OPTIMAL MONETARY POLICY IN A DOLLARIZED ECONOMY

Dung Nguyen-Tri (Tri Dung Nguyen)

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Department of Economics

Birmingham Business School

College of Social Sciences

University of Birmingham

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Abstract

This thesis attempts to specify an optimal monetary policy for a developing country with partial dollarization. In particular, optimal monetary policy options are investigated within the context of an estimated Dynamic Stochastic General Equilibrium (DSGE) model in order to examine the interaction of the monetary authority with the rest of the economy.

The first chapter introduces the research questions, scope of the research, and a brief illustration of dollarization in developing countries in general and Vietnam in particular. Chapter 2 begins with the formulation of an open economy DSGE model with typical features of a developing country, including partial dollarization and a chronic budget-deficit fiscal rule. A parameterized version of the model is then simulated to investigate the sensitivity and significance of various nominal and real frictions in the model. The remainder of the chapter is devoted to an examination of the role of partial dollarization feature in the model economy.

Chapter 3 estimates the theoretical model using a dataset from Vietnam - a developing country in the Southeast Asia. This is the first estimated DSGE model for Vietnam, as far as my supervisors and I are aware. In particular, the model is estimated with Bayesian

estimation method based on thirteen macroeconomic series with the presence of orthogonal structural shocks on both supply and demand side. The final section of this chapter is devoted to a discussion of the relative importance of various shocks and frictions for explaining the dynamics of the model economy and an evaluation of the model's empirical properties using standard validation techniques.

Estimation results are then integrated in the fourth chapter in order to specify an optimal monetary policy under the estimated model. The first section introduces Currie & Levine (1993)'s method to optimal policy problem which uses standard control state-space approach and Bellman's principle of optimality. Following the derivation of the optimal monetary policy problem with linear, forward-looking constraints and a quadratic objective, asymptotic losses are then presented for a Ramsey policy, a discretionary policy, and an *ex ante* optimized simple Taylor-type rule. The last section of the chapter discusses the dynamics of the model economy under optimal policy. Among the major findings was the superiority of the optimal Ramsey policy over other policy options.

Chapter 5 concludes with some brief comments on the results of general interest, the contributions of the research, and then proposes a potential agenda for future research.

Acknowledgements

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Chapter 1

Introduction

The role of monetary policy in the stabilization of the business cycle has been centre stage in discussions between economists and policymakers over the past few decades. Monetary policy design and implementation are also a crucial part of the economic agenda in almost all countries. However, as we discuss below, there is a clear sense that the effectiveness and transmission mechanism of monetary policy in developing countries are more or less impaired by the presence of partial dollarization in the economy. Therefore, a large literature has focused on the emergence of official and *de facto* dollarization in developing countries, theoretical aspects of dollarization, and empirical evidence on the impact of dollarization in the economy.

1.1 Question for the Thesis

Departing from the common interests in the literature, the present thesis pursues research on the design and implementation of monetary policy in a developing country with particular reference to Vietnam, with an emphasis on the dollarization feature of the economy. In this context, the research question here is how should the central bank formulate an optimal monetary policy that aims at maximizing the social welfare?

In order to answer this question, an open-economy Dynamic Stochastic General Equilibrium (DSGE) model developed by Adolfson et al. (2005) is extended to incorporate specific features of a developing country to further examine how partial dollarization impacts the formulation and working of monetary policy in practice. Qualitative and quantitative analyses are conducted using modern econometric methods and Bayesian estimation techniques, given a macro dataset from Vietnam. As far as my supervisors and I are aware, this is the first estimated DSGE model for Vietnam. Meaningful results obtained from the thesis should be useful for macro-economists and policymakers in developing countries in general, and for Vietnam's central bank in particular.

1.2 Structure of the Thesis

Following the introduction in Chapter 1, Chapter 2 sets out a New Keynesian model for a small open economy with key blocks from standard DSGE models. A number of nominal and real frictions are included in the model, including price/wage stickiness in the tradition of Calvo (1983), variable capital utilization, capital adjustment costs, and habit persistence. The model differs from classic models in the field in two features which are less usual in standard DSGE models but more relevant to a developing country like Vietnam: partial dollarization and a budget-deficit fiscal rule which has been in use since the 1990s. Sensitivity analysis of the real/nominal frictions and dollarization feature is then conducted in order to examine the role of each friction in the simulated model. Especially, impulse responses from different scenarios of dollarization are consistent with predominant findings that dollarization hinders the effectiveness of monetary policy.

In Chapter 3, Bayesian estimation methods, which are a type of hybrid approach between informal calibration and Maximum Likelihood, are applied to estimate key parameters of the model. In particular, a dataset consisting of 13 observables on Vietnam's economy and the US output, inflation, and interest rate as world economy's indicators has been used in the estimation. Multivariate convergence of the estimated parameters has been achieved and estimation outputs also show that the data provide useful information to update the priors. Properties of the estimated model and a model fit analysis are also investigated after estimation. In addition, by comparing log likelihoods and second moments of key variables with those of the data, the hypothesis of high dollarization scenario is found to outperform baseline and low dollarization variants of the model.

Furthermore, optimal monetary policy options are then characterized and set up in Chapter 4 in a standard control state-space approach, Bellman's principle of optimality as in Currie & Levine (1993). In particular, a Ramsey policy, a discretionary policy, and an *ex ante* optimized simple Taylor-type rule are evaluated on the basis of related asymptotic losses. The nature of dynamic responses of the macroeconomy under optimal monetary policy is also discussed in detail. Results show that in an economy characterized by partial dollarization, the optimal Ramsey policy compares very well with other policy options, while the *ex ante* optimized simple Taylor-type rule is almost as good as the optimal discretionary policy.

Finally, Chapter 5 concludes and summarizes important results of general interest and key contributions of the present research. A potential agenda of further research is also proposed at the end of the thesis.

1.3 Overview of Vietnam's Economy

1.3.1 Doi Moi Process

After the war with the US ended in 1975, the whole country was formally reunified but, as argued by Van Arkadie & Mallon (2004), the integration of two very different economic systems presented a formidable challenge. North Vietnam had sought to implement a Soviet approach to economic development in which the economy, mainly the allocation of inputs and outputs, was centrally planned with prices fixed and regulated, and state enterprises were the main vehicle for state-led growth, while in the south, a market economy was firmly established under the old regime where goods and services were freely distributed following the demand and supply of the market, and the private sector was the main player in the economy. Nevertheless, a large socialist transformation program, which according to Riedel & Comer (1997) meant public ownership of the means of production under the form of state ownership of industry, collectivisation of agricultural and handicraft sectors, and a state monopoly on foreign trade, was tightly implemented throughout the south. The commercial and industrial sectors were nationalised, while agriculture was collectivised. Human capital also faced a serious loss due to the leaving of hundred thousands of boat people to other countries. The motivation system and benefit division of the new regime directly had adverse effects on the economy. Productivity promptly fell, output in agricultural sectors went down, industry idled and commerce was immobilised. Furthermore, the US aid to the south stopped in 1975 and was replaced by the US sanctions, according to Dang (2009), and aid from Soviet bloc and China to the north was gradually cut down while the two countries were in tense political conflicts until 1978[1].

The socialist transformation program in the south, accompanied by the drive of the "socialist relation of production" throughout the country, had a disappointing impact to the economy, particularly in the agricultural sector. As Vietnam was characterised as an agricultural economy where agriculture accounted for about half of GDP and nearly 80 percent of the population was rural, as pointed out by Irvin (1995), the decline in agricultural output in this period had a serious impact on the living standard of the majority of the people. As shown by Irvin (1995), per capita national output was estimated to have fallen by 10 percent while inflation had risen to 140 percent in 1980. Public spending on the war with China in 1979 put a heavy burden on a poor economy which had just got out of another long war. In the absence of a coherent strategy, as argued Irvin (1995), government policy was essentially of an *ad hoc* nature and, thus, the implementation of two price-wage-currency reforms in 1982 and 1985 failed to meet the targets of controlling inflation and the scarcity of goods as they were all directed at the symptoms rather than the economy's fundamental problems. As a result of these unsuccessful reforms, the consumer price level rocketed up by 774.7% in 1986, as shown in figure 1.1, in spite of the state price controls, while the increase of wages lagged far behind, as argued by Tran (1996). Until 1986, Vietnam experienced a sharp economic downturn in which, as pointed out by Mallon (1999), the government's revenues were small, the fiscal deficit was large and persistent, while budget reserves were scarce owing to high military spending and support provided to unprofitable state-owned enterprises. There was almost no foreign investment, the technology gap between Vietnam and other countries in the region was

[1] According to Dang (2009), aid from China which averaged 300-400 million USD per annum in previous periods was also stopped completely in 1977.

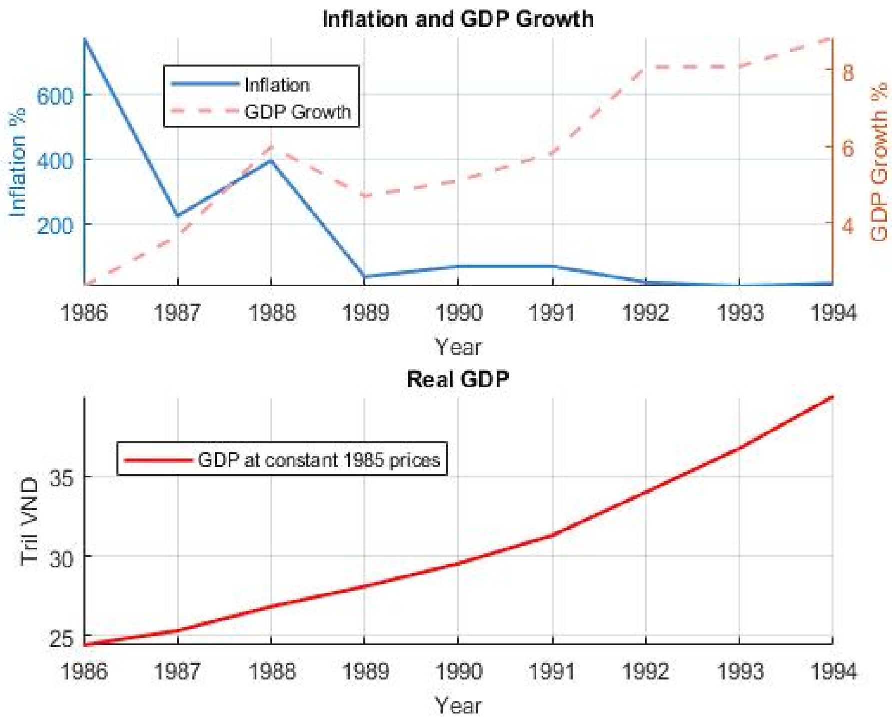
becoming larger. These pressing challenges made the “Doi Moi” in 1986 unavoidable.

Doi Moi was formally approved by the Sixth Party Congress of the Vietnam Communist Party (VCP) in December 1986. Doi Moi comprised an agenda on policy reformation in order to stabilise the economy, to restore, and to speed up economic growth. The congress decided to eliminate the system of bureaucratic and centralised management based on state subsidies, and to shift to a multi-sector, market-oriented economy with a higher role for the private sector. Capital resources were directed to developing the agriculture sector, expanding consumer goods production, and improving foreign trade as well as foreign investment. Van Arkadie & Mallon (2004) argue that the policies approved at the Sixth Party Congress represented a breakthrough from earlier policies and were the conclusion of internal argument about the breakdown of the previous system to result in tangible effects on the welfare of the Vietnamese people. Following this event, a specific plan of Doi Moi was considered in a series of the VCP meetings, undoubtedly indicating the demand to change from state control towards indicative planning and macroeconomic policy levers.

Moreover, significant micro-level reforms implemented after 1986 brought about a strong supply response that significantly developed the environment for an effective implementation of macro level reforms that followed. In 1987, large-scale price reforms were implemented with the formal price of non-essential consumer products being increased to meet the market prices and the scope of controlling being considerably diminished. Simultaneously, there was a significant devaluation of the Vietnamese official currency (VND). The approval of the VCP Resolution No. 10, together with the enactment of the

Land Law and Foreign Investment Law in 1988, presented a milestone of the reform and a turning point in the economic development of the country. As argued by Tran (1998), this brought about a bold institutional change to the agricultural sector and marked the end of bureaucracy and the centrally planned targets mechanism in Vietnam. The abolition of an old goods-based relationship between farmers and their cooperatives also led to a sustainable development of the economy in general and the agricultural sector in particular since 1988. As a result, the hyperinflation during 1986-88 was brought down to 34.7 percent in 1989 and stabilised at the one-digit level from 1992 onward, as illustrated in figure 1.1[2].

Figure 1.1: Key Macroeconomic Indicators, 1986-1994



Source: General Statistics Office of Vietnam.

As argued by Irvin (1995), after experiencing a difficult period of reform and adjust-

[2]Figure 1.1 shows real GDP at constant 1985 prices in trillions of VND as there was no official market exchange rate between VND/USD in this period.

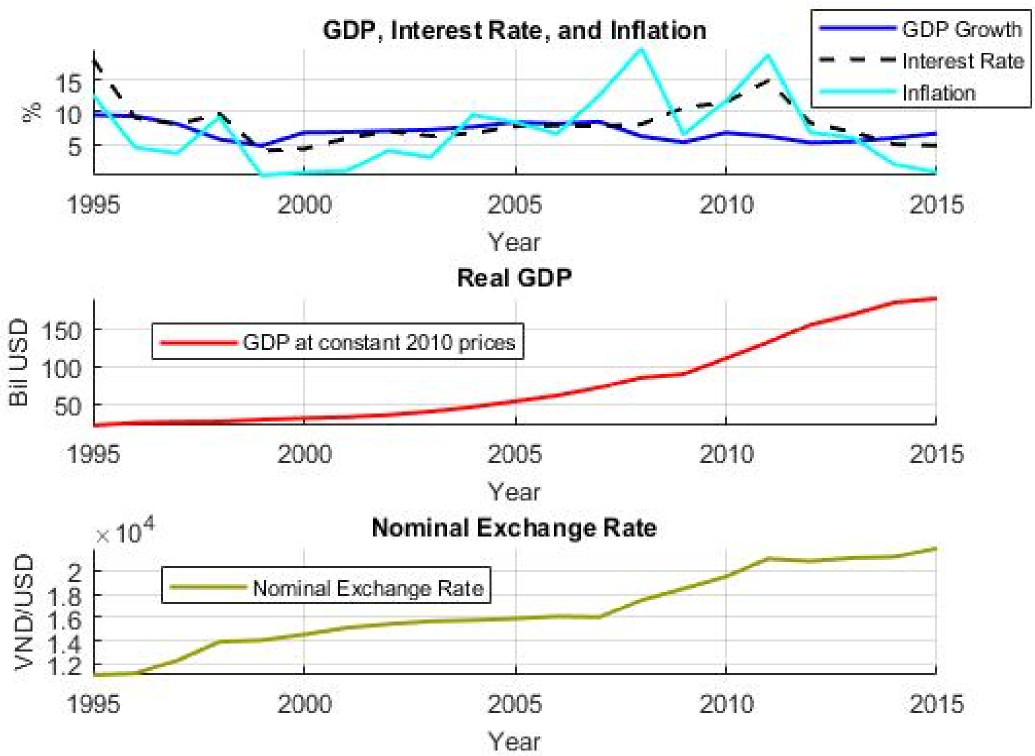
ment in the 1980s, Vietnam in the early 1990s enjoyed rapid, export-led growth which averaged 7.6 percent a year (Griffin (2016)). Furthermore, the acceleration of international economic integration in the first half of the 1990s also played a key role in expanding markets for exports, attracting foreign investors, and enhancing economic efficiency. The first Vietnam's western trading partner was the European Union (EU) after a bilateral trade agreement was signed in 1992. In 1995, Vietnam normalised diplomatic relation with the US and joined the Association of Southeast Asian Nations (ASEAN). Having joined the Asia-Pacific Economic Cooperation (APEC) in 1998, Vietnam and the US finally concluded the first ever bilateral trade agreement in 2000. This agreement came into effect in 2001 and immediately generated significant impetus to the country's export, investment, and economic growth. In addition, as an active member of ASEAN, Vietnam has also been engaged in many Free Trade Agreements (FTAs) between ASEAN and trading partners such as ASEAN-China Free Trade Area (2002), ASEAN-Japan Comprehensive Economic Partnership (2003), and ASEAN-South Korea FTA (2005)[3].

This open-door policy has provided sharper supply-side incentives to the economy, creating a huge market access for Vietnamese entrepreneurs, significant "vent for surplus" of cheap labour and agricultural products, and thus played a key role in the boom in exports which is one of the key engines of economic growth in Vietnam. Van Arkadie et al. (2010) argue that the key drivers of economic growth in Vietnam in this period also include agricultural growth, export growth, and booming foreign investment. However, the foundations of economic growth were mainly capital expansion and labour inputs but,

[3] In recent years, Vietnam has signed FTA on goods with India in 2010, Agreement on Comprehensive Partnership with Japan in 2008, FTA with Chile in 2013, and most importantly Free Trade Agreement and Investment Protection Agreement with the EU in 2019.

as argued by Nguyen & Nguyen (2009), not accompanied by technological progress. While capital expansion in Vietnam was attributed to foreign investment and remittances from Vietnamese living abroad ("Viet-kieu"), the expansion of labour inputs was primarily due to labour migration from rural to urban areas. In the second half of the 1990s, economic growth slowed down and inflation remained subdued due to the impact of the Asian financial crisis in 1997-98 as well as the increasingly unsustainable composition of growth. However, as argued by Jenkins (2004), it is the increased global integration of Vietnamese economy during the 1990s that helped the economy to weather the crisis rather well, and economic growth continued to increase at an average of 7.6 percent per annum between 1990 and 2000.

Figure 1.2: Key Macroeconomic Indicators, 1995-2015



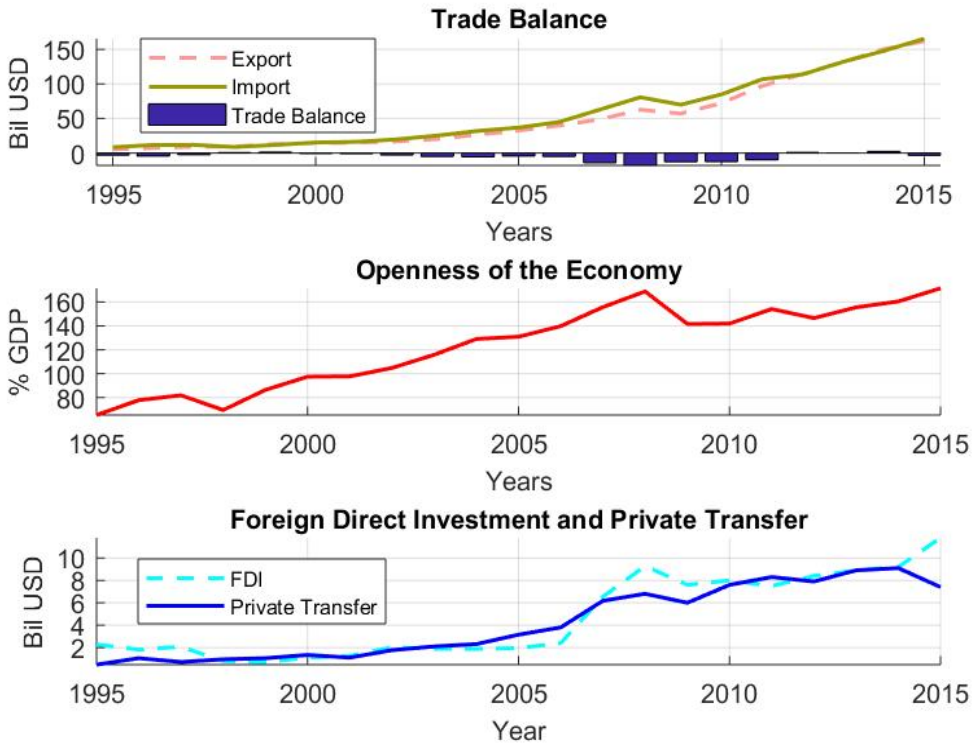
Source: General Statistics Office of Vietnam and the IMF.

According to Vuong (2014), Vietnam's economic growth averaged 7.5 percent in the years leading to the global financial crisis. However, the rapid rate of growth in this period succumbed to the "resource curse" problem as there appeared more evidence that economic growth relied heavily on excess consumption, natural resource exports, and capital endowments, while innovation and productivity were not the main engines of growth[4]. As shown in figure 1.4, natural resources-related products such as crude oil, coal, or rubber represent a large share of the overall exports from Vietnam. Following the WTO accession in early 2007, Vietnam experienced a surge in foreign direct investment and domestic credit growth, as shown by Takeda & Ghura (2010), galloped to 53.9 percent per annum. As a result, double-digit inflation recurred and reached 19.9 percent in 2008, as depicted in figure 1.2, the highest level since the early 1990s. While monetary policy tightening and fiscal austerity were being implemented to curb rising inflation, according to Nguyen et al. (2011), the global crisis unfolded causing economic growth to fall from over 8 percent in 2007 to 6.3 in 2008 and even down to 3.9 percent in the first half of 2009 as a consequence of a sharp fall in demand for export and capital inflows. Within a short space of time, the economy experienced another sharp policy reversal when the government launched a large fiscal stimulus package of 8 billion USD, while the monetary authority at the same time pumped large amounts of credit into the economy in 2009 and 2010. Credit grew at 39.6 and 32.4 percent, respectively, as in Rodlauer & Salgado (2014), and the VND was sharply devalued by 10.1% in 2011 (SBV (2011)). The government's efforts helped push GDP growth back to 6.78% but at the price of galloping inflation at 11.75 percent in 2010, and 18.13 percent in 2011 (SBV (2010), SBV (2011)). In a nutshell,

[4]According to Vuong (2014), Vietnam had a very high incremental capital to output ratio (ICOR) of 7-8 times compared to other Southeast Asian economies of 3-4. Investment to GDP ratio rose from an average of 4.9 percent during 1996-2000 to 39.1 percent in 2001-05.

the period from 2007-11 was the second vicious circle since the Doi Moi, given the impacts of the global crisis, that propelled the economy toward another macroeconomic crisis, the outcome of which was serious inflationary pressures and a heavy burden on budget and trade deficits.

Figure 1.3: Trade Indicators, Foreign Direct Investment, and Private Transfer, 1995-2015



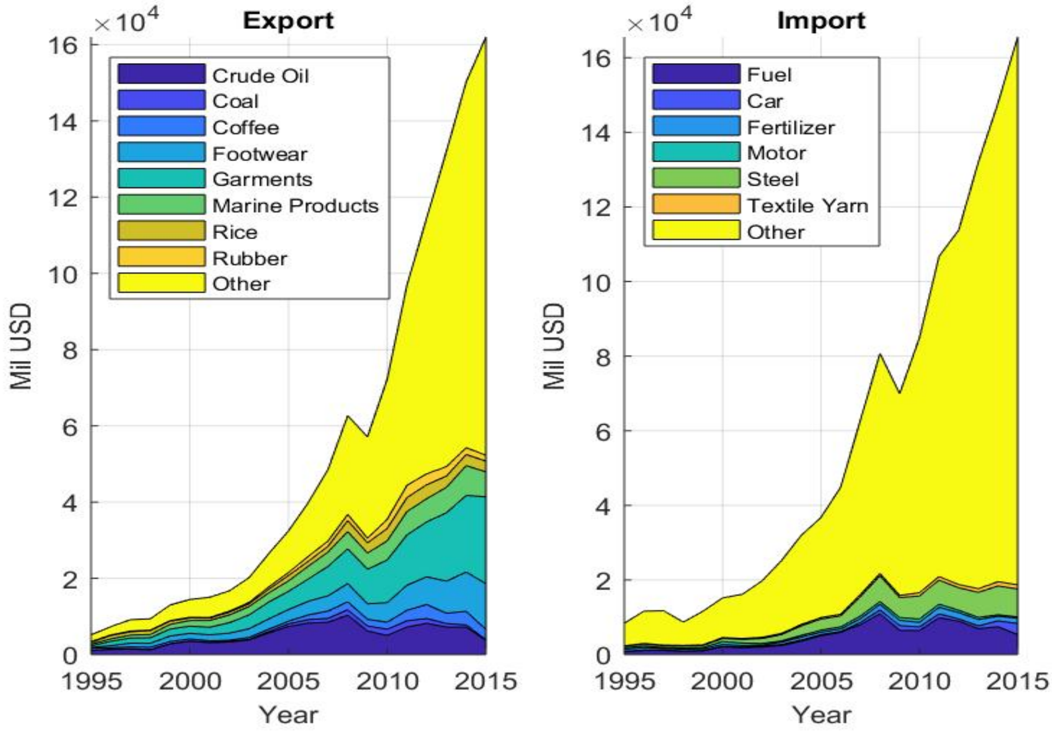
Source: General Statistics Office of Vietnam and the IMF.

Another milestone in Vietnam's contemporary economic history was the issuance of the government's Resolution No. 11 in early 2011, a policy package which specifies a wide range of monetary, fiscal, and structural reforms, aiming to restore macroeconomic stability and cool down an overheated economy. This resolution helped address high levels of inflation, tensions in the foreign exchange market, high nominal interest rates, and declining foreign exchange reserves. Its results proved that this policy remained one of the most important macroeconomic policies in the government's agenda in recent years. In spite of structural challenges that create a headwind for growth[5], figure 1.2 shows that inflation was on a downward trend to 6.8, 6.0, and 1.8 percent in 2012, 2013, and 2014, respectively. Vietnam's economic growth gradually recovered and, as shown by SBV (2015), reached 6.68 percent in 2015, the highest level since 2008, while inflation was 0.63 percent, the lowest level in a 15-year period. The recovery of economic growth in this period was mostly led by a strong FDI-driven export sector which is in turn underpinned by further recovery in domestic demand, robust private consumption, and investment growth[6].

[5]Such as SOE reforms, banking sector weaknesses, diminishing fiscal space, and SOE restructuring costs etc.

[6]Exports increased at 12.6 percent and final consumption increased at 9.1 percent, the highest level since 2008.

Figure 1.4: Composition of Trade, 1995-2015



Source: General Statistics Office of Vietnam and the IMF.

1.3.2 Monetary and Fiscal Framework

Monetary Policy Framework. Since the Doi Moi, Vietnam has implemented sweeping banking sector reforms, of which the liberalisation of the banking system was one of the most important policies. Turning points include the transformation of the mono-banking system into a two-tier system in 1990, allowing commercial banks to be in operation, the restructuring of state-owned commercial banks, and the development of financial markets. As regulated by the Law on State Bank of Vietnam, the State Bank of Vietnam (SBV), a government agency with the governor being a member of the cabinet, functions as a typical central bank. However, it was not until the mid-1990s that SBV-controlled deposit/lending interest rates were replaced by market-determined interest rates with the lifting of floors for deposit rates and ceilings on lending rates. Also, another thorny issue facing monetary authority is the usual structural changes that could destabilise the link between monetary policy and the real economy. The relatively underdeveloped monetary market also limits the scope of standard open-market operations which, according to Camen (2006), were not introduced until 2000, making it more difficult to fine tune monetary policy. Other macroprudential policy tools, such as countercyclical capital buffers or countercyclical liquidity requirements, which are seen as useful complement to standard monetary policy in emerging economies in dealing with capital inflows and asset price inflation pressures, have not been in effective operation in Vietnam.

As argued by Anwar & Nguyen (2018), one of the distinct features of Vietnam's monetary policy is the mechanism which distinguishes between the functions of SBV and those related to national monetary policy. Decisions regarding the monetary policy plan are principal functions of the government. A projection of the annual inflation rate is prepared by the SBV and then discussed among the cabinet. This projection, once submitted to the National Assembly for approval, will be the official annual inflation target, according to SBV (2015). As pointed out by Anwar & Nguyen (2018), the SBV, an integral part of the government, implements monetary policy through standard instruments such as interest rate, reserve requirement ratio, and exchange rate[7]. In addition, as argued by Camen (2006) and IFS (2017), although the SBV has two lending facilities, which are

[7]With regard to exchange rate policy, the SBV announces an official VND/USD exchange rate with bands and manages the supply of foreign exchange to keep the interbank exchange rate within the bands.

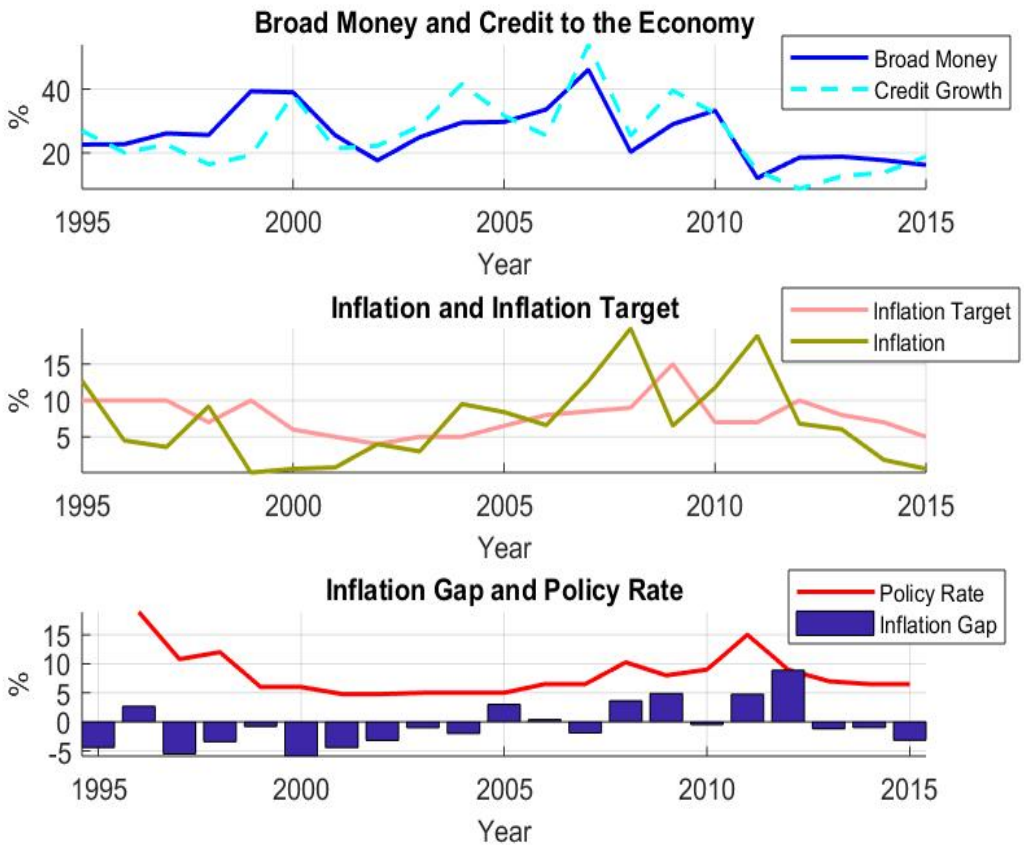
refinancing and discount rates[8], its refinancing rate is mainly used as the policy rate.

Another notable feature of the Vietnamese economy is its high level of dollarization since the Doi Moi which, according to Zamaróczy & Sa (2003), presents a pressing challenge for monetary policy makers to devise an optimal monetary policy. As argued by Zamaróczy & Sa (2003), dollarization was neither sought nor encouraged by monetary authorities but it came from a sudden and massive foreign currency inflows on the "supply side" after the open-door policy, stemming mainly from sizable foreign direct investment, private transfers, and export earnings, as shown in figure 1.3, and a lack of confidence in the Vietnamese currency on the "demand side". Figure 1.7 shows that dollarization in Vietnam reached its highest level of 41.2% in 1991 and remained in the range from approximately 10-30% during the whole period of study. Despite the fact that dollarization can send an important positive signal to foreign investors, signifying increased stability and reduced risk, as argued by Menon (2008), it poses a serious threat to the formulation and conduct of monetary policy in Vietnam.[9]

[8]According to Camen (2006), both are collateralised and the latter gives commercial banks access to funds subject to quotas. The refinancing rate is the upper rate and the discount rate the lower one for lending from the SBV, defining the band within which the rate for open market operations moves.

[9]A detailed discussion on dollarization is presented in the next section.

Figure 1.5: Monetary Policy in Vietnam, 1995-2015



Source: National Assembly, General Statistics Office of Vietnam and the IMF.

As a country in transition, Vietnam experienced a bout of hyperinflation in the second half of the 1980s and early 1990s, but major stabilisation efforts have brought inflation under control, as shown in the previous section. Figure 1.5 shows key indicators of Vietnam's monetary policy from 1995 to 2015. The dashed green curve and the pink curve illustrate the annual growth rate of credit to the economy and the inflation target[10], while the blue bars, blue curve, dark yellow curve, and red curve show the inflation gap, broad money growth, inflation rate, and policy rate, respectively. As shown in figure 1.5, the

[10]These annual inflation targets are extracted from the National Assembly's annual resolutions on socio-economic development plan during the period 1995-2015. They are set for the next 12 months.

period after 1994 has been characterised by modest inflation, except a turbulence from 2007-11. A striking feature during this period, as argued by Camen (2006), is a lack of a close relationship between inflation rates and the growth rates of money and credit to the economy. While the average annual rates of money and credit growth were 26.1 and 25.4 percent, respectively, inflation averaged approximately 7 percent during the period 1995-2015. This disconnect between money growth and inflation, as Camen (2006) points out, reflects a rapid rate of monetisation of assets in Vietnam in this period.[11]

However, figure 1.5 also shows that there seems to be a positive correlation between the policy rate and inflation gap. The improved behaviour of inflation during the period of study can be attributed to the SBV's increased aggressiveness in responding to realised inflation and, to some extent, to the amount of capacity in the economy, as shown in figure 1.2. While the policy rate followed a downward trend from 1995 to 2007 as the inflation gap was mostly negative, it picked up during the 2008-11 turbulence with an average inflation rate of 14.3 percent in this period, before dropping in recent years when the inflation gap turned negative again and economic growth slackened considerably. As argued by Hetzel (2000), Taylor's original rule is deduced from two aspects of monetary policy in which the central bank uses a short-term interest rate as its policy instrument and sets its

[11] The ratio of broad money to GDP, which is one of the main indicators of financial deepening and also an inverse ratio of the velocity of money, has increased from about 20 percent in the mid-1990s to around 138 percent in 2015. Commercial banks have in many years been major owners of government's bonds, which was up to 80 percent in 2015, according to the Ministry of Finance. From 2012-14, while fiscal deficits increased to 5.4, 6.6, and 6.3 percent, respectively, large shares of money growth were directed toward financing fiscal deficits as the growth rates of credit to the economy were relatively smaller than money growth rates in the same year (8.7 and 18.5 percent in 2012, 12.7 and 18.8 percent in 2013, and 13.8 and 17.7 percent in 2014.

interest rate peg based on the observed behaviour of the economy. Any characterisation of a "leaning against the wind" monetary policy in which the central bank raises its interest rate peg when economic activity is strong and inflation undesirably high, and conversely, poses the flavour of a Taylor rule. From this point of view, a simple Taylor-type rule could be appropriate to describe the formulation and conduct of monetary policy by the central bank in Vietnam in the relevant period. This argument is in line with Huynh et al. (2017) where a "leaning against the wind", or a Taylor rule, is shown to be applicable to stabilise and improve Vietnamese economy performance.[12]

Fiscal Policy Framework. Fiscal policy in Vietnam before Doi Moi, as argued by Nguyen (2017), was characterised by an expansionary setting and the state budget was not only a pool of fund for the state sector but also the planned distributor for the whole economy. The main internal budget sources were from state-owned enterprises' revenues which were required to be transferred entirely to the state budget[13]. However, as the majority of state budget came from external funding from the Eastern Bloc, a sudden drop in capital inflows in the end of the 1980s triggered a shock to the government's fiscal position. As a result, the government ran large budget deficits which were mostly financed by printing money, while revenues from taxes and fees played a secondary role during this

[12]There is also a coordination between monetary and fiscal policies. The objective of monetary policy, as stated by the State Bank of Vietnam (https://www.sbv.gov.vn/monetarypolicyobjectives), is to manage monetary policy in a proactive and flexible manner in close association with the fiscal policy to control inflation, stabilise macro-economy, support economic growth at a reasonable level, and ensure the liquidity of credit institutions. The SBV (2011)'s annual report also states in page 19 that low investment growth, which was largely public investment, "helped tighten monetary policy, decrease money supply, and control credit growth at low level."

[13]And their losses, if any, were also financed by the state budget.

period.

From the beginning of the Doi Moi, Vietnam embarked on extensive fiscal reforms with fiscal decentralisation being a fundamental cornerstone of the whole program, as argued by Martinez-Vazquez (2004), in order to facilitate efficient allocation of resources and to meet the needs of local population. According to Vu (2016), fiscal decentralisation in Vietnam started in 1989[14] and accelerated with the promulgation of the first State Budget Law in Vietnam's history in 1996. Given the current administrative structure[15], as argued by Martinez-Vazquez (2004), Vietnam's budget structure is highly hierarchical and follows a nested or Matruska doll model which was common in the former Soviet Union[16]. In addition, according to WB (2015), the administrative agencies at local governments operate on the basis of a unique nested system of dual administrative subordination, meaning that they are horizontally accountable to the corresponding People's Council, but at the same time vertically accountable to their functional line department at the immediate higher level, and ultimately central line ministries. Therefore, the budget at each local level has to be approved not only by the corresponding People's Council, but also by the upper level government, and will be submitted through a bottom-up process before all sub-national budgets are consolidated into the state budget for approval by the National Assembly.

Furthermore, at central level, a legacy of central planning has been the separation of managerial responsibilities for recurrent and capital expenditures, as argued by Leung

[14]With Council of Ministers/Government's Resolution No, 186.

[15]Vietnam's territorial administrative structure is organized into four levels, including central government, 63 provinces with 5 large cities being granted provincial status, districts, and communes.

[16]But now this mechanism is only present in China, Lao PDR, and Vietnam, as argued by WB (2015).

(2010). The budget planning is managed according to a dual budgeting process in which the Ministry of Finance leads the planning of the recurrent budget and the Ministry of Planning and Investment is responsible for the preparation of the capital budget. While local expenditure constitutes over half of total government spending, the hierarchical nature of the budgeting process undermines local authorities' policy autonomy over local budgets as they are highly subject to changes and revision requests by higher levels of government[17], as pointed out by Morgan & Trinh (2017). This budgeting system induces an asymmetry in fiscal relations between central and local authorities and even within provinces, leading to what we considered to be inequitable outcomes in resource allocation across tiers of governments as well as a mismatch with spending responsibilities, as argued by Martinez-Vazquez (2004). However, according to Nguyen & Anwar (2011), despite these shortcomings, fiscal decentralisation in Vietnam has improved transparency and brought the decision-making process closer to beneficiaries.

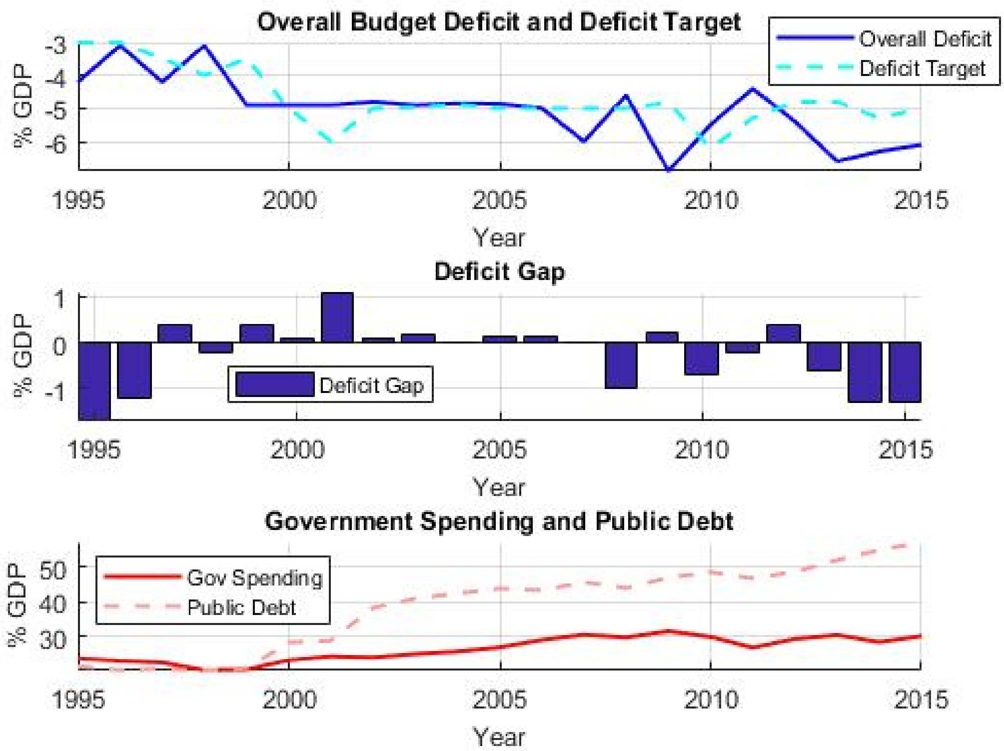
Therefore, in recent years, Vietnam has implemented various policies to reform the fiscal framework, including a greater fiscal decentralisation throughout the country and the modernisation of the tax system[18]. The government also reduced and eventually stopped subsidising state-owned enterprises, and imposed a hard budget constraint on state companies. As a result, Vietnam's fiscal position has gradually been improved and

[17]There are three types of revenues: (i) 100 percent central; (ii) 100 percent local; and (iii) shared revenue to close the gap between 100 percent local revenue and local spending needs. The horizontal fiscal gap which remains after shared revenue are added to 100 percent local revenue are filled by intergovernmental transfers.

[18]A series of tax laws were passed during this period, including laws on revenue tax, special consumption tax, export-import tax, agricultural land-use tax, land-use-right transfer tax, natural resources tax, income tax, house-land tax.

an active fiscal policy mechanism has been put in operation through spending adjustments as well as maintaining an efficient tax system. However, the most pressing challenge has been budget deficits that Vietnam continued to run during the whole period and remained unchanged in recent years. As argued by Le (2016), the key reason underlying Vietnam's chronic fiscal deficit position is the government's ever-increasing recurrent expenditures, which could account for roughly two thirds of the government's annual spending and could undermine Vietnam's fiscal sustainability in the medium and long run.

Figure 1.6: Fiscal Policy in Vietnam, 1995-2015



Source: National Assembly, General Statistics Office of Vietnam and the IMF.

As a chronic budget-deficit running country, one of the most important indicators of the government's state budget proposal submitted to the National Assembly is an estimate for the annual budget deficit, which will then be discussed and approved by the

National Assembly as a deficit target. Figure 1.6 shows key indicators of Vietnam's fiscal policy during the period from 1995 to 2015. The dashed green curve depicts the annual deficit targets[19], while the blue bars, dashed pink curve, and red curve illustrate the deficit gap, public debt, and government spending, respectively. As shown by the third subplot of figure 1.6, public debt and government spending have been on increasingly divergent paths since the late 1990s. Public debt continuously increased from around 20 percent of GDP in late 1990s to 57.1 percent in 2015, while government spending outturns were characterised by modest rates in the years from 1995-99, around 20 percent per annum, before it picked up slightly during the period 2000-05 and remained stable at around 30 percent of GDP since 2006. In addition, the second subplot of figure 1.6 also shows a tendency of negative correlation between the deficit gap and government spending, especially during the late 1990s and the period from 2006-15.

As a country in transition, capital investment for Vietnam by state-owned enterprises and government's recurrent expenditure on public wage bill remain a significant part of annual government spending. However, one of the core challenges facing the Vietnamese dual economy[20], as argued by Perkins & Vu (2010), is the inefficiency of state-owned enterprises' public investments, which skyrocketed almost 60 percent in 2007, resulting in persistently high budget deficit. Therefore, the government has implemented a public investment restructuring programme in response to the increasing level of public debts

[19]These annual budget deficit targets are extracted from the National Assembly's annual resolutions on state budget estimate from 1995-2015. They are set for the next 12 months.

[20]As argued by Perkins & Vu (2010), while the private sector which performs far better a job in terms of job creation and economic growth has been strangled, state-owned enterprises continued to receive a great deal of credit in the economy and executed a lion's share of public investments.

and to meet the annual budget deficit target. In particular, as the deficit gap widened following the economic slump in 2007-11, as argued by Groom et al. (2018), the government reduced capital spending in the years that followed, from 13.1 percent of GDP in 2009 to 7.8 percent in 2012, while non-wage recurrent spending was also cut down by 10 percent. Furthermore, according to Takeyama (2018), when public debt outstanding increased to 63.3 percent in 2016, the government also made a bold decision to control the public debts by stopping government guarantees to new state-owned enterprise's borrowings, which by the end of 2016 accounted for 20 percent of government guaranteed debts.

In addition, the government also actively responds to the deficit gap by rationalising the public sector headcount as public wage bill is significantly higher than in other countries in the region, as argued by Rodlauer & Lane (2016). In Vietnam, the wage bill takes a large share of the government's recurrent expenditure, mounting up to approximately 20 percent of total budget spending, as argued by Dinh et al. (2017). In the period from 2014-17 when the deficit gap was in a surge, wages in the public sector were managed to grow only by an average of 3 percent, reflecting the government's effort in reining in the public sector wage bill in order to reduce the deficit gap, while nominal wages in the private sector grew rapidly at an average rate of 8.4 percent, according to Dinh et al. (2017). Given these developments, a simple fiscal rule could be appropriate to capture the behaviour of fiscal authority in response to the movements of deficit gap and public debt during the relevant period.

1.4 Dollarization: the Issue and the Literature

The fundamental roles of a domestic currency include a *store of value*, a *means of payment/exchange*, and a *unit of account*. Therefore, *dollarization*, as defined by Feige (2003), is the process of substituting the United States dollar (USD) for a domestic currency to fulfill these fundamental functions of the domestic currency. Viseth (2002) explains that this is the phenomenon in which USD is used as a store of value, unit of account as prices of goods being quoted in USD, and later as a means of payment in the economy. In practice, the term *dollarization*, as emphasized by Calvo & Gramont (1992), indicates that USD serves as a unit of account or as a store of value, and not necessarily as a medium of exchange. Thus, an economy in which USD serves only as unit of account and/or store of value is dollarized, but may not be subject to "currency substitution", another term which has also been in widespread use in the literature to describe dollarization. In other words, currency substitution is normally the last stage of the dollarization process. This is in line with Feige (2003) who distinguishes *currency substitution* where USD is used as a medium of exchange from *asset substitution* in which USD is just a store of value and/or a unit of account. Feige (2003) shows that this phenomenon can be classified as either *de jure* dollarization if the policy is officially adopted by the government or *de facto* dollarization when agents voluntarily substitute USD for the domestic currency as a means of payment and unit of account and/or hold USD-denominated assets as a store of value.

This is a common characteristic of developing and transitional countries as these economies usually have to face with prolonged periods of high and variable inflation and expectation of high exchange rate depreciation, as argued by Ra (2008). In general, the level of dollarization in an economy can be measured by either foreign currency deposits (FCD) as a proportion of the overall monetary supply[21] or the amount of foreign currency in circulation (FCC). However, in developing/transitional economies, measuring FCC is empirically impossible because of the significant operation of the unofficial sector in the economy. The ratio of FCD over broad money (M2) as in Viseth (2002), or the proportion of M2 denominated in USD as in Calvo & Gramont (1992), is thus the most common proxy for the dollarization status of a country.

As shown by Ra (2008), dollarization[22] has been one of the thorniest issues in Latin America such as Argentina, Bolivia, Brazil, Mexico, Peru, and Uruguay, especially in the 1980s. From the early 1990s, dollarization has come out of the region to become a typical feature in transitional economies in Eastern Europe and the former Soviet Union. Vietnam, a transitional economy in Indochina, is not an exception. Dollarization in Vietnam can be traced back as far as the 1960s when the US officially participated in the Vietnam war and as a result, the USD was in wide use and circulation in the South in parallel with the South Vietnamese government's currency, while all foreign currencies were completely forbidden in Northern Vietnam. After the reunification of the country in 1975, however, holding of foreign currency was then ruled out in the whole country. It was not until the late 1980s that dollarization has come back as a nationwide phenomenon following the government's effort to curb hyperinflation during 1986-88. This bout of hyperinflation, which peaked 350% in 1988, was fuelled by a mistaken policy package called "Price - Salary - Currency"[23] and the corresponding dramatic depreciation of the exchange rate[24].

[21]Total liquidity, M2.

[22]A similar phenomenon is euroization.

[23]"Gia - Luong - Tien" in Vietnamese.

[24]Goujon (2006) shows that the depreciation is 20,000%, from 15 VND/USD at the end of 1985 to 3000

Dollarization in Vietnam in the early years of the open-door policy came from the "supply side" in the form of massive foreign currency inflows stemming mainly from sizable foreign direct investment, private transfers, and export earnings, as argued by Zamaróczy & Sa (2003). In addition, given a lack of confidence in the Vietnamese currency on the "demand side" and an inappropriate conduct of monetary policy, these large foreign currency inflows provided the impetus for speedy dollarization in the following years. In particular, the milestone in the dollarization process was in 1989 when bold reform policies were implemented with the removal of price controls, the unification of the exchange rate regimes, and especially the introduction of foreign currency deposits. At the same time, interest rate on domestic currency-denominated deposits was raised accordingly which helped bring about a spectacular decline in inflation, back to 35% in 1989. It is this open-door policy for USD deposits and the loss of public confidence in the local currency that marked the beginning of a period of serious dollarization in Vietnam.

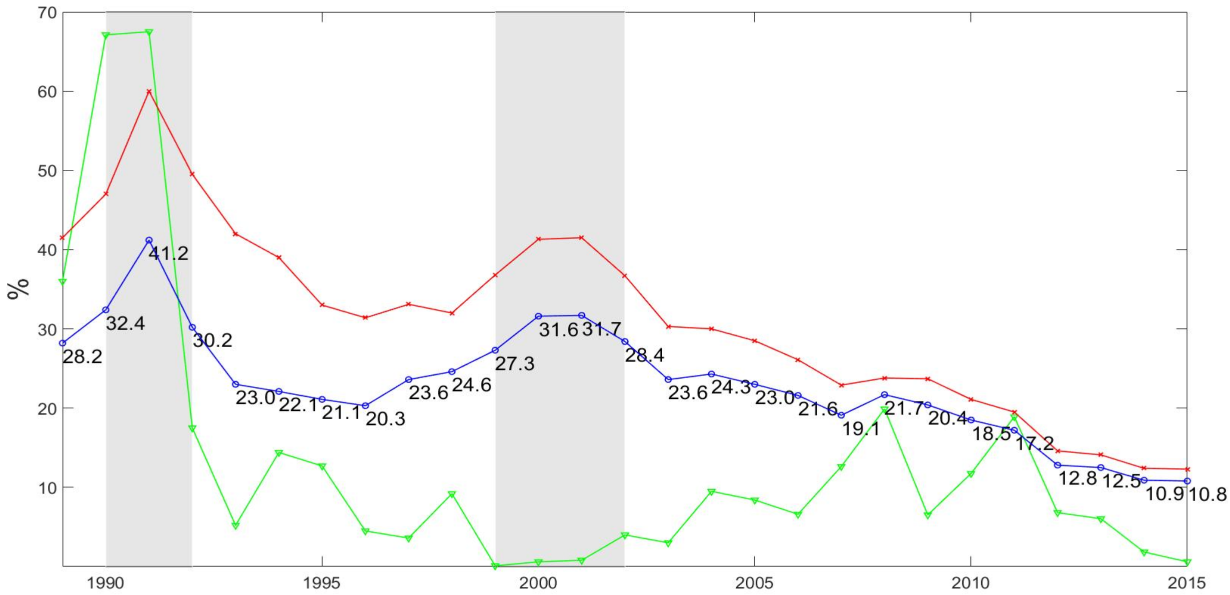
However, as argued by Goujon (2006), a rapid money creation in the next three years (1989-91) under weak domestic credit control resulted in another surge of inflation[25], a further 367% depreciation of the domestic currency against the USD, and notably a widespread dollarization in the economy. The following years witnessed a short period of dollarization reversion together with a moderate average inflation rate of about 10% as a result of, as pointed out by Guillaumont Jeanneney (1994), a large USD sales in the foreign exchange market in 1992 which led to a 25% appreciation of the domestic currency, better control of monetary expansion, and the official policy of pegging the domestic currency to

VND/USD at the end of 1988.

[25]67% in 1990 and 72% in 1991.

the USD. Figure 1.7 shows a general picture of dollarization in Vietnam since the Doi Moi in the 1980s. The blue line depicts the ratio between foreign currency deposits (FCD) and the overall money supply (M2) as a proxy for dollarization status in Vietnam, while the red line shows the ratio of FCD over total deposits. The green line indicates annual inflation rates in the same period. The level of dollarization basically reached its peak of 41.2% in 1991, and remained in the range from 10-30% during the whole period except another comeback at 31.7% in 2001.

Figure 1.7: Dollarization in Vietnam



Source: International Monetary Fund and Fulbright Economics Teaching Program.

While figures on FCD tell us an official picture of dollarization in Vietnam over the past three decades, the use of foreign currencies in reality, mostly USD, has become more and more substantial in the economy over the same period as a medium of exchange and unit of account throughout the country. Pham (2018) shows that in Vietnam, the USD has been used in quotation for most durable goods such as automobiles, motorbikes, radios, televisions, laptops, and real estate. Services in many cases are also quoted in USD, including hotel rooms, spa services, and tuition fees at some universities.

In theory, dollarization can help prevent currency and balance of payments crises, and reduce the costs of further integration into the world economy. In addition, dollarization can be employed in some countries as a means to restrain inflation and promote the roles of domestic financial institutions in stimulating domestic as well as foreign investment. On the other hand, however, dollarization in developing countries, to some extent, causes macroeconomic instabilities and deprives the central bank of its lender of last resort function, as pointed out by Hauskrecht & Hai (2004). In other words, the central bank's role as a money controller weakens, posing more systemic risks to the financial system.

Dollarization in developing and transitional economies has attracted a number of researchers in the field. Two main groups of research questions include, first, what are the determinants of dollarization or the use of the USD in a country? And second, what are the differences in key economic/monetary policy performance[26] between a non-dollarized and a dollarized economy? A majority of studies on dollarization pay attention to the reasons why a country has to take to the use of USD in place of domestic currency. Most of them agree that dollarization is usually one of the ultimate consequences of high inflation, as in Calvo & Gramont (1992), Civcir (2005), and Hauskrecht & Hai (2004). Another key determinant, as shown by Vega et al. (2012) and Basso et al. (2007), is the interest rate differentials between USD and the domestic currency. The trade-off between inflation and the real exchange rate variability is also found by Basso et al. (2007) to be

[26]Key variables in consideration include inflation, growth, exchange rate/interest rate pass-through, determinants of inflation.

a significant factor explaining dollarization. Other important determinants, as shown by Civcir (2005) and Terrones & Catão (2000), include relative rates of return of domestic and foreign currency denominated assets, expected change in the exchange rate, structural factors related to costly banking, credit market imperfections, the availability of tradable collateral, and the credibility of current economic policies.

On the consequence of dollarization, Vega (2012) shows that in an economy characterized by partial dollarization, the wedge between foreign and domestic currency lending rates is decreasing while the degree of correlation between negative returns and the exchange rate is increasing in periods of exchange rate volatility. As argued by Calvo & Gramont (1992), dollarization aggravates the initial recession in a money-based stabilization and makes the initial boom more pronounced in an exchange rate-based stabilization. Furthermore, dollarization is likely to increase financial instability in dollarized economies, as found by De Nicoló et al. (2003), and also depresses bank performance as well as lowers bank profitability, as in Kutan et al. (2012). Edwards (2001) uses a panel from 1970-98 to compare the economic performance of 11 dollarized economies with non-dollarized counterparts in Latin America. He finds that, except for experiencing significantly lower inflation, dollarizers have not had a more successful fiscal performance than non-dollarizers, nor been spared major current account reversals, and their growth rates have also been lower. Similar results are found by Edwards & Magendzo (2003) where dollarized countries have had lower rates of inflation but, at the same time, statistically lower rates of GDP per capita growth. In addition, exchange rate pass-through to prices in highly dollarized countries is significantly larger and more persistent, reinforcing the claim that "fear of floating" is a greater problem for highly dollarized economies, as argued by Reinhart et al. (2003).

Also, Alvarez-Plata & Garcia-Herrero (2008) show that dollarization can actually increase the pass-through of the exchange rate to prices and require larger monetary aggregates, including foreign currency, to be monitored. It leads to large currency mismatches due to the immediate impact of exchange rate depreciation on foreign currency-denominated liabilities. Watanabe (2007) argues that higher dollarization deters financial development through two channels. The first one is a standard self-insurance mechanism that weakens the incentives of commercial banks to screen and monitor private borrowers. The second one is an informal remittance channel that keeps dollarization higher. Bellocq & Silve (2008) also warns that in this case, the capacity of the central bank to regulate the foreign currency liquidity risk and stabilise the financial system is very limited.

In line with other developing countries, inflation and international reserves are found to be the first and foremost determinants of dollarization in Vietnam, as in Anh (2018), Hauskrecht et al. (2004), and Nguyen & Pfau (2010). However, as Vietnam is a country in transition from a centrally planned toward a market economy, dollarization has also been associated with some specific causes. Hauskrecht & Hai (2004) show that the limited ability to borrow domestically and abroad in domestic currency has been causal for the dollarization of the Vietnamese economy. Pham (2018), by employing a VAR model, argues that parallel market premium is the key determinant of dollarization in Vietnam, in addition to the ceiling interest rate for USD deposit and interest rate differentials in the economy. The rate of domestic currency depreciation is also found to be significant in the shift toward USD with 3-4 month lag, as in Vuong (2003).

In a nutshell, while a large literature has focused on the determinants of dollarization in developing countries and its impact on the performance of key macroeconomic variables, little has paid attention to empirical assessment of the design and transmission mechanism of monetary policy in dollarized economies. This study not only adds empirically significant evidence on the optimal conduct of monetary policy, but also contributes to providing a detailed analysis on the dynamics of the economy under optimal policy framework with particular reference to Vietnam.

Chapter 2

A DSGE Model for Developing Countries

In this chapter, a Dynamic Stochastic General Equilibrium (DSGE) model for an open, developing economy is set up with key blocks from standard DSGE models. A number of nominal and real frictions, including sticky prices, sticky wages, variable capital utilization, capital adjustment costs and habit persistence, are also included in the model. Workers, importers and domestic intermediate good producers are monopolistic competitors, facing wage and price sluggishness *à la* Calvo (1983). It should be noted that partial dollarization, a common characteristic in developing world, is a key feature of the present model. The theoretical model is also simulated to analyse the role of each friction and dollarization feature in driving the development of the model economy.

2.1 Introduction

Christiano et al. (2018) argue that the outcome of any important macroeconomic policy change is the net effect of forces operating on different parts of the economy. A central challenge, therefore, is how to evaluate the relative strength of those forces and Dynamic Stochastic General Equilibrium (DSGE) models, among a range of tools, are the leading tool for such exercises in an open and transparent manner. Since the seminal paper published in 1982 by Kydland & Prescott (1982) in which, as shown by Fernández-Villaverde (2010), for the first time macroeconomists could build a small and coherent dynamic model of the economy with optimizing agents, rational expectations, and market clearing, New Keynesian DSGE models have become the standard workhorse for quantitative analysis of policies as they can help evaluate the desirability of different policy strategies and of institutional development.

A large literature has been devoted to elaborating the significant role of economic policy in minimizing welfare distortions caused by imperfections and rigidities, which are the crucial distinction of the Keynesian perspective that real economies are not perfectly flexible nor perfectly competitive. As shown by Tovar (2009), DSGE models can help to identify sources of fluctuations, answer questions about the impact of structural changes, forecast and predict the effect of policy changes, elucidate the sources of macroeconomic fluctuations, and perform counterfactual experiments. However, benchmark DSGE models have paid little attention to specific features of developing countries which, as a consequence, may fail to explain important regularities of the business cycle in developing world.

In developing key building blocks of a DSGE model for developing countries, we have borrowed important insights from Christiano et al. (2005), Adolfson et al. (2005), Adolfson et al. (2007), Altig et al. (2011), and Erceg et al. (2000). Particularly, price-maker feature of firms, which is a prerequisite for price stickiness and is opposed to price-taker property in a perfectly competitive world, can be achieved in a tractable way by the monopolistic competition approach of Dixit & Stiglitz (1977). Nominal price inertia also requires some form of staggered prices and the Calvo (1983) formulation is used as it has become standard because of its advantage in aggregation. A similar persistence mechanism on wage setting suggested by Erceg et al. (2000) is also used for the formulation of the present model. Additional frictions which have been proposed in recent years to improve the theoretical and empirical adequacy of DSGE models such as habit formation in consumption preferences, adjustment costs in investment, and variable capital utilization by Christiano et al. (2005) are also explicitly incorporated in the model.

Therefore, the demand side of the model is composed of households consuming a basket of domestically produced goods and imported goods supplied by domestic and importing firms. Households can save in domestic bonds, foreign bonds and hold cash and their choice can be balanced into an arbitrage condition[1]. In the domestic market, they can also rent capital to domestic firms subject to a capital investment adjustment cost. In addition, households are monopoly suppliers of their differentiated labour services so they can set their own wage, leading to an explicit wage equation. On the supply side, there are firms in the domestic and external sectors. Intermediate good firms produce differentiated goods and index their prices with Calvo-type setting, given their capital and labour inputs. Following Adolfson et al. (2005), we include in the model a stochastic unit-root

[1] The uncovered interest rate parity (UIP) condition.

technology shock which can induce a common stochastic trend in aggregate quantities so as to render stationarity of all model variables.

However, we depart from Adolfson et al. (2005), Christiano et al. (2005), and Altig et al. (2011) by incorporating characteristics typical of a developing economy. In Vietnam as well as in many other developing countries, there is an "original sin" that governments have to take to official borrowing in foreign currency, mostly USD, because of domestic fiscal strains. This problem is acute and particularly so when oil prices are high. Fiscal strains and reputational concerns create the need for fiscal rule. In addition, fiscal strains also create suspicion that the authorities might turn to the printing press, in emergencies, strengthening firms' and households' demand for assets in foreign currency as a hedge, leading to partial dollarization. Therefore, the present model will first feature, as in Vietnam, a chronic budget-deficit fiscal rule where fiscal authority pursues an explicit target for the primary deficit-to-GDP ratio. Second, partial dollarization is also explicitly included in the model. Recent literature has identified two forms of partial dollarization which are *currency substitution* and *asset substitution*. In a currency substitution regime, the domestic currency is partially replaced by a foreign currency in its function as a medium of payment, while with asset substitution, the domestic currency is partially substituted by USD as unit of account/store of value. We introduce asset substitution to the model economy by exogenously assuming that a subset of domestic firms set their prices in USD. Thus in this case, the domestic Phillips curve will be a weighted combination of the Phillips curves for firms setting their prices in domestic currency and those setting their prices in USD.

This chapter is organized as follows. We present the theoretical DSGE model with particular emphasis on developing economy aspects in Section 2. In Section 3, the theoretical model is parameterized and the conditions for determinacy is examined. Section 4 simulates the model so as to facilitate the sensitivity analysis of frictions as well as dollarization feature set up in the theoretical model. A summary of the chapter is given in Section 5.

2.2 The DSGE Model for Developing Countries

2.2.1 Households

The households in the economy enjoy utility from consumption while receiving disutility from working. They are also suppliers of labour in the economy. Households rent capital to the domestic firms and decide how much to invest in the capital stock given certain capital adjustment costs. These are costs to adjusting the investment rate as well as costs of varying the utilization rate of the capital stock. Each household is a monopoly supplier of a differentiated labour service which implies that they can set their own wage.

First, following Adolfson et al. (2007), a permanent technology shock ϵ_t^z is explicitly specified in the model in order to render stationarity in all model variables. The growth process for this trend level of technology is exogenously given by

$$\mu_t^z = \rho_{\mu_z} \mu_{t-1}^z + \varepsilon_t^z \tag{2.1}$$

where $\mu_t^z = \epsilon_t^z / \epsilon_{t-1}^z$ and ε_t^z is the innovation.

Second, the model economy is comprised of a continuum of households indexed by $j \in (0,1)$. Following Christiano et al. (2005), the households are assumed to obtain utility from consumption, leisure and cash balances and thus have to choose between their current level of consumption, amount of cash holdings, foreign bond holdings, and domestic deposits in order to maximize their intertemporal utility. In addition, the households need to make decisions on the level of capital services provided to the firms, level of investment and capital utilization. The preferences of a generic household j are given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[u(C_{j,t} - bC_{j,t-1}) - f^1(h_{j,t}) + f^2(q_{j,t})\right] \tag{2.2}$$

with the corresponding functional forms for utility, labour, and cash holdings as follows

$$u(.) = \ln(.); f^1(.) = \Lambda_h \frac{(.)^{1+\sigma_h}}{1+\sigma_h}; f^2(.) = \Lambda_q \frac{(.)^{1-\sigma_q} - 1}{1-\sigma_q} \tag{2.3}$$

In addition, following Adolfson et al. (2007), consumption preference shocks ϵ_t^c, labour supply ϵ_t^h, and money demand shock ϵ_t^q assumed to follow AR(1) processes are also included to obtain the household's preferences of the form

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\epsilon_t^c \ln(C_{j,t} - bC_{j,t-1}) - \epsilon_t^h \Lambda_h \frac{h_{j,t}^{1+\sigma_h}}{1+\sigma_L} + \epsilon_t^q \Lambda_q \frac{q_{j,t}^{1-\sigma_q} - 1}{1-\sigma_q}\right] \tag{2.4}$$

where utility of the j^{th} household depends positively on consumption $C_{j,t}$, which is also relative to an internal habit persistence included through parameter b, and $q_{j,t}$ - the household's real assets in non-interest bearing form/cash holdings in domestic currency after being rendered stationary[2] with the trend level of technology ϵ_t^z. It also depends negatively on the household's labour supply $h_{j,t}$. Parameters σ_h and σ_q show the labour supply and cash holding elasticities. Λ_h and Λ_q are constants in the labour disutility function and cash holding utility function.

[2] $q_{j,t} = Q_{j,t}/P_t\epsilon_t^z$, where $Q_{j,t}$ are nominal assets, $Q_{j,t}/P_t$ are real assets.

Third, following Arrow et al. (1961), which proposed the CES function, and Dixit & Stiglitz (1977) who proposed the variety of models of consumer behaviour, the aggregate consumption of households is assumed to be given by a Constant Elasticity of Substitution (CES) index of a continuum of differentiated goods, produced, respectively, domestically and from abroad as follows:

$$C_t = \left[(1-\omega_c)^{1/\eta_c}(C_t^d)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c}(C_t^m)^{(\eta_c-1)/\eta_c}\right]^{\eta_c/(\eta_c-1)} \tag{2.5}$$

where C_t^d and C_t^m are domestic and imported consumption; ω_c and η_c are the share of imports in consumption and the elasticity of substitution between domestic and imported consumption goods, respectively.

Demands for domestic consumption and imported consumption are given by

$$C_t^d = \left(\frac{P_t}{P_t^c}\right)^{-\eta_c} C_t \tag{2.6}$$

$$C_t^m = \left(\frac{P_t^{m,c}}{P_t^c}\right)^{-\eta_c} C_t \tag{2.7}$$

Fourth, the households' capital services K_t can be increased by their investing an amount of I_t in the additional physical capital $\bar{K}_t$, taking one period to come into action, or by directly raising the utilization rate of the *physical*[3] capital stock at hand ($K_t = u_t\bar{K}_t$) where u_t is the utilization rate[4] that, according to Christiano et al. (2005), satisfies $u = 1$

[3] *Physical* capital implies all factors of production (or input into the process of production), such as machinery, buildings, or computers etc. Thus, capital services are the factors of production in operation which define the capital utilization rate u_t.

[4] Christiano et al. (2005) point out three measures of capital utilization: first, the intensity with which all factors of production are used in the industrial production sector; second, time series on electricity

at steady state[5] and the capital utilization cost function $f(u_t)$ at steady state is zero[6].

Total investment is also assumed to follow a CES function of the form

$$I_t = \left[(1-\omega_i)^{1/\eta_i}(I_t^d)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i}(I_t^m)^{(\eta_i-1)/\eta_i}\right]^{\eta_i/(\eta_i-1)} \tag{2.8}$$

where I_t^d and I_t^m are purchases of domestic and imported investment goods, respectively; ω_i is the share of imports in investment; η_i is the elasticity of substitution between investment goods.

Demands for domestic investment and imported investment are given by

$$I_t^d = \left(\frac{P_t}{P_t^i}\right)^{-\eta_i} I_t \tag{2.9}$$

$$I_t^m = \left(\frac{P_t^{m,i}}{P_t^i}\right)^{-\eta_i} I_t \tag{2.10}$$

Following Altig et al. (2011), the technology that transforms current and past investment into physical capital is specified by the equation

$$f^3(I_t, I_{t-1}) = [1 - f^4(I_t/I_{t-1})]I_t \tag{2.11}$$

where function f^4 is assumed to be increasing, convex and satisfies $f^4(1) = f^{4\prime}(1) = 0$, $f^{4\prime\prime}(1) > 0$ at steady state. We define $\zeta \equiv f^{4\prime\prime}(1)$ as the investment adjustment cost

consumption in the industrial production (assuming capital services and electricity are used in fixed proportions); and third, the workweek of capital as measured by average hours worked.

[5] All capital in operation.

[6] $f(1) = 0$ or no cost for capital utilization at steady state. The fraction $1/\sigma_a$, as argued by Altig et al. (2011), is taken as the elasticity of capital utilization with respect to the rental rate of capital R_t^k, where $\sigma_a = f''(1)/f'(1)$.

parameter.

Therefore, the law of motion for the households physical capital stock is given by

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \epsilon_t^i f^3(I_t, I_{t-1}) + \vartheta_t \tag{2.12}$$

where ϵ_t^i is a stationary investment-specific technology shock[7]; and ϑ_t is the capital-market-access variable reflecting the accessibility to a market where households can purchase new physical capital $\bar{K}_{t+1}$. As argued by Adolfson et al. (2007), as all households are identical, the model is in equilibrium only if $\vartheta_t = 0$.

Fifth, in addition to accumulating physical capital and holding cash, following Adolfson et al. (2005), the households can save in domestic and foreign bonds with a *risk-premium* on foreign bond holdings defined as $f^5(a_t, \varphi_t)$, where the *real aggregate net foreign asset position* of the domestic economy after being scaled with the trend level of technology ϵ_t^z is given by

$$a_t \equiv \frac{S_t B_{t+1}^*/P_t}{\epsilon_t^z} \tag{2.13}$$

Following Schmitt-Grohé & Uribe (2001) and Adolfson et al. (2005), a risk-premium on foreign bond holdings is also assumed to be strictly decreasing in net foreign asset

[7]Greenwood et al. (1992) introduced an investment-specific technology shock to describe the technological change that is specific to or embodied in new capital goods to distinguish with the "newtral" (disembodies) technology shock, proposed by neoclassical growth models as in Solow (1957), Kydland & Prescott (1982), and Long Jr & Plosser (1983), that allows all goods to be produced more efficiently.

position, to satisfy $f^5(0,0)=1$, and is given by the functional form of[8]

$$f^5(a_t,\varphi_t)=\exp(-\Lambda_a(a_t-\bar{a})+\varphi_t) \tag{2.14}$$

where φ_t is a time-varying shock to the risk premium; Λ_a is a constant[9]. This implies that domestic households have to pay a premium over the exogenous foreign interest rate R_t^* if the domestic economy as a whole is a net borrower ($B_t^*<0$), and receive a lower remuneration on their savings if the domestic economy is a net lender ($B_t^*>0$).

Therefore, the evolution equation for the households' assets is given by

$$\begin{aligned} M_{j,t+1}+S_tB_{j,t+1}^*+P_t^cC_{j,t}(1+\tau_t^c)+P_t^iI_{j,t}+P_t[f(u_{j,t})\bar{K}_{j,t}+P_t^k\vartheta_t] \\ =R_{t-1}(M_{j,t}-Q_{j,t})+Q_{j,t}+(1-\tau_t^k)\Pi_t+(1-\tau_t^y)\frac{W_{j,t}}{1+\tau_t^w}h_{j,t} \\ +(1-\tau_t^k)R_t^ku_{j,t}\bar{K}_{j,t}+R_{t-1}^*f^5(a_{t-1},\varphi_{t-1})S_tB_{j,t}^*+D_{j,t} \\ -\tau_t^k\left[(R_{t-1}-1)(M_{j,t}-Q_{j,t})+(R_{t-1}^*f^5(a_{t-1},\varphi_{t-1})-1)S_tB_{j,t}^*+B_{j,t}^*(S_t-S_{t-1})\right] \end{aligned} \tag{2.15}$$

The left-hand-side of the equation shows how the households use their resources. $M_{j,t+1}$ denotes total assets including both interest-bearing and non-interest-bearing assets while $S_tB_{j,t+1}^*$ is foreign bond holdings. These stock variables are calculated at end-of-period

[8]According to Schmitt-Grohé & Uribe (2001), the size of the risk premium is assumed to be increasing in the country's stock of foreign debt. Thus, the "No Ponzi Game" condition in which the households are solvent if the present value of their long-run liability is non-positive can be satisfied.

[9]It is assumed that $\Lambda_a,\bar{a}\geq 0$. According to Schmitt-Grohé & Uribe (2001), the introduction of the risk-premium on foreign bond holdings is also for a technical issue because, as is well known, small open economies facing a purely exogenous world interest rate display nonstationary dynamics in response to stationary exogenous shocks. As a result, the solution to the log-linearized equilibrium conditions may not be a valid approximation to the exact, nonlinear equilibrium system. Thus, the introduction of a variable risk-premium on foreign bond holdings can eliminate any source of nonstationarity.

so value at time t should be indexed $t+1$. $P_t^c C_{j,t}(1+\tau_t^c)$, $P_t^i I_{j,t}$, and $P_t f(u_{j,t})\bar{K}_{j,t}$ are the households' consumption, nominal resources spent on investment goods, and capital adjustment costs with the utilization cost function $f(u_t)$, respectively, where τ_t^c is a consumption tax. As physical capital stock is owned by the households, they have to pay capital adjustment costs[10]. The cost for capital market access paid by households is $P_t P_t^k \vartheta_t$ where P_t^k is the price of capital.

On the right-hand-side of the equation, the resources households have at their disposal include $Q_{j,t}$, Π_t, and $D_{j,t}$ which are non-interest bearing assets, profits, and net cash income (from holding cash) from participating in state contingent securities at time t, respectively. In addition, the households' current assets can also be financed by income on interest-bearing assets from previous period $R_{t-1}(M_{j,t} - Q_{j,t})$, in which $(M_{j,t} - Q_{j,t})$ is nominal domestic assets that are not held in cash[11], and $R_t = 1 + r_t$ is the gross interest rate; labour income $(1-\tau_t^y)\frac{W_{j,t}}{1+\tau_t^w}h_{j,t}$ where τ_t^w and τ_t^y are pay-roll tax and labour-income tax; income from physical capital stock $(1-\tau_t^k)R_t^k u_{j,t}\bar{K}_{j,t}$, with capital-income tax τ_t^k and gross nominal rental rate per unit of capital services R_t^k; and income from foreign bond holding from previous period $R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) S_t B_{j,t}^*$, where $R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1})$ is the risk-adjusted pre-tax gross interest rate from holding foreign bond. However, households also have to pay capital-income taxes for the profits they gain from interest-bearing assets $\tau_t^k(R_{t-1} - 1)(M_{j,t} - Q_{j,t})$, foreign bond holding from previous period $\tau_t^k(R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) - 1) S_t B_{j,t}^*$, and what they get from exchange rate differences

[10] $f(u_t)P_t$ in the budget constraint.

[11] The households' nominal domestic assets not held in cash earn the interest rate of R_{t-1} as we can think of an asset paying out a nominal amount with certainty, or as argued by Adolfson et al. (2005), simply a zero-coupon bond paying out B_t at period t that can be bought at price $1/R_{t-1}$ at period $t-1$.

$\tau_t^k B_{j,t}^*(S_t - S_{t-1})$.

Finally, following Erceg et al. (2000), Adolfson et al. (2005), and Christiano et al. (2005), we assume that each household is a monopoly supplier of a differentiated labour service. The wage is then determined by the households, given a stickiness that is introduced *à la* Calvo (1983).

Each household sells its labour, $h_{j,t}$, to an aggregator which transforms household labour into a homogeneous input good H_t using the production function of Dixit & Stiglitz (1977)'s form as follows

$$H_t = \left[\int_0^1 (h_{j,t})^{1/\lambda^w} dj \right]^{\lambda^w} \tag{2.16}$$

where the wage markup $\lambda^w \geq 1$.

In any period t, there is a probability of $1 - \xi^w$ that a household j can renegotiate its wage contract to have a new wage rate $\tilde{W}_t$. However, in the case when it is not allowed to renegotiate, following Adolfson et al. (2005), household j automatically indexes its wage to last period's CPI inflation, current inflation target, and current permanent technology growth rate as follows

$$\tilde{\tilde{W}}_t = (\pi_{t-1}^c)^{\kappa^w} (\bar{\pi}_t)^{1-\kappa^w} \mu_{z,t} W_{t-1} \tag{2.17}$$

where κ^w is the indexation parameter; π_t^c is the CPI inflation rate in the previous period; $\bar{\pi}_{t+1}$ is the current inflation target.

If household j is not allowed to change its wage during s periods ahead, the index for its wage in period $t+s$ is given by

$$\tilde{W}_{t+s} = (\pi_t^c ... \pi_{t+s-1}^c)^{\kappa^w} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^w} (\mu_{z,t+1} ... \mu_{z,t+s}) \tilde{W}_t \tag{2.18}$$

Therefore, in any period t, the j^{th} household that is able to reset its contract wage maximizes its utility function with respect to the wage rate $\tilde{W}_{j,t}$ as follows

$$\max_{\tilde{W}_t} E_t \sum_{s=0}^{\infty} (\beta \xi^w)^s \left\{ \begin{array}{l} -\epsilon_{t+s}^h \Lambda_h \frac{(h_{j,t+s})^{1+\sigma_h}}{1+\sigma_h} \\ +n_{t+s} \frac{1-\tau_{t+s}^y}{1+\tau_{t+s}^w} \left[(\pi_t^c ... \pi_{t+s-1}^c)^{\kappa_w} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa_w} (\mu_{z,t+1} ... \mu_{z,t+s}) \tilde{W}_t \right] h_{j,t+s} \end{array} \right\} \tag{2.19}$$

taking into account the possibility ξ^w that the wage will not be reoptimized in the future and the marginal disutility of labour.

However, following Smets & Wouters (2003) and Adolfson et al. (2005), we need to use employment instead of aggregate hours worked due to the lack of these data for the sample country in order to perform Bayesian estimation. In addition, as they also point out, employment is in general likely to be stickier than total hours worked in response to macroeconomic shocks. Thus, we assume that only a constant fraction $1-\xi^e$ of the firms is assumed to be able to adjust the level of employment to their desired amount of total labour input $\tilde{\Psi}_{i,t}$. The rest of the firms ξ^e have to maintain the level of employment as in the previous period. The difference will be taken up by unobserved hours worked per employee because, following Adolfson et al. (2005), each employee is assumed to supply his labour inelastically after having set his wage. Or as argued by Smets & Wouters (2003), as hours worked is assumed to be perfectly flexible, the overall labour input will not be affected by the rigidity in employment.

Firm i is assumed to face the following optimization problem

$$\min_{\tilde{\Psi}i,t} \sum_{s=0}^{\infty} (\beta\xi^e)^s (l_i \tilde{\Psi}_{i,t} - H_{i,t+s})^2 \tag{2.20}$$

where l_i is hours per worker in firm i, $H_{i,t+s}$ is hours worked.

2.2.2 Firms

The model consists of two main categories of firms. There are intermediate good and final good firms, operating in two sectors of an open economy. In the domestic sector, intermediate firms produce differentiated goods, using capital services and labour inputs only, and set their prices according to an indexation variant of the Calvo (1983)'s model, while final good firms use a continuum of these intermediate goods to produce a homogeneous final good which can be used for consumption and investment by the households.

In the external sector, each of the intermediate importing firms buys a homogeneous good at price P_t^* in the world market and converts it into a differentiated import good, either for consumption or investment, through a brand naming technology before selling it to final imported-consumption firm or final imported-investment firm. Also, each of the exporting firms buys the domestic final good at price P_t and differentiates it by brand naming to sell to foreign households. Thus, each of them is a monopolistic supplier of a specific product in the world market.

Domestic Sector

Final Good Firms

Following Erceg et al. (2000), it is convenient to abstract from the household's problem of choosing the optimal quantity of each differentiated good $Y_{i,t}$ as households are assumed to have identical preferences. At time t, final good firms combine and transform a continuum of intermediate goods, indexed by $i \in (0, 1)$ into a homogeneous final good, which can be used for consumption and investment by the households[12]. The final good sector is perfectly competitive.

The output is assembled by a representative final good firm using a technology of the Dixit & Stiglitz (1977) form as follows

$$Y_t = \left[\int_0^1 Y_{i,t}^{1/\lambda_t^d} di \right]^{\lambda_t^d} \tag{2.21}$$

in which $Y_{i,t}$ denotes time t input of intermediate good i and $\lambda_t^d \geq 1$ is the time-varying markup of firm i in the domestic market.

In a perfectly competitive market, the final good firm is a price taker and thus output price P_t as well as input prices $P_{i,t}$ are taken as given. The final good firm sells units of the output at their unit cost P_t given by

$$P_t = \left[\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_t^d}} di \right]^{1-\lambda_t^d} \tag{2.22}$$

where, as argued by Erceg et al. (2000), P_t can be interpreted as the aggregate price index.

The final good firm's demand for each good $Y_{i,t}$, or also as argued by Erceg et al.

[12]Also according to Erceg et al. (2000), the final good firm combines intermediate goods in the same proportions as the households would choose. Thus, the final good firm's demand for each differentiated good is equal to the sum of household demand.

(2000), equivalently total household demand for the good, is given by

$$Y_{i,t} = \left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_t^d}{\lambda_t^d - 1}} Y_t \tag{2.23}$$

Intermediate Goods Firms

The domestic market is also composed of a continuum of intermediate goods firms. These firms operate in a monopolistic competitive market and thus each of them is a monopolist in producing its own good and competitive in the markets for inputs.

Following Christiano et al. (2005) and Adolfson et al. (2005), an intermediate good $i \in (0,1)$ is produced given a Cobb-Douglas production function of the form

$$Y_{i,t} = (\epsilon_t^z)^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - \epsilon_t^z \phi \tag{2.24}$$

where $0 < \alpha < 1$ and a fixed cost of production ϕ is included and set to make sure that profits are zero in steady state. $H_{i,t}$ and $K_{i,t}$ are homogeneous labour and capital services[13] used by the ith firm in its production, respectively. The function also includes the permanent technology shock ϵ_t^z and a domestic covariance stationary technology shock ϵ_t.

As workers must be paid in advance of production, a fraction ν_t of the intermediate firms is assumed to have to borrow to pay their wage bills at a gross nominal interest rate R_t^w. Thus, the end-of-period labour costs of the firm are $W_t R_t^w H_{i,t}$, where W_t is the nominal wage rate per unit of aggregate and homogeneous labour $H_{i,t}$ and R_t^w is given by an index of the economy-wide gross nominal interest rate R_{t-1} as follows

$$R_t^w = \nu_t R_{t-1} + 1 - \nu_t \tag{2.25}$$

[13]Which can be different from physical capital stock as there is also variable capital utilization in the model.

The capital costs of firm i are $R_t^k K_{i,t}$ where $K_{i,t}$ is the capital services stock and R_t^k, as defined above, is the gross nominal rental rate per unit of capital services. Given the assumption that price for intermediate good i, $P_{i,t}$, is given, the firm is then constrained to produce $Y_{i,t}$ and thus the cost minimization problem faced by the i-th intermediate firm is given by[14]

$$\min_{K_{i,t},H_{i,t}} W_t R_t^w H_{i,t} + R_t^k K_{i,t} + \lambda_t P_{i,t}\left[Y_{i,t} - (\epsilon_t^z)^{1-\alpha}\epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} + \epsilon_t^z \phi\right] \quad (2.26)$$

Each of the intermediate good firm is subject to price stickiness through an indexation variant of Calvo (1983). In any period, the probability that a given price can be reoptimized at a new price, denoted by $\tilde{P}_t$, is constant and equal to $1 - \xi^d$.

For the remaining firms with probability ξ^d of being not allowed to reoptimize, following Adolfson et al. (2005), their price is then indexed to last period's inflation π_{t-1} and the current inflation target $\bar{\pi}_t$

$$\tilde{\tilde{P}}_t = \pi_{t-1}^{\kappa^d}(\bar{\pi}_t)^{1-\kappa^d} P_{t-1} \quad (2.27)$$

where κ^d is a parameter for partial indexation. From equation 2.22, the aggregate price in period t is now given by an index of $\tilde{P}_t$ and $\tilde{\tilde{P}}_t$ as follows[15]

$$P_t = \left[\left(\int_0^{\xi^d} (\tilde{\tilde{P}}_t)^{\frac{1}{1-\lambda_t^d}} + \int_{\xi^d}^1 (\tilde{P}_t)^{\frac{1}{1-\lambda_t^d}}\right) di\right]^{1-\lambda_t^d} \quad (2.28)$$

[14]The Lagrangian $\lambda_t P_{i,t}$ can be taken as the nominal marginal cost of firm i, $MC_{i,t}$ and λ_t is the real marginal cost.

[15]All firms that can reoptimize will always set the same price $\tilde{P}_t$ and those that are not allowed to reoptimize set the same price $\tilde{\tilde{P}}_t$. While aggregate price P_t is an index of prices of optimized and non-optimized firms, prices for consumption and investment (P_t^c and P_t^i) are indices of domestic consumption/investment goods prices (both at P_t) and imported consumption/investment goods prices $P_t^{m,c}$ and $P_t^{m,i}$.

If a firm is not allowed to change its price during s periods ahead, or in other words, the optimized price today will remain the same in the next s periods, then the price in period $t+s$ will be an index of the optimized price today as follows

$$\tilde{P}_{t+s} = (\pi_t ... \pi_{t+s-1})^{\kappa^d} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^d} \tilde{P}_t \tag{2.29}$$

Thus, firm i has to solve the following optimization problem, taking into account that there might not be a chance to optimally adjust the price, in order to set its price

$$\max_{\tilde{P}_t} E_t \sum_{s=0}^{\infty} (\beta\xi^d)^s n_{t+s} \left\{ \left[(\pi_t ... \pi_{t+s-1})^{\kappa^d} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^d} \tilde{P}_t \right] Y_{i,t+s} - MC_{i,t+s}(Y_{i,t+s} + \epsilon^z_{t+s}\phi) \right\} \tag{2.30}$$

This problem shows that the price set at time t by firm i is a function of expected future marginal costs and the firm also uses the stochastic discount factor $(\beta\xi^d)^s n_{t+s}$ to make profits conditional upon utility, in which β denotes the constant discount factor while n_{t+s} is the marginal utility of the households' nominal income in period $t+s$ that is exogenous to the intermediate firm.

External Sector

Final Importing Firms

The final homogeneous imported-consumption good or final homogeneous imported-investment good is produced by final imported-consumption firms or final imported-investment firms by combining and transforming a continuum of differentiated imported consumption/investment goods. The relationships between the demands for final homogeneous imported-consumption good and final homogeneous imported-investment good and the corresponding demands faced by each individual firm are given by CES functions of the form

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_t^{m,c}}} di \right]^{\lambda_t^{m,c}} \tag{2.31}$$

$$I_t^m = \left[\int_0^1 (I_{i,t}^m)^{\frac{1}{\lambda_t^i}} di \right]^{\lambda_t^{m,i}} \tag{2.32}$$

where $\lambda_t^{m,c}$ and $\lambda_t^{m,i}$ are time varying markups on the imported consumption/investment good.

Intermediate Importing Firms

Each of a continuum of intermediate importing firms imports a homogeneous good at price P_t^* in the world market and converts it into a differentiated good, either a consumption good $C_{i,t}^m$ or an investment good $I_{i,t}^m$, through a brand naming technology before selling it to the final imported-consumption firm or final imported-investment firm.

Price stickiness in the external sector is also introduced following Calvo (1983). Each imported-consumption/investment firm faces a probability $1 - \xi^{m,c}$ and $1 - \xi^{m,i}$ that it can reoptimize its price at $\tilde{P}_t^{m,c}$ and $\tilde{P}_t^{m,i}$, respectively, in any period[16]. As in the case of domestic intermediate firms, following Adolfson et al. (2005), importing firms that are not allowed to reoptimize with probability $\xi^{m,c}$ and $\xi^{m,i}$ then index their price to last period's inflation and current inflation target as follows

$$\tilde{\tilde{P}}_t^{m,c} = (\pi_{t-1}^{m,c})^{\kappa^{m,c}} (\bar{\pi}_t)^{1-\kappa^{m,c}} P_{t-1}^{m,c} \tag{2.33}$$

[16]Note that the superscript i denotes investment while the subscript i denotes firm i. Given the stickiness of the domestic prices, incomplete exchange rate pass-through to the prices of imported consumption/investment goods is assured even though importing firms of both kinds buy a homogeneous foreign good at the same price.

$$\tilde{\tilde{P}}_t^{m,i} = (\pi_{t-1}^{m,i})^{\kappa^{m,i}}(\bar{\pi}_t)^{1-\kappa^{m,i}} P_{t-1}^{m,i} \tag{2.34}$$

where $\kappa^{m,c}$ and $\kappa^{m,i}$ are indexation parameters.

If a period t optimizing firm is not allowed to change its price during s periods ahead, the price in period $t+s$ will be indexed to today's optimized price as follows

$$\tilde{P}_{t+s}^{m,c} = (\pi_t^{m,c}...\pi_{t+s-1}^{m,c})^{\kappa^{m,c}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,c}} \tilde{P}_t^{m,c} \tag{2.35}$$

$$\tilde{P}_{t+s}^{m,i} = (\pi_t^{m,i}...\pi_{t+s-1}^{m,i})^{\kappa^{m,i}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,i}} \tilde{P}_t^{m,i} \tag{2.36}$$

The intermediate imported-consumption firms have to solve the following profit optimization problem to set prices

$$\begin{aligned} \max_{\tilde{P}_t^{m,c}} E_t \sum_{s=0}^{\infty} (\beta\xi^{m,c})^s n_{t+s} [(\pi_t^{m,c}...\pi_{t+s-1}^{m,c})^{\kappa^{m,c}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,c}} \tilde{P}_t^{m,c} C_{i,t+s}^{m,c} \\ -S_{t+s} P_{t+s}^{*}(C_{i,t+s}^{m,i} + \epsilon_{t+s}^{z}\phi^{m,c})] \end{aligned} \tag{2.37}$$

And the objective function of intermediate imported-investment firms is

$$\begin{aligned} \max_{\tilde{P}_t^{m,i}} E_t \sum_{s=0}^{\infty} (\beta\xi^{m,i})^s n_{t+s} [(\pi_t^{m,i}...\pi_{t+s-1}^{m,i})^{\kappa^{m,i}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,i}} \tilde{P}_t^{m,i} I_{i,t+s}^{m,i} \\ -S_{t+s} P_{t+s}^{*}(I_{i,t+s}^{m,i} + \epsilon_{t+s}^{z}\phi^{m,i})] \end{aligned} \tag{2.38}$$

where the stochastic discount factors are $(\beta\xi^{m,c})^s n_{t+s}$ and $(\beta\xi^{m,i})^s n_{t+s}$, respectively; $\phi^{m,c}$ and $\phi^{m,i}$ are fixed costs.

Exporting Firms

A continuum of exporting firms buy the homogeneous domestic final good at price P_t, turn this into differentiated export goods and sell them to the households in the world market. In addition, we apply the same price-setting framework *à la* Calvo (1983) as in the intermediate importing firms. The new optimized export price is $\tilde{P}_t^x$, while an export firm which is not allowed to reoptimize its price is assumed to index the price to last period's export price inflation and the domestic inflation target as follows

$$\tilde{\tilde{P}}_t^x = (\pi_{t-1}^x)^{\kappa^x} (\bar{\pi}_t)^{1-\kappa^x} P_{t-1}^x \tag{2.39}$$

The i^{th} export firm's objective function is given by

$$\max_{\tilde{P}_t^x} E_t \sum_{s=0}^{\infty} (\beta\xi^x)^s n_{t+s} \left[(\pi_t^x ... \pi_{t+s-1}^x)^{\kappa^x} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^x} \tilde{P}_t^x X_{i,t+s} - \frac{P_{t+s}}{S_{t+s}} (X_{i,t+s} + \epsilon_{t+s}^z \phi^x) \right] \tag{2.40}$$

where $X_{i,t+s}$ is the demand for exporting firm i at t+s.

In a nutshell, after solving the objective functions of each type of firms and replacing individual demand by the aggregate demand, we come up with four New Keynesian Phillips curves which determine inflation in the domestic market, imported-investment market, imported-consumption market, and export market.

2.2.3 The Rest of the World

Also following Adolfson et al. (2005), the model is assumed to be a small open economy in comparison with the rest of the world and thus it plays a negligible part in the world's aggregate consumption and investment. The world's demands for aggregate domestic consumption and investment goods are given by

$$C_t^x = \left(\frac{P_t^x}{P_t^*} \right)^{\eta_f} C_t^* \tag{2.41}$$

$$I_t^x = \left(\frac{P_t^x}{P_t^*} \right)^{\eta_f} I_t^* \tag{2.42}$$

where C_t^*, I_t^* and P_t^* are foreign consumption, investment goods and price level, respectively. Parameter η_f denotes the elasticity of substitution which is assumed to be the same for consumption and investment so that we can take the world's output $Y_t^* = C_t^* + I_t^*$ as the only demand, be it consumption or investment, and do not have to take into consideration the amount of exporting goods used for consumption and investment in the world economy.

Foreign variables are assumed to follow AR(1) processes which describe the dynamics of the world income Y_t^*, inflation π_t^* and interest rate R_t^*. A stationary asymmetric technology shock $\tilde{\epsilon}_t^{z*} = \epsilon_t^{z*}/\epsilon_t^z$ is included to allow for temporary differences of technological progress domestically and abroad, where ϵ_t^{z*} is the total technology level abroad. Y_t^* is also scaled with ϵ_t^{z*} to get the stationarized version.

The equations for the rest of the world are as follows

$$\ln y_t^* = \ln y^* + \rho_Y(\ln y_{t-1}^* - \ln y^*) + \varepsilon_t^{Y^*} \tag{2.43}$$

$$\pi_t^* = \pi^* + \rho_{\pi^*}(\pi_{t-1}^* - \pi^*) + \varepsilon_t^{\pi^*} \tag{2.44}$$

$$R_t^* = R^* + \rho_{R^*}(R_{t-1}^* - R^*) + \varepsilon_t^{R^*} \tag{2.45}$$

2.2.4 Identities of the Open Economy

In an open economy, *real exchange rate (RER)* is one of the key relative prices between the domestic economy and foreign economy, which is given by

$$RER_t = \frac{S_t P_t^*}{P_t^c} \tag{2.46}$$

where S_t is the nominal exchange rate (the price of foreign exchange in units of home currency). So an increase in RER reflects increased international competitiveness of the home economy.

The standard *Uncovered Interest Rate Parity* shows that the difference in interest rates equals the expected change in the nominal exchange rate. It is also sensitive to the net foreign asset position when being derived from the first order conditions with respect to foreign bond holding:

$$\hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{S}_{t+1} - \Lambda_a \hat{a}_t + \hat{\varphi}_t \tag{2.47}$$

where all variables are in log-linearized form and the functional form for the premium on foreign bond holdings is assumed to be

$$f^5(a_t, \varphi_t) = \exp(-\Lambda_a(a_t - \bar{a}) + \varphi_t)$$

2.2.5 The Government

Fiscal policy in the model reflects the actual regime that has been implemented in Vietnam since the Doi Moi in the 1990s. The deficit targets need to be met annually and government spending is the key fiscal instrument. Following Gouvea et al. (2008), a feedback rule for the fiscal authority in which government spending responds to the deviation of the primary deficit[17] from its target and the public debt-to-GDP ratio is given by

$$G_t = \phi_G G_{t-1} + (1 - \phi_G)\left[\phi_\Omega(\Omega_{t-1} - \bar{\Omega}_{t-1}) - \phi_B B_t^r\right] + z_t^G \tag{2.48}$$

[17] Primary budget deficit excludes interest payments for government debts, if compared with budget deficit.

where G_t is government expenditure as proportion of GDP. Ω_t and $\bar{\Omega}_t$ denote the primary deficit as proportion of GDP and primary deficit target, respectively[18]. B_t^r is the debt-to-GDP ratio and z_t^G is taken as a shock to government spending.

The stationary version of the law of motion for the government's debt is as follows

$$
\begin{aligned}
B_{t+1}^r &= R_t \left(\frac{B_t^r}{\pi_t} \frac{Y_{t-1}}{Y_t} - \Omega_t \right) \\
&= R_t \left(\frac{B_t^r}{\pi_t} \frac{y_{t-1}}{y_t \mu_{z,t}} - \Omega_t \right) \qquad (2.49)
\end{aligned}
$$

Following Smets & Wouters (2003), we assume that the central bank follows a Taylor-type rule which, in principle, responds to the deviations of inflation from its target and output gap. However, for the sake of simplicity, following Cantore et al. (2016), the rule is assumed to be a so-called *implementable* Taylor rule. This rule is implementable because monetary authorities do not have to take into consideration the output gap which is an essential part of a *conventional* Taylor rule.

$$
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)[\hat{\bar{\pi}}_t + \theta_\pi (\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t) + \theta_y \hat{y}_{t-1}] + z_t^R \qquad (2.50)
$$

where z_t^R denotes a shock to monetary policy; $\hat{R}_t$, $\hat{\bar{\pi}}_t$, $\hat{\pi}_{t-1}^c$, and $\hat{y}_t$ are interest rate, inflation target, inflation, and output, respectively; $\rho_R, \theta_\pi, \theta_y$ are interest rate smoothing, inflation response, and output response parameters, respectively.

2.2.6 Equilibrium

The goods market clears when demands from domestic households, the government and the world market are met by the output of the final good firm or the aggregate production

[18] Ω_t is the budget balance and negative in case of deficit. Thus, ϕ_Ω is positive, reflecting the fact that any deviations of the primary deficit from the target will be corrected by government spending.

of the intermediate good firms:

$$C_t^d + I_t^d + G_t^n + C_t^x + I_t^x = \epsilon_t(\epsilon_t^z)^{1-\alpha} K_t^\alpha H_t^{1-\alpha} - \epsilon_t^z \phi - f(u_t)\bar{K}_t \tag{2.51}$$

where G_t^n is government consumption before being rendered stationary by ϵ_t^z.

The foreign bond market clears when the trade balance of the external sector is equal to the households' position on foreign bond holdings:

$$S_t B_{t+1}^* = S_t P_t^x (C_t^x + I_t^x) - S_t P_t^* (C_t^m + I_t^m) + R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) S_t B_t^* \tag{2.52}$$

where $f^5(a_{t-1}, \varphi_{t-1})$ is the risk-adjusted gross nominal interest rate.

The credit market is in equilibrium when the demand for credit from domestic intermediate firms to pay their wage bills equals the available assets:

$$\nu W_t H_t = M_{t+1} - Q_t$$

or

$$\nu W_t H_t = \mu_t M_t - Q_t$$

where $\mu_t = \frac{M_{t+1}}{M_t}$ is defined as money growth. The stationarized form is given by

$$\nu w_t H_t = \frac{\mu_t m_t}{\pi_t \mu_{z,t}} - q_t \tag{2.53}$$

where, following Altig et al. (2003) and Adolfson et al. (2005), $q_t = Q_t / P_t \epsilon_t^z, w_t = W_t / P_t \epsilon_t^z, m_t = M_t / P_{t-1} \epsilon_{t-1}^z$.

2.2.7 Partial Dollarization

Dollarization is a distinct feature of many developing countries in general, and Vietnam in particular, as we have witnessed from the data. DSGE models are, as argued by Castillo et al. (2006), regarded as useful tools to deal with this issue since they can make explicit the operating mechanism of dollarization.

It is assumed that among all types of firm in the model, only domestic final goods firms dollarize. Following Castillo et al. (2013), partial dollarization is introduced through the form of asset substitution in which we exogenously assume that a subset of domestic final goods firms set their prices in USD, $0 < \delta^{pd} < 1$. Therefore, the real marginal costs of domestic firms with domestic currency-denominated prices (P_t^D) and of domestic firms with USD-denominated prices (P_t^{USD}) are adjusted with the relative price of goods in domestic currency to the consumer price index (t_t^D) and the relative price between domestic currency and USD (e_t) as follows[19]

$$mc_t^D = \frac{mc_t}{t_t^D} \tag{2.54}$$

$$mc_t^{USD} = \frac{mc_t}{t_t^D e_t} \tag{2.55}$$

The relative price between domestic currency and USD[20] and the relative price of goods in domestic currency to the consumer price index are given by

$$e_t = \frac{P_t^D}{S_t P_t^{USD}} \tag{2.56}$$

$$t_t^D = \frac{P_t^D}{P_t^c} \tag{2.57}$$

In both cases, each group of firms faces the same optimization problem as in the baseline framework. Therefore, there arise two different Phillips curves determining the inflation rates for the two groups of firms, π_t^D and π_t^{USD}, respectively.

---

[19]Dollarization is modelled as asset substitution, so it only captures the unit of account/store of value functions of money but does not affect the medium of exchange function of money.

[20]Deviations from the law of one price.

Finally, the domestic inflation is then a weighted average of inflation in domestic currency and foreign currency plus the variation in the exchange rate, which is given by

$$\hat{\pi}_t = (1 - \delta^{pd})\hat{\pi}_t^D + \delta^{pd}(\hat{\pi}_t^{USD} + \Delta S_t) \tag{2.58}$$

We can see from this equation that partial dollarization not only increases the sensitivity of domestic inflation to the depreciation of the nominal exchange rate but also adds endogenous persistence to inflation.

2.2.8 Structural Shock Processes

All of the structural shocks and tax rates in the model are assumed to follow AR(1) processes, in linearized form, as follows

$$\hat{x_t} = \rho_x \hat{x}_{t-1} + \varepsilon_t^x, \quad \varepsilon_t^x \overset{iid}{\sim} N(0, \sigma_x^2) \tag{2.59}$$

where

$$\hat{x_t} = \{\hat{\mu}_{z,t}, \hat{\lambda}_t^d, \hat{\lambda}_t^{m,c}, \hat{\lambda}_t^{m,i}, \hat{\lambda}_t^x, \hat{\epsilon}_t, \hat{\epsilon}_t^c, \hat{\epsilon}_t^h, \hat{\epsilon}_t^q, \hat{\epsilon}_t^i, \hat{\tilde{\epsilon}}_t^{z*}, \hat{\varphi}_t, \hat{\bar{\pi}}_t, \hat{\tau}_t^k, \hat{\tau}_t^c, \hat{\tau}_t^w, \hat{\tau}_t^y\}$$

and

$$\mu_{z,t} = \frac{\epsilon_t^z}{\epsilon_{t-1}^z}; \tilde{\epsilon}_t^{z*} = \frac{\epsilon_t^{z*}}{\epsilon_t^z}; \hat{\epsilon}_t = \frac{(\epsilon_t - \epsilon)}{\epsilon}; \hat{\epsilon}_t^c = \frac{(\epsilon_t^c - \epsilon^c)}{\epsilon^c}; \hat{\epsilon}_t^h = \frac{(\epsilon_t^h - \epsilon^h)}{\epsilon^h}; \hat{\epsilon}_t^q = \frac{(\epsilon_t^q - \epsilon^q)}{\epsilon^q}; \hat{\epsilon}_t^i = \frac{(\epsilon_t^i - \epsilon^i)}{\epsilon^i}$$

2.3 Model Parameterization

2.3.1 Parameterization

The limited availability of Vietnam's macroeconomic data is the key challenge for the calibration of the model. In addition, the limited availability of studies using DSGE framework on Vietnam data also hinders the use of prior information from previous papers at both macro and micro level, which is considered a formal practice to calibrate and estimate DSGE models with Bayesian technique. Therefore, the model parameters of the baseline scenario are calibrated on the basis of observations for Vietnam during the period 1996-2012 and estimates established by relevant previous studies[21] for the purposes of the simulations and sensitivity analysis exercises of the chapter.

First, the dollarization parameter δ^{pd} is set to the low, medium and high rates of dollarization which are 0.01, 0.30, and 0.46 respectively. Consistent with earlier studies, the discount factor β is set to 0.98 that implies a steady state value of inflation at around 6 percent. Second, following Christiano et al. (2005), the Frisch labour supply elasticity $1/\sigma_h$ is set to 1, which is also strongly supported by micro evidence on the labour market as in Gomme & Lkhagvasuren (2013). The markup power in the wage setting λ^w is set to 1.05 as it is argued that this elasticity is low in comparison with the values assumed in the real business cycle literature, but is well within the rage of point estimates reported in the labour literature. Following Adolfson et al. (2005), the constant in the labour disutility function (steady state hours worked) Λ_h is set to 7.5[22]. The share of imported consumption in aggregate consumption ω_c and the share of imported investment in aggregate investment ω_i are calibrated at 0.31 and 0.55, respectively.

Third, following Adolfson et al. (2005), the constant in the cash holding disutility function Λ_q is set to 0.38 and the cash holding elasticity σ_q to 10.62 which is also consistent with Smets & Wouters (2007) and Christiano et al. (2005). The depreciation rate δ is calibrated at 0.013 and the share of capital in production α is 30 percent. The baseline model is assumed to have zero capital utilization rate by setting σ_a, a parameter

[21] Christiano et al. (2005), Adolfson et al. (2005), Altig et al. (2011), and Erceg et al. (2000).

[22] This indicates roughly 30 percent of the households' time is spent on work in steady states.

determining the capital utilization rate[23] $\hat{u}_t$, at a very large number 10^6. For the elasticities and markups in the model, the gross markup for the domestic sector λ^d is set to 1.2 while the external sector markups $\lambda^{m,i}, \lambda^{m,c}$ to 1.3, implying a substitution elasticity *within* domestic goods $\eta^d \equiv \lambda^d/(\lambda^d - 1)$ at 6 and a substitution elasticity *within* imported consumption/investment goods at 5. These are approximately the standard mark-up on prices as in Corsetti et al. (2012). In addition, as argued by Chari et al. (2002), the most reliable studies seem to indicate a range for the elasticity between 1 and 2. The elasticities *between* domestic and imported-consumption goods η^c, domestic and imported-investment goods η^i as well as the elasticity of substitution *within* goods in the foreign economy η^f[24] are thus set to 1.5.

Fourth, all nominal friction parameters ξ are set at 0.6 - 0.8 in order to ensure a 3-quarter average length of price and wage adjustments, which are also consistent with Corsetti et al. (2012). For other real friction parameters, habit formation b is set to 0.7 which is the point estimate by Boldrin et al. (2001), Christiano et al. (2005), and Smets & Wouters (2007). The share of firms that have to finance wage in advance[25] ν is set to unity and investment adjustment cost ζ to 8.7. Likewise, the indexation parameters κ for the wage level and for domestic and external sector prices are set according to Smets & Wouters (2007) and Adolfson et al. (2007) at 0.5 and 2, respectively.

Fifth, for the specification of monetary policy parameters, we rely on estimates reported by Clarida et al. (2000) so as to meet the Blanchard-Kahn conditions for determi-

[23]See details in appendix C.

[24]Which is assumed to be the same for both foreign consumption and investment.

[25]Working capital channel.

nacy when solving the model. The interest rate smoothing is set to 0.9, inflation response to 1.7 and output response 0.1. As for fiscal policy, we rely on estimates reported by De Castro et al. (2011) for a developing country in Latin America. Specifically, the feedback parameters on government spending ϕ^G, deviation of the primary budget deficit from its target ϕ^Ω, and government debt ϕ^B are calibrated at 0.7, 0.46, and 0.17, respectively. The government long-term debt B^r is calibrated at 0.65 in order to match the average level during the sample period.

Finally, for the shocks which are serially correlated, the autoregressive parameters ρ are set from 0.8 - 0.9, implying that all of them are stationary processes.

2.3.2 Conditions for Determinacy

In the context of the economic reforms that are under way over the last decades, the role and content of monetary policy have evoked larger attention. According to Friedman (1968), the first and the foremost lesson that history teaches about what monetary policy can do is that it can be prevented from being a major source of economic disturbance and provide a stable foundation for the economy, contributing to offsetting major disturbances arising from other sources. Also, as shown by Woodford (2003), it is sometimes argued that maintaining a low and stable rate of inflation should be the exclusive goal of monetary policy on the basis that money is neutral in the long run. At the same time, other discussions of actual monetary policy assume at least some degree of concern with the stabilization of economic activity as well, which was often treated as the primary goal of monetary policy in the early literature. Furthermore, having lived through the economic turmoil over the last decades, as pointed out by Fender (2012), whether monetary policy had an important role in causing the crises is debatable, but it is shown that monetary policy can make a contribution in preventing a crisis from having too large an effect on output and employment, and in facilitating recovery from any such effects it might have. Therefore, it is of intrinsic interest to examine to what extent a developing economy with partial dollarization can be stabilized with a simple Taylor-type rule that only tracks a few nominal variables.

The exercise is conducted through a method introduced by Blanchard & Kahn (1980) in a seminal paper published in 1980, which focuses on a procedure to find the explicit solution for a class of general linear difference models together with the conditions for existence and uniqueness. The present model can be put into its state space form as follows

$$\begin{bmatrix} x^1_{t+1} \\ E_t x^2_{t+1} \end{bmatrix} = \Gamma \begin{bmatrix} x^1_t \\ x^2_t \end{bmatrix} + \Theta R_t + \Pi \varepsilon_{t+1}; \qquad o_t = \Xi \begin{bmatrix} x^1_t \\ x^2_t \end{bmatrix} \tag{2.60}$$

where $(n-m) \times 1$ vector x^1_t consists of predetermined variables at time t with x^1_0 given; x^2_t is a $m \times 1$ vector of non-predetermined variables; and o_t is a vector of outputs. All of the variables are absolute or proportional deviations about their steady states. Matrices Γ, Θ, Π and Ξ are fixed and ε_t is a vector of random zero-mean shocks.

Rational expectations are then formed assuming a full information set $\{x^1_s, x^2_s, \varepsilon_s\}$ with $s \leq t$, the model equations, and a monetary policy rule of the form

$$R_t = D \begin{bmatrix} x^1_t \\ x^2_t \end{bmatrix} \tag{2.61}$$

where matrix D selects a subset of the model's variables from which monetary authority feedbacks.

According to Blanchard & Kahn (1980), when we substitute the monetary policy rule 2.61 into the state space representation 2.60, the condition for a stable and unique equilibrium of the model is dependent on the magnitude of the eigenvalues of the matrix $\Gamma + \Theta D$. If the number of eigenvalues of matrix $\Gamma + \Theta D$ outside the unit circle equals the number of the model's non-predetermined variables, the model has a *unique and stable solution*. *Instability* happens if the number of eigenvalues of matrix $\Gamma + \Theta D$ outside the unit circle is larger than the number of non-predetermined variables. In this case, when the economy is pushed off its long-run growth level after a shock, it cannot ever converge back to it, but rather finishes up with explosive inflation dynamics.

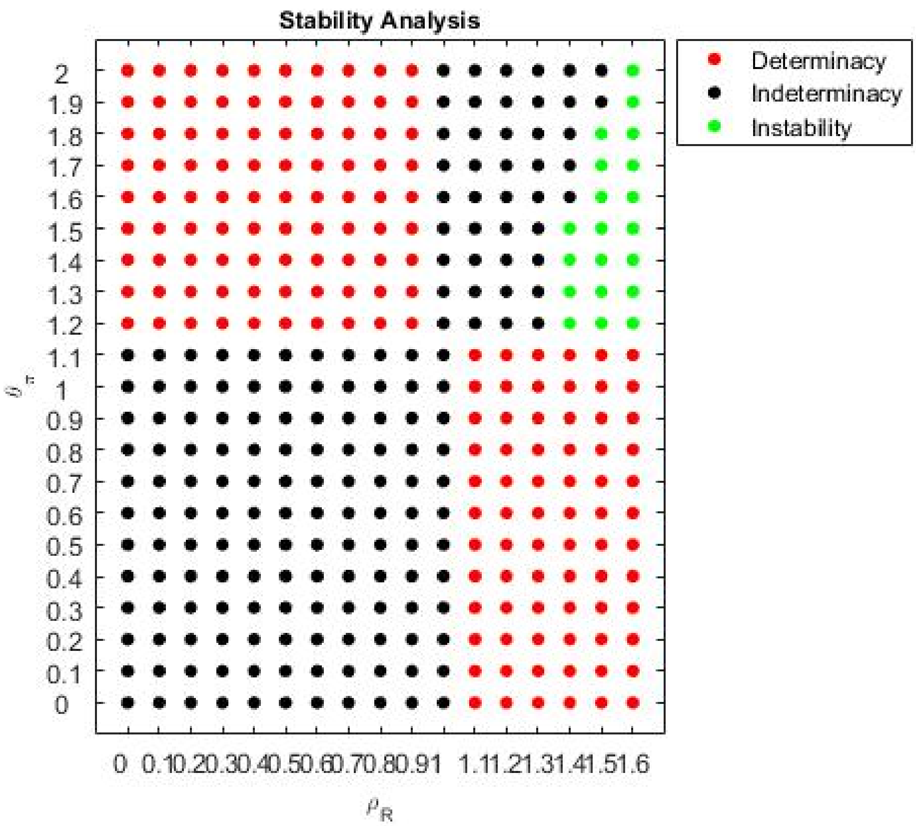
On the other side, when the number of eigenvalues of $\Gamma + \Theta D$ outside the unit circle is smaller than the number of non-predetermined variables, the model is of *indeterminacy*. In this case, Cantore et al. (2013) argue that if a shock displaces the economy from its steady state, there are many possible paths leading back to equilibrium, or in other words, there are multiple well-behaved rational expectations solutions to the model economy. Given forward-looking rules, indeterminacy can occur when central banks respond to private sector's inflation expectations which are driven by non-fundamental exogenous random shocks on the demand or supply[26]. Therefore, the monetary policy rule itself can

[26]They are shocks which are not based on economic fundamentals such as preferences or technology, and are named *sunspots*. When central banks adjust the rule's coefficients in order to accommodate the private sector's inflation expectations, those expectations become self-fulfilling. Therefore, the monetary policy rule will not be able to uniquely pin down the behaviour of at least one real/nominal variable in the model, making various paths compatible with the equilibrium.

introduce indeterminacy, leading to "sunspot equilibria". It should be noted that sunspot fluctuations, as argued by Cantore et al. (2013), are typically welfare-reducing at potentially large magnitude.

Figure 2.1 plots the results of the Blanchard-Kahn determinacy and stability conditions for the present model with respect to inflation response and interest rate smoothing parameters. The results show different values of the key parameters of the monetary policy rule, θ_π and ρ_R, with which the model has a unique solution. The red regions are the combination of the parameters of interest rate and inflation that introduce determinacy. The parameter step is set at 0.1 and the maximum values of 2 and 1.6 are set for θ_π and ρ_R, respectively. These values have been well-identified by previous VAR models.

Figure 2.1: Conditions for Determinacy



The results also show a lower bound of 1.2 for the inflation response parameter beyond which the interest rate rule leads to indeterminacy when the interest smoothing parameter is set within a range of 0 and 0.91. Put differently, the specified Taylor-type rule will generate a saddle-path stability or unique solution when the inflation response parameter θ_π is set between 1.2 and 2 in the current exercise, while interest rate smoothing parameter lies within the range from 0 and 0.91. This is more or less in line with the results reported by Batini et al. (2008) who use Chilean macroeconomic dataset. In addition, the model can also generate a unique solution when inflation response parameter is set less than 1.2 combined with any value of the smoothing parameter from 1.1 to 1.6.

2.4 Model Simulation

2.4.1 Properties of the Simulated Model

In this section, the calibrated model is simulated to analyse the impulse responses to a monetary policy shock which is an important feature of the developing country scrutinized in this dissertation.

The impulse response functions of key endogenous variables to a one-standard deviation monetary policy shock are reported in figure 2.2. The figure exhibits the response of inflation, output, consumption, investment, and employment where all variables are expressed as a percentage deviation from their steady states and most of the responses lie within three standard deviations. The simulation results show that the baseline model succeeds in accounting for the dynamic response of an emerging economy to a monetary policy shock. The U-shaped responses, which are pervasive features of estimated VAR models, are also consistent with most of the aggregate variables in consideration, with the exception of the price level, the real wage, and the real exchange rate. In particular, as money wage rates are relatively stickier than prices in the present model, a real wage increase after a contractionary monetary shock reflects the effects of a decrease in employment and is broadly consistent with the results in Altig et al. (2011) and Adolfson et al. (2005). As argued, at least implicitly, by Keynes (1936), goods prices were much more flexible than the money wage rate and firms should be regarded, approximately, as profit maximising competitors. In this case, the assumption of positive but diminishing marginal product of labour implied a negative association between the real wage and the level of employment.[27]

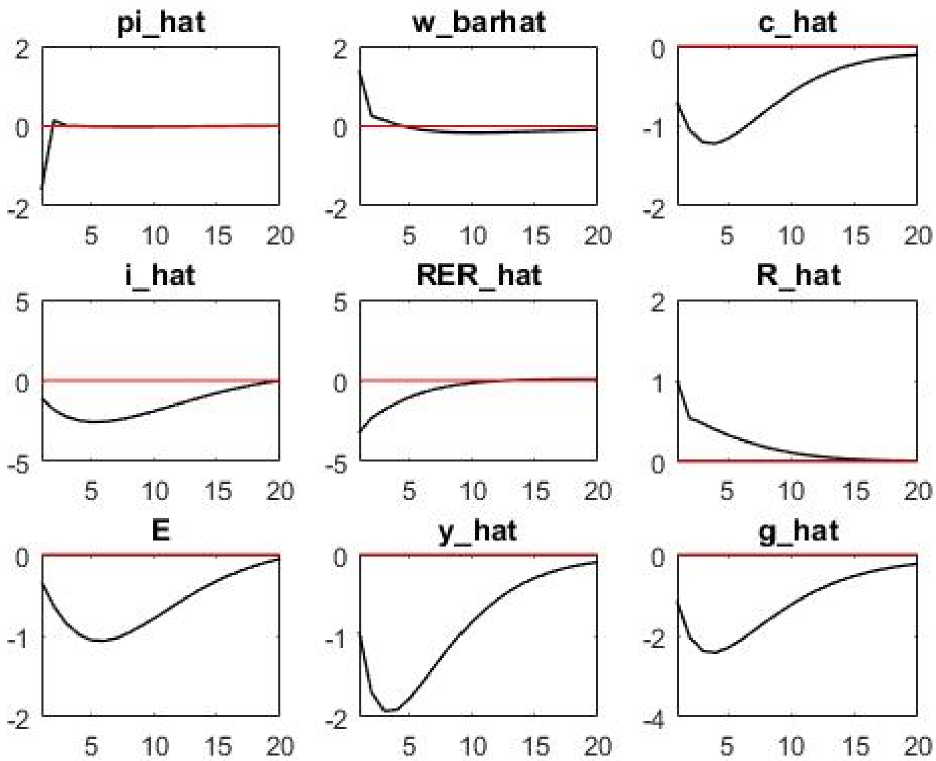
The effect of the shock on aggregate quantities are also well in line with Adolfson et al. (2007) except for the domestic inflation. A contractionary monetary policy shock induces a sharp decline in the price level, which then returns to its preshock level after approximately 2 quarters. It should be noted that while inflation is already shown by Adolfson et al. (2005) to be less persistent than the typical estimates in the VAR literature[28], the effect of a policy shock on the price level in the present model is even far less inertial. This is particularly a notable impact of partial dollarization in the economy. The expected effect of a tightened monetary policy aiming at curbing high inflation in dollarized economies may not be as effective as we would see in developed countries as

[27]As explained by Wells (1987), Keynes (1936) postulates that it is the level of employment which determines the real wage, not the other way around.

[28]As in Christiano et al. (2005), Altig et al. (2003), Altig et al. (2011). Adolfson et al. (2007) argue that this may be due to the fact that the simulated model allows for a very high capital utilization rate when σ_a is set so as to get a very small elasticity of capital utilization with respect to the rental rate of capital.

the policy just induces a short-term effect due to partial dollarization. In addition, a monetary policy shock leads to a much more persistent effect on output as the response exhibits a prolonged slump that lasts very much longer after the price has returned to its steady state value. The lowest level is seen about 3 quarters after the shock and the output response is negative for almost 20 quarters. The magnitude of the effect on output is almost twice as that of employment and consumption.

Figure 2.2: Impulse Response Functions to a Contractionary Monetary Policy Shock



Another result that is worth emphasizing is that the shock has even a more persistent effect on key endogenous variables than on the interest rate itself. While output, consumption, investment, and employment are not back to where they started after roughly 20 quarters, nominal interest rate returns to its steady state after just around 15 quarters. This phenomenon where the shock's effects on key endogenous variables last longer than that on the policy variable itself reflects a strong internal propagation mechanism within the model which, as argued by Bouakez et al. (2005), can explain the persistent and hump-shaped response of key endogenous variables to monetary shocks. Figure 2.2 also shows that a contractionary policy shock generates a corresponding rise in the real wage, though not in a hump-shaped manner. As the effectiveness of monetary policy is shown to be hindered by the presence of partial dollarization in the model, so too is the effect of the policy shock on real wage when its positive response seems to fade away after just a year.

2.4.2 Sensitivity Analysis of Frictions

The calibrated model is also alternatively simulated in order to examine the quantitative contributions of relevant frictions in the model to its performance. The exercise is carried out by assessing the behaviour of the impulse response functions of key endogenous variables following a positive monetary shock when some of the nominal/real frictions are switched off while others are retained during the process.

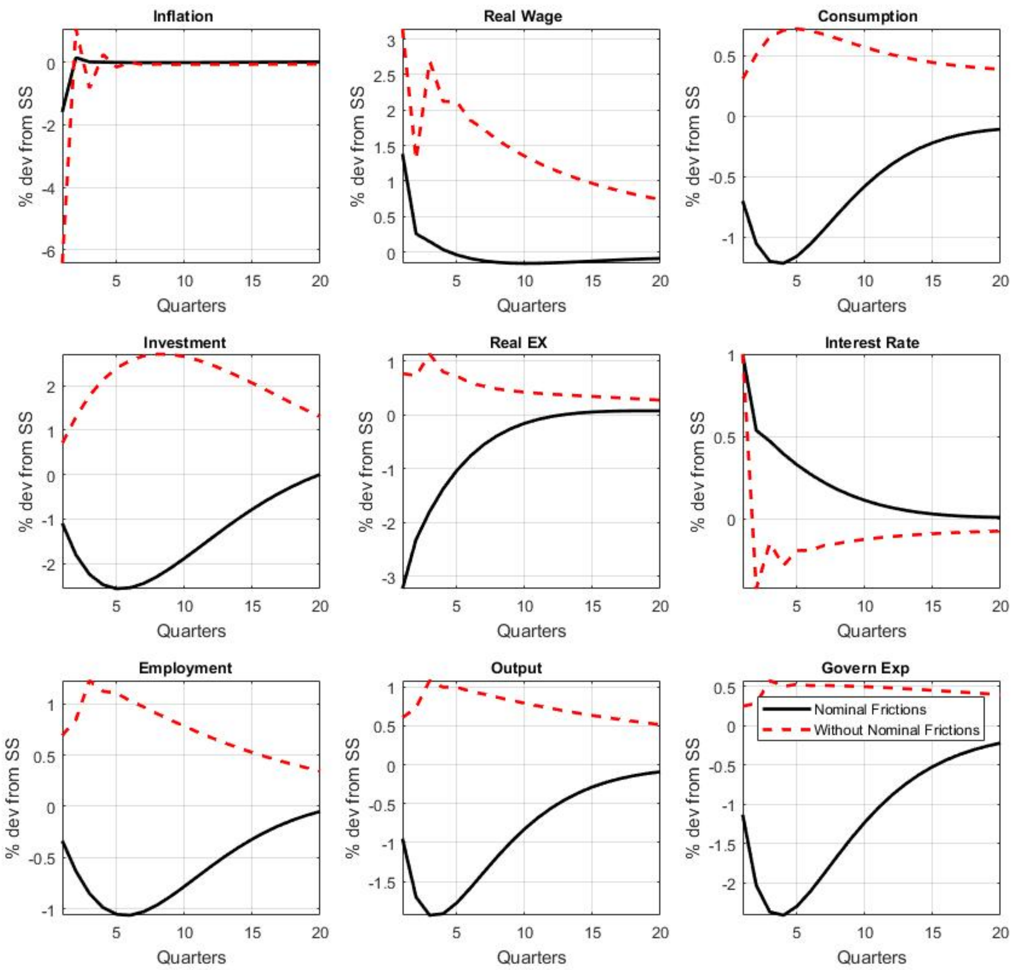
In the model, we feature two sources of nominal rigidities which are price stickiness according to the staggered contracts in Calvo style and price/wage indexation. This type of contract is modeled for all firms in the domestic and external sectors, and also for the wages. In addition, the model also incorporates four sources of real rigidity, which are internal habit formation in households' consumption, investment adjustment costs, variable capital utilization, and working capital channel.

Nominal Frictions

Price/wage Stickiness. In order to show how the nominal frictions in the model actually shape the impulse response functions, the calibrated model is simulated for the variant where all nominal frictions, including Calvo indices for domestic firms, consumption/investment imports, exports, and wages, are all set approximately to zero and the remaining parameters are held at their benchmark values so that all prices are considered flexible ($\xi^d = \xi^{m,c} = \xi^{m,i} = \xi^x = \xi^w = 0.01$). The responses predicted by the baseline model are plotted in continuous lines, while the dotted lines show responses from the restricted model.

The impulse response functions of key aggregate variables are displayed in figure 2.3. It is shown that the introduction of nominal frictions plays a critical role in the model's performance and all the effects on price, output, consumption, investment would be completely different when price and wage stickiness is dropped from the model. First, a key result of the comparison with the model without nominal frictions is that the presence of nominal frictions helps create the U-shaped impulse responses of key aggregate variables after a monetary policy shock, which is known to be a typical feature of VAR and New Keynesian models. Second, as in previous studies, the policy operates effectively through an interest rate channel that has become a standard framework for investigating monetary policy issues. A monetary shock in the model with nominal frictions not only induces plausible movements of inflation and other aggregate variables, but also creates a substantial trade-off between stabilizing inflation and output in the short-term which also prevails in previous VARs and DSGE models.

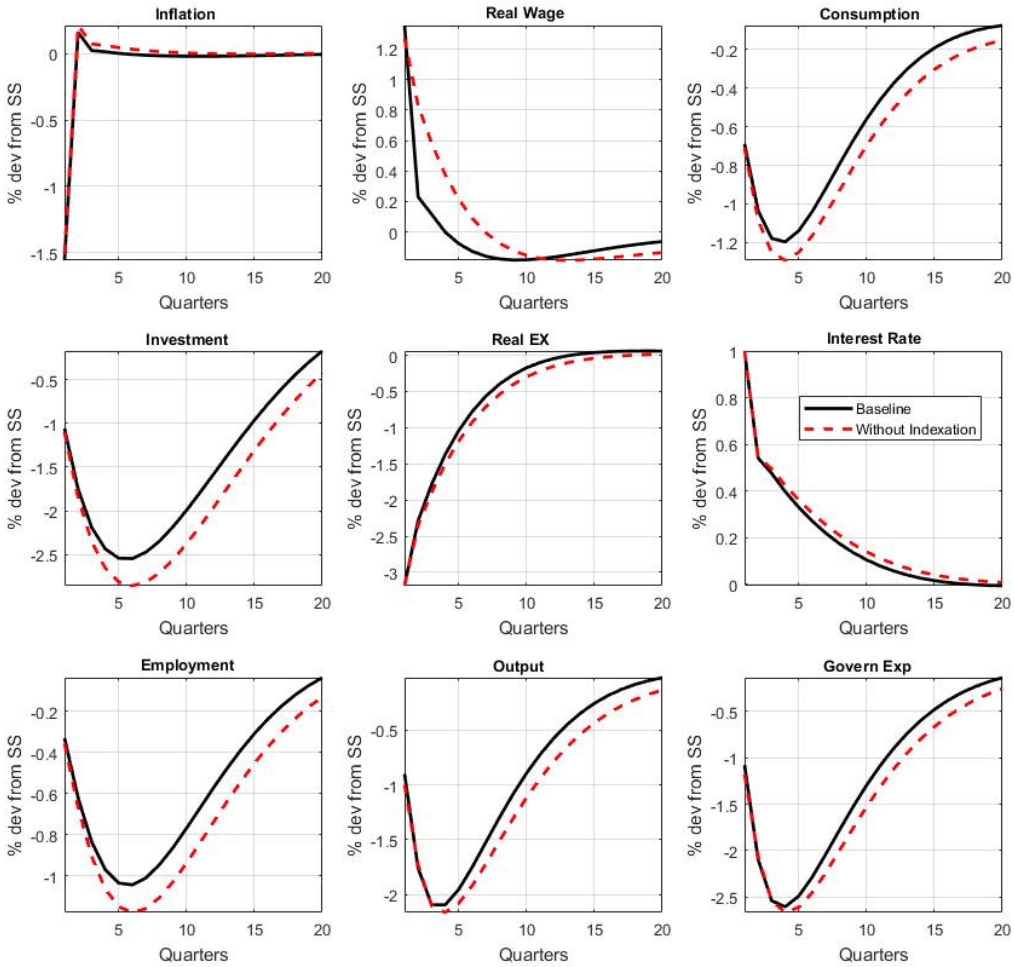
Figure 2.3: Sensitivity Analysis of Nominal Frictions



Finally, the presence of nominal frictions increases the sensitivity of aggregate variables to the interest rate with considerably higher magnitudes. Furthermore, the "puzzle" in the behaviour of real exchange rate after a monetary shock in the variant without nominal frictions is replaced by an increasing function which returns to the steady state after the shock in roughly 2 years. These results are highly in line with Christiano et al. (2018) who show that Calvo-style frictions generate implications which are consistent with aspects of micro data, and are believed to be an essential feature of business cycles in economies where inflation is moderate.

Price Indexation. While price/wage stickiness allows the firms and households to adjust prices/wages in response to a wide variety of information, dynamic indexation in contrast restricts the adjustment to a single source of information which is the inflation in the previous period. Figure 2.4 shows how aggregate variables respond to a monetary policy shock in the baseline and restricted models (all indexation parameters κ are set to 0.01). The black lines show impulse responses based on the fully-calibrated model, while the dotted red lines are impulse responses from the restricted model.

Figure 2.4: Sensitivity Analysis of Indexation



It is shown that the responses to a monetary policy shock are qualitatively similar across the whole simulated period. A contractionary shock produces an immediate decline in inflation and negative, U-shaped responses of output, consumption, employment, and investment in both models. The presence of domestic price and wage indexation generates a relatively smaller magnitude of the responses and induces a slightly lower persistence of output, consumption, employment, and investment. The impulse responses of inflation also indicate that price/wage indexation has seemingly little effect on the dynamics of inflation in the model. However, it should be noted that indexation brings the real wage rate back down faster, softening the blow to output to some extent.

Real Frictions

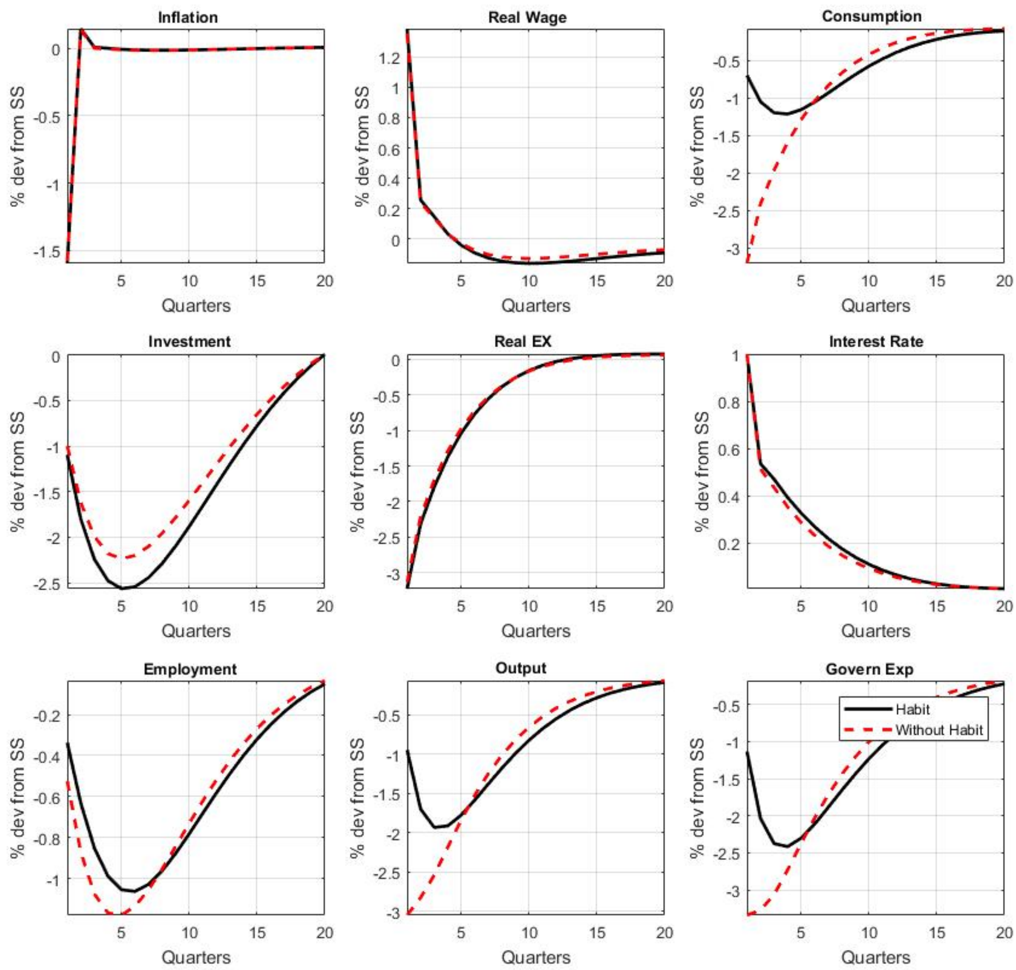
In order to examine the role of real frictions, the model is simulated to compare the impulse response functions of the baseline model with a model variant without habit persistence and a variant without investment adjustment cost. The contributions of variable capital utilization and working capital channel are also analysed and included.

Habit Persistence. Habit formation, as argued by Bouakez et al. (2005), induces agents to adjust their labour supply more gradually to attain a smoother and more persistent consumption profile than under time-separable preferences. This exercise examines the contribution of habit formation to the propagation mechanism of the model. To that end, a restricted version of the model without habit formation ($b = 0.01$) is perturbed and simulated.

The impulse response functions of key aggregate variables of the baseline model and the restricted model are plotted in figure 2.5. While inflation and nominal interest rates in the restricted model respond in a qualitatively similar manner to a monetary policy shock in both cases, output and consumption decrease sharply after a contractionary monetary shock and return fairly slowly to their steady state levels. The key result here is that without habit formation, the dynamics of the responses are just monotone and do not exhibit U-shaped pattern, a property that has been widely predicted by previous studies featuring general equilibrium models, as we can see in the baseline model.

In addition, the presence of habit formation makes almost no difference to the dynamics of inflation in the model. These results are largely consistent with Bouakez et al. (2005). However, it is shown that habit formation can to some extent magnify the persistence of output response after a monetary shock. The reason here is that, as explained by Bouakez et al. (2005), habit-forming agents allocate their resources so as to get a smoother and more persistent consumption profile than agents in models with time-separable preferences.

Figure 2.5: Sensitivity Analysis of Habit Formation



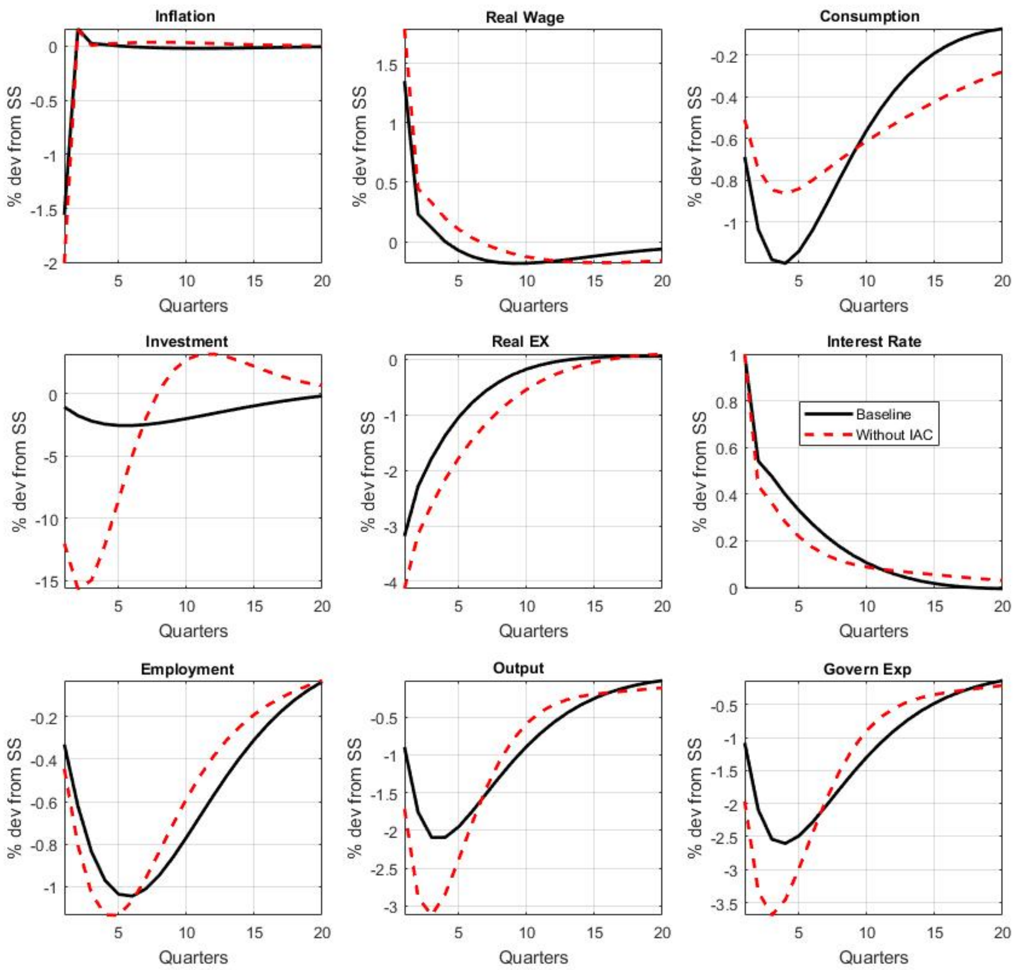
Investment Adjustment Costs. In the first generation of DSGE models, the capital stock was assumed to be freely changed from period to period without any restriction via the investment process. However, in reality, the investment process is restricted by adjustment costs which lead to additional real rigidities in the capital accumulation process. In other words, investment is not fully and costlessly transformed into next period's capital stock[29].

[29]Two approaches have been developed to examine the adjustment costs in the investment process.

Figure 2.6 shows how the presence of investment adjustment costs influences the effects of a contractionary monetary shock in the model. The impulse response functions of key aggregate variables of the baseline model and the restricted model are plotted. The black lines indicate impulse responses from the baseline with ζ being calibrated at 8.67, which is then compared with the impulse responses of the model restricted at almost zero for the investment adjustment cost parameter.

First, if we assume that the firms in the model are the owners of the capital stock and decide the level of investment in each period, then the so-called Tobin's Q is used to examine the dynamics of the Q ratio that represents the ratio between the market value of the firm and the replacement cost of its capital stock. Second, if the households are assumed to be the owners of the capital stock, as in the present model, investment adjustment costs are imposed upon the households who decide the investment level.

Figure 2.6: Sensitivity Analysis of Investment Adjustment Costs

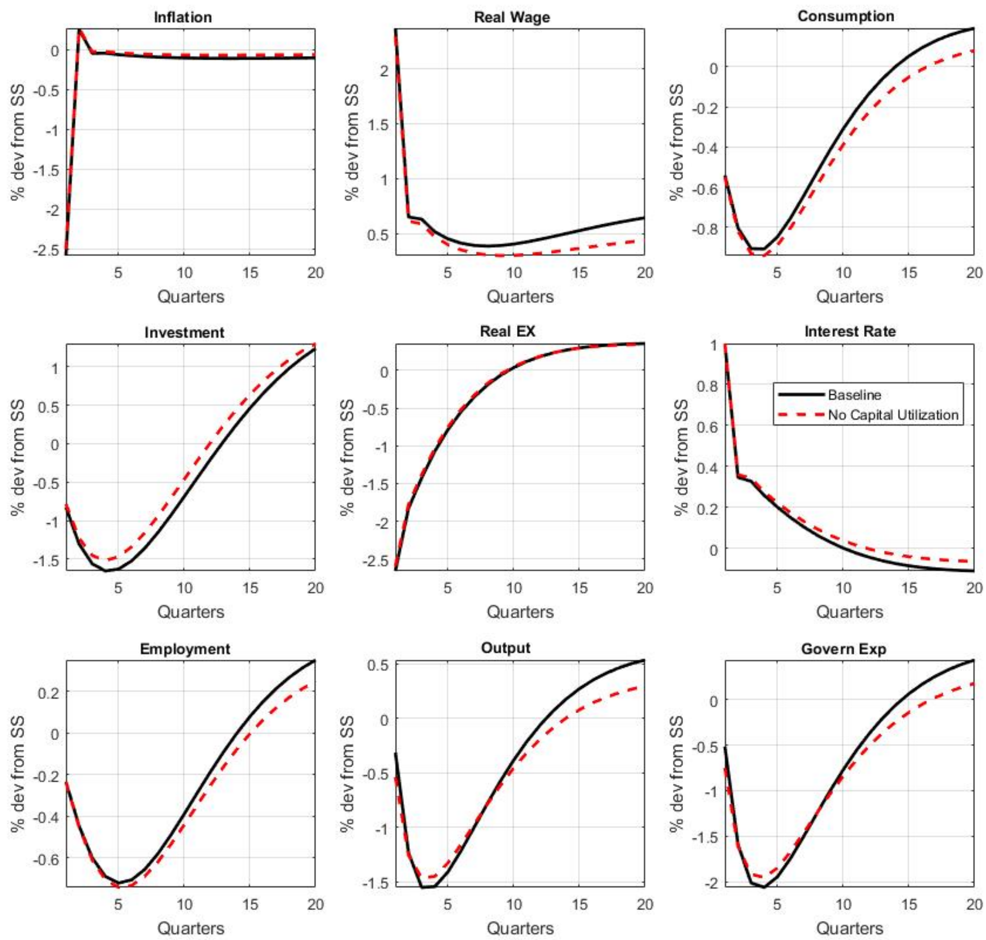


While the dynamic responses of the aggregate quantities are qualitatively almost the same, they exhibit some notable differences between different variants of the model. The impulse responses of aggregate investment and output are U-shaped at considerably larger magnitude than they are in the baseline model where adjusting investment is costly. However, the presence of adjustment costs associated with investment induces much more persistent response of investment to a monetary policy shock in the model. These results imply that the existence of investment adjustment costs leads to a different transmission mechanism of the monetary shock to the capital stock and apparently changes the amount of investment in each period. Therefore, a cost to investment magnifies the persistence in the capital stock accumulation process. A similar result is found in Christiano et al. (2005) who show that a sticky price model can generate hump-shaped investment dynamics and induce inertia in investment when investment costs are included. Investment adjustment costs, thus, have important contributions to understanding the aggregate dynamics of the model economy.

Capital Utilization. Christiano et al. (2005) consider two scenarios where the capital utilization rate is fixed at the value of $\sigma_a = 0.01$ for the baseline and a variant with $\sigma_a = 100$, implying the case without variable capital utilization. However, in order to achieve determinacy, we set the elasticity of capital utilization with respect to the rental rate of capital at 0.1 ($\sigma_a = 10$) for the benchmark calibration and evaluate the sensitivity of the capital utilization decision when there is no variable capital utilization, $\sigma_a = 10^6$, following Adolfson et al. (2007). Figure 2.7 provides an illustration of the role that capital utilization plays in the model.

In particular, while the magnitude of the responses of the endogenous variables in consideration do not change significantly, the presence of capital utilization only induces a slightly less persistent effect on output, employment, consumption, and investment, but a stronger effect on the magnitude and persistence with respect to investment. As argued by Christiano et al. (2005), in a model with moderate wage and price stickiness, it is crucial to allow for variable capital utilization in order to generate inertia in inflation and persistence in output as it helps dampen the large rise in the rental rate of capital that would otherwise occur. However, given a model for developing countries where the domestic prices are also determined by a certain level of partial dollarization, the presence of variable capital utilization seems to have little effect on the dynamics of inflation. This result is broadly consistent with Adolfson et al. (2007) where variable capital utilization does not seem to play a critical role in the model's performance and capital utilization cost parameter is thus switched off during the estimation.

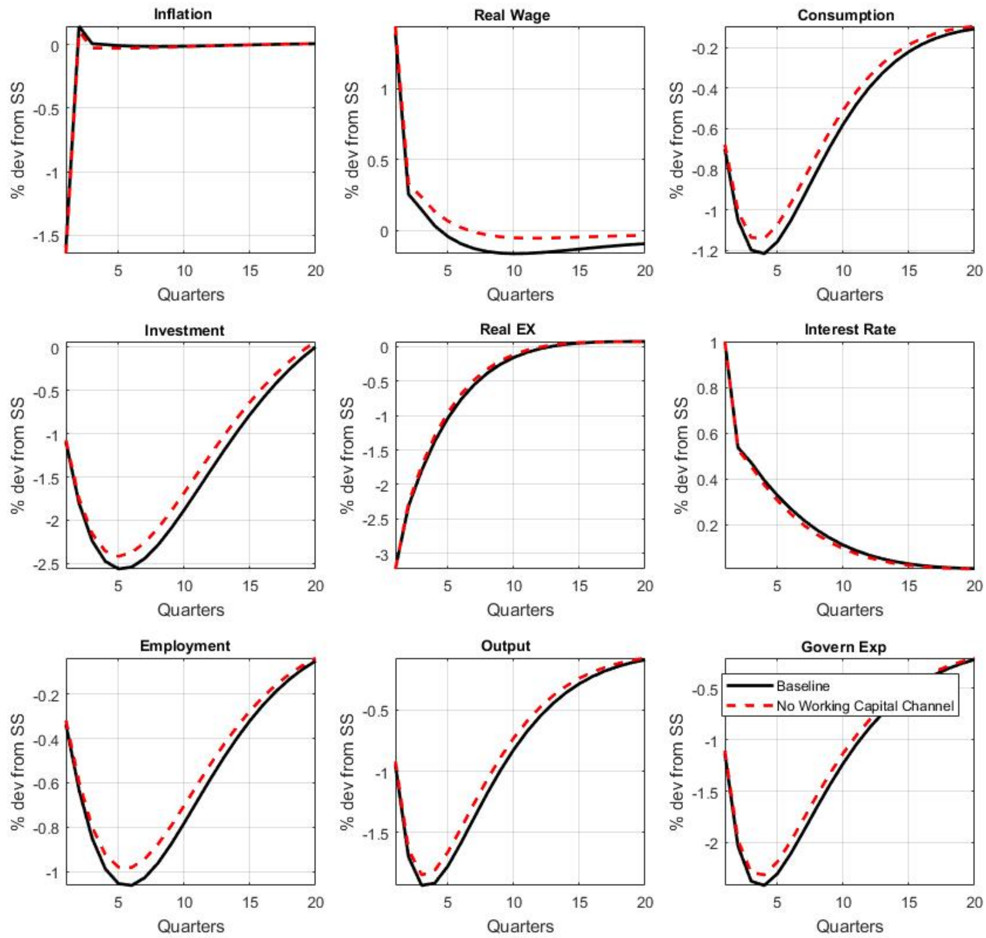
Figure 2.7: Sensitivity Analysis of Variable Capital Utilization



Working Capital Channel. Working capital is defined by Corugedo et al. (2011) as the difference between current assets, which are firms' resources in cash or readily convertible into cash such as inventories, and current liabilities or firms' cash requirements. Firms thus have a funding gap between when they pay the costs of inputs and when they receive the revenues from the sales of output. Thus, working capital represents operating liquidity available to firms and having the right amount of working capital at the right time is crucial for the efficient operation of firms. Therefore, frictions in working capital channel can affect the supply side of the economy, leading to lower employment and output while pushing up inflation. This is the reason why working capital channel is assumed to play a significant role as a transmission channel of economic shocks into the economy, especially shocks on the financial and banking sector.

Figure 2.8 plots the impulse responses of aggregate variables in the case where there is no working capital channel ($\nu = 0.01$) compared with results from the benchmark model. It is shown that introducing working capital channel slightly magnifies the responses of real variables, including output, consumption, and investment, to a monetary policy shock. The likely reason is that the presence of working capital channel pushes up the sensitivity of output to the interest rate, as the firms' marginal costs are also increased.

Figure 2.8: Sensitivity Analysis of Working Capital Channel

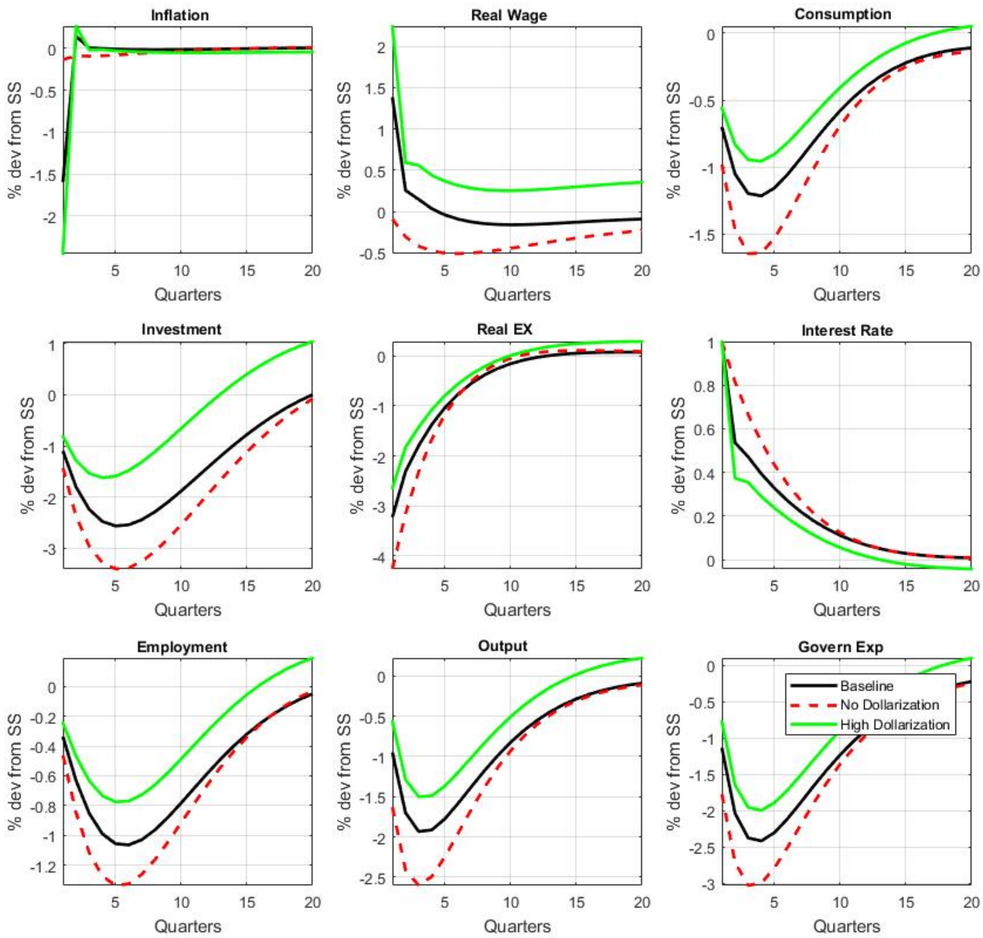


However, the presence of working capital channel is also shown to have very little impact on inflation. In other words, there is almost no difference in the response of inflation in the model economy as inflation behaves almost the same in the two variants of the model. This result is consistent with Cabezon (2014) in which it is shown that working capital does not have critical implications for monetary policy in a relatively open economy, though this finding differs from that of Ravenna & Walsh (2006) for a closed-economy model, where the working capital channel is found to create a trade-off between output and price stabilization. The main finding here is that even though working capital can be an efficient cost channel in which monetary policy can affect the economy, it plays a negligible role in the determination of domestic inflation in the present model because, as argued by Cabezon (2014), the effect of a monetary shock on the firms' marginal costs may be neutralized through the exchange rate channel.

2.4.3 Sensitivity Analysis of Dollarization

Figure 2.9 illustrates how different levels of partial dollarization influence the performance of the model after a monetary policy shock. The impulse responses of key aggregate variables of the baseline model ($\delta^{pd} = 0.3$) are compared with the impulse responses from two other variants of the model in which dollarization rates are imposed at a higher level of 0.46 and almost no dollarization ($\delta^{pd} = 0.01$). The black lines show the impulse responses from the baseline model, while the impulse responses of aggregate variables of the high and low dollarization scenarios are plotted in the green and red dotted lines, respectively.

Figure 2.9: Sensitivity Analysis of Partial Dollarization



A number of results are worth emphasizing in this case. First, it is shown that the impulse response functions of aggregate variables to a shock on monetary policy in all scenarios are produced in U-shaped manner as in previous VAR or DSGE-based studies. However, with the presence of dollarization in the model, a shock on monetary policy has considerably larger effects on inflation dynamics. The higher degree of dollarization, the larger the magnitudes of the responses to a contractionary monetary policy shock. The initial decline in the price level when there is serious dollarization in the economy is almost twice as big as the decline in the model with moderate dollarization. The effect is even more notable when compared with the restricted model without dollarization.

Second, as the level of dollarization increases, the model generates weaker responses of consumption, investment, and output accordingly. In addition, the persistence of the responses of consumption, investment, and output is also inversely proportional to the level of dollarization, though the peak effects in all three variants of the model occur roughly three quarters after the shock. Finally, it has been shown that if a monetary shock produces more persistent effects on output than that of the instrument itself, there should be a strong internal transmission mechanism in the model economy. The results show that a monetary policy shock in the present model induces a more persistent effect on output than on interest rate in model variants with lower level of dollarization. Therefore, the presence of dollarization in the economy can substantially hinder the effectiveness of monetary policy.

These findings are broadly consistent with Baliño et al. (1999) who argue that dollarization makes the demand for domestic money more volatile as agents shift frequently between currencies for a number of not easily identifiable reasons in order to avoid the negative effects of inflation. Yeyati (2006) also argues that if the switch to available foreign-currency assets becomes less costly, the demand for reserve money in an economy with partial dollarization would be more sensitive to a monetary expansion or an exchange rate fluctuation. This poses challenges to the pursuit of a coherent and independent monetary policy and of seigniorage, as in Chang & Velasco (2002). Figure 2.9 clearly shows that by eliminating dollarization in the economy, the magnitudes of the impulse responses of aggregate variables to a monetary policy shock are stronger, making the transmission mechanism of monetary policy more effective. This result is also in line with Ohanyan (2016) who shows that with prices and interest rates largely influenced by foreign factors, the economy appears to have smoother dynamics and stabilized domestic shocks, but a weaker interest rate channel of monetary policy.

Furthermore, Calvo & Gramont (1992) point out that exchange rate volatility is expected to be higher in dollarized economies. This strong positive correlation between exchange rate volatility and dollarization as well as unstable domestic demand are also the key implication of earlier literature on the challenge posed by dollarization to the effectiveness of monetary policy. In addition, as argued by Schaub (2009), it is the higher elasticity of inflation to a monetary shock that limits the scope for monetary policy effectiveness because of a lack of self-stabilizing effects in dollarized economies.

2.5 Summary

Following standard DSGE models, the DSGE model set up in this chapter shares the essential features developed by the New Keynesian school. A relatively large number of nominal and real frictions are incorporated in order to capture the empirical dynamics of the macroeconomic variables, including price and wage stickiness, investment adjustment costs, variable capital utilization, and habit formation in consumption. In addition, the model is extended with two typical features of many developing countries which are partial dollarization in the supply sector and a chronic budget-deficit fiscal rule on the part of the government.

In the theoretical model, the stochastics of the variables are explained by various structural shocks, with technology and labour supply shocks on the supply side and consumption, investment, and policy shocks on the demand side. The model is then fully calibrated and simulated to examine the sensitivity of each of the nominal and real frictions in explaining the dynamic development of the model economy. The results of the exercise show that, with the exception of working capital channel and variable capital utilization, nominal and real frictions play a significant role in the theoretical model in generating the standard U-shaped dynamics of real variables.

Different scenarios of partial dollarization are also simulated so as to assess the importance of dollarization in the performance of the model. Sensitivity analysis on partial dollarization establishes that partial dollarization plays a significant role in shaping the dynamics of the model following a monetary policy shock. The simulated results show that the responses of real variables are larger in magnitude as the degree of partial dollarization in the model economy decreases. Therefore, the presence of partial dollarization is believed to substantially impede the effectiveness of monetary policy.

Chapter 3

Bayesian Estimation of the DSGE Model for Vietnam

This chapter estimates the Dynamic Stochastic General Equilibrium (DSGE) model for developing countries developed in the preceding chapter for a macroeconomic dataset of Vietnam. The model is estimated with Bayesian techniques using ten key macroeconomic variables, including GDP, household consumption, government consumption, investment, employment, interest rate, real exchange rate, inflation, export, and import. The U.S. time series for output, interest rate, and inflation are used as the world indicators. The presence of orthogonal structural shocks on both supply and demand side allows for an empirical investigation and comparison of the effects of the shocks as well as their contribution to the development of the model economy. The estimated model is also used to compare variants of the model at different levels of partial dollarization to find out the most favourable setup that captures the properties of the data.

3.1 Introduction

Before the Lucas critique, structural economic models which are theoretically constructed and represent economic processes by a set of variables and a set of logical and/or quantitative relationships between them were used to derive optimal decision rules of economic agents. The decision rule was thought to embody constant coefficients and could be used to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, and then used for macroeconomic policy selection. Traditional structural models, however, fail to recognize that the decision rules depend on parameters of which many are variable *reduced-form parameters* depending on stochastic *policy parameters*. Thus, any changes in the policy will systematically alter the structure of the econometric model, making it impossible to use those optimal decision rules, which consequently also vary systematically with changes of the decision maker, for the purpose of policy evaluation and forecast.

DSGE models with micro-foundations have been developed to address the Lucas critique. At the centre of DSGE models are the *structural/deep parameters* describing the preferences of the agents and the constraints on technology and resources in the economy. These parameters, as pointed out by Fernández-Villaverde et al. (2007), are called structural in the sense that they are invariant to interventions from policy makers, including shocks by nature, as these parameters have been fully interpretable from the perspective of economic theory and micro evidence. Thus, DSGE models avoid being subject to the

Lucas critique and can be used to quantitatively evaluate policy. The separation between structural parameters and reduced-form parameters[1] leads to the development of non-linear estimation techniques such as generalized method of moments, indirect inference, maximum likelihood or Bayesian methods that are technically not dependant on the reduced forms of the parameters themselves.

This chapter presents the estimation of the present theoretical DSGE model using Bayesian estimation methods and macroeconomic data from Vietnam during the period 1996-2012. Traditionally, a *calibration method* is a good choice to pin down parameters in such complicated structural models. The deterministic steady states are used to solve for parameter values that result in observed long-run outcomes for macroeconomic variables such as hours worked, the great ratios, or the real interest rate. This is called *first-moment matching method* and it can be extended to a *second-moment matching method*, including variances, correlations and autocorrelations to calibrate shock processes. However, given the development of macroeconometrics in recent years, economists tend to replace this informal moment matching method with a formal *systems estimation method.* As argued by Cantore et al. (2016), Maximum Likelihood (ML), Generalized Method of Moments (GMM), and Bayesian Estimation are three widely-used approaches that deploy systems estimation methods for DSGE models estimation.

Bayesian approach, among others, allows us not only to take advantage of prior information to identify key structural parameters, but also to utilize all of the cross equation restrictions implied by the DSGE setup, making the estimation process more efficient[2].

[1] Reduce-form parameters are normally estimated by regression analysis.

[2] Partial equilibrium models only consider the clearance of a specific market of interest, while prices

Bayesian estimation is also a full information systems estimation method, but to some extent a *hybrid* approach between informal calibration and ML. As argued by Cantore et al. (2016), when there is no prior information, Bayesian approach converges to ML and if the priors are believed to be correct, we come back to calibration. Given the priors, Bayesian method enables the utilization of additional sources of information to help solve the problem of *flat likelihood surface* in some directions[3] by adding "curvature" to the likelihood. In addition, Fernández-Villaverde (2010) shows that Bayesian approach provides a straightforward method of evaluating the ability of the models in capturing the cyclical features of the data, while allowing for a fully structural approach to analyse the sources of fluctuation. It also provides facilities for the construction of confidence intervals for estimated parameters, computing impulse response functions, forecasting, and performing model comparison.

In a practical exercise, as argued by Cantore et al. (2016), the Bayesian estimation method will first employ the log-linearized approximation of the original model's non-linear optimality conditions, normally the first-order conditions, using Taylor expansion about a deterministic steady state to obtain a linear rational expectation system. This system is then solved for its state-space form to derive the policy functions in the model's predetermined variables. Standard Kalman filter is then applied to evaluate the likelihood function under the assumption of Gaussian distribution of the disturbances. Finally, this likelihood function will be combined with the prior assumptions about the parameters to compute their posterior probability.

of all other markets such as wages, interest rate are held fixed during the analysis.

[3]This is a major problem in likelihood approaches.

The rest of the chapter is structured as follows. Section 2 presents the basic principles of the Bayesian estimation method and procedure. In Section 3, we first discuss the detailed methodology for the transformation of the data before being taken to estimation process, then present the derivation of measurement equations which must be included in the system of model equations in order to match the model's variables with the observables. Section 4 describes the prior distributions and discusses identification problems related to the estimated parameters. The estimation results are reported in Section 5. In Section 6 and 7, properties and the fit of the estimated model are in turn analysed and discussed in detail before we have some concluding remarks in Section 8.

3.2 Bayesian Estimation Method

Bayesian econometrics is named after Reverend Thomas Bayes (1702-1761), an English statistician, with important contributions to the original ideas by Pierre-Simon Laplace (1749-1827), for the idea of "inverse probability". However, according to Greenberg (2012), it was not until the early 1960s that the application of the Bayesian viewpoint to econometric models was realized by A. Zellner. The most important breakthrough of the approach occurred in the early 1990s with the application of Markov Chain Monte Carlo simulation to statistical and econometric models, given its reliance on simulation techniques rather than standard probability theory and statistics. Sims (2007) in his seminal paper "Why Econometrics Should Always and Everywhere Be Bayesian?" argues that Bayesian inference is a way of thinking, not just a basket of methods, and Bayesian approaches might become more practical and prevalent.

This section presents the principles of Bayesian estimation method and detailed procedure for Bayesian estimation which is used for direct estimation and evaluation of the DSGE model developed in the previous chapter.

3.2.1 Principles of Bayesian Estimation

The principle of Bayesian analysis is broadly based on the *Bayes's Rule* which is specified as follows

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)} \tag{3.1}$$

where $p(A)$ is the probability of A, $p(A|B)$ the probability of A given B. And

$$p(A|B) \equiv \frac{p(A,B)}{p(B)} \tag{3.2}$$

where $p(A,B)$ is the probability of A and B happening together.

Suppose that we estimate a vector of the model's parameters $\theta = \theta_1, \theta_2, ..., \theta_n$ as a subset of the model's parameters Θ given the full model's variables y, Bayesian econometrics uses the full dataset of all observed variables ($Y^T = y_1, y_2, ..., y_T$) and applies Bayes' Rule to get

$$p(\theta|Y^T) \equiv \frac{p(Y^T|\theta)p(\theta)}{p(Y^T)} \tag{3.3}$$

where $p(\theta)$ is the prior density of the parameters and $p(Y^T|\theta)$ is the likelihood density of the data given the model's parameters[4].

[4]This density is also the likelihood function of the parameters to be maximized, which is calculated by the formula

$$p(Y^T|\theta) = L(\theta; Y^T) = p(y_1|\theta)\prod_{t=2}^{T} p(y_t|Y^{t-1}, \theta)$$

Equation 3.3 shows that the posterior density $p(\theta|Y^T)$ depends only on $p(Y^T|\theta)p(\theta)$ as $p(Y^T)$ is already given. This key part of the posterior density is defined as posterior kernel $\mathcal{K}(\theta|Y^T)$[5] and denoted by

$$p(\theta|Y^T) \propto p(Y^T|\theta)p(\theta) \equiv \mathcal{K}(\theta|Y^T) \tag{3.4}$$

According to Miao (2014), *the Bayesian estimation method is to choose a parameter value so as to maximize the posterior density* $p(\theta|Y^T)$*, or more simply the posterior kernel* $\mathcal{K}(\theta|Y^T)$. It should be noted that the posterior density is the likelihood density of the data given the model's parameters $p(Y^T|\theta)$, which is the density that Maximum Likelihood method aims to maximize and computed using the Kalman Filter, updated by the prior information of the parameters $p(\theta)$ which gives the surface more curvature[6].

In addition, the posterior distribution $p(\theta|Y^T)$ which contains important information about the parameters θ we are estimating, including distributional means, medians,

[5] Note that the difference between posterior density $p(\theta|y)$ and posterior kernel $\mathcal{K}(\theta|y)$ is

$$p(\theta|Y^T) \equiv \frac{\mathcal{K}(\theta|Y^T)}{p(Y^T)}$$

[6] While Maximum Likelihood (ML) procedure stops at maximizing $p(y|Y^T)$, the Bayesian method extends ML method further by updating the results with prior information of the parameters. The result of the Bayesian procedure combines information from data and prior information we already know about the parameters in estimation themselves.

We can also distinguish a key difference between Bayesian (as well as ML) and classical/frequentist econometrics. While the former assumes that if we need to estimate unknown parameters θ, then given the data we should find the probability of getting those parameters of the model $p(\theta)$ (not fixed but rather random objects), the latter treats θ as some unknown but fixed parameters and uses frequentist method such as OLS, given the data, to find those fixed values. The point estimates of those parameters can be the mode or median of the density.

modes, and respective standard deviations, also allows us to implement Bayesian inference as in the frequentist approach. Suppose Bayesian inference is expressed as $E[g(\theta)|Y^T)]$ where $g(\theta)$ is the inferred function we are interested in such as confidence interval etc., given the data Y^T, the *Numerical Integration* method is used to compute the conditional distribution of the target function as follows

$$E[g(\theta)|Y^T)] = \int g(\theta)p(\theta|Y^T)d\theta \tag{3.5}$$

However, both Bayesian estimation (as well as ML method) and Bayesian inference involve the evaluation of the likelihood function which, as argued by Miao (2014), typically has no analytical solution. In other words, it is often impossible to evaluate the integral in the likelihood function or the inference function analytically. Therefore, in practical exercises, the Bayesian estimation requires a two-stage procedure as follows

- Stage 1: Solve the model and then use a filtering procedure[7] to evaluate the likelihood function $p(y|Y^T)$, maximize $p(y|Y^T)$ numerically to arrive at the mode of the estimated parameters θ;

- Stage 2: A Markov Chain Monte Carlo (MCMC) simulation method[8] is used to sample from the estimated mode in the first step which wanders over the posterior distribution of the parameters. As the number of draws N increases, we can invoke the Law of Large Numbers and the Central Limit Theorem to derive the posterior distribution of the parameters.

[7]Such as the linear Kalman Filter assuming linear transition and measurement equations and Gaussian shocks.

[8]Such as Metropolis-Hastings MCMC.

3.2.2 Estimation Procedure

Analytical evaluation of the likelihood function is a great challenge. In practice, as argued by Fernández-Villaverde (2010), DSGE models do not have a "paper and pencil" solution, with very few exceptions. Therefore, numerical methods are normally used to solve the problem in order to arrive at the estimated mode as output for Stage 1. While in a general case, Bayesian Maximum Likelihood is used to compute the likelihood function, the Kalman filter is the alternative to evaluate the likelihood function when Gaussian assumptions, which are prevalent in the DSGE literature, of the distribution of the shock processes are made[9].

3.2.2.1. Stage 1 - Computation of the Posterior Mode

The maximization of the likelihood function of DSGE models, which is the essential part of Stage 1, normally involves the evaluation of a complicated, highly multidimensional object. In addition, as argued by Fernández-Villaverde (2010), the likelihood function of DSGE models is full of local maxima and minima and of nearly flat surfaces due to the sparsity of the data[10] and the flexibility of DSGE models in generating similar behaviour with relatively different combination of parameter values. In a general case, the Bayesian Maximum Likelihood method can be used to deal with this problem. However, as stressed above, the Kalman filter is more efficient in computing the likelihood function when Gaussian distributional assumptions for the shocks are added.

[9] For the sake of simplicity, we skip the discussion on Particle Filter for the case of non-linear state space representation or the case in which the shocks are not normally distributed.

[10] As quarterly data are often not comprised of many observations that micro panels provide.

Bayesian Maximum Likelihood: The General Case

Numerical methods such as perturbation or numerical projection produce a policy function/decision rule with respect to the whole set of model variables y_t which is expressed as a polynomial of state variables (at time $t-1$) and model shocks ε_t as follows

$$y_t = h(y_{t-1}, \varepsilon_t; \theta) \tag{3.6}$$

When it comes to estimation, however, not all of the model's variables in y_t are observable. In practice, suppose only a subset of the variables y_t^* can be observed[11] with a dataset $Y^{*,T}$, which is "smaller" than a full dataset Y^T. Thus, the observed variables y_t^* are just a subset of the model's variables y_t and they are linked by a *measurement equation* of the form

$$y_t^* = l(y_t, v_t; \theta) \tag{3.7}$$

where v_t is a vector of shocks to the observables such as measurement error.

As mentioned above, our objective is to evaluate the likelihood function of the data given the model's parameters $p(Y^{*,T}|\theta)$ so as to get to the target likelihood function of the parameters given our data $p(\theta|Y^{*,T})$. As shown by Fernández-Villaverde (2010), we can take advantage of the Markov structure of the state space representation to write the likelihood density of the data given the model's parameters as

$$p(Y^{*,T}|\theta) = p(y_1^*|\theta) \prod_{t=2}^{T} p(y_t^*|Y^{*,t-1}, \theta) \tag{3.8}$$

where y_1^*, y_t^* are observations number 1 and number t in the data $Y^{*,T}$.

[11]Recall that all of the arguments in the previous section are applied for a full dataset Y^T which means all of the model's variables are observed.

Deriving this equation further using Lemma 3 given in Appendix D we have

$$p(Y^{*T}|\theta) = \int p(y_1^*|y_1, \theta)p(y_1|\theta)dy_1 \times \prod_{t=2}^{T} p(y_t^*|Y^{*,t-1}, y_1, \theta)p(y_t|Y^{*,t-1}, \theta)dy_t \qquad (3.9)$$

On the right-hand side, $p(y_1^*|y_1, \theta)$ and $p(y_1|\theta)$ are given as initial conditions. Also, as $Y^{*,t-1} \in y_t$, we have

$$p(y_t^*|Y^{*,t-1}, y_1, \theta) = p(y_t^*|y_t, \theta) \qquad (3.10)$$

which is proved in Lemma 2.

Therefore, the evaluation of the posterior likelihood function in the general case is equivalent to the computation of the last term $\{p(y_t|Y^{*,t-1}, \theta)\}_{t=1}^{T}$. For this task, as shown by Fernández-Villaverde (2010), we need to rely on two fundamental tools which are the Chapman-Kolmogorov equation and Bayes' theorem. First, the Chapman-Kolmogorov equation describes the forecasting rule for the evolution of states. It shows that the distribution of states tomorrow given an observation until today, $p(y_{t+1}|Y^{*,t}, \theta)$, is equal to today's distribution $p(y_t|Y^{*,t}, \theta)$ times the transition probabilities $p(y_{t+1}|y_t, \theta)$:

$$p(y_{t+1}|Y^{*,t}, \theta) = \int p(y_{t+1}|y_t, \theta)p(y_t|Y^{*,t}, \theta)dy_t \qquad (3.11)$$

where among two terms on the right-hand side of this equation, the former $p(y_{t+1}|y_t, \theta)$ can be induced from Lemma 1, while the later $p(y_t|Y^{*,t}, \theta)$ is computed using Lemma 4.

Second, Bayes's theorem helps, as proved in Lemma 4, to update the distribution of states $p(y_t|Y^{*,t}, \theta)$ whenever a new observation y_t^* arrives with corresponding probability $p(y_t^*|y_t, \theta)$ before Chapman-Kolmogorov equation comes in as a rule to forecast the evolution of the state from our initial condition $p(y_1, \theta)$ up to $p(y_t|Y^{*,t-1}, \theta)$, which is the target distribution we are interested in.

Kalman Filter: The Special Case of Gaussian

As argued by Miao (2014) and Fernández-Villaverde (2010), even though Bayes' theorem and the Chapman-Kolmogorov equation are mathematically straightforward, the numerical computation of the term $\{p(y_t|Y^{*,t-1},\theta)\}_{t=1}^{T}$ in practice is almost impossible as it would require the computation of numerous integrals. However, given the assumptions of normal distribution of the shocks[12] which are prevalent in the literature and also the case of the present model, Fernández-Villaverde (2010) argues that we can take advantage of the observation that all of the relevant conditional distributions are thus also Gaussian[13]. If so, we need only to keep track of the mean and variance of these conditional normal distributions. In this case, Ricatti equations of the Kalman Filter are used to compute the log likelihood density of $p(Y^{*,T}|\theta)$[14].

In particular, Miao (2014) shows that in the first instance, a perturbation method is used to solve the DSGE model in order to arrive at the decision rules which can be expressed in *a linear state space representation* including a decision rule of the form

$$\tilde{y}_t = A(\theta)\tilde{y}_{t-1} + C(\theta)\varepsilon_t \tag{3.12}$$

and a transition equation for observed variables given by

$$y_t^* = G\bar{y}(\theta) + G\tilde{y}_t + v_t \tag{3.13}$$

where $\tilde{y}_t$ is the deviation from steady state $\bar{y}(\theta)$ and $E\varepsilon_t = Ev_t = 0$; $E\varepsilon_t\varepsilon_t' = Q(\theta)$; $Ev_tv_t' = V(\theta)$. The coefficients A, C, G and the steady state values are dependent on the

[12]Another additional assumption is the linearity of the transition and measurement equations.

[13]Because the space of normal distributions is a vector space.

[14]This is also the case that is currently run by Dynare.

structural parameter θ we are estimating. Coefficient G of the transition equation collects the vector of observables from the vector of all equilibrium variables y_t.

If we define the linear projections as

$$\hat{y}_{t|t-1} = E_t[\tilde{y}_t|Y^{*,t-1}]$$

where $\hat{y}_t$ is the projected values and the subscript $t|t-1$ indicates a draw at t conditional on information until $t-1$; and the variance-covariance matrices as

$$P_t = E\left[(\tilde{y}_t - \hat{y}_{t|t-1})(\tilde{y}_t - \hat{y}_{t|t-1})'|Y^{*,t-1}\right]$$

Then, given the initial values $\hat{y}_{1|0}$, P_1, and coefficients A, C, G, Q, V, P_t, Miao (2014) shows that the following recursive identities can be achieved using the Kalman Filter

$$\begin{cases} a_t & = y_t^* - \hat{y}^*_{t|t-1} = y_t^* - G\bar{y}(\theta) + G\tilde{y}_{t|t-1} \\ \Omega_{t|t-1} & = GP_tG' + V(\theta) \\ K_t & = A(\theta)P_tG'\Omega^{-1}_{t|t-1} \\ P_{t+1} & = A(\theta)P_t[A(\theta) - K_tG]' + C(\theta)Q(\theta)C(\theta)' \\ \hat{y}_{t+1|t} & = A(\theta)\hat{y}_{t|t-1} + K_t a_t \end{cases}$$

Finally, using the first two identities a_t[15] and $\Omega_{t|t-1}$, the log likelihood density of the data given the model's parameters can be computed as follows

$$\begin{aligned} \ln p(Y^{*,T}|\theta) & = \sum_{t=1}^{T} \ln p(y_t^*|Y^{*,t-1},\theta) \\ & = -\frac{n^*T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln|\Omega_{t|t-1}| + a_t'\Omega^{-1}_{t|t-1}a_t\right) \end{aligned} \tag{3.14}$$

where n^* is the number of observables.

[15]Which can be interpreted as the innovation of the observables.

In practical exercises, the detailed computation algorithm[16] for Stage 1 is suggested by Cantore et al. (2016) as follows:

Step 1: Solve the calibrated model numerically for *a particular* parameter vector θ_i[17]. Results are corresponding *policy functions* and a *decision matrix* with all numerically-solved values of the coefficients of θ_i which are used as the initial values for the Maximum Likelihood algorithm.

Step 2: Given the estimated values of the model parameters in θ_i from step 1, the likelihood density of the data, $Y^{*,T}$, which is $p(Y^{*,T}|\theta_i)$ is evaluated using the linear Kalman Filter, given the necessary assumption that all shocks are Gaussian.

Step 3: Return to step 1, solve the model again for *another particular* parameter vector θ_{i+1}[18] and then evaluate the corresponding likelihood density $p(Y^{*,T}|\theta_{i+1})$ using Kalman filter as in Step 2. The algorithm then just repeats steps 1 and 2 to get a new $p(Y^{*,T}|\theta_{i+1})$ so as to maximize $p(Y^{*,T}|\theta)$ numerically. The converged results θ^0 are the *posterior mode* of the parameters[19].

Step 4: The posterior mode θ^0 will be used as the starting point for the second stage. We can also use this posterior mode to estimate the covariance matrix of all relevant

[16] Similar to Maximum Likelihood to estimate DSGE models.

[17] This is currently a first-order linear solution from perturbation technique implemented by Dynare.

[18] Specified by the algorithm.

[19] The algorithm sets up a threshold and a number of iterations which is large enough to get the maximum likelihood numerically.

parameters by the negative inverse Hessian matrix evaluated at the posterior mode

$$\hat{\sum}_{\theta \text{ at the mode}} = \left(-\frac{\partial^2 \log(\mathcal{K}(\theta|Y^{*,T}))}{\partial\theta\partial\theta'} \right)^{-1} \tag{3.15}$$

where the term in brackets is the Hessian[20].

Outputs from Stage 1 include the prior mean, estimated mode, standard deviation, and a t-test. The two popular mode compute methods which are currently used by Dynare are mode_compute = 4 or 6.

3.2.2.2. Stage 2: Computation of the Posterior Distribution

In Stage 2, the Markov Chain Monte Carlo Metropolis-Hastings algorithm is used to find the posterior distribution of the parameters. Cantore et al. (2016) show that the procedure is developed with theoretical contributions from each school named in the algorithm. First, the Markov Chain postulates that any draw θ_s depends on its previous draw θ_{s-1}. Second, Monte Carlo adds that a given draw θ_s^*, which is also our *target draw* in this context, can be drawn at random from a *candidate/transition distribution* which is called $\alpha(\theta_{s-1}, \theta_s^*)$. That means that θ_s^* can only be either the previous draw or itself. Thus, a new draw θ_s will either be our new target draw for the current round of the process θ_s^* or stand still at the previous point θ_{s-1}. In the later case, we add nothing to the density that we are interested in. Thus, the Markov Chain Monte Carlo approach starts from an initial value θ^0 which is normally the estimated mode from the first stage to sketch out a limiting distribution which is also our *target distribution*[21].

[20]The square matrix of second-order partial derivatives of the likelihood density function $\log(p(y|\theta))$ with respect to all values of the parameters at the mode of $\theta = (\theta_1, \theta_2, ..., \theta_n)$.

[21]In practice, we need to discard the first several thousand draws to ensure that the sequence is not

Third, as there always exists a discrepancy between the candidate and target distribution, however, if we take the whole bunch of draws from MCMC, there may be a lot of wrong draws from our target draws while we still want to sketch out the whole parameter space as much as possible. Thus, as argued by Cantore et al. (2016), the Metropolis-Hastings rule contributes to this process by calculating an *acceptance rate* which allows us to discard the wrong draws from the MCMC process and, more importantly, suggests the next candidate to continue the procedure. Accordingly, the next candidate θ_s^* for the process should be chosen given a Random Walk rule as follows

$$\theta_s^* = \theta_{s-1} + z$$

where the shock z has mean θ_{s-1} and variance $c\sigma^2$, or $z \sim N(\theta_{s-1}, c\sigma^2)$.

In a nutshell, the full Markov Chain Monte Carlo Metropolis-Hastings algorithm in the second stage of Bayesian estimation method is specified as

Step 1: Choose a starting point θ^0 for the process, which is the posterior mode from the first stage.

Step 2: Apply the Metropolis-Hastings rule to find the next candidate θ_i^* for the distribution.

$$\theta_i^* = \theta^0 + z$$

Step 3: Given θ_i^*, come back to solve the model and use the Kalman Filter to compute the likelihood density $p(\theta_i^*|y)$, given the available data.

dependent on the starting point θ^0.

Step 4: Given $p(\theta_i^*|y)$, calculate the acceptance rate by the general formula

$$r = \min\left[\frac{p(\theta_i^*|y)}{p(\theta_{i-1}|y)}, 1\right]$$

where θ_{i-1} is just θ^0 in the first round.

Step 5: Come back to step 2 to find the next candidate. Keep doing this process until sufficient draws are generated. We can then sketch out the whole posterior distribution from accepted draws for parameters θ.

It is shown by Cantore et al. (2016) that the jump step from θ_{s-1} to θ_s^* in step 4 depends on z, which is then dependent on the constant c[22]. It should also be noted that, first, if c is too small, z's variance as well as the jump step become too small, the rate of accepted draws will be too high and approach 1, and the chain thus gets stuck around a local maximum and does not visit the tails. Second, if c is too large, the acceptance rate will be low because z's variance is now too large, θ_i^* goes very far from $\theta_{i-1} = \theta^0$, which is already an *accepted draw* at the mode, then its likelihood $p(\theta_i^*|y)$ becomes very much smaller compared with the mode. The draws will come to regions of lower probability and the chain again gets stuck in the tails of the target distribution[23].

[22]The constant c is chosen by user at the beginning of the Bayesian process.

[23]Therefore, the ideal acceptance rate should be from 20-40% during the whole process in order to ensure that the entire domain of the target distribution is visited. We need to choose c from the beginning of the estimation so as to make sure the acceptance rate in the range from 20-40%.

3.3 Data Transformation and Measurement Equations

The Bayesian estimation method allows the use of all available information in the form of prior distributions, as well as the use of observed data to update these priors, in order to obtain the posterior distributions of the estimated parameters. However, one of the fundamental issues in the estimation process is that the observed data usually do not exactly match the model's variables. This could be because of a stochastic trend in the data, while the model's variables are stationary since the model is solved using perturbation techniques. The mapping of data onto model's variables is expressed in measurement equations section incorporated into the Dynare file.

3.3.1 Data Transformation

While many models are agnostic about the source of the trend in the data, there are models where the source of the trend is explicitly specified, as in the present model. Typically, there are two sources of trend: (i) growth in population and (ii) technological growth. Both are assumed to be a stochastic growth trend in the form of a random walk with drift. As specified by Pfeifer (2018), for log-linearized DSGE models with a specified trend, the respective variables are integrated, i.e. $I(1)$ and differencing makes them stationary, or $I(0)$.

In the present model, *original* variables used to derive the firms and households' problems are capital letters $(Y_t, C_t, ...)$ which need to be mapped with the original data $(Y_t^{data}, C_t^{data}, ...)$ when it comes to estimation. However, by incorporating a specified trend, we have transformed the model's original variables by ϵ_t^z to obtain lower case model's *transformed* variables $(y_t, c_t, ...)$. In order to estimate the model, transformed variables which are then log-linearized around a steady state ($\hat{y}_t, \hat{c}_t, ...$) have to be mapped with *observed* variables ($y_t^{obs}, c_t^{obs}, ...$).

First, for *integrated (non-stationary) variables* such as output, the model's original variables are first detrended by dividing the trend ϵ_t^z to get the intensive form of the transformed variable as follows

$$y_t = \frac{Y_t}{\epsilon_t^z} \tag{3.16}$$

Then,

$$Y_t^{data} = Y_t = y_t \epsilon_t^z \tag{3.17}$$

This equation cannot be entered into Dynare because ϵ_t^z is not a stationary variable. However, its growth rate in the detrended law of motion is stationary. Thus, in order to get rid of ϵ_t^z, we can simply work with its growth rate, $\mu_{z,t}$. This term only enters the measurement equations of variables that have a real unit root in the model.

Differencing the logs of Y_t^{data} gives exactly the observed variable y_t^{obs} because

$$y_t^{obs} = \log(Y_t^{data}) - \log(Y_{t-1}^{data})$$

$$= \log(Y_t) - \log(Y_{t-1})$$

$$= \log(y_t) - \log(y_{t-1}) + [\log(\epsilon_t^z) - \log(\epsilon_{t-1}^z)]$$

$$= [\log(y_t) - \log(\bar{y})] - [\log(y_{t-1}) - \log(\bar{y})] + \left[\log(\epsilon_t^z) - \log(\epsilon_{t-1}^z)\right]$$

so we have

$$y_t^{obs} = \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_{z,t}$$

Second, for *stationary variables*, such as inflation and interest rates, we can simply take difference of the log of data

$$\pi_t^{obs} = \log(\pi_t^{data}) - \log(\pi_{t-1}^{data})$$

$$R_t^{obs} = \log(R_t^{data}) - \log(R_{t-1}^{data})$$

And the measurement equations should be

$$\pi_t^{obs} = \hat{\pi}_t - \hat{\pi}_{t-1}$$

$$R_t^{obs} = \hat{R}_t - \hat{R}_{t-1}$$

In order to estimate the parameters of the DSGE model presented in Chapter 2, a quarterly macroeconomic dataset published by the General Statistics Office of Vietnam over the period 1996:1 - 2012:2 is then collected which covers 66 data points. The vector of observables consists of 13 variables, including 10 domestic and 3 foreign variables. The US output, inflation, and the federal funds rate series are used as the world indicators. Thus, the data vector is given by

> $Data_t$ = {GDP (Y_t), household consumption (C_t), investment (I_t), real exchange rate (RER_t), lending rate of four biggest commercial banks (R_t), inflation (π_t), government consumption (G_t), employment (E_t), exports (X_t), imports (M_t), foreign output (Y_t^*), foreign inflation (π_t^*), foreign interest rate (R_t^*)}

By construction, DSGE models focus on explaining business cycle fluctuations around steady state values and model solutions imply stationarity of the variables. However, original macroeconomic series often involve both trends and cycles, making it necessary to transform the series so that we can focus on cyclical behaviour. Thus, actual series are seasonally adjusted with the X13-method of the U.S. Census Bureau and growth rates are computed as quarterly log differences, while inflation and interest rates are expressed at quarterly rates. A full description of the data transformation is given in appendix A.

3.3.2 Measurement Equations in Special Cases

It should be noted that, as argued by Adolfson et al. (2005), while data on consumption, investment, import, and export are collected as simple summation of their components, the corresponding variables in the present theoretical model are CES aggregates of domestic and foreign components in the case of consumption and investment, or weighted indices of consumption and investment goods in export and import. For example, while data for investment are observed as $I_t^{data} = I_t^d + I_t^m$, the original variable for investment I_t is not just a simple summation of I_t^d and I_t^m as in the data but rather a CES index of domestic and imported components.

Therefore, an *adjusted* original variable for investment $\tilde{I}_t$, which is the sum of I_t^d and I_t^m adjusted with corresponding relative prices, needs to be derived and then used to map with data on consumption ($\tilde{I}_t = I_t^{data}$). In other words, the adjusted original variable plays an intermediate role in this context to link the model's original variable (I_t) with observed variable (i_t^{obs}). The adjusted variable for investment is given by

$$\begin{aligned}
\tilde{I}_t &= I_t^d + I_t^m \\
&= \left[(1-\omega_i)\left(\frac{P_t}{P_t^i}\right)^{-\eta_i} + \omega_i\left(\frac{P_t^{m,i}}{P_t^i}\right)^{-\eta_i}\right] I_t \\
&= \left[(1-\omega_i)\left(\frac{1}{\chi_t^{i,d}}\right)^{-\eta_i} + \omega_i(\chi_t^{mi,i})^{-\eta_i}\right] I_t \qquad (3.18)
\end{aligned}$$

where $\chi_t^{mi,i} = \frac{P_t^{m,i}}{P_t^i} = 1/\chi_t^{i,mi}$, which is the price of domestic investment relative to price of imported investment.

The same observations apply to consumption that lead to the adjusted variable as follows

$$\begin{aligned}
\tilde{C}_t &= C_t^d + C_t^m \\
&= \left[(1-\omega_c)\left(\frac{P_t}{P_t^c}\right)^{-\eta_c} + \omega_c\left(\frac{P_t^{m,c}}{P_t^c}\right)^{-\eta_c}\right] C_t \\
&= \left[(1-\omega_c)\left(\frac{1}{\chi_t^{c,d}}\right)^{-\eta_c} + \omega_c(\chi_t^{mc,c})^{-\eta_c}\right] C_t \qquad (3.19)
\end{aligned}$$

where $\chi_t^{mc,c} = \frac{P_t^{m,c}}{P_t^c} = 1/\chi_t^{c,mc}$, which is the price of domestic consumption relative to price of imported consumption.

Adjusted variables for total imports and exports are given by

$$\begin{aligned}
\tilde{M}_t &= C_t^m + I_t^m \\
&= \omega_c(\chi_t^{mc,c})^{-\eta_c} C_t + \omega_i(\chi_t^{mi,i})^{-\eta_i} I_t \\
&= \omega_c\left(\frac{\chi_t^{mc,d}}{\chi_t^{c,d}}\right)^{-\eta_c} C_t + \omega_i\left(\frac{\chi_t^{mi,d}}{\chi_t^{i,d}}\right)^{-\eta_i} I_t \qquad (3.20)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{X}_t &= \tilde{C}_t + \tilde{I} \\
&= \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^* \\
&= (\chi_t^{x,*})^{-\eta_f} Y_t^* \qquad (3.21)
\end{aligned}$$

Finally, log-linearization techniques are used to link the data (C_t^{data}, I_t^{data}, M_t^{data}, X_t^{data}) with the observed variables (c_t^{obs}, i_t^{obs}, m_t^{obs}, x_t^{obs}), and then to map observed variables with the model's original variables (C_t, I_t, M_t, X_t), or more directly with their log-linearized counterparts during the estimation process ($\hat{c}_t$, $\hat{i}_t$, $\hat{m}_t$, $\hat{x}_t$), through the intermediate adjusted original variables ($\tilde{C}_t$, $\tilde{I}_t$, $\tilde{M}_t$, $\tilde{X}_t$). The corresponding log-linearized measurement equations for consumption, investment, imports, and exports are as follows

$$c_t^{obs} = \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_{z,t} \tag{3.22}$$

$$i_t^{obs} = \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_{z,t} \tag{3.23}$$

$$m_t^{obs} = \frac{\varrho_4}{\varrho_3}(\hat{c}_t - \hat{c}_{t-1}) + \frac{\varrho_5}{\varrho_3}(\hat{i}_t - \hat{i}_{t-1}) + \hat{\mu}_{z,t} \tag{3.24}$$

$$x_t^{obs} = \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\mu}_{z,t} \tag{3.25}$$

where specific parameters ϱ_3, ϱ_4, and ϱ_5 are derived as in appendix C.

3.3.3 Stochastic Singularity and Remedies

As pointed out by Fernández-Villaverde (2010), if the number of observables is more than the number of shocks, the DSGE model will be up to first-order, stochastically singular, that is, the extra observables would be a *deterministic function* of the other observables and the likelihood of observing those added observables would be $-\infty$ with probability 1. In other words, there will be two or more linear combinations of the variables that hold without noises, voiding any inference exercise. Thus, stochastic singularity, as argued by Ruge-Murcia (2007), is one of the most important features of DSGE models which has implications for all estimation procedures, limiting the number of variables that can be exploited for the estimation.

Ruge-Murcia (2007) also suggests three strategies to deal with singular DSGE models. First, the model can be estimated with a number of shocks at least as many as the number of observed variables. Second, error terms can be added to observation equations of the state-space representation which contain no information about current or future structural shocks. Finally, the model can be extended to permit additional structural shocks and this strategy seems to be attractive because it increases the realism of the model and allows the use of more observables in the estimation process.

In a nutshell, in order to have well-defined econometric estimates, it is necessary that for every observed variable, there should be at least one unobservable stochastic shock driving its evolution, making it possible to generate the forecast error for every observed variable and more importantly, a non-zero likelihood of fitting the predicted values from our theoretical model with the observed data. In the present model, even though stochastic singularity is not a serious problem during the estimation as the number of observables is less than the number of structural shocks, measurement errors are still added to measurement equations due to the fact that time series in developing countries in general and Vietnam in particular are noisy and presumably poorly measured. In addition, measurement errors are technically a way of accounting for model misspecification when the data violate the cross-equation restrictions implied by the theoretical model.

3.4 Prior Distributions and Identification

3.4.1 Prior Distributions

One of the advantages of Bayesian estimation and inference is that the technique allows us to use pre-sample information, which is often very rich and considerably useful, from previous studies at both macro and micro levels in a formal way. Estimates of macro parameters from different countries are one of the best sources of pre-sample information given a default belief, as argued by Fernández-Villaverde (2010), that individuals are broadly the same across countries and that differences in behaviour can largely be accounted for by differences in relative prices. In addition, microeconometric evidence such as estimates on discount factor is another source of pre-sample information which can be very useful for the specification of prior distributions. Pre-sample information is particularly convenient when it comes to the estimation of DSGE models for developing countries due to the limited source of data or the possibility of a change in policy regime.

During the estimation, prior distributions show us all the available information about the parameter prior to observing the data. On the other hand, the data, or observed variables, are then used to update the prior information in order to arrive at the posterior distribution of the parameters which is characterized by its measures of location (mode and mean) and spread (standard deviation and probability intervals). The present model is estimated for 12 key structural parameters and 8 shock-related parameters. Table 1 summarizes the prior and posterior distributions of the estimated parameters. For prior information, following Adolfson et al. (2005), a beta distribution is first used for parameters bounded between 0 and 1, including nominal stickiness parameters ξ, indexation κ, habit persistence b, and the persistence parameters of the shock processes ρ. In the utility function, habit parameter is assumed to fluctuate around 0.7 with a standard error of 0.1, while the indexation to previous price is set around 0.5 with a standard error of 0.15. Also, the Calvo and indexation parameters for wages are assumed to fluctuate around 0.7 and 0.5 with standard errors of 0.05 and 0.15, respectively. For the shocks which are serially correlated, the prior mean of the autoregressive coefficient is set at 0.50 with standard errors of 0.2.

Second, the normal distribution is used for all unbounded parameters such as investment adjustment costs ζ, which is assumed to be around 2 with a standard error of 1.5. Third, inverse gamma distribution is used for parameters assumed to be positive such as the standard deviations of the shocks, domestic markup, and the substitution elasticities. The domestic markup is assumed to be around 1.2 with a standard error of 2.0, while the priors for substitution elasticities are set at 1.5 with a standard error of 4.0. The means for the standard deviations of the monetary policy rule, government spending rule, and domestic mark-up shocks, which are of interest of the present study, are set at 0.1, 0.5, and 0.1, respectively. Two degrees of freedom (standard deviation 2) are also admitted for all of them, which is considered by Smets & Wouters (2007) to be a rather loose prior.

Finally, the parameters describing monetary policy rule, which is the focus of the present research, are based on a standard Taylor rule. While the long-run reaction on inflation and output are described by a normal distribution with mean 2.0 and 0.125 and standard errors 0.25 and 0.05, respectively, the persistence of the policy rule is determined by the coefficient of the lagged interest rate, which is also assumed to be normally distributed around a mean of 0.75 with a standard error of 0.1.

3.4.2 Identification Analysis of the Parameters

Before taking the model to data, we need to confront the question of parameter identifiability as parameter identification is a prerequisite for the informativeness of the estimators. This section seeks to answer two questions: (i) how to check whether a parameter is identified or not? and (ii) if it is identified, what is the strength of that identification? In order to answer these questions, following Iskrev (2010) and Iskrev & Ratto (2011), the classical rank condition approach[24] is used to analytically evaluate the information matrix of the reduced-form model and check for rank deficiency of the gradient matrix to get information on the identification of the parameters given the priors and observables.

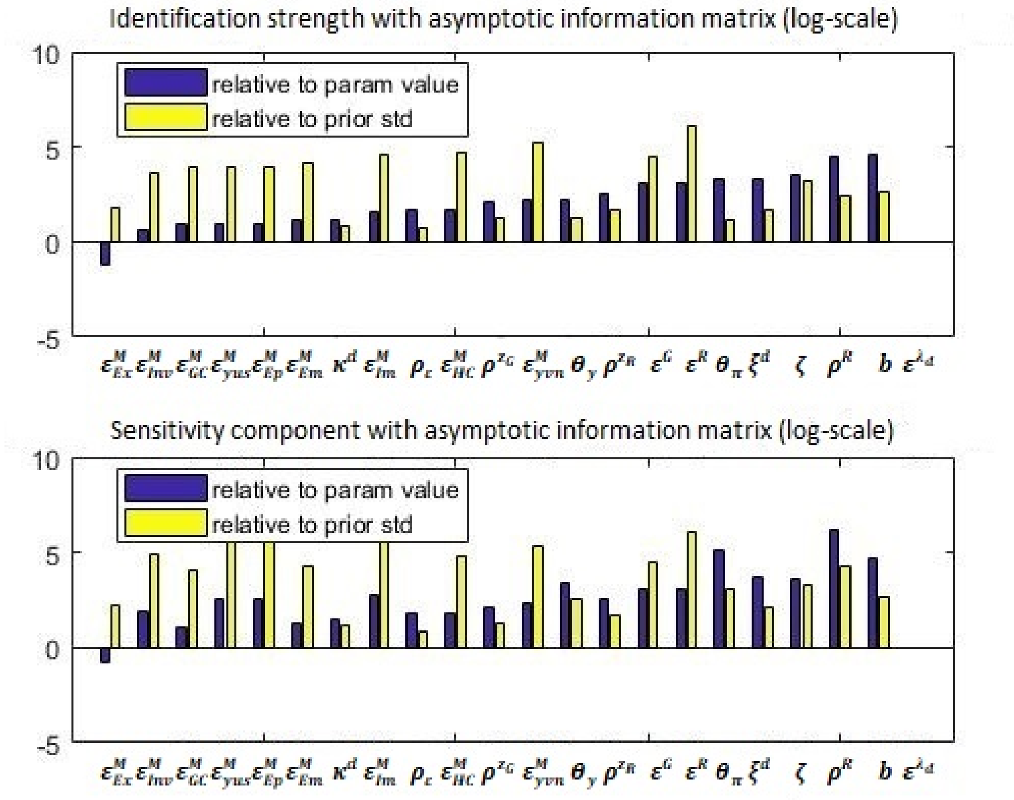
The parameters passing the identification check are then ranked in terms of strength of identification. As defined by Pfeifer (2017), *weak identification* in general can be due to (i) a *correlation* of the parameters in which other parameters also having exactly the same effect on the likelihood and the collinearity dampens the effect of the related parameter; or (ii) a *sensitivity* that the likelihood does not change at all, or the moments do not change, with the related parameter. Correlation of the parameters leads to a flat likelihood at the local point which can, however, be diagnosed and completely ruled out by checking collinearity between the effects of different parameters on the likelihood. If there exists an exact linear dependence between a pair and among all possible combinations, their effects on the moments are not distinct and the violation of this condition must indicate a

[24]It should be noted that the sources of identification failure could, as argued by Cantore et al. (2016), be (i) marginalisation (omitting relevant explanatory variables from the analysis) which is caused by the model structure, i.e. mapping deep parameters to the reduced form coefficients of the solution and mapping the solution to the population objective function (only this issue is considered in the present study); and (ii) lack of information which comes from the data, i.e. mapping the population to the sample objective. Current approaches to deal with identification problem in the literature can be categorized into three main branches. The first group uses classical rank condition approach that checks rank deficiency of the gradient matrices based on model-implied moments. Second, objective function method focuses on population moments of the data. The last approach is based on observational equivalence focusing on (non-linear) system matrices in the solution model.

flat likelihood and weak identification. Furthermore, in order to elaborate the sensitivity effect of the parameter, a *moment information matrix* is introduced and computed either by using the parameter values at the prior means or by normalizing the identification strength measures relative to the parameters' priors on standard deviation.

Figure 3.1 plots the measures described in the previous section, showing the identification strength and sensitivity component in the moments of the model where large bars in absolute value (in the log-scale plot) imply strong identification for the related parameter. In the upper panel, the bars show the identification strength of the parameters based on the information matrix normalized by either the parameter at the prior mean (blue bars) or the standard deviation at the prior mean (yellow bars). As explained by Pfeifer (2017), the graphs generally use a log-scale except for parameters that are unidentified, which are shown with a bar length of exactly 0. Intuitively, these bars represent the normalized curvature of the log likelihood function in the direction of the parameter at the prior mean. If the strength is 0 for both bars, the parameter is not identified as the likelihood function is flat in this direction. The larger the absolute value of the bars, the stronger the parameter's identification.

Figure 3.1: Identification Strength and Sensitivity Component



The lower panel decomposes further the effect on the parameters shown in the upper panel. As pointed out in the previous section, any instance of weak identification can be due to either the fact that other parameters linearly compensate (or replace) the effect of a parameter or the likelihood does not change at all with the related parameter. The weighting can be conducted with either prior means (blue bars) or prior standard deviations (yellow bars). The results show that almost all of the parameters of interest, except for λ^d, are well identified given our vector of observables. Moreover, the key parameters of the monetary policy rule θ_π, θ_y, and ρ_R are strongly identified both in terms of overall identification and sensitivity component.

Furthermore, in order to disaggregate the impact of a change of the elements of the parameter vector θ on the model's moments, as suggested by Iskrev & Ratto (2011), a normalization/standardization procedure is implemented to plot three different measures of sensitivity either at the prior mean or standard deviation using Jacobian/local derivatives. As argued by Pfeifer (2017), since derivatives are not scale-invariant and thus not easily comparable, the derivative of the jth moment of the moment vector m_j with respect to the parameter entry i, $\partial m_j / \partial \theta_i$, is then normalized by the corresponding ratio of standard deviation, $std(\theta_i)/std(m_j)$, to have

$$\frac{\partial m_j / std(m_j)}{\partial \theta_i / std(\theta_i)}$$

The normalization of the change in the i^{th} parameter $\partial \theta_i$ by its variance $std(\theta_i)$ technically accounts for different parameter uncertainty by ascribing, ceteris paribus, more importance to more variable parameters as they induce higher changes in the moments, while the normalization of a change in the moment ∂m_j with its standard deviation $std(m_j)$ allows for the comparison of the impact of the i^{th} parameter on differently volatile moments, as explained by Pfeifer (2017).

The bars in figure 3.2 show the norm of the columns of three standardized Jacobian matrices for the related parameter shown on the horizontal axis, including the moments matrix[25], the solution matrix[26], and the Linear Rational Expectation (LRE) model[27]. Consistent with the aggregate results, the key parameters of interest, especially monetary policy rule parameters, have relatively good identifiability strength with respect to any of

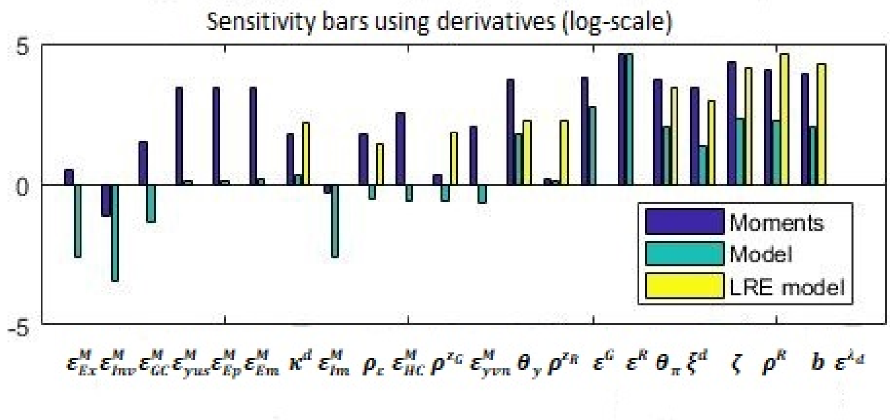
[25] $\partial \mathbf{m}_T / \partial \theta'$. This matrix tells us how well a parameter can be identified due to the strength of its impact on the observed moments, as in Pfeifer (2017).

[26] $\partial \tau / \partial \theta'$. This matrix shows how well a parameter could in principle be identified if all state variables were observed, as in Pfeifer (2017).

[27] $\partial \gamma / \partial \theta'$. This shows trivial cases where non-identifiability is due to, for example, the fact that some parameters always show up as a product in the model equations, as in Pfeifer (2017).

the sensitivity measures.

Figure 3.2: Sensitivity Analysis using Derivatives



Finally, figures 3.3 and 3.4 help to check the collinearity between the effects of different parameters and show, as guided by Pfeifer (2017), which linear combination of parameters in the columns is able to best replicate (or replace) the effect of the parameter in the row on the moments of the observed variables. These are the results of the brute force search for the groups of parameters whose columns in the Jacobian matrix best explain each column of the Jacobian, which means that those groups of parameters best reproduce the behaviour of each single parameter in the model. As the implied values rises, the related parameter is more of relative redundancy, meaning weak- or un-identifiability. In these figures, the darker red squares show the more critical collinearity between parameters.

The figures spot some concerns over possible collinearity of the parameters in the present model. For example, the row of θ_π suggests that there should be a correlation between the effect of θ_π on the model moments and the effect of a linear combination of ρ_R and θ_y. However, as shown in Dynare outputs, the *cosn* of the collinearity pattern of θ_π and a combination of ρ_R and θ_y is 0.9787, which is not identical to 1, indicating a high but not perfect collinearity and thus not problematic in this case, as argued by Iskrev & Ratto (2010) and Iskrev & Ratto (2011).

Figure 3.3: Collinearity Patterns with 1 Parameter

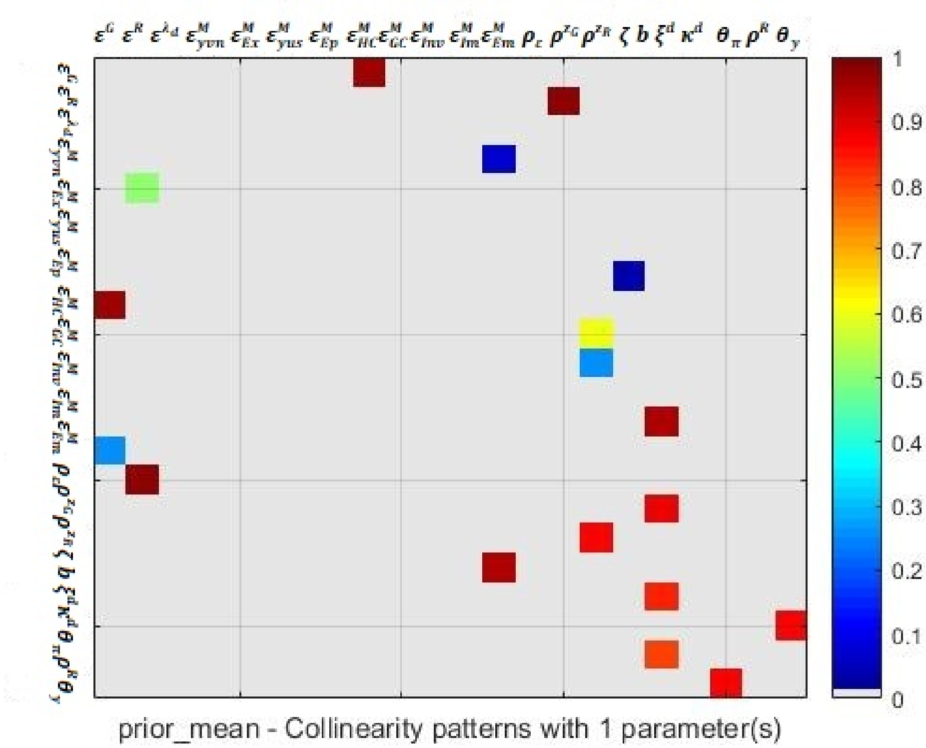
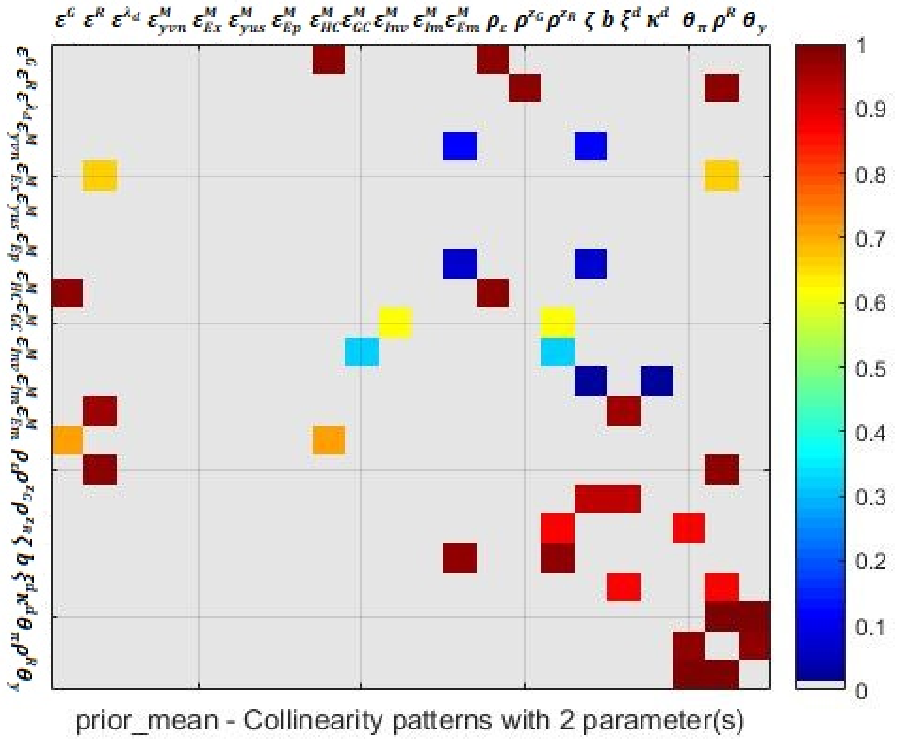


Figure 3.4: Pairwise Collinearity Patterns



3.5 Estimation Results

The parameters which are chosen for estimation relate mostly to monetary policy and to deep parameters (such as the nominal and real frictions in the model). Parameters that are weakly identified by the available dataset or related to the steady states of the observables are calibrated and kept fixed throughout the estimation process. As discussed in the technical section above, during the estimation process, the joint posterior distribution of all estimated parameters is obtained in two stages. In the first stage, the posterior mode and the approximated standard errors based on the Hessian matrix evaluated at the mode are computed using the Monte-Carlo based optimization routine (mode_compute = 6), which is not a standard numerical optimization routine such as Matlab's fmincon (mode_compute = 1) or Christopher Sims' optimizer csminwel (mode_compute = 4) in terms of time efficiency. However, the latter is a derivative-based Newton-type and thus a local optimizer, while the former should work more globally.

In the second stage, draws for the joint posterior distribution are generated using the Metropolis-Hastings sampling algorithm, based on 300,000 draws, and the proposal distribution is taken to be the multivariate normal density centred at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode. Table 3.1 shows the prior distribution information and the posterior means of all estimated parameters along with the approximate posterior standard deviation obtained from the inverse Hessian. In addition, the table also presents the 90% highest probability density (HPD) intervals for the parameters of interest. It is noteworthy that all parameters and standard errors of the shocks are estimated to be significantly different from zero. As argued by Smets & Wouters (2003), this tells us that they play a significant role in the estimated model.

Parameters		**Prior Distribution**			**Posterior Distribution**		
		type	mean	std	mean	90% HPD interval	std
Monetary Policy Parameters							
Inflation Response	θ_π	norm	2.000	0.25	1.6353	1.3316 1.9185	0.1326
Interest Rate Smoothing	ρ_R	beta	0.750	0.10	0.8458	0.8091 0.8829	0.0113
Output Response	θ_y	norm	0.125	0.05	0.1558	0.0945 0.2136	0.0179
Structural Parameters							
Inv. Adj. Cost	ζ	norm	2.000	1.50	2.7495	1.3033 4.2433	0.4365
Habit Formation	b	beta	0.708	0.10	0.6448	0.4621 0.7664	0.0192
Calvo Domestic Prices	ξ^d	beta	0.500	0.10	0.3205	0.1299 0.4997	0.0604
Calvo Wages	ξ^w	beta	0.675	0.05	0.6963	0.6232 0.7601	0.0321
Domestic Prices Index.	κ^d	beta	0.5	0.15	0.0624	0.0004 0.1282	0.0268
Wages Indexation	κ^w	beta	0.5	0.15	0.4608	0.2310 0.7192	0.0659
Domestic Markup	λ^d	inv. gamma	1.2	2.00	1.9571	0.5690 3.6867	1.0688
Inv. Sub. Elasticity	η^i	inv. gamma	1.5	4.00	0.9937	0.3759 1.6948	0.7280
Con. Sub. Elasticity	η^c	inv. gamma	1.5	4.00	0.5351	0.3104 0.7590	0.2396
Con. Pref. Shock	ρ_{ϵ^c}	beta	0.5	0.20	0.5834	0.3114 0.8521	0.0530
Labour Supply Shock	ρ_{ϵ^h}	beta	0.5	0.15	0.8755	0.8034 0.9434	0.0533
Invest-spe. Tech. Shock	ρ_{ϵ^i}	beta	0.5	0.20	0.9587	0.9328 0.9992	0.0176
Unit Root Tech. Shock	ρ_{μ_z}	beta	0.50	0.20	0.6796	0.5175 0.8011	0.0038
Stationary Tech. Shock	ρ_ϵ	beta	0.500	0.20	0.3103	0.0427 0.5160	0.0651
Standard Deviation of Shocks							
Fiscal Pol. Shock	σ_{z^G}	inv. gamma	0.5	2.00	0.0074	0.0057 0.0091	0.1458
Monetary Pol. Shock	σ_{z^R}	inv. gamma	0.1	2.00	0.1794	0.1721 0.1845	0.0480
Domestic Markup Shock	σ_λ	inv. gamma	0.1	2.00	0.0715	0.0245 0.1209	0.0234

Table 3.1: Prior and Posterior Distributions

Furthermore, estimation results are also presented visually in figures 3.5 to 3.7 where both prior and posterior distributions are plotted. The horizontal axis shows part of the support of the prior distribution, while the vertical axis indicates the corresponding density. The grey line depicts the prior density and the black line indicates the density of the posterior distribution. The dashed green vertical line shows the posterior mode. As argued by Pfeifer (2017), if the posterior looks like the prior, then either our prior was an accurate reflection of the information in the data or, more often, the parameter of interest is only weakly identified and the data do not provide much information to update our priors.

Figure 3.5: Priors and Posteriors - A

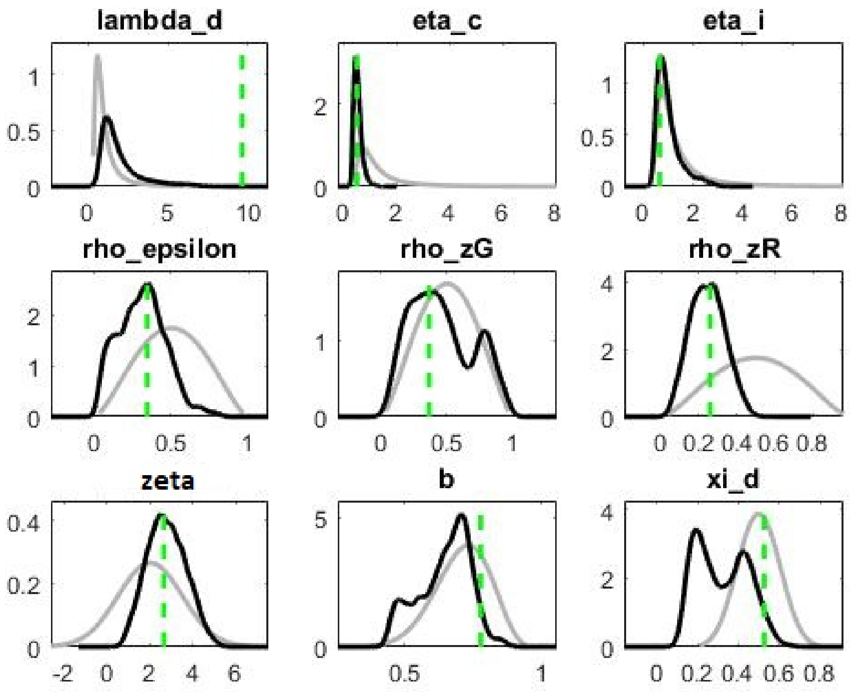


Figure 3.6: Priors and Posteriors - B

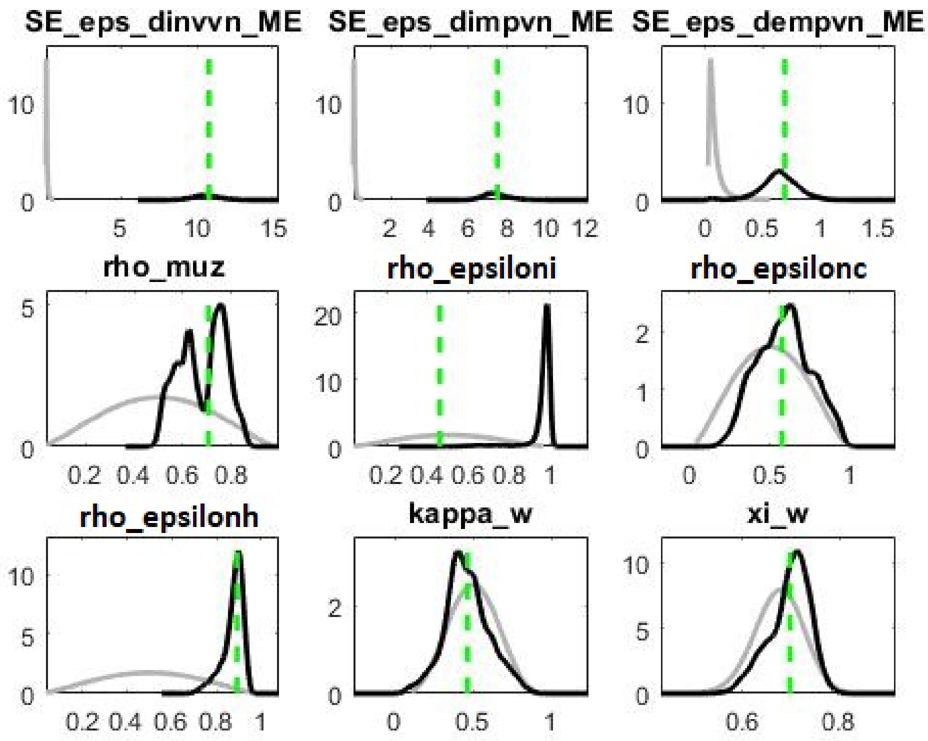
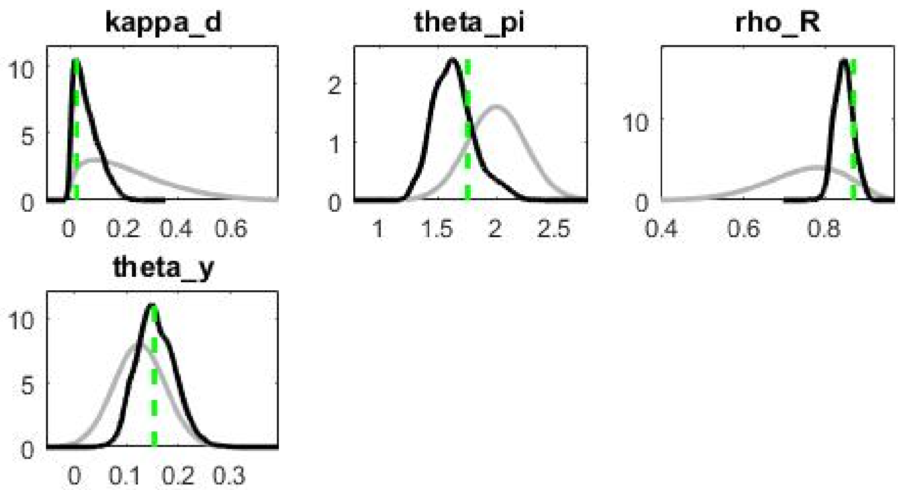


Figure 3.7: Priors and Posteriors - C



A number of interesting results merit attention. First, the point estimate for the domestic sticky price parameters ξ^d at 0.3205 is substantially lower than that of recent estimated DSGE models where Calvo estimates suggest that firms re-optimize their prices around every six quarters. However, the implication of an estimated average price duration of roughly 1.5 quarters[28] is fully consistent with Altig et al. (2004) and Altig et al. (2011) who show that firms re-optimize prices on average once every 1.5 quarters. The difference is that while Altig et al. (2004) find an aggregate inflation inertia despite the fact that firms change prices frequently because they do so by a small amount, the high frequency and large magnitude of price change in the present model for developing countries can be the reasons accounting for the lack of inflation inertia in the model. Also, as argued by Altig et al. (2004), this reflects conflicting pictures of price behaviour in

[28]The average price duration is $1/(1-\xi^d)$.

which microeconomic data indicate that firms change prices frequently while macroeconomic data suggest that inflation is inertial and these conflicting results are also present in recent macroeconomic models. In addition, the estimate of the sticky wage parameter is 0.6963, indicating a considerable degree of wage stickiness where wages can be re-optimized only after 3.3 quarters. The wage indexation parameter is estimated at 0.4608, which is broadly consistent with Adolfson et al. (2007) and Dutu (2016) where their estimates are 0.497 for the Euro area and 0.50 using Indonesian macro data.

Second, the point estimate of the indexation parameter κ^d at 0.0624 indicates a substantially forward-looking Phillips curve in the estimated model. This result is empirically robust as it is broadly in line with recent estimated DSGE models in which estimates suggest that the Phillips curves in most cases are forward-looking. It should also be noted that although the results are estimated based on a dataset from a developing country, it is shown to be consistent with those from estimated models using developed countries' data such as Adolfson et al. (2007) or Galí et al. (2001).

This result also suggests an intrinsic inflation persistence in the model economy which is strongly supported by Levin et al. (2004)'s finding that if the central bank's inflation objective is not transparent or credible, which is a prevalent feature of many developing countries, the private sector's rational forecast of medium- to long-run inflation depends on the recent behaviour of actual inflation. In addition, the intrinsic inflation persistence property of the estimated model is also robust and highly consistent with Sheedy (2010) or Fuhrer (2005) where empirical evidence suggests that inflation determination is found not to be purely forward-looking but intrinsic inflation somehow can constitute the dominant source of persistence.

Third, in addition to indexation, habit formation is typically regarded as an essential feature in rational expectations models in order to match the persistence in the data, to account for the dynamics of the model's variables as well as to improve its fit. The estimated mean of habit parameter b at 0.6448 is also highly consistent with estimates found by standard DSGE models. For example, Christiano et al. (2005) estimate habit formation parameter at 0.65, Smets & Wouters (2005) at 0.69, Fuhrer (2000) at 0.8, and Boldrin et al. (2001) at 0.7. Though it is admittedly hard to agree on the role of habits in DSGE models, particularly in developing economies, the estimated result appears supportive of the importance of additional sources of endogenous persistence in DSGE models in explaining the inertial behaviour of macroeconomic variables. Also, as shown in figure 3.12, the slow contraction in demand for goods reflects the presence of habit formation in preferences and adjustment cost in investment, as argued by Christiano et al. (2005).

Fourth, for the present DSGE model, the investment adjustment costs parameter is estimated at 2.75, which is slightly higher than the estimate using the US data at 2.48 in Christiano et al. (2005) but lower than the estimate of 6.9 for the Euro area reported in Smets & Wouters (2003). The elasticity of investment with respect to 1 percent temporary increase in the current price of installed capital is thus 0.36,[29] while the elasticity of investment with respect to 1 percentage point change in the permanent price of capital, which is adjusted by the discount factor, is 0.18, indicating that a permanent 1 percentage point change in the price of capital induces a 0.18 percent change in investment. As

[29]The elasticity of investment with respect to 1 percent *temporary* increase in the current price of installed capital is $1/\zeta$; with respect to 1 percent *permanent* increase in the price of capital is $1/\zeta(1-\beta)$.

argued by Christiano et al. (2005), a more persistent change in the price of capital causes a larger percentage change in investment because adjustment costs induce agents to be forward-looking.

Fifth, the posterior means of persistence parameters of the supply-side technology and labour supply shocks are estimated at 0.68 and 0.87, respectively. The demand-side shock coefficient for consumption preference is estimated at 0.58. However, the persistence coefficient of the monetary policy shock estimated at 0.18 is substantially lower than the estimates reported in standard DSGE models. This low level of persistence in the monetary policy rule is also reflected in the *inertia puzzle* behaviour of prices in the estimated model[30]. The estimates of the substitution elasticity of domestic and foreign investment and consumption goods are 0.994 and 0.535 respectively, which are less than 1 and not far from the estimate of 0.477 for Indonesia, a neighbouring country in the same region, in Dutu (2016). This may reflect the fact that in many developing countries, the short-run demand elasticity for oil import, a foreign consumable which usually takes a large share of consumption, is very low.

Finally, the estimates for monetary policy parameters are 1.63, 0.84 and 0.15 for inflation response, interest rate smoothing, and output response, respectively. In general, the estimation delivers plausible parameters for the reaction rule for the monetary authorities both in the short term and long term. The estimate for the inflation response is broadly in line with Taylor (1993) where the estimated result shows a long-term response of interest rates to inflation greater than 1. The substantial degree of the estimated in-

[30]See detailed discussion in the impulse response functions section.

terest rate smoothing is also in line with a common finding from the literature that, as argued by English et al. (2003), the lagged interest rate enters estimated policy rules with overwhelming significance[31]. The short-term reaction to output in the estimated results is also significantly positive and different from zero, which is consistent with the standard rule suggested by Taylor (1993). It should be noted that our estimates for a model for developing countries with partial dollarization also reflect the fact that policy-makers have to take into account the policy movements from foreign central banks. Thus, monetary authorities appear to put lower weight on output in order to achieve its own targets such as exchange rate stabilization. This result, surprisingly, fits the actual policy performance of Vietnam in recent years remarkably well.

3.6 Properties of the Estimated Model

3.6.1 Impulse Response Analysis

The Bayesian impulse response functions of the macro variables to key demand-side and supply-side shocks are plotted in figures 3.8 - 3.12. The grey-shaded areas indicate the *Highest Posterior Density Interval* (HPDI) of the estimated parameters, which is the key difference in the computation of impulse response functions between Bayesian estimation and the frequentist approach. While classical IRFs are computed at the calibrated parameter combination, Bayesian impulse response functions show the mean impulse responses, that is, parameters are set to the posterior means before endogenous variables are then IRF simulated corresponding to this interval of the parameters[32].

[31] Sack (1998), Clarida et al. (2000), Amato & Laubach (1999), and Goodhart (1996), for example.

[32] In case of Maximum Likelihood, parameters are set to the posterior modes.

There are some typical as well as specific features from the Bayesian impulse responses which are worth considering. On the supply side, the first result that stands out, as shown in figure 3.8, is that there is a decline of employment in response to a positive technology shock. This result is largely consistent with Galí (1999) who shows that a positive technology shock will have a negative short-run effect on employment. This is because in this case, as argued by Galí (1999), the combination of price rigidities and demand constraints leads firms to contract employment in the face of an exogenous increase in multifactor productivity. Put simply, producing the same output will require less labour input as the technology shock is positive. Therefore, we can observe a decline in employment, which is in stark contrast with the basic RBC model[33]. A similar result is also found by Rotemberg (1996), King & Wolman (1996), and King & Watson (1996). Furthermore, the strong positive comovement of output and employment, which is generally viewed as central characteristic of business cycle, then leads to a decrease in output. In addition, in the face of a decrease in output which is triggered by the decline in employment, it also brings about a decline in investment and consumption in the following quarters.

The decrease of output is largely consistent with Tervala (2007) who argues that in an open economy, there is an additional factor that can cause a decline in employment and output in the short run which is named "the expenditure-switching effect of an exchange rate change". It is because when the country's currency appreciates, the relative prices of exports also increase and world consumption shifts away from the country's goods. Furthermore, given a negative position on foreign bond holdings, as is well known in de-

[33] King & Rebelo (1999), for example.

veloping countries, a positive technology shock will reduce the risk-premium on foreign borrowing, inducing real exchange rate appreciation, as featured in the variable risk-premium identity. As shown in figure 3.8, a technology shock leads to an appreciation of the real exchange rate[34], causing a decline in output in the short run.

Also, as argued by Adolfson et al. (2011), a positive technology shock in this case creates a trade-off between a decline in the output and the induced increase in inflation, leading to an increase in the interest rate. Therefore, as shown in the next section on shock decomposition, technology shocks in the estimated model play a dominant role in explaining the business cycle variations in Vietnam and these shocks can help explain the negative correlation between output growth and inflation in the sample. Technology shocks in Adolfson et al. (2011) produce similar impacts on aggregate quantities. This result is also in line with Francis & Ramey (2005) who argue that the sign of the response of inflation and the interest rate in a model with nominal rigidities is a priori indeterminate. However, it is shown that both variables always move in the same direction on impact following a technology shock, while a monetary shock would normally induce a negative correlation between these two variables. In addition, the real wage also decreases because, as argued by Cantore et al. (2013), employment decreases less than proportionally with output. The impulse response functions also show that the exchange rate appreciates in

[34]The appreciation of real exchange rate in the present model following a technology shock is largely consistent with Corsetti et al. (2008) where a positive technology shock appreciates the real exchange rate for at least a year (using US data). Similar results are also reported in Enders & Müller (2009) and Enders et al. (2011). In Armington-type models, such as Raffo (2010), an unanticipated jump in domestic technology leads to a depreciation of the real exchange rate. However, as the model economy is assumed to be a very small open economy, it should not influence world relative prices.

response to a positive technology shock, even though the key driver of the exchange rate volatility, as shown by Eichenbaum & Evans (1995) and Clarida & Gali (1994), should be a monetary disturbance[35]. This result is largely consistent with Enders et al. (2011).

As shown in figure 3.9, an increase in the disutility of work seems to produce qualitatively similar effects on real variables as a positive technology shock. As expected, a negative shock to labour supply leads to a decline in employment and other real variables, including consumption and investment, inducing a fall in output. This result is broadly consistent with Altig et al. (2011) and Adolfson et al. (2007). Real wage increases, as argued by Cantore et al. (2013), because labour supply decreases more than proportionally with that of output in the first several quarters. It is this increase in the real wage that leads to a rise in the marginal cost and a rise in inflation following a labour supply shock, as argued by Smets & Wouters (2003). The behaviour of the aggregate variables after a labour supply shock in the present model is largely in line with the result shown by Adolfson et al. (2005) that nominal rigidities are very important in shaping the dynamics of the labour supply shock. If prices and wages are flexible, the labour supply shock has limited effects on the development of the model economy.

[35] In particular, Clarida & Gali (1994) show that demand shocks explain the majority of the variance in real exchange rate fluctuations, while supply shocks explain very little.

Figure 3.8: Impulse Responses to a Technology Shock

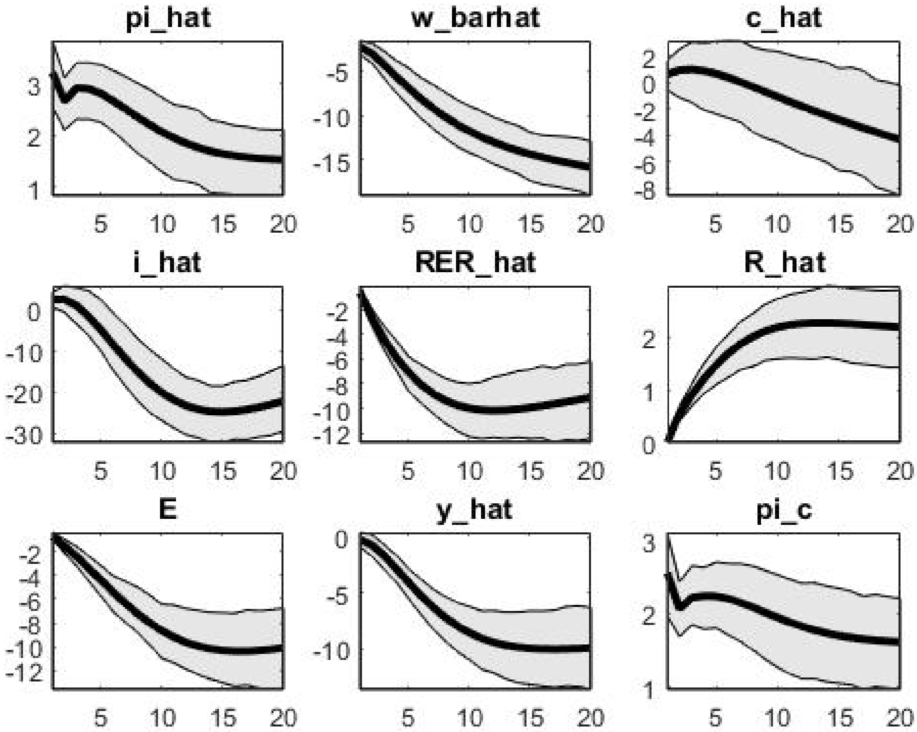
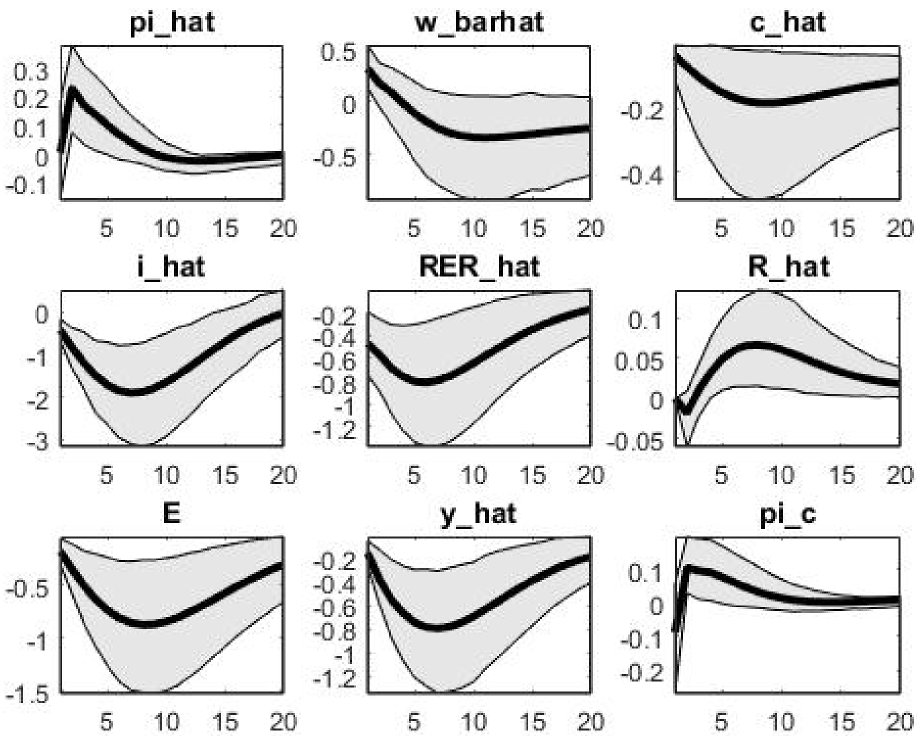


Figure 3.9: Impulse Responses to a Labour Supply Shock



On the demand side, it is shown in figure 3.10 that a positive consumption preference shock induces significant effects on aggregate variables, which is also in line with Adolfson et al. (2005). In particular, while an increase in the utility of the households leads to a drop in investment, households have to increase their labour effort in order to meet their desired increase in consumption. In consistence with Adolfson et al. (2005) and Khan & Tsoukalas (2012), consumption preference shock explains for a substantial amount (10.88 percent) of the cyclical behaviour of consumption in the short run, as can be seen in table 3.2. In addition, as argued by Cantore et al. (2013), as labour supply increases slightly less than proportionally with output following a preference shock, real wage also increases slightly in the first few quarters.

Figure 3.11 displays the pattern of the dynamic responses of output and its three demand components, including government spending, investment, and consumption, after a positive government spending shock. An increase in government purchases, as explained by Galí et al. (2007), has a negative wealth effect which is reflected in lower consumption and a rise in labour supply. The latter effect leads to a lower real wage in equilibrium but a rise in employment and output. Given a constant money supply, the rise in consumption is accompanied by an investment decline due to a higher interest rate. This result is strongly supported by Christiano & Eichenbaum (1992), Baxter & King (1993), and Fatás & Mihov (2001). However, the variance decomposition shows that with a chronic budget-deficit rule in the present model, a fiscal expansion only has a limited effect on output and the effects on other aggregate economic activities are also gradually transmitted[36].

[36] A positive fiscal shock accounts for only 4.88 percent of output variation in the first quarter and dies out rapidly in the following quarters.

Figure 3.10: Impulse Responses to a Consumption Preference Shock

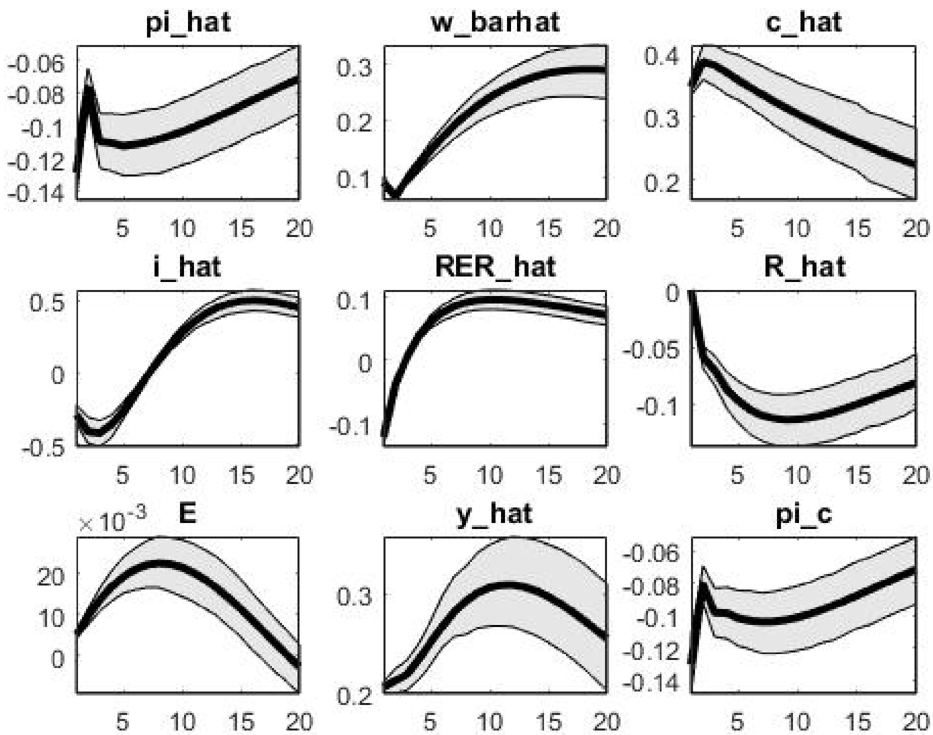
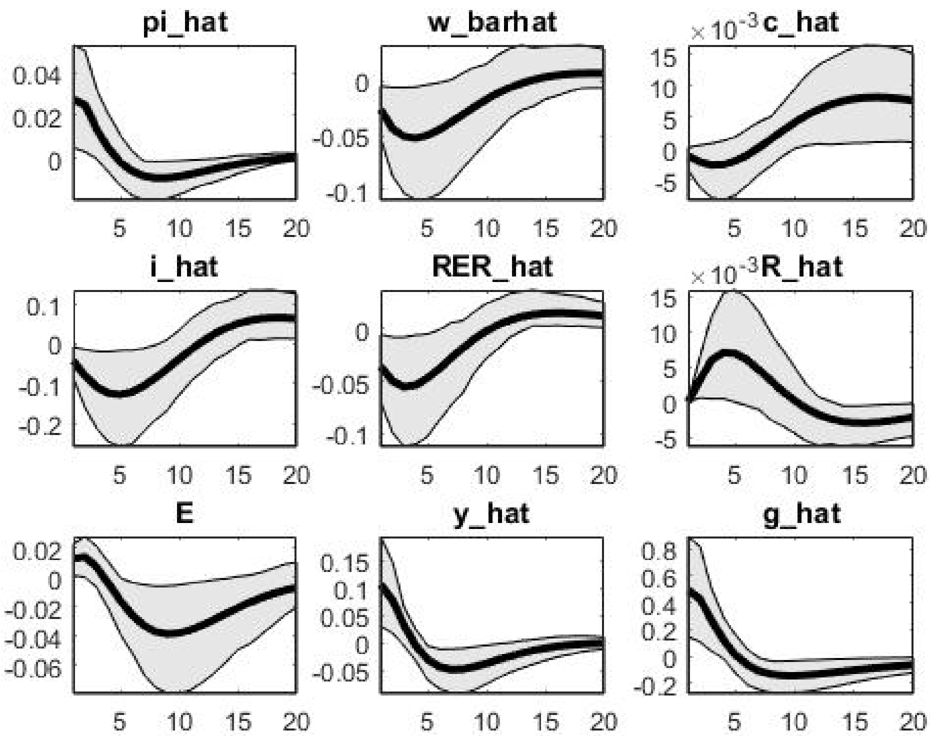


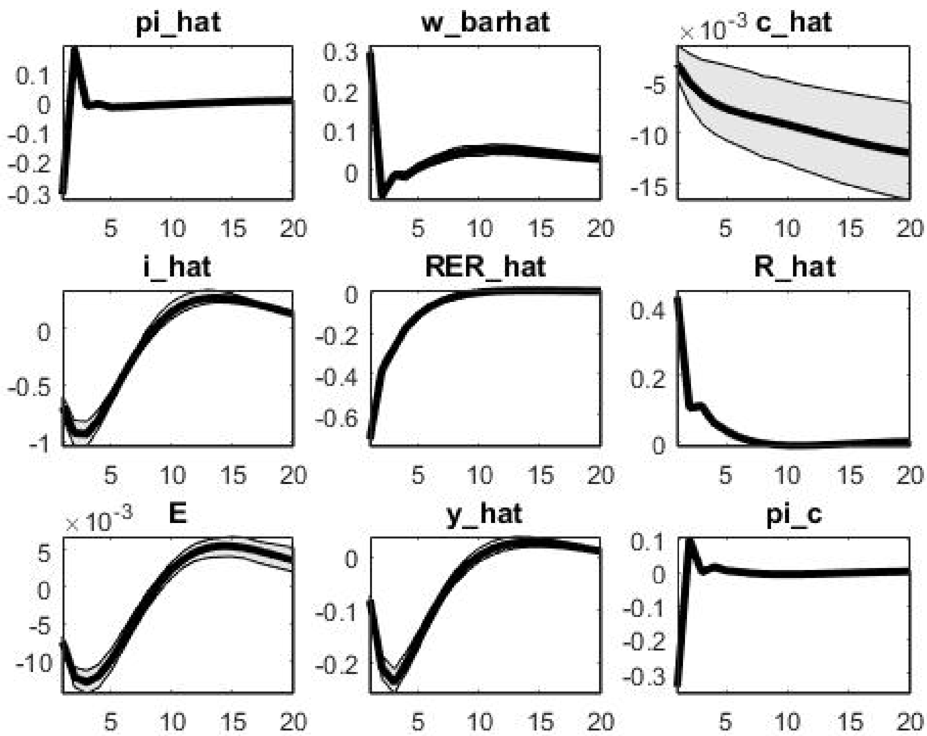
Figure 3.11: Impulse Responses to a Government Spending Shock



Finally, figure 3.12 shows the dynamic effects on aggregate quantities in the economy following a shock to monetary policy. It is clearly shown from the results that an unanticipated temporary increase in the nominal interest rate reduces consumption and investment, leading to a decline in employment and output in a U-shaped manner. The impact reaches its bottom after roughly one year and returns to the pre-shock levels after approximately two years. While the fall in investment occurs because of a sharp jump in the interest rate, the increase in real wage reflects the effects of a decrease in employment. Keynes (1936) argued, at least implicitly, that goods prices were much more flexible than the money wage rate and that firms should be regarded, approximately, as profit maximising competitors. In this case, the assumption of positive but diminishing marginal product of labour implied a negative association between the real wage and the level of employment. Therefore, the behaviour of real wage in the estimated model is largely in line with this argument as money wage rates are estimated to be relatively stickier than prices, which are roughly 3.3 and 1.5 quarters, respectively. The responses of the model's real variables are broadly consistent with the results reported in Altig et al. (2011) and Adolfson et al. (2005).

The estimated model is also successful in accounting for a trade-off between inflation and output when a fall in aggregate demand leads to a lower level of inflation. However, it should be noted that one of the distinct features of the present theoretical model for developing countries is the *inertia puzzle* in the response of inflation where it falls sharply after a positive monetary policy shock and then quickly returns and even slightly overshoots the steady state. A similar result is also found by Altig et al. (2011). This is because, as argued by Altig et al. (2011), the firms re-optimize prices very frequently at estimated intervals of 1.5 quarters, given a relatively high level of capital utilization cost, as further stressed by Adolfson et al. (2005).

Figure 3.12: Impulse Responses to a Monetary Policy Shock



3.6.2 Variance Decomposition

Table 3.2 compares the contribution of respective structural shocks to the forecast error variance of key macroeconomic variables of the economy in the short run, medium run, and long run. Particularly, it shows the percentage of variance of the n-step-ahead forecast error of the variables. Results, which are reported for 1, 4, 20, and 50 periods of forecast horizon, show that macro variables in the estimated model behave differently after various shocks from the demand side and supply side. Some important insights emerge.

First, the effects of an investment-specific technology shock on most of macroeconomic variables except for the real wage build up as the horizon lengthens, which is broadly in line with Smets & Wouters (2003). In addition, the results show that the investment-specific technology shock is the key driver of the business cycle as it accounts for 70-76 percent of the long-run forecast variances in macroeconomic variables, including output, inflation, government spending, employment, and the interest rate. In the medium run, the investment-specific technology shock also accounts for more than 50 percent of the variability of key aggregate variables in the economy. On the other hand, the unit root technology shock only plays a limited role in driving the variance of output in the long run, accounting for 12.15 percent, while it has more significant impacts in the short run, inducing 26.15 percent of output variance in the first quarter and 27.12 percent after one year.

Second, the results show that consumption preference shock plays a very limited role in the present estimated model. In the short run, a shock on consumption preference only accounts for 0.23 and 0.26 percent of the variation of inflation in the first quarter and after one year, while a unit root technology shock captures 13.15 and 18.34 percent, respectively. The consumption preference shock only makes a small contribution in explaining movements in output growth and labour inputs, accounting for 4.08 and 0.09 percent in the short run, respectively, while results reported by Ireland (2004) show a preference shock's contribution of 20 percent and by Galí & Rabanal (2004) 57.1 percent to the variability of output. However, the insignificance of consumption preference shock in driving the business cycle is broadly consistent with Adolfson et al. (2005) who show that preference shock only contributes less than 0.2 percent to the variation of both output growth and employment in any time horizon. Likewise, on the supply side, labour supply shock also plays a very limited role in driving the movements of aggregate variables, which is largely in line with Smets & Wouters (2003). Inflation, for example, is estimated to respond very sluggishly to both consumption preference and labour supply shocks at all horizons.

Third, it is shown that monetary policy shock explains less than 1 percent of output growth and labour movements. The limited contribution from a monetary policy shock to the variability of the business cycle is not dissimilar to the findings reported by Galí & Rabanal (2004) where monetary policy shock accounts for only 4.8 and 0.4 percent of output growth and labour input variation, respectively. However, while Galí & Rabanal (2004) show that monetary shock is an important determinant of inflation variability, contributing to 27.1 percent of total volatility, a shock to monetary policy in the present model contributes only 1.36 percent to the variation of inflation in the very short run and its contribution is even less significant at longer horizons. This result is consistent with Christiano et al. (2005) in which policy shocks are found to account for only a small fraction of inflation's variation and, at the same time, for a nontrivial fraction of the variation in other real variables, except for the real wage. This effect, as explained by Smets & Wouters (2003), could well be due to the fact that under the estimated monetary policy rule, the interest rates respond strongly to a policy shock, thus preventing inflationary or deflationary pressures from arising.

Finally, all markup shocks in the external sector except for the imported consumption markup shock only explain a small amount of the variability of aggregate quantities in the short, medium, and long term. Specifically, the imported consumption markup shock accounts for more than a quarter of the variability of the real exchange rate in the short run, which is broadly consistent with Adolfson et al. (2005). However, unlike Adolfson et al. (2005), the imported consumption markup shock is shown to be an important supporting source driving the business cycle of the model economy, contributing roughly 7 and 3 percent of the variability of output growth and inflation, respectively, in the short run.

	Inf.	**Wage**	**Con.**	**Inv.**	**Exc.**	**Rate**	**Emp.**	**Out.**	**Spen.**
Technology shock									
1 period	13.15	10.94	87.17	9.83	3.27	0.00	14.10	26.15	14.11
4 periods	18.34	16.28	89.97	2.77	6.07	16.32	11.78	27.12	29.50
20 periods	17.60	31.32	70.91	4.50	6.51	18.04	7.94	22.15	23.70
50 periods	11.31	30.36	45.42	1.65	2.56	11.35	11.48	12.15	12.94
Investment-spe. tech. shock									
1 period	10.77	0.54	1.40	32.35	0.53	0.00	36.12	2.83	2.39
4 periods	19.66	0.50	3.54	9.63	12.95	18.48	56.72	1.31	1.06
20 periods	54.76	4.99	20.33	59.44	57.42	57.79	73.57	44.15	43.92
50 periods	73.50	27.86	45.15	86.50	82.81	76.13	71.91	73.05	72.81
Labour supply shock									
1 period	1.12	0.00	0.02	0.02	0.07	0.00	0.03	0.67	0.25
4 periods	1.65	0.51	0.07	0.39	2.11	1.28	0.03	2.01	1.64
20 periods	1.08	1.99	0.37	1.20	1.27	1.03	0.24	1.53	1.44
50 periods	0.60	1.60	0.50	0.40	0.49	0.55	0.58	0.69	0.65
Cons. preference shock									
1 period	0.23	0.17	10.88	0.38	0.20	0.00	0.09	4.08	2.06
4 periods	0.26	0.14	5.01	0.15	0.08	0.21	0.09	1.19	1.13
20 periods	0.26	0.27	1.93	0.05	0.06	0.23	0.01	0.43	0.40
50 periods	0.17	0.30	0.90	0.02	0.03	0.15	0.02	0.25	0.23
Monetary policy shock									
1 period	1.36	1.80	0.00	2.24	7.18	99.94	0.20	0.64	0.72
4 periods	0.73	0.34	0.00	0.81	3.42	2.93	0.08	0.93	1.62
20 periods	0.18	0.03	0.00	0.07	0.34	0.28	0.00	0.06	0.11
50 periods	0.09	0.01	0.00	0.02	0.13	0.13	0.00	0.02	0.05
Imp. Con. markup shock									
1 period	2.94	3.57	0.06	0.52	18.94	0.00	0.04	6.66	2.98
4 periods	1.23	2.52	0.15	0.19	26.73	0.05	0.02	1.86	1.48
20 periods	0.33	0.28	0.72	0.02	8.59	0.09	0.00	0.15	0.10
50 periods	0.20	0.17	0.86	0.01	4.04	0.08	0.01	0.06	0.04
Imp. Inv. markup shock									
1 period	0.02	0.04	0.00	0.03	0.05	0.00	0.00	0.04	0.02
4 periods	0.02	0.04	0.00	0.03	0.03	0.03	0.00	0.03	0.03
20 periods	0.01	0.04	0.02	0.02	0.03	0.01	0.01	0.01	0.01
50 periods	0.01	0.02	0.02	0.01	0.03	0.01	0.01	0.00	0.00
Export markup shock									
1 period	0.07	0.07	0.00	0.00	0.08	0.00	0.00	0.28	0.13
4 periods	0.08	0.07	0.00	0.03	0.03	0.08	0.00	0.26	0.24
20 periods	0.05	0.10	0.01	0.07	0.01	0.04	0.02	0.09	0.08
50 periods	0.03	0.08	0.02	0.02	0.00	0.02	0.03	0.04	0.04

Note: *Con.* consumption; *Inv.* investment; *Exc.* exchange rate; *Emp.* employment; *Spen.* spending.

Table 3.2: Variance Decomposition (in percent)

3.6.3 Historical Decomposition

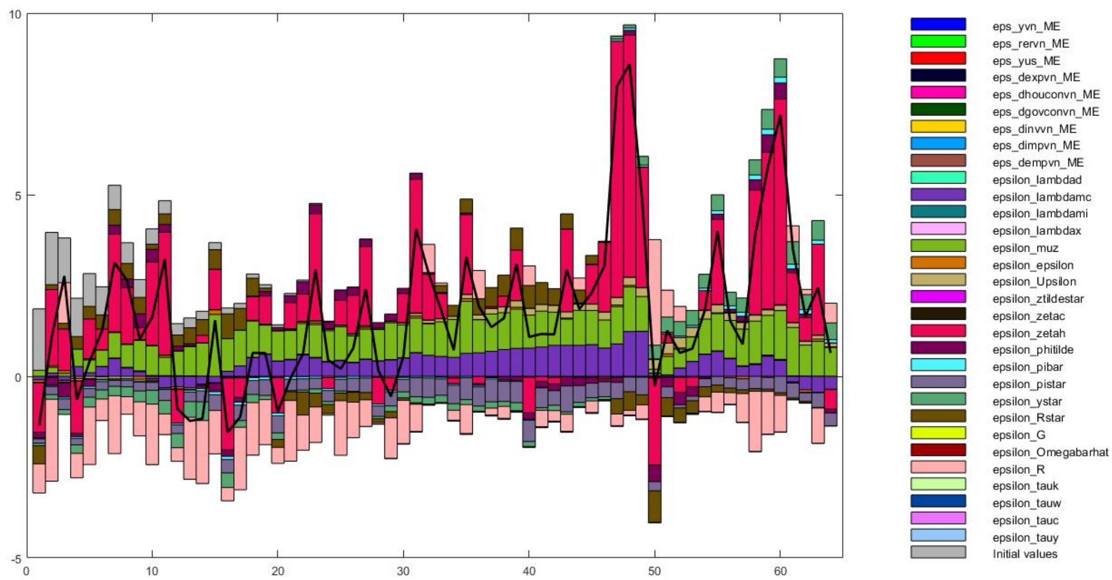
While smoothed variables and smoothed shocks plots generate the *best guess* for the path of all observed variables and the *best guess* for the structural shocks of the theoretical model, given our set of observables (mean estimate of the smoothed variables and the smoothed structural shocks), following Pfeifer (2017), figures 3.13 and 3.14 go one step further to decompose the contribution of the respective smoothed shocks to the deviation of the smoothed endogenous variables from its steady state[37]. The black line shows the deviation of the smoothed value of the corresponding endogenous variable from its steady state at the specified parameter set. This parameter set is, by default, the posterior mean when Bayesian estimation is performed and posterior mode in case of Maximum Likelihood. The coloured bars describe the contribution of the respective smoothed shocks to the deviation of the corresponding smoothed variable from its steady state. In fact, this can be taken as the *best guess* of the shocks that lead to the *best guess* of the deviation of the unobserved endogenous variables. As argued by Pfeifer (2017), the *initial values* indicate the part of the deviations from steady state which is not explained by the smoothed shocks, but rather by the unknown initial value of the state variables and this impact usually goes away relatively quickly.

Figure 3.13 decomposes the historical inflation time series using the estimated parameters. It is clearly shown that the variability of inflation during the sample period was to some extent driven by both demand and supply shocks. The historical decomposition results indicate that technology and labour supply shocks are the key drivers of the vari-

[37]The difference between the smoothed variables plot and shock decomposition graph is that in the former, steady state values are added to the smoothed variables while the later excludes steady state values.

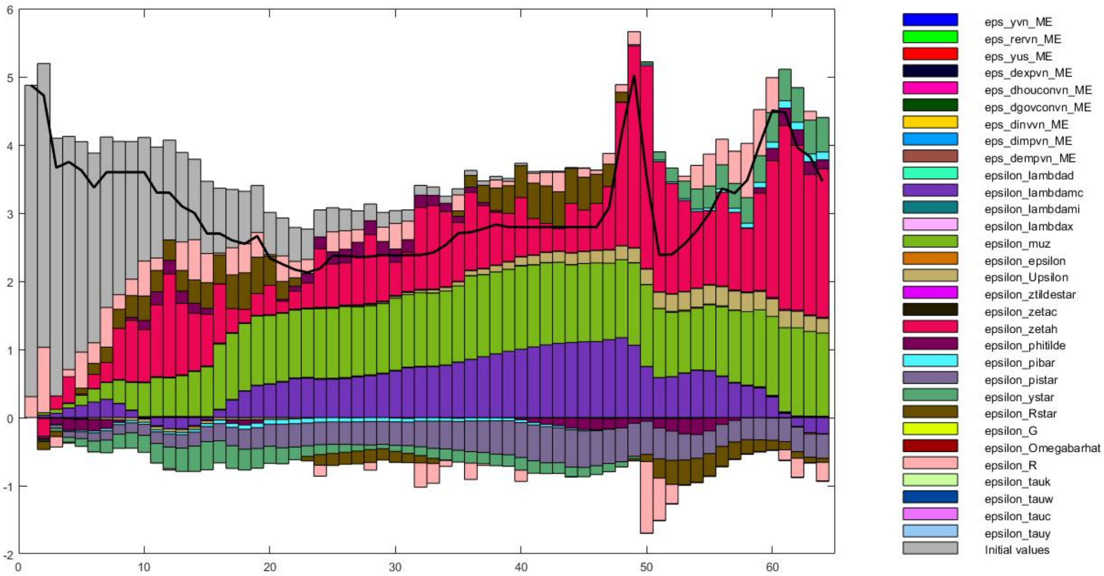
ation of inflation during the whole sample period. Monetary policy also played an active role in affecting the variability of inflation in the first half of the sample period, but a very limited role in the determination of the level of inflation before and after the burst of the global financial crisis in 2007. Initial conditions, which refer to the part of the deviations from its steady state explained by the unknown initial value of the state variables, are also included in the plots. The influence of the initial values, by construction, usually dies out relatively quickly as shown in the figures, but its persistence depends on how much persistence the model has.

Figure 3.13: Historical Decomposition of Inflation



The contributions of various shocks to the variation of interest rate during the sample period are depicted in figure 3.14. The figure shows that, according to the estimated model, shocks to technology and labour supply played a dominant role in explaining the variation in interest rates during the sample period. This result is broadly consistent with the variance decomposition analysis in the previous section and reflects the fact that the dynamics of the model economy depends more heavily on the supply side. It is shown that a shock to labour supply was the key factor in driving the variability of interest rate during the financial crisis, other than a relatively large contribution from the initial values of the state variable in the beginning of the sample period. In addition, the monetary policy manipulation of the authority also plays a non-trivial role in the variance of the interest rate during the whole sample period.

Figure 3.14: Historical Decomposition of Interest Rate



3.7 Model Fit

It is shown that the present model is able to produce reasonable and significant estimates of the model parameters. This section discusses the smoothed variables and smoothed shocks from the estimated model and a comparison of different versions of the theoretical model with respect to different degrees of partial dollarization in the economy. In addition, the overall significance of the estimates is also analysed through the assessment of the multivariate convergence diagnostics.

3.7.1 Smoothed Variables and Smoothed Shocks

Figures 3.15 and 3.16 show the smoothed shocks plots during the estimation process. The black lines show the estimate of the smoothed structural shocks, or the best guess for the structural shocks given all observables. These are derived from the Kalman smoother at the posterior mean in the Bayesian estimation, as explained by Pfeifer (2017).

Figure 3.15: Smoothed Shocks - A

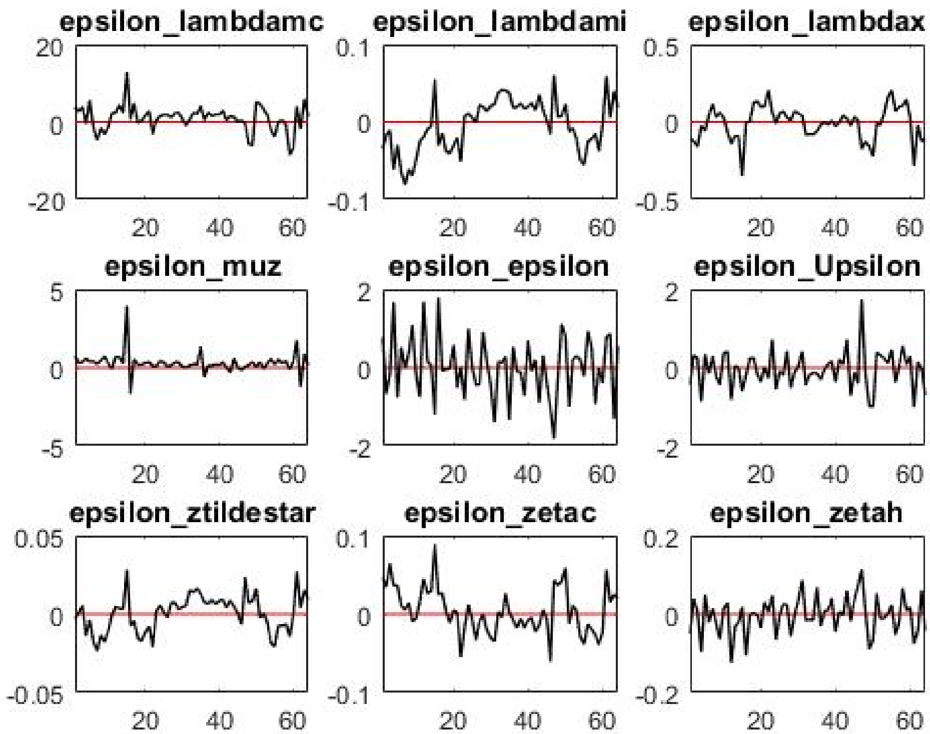
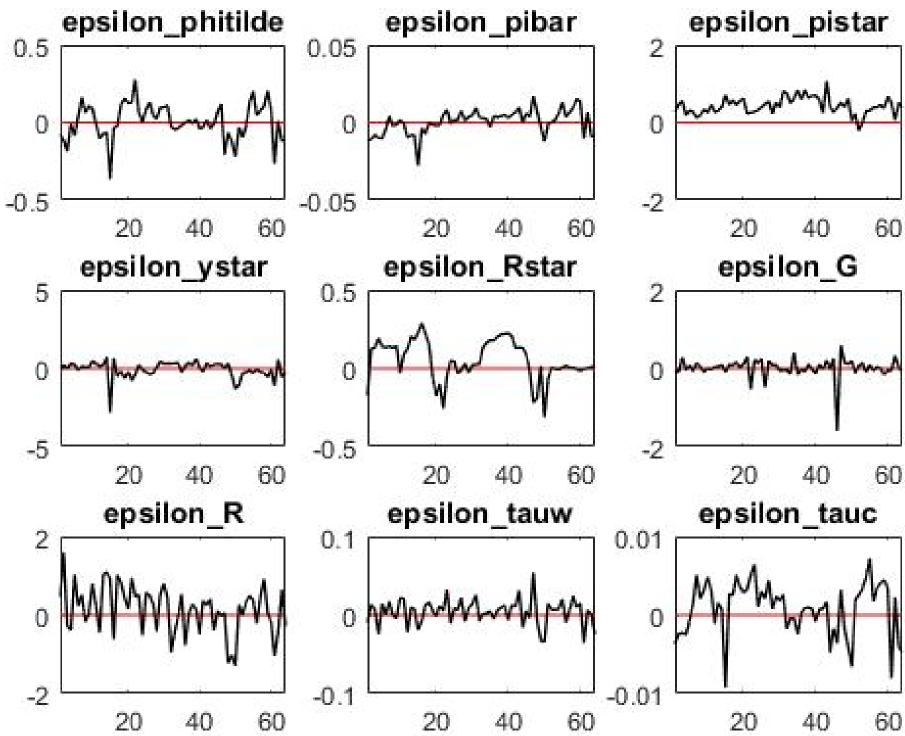
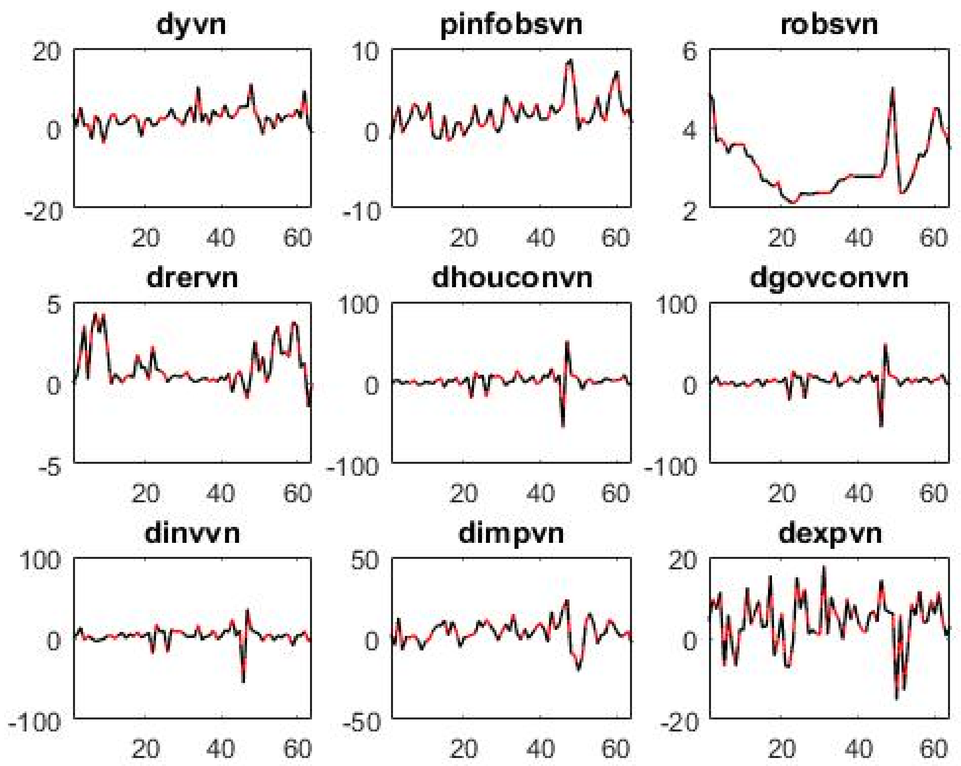


Figure 3.16: Smoothed Shocks - B



In addition, figure 3.17 plots the historical and smoothed variables, which are the best guess for observed variables used in the estimation in comparison with their actual data. While the dashed black lines show the actual data, the red lines indicate the estimate of the smoothed variable, or the best guess for the observed variables given all observed data, as guided by Pfeifer (2017). These are also derived from the Kalman smoother at the posterior mean in the Bayesian estimation.

Figure 3.17: Historical and Smoothed Variables



These figures show that the estimated model succeeds in producing the in-sample fit, given the conformity of the data and the artificial time series generated from the estimated model. It is also noticeable that the smoothed shock plots show that most of the shocks are also in line with our *a priori* expectations.

3.7.2 Model Comparison and Validation

As argued by Cantore et al. (2016), the plausibility of the model can be assessed by comparing the *marginal likelihood* associated with different variants of the theoretical model, the vector autocovariance or the unconditional second moments with those of the data. In this section, the role of partial dollarization is further investigated by comparing the log marginal likelihood of the baseline model with the log marginal likelihood for a specification of the model at high level of dollarization, where $\delta^{pd} = 0.46$ and a low dollarization scenario where $\delta^{pd} = 0.01$.

Bayesian inference, as argued by Cantore et al. (2016), provides a framework for the comparison of alternative and potentially misspecified models based on their marginal likelihood. Recall the Bayes's Rule

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta)$$

Under the framework of Bayesian estimation so far, the unconditional density of the data $p(y)$ has not been used/needed in order to maximize the likelihood $p(\theta|y)$ with respect to θ. However, the unconditional density of the data is of importance in performing Bayesian model comparisons. It is computed by integrating over the prior distribution space Θ as follows

$$p(y) = \int_{\Theta} p(y|\theta)p(\theta)d\theta$$

As argued by Smets & Wouters (2003), the marginal likelihood of a model reflects its prediction performance and the Bayes factor compares abilities of alternative models in predicting out of sample. For a particular model $m_i \in M$ among a number of alternatives M models, the *marginal likelihood* of the corresponding model and the *Bayes Factor* between two competing models are given by

$$p(y|m_i) = \int_{\Theta} p(y|\theta, m_i)p(\theta|m_i)d\theta$$

$$BF_{i,j} = \frac{p(y|m_i)}{p(y|m_j)} = \frac{\exp[LL(y|m_i)]}{\exp[LL(y|m_j)]}$$

where $p(\theta|m_i)$ is the prior, $p(y|m_i)$ is the data density for model m_i given parameter vector θ, and the log-likelihood $LL(y|m_i) \equiv \log[p(y|m_i)]$ can be obtained from the log of the marginal likelihood mentioned above.

Given the Bayes factors, the corresponding model probabilities are given by

$$p_i = p_1 BF_{i,1}$$

where $p_1 = 1/\sum_{i=2}^{n} BF_{i,1}$ given the fact that $\sum_{i=1}^{n} p_i = 1$.

Table 3.3 reports the log marginal likelihood of alternative variants of the original model with different levels of partial dollarization together with the corresponding Bayes factors related to the baseline model. The probability of each variant of the model is computed based on its log-likelihood values obtained from the first and second stage of the estimation. The log marginal likelihood results show that the sample data prefer the high-dollarization scenario with a log marginal likelihood of -1934.51, larger than that of the baseline model (-2229.73) and the low-dollarization version of the model (-1953.30). In other words, the estimated model with high level of partial dollarization is the most favourable variant of the theoretical model with respect to its ability to capture the main features of the macroeconomic data of Vietnam and entertain the structural shocks to capture the stochastics.

Model	**Low Dol.**	**Baseline**	**High Dol.**
LLs (Stage 1)	-1974.83	-2325.93	-1965.18
LLs (MCMC)	-1953.30	-2229.73	-1934.51
Bayes Factors to Baseline	1.13	1	1.63
Prob (MCMC)	0	0	1

Table 3.3: Marginal Log-likelihood Values

Furthermore, table 3.3 also shows the computed results of the Bayes factors of the model with high and low levels of partial dollarization in relation to the baseline model. While the results indicate that the Bayes factor of 1.13 is in favour of the model with low level of partial dollarization, the model with high level of partial dollarization is even more favourable than the baseline model with the Bayes factor at 1.63. In addition, the results on model probabilities also suggest that the model with high level of partial dollarization does the best in predicting all of the observables over the sample period. This specification of the model will thus be the one to be used in the qualitative and quantitative analyses as well as the computation of optimal policies in the next chapter.

However, the limitation of this likelihood comparison methodology is that the assessment of model fit can only be implemented within the range of variants of the same model with different restrictions. The outperforming model in the space of competing models may, as argued by Cantore et al. (2016), still be inadequate in capturing the key dynamics of the data. Sims (2002) argues that weighting the evidence in favour of a particular characteristic of the model but failing to account for other aspects of the specification of the model may lead to *disparate inference* which is the comparison of things belonging to totally different kinds and not in the same class. As the models being considered are too sparse, thus using posterior odds can lead to extreme outcomes and may also be highly dependent on the prior distribution. As proposed by Sims (2002), the possible solution to solve this problem is to "fill the space of models" in order to address the lack of continuity in the model space and to make the model comparison more robust.

Therefore, while Bayes Factors can be used to assess the *relative fit* among different variants of the model, *absolute fit* of each variant of the model can be evaluated by comparing the corresponding model-based test quantities with those in the data. Following Schorfheide (2000), the absolute fit of the estimated model can be evaluated by comparing the test quantity $T(y)$, which is a test criteria that reflects any aspects of the data such as second moment, with the corresponding predictive quantity $T(y^{sim})$. This predictive quantity $T(y^{sim})$ is obtained from the simulation of the estimated model in which the path of the variables over the number of simulated periods is first computed by solving all of the model equations for every period, and then the result will be used to compute the empirical moments of the simulated variables y^{sim}.

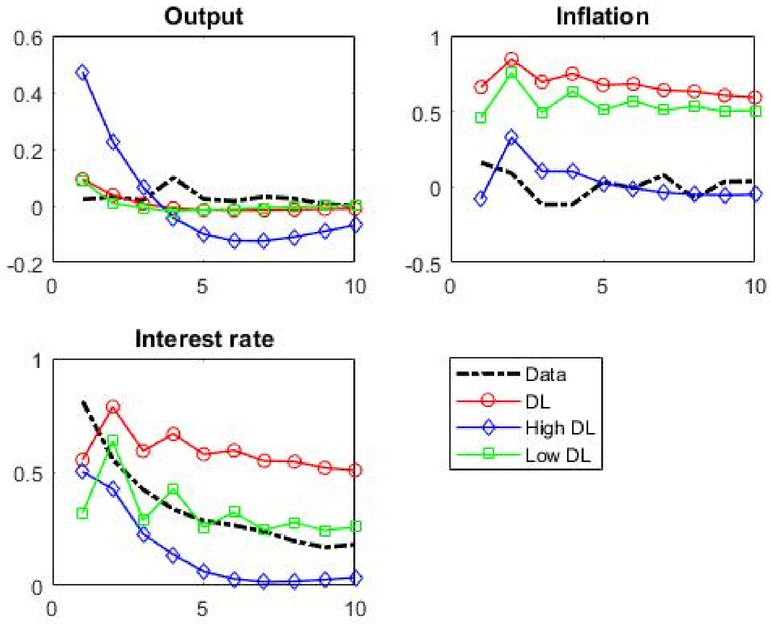
Table 3.4 shows some selected second moments of three key variables (output, inflation, and interest rate) implied by the Bayesian estimation procedure, including standard deviation, cross-correlation with output, and autocorrelation of order 1. These are computed by solving the models at the posterior means obtained from the estimation. The statistics are then compared with the second moments in the actual data in order to evaluate the corresponding estimated model's empirical performance.

	Output	Inflation	Interest Rate
Standard Deviation			
Data	22.3876	2.0986	0.7119
Low DL	10.4029	6.9772	13.8640
Baseline	12.4452	4.9774	7.8939
High DL	22.1376	44.4151	24.0392
Cross-correlation with Output			
Data	1	-0.2116	0.0418
Low DL	1	-0.0504	-0.1770
Baseline	1	-0.0931	-0.1012
High DL	1	-0.0968	0.0260
Autocorrelation (Order 1)			
Data	-0.74	0.5379	0.8124
Low DL	0.0880	0.4587	0.3174
Baseline	0.0943	0.6604	0.5534
High DL	0.4731	-0.0766	0.5011

Table 3.4: Selected Second Moments of the Model Variants

Finally, in order to investigate further how the estimated models capture the data's statistics and especially the persistence, following Fuhrer & Moore (1995), the vector autocorelation functions of the respective models with low, high, and baseline levels of partial dollarization are compared with those in the data. Figure 3.18 compares the autocorrelation functions up to order 10 with respect to output, inflation, and interest rate generated by three variants of the model with those in the observables.

Figure 3.18: Autocorrelation Functions Comparison



A number of results are worth noting. First, the cross-correlation of inflation and the interest rate with output statistics indicate that only the high-dollarization version is indeed able to mimic the sign and the magnitude of the cross-correlation in the data. Second, the upper part of table 3.4 shows that the standard deviations of output generated from the model with high dollarization level are the closest to those of the data, though this variant of the model is not really successful in replicating the standard deviations for inflation and the interest rate in the data. Third, the high-dollarization model also does at least as good a job as the baseline model in predicting the first-order autocorrelation functions of the interest rate, while the first-order autocorrelation functions statistics for inflation are in favour of the baseline model. However, all three variants of the model face difficulties in accounting for the negative first-order autocorrelation functions in output.

Finally, the comparison of autocorrelation functions up to order 10 for inflation and the interest rate depicted in figure 3.18 indicates that the model with high level of partial dollarization is the most successful variant of the estimated model which is able to mimic the autocorrelation functions of inflation and the interest rate up to order 10. However, as mentioned above, all three variants of the model are not really successful in replicating the autocorrelation functions of observed output. In general, even though there is quite a room for further improvements in some respects, the model with high level of partial dollarization is shown to be the most favourable version of the model which is able to capture the persistence and volatility of key endogenous variables relatively well.

3.7.3 Convergence Diagnostics

The Markov Chain Monte Carlo (MCMC) algorithms that we are using in the estimation of the present model are also in wide use for fitting economic models in situations where the application of traditional estimation techniques is not easy[38]. However, one of the stumbling blocks in using a MCMC algorithm is to determine the convergence of the algorithm as it is not only that of a scalar quantity to a point, but a distribution of another distribution. Gelman & Shirley (2011) argue that MCMC methods are wonderfully convenient and flexible but, compared to more simple methods of statistical computation, they face two difficulties: first, running the Markov chains long enough for convergence, and second, having enough simulation draws for suitably accurate inference. Sinharay (2003) argues that we can safely assume convergence of a MCMC algorithm if we are certain that the sample generated from the algorithm is indeed from the posterior distribution of interest. Technically speaking, the convergence occurs when the generated Markov chain

[38]The key idea of an MCMC algorithm is to create a stationary time series that is the same as a posterior distribution of interest.

converges in distribution to the posterior distribution of interest.

In order to perform statistical analysis on the convergence of the MCMC algorithm[39], Gelman & Rubin (1992) suggest running multiple chains with overdispersed starting values to compute and estimate the posterior distribution and a reduction factor reflecting how much sharper the distributional estimate might become if the simulations were continued indefinitely. This is in essence a comparison of the between-chain and within-chain moments of the second order and quantitatively represented by a *potential scale reduction factor* (PSRF). However, the limitation of the original convergence diagnostic, as argued by Brooks & Gelman (1998), is the assumption of normality of the marginal distribution of each scalar quantity. This assumption is unappealing as MCMC methods are often used for highly non-normal, and even multimodal, densities.

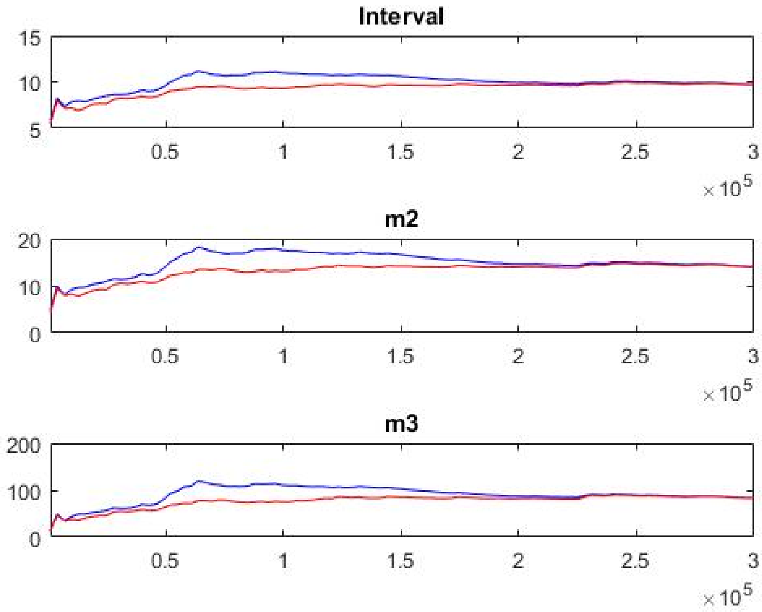
Therefore, following Brooks & Gelman (1998), the PSRF is further refined to compute an *interval* PSRF based on the empirical interval lengths as a measure of information,

[39]A number of diagnostics have also been suggested. Some of them apply the theory of Markov chains to the sampled values to detect if the sampled distribution has reached stationarity. Other approaches compare the sampled distributions obtained from the MCMC for different runs, in which one run may be the subset of another, and convergence can be concluded when the difference between some aspects of the empirical distributions (mean, quantile, cumulative density function etc.) over different runs is negligible in some sense. However, the key disadvantage of these approaches is that they can measure the distance between the sampled distribution and the target posterior distribution rather than examining one or two approximations of the latter. In addition, there are also a number of simple tools for MCMC convergence assessment such as the time-series plots, running mean plots, plot of autocorrelation functions etc. These methods examine the chains of values generated for each parameter to determine if the simulation process stabilizes in some sense.

rather than variance estimates. Thus, the interval PSRF is a ratio of interval lengths instead of a variance ratio and in fact it does not require normality assumption. As shown by Pfeifer (2017), while the univariate convergence diagnostics are computed based on individual parameters' distributions, the multivariate convergence diagnostics are generated based on the range of the posterior likelihood function and the posterior kernel is used to aggregate the parameters.

Figure 3.19 shows the multivariate convergence diagnostics of the model's estimated parameters. As guided by Pfeifer (2017), the first subfigure with the appended "Interval" indicates the Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the 80% interval/quantile range based on the pooled draws from all sequences and the red line indicates the mean interval range based on the draws of the individual sequences. The second and third subfigures with the appended "m2" and "m3" show an estimate of the same statistics for the second and third central moments respectively, which are the squared and cubed absolute deviations from the pooled and the within-sample means. As shown in figure 3.19, the blue and red lines in all measures of interval, second, and third moments stabilize horizontally and are close to each other, meaning that the potential scale reduction factors in any measures are close to 1. This suggests that the parameters' convergence has been obtained after 300,000 iterations.

Figure 3.19: MCMC Multivariate Convergence Diagnostics (Brooks and Gelman, 1998)



3.8 Summary

In this chapter, the theoretical DSGE model is estimated using a full macro dataset of 13 observed variables from Vietnam. As far as my supervisors and I are aware, this is the first estimated DSGE model for Vietnam. As the theoretical model features a non-stationary technology shock that induces a common stochastic trend in the real variables, the observed variables are then first differenced in order to achieve stationarity. Also, given the limited quality of the dataset collected from a developing country, measurement equations are also added to account for measurement errors in the data.

The model is estimated for 20 parameters using Bayesian estimation methods including structural, policy, and shock parameters after an extensive identification diagnostics. The results show that all estimated parameters are significantly different from zero. The data do provide useful information to update the priors so as to arrive at the posterior distributions. Impulse response functions of key macroeconomic variables to structural shocks show that the introduction of frictions in the model induces the empirical persistence and covariances in the main macroeconomic data. The transmission of structural shocks as well as monetary policy shocks are also examined thoroughly by a variance decomposition at various time horizons. The estimates of the effects of the respective shocks show that the technology shock on the supply side is the key source driving the business cycle fluctuations.

Finally, smoothed variables and shocks are generated, and convergence diagnostics are assessed to confirm the model's ability to produce reasonable and significant estimates of the parameters. Results from the model validation and comparison exercise are in favour of the model with high level of partial dollarization, which is able to capture the persistence and volatility of key variables relatively well. By and large, there is strong support for the use of the present estimated model for optimal monetary policy analysis in an empirically plausible setup.

Chapter 4

Optimal Monetary Policy for Vietnam

This chapter summarizes the methodology to derive an optimal monetary policy problem with linear, forward-looking constraints and a quadratic objective that yields a correct linear approximation to the optimal policy rules under the estimated Dynamic Stochastic General Equilibrium (DSGE) model. A quadratic loss function is chosen so as to facilitate the comparison of corresponding asymptotic losses from an optimal Ramsey policy, an optimal discretionary policy, and an *ex ante* optimized simple Taylor-type rule. A detailed discussion on the dynamics of the model economy under optimal policy is also presented in the final section.

4.1 Introduction

In a Dynamic Stochastic General Equilibrium (DSGE) model subject to nominal rigidities, it is shown that monetary policy can actively contribute to some extent to stabilising the real and nominal economy. Therefore, a large body of studies has focused on examining optimal monetary policy in both backward-looking and forward-looking environments. Generally speaking, in an optimal policy problem the policymaker sets out a number of instruments to maximize a *general discounted welfare criterion*[1] subject to constraints of a suitable DSGE model. Traditionally, when the expected discounted household utility is set up as the welfare criterion, this becomes the well-known Ramsey problem in which we get a complex solution in the form of a sequence of variables in the best equilibrium, even in a simple New Keynesian setup.

In this study, we consider the case of a Ramsey policy, a discretionary policy, and a simple Taylor-type rule for the monetary authority to gauge the feasibility of such policy framework in the context of developing world with partial dollarization. When it comes to solving this optimal policy problem, according to Cantore et al. (2016), only the Ramsey solution is available for a non-linear model and for a general objective function which is the expected discounted household's utility. Thus, a linear-quadratic problem is, in general, manageable and can be used for the computation of all policy options of interest. The central bank's optimal monetary policy problem is thus to minimize an intertemporal loss function subject to constraints with forward-looking rational expectations.

The remainder of the chapter is organized as follows. Section 2 reviews some of the early papers and arguments on the optimal monetary policy problem. Section 3 describes in detail the optimal monetary policy problem in which we assume the central bank's ability to follow a commitment rule, whether in the form of a Ramsey problem or with

[1]Discounted household utility, discounted losses etc.

an *ex ante* optimized Taylor-type simple rule, or a discretionary policy. A general framework for the loss function is given in Section 4, focusing on a standard *ad hoc* quadratic loss function in deviation form. Section 5 sets out the detailed mathematical foundation of a linear-quadratic problem which facilitates the understanding of the central bank's stabilization goals and trade-offs and summarizes the computational methodology for the linear-quadratic problem. Section 6 presents the results from the estimation of the optimal monetary policy. The dynamics of the model economy under optimal policy is discussed in Section 7. Finally, Section 8 contains the concluding remarks.

4.2 Optimal Monetary Policy in Retrospect

4.2.1 Early Discussions on Optimal Policy

The debate on optimal monetary policy can be dated back to the arguments between rule and discretion schools. Fischer (1990) in a review shows that the debate started with the rules of game of the gold standard under which rules were not well-defined, while active discretionary policy was pursued to defend the gold standard. On the other side, Milton Friedman was the pioneer economist to develop initial arguments for rules, followed by a series of analysis on alternative rules such as constant money growth rate rule, interest rate rule, or nominal GDP targeting. However, these arguments were under serious criticism and suffered from the dominance of discretion.

It was not until 1977 with the introduction of *dynamic/time inconsistency* by Kydland & Prescott (1977) that the case for rules rather than discretion was strongly supported by economic literature in the field. "A policy is dynamically inconsistent when a future policy decision that forms part of an optimal plan formulated at an initial date is no longer optimal from the viewpoint of a later date, even though no relevant new information has appeared in the meantime", as paraphrased by Blanchard & Fischer (1989). In other words, Kydland & Prescott (1977) highlight the manner in which rational expectations can fundamentally alter the nature of optimal policies by showing that "optimal, welfare-maximising policies which are optimal *ex ante* become sub-optimal *ex post* or with the passage of time." The term *time-inconsistency* is generally used to describe this property. The cost for this type of policy is a problem called *inflation bias* where monetary authority, with a discretionary policy, can print more money and create more inflation than people expect. However, though these inflation surprises can have some benefits, they cannot arise systematically in equilibrium when people with rational expectations understand the central bank's incentives and form their expectations accordingly. As the central bank has the authority to create inflation shocks *ex post*, the equilibrium growth rates of money and prices turn out to be higher than otherwise.

Therefore, a significant welfare gain can be achieved if policymakers precommit to a stabilization plan and the *precommitment* is made with *credibility*. Kydland & Prescott (1977) specifically suggest that discretionary policy for which policymakers select the best action, given the current situation, will not typically result in the social objective function being maximized. Such behaviour results in suboptimal planning or economic instability. Rather, policymakers should follow rules which specify current decisions as a function of the current state, and economic performance will then be improved. Blanchard & Fischer (1989) also argue that a precommitted government, such as one following policy rules, will carry out the policy that is optimal given that it is expected. A discretionary authority may be, under rational expectations, expected to take short-run optimal decision every time it can and the consequence is that it not only gains nothing from the action, but on average produces a worse outcome than an authority with a rule.

Time-inconsistency problem of a policy can thus be understood as the case where the central bank takes advantage of creating *ex post* monetary shocks to renege and re-optimize the optimal policy to achieve its own target, leaving the overall social welfare to be suboptimal due to an associated cost. In mathematical terminology, if the optimal rule is recomputed at a later time $t_0 > 0$ with the integral evaluated over $[t_0, t]$ instead of $[0, t]$, the rule is said to be changing with the passage of time and it can be time-inconsistent, as in Currie & Levine (1993). Rogoff (1987) argues that the government might be able to increase its own welfare, and in some instances social welfare, if it could precommit to a path/rule for the money supply even if there are no exogenous disturbances. Dennis (2007) shows that in the absence of a commitment mechanism, optimal monetary policies in general are time-inconsistent. Time-inconsistency problem can be addressed either by simply assuming that the central bank commits not to reoptimize, but rather just undertakes a single optimization and implements the chosen policy in all subsequent periods; or by explicitly modeling the strategic interactions that occur between the various economic agents in the model[2].

[2]It is a subgame-perfect equilibrium problem which commonly solves for Markov-perfect Stackelberg-Nash equilibria. "Timeless perspective" is an alternative approach which is developed by Woodford (1999). In this framework, the policymaker during its optimization process behaves as it would have chosen to if it had optimized at a time in the past.

Further development in mid-1980s has contributed considerably to the game-theoretic discussion on optimal monetary policy. However, these early arguments rest on the assumption that the game between policymakers and private sector is a one-shot game. Under a repeated stochastic policy game framework, the problem of time-consistency but non-optimality of discretionary policy due to inflation bias[3] can be mitigated by an appropriate design of monetary policy institutions. Thus, Barro & Gordon (1983) first propose a reputational equilibrium framework in which the outcomes are weighted averages of those from discretion and those from the ideal rule. While Kydland & Prescott (1977) show that commitments (rules) for monetary behaviour outweigh discretionary policy, Barro & Gordon (1983) argue that it is possible for reputational forces to substitute for formal rules given the repeated interaction between policymakers and private agents. *Reputation* is modeled as the use of a "trigger strategy" in which the private sector sets low wages (or inflation expectations) and the policymakers set low inflation. The private sector can punish the policymakers by setting high wages in the next period if they set high inflation. It is this punishment strategy that creates incentives for policymakers to preserve their reputation of setting low inflation.

Blanchard & Fischer (1989) comment that reputation is the most interesting and persuasive explanation of how governments avoid dynamic inconsistency. By acting consistently over long periods, governments can build up a reputation that causes the private sector to believe their announcements. However, solutions from a reputational framework when dealing with time-inconsistency problem can become complex and give rise to multiple equilibria. It is, thus, not easy to settle down the coordination between governments

[3]Expected inflation is positive and exceeds the socially optimal zero rate and is detrimental to welfare.

and the private sector within one particular equilibrium.

Second, Rogoff (1985) argues that inflation bias as mentioned above can be reduced considerably by appointing *conservative policymakers*. This type of central banker, as argued by Rogoff (1985), does not share the social objective function. When there is a distortion leading to a high time-consistent rate of inflation, the society can make itself better off by having the central bank place more weight on inflation-rate stabilization relative to employment stabilization. Blanchard & Fischer (1989) also agree that the more conservative policymakers are, the closer the society comes to achieving the precommited equilibrium. Among the optimal designs of monetary institutions which have been suggested over the past three decades to deal with time-inconsistency problem, delegation of monetary policy to an independent central bank has been the focus of large amount of literature in the field.

Finally, *optimal contracts* are also a possible solution to time-inconsistency problem of optimal policy. Walsh (1995) adopts a principal-agent framework and considers the relationship between governments and central banks as a principal-agent problem. Central banks are taken as the agent of the government which attempts to maximize an objective function that depends on contingent transfers from the government. The inflation bias inherent to discretionary policy is eliminated, as argued by Walsh (1995), and an optimal response to shocks is achieved by the optimal incentive contract.

In general, these initial arguments on optimal policy focus mainly on the debate of rules versus discretion. However, most of the early discussions are based on *game-theoretic* analysis with one-shot game problem and hence may have overstated the government's credibility[4] which can help mitigate or even eliminate time-inconsistency problem. Currie & Levine (1993) suggest that it is, therefore, much more natural to consider this optimal policy problem as a continuing game, not in the restricted two-period setting.

4.2.2 Recent Discussions on Optimal Policy

The topic of time inconsistency is of central importance in explaining many of the institutions of policy making, as argued by Fender (2012). As time-inconsistent policies will generally not be pursued, policy makers are constrained to pursue time-consistent or credible policies. However, as suggested by Fender (2012), there is a possibility that many optimal monetary policies may be time inconsistent, so that the outcome will be inferior to what would be obtained if the optimal monetary policy were pursued. The time-inconsistency of optimal policies in models with forward-looking rational expectations has long been discussed and the arguments, as pointed out by Currie & Levine (1993), are mainly formulated on the basis of *Pontryagin's maximum principle* in which the passage of time leads to an incentive to renege on the initial optimal plan and to adopt a new one. The private sector with rational expectations can anticipate this and the initial policy becomes incredible and the consequence is an inflation bias, resulting in suboptimal monetary policy from a social welfare standpoint.

Woodford (2003) argues that the central bank's stabilization goals can be most effectively achieved only to the extent that the central bank not only acts appropriately, but is also understood by the private sector to predictably act in a certain way. Thus, a

[4]Time-consistency, inflation bias, rules vs. discretion, commitment, credibility are key issues in the centre of the policy debate in this period.

decision procedure based on a rule is able to successfully steer private-sector expectations as systematic character of the central bank's actions can be most easily made apparent to the agents. Furthermore, Woodford (2003) points out that an optimal monetary policy is generally not correspondent to what would result from discretionary optimization - a procedure under which at each time an action is to be taken, the central bank evaluates the economy's current state and hence its possible future paths from now on and to choose the optimal current action in the light of this analysis, which no advance commitment about future actions, except that they will similarly be the ones that seem best in whatever state may be reached in the future. Therefore, much of the recent literature has focused on the type of policy rule that a central bank should commit itself in order to reap the benefits of policy commitment and, as shown by Woodford (2003), a commitment to a relatively simple rule is supposed to represent the only feasible form of commitment because of difficulties of explaining the nature of the central bank's commitment to the public some more complex case would otherwise have to face.

Currie & Levine (1993) develop a new approach to address the optimal policy problem in which optimal policies are examined under a standard control state-space approach with a stochastic noise[5] and the solution procedure for obtaining an optimal policy in-

[5]The significance of the stochastic noise in the model also helps transform the nature of the policy problem from one-shot deterministic policy game into a repeated stochastic policy game.

stead satisfies *Bellman's principle of optimality*[6]. By definition, time-consistent optimal policy in this framework must not depend on past values of the state vector, while it will for time-inconsistent policies. Central banks have to make a choice between two *types of policy* which are time-consistent policies where the central bank has no credibility; or time-inconsistent ones with credibility where the central bank does not succumb to any short-run incentive to renege.

As argued by Currie & Levine (1993), Ramsey, discretionary, and simple rule policies are in essence all *rules* in the sense of a state-contingent or feedback procedure in which monetary policy instruments are changed period-by-period in response to changes in target variables such as output and inflation. Optimal policy in form of a Ramsey commitment rule, as pointed out by Cantore et al. (2013), is formulated only once at time $t = 0$ when policymakers simply set the co-state vector associated with the forward-looking variables in the first-order conditions at 0. Thus, optimal Ramsey policy will not change from period to period and is, by definition, time-consistent. However, this policy is essentially a complex rule optimized at time $t = 0$ and future predetermined components of the state vector at time $t > 0$ will not be taken into consideration in future

[6]As shown by Currie & Levine (1993), stochastic optimal control problem aims at maximizing/minimizing, for example, the function

$$J(u) = Var[x(T)] = E[x^2(T)] - (E[x(T)])^2$$

Then, optimal control theory helps to, first, verify the existence of a minimum/maximum of the performance functional J, and second, implement explicit computation/characterization of such a minimum/maximum using either (i) Bellman's principle which yields the Bellman equation for the value function; or (ii) Pontryagin's maximum principle which yields the Hamiltonian system for "the derivative" of the value function.

periods. According to Currie & Levine (1993), this is the key feature of commitment policy contrasting with discretionary policy[7].

While optimal Ramsey policy targets all endogenous variables in the model, an *ex ante* optimized simple rule is "simple" in the sense that policymakers just assign the instruments to a limited number of target variables. Thus, simple monetary rules can be characterized by the solution $r_{n,t} = Dy_t$ where policy instrument $r_{n,t}$ is managed to achieve the policy's targets and matrix D helps select just a subset of endogenous variables y_t in order to achieve simplicity. Simple rules, as argued by Cantore et al. (2013), are time-inconsistent even in non-rational expectation models because the rules will change over time as the optimized matrix D^* at period $t > 0$ will depend on the state vector at time t and so the new rule will differ from that at $t = 0$. Technically speaking, unlike optimal commitment or discretionary counterparts, optimized simple rules are, as explained by Currie & Levine (1993), not certainty equivalent in the sense that they are not robust, or in other words, not independent of the variance-covariance matrix of additive disturbances. Thus, this period's optimized simple rule is not the same as next period's, or this period's optimal rule will become sub-optimal next period, leading to the time-inconsistency problem.

Finally, discretionary policy which is defined as decisions based on the *ad hoc* judgment of policymakers, without any predetermined rules, is also a stationary solution of

[7]The main problem with optimal Ramsey policy is the classic time-inconsistency problem facing stabilization policy. As shown in section 4.3.1, in the absence of some institutional arrangements which effectively deter any change in the previously announced optimal Ramsey rule, there is an irresistible incentive to renege by resetting $\lambda_{2,t} = 0$ at any period $t > 0$ along the trajectory of the optimal policy, causing a time-consistent optimal Ramsey policy to become time-inconsistent.

the form $r_{n,t} = -Az_t$, where A is the coefficient matrix and z_t is the current period's state vector. In principle, as argued by Cantore et al. (2013), this is also a feedback rule but the optimal monetary policy is "discretionary" because at each period t along the trajectory of the optimal policy, policymakers will set the instrument $r_{n,t}$ based on the same period's state vector z_t. This rule is thus also unchanged over time in the sense that the coefficient A is the solution of an iterative process starting with some initial values and the process then converges to stationary values of the coefficients independent of their initial values. Thus, optimal discretionary policy is also a time-consistent policy.

In conclusion, recent literature using a state-space approach has agreed that optimal Ramsey policy and optimal discretionary policy are taken as time-consistent, while *ex ante* optimized simple rules are time-inconsistent. The comparison of different optimal monetary policy setups should be conducted based on a standard control state-space approach with Bellman's principle of optimality, as proposed by Currie & Levine (1993). The rest of the chapter evaluates and suggests the normative options for policymakers to select from a Ramsey commitment rule, discretionary policy, and an *ex ante* optimized Taylor-type rule by comparing their outcomes/related asymptotic loss values.

4.3 The Optimal Monetary Policy Problem

4.3.1 The Ramsey Problem

The Ramsey problem in a dynamic, stochastic, and non-linear form, as set out in Cantore et al. (2016), can be expressed in state-space representation as follows

$$z_t = h(z_{t-1}, x_t, w_t, \varepsilon_t) \tag{4.1}$$

with transition rule

$$\mathbb{E}_t x_{t+1} = g(z_t, x_t, w_t) \tag{4.2}$$

where z_t is a $(n-m)\times 1$ vector of backward-looking/lagged/state variables, x_t is a $m \times 1$ vector of forward-looking/lead variables/jumpers, ε_t is a $l \times 1$ vector of *i.i.d* shock variables, and w_t is a vector of policy instruments, with size $r \times 1$[8].

If we define the vector of all variables in the model as

$$y_t \equiv \begin{bmatrix} z_t \\ \\ x_t \end{bmatrix}$$

Then, the model equations from 4.1 and 4.2 can be rewritten in a compact form as

$$\mathbb{E}\left[f(y_t, y_{t+1}, y_{y-1}, w_t, \varepsilon_{t+1})\right] = 0 \tag{4.3}$$

where $\mathbb{E}_t[\varepsilon_{t+1}] = 0$ and $\mathbb{E}_t\left[\varepsilon_{t+1}\varepsilon'_{t+1}\right] = \Sigma_\varepsilon$.

The optimal policy with Ramsey problem is to maximize the *households' discounted utility* at time 0, which is

$$\Omega_0 = E_0\left[\sum_{t=0}^{\infty} \beta^t u(y_t, y_{t-1}, w_t)\right] \tag{4.4}$$

subject to all model equations as constraints and given the initial values of the elements of the state variables z_0.

The Lagrangian is specified as follows

$$L = E_0\left\{\sum \beta^t \left[u(y_t, y_{t-1}, w_t)\right] + \lambda' f(y_t, y_{t-1}, w_t, \varepsilon_{t+1})\right]\right\} \tag{4.5}$$

[8]Note that under *perfect information*, all variables dated t or earlier are observed at time t, including shocks.

where λ_t is a column vector of multipliers associated with n constraints defining the model[9].

As shown by Cantore et al. (2016), the multiplier λ_t can be partitioned into two components $[\lambda_{1,t}, \lambda_{2,t}]$ in which $\lambda_{1,t}$ is the co-state vector[10] associated with the backward-looking component of the first-order condition and $\lambda_{2,t}$ is the co-state vector associated with the forward-looking component of the first-order condition, which is x_t. Thus, from initial n of λ_t, we now need to meet $2n$ boundary conditions. For the first n conditions related to the state variables, we know that z_0 is already given, so they are satisfied. In the present model, the only instrument is the nominal interest rate $R_{n,t}$ and according to Currie & Levine (1993), the optimal Ramsey rule can thus be characterized by

$$R_{n,t} = D \begin{bmatrix} z_t \\ \lambda_{2,t} \end{bmatrix} \tag{4.6}$$

where

$$\begin{bmatrix} z_{t+1} \\ \lambda_{2,t+1} \end{bmatrix} = F \begin{bmatrix} z_t \\ \lambda_{2,t} \end{bmatrix} \tag{4.7}$$

[9]Note that the total number of variables/equations in the model is n. Each model equation is taken as a constraint of the maximization problem of Ω_0, corresponding to one of the λ.

[10]Co-state vector is a vector of first-order differential equations/partial derivatives of the negative of the Hamiltonian with respect to the state variables. Co-state variables can be interpreted as Lagrange multipliers associated with the state equations. While the state equations represent constraints of the minimization problem, the co-state variables represent the marginal cost of violating those constraints. State equations are solved forward in time as they are subject to an initial condition. However, co-state equations must satisfy a terminal condition and are solved backward in time, from the final point toward the beginning. In our case, z_0 is given as initial condition, so co-state vector can be solved. However, as x_t is not given or a free variable, *transversality condition* requires that the co-state vector to be 0.

and D, F are fixed matrices.

In the equation of first-order derivatives, all components need to be zero[11]. Since x_t is a free variable, as argued by Currie & Levine (1993), the *transversality condition*[12] requires the remaining n conditions related to these forward-looking variables to satisfy

$$\lambda_{2,0} = 0 \tag{4.8}$$

Thus, in the context of commitment, in order to achieve optimality, the policymaker just simply sets this component of λ_t, which is $\lambda_{2,0}$ equal to 0 at time $t = 0$. In periods at $t > 0$, with this form of *ex ante* optimal policy, there exists a gain from reneging by resetting $\lambda_{2,t} = 0$. As emphasized by Cantore et al. (2016), it is this incentive which exists at all points along the trajectory of the optimal policy that encourages the policymaker to re-optimize in this fashion. In other words, this is exactly the *time-inconsistency problem* we have to deal with in pursuing a stabilization policy with an optimal Ramsey setup.

[11]Kuhn-Tucker complementary slackness conditions.

[12]Currie & Levine (1993) argue that in the first-order condition of an optimal problem, usually defined as an Euler equation, when we do the partition as above for the state and forward-looking variables, the second part with forward-looking variables (usually with wealth) induces the *transversality condition* in an infinite horizon case as follows

$$\lim_{T\to\infty} \beta^T[-\lambda_{2,t}(x_T, x_{T+1})]x_{T+1} = 0$$

that can be interpreted as "the present discounted value of wealth at infinity must be zero, or wealth x_{T+1} should not grow too fast compared to its marginal value $\beta^T[-\lambda_{2,t}(x_T, x_{T+1})]$." This basically says that $\lambda_{2,t}$ must be zero. In other words, nothing should be saved in the last period unless it is costless to do so, which is $\beta^T[-\lambda_{2,t}(x_T, x_{T+1})] = 0$ (Kamihigashi, 2006).

4.3.2 Optimized Simple Rules

As shown above, the optimal policy in the form of the Ramsey solution for the instruments can be expressed as a function of state variables and co-state vector of the jumpers as follows

$$w_t = f(z_t, \lambda_{2,t}) \tag{4.9}$$

As argued by Cantore et al. (2016), the implementation of this solution can, however, be problematic because of its complexity and the observability of elements of z_t[13]. Therefore, the literature also pay a lot of attention to simple rules, taking Ramsey solution as a benchmark to compare and evaluate the feasibility of the simple rules. In this case, the optimal policy problem solves for an optimized simple rule in which the vector of instruments w_t[14] responds to an observed subset of the variables y_t as follows

$$w_t = H \begin{bmatrix} z_t \\ x_t \end{bmatrix} \tag{4.10}$$

where the matrix H selects a subset of y_t from which to feedback[15].

Given this optimized simple rule, following Cantore et al. (2016), we can redefine the inter-temporal welfare loss at time t in Bellman form as

$$\Omega_t(z_t) = u(y_t, y_{t-1}, w_t) + \beta \mathbb{E}_t[\Omega_{t+1}(z_{t+1})] \tag{4.11}$$

Given the steady-state values for the instrument w, the optimized simple rule problem then computes a second-order solution for a particular setting of w and solves the maximization problem at $t = 0$

$$\max_{w,H} \Omega_0(z_0, \Sigma, w, H) \tag{4.12}$$

[13]Technology process or $\lambda_{2,t}$, for example

[14]In the present model, w_t includes the nominal interest rate $R_{n,t}$ only.

[15]In the present model, they are lagged interest rate, output, and inflation.

given the initial values of the state variables z_0 and the variance-covariance matrix of shocks Σ[16].

In the present DSGE model, we set initial values equal its steady state $z_0 = z$ and maximize the conditional value of social welfare at this steady state, given the variance-covariance matrix of the shocks[17]. The simple Taylor rule examined in this chapter is of the form

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t + \theta_\pi(\hat{\pi}^c_{t-1} - \hat{\bar{\pi}}_t) + \theta_y \hat{y}_{t-1}\right] + z^R \tag{4.13}$$

In order to maximize efficiently with respect to feedback parameters, following Cantore et al. (2016), we re-parameterize the feedback parameters as follows

$$\alpha_{\bar{\pi}} \equiv (1-\rho_R)$$

$$\alpha_\pi \equiv (1-\rho_R)\theta_\pi$$

$$\alpha_y \equiv (1-\rho_R)\theta_y$$

Then the simple Taylor rule now becomes

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + \alpha_{\bar{\pi}} \hat{\bar{\pi}}_t + \alpha_\pi(\hat{\pi}^c_{t-1} - \hat{\bar{\pi}}_t) + \alpha_y \hat{y}_{t-1} + z^R \tag{4.14}$$

This type of simple policy rule, as suggested by Taylor (1993), does not necessarily mean either a fixed setting for the policy instruments or a mechanical formula. In fact, this algebraic formula just sketches out a contingency plan that lasts until the monetary authority officially cancels the plan.

[16] A diagonal matrix in all the codes.

[17] In a purely dynamic and deterministic model, there is no exogenous shock and the optimization problem is driven by the need to return from z_0 to its steady state z.

4.3.3 Discretionary Monetary Policy

In order to evaluate the optimal discretionary policy, following Cantore et al. (2016), the expected loss at time t given the observed state variables z_t is expressed in Bellman form as follows

$$\begin{aligned}\Omega_t(z_t) &= \mathbb{E}_t\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}u(y_t, y_{t-1}, w_t)\right] \\ &= u(y_t, y_{t-1}, w_t) + \beta\mathbb{E}_t[\Omega_{t+1}(z_{t+1})]\end{aligned} \tag{4.15}$$

If the value function is defined as

$$V(z_t) = \max_{w_t}\{u(y_t, y_{t-1}, w_t) + \beta\mathbb{E}_t[\Omega_{t+1}(z_{t+1})]\}$$

Then, the Bellman equation should be

$$V(z_t) = \max_{w_t}\mathbb{E}_t\{u(y_t, y_{t-1}, w_t) + \beta\mathbb{E}_t[\Omega_{t+1}(z_{t+1})]\}$$

As argued by Cantore et al. (2016), the computation of the discretionary solution involves the minimization of Ω_t at time t, subject to all of the model constraints, under the knowledge that the same procedure will be implemented to minimize Ω_{t+1} at time $t+1$, and so on. This process should end up with a stationary Markov Perfect solution of the form

$$w_t = f(z_t)$$

$$x_t = g(z_t)$$

which show a policy function for the instruments and the transition rules for forward-looking variables as functions of the state variables.

4.4 Quadratic Loss Function

King & Wolman (1999) argue that a loss function for the monetary authority plays a key role in determining the details of optimal monetary policy. Stabilizing output by eliminating the "Okun gaps"[18] as well as keeping a low and stable level of inflation are widely viewed as important monetary policy goals. Thus, the loss function helps balance different targets of an optimal policy and, in general, our presumption is that the optimal policy should involve variability in both inflation and real economic activity.

Traditionally, it has been assumed that monetary authority should minimize some *ad hoc* quadratic loss function, where losses are caused by key variables being away from their respective targets. According to Woodford (2003), one of the reasons for resorting to a quadratic loss function that represents a quadratic approximation[19] is a mathematical convenience as we can address the nature of optimal policy within a linear-quadratic optimal control framework in which the characterization of optimal policy is relatively simple. This quadratic *ad hoc* loss function is of the form

$$\Omega = E_t \sum_{i=0}^{\infty} \beta^i [y'_{t+i} W y_{t+i}]$$

where the monetary policy discount factor β is fixed to the value chosen for the households' discount factor. The vector $y_t = [x'_t u'_t]$ contains x_t: a $n \times 1$ vector of endogenous and exogenous variables; and u_t: a $p \times 1$ vector of control variables. W is a time-invariant symmetric, positive semi-definite matrix of policy weights reflecting the central bank's preferences over the target variables.

An alternative approach toward optimal monetary policy which is, however, less commonly used in the literature is a welfare-based loss function. King & Wolman (1999) also develop a so-called welfare-based loss function which involves maximizing the representa-

[18]Between output and its potential level, as in Okun (1963).

[19]Second-order Taylor series.

tive agent's welfare, that is, the agent's lifetime utility as follows

$$\Omega = \max \sum_{i=0}^{\infty} \beta^i u(.)$$

However, Ilbas (2008) argues that adopting a welfare-based loss function would make our results rely more heavily on the specific assumptions about the underlying model than would be the case under an *ad hoc* loss function. Adolfson et al. (2011) also point out that the quadratic approximations of the welfare of a representative household are very model-dependent and reflect the particular distortions assumed in any given model because, as shown by Clarida et al. (1999), in this instance, the relative weights we assign to respective variables in the quadratic loss function are also functions of the model's primitive parameters. The households' welfare is thus, in any case, hardly a central bank's operational objective, although it may be of interest and relevance to examine how households' welfare in particular models is affected by the central bank's policy.

Therefore, following Cantore et al. (2016), a standard *ad hoc* quadratic loss function[20] in deviation form is therefore used to evaluate optimal policy choices in this chapter,

[20] Intertemporal loss function has a discounted sum of all periods' losses, while one-period loss function incorporates only one period loss.

which is of the form[21]

$$\Omega_0 = (1-\beta)E_0\left[\sum_{t=0}^{\infty}\beta^t(16\pi_t^2 + \lambda_1 y_t^2 + \lambda_2 r_{n,t}^2\right] \tag{4.16}$$

$$\simeq 16\text{var}(\pi_t) + \lambda_1\text{var}(y_t) + \lambda_2\text{var}(r_{n,t}) \quad \text{as} \quad \beta \to 1$$

where the variances are unconditional[22].

It is stressed by Batini et al. (2006) that this is the central bank's loss function, not a welfare function, which describes the actual policy objectives the central bank has rather than what they should have. Also, it is desirable, as argued by Dieppe et al. (2005), to have a penalty on the instrument variability. This is reflected by a positive weight on interest rate variability in the loss function[23]. Therefore, the loss function of the central bank penalizes the deviations of inflation, output, and the interest rate. In the simplest case, all the weights are set to be unity or the central bank gives equal priority to the

[21]As argued by Vredin (2015), Smets (2014), and Froyen & Guender (2016), there should be an inclusion of financial stability indicators in the central bank's loss function as they are welfare-improving. Real exchange rate, among others, is a major shock of financial stability and can be included. As shown by Froyen & Guender (2016), when the exchange rate is given a non-zero weight in the loss function, the loss under a Taylor rule at optimal policy is diminished considerably. In addition, as shown in Curdia & Woodford (2011) and Woodford (2012), the presence of frictions in the financial intermediation sector also leads to the inclusion of a financial stability objective in the loss function. However, for the sake of simplicity and computational efficiency, the loss function is simplified by excluding the exchange rate in the present model and this property would be featured in our further studies in the future.

[22]As the model is quarterly, the annualized rate of inflation entering the loss function should be $\pi^a \approx 4\pi_t^q$, then the period loss function is given by $16\text{var}(\pi_t)$.

[23]We should distinguish between discount factor (β) and discount rate (r). The former tells us "how much agents care about a period in the future as compared to today". If $\beta \to 1$, agents take the next period as equal. The latter shows the rate at which agents discount future utility in a multi-period model. The relationship between two notions is $\beta = \frac{1}{1+r}$.

variance of annualized inflation, output, and also the instrument. However, in the present research, following Yellen (2012) and Reifschneider et al. (2015), the weights for the annualized rate of inflation, interest rate, and unemployment gap are all set to unity. Based on the widely spread empirical specification of the Okun's law, as argued by Yellen (2012), the unit weight on the unemployment gap accordingly converts into a weight of $\lambda_1 = 0.25$ for the variation of output in the loss function. Also following Yellen (2012), the variance of the change in short-run interest rate is weighted at $\lambda_2 = 1$.

In addition, as the discount factor $\beta \to 1$, agents care about future period the same as today. Then the *ad hoc* loss function used in the evaluation is of the form

$$\begin{aligned} \Omega_0 &= (1-\beta)E_0\left[\sum_{t=0}^{\infty}\beta^t(16\pi_t^2 + 0.25y_t^2 + r_{n,t}^2)\right] \\ &\simeq 16\text{var}(\pi_t) + 0.25\text{var}(y_t) + \text{var}(r_{n,t}) \quad \text{as} \quad \beta \to 1 \end{aligned} \tag{4.17}$$

4.5 Computational Methodology

The linear-quadratic (LQ) approximation approach is applied to this non-linear dynamic optimization problem for a number of reasons. As argued by Cantore et al. (2013), the characterization of time-consistent Ramsey equilibria for a single policymaker, and even so for many interacting central banks, is well understood. Besides, policy can be decomposed into deterministic and stochastic components and this is a very convenient property as it enables the stochastic stabilization component to be pursued using simple Taylor-type feedback rules rather than the exceedingly complex optimal counterpart. In addition, for sufficiently simple models, LQ approximation allows analytical rather than numerical solution. Benigno & Woodford (2012) also prove that the most we can obtain with any generality would be for the solution to the LQ problem to represent a local linear approximation to the actual optimal policy, that is, a first-order Taylor approximation to the true, nonlinear optimal policy rule. Therefore, this section considers the stochastic linear-quadratic problem which can usefully be employed to approximate the optimal policy choices and summarizes the solution method given by Heer & Maussner (2009).

4.5.1 Stochastic Linear-Quadratic Problems

Linear-quadratic models have two essential features, including (i) a quadratic *reward/return functions*[24]; and (ii) linear constraints. In other words, it is a class of problems with a quadratic return function and a linear transition function. This specification of the model leads to a wide application of the optimal linear regulator problems for which the Bellman equation can be solved quickly using linear algebra. This is also, in the class of DSGE models, used to solve the problem of optimal monetary policy through the quadratic loss function.

Heer & Maussner (2009) provide a detailed framework of the stochastic linear-quadratic model and derive its important properties. Suppose the economy is characterized by a stochastic linear model

$$\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t \tag{4.18}$$

where $\mathbf{x}_t$: $n \times 1$ vector of predetermined/state variables, $\mathbf{u}_t$: $m \times 1$ vector of control variables of the model (which are set by the agents/fictitious social planner), ϵ_t: $n \times 1$ vector of shocks which has a multivariate normal distribution with $E(\epsilon_t) = 0$ and $E(\epsilon\epsilon') = \Sigma$.

[24] Describing how the agent "ought" to behave, or in its normative content, stipulating what we want the agent to accomplish.

With the presence of shocks, the social planner cannot control the economy perfectly and has to choose $\mathbf{u}_t$ before he can ascertain the size of the shocks.

Given the initial values of the state variables $\mathbf{x}_0$, the planner's (intertemporal) objective is to solve the maximization problem at $t = 0$, which is basically a quadratic function expressed in Lyapunov form:

$$\max_{\mathbf{x}_t,\mathbf{u}_t} E_0 \sum_{t=0}^{\infty} \beta^t [\mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t] \tag{4.19}$$

subject to the model equations in 4.18. This expression shows that the objective function is quadratic in $\mathbf{x}_t, \mathbf{u}_t$ and there is also a product of $\mathbf{x}_t$ and $\mathbf{u}_t$. The equation is by definition concave in $\mathbf{x}_t, \mathbf{u}_t$[25]. This also requires that both the symmetric $n \times n$ matrix Q and $m \times m$ matrix R are negative semidefinite, so both quadratic terms are negative: $\mathbf{x}_t' Q \mathbf{x}_t \leq 0$ and $\mathbf{u}_t' R \mathbf{u}_t \leq 0$.

The current period objective function is given by

$$g(\mathbf{x}_t, \mathbf{u}_t) := \mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t$$

In matrix form we have[26]

[25] π_t^2, y_t^2, r_t^2 in the present model.

[26] We can prove this by deriving the matrix form

$$g(\mathbf{x}_t, \mathbf{u}_t) := \begin{bmatrix} \mathbf{x}_t' & \mathbf{u}_t\prime \end{bmatrix} \begin{bmatrix} Q & S' \\ S & R \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix}$$

$$:= \begin{bmatrix} (\mathbf{x}_t' Q + \mathbf{u}_t' S)\mathbf{x}_t + (\mathbf{x}_t' S' + \mathbf{u}_t' R)\mathbf{u}_t \end{bmatrix}$$

$$:= \mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t\prime S \mathbf{x}_t + \mathbf{x}_t' S' \mathbf{u}_t + \mathbf{u}_t' R \mathbf{u}_t$$

And we have

$$\mathbf{x}_t' S' \mathbf{u}_t = \mathbf{u}_t' S \mathbf{x}_t$$

$$g(\mathbf{x}_t, \mathbf{u}_t) := \begin{bmatrix} \mathbf{x}_t' & \mathbf{u}_t\prime \end{bmatrix} \begin{bmatrix} Q & S' \\ S & R \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{bmatrix} \tag{4.20}$$

4.5.2 Solving the Model

There are two popular approaches to solve for the solution of the linear-quadratic problem (Ramsey problem), based on *Euler Equations* method or *Dynamic Programming* method. While the former relies on a linear approximation of the model's Euler equations and then solves the resulting system of linear stochastic difference equations to get the solution using a *Linear Approximation* technique, the latter performs the *LQ Approximation* technique in order to approximate the policy function by value function iteration.

The second approach which is used for computational purpose in this chapter is also explained in detail by Heer & Maussner (2009). Suppose the Bellman equation for the stochastic linear-quadratic problem is given by

$$v(\mathbf{x}_t) := \max_{\mathbf{u}_t} \{\mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t + \beta E[v(\mathbf{x}_{t+1})]\} \tag{4.21}$$

If we use the model representation given by 4.18 to replace the next period's variables, the Bellman equation becomes

$$v(\mathbf{x}_t) := \max_{\mathbf{u}_t} \{\mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t + \beta E[v(A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t)]\} \tag{4.22}$$

The expectations are taken conditional on the information contained in the current state $\mathbf{x}_t$. Let us guess the value function is given by

$$v(\mathbf{x}_t) := \mathbf{x}_t' P \mathbf{x}_t + d \tag{4.23}$$

Then

$$g(\mathbf{x}_t, \mathbf{u}_t) := \mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t$$

where P is a n dimensional symmetric, negative semidefinite square matrix and $d \in R$.

Then the following identity is also true

$$v(\mathbf{x_{t+1}}) := \mathbf{x_{t+1}}' P \mathbf{x_{t+1}} + d \tag{4.24}$$

$$:= (A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t)' P (A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t) + d$$

From 4.21, 4.23, 4.24, the Bellman equation can be rewritten as

$$\mathbf{x}_t' P \mathbf{x}_t + d := \max_{\mathbf{u}_t} \{\mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t + \beta E[(A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t)' P (A\mathbf{x}_t + B\mathbf{u}_t + \epsilon_t) + d]\} \tag{4.25}$$

Evaluating the conditional expectation of this equation we have

$$\mathbf{x}_t' P \mathbf{x}_t + d := \max_{\mathbf{u}_t} \{\mathbf{x}_t' Q \mathbf{x}_t + \mathbf{u}_t' R \mathbf{u}_t + 2\mathbf{u}_t' S \mathbf{x}_t \tag{4.26}$$

$$+\beta \mathbf{x}_t' A' P A \mathbf{x}_t + 2\beta \mathbf{x}_t' A' P B \mathbf{u}_t + \beta \mathbf{u}_t' B' P B \mathbf{u}_t + \beta tr(P\Sigma) + \beta d\}$$

Differentiating the right-hand side of 4.26 with respect to $\mathbf{u}_t$ and solving the first-order condition, we get the **policy function** for $\mathbf{u}_t$, which describes the vector of control variables $\mathbf{u}_t$ as a function of the state variables $\mathbf{x}_t$, as follows

$$\mathbf{u}_t = -(R + \beta B' P B)^{-1} (S + \beta B' P A) \mathbf{x}_t$$

or

$$\mathbf{u}_t = -F \mathbf{x}_t \tag{4.27}$$

where

$$F = (R + \beta B' P B)^{-1} (S + \beta B' P A)$$

In this expression, F, A, B are given from the model equation, R from the quadratic planner's objective function. In order to pin down F, the only term we need to specify is

P. Putting $\mathbf{u}_t$ back into the Bellman equation 4.26 we have

$$\mathbf{x}_t'P\mathbf{x}_t+d:=\mathbf{x}_t'[Q+\beta A'PA-(S+\beta B'PA)'[R+B'PB]^{-1}(S+\beta B'PA)]\mathbf{x}_t+\frac{\beta}{1-\beta}tr(P\Sigma) \tag{4.28}$$

Two identities can be inferred by comparing the constant terms in both sides of the above equation. First,

$$d=\frac{\beta}{1-\beta}tr(P\Sigma) \tag{4.29}$$

and second, an equation for P, also known as the matrix Riccati equation[27] which is a difference equation with quadratic unknowns, is given by

$$P=Q+\beta A'PA-(S+\beta B'PA)'[R+B'PB]^{-1}(S+\beta B'PA) \tag{4.30}$$

Solving 4.30 by iterating on the matrix Riccati difference equation, or matrix factorization, we get a solution for P which is needed to pin down F.

Thus, from equations 4.18 and 4.27, the dynamics of the economy is, finally, given by

$$\mathbf{x}_{t+1}=(A-FB\mathbf{u}_t)\mathbf{x}_t+\epsilon_t \tag{4.31}$$

4.5.3 Linear-Quadratic Approximation

Linear-Quadratic (LQ) approximation is the appropriate method to solve the linear-quadratic problem specified in the previous section. According to Heer & Maussner (2009), suppose the model consists of a n vector of state variables $\mathbf{x}$, a m vector of control variables $\mathbf{u}$, and a current period *return function* $g(\mathbf{x},\mathbf{u})$ which includes all the non-linear relations of the model[28]. The model's (linear) equations in this section are, for

[27] Riccati (1724) is the first official document on Riccati equation.

[28] The loss function in the present model.

the sake of simplicity, redefined as

$$\mathbf{x}' = A\mathbf{x} + B\mathbf{u} \tag{4.32}$$

where $\mathbf{x}'$ means $\mathbf{x}_{t+1}$.

If we define a $n+m$ column vector $\mathbf{y} = [\mathbf{x}^T, \mathbf{u}^T]$, then there is usually a stationary solution of the deterministic counterpart of the model $\bar{\mathbf{y}} = [\bar{\mathbf{x}}^T, \bar{\mathbf{u}}^{\mathbf{T}}]$, which is also the *steady states* of the model having the property that $\bar{\mathbf{y}} = h(\bar{\mathbf{y}}, 0)$ where h is the policy function.

Following Heer & Maussner (2009), the procedure for performing a linear-quadratic approximation is specified as follows:

Stage 1: Approximation of the Bellman/Value Equation

If $Q \in \mathbb{R}^{(n+m)\times(n+m)}$ is the symmetric matrix that characterizes the linear-quadratic approximation of the current period return function g, we can apply the Taylor's theorem to expand this return function at order $n+m$ about the steady states $\bar{y}$ to get

$$\mathbf{y}^T Q\mathbf{y} = g(\bar{\mathbf{y}}) + \sum_{i=1}^{n+m} g_i(y_i - \bar{y}_i) + \frac{1}{2}\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} g_{ij}(y_i - \bar{y}_i)(y_j - \bar{y}_j) \tag{4.33}$$

where g_i and g_{ij} are the first and second-order partial derivatives of g at $\bar{\mathbf{y}}$.

From this equation, by comparing Q and the coefficients on the right-hand side, we can find out all elements of Q as follows

$$q_{11} = g(\bar{\mathbf{y}}) + \sum_{i=1}^{n+m} g_i\bar{y}_i + \frac{1}{2}\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} g_{ij}\bar{y}_i\bar{y}_j$$

$$q_{1i} = q_{i1} = \frac{1}{2}g_i - \frac{1}{2}\sum_{j=1}^{n+m} g_{ij}\bar{y}_j$$

$$q_{1j} = q_{ji} = \frac{1}{2}g_{ij}$$

with $i, j = 2, 3, ..., n+m$.

At the end of this stage, we get the matrix Q which is the core of a matrix R characterizing the Bellman/value function.

Stage 2: Reduction of the Bellman/Value Function

In this stage, we begin to eliminate all future values of the state variables so that the resulting quadratic form of the Bellman/value equation is reduced to a function of the current state variables and the control variables. In order to achieve this target, we need to implement the iteration all the way backward from the final point in the time frame (n in this model).

After each step of the iteration process, we get a new matrix R which characterizes a new Bellman equation. Suppose a matrix R^s characterizing the quadratic form of the right-hand side of the Bellman equation at reduction step s, where

$$R^1 := \begin{bmatrix} Q_{(n+m)\times(n+m)} & 0_{(n+m)\times n} \\ 0_{n\times(n+m)} & \beta V^0_{n\times n} \end{bmatrix}$$

Let $\mathbf{c}_s^T$ denote the $n+1-s$ row of the matrix

$$C_s = [A \quad B \quad 0_{n\times(n-s)}]$$

Then, for $s = 1, 2, ..., n$, we iterate on

$$R^{s+1} = \begin{bmatrix} I_{2n+m-s} \\ \mathbf{c}_s^T \end{bmatrix}^T R^s \begin{bmatrix} I_{2n+m-s} \\ \mathbf{c}_s^T \end{bmatrix}$$

The iteration continues until all future values of the state variables are eliminated from the value function. At this point, we can move to the next stage of maximization.

Stage 3: Maximization of the Bellman/Value Function

It is worth noting that after stage 2, there are only the current state of the state variables and control variables in the Bellman equation. We can now solve the maximization problem by taking first-order derivative of the value function with respect to the control variables. Doing this implies eliminating the control variables from the right-hand side of the Bellman equation. In addition, by setting the first-order derivatives to 0, we can find an expression for the control variables as functions of the state variables. Though this is exactly the policy function at this point of the procedure, it will not be the solution until we get *the policy function associated with the maximum of the value function when convergence has been achieved.*

After the last reduction step, matrix R is reduced to a square matrix of $n + m$, which is associated with a value function V^0. We therefore need another m maximization steps so as to get matrix R to be further reduced down to a square matrix of size n where at this point we also get a new guess of the value function V^1.

During the maximization stage, at any step $s = 1, 2, ..., m$, the optimal choice of the control variable u_{m+1-s} as a linear function of the state variables and the remaining control variables in the model $[x_1, x_2, ..., x_n, u_1, u_2, ..., u_{m-s}]$ is given by the row vector

$$\mathbf{d}_s^T = \left[-\frac{r_{1k}}{r_{kk}}, -\frac{r_{2k}}{r_{kk}}, ..., -\frac{r_{k-1,k}}{r_{kk}} \right], \quad k = n + m - s$$

Thus, we now continue to iterate on

$$R^{s+1} = \begin{bmatrix} I_{n+m-s} \\ \mathbf{d}_s^T \end{bmatrix}^T R^s \begin{bmatrix} I_{n+m-s} \\ \mathbf{d}_s^T \end{bmatrix}, \quad s = 1, 2, ..., m$$

As mentioned above, a new guess of the value function $V^1 = R^{m+1}$ is achieved when matrix R is reduced to size n. At this point, we need to compare elements of the new guess to those of the previous one V^0. The iteration stops if and only if the maximal element in $|V^1 - V^0|$ is smaller than a threshold of $\epsilon(1 - \beta)$ for some small positive ϵ:

$$\max_{ij} |v_{ij}^0 - v_{ij}^1| < \epsilon(1 - \beta)$$

with $i, j = 1, 2, ..., n$.

In this case, we have arrived at the maximum of the value function. Then we stop the iteration and jump to the last stage to compute the solution policy function. If it is still over the threshold, we replace V^0 with V^1 and continue to iterate.

Stage 4: Computation of the Policy Function

During the maximization stage, vectors $\mathbf{d}_s$ are stored in a matrix D of the size $m \times (n + m + 1)$. At the end of the maximization stage, convergence has been achieved and the associated matrix $D = (d_{ji})$ can be used to derive the policy functions for all of the control variables in the model. The last row of matrix D consists of the coefficients that help shape the policy function for the first control variable as a function of all n state variables, which can be written as

$$u_1 = \sum_{i=1}^{n} d_{mi} x_i$$

Similarly, the row before the last row consists of the coefficients for the policy function of the second control variable as a function of all n state variables and the previous control variable.

$$u_2 = \sum_{i=1}^{n} d_{m-1,i} x_i + d_{m-1,n+1} u_1$$

and so on.

Finally, we can construct a $m \times n$ matrix of coefficients of the policy functions $F = (f_{ji})$ for all m control variables which are dependent on n state variables. The i column of matrix D shapes the i column of matrix F by setting

$$f_{1,i} = d_{m,i}$$

$$f_{2,i} = d_{m-1,i} + d_{m-1,n+1} f_{1,i}$$

$$f_{j,i} = d_{m+1-j,i} + \sum_{k=1}^{j-1} d_{m+1-j,n+k} f_{k,i}$$

where $j = 1, 2, ..., m$ and $i = 1, 2, ..., n$.

In matrix form, we have

$$\begin{bmatrix} ... & d_{1,i} & ... \\ & ... & \\ ... & d_{m-1,i} & ... \\ ... & d_{m,i} & ... \end{bmatrix} \rightarrow \begin{bmatrix} ... & f_{1,i} = d_{m,i} & ... \\ & ... & \\ ... & f_{m-1,i} = d_{2,i} + \sum_{k=1}^{m-2} d_{2,n+k} f_{k,i} & ... \\ ... & f_{m,i} = d_{1,i} + \sum_{k=1}^{m-1} d_{1,n+k} f_{k,i} & ... \end{bmatrix}$$

4.6 Estimation Results

As shown in chapter 3, the marginal likelihood values[29] speak clearly in favour of the high dollarization scenario, compared to the baseline and low dollarization ones[30]. The fit of the model is also considerably better with a high dollarization rate when comparing second moments implied by the Bayesian estimation procedure with those of the data. Therefore, in this section, numerical results for optimal monetary policies are presented for the case of highly-dollarized economy ($\delta^{pd} = 0.46$). Results are from a linear-quadratic problem with a quadratic loss function on the part of the monetary authority. The results are formally quadratic approximation about the steady state of the Lagrangian, representing the true approximation about the fully optimal solution.

Table 4.1 reports the asymptotic losses for an optimal Ramsey policy, an optimal discretionary policy, and an *ex ante* optimized simple Taylor-type rule together with the rule's optimized parameters' values.

Setup	**Asymptotic Loss**	ρ_R	α_π	α_y	**s.d**
Ramsey	24.2043	n.a	n.a	n.a	1.9190
Discretion	37.7316	n.a	n.a	n.a	2.4322
Simple Rule	35.7184	0.9999	2.1281	-0.0609	1.7369

Table 4.1: Outputs from the Optimal Monetary Policy Problem

Several important features are worth noting. First, it is very clear that, using the loss measure, the optimal Ramsey policy dominates the policy options, followed by the

[29]From Laplace approximation as well as from MCMC.

[30]-56244, -58887, and -60426, respectively.

simple rule, and last, the discretionary option. The asymptotic loss is largest in case of discretion which is 37.7316, slightly bigger than 35.7184 in the scenario of simple rule. The loss is smallest at 24.2043 when Ramsey commitment policy is chosen. This result has also been affirmed by Boehm & House (2014) that optimal monetary policy depends in complicated ways on the underlying state variables and is often history-dependent, as is the case of Ramsey commitment.

Second, in line with the previous literature, it is found that interest rate smoothing plays an important role in monetary policy setting. The smoothing parameter is estimated at around 0.9999 which implies a very slow adjustment of the policy rate to its benchmark level. This is also consistent with Taylor (1993)'s original rule. In addition, the estimated inflation reaction parameter is 2.1281, considerably higher than the average estimated value of 1.5 for emerging economies by Hofmann & Bogdanova (2012) or the same benchmark of inflation coefficient in Taylor (1993). However, rather than indicating a genuine violation of the Taylor principle, this result may just be a reflection of the central bank's difficulties in coping with chronically high levels of inflation in a heavily-dollarized economy.

Third, deviating from the standard Taylor rule, the estimated response of the policy rate to output is estimated at -0.0609. This finding, on the one hand, implies a higher preference for inflation stabilisation. On the other hand, the negative sign of output parameter also shows that under a heavily-dollarized economy where the autonomy of the central bank is always the thorniest issue because of its dependence on the foreign central bank's policy, a monetary authority following a simple rule may still run an expansionary monetary policy to some extent to achieve its own targets[31] when output increases in the short run. This result is strongly supported by Hendry et al. (2003) who argue that in a model where the key shocks driving the business cycles of the economy, which are technology shocks in the present model, produce negatively correlated pressures on inflation and output, the second-round effects of these negative responses would facilitate rather than undermine stabilisation[32]. In this case, while the increase in inflation requires the central bank to raise nominal interest rates, an original decline in output also leads the central bank to increase nominal interest rates in order to reduce the upward pressures on inflation, seemingly at the expense of output[33].

Finally, one way to assess the scope of the monetary policy activism or the ranges in which monetary authority can manipulate the instrument to achieve its targets is to evaluate the steady-state variances of the nominal interest rate, which is the central bank's instrument in this case. These variances also highlight possible *zero lower bound* (ZLB) problems which constrain the central bank's ability to stimulate the economy during downturns. As we can see from the results reported in the last column of table 4.1, the

[31] For example, exchange rate stabilization in case of Vietnam.

[32] As a priori perception, raising interest rates when output is already low would exacerbate fluctuations and make monetary policy to have a destabilising effect on the economy.

[33] According to Nikolsko-Rzhevskyy et al. (2019), the Taylor principles, which have become a central tenet of monetary policy, consist of four elements: (i) the coefficient on inflation should be greater than and significantly different from 1 (the nominal interest rate should be raised more than point for point when inflation rises, so that the real interest rate increases); (ii) the coefficient on the output gap should be greater than 0, less than 1, and significantly different than both; (iii) the inflation target should equal 2.0; and (iv) the equilibrium real interest rate should also equal 2.0. Nikolsko-Rzhevskyy et al. (2019) also find evidence that violations of one or more of the elements of Taylor principles, which should be the case for many developing countries, can lead to the Taylor rule deviations.

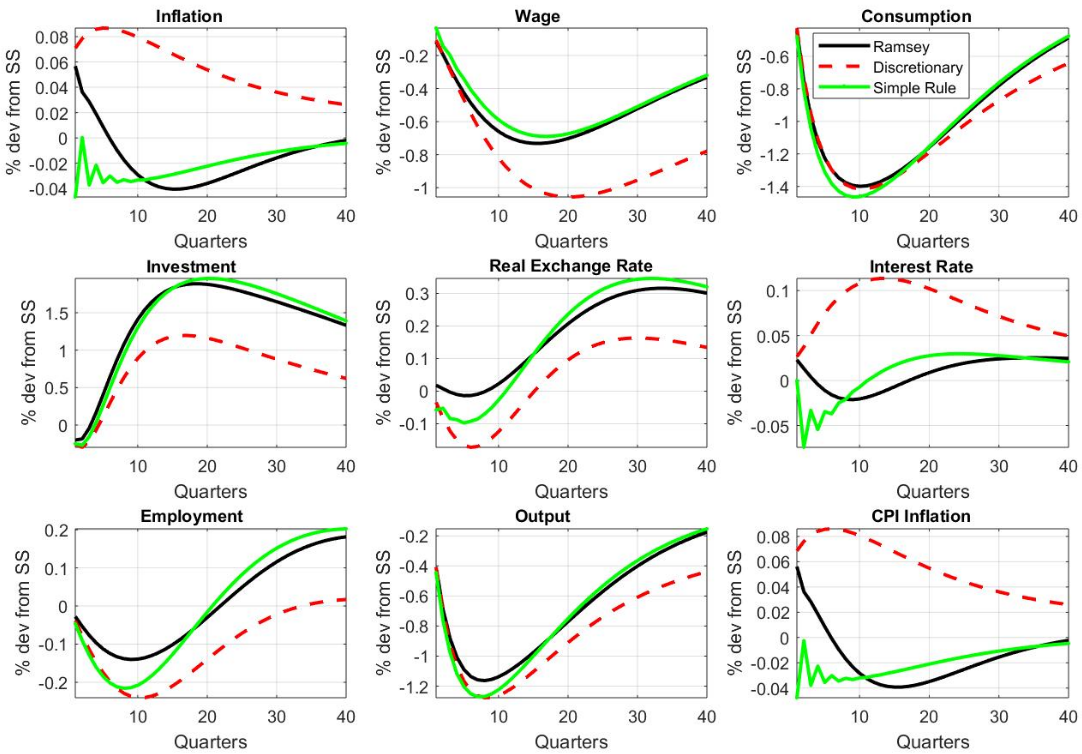
central bank cannot have much room for policy manipulation with quite small standard deviation of the nominal interest rate when following a simple rule.

4.7 Dynamics under Optimal Policy

4.7.1 Supply Shock

Figure 4.1 shows the impulse response functions (IRFs) to a positive (one standard deviation) supply shock, particularly a technology shock, under the optimal Ramsey policy, optimal discretionary policy, and *ex ante* optimized simple Taylor rule. The plots show the IRFs as proportional deviations about the steady state for key variables, including output, consumption, investment, the real wage, the exchange rate, employment, and inflation. The nominal interest rate is used as the central bank's instrument in this problem.

Figure 4.1: Impulse Responses to a Technology Shock



The dynamics of the economy under the optimal Ramsey rule and the optimal discretionary policy, which are depicted by the black and dashed red curves, respectively, are largely in line with Galí (1999) and the previous analysis in section 3.6. As argued by Tervala (2007), in an open economy, there is an "expenditure-switching effect of an exchange rate change" in the model that causes a decline in employment and output in the short run, following a positive technology shock. It is because when the country's currency appreciates after the shock, as shown in figure 4.1, the relative prices of exports also increase and thus world consumption shifts away from the country's goods. In addition, the positive response of inflation is due to the trade-off, which is, as argued by Adolfson et al. (2011), created by a positive technology shock[34], between the decline in the output gap

[34]Unlike the traditional/short-run trade-off between the level of output/unemployment rate and the

and balancing the induced increase in inflation. We see that inflation jumps up and output declines right after the shock and it takes quite a long time, almost 40 quarters, before inflation can be brought back to its steady state as the shock is literally persistent in this case. If the technology shock had been less persistent, the inefficient trade-off between inflation and output stabilization would be less pronounced. In addition, for all three policy options, an increase in domestic inflation implies a real exchange rate appreciation, which is broadly in line with the result found by Chang et al. (2015) using China's macro data. It should be noted that a positive technology shock is traditionally understood to induce an increase in employment and output. Therefore, the opposite behaviour in the present model is named by economists in the field the "technology-hours/employment debate" as in Cantore et al. (2014), Cantore et al. (2017), Francis & Ramey (2009), Rujin (2017), and Pelgrin & Corrigan (2005), and is further explained in the next section.

Impulse responses under the optimized simple Taylor rule are shown by the green curves. It is clear that the optimized simple Taylor rule stabilizes inflation at a relatively higher cost in terms of output, not to mention an initial puzzle in some quarters. This is an unfavourable trade-off between stabilizing inflation and output after a technology shock, compared with the optimal Ramsey policy. We also see that the optimized simple Taylor rule is slightly less successful in stabilizing consumption and employment or stipulating investment, whereas an appreciation of the real exchange rate remains for a long

level of inflation, optimal policy problems consider a new trade-off which, as defined by Fuhrer (1997), focuses on an efficient long-run trade-off between the variability in inflation and in the output gap. The research on the new trade-off suggests that attempting to keep inflation within a very narrow band may increase fluctuations in real output and employment. Conversely, attempts to smooth business cycle fluctuations more actively will lead to wider fluctuations in inflation.

time. The plot shows that the optimal Ramsey policy is somehow more effective than the optimized simple rule in terms of stabilizing inflation in the long run, although the former's initial response is slightly larger in magnitude. A qualitatively similar result is also found by Adjemian et al. (2007). In addition, comparing with the optimal Ramsey rule or optimal discretionary policy, inflation in the optimized Taylor rule falls sharply and then behaves as a "puzzle" following a technology shock. Given the chosen simple rule, this consequently calls for a puzzled fluctuation of the nominal interest rate in the opposite direction.

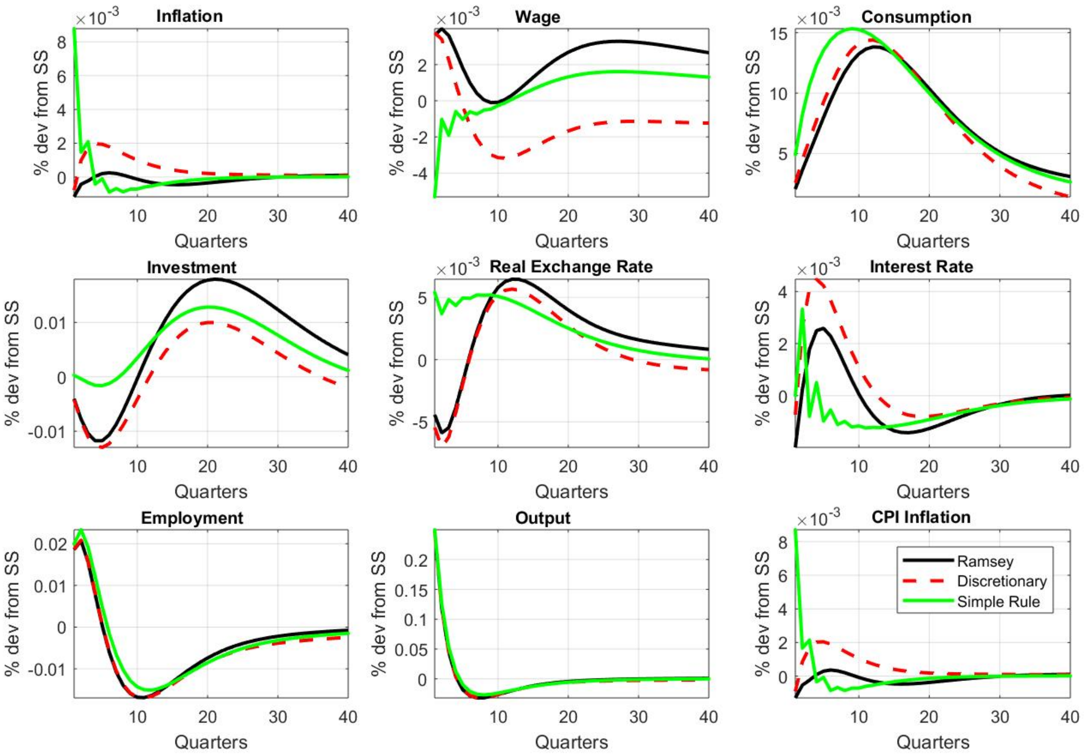
Impulse responses under the optimal discretionary policy shown by the dashed red curves behave almost the same as in the case of the optimal Ramsey policy. Similarly, the optimal discretionary policy also outperforms the optimized simple rule by inducing a more efficient trade-off between the variability of inflation and output. However, compared with the optimal Ramsey rule, there is a larger initial jump of inflation after a technology shock because, as argued by Ravenna & Walsh (2006), the central bank cannot commit to producing a future deflation under discretion, making it less able to stabilize current inflation. This obviously makes the optimal discretionary option less preferable for the central bank. A lower nominal interest rate response also allows for larger scope of monetary activism with the optimal Ramsey rule. Furthermore, as shown in figure 4.1, it requires a longer time for real quantities including consumption, output, and employment to come back to steady state under the optimal discretionary policy.

Finally, from a labour market perspective, figure 4.1 also compares the impulse responses of key variables of the labour market after a positive supply shock. We can see that wages, employment, and output fall and rise back to the steady state as the shock fades away. As argued by Cantore et al. (2013), the real wages decrease because labour supply decreases less than equiproportionately with output. We can also see from figure 4.1 that the optimal Ramsey rule, optimal discretionary policy, and optimized simple rule lead to different levels of impact on the labour market with the smallest being from the optimal Ramsey policy both in the short run and long run. Therefore, the optimal Ramsey policy induces not only the smallest asymptotic loss but also the most efficient responses compared with the other options.

4.7.2 Demand Shock

Figure 4.2 shows the impulse response functions for the same set of endogenous variables following a positive demand-side shock (one standard deviation) to government expenditure with persistence parameters being approximately 0.7. The dashed red curve depicts the responses under the optimal discretionary policy, while the black and green curves show the responses under the optimal Ramsey policy and the *ex ante* optimized Taylor rule, respectively.

Figure 4.2: Impulse Responses to a Government Spending Shock



As expected, the shock materially affects real variables in the economy where, with sticky prices in the model, firms respond to an increase in demand by raising output, leading to an upward movement of the aggregate output. Output then decreases and overshoots the steady state before rising back to the steady state in the long term. Furthermore, impulse responses from other real variables in the model show that a shock to government spending to some extent causes a crowding-in effect on consumption, but crowds out investment in all policy options and the effect is much smaller in the simple rule setup.

The major difference, however, occurs in the behaviour of inflation, which is shown to be much more sensitive to a demand shock under the optimized simple rule. The plots under the optimal Ramsey rule and optimal discretionary policy show the same trade-off between balancing a slightly induced decline in inflation and the variability of output, which is in line with Adolfson et al. (2011), while there is a sharp rise of inflation under the optimized simple rule following a positive demand shock. Contrary to the supply shock, however, a demand shock under optimal policies results in a relatively less persistent trade-off, which is approximately after two years before coming back to the steady state in the medium term when, as argued by Khan et al. (2003), the price level should be largely stabilized. The behaviour of key aggregate variables following a government spending shock under the optimal Ramsey and discretionary policies in the present model is largely consistent with Adjemian et al. (2007).

It should be noted that the response of inflation, under a technology shock and government spending shock alike, shows that commitment under the optimal Ramsey option can enable the central bank to contain inflation more efficiently, which under discretion rises substantially. This is, as argued by Cantore et al. (2013), a major source of welfare differences which are highlighted in table 4.1. In addition, Cantore et al. (2013) also argue that only the policies generating an increasing interest rate path after the initial fall are time-consistent. The time-consistent optimal options, Ramsey and discretionary policies, see this happening with a slightly larger drop of interest rate and inflation in case of the optimal Ramsey policy. This can allow for an expansionary monetary policy when inflation goes down, while it is not the case with the optimized simple rule.

In the labour market, as argued by Cantore et al. (2013), a rise in consumption decreases the marginal utility of consumption, inducing the households to switch from leisure and consumption. This feature is reflected in figure 4.2 by the opposite movements of employment and consumption. While employment decreases sharply over 10 quarters after the initial increase, there is a strong upward movement in consumption over the same period. Also, as labour supply increases much less than proportionally with output, the substitution effect outweighs the income effect to cause the real wage to go up, except the case of the simple rule.

4.7.3 Technology-Employment Debate

The behaviour of the IRFs for employment and output under optimal monetary policy in most cases, as shown by our simulations, is the major point of distinction in this model. We can see from the IRFs that there is a short-run decline of employment after a positive technology shock together with a corresponding decline in output[35]. This result lies in the centre of the technology-hours/employment debate which is, as argued by Cantore et al. (2013), probably one of the most controversial issues in business cycle theory. In general, canonical real business cycle (RBC) models[36] predict that positive technology shocks should generate a short-run increase in hours worked, while Keynesian models in which output is largely determined by aggregate demand in the short run due to price/wage rigidities argue that such shocks temporarily reduce hours (employment).

Proponents of RBC school, after the seminal work of Kydland & Prescott (1982) and

[35]Demand-side shocks are also shown to generate a strong positive comovement between GDP and employment, which is generally taken by much of the literature as *a central, uncontroversial characteristic of any theory of business cycles*, as argued by Galí (1999) and Galí & Rabanal (2004).

[36]In canonical RBC models, output is largely driven by supply-side shocks.

Prescott (1986), have claimed a central role for exogenous variations in technology as a source of economic fluctuations in industrialized economies. As explained by Galí (1999), the RBC's key argument is that aggregate fluctuations can be explained, at least to a first approximation, as the economy's response to exogenous variations in technology with a highly positive correlation between the shock and hours (employment). This argument has been strongly supported by the ability of RBC models to generate unconditional moments for a number of macroeconomic variables that display patterns similar to the data. In particular, RBC models show that there is an increase of hours (employment) after a technology shock which reflects the shifts in the labour demand schedule caused by technology shocks. However, those dynamics have been interpreted by RBC economists in an economy with perfect competition and intertemporally optimizing agents, especially in which the role of nominal frictions and monetary policy is, as stressed by Galí (1999), at most, secondary.

On the contrary, Galí (1999), in line with a class of New Keynesian models with imperfect competition and sticky prices, shows that there is a persistent decline of hours (employment) in response to a positive technology shock. The movements in output and hours (employment) attributed to demand shocks are strongly positively correlated and account for the bulk of the business cycles, while neither is true for the fluctuations attributed to technology shocks. Thus, technology shocks cannot be a quantitatively important and, even less, a dominant source of observed aggregate fluctuations. Instead, their caveats notwithstanding, New Keynesian models point to demand factors as the main force behind the strong positive comovement between output and employment that is the hallmark of the business cycle.

The pattern of the impulse response of employment generated by the present model is largely consistent with the class of New Keynesian models, where it also shows a persistent decline in employment after a positive technology shock. Over the past decades, most of the rapid growth in many emerging economies has been propelled by export-oriented industries which are highly labour-intensive in nature, as argued by Mottaleb & Kalirajan (2014). As the majority of labour-intensive sectors which have been reallocated from developed to developing countries lead to a considerably higher share of labour inputs in GDP, productivity growth becomes less a significant factor of output creation. Evidence from Vietnam shows that an expanding labour pool and a structural shift away from agriculture contributed more than two-thirds of Vietnam's GDP growth in the period 2005-10, as in Breu et al. (2012). Thus, when labour inputs play a key role in the variation of output, a sharp decline of employment after a positive technology shock, as established by Galí (1999), should also be responsible for the short-run downward movement of output.

In recent years, a large empirical literature has been developed to identify technology shocks as well as analyse the technology-hours correlation in order to provide empirical validity for the above competing theories. However, as argued by Cantore et al. (2017), its results rely on two strict assumptions that the impact of technology shocks not only remains constant over time but also depends crucially on the speed of price adjustments on the demand side. Cantore et al. (2014) relax these assumptions and show that the response of hours depends on the factor-augmenting nature of technology and the capital-labour substitution elasticity in either RBC or New Keynesian models. The results also show that both of them can generate technology-hours responses of positive or negative signs.

Furthermore, Cantore et al. (2017) provide empirical evidences using the US data in the post-war period to reaffirm that the change in the response of hours to technology shocks relates to changes in key supply-side parameters. There is a significant sign variation of the response of hours worked to a positive technology shock and the sign of the response of hours is crucially driven by a change in the magnitude of the elasticity of capital-labour substitution. Therefore, to what extent technology shocks are responsible for the pattern of labour and GDP fluctuations associated with business cycles remains an open question.

4.8 Summary

This chapter discusses the basic framework of an optimal monetary policy problem and the linear-quadratic approximation approach that facilitates the estimation of the problem. In addition, the nature of the dynamic responses of the economy under optimal monetary policy is also analysed with some important findings. It is shown that the problem of the central bank is to choose a time path for the instrument (interest rate in this case) to engineer the time paths of the target variables that minimize the objective/loss function. A central bank operating under discretion chooses the current interest rate by reoptimizing every period. Any promises made in the past do not constrain current policy. With rules, whether a Ramsey commitment or a simple one, the central bank chooses a plan for the path of the interest rates that it sticks to for ever.

The key distinction between discretion and rules is whether current commitments constrain the future course of policy in any credible way, as argued by Clarida et al. (1999). The two approaches also differ in their implications for the transmission mechanism from policy to private sector beliefs. Under a framework of commitment, it is simply the binding commitment that makes the policy believable in general. However, within a discretion framework, a perceptive private sector forms its expectations taking into account how central bank may adjust its policy.

The estimation results show that the optimal Ramsey commitment policy compares very well with other policy options, while of the other two, the *ex ante* optimized simple rule is almost as good as the optimal discretionary policy. The outperformance of the optimal Ramsey policy is also depicted by the impulse response functions of key endogenous variables of the model following a supply-side shock or a demand-side shock. The optimal Ramsey policy allows the monetary authority to contain inflation more efficiently than with other options. Therefore, the key implication from this chapter should be a specified policy rule that could be recommended for practical use with great confidence. However, as suggested by Clarida et al. (1999), it is not less useful to take a close look at the cases of discretion and simple rules in order to develop a set of *normative guidelines* for the policy-making process.

Chapter 5

Conclusion

This chapter presents the findings and contributions of the thesis and also relates the findings of the study to the objective of monetary policy within the macroeconomic framework of a developing country. The chapter is organized into three sections. The first section is devoted to the research results of general interest. Key contributions of the research are presented in Section 2. The last section briefly indicates some limitations and major further questions which are prompted by this study. They form a potential agenda of research beyond the scope of this thesis.

5.1 The Thesis in Summary

This thesis sheds a new light on the analysis of macroeconomics in developing countries thanks to the use of latest macroeconomic theories and methodological tools. While new classical school culminated in real business cycle theory, modern macroeconomists formulate models with the crucial element of a Keynesian perspective which is the view that real economies are neither perfectly flexible nor perfectly competitive. This provides a powerful cause for (inefficient) business cycle fluctuations and some indications of the temporary non-neutrality of monetary policy. Thus, imperfect competition/information and nominal price rigidities later came to make their way into the current class of DSGE models.

After the introduction of a general picture of dollarization and a thorough review of the theoretical as well as empirical literature on dollarization in developing countries in chapter 1, a DSGE model of a developing economy which pays a special attention to important characteristics of developing countries is set up in chapter 2. This model incorporates key features of New Keynesian economics and develops a setup for partial dollarization on the supply side. Furthermore, a chronic budget-deficit rule which best describes Vietnam's fiscal policy formulation over the past decades is also examined in the context of this model. These two features appear to be typical characteristics of the macroeconomic background in many developing countries. The chapter also offers a simulation of the theoretical model, including assessments of the relative importance of real and nominal frictions to explain the dynamic development of the model economy.

In addition, chapter 2 employs modern econometrics and computational methods to obtain model solutions. In particular, perturbation methods are first used to derive the decision rules and transition equations for forward-looking variables from the first order conditions of the dynamic optimization problem for a representative agent. Calibration is then used to pin down model parameters based on previous studies in the field, especially studies on developing countries in the Southeast Asia and Latin America which share a lot of common characteristics with Vietnam. Simulated impulse responses after a monetary shock show that the U-shaped responses which are pervasive features of estimated

VAR models are also displayed for most of the key variables in the model. Monetary policy effectiveness seems to be only temporary due to a high level of dollarization in the economy. This finding is consistent with previous studies.

In chapter 3, the econometric model is further refined and then brought to a macroeconomic data set of Vietnam which includes 13 observables. Departing from previous studies which are mostly based on standard econometric and informal moment matching methods, the research takes advantage of formal systems estimation, particularly Bayesian estimation techniques which are now one of the most widely used system estimation methods for DSGE models. Given the derivation of model variables as deviations about steady states and the explicitly-specified trend, measurement equations play an important role in matching model variables to the data. A possible linear relationship among observables is eliminated in order to avoid singularity problems that can arise with Bayesian estimation. According to the estimated model, there is a considerable number of shocks that play significant roles in the fluctuations of key variables under consideration. It is also found to be a fitted model to explain the variations in the data, especially the dynamics of key variables including inflation, output, and consumption following a monetary policy disturbance.

The study also benefits from the use of a linear-quadratic framework to design an optimal monetary policy mechanism in chapter 4 within the estimated DSGE model. Linear-quadratic approximation methods are also utilized to approximate the asymptotic losses for an optimal Ramsey commitment rule, an optimal discretionary policy, and an *ex ante* optimized Taylor-type rule. Results show that a commitment rule dominates the other policy options and would be a useful benchmark or normative policy for monetary policy design. While the external validity of the findings from this research remains to be tested, its internal validity is supported by the multivariate convergence estimation results and the robustness of the optimal monetary policy setup.

5.2 Contributions of the Thesis

The dissertation makes several contributions to the study of macroeconomic policies in developing countries. First, the study establishes a comprehensive picture of dollarization in Vietnam which is the country in focus of the research. It is shown that the rationale for dollarization in developing countries, in the first place, lacks a standard theoretical justification. Additionally, it emphasizes the need for empirical evidence on the behaviour of the monetary authority under dollarization, in order to ascertain the foundation of macroeconomic policies in officially dollarized economies. The model economy of the study is also the first estimated DSGE model for Vietnam, as far as my supervisors and I are aware.

Second, this is one of the early attempts to incorporate key features of a developing economy[1] in a DSGE model which includes a dollarized supply side and a chronic budget-deficit fiscal policy on the part of the government. As an "original sin", the need for official borrowing in foreign currency reflects domestic fiscal strains, which are acute in many emerging economies, and particularly so when they import oil at times when oil prices are high. Fiscal strains and reputational concerns create the need for fiscal rules, of which Vietnam provides an example. Fiscal strains also create suspicion that the

[1] They are widely recognized as dollarization, budget-deficit fiscal policy, oil-dependent fiscal policy etc.

authorities might turn, as many have, to the printing press in emergencies. This strengthens firms' and households' demand for assets and transactions in foreign currency, as a hedge, when permitted to do so. Hence "dollarization". However, elaborating dollarization aspects of an emerging economy is challenging and the approach of the present study focuses on, among other dimensions, partial dollarization in the form of asset substitution of domestic firms which eventually leads to different Phillips curves for the group of firms setting prices in foreign currency and the others, which set prices in domestic currency. Various policy implications from three scenarios of dollarization (low, baseline, and high) are addressed throughout the analysis.

The study reveals how to incorporate features from non-standard market economies into standard DSGE models which are first designed to describe model economies with rational expectations and rationally-behaved agents. When it is taken to Vietnamese macro data, the estimated model shows that different levels of dollarization in the economy lead to different policy options for the authority. However, the high-dollarization scenario variant of the model is the best fit with data and the behaviour of key endogenous variables is also highly consistent with results from other seminal DSGE models. This is one of the key technical contributions from the research.

Third, simulation results under the estimated model show that dollarization hinders the effectiveness of monetary policy, which accords closely with predominant findings from previous studies. Partial dollarization makes the demand for domestic money more volatile and more sensitive to a monetary expansion or an exchange rate fluctuation. The strong positive correlation between exchange rate volatility and dollarization is also a serious challenge to monetary authorities in dollarized economies. Furthermore, it is the high elasticity of inflation to a monetary shock that limits the scope for monetary policy effectiveness because of a lack of self-stabilizing effects in dollarized economies.

The fourth major contribution of the study is the empirical assessment of optimal policies under the framework of a DSGE model for developing countries. An optimal monetary policy is further unpackaged into three standard options. These are (i) an optimal Ramsey commitment rule, (ii) an optimal discretionary policy, and (iii) an *ex ante* optimized Taylor-type rule. Departing from previous studies, the state-space approach is utilized to facilitate the debate on optimal policy options. A standard *ad hoc* quadratic loss function where the central bank is assumed to be concerned with both long-run distortion and short-run non-optimal fluctuations in inflation is also used in the evaluation.

Asymptotic losses are computed and compared in three scenarios of policy options for Vietnam. Impulse responses of key variables under a supply shock and demand shock are also analysed carefully in order not to conflate all kinds of policy options. Evidence from the study shows that dollarization in the economy leads to important differences in the responses to both supply-side and especially demand-side shocks. Key characteristics in the trade-off between the variability in inflation and output are largely consistent with those in standard New Keynesian models for non-dollarized economies. The estimation results are in favour of the optimal Ramsey commitment policy, which could be used as normative guidelines for the central bank's policy decisions.

Finally, it is important to acknowledge a distinctive behaviour of macroeconomic variables in the present DSGE model for a dollarized economy. While proponents of real business cycle models argue that technology shocks play a key role in driving the economic fluctuations in industrialized economies and there should be a positive correlation between technology shocks and hours/employment, New Keynesian economists pioneered by Galí (1999) show that with imperfect competition and sticky prices in the model, the relationship between a technology shock and hours/employment is negative. However, recent empirical studies, such as Cantore et al. (2014) and Cantore et al. (2017), show that there is a significant sign variation of the response of hours worked to a positive technology shock, which is crucially driven by a change in the magnitude of the elasticity of capital-labour substitution.

The pattern of the impulse response of employment in the present study is largely in line with Galí (1999). Results show a significant finding from the analytical simulation that there is a persistent decline in employment following a positive technology shock. As it has been widely established that labour inputs play a key role in the variation of output and most of the rapid growth over the past few decades in many emerging economies has been led by highly labour-intensive sectors, a negative relationship between a supply shock and hours/employment in the present model is also considered to be responsible for the short-run decrease of output.

5.3 A Potential Research Agenda

Having shown that the estimated model, to a large extent, fits Vietnam's macroeconomic data and following the broad lines of objectives set forth in the present study, a number of key macroeconomic issues can be suggested for further research. Apart from a quantitative assessment of different choices of optimal monetary policies which has already been analysed in the study, the first potential research question would be an investigation in the main driving forces of output in a dollarized economy. The impact of various levels of dollarization on output should be analysed, distinguishing developing countries from their developed counterparts.

The second type of further investigation would involve the determinants of inflation and the output-inflation cross-correlation. This issue has been addressed in a number of previous studies. Nguyen et al. (2012) show that money supply and other supply-side factors such as crude oil and rice prices present dominant elements in the inflation process. Elgammal & Eissa (2016) find a significant relationship linking inflation and three macro variables including past inflation, real income, and exchange rate. Moreover, past inflation plays the most important role in explaining inflation in Vietnam. It is also found by Nguyen & Nguyen (2010) that public expectations and memory of inflation are crucial factors in shaping the current rate of inflation. While most of these studies use standard econometric models, the estimated model of the present study can be an alternative and feasible option to implement empirical validation tests, compared with results from identified VAR or VECM models in previous studies.

Third, although the DSGE setup for a dollarized economy in this study has, in general, been shown to be an empirically plausible model for the purpose of policy analysis, a well-developed banking sector with relevant financial frictions, which has recently been seen to be an important feature of the state-of-the-art DSGE models, is still absent in this model. In particular, seminal improvements in this area include, first, a *financial* *accelerator* by Bernanke et al. (1999), which is later developed by Gertler & Kiyotaki (2010) in a number of respects, allowing endogenous developments in credit markets work to amplify and propagate shocks to the macroeconomy. The second is a *collateral constraint* in the spirit of Kiyotaki & Moore (1997) where the dynamic interaction between credit limits and asset prices plays a powerful role in the transmission mechanism by which the effects of shocks persist, amplify, and spill over to other sectors. While these financial frictions are embedded into a RBC framework, the DSGE model in the present study can be extended to incorporate a financial friction either between banking sector and firms/entrepreneurs or between households and the banking sector[2].

Fourth, the policy arrangements for monetary authority in this study are basically Taylor-type rules for the nominal interest rate. Policy recommendations are thus based on quantitative analysis on nominal interest rate as the main policy instrument. However, while whether low interest rates are deflationary or inflationary is still an on-going debate[3], a number of other instruments are considered not to be less feasible solutions and should be taken into account in further research. The Singaporean exchange-rate-based

[2] *Credit frictions*, as proposed by Cúrdia & Woodford (2016) and Del Negro et al. (2010) is also a possible option.

[3] The debate was initiated by Cochrane (2015) who supports the "neo-Fisherian" proposition that low nominal interest rates can cause inflation to be lower. Schmitt-Grohé & Uribe (2014), under such a view, also suggest that "when inflation falls below a threshold, the central bank should temporarily deviate from the traditional Taylor rule by following a deterministic path for the nominal interest rate that reaches the intended target for this policy instrument in finite time." However, García-Schmidt & Woodford (2019) in their latest publication in the American Economic Review (2019) argue that a commitment to maintain a low nominal interest rate for longer should always be expansionary and inflationary, rather than causing deflation.

monetary system with the exchange rate as the key instrument for monetary policy is a prominent example. And Malaysia is now another. Chow et al. (2014) use a DSGE-VAR model to argue that the exchange rate rule has a comparative advantage over the Taylor-type rule when export price shocks are the major sources of real volatility. A Taylor rule should be preferable only if domestic productivity shocks are dominant. The exchange rate rule is also shown to dominate Taylor rule for reducing inflation persistence. As Vietnam is an export-oriented economy, the present DSGE model can be extended in this dimension and then taken to independent empirical validation methods.

Fifth, Castillo (2006) finds that the combination of sticky prices and sector specific productivity shocks makes it optimal for some firms to use a foreign currency as unit of account and this equilibrium can be sustained by the central bank's actively manipulating the nominal exchange rate as a policy instrument. A certain level of dollarization in this case to some extent can be a sustainable macroeconomic option. Therefore, given the fact that dollarization in a broad sense is increasingly a defining characteristic of developing economies, the discussion can be moved forward beyond the importance of this trend quantitatively as well as theoretically to endogenize dollarization parameter and examine the optimal level of dollarization in developing countries.

Finally, given the estimated DSGE model, it should be useful to look at important questions for developing countries that remain unanswered such as "Is high dollarization potentially reversible and the corresponding welfare implications?" "Does dollarization, given the fact that domestic agents can switch currencies quickly, make a policy of lower inflation more credible?" "How does partial dollarization compare with 100% dollarization

(Panama or Ecuador)?" "Will dollarization be replaced or challenged by Renminbization?" In addition, it should also be worth improving the current model by incorporating more advanced features such as heterogeneous agents, bounded rationality, and the specification of the steady-state growth path for developing countries[4], given a possible future slowdown after a long period of rapid growth[5] or even a preclusion of the long-run steady-state growth path unless all technical progress is labour-augmenting[6], which is, as argued by Leon-Ledesma & Satchi (2018), difficult to justify. While the first generation of DSGE models was rooted in the rational expectations revolution and built on the representative agent paradigm, the development of information technology and improvements in numerical methods have made it possible to study rich heterogeneous agent models. These models, once matched to micro data, are able to deliver empirically realistic results which matter a great deal for the conduct of monetary policy, as argued by Kaplan et al. (2018) in their latest publication in the American Economic Review.

[4]The loglinearization of the variables about a zero inflation steady state is a standard practice in DSGE models. However, it could be problematic when the model is taken to developing countries' data with high inflation.

[5]As shown by Barro & Sala-í Martín (1992), there is a growth convergence in the sense that developing countries tend to grow faster than developed countries when they are below the steady-state position. Therefore, Vietnam's current rapid growth would not last for ever and even slow down when public investment share tends to be larger, as argued by Sala-í Martín et al. (2004).

[6]Technical progress can be incorporated into the Solow growth model by taking it as a labour-augmenting factor. As the state of technology improves, technical progress makes each worker more productive by augmenting their labour. In addition, as shown by Leon-Ledesma & Satchi (2018), the steady-state growth theorem states that balanced growth requires either all technical progress to be labour-augmenting or the elasticity of substitution between capital and labour to equal one in the long run.

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Appendices

Appendix A

Data for the Estimation

The data are quarterly and provided by the General Statistics Office of Vietnam (GSO). They cover the period from 1996:Q1 to 2012:Q2. The U.S. output, inflation, interest rate, and population index (LNSindex) are from the U.S. Department of Commerce, Bureau of Economic Analysis, the Board of Governors of the Federal Reserve System, and the U.S. Bureau of Labor Statistics. Following Smets & Wouters (2007), observed variables are scaled with a factor of 100. As the model is linearized before being taken to estimation, the time series are constructed as follows[1]

- **Real Output Growth**.

$$\text{Output} = [\log(GDP_t) - \log(GDP_{t-1})] * 100$$

- **Inflation**.

$$\text{Inflation} = [\log(CPI_t) - \log(CPI_{t-1})] * 100$$

[1]Three stochastic trends that affect the genuity of data on the development of an economy include: (i) price factor or inflation which will be eliminated by using real variables in the estimation; (ii) population growth; and (iii) technological progress. The latter two factors are dealt with by modelling techniques.

- **Interest Rate**.

$$\text{Interest Rate} = \text{LENDRATE}/4$$

- **Real Household Consumption Growth**.

$$\text{Household Consumption} = [\log(CONHH_t) - \log(CONHH_{t-1})] * 100$$

- **Real Government Consumption Growth**.

$$\text{Government Consumption} = [\log(CONGOV_t) - \log(CONGOV_{t-1})] * 100$$

- **Real Investment Growth**.

$$\text{Investment} = [\log(INV_t) - \log(INV_{t-1})] * 100$$

- **Real Exchange Rate Growth**.

$$\text{RER} = [\log(RER_t) - \log(RER_{t-1})] * 100$$

- **Employment Growth**.

$$\text{Employment} = [\log(EMP_t) - \log(EMP_{t-1})] * 100$$

- **Real Export Growth**.

$$\text{Exports} = [\log(EXP_t) - \log(EXP_{t-1})] * 100$$

- **Real Import Growth**.

$$\text{Imports} = [\log(IMP_t) - \log(IMP_{t-1})] * 100$$

- **U.S. Real Output Growth**.

$$\text{U.S. Output} = \left[\log\left(\frac{GDPC_t}{LNSindex_t}\right) - \log\left(\frac{GDPC_{t-1}}{LNSindex_{t-1}}\right)\right] * 100$$

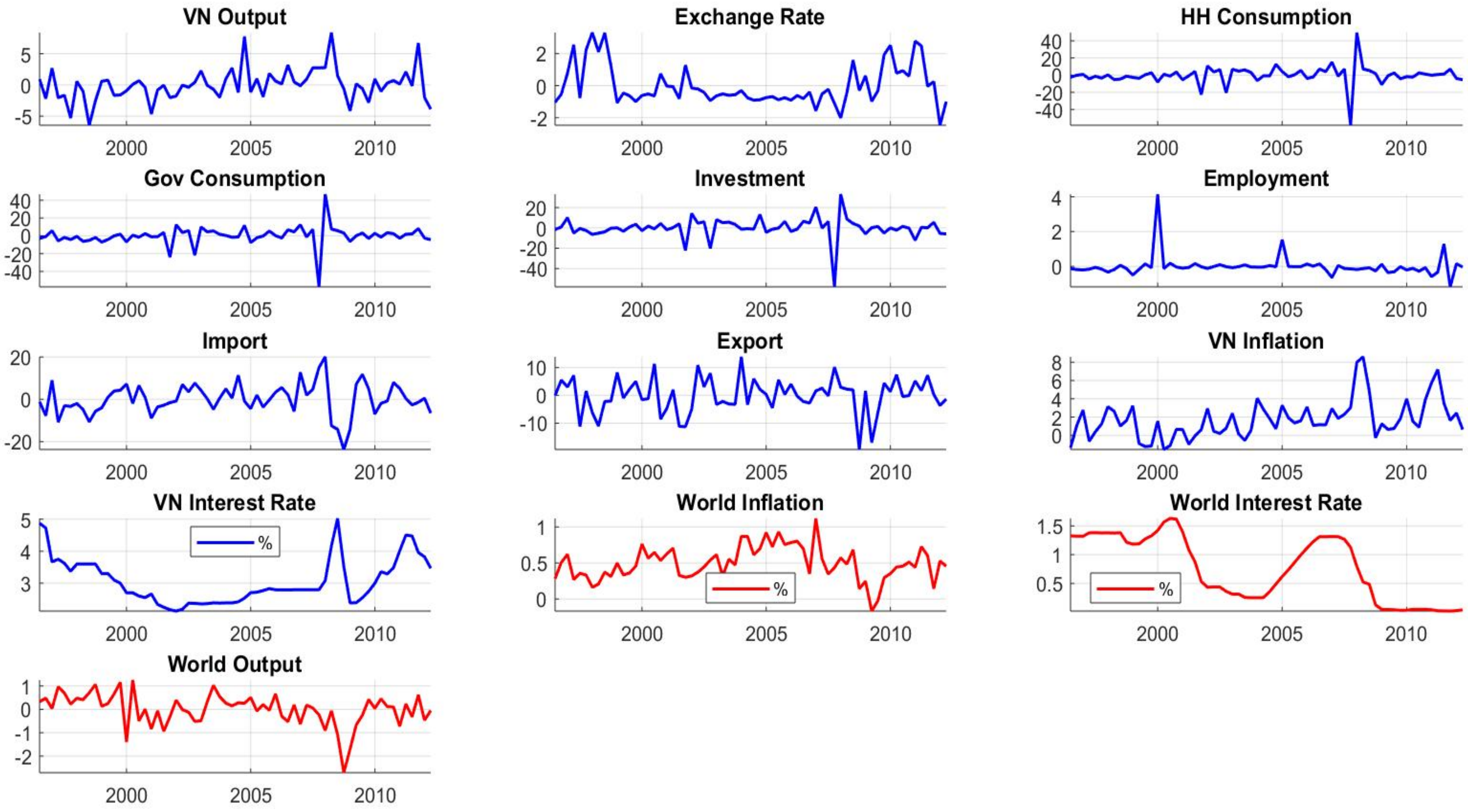
- **U.S. Inflation**. Gross Domestic Product - Implicit Price Deflator (GDPDEF).

$$\text{U.S. Inflation} = \log\left(\frac{GDPDEF_t}{GDPDEF_{t-1}}\right) * 100$$

- **U.S. Interest Rate**. Federal Funds Rate (FFR) is measured as the averages of daily figures, percent.

$$\text{U.S. Interest Rate} = \text{FFR}/4$$

Figure A.1: Data for the Estimation



Appendix B

Model Derivation

The decision problems of firms and households as well as equilibrium conditions are derived as follows[1]

Households

First, by maximizing consumption (C_t) given by equation 2.5 subject to the budget constraint

$$P_t C_t^d + P_t^{m,c} C_t^m = P_t^c C_t$$

we obtain the following consumption demand function

$$C_t^d = (1 - \omega_c) \left[\frac{P_t}{P_t^c} \right]^{-\eta_c} C_t$$

$$C_t^m = \omega_c \left[\frac{P_t^{m,c}}{P_t^c} \right]^{-\eta_c} C_t$$

where the consumer price index is given by

$$P_t^c = \left[(1 - \omega_c) P_t^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c} \right]^{1/(1-\eta_c)}$$

[1] This section is adapted from Adolfson et al. (2005).

As for investment, we know that the prices of the domestically produced investment goods and domestically produced consumption goods are the same (P_t). By maximizing investment (I_t) given by equation 2.8, subject to the budget constrain to investment

$$P_t I_t^d + P_t^{m,i} I_t^m = P_t^i C_t$$

we obtain the following investment demand function

$$\begin{aligned} I_t^d &= (1-\omega_i)\left[\frac{P_t}{P_t^i}\right]^{-\eta_i} I_t \\ I_t^m &= \omega_i \left[\frac{P_t^{m,i}}{P_t^i}\right]^{-\eta_i} I_t \end{aligned} \tag{B.1}$$

where the aggregate investment price index is given by

$$P_t^i = \left[(1-\omega_i)P_t^{1-\eta_i} + \omega_i (P_t^{m,i})^{1-\eta_i}\right]^{1/(1-\eta_i)}$$

Second, taking partial derivatives of equation 2.11 with respect to I_t and I_{t-1} we have

$$f_1^3(I_t, I_{t-1}) \equiv \frac{\partial f^3(I_t, I_{t-1})}{\partial I_t} = -f^{4\prime}(I_t/I_{t-1})I_t/I_{t-1} + [1 - f^4(I_t/I_{t-1})] \tag{B.2}$$

$$f_2^3(I_t, I_{t-1}) \equiv \frac{\partial f^3(I_t, I_{t-1})}{\partial I_{t-1}} = f^{4\prime}(I_t/I_{t-1})\left(\frac{I_t}{I_{t-1}}\right)^2 \tag{B.3}$$

Third, in order to maximize the intertemporal utility subject to the given constraints, the representative household j solves the following Lagrangian problem[2]

$$\max_{C_{j,t};M_{j,t+1};\vartheta_t;\bar{K}_{j,t+1};I_{j,t};u_{j,t};Q_{j,t};B^*_{j,t+1};h_{j,t}} E_0^j \sum_{t=0}^{\infty} \beta^t (L_t)$$

where

$$L_t = \left\{ \epsilon_t^c \ln(C_{j,t} - bC_{j,t-1}) - \epsilon_t^h \Lambda_h \frac{h_{j,t}^{1+\sigma_h}}{1+\sigma_h} + \Lambda_q \frac{(Q_{j,t}/\epsilon_t^z P_t)^{1-\sigma_q} - 1}{1-\sigma_q} \right\}$$

$$+n_t \left\{ \begin{array}{r} R_{t-1}(M_{j,t} - Q_{j,t}) + Q_{j,t} + (1-\tau_t^k)\Pi_t + (1-\tau^y)\frac{W_{j,t}}{1+\tau_t^w} h_{j,t} \\ +(1-\tau_t^k)R_t^k u_{j,t}\bar{K}_{j,t} + R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) S_t B_{j,t}^* \\ -\tau_t^k \left[(R_{t-1}-1)(M_{j,t} - Q_{j,t}) + (R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) - 1) S_t B_{j,t}^* + B_{j,t}^*(S_t - S_{t-1})\right] \\ -\left[M_{j,t+1} + S_t B_{j,t+1}^* + P_t^c C_{j,t}(1+\tau_t^c) + P_t^i I_{j,t} + P_t(f(u_{j,t})\bar{K}_{j,t} + P_t^k \vartheta_t)\right] \end{array} \right\}$$

[2]The money demand shock is not considered in this case.

$$+\omega_t \left\{(1-\delta)\bar{K}_t + \upsilon_t[1 - f^4(I_t/I_{t-1})]I_t + \vartheta_t - \bar{K}_{j,t+1}\right\}$$

For the sake of notation simplicity, we define $\psi_{z,t} \equiv \epsilon_t^z \psi_t = \epsilon_t^z n_t P_t$ (the Lagrangian multiplier being scaled with ϵ_t^z) which is used in all of the first order conditions. The first-order conditions for the household's problem after dividing all quantities with the trend level of technology (ϵ_t^z) in order to render stationarity of all variables are given by

Consumption c_t:

$$\frac{\epsilon_t^c}{c_t b c_{t-1}\frac{1}{\mu_{z,t}}} - \beta b E_t \frac{\epsilon_{t+1}^c}{c_{t+1}\mu_{z,t+1} - bc_t} - \psi_{z,t}\frac{P_t^c}{P_t}(1+\tau_t^c) = 0 \tag{B.4}$$

Total assets m_{t+1}:

$$-\psi_{z,t} + \beta E_t \left[\frac{\psi_{z,t+1}}{\mu_{z,t+1}}\frac{R_t}{\pi_{t+1}} - \frac{1}{\mu_{z,t+1}}\frac{\psi_{z,t+1}}{\pi_{t+1}}\tau_{t+1}^k(R_t - 1)\right] = 0 \tag{B.5}$$

Access to market ϑ_t:

$$-\psi_t P_t^k + \omega_t = 0 \tag{B.6}$$

Capital stock $\bar{k}_{t+1}$:

$$P_t^k \psi_{z,t} + \beta E_t \left\{\frac{\psi_{z,t+1}}{\mu_{z,t+1}}\left[(1-\delta)P_{t+1}^k + (1-\tau_{t+1}^k)r_{t+1}^k u_{t+1} - f(u_{t+1})\right]\right\} = 0 \tag{B.7}$$

Investment i_t:

$$-\psi_{z,t}\frac{P_t^i}{P_t} + P_t^k \psi_{z,t}\epsilon_t^i f_1^3(i_t, i_{t-1}, \mu_{z,t}) + \beta E_t\left[P_{t+1}^k \frac{\psi_{z,t+1}}{\mu_{z,t+1}}\epsilon_{t+1}^i f_2^3(i_{t+1}, i_t, \mu_{z,t+1})\right] = 0 \tag{B.8}$$

Utilization rate u_t:

$$\psi_{z,t}\left[(1-\tau^k)r_t^k - f'(u_t)\right] = 0 \tag{B.9}$$

Cash holdings q_t:

$$\epsilon_t^q \Lambda_q q_t^{-\sigma_q} - (1-\tau_t^k)\psi_{z,t}(R_{t-1} - 1) = 0 \tag{B.10}$$

Foreign bond b^*_{t+1}:

$$-\psi_{z,t}S_t + \beta E_t\{\frac{\psi_{z,t+1}}{\mu_{z,t+1}\pi_{t+1}}[S_{t+1}R^*_t f^5(a_t,\varphi_t) \tag{B.11}$$

$$-\tau^k_{t+1}S_{t+1}(R^*_t f^5(a_t,\varphi_t)-1) - \tau^k_{t+1}(S_{t+1}-S_t)]\} = 0$$

where real variables already rendered stationary are denoted by lower-case letters ($x_t = X_t/\epsilon^z_t$).

Fourth, the first-order condition for the optimization problem is given by

$$E_t\sum_{s=0}^{\infty}(\beta\xi^w)^s h_{j,t+s}\left\{\begin{array}{l} -\epsilon^h_{t+s}\Lambda_h(h_{j,t+s})^{\sigma_h}+ \\ \tilde{W}_t\frac{P_{t+s}}{P_t}\frac{\epsilon^z_{t+s}}{\epsilon^z_t}\frac{n_{t+s}}{\lambda^w}\frac{1-\tau^y_{t+s}}{1+\tau^w_{t+s}}\left[\frac{(\frac{P^c_{t+s-1}}{P^c_{t-1}})^{\kappa^w}(\bar{\pi}_{t+1}\ldots\bar{\pi}_{t+s})^{1-\kappa^w}}{\frac{P_{t+s}}{P_t}}\right]=0 \end{array}\right\} \tag{B.12}$$

Firms

Domestic Sector

Taking first-order conditions of equation 2.26 gives us the following relationships

$$\text{With respect to } H_{i,t}: W_tR^w_t = (1-\alpha)\lambda_tP_{i,t}(\epsilon^z_t)^{1-\alpha}\epsilon_tK^{\alpha}_{i,t}H^{-\alpha}_{i,t} \tag{B.13}$$

$$\text{With respect to } K_{i,t}: R^k_t = \alpha\lambda_tP_{i,t}(\epsilon^z_t)^{1-\alpha}\epsilon_tK^{\alpha-1}_{i,t}H^{1-\alpha}_{i,t} \tag{B.14}$$

Non-stationary variables[3] are stationarized, as in Altig et al. (2003) and Adolfson et al. (2005), as follows

$$r^k_t \equiv \frac{R^k_t}{P_t}; w_t \equiv \frac{W_t}{P_t\epsilon^z_t}; k_{t+1} \equiv \frac{K_{t+1}}{\epsilon^z_t}; \bar{k}_{t+1} \equiv \frac{\bar{K}_{t+1}}{\epsilon^z_t} \tag{B.15}$$

[3]Because of a nominal and real stochastic trend due to the permanent technology shock and the unit-root in the price level.

where $\bar{K}_{t+1}$ is the physical capital stock at date $t+1$ and K_{t+1} is the capital services generated by the former[4].

From equations B.13, B.14, and B.15 we obtain the following solution for the scaled equilibrium rental rate of capital

$$r_t^k = \frac{\alpha}{1-\alpha} w_t \mu_{z,t} R_t^w k_t^{-1} H_t \tag{B.16}$$

From equations B.13 and B.14, the equilibrium real marginal cost is given by

$$mc_t \equiv \lambda_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right) (r_t^k)^{\alpha} (w_t R_t^w)^{1-\alpha} \frac{1}{\epsilon_t} \tag{B.17}$$

Using 2.23, the first-order condition of the intermediate firms' optimization problem is given by

$$E_t \sum_{s=0}^{\infty} (\beta \xi^d)^s n_{t+s} \left[\frac{(P_{t+s-1}/P_{t-1})^{\kappa^d} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^d}}{(P_{t+s}/P_t)} \right]^{-\frac{\lambda_{t+s}^d}{\lambda_{t+s}^d - 1}} Y_{t+s} P_{t+s}$$
$$\left[\frac{(P_{t+s-1}/P_{t-1})^{\kappa^d} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+s})^{1-\kappa^d}}{(P_{t+s}/P_t)} \frac{\tilde{P}_t}{P_t} - \frac{\lambda_t^d MC_{i,t+s}}{P_{t+s}} \right] = 0 \tag{B.18}$$

The aggregate domestic price for the final good in equation 2.22 can be rewritten in a CES form of the reoptimized price $\tilde{P}_t$ (for all firms that are allowed to reoptimize) and indexed price $\tilde{\tilde{P}}_t = P_{t-1} \pi_{t-1}^{\kappa^d} (\bar{\pi}_t^c)^{1-\kappa^d}$ (for the rest which are not allowed to reoptimize at that date) as follows

$$P_t = \left\{ \left[\int_0^{\xi^d} (P_{t-1} \pi_{t-1}^{\kappa^d} (\bar{\pi}_t)^{1-\kappa^d})^{\frac{1}{1-\lambda_t^d}} + \int_{\xi^d}^1 (\tilde{P}_t^{)^{\frac{1}{1-\lambda_t^d}}} \right] di \right\}^{1-\lambda_t^d}$$
$$= \left[\xi^d (P_{t-1} \pi_{t-1}^{\kappa^d} (\bar{\pi}_t)^{1-\kappa^d})^{\frac{1}{1-\lambda_t^d}} + (1-\xi^d)(\tilde{P}_t)^{\frac{1}{1-\lambda_t^d}} \right]^{1-\lambda_t^d} \tag{B.19}$$

[4]Following Adolfson et al. (2005), both variables are rendered stationary by ϵ_t^z, though only the physical capital stock $\bar{K}_{t+1}$ is a stock variable which is determined in period t, while the capital services K_{t+1} is determined in period $t+1$.

Log-linearizing B.18 using B.19 yields the following log-linearized domestic Phillips curve:

$$(\hat{\pi}_t - \hat{\bar{\pi}}_t) = \frac{\beta}{1+\kappa^d\beta}(E_t\hat{\pi}_{t+1} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^d}{1+\kappa^d\beta}(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t) \\ -\frac{\kappa^d\beta(1-\rho_\pi)}{1+\kappa^d\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^d)(1-\beta\xi^d)}{\xi^d(1+\kappa^d\beta)}(\hat{mc}_t + \hat{\lambda}_t^d) \tag{B.20}$$

External Sector

From equations 2.31 and 2.32, each intermediate importing firm i faces demands for imported consumption/investment goods as follows

$$C_{i,t}^m = \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{\frac{\lambda_t^{m,c}}{\lambda_t^{m,c}-1}} C_t^m$$

$$I_{i,t}^m = \left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}}\right)^{\frac{\lambda_t^{m,i}}{\lambda_t^{m,i}-1}} I_t^m$$

Using these identities in the first order condition of the importing firms' profit optimization problems set up in 2.37 and 2.38 we have

$$E_t\sum_{s=0}^{\infty}(\beta\xi^{m,c})^s v_{t+s}\left[\frac{(P_{t+s-1}^{m,c}/P_{t-1}^{m,c})^{\kappa^{m,c}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,c}}}{(P_{t+s}^{m,c}/P_t^{m,c})}\right]^{-\frac{\lambda_{t+s}^{m,c}}{\lambda_{t+s}^{m,c}-1}} C_{t+s}^m P_{t+s}^{m,c} \\ \left[\frac{(P_{t+s-1}^{m,c}/P_{t-1}^{m,c})^{\kappa^{m,c}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,c}}}{(P_{t+s}^{m,c}/P_t^{m,c})}\frac{\tilde{P}_t^{m,c}}{P_t^{m,c}} - \frac{\lambda_{t+s}^{m,c}S_{t+s}P_{t+s}^*}{P_t^{m,c}}\right] = 0 \tag{B.21}$$

$$E_t\sum_{s=0}^{\infty}(\beta\xi^{m,i})^s v_{t+s}\left[\frac{(P_{t+s-1}^{m,i}/P_{t-1}^{m,i})^{\kappa^{m,i}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,i}}}{(P_{t+s}^{m,i}/P_t^{m,i})}\right]^{-\frac{\lambda_{t+s}^{m,i}}{\lambda_{t+s}^{m,i}-1}} I_{t+s}^m P_{t+s}^{m,i} \\ \left[\frac{(P_{t+s-1}^{m,i}/P_{t-1}^{m,i})^{\kappa^{m,i}}(\bar{\pi}_{t+1}...\bar{\pi}_{t+s})^{1-\kappa^{m,i}}}{(P_{t+s}^{m,i}/P_t^{m,i})}\frac{\tilde{P}_t^{m,i}}{P_t^{m,i}} - \frac{\lambda_{t+s}^{m,i}S_{t+s}P_{t+s}^*}{P_t^{m,i}}\right] = 0 \tag{B.22}$$

The aggregate price indices for imported consumption and imported investment are given as CES forms as follows

$$P_t^{m,c} = \left[\int_0^1 (P_{i,t}^{m,c})^{\frac{1}{1-\lambda_t^{m,c}}} di\right]^{1-\lambda_t^{m,c}}$$

$$= \left\{ \xi^{m,c} [P_{t-1}^{m,c} (\pi_{t-1}^{m,c})^{\kappa^{m,c}} (\bar{\pi}_t)^{1-\kappa^{m,c}}]^{\frac{1}{1-\lambda_t^{m,c}}} + (1-\xi^{m,c})(\tilde{P}_t^{m,c})^{\frac{1}{1-\lambda_t^{m,c}}} \right\}^{1-\lambda_t^{m,c}} \tag{B.23}$$

$$P_t^{m,i} = \left[\int_0^1 (P_{i,t}^{m,i})^{\frac{1}{1-\lambda_t^{m,i}}} di \right]^{1-\lambda_t^{m,i}}$$

$$= \left\{ \xi^{m,i} [P_{t-1}^{m,i} (\pi_{t-1}^{m,i})^{\kappa^{m,i}} (\bar{\pi}_t)^{1-\kappa^{m,i}}]^{\frac{1}{1-\lambda_t^{m,i}}} + (1-\xi^{m,i})(\tilde{P}_t^{m,i})^{\frac{1}{1-\lambda_t^{m,i}}} \right\}^{1-\lambda_t^{m,i}} \tag{B.24}$$

Log-linearizing equation B.21 using B.23 yields the log-linearized Phillips curve for imported-consumption firms

$$(\hat{\pi}_t^{m,c} - \hat{\bar{\pi}}_t) = \frac{\beta}{1+\kappa^{m,c}\beta}(E_t\hat{\pi}_{t+1}^{m,c} - \rho_\pi \hat{\bar{\pi}}_t) + \frac{\kappa^{m,c}}{1+\kappa^{m,c}\beta}(\hat{\pi}_{t-1}^{m,c} - \hat{\bar{\pi}}_t)$$
$$-\frac{\kappa^{m,c}\beta(1-\rho_\pi)}{1+\kappa^{m,c}\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^{m,c})(1-\beta\xi^{m,c})}{\xi^{m,c}(1+\kappa^{m,c}\beta)}(\hat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c})$$

where

$$\hat{mc}_t^{m,c} = \hat{p}_t^* + \hat{s}_t - \hat{p}_t^{m,c}$$

Log-linearizing equation B.22 using B.24 yields the log-linearized Phillips curve for imported-investment firms

$$(\hat{\pi}_t^{m,i} - \hat{\bar{\pi}}_t) = \frac{\beta}{1+\kappa^{m,i}\beta}(E_t\hat{\pi}_{t+1}^{m,i} - \rho_\pi \hat{\bar{\pi}}_t) + \frac{\kappa^{m,i}}{1+\kappa^{m,i}\beta}(\hat{\pi}_{t-1}^{m,i} - \hat{\bar{\pi}}_t)$$
$$-\frac{\kappa^{m,i}\beta(1-\rho_\pi)}{1+\kappa^{m,i}\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^{m,i})(1-\beta\xi^{m,i})}{\xi^{m,i}(1+\kappa^{m,i}\beta)}(\hat{mc}_t^{m,i} + \hat{\lambda}_t^{m,i})$$

where

$$\hat{mc}_t^{m,i} = \hat{p}_t^* + \hat{s}_t - \hat{p}_t^{m,i}$$

The demand for exporting firm i is given by

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x} \right)^{\frac{\lambda_t^x}{\lambda_t^x - 1}} X_t \tag{B.25}$$

where X_t is the demand for final export good and $X_{i,t}$ is demand of individual intermediate exporting firms. λ_t^x denotes a stochastic/time-varying markup on the exported goods.

The log-linearized first-order condition of the optimization problem implies the aggregate export Phillips curve of the same form

$$
\begin{aligned}
(\hat{\pi}_t^x - \hat{\bar{\pi}}_t) = &\frac{\beta}{1+\kappa^x\beta}(E_t\hat{\pi}_{t+1}^x - \rho_\pi \hat{\bar{\pi}}_t) + \frac{\kappa^x}{1+\kappa^x\beta}(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t) \\
&-\frac{\kappa^x\beta(1-\rho_\pi)}{1+\kappa^x\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^x)(1-\beta\xi^x)}{\xi^x(1+\kappa^x\beta)}(\hat{mc}_t^x + \hat{\lambda}_t^x) \qquad \text{(B.26)}
\end{aligned}
$$

where

$$
\hat{mc}_t^x = \hat{p}_t + \hat{s}_t - \hat{p}_t^x
$$

In general, there are thus four specific Phillips curve relations determining inflation in the domestic, imported-consumption, imported-investment, and export sectors. In case of price dollarization, the domestic Phillips curve will be a weighted combination of the Phillips curves for firms setting their prices in domestic currency and those setting their prices in USD.

Identities of the Open Economy

There are basically two different types of *relative prices* for an open economy. In the internal sector, the domestic households care about relative prices between imported consumption/investment goods and the domestic final good:

$$
\chi_t^{mc,d} \equiv \frac{P_t^{m,c}}{P_t} = \frac{P_t^{m,c}}{P_t}\frac{P_{t-1}^{m,c}}{P_{t-1}}\frac{P_{t-1}}{P_{t-1}^{m,c}} = \frac{\chi_{t-1}^{mc,d}\pi_t^{m,c}}{\pi_t}
$$

$$
\chi_t^{mi,d} \equiv \frac{P_t^{m,i}}{P_t} = \frac{\chi_{t-1}^{mi,d}\pi_t^{m,i}}{\pi_t}
$$

And the relative prices between the aggregate consumption/investment goods and the domestic final good:

$$
\chi_t^{c,d} \equiv \frac{P_t^{c}}{P_t} = \frac{\chi_{t-1}^{c,d}\pi_t^{c}}{\pi_t}
$$

$$\chi_t^{i,d} \equiv \frac{P_t^i}{P_t} = \frac{\chi_{t-1}^{i,d}\pi_t^i}{\pi_t}$$

where P_t denotes price for the domestic final good. As the final good can be consumed or invested by the households, P_t is taken as both the domestic consumption price and the domestic investment price. P_t^c and P_t^i are aggregate consumption and aggregate investment prices, respectively, which are indices of domestic/imported consumption/investment prices.

In the external sector, foreign households and exporting firms take into account relative prices between the final exported good and the foreign good:

$$\chi_t^{x,*} \equiv \frac{P_t^x}{P_t^*} = \frac{\chi_{t-1}^{x,*}\pi_t^x}{\pi_t^*}$$

The deviations from the law of one price for the exported good are given by

$$mc_t^x \equiv \frac{P_t}{S_t P_t^x}$$

or we have

$$\frac{mc_t^x}{mc_{t-1}^x} = \frac{P_t/P_{t-1}}{S_t/S_{t-1}P_t^x/P_{t-1}^x} = \frac{\pi_t}{S_t/S_{t-1}\pi_t^*} \tag{B.27}$$

And importing firms look at the following relative price

$$\chi_t^f \equiv \frac{P_t}{S_t P_t^*} = mc_t^x \chi_t^{x,*} \tag{B.28}$$

So, the deviations from the law of one price for imported consumption/investment goods are given by

$$mc_t^{m,c} \equiv \frac{S_t P_t^*}{P_t^{m,c}} = \frac{1}{\chi_t^f \chi_t^{mc,d}} = \frac{1}{mc_t^x \chi_t^{x,*} \chi_t^{mc,d}} \tag{B.29}$$

$$mc_t^{m,i} \equiv \frac{S_t P_t^*}{P_\chi} = \frac{1}{\chi_t^f \chi_t^{mi,d}} = \frac{1}{mc_t^x \chi_t^{x,*} \chi_t^{mi,d}} \tag{B.30}$$

From equation 2.46, replace the formula for P_t^c given by

$$P_t^c = \left[(1-\omega_c)P_t^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c}\right]^{1/(1-\eta_c)} \tag{B.31}$$

into the definition of real exchange rate to get

$$RER_t = \left[(1-\omega_c)\left(\frac{P_t}{S_t P_t^*}\right)^{1-\eta_c} + \omega_c\left(\frac{P_t^{m,c}}{S_t P_t^*}\right)^{1-\eta_c}\right]^{-1/(1-\eta_c)}$$

Given the relative prices, we have

$$RER_t = \left[(1-\omega_c)\left(\chi_t^f\right)^{1-\eta_c} + \omega_c\left(\frac{1}{mc_t^{m,c}}\right)^{1-\eta_c}\right]^{-1/(1-\eta_c)}$$

or

$$RER_t = \left[(1-\omega_c)\left(mc_t^x \chi_t^{x,*}\right)^{1-\eta_c} + \omega_c\left(mc_t^x \chi_t^{x,*}\chi_t^{mc,d}\right)^{1-\eta_c}\right]^{-1/(1-\eta_c)} \quad \text{(B.32)}$$

Fiscal Policy

The nominal deficit is defined as the difference between non-interest government revenues and expenses which is given by the nominal tax revenues and nominal government expenditure

$$\Omega_t^n = TAX_t^n - P_t G_t^n$$

where TAX_t^n is given by

$$TAX_t^n = \tau_t^c P_t^c C_t + \frac{\tau_t^y + \tau_t^w}{1+\tau_t^w} W_t H_t$$

$$+\tau_t^k \left\{(R_{t-1}-1)(M_t - Q_t) + R_t^k u_t \bar{K}_t + [R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) - 1]S_t B_t^* + \pi_t\right\}$$

However, for the sake of simplicity, following De Castro et al. (2011), we assume that total taxes are a time-varying proportion of nominal GDP of the form

$$TAX_t^n = T_t(P_t Y_t)$$

Therefore, the primary deficit-to-GDP ratio can be written as

$$\Omega_t = \frac{\Omega_t^n}{P_t Y_t} = \frac{T_t(P_t Y_t)}{P_t Y_t} - \frac{P_t G_t^n}{P_t Y_t}$$

Then,

$$P_t G_t^n = P_t Y_t (T_t - \Omega_t)$$

If we assume that government deficit is financed only by domestic debt, the budget constraint for the government is then given by

$$P_t G_t^n + B_t = \frac{B_{t+1}}{R_t} + T_t(P_t Y_t)$$

where B_t is the level of debt.

Dividing the above equation by current nominal GDP using the above identity for G_t^n, the law of motion for the government debt as proportion of GDP is given by

$$B_{t+1}^r = R_t \left(\frac{B_t^r}{\pi_t} \frac{Y_{t-1}}{Y_t} - \Omega_t \right) \tag{B.33}$$

General Equilibrium

From equations 2.6, 2.9, 2.41, and 2.42 we substitute all disaggregated variables of the goods market equilibrium condition given by equation 2.51 by aggregate variables to get

$$(1-\omega_c)\left(\frac{P_t^c}{P_t}\right)^{\eta_c} C_t + (1-\omega_i)\left(\frac{P_t^i}{P_t}\right)^{\eta_i} I_t + G_t^n + \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} (C_t^* + I_t^*)$$
$$= \epsilon_t (\epsilon_t^z)^{1-\alpha} K_t^\alpha H_t^{1-\alpha} - \epsilon_t^z \phi - f(u_t)\bar{K}_t \tag{B.34}$$

Assuming that world output is just composed of consumption and investment, $Y_t^* = C_t^* + I_t^*$, the stationarized version of the goods market clearing condition after scaling Y_t^* with the world's stationary technology shock ϵ_t^{z*} as well as all other real variables with ϵ_t^z is as follows

$$(1-\omega_c)\left(\frac{P_t^c}{P_t}\right)^{\eta_c} c_t + (1-\omega_i)\left(\frac{P_t^i}{P_t}\right)^{\eta_i} i_t + g_t + \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} y_t^* \frac{\epsilon_t^{z*}}{\epsilon_t^z}$$

$$= \epsilon_t \left(\frac{1}{\mu_{z,t}}\right)^{\alpha} k_t^{\alpha} H_t^{1-\alpha} - \phi - f(u_t)\bar{k}_t \frac{1}{\mu_{z,t}} \tag{B.35}$$

In the foreign bonds market, we divide both sides of the equilibrium condition given by equation 2.52 by $P_t\epsilon_t^z$, using $\frac{C_t^x}{\epsilon_t^z} + \frac{I_t^x}{\epsilon_t^z} = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} \frac{Y_t^*}{\epsilon_t^{z*}} \frac{\epsilon_t^{z*}}{\epsilon_t^z}$ to get

$$a_t = \frac{1}{mc_t^x}(\chi_t^{x,*})^{\eta_f} y_t^* \tilde{\epsilon}_t^{z*} - (\chi_t^f)^{-1}(c_t^m + i_t^m) + R_{t-1}^* f^5(a_{t-1}, \varphi_{t-1}) \frac{a_{t-1}}{\phi_t \mu_{z,t}} \frac{S_t}{S_{t-1}} \tag{B.36}$$

where $\tilde{\epsilon}_t^{z*} = \epsilon_t^{z*}/\epsilon_t^z$[5].

Monetary growth in the credit market is given by

$$\mu_t = \frac{M_{t+1}}{M_t} = \frac{m_{t+1}\epsilon_t^z P_t}{m_t \epsilon_{t-1}^z P_{t-1}} = \frac{m_{t+1}\mu_{z,t}\pi_t}{m_t} \tag{B.37}$$

where $m_t = M_t/P_{t-1}\epsilon_{t-1}^z$.

Partial Dollarization

The relative price of goods in domestic currency and consumer price index is given by

$$t_t^D = \frac{P_t^D}{P_t^c} = \frac{P_t^D}{P_t^c} \frac{P_{t-1}^D}{P_{t-1}^c} \frac{P_{t-1}^c}{P_{t-1}^D}$$

$$= \frac{t_{t-1}^D \pi_t^D}{\pi_t^c} \tag{B.38}$$

The relative price between domestic currency and USD is given by

$$\frac{e_t}{e_{t-1}} = \frac{P_t^D / P_{t-1}^D}{S_t/S_{t-1} P_t^{USD} / P_{t-1}^{USD}}$$

$$= \frac{\pi_t^D}{S_t/S_{t-1}\pi_t^{USD}} \tag{B.39}$$

[5]The fraction $\tilde{\epsilon}_t^{z*} = \epsilon_t^{z*}/\epsilon_t^z$ can be taken as a shock measuring the asymmetric level of technological progress between the domestic economy and the rest of the world.

Appendix C

The Log-linearized Model

By logging both sides of the equation in the non-linear setup and using Taylor expansions about the steady state, we can derive the corresponding linear one which is also the deviation of the variable around its steady state. The hat variables in the log-linearized equations are the deviations from their steady states[1]. Log-linearized equations include[2]:

Households

Log-linearized law of motion for capital given by 2.12[3]

$$\hat{\bar{k}}_{t+1} = (1-\delta)\frac{1}{\mu_z}\hat{\bar{k}}_t - (1-\delta)\frac{1}{\mu_z}\hat{\mu}_{z,t} + [1-(1-\delta)\frac{1}{\mu_z}\hat{\epsilon}^i_t + [1-(1-\delta)\frac{1}{\mu_z}]\hat{i}_t \quad \text{(C.1)}$$

Log-linearized real wage equation from the optimization problem with respect to real wage given by equation B.12

[1]This section is adapted from Adolfson et al. (2005).

[2]Log of the deviation from steady state of a constant is 0.

$$\hat{x}_t = \frac{X_t - X}{X} = \log(1 + \frac{X_t - X}{X}) = \log(\frac{X_t}{X}) = \log X_t - \log X$$

where $\log(1+X) = X$ if X is small. Thus, a constant c has the identity $\hat{c} = \log(1 + \frac{c-c}{c}) = \log 1 = 0$

[3]Note that $\vartheta_t = 0$ for all t in equilibrium.

$$\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\xi^w\hat{w}_{t-1}+\sigma_h\lambda^w-\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}(1+\beta\xi^{w2})\hat{w}_t$$
$$+\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\beta\xi^w\hat{w}_{t+1}-\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\xi^w(\hat{\pi}_t-\hat{\bar{\pi}}_t)$$
$$+\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\beta\xi^w(\hat{\pi}_{t+1}-\rho_\pi\hat{\bar{\pi}})+\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\xi^w\kappa^w(\hat{\pi}^c_{t-1}-\hat{\bar{\pi}}_t)$$
$$-\frac{[\lambda^w\sigma_h-(1-\lambda^w)]}{[(1-\beta\xi^w)(1-\xi^w)]}\beta\xi^w\kappa^w(\hat{\pi}^c_t-\rho_\pi\hat{\bar{\pi}})+(1-\lambda^w)\hat{\psi}^\tau_{z,t}-(1-\lambda^w)\sigma_h\hat{H}_t$$
$$-(1-\lambda^w)\frac{\tau^y}{(1-\tau^y)}\hat{\tau}^y_t-(1-\lambda^w)\frac{\tau^w}{(1+\tau^w)}\hat{\tau}^w_t-(1-\lambda^w)\hat{\epsilon}^h_t=0$$

Log-linearized Euler equation from the optimizing with respect to consumption given by B.4

$$-b\beta\mu_z\hat{c}_{t+1}+(\mu_z^2+b^2\beta)\hat{c}_t-b\mu_z\hat{c}_{t-1}+b\mu_z(\hat{\mu}_{z,t}-\beta\hat{\mu}_{z,t+1})+(\mu_z-b\beta))(\mu_z-b)\hat{\psi}_{z,t}$$
$$+\frac{\tau^c}{1+\tau^c}(\mu_z-b\beta)(\mu_z-b)\hat{\tau}^c_t+(\mu_z-b\beta)(\mu_z-b)\hat{\chi}^{c,d}_t-(\mu_z-b)(\mu_z\hat{\xi}^c_t-b\beta\hat{\xi}^c_{t+1})=0 \quad \text{(C.2)}$$

Log-linearized equation from the optimizing with respect to total assets given by B.5

$$-\mu\hat{\psi}_{z,t}+\mu\hat{\psi}_{z,t+1}-\mu\hat{\mu}_{z,t+1}+(\mu-\beta\tau^k)\hat{R}_t-\mu\hat{\pi}_{t+1}+\frac{\tau^k}{1-\tau^k}(\beta-\mu)\hat{\tau}^k_{t+1}=0 \qquad \text{(C.3)}$$

Log-linearized equation from the optimizing with respect to capital stock given by B.7

$$\hat{\psi}_{z,t}+\hat{\mu}_{z,t+1}-\hat{\psi}_{z,t+1}-\frac{\beta(1-\delta)}{\mu_z}\hat{P}^k_{t+1}+\hat{P}^k_t-\frac{\mu_z-\beta(1-\delta)}{\mu_z}\hat{r}^k_{t+1}+\frac{\tau^k}{1-\tau^k}\frac{\mu_z-\beta(1-\delta)}{\mu_z}\hat{\tau}^k_{k+1}=0 \quad \text{(C.4)}$$

Log-linearized equation from the optimizing with respect to investment given by B.8

$$\hat{P}^k_t+\hat{\epsilon}^i_t-\hat{\chi}^{i,d}_t-\mu_z^2\zeta[(\hat{i}_t-\hat{i}_{t-1}-\beta(\hat{i}_{t+1}-\hat{i}_t)+\hat{\mu}_{z,t}-\beta\hat{\mu}_{z,t+1}]=0 \qquad \text{(C.5)}$$

Log-linearized equation from the optimizing with respect to capital utilization rate given by B.9[4]

$$\hat{u}_t=\frac{1}{\sigma_a}\hat{r}^k_t-\frac{1}{\sigma_a}\frac{\tau^k}{1-\tau^k}\hat{\tau}^k_t \qquad \text{(C.6)}$$

[4]We know that the utilization cost function $a(u_t)$ is assumed to satisfy $f(1)=0, f'(u)=(1-\tau^k)r^k$ at steady state value $u=1$.

where, as argued by Altig et al. (2011), $\frac{1}{\sigma_a}$ is taken as the elasticity of capital utilization rate to the rental rate of capital. A smaller value of σ_a corresponds to a larger elasticity.

Log-linearized equation from the optimizing with respect to cash holdings given by B.10

$$\hat{q}_t = \frac{1}{\sigma_q}\left(\hat{\epsilon}_t^q + \frac{\tau^k}{1-\tau^k}\hat{\tau}_t^k - \hat{\psi}_{z,t} - \frac{R}{R-1}\hat{R}_{t-1}\right) \tag{C.7}$$

Log-linearized aggregate employment equation is given by log-linearizing the first order condition of the optimization problem 2.20, using the log-linearized employment aggregator. This can be taken as an auxiliary equation that links $\hat{\Psi}_t$ with $\hat{H}_t$:

$$\Delta\hat{\Psi}_t = \beta E_t \Delta\hat{\Psi}_{t+1} + \frac{(1-\xi^e)(1-\beta\xi^e)}{\xi^e}(\hat{H}_t - \hat{\Psi}_t) \tag{C.8}$$

where $\hat{\Psi}_t = (\tilde{\Psi}_t - \tilde{\Psi})/\tilde{\Psi}$ and $\hat{\Psi}_t = \hat{H}_t$ if $\xi^e = 0$.

Firms

The *Domestic Phillips curve* or the aggregate supply curve for domestic firms takes the form

$$(\hat{\pi}_t - \hat{\bar{\pi}}_t) = \frac{\beta}{1+\kappa^d\beta}(E_t\hat{\pi}_{t+1} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^d}{1+\kappa^d\beta}(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t) \tag{C.9}$$
$$-\frac{\kappa^d\beta(1-\rho_\pi)}{1+\kappa^d\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^d)(1-\beta\xi^d)}{\xi^d(1+\kappa^d\beta)}(\hat{mc}_t + \hat{\lambda}_t^d)$$

Key related identities of domestic firms include the *real rental rate of capital* (from equation B.16)

$$\hat{r}_t^k = \hat{\mu}_{z,t} + \hat{w}_t + \hat{R}_t^w + \hat{H}_t - \hat{k}_t \tag{C.10}$$

Real marginal costs for intermediate good firms (from equation B.17)

$$\hat{mc}_t = \alpha\hat{r}_t^k + (1-\alpha)(\hat{w}_t + \hat{R}_t^w) - \hat{\epsilon}_t \tag{C.11}$$

or

$$\hat{mc}_t = \alpha(\hat{\mu}_{z,t} + \hat{H}_t - \hat{k}_t) + \hat{w}_t + \hat{R}^w_t - \hat{\epsilon}_t \tag{C.12}$$

And the *nominal rate of interest* intermediate firms borrow to pay for wages (from equation 2.25)

$$\hat{R}^w_t = \frac{\nu R}{\nu R + 1 - \nu}\hat{R}_{t-1} + \frac{\nu(R-1)}{\nu R + 1 - \nu}\hat{\nu}_t \tag{C.13}$$

The *Phillips curves* for the consumption-importing firms and investment-importing firms

$$\begin{aligned}(\hat{\pi}^{m,c}_t - \hat{\bar{\pi}}_t) &= \frac{\beta}{1+\kappa^{m,c}\beta}(E_t\hat{\pi}^{m,c}_{t+1} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^{m,c}}{1+\kappa^{m,c}\beta}(\hat{\pi}^{m,c}_{t-1} - \hat{\bar{\pi}}_t)\\ &-\frac{\kappa^{m,c}\beta(1-\rho_\pi)}{1+\kappa^{m,c}\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^{m,c})(1-\beta\xi^{m,c})}{\xi^{m,c}(1+\kappa^{m,c}\beta)}(\hat{mc}^{m,c}_t + \hat{\lambda}^{m,c}_t)\end{aligned} \tag{C.14}$$

$$\begin{aligned}(\hat{\pi}^{m,i}_t - \hat{\bar{\pi}}_t) &= \frac{\beta}{1+\kappa^{m,i}\beta}(E_t\hat{\pi}^{m,i}_{t+1} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^{m,i}}{1+\kappa^{m,i}\beta}(\hat{\pi}^{m,i}_{t-1} - \hat{\bar{\pi}}_t)\\ &-\frac{\kappa^{m,i}\beta(1-\rho_\pi)}{1+\kappa^{m,i}\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^{m,i})(1-\beta\xi^{m,i})}{\xi^{m,i}(1+\kappa^{m,i}\beta)}(\hat{mc}^{m,i}_t + \hat{\lambda}^{m,i}_t)\end{aligned} \tag{C.15}$$

where the corresponding marginal costs are given by log-linearized versions of equations B.29 and B.30

$$\hat{mc}^{m,c}_t = -\hat{mc}^x_t + \hat{\chi}^{x,*}_t - \hat{\chi}^{mc,d}_t \tag{C.16}$$

$$\hat{mc}^{m,i}_t = -\hat{mc}^x_t + \hat{\chi}^{x,*}_t - \hat{\chi}^{mi,d}_t \tag{C.17}$$

The *Phillips curve* for exporting firms

$$\begin{aligned}(\hat{\pi}^{x}_t - \hat{\bar{\pi}}_t) &= \frac{\beta}{1+\kappa^{x}\beta}(E_t\hat{\pi}^{x}_{t+1} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^{x}}{1+\kappa^{x}\beta}(\hat{\pi}^{x}_{t-1} - \hat{\bar{\pi}}_t)\\ &-\frac{\kappa^{x}\beta(1-\rho_\pi)}{1+\kappa^{x}\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^{x})(1-\beta\xi^{x})}{\xi^{x}(1+\kappa^{x}\beta)}(\hat{mc}^{x}_t + \hat{\lambda}^{x}_t)\end{aligned} \tag{C.18}$$

where by definition

$$mc^x_t \equiv \frac{P_t}{S_tP^x_t}$$

or

$$mc^x_t = \frac{P_t}{S_tP^x_t}\frac{P_{t-1}}{S_{t-1}P^x_{t-1}}\frac{S_{t-1}P^x_{t-1}}{P_{t-1}} = \frac{P_{t-1}}{S_{t-1}P^x_{t-1}}\frac{P_t}{P_{t-1}}\frac{P^x_{t-1}}{P^x_t}\frac{S_{t-1}}{S_t}$$

$$mc_t^x = \frac{mc_{t-1}^x \pi_t}{\pi_t^x} \frac{S_{t-1}}{S_t}$$

The log-linearized version of this equation is

$$\hat{mc}_t^x = \hat{mc}_{t-1}^x + \hat{\pi}_t - \hat{\pi}_t^x - \Delta \hat{S}_t \tag{C.19}$$

General Equilibrium

Log-linearized aggregate resource constraint in the goods market, given by B.35

$$(1-\omega_c)(\chi^{c,d})^{\eta_c} \frac{c}{y} (\hat{c}_t + \eta_c \hat{\chi}_t^{c,d}) + (1-\omega_i)(\chi^{i,d})^{\eta_i} \frac{i}{y} (\hat{i}_t + \eta_i \hat{\chi}_t^{i,d}) + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} (\hat{y}_t^* - \eta_f \hat{\chi}_t^{x,*} + \hat{\tilde{\epsilon}}_t^{z*})$$

$$= \lambda_d [\hat{\epsilon}_t + \alpha(\hat{k}_t - \hat{\mu}_{z,t}) + (1-\alpha)\hat{H}_t] - (1-\tau^k) r^k \frac{\bar{k}}{y} \frac{1}{\mu_z} (\hat{k}_t - \hat{\bar{k}}_t) \tag{C.20}$$

Log-linearized law of motion for net foreign assets, given by B.36

$$\hat{a}_t = -y^* \hat{mc}_t^x - \eta_f y^* \hat{\chi}_t^{x,*} + y^* \hat{y}_t^* + y^* \hat{\tilde{z}}_t^* + (c^m + i^m) \hat{\chi}_t^f - c^m [-\eta_c (1-\omega_c)(\chi^{c,d})^{-(1-\eta_c)} \hat{\chi}_t^{c,d} + \hat{c}_t]$$

$$+ i^m [-\eta_i (1-\omega_i)(\chi^{i,d})^{-(1-\eta_i)} \hat{\chi}_t^{mi,d} + \hat{i}_t] + \frac{R}{\pi \mu_z} \hat{a}_{t-1} \tag{C.21}$$

Log-linearized version of the loan market condition, given by 2.53

$$\nu w H (\hat{\nu}_t + \hat{w}_t + \hat{H}_t) = \frac{\mu m}{\pi \mu_z} (\hat{\mu}_t + \hat{m}_t - \hat{\pi}_t - \hat{\mu}_{z,t}) - q \hat{q}_t \tag{C.22}$$

where the log-linearized relationship between money growth and real balances given by B.37 is

$$\hat{\mu}_t - \hat{m}_{t+1} - \hat{\mu}_{z,t} - \hat{\pi}_t + \hat{m}_t = 0 \tag{C.23}$$

Foreign Economy

Output

$$\hat{y}_t^* = \rho_{Y^*} \hat{y}_{t-1}^* + \varepsilon_t^{Y^*} \tag{C.24}$$

Inflation

$$\hat{\pi}_t^* = \rho_{\pi^*}\hat{\pi}_{t-1}^* + \varepsilon_t^{\pi^*} \tag{C.25}$$

Interest rate

$$\hat{R}_t^* = \rho_{R^*}\hat{R}_{t-1}^* + \varepsilon_t^{R^*} \tag{C.26}$$

Identities of the Open Economy

By definition, the relative price

$$\chi_t^{mc,d} \equiv \frac{P_t^{m,c}}{P_t}$$

or equivalently

$$\chi_t^{mc,d} = \frac{P_t^{m,c}}{P_t}\frac{P_{t-1}^{m,c}}{P_{t-1}}\frac{P_{t-1}}{P_{t-1}^{m,c}} = \frac{\chi_{t-1}^{mc,d}\pi_t^{m,c}}{\pi_t}$$

The log-linearized version of this equation is

$$\hat{\chi}_t^{mc,d} = \hat{\chi}_{t-1}^{mc,d} + \hat{\pi}_t^{m,c} - \hat{\pi}_t$$

The same derivation is applied to have

$$\hat{\chi}_t^{mi,d} = \hat{\chi}_{t-1}^{mi,d} + \hat{\pi}_t^{m,i} - \hat{\pi}_t$$

$$\hat{\chi}_t^{x,*} = \hat{\chi}_{t-1}^{x,*} + \hat{\pi}_t^{x} - \hat{\pi}_t^*$$

The uncovered interest rate parity

$$\hat{R}_t - \hat{R}_t^* = E_t\Delta\hat{S}_{t+1} - \Lambda_a\hat{a}_t + \hat{\varphi}_t \tag{C.27}$$

Government

The fiscal rule is given by

$$\hat{G}_t = \phi_G\hat{G}_{t-1} + (1-\phi_G)(\phi_\Omega\hat{\Omega}_{t-1} - \phi_B\hat{B}_t^r) + z_t^G \tag{C.28}$$

The law of motion for government debt

$$\hat{B}_t^r = R[\hat{B}_{t-1}^r + \hat{G}_t - B^r(\hat{y}_t - \hat{y}_{t-1} + \hat{\pi}_t - \hat{\mu}_z)] + B^r\hat{R}_t \tag{C.29}$$

Monetary Rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\hat{\bar{\pi}}_t + \theta_\pi(\hat{\pi}_t^c - \hat{\bar{\pi}}_t) + \theta_y\hat{y}_t] + z_t^R \tag{C.30}$$

Partial Dollarization

The log-linearized versions of the real marginal costs and relative prices in equations 2.54, 2.55, B.38, and B.39 are given by

$$\hat{mc}_t^D = \hat{mc}_t - \hat{t}_t^D$$

$$\hat{mc}_t^{USD} = \hat{mc}_t - \hat{t}_t^D - \hat{e}_t$$

$$\hat{t}_t^D = \hat{t}_{t-1}^D + \hat{\pi}_t^D - \hat{\pi}_t^c$$

$$\hat{e}_t = \hat{e}_{t-1} + \hat{\pi}_t^D - \hat{\pi}_t^{USD} - \Delta\hat{S}_t$$

The Phillips curve for the firms that set prices in domestic currency

$$\begin{aligned}(\hat{\pi}_t^D - \hat{\bar{\pi}}_t) = {} & \frac{\beta}{1+\kappa^d\beta}(E_t\hat{\pi}_{t+1}^D - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^d}{1+\kappa^d\beta}(\hat{\pi}_{t-1}^D - \hat{\bar{\pi}}_t) \\ & - \frac{\kappa^d\beta(1-\rho_\pi)}{1+\kappa^d\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^d)(1-\beta\xi^d)}{\xi^d(1+\kappa^d\beta)}(\hat{mc}_t^D + \hat{\lambda}_t^d)\end{aligned} \tag{C.31}$$

The Phillips curve for the firms that set prices in USD

$$\begin{aligned}(\hat{\pi}_t^{USD} - \hat{\bar{\pi}}_t) = {} & \frac{\beta}{1+\kappa^d\beta}(E_t\hat{\pi}_{t+1}^{USD} - \rho_\pi\hat{\bar{\pi}}_t) + \frac{\kappa^d}{1+\kappa^d\beta}(\hat{\pi}_{t-1}^{USD} - \hat{\bar{\pi}}_t) \\ & - \frac{\kappa^d\beta(1-\rho_\pi)}{1+\kappa^d\beta}\hat{\bar{\pi}}_t + \frac{(1-\xi^d)(1-\beta\xi^d)}{\xi^d(1+\kappa^d\beta)}(\hat{mc}_t^{USD} + \hat{\lambda}_t^d)\end{aligned} \tag{C.32}$$

The domestic Phillips curve in case of partial dollarization

$$\hat{\pi}_t = (1-\delta^{pd})\hat{\pi}_t^D + \delta^{pd}(\hat{\pi}_t^{USD} + \Delta\hat{S}_t) \tag{C.33}$$

where $0 < \delta^{pd} < 1$ is the degree of preference for USD.

Measurement Equations in Special Cases

First, the relationship between steady states of the original and adjusted original consumption variables is given by

$$\tilde{C} = \left[(1-\omega_c)\left(\frac{1}{\chi^{c,d}}\right)^{-\eta_c} + \omega_c(\chi^{mc,c})^{-\eta_c}\right] C$$

or

$$\frac{\tilde{C}}{C} = \varrho_1 \tag{C.34}$$

where

$$\varrho_1 = (1-\omega_c)(\chi^{c,d})^{\eta_c} + \omega_c(\chi^{mc,c})^{-\eta_c}$$

Equation 3.18 is log-linearized as follows

$$\tilde{C}_t = (1-\omega_c)\left(\frac{1}{\chi_t^{c,d}}\right)^{-\eta_c} C_t + \omega_c(\chi_t^{mc,c})^{-\eta_c} C_t$$

$$\tilde{C}(1+\hat{\tilde{c}}_t) = (1-\omega_c)(\chi^{c,d})^{\eta_c} C(1+\eta_c\hat{\chi}_t^{c,d} + \hat{c}_t) + \omega_c(\chi^{mc,c})^{-\eta_c} C(1-\eta_c\hat{\chi}_t^{mc,c} + \hat{c}_t)$$

$$\frac{\tilde{C}}{C} = \frac{(1-\omega_c)(\chi^{c,d})^{\eta_c}(1+\eta_c\hat{\chi}_t^{c,d} + \hat{c}_t) + \omega_c(\chi^{mc,c})^{-\eta_c}(1-\eta_c\hat{\chi}_t^{mc,c} + \hat{c}_t)}{(1+\hat{\tilde{c}}_t)}$$

$$\frac{\tilde{C}}{C} = \frac{\hat{c}_t\varrho_1 + \varrho_{2,t}}{(1+\hat{\tilde{c}}_t)} \tag{C.35}$$

where

$$\varrho_{2,t} = (1-\omega_c)(\chi^{c,d})^{\eta_c}(1+\eta_c\hat{\chi}_t^{c,d}) + \omega_c(\chi^{mc,c})^{-\eta_c}(1-\eta_c\hat{\chi}_t^{mc,c})$$

From identities C.34 and C.35 we have

$$\varrho_1 = \frac{\hat{c}_t\varrho_1 + \varrho_{2,t}}{(1+\hat{\tilde{c}}_t)}$$

or the link between the log-linearized original variables and the log-linearized adjusted original variables is given by

$$\hat{\tilde{c}}_t = \hat{c}_t + \frac{\varrho_{2,t} - \varrho_1}{\varrho_1}$$

In order to facilitate the estimation of the model, it is assumed that the differences of relative inflation rates are negligible and can be accounted for by measurement errors. The measurement equation for consumption is, therefore, defined by

$$\begin{aligned} c_t^{obs} &= \log(C_t^{data}) - \log(C_{t-1}^{data}) \\ &= \log(\tilde{C}_t) - \log(\tilde{C}_{t-1}) \\ &= \hat{\tilde{c}}_t - \hat{\tilde{c}}_{t-1} + \hat{\mu}_{z,t} \\ &= \left(\hat{c}_t + \frac{\varrho_{2,t} - \varrho_1}{\varrho_1}\right) - \left(\hat{c}_{t-1} + \frac{\varrho_{2,t-1} - \varrho_1}{\varrho_1}\right) + \hat{\mu}_{z,t} \\ &= \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_{z,t} \end{aligned} \tag{C.36}$$

Second, the same observations apply to investment which leads to the measurement equation for investment as follows

$$i_t^{obs} = \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_{z,t} \tag{C.37}$$

Third, total imports are given by

$$\begin{aligned} \tilde{M}_t &= \omega_c(\chi_t^{mc,c})^{-\eta_c} C_t + \omega_i(\chi_t^{mi,i})^{-\eta_i} I_t \\ &= \omega_c \left(\frac{\chi_t^{mc,d}}{\chi_t^{c,d}}\right)^{-\eta_c} C_t + \omega_i \left(\frac{\chi_t^{mi,d}}{\chi_t^{i,d}}\right)^{-\eta_i} I_t \end{aligned}$$

Log-linearizing both sides to get

$$\tilde{M}(1 + \hat{\tilde{m}}_t) = \omega_c(\chi^{mc,c})^{-\eta_c} C(1 - \eta_c \hat{\chi}_t^{mc,c} + \hat{c}_t) + \omega_i(\chi^{mi,i})^{-\eta_c} I(1 - \eta_i \hat{\chi}_t^{mi,i} + \hat{i}_t)$$

A simplified version of the equation is given by

$$\varrho_3(1 + \hat{\tilde{m}}_t) = \varrho_4 \hat{c}_t + \varrho_5 \hat{i}_t + \varrho_{6,t}$$

where the steady state of imports is defined as

$$\varrho_3 = \tilde{M} = \omega_c(\chi^{mc,c})^{-\eta_c}C + \omega_i(\chi^{mi,i})^{-\eta_i}I$$

and

$$\varrho_4 = \omega_c(\chi^{mc,c})^{-\eta_c}C$$

$$\varrho_5 = \omega_i(\chi^{mi,i})^{-\eta_i}I$$

$$\varrho_{6,t} = \omega_c(\chi^{mc,c})^{-\eta_c}C(1-\eta_c\hat{\chi}_t^{mc,c}) + \omega_i(\chi^{mi,i})^{-\eta_c}I(1-\eta_i\hat{\chi}_t^{mi,i})$$

Then,

$$\hat{\tilde{m}}_t = \frac{\varrho_4\hat{c}_t + \varrho_5\hat{\tilde{i}}_t + \varrho_{6,t} - \varrho_3}{\varrho_3}$$

As argued above, the measurement equation for imports is specified as

$$m_t^{obs} = \hat{\tilde{m}}_t - \hat{\tilde{m}}_{t-1} + \hat{\mu}_{z,t}$$

$$= \left(\frac{\varrho_4\hat{c}_t + \varrho_5\hat{\tilde{i}}_t + \varrho_{6,t} - \varrho_3}{\varrho_3}\right) - \left(\frac{\varrho_4\hat{c}_{t-1} + \varrho_5\hat{\tilde{i}}_{t-1} + \varrho_{6,t-1} - \varrho_3}{\varrho_3}\right) + \hat{\mu}_{z,t}$$

$$= \frac{\varrho_4}{\varrho_3}(\hat{c}_t - \hat{c}_{t-1}) + \frac{\varrho_5}{\varrho_3}(\hat{\tilde{i}}_t - \hat{\tilde{i}}_{t-1}) + \hat{\mu}_{z,t} \tag{C.38}$$

Finally, exports are taken as

$$\tilde{X}_t = \tilde{C}_t + \tilde{I}_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*$$

$$= (\chi_t^{x,*})^{-\eta_f}Y_t^*$$

Log-linearize this equation to get

$$\tilde{X}(1+\hat{\tilde{x}}_t) = (\chi^{x,*})^{-\eta_f}Y^*(1-\eta_f\hat{\chi}_t^{x,*} + \hat{y}_t^*)$$

where the steady state position is given by

$$\tilde{X} = (\chi^{x,*})^{-\eta_f}Y^*$$

Then,

$$(\chi^{x,*})^{-\eta_f} Y^*(1 + \hat{\tilde{x}}_t) = (\chi^{x,*})^{-\eta_f} Y^*(1 - \eta_f \hat{\chi}_t^{x,*} + \hat{y}_t^*)$$

$$\hat{\tilde{x}}_t = \hat{y}_t^* - \eta_f \hat{\chi}_t^{x,*}$$

As argued above, the measurement equation for exports is given by

$$x_t^{obs} = \hat{\tilde{x}}_t - \hat{\tilde{x}}_{t-1} + \hat{\mu}_{z,t}$$

$$= (\hat{y}_t^* - \eta_f \hat{\chi}_t^{x,*}) - (\hat{y}_{t-1}^* - \eta_f \hat{\chi}_{t-1}^{x,*}) + \hat{\mu}_{z,t}$$

$$= \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\mu}_{z,t} \tag{C.39}$$

Appendix D

Bayesian Maximum Likelihood

The following lemmas are needed to evaluate the likelihood of the data given the parameter $p(Y^{*,T}|\theta)$:

Lemma 1: Given the decision rule 3.6, we can compute the density of the model variables given state variables

$$p(y_t|y_{t-1}, \theta) \tag{D.1}$$

Lemma 2: From the measurement equation 3.7, we can compute the density of the observables given model variables

$$p(y_t^*|y_t, \theta) \tag{D.2}$$

Lemma 3: By definition of conditional probability

$$p(y_1^*, y_1) = p(y_1^*|y_1)p(y_1)$$

However, because $y_1^* \in y_1$, then we have

$$p(y_1^*, y_1) = p(y_1^*) = p(y_1^*|y_1)p(y_1)$$

And if we add in the known θ in both sides, the equation remains the same

$$p(y_1^*|\theta) = p(y_1^*|y_1, \theta)p(y_1|\theta) \tag{D.3}$$

Lemma 4:

If we take y_t as A, $Y^{*,t}$ as B, applying Bayes' Rule we have

$$p(y_t|Y^{*,t}) = \frac{p(Y^{*,t}|y_t)p(y_t)}{p(Y^{*,t})}$$

Note that $p(Y^{*,t}) = p(y_t^*)$ and $p(y_t) = p(y_t|Y^{*,t-1})$, then we have

$$p(y_t|Y^{*,t}) = \frac{p(y_t^*|y_t)p(y_t|Y^{*,t-1})}{p(y_t^*)}$$

Also note that $p(y_t^*) = p(y_t^*|Y^{*,t-1})$, then we have

$$p(y_t|Y^{*,t}) = \frac{p(y_t^*|y_t)p(y_t|Y^{*,t-1})}{p(y_t^*|Y^{*,t-1})}$$

Finally, adding θ to both sides, we have

$$p(y_t|Y^{*,t}, \theta) = \frac{p(y_t^*|y_t, \theta)p(y_t|Y^{*,t-1}, \theta)}{p(y_t^*|Y^{*,t-1}, \theta)} \tag{D.4}$$

where $p(y_t^*|y_t, \theta)$ is identified by Lemma 2, $(y_t|Y^{*,t-1}, \theta)$ is identified by the Chapman-Kolmogorov equation from the latest round of forecasting.

Applying Lemma 3 for all observations up to t, the conditional likelihood in the denominator is also identified by an integral of

$$p(y_t^*|Y^{*,t-1}, \theta) = \int p(y_t^*|y_t, \theta)p(y_t|Y^{*,t-1}, \theta)dy_t \tag{D.5}$$