:

> *(The teacher has asked six girls to stand at the front of the class and to hold one hand up each*).
> Teacher: I have five fingers, which is five, five, five, five, five, five (*touches one hand at a time*). And they are all the same. I have six girls. So I take just five and I multiply it by six. (…)
> Pupils: Thirty!
> Teacher: (*Writes $5 \times 6 = 30$*)
> [MD1minute13]

A semiotic representation is shown in Figure 9.6, which also includes the associated word *multiply*:

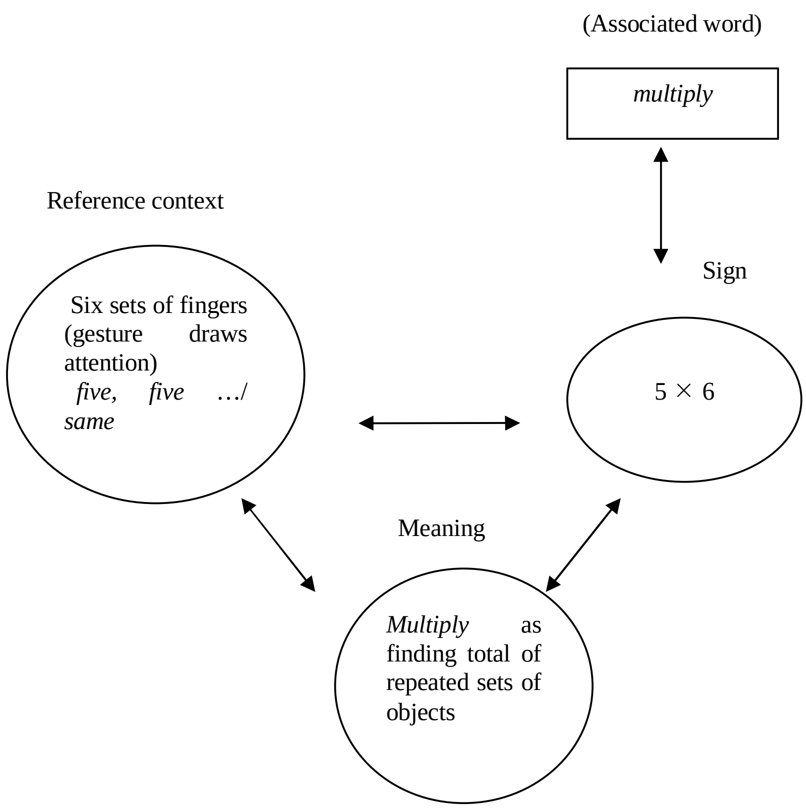


Figure 9.6 Relating groups of objects to notation

The reference context consisted of the pupils' fingers to which attention was drawn by touching. The teacher encouraged a quantative interpretation by focusing on the number of fingers ('five, five …') rather than talking about say, how small or clean the girls' hands were. By stating that the hands 'are the same', Rose encouraged the girls to focus on the sameness of the object, rather than, say, the fact that each hand belonged to a different girl. Thus the hands became a reference context for multiplication by virtue of the language used in conjunction with them that helped to focus attention on the elements of the situation that were necessary for a multiplicative interpretation. These elements were represented through the notation 5×6 (here Rose once again departed from the textbook convention by which this situation would have been represented as 6×5). The word *multiply* was used in conjunction with the notation, so that the apparent meaning for the word was one of finding the total number, given similar sets.

In the next illustration, Rose referred to notation written in the pupils' copybook and on the whiteboard, linking notation for repeated addition / counting with the notation for multiplication. Rose drew attention to the repetition of the number and, together with a pupil, offered the multiplication notation as an alternative by way of the words 'instead of'. The topic-related words used in conjunction with this situation were *multiply* and *multiplication*:

> *(The teacher is introducing the topic and has asked the girls to open their copybook on the pages dated three months earlier, where they had listed words and symbols associated with multiplication and division. One of the entries is '2 + 2+ 2 +2'*).
>
> Teacher: We said that when we multiply we are repeating the number. For example, if I have two, two and two, instead of counting two plus two plus two, I can …?
>
> Pupil: Multiply.
>
> Teacher: MULTIPLY
>
> (…)
>
> Teacher: (*There is 2+2+2 written on the board*). I cannot do a multiplication if the numbers are not all the same (…) (*Addressing Nadia*) Can you come and show us the sign of the multiplication? (…) (*shortly afterwards, Rose writes 3 × 2 on the board*).
>
> [MD1minute0]

Diagrammatically:

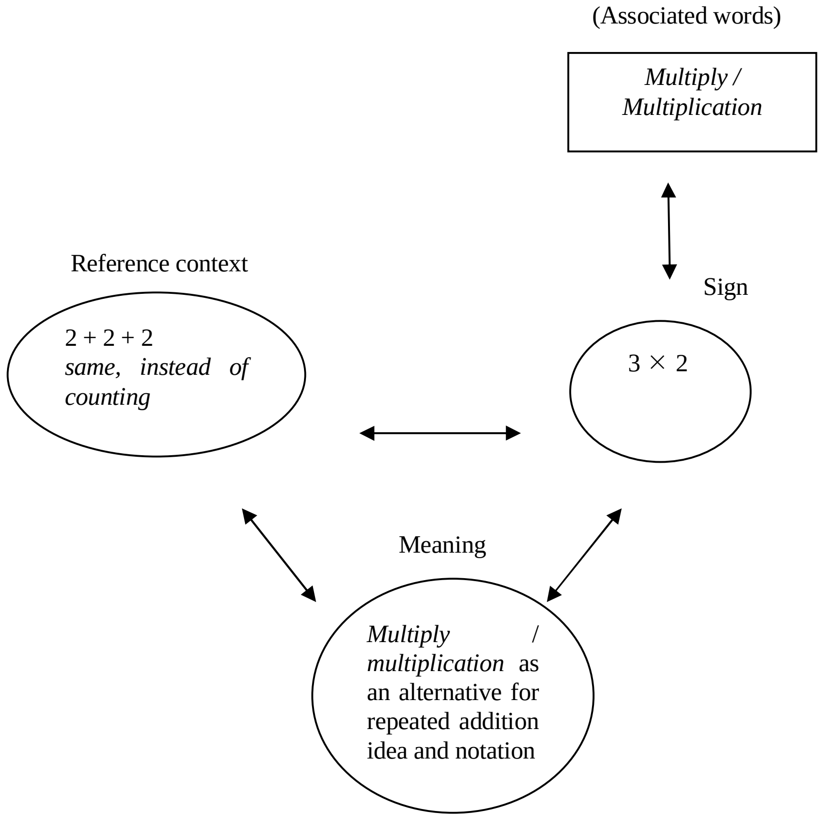


Figure 9.7 Relating addition and multiplication notation

In the third excerpt, Rose drew attention to repeated addition notation by circling it while saying ‘group all this’. Once again, she offered the multiplication notation as an alternative for the addition notation through the expression ‘instead of’:

> (*The class is focusing on a textbook example showing four monsters with three legs each. The teacher has written 3 + 3 + 3 + 3 on the whiteboard*).
>
> Teacher: We can multiply instead of having addition. Instead of adding three plus three, plus three, plus three, what can I do? I can group all this (*draws a large oval around the repeated addition notation*). Each monster has three legs. So instead of adding three for four times, what can I do? I can work it out as …?
>
> Pupil: Four times three /.
>
> Janica: /Three and three and …/
>
> Teacher: (*Says yes, in this case you can add because there are only four threes. Later they will need to work out more and larger numbers*). So instead of adding three for four times, what do I do? I multiply …?
>
> Pupils: Three.
>
> Teacher: Three by …?

Pupils: Four.

Teacher: Three by four (*writes* $3 \times 4 =$ *on the board*). And how many legs is that?

Pupils: Twelve.

Teacher: Twelve (*writes 12 on the board to complete equation* $3 \times 4 = 12$)

[MD5minute7]

I would like to use this excerpt to offer a theoretical explanation for a chain that can be set up between pictures, addition notation and multiplication notation, which I show in Figure 9.8. I consider the chain to be set up in two links: A first link appeared to be a relationship between the pictures and the addition notation which was created by way of the language ‘How many legs is that?’, ‘each’ and so on. This language led the pupils to give a quantitative interpretation to the pictures, unlike other possible questions such as ‘Are the monsters happy?’ or ‘Are they wearing shoes?’ In particular, the language drew attention to similar groupings and hence establishing a meaning for repeated addition as similar groupings. This was represented by the symbols 3 + 3 + 3 + 3. In the second link, this same notation was then used to support a meaning for multiplication. The language used drew attention to the four-fold presence of the number 3 and suggested an interpretation for multiplication notation as an alternative to addition notation. This was offered by way of the words 'instead of', 'four times' and 'an easier way', 'I can group all this' (concurrently circling a set of numbers). Thus the role of the addition notation in the semiotic model ‘changed’ from the symbol representing a situation, to form part of a ‘new’ reference context supporting a meaning for multiplication. In the complete chain (Figure 9.8), I also include the word *multiply* as a name for the procedure which now brings with it more ‘baggage’ than when it was used in relation to the recall of the tables.

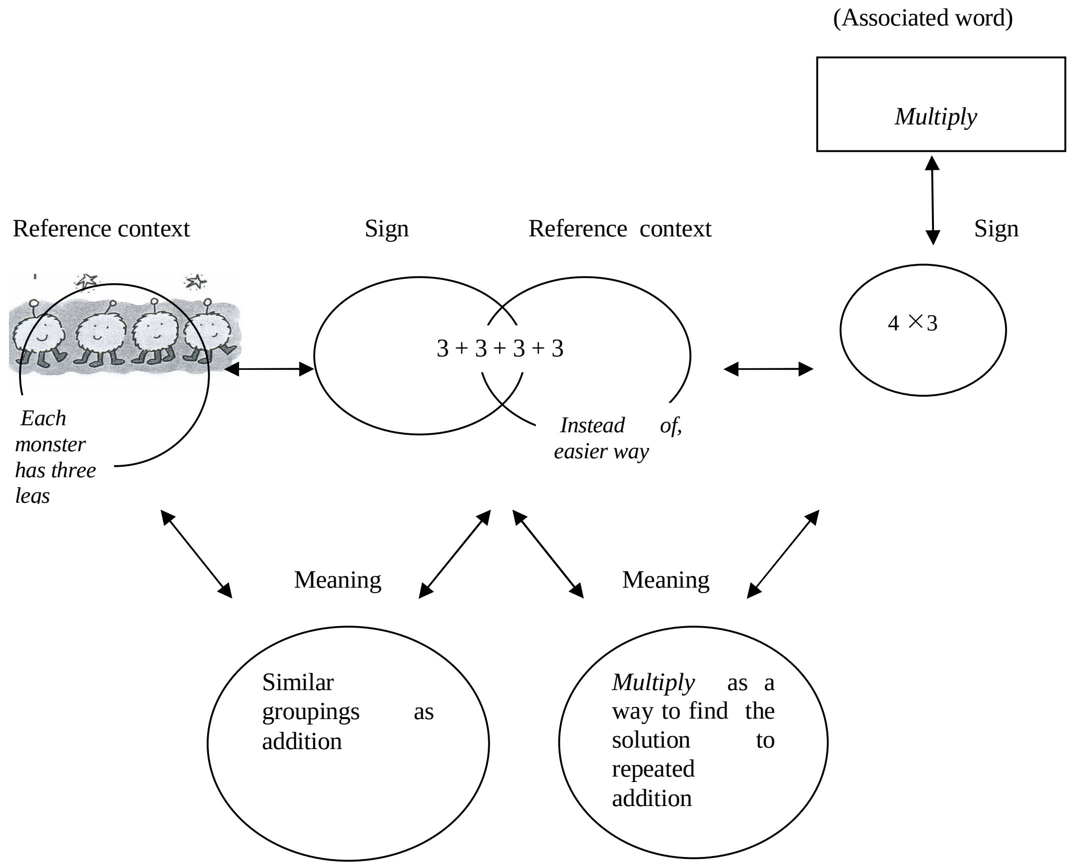


Figure 9.8 A concept for multiplication / multiply

The changing role of the addition notation can be considered an illustration of how unlimited semiosis (Eco, 1976) can occur, as chains of meanings are set up between various signs. It should be noted that the pictures of the monsters are themselves symbols representing a tangible (albeit fictitious in this case) situation, and the chain could be extended to the left to indicate this representation if the monsters were real-life creatures.

The combination of expressions, pictures and use of notation varied over the week. However, although the situations were different (fingers, monsters legs, skittle points, computer blasts and so on) the presence of a repetition of a similar set was always evident, thus conveying a sense of consistency across the examples. The concept was addressed in terms of the words *multiplication* and/or *multiply by* and/or *times* and I believe that all these words came to be associated with the meaning of an alternative for repeated addition. I considered that 'clarity' was ensured by the ‘proximity’ of the language and the pictures/notation, in the sense that to what the language was referring was evident. Hence together, the language and objects offered a supportive reference context for multiplication as repeated addition, thus ‘gluing’ (Hewitt, 2001) the words with the mathematical notion.

As already mentioned, the apparent existence of the links represented above was evidenced in the pupils’ explanations during the interviews, although the pupils did not actually use the expression ‘instead of’ but gave an indication of the same semantic idea through different terms (Chapman, 2003). Hence, I concluded that Rose had achieved the objective she had expressed during the interviews:

> “It is a repetition, so now I want them to work out by multiplying instead of adding, adding, adding (…) It’s an easy way out.” [RoseM&D(2)Q2]

On the other hand, I think that a concept for division was not so successfully shared. In the next section of this chapter I will explore why this may have been so by reflecting on the use of the words *sharing* and *grouping*, and the expression *repeated subtraction*.

9.5 Developing a concept for division

As was the case for *multiplication*, opinions varied regarding the previous familiarity of the word *division*, while the teacher thought it to be ‘fairly new’. At Grade 3 level, division can be addressed through the different actions of either sharing or grouping. In each case, the language used would need to be different to reflect the different situation. For example, in a sharing situation, I would expect attention to be drawn to the two elements of the situations (e.g. children and stickers) and words such as ‘share’, ‘give out’, ‘each’ to be used. On the other hand, in a grouping situation, I would expect attention to be drawn to the necessity of creating several sets of a given size through expression such as ‘how many groups of *x* items can be made?’ I conjecture that if the actions of sharing or grouping are clear to the pupils, then it may be possible to link the elements of the situations (e.g. original set size, size of a group, number of children and so on) to the numbers forming part of the division notation. However, during my observations, I felt that the ideas for sharing and grouping were not clearly set out by the teacher.

First of all, no clear distinction was made between the two words/actions since Rose often used the words as though they were synonyms of each other, as in the following example (Rose here mentions ‘cents’ rather than pence, a similar subdivision of a Maltese Lira or pound):

(*Pupils are working out a textbook exercise were they are to find how many 5 pence coins are needed to buy stamp amounts that are multiples of five e.g. a 30p stamp. As the pupils write the appropriate division operations, the teacher walks round the class looking over the pupils' shoulders*).

Teacher: (*Speaking to the class in general*). We are sharing it [the stamp value], we are grouping it in five cents coins.

[MD2minute39]

Furthermore, Rose did not use the words *sharing* and *grouping* consistently or indeed at times, appropriately, so that their meaning was not altogether evident in relation to what the girls were perceiving. I will now outline the uses of each word in turn.

9.5.1 Sharing

Rose used the idea of sharing as an introduction to division. She asked the pupils whether they would end up with less sweets if they were to share them with their sister, to which the girls answered in the affirmative. Rose then called out three pupils and asked them to raise one hand:

(*Three children are standing at the front of the room, each holding one hand up*).

Teacher:	I have fifteen fingers (…) and three hands. So I'm dividing, I'm SHARING the fingers among three hands. And what do I get as an answer?
Pupils:	Five.
Teacher:	Five. So if we take fifteen fingers (*writes 15 on the whiteboard*), how many fingers on each hand?
Pupils:	Five / fifteen.
Teacher:	Five on each hand. So I divide by five (*writes ÷5 on the board*), and how many hands is that?
Pupils:	Three.
Teacher:	Three. (*Completes the notation 15 ÷ 5 = 3 on the board*).

[MD2minute20]

In this situation, the action of sharing was not perceived, nor indeed possible, and as Rose wrote the notation on the board, she changed the interpretation from a sharing structure to a grouping one. Hence, no difference was made between dividing by the number of children and dividing by the number of fingers on each hand. Rose then called out another child and asked all four pupils to hold up *two* hands each. In answer to her question 'What are we going to do?', Sandra suggested dividing by ten, and a similar discussion ensued regarding 40 fingers and 4 children.

The only other time when the structure of sharing was addressed during the week was for one particular textbook exercise. Although the authors of the book intended a grouping structure, a misprint in some of the textbooks prompted Rose to interpret the structure as a sharing one. That is, for $x \div y$, Rose led the pupils to state the number of groups to be formed and to find how many sweets there would be in each group[6]:

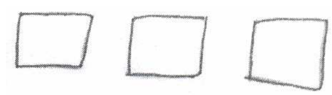
Teacher: (*Writes 12 ÷ 3 on the whiteboard).* How many sweets do I have? (*Touches the 12*).

Pupils: Twelve.

Teacher: How many groups I am going to have? (*Touches the 3*).

Pupils: Three.

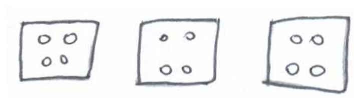
Teacher: So I need three groups here. (*Draws three' squares' on the board*).



Now let me work out this division, so that I will know how much I have to give each of them. Annemarie. Twelve division by three.

Annemarie: (O*pens up one finger at a time, the teacher does the same*). One three is three, two threes are six, three threes are nine, four threes are twelve.

Teacher: **So** goes four (*completes the equation 12 ÷ 3 = 4 and draws four 'sweets' in each 'square'*).



[G3M&D4minute19]

It is interesting to note that although the whole exercise lasted 33 minutes, that is, roughly half the lesson, the word *share* was only used twice towards the beginning ("I want to share them among five girls, five boxes") and *sharing* was used in the very last statement ("Yes, because here we are sharing") in answer to a pupil's inaudible comment. Rather, throughout the exercise, it was the word *groups* that was used frequently. I will revisit this point shortly since I believe it may have had some bearing on the pupils' development of meanings for *grouping* and *sharing*.

Although Rose sometimes stressed the word *sharing* with her voice, she did not specifically explain it. She seemed to assume that the girls would know its meaning, an assumption I believe

[6] The picture-example for the first division notation showed fish, but Rose reinterpreted the subsequent examples in terms of sweets in order to simplify the whiteboard diagram.

was correct. In fact, during the interviews, the pupils offered appropriate examples of sharing situations, which they seemed to base on personal experiences:

> Sandra: Sharing means, means, em … you have … you have a … six yo-yos. And there are two persons. So I don't want to take those six yo-yos by myself. I get two for me, two for her and two for her. No. One for her, one for me and that. You do that. And we have … we have equally. [G3M&D(A) Q1]

> Ramona: Sharing, **you've got** … ten sweets **then you do, you take** five **yourself** *(points to her classmate Kelly)* **and I take** five **myself** (*points to herself*). [G3M&D(C)Q1]

None of the pupils explained *sharing* in terms of the division operation. Therefore, while there was evidence from their explanations of the link shown in Figure 9.9a the association with division shown in Figure 9.9b was not expressed.

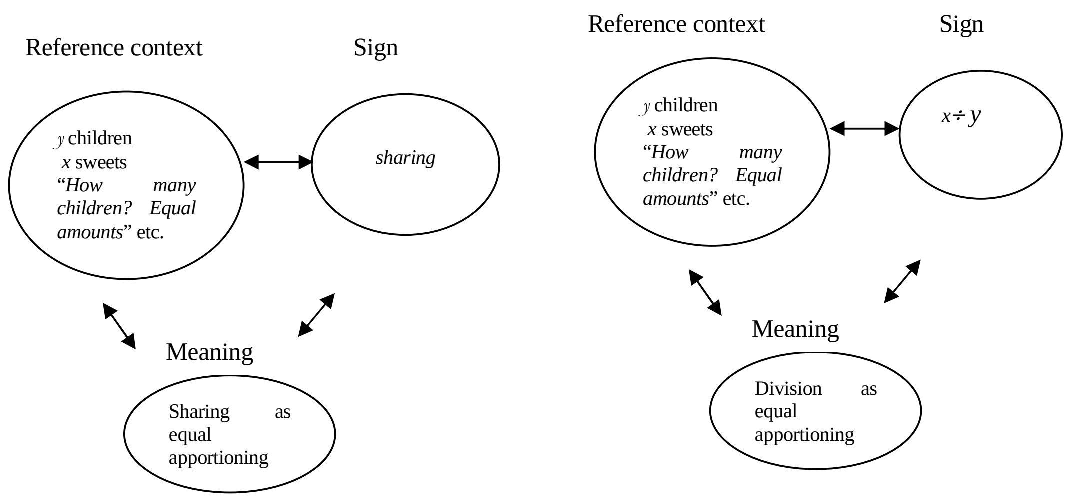


Figure 9.9.a &b. Meanings for *sharing*

Not much attention had been given to division as sharing during the week and this might explain why the pupils did not spontaneously link sharing with the division operation. However, the girls did not explain *grouping* either in terms of division, even though grouping was in fact focused on more than sharing as a structure.

9.5.2 Grouping

All the pupils stated that the word *grouping* was already familiar to them. However, during the interviews, they did not distinguish between *grouping* and *sharing*, and this illustrates that even words that were considered familiar pupils are worth considering in the light of the observed lessons. In this section I attempt to explain why a meaning for the word was not shared.

First, over the week Rose used the word *grouping* 19 times, making it the second least used topic-related word (see Figure 7.1a). Second, as already stated, part of the difficulty appeared to be Rose's occasional slippage between the two words *sharing* and *grouping.* Third, I noted that *grouping* was used in different ways. I present the variations in the order that they were carried out in the class. The first three examples formed part of the second lesson with the first instance occurring shortly after the word *grouping* had been read out from the pupils' copybooks as one of the words listed under the heading 'Division'.

(a) <u>*Grouping* used in a context where the word appears to mean a repeated quantity.</u>

(i) <u>No action evident</u>. The children were working out an exercise that required them to find the number of 5p coins needed to buy stamps of 30p, 45p etc. The teacher talk included "we want it in 5p coins" and "we're going to give it to the post office in 5 cent [Maltese denomination] coins". [G3M&D2minute35]. The first example dealing with a 30p stamp was shown on the textbook as '30p ÷ 5p = 6' and Rose encouraged the girls to use the same format for the other examples, and to use the tables to find the answers. Although the word *grouping* was not used during the initial explanation, Rose walked around the class as the girls worked and at one time said: "We are grouping the 5 pence coins" [G3M&D2minute39]. Strictly speaking, no items had been physically grouped, although Rose may have wished to infer that the five pence coins embodied a 'group' of five 1p value.

(ii) <u>An action evident, but no group is evident</u>. In the first story sum tackled, the word *grouping* was used in the sense of 'putting 5p aside repeatedly':

> (*The class has just read a story sum about a girl saving 5p a day in order to buy a notebook costing 45p*).
> Teacher: So she saves 5p on one day (*gestures putting something aside*), 5p on another (*same gesture*), 5p on another (*same gesture*) … until she manages to save 45p. First of all, if you look at me, I'm doing an action, which tells you what's happening. (…) what are we doing here?
> Pupils: (*Put hands up eagerly*).
> Teacher: Lara?
> Lara: We are grouping.
> Teacher: We are grouping them.
> [G3M&D2minute52]

Here an action was in fact evident, one that implied putting 5p coins aside, although no ‘group’ of objects was perceived. The gesture used in this context was used by Rose almost whenever she used the word *grouping* and indeed, came to serve the purpose of prompting the girls to say that ‘they were grouping’. The gesture consisted of the finger-tips of one hand touching the fingertips of the other hand as though putting things together, as in Figure 9.10. This action was repeated according to the number of groups, each time further to the left in relation to Rose’s body.

Figure 9. 10. Rose’s ‘grouping’ gesture

(b) <u>Grouping action evident; the word appears to mean putting items together, then putting them aside.</u> The subsequent story sum dealt with a little boy packing cans into boxes. The language of the story sum “Mick packs cans into boxes …” and the picture shown in the textbook (Figure 9.11) gave a sense of an action taking place:

Figure 9.11 Textbook picture for grouping cans

Teacher:	I’m going to act it out (...) How many cans has he got? (*makes a gesture indicating putting things together and putting them aside*).
Pupil:	Twenty.
Teacher:	Twenty. (*Opens her arms out wide*). Now they do not fit all in one box. He takes five of them and puts them in a box (*brings arms closer and mimics putting something aside. This*

gesture is done four times as the teacher talks). [So] Five in one box, another five in a box, another five in a box, another five in a box. We want to know how many boxes we need. Annemarie?

Annemarie: (*Silent*).

Teacher: (*Repeats above explanations and gestures*). What is happening here?

Annemarie: Grouping.

(…)

Teacher: How many boxes do I need?

Pupil: Four.

Teacher: Four. How did you work that out?

Pupil: Five, ten, fifteen, twenty (*opens fingers out*).

Teacher: We already know the answer. Now I would like to work it out with a 'statement' and a division (*touches notation written on the board for the previous story sum to indicate the approach required*).

Nadia: "Five cans equals one box (…) twenty cans equals how many?"

Teacher: (*Writes the following on the whiteboard*:

5 cans = 1 box
20 cans = ?

(*Touches the board underneath the 'statement'*) What do I write here?

[G3M&D2minute66)

The pupil Rita then suggested 5 ÷ 20 and Rose guided her to giving the correct notation 20 ÷ 5 (see Section 9.3.2), which Rose wrote on the board. She then asked if someone could work it out, and Melissa counted in fives up to twenty.

Teacher: And how many boxes is that?

Melissa: Four.

Teacher: Four. It is very important that you make use of your fingers. Then I write my answer. (*Completes equation on whiteboard as shown*):

20 ÷ 5 = 4 boxes

[G3M&D2minute66)

The packing situation seems to have included two parts. First, the action of grouping cans, the solution of which was found by Annemarie by counting up in fives. Second, the manipulation of the notation, which however, I feel was left somewhat disconnected from the first part. The various parts of the notation (20, 5 etc.) were not focused on explicitly, in the sense of stressing that the 20 represents the original set to be acted on, the five represents the size of each group, and so on. The writing of the 'statement' and subsequent manipulation of the notation *per se* seemed to hinder the setting up a link between the physical action of grouping and the division concept and notation.

It was interesting to note that the above quoted examples became progressively more 'concrete' in the sense that they offered progressively more opportunity to perceive an action with actual groups being formed in the cans example. I might have expected them to be offered in the reverse order. I say this based on a taken-for-granted assumption that younger children may benefit from tangible experiences. However, I will consider the point more theoretically in Section 9.6. The final example below illustrates another use of grouping in yet another context, this time in conjunction with a number line in the fifth lesson.

(c) <u>*Grouping* appears to refer to equal sets of numbers on a number line, but the diagram is more suggestive of multiplication</u>. The class was focusing on a textbook exercise that depicted a kangaroo jumping in threes on a number line. The numbers up to 30 were represented as 'stones' and the first two hops were indicated (I reproduce the line up to 16 here):

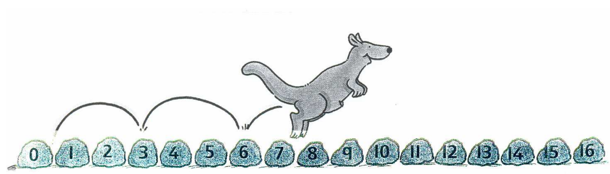


Figure 9. 12. Textbook depiction of a number line.

The first exercise that was presented in relation to this diagram had focused on multiplication ("find what number the kangaroo lands on if he jumps 4 hops"). The aim of the subsequent exercise was to find how many hops the kangaroo had taken to land on 9, 15 etc. that is, to offer a division interpretation for the number line activity. Rose asked 'What are we doing?', which usually meant that she was expecting an operation to be suggested, but the division operation she hoped for was only offered by the girls after much prompting, and after they had suggested counting, jumping, grouping, multiplication and addition. In the course of the interaction, Rose mentioned 'sharing' but this was not evident in the picture; furthermore, although 'groupings' could indeed be perceived (hops of threes), the diagram appeared to be more suggestive of multiplication, hence repeated addition, than of division, since the kangaroo was jumping 'up' the number line (left to right). Even having established that the operation was division, it was only with great difficulty that Rose led the girls to conclude that the operation to be worked out was 9 ÷ 3. The full transcription illustrating the interaction may be viewed in Appendix J.

Reflecting on the various uses of the word *grouping*, I concluded that the only apparently consistent feature of the contexts was the word itself and the gesture that was usually used to accompany it. During the lessons, pupils were encouraged to associate the word *grouping* with the word *division*, and with guidance, they formulated the appropriate operation. However, Rose did not draw attention to how the elements of the contexts related to the division operation (i.e. *explicitly* discuss why a certain number is written first in the notation, the significance of the divisor and so on), so that possibly the relationship between the respective situations and division was not rendered clear.

When I asked the pupils to explain the word *grouping*, none of them used the idea of division in their explanations. Furthermore, although the pupils stated that the two words *sharing* and *grouping* had already been known to them - an opinion also held by Rose - the interview data suggests that the girls failed to make a distinction between the two. For the word *grouping*, the explanations offered described yet again sharing situations, although the pupils sometimes included the word *groups* and/or a gesture similar to Rose's grouping gesture. For example, Ramona and Kelly, in their joint explanation, gave a sharing situation with Kelly concluding:

> " (…) **Into two bags. And that's** five sweets **in a bag** (*makes grouping gesture*) **and another** five sweets **in a bag** (*makes grouping gesture again this time further to the right*). [G3M&D(C)Q1]

The lack of distinction between *sharing* and *grouping* is illustrated by the joint explanation given by Melissa and Lara:

Lara:	Grouping, em … when we have a … twelve sweets and we grouping.
Melissa:	We have to share them.
I:	How do you group twelve sweets?
(…)	
Lara:	We share, em, grouping (*joins fingers of one hand together*).
I:	OK. How would you group them?
Lara /Mel:	(*Silence*).
I:	OK what about sharing? What does sharing mean?
Lara:	When we have em … a six girls and, and I have a sweets, and we share it; you have five, five, five … (*gestures giving*

something out to various people).
Melissa: Sharing means that we have … (*long pause*) when we have three, three girls and we have three, three children you have to share them with the three girls.
I: So give me an example of grouping, Melissa.
Melissa: Grouping is em … when we have … ten sweets and we have sh… and we have to group them for two children.
And how many would they get each?
Melissa: Five!
[G3M&D(B)Q2]

Similarly, Maria appeared at first to be talking about a grouping situation, but in her Maltese explanation later, described a sharing situation:

> Maria: Em, you have eight, em and you … group them in two (*joins fingers of one hand*). You make two, two, two, two, two, two (sic) (*moves bunched fingers from right to left*). [G3M&D(A)Q2]
>
> [Grouping is] **when you've got eight dolls and you share them. And so I'll have to share them between two and they have to be enough [to go round] for everyone.** [G3M&D(A)Q1]

I concluded that pupils developed an interpretation for both words *as the formation of equal groups through a sharing action*. Perhaps Rose did not give importance to the distinction between the structures I myself give, or was not sufficiently aware of it. Whatever the case, I think that the pupils' ideas may have been a result of Rose's occasional use of the words as though they were synonyms of each other, the various uses of the word *grouping* (sometimes in contexts were the formation of groups was not evident) and Rose's frequent use of the word *groups* in both sharing and grouping situation. Indeed, during the week, the word *group/s* was used 46 times by the teacher and 5 times by the pupils, more frequently than the word *grouping* itself which was used 19 times.

9.5.3 The notion of repeated subtraction

The concept of *multiplication* as presented to young children is based primarily on the idea of a repeated addition and a parallel can be developed for division as repeated subtraction. I believe that had this key aspect been addressed in more detail, as it had been done for multiplication, appropriate chains of meaning may have been set up. However, the idea of repeated subtraction was mentioned only very briefly twice. The first instance was at the start of the topic when the girls read out the phrase 'to divide is a repeated subtraction' from the notes written in their copybooks. The other instance was when it was used was by the teacher during the correction of the ‘kangaroo’ exercise in the fifth and final lesson. As the exercise progressed, the teacher wrote the following notation on the board:

11. 9 ÷ 3 = 3 hops
12. 30 ÷ 3 = 10
13. 21 ÷ 3 = 7

Rose explained to the class:

> T: In the division, the number **look**, becomes SMALLER. See? (*The teacher runs her finger down the ‘answers’ column - 3, 10, 7 - then up the dividends column 21, 30, 9*). It always becomes smaller because we are dividing, grouping, we are sharing. It is a repeated subtraction.
> [MD5minute58]

I myself was able to interpret the gesture to mean that 3 was smaller than 9 (reading horizontally right to left across the whiteboard example numbered 11 above), 10 was smaller than 30 (second example) and 7 was smaller than 21 (third example). However, I suggest that this relationship was not clear to the pupils, since it was not clear to what the teacher's language was referring. The notation did not show subtractions, and the only ‘repeated’ things were the divisor 3 and the symbols ÷ and = present in each example. As she moved her finger down and up the numbers, no obvious pattern of something getting smaller could be perceived. The kangaroo had been presented by the textbook as jumping ‘up’ the number line, since it was needed for the previous multiplication exercise; however, a more appropriate representation for division as repeated subtraction would have been the kangaroo jumping ‘down’ (i.e. right to left). The repeated subtraction notation 9 -3 - 3 - 3 = 0 was not used at any time by Rose.

As she herself explained, Rose did not give this idea as much importance as repeated addition:

> “At this age, I think they will find more work added up, for example, in an addition you can have four, five, six numbers on top of each other. In a subtraction you don’t find that much at this stage, because you can find it later on, for example 36 minus 5, minus 2, minus 1. With regards division you have the same numbers. But it hardly happens. I just mention it because of subtraction so that it clicks that something is becoming smaller and smaller every time.” [RoseM&D2(025)]

However, this relation with subtraction (both ‘make less’) was not actually stated explicitly by Rose at any time during the week.

During the interviews, an unexpected interpretation for the idea of ‘repeated subtraction’ was offered by three of the pupils (the other three did not offer explanations). I asked the girls what the teacher had meant by saying that 'division is repeated subtraction' and they answered as follows:

> Sandra: [Repeated subtraction] **means** if you have three minus three, minus three, minus three … (*trails off*).
> Maria: **And you do them** division.
> [G3M&D(A)REFERENCE]
>
> Melissa: **Instead of us doing** six, minus six, minus six, minus six, **you do** six … division … by … three equals … (trails off).
> [G3M&D(B) REFERENCE]

The interpretation the girls were apparently giving was that of 3 – 3 – 3 – 3 or 6 – 6 – 6, possibly drawing on their knowledge of repeated addition as 3 + 3 + 3 + 3. However, this is not what is normally meant by the statement ‘division is repeated subtraction’, and therefore, although a plausible interpretation, it is not an appropriate one. Such an interpretation may be considered an inappropriate link between signs, or what is commonly referred to as a misconception.

I suggest that the use of repeated subtraction notation (e.g. 12 – 3 – 3 – 3 – 3 = 0) may need more detailed attention than that for repeated addition (e.g. 3 + 3 + 3 + 3 = 12), since it is more complex. The starting point in the division notation is the original quantity and the number of 3s is not optional but must reduce 12 to 0; the solution is not the ‘answer’ (0), but the number of repetitions. A possible semiotic chain for division as repeated subtraction – which however was not set up and hence not shared - is shown in Figure 9.13.

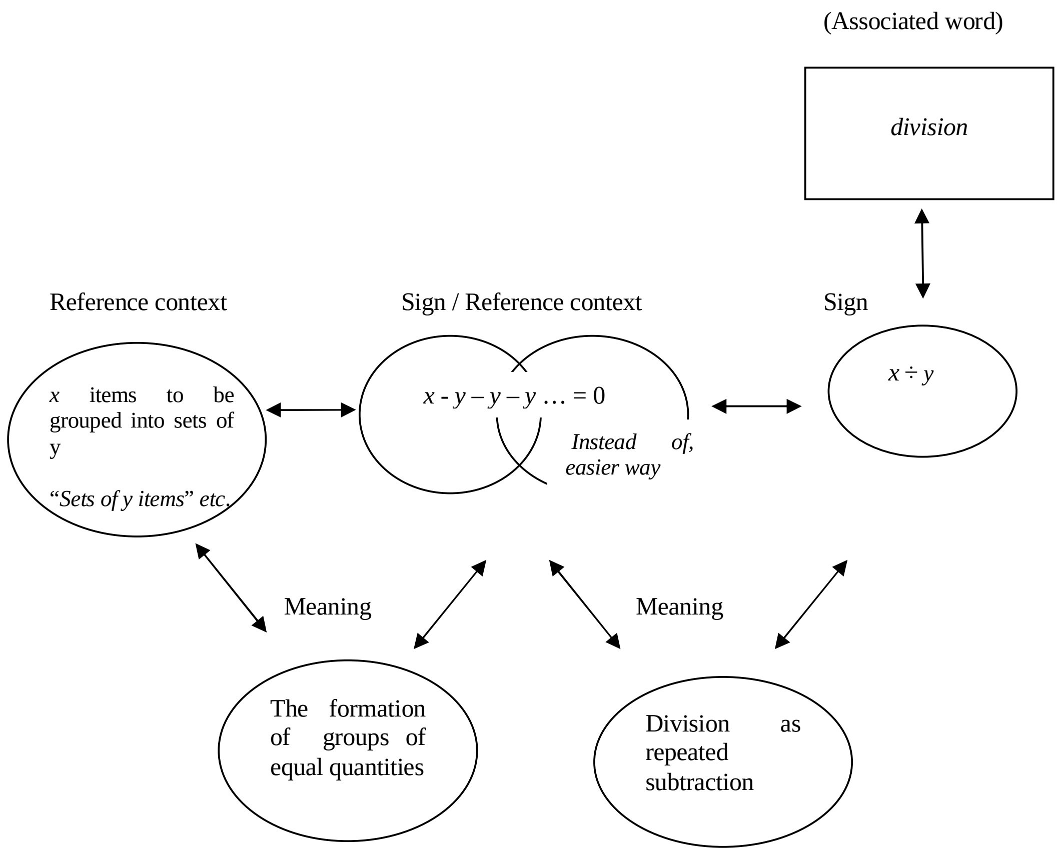


Figure 9.13 A concept for division as grouping / repeated subtraction

Division is often said to be the 'hardest' of the four operations to teach (Frobisher, 1999, van de Walle, 2004) and using my data as background, I would now like to reflect on why this might be so from a semiotic point of view.

9.6 Comparing possible representations for multiplication and division

For this discussion, I consider multiplication and division through four different structures as indicated in Table 9.2. The first three structures - equal groupings, number line, allocation - are adapted from Anghileri and Johnson (1988, p.158-162) and number triples is adapted from Anghileri (2000, p.76). It should be noted that I include only aspects relevant to Grade 3 level and thus I do not consider the structures of scale (for both multiplication and division) nor the idea of Cartesian products for multiplication.

Structures	Multiplication	Division
Equal groupings	Finding the total of a number of equivalent disjoint sets. e.g. three pairs of boots.	Rearranging member of a set to create equal subsets. E.g. six apples to be arranged into twos.
Number line	Movement 'up' a number line in equal steps. (e.g.$0 \rightarrow 3 \rightarrow 6 \rightarrow 9$)	Movement 'down' a number line in equal steps. (e.g.$0 \leftarrow 3 \leftarrow 6 \leftarrow 9$)
Allocation (rate / sharing)	Equal sets (portions) are matched using a tally of objects (owners). E.g. Three boys get four stickers each.	Sharing out items into a given number of partitions. E.g. sharing twelve stickers among three boys.
Number triples	Appreciation of a number triple independently of a practical procedure. E.g. $3 \times 4 = 12$ and $4 \times 3 = 12$	Appreciation of a number triple independently of a practical procedure e.g. $12 \div 3 = 4$ and $12 \div 4 = 3$.

Table 9. 2 Comparing multiplication and division structures

All the above multiplication and division situations can be represented as $m \times n$ and $x \div y$ respectively, but in order to allow for the various interpretations, different reference contexts must be available comprising different actions (real or imagined), pictures and associated language. I suggest that the first three structures can be represented by either active or static representations. By 'active' representations I mean that an action takes place, while a static representation is a picture or diagram that captures the situation at some moment in time. In the following sections I will argue that while active representations can be easily perceived for both multiplication and division, it is harder to present and interpret static representations for division than for multiplication. Finally, I will reflect on how the number triples come to exist independent of perceivable objects, and how in this case mathematical language serves as its own context.

9.6.1 'Active' representations

For multiplication, a child might actively create sets of objects (e.g. buns) or allocate items to a given number of friends. In the absence of tangible objects, a child can *draw* the objects one by one, and I can still consider this to be an active representation. Similarly, marking jumps on a number line can also be considered to be an active representation. The language used in close conjunction with the action and objects has the potential to draw attention to features of the situation: ‘groups of 4’, ‘how many in each group / box / jump [on number line]?’, ‘groups of the same size’, ‘how much altogether?’, ‘how big is the jump?’, ‘each friend gets four stickers’ and so on, depending on the situation.

The first three structures for division listed in Table 9.2 can also be represented actively. In a situation where someone is physically grouping, sharing or moving down a number line, attention can be drawn to features of these actions that can relate them to the operation of division. For example, attention might be drawn to the original set to be acted on, the number of sets or number of items in a set, or the size of the step on the number line.

Division as grouping and on the number line can be viewed as repeated subtraction although items may not actually be ‘taken away’ (the common interpretation of subtraction for young children), but simply ‘utilised’. So for example, if a boy is packing cans into boxes, 5 per box, he is *utilizing* the cans (i.e. completing sets), rather than taking them away. This in itself may present a language difficulty for young children since it is necessary to give a different interpretation of subtraction in relation to the objects being acted upon. For sharing, the action involves two tangible elements (e.g. children and sweets), and the language needs to deal with apportioning items.

As in the case of multiplication, a picture can be used to prompt an active representation of division, as in the case of circling items for grouping or indicating a ‘giving out’ action:

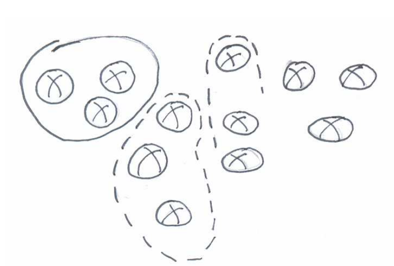
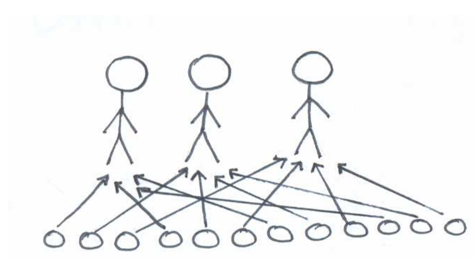



Figure 9.14. a & b ‘Active’ representations of grouping and sharing through pictures sharing adapted from Anghileri & Johnson (1988, p.160).

During my week of observations, no active representations were carried out for multiplication, nor for division as grouping. The only active representation for sharing was the exercise where sweets were placed into a given number of boxes through drawing. Here Rose did ask 'how many sweets do I have?' while she touched the relevant numbers on the whiteboard. However, this was the only exercise of its kind, and the words *share* / *sharing* were hardly used at all and therefore the potential of the situation was not realised. I suggest that the advantage of an active representation is that language can accompany the unfolding situation, and hence support the setting up of semiotic chains.

9.6.2 'Static' representations

Multiplication situations can be shown pictorially, in which case the representation is static. Referring back to the structures identified by Anghileri and Johnson (1988) (see Table 9.2) I noted that the examples provided by the textbook were of the 'allocation' type. Two examples of static representations for multiplication offered by the textbook are shown below:

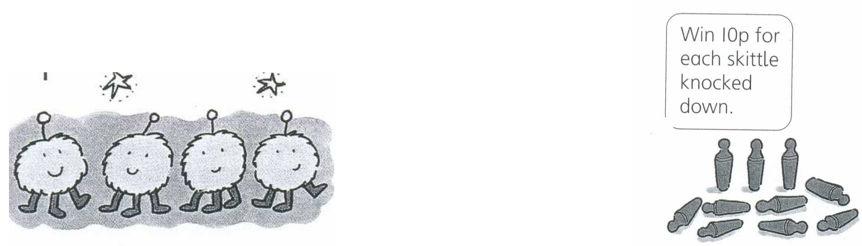


Figure 9.15 a &b. Textbook static depictions for multiplication

Pictures capture a moment in time and do not actually involve the physical creation of groups. The sets are concurrently present and the language used in conjunction with these diagrams can aid the child to perceive the idea of repeated addition in the sense of 'yet another set'. Several such activities were carried out during the week and the semiotic links shown in Figure 9.7 - 9.8 were possible.

On the other hand, I suggest that it is harder to perceive a division situation through a static picture. For example, the textbook offered the diagram reproduced in Figure 9.16 beneath the page title 'Multiplying and dividing'.

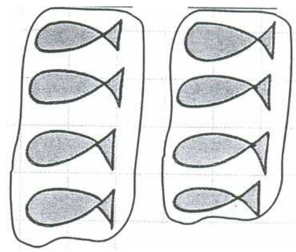


Figure 9. 16. Textbook static representation for division

In an apparent aid to interpretation, the authors offered both the multiplication and division (as grouping) notation in relation to the diagram:

$$8 \div 4 = 2$$

$$2 \times 4 = 8$$

However, I suggest that is more likely for a child to perceive the sets as 2 sets of 4 fish (i.e. a multiplicative situation) rather than a total of 8 subdivided into 2 groups of 4. In order to focus on the original set of 8 fish, the child must temporarily ignore the lines that create the subsets. Similarly, if a diagram is attempted for sharing, the perceiver may 'see' the diagram as repeated similar sets, rather than an original whole set now apportioned out.

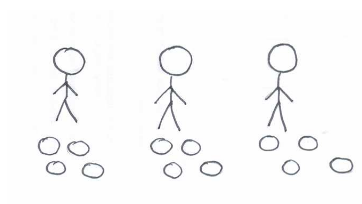


Figure 9.17 Static representation of sharing

Rose did not offer many static representations and when she did, the language used in conjunction with them was not appropriate, for example 'sharing hands', 'grouping 5p coins for a stamp' and division on a number line showing upward jumps. Thus, the reference contexts were not altogether helpful to support a concept for division as grouping /sharing. I conjecture that it is harder for a teacher to combine language with static diagrams to create supportive reference contexts for division and yet, by its very nature, a textbook tends to offer static representations which therefore, are limited. Due to this apparent limitation, teachers may have less representations to turn to as a support for the concept of division than for multiplication.

9.6.3 Number triples: mathematical language as its own context

The final structure for multiplication given in Table 9.2 is that of number triples. According to Anghileri and Johnson (1988), arrays are useful to support a meaning for these relationships. Language can be used to draw attention to the number of dots/stars/pegs etc. in a row, column or all the array. During my observations, arrays were presented as stamp sheets and peg boards which served as static representations of repeated sets.

However, ultimately, it is important for children to appreciate the triples independently of any perceivable object or situation (Anghileri, 2000). For this layer of meaning, language serves as its own context (Vygotsky, 1962). More specifically, *mathematical* language serves as its own context, since multiplication is talked about as a relationship between three numbers m, n and p, such that p is the product of two factors m and n. Figure 9.18 shows a possible semiotic chain for this idea:

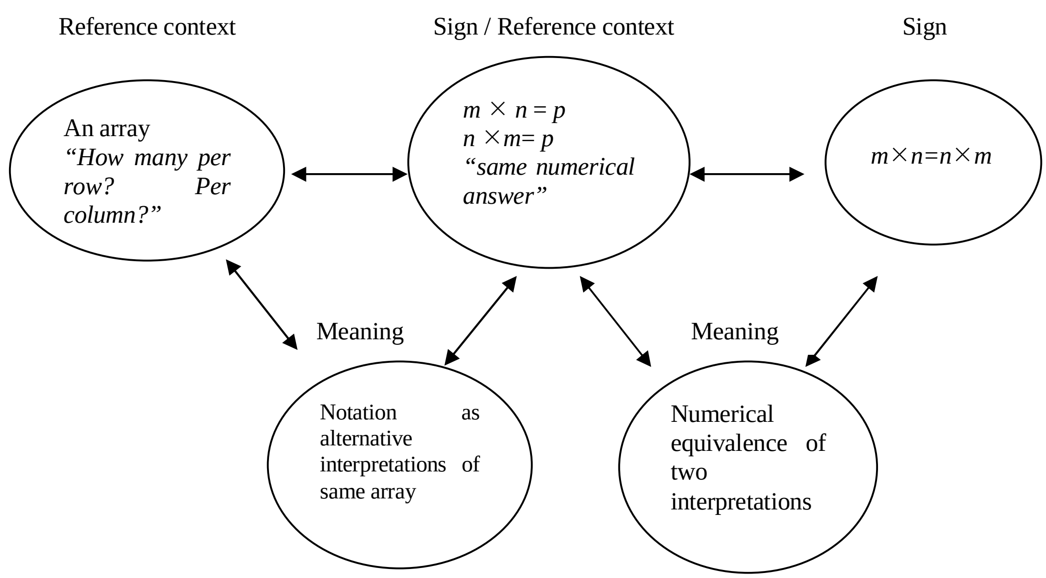


Figure 9.18. The multiplication triples

As already discussed, the idea of commutativity was addressed during the lessons, not only through the arrays, but also by virtue of the tables the pupils knew so well. An awareness of commutativity contributes to the development of meaning for the triples relationship with respect to multiplication.

The number triples can be extended to division and once again arrays might be used to support the relationships $x \div y = z$ and $x \div z = y$. A semiotic interpretation is as follows:

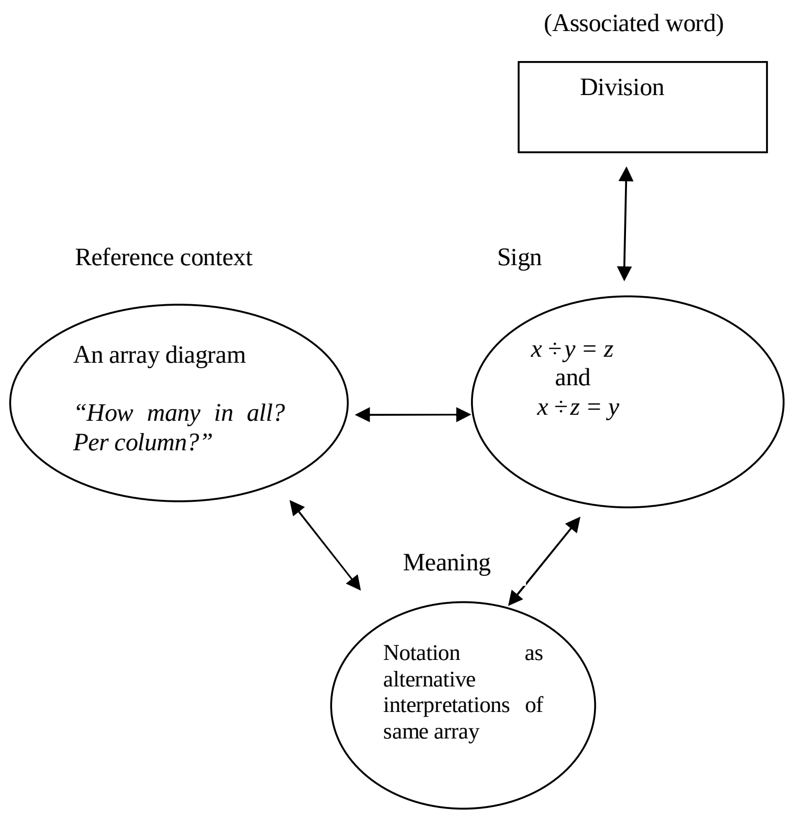


Figure 9.19 Number triples for division

However, the use of an array as a representation brings with it the same difficulties mentioned previously for static repeated groups.

The strength of the triples lies in the inverse relationship between multiplication and division that can be expressed as: 'IF $m \times n = p$ (and therefore $n \times m = p$) THEN $p \div n = m$ and $p \div m = n$'. A semiotic representation of the inverse relationship is shown in Figure 9.20:

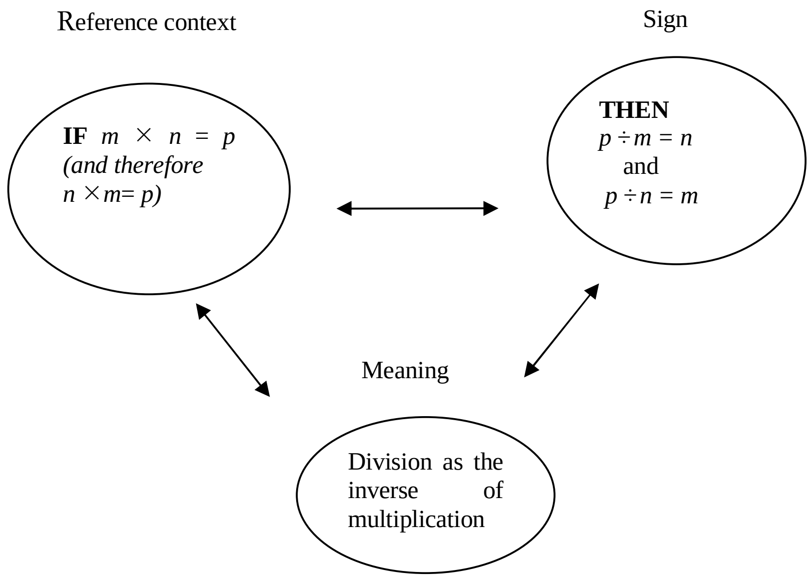


Figure 9.20.Number triples

In this interpretation, both the 'reference context' and the 'symbol' elements of the semiotic model consist of notation and mathematical words together. The left hand side and right hand side can easily be inverted (e.g. if $p \div m = n$, then $m \times n = p$) and Steinbring's (2002, p.8) point that "sign systems and reference contexts are (…) temporarily equal, none precedes the other" is very evident here. According to Chapman (2003) statements of the kind "If $m \times n = p$ then $p \div m = n$" constitute 'mathematical language' since they are characterised by a metonymic rather than metaphoric orientation (e.g. monsters and stamps) and high modality (that is, a strong sense of certainty). Here meaning is created and expressed within the realm of mathematics itself.

Apart from appreciating a mathematical relationship for its own sake, the triples relationships are useful to understand since it is not common practice to learn to recite 'division tables' ("3 divided by 3 is 1, 6 divided by 3 is 2, 9 divided by 3 is three …"), whose quick recall would provide a handy strategy for finding solutions. Thus knowledge of the inverse relationship with multiplication allows me to apply the relationship to a variety of contexts. So, for example, given the question "*How many days to save 45p, if 5p is saved per day*?" one can reason out using the triples as follows: "if $9 \times 5 = 45$, then $45 \div 5 = 9$', therefore the solution is 9 days". In fact, I suggest that a 'division' solution of this question can only be found using the relationship: no repeated subtractions or groupings are taken or made from the 45p and it is hard to bring to mind an image that specifically depicts ‘division’ (putting 5p coins in a money-box is likely to be more readily interpreted as a depiction of repeated addition). Similarly, if asked to find the number of 5p coins needed to buy stamps of values 30p etc, it is difficult to draw a diagram to depict this (repeated 5p coins would be a depiction of repeated addition). Again, the triples would be a useful interpretation. While Rose guided the pupils to offer appropriate triples, it is interesting to note that these two exercises (saving 5p / stamp values) were the *first* to be carried out by the class regarding division. However, I suggest that in view of the points raised in this discussion, it may be useful for a teacher to bear in mind that such exercises are different to active and static representations and possibly ‘harder’, since they depend solely on metonymic mathematical language for their interpretation.

9.7 Conclusion

It appeared that when words were used in a referential role, it was easy to achieve clarity by virtue of the close perceptual association between the word and to what it referred. Furthermore, the procedures of *multiplying* and *dividing* also became apparent to the pupils as these words were used frequently, and the proximity of their use with notation/actions appeared to render the meaning of the words clear. When a layer of meaning goes beyond reference or procedure, it may be harder for clarity to be achieved, since it is a more complex task for an association to be set up between language and that to which it refers. In this chapter, I discussed various potential language difficulties related to Multiplication and Division, and developed a discussion regarding how the concepts can be represented through ‘active’, ‘static’ and metonymic representations.

One of the aims of my study was to tease out issues that may be particular to immersion classrooms and which might be general language aspects. I suggest that all the points discussed in this chapter may very well have been identified in a first language classroom. Of course, I cannot exclude the possibility that Rose structured her explanations because she was ‘obliged’ to use English. However, I do not have evidence that indicates with any certainty that the use of English was detrimental to the pupils’ understanding of the mathematical words discussed – the pupils’ explanations indicated that they had followed the teacher’s line of thought for multiplication; for division, incomplete or inappropriate ideas expressed by the pupils can be explained by way of the general approach taken by the teacher, an approach that may very well have been replicated even if the lessons had been done through Mixed Maltese English.

However, as a point of reflection on the language issue, I would like to raise the possibility that one of the pupils did not understand the word *repeated*.

Ramona was a pupil who was described by her teacher as ‘weak’. When asked to explain 'repeated subtraction', Ramona had her copybook in front of her, open on the page showing the multiplication / division words. She glanced briefly at the notes and replied:

> “**Em … you’ve got a** number **and em … you keep … for example, you’ve got a number and you’ve got to keep … em … and** (long pause) **you’ve got to revise it”** [G3M&D(C)Q6].

It is possible that Ramona meant to say **tirrepetiha** (**you repeat it**) rather than what she actually said **- tirrivajżjaha** (**you revise it**). I say this because of her use of '**you keep …**' by which she may have intended to say '**you keep repeating it**' but then could not find the right word and used '**revise**' instead. The choice of '**revise**' here may have been influenced by her teacher's occasional comments that they should revise their notes and their tables. However, another possible interpretation could be that Ramona had not understood the word *repeated* during the lessons: indeed during the interviews there was no evidence that she viewed multiplication /division as repetitions, nor that she understood the word *repeated* (except for the possible implication of ‘you keep …’). This was not the case for the other five pupils, who through some statement or another indicated that they knew what *repeated* meant. Obviously, if Ramona had not understood the word *repeated*, she would have missed out on a crucial point in the topic’s development.

Clear semiotic links are an important language issue that must not be overlooked whichever language is chosen as a medium of instruction. Therefore, it is worth reflecting on the exclusive use of English with respect to various representations. When dealing with active and static representations, it is important that a teacher uses language that is easily understood by the pupils, in order that the objects and pictures being utilised lend the necessary support to pupil interpretation. Perhaps it may in fact be helpful for the pupils to use some Maltese in these situations. With regard to the number triples, these are expressed through notation, hence would retain their format even in a classroom where Mixed Maltese English is used. However, in such a classroom, one can reflect on the move from potential expression of the ‘if …then’ idea in Maltese, to expressing the idea through mathematical English.

As I looked out for ‘clarity’, which I assumed necessary for sharing meaning, I remained open to any other features that might have had a bearing on sharing meaning. However, at this stage, I have not identified any other feature other than frequency, previously discussed.

CHAPTER 10

Graphs

10.1 Introduction

The topic ‘Graphs’ was carried out in the Grade 6 classroom. Gina’s intentions for the week were expressed as follows:

> Gina: “Most important is reading from a graph (…) They [pupils] are used to plotting block graphs. (…) What they didn’t tackle [in the past] was that from a graph you can be asked questions, and derive answers by looking at the graph”. [GinaGraphs(1)Q1]]

I must express some doubt about the fact that the girls would not have experienced answering questions about a graph, since this is generally included in the teaching and learning of graphs at the earliest stages of graphical representation, usually Grade 3 level. However, this is what Gina believed and indeed, a lot of lesson time was spent reading off information from graphs. The graphs tackled were block/bar graphs, straight-line graphs and pie charts. The general approach taken was for the girls to draw, or copy, a graph and then to answer related questions. Some sample questions are listed in Table 10.1:

Graph		**Sample of questions asked**
Bar graph showing a pupil’s test results per month.		What was the average mark for the five tests? Between which two months was there most improvement?
Straight line graph showing the distances covered by a man walking at 5 km/h.		At what time did the man start walking? What distance did he cover between 10.30 a.m. and noon?
Pie chart showing preferred sports of 16 children.		How many children voted for swimming? Which sport was least popular?

Table 10.1. Sample of questions asked in relation to graphs

The girls used a copybook which had alternating squared (2mm) and lined pages. Graphs were drawn on the squared pages, while information or any workings were written on the facing lined pages. Class corrections were lengthy since the teacher drew each graph on the board and then went through the working of every question together with the pupils. The spread of the work is summarised in the table below and includes the correction of homework tasks:

	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**	**Lesson 5**
Bar graphs	3	2	-	-	2
Line graphs	-	2	4	3	1
Pie charts	-	-	-	1	4

Table 10.2. Distribution of graphs over the week

As explained in Chapter 8, one limitation I am aware of for the topic ‘Graphs’ is my uncertainty about the familiarity of some of the topic-related words. During the interviews, I showed the pupils the list of words and asked them to point out which ones were new. However, with hindsight I realise that this may not have been a very reliable method since some initial conclusions did not tally with other interview data. For example, the word *data* was not mentioned as new by one of the pupils, Stefania, leading me to conclude that this word had already been familiar to her from before. However, later on in the interview, she said she did not recall the word at all. Another two pupils (interviewed together) said they were not sure whether they had known the word *data* prior to Grade 6, giving me the impression that they knew the word but could not identify when they had learnt it. However, later on in the interview, they went on to say that they could not recall the word being used during the week and could not give an explanation of it. The same situation arose for the same two pupils for the expression *drop a perpendicular*. Although these were the most obvious discrepancies, I was left with some doubt about the way I collected this information for ‘Graphs’. Hence, I took all the evidence into consideration when commenting about whether words were new or not in the course of my discussions.

I start by considering the use of the words *graphs*, *axes* and *data* as reference, then examine in more detail the meanings for graphs. I then consider the two topic-related verbs, *plot* and *drop a perpendicular*, and finally reflect on the interrelated meanings for *scale* and *representation*.

10.2 Words as references: *graphs, axes* and *data*

Just as I had identified words that named notation for the topic ‘Multiplication and Division’, so for the topic ‘Graphs’ there were words that named the diagrammatic representations as shown in Figure 10.1.

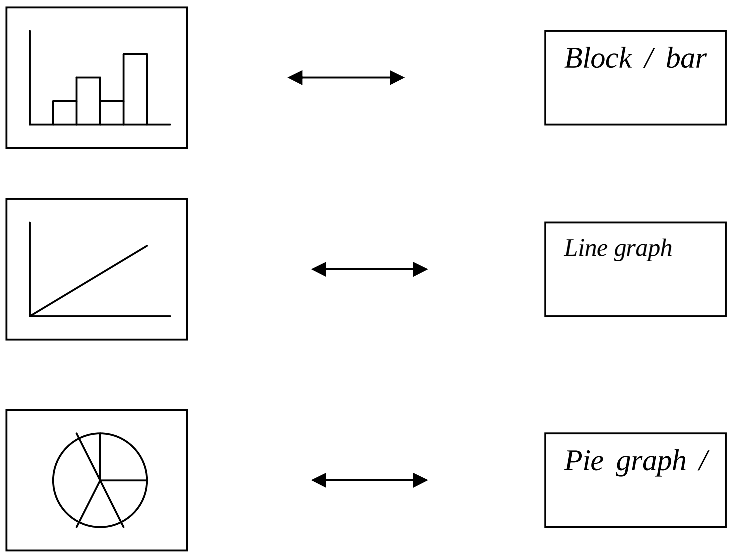


Figure 10.1. Names for different types of graphs.

The first type of graph was usually drawn as bars as shown in the figure, although Gina tended to refer to such graphs as *block graphs.* The name *block graph* comes from an earlier version of this type of graph where ‘blocks’ (often squares) are used to represent items in a one-to-one or one-to-many relationship. The pupils used *bar graph* more than *block graph* during the lessons, although my impression over the week was that the pupils were well aware of the interchangeability of the two terms. Both *pie graph* and *pie chart* were used by the teacher and the pupils for the third type of graph. The name *line graph* was used consistently. Very often, any one of the above graphs was simply referred to as a ‘*graph*’.

All the six girls interviewed said that they had known the word *graph* previously, and from the conclusive interview data, I can say that *block graph* had already been known to 4 pupils, *bar graph* to 5 of them and *pie chart* to 4. The least familiar name appeared to be *line graph,* since four of the interviewed girls said they had not known it previously. All the girls were successful in expressing the three names during the interviews. I attributed this to the possible previous knowledge of the meaning of the words but also to the fact that during the lessons, the words had been used in close conjunction with the diagrammatic representation to which they referred.

Similarly, the expressions *x*- and *y-axis* (which were new expressions to all six pupils interviewed) were used very frequently during the lessons in reference to the vertical and horizontal lines drawn on the board and on the pupils' copybooks. The 'naming' role of the expressions was highlighted by Gina early on in the development of the topic, when the names were offered by one of the pupils Daniela:

> (*This is the first class discussion regarding graphs. The class is discussing a way to represent information regarding the months in which their birthdays fall. Up to now the pupils have suggested a 'graph' and the need for a vertical and horizontal line, which the teacher has drawn on the board*).
>
> Teacher: Listen girls! Somebody said the horizontal line and the vertical line. They have names. And Daniela named the names. She called them …? (*Turns to Daniela*). Tell us.
>
> Daniela: Y-axis and x-axis.
>
> Teacher: Now which is the x-axis?
>
> Daniela: The horizontal (*gestures a horizontal orientation*).
>
> Teacher: The horizontal line, she called it an 'x-axis' (*writes* 'x axis' *at the end of the horizontal line drawn on the board*).
>
> Clare: Can you name it the 'b-axis'?
>
> Teacher: Usually there's a standard form. It IS standard. And the vertical line, we call it the 'y-axis'.
>
> [G6Graphs1minute16]

Over the week, attention was frequently drawn to the axes, since they formed an important part of the diagrams being drawn. For example Gina might say 'We have pounds on the x-axis' [G6Graphs2minute127]. For every bar graph or line graph drawn, Gina and the girls would always label the axes as 'x axis' and 'y axis' (not hyphenated) as shown:

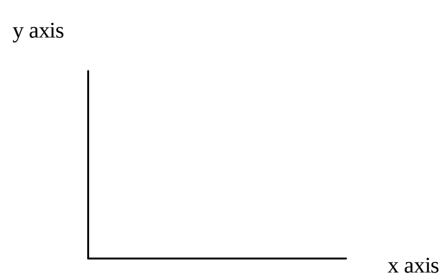


Figure 10. 2. Labelling the axes

During the interviews, the girls used the words appropriately as names, for example:

> Stefania: (*She has her copybook open in front of her*). Here's the x-axis (*touches the x-axis of the graph shown on the page*) and here's the y-axis (*indicates y-axis*). [G6Graphs(B)Q2]

I concluded that the names for the axes were successfully shared and I attributed this success to frequency of use and to the fact that what the word referred to was perceptually evident. However, there was another mathematical noun that was used for which for which I believe meaning was not so clear. This was the word *data* which can be defined in a singular sense as a "collection of information" (Haylock, 2001, p.176). The class 'collected' data only for the first two graphs, that is, when Gina asked about their birthdays and favourite animals and wrote the frequencies on the board. On one occasion, data regarding a baby's increasing weight over time was provided in a table printed in the textbook. However, in all other cases, the girls were presented with an already-printed graph, so that the data had already been depicted. Gina introduced the word *data* during the first lesson as follows:

> (*The teacher is about to start a class correction of the drawing of the first bar graph of the week – "birthdays". Some girls have coloured the bars and the teacher shows the class a copybook*).
>
> Teacher: Look, girls, when you use different colours, how clearly you can spot the data.
>
> Pupil 1: Data?
>
> Teacher: The information. Data. Information.
>
> Charmaine: **What does** data **mean**, Miss?
>
> Teacher: It's information. Things you need to know to work.
>
> Dorianne: (*Inaudible*) find the data? [appears to be asking what sort of question they might be asked about 'data'].
>
> Teacher: No, [a question could be] "from your data, find, show me, tell me …"
>
> [G6Graphs1minute57]

Although not altogether clear, Gina seemed to be referring to values or information shown on the axes. In the next lesson, the word *data* appeared to be used with another meaning, as she explained to the girls how to organise their copybook work:

> Teacher: Remember! The graph is on your page (*shows up the squared side of copybook*) and the DATA on the other (*indicates facing lined page*) (…) A graph … and ALL the data next to it [i.e. on facing page]; (*turns page to show two new pages*) a new graph … and the data next to it.
>
> [G6Graphs2 minute7]

In this context, *data* appeared to refer to what was written on the right hand (lined) page which in fact was the scale and the answers to the questions asked in relation to the graph. So for the graph depicting students' test marks, the written component on the lined page consisted of "y axis 1cm represents 1 mark", "Ron scored 100%", "Ron came first" etc. and the calculation of the average mark of the test. Admittedly, this information is data too, but in the process known as handling data, it is usually the initial, researched information that is referred to as data. After being collected, this is then analysed and represented on a graph.

Two points emerged from my reflections on the sharing of meaning of the word *data*. First, I noted that when a mathematical object is not immediately perceived, it is harder to name it. While this may seem like an obvious statement, the implication for teaching is that it may be helpful to render perceivable anything that in fact can be. In this case, because 'data' (in the sense of actively collected information, written frequency tables etc) was not available for most of the graphs, Gina's use of the word in the second lesson appeared to take on another meaning, that is 'data is the written element of the graph work'. I cannot tell whether it was actually Gina's intention to convey this meaning, but it is worth noting the pupils' explanation for the word. Rachel and Claudette - who had said that they had already known the word - used it in the sense of the written elements on the right-hand-side of the copybook:

> Rachel: This is the data. (*She indicates a lined copybook page facing a line graph which shows the written scale and solutions to quesitons*).
> Claudette: It will be written.
> [G6Graphs(A)Q2]

A different interpretation was Dorianne's. She interpreted the word to mean the written values on the graph:

> (*Dorianne has her copybook open on a page showing a straight-line graph. It shows the increasing weight of a baby up to 10 weeks*).
> Dorianne: Information. Data. Em, instead you find [i.e. instead of finding] the data by reading, you find the data by looking at the graph. For example, 'five and a half kilogrammes' (*reading the x-axis*) is the data of … ten (*touches the '10' written on the y-axis*). [
> [G6Graphs(B)Q2].

I conjecture that Dorianne may have been prompted to focus on the data as depicted on the graph because of Gina's comment to the class: "When you use different colours, [look] how quickly you can spot the data" and Gina's response directed to Dorianne herself during the lesson: "From your data, find, show me, tell me …". After all, most of the time, the girls were finding solutions directly from the graph rather than from any previously given information.

A second point of interest regards the frequency of use of the word and the pupils' ability to recall its meaning. In Section 8.3, I suggested that a word which was used very little was unlikely to be recalled by the pupils, and this was particularly true for the Grade 3 pupils. In Grade 6, the word *data* was not used very much over the week – 12 times by the teacher and twice by the pupils. Indeed, three of the pupils interviewed did not recall the word, including Charmaine who had asked the teacher what the word meant. However, three pupils *did* in fact recall the word and offered a meaning for it. This seems to suggest that the Grade 6 pupils may be able to recall words more easily (that is, after less exposure to the words) than their Grade 3 counterparts. If this is the case, it may be particularly important for words to be used appropriately with older pupils, since they might take a one-off statement to be the intended meaning, even if it is not. I will revisit the point later on in the chapter.

10.3 Further layers of meaning for *graphs*

I would now like to reflect on what meaning Gina tried to share with her pupils beyond naming. I will comment on three aspects related to 'knowing' graphs: procedures for drawing and reading graphs; the idea that a graph is a 'picture' and the awareness of similarity / differences between graphs.

10.3.1 Reading graphs

Sometimes Gina modelled the drawing and reading of a graph, which the girls then copied. On other occasions, the girls worked the exercise out on their own in class or at home and this was then followed by a class correction. Since each type of graph required a different method for drawing and reading, I will not go into detail regarding Gina's instructions or modelling, but suffice it to say that Gina was consistent in her instructions for the respective graphs and during the interviews, the girls were able to explain how to draw and read off the graphs in order to find the solutions to the questions asked. When I asked the girls to 'tell me about' a graph, they explained the procedures for creating the graph and / or for finding required solutions to the questions. The following excerpts are illustrations:

(*I have asked Stefania to tell me about graphs. She has opened her copybook on a page showing a bar graph*).

Stefania: Here are the months, you write them out in the x-axis (*touches x-axis*). And on the y-axis there are the marks. In January a boy got 40 marks (*touches the 40 shown on the y-axis*). And then in February he got 30. And in March he improved and he got over 80 (…) Then, in the month that he, em improved much it was March.

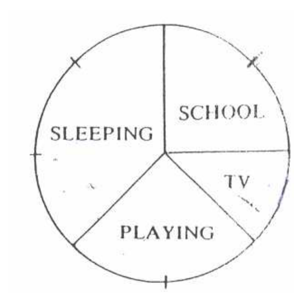
[G6Graphs(B)Q2]

(*I have asked Claudette and Rachel to tell me something about 'these' sort of graphs, while indicating a line graph on their copybook*)

Rachel: You mark like that (*touches small plotted dots on her graph*). You just put a dot, to mark all of them [given points], and then you do a straight line upon them.

[G6Graphs(A) Q2]

(*I point to a diagram of a pie chart drawn in their copybooks and ask them to tell me about it).*



Charmaine: He tells you for example "How many … how much time did the children spend sleeping? You see how many, how many pieces there is over … and for example, he tells you to do it in a fraction. You have three (*counts the marked out sections*) three on eight, 'cause here you have three pieces and eight all of them.

[G6Graphs6(C) Q2]

I concluded that the girls were successful in recalling how to draw and read the various graphs (although it must be said that the girls probably already knew how to draw and read bar graphs and may have had previous experience of pie charts). Copying and reading graphs contributes to pupils' knowledge of the topic in the sense that the function of a graph as a source of information is appreciated.

10.3.2 Graphs as pictures

One idea that Gina liked to repeat was that ‘a graph is a picture’. The following two excerpts illustrate this point:

> (*The class has drawn their first bar graph for the week depicting birthday across the months*).
>
> Teacher: We have talked about our birthdays and we have DRAWN them. Which is the EASIEST way to get a quicker information? Having them listed down or having it in graph form?
> Federica: In graph form.
> Pupils: Graph.
> Teacher: **So**, what is the use of a graph?
> Federica: To make it easier.
> Pupil 1: To show (*inaudible*).
> Pupil 2: Simplify.
> Teacher: Yes, to SIMPLIFY, to GROUP and picture all the information you can have.
> (…)
> Claudette: It’s smarter, because if you draw a chart you can draw, and if you have just this … (*gesture off camera, possibly Claudette is referring to the written data*)
> Teacher: Right! This is more attractive! (*touches bar graph drawn on the board*).
> [G6Graphs1minue68]

> (*The class is correcting a straight-line graph that they had done for homework. One question required the graph to be extended, something that not all pupils did, since the scale they had used did not allow them to. These pupils either left the question out or found the solution – 3 ½ kg - by simple proportion*).
>
> Teacher: Those of us [you] who had space [on the copybooks] will realise that it is true [that the answer is] three and a half kilos. **So**, (…) **the** teacher **was right yesterday when she said** “A graph is a picture way of working out sums”.
> [G6Graphs3minute71]

The idea that a graph is a picture appeared to be one of the key meanings that Gina wished to share. The meaning here is a metaphoric one in the sense that a name is applied to an object to which it is not literally applicable (Allen, 1990). In the previous chapter, I dealt with the idea of metaphor in the sense that items such as monsters, skittle points etc. can be utilised in a reference context to provide support that is crucial for understanding a mathematical idea. However, I consider that the statement ‘a graph IS a picture’ to be a different type of metaphor, one that

Pimm (1987) called an ‘extra-mathematical’ metaphor or what Nolder (1991) referred to as a pedagogic metaphor.

In Chapter 9 I argued that what is relevant to, say, a monster picture can be indicated through the accompanying language. In the case of a metaphoric statement such as ‘is a picture’, the meaning is expressed through language alone and consists of two domains: the ‘tenor’, in this case a graph and the ‘vehicle’, in this case a picture (Presmeg, 1997, p.269). However, the analogy refers only to *some* elements of the two domains. As explained by Presmeg (*ibid*), the similar elements constitute the ‘ground’ of the comparison, while the dissimilar elements constitute the ‘tension’. The key common ground between graphs and pictures is the idea of diagrammatic representation. Other commonalities I note are: (a) for the most basic block graph where the scale is 1:1, the one-to-one correspondence can be likened to a picture drawn of a real life situation were say, a girl in a picture represents one real girl, two trees represent two real trees and so on; (b) children may colour a block or bar graph, and this colour element may also be common to a (coloured) picture; this seemed to be the idea of ‘attractiveness’ alluded to in the classroom and many of the graphs were coloured; (c) a graph substitutes or complements writing, as does a picture in a story.

On the other hand, I can identify tensions between the domains as follows: (a) line-graphs are often not coloured; (b) drawing and plotting graphs is expected to be done precisely and within standard conventions, while pictures are drawn freely - indeed, idiosyncratic representation is often encouraged within the discipline of art; (c) interpretation of a graph is often delimited, but a picture can prompt a variety of interpretations, all of which may be considered valid.

The ground and tension aspects of the metaphor were not made explicit by Gina, and I felt that she focused mainly on the common idea of diagrammatic representation and the features of complementarity / alternative and attractiveness. Indeed, the pupils’ explanations centred on these ideas:

Claudette: **The** bar line graph (sic) **is sort of .. instead of writing, em** just **writing the words**, **we make a** graph, **that looks more attractive, and simpler. It’s more pleasing to the eye**.
[G6Graphs(A)Q1]

Dorianne: Instead of reading (*indicates lined side of copybook*), we look at the picture and it, it show you /
Stefania: **Because you’re not going to remember anything in your**

head. So you draw … it gives you the answers**.**

Dorianne: It shows you the writing. The writing is the picture (*runs her hand across facing copybook pages – lined and squared*).

[G6Graphs(B)Q5]

Josephine: **[The teacher said it's a picture way to work out sums**] **so that we'll study off. Because if we see it, we can imagine it better.**

[G6Graphs(C)Q5]

Hence, I believe that Gina was successful in what she wished to convey, although I suggest that more reflection on the metaphor could have been useful to extend meaning.

If I apply the triadic semiotic model to illustrate the meaning of graph in the sense of a picture, the reference context is language itself. I cannot place a *particular* picture as a reference context, since the statement 'a graph is a picture' is general. The interpretation ideally includes a combination of elements of the ground and the tension. This is illustrated in Figure 10.3:

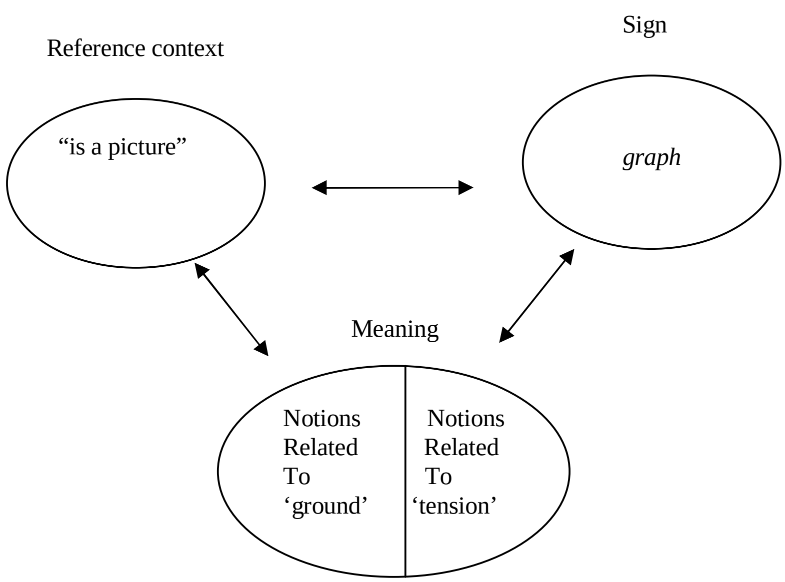


Figure 10.3. A meaning for a graph as a picture

Obviously, a basic requirement is being familiar with the word *picture.* I believe the Grade 6 girls were in fact familiar with the word, but generally speaking, a vehicle which is *not* familiar could lead to what Nolder (1991) called a pedagogic disaster. This is important to keep in mind for any classroom, and needs particular care in an immersion classroom. Finally, I suggest that an interesting question to investigate - but one that I cannot answer within the scope of my project - is whether a teacher in an immersion classroom might avoid pedagogic metaphor or, on the other hand, make more use of this linguistic aid.

10.3.3 Similarities and differences between graphs

Just as Gina focused on some of the common ground between domains involved in the 'picture' metaphor, she also focused on the common ground *within* the domain of mathematics, that is, she tended to focus on what was common for all the three types of graphs tackled and not the differences.

First of all, Gina used the metaphor of 'picture' for all graphs, and this may have implied a similarity between the various types. Secondly, Gina uttered a couple of statements that may have emphasised similarity. The first instance of this was when, on Gina's instructions, the pupils had represented that following data on a bar graph:

Number of books	1	2	3	4	5	6
Cost	2	4	6	8	10	12

Figure 10.4. Data for a straight-line graph written in pupils' copybooks

Gina then said:

> What can be done in a block graph, can be done as well in a line graph. [G6Graphs2minute88].

The class went on to draw their first line graph using this same data. I suggest that this may have given the impression that both types of graphs can serve the same purpose, that is, either can be used to represent some given information. The other statement highlighting similarity was the following:

> [The pie chart] is a graph like the other graphs, but in a circular way. It doesn't have axes. [G6Graphs4minuteCHECK 114]

A third interesting point is that several (though not all) pupils in the class had the habit of colouring beneath the slope on a straight-line graph as shown in Figure10.5, usually using a 1cm width as they had done for bar graphs, that is, not necessarily related to the actual scale being used on the x-axis (here the diagram is reproduced in black-and-white and reduced in size):

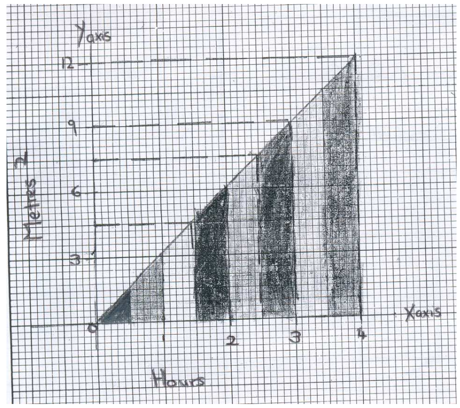


Figure 10.5. Sample of a pupil's line graph

I cannot say what prompted them to do this; perhaps one or two pupils started using colour and the others followed. Gina had certainly never encouraged the habit explicitly, and once even told the girls that it was not necessary. When I asked her about it, she replied:

> Gina: I DID try to emphasise that there is no need to colour. But they felt that it was easier (…) so I let them. [G6GinaGraphs(2)6e].

When I asked the interviewed girls about why they used colour in this way, they generally remarked that they 'could understand the graphs better' when they did. I concluded that what was 'easier' was following particular vertical lines that I think perhaps stood out more clearly because of the colour divisions. I suggest that the colouring was yet another element that promoted the idea of similarity between block and line graphs, because they were rendered perceptually more similar.

Later, the element of similarity was echoed by the pupils. I asked them what all the graphs had in common and why they were different:

Claudette:	**They're all** graphs, **for example, you put this** data **into a** pie chart (*referring to a pie chart example on her copybook*). **But I can put this data into a** bar, line graph **too**.
I:	**Why are they different? Why do all three types?**
Rachel:	**For example, this is round** (*indicates the pie chart in the copybook*), **it's** shape **is round**. This one has bars (*flips pages of copybook to show a bar graph*). **And this one** [**has**] lines (*shows a line graph*).

[G6Graphs (A)Q4]

Dorianne: **They all** (…)**represent what you've done in the day.** (*She indicates a pie-chart showing distribution of activities over a day*) **Because even for this one, for example you can also make a** graph, **em** block graph, **because they've got sort of the same. You can represent them in a** block graph, **in a** line graph, bar graph.

Stefania: **I don't think those are any different. The only thing is that the** pie chart **is different because it is round. But the** line graph **and the** bar graph **don't have much that is different. The only thing that's different is that the** bar graph **is with** blocks, **and the** line chart **you do the** line, **you don't build it.**

[G6Graphs (B)Q4]

The two teacher-statements regarding similarity quoted previously were the only two such statements suggesting graphs were the 'same' and I was struck once more by how two isolated statements (perhaps however, coupled with the picture metaphor common to all graphs, and the lack of discussion of any *difference* between the graphs) appeared to have an impact on the pupils' expression of meaning.

The three types of graphs are different in the sense that each is useful to depict a particular type of data, but it is interesting to note that any differences between the graphs were expressed by the girls in terms of perceptual elements ('round', 'has bars', 'has axes' etc). This is perhaps not surprising since the names of the graphs owe their names to what is perceptually evident. So, for example, the line representing a relationship on a 'line graph' can be seen, as can the 'bars' on a bar graph (I noted through the interviews that the girls were well aware of the interchangeability of the words *bar/block graph* possibly through previous experience). The perceptual elements of the bar / line graphs had in fact been emphasised by Gina as illustrated in the following excerpts:

(*The class is discussing a graph to show birthdays. Through discussion, both teacher and pupils have prepared the axes on the board / copybooks.*

Teacher: [This is] A block graph because we are building in blocks. January, one child (*draws a bar I unit high*); February, one child (*draws a bar 1 unit high*); March, three children (*draws a bar 3 units high*)...

[G6Graphs1minute46]

(*The teacher is explaining to the class how to plot a line graph on the board. She has plotted some points, marking them with crosses*).

Teacher: [For] the line graph, we join the points and as you notice, we get a straight line (*joins the crosses on the board*) and there's

your line graph!
[G6Graphs2minute106]

Perceptual elements of a pie chart were emphasised through the use of imagery where the pie chart was imagined to be a round pie. This imagery prompted an animated discussion as follows:

(*The teacher is conducting a quick revision of the types of graph they have tackled up to now. The pupils have recalled bar graphs, offered block graphs as 'the same' thing and also mentioned line graphs).*

Teacher: Is there another type of graph?
Claudette: Pie chart!
Teacher: Pie chart. What is a pie?
Pupils: (*Giggle*).
Pupil 1: The circle graph!
Teacher: Somebody mentioned the pie chart. Did you hear of a pie?
Pupils: Yes!
Teacher: I can imagine an apple pie … a lemon pie … a meat pie … a very good meat pie …
Pupils: Aaaah!
(…)
Teacher: Can you imagine a pie graph?
Claudette: Like this, Miss! (*Claudette indicates a pie chart printed on the handout on her desk).*

(…)
Teacher: (*Instructs the girls to look at the diagram on their handout*). What does it remind you of?
Pupils: A pie.
Teacher: It reminds me of a cake!
Clare: It reminds me of hungry!
(…)
Pupil 2: A circle.
Claudette: A clock.
[G6Graphs4minute111]

The pie chart was set as a homework task and was corrected the following day. Gina started off the correction by sketching the diagram shown in Figure 10.6 on the board and asking the girls: "This is in the form of … what?" [G6Graphs5minute74].

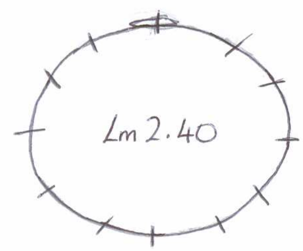


Figure 10.6. Whiteboard sketch of pie chart

The girls answered by offering various images, apparently finding the activity amusing: a clock, a moneybox, a cake, a circle, a frog, a pin-cushion and a prickly-pear leaf. It was with some effort that Gina moved the girls away from these idiosyncratic images to focus on the 12 subdivisions as fractions of the total amount of money.

Perceptual elements may be useful to help memorization as implied by Rachel and Claudette:

Rachel:	**We call them that because we understand them better. If you tell me a different name, I'm certainly not going to take any notice.** [Block graphs **are called that**] **because they're like** blocks.
Claudette:	**They're like** bars.
Rachel:	**And a** line graph **because [it's] a** line!
Claudette:	[**and**] pie graph **because you think it's a** pie!

[G6Graphs(A)Q4]

However, an important idea regarding the differences between the graph types is the appropriateness for the type of data that they are used to represent. As already pointed out, the girls seemed to think that any graph could be used for any type of data. Since they were guided by the teacher or the textbook regarding which graph to draw, I did not observe the pupils actually take decisions about which of the three graphs they should draw for collected or given data. Hence, they did not have the opportunity to reflect on the appropriateness of each and on the 'difference' between one type of representation and another. This awareness is an important aspect for developing a meaning for graphs. Thus although Gina appeared to be successful in getting the girls to practise drawing and reading off graphs, I noted that throughout the week there was an emphasis on the perceptual differences of the graphs. I conjecture that this may have been encouraged by the close association of the name of the graph with its features, the time spent on drawing the graphs and the imagery offered. An appreciation of why they are ‘mathematically’ different may require more explicit consideration of the data in question and goes beyond what is immediately evident.

10.4 Verbs related to graphs

For the topic ‘Graphs’, I identified two verbs: *to plot* and *to drop [a perpendicular]*. I will discuss each in turn.

10.4.1 *To plot*

In Gina’s language, pie charts were ‘drawn’ but block/bar and straight-line graphs were ‘plotted’. The word *plot* was used in a variety of ways. In the first excerpt below, Gina used the word *plot* at an early stage of the construction of a graph, and the meaning implied in the joint conversation between the teacher and the pupils appeared to be ‘add something to the diagram’:

> (*This is the introductory discussion about graphs. The class has established that to draw a graph they need squared paper and axes. The teacher has drawn vertical and horizontal lines on the board*).
> Teacher: Now can we draw our graph?
> Pupils: Yes!
> Teacher: First of all, what are we going to plot?
> Pupil 1: How many lines we need. [referring to how many centimetres they need to mark on the *y*-axis].
> Katia: **The** y-axis, **you do that from** one **to** twelve.
> [G6Graphs1minute11]

In the following excerpt, what was added in was values for *x*:

> (*The class is focusing on a textbook exercise where data given in a table indicates a baby’s increasing weight from birth to 10 weeks. The teacher has sketched the axes on the board).*
> Teacher: How am I going to plot these? [referring to given *x* values] (*Runs hand along horizontal line on the board*). Where is your ‘birth’?
> [G6Graphs5minute51]

On several occasions the word *plot* seemed to mean ‘draw’ or ‘create part/all of the graph’:

(*The teacher refers the pupils to a textbook diagram of a bar graph showing hours of sunshine over the months*).

Teacher: Can you explain that picture? Dorianne?

Dorianne: That picture tells you that, em, in the month of January, em, hours of sunshine/

Teacher: (*Interrupts*)/ [Wait] a second. If you had to tell me what kind of graph … it is a block graph we're plotting … what?

Dorianne: We're pl … (*stops*).

Teacher: Plotting.

Dorianne: We're plotting the hours of sunshine.

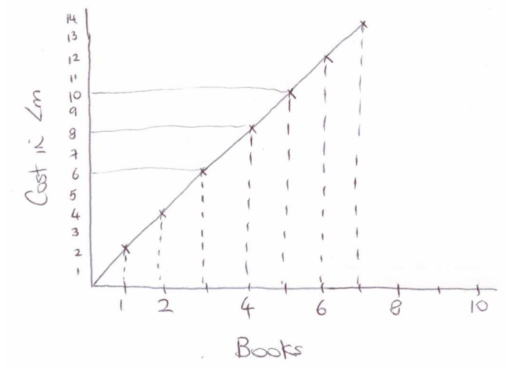
[G6Graphs1minute114]

On one occasion Gina used the word *plot* in the sense of mapping out a relationship between two variables. This was the first example of a line graph to be carried out:

(*The class is looking at a textbook example showing number of books and cost in Maltese Liri [pounds]. This same data had already been shown on a block graph).*

Teacher: We are plotting the number of books against the cost. (…) one book is going to cost two pounds. So I look at the number 1 (*writes on x-axis*) and I go up one, two pounds (*draws a dotted line vertically upwards and marks a cross at (1,2) – See diagram below*). The same one we did in block, we're doing it in line graph. Two books cost four pounds, so we go up … and four (*draws a dotted line and cross.*

The teacher goes around the class to monitor the girls' work as they copy the whiteboard work. Then goes back to whiteboard). Let me plot mine. Three books, six (*draws a dotted line upwards, and 'checks' position of cross by drawing a faint horizontal line from y-axis towards the right)*; four books, eight (*dotted line upwards, faint horizontal line, cross*); five books, ten (*dotted line, horizontal line, cross*); six … (*dotted line, cross – diagram getting out of reach at this stage*); seven books (*dotted line, cross*). (*At this point one of the pupils stops her, telling her that the textbook showed only up to 6 books*). The line graph, we join points and we're supposed to get a straight line. (*She joins the crosses as shown*:



And there's your line graph!
[G6Graphs2minute88 &94]

Although the uses of the word *plot* in the class were varied, there was a consistent element which was the fact that *plotting* always involved some contribution to the construction of a bar or line graph. Furthermore, the word was used in close conjunction with the actual drawing or writing on the graphs. *Plot* is one of the words for which I cannot state with certainty whether it was familiar to the girls, although Charmaine and Josephine clearly stated that it was new. The pupils expressed similar meanings as *making* and *drawing* as follows:

Rachel: To plot, like 'to plot the graph'. To make the graph.
[G6Graphs(A) Q2]

Josephine: **By** 'plot the graph' **I understood you do it as a graph like this** (*gestures a horizontal orientation*).
[G6Graphs(C) Q1]

Stefania: [Plotting is] like building … to draw … em to draw how much they (*flicks through copybook*) like this (*indicates a bar graph*). You plot it /
Dorianne: You have the towers - They're like building the towers (*gestures placing one thing/block on another*). But you're drawing.
[G6Graphs(B) Q2]

I noted yet again that an idea that was mentioned only once was later recalled by a pupil: Charmaine gave an interpretation of plotting as 'extending' the graph, apparently recalling Gina's action for just one of the graphs (books/cost) as quoted previously.

Charmaine: ‘Plot the graph’ [means] for example, like this (*touches the copybook page*). First you had till the number six, till here (*touches the 6 on the x-axis*). You do it until the twelve. Then he tells you ‘plot the graph’ and you have to continue it. For example, seven to fourteen, eight to sixteen (*each time touches the axes where each number is written*).

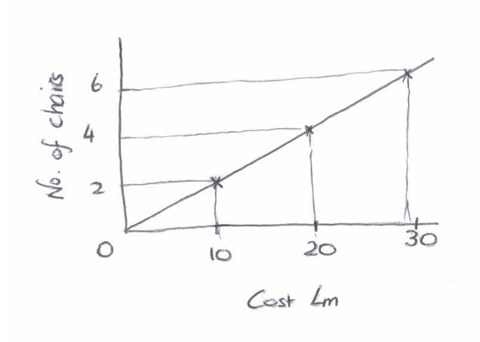
[G6Graphs(C)Q2]

None of the girls talked about *plotting* in the sense of representing a relationship, and I believe that the general use of the word in the sense of *drawing* / *creating* /*adding to a graph* used in the lessons obscured its specific meaning. As pointed out by Halliday (1978), words within a register are created to refer to specific notions; the word *plot* is not simply an alternative for the more informal *draw* or *make*, but implies a relationship between two variables that is defined by a series of points. So for example, while Van de Walle (2004, p.393) uses the word ‘make’ in relation to bar and ‘circle graphs’, on the other hand he talks about ‘plotting points’ on a line graph (p.440). Similarly, Frobisher *et al* (1999, p.282) talk about ‘plotting’ points when representing a function. Hence, the word *plot* is a good example of how the use of a mathematical word can help to convey a very specific mathematical meaning. Just as in Grade 3, the use of *sharing* and *grouping* in different situations can indicate a difference in the relationship between the elements of a division situation, so too, the differentiated use of the words *draw* and *plot* has the potential to indicate a difference between types of graphs.

10.4.2. *To drop a perpendicular*

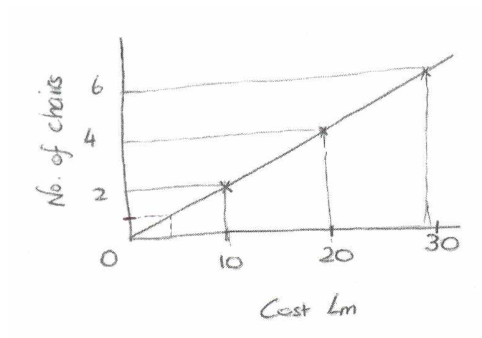
Gina used the expression *drop a perpendicular* when she wished to find a corresponding value of *x* for a given value of *y* on a straight line graph. The expression was not always used in its entirety. On seven occasions, the expression was shortened to ‘you drop’ and twice Gina used ‘perpendicular’ on its own. For example:

(*Gina has copied a straight line graph from a handout onto the whiteboard, using faint vertical and horizontal lines to guide her sketch).*



The handout question now requires them to find the cost of 1 and 3 chairs)

Teacher: One chair *(marks a point half way between 0 and 2 on the vertical axis*). To find the cost, we move the line (*draws a faint horizontal line starting at the marking to join the slope. Stops momentarily at the slope*). Where it meets, we drop (*draws a faint vertical line to meet the x-axis*). It is halfway between zero and ten. So [the answer is] five.



(*The teacher now explains again*): One chair (*places her finger at 1 on the y-axis*), move to the line (*moves finger along the faint horizontal line she has just drawn*). Where you meet the line, drop a perpendicular (*moves finger downwards along the vertical line down to the x-axis*). Five.

[G6Graphs2minute120]

During the interview, Rachel and Claudette did not specify whether the expression was new or not, although the other four stated clearly that they had not known it before. When I asked the girls about the meaning of the expression, Claudette and Rachel offered a joint explanation as follows:

Rachel: (*Flips through her copybook to a page showing a line graph that depicted time in weeks (x-axis) and baby's weight in kilos (y-axis))*. For example, you want to know between the four-and–a-half and the five (*touches the y-axis of the graph*). And let's say, there were six children (sic) and it drops till here, for example the line (*touches x-axis*). And you 'drop a perpendicular', like a box (*referring to the 'rectangle' created by the horizontal and vertical dotted lines*).
I: Hmm, I see. What do you think Claudette?
Claudette: To go directly down, vertical.
I: So what does the word perpendicular mean?
Rachel: (*Looks at Claudette*)
Claudette: Vertical.
[G6Graphs(A)Q2]

I suggest that through the words *you drop*, Claudette and Rachel had got the sense of a downward direction and in the Maltese explanation they stated:

Rachel: **You go down … that falls** [Rachel did not specify what 'that' refers to].
Claudette: **A vertical line falls.**
[G6Graphs(C)Q1]

It is interesting to note that the girls translated *drop a perpendicular* as 'you go down' and 'a line falls'. Both phrases are grammatically and epistemologically different to saying 'drop a perpendicular' which is the more usual way of using the expression as part of the English mathematics register. Thus, this is another variation in expression that I can add to those outlined in Table 7.3.

None of the other pupils were able to offer an explanation. Dorianne recalled hearing the expression but could not remember what it meant, while the other three pupils had no recollection of it ever being used. During the lessons, I had noted that all the class drew the lines appropriately and hence have evidence that the girls could 'match' corresponding x and y values. The point here is that the girls could not give an explanation of the *expression* when asked. I will explore why this may have been so, given that Gina was very much aware that this expression had been new to her class.

Reflecting on the classroom interaction I noted that the expression was not used at all by the pupils. As discussed in Chapter 7, the Grade 6 class, the girls 'got away' with using informal language. So for example, a direct reference to the idea was when a pupil said "And then you go down” [G6Graphs3minute4] or when Federica said “**Like this**” using gestures to indicate the necessary orientations [G6Graphs3minute65]. Even during the interviews, the girls could explain the procedure of matching values, but without using the expression *drop a perpendicular*. For example:

> (*I have opened Dorianne’s copybook on a page showing a straight-line graph and asked the girls to talk about those type of graphs. The graph shows the relationship between weight of sugar (x-axis) and cost in cents (y-axis). Dorianne is explaining how to read the graph in order to find how much sugar one can buy with 35c).*
>
> Dorianne: For example, [to answer the question] “How much can you buy with thirty-five cents?” You go to the thirty-five cents (*indicates the point on the y-axis*), you do a line not be mistake, ‘cause you can mistake. You do a line (*gestures a horizontal line*) em, **but** the line stop here (*indicates where the horizontal line she had drawn met the plotted slope*). So I go down and I say “Which one is here?” (*touches x-axis*). So this is one-and-one-half (*indicates the point marked 1½ on x-axis*), this is two (*indicates the point marked 2 on x-axis*) and between there is one and three fourths. So you can buy one and three fourths kilogramme of sugar.
>
> [G6Graphs(B)Q2]

However, the lack of use on the pupils’ part during the lessons does not offer a full explanation of why the girls did not recall the meaning of the expression: there were many words that the pupils in both Grades hardly used and for which they could in fact offer appropriate explanations.

A possible reason may have been that Gina herself did not use the expression very much. It was only used in full 7 times and only to match a y-value with an x-value. Different language was used when an x-value had to be matched to a y-value. For example:

> (*There is a graph drawn on the board showing the cost of oil per litre*. *The girls are to find the cost of 2.75litres of oil*).
>
> Teacher: We go up in a bold dotted line (*draws a vertical dotted line starting at 2.75 L on the x-axis until it reaches the slope*) and then you read it off in cents (*she moves her hand horizontally to the left to meet the y-axis*) and your answer should be…?
>
> Pupils: Fifty-five cents.
>
> [G6Graphs3minute22]

However, as already discussed, I have some evidence that infrequent use of a word did not appear to be such an influential factor as it had been for the younger pupils, so this again might not offer a full explanation.

Therefore, I would like to suggest that part of the difficulty in the expression being 'glued' (Hewitt, 2001) to the action of drawing a line between the slope and the x-axis may lie in the nature of the expression itself, which incorporates the notion of perpendicularity and the action of purposeful dropping which in this case requires a re-interpretation from its everyday meaning.

The word *perpendicular* suggests a relative orientation between two lines and a meaning can be illustrated as follows:

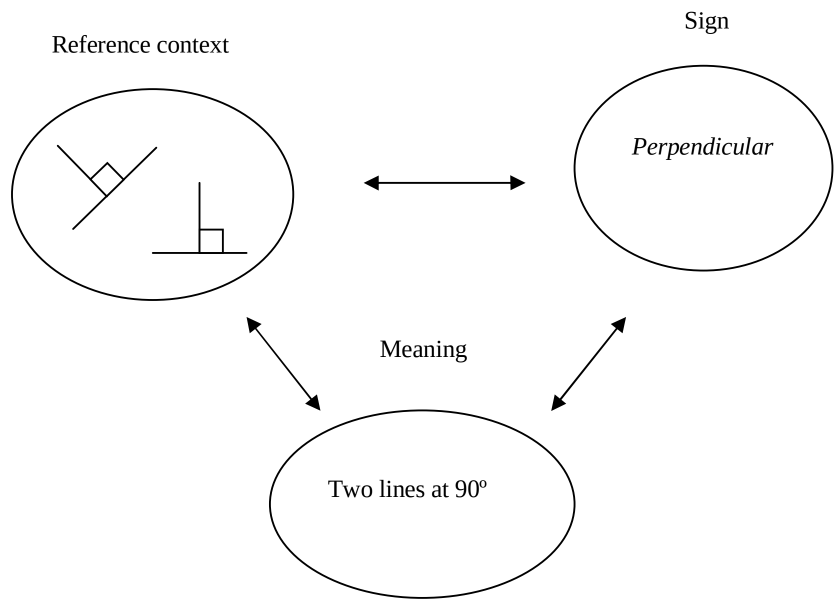


Figure 10.7. A meaning for perpendicularity

Gina did not focus on the general property of perpendicularity. It is interesting to note that *perpendicular* is one of those words that can be used as both a property (adjective) and an object (noun) just like the example *diagonal* offered by Pimm (1987) but in the Grade 6 classroom, the word was used as a noun. The meaning for [*a*] *perpendicular* that Gina appeared to convey to Claudette was that of an alternative for a *vertical line* (the word *vertical* had previously been used to describe the orientation of the y-axis, and the girls seemed familiar with the word). Since the girls modelled the downward dotted line drawn by Gina, and used it appropriately to find the required solution, it was not necessary for them to fully appreciate the expression being used. The vertical line drawn along the copybook lines always met the x-axis at 90°, so the girls did not actually need to consider degrees or orientation. I can contrast this situation with other possible ones, illustrated in Figure 10.8, where it is more important to appreciate the significance of the expression in order to construct appropriate lines:

Figure 10.8 a&b. Dropping a perpendicular in a geometry context.

With regard to the action of *dropping* I note that this is an example of a word that has a different meaning across everyday and mathematical contexts. Durkin and Shire (1991) explain that this characteristic, known as polysemy (different but related meanings), is common among spatial terms (see Section 3.4). Here *dropping* is not the everyday action of letting something fall to the floor, but rather, it refers to drawing a line in a particular orientation with respect to another. If the base line is horizontal as it would be when drawn on a whiteboard, then the direction is actually downwards. Even when drawn on a copybook, 'dropping' may still be considered 'vertical'. However, if the base line is not horizontal then the orientation of the perpendicular is not vertically downwards, in which case a new interpretation needs to be given to the verb *to drop*.

As part of the mathematics register, the expression *drop a perpendicular* is a style of meaning whereby different elements are combined to give a new meaning (Halliday, 1978). The combined meaning can be depicted as shown in Figure 10.9:

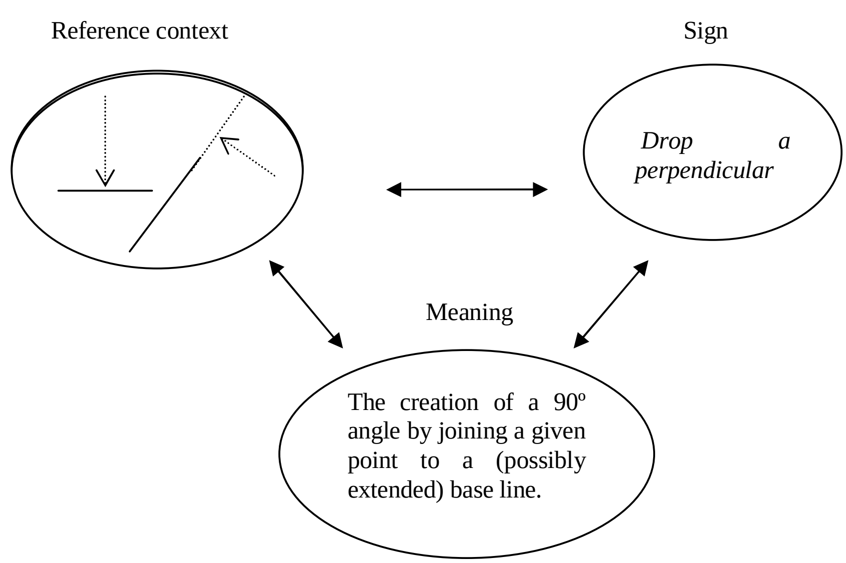


Figure 10.9. A meaning for the expression drop a perpendicular

As in the case of the picture metaphor, Gina did not specifically focus on the similarity and dissimilarity between the everyday word and the mathematical application. The complexity of the interpretation of the expression, together with the fact that the pupils did not actually 'need' to know the expression, as long as they could carry out the necessary action, may explain why four of the pupils were not able to offer an explanation. I suggest that the sharing of meaning of an expression such as *drop a perpendicular* requires explicit attention to be given to each of the separate elements *and* their combination. Hence, this renders the verb a different type of verb to *multiply*, *divide*, *share*, *group* or *plot.*

As with other issues that I have discussed regarding meaning, this is one that applies to all classrooms, but once again, may need particular attention in an immersion classroom, since knowledge of the everyday word *drop* may be harder to assume.

10.5 Expressing relationships/concepts: *scale* and *representing*

A key relationship expressed during the week was that of scale and representation and the words were used in close conjunction with each other. For every bar or line graph drawn the girls wrote down two scales, one for the y-axis, and another for the x-axis. In the latter case, the scale was used even for bar graphs, where it referred - inappropriately - to the width of the bars ("1cm represents 1 student"). Gina first used the word *represent,* with respect to the 'birthdays' graph:

(*The class has been discussing how many birthdays there were in the class per month of the year. The frequencies are written on the board. The teacher has prepared the axes on the board and the class has established that on the x-axis bars will be 1cm wide. The teacher says that the x-axis was 'easy', but now they need to take some decisions).*

Teacher:	We look at our groups – we have one [birthday for January], one [for February], three [March], three [April]… and the highest number is what?
Pupils:	Four.
Teacher:	Four. So we need only four spaces.
(…)	
Teacher:	Now, if we use one box to represent one child … can we do that?
Federica:	Yes.
Clare:	But it will be too small.
(…)	
Teacher:	Somebody said it would be too small. So what can we do?
Pupil 1:	Two centimetres, one child.
Teacher:	And we get how many boxes in all?
Pupils 1 & 2:	Eight.

Teacher: Eight. Can we afford more?

Clare: Yes. Three centimetres.

Pupil 3: **Yes**, Miss. Twelve.

Teacher: Shall we have three? Do you prefer to have three boxes for one child?

Pupils: Yes.

Pupil 4: **It's better** [**to use**] one box, Miss.

Pupil 5: No, two.

Federica: It's too small.

Pupils: Three / Four.

Teacher: Let's go midway. Let's say two centimetres represents one child. So, y-axis: two centimetres represents one child. (*writes on board*:

y axis 2cm rep. one child

Pupils: (*The pupils copy what the teacher has drawn /written on the board into their copybooks*).

[G6Graphs1minute35]

The word *scale* then started being used the following day. Writing the scale ("y-axis 1cm represents …" etc.) was an essential part of every graph block and line graph drawn, with Gina emphasising the consequence of using different scales:

(*The teacher is showing up two copybooks as illustrations of two different ways of drawing the graph the girls had to do as homework*).

Teacher: They are both right. In one case, one centimetre is representing two marks. In the other case, one centimetre is representing one mark. (…) The difference is in size only, but the results eventually will come out the same. The only difference is THIS is clearer (*indicates the graph with the larger scale*), perhaps because it's bigger, while this is on a smaller scale (*opens thumb and finger about 10cm apart*).

[G6Graphs2minute10]

During the interviews, three of the pupils had suggested that the words *represent* and *scale* were new to Grade 6. Dorianne stated that *represent* was new to her, but she had known *scale* previously. Claudette and Rachel did not specifically point out whether the words were new or not. Whatever the previous level of familiarity, it appeared that all the pupils came to share similar meanings. For example:

Rachel:	Well, if the chart of the graph is big, you can do it a bit small like … for example, if it's like till hundred … let's say it's till thirty and you don't have thirty boxes here (*touches copybook page*), so we can do it till twenty **for example**.
Claudette:	You put it to scale.
I:	What does it mean 'to put it to scale'?
Claudette:	For example the teacher told us really WE can decide. We can do one box for twenty children and we can do TWO boxes for twenty children.
Rachel:	It depends how we like it … scale, it can be different, but the working it has to be the same.
(…)	
Rachel:	Scale is that (*points to written work on the lined page e.g. x-axis 2cm = 1 child*).

[G6Graphs(A)Q2]

I:	What can you tell me about scale?
Charmaine:	To represent. (…) (*indicates a block graph showing number of boxes of apples on the y-axis*) on the y-axis we have one centimetre representing one box [of apples].
(…)	
Josephine:	[Scale is] you say the box how many you have to do.

[G6Graphs(C) Q2]

It seemed that the chains of meaning at play were as follows: *represents* implied a relationship between multiples and familiar notions such as *marks*, *centimetres*. Depicted on an axis, these constituted a reference context (Figure 10.10).

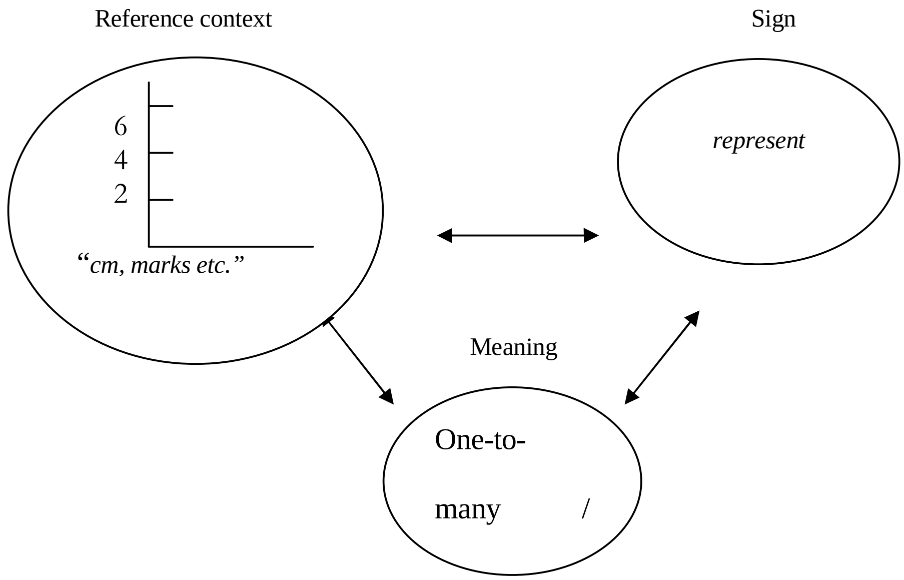


Figure 10.10. A meaning for *represent*

The word *scale* was then used as a 'name' for the statement expressing the relationship of representation:

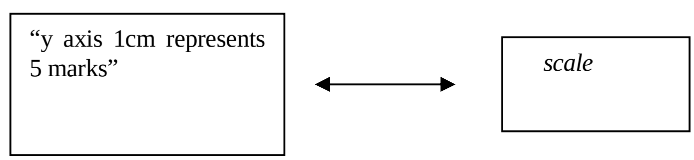


Figure 10.11. *Scale* as a reference to particular statements.

I concluded that Gina was successful in sharing the meaning of the relationship. This may have been a result of frequent use in the classroom by both the teacher and the pupils and of the perceptually obvious 'effect' of scale.

10.6 Conclusion

For the topic 'Graphs' I concluded that in a similar way to 'Multiplication and Division' words, it was easy to share the meaning of a word if this played a referential role. These included the words *graph*, *block graph, line graph*, *pie graph*, *x*- and *y-axis*. On the other hand, the word *data* appeared to mean any writing related to the diagram. I am not sure if this was Gina's own meaning for the word *data*, but the situation suggested to me that when what is being referred to is not immediately perceived or experienced (the girls did not collect data themselves and only one table of data was given) it becomes harder to share a meaning for a word.

Language alone was used in the sharing of the meaning of graphs as ‘pictures’, which offered a metaphoric meaning for the graphs. The use of a pedagogic metaphor was a new element to the topic of ‘Graphs’ and one not found in the topic ‘Multiplication and Division’. I reflected on how the interpretation of such a metaphor ideally requires an appreciation of elements related to the ground and the tension between the domains, but noted that Gina did not focus explicitly on these. I conjectured that this metaphor, which was common to all the three types of graphs tackled, encouraged the pupils to view graphs as similar, rather than different. Similarity was also implied in two of the teacher’s statements and possibly by the fact that many of the girls coloured the area beneath a straight-line graph just as they would the bars of a bar graph. Differences between the graphs were identified by the girls in terms of perceptual aspects and I suggest that this was a result of the names of the graphs themselves, the emphasis on the drawing of the graphs and the imagery invoked during the lessons. While appreciating similarities between graphs is useful, I believe that focusing on the differences is also an important part of understanding this topic.

Some mathematical words are verbs. For the Grade 3 topic ‘Multiplication and Division’, I noted that the pupils did not seem to appreciate the difference between *sharing* and *grouping* as division, because the actions to which the words referred to were not clear. In the case of Grade 6, the action of *plotting* was clear, but generalised to mean *make* / *draw* / *add to diagram*. Its specific mathematical meaning with respect to charting out points depicting some relationship between two variables was not brought out by the teacher; *plotting* is a ‘more mathematical’ word than *sharing* and *grouping* in the sense that its application is more common in what I might refer to as ‘mathematical contexts’ than in everyday ones. Hence, attention may need to be given to sharing its meaning as part of the mathematics register. The same can be said of the expression *dropping a perpendicular,* which involves a reinterpretation of the verb *dropping* from its everyday meaning, and recognition of perpendicularity. Gina appeared to assume that the girls would follow what she meant if she accompanied the words by gestures, which in fact they did. However, four of the pupils had no recollection of the expression itself. It seemed to me that during the lessons the girls could get by, not only without using the language themselves, but also without focusing on the language that described the action.

A key concept for this topic was the relationship of scale and representation. Both ideas were successfully shared with the pupils, possibly because of the frequent use, their ‘necessity’ given the task at hand and the obvious effect scale had on the size of the pupils’ graphs. The development of this concept involved a semiotic link between a feature of the diagram and familiar words such as ‘big’, ‘centimetre’ and so on, that lent support to the meaning of the word *represent* as a ‘one to one’ ‘one to many’ or ‘many to one’ relationship. *Scale* then referred to the explicit stating of this relationship. Since the link appeared clear, this idea was shared successfully as had a meaning for *multiplication* in Grade 3. However, I also noted that the idea of scale was extended unnecessarily beyond its normally accepted meaning, to refer to the width of the bars on a bar graph.

One point that came to light was that it seemed that the Grade 6 girls were more successful in recalling a word or statement that was used little, unlike their Grade 3 counterparts. The pedagogic implication here is that two-fold: first of all it may be necessary to use new mathematical vocabulary relatively more with younger pupils, and secondly that care should be taken when offering a statement to older pupils, since some individuals may very well take this statement to be the intended meaning to be shared, even if it is not.

My examination of expression of meaning in the class, and during the interviews, highlights the complexity of using and understanding language within mathematics classrooms. I appreciate that it is very difficult in practice for a teacher to maintain a level of self-awareness suggested by the detail of the present written analysis, however, my aim is to draw out some general key points that can offer guidance for the use and development of mathematical vocabulary. As for ‘Multiplication and Division’, I noted that many of the points are applicable to ‘any’ classroom, although, of course, I cannot exclude the possibility that clarity was hindered because of the obligation to use English. That is, it is possible – although I have no way of knowing – that points may have been rendered more clear had Gina given the lesson in Mixed Maltese English.

As I reflected on this topic, I continued to remain open to other features of word use, other than clarity, that appeared to help or hinder sharing of meaning. However, no other feature became evident apart from frequency that I have already discussed. I will now consider the last topic ‘Length’ for both Grade 3 and 6. My discussions will reinforce ideas already presented and add new elements for reflection.

C H A P T E R 1 1

The Topic 'Length' and the Notion of Significance

11.1 Introduction

In this chapter, I discuss the topic 'Length' as it was taught in both Grades. The work carried out during the weeks is summarised in Table 11.1:

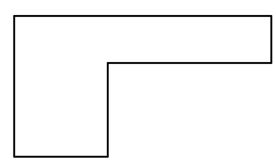
	Grade 3 (5 lessons of average duration 50 minutes)	**Grade 6 (8 lessons of average duration 97 minutes)**
1	Using the measuring tape to measure various items.	Discussion about non-standard units, including some practical measuring with body parts.
2	Measuring pupils' heights; fractional parts of the metre; relationship between metres and kilometres.	Unit conversions with centimetres and millimetres.
3	Practical activities involving estimation; using a ruler to draw straight lines of given lengths.	As above, and also conversions involving metres and centimetres; measuring pupils' height.
4	Unit conversions such as 2m50cm = ____.	The kilometre unit; conversions km / m; measuring various parts of the classroom (walls, windows etc).
5	Same as above and measuring lines that formed part of a diagram of a space shuttle.	Finding the perimeter of polygons, in particular (rectangles, U- and L-shaped figures). For example: 
6	-	Same as above.
7	-	Various word problems dealing with 'length' situations.
8	-	Same as above.

Table 11.1. Overview of the main activities of the topic 'Length' for Grades 3 & 6

In Grade 6, both Gina and the pupils agreed that a lot of the work was similar to that of the previous year's. Gina stated that all the words in my interview list (see Table 8.1b) had already been familiar to the girls. On the other hand, when I went through the list of words with the Grade 6 pupils, all pupils admitted they did not recall the word *metric* while opinion varied regarding the familiarity of the words *spans* and *regular* / *irregular*. In Grade 3 there was more that was new, with pupils suggesting that they had only been familiar with ideas of *longer*, *shorter*, *centimetre* and *metre,* with differing opinions given for the verb *measure*.

I start my discussion by reflecting on clarity or otherwise of particular words. For this topic, I identified references, verbs, concepts and also a property. In order to avoid repetition of previous discussions, I will not go into detail about all the words as I did for the other topics, but offer only a selection. I then present a collection of points of interest, including a reflection on the potential use of translation for this topic. Finally I introduce and discuss a third feature that appeared to support sharing of meaning, a feature that I call 'significance'.

11.2 Reflections on clarity of meaning

11.2.1 References and verbs

Words that were used as references in close association with the perceivable object they denoted were rendered clear. An example of such a word was *perimeter* in Grade 6. This was used to refer to the boundary of a rectangle, and usually found by calculating (length + breadth) × 2 or (length × 2) + (breadth × 2). The word was also used to refer to the boundary of U- and L-shaped regions such as the one illustrated here:

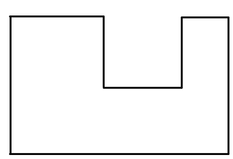


Figure 11.1. U-shaped region considered in Grade 6

During the interviews, some pupils expressed meanings for the word *perimeter* that reflected reference. For example:

Joanne: The outside of the shape.
Federica: (*Gestures a circular motion with her hand in the air*).
[G6Length(A)Q2]

With regards to verbs, the main one used in both classes was *to measure*. In Grade 6, my observations confirmed Gina's and the pupils' statements that the word and the action it denoted were already familiar to the pupils. On the other hand, in Grade 3, not all pupils had known the word previously, but I believe that its meaning was rendered clear by the carrying out of the action on various occasions. The girls measured successfully, and during the interviews, all of them expressed an appropriate meaning for the word. They tended to translate it into the Maltese **kejjel** or the loan-shift **immexerja.** The Grade 6 girls also translated *measure* when offering a meaning. The link with the Maltese equivalents may have been possible because the Maltese word was available to the children through experiences outside the classroom. At no time did the teachers themselves offer the translation.

11.2.2 Reference and concept: variations in the meaning of *length* and *height*

The word *length* can be used in different ways. It can serve as a general word that refers to a "measurement or extent from end to end" (Allen, 1990, p.678); it can also be used in the sense of a property of some object ("the length of a road"). In particular, it can refer more specifically to one particular dimension of an object. In this sense, it is customary to consider the '*length*' to be the greater of two[7], (or greatest of three) dimensions, the *width* as the shorter, and the *height* the vertical length. For example, I can talk about the length, width and height of a desk.

The words *width* and *height* were used briefly in Grade 3, although as stated in Chapter 8, the pupils did not recall them at all. Hence, I will focus on the words as used in Grade 6, where generally the words were used as names for particular dimensions of a rectangular shape and /or a desk. During the interviews, all the six pupils were able to use the words in this referential sense:

> Monica: **This week we talked about** length **and** breadth. **Em, the** sides **of the** shapes [rectangles], **and with those we can find the** perimeter. [G6Length(C)Q1]

However, an interesting and unexpected interpretation by one of the pupils involved the interchange of the name of the dimension height with length. I will trace what I believe to be the

[7] Strictly speaking, an investigative exercise may require that the 'length' of say, a given rectangle be varied until it is actually shorter that the 'width', but it is common practice to start by referring to the side which is originally longer as the 'length'.

cause of this by explaining the development of the discussion surrounding the dimensions. To begin with, an interchangeability of the words *width* and *breadth* was suggested by Gina on more than one occasion:

> Teacher: (*With reference to the dimensions of the classroom*): We called one the 'length' and one the 'width' or 'breadth', call it what you like.
>
> [G6Length2minute12]

However, Gina also suggested that *length* and *width/breadth* were interchangeable as names:

> (*The class' attention is focused on the rectangular desks*).
>
> Teacher: The LONGER side of an object we call the length. The SHORTER side we give it another name … They are both lengths, but to distinguish which side we are talking about, we call one being the 'length' and one being the 'width', or 'breadth' … If you decide that this is going to be your length (*touches one of the shorter sides of a pupils desk*) and this is going to be your width (*touches on of the longer sides*), it doesn't matter (…) [but] you have to DISTINGUISH which name you're going to give to what side.
>
> [G6Length2minute12]

Gina likened the interchangeability of *length* and *width* to the girls' own names which may be different. She explained that two girls had their own particular names, for example Clare and Charmaine, and what was important was to distinguish clearly which name you have chosen for which girl. As she had done for the metaphor 'a graph is a picture', here too Gina appeared to focus on the similarity of the analogy. Naming sides and girls allows reference, and both acts of naming have a certain element of arbitrariness: Clare could have been named Jennifer and the reference words *length* and *width* may have historically developed differently. However, there is a difference in the analogy: Clare's parents were completely free to name their daughter anything they wished, and Clare – even once named - may choose to be called 'C.J.' by her friends. On the other hand, mathematical references - once established within a community - come to be used in the same way by everyone to facilitate communication. Thus, the interchangeability suggested by Gina for the rectangles under consideration was not altogether appropriate.

The idea of interchangeability was reflected in what the girls explained during the interviews. For example:

Katrina: Length **is like this** (*indicates one of the shorter sides of the coffee table in front of her*) **and** breadth **is like this** (*indicates one of the longer sides*).

Celia: **You can say for example that THIS is** length (*indicates a longer side*) **and THIS is** breadth (*indicates a shorter side*).

[G6Length(B)Q1]

Explanations like this were not surprising since this is what Gina had suggested several times during the lessons. However, what is of interest is that one of the pupils, Monica, also extended the idea of interchange to the dimensions *length* and *height*:

Monica: We said that breadth and width are the same thing. And length and height are the same thing too. (…) This is the length and the height (*she touches one of the longer sides of the coffee table*) and this is the breadth and the width (*touches one of the shorter sides*).

[G6Length(C)Q2]

Gina had not suggested *this* interchangeability, so I wondered what had been said in the class that may have influenced Monica's expression of meaning. I identified two instances. One was a word problem that the class had worked out used the word *length* instead of *height* in relation to a building:

The length of one storey is 3.17 metres. In a high modern building there are 9 storeys. What is the height of this modern building? [G6Length7minute47]

Throughout the discussion of this story sum, Gina used the word *length* instead of *height*. Another instance was when Gina explained that height was a special case of length:

Teacher: (*The teacher asked the class what they understood by the word* height).

Celia: [Height is] the, the length … the length of something standing up.

Clare: But Miss, you told us that it's length [not clear what 'it' refers to]. How can it be length and height?

(…)

Teacher: Height… it starts from the floor (*points to the floor*); it is length as well - we're still measuring the length, but instead of being horizontal or flat, it goes up from the floor upwards. You have to lift your head. Look up. We call that height.

[G6Length1minute30]

It seemed that on these latter occasions, Gina used the words *length* and *height* in a 'general' sense, but she did not distinguish explicitly between names for specific dimensions and general spatial concepts. Monica may have confused the two uses and this suggests that it is helpful for a teacher to explicitly point out the different ways in which the word *length* may be used.

11.2.3 Standard and non-standard units

A key part of learning about length includes the indirect comparison of two or more items through a unit (Askew, 1998). Units link the general notions of length with measurement, since through the action of measuring, I can establish that say, the length of a table is 90cm long. The units in use in Grade 3 were the metre, centimetre, and kilometre, while millimetres were also used in Grade 6. In Grade 6, all the units had been familiar, while in Grade 3, *kilometre* was new and opinions regarding *metre* and *centimetre* varied. The notion of a unit involves both spatial and numerical ideas since a unit can be considered from two inter-related aspects: the 'size' of the unit and its numerical relationship with other units. The latter was rendered 'clear' through listings in the pupils' copybooks (Figure 11.2) and the actual perceiving of the subdivisions on a ruler or tape where possible.

Grade 3		**Grade 6**
1km = 1000cm 1 m = 100cm 1/2 m = 50cm 1/4 m = 25cm		(…) 10mm → 1cm 10cm → 100cm 100cm → 1000mm 100cm → 1m 1/2 cm → 5mm (…)

Figure 11.2. Sample of pupils' written notes for unit conversions.

Many exercises were carried out where pupils were expected to express a measurement using a different unit, as in the examples '346cm = ___ m ___ cm' (Grade 3) and '64mm = ___ cm' (Grade 6), and therefore the relationships and unit names were utilised frequently. Consequently, during the interviews, all pupils appeared confident in working with the numerical relationships.

On the other hand, appreciation of the *size* of a unit requires experiences with a variety of reference contexts which must, however, indicate the length in question consistently. For example, a *metre* length may be indicated by placing two hands apart, by showing up a strip of paper which is exactly 1 metre long, or by means of markings on a piece of wood. The word *metre* does not name the object perceived, but refers to a particular length that is represented by the object. A semiotic representation of this is shown below, and this can be adapted for any unit:

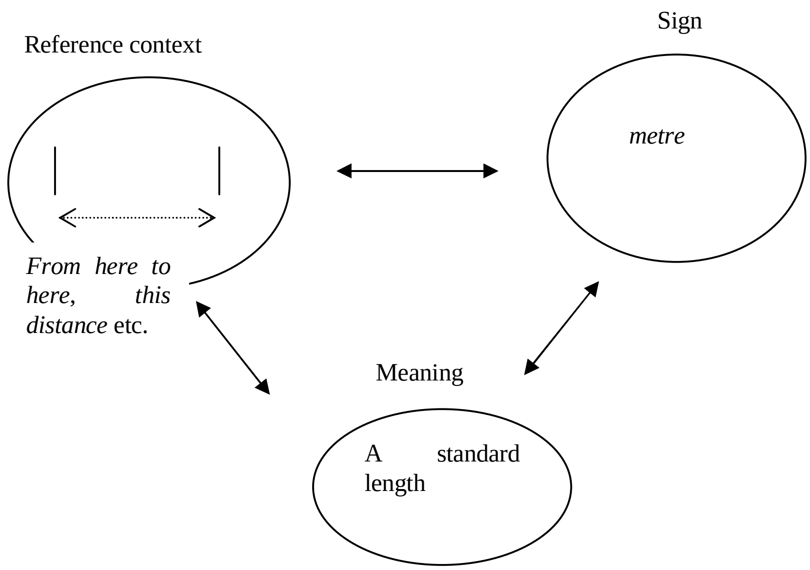


Figure 11.3. A semiotic model for *metre*

The Grade 6 pupils were able to indicate metre, centimetre and millimetre lengths easily, so I will discuss Grade 3 experiences, since the units were newer to the younger pupils. I noted that the more experience the Grade 3 pupils had with perceiving a unit in class, the more able they were to later illustrate its size. So, for example, the girls had a lot of 'experience' with the centimetre unit. First, Rose used her finger width as a reference, a strategy that Haylock (2001) suggested is helpful. The girls also had ample opportunity to look at representations of a centimetre length since they often used their measuring tape or ruler to measure things. During the interviews, the pupils were generally successful in offering size as an explanation for a *centimetre*, indicating the approximate length in one way or another. On the other hand, the girls had less opportunity to view the metre length, since they themselves did not measure in metres. The unit size was first identified when the girls were finding various lengths on their measuring tapes (25cm, 35cm and so on), holding the tape up as they did. When they identified 100cm, their outstretched arm position became the reference for the metre:

Teacher: (*Teacher opens her arms out wide, coping the girls' gesture*). It's about one metre. About one hundred centimetres. [G3Length1minute35]

Further to this representation, one of the pupils, Nadia, suggested that *one* outstretched arm is about half a metre, while the length from the elbow downwards, a quarter of a metre. Rose appeared to consider these positions helpful, and the gestures were used a few times in the lesson to lend meaning to the idea of a metre length and its fractional parts. Rose also showed up a metre ruler to indicate the metre length. During the interviews, only three of the pupils were able to indicate approximate metre lengths and I concluded that although the size of the metre was rendered clear in the classroom, less opportunities to visualize it may have had some impact on some of the pupils' ability to indicate it later. With regard to the kilometre, I could not expect the pupils to indicate the unit size directly, since this is not practical. However, I will return to this unit in Section 11.4.1.

My observations suggested that the more experience the Grade 3 pupils had with a unit, the more able they were to illustrate its size later. I note a parallel here with my argument that frequency of word use aids recollection, and also note that my observation supports Blinko and Slater's (1996, p.ix) suggestion that "the ability to 'visualise' and estimate quantities takes a great deal of time and requires a broad range of experiences".

One aspect addressed in Grade 6, but not in Grade 3, was the idea of measuring with non-standard units. Non-standard units are generally considered useful prior to using standard ones. Askew (1998) suggested that when children use non-standard units, they come to appreciate that some types of units are more effective than others. The units used in Grade 6 were the hand span, foot length (Gina called this a *foot span*), arm span, the length of a finger, wrist-to-elbow, the length of a leg and body width. Some practical examples of measuring their desks and the classroom wall were carried out by some the pupils.

The word *span* was used fairly frequently during these activities and was recalled by all the pupils. During the interviews, the girls were able to demonstrate a hand span and a 'foot span', and they were also able to offer other non-standard units like a finger and so on. I suggest that the clarity of the meaning for, say, *hand span* was ensured by its referential role. However, it was interesting to note that the three pupils who gave an explanation for an *arm span* (in separate interviews) interpreted it to mean the length of the arm from wrist to elbow, or wrist to shoulder.

Gina had, in fact, opened both arms out wide to demonstrate an arm span, and had counted units as two girls measured the classroom wall, so that my first impression had been that the meaning of the word had been clear. However, on examining the classroom interaction, I realised that, except in the case of *arm span*, the non-standard units used were *single* body parts, with one stretched out hand being called a *hand span* and the length of one foot a *foot span*. It is therefore possible that the girls associated the word *span* with the length of one part of the body. In fact, during the interview, Clare suggested that you could measure with a ‘finger span’ while Celia suggested a ‘leg span’ [G6Length(C)Q2 and G6Length(B)Q2 respectively]. Although this unusual nomenclature may not have been detrimental to the girls’ appreciation of units, I feel that this example highlights the intricacies of mathematical language use.

11.2.4 The property of regularity

One ‘type’ of word that had not featured in the previous topics considered was properties. In Grade 6 a property addressed was regularity of shapes. During the interview, Clare and Monica were the only two who stated that they had known the meaning of the words *regular* / *irregular* prior to Grade 6. The girls offered a variety of explanations for the words and these are summarized in Table 11.2. Their responses are presented as the girls were paired, and it should be noted that I specifically asked them to classify a rectangle.

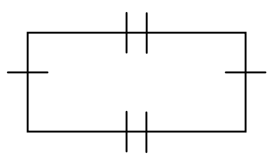
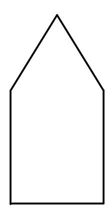
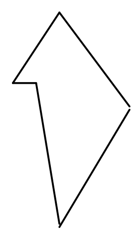
	Regular shapes	**Irregular shapes**
Clare	Regular shapes are ones with all sides equal.	Irregular shapes do NOT have all sides equal. If an octagon has seven sides equal and one different, that's irregular. A rectangle is irregular because not all sides are equal.
Monica	Regular is when the shape has four sides equal. No, there can be eight sides, an octagon or hexagon etc.	A rectangle is irregular.
Joanne	A shape with all sides equal, like a square. Also, we know them well. Up to know we've learnt mostly regular shapes. A rectangle is regular.	An irregular shape is drawn as you wish. A scalene triangle is an irregular shape. (*I indicate the L- and U-shaped regions in their copybooks*). Those are irregular.
Federica	A rectangle is regular.	A trapezium is irregular, because it's got one side that is not the same as the others, 'sideways' (gestures a slanting orientation).
Celia	A regular shape is one that has all sides the same. A regular shape has to have at least two sides equal. Actually, two pairs. For example like these:  	An irregular shape is one where one side does not match the others at all. For example: 
Katrina	(*She recalled the words being used, but not their meanings. She asked if they had anything to do with division, then left the talking up to Celia*)	

Table 11.2. A summary of the Grade 6 pupils' explanations for *regular / irregular*

Clare and Monica were the only two who appeared confident in their explanations, although of course they did not define the property correctly. Perhaps they had learnt it in this way in a previous Grade. Joanne and Celia contradicted themselves, and Joanne seemed to associate regularity with the shapes they were mainly familiar with – at the time I understood her comment to mean the 'basic' shapes that young pupils are often presented with as an introduction to geometry. I concluded that the meaning for *regular*/*irregular* had not been indicated clearly, and I examined the classroom data to explore why. The words *regular*/*irregular* were first mentioned as follows:

(*The class is discussing finding perimeter of a rectangular shape; methods of addition and/or multiplication have been suggested*).

Claudette:	You can't always do 'times', because when you have a shape and … (*trails off*) … that one (*points to the rectangle drawn on the board to represent their desks*) the sides, they were equal. If I have a shape and they're not equal, you can't multiply because they're not the same.
(…)	
Clare:	An irregular shape!
Teacher:	(*Ignores Clare's comment and continues talking about methods for finding the perimeter of a rectangle*).
Clare:	But Miss, if you have an IRREGULAR shape ...!
Teacher:	Wait, wait! We're doing regular shapes right now.

[G6Length5minute15]

The implication that a rectangle was a regular shape was later reinforced when Gina asked the class: "What if my table isn't regular?" [minute 27]. She sketched one of the desk arrangements as shown, filling in the dimensions as the girls suggested them (they had measured these themselves).

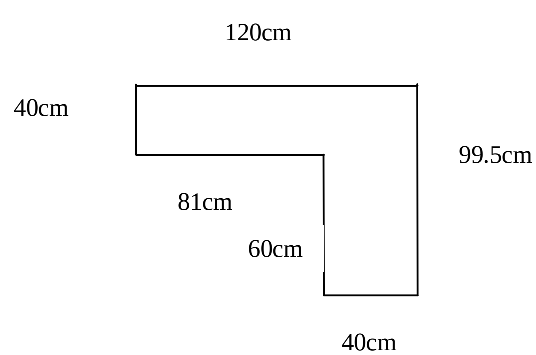


Figure 11.4 Finding the perimeter of an irregular shape

Later in the lesson, Gina asked if a sketched rectangle was a 'regular shape' [minute55]. The girls answer 'yes' in chorus, an answer that Gina accepted. The following day the same idea was once more reinforced:

Teacher:	Yesterday we found that we can have perimeter of irregular shapes and we noticed that the way our classroom [layout] is, we have a lot of irregular shapes. We HAVE regular ones like the teacher's desk, the door, the windows/
Pupil:	/the board.
Teacher:	The board. But the setting of our classroom, of our tables is rather irregular. Whether it's regular or irregular, we can still find the length [perimeter], right?

[G6Length6minute14]

I noted that the idea that a rectangle was regular was always implied, although not stated explicitly by the teacher. What it was exactly that made a rectangle 'regular' - and hence other shapes irregular - was not stated. Such a focussing would have served to stress some aspects of the shape while ignoring others (Hewitt, 2006). For regular shapes, features to be stressed are lengths of sides and size of angles, while features to be ignored include size of the shape and orientation. Since Gina did not specifically stress anything in the reference context, the girls were 'free' to focus on features of their choice and this may have prompted the association of regularity with 'basic' shapes (Joanne), shapes with two pairs of equal sides (Celia) and shapes with angles that are not 90 degrees (Federica). Katrina was unable to offer any explanation. Finally, I found Gina's statement 'the setting of our tables is rather irregular' an interesting one, since it seemed to imply that regularity/irregularity may lie on a continuum, rather than be mutually exclusive. Perhaps Gina herself was unsure of the exact definition: after all, the subject came up only because Clare insisted on the property being discussed.

11.3 A collection of language-related points for the topic 'Length'

In this section, I would like to offer a collection of language-related points that came to light through this topic.

11.3.1 One-off statements in Grade 6

For the topic 'Graphs' I noted that the girls repeated an interpretation that had only been uttered once by their teacher in class. In this topic I also noted such an example, and this appeared to reinforce the point that older pupils may attend to a one-off interpretation (a phenomenon I did not observe for the younger pupils). In the case of the Grade 6 topic ‘Length’, the idea was that a *measurement* was a measuring tool or instrument, implied only twice by Gina:

> (*Kirsty has just commented that her uncle uses a ruler a lot in his job).*
> Teacher: Rulers, measurements are very important.
> [G6Length2minutes18&22]

> (*An L-shape polygon is drawn on the whiteboard. The measurements are written next to the respective sides except for two sides that are missing*).
> Teacher: Can we find the lengths without using measurements?
> [G6Length6minutes18]

Clare and Monica later suggested the following explanation for *measurement*:

> Clare: ‘Measurement’ **(…) could be the TOOL that you used.**
> (…)
> Monica: (*Picks up a ruler*). This is the measurement to measure something.
> (…)
> I: **So an architect’s measuring tape can be a** measurement?
> Clare: **Exactly.**
> [G6Length(C)Q1]

Later, Clare also referred to metres, centimetres, millimetres and kilometres as ‘types of measurements’ [(C)Q2]. That is, she also considered *measurement* as a quantification of length by means of a unit, but the point here is that she repeated the ‘tool’ point stated just twice by Gina.

11.3.2 An unexpected interpretation for *measurement* and *length*

For the topic ‘Multiplication and Division’ I came across an unexpected interpretation of the expression *repeated subtraction*. For the Grade 3 topic ‘Length’, Kim offered an unexpected interpretation of the words *length* and *measurement.* Kim first stated that she did not recall either *length* or *measurement;* furthermore, when I pointed to the word *measurement* printed on a handout they had worked out, she said that she did not know what the word *measurement* was referring to (although she did recall what they had been expected to do in the exercise). However, as I asked her about the word *length* in the second part of the interview, Kim happened to have her copybook open on the page where she had the following note written:

Length		Measurement
kilometre	=	km
metre	=	m
centimetre	=	cm

Kim suggested that the words listed on the right hand side were ‘length’, while the shortened form of the units were referred to as ‘measurement’. I find this point particularly interesting because it highlights the influence of perceivable objects (in this case, the written notes suggestive of two labelled lists). I believe that Kim’s interpretation was not what Rose had intended when she had written these notes on the board: Rose had written *measurement* as an alternative title to ‘Length’, and the shortened forms just happened to be placed exactly under the word ‘measurement’. Of course, it is not possible to anticipate all possible interpretations of what is said and written in the classroom, but examples such as these highlight Eco’s (1976) suggestion that a meaning is a possible interpretation by a possible interpreter, and heightens my awareness that we can never be sure that the language and other signs will be interpreted in the way we intend.

11.3.3 Lexical ambiguity

Garbe (1985), Miller (1993) and Olivares (1996) discussed the problems second language learners may have with English words in terms of what Durkin and Shire (1991) called lexical ambiguity. For the topics ‘Multiplication and Division’ and ‘Graphs’, I did not note any examples of ambiguity, but for the topic ‘Length’ I noted three examples of homophones, that is, words that sound the same, but have distinct meanings. In Grade 6, the homophones that cropped up were *breadth* and *breath*. However, the classroom interaction and interview data suggest that this did not create any difficulties, since the girls seemed fully aware that they were different words with different meanings, and in the classroom, the pupil Ritienne even stated that the words were homophones [G6Length1minute69], a grammar point they had addressed during their English lessons.

Another word was *width*. The Grade 6 girls were well aware of this word, but in Grade 3, although all the pupils stated that they had never heard the word before, Charlotte momentarily confused it with the word *with*. However, on seeing its written form, she realised that this was in fact a different word to the one she knew. Another Grade 3 pupil, Fiona, associated *height* with *hide*:

Fiona: Height? **You don't mean** 'hide' **to hide one-self, do you?**
[G3Length(C)Q2]

I am not in a position to say whether the similarity of sound confused the pupils during the lessons - the words *width* and *height* were in fact used very little in Grade 3. One interesting point indicated by Kim, was the linking of the English word *height* with a Maltese word that is sounded in the same way, **ħajt.** During the interview with Kim and Fiona, the girls stated that they had no recollection of the word *height*. However, Kim later suggested:

Kim: 'Height' is in Maltese – **ħajt! [wall].**
[G3Length(C)Q2]

I think that during the interview, Kim and Fiona who suggested **ħajt** and *hide* respectively were just suggesting different possibilities to help out in the interview. However, their suggestions bring out further possible interpretations and illustrates how similar sounding words may be confused not only *within* a language (*width*/*with*, *height/hide*, *breadth/breath*), but also *across* languages. Off hand, would not think that there are many mathematical words that present this situation. However, exploring the point may form part of our reflections on the use of the English mathematics register in order to pre-empt any possible ambiguity that might arise in a classroom.

11.3.4 Reflections on the potential role of translation

Unlike the topics previously discussed, 'Length' is a topic for which Maltese equivalents for many of the words are common, such as *height*, *measure* and vocabulary related to direct comparison. (Words such as *perimeter* and *regular* are more likely to be met with in a mathematics classroom and translations not so commonly used). By Grade 3 level it seemed that the pupils were familiar with *long*/*longer*, and *short*/*shorter*. The pupils stated very confidently that they knew these words of direct comparison, which are usually introduced in Grade 2:

'**Those are easy**!' [Jessica, G3Length(A)Q6]

The pupils in both Grades were also able to offer translations for *longer* / *shorter* (**itwal** / **iqsar**) although as explained in Chapter 6, the younger ones were sometimes less precise in their translation (e.g. **small** for *short*, **long** for *longer*).

Although I cannot be sure, I might assume that Maltese speaking children would first learn the Maltese words through everyday experiences and then come across the English ones as part of their school experiences. An illustration of this cropped up with the word *height*. This word was not recalled by any of the Grade 3 pupils, but this is not to say that the girls did not have a concept of height. For example, during my interview with Petra and Charlotte I decided to pursue whether the girls could talk about the notion in Maltese and asked them if they had talked about **għoli** (**height**) during the week. (The same word is used in Maltese for *height* and *high,* although the former is often preceded by the definite article). Petra and Charlotte explained as follows:

Petra:	**No, we didn't talk much about that. But we know something about height [għoli]. You can have high [għoli] and low [baxx].**
Charlotte:	**Something high and another low.**
[G3Length(B)Q1]	

Petra then went on to give an explanation of how you could stand on a chair to measure the nearby wall. By offering the opposite of *high* as *low*, and explaining how to measure the appropriate dimension of a wall, Petra indicated an awareness for height even though she may not have had available to her the English word to express the idea.

This example supports the argument I presented in Chapter 6, where I suggested that one way of increasing pupils' knowledge of English could be to explicitly provide vocabulary for already existing concepts. I now offer a semiotic representation for the translation of *longer/shorter* in Figure 11.5. In this representation, the Maltese words change role from sign to forming part of the reference context; the English versions serve to rename the 'same' concept and hence are 'equivalent' to the Maltese ones. I acknowledge that I use the term 'equivalent' rather loosely, since there may be subtle differences in the status attributed to the words by virtue of the contexts in which they are used (everyday / academic). Evans (1999) suggested that such a difference constitutes an emotional charge that forms part of the meaning attached to the words. This in itself is an important aspect of meaning, which however, is beyond the scope of my study. In the diagram, I account for the possible variation in meaning due to affect by suggesting two overlapping bubbles.

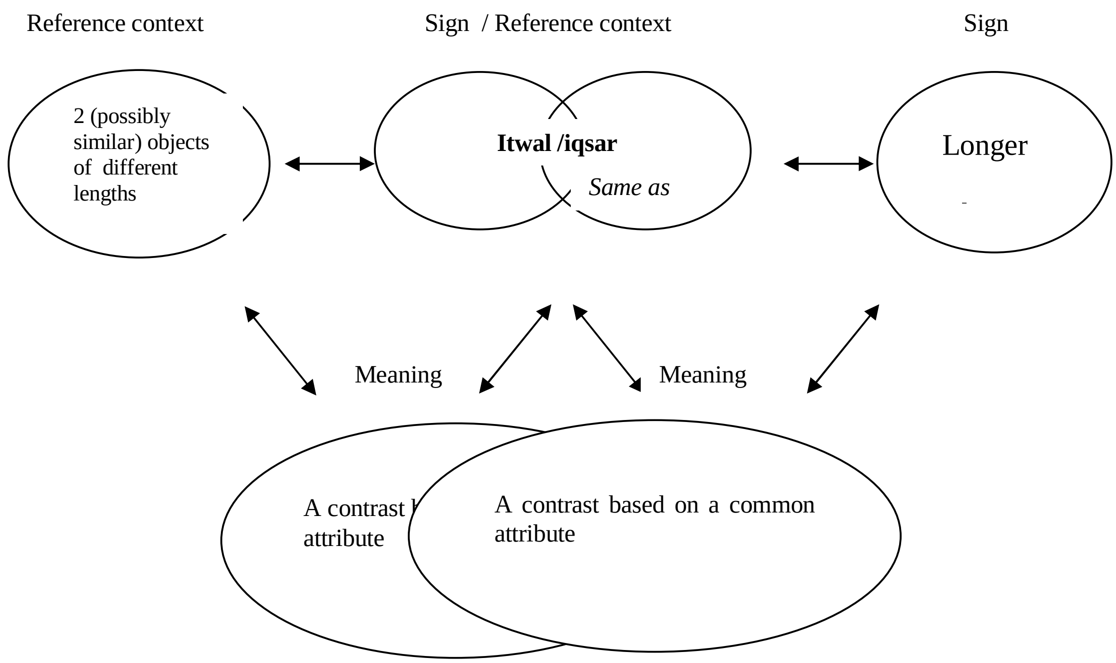


Figure 11.5. A semiotic chain for translation

This and similar chains can be set up explicitly in a classroom wherein code-switching is considered to be a resource rather than a problem (Adler, 2001).

11.4 Significance

My third research question asked what 'conditions' appear necessary for successful sharing of meaning. I assumed that clarity was one of the conditions, and in my discussions have attempted to qualify in more detail what rendered meanings 'clear'. I also noted that frequent use of a word in class appeared to aid recollection, a first step in the pupils' expression of meaning. As part of my reflections on the topic 'Length', I identified a third feature of word use that I believe may have contributed to bringing mathematical words to the fore and hence had some bearing on sharing meaning. I call this feature 'significance' by which I mean how crucial a word appeared to be in a context. Two types of situations appeared to have the potential to render a word significant: first, when the word was the focus of the talk or centre of the teacher's or pupils' attention; second, the fact that the word could not be replaced by another one. I will discuss each in turn.

From the outset of this discussion, I must state that of the three conditions, the idea of significance is the one which is most conjectural. Frequency is the most ‘concrete’, since I identified it by counting words; I interpreted clarity as the ‘proximity’ between spoken words and perceivable notation and objects and expressed the notion through a semiotic model. Significance, on the other hand, is based on my assumption of where the teacher’s and pupils’ attention may have been and I acknowledge that it is difficult for an observer to judge where exactly a person’s attention is placed at any moment in time.

11.4.1 A word at the focus of attention

There were several examples of words that appeared to be at the centre of attention and in these cases, I considered the use of the word to be significant. For example, in unit conversion activities, words such as *metre* and *centimetre* were significant. If pupils were asked to find the boundary of a region, then the word *perimeter* played an important role since the subject at hand *was* the idea of perimeter (“Find the perimeter of …”). I found that words that were used in a significant way were later recalled and explained appropriately by the pupils.

On the other hand, there were other words, particularly in the Grade 3 classroom that were used in a way that I considered *not* significant, because as the teacher spoke, she seemed to direct attention to something other than the idea denoted by the word. For example, in the following excerpt, Rose asked the girls to measure the height of their desks. Although the word *height* was temporarily stressed by the teacher’s intonation, the main focus of attention appeared to be the use of the measuring tape.

> (*The pupils have just measured the top edges of their desks*).
>
> Teacher: Now we are going to measure the HEIGHT of our desks. Remember we start from the ‘one’ and you let your tape reach the floor (*places her measuring tape touching the top edge of the desk. The girls eagerly kneel down to start measuring*). Now listen … we are going to measure from the top of the table until you reach the floor. (*The girls measure in twos. The teacher suggests that if they find it difficult to read off the number close to the floor, then they can measure ‘upwards’ with the end of the tape touching the floor*).
>
> [G3Length1minute24]

Van de Walle (2004) suggested that measuring with a standard unit like the centimetre may draw attention away from the attribute itself and indeed, I noted that during this activity, attention was on the measuring action and on how to be as accurate as possible, rather than on the actual dimension. Thus I suggest that the word used to denote it was not significant.

Another word that did not seem significant was *width;* it seemed to me that when it was used, attention may have been on say, a measuring activity or centimetre measurements. For example, in the following excerpt, it was the kilometre that was at the centre of attention:

> (*A discussion is going on about the unit kilometre*)
> Teacher: Do we use kilometres to measure our desks (…) or furniture? Or the width of the door? Or our height? For what do we use kilometres, Angela?
> [G3Length2minute31]

The interview data later revealed that none of the pupils recalled the words *height* or *width*:

> Jessica: Height? I don't remember the word 'height'. [G3Length (A)Q2]

> Petra: (*Referring to the word* width). **We didn't use it**. [G3Length(B)Q1]

As previously noted, the words *height* and *width* were not used very often in Grade 3. Hence, the lack of frequency on its own may very well explain why the pupils did not recall the words later. I will reflect on the inter-relationship between frequency and significance in Section 11.5, but tentatively suggest that the lack of significance of the words in the situation they were used may also have contributed to the pupils' inability to recall and hence explain the words. Looking back at the topics already discussed, I was reminded of the expression *drop a perpendicular* as used in Grade 6, and can re-interpret my previous discussion by stating that the expression had not been rendered significant in the course of the graph activity, since the teacher's and pupils' attention had been on the *y* and *x*-values.

11.4.2 Substitution of a mathematical word

I suggest that another way that a word may lose significance is if it can be replaced. The possibility of replacing a word by another one was originally brought to my attention by the Grade 3 teacher, Rose. I had commented to her that she had started using the names for units (*metre* etc.) immediately as she started the work on measuring, and yet when tackling estimation, she had introduced the word *estimate* as the activity unfolded. Rose explained:

Rose: With regards to *metres* and *centimetres*, well, that is the word. I couldn't replace those words. (…) [but] when we talk about verbs, I can say [an] action, doing something, a 'doing' word". [RoseLength(2)Q3d)]

Rose replaced the word *estimate* by expressions like *guess* or *say about how much*, possibly to help the children understand what was meant by the mathematical word. The following excerpt is an example of how the word *estimate* was used (I note that here that the word was used both as a verb and as a noun but will not dwell on this point):

(*The class is looking at a textbook page that shows some objects e.g. pencil holder, plate etc. She is reading the 'bubble speech' printed on the textbook*).

Teacher: "Find one of each object. Estimate its length in centimetres – use a ruler to measure it". Now when you estimate, we are going to say ABOUT how much. (…) For example, when I open my hand (*stretches out fingers of one hand*) I can say it's about 10 or 11 centimetres. You have to first say about how much it is, but then in the other bubble speech we have "Use a ruler to measure it". So first we write what we think - about, roughly, we say roughly, the estimate, then we have to measure it with the ruler.

[G3Length3minute14]

After the girls had worked together, Rose elicited answers by asking pupils questions such as: "How long did you think …?". Consequently, the word *estimate* was not used any more for the rest of the lesson, and indeed the week. Later, none of the girls recalled the word and hence they could not offer an explanation for it.

Kim: **The** teacher **hasn't mentioned it to us yet**. [G3Length(C)Q1].

Yet, the Grade 3 girls could in fact recall the estimation *activity* they had carried out in class, with four of the pupils even offering details about what had gone on in the classroom. For example:

(*Petra and Charlotte have just stated that they do not remember the word* estimate *that is printed on my sheet. Petra looks through some textbook pages, finds the word and indicates it).*

Petra: (*Reading*) "Estimate its length in centimetres".

I: Do you remember what it means?

Petra: (*Scratches her head, looks puzzled*). Not a lot.

I: Do you remember what you had to do in this exercise?

Petra: Yes! Yes! This one was my shoe … [referring to the picture of a shoe on the page]. It was funny! These were the hand of Sonia [referring to the picture of a hand] and they got it all

	right [presumably referring to all the estimates offered in Sonia's group]. It was fourteen centimetres.
Charlotte:	Fourteen and a half.
Petra:	We … the teacher find it, all these things [pictured on the page] and gave it to four groups. One group we have to measure/
Charlotte:	/this things (*points to pictures*).
Petra:	This thing. But first we have to guess what they … (*trials off*).

[G3Length(B)Q2]

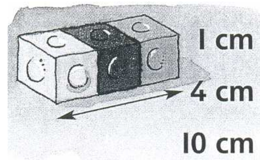
I suggest that the contexts when the word *estimate* was used lent enough support to allow the pupils to carry out the action without actually focusing on the word used to denote it. Possibly, the Grade 3 girls had been able to make sense of the activity by way of the other words that were (presumably) familiar to them such as *say roughly*, *guess* and *think*. Of course, paraphrasing a word or expression is one way of expressing its meaning, and indeed, I have suggested that use of familiar words is a useful, even necessary, aspect of sharing meaning. However, excerpts like the above also make me aware that *because* a word can be substituted, then the mathematical word may be used less and also be rendered less significant in the given context. Thus, a fine line exists between using alternatives to aid understanding and actually bringing the new mathematical word to the fore. I tentatively conjecture that this may be exacerbated in an immersion situation because children may focus their attention on the more familiar substitutions in order to follow the general flow of the lesson, thus perhaps 'ignoring' the new mathematical words.

Another 'replacement' that occurred, once again in Grade 3, was between *length* and *measurement*, that is between two mathematical words. The word *measurement* can be used to quantify a distance with respect to a unit, so that a length can be talked about as a 'measurement'. Hence length and measurement are very closely related, as can be seen in the following textbook statements:

> (*With reference to examples of the type: 1m 35cm = ___cm*). Write these lengths in centimetres.
>
> (*With reference to pictured objects having one dimension marked and a set of three measurements suggested*). Choose the nearest length.

Rose used the words interchangeably, for example:

(*The class is looking at a textbook page. Pictures show a mug, a pencil, an ant and a stick of blocks. A double arrow marks a length for each item. Three alternative measurements are given for each item. The teacher is here referring to the picture of the stick of blocks shown below:*).



Teacher: Now we have three measurements: one centimetre, four centimetres, ten centimetres. Which do you think is the correct length? (…) We're going to mark what we think is the right measurement.
[G3Length3minute36]

The words *length* and *measurement* were used a moderate number of times (37 and 19 times respectively by the teacher, once each by the pupils). I had considered that when the words were used, their meaning was in fact clearly indicated. For example, "Find the length" was printed near pictures of objects whose length was marked with a double sided arrow; the word *measurement* was used in close conjunction with centimetres, metres etc. *Length* was new to all six pupils except Jessica, while *measurement* was new to four pupils, with Petra and Charlotte not indicating familiarity. It may be that the interchangeability of the words rendered each less significant and this may have had some bearing on the pupils' later expression of meaning. During the interviews, only three of the pupils recalled and gave an explanation for *length* in terms of measuring something (e.g. a line). Only two girls explained *measurement,* and these somewhat doubtfully. For example, Jessica first stated that she could not remember the word, then suggested that 'it's like when you measure' [G3Length(A)Q2]. Petra suggested **kejjel** (**to measure**) but she seemed to have some doubt that this could be correct since she had already offered **kejjel** as an explanation for *to measure* [G3Length(B)Q1]. (I cannot tell whether Jessica and Petra were actually recalling the association of measurement with length / measuring or simply drawing spontaneously on the similarity of the sound of the words).

Of course, *length* and *measurement* are in fact so closely related, that perhaps making a distinction is not important in practice. However, I was reminded of the interchange between *sharing* and *grouping*, again two mathematical words that are closely related, but in this case denoting different ideas that are in fact, important to distinguish. I can now add to my discussion on sharing and grouping by stating that the interchange may have rendered each of the words less significant. As already noted in Chapter 9, pupils' explanations were then based on sharing, an idea that they were apparently very familiar with.

11.5 Inter-relatedness of frequency, clarity and significance

Having identified three 'conditions' that appeared helpful for sharing meaning, that is, frequency, clarity and significance, I decided to revisit the use of all the words I have considered in this study and summarise the conditions for each. I thought that this exercise would be useful in order to check out the inter-relatedness of the conditions. That is, to note whether a word that was frequently used was also used in a significant or clear way, whether lack of clarity accompanied lack of significance and so on. I present the summary in Table 11.3 for which it should be noted that:

- The words are grouped according to topic and Grade 6 'Length' includes some words that I did not discuss in this chapter for the sake of brevity.

- The symbol ✓ indicates frequency (F), clarity (C) or significance (S), while × indicates lack thereof. Some entries are marked with both symbols. For example, *grouping* was significant by being the focus of attention at the time ("we are grouping"); on the other hand, it was not significant in the sense that it was interchanged with *sharing*, and hence not distinguished from it.

- The numbers given in brackets after each word are the combined frequency of teacher use and pupil use. However, as already discussed, it was the teacher who tended to use the words most, therefore for several of the entries, the 'combined' frequency is actually the teacher's use. To distinguish between frequently and infrequently used words I chose an arbitrary cut-off point of 30. My choice was influenced by the difference in ability of Grade 3 pupils to explain the words *divide by* (42) and *grouping* (19) and the Grade 6 pupils' difference in recollection of *pie-chart* (33) and *drop a perpendicular* (16).

- I classified clarity as explained in previous discussions; the idea expressed by the teacher may not necessarily have been altogether appropriate (as, say, in the case of *regular*).

- I classified significance as explained earlier in this Chapter, that is, in terms of the word being at the focus of attention, or not being replaced by another word.

Grade 3				Grade 6			
Word	**F**	**C**	**S**	**Word**	**F**	**C**	**S**
Multiply by(185)	✓	✓	✓	Graph (168)	✓	✓	✓
Division (139)	✓	✓	✓	x- / y- /axis (165)	✓	✓	✓
Times (105)	✓	✓	✓	Represents (100)	✓	✓	✓
Tables (61)	✓	✓	✓	Scale (38)	✓	✓	✓
Multiplication (54)	✓	✓	✓	Plot (37)	✓	✓	✓
Divide by (42)	✓	✓	✓	Pie-chart (33)	✓	✓	✓
Grouping (19)	×	×	✓×	Line-graph (23)	×	✓	✓
Sharing (13)	×	×	×	Drop a perpendicular(16)	×	✓	×
				Block-graph (16)	×	✓	✓
				Data (14)	×	×	✓
				Bar-graph (10)	×	✓	✓
Centimetre(664)	✓	✓	✓	Centimetre (560)	✓	✓	✓
Metre (344)	✓	✓	✓	Metre (356)	✓	✓	✓
Measure (99)	✓	✓	✓	Millimetre (295)	✓	✓	✓
Long/er (68)	✓	✓	✓	Measure (227)	✓	✓	✓
Kilometre (53)	✓	✓	✓	Length (190)	✓	✓	✓
Length (38)	✓	✓	✓×	(various) Spans (72)	✓	✓×	✓
Short/er (21)	×	✓	✓	Long/er (62)	✓	✓	✓
Measurement (19)	×	✓	✓×	Perimeter (64)	✓	✓	✓
Width (9)	×	✓	×	Width (62)	✓	✓	✓
Estimate (7)	×	✓	×	Kilometre (62)	✓	✓	✓
Height (5)	×	✓	×	Height (51)	✓	✓	✓
				Measurement (34)	✓	✓	✓
				Breadth (43)	✓	✓	✓
				Short/er (22)	×	✓	✓
				Irregular (13)	×	×	✓
				Regular (9)	×	×	✓
				Metric (4)	×	×	✓

Table 11.3. Interrelatedness of frequency, clarity and significance.

As can be seen from Table 11.3, various combinations of the conditions occurred. Words that had been used with all three conditions were later recalled and explained appropriately by the pupils. One such word was *perimeter*. It was used 47 times by the teacher and 17 times by the pupils, a number of times I might consider moderately frequently for the teacher and certainly frequent for the pupils, considering their general use of mathematical vocabulary. It was clear to what the word referred since the boundary was perceptually evident and often drawn by the teacher and pupils themselves. The word was significant because it was the focus of attention for the several examples carried out ("find the perimeter" / "the perimeter of [shape] number 1 is 84 metres" [G6Length6minute2]). Furthermore, the word it was not replaced by any other.

It is important to note that the word *perimeter* was already familiar to four of the pupils I interviewed and this might account for their successful explanation. However, the word was in fact new for two of the pupils. One of them was Katrina, who was generally the pupil least confident during the interviews. Yet, even she explained in what was for her great detail (albeit offering an inappropriate measurement for the width of the table and working out the answer incorrectly):

> Katrina: Perimeter … you have a, this shape (*touches the coffee table in front of her*). This is two centimetres (*touches one of the shorter sides of the table*), and that [is] two (*indicates the side opposite to it*). This is hundred and hundred (*touches the longer sides*). You add them up. And it come three hundred. (*gestures around the four edges of the table*).
> [G6Length(B)Q2]

Similarly, in Grade 3 one word that was used with all three conditions was *multiply*. It was used with all conditions, and was shared successfully (see Chapter 9). Obviously, previous familiarity is important to keep in mind when interpreting the presence of the conditions. For example, although *sharing* in Grade 3 was used *in*frequently, not clearly and not in a significant way, yet the girls were still able to explain it appropriately, since they had already known the meaning of the word.

Some words were significant but not clear. For example, *data* had been the focus of attention more than once, but Gina was not specific enough about to what it was exactly that the word referred. On the other hand, some words were clear (in the sense apparently intended by the teacher) but not significant. This was the case for *drop a perpendicular*, which at the time was clearly associated with the specific drawing of vertical lines, with clarity was being assisted by Gina's gestures. On the other hand, the expression seemed to have lost significance as attention appeared to be more on actually drawing the line along the copybook page and finding the relevant x-values. By examining the pupils responses during the interviews, I noted that the girls were less successful in recalling and / or explaining words that had lacked either clarity or significance. (The details of recollection or explanations have been amply developed in previous chapters; I have refrained from inserting details of the girls' ability to recall/explain in Table 11.3 due to the complexity of presenting the wide variety of combinations. Words could be classified as new / familiar to some /all the pupils; a word might have been recalled by some (differing numbers) / all of the pupils; a recalled word might have been given an appropriate (possibly different) explanation by some / all the pupils and so on.

I noted that infrequently used words tended to also lack either clarity or significance, and therefore it is not possible to pin-point which of the conditions had most bearing on the pupils' inability to recall and/or explain the words. What I can suggest however, is that in an attempt to share the meaning of words that make up the mathematics register, it may be helpful if a teacher works towards maximising each condition, that is frequency, clarity and significance in her use of a word. Furthermore, in Chapter 7, I stated that pupils themselves should be encouraged to use mathematical vocabulary to promote effective communication and to allow a teacher to gauge understanding. I can now suggest that increasing pupil-use of the vocabulary can be viewed as an opportunity to increase the general frequency of word use in class and promote the significance of the words since, presumably, a pupil needs to 'focus' on a word in order to use it. Finally, pupil-use of vocabulary exposes how 'clear' pupils are about the ideas that words denote.

11.6 Conclusion

For the topic 'Length' I confirmed my previous observations that close association between something that is perceived and the language used to refer to it, is very helpful for sharing mathematical meaning. For the sake of brevity I did not discuss all the topic-related words, but gave some detail regarding *perimeter* and *to measure*, meanings for which were successfully shared. I also discussed units and noted that the girls were able to offer the numerical relationships between various units, since these had been rendered clear during the lessons. However, understanding units also includes an appreciation of the size of the unit and my data supports the idea that this appreciation depends heavily on experience. The topic 'Length' offered an opportunity to discuss a mathematical property, that of regularity. I noted that lack of clarity in this respect was a result of the teacher not stressing the features of the reference context that imply regularity. Hence, the pupils stressed and ignored features at will, and this resulted in a variety of explanations of the property.

Some interesting examples came to light that illustrate the intricacies of language use during mathematics lessons. For example, in Grade 6, one pupil interchanged *height* and *length* as dimensions, apparently basing this possibility on the teacher-suggested interchanges *width/breadth* and *length/width* and the fact that height as a general spatial concept is a particular orientation of length. Another example was pupils' interpretation of a *span* as a single body part, due to the way the word was used in class and the fact that most non-standard units used were in fact, single body parts. In Grade 3, I noted a pupil's unexpected interpretation for *length* and *measurement*, and the possibility of lexical ambiguity across languages. I also confirmed that the Grade 6 pupils sometimes recalled one-off statements uttered by the teacher, implying that perhaps a teacher of older pupils may need to be more careful to use mathematical words appropriately.

This topic highlighted the importance of a primary school teacher's own mathematical knowledge: Gina defined *regular* shapes incorrectly and suggested an interchangeability of the names for the dimensions *length*/*width* for a rectangular shape. She also used an unusual expression *foot span* which I believe played a part in misleading the pupils to think of a *span* as the length of a single body part. Since Shulman's (1986) seminal work on the area, interest in the complex relationship between subject knowledge and pedagogy has grown (Banks *et al*, 1999). Although the issue is beyond the scope of my study, it is of course relevant to me as a mathematics educator, since as suggested by McNamara (1991), it is important for teachers to have a sound knowledge of subject areas.

Although the points I have mentioned above appear to be generally applicable to 'any' mathematics classroom, 'Measurement' is a mathematical area where several words are available in Maltese. Hence I took the opportunity to use my semiotic model to offer a theoretical representation of translating from one language to another. Without wishing to underestimate the affective element involved in translation, I suggested that in a classroom where code-switching is used, the new English word can be introduced as an alternative 'naming' of an already familiar idea.

Through this last topic, I developed the idea of significance. This refers to how crucial a word appears to be in a given context. I suggested that significance is a result of the word (and hence the idea it denotes) being in focus; significance is also ensured by the fact that a word is not easily replace. Hence, *lack* of significance may occur when a word/idea is not the centre of attention and also when a word is replaced by another (everyday or mathematical) word. I concluded that ultimately, three conditions appear to be necessary for sharing of meaning: frequency, clarity and significance. The inter-relatedness of the three makes it difficult to state which has most bearing on sharing meaning, and therefore I suggest that, whatever language is used as a medium of instruction, it may be helpful for teachers to try to maximise the ‘conditions’, both in their own usage and that of the pupils.

At this stage, I can only wonder what similarities and differences may have resulted in the three conditions listed in Table 11.3 had the lessons been given in Mixed Maltese English. Furthermore, I wonder if code-switching in general might offer a third way of promoting significance, namely by the highlighting of mathematical words by virtue of them being in another language.

CHAPTER 12

Conclusion

12.1 Introduction

In this final chapter, I retrace the origin of my study and outline the underlying theoretical and methodological foundations that guided my approach. I discuss limitations, then summarise my findings and reflections. I end by recommending further studies and suggesting a way forward regarding the language debate in Malta.

12.2 Retracing theoretical and methodological foundations

My interest in the present project was sparked by the National Minimum Curriculum recommendation that mathematics be taught through English. At first this seemed to me a sensible idea, since mathematical vocabulary in Malta tends to be retained in English, and textbooks and exams are always printed in English. However, conversations with linguists prompted me to reflect on the pedagogic value of code-switching between Maltese and English. My original aim for the study was to focus on how code-switching was used in two classrooms to share meanings for English mathematical words. However, unexpected circumstances dictated that I observe lessons where English was being used as a medium of instruction. I ser out to address the following questions:

> *(1) How does the NMC recommendation regarding the use of English for mathematics fit in with other educational principles promoted in the same document?*
>
> *(2)(a) How much, and with what ease, do pupils talk in immersion classrooms? (b) How 'mathematical' is their talk, in terms of the inclusion of mathematical vocabulary?*
>
> *(3) What conditions appear to be helpful for a teacher to 'share' the meaning of (a selection of) mathematical words with the pupils?*

Although the scope of the study seemed quite wide, I viewed this as a positive thing since I believed that a wider base could potentially offer a good spring board to extend the local discussion regarding the use of English as a medium of instruction for mathematics. I hoped to tease out what is particularly relevant to an immersion classroom, and what might be applicable to 'any' primary mathematics classroom.

One of the challenges I faced in the development of this study, was to find an approach whereby the various strands were kept together. I found Halliday's (1978) notion of register helpful, since it considers simultaneously the use and the meanings of specific vocabulary. Pimm (1987) suggested that a register serves as both a medium and a message and I used this idea to structure my discussions. As part of my reflections on language as a medium, I considered tensions arising within the NMC document. I also considered the frequency of word use, the extent of talk and variations in the use of mathematical words (as compared to how the words are generally used in English). With regard to language as a message, I considered the meanings of a selection of topic-related words. I shifted from medium to message by reflecting on the relationship between frequency of use and the familiarity of the words.

I approached the data collection phase with certain assumptions in mind. I believed that learning mathematics took place as a social activity wherein language was crucial to teaching and learning. Hence, I viewed the classroom as a 'community of practice' (Lave and Wenger, 1991) where speaking mathematically constituted part of the learning of the subject. I assumed that mathematical words functioned not only as a means of communication as such, but as the objects of the communication activity itself. I viewed the teacher as an 'expert' who shared her knowledge with the pupils. Since knowledge was shared through explicit instruction, the word meanings and their inter-relationships could be considered as scientific concepts (Vygotsky, 1962). I assumed that through participation in lessons (including listening), pupils may have come to express similar meanings to those expressed in the classroom. The meanings were not passed on intact from teacher to pupil, but rather, were flexible. Following Eco (1976), I viewed meaning as a possible interpretation by a possible interpreter.

The detail necessary for my reflections meant that the most appropriate research method would be an interpretative small scale study. As stated by Wolcott (1995), a case study is unique, but not so unique that we cannot learn from it and apply its lessons to other situations. In part, my study was 'grounded', in the sense that I generated theory through the research data (Glaser and Strauss, 1967). The reflections I presented in my study emerged from my consideration of transcriptions 'surrounding' the mathematical words in the classroom, and the teachers' and children's responses during interviews.

Two key ideas regarding meaning were, however, in place before I collected the data. First, I started with an assumption that clarity was important, following Mercer's (2000a) stand that teachers introduce pupils to technical words by using them in contexts that make their meanings clear. As part of my study, I then attempted to qualify in more detail what rendered meaning clear or unclear. Second, I needed a way to talk about meaning, and after the pilot-study I devised a semiotic model based on the one presented in Steinbring (1997). My model allowed me to consider diagrams and notation along with language as shown in Figure 12.1.

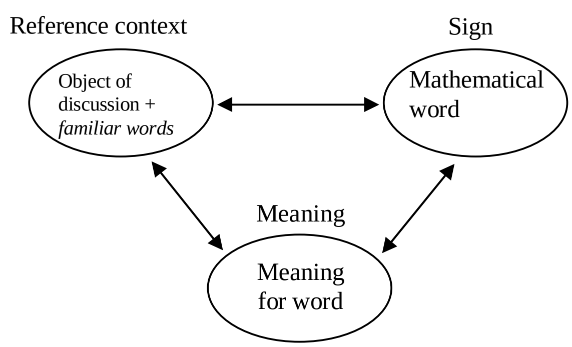


Figure 12.1. General semiotic model used in my analysis.

Extending this model as a sequence of 'triangles' also enabled me to illustrate how understanding may develop as a series of semiotic chains.

12.3 Limitations of the study

Any study undertaken comes with limitations. Traditionally, one characteristic that has been associated with limiting a case study like my own is its lack of generality. However, the point of my study was to show that certain situations or interpretations can, and do, occur. My research method allowed me to recognise such situations and use them as a starting point to reflect on particular issues. Indeed, I consider the small scale element to be a strength, not a weakness, of my study.

Another possible restriction was the 'authenticity' of the observed situations: traditionally, this would be questioned, since the teachers knew my research intentions, and consequently they may have changed the way in which they taught. I acknowledge that my presence may have influenced not only the teacher, but even the pupils' input, but I believed that any classroom episode or interview conversation can unfold in an infinite number of ways, and for the purpose of my study, 'any' situation offered a valuable source for reflection.

Similarly, during the interviews, the teachers and pupils may have told me what they thought (for some reason) I might want to hear. This might have happened while we talked about the use of English as a medium of instruction and in the teachers' case, their intentions regarding the teaching of the topics. My analysis of their thoughts is based on what they stated, and I can only assume that what they expressed was what they really believed.

However, there are some aspects of the collection and analysis of data that I do consider as limitations and I outline these below:

- My recording method in the classroom did not pick up all verbal and non-verbal communication: some missed data may have been potentially relevant in shaping my analysis;

- During the interviews, I asked for meanings of words outside of any supporting situation, and this might account for the pupils not being able to offer an explanation for a word. Perhaps had I presented the words within a situation, word problem, story and so on, their responses may have been different;

- Another possible limitation was that the pupils may not have said everything they 'could' have said regarding the meaning they held for a word. They may have left the talking up to their friend, or gone along with a particular explanation initiated by the friend;

- In my analysis, I made certain assumptions whether the selected words were new to the pupils or not, based on their own judgments. I realise that it may have been rather difficult for a pupil to be sure exactly when she got to know a word. On the other hand, even if I was confident that a word had been familiar to a pupil, this did not tell me anything about what it was that she knew regarding the meaning for that word prior to the lessons;

- The notion of sharing meaning by comparing classroom and interview data is always open to doubt, since I could never be sure that the girls would not have expressed a similar meaning prior to the lessons, or that their expression of meaning had not been influenced by other experiences other than the classroom situation.

While I did my best to minimise limitations, I have accepted them as an integral part of the study and reflected on them as part of the ongoing analysis. I will now summarise the main points of my analysis.

12.4 Reflections on language as a medium

12.4.1 Tensions between the immersion recommendation and other NMC principles

I believe that the use of English brought with it tensions with other National Minimum Curriculum principles or recommendations. The first tension related to the policy of inclusion: my data indicated that the pupils who were considered ‘weak’ in mathematics by their teachers felt hindered by the immersion approach. They reported that they did not always understand the lessons, and would prefer them to be in Maltese.

In the classrooms I observed, the pupils always communicated with each other in Maltese, whether for social reasons or to talk about a mathematical idea. This rendered the language of the classroom inconsistent (in the sense of sticking to English) creating another tension with the NMC wish for consistency of language. I also conjectured that the immersion approach might hinder co-operative learning, yet another ideal expressed in the NMC document.

Using English for mathematics may work against the NMC ideal of strengthening the Maltese language, since it might send a hidden message that Maltese is not a suitable language for the discipline. Indeed, I noted that some pupils had difficulty in offering translations from English to Maltese, even for words that I would have considered quite ‘common’. I cannot conclude that they did not know the Maltese word or its associated concept, but only that the connection between Maltese and school mathematics was not made when requested. I believe that one way to ‘strengthen’ a language is to develop its various registers, and yet an immersion approach actually reduces the need for discussion about a Maltese mathematics register to take place at all. Furthermore, if the NMC recommendation is adopted on a large scale in Malta, then strategies that teachers may have developed over time to help them cope with the code-switching situation will be side-lined.

12.4.2 The use of an English mathematics register

The English immersion approach needs to be considered in parallel with the trend in mathematics education to promote talk in the classroom. My data suggested that the extent of pupil talk depends partly on the teacher's preferred pedagogic style. The IRF style used in Grade 3 did not support much pupil talk and, presumably, this style would inhibit pupil talk even if a first language is used. However, for teachers wishing to encourage talk, as in the Grade 6 class, using an immersion approach may hamper their efforts, since the pupils may find it difficult to express themselves at length. Indeed, the Grade 6 teacher often filled in language herself to help the girls along ("Angelica is saying …"). Hence, the extent of talk observed appeared to depend on both pedagogic preference (which I suggest is in part a reflection of the relationship between teacher and pupils) and the fact that English was used as a medium of instruction.

The practice of including mathematical words can render communication more effective and give an indication of pupils' understanding. From my observations, it transpired that increased talk did not necessarily imply increased use of mathematical words. The Grade 6 pupils, although encouraged to talk more than their Grade 3 counterparts, tended to 'get by' without using mathematical words. To a certain extent this may have been a result of the use of English, since the pupils often used gestures or verbal pointers to express themselves. The teachers allowed the pupils to use ordinary English (correct or incorrect) and gestures, so as not to discourage their efforts to use the language. (I have to state, of course, that I cannot be sure that they would have given mathematical vocabulary importance in a mixed-code setting). However, the inclusion of mathematical words in pupils' talk also seemed to depend on the type of activities in which the pupils were engaged. Words that were 'needed' for an exercise at hand were in fact used. The implication for teaching is that if we are to consider the increase of mathematical vocabulary to be something desirable, then we may need to design activities that promote the use of mathematical words, irrespective of whether English or code-switching is used. Hence, as in the case of extent of talk, both the medium of instruction and pedagogical approaches appear to have influenced the inclusion of mathematical words.

The pupils (and sometimes teachers) used expressions such as ‘do multiply’, ‘the multiply’, ‘do plus’, ‘I did them plus’, ‘three multiply by four’ and ‘twelve division by four’, which vary from what I might normally expect as part of the English mathematics register. I cannot say how widespread these expressions are, but I suggest that if we are to promote English as a medium of instruction, then it may be appropriate to reflect on our ‘ways of saying’, especially since some variations may imply epistemological differences in terms of how mathematics comes into being. Attention to mathematical English may in itself contribute to the pupils’ overall knowledge of English, the ideal so desired by the writers of the NMC. Ironically, it may be the case that insisting on spoken English may actually work against the ideal of giving explicit attention to the English register: the Grade 6 teacher seemed to think that the pupils would ‘pick up’ the appropriate ways of saying in time.

Although I am generally in favour of developing code-switching as a resource, I acknowledge that a mixed code approach brings with it its own issues. When we accept to use a mixed spoken code in class, we essentially have two different registers in use: an informal spoken mixed one, and a formal written English one. Hence, if we are to believe in the importance of focusing on mathematical language, then we have two very different registers to attend to, rather than closely related ones (English spoken and English written).

Furthermore, even if we retain a spoken mixed register, this in itself may need to be reflected on by both linguists and mathematics educators. Unlike mathematical English, which although varies according to context, nonetheless ‘exists’, we cannot claim to have an established Maltese or Mixed Maltese English academic register. Reflections on this register may need to include heightened awareness of which English words actually have common Maltese translations (e.g. *height*) and ones which do not (e.g. *drop a perpendicular*). Furthermore, I noted that English nouns and adjectives (e.g. *shape*, *regular*, *regular shape*) ‘fitted’ well into Maltese speech, but that verbs were harder to incorporate since in Maltese, verbs need to be conjugated. In cases when the English word was retained, the pupils used expressions such as ‘**għamel** multiply’ [**do** multiply], which may explain why pupils then said in English ‘do multiply’. Hence, whichever language we choose to use for classroom interaction, reflection is necessary in terms of *mathematical* language.

12.5 Linking medium to message through frequency and familiarity of words

'Knowing' the meaning for a mathematical word implies being familiar with it to some extent. Despite the limitation attached to establishing familiarity, I felt that an attempt at doing this was important since my interpretation of the success or otherwise of shared meaning could only be carried out in the light of whether the words were known beforehand or not.

I noted an occasional discrepancy between what the teacher believed was new vocabulary and what the pupils stated was new, and I suggest that a close correspondence between these two opinions may be useful as a starting point for the sharing of meaning. The difference in opinion is potentially more problematic if the teacher inadvertently assumes familiarity.

I had expected that new mathematical words might have been used more than others by the teachers, but in fact several new words were used quite infrequently. This seemed to be because the frequency of use of mathematical words depended on whether they were 'needed' for an activity at hand and hence new words were not necessarily used more than previously known ones.

I noted that the frequency of use of a word had some bearing on the pupils' ability to later recall the word. Those words that were used very little by the teacher were not recalled by the pupils, and hence the girls could not offer any explanation for them. I consider that the first indicator of shared meaning is recollection of the word, and hence frequency of use seems to be related to this first layer of meaning (Roberts, 1998). The relationship between frequency and familiarity allowed me to shift from viewing language as a medium (the inclusion of the word in the classroom talk) to language as a message (knowledge of the word).

12.6 Language as a message

It is sometimes suggested locally that one reason why mathematics should be taught through our second language is because mathematical words are in English. The suggestion seems to be based on what is common to the words i.e. that they are all in English. However, words are 'different' in the sense that some are used as references, some denote properties and concepts, while others are verbs. In this study I have reflected on if, and how, clarity was established for a selection of topic-related words. I believe that the points I noted in this respect can be applied to 'any' primary mathematics classroom, in the sense that the difficulties I noted seemed to be more related to pedagogic approaches, and at times teacher knowledge, than to the fact that the lessons were done in English. Of course, I cannot exclude the fact that the teachers' approach itself was influenced by the use of English, but I am not in position to say to what extent this may have happened. Furthermore, although some pupils expressed reservation about the use of English, my study did not allow me to draw conclusions regarding the extent to which childen's understanding was inhibited, if at all, by the approach. Generally, the pupils' appeared to 'follow' the lessons and the dissimilarity of meanings expressed during the interviews could be explained by factors that went beyond the use of English itself. These factors were: frequency of use of the mathematical word, clarity and the significance attributed to the word when it was used. I now summarise my conclusions regarding each condition.

12.6.1 Frequency

The teachers used mathematical words much more than the pupils, but I also noted that there was a big discrepancy between the number of times some words were used when compared to others. The words that were used most were those related to the types of tasks carried out, so for example, *centimetre* was used a lot thanks to the unit conversion exercises, *multiply by* was used frequently since many examples of the type $m \times n$ were carried out. Thus, words that were 'needed' were used, while those that were not needed - such as *width* in Grade 3 or *data* in Grade 6 - were not used so frequently. While this might seem like an obvious observation, the implication is that if we wish to encourage the use and development of meaning of other words, then we need to design activities that encourage their use.

I found that words that were used very little collectively by the teacher and pupils (say, 4 or 7 times) were not recalled by the pupils at all after the lessons. Even 19 instances of measurement did not seem 'enough' for the Grade 3 pupils, although two or three Grade 6 girls did recall words used 12 and 16 times (*data* and *drop a perpendicular* respectively). On the other hand, those words that were used much more (e.g. 74, 162, 464 times) were recalled by the pupils and appropriate explanations were given. It is not possible to draw a clear line between what is frequent use and what is not, but the implication seems to be that if a teacher wishes to share the meaning of a word, then it would be best to maximise her use of the words, especially with younger pupils.

Having said this, I also noted that the Grade 6 pupils were able to recall words more easily than their younger counterparts, possibly because of their age. Indeed, this ability to recall sometimes surprised me, in that some pupils recalled a one-off statement mentioned by the teacher, even if the statement was not altogether correct or appropriate. This seems to imply that care should be taken when offering statements to older pupils, since some individuals may very well take this statement to be the intended meaning to be shared, even if it is not. Of course, it is not practical to expect teachers to be conscious of every word or statement they utter, so all I can suggest is heightened awareness in this regard.

12.6.2 Clarity

Vygotsky (1981c) suggested that a primary role for a word is that of reference. I noted that words that served as 'references' for something perceived were easily shared. These included words such as *times* for the symbol $\times$, *axes* for the relevant parts of a graph diagram and *width* as a name for one of the dimensions of a rectangle. In such a role, the word and the object to which it referred were temporary and spatially co-present (Wertsch, 1985). On the other hand, it was 'harder' to share the meaning for words for which it was not so evident to what they referred. For example, a meaning for *data* was not so easily shared because no actual data was collected by the pupils. Pupils who did recall the word *data*, associated it with the written aspects on their copybook (e.g. the scale and answers to textbook questions regarding the graph), possibly because this is what was 'tangibly' available. The implication is that it may be helpful to render perceivable or tangible anything that can be. Furthermore, examples like this illustrate the complexities of sharing the meaning for words.

Of course, a meaning for a word generally goes beyond reference so that understanding *multiplication*, for example, goes beyond recognising and naming the notation. I explored clarity for concepts through semiotic chains. For example, the concept of multiplication as repeated addition appeared clear because the language used was closely associated with the objects, pictures or notation available: the teacher might say 'each monster has three legs each' when the monsters and legs were evident; she might say 'repeated addition' in association with the written notation $n+n+n+n$. Several ideas appeared clearly presented including the numerical relationship between units (metre / centimetre etc.) in both classes, the centimetre 'size' in Grade 3 and *scale* as a one-to-many / many-to-many relationship (Grade 6).

On the other hand, some other meanings for words or expressions were not rendered clear. For example in Grade 3, *repeated subtraction* for division was not clear, since the language was not used alongside relevant notation or a repeated taking away action. Lack of clarity for division was also a consequence of the fact that when the verbs *share* and *group* were used, the respective actions were not evident, unlike the action of *measuring* that was clearly indicated and reinforced through the pupils' own engagement in the activity. Generally speaking, clarity seemed to be related to an element of 'proximity' between the language used and other aspects of the reference context. Beyond this general idea, it is difficult to give a summary of clarity that covers all words discussed in this study, so I will briefly summarise the main points for the respective topics.

Multiplication and Division (Grade 3)

- If the maxim 'multiplication makes bigger/division makes smaller' is used (although not generally true, so strictly speaking it is not recommended that the maxim is used at all), then understanding *what* is bigger/smaller *than what* may help children appreciate multiplication and division relationships in the particular situation.

- The idea of repeated subtraction may need to be reinterpreted when objects are 'utilised' rather than 'taken away', since taking away is the most common interpretation of subtraction at a young age.

- Care should be taken when using expressions such as 'the same' in reference to 4×5 / 5×4. 'Sameness' is only applicable in an array situation, since 4 monsters with 5 legs each are not the same as 5 monsters with 4 legs each. The use of 'the same' in such cases, and the phrase 'the other way round' for the relationship between the triples $4\times5 = 20$ and $20 \div 5 = 4$ may have led pupils to interchange the notations $20 \div 5 = 4$ and $5 \div 20 = 4$. Hence, I recommend careful use of language in these situations.

- Prompted by the lack of clarity observed for the meaning of a concept for division, I developed the idea of active and static representations for multiplication and division. Active representations are equally easy to provide for both operations, where accompanying language can support the actions going on (repeated addition/subtraction, sharing). However, it is harder to create a helpful static reference context for division because the picture may be perceived as repeated sets (i.e multiplication) rather than a large set already subdivided. Thus semiotic links for division may be harder to establish than for multiplication, supporting the view that division is the hardest of the four operations to teach and learn (van de Walle, 2004) .

- Moving from situations involving objects/pictures to expressing general number relationships such as 'if $m \times n = p$, then $p \div n = m$ and $p \div m = n$', illustrates a shift from metaphoric to metonymic language. Chapman (2003) has suggested that this shift renders language more 'mathematical' and encourages deeper understanding of the subject. However, I noted that tasks that required this relationship were presented *first* during the week, prior to static representations (active representations were not used). I suggest that it may be more appropriate for active and static representations to be carried out first, since the metonymic relationship above depends solely on mathematical language for its interpretation and not on perceived objects/pictures.

Graphs (Grade 6)

- The teacher used the pedagogic metaphor 'a graph is a picture'. When such a metaphor is used, an obvious requirement is that the pupils understand the meaning of the word *picture*. However, it may also be helpful to discuss explicitly the similar/dissimilar features of the domains (graph/picture) in order that the metaphor is fully appreciated. The role of language in this context is crucial since the reference context consists entirely of language (no tangible objects are present) and hence meaning is to be found within the language itself.

- *Bar/line* and *pie graphs* owe their names to what is perceptually obvious. The teacher's occasional comments that 'they are the same', 'what can be shown on a bar graph can be shown on a line graph' and the pupils habit of colouring 1cm widths underneath the line-graph, seemed to encourage pupils to focus on what was perceptually similar/different for the graphs. However, graphs are different in the sense that they are used to represent different types of data and in order to appreciate this, more explicit consideration of the data in question may be required.

- This topic included the expression *drop a perpendicular*. The everyday usage of *drop* and the classroom emphasis on a vertical line appeared to restrict the pupils' interpretation of this expression which in a mathematical context requires an appreciation of perpendicularity and a reinterpretation of the word *drop*.

- The word *plot* was used by the teacher in the sense of *make*/*draw*/*add to diagram* and its specific meaning with respect to a relationship between two variables was not brought out. I suggest that distinguishing between these verbs may strengthen the pupils' knowledge of the mathematics register.

Length (Grade 3 and 6)

- Knowing a unit involves an awareness of its size and also its relationship with other units. The pupils got plenty of practice converting units, but their classroom experiences with focusing on the 'sizes' of the units varied. Although the Grade 6 girls were very familiar with the sizes, these were newer for the younger pupils. My reflections on the pupils' ability to indicate the sizes during the interviews confirmed other researchers' (e.g. Blinko and Slater, 1996) comments that knowledge of the measuring units depend heavily on experience.

- In Grade 6, the teacher suggested a freedom to interchange the names of the dimensions *length* and *width* of an object. This idea was later extended by one pupil to the interchange between the dimensions *length* and *height*. I suggest that this was because the teacher did not differentiate between *length* as a general attribute and *length* as a specific dimension, implying that it may be helpful for subtle differences in meaning to be made explicit.

- While none of the Grade 3 girls remembered the word *height*, two of them offered an explanation for the idea when I suggested the Maltese word for height (**għoli**). Since some aspects of the mathematical area of Measurement are closely tied to everyday experiences, pupils may already be familiar with the ideas and the Maltese words that denote them. Hence, drawing on this knowledge may be helpful to share meaning for the English word, but this is not possible in a classroom where immersion is strictly adhered to.

- The same word *height* also brought my attention to the fact that lexical ambiguity may occur not only within a language (e.g. *breadth* / *breath*), but also across languages – *height* sounds the same as the Maltese **ħajt** [wall]. Although I do not have evidence that this similarity caused any confusion during the lessons, it is worth identifying English/Maltese words for which similarity of sound may be the case.

One thing that was highlighted for me in this study was the fact that a meaning is a *possible* interpretation and that we can never really be sure that what we try to convey will be understood as we intend. This was evident throughout, but three examples that struck me in particular were the following:

- The three Grade 3 pupils who offered an explanation for the expression *repeated subtraction* suggested that it meant say, 3 – 3 – 3 – 3… ;

- A Grade 3 pupil suggested that *measurements* were the units written in short (m, cm, km), as opposed to *length* which she said meant the units written in full (metre etc.);

- As already mentioned, in Grade 6, one pupil extended the idea of height as a particular orientation of length, to an interchangeability of *length* and *height* as dimensions of an object.

By examining the classroom talk, I could find explanations for why this might happen, and these situations illustrated the intricacies of linking language with other elements such as notation, diagrams and real objects.

12.6.3 Significance

A third condition for sharing meaning that I have discussed is significance. Two types of situations appeared to render a word significant. The first was when a word was the focus of the talk , or in Halliday's (1978) terms, when the field of talk was the idea that the word denoted. For example, if the activity was a conversion activity, then the words *centimetre* and *metre* were very significant – the talk was actually *about* these units. Significance was the case for many of the mathematical words that I considered. On the other hand, a mathematical word may have been used in passing, or assumed familiar in a context where the focus of attention is on something else. For example, in Grade 3, when the words *width* and *height* were used in relation to a table, the main point of the task at hand was using the measuring tape accurately, so that the words *height* and *width* themselves were not significant at that point.

Another element that appeared to render a word significant was the fact that it ‘could not’ be replaced by another word. So for example, the word *kilometre* was not easily replaced, nor was *multiplication*. On the other hand, in Grade 3, *estimate* was replaced by the everyday word *guess*. Alternatives play an important role in supporting the meaning for a new word, so that a fine line exists between using alternatives and bringing the new word to the fore. While researchers (e.g. Harvey, 1982; Adler, 2001) have argued that it is useful for teachers to encourage children to move from informal to formal language, I further suggest that teachers themselves may need to be aware of their own shift in this regard.

I also highlighted the fact that sometimes it may be another mathematical word that is used as a replacement rather than an everyday word. For example, the words *length* and *measurement* were interchanged by the teacher in Grade 3. While admittedly, these two words are very closely related and therefore the interchange may not have posed any problems in this class, the same teacher also interchanged *sharing* and *grouping*; in this case a distinction between the words was important since they denote different division structures.

12.6.4 The inter-relationship between frequency, clarity and significance

I noted that for many words, frequency, clarity and significance went together, so that for example, the word *graph* was used frequently, clearly *and* in a significant way. I noted that for a word used with all three conditions satisfied, all pupils recalled the word and gave an appropriate explanation, even if the word was ‘new’ to them. On the other hand, if a word lacked either one or the other of these conditions when used in class, then the word was not recalled by all the pupils, or not explained satisfactorily. Since lack of clarity or significance was generally accompanied by lack of frequency, it is difficult to say with certainty which condition had the most bearing on sharing of meaning. However, what I can suggest is that whatever language is used as a medium of instruction, it may be helpful for a teacher to consciously maximise each condition in order to aid the sharing of meaning.

12.7 Suggestions for further research

This study may act as a catalyst for further research and the following are some suggestions that may be worth pursuing:

- Similar studies can be carried out with different age groups and topics, in order to focus on the development of other mathematical ideas;
- The notions of frequency, clarity and significance as conditions for sharing meaning may be further explored;
- Chains of meaning may be studied within classrooms where code-switching is used;
- Teachers can be encouraged to reflect on their own practice with a particular emphasis on aspects of language;
- Talk in local mathematics classrooms can be studied more specifically in terms of a *discourse* wherein reflections could centre on social inclusion and participants' roles in the creation of mathematics.

12.8 The way forward

Having completed this study, I now consider the way forward to be three-pronged. First, my reflections can help me in my own practice as a mathematics primary-teacher educator. What I have learnt in terms of knowledge about mathematical language and also about the practice of research can now influence me in terms of what I focus on with my students. Increased discussion regarding mathematical language is important since, as Zaskis (2000) pointed out, trainee teachers may not feel confident with it, or may initially not view it as necessary and resist its use.

Furthermore, my observations have confirmed my belief that a primary teacher's knowledge of the subject is important. The classroom interaction I observed revealed some inadequacies in the teachers' knowledge. I suspected that the Grade 3 teacher was not sufficiently aware herself of the distinction between *sharing* and *grouping* as division structures, and for the topic 'Length', she inappropriately called both dimensions of a table the *width*. For the topic 'Graphs', the Grade

6 teacher used the expression *drop a perpendicular* in a restricted way, encouraged an inappropriate use of *scale* on the x-axis and used *plot* in the general sense of drawing/making a graph; for 'Length', she implied an incorrect definition of *regular* shapes and suggested an interchangeability of the names for the dimensions *length*/*width* of a rectangle.

Except perhaps for the Grade 3 teacher's use of *width* for the length of the desk, the other instances were too consistent for me to consider them 'slips' that sometimes occur as people talk, and that perhaps I might expect if a teacher is using a second language (although of course, these possibilities cannot be excluded). Since teachers are likely to share the knowledge that they have, it is therefore important that this knowledge is similar to that of the general mathematics community. The implication for me, as a primary teacher educator, is to strive to find ways to strengthen this knowledge.

A second path worth pursuing is creating a dialogue between mathematics educators and linguistics in order to reflect on the registers in use, whether English or Mixed Maltese English. A Council was set up in April 2005 whose aim is to focus on the strengthening of the Maltese language, including focusing on technical language for various disciplines and areas. This may prove to be a suitable channel for encouraging discussion on the Mixed Maltese English mathematics register. Whether a need will be felt to establish an academic mathematics register has yet to be seen. Of course, if code-switching is retained, my original interest in how Maltese helps in the sharing of meaning of English mathematical words remains an area worth researching.

Third, it is important that my reflections are shared with fellow mathematics educators, policy makers and classroom teachers in Malta. This would help me to achieve my goal of extending the language debate. Although over the recent years I have had the opportunity to discuss informally, four particular events have offered more structured fora for discussion. The first opportunity was prior to the start of this project, during the interim period between the draft National Minimum Curriculum and the publication of its final version. In October 1998, I organised a meeting for mathematics teachers (these were mostly secondary subject co-ordinators since they were the ones able to leave school during school hours), education officers, members of the NMC steering committee and University mathematics educators and linguistics. The meeting was small (approximately twenty people), but the group's feedback resulted in an important amendment in the NMC document: the immersion suggestion for mathematics, science and technical education was changed from a dogmatic directive in the draft document "**the NMC *obliges* the teachers in this [primary] sector** …" (Ministry of Education and National Culture, 1998, p. 21, translation and italics mine) to a recommendation phrased as "the NMC *encourages* teachers …" in the final publication (Ministry of Education, 1999, p.79, italics mine). I believe that the amendment was a first step to a more open consideration of the issue.

By September 2004 I had done a fair amount of work on my project, and I therefore outlined the tensions I had identified within the NMC in an international conference on Early Childhood that was held in Malta (Farrugia, 2004). My aim at this point was to problematise the NMC recommendation publicly. More recently (Farrugia, 2006), I addressed a conference attended by different people involved in the sphere of education as part of a local school's centenary anniversary celebrations. Since the audience was varied, and also included teachers who taught mathematics through English to *English* speakers, I spoke about mathematical vocabulary in general, focusing on the ideas of frequency and significance. This opportunity served as a first attempt to widen the language discussion beyond the Maltese/English debate.

An event that helped to bring the medium of instruction discussion to the fore, was a National Conference for Mathematics organised by the Ministry of Education in collaboration with the University of Malta in October 2004. One of the themes discussed in the various workshops was the language issue. I was involved in coordinating a workshop on the day and later formed part of the Action Group set up to compile the various workshops' notes and make recommendations for public dissemination. In the resulting document published a year later (Ministry of Education, Youth and Employment, 2005), the recommendation for language was written by the Group as follows:

> "The language used should be the one that helps children understand mathematical concepts. The use of English [for mathematics] should not be discussed in isolation for mathematics (…) but should be part of a whole school policy regarding bilingualism. It should be made clearer that when the NMC encourages teachers to teach mathematics in English, it is not making an imposition, but recommending that this policy needs to be implemented on a school and class level. (*ibid*, p.11)

The significance of this statement is that it paves the way for further discussion, and class or school-based reflections on the use of language.

One of the main aims of this study was for me to be in a better position to extend the local debate regarding the medium of instruction for mathematics, by bringing out issues related to immersion and to direct attention to mathematical language as a register. From my personal experience, I can say that ultimately all mathematics educators appear to have one common objective: that language be used in the best possible way in order to offer our children opportunities to understand and appreciate mathematics. I hope that the reflections I offer here may contribute towards finding this way.

APPENDIX A

Summary of two language-related studies carried out in Malta with respect to Science (cf. Section 3.8)

As is the case for mathematics, the teaching and learning of science in Malta involves the use of English written texts. Classroom interaction is likely to include code-switching between Maltese and English. The following two studies focused on the language aspect of science education in Malta.

Ventura, F. (1991). Language and the science curriculum. *Education (Malta)*, 4 (2), 15–18.

This study was carried out in 1984 with a group of about 300 First and Second Form students (ages 11-13) and their teachers. The teachers filled in a questionnaire regarding how they used language in class. From their responses, it resulted that they used more Maltese with the 'less able' pupils than with the 'more able' ones. The teachers predicted that all pupils would do better in a science test presented in Maltese rather than in English, as was the norm. However, they expected the 'less able' ones to gain more. A test consisting of 50 items was constructed in two versions – one in English and one in Maltese. In the latter, most scientific words were presented in a Maltese translation, although some were presented in English within inverted commas. (This detail is not stated in the paper, but was made available to me through a personal communication with the author). The pupils were given either one test or the other. Statistical analysis of the responses showed that there was a significant difference in favour of those who sat for the Maltese test (t=7.875, $p< .001$). The results suggested that as far as the 'weaker' students were concerned, those taking the English version were disadvantaged with respect to their peers taking the English version. The weak students obtained significantly higher marks in the Maltese test, although their performance was still very weak. In the case of the 'more able' students, those taking the Maltese version did not gain advantage over those taking the English version. Ventura concluded that there appeared to be a cut-off point in the effect of language on achievement and recommended that further studies be carried out to guide a decision whether a official language policy for science would be appropriate or whether policies should be school or class based.

*

Farrell, M.P. and Ventura, F. (1998). Words and understanding in Physics. *Language and Education*, *12* (4), 243-253.

The aim of this study was to assess the understanding of about 300 Sixth Form students' (average age 17) understanding of frequently used non-technical words (e.g. random, mutual etc.) and technical ones (e.g. monochromatic, dielectric etc.) in the Physics register. Furthermore, the authors wished to investigate any differences between perceived and actual comprehension. In the first part of the study, 75 words were presented to the students (50 non-technical, 25 technical), and the students were asked to state whether they knew their meanings, by indicating 'yes' or 'no'. In the second part, each word under consideration was included in a full sentence and the students were asked what it meant. They could answer in Maltese or draw if they so wished. Using a paired *t*-test, the authors found that there was a very low correlation between the claimed and actual results, with the mean of the actual results being notably smaller. Incorrect explanations were the result of various misinterpretations, for example, confusing the word's scientific meaning with its 'everyday' meaning, phonetic interference (e.g. taking *finite* to mean fine) and many other interesting reasons. Farrell and Ventura recommended the use of simple and familiar vocabulary by teachers, while admitting that it a counter-argued could be brought that it is better to help students *through* the language than *around* it. They concluded that ultimately, it seemed that even at this level of education, word understanding was a matter that needed attention.

A P P E N D I X   B

Examples of studies that utilise a theory of semiotics (cf. Section 4.5.1)

Various researchers have utilised a theory of semiotics in their research within mathematics education. The list below gives but an indication of such work. The studies are presented in chronological order, since I believe this reflects the ever-increasing scope of the theory's application to a wide range of interests.

Walkerdine	1988	Described how a mother led a child to counting the number of drinks to be poured for a group of persons at a social gathering: the persons were represented by names, names by fingers, and fingers by spoken numerals.
Cobb *et al*	1997	Described how a teacher and her pupils established relationships between candies, unifix cubes, pictures and verbal enumerations.
Steinbring	1997	Explored the development of children's arithmetical relationships through blackboard images and standard addition notation.
Presmeg	2001	Focused on developing mathematical ideas from cultural items, and reported the chains identified by a teacher who, using a mountain bicycle as a starting point, focused on gear ratios and then went on to develop a hyperbolic function.
Duval	2001	Analysed the use of diagrams in meaning-making.
Dörfler	2001	Also reflected on diagrams.
Radford	2001	Described a project in which students used algebraic signs and endowed them with meaning.
Carreira *et al*	2002	Discussed the role of metaphors in chains of signification that served to bridge mathematics and other conceptual domains such as geography and economics.
Sáenz-Ludlow	2003	Discussed how signs may include idiosyncratic elements such as personal and collective metaphors, informal diagrams and gestures.
Cerullli	2003	Explored the evolution of concepts that originated through an interplay between two 'worlds' namely mathematics within the software *L'Algebrista*, and mathematics outside it ('class' mathematics).
Schreiber	2005	Described how students generated alphanumerical and/or graphical notation to externalise their problem-solving ideas in an internet-chat context.

A P P E N D I X C

Letter to Parents – English version (cf. Section 5.4)*

Department of Primary Education
Faculty of Education
University of Malta

16 January 2003

Dear Parents,

I am researching the teaching and learning of some Mathematics topics in Year Three and Year Six. [HEAD OF SCHOOL] and your child's teacher have kindly agreed to allow me to observe and video-tape the Mathematics lessons for about two weeks.

After each topic, I would like to talk to a few children about the work they would have done during that week; I will plan a suitable time with the teacher. I would need to video-record the conversations, so that I can listen to them again later. <u>Please note that I will not be testing the children in any way and that the videos will not be watched by anyone else</u>.

If you prefer that your daughter does not take part, please fill in the form below and return it through your class teacher. In case of any difficulty, please contact me on [TELEPHONE] or send a message through your child for me to contact you. Thank you for your cooperation.

Sincerely,

Marie Therese Farrugia

- -

I understand that Mrs. M. T. Farrugia would like to record some of the children while she speaks to them about the Mathematics work covered during the week. However, I would prefer that my daughter ______________________________ does not take part in this activity.

Parent's / Guardian's signature : ______________________________

* English and Maltese versions were printed back-to-back

A P P E N D I X D

The participating pupils (cf. Section 5.5.2)

The pupils selected for participating in the interviews had been described as follows by their respective teachers:

Grade 3		
'High Achievers'	**'Average pupils'**	**'Low achievers'**
Maria Petra Sandra Sonia	Charlotte Jessica Kelly Kim Lara	Fiona Melissa Ramona

Grade 6			
'Very Good'	**'Good'**	**'Fair'**	**'Weak'**
Clare Claudette Joanne	Celia Dorianne Federica Rachel	Charmaine Monica Stefania	Josephine Katrina

Four topics were observed in all. I paired the pupils for the interviews as follows, being guided by their teacher's suggestions regarding different personalities:

Grade 3 Interview pairs	
Length	**Multiplication and Division**
(A) Sonia + Jessica (B) Petra + Charlotte (C) Kim + Fiona	(A) Maria + Sandra (B) Lara + Melissa (C) Kelly + Ramona

Grade 6 Interview pairs	
Length	**Graphs**
(A) Joanne + Federica (B) Celia + Katrina (C) Clare + Monica	(A) Claudette + Rachel (B) Dorianne + Stefania (C) Charmaine + Josephine

A P P E N D I X E

Mathematical words selected per topic (cf. Section 5.5.3)

Year 3 Multiplication & Division	**Year 6 Graphs**	**Year 3 Length**	**Year 6 Length**
Multiplication	Graph	Centimetre/s	Centimetre /s
Multiply by	Block graph	Metre /s	Metre /s
Times	Bar graph	Kilometre /s	Millimetre /s
Tables	(Straight) line graph	Measure/s/ed/ing	Kilometre /s
Division	Pie-graph/chart	Measurement/s	Measure /s/ed/ing
Divide by	x-/y-axis /axes	Estimate	Measurement /s
Grouping	Represent / ing	Long/er/est	Longer / er /est
Sharing	Plot /ing /ed	Short/er/est	Short / er /est
	Scale	Length/s	Length /s
	Drop a perpendicular	Width	Width
	Data	Height	Breadth
			Height
			Perimeter
			Spans(hand/arm/foot)
			Regular
			Irregular
			Metric

A P P E N D I X F

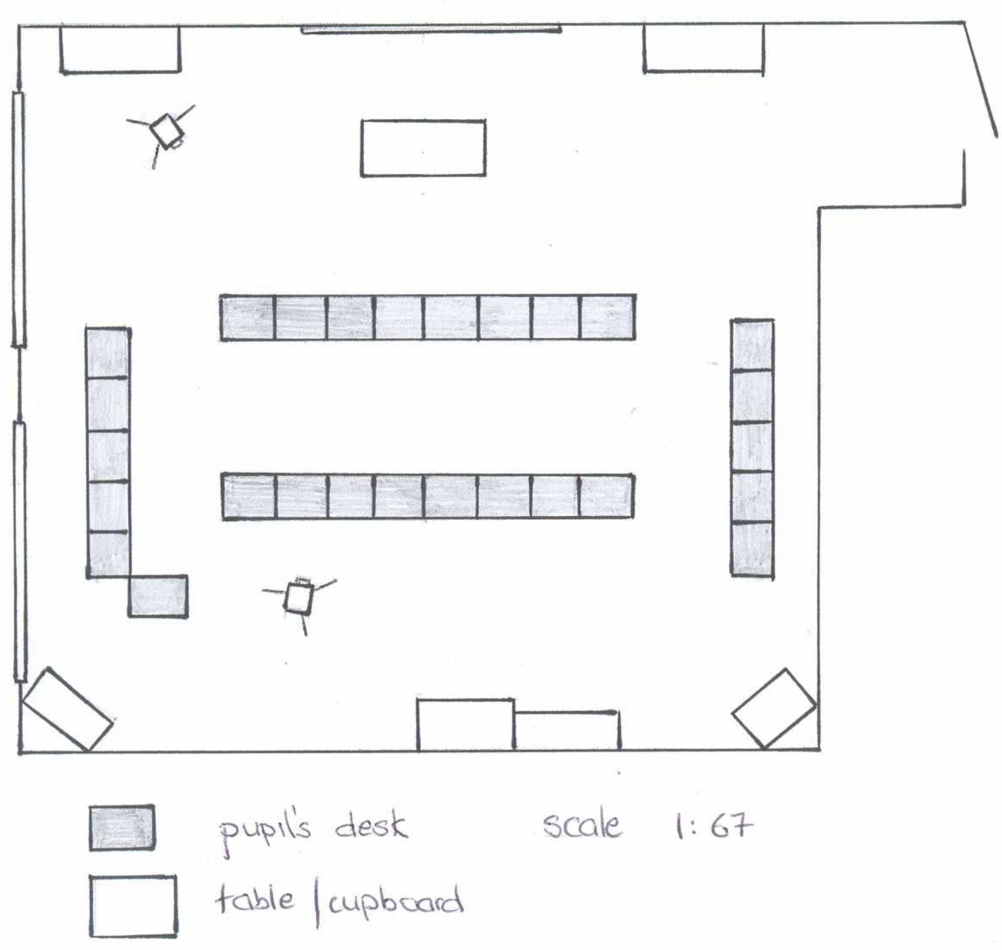
Time-frame for Data Collection 2002 - 2003 (cf. Section 5.6.1)

December 2002		
Wednesday 4	Meeting with Head of School	
Wednesday 11	Telephone contact with teachers Rose and Gina	
Wednesday 18	Meeting with teachers to explain research project; choice of topics finalised over the phone mid-January.	
	Christmas Holidays + waiting time	
January 2003		
Monday 20	Letter to Parents	
Tuesday 21	First trial lesson Years 3 & 6 (familiarisation & camera trial)	
Wednesday 22	--	
Thursday 23	Second trial lesson Years 3 & 6 (familiarisation & camera trial) *(Union Directive)*	
Friday 24	Trial Interview with three Year 3 pupils Teacher Interview (General discussion re use of language)	
Monday 27	--	
Tuesday 28	Trial Interview with pupils Year 3 and Year 6 pupils (pairs)	
Wednesday 29	Extra observation of Year 6 (familiarisation) Teacher Interview Length1 Year 6	
Thursday 30	--	
Friday 31	Extra observation of Year 3 (familiarisation) Teacher Interview Length1 Year 3	
	Revision and Exams	
	(GRADE 3 lessons & interviews)	**(GRADE 6 lessons & interviews)**
February 2003		
Monday 17	Length 1	
Tuesday 18	Length 2	
Wednesday 19	Length 3	
Thursday 20	Length 4	
Friday 21	Length 5	
Monday 24	Children Interview (A)	Length 1
Tuesday 25	Children Interviews (B) & (C)	Length 2
Wednesday 26		Length 3
Thursday 27	Teacher Interview Length (Part1)	Length 4
Friday 28	*School Activity*	
March 2003		
Monday 3	*MIDTERM*	
Tuesday 4		
Wednesday 5	Teacher Interview Length (Part 2) Multiplication & Division 1	Length 5
Thursday 6	Teacher Interview M&D1 Multiplication & Division 2	Length 6
Friday 7	*School Activity*	

Monday 10	Multiplication & Division 3	Length 7
Tuesday 11	Multiplication & Division 4	Length 8
Wednesday 12	Multiplication & Division 5	Children Interview (A), (B), (C)
Thursday 13		Teacher Interview Length2 (Part 1)
Friday 14		Teacher Interview Length2 (Part 2)
Monday 17	Children Interview (A) Children Interview (B)	Teacher Interview Graphs 1
Tuesday 18	Children Interview (C) Teacher Interview M&D2	
Wednesday 19	*Public Holiday*	
Thursday 20		
Friday 21	*School Activity*	
Monday 24		Graphs 1
Tuesday 25		Graphs 2
Wednesday 26		Graphs 3
Thursday 27		Graphs 4
Friday 28		Graphs 5
Monday 31	*Public Holiday*	
Tuesday 1 **(April)**		Children Interview (A) Children Interview (B) Children Interview (C) Teacher Interview Graphs2

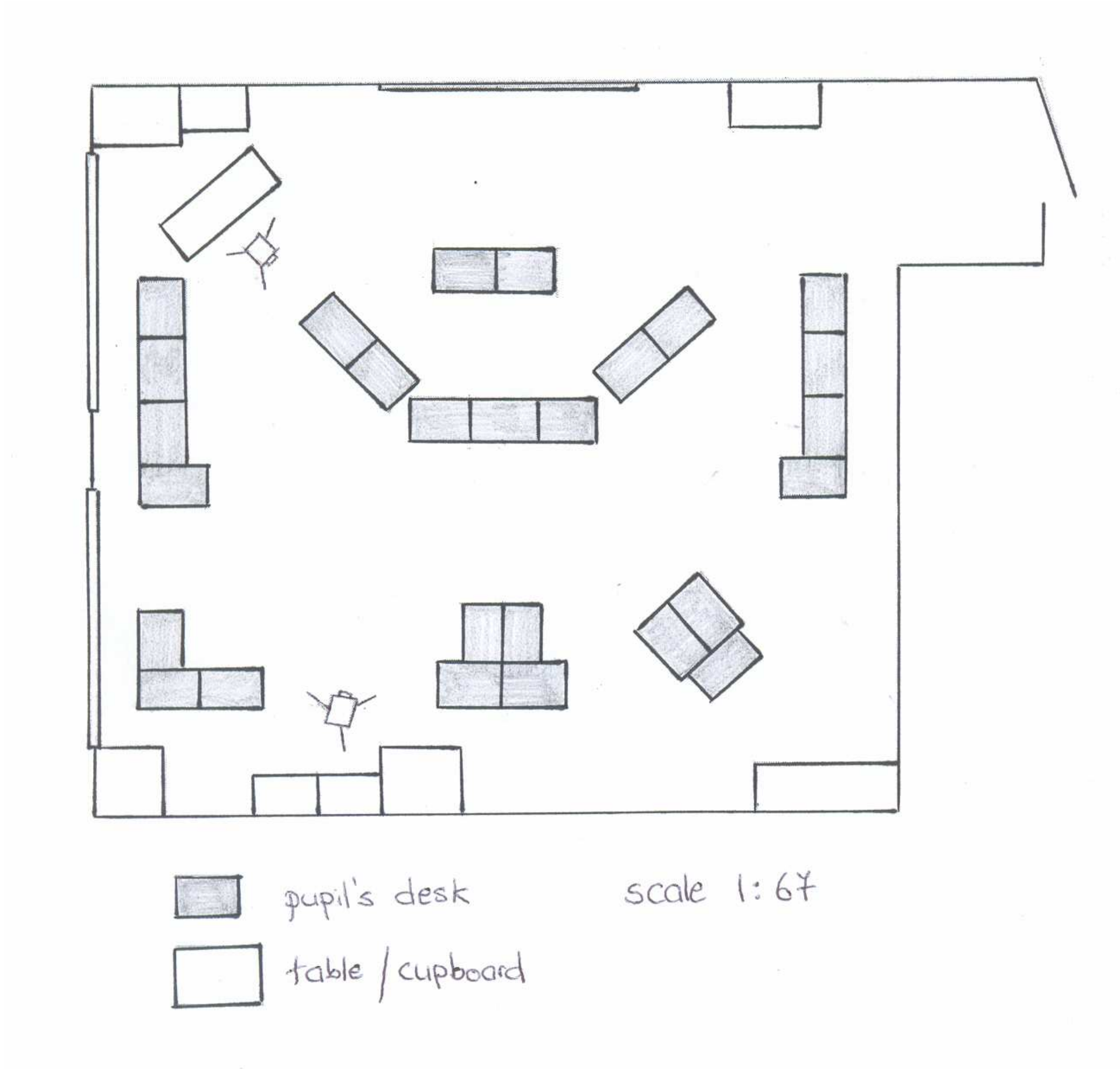
A P P E N D I X   G 1

Floor Plan Year 3 (cf. Section 5.6.2)



A P P E N D I X   G 2

Floor Plan Year 6 (cf. Section 5.6.2)



A P P E N D I X H 1

Teacher interview: general discussion on language (cf. Section 5.6.3)

General Language Use

1. One of my own interests in language was to observe how teachers might use Maltese to teach Maths. I noticed that you hardly used Maltese at all. Tell me about how you choose to use language.

2. How was the school policy for language for mathematics decided upon? Are there other subjects that you teach through English?

3. I noticed that many of the children are Maltese speaking.
    a. What are advantages and disadvantages of using English in class for these children?
    b. Does their knowledge of English limit you in the way you explain, or give examples?
    c. Do you think that using English affects children's participation?
    d. Do you think that using English might limit their explanations (e.g. length or complexity).
    e. What would be advantages and disadvantages of using *Maltese* throughout?

Mathematical Vocabulary

1. My main objective is to focus on the teaching of mathematical words.
    a. What would you consider to be mathematical vocabulary? Give some examples.
    b. Would you normally refer to words as 'mathematical', or do you refer to them in some other way to yourselves?
    c. Do you indicate such words in any way to your pupils?

2. Do you ever find yourself translating specific words into Maltese?

A P P E N D I X   H 2

Teacher interview Year 3 Length 1 (cf. Section 5.6.3)

1. What would you say are the main teaching points for this topic?
2. What would you consider to be the 'key' mathematical vocabulary for this topic?
3. I've also noted these words: what do you think? (see list below)
4. Do you think that the girls are already familiar with the words you've mentioned?
5. On the other hand, perhaps there are 'new words': Which ones might these be?
6. Are some of the meanings of the words of this topic harder to teach than others?
7. Are there instances when perhaps Maltese may come in handy during the teaching of this topic?

(presented on separate sheet)

Vocabulary (Length : taken from textbook)
Length
Metre
Centimetre
Kilometre
The nearest
Estimate
Measure

A P P E N D I X H 3

Teacher interview Year 3 Length 2 (cf. Section 5.6.3)

1. I identified these topic related words this week: which ones do you think were new and which ones familiar? (*see list*)

2. What meaning do you think you shared with the children regarding the following:
   - metre - centimetre - kilometre
   - length - width - height
   - Measure - measurement
   - Estimate
   - Measure

3. Tell me about these instances from your lessons:
   a. Using your finger in relation to the centimetre.
   b. When you asked them to measure a table etc. for home, you left them free to choose any part of the item that they preferred.
   c. You asked children to open their arms wide, and even said you're like a scarecrow at one time! Why did you do this?
   d. I noticed that for the idea of estimation you talked about 'roughly', 'nearest' etc. in relation to their measuring activity (desk etc) and then introduced the *word itself* in the third lesson "*Now when you estimate it means that we are going to say ABOUT how much*". On the other hand, for 'metre' and 'centimetre' you started the first lesson with the words "*When we talk about length, measurement, we talk about metres and centimetres. Now if we want to know how long something is or how wide an object is, we have to measure with a measuring tape or a ruler. Then have a look at the measuring tape and we will look at the centimetres*" (You then wrote metres and centimetres on the board etc). Tell me about the difference in approach.
   e. You had said in our first discussion that the word 'estimation' was a "scary one". After the week of lessons, tell me more about that.

4. When talking to the children, I noticed that they did not recall the words 'width', 'height' and 'estimate'; one of them remembered 'height'.
   a. Does this surprise you or not?
   b. Is it something to be concerned about at all?

Contd.

5. I noticed that two children drew on their experiences when I pressed them about width and height. Two girls thought I meant ‘with’, one girl thought of ‘height’ as the Maltese ‘hajt’(cotton thread) while another thought of Maltese ‘hajt’ in the sense of wall. Do you think that the fact that other words sound the same could be a problem?

6. Many of the words relevant to this topic have Maltese equivalents. I noticed you didn’t use them. Tell me about this.

7. I noticed that children talked to each other in Maltese, both socially and about Maths (in groups). What is your comment about this considering your opinion about the importance of doing Maths in English? Can you clarify what you feel should be the English aspects of the lesson therefore?

8. I’m wondering if you could actually teach the written form explicitly say, by translation while conducting lessons in mixed Maltese and English. What do you think about this? (Did you ever do something like this or have you always taught in English as you do now?)

Contd.

metre

centimetre

kilometre

length

width

height

longer

shorter

estimate

measure

measurement

A P P E N D I X H 4

Teacher interview Year 3 Multiplication & Division 1(cf. Section 5.6.3)

1. What would you say are the main teaching points for this topic?
2. What would you consider to be the ‘key’ mathematical vocabulary for this topic?
3. Do you think that the girls are already familiar with the words you’ve mentioned?
4. On the other hand, perhaps there are ‘new words’:Which ones might these be?
5. Are some of the meanings of the words of this topic harder to teach than others?
6. Are there instances when perhaps Maltese may come in handy during the teaching of this topic?

A P P E N D I X H 5

Teacher interview Year 3 Multiplication & Division 2 (refer section 5.6.3)

1. I identified these topic related words this week: which ones do you think were new and which ones familiar? (*see list below*)

2. What meaning do you think you shared with the children regarding these words:
   a. Multiplication – multiply by
   b. Times
   c. Division – divide by
   d. Sharing - grouping
   e. tables

3. You said the following about multiplication. Tell me about them.
   a. “multiplication is instead adding”
   b. “multiplication is repeated addition”
   c. “multiplication is repeating a number/ repetition of the same number”
   d. “Multiplication is instead of counting”
   e. “for multiplication the number becomes larger/bigger”
   f. “multiplication is ‘the easy way out’”

4. You said the following about division. Tell me about them.
   a. “division is repeated subtraction”
   b. “division is the opposite of multiplication”
   c. “when we divide, the number becomes smaller/less”
   d. “when we are grouping, we are dividing”

5. Let’s take ‘3 x 4’.
   a. What would you wish to share with the children regarding ‘3 x 4’?
   b. At one time you told the children ‘we must make a difference – we must know what the numbers stand for’. Tell me about this.

6. At one time with reference to a set of stamps (array in book) you said “it’s the same, they’re going to give the same answer”. What did you mean by this?

Contd.

7. You had done some similar work in December.
    a. Tell me about what you had done.
    b. The significance of the written notes (4th December)
    c. I noticed that the girls already knew the ‘tables’ of 3, 5, 10, which you used for this week’s groupings. Tell me about the order in which you plan the work.

8. We could use Maltese equivalents (timmultiplika, tiddividi, taqsam…). Do you think this would be useful?

9. In general I felt that you used the Maths words more than the girls.
    a. What do you think?
    b. Is it useful for the girls to use the words (e.g. multiply, share etc)

10. Now that the process is ended, tell me how you feel about having me in the class.

(presented on a separate sheet)

multiplication

multiply by

times

division

divide by

sharing

grouping

tables

APPENDIX H6

Teacher interview Year 6 Length 1 (cf. Section 5.6.3)

1. What would you say are the main teaching points for this topic?
2. What would you consider to be the ‘key’ mathematical vocabulary for this topic?
3. I’ve also noted these words: what do you think? (see list)
4. Do you think that the girls are already familiar with the words you’ve mentioned?
5. On the other hand, perhaps there are ‘new words’: Which ones might these be?
6. Are some of the meanings of the words of this topic harder to teach than others?
7. Are there instances when perhaps Maltese may come in handy during the teaching of this topic?

(presented on a separate sheet)

Vocabulary (Length and Perimeter)
Length
width
perimeter
metre
centimetre
millimetre
kilometre
Unit
To the nearest
Distance
Trundle wheel
Measuring/inch tape
Approximate
Estimate
Measure
Convert

APPENDIX H7

Teacher interview Year 6 Length 2 (cf. Section 5.6.3)

1. I identified these topic related words this week: which ones do you think were new and which ones familiar? (*see list below*)

2. What meaning did you wish to share with the children regarding the following:
    a. metre - centimetre – millimeter - kilometre
    b. length - width - height – perimeter
    c. Measure - measurement
    d. estimate
    e. regular / irregular
    f. metric

3. I noticed that the girls in general did not recall the word 'metric'. Would this worry you?

4. I think that in general, you used mathematical words more than the girls. How important is it for the girls to use the mathematical words themselves?

5. Tell me about these instances from your lessons:
    a. I noticed you used a lot of 'why' and 'what do you think' type of questions. Tell me about this approach to teaching Maths.
    b. I noticed that in the first lesson, you introduced a lot of vocabulary and said "today we discussed the words, tomorrow we will measure". Tell me about this approach.
    ~~c.~~ You mentioned that 'length' and 'width' are names that are given like 'Jessica' is a name for a girl. Tell me about this. At one time you told someone that the words length and width are interchangeable. Tell me about this.

6. I noticed that it was during the story sums lesson that you used some Maltese, in particular to explain to a few of the pupils. Tell me about this.

7. Many of the words relevant to this topic have Maltese equivalents. I noticed you didn't use them. Tell me about this.

8. I noticed that children talked to each other in Maltese, both socially and about Maths (in groups). What is your comment about this considering your opinion about the importance of doing Maths in English? Can you clarify what you feel should be the English aspects of the lesson therefore?

Contd.

9. Girls seem to manage OK with English. I'm wondering whether it's because by this age, children have had much more exposure to the language, not only from Maths but from other experiences. I'm wondering about how an approach similar to yours could be used with Year 1 or Year 2 children.

10. I'm wondering if you could actually teach the written form explicitly say, by translation while conducting lessons in mixed Maltese and English. What do you think about this? (Did you ever do something like this or have you always taught in English as you do now?)

Contd.

metre

millimetre

centimetre

kilometre

hand span, arm span etc.

metric

length

width

breadth

height

perimeter

long / longer

short / shorter

measure

measurement

regular

irregular

APPENDIX H8

Teacher interview Year 6 Graphs 1 (cf. Section 5.6.3)

1. What would you say are the main teaching points for this topic?
2. What would you consider to be the ‘key’ mathematical vocabulary for this topic?
3. Do you think that the girls are already familiar with the words you’ve mentioned?
4. On the other hand, perhaps there are ‘new words’: Which ones might these be?
5. Are some of the meanings of the words of this topic harder to teach than others?
6. Are there instances when perhaps Maltese may come in handy during the teaching of this topic?

A P P E N D I X H 9

Teacher interview Year 6 Graphs 2 (cf. Section 5.6.3)

1. I identified these topic related words this week: which ones do you think were new and which ones familiar? (*see list below*)

2. What meaning did you wish to convey regarding the words?

3. These are all called ‘graphs’. What is the same and what is different about block/bar/line graphs/pie-charts.

4. You chose to use copybooks with 2mm markings. Tell me about this.

5. Sometimes the girls had difficulties in working out the questions. What is it that they find difficult?

6. Tell me about these instances from the lessons:
    a. What did you mean when you said “a graph is to see things more clearly / more easily”?
    b. What did you mean when you said “a graph is a picture way of doing our sums”?
    c. You used the expression ‘drop a perpendicular’ while the girls did not use it. What do you think about this?
    d. You asked the girls what the pie-chart reminded them and mentioned yourself: cake, pie, purse. The girls mentioned ‘prickly pear’, circle, alarm clock, my glasses. What was the significance of this discussion?
    e. I noticed that some girls (Daniela, Clare, Monica, Joan) coloured cm widths for the line graph. What do you think of this?

7. I felt that that you used more Maltese in this topic than for Length. Are you aware of this? Why do you think this happened?

8. What is your opinion about sentences like this that were used by the girls:
    a. “I did them plus”
    b. “the y-axis, I did them 4cm”
    c. “Each piece three hours, mela six hours. School it came six hours”.

9. Now that the process is ended, tell me how you felt about having me in the class.

Contd.

graph

block graph

bar graph

line graph

pie-graph (chart)

x-axis

y-axis

data

plot

represent

scale

drop a perpendicular

APPENDIX I1

Pupil interview Year 3 Length (cf. Section 5.6.4)

1. There's a new friend called Jessica going to join your class. Tell you new friend about what you learnt this week that's important for her to know. She doesn't understand English so we'll have to explain everything in MALTESE. (Let girls talk freely, then mention words that do not crop up in conversation, see list)

2. Now another friend comes along, Rosie, this time she only understands English! Let's explain to her again in ENGLISH.

3. Tell me about the homework when the teacher asked you to measure things at home.

4. Nadia mentioned a way to remember half a metre, then teacher used it and also a way to remember the ¼ metre? Do you recall this? Did it help?

5. Which of these words (mention from list) were new to you this year?

6. Are these words (mention from list) in Maltese or English? Do you know Maltese words for these?

7. What do you think about the fact that the teacher uses English in class for Maths? How do you prefer to speak to teacher, to classmates?

Contd.

metre

centimetre

kilometre

length

width

height

longer

shorter

estimate

measure

measurement

A P P E N D I X I 2

Pupil interview Year 3 Multiplication & Division (cf. Section 5.6.4)

1. There's a new friend called Jessica going to join your class. Tell your new friend about what you learnt this week that's important for her to know. She doesn't understand English so we'll have to explain everything in MALTESE. (Let girls talk freely, then mention words that do not crop up in conversation, see list)

2. Now another friend comes along, Rosie, this time she only understands English! Let's explain to her again in ENGLISH.

3. Which of these words (list) were new to you this year?

4. Are these words (mention from list) in Maltese or English? Do you know Maltese words for these?

5. Here are some things that the teacher said about multiplication this week. I wonder what she meant?
    a. "multiplication is instead of adding"
    b. "multiplication is repeated addition"
    c. "multiplication is instead of counting"
    d. "for multiplication the number becomes larger/bigger"
    e. "multiplication is 'the easy way out'"

6. Here are some things that the teacher said about division this week. I wonder what she meant?
    a. "division is repeated subtraction"
    b. "division is the opposite of multiplication"
    c. "when we divide, the number becomes smaller/less"
    d. "when we are grouping, sharing we are dividing"

7. Let's look at your 4th December notes [*the list of key words for multiplication & division*]. Tell me about them.

Contd.

8. Look at this card: [flashcard 6 x 5]
9. Tell me about this is all about.
10. Are 6 x 5 and 5 x 6 the same?
11. (*Refer to stamps diagram [a rectangular array]*). How can you find the number of stamps? At one time with reference to the set of stamps the teacher said “it’s the same, they’re going to give the same answer”. What did she mean by this?

12. Tell me about this card [12 ÷ 3] (children to read and explain). Is it same as 3 ÷ 12?

13. What do you think about the fact that the teacher uses English in class for Maths? How do you prefer to speak to teacher, to classmates?

(words presented on a separate sheet)

multiplication

multiply by

times

division

divide by

sharing

grouping

tables

A P P E N D I X I 3

Pupil interview Year 6 Length (cf. Section 5.6.4)

1. There's a new friend called Jessica going to join your class. Tell you new friend about what you learnt this week that's important for her to know. She doesn't understand English so we'll have to explain everything in MALTESE. (Let girls talk freely, then mention words that do not crop up in conversation, see list) *[Check out interchangeability of length and width]*.

2. Now another friend comes along, Rosie, this time she only understands English. Let's explain to her again in ENGLISH. (*Go through list systematically*).

3. Which of these words (mention from list) were new to you this year?

4. Are these words (*list*) in Maltese or English? Do you know Maltese words for these?

5. What do you think about the fact that the teacher uses English in class for Maths? What would you think of lessons completely in Maltese? How do you yourself prefer to speak to teacher, to classmates?

Contd.

metre

millimetre

centimetre

kilometre

hand span, arm span etc.

metric

length

width

breadth

height

perimeter

long / longer

short / shorter

measure

measurement

regular

irregular

A P P E N D I X I 4

Pupil interview Year 6 Graphs (refer Section 5.6.4)

1. There's a new friend called Jessica going to join your class. Tell your new friend about what you learnt this week that's important for her to know. She doesn't understand Maltese so we'll have to explain everything in MALTESE. (Let girls talk freely – remember *Focus on cm representation, axes etc. How did you decide on how big to draw the graph?* then mention words that do not crop up in conversation, see list).

2. Now another friend comes along, Jessica, this time she doesn't understand English. Let's explain to her again in ENGLISH.

3. Which ideas / words were new to you this year?

4. Why do you think the graphs are given these names?

5. At one time the teacher said "a graph is a picture way to work out our sums" and "a graph helps us to see things more clearly"? What did she mean by this?

6. The teacher used the expression 'drop a perpendicular' a few times. What did she mean by this?

7. You coloured cm widths on your line graphs. Tell me about this.

8. At one time the teacher asked the class to think about what the pie-chart reminds you of. Did you suggest anything? Why do you think she asked you to do this?

9. What do you think about the fact that the teacher uses English in class for Maths? What would you think of lessons completely in Maltese? How do you yourself prefer to speak to teacher, to classmates?

Contd.

graph

block graph

bar graph

line graph

pie-graph (chart)

x-axis

y-axis

data

plot

represent

scale

drop a perpendicular

A P P E N D I X J

'Kangaroo' Transcription, Multiplication and Division Lesson 5 (cf. Section 9.5.2)

(*There is a number line drawn on the whiteboard. The pupil Angela has marked arcs representing kangaroo hops: from 0 to 3, 3 to 6 and 6 to 9 to illustrate the textbook question that states that the kangaroo has landed on 9*).

Teacher:	In order to reach the number nine, the kangaroo has to do THREE hops. What are we doing to the numbers here? Sonia?
Sonia:	Counting in threes.
Teacher:	Counting in threes. Very good. Can I have another em, answer (…) What are we doing?
Pupil 1:	Grouping in threes.
Teacher:	**Good girl**! We are grouping in threes. Very good. We are grouping in threes (…). And instead of grouping, what can we say as well?
Katia:	Adding.
Teacher:	[For] adding we are multiplying really. When we are grouping in threes, what can we say?
Melissa:	Jumping in threes.
Pupil 1:	Times.
Teacher:	No, it's not times. We are GROUPING in threes. Sharon?
Sharon:	Dividing.
Teacher:	We are DIVIDING. When we are grouping, dividing (…) sharing, they all mean division. So, when we are grouping in threes, we are dividing. Now if your number is too big, we cannot go on writing all the numbers here (*touches number line*) and doing all these arches (*touches the arches that Angela had drawn on the whiteoard*) and trying to group them, all right? What can we do instead?
Pupils:	(*Some hands go up*)
Teacher:	Jessica?
Jessica:	Times.
Teacher:	No! Sharon just said [it]. What can I do? (*Looking at Sharon*)
Sharon:	Division.
Teacher:	Division! Now can you work out a division for me from this?
Pupils:	(*Some hands go up*)
Pupil 1:	Three division by three.
Teacher:	Three division by three is going to give me one. The kangaroo jumped until it reached nine.
Pupils:	Miss! Miss!
Teacher:	Petra?
Petra:	Six division by three.
Teacher:	(*Sounds irritated*). Oh, where are you getting the six from?!
Pupil 2:	Nine division by three.
Teacher:	(*Teacher does not appear to hear this response*).
Pupils:	Miss! Miss!
Teacher:	**Let's see** … Kim.
Kim:	Three division by nine.
Teacher:	We can't divide three by nine. Turn the numbers.
Kim:	Nine division by three.
Teacher:	Nine division by three. (…) one hop is three numbers OK? (…) I have to take the number and divide it by three, because the kangaroo jumps in threes.

[G3M&D5minute46]

R E F E R E N C E S

Adetula, L. O. (1990). Language factor: does it affect children's performance on word problems? *Educational Studies in Mathematics, 21*, 351-365.

Adler, J. (1998). Lights and limits: recontextualising Lave and Wenger to theorise knowledge of teaching and learning to school mathematics. In A. Watson (Ed.), *Situated Cognition and the Learning of Mathematics*. Oxford: Centre for Mathematics Education Research.

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dortrecht: Kluwer Academic Publisher.

Aiken, L. R. (1972). Language factors in learning mathematics. *Review of Educational Research, 42*(3), 359-385.

Allen, R. E. (Ed.). (1990). *The Concise Oxford Dictionary*. Oxford: Clarendon Press.

Anderson, L. W., & Burns, R. B. (1989). *Research in Classrooms:The Study of Teachers, Teaching and Instruction*. Oxford: Pergamon Press.

Anghileri, J. (1991). The language of multiplication and division. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education*. Milton Keynes: Open University Press.

Anghileri, J. (1995). Making sense of symbols. In J. Anghileri (Ed.), *Children's Mathematical Thinking in the Primary Years*. London: Cassell.

Anghileri, J. (2000). *Teaching Number Sense*. London: Continuum.

Anghileri, J., & Johnson, D. C. (1988). Arithmetic operations on whole numbers: multiplication and division. In T. R. Post (Ed.), *Teaching Mathematics in Grades 4 - 8: Research Based Methods*. Mass.: Allyn & Bacon.

Appleby, J. (2003). Solving word problems in the primary classroom. *Primary Mathematics, 7*(1), 8-11.

Aquilina, J. (1999). *English-Maltese Dictionary*. Malta: Midsea Books Ltd.

Arthur , J. (1996). Code switching and collusion: classroom interaction in Botswana primary schools. *Linguistics and Education, 8*(17-33).

Askew, M. (1998). *Teaching Primary Mathematics*. London: Hodder and Stoughton.

Austin, J. L., & Howson, A. G. (1979). Language and mathematical education. *Educational Studies in Mathematics, 10*, 161-197.

Azzopardi, C. (2003). *Gwida Għall-Ortografija: Għajnujiet għall-kitba bil-Malti [A Guide to Orthography: An Aid to Maltese Writing]*. Malta: Klabb Kotba Maltin.

Baker, C. (2001). *Foundations of Bilingual Education and Bilingualism* (3rd Edition). Clevedon: Multilingual Matters.

Banks, F., Leach, J., & Moon, B. (1999). New understandings of teachers' pedagogic knowledge. In J. Leach & B. Moon (Eds.), *Learners and Pedagogy*. London: Paul Chapman in association with The Open University.

Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga reo tātai: Issues in the development of a Māori mathematics register. *For the Learning of Mathematics, 18*(1), 3-9.

Barwell, R. (2003a). Patterns of attention in the interaction of a primary school mathematics student with English as an additional language. *Educational Studies in Mathematics, 53*, 35-59.

Barwell, R. (2003b). Linguistic discrimination: an issue for research in mathematics education. *For the Learning of Mathematics, 23*(2), 37-43.

Bassey, M. (1995). *Creating Education Through Research*. Newark, U.K.: Kirklington Moor Press in conjunction with the British Educational Research Association.

Bassey, M. (1999). *Case Study Research in Educational Settings*. Milton Keynes: Open University Press.

Bauersfeld, H. (1998). Interaction, construction and knowledge: alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney & D. Jones (Eds.), *Perspectives on Research on Effective Mathematics Teaching, Volume 1*. Virginia: National Council of Teachers of Mathematics.

Blinko, J., & Slater, A. (1996). *Teaching Measures*. London: Hodder and Arnold.

Blouet, B. (1981). *The Story of Malta*. Malta: Progress Press.

Borba, M. C. (2005). Humans-with-media: transforming communication. In A. Chronaki & I. M. Christiansen (Eds.), *Challenging Persectives on Mathematics Classroom Communication*. Greenwich, Connecticut: Information Age Publishing.

Borg, A. (1980). Language and Socialisation in Developing Malta. *Work in Progress, Department of Linguistics, University of Edinburgh, 13.*

Bradford, K., & Brown, T. (2005). Ceci n'est pas un “circle”. *For the Learning of Mathematics, 25*(1), 16-19.

Brincat, J. M. (2006). *Il-Malti: Elf Sena ta' Storja [The Maltese Language: One Thousand Years of History] (2nd Edition)*. Malta: Pubblikazzjonijiet Independenza (PIN).

Brissenden, T. (1988). *Talking About Mathematics: Mathematical Discussion in Primary Classrooms.* Oxford: Blackwell.

British Educational Research Association (2004). *Revised Ethical Guidelines for Educational Research*. Southwell: B.E.R.A.

Brodie, K. (1989). Learning mathematics in a second language. *Educational Review, 41*(1), 39-53.

Bruner, J. (1986). *Actual Minds, Possible Worlds*. Cambridge, Mass.: Harvard University Press.

Bunyi, G. (1997). Multilingualism and discourse in primary school mathematics in Kenya. *Language, Culture and Curriculum, 10*(1), 52-65.

Burgess, R. G. (1989). Grey areas: ethical dilemmas in educational ethnography. In R. G. Burgess (Ed.), *The Ethics of Educational Research*. London: Falmer Press.

Burton, L. (2002). Methodology and methods in mathematics education research: where is ‘the why’? In S. Goodchild & L. English (Eds.), *Researching Mathematics Classrooms: A Critical Examination of Methodology*. London: Praeger.

Burton, L., & Morgan, C. (2000). Mathematicians writing. *Journal for Research in Mathematics Education, 31*(4), 429-453.

Cameron, D., Frazer, E., Harvey, P., Rampton, M. B. H., & Richardson, K. (1992). *Researching Language: Issues of Power and Method*. London: Routledge.

Camilleri, A. (1995). *Bilingualism in Education: The Maltese Experience*. Heidelberg: Groos.

Camilleri, G. (2004). Negative transfer in Maltese students' writing in Engish. *Journal of Maltese Education Research, 2*(1), 3-12.

Camilleri-Grima, A. (2003). “Do as I say, not as I do”. Legitimate language in bilingual Malta! In L. Huss, A. Camilleri-Grima & K. A. King (Eds.), *Transcending Monolingualism. Linguistic Revitalization in Education.* Lisse, The Netherlands: Swets and Zeitlingee.

Campbell, D. R. (1986). Developing mathematical literacy in a bilingual classroom. In J. Cook-Gumperz (Ed.), *The Social Construction of Literacy.* Cambridge: Cambridge University Press.

Capps, L. R., & Pickreign, J. (1993). Language connections in mathematics: A critical part of mathematics instruction. *Arithmetic Teacher, 41*(September), 8-12.

Carreira, S., Evans, J., Lerman, S., & Morgan, C. (2002). Mathematical thinking: studying the notion of ‘transfer’. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education, East Anglia, U.K.,21-26 July, 2002.Volume 2.* Norwich: University of East Anglia.

Cerulli, M. (2003). *Instruments of semiotic mediation in algebra, an example.* Paper presented at the Third Conference of the European Society for Research in Mathematics Education

(CERME3), Bellaria, Italy, 28 February - 3 March 2003. http://www.dm/unipi.it~didattica/CERME3/proceedings/Groups/TG8. Downloaded 21/09/04.

Chapman, A. P. (2003). *Language Practices in School Mathematics: A Social Semiotic Approach*. Lewiston: Mellen Studies in Education.

Charmaz, K. (2000). Grounded theory: objectivist and constructivist methods. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research*. London: Sage Publications.

Clarkson, P. C. (1991). Mathematics in a multilingual society. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education*. Milton Keynes: Open University Press.

Clemson, D., & Clemson, W. (1994). *Mathematics in the Early Years*. London: Routledge.

Cobb, P., & Bauersfeld, H. (1995). Introduction: the coordination of psychological and sociological perspectives in mathematics education. In P. Cobb & H. Bauersfeld (Eds.), *The Emergence of Mathematical Meaning*. New Jersey: Lawrence Erlbaum Associates.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: the emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. A. Whitson, A. (Eds.), *Situated Cognition: Social, Semiotic, and Psychological Perspectives*. Mahwah, N.J.: Lawrence Erlbaum Associates Publishers.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research Methods in Education*. London: RoutledgeFalmer.

Cooper, P. W. (1993). Field relations and the problem of authenticity in researching participants' perceptions of teaching and learning in classrooms. *British Educational Research Journal, 19*(4), 323-338.

Creese, A. (2004). Bilingual teachers in mainstream secondary school classrooms: using Turkish for curriculum learning. *Bilingual Education and Bilingualism, 7*(2 & 3), 189-203.

Cuevas, G. J. (1991). Developing communication skills in mathematics for students with limited English proficiency. *The Mathematics Teacher, 84*(3), 186-189.

Denzin, N. K., & Lincoln, Y. S. (2000). The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research (Second Edition)*. London: Sage Publications.

Department of Information (2004). *The Maltese Language Act (Parliamentary Act passed 14th July 2004)*. Malta: Department of Information.

DES. (1982). *Mathematics Counts.* London: HMSO.

Dey, I. (2004). Grounded theory. In C. Seale, G. Gobo, J. F. Gubrium & D. Silverman (Eds.), *Qualitative Research Practice*. London: Sage.

Department for Education and Skills. (2002). *Access and Engagement in Mathematics - EAL.* Http//www.standards.dfes.gov.uk/keystage3/respub/ma_eal. Downloaded 15 December 2004.

Dörfler, W. (2001). *Instances of diagrammatic reasoning.* Paper presented to the Discussion Group on Semiotics in Mathematics Education at the 25th PME international conference, The Netherlands, University of Utrecht, 12 – 17 July, 2001. Http://www.math.uncc.edu/~sae/dg3/dg3.html. Downloaded 2 August 2002.

Douek, N. (2005). *The role of language in the relation between theorisation and the experience of activity*. Paper presented at the Fourth Conference of the European Society for Researach Education, Sant Feliu de Guíxols, Spain, 17 - 21 February 2005. http://cerme.crm.es//papers. Downloaded 03/02/05.

Dunne, E., & Bennett, N. (1990). *Talking and Learning in Groups*. London: Routledge.

Durkin, K., & Shire, B. (1991). Lexical ambiguity in mathematical contexts. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education*. Milton Keynes: Open University Press.

Duval, R. (2001). *The cognitive analysis of problems of comprehension in the learning of* *mathematics*. Paper presented to the Discussion Group on Semiotics in Mathematics Education at the 25th PME International Conference, The Netherlands, University of Utrecht, 12-17 July, 2001. Http://www.math.uncc.edu/~sae/dg3/dg3.html. Downloaded 2 August 2002.

Earp, N. W., & Tanner, F. W. (1980). Mathematics and language. *The Arithmetic Teacher, 28*(December), 32-34.

Eco, U. (1976). *A Theory of Semiotics*. London: Macmillan.

Eder, D., & Fingerson, L. (2002). Interviewing children and adolescents. In J. F. Gubrium & J. A. Holstein (Eds.), *Handbook of Interview Research: Context and Method*. London: Sage.

English, M. (1981). Talking: does it help? In C. Sutton (Ed.), *Communicating in the Classroom*. London: Hodder and Stoughton.

European, C. (2004a). *Many Tongues, One Family*. Luxembourg: Office for Official Publications of the European Communities.

European, C. (2004b). *Promoting Language Learning and Linguistic Diversity*. Luxembourg: Office for Official Publication of the European Communities.

Evans, J. (1999). Building bridges: reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics, 39*, 23-44.

Farrell, M. P., & Ventura, F. (1998). Words and understanding in Physics. *Language and Education, 12*(4), 243-253.

Farrugia, M. T. (1995). *Ten-Year-Old Children's Understanding of Fractions.* Unpublished Masters Dissertation, University of Malta, Malta.

Farrugia, M. T. (2004). *Learning mathematics through English: tensions arising in the implementation of the language policy in Malta*. Paper presented at the 14th annual conference of the European Early Childhood Education Research Association (EECERA), University of Malta, 1 - 4 September 2004.

Farrugia, M. T. (2006). *Focusing on Mathematical Language in Primary Classsrooms.* Unpublished paper presented at the 100th Anniversary conference of Chiswick House School, Malta, 14-15th March, 2006.

Fontana, A., & Frey, J., H. (1994). Interviewing. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research*. London: Sage.

Frendo, H. (1975). Language and nationality in an island colony: Malta. *Canadian Review of Studies in Nationalism, 3*, 22-33.

Frobisher, L., Monaghan, J., Orton, A., Orton, J., Roper, T., & Threlfall, J. (1999). *Learning to Teach Number*. Glos., England: Stanley Thornes.

Furlong, V. J., & Edwards, A. D. (1993). Language in classroom interaction: theory and data. In M. Hammersley (Ed.), *Controversies in Classroom Research (2nd Edition)*. Buckingham: Open University Press.

Garbe, D. G. (1985). Mathematics vocabulary and the culturally different student. *Arithmetic Teacher, 33*(2), 39-42.

Gee, J. P. (1997). Thinking, learning and reading: the situated sociocultural mind. In D. Kirshner & J. A. Whitson, A. (Eds.), *Situated Cognition: Social, Semiotic and Psychological Perspectives*. Mahwah, N.J.: Lawrence Erlbaum Associates, Publishers.

Gfeller, E., & Robinson, C. (1998). Which language for teaching? The cultural messages transmitted by the languages used in education. *Language and Education, 12*(1), 18-32.

Gibbons, P. (1998). Classroom talk and the learning of new registers in a second language. *Language and Education, 12*(2), 99-118.

Glaser, B. G., & Strauss, A. L. (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. New York: Aldine.

Goldin-Meadow, S., Kim, S., & Singer, M. (1999). What the teacher’s hands tell the student's

mind about math. *Journal of Educational Psychology, 91*(4), 720-730.
Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multicultural classrooms. *Educational Studies in Mathematics, 47*, 7-33.
Greenlee, D. (1973). *Peirce's Concept of Sign*. The Hague: Mouton.
Griffiths, R., & Clyne, M. (1994). *Language in the Mathematics Classroom: Talking, Representing, Recording*. Melbourne: Eleanor Curtain Publishing.
Hadar, U., & Butterworth, B. (1997). Iconic gestures, imagery, and word retrieval in speech. *Semiotica, 115*(1 & 2), 147-172.
Halliday, M. A. K. (1973). *Explorations in the Functions of Language*. London: Edward Arnold.
Halliday, M. A. K. (1976). *System and Function in Language. Selected Papers edited by G. Kress.* London: Oxford University Press.
Halliday, M. A. K. (1978). *Language as Social Semiotic*. London: Edward Arnold.
Halliday, M. A. K. (1988). On the language of physical science. In M. Ghadessy (Ed.), *Registers of Written English: Situational Factors and Linguistic Features*. London: Pinter.
Halliday, M. A. K. (with R. Hasan). (1985). *Language, Context, and Text: Aspects of Language in a Social-Semiotic Perspective*. Victoria: Deakin University.
Hammersley, M. (1994). Introducing ethnography. In D. Graddol, J. Maybin & B. Stierer (Eds.), *Researching Language and Literacy in Social Context*. Clevedon: Multilingual Matters in association with The Open University.
Hanley, A. (1978). Verbal mathematics. *Mathematics in School, 7*(4), 27-30.
Harvey, R. (1982). 'I can keep going up if I want to': one way of looking at learning in mathematics. In R. Harvey, D. Kerslake, H. Shuard & M. Torbe (Eds.), *Mathematics*. London: Ward Lock Educational.
Hatch, E., & Brown, C. (1995). *Vocabulary, Semantics, and Language Education*. Cambridge: Cambridge University Press.
Hausman, C. R. (1993). *Charles Peirce's Evolutionary Philosophy*. Cambridge: Cambridge University Press.
Haylock, D. (2001). *Mathematics Explained for Primary Teachers*. London: Paul Chapman.
Hewitt, D. (1996). Mathematical fluency: the nature of practice and the role of subordination. *For the Learning of Mathematics, 16*(2), 28-35.
Hewitt, D. (2001). Arbitrary and Necessary: Part 2 Assisting Memory. *For the Learning of Mathematics, 21*(1), 44-51.
Hewitt, D. (2006). Subject-specific pedagogy: global and local aspects. *For the Learning of Mathematics, 26*(1), 20-21.
Hodge, R., & Kress, G. (1988). *Social Semiotics*. Cambridge: Polity Press.
Jones, P. L. (1982). Learning mathematics in a second language: a problem with more and less. *Educational Studies in Mathematics, 13*, 269 - 287.
Jones, D. V. (1997). Bilingual mathematics: development and practice in Wales. *The Curriculum Journal, 8*(3), 393-410.
Jones, D. V. (1998). National curriculum tests for mathematics in English and Welsh: creating matched assessments. *Assessment in Education, 5*(2), 193-211.
Jones, D. V. (2000). Talk and texts in bilingual mathematics lessons in Wales. *The Welsh Journal of Education, 9*(2), 102-119.
Jones, D., & Martin-Jones, M. (2004). Bilingual education and language revitalisation in Wales: past achievements and current issues. In W. Tollefson & A. Tsui (Eds.), *Medium of Instruction Policies: Which Agenda? Whose Agenda?* New Jersey: Lawrence Erbaum Associates.
Kamii, C. (1994). *Young Children Continue to Reinvent Arithmetic*. New York: Teachers College Press.
Kane, R. B. (1968). The readability of mathematical English. *Journal of Research in Science*

Education, 5, 296-298.
Keenan, P. J. (1879). *An Inquiry Into the System of Education in Malta.* London: Charles Thom.
Kelly, M., Grenfell, M., Allan, R., Kriza, C., & McEvoy, W. (2004). *European Profile for Language Teacher Education - A Frame of Reference.* A report to the European Commission Directorate General for Education and Culture.
Kerry, T. (1981). Talking: the teacher's role. In C. Sutton (Ed.), *Communicating in the Classroom*. London: Hodder and Stoughton.
Kerslake, D. (1982). Talking about Mathematics. In R. Harvey, D. Kerslake, H. Shuard & M. Torbe (Eds.), *Mathematics*. London: Ward Lock Educational.
Kilpatrick, J., Hoyles, C., & Skovsmose, O., in collaboration with P. Valero (Eds.). (2005). *Meaning in Mathematics Education*. New York: Springer.
Lave, J. (1997). The culture of acquisition and the practice of understanding. In D. Kirshner & J. Whitson, A. (Eds.), *Situated Cognition: Social, Semiotic and Psychological Perspectives.* Mahwah, N.J.: Lawrence Erlbaum Associates Publishers.
Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
Le Compte, M., & Preissle, J. (1993). *Ethnography and Qualitative Design in Educational Research (Second edition).* London: Academic Press.
Lemke, J. L. (2003). Mathematics in the middle: measure, picture, gesture, sign and word. In M. Anderson, A. Sáenz-Ludlow, S. Zellweger & V. Cifarelli (Eds.), *Educational Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing*. Brooklyn, N.Y.: Legas.
Lerman, S. (1994). Articulating theories of mathematics learning. In P. Ernest (Ed.), *Constructing Mathematical Knowledge: Epistemology and Mathematical Education*. London: The Falmer Press.
Lerman, S. (1996). Intersubjectivity in mathematics learning: a challenge to the Radical Constructivist paradigm. *Journal for Research in Mathematics Education, 27*(2), 133-150.
Lidov, D. (1999). *Elements of Semiotics.* New York: St. Martin's Press.
Lin, A. M. Y. (1996). Bilingualism or linguistic segragation? Symbolic domination, resistance and code switching in Hong Kong schools. *Linguistics and Education, 8,* 49-84.
MacGregor, M. (1993). Teaching mathematics in English to students of non-English-speaking background. *Multicultural Teaching, 11*(3), 31-34.
Marks, G., & Mousley, J. (1990). Mathematics education and genre: dare we make the process writing mistake again? *Language and Education, 4*(2), 117-135.
Marsh, D. (2002). *CLIL/EMILE - The European Dimension: Actions, Trends and Foresight Potential*. Report compiled and authoured by D. Marsh, University of Jyvaskyla, Finland.
Marshall, D. (1971). *History of the Maltese Language in Local Education*. Malta: University Press.
Marshall, C., & Rossman, G. B. (1989). *Designing Qualitative Research*. London: Sage.
Mason, J. (1998). Researching from the inside in mathematics education. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a Research Domain: A Search for Identity: An ICMI Study, Book 2*. Dortrecht, The Netherlands: Kluwer Academic Publishers.
Mason, J. (2002). *Researching Your Own Practice: The Discipline of Noticing*. London: RoutledgeFalmer.
McNamara, D. (1991). Subject Knowledge and its application: problems and possibilities for teacher educators. *Journal of Education for Teaching, 17*(2), 113-128.
McNeill, D. (1992). *Hand and mind: what gestures reveal about thought*. Chicago: Chicago University Press.
Meaney, T. (2005). Mathematics as text. In A. Chronaki & I. M. Christiansen (Eds.),

Challenging Perspectives on Mathematics Classroom Communication. Greenwich, Connectincut: Information Age Publishing.
Mellin-Olsen, S. (1987). *The Politics of Mathematics Education*. Dordrecht: Kluwer Academic Publishers.
Mercer, N. (2000a, first published 1995). *The Guided Construction of Knowledge*. Clevedon: Multilingual Matters.
Mercer, N. (2000b). *Words and Minds*. London: Routledge.
Merritt, M., Ailie, C., Abagi, J. O., & Bunyi, G. (1992). Socialising multilingualism: determinants of codeswitching in Kenyan primary classrooms. *Journal of Multilingual and Multicultural Development*, *13*(1 & 2), 103 -121.
Mertens, D. M. (1998). *Research Methods in Education and Psychology*. London: Sage Publishers.
Merttens, R., & Kirkby, D. (1999). *Abacus (Mathematics Scheme)*. Oxford: Ginn.
Miller, L. D. (1993). Making the connection with language. *The Arithmetic Teacher, 41*(2), 311-316.
Ministry of Education and National Culture (1998). *Kurrikulu Nazzjonali Ġdid għall-Edukazzjoni f'Malta bejn l-eta ta' 3 u 16-il Sena [A New National Curriculum for Education in Malta 3 -16]*. Malta: Ministry of Education and National Culture.
Ministry of Education (1999). *Creating the Future Together: National Minimum Curriculum*. Malta: Ministry of Education.
Ministry of Education, Youth and Employment (in collaboration with the University of Malta). (2005). *National Conference 2004: The Teaching and Learning of Mathematics in Malta.* Malta: Department for Curriculum Management, Ministry of Education, Youth and Employment.
Monaghan, J. (1991). Problems with the language of limits. *For the Learning of Mathematics, 11*(3), 20-24.
Morgan, C. (1998). *Writing Mathematically: The Discourse of Investigation*. London: Falmer Press.
Morgan, C. (2001). Mathematics and human activity: representation in mathematical writing. In C. Morgan & K. Jones (Eds.), *Research in Mathematical Education, Volume 3.* London: BSRLM.
Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11-19.
Munro, J. (1990). Mathematics and language: a subset of language? In G. Davis & R. P. Hunting (Eds.), *Language Issues in Learning and Teaching Mathematics*. Bundoora, Australia: La Trobe University.
Myers-Scotton, C. (1993). *Duelling Languages: Grammatical Structure in Codeswitching*. Oxford: Clarendon Press.
Ndayipfukamiye, L. (1996). The contradictions of teaching bilingually in post-colonial Burundi: From nyakatsi to maisons en étage. *Linguistics and Education, 8*, 35-47.
Nolder, R. (1991). Mixing metaphor and mathematics in the secondary classroom. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education*. Milton Keynes: Open University.
Ogden, C. K., & Richards, I. A. (1923). *The Meaning of Meaning*. London: Routledge & Kegan Paul Ltd.
Olivares, R. A. (1996). Communication in mathematics for students with limited English proficiency. In P. C. Elliot & M. J. Kenney (Eds.), *Communication in Mathematics K - 12 and Beyond*. Reston,Virginia: National Council of Teachers of Mathematics.
Orton, A. (1992). *Learning Mathematics: Issues, Theory and Classroom Practice (2nd edition).* London: Cassell.

Otte, M. (2005). Meaning and mathematics. In J. Kilpatrick, C. Hoyles, & O. Skovsmose in collaboration with P. Valero (Eds.), *Meaning in Mathematics Education*. New York: Springer.

Pakir, A. (2004). Medium-of instruction policy in Singapore. In J. W. Tollefson & A. Tsui (Eds.), *Medium of Instruction Policies: Which Agenda? Whose Agenda?* Mahwah, N.J.:Lawrence Erbaum Associates.

Phillips, L., & Jorgensen, M. W. (2002). *Discourse Analysis as Theory and Method*. London: Sage.

Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. London: Routledge.

Pimm, D. (1991). Communicating mathematically. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education: Research and Practice*. Milton Keynes: Open University Press.

Pimm, D. (1992). Mathematics? I speak it fluently. In K. Norman (Ed.), *Thinking Voices: The Work of the National Oracy Project.* London: Hodder and Stoughton.

Pimm, D. (1994). Spoken mathematical classroom culture: artifice and artificiality. In S. Lerman (Ed.), *Cultural Presepctives on the Mathematics Classroom*. Dordrecht: Kluwer Academic Publishers.

Pimm, D. (1995). *Symbols and Meanings in School Mathematics*. London: Routledge.

Potter, J. and Wetherell, M. (1987). *Discourse and Social Psychology: Beyond Attitudes and Behaviour*. London: Sage

Presmeg, N. C. (1997). Reasoning with Metaphors and Metonymies in Mathematics Learning. In L. D. English (Ed.), *Mathematical Reasoning: Analogies, Metaphors and Images*. N.J.: Lawrence Erlbaum Publishers.

Presmeg, N. (2001). *Progressive mathematizing using semiotic chaining.* Unpublished paper presented to the Discussion Group on Semiotics in Mathematics Education at the 25th PME International Conference, The Netherlands, University of Utrecht, 12-17 July, 2001. Http://www.math.uncc.edu/~sae/dg3/dg3.html. Downloaded 2 August 2002.

Radford, L. (2001). Signs and meanings in students' emerging algebraic thinking: a semiotic analysis. *Educational Studies in Mathematics, 42,* 237-268.

Roberts, T. (1998). Mathematical registers in Aboriginal languages. *For the Learning of Mathematics*, *18*(1), 10-16.

Robson, C. (1997, first published 1993). *Real World Research: A Resource for Social Scientists and Practitioner-Researchers*. Oxford: Blackwell.

Rogoff, B. (1984). Introduction: thinking and learning in social context. In B. Rogoff & J. Lave (Eds.), *Everyday Cognition: Its Development in Social Context*. Mass: Harvard University Press.

Rogoff, B. (1995). Observing sociocultural activity on three planes: participatory appropriation, guided participation, and apprenticeship. In J. V. Wertsch, P. Del Rio & A. Alvarez (Eds.), *Sociocultural Studies of Mind*. Cambridge: Cambridge University Press.

Rosensohn, W. L. (1974). *The Phenomenology of Charles S. Peirce*. Amsterdam: B.R.Gruner B.V.

Roth, W. M. (2000). From gesture to scientific language. *Journal of Pragmatics, 32*, 1683-1714.

Roth, W. M. (2001). Gestures: their role in teaching and learning. *Review of Educational Research*, *71*(3), 365-392.

Roth, W.M., & Welzel, M. (2001). From activity to gestures and scientific language. *Journal of Research in Science Teaching, 38*(1), 103-136.

Rothman, R., W, & Cohen, J. (1989). The language of math needs to be taught. *Academic Therapy*, *25*(2), 133-142.

Sáenz-Ludlow, A. (2003). Classroom discourse in mathematics as an evolving interpreting game. In M. Anderson, A. Saenz-Ludlow, S. Zellweger & V. V. Cifarelli (Eds.), *Educational*

Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing. Brooklyn, N.Y.: Legas.

Sáenz-Ludlow, A. (2004). Metaphor and numerical diagrams in the arithmetical activity of a fourth-grade class. *Journal for Research in Mathematics Education, 35*(1), 34-56.

Sanger, J. (1996). *The Compleat Observer? A Field Research Guide to Observation*. London: Falmer.

Saussure, F. d. (1983). *Course in General Linguistics. Translated and annotated by Roy Harris*. London: Duckworth.

Schliemann, A., D., Araujo, C., Cassunde, M. A., Suzana, M., & Niceas, L. (1998). Use of multiplicative communtativity by school children and street sellers. *Journal for Research in Mathematics Education, 29*(4), 422-435.

Schreiber, C. (2005). *Semiotic processes in a mathematical internet-chat.* Paper presented at the Fourth Conference of the European Society for Research in Mathematics Education (CERME4). Sant Feliu de Guíxols, Spain, 17-21 February 2005. http://cerme.crm.es//Papers. Downloaded 03/02/05.

Seeger, F. (1998). Discourse and beyond: On the ethnography of classroom discourse. In H. Steinbring, M. G. Bartolinin Bussi & A. Sierpinski (Eds.), *Language and Communication in the Mathematics Classroom*. Reston, Virginia: National Council of Teachers of Mathematics.

Setati, M. (1998). Code-switching in a senior primary class of second-language learners. *For the Learning of Mathematics, 18*(1), 34-40.

Setati, M. (2003). Language use in a multilingual mathematics classroom in South Africa: a different perspective. In N. A. Pateman, B. J. Dougherty & J. Zillox (Eds.), *Proceedings of the 2003 Joint Meeting of the PME and PMENA, Vol 4.* Honolulu: University of Hawaii.

Setati, M., & Adler, J. (2000). Between languages and discourses: language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*, 243-269.

Shimahara, N. (1990). Anthroethnography: a methodological consideration. In R. Sherman, R. & R. B. Rodman (Eds.), *Qualitative Research in Education: Focus and Methods*. London: Falmer.

Shipman, M. (1997). *The Limitations of Social Research (4th Edition).* London: Longman.

Shuard, H., & Rothery, A. (1984). *Children Reading Mathematics*. London: John Murray.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Research Review, 57*(1), 4-14.

Sierpinska, A. (1994). *Understanding in Mathematics*. London: The Falmer Press.

Sierpinska. (1998). Three epistemologies, three views of classroom communication: Constructivism, sociocultural approaches, interactionism. In H. Steinbring, M. G. Bartolini Bussi & A. Sierpinski (Eds.), *Language and Communication in the Mathematics Classroom*. Reston, Virginia: National Council of Teachers of Mathematics.

Sierpinska, A. (2005). Discoursing mathematics away. In J. Kilpatrick, C. Hoyles, & O. Skovsmose in collaboration with P. Valero (Eds.), *Meaning in Mathematics Education*. N.Y.: Springer.

Sinclair, J., & Coulthard, M. (1975). *Towards and Analysis of Discourse: The English used by Teachers and Pupils*. Oxford: Oxford University Press.

Sowder, J. T. (1998). Ethics in mathematics education research. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a Research Domain: A Search for Identity: An ICMI Study, Book 2*. Dortrecht, The Netherlands: Kluwer Academic Publishers.

Steffe, L. P., & Tzur, R. (1994). Interaction and children's mathematics. In P. Ernest (Ed.), *Constructing Mathematical Knowledge: Epistemology and Mathematical Education*.

London: The Falmer Press.
Steffe, L. P., & Thompson, P. W. (2000). Interaction or intersubjectivity: a reply to Lerman. *Journal for Research in Mathematics Education, 31*(2), 191-209.
Steinbring, H. (1997). Epistemological investigation of classroom interaction in elementary mathematics teaching. *Educational Studies in Mathematics, 32*, 49-92.
Steinbring, H. (2002). *What makes a sign a mathematical sign? - An epistemological perspective on mathematical interaction.* Paper presented to the Discussion Group on Semiotics in Mathematics Education at the 26th PME International Conference, England, University of East Anglia, 21-26 July, 2002. Http://www.math.uncc.edu/~sae/dg3/dg3.html. Downloaded 2 August 2002.
Steinbring, H. (2005). *The Construction of New Mathematical Knowledge in Classroom Interaction*. New York: Springer.
Sturman, A. (1994). Case study methods. In K. P. Keeves (Ed.), *Educational Research, Methodology, and Measurement: an International Handbook (2nd Edition).* Oxford: Pergamon.
Sultana, R. (1997). Education as struggle. In R. Sultana (Ed.), *Inside/Outside Schools: Towards a Critical Sociology of Education in Malta*. Malta: Publishers Enterprises Group.
Swann, J. (1994). Observing and recording talk in educational settings. In D. Graddol, J. Maybin & B. Stierer (Eds.), *Researching Language and Literacy in Social Context*. Clevedon: Multilingual Matters in association with The Open University.
Tobin, Y. (1990). *Semiotics and Linguistics*. Essex: Longman.
Tollefson, J. W., & Tsui, A. B. M. (2004). *Medium of Instruction Policies: Which Agenda? Whose Agenda?* Mahwah, N.J.: Lawrence Erlbaum Associates.
Torbe, M., & Shuard, H. (1982). Mathematics and language. In R. Harvey, D. Kerslake, H. Shuard & M. Torbe (Eds.), *Mathematics*. London: Ward Lock Educational.
Tymchuk, A. J. (1992). Assent processes. In B. Stanley & J. E. Sieber (Eds.), *Social Research on Children and Adolescents: Ethical Issues.* London: Sage.
Usiskin, Z. (1996). Mathematics as a Language. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in Mathematics K - 12 and Beyond*. Reston,Virginia: National Council of Teachers of Mathematics.
Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Pearson.
van Eerde, D., & Hajer, M. (2005). *Language sensitive math teaching in a multicultural classroom: Students' talking and writing enlightens hidden problems.* Paper presented at the Fourth Congress of the European Society for Research in Mathematics Education (CERME 4), 17 - 21 February 2005, Sant Feliu de Guíxols, Spain. Http://cerme4.crm.ed/papers. Downloaded 03/02/05.
Van Valin, R. D. (1980). Meaning and interpretation. *Journal of Pragmatics, 4*, 213-231.
Ventura, F. (1991). Language and the science curriculum. *Education (Malta), 4*(2), 15-18.
Vile, A. (1997). *What can semiotics offer mathematics education?* Unpublished proceedings of the day conferences of the British Society for Research into Learning Mathematics (BSRLM) Volume 17 (1&2), Universities of Nottingham & Oxford, U.K. 1 March & 7 June 1997.
Vile, A. (2003). Mathematics in flatland: a Peircean, trialectic view of the nature of mathematics. In M. Anderson, A. Saenz-Ludlow, S. Zellweger & V. V. Cifarelli (Eds.), *Educational Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing*. Brooklyn, N.Y.: Legas.
von Glasersfeld, E. (1990). Environment and communication. In L. P. Steffe & T. Wood (Eds.), *Transforming Children's Mathematics Education: International Perspectives*. New Jersey: Lawrence Erlbaum Associates Publishers.

von Glasersfeld, E. (1995). *Radical Constructivism: A Way of Knowing and Learning*. London: The Falmer Press.

von Glasersfeld, E. (1996). Aspects of Radical Constructivism and its educational recommendations. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of Mathematical Learning*. New Jersey: Lawrence Erlbaum Associates Publishers.

Vygotsky, L. S. (1962). *Thought and Language (edited and translated by E. Hanfmann and G. Vakar)*. Cambridge, Massachusetts: Massachusetts Institute of Technology.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes. (Edited by M.Cole, V. John-Steiner, S. Scribner and E. Souberman).* Cambridge, Massachusetts: Harvard University Press.

Vygotsky, L. S. (1981a). The genesis of higher mental functions. In J. Wertsch (Ed.), *The Concept of Activity in Soviet Psychology*. Armonk, N.Y.: M.E. Sharpe.

Vygotsky, L. S. (1981b). The instrumental method in psychology. In J. Wertsch (Ed.), *The Concept of Activity in Soviet Psychology*. Armonk, N.Y.: M.E. Sharpe.

Vygotsky, L. S. (1981c). The development of higher forms of attention in childhood. In J. Wertsch (Ed.), *The Concept of Activity in Soviet Psychology*. Armonk, N.Y.: M.E. Sharpe.

Wagner Alibali, M., Flevares, L. M., & Goldin-Meadow, S. (1997). Assessing knowledge assessed in gesture: do teachers have the upper hand? *Journal of Educational Psychology, 89*(1), 183-193.

Walkerdine, V. (1988). *The Mastery of Reason: Cognitive Development and the Production of Rationality*. London: Routledge.

Watson, A. (2002). Reaction [to David and Lopes]. In S. Goodchild & L. English (Eds.), *Researching Mathematics Classrooms: A Critical Examination of Methodology*. London: Praeger.

Wellington, J. (2000). *Educational Research: Contemporary Issues and Practical Approaches*. London: Continuum.

Wells, G. (1999). *Dialogic Inquiry: Towards a Sociocultural Practice and Theory of Education*. Cambridge: Cambridge University Press.

Wertsch, J. (1985). *Vygotsky and the Social Formation of Mind*. Cambridge, Massachusetts: Harvard University Press.

Wertsch, J., & Stone, C. A. (1985). The concept of internalisation in Vygotsky's account of the genesis of higher mental functions. In J. Wertsch (Ed.), *Culture, Communication and Cognition: Vygotskian Perspectives.* Cambridge: Cambridge University Press.

Wertsch, J., & Tulviste, P. (1996). L.S. Vygotoksy and contemporary developmental psychology. In H. Daniels (Ed.), *An Introduction to Vygotsky*. London: Routledge.

Whitebread, D. (1995). Emergent mathematics or how to help young children become confident mathematicians. In J. Anghileri (Ed.), *Children's Mathematical Thinking in the Primary Years:Persectives on Children's Learning*. London: Cassell.

Whitson, J., A. (1997). Cognition as a semiosic process: from situated mediation to critical reflective transcendence. In D. Kirshner & J. A. Whitson, A. (Eds.), *Situated Cognition: Social, Semiotic, and Psychological Perspectives*. Mahwah, N.J.: Lawrence Erlbaum Associates Publishers.

Wiliam, D. (1998). A framework for thinking about research in mathematics and science education. In J. Malone, A., B. Atweh & J. Northfield, R. (Eds.), *Research and Supervision in Mathematics and Science Education*. Mahwah, N.J.: Lawrence Erlbaum Associates Publishers.

Wolcott, H. F. (1995). *The Art of Fieldwork*. London: Sage Publications.

Wood, T. (1994). Patterns of interaction and the culture of mathematics classrooms. In S. Lerman

(Ed.), *Cultural Perspectives on the Mathematics Classroom*. Dortrecht: Kluwer Academic Publishers.
Xerri, J., A. (1994). Major developments in Maltese primary education. In C. Farrugia (Ed.), *A New Vision for Primary Schools*. Malta: Malta Union of Teachers.
Zammit Mangion, J. (2000). *L-Istorja ta' l-Edukazzjoni f'Malta [The History of Education in Malta]*. Malta: Pubblikazzjonijiet Independenza (PIN).
Zaskis, R. (2000). Using code-switching as a tool for learning mathematical language. *For the Learning of Mathematics, 20*(3), 38-43.
Zevenbergen, R. (1998). Ethnography in the classroom. In J. Malone, A., B. Atweh & J. Northfield, R. (Eds.), *Research and Supervision in Mathematics and Science Education*. Mahwah, N.J.: Lawrence Erlbaum Associates Publishers.