Metasurface Waveguide Leaky-Wave Antennas

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***Abstract*—Metasurfaces can be engineered to guide surface waves in a homogeneous path, where sub-wavelength size printed patches are etched on a grounded high-frequency laminate.**

When the homogeneity of the patches is compromised or it is inappropriately excited, leakage takes place. This effect can be exploited to design leaky-wave antennas, by introducing perturbations on the pattern so radiation occur in a controlled manner. The aim of this paper is to propose certain candidate antenna designs. The engineering method to derive effective refractive index is comprehensively investigated to guide and radiate surface waves at a centre operating frequency of $10GHz$ as a validation of the theory proposed.

***Index Terms*—Metasurface, leaky-lave, antenna, surface waves, high impedance surface.**

I. Introduction

METASURFACES are artificially structured materials with variable impedance, formed by highly subwavelength inclusions embedded in a host medium, which yields homogenised permittivity and permeability values with unusual electromagnetic responses not generally available in nature. Especially a negative refractive index [1].

Metasurfaces can be studied as a distinctive model of periodic structures for surface guiding, when the spacing between patches w is uniform [2].

These surfaces can be engineered to have very high performance with low loss and low dispersion, they also have the prospective to be inexpensive for a wide range of applications, including systems which are intended of radiating fields, as it has been reported in [3], [4].

Leaky-wave antennas are simple to fabricate and can generate high antenna efficiency with a low-cross polarisation.

The first goal of this paper is to trap the wave along the surface, and create inhomogeneous plane waves. Followed by an exhaustive exploration on new techniques in furtherance of transforming a metasurface waveguide into a leaky-wave antenna.

II. Metasurface Waveguide Design

A. Surface Waves and Impedance Modulation

H. M. Barlow and J. Brown [5] modelled the properties of surface waves back in 1961. They defined a surface wave as: *"…one that propagates along an interface between two different media without radiation; such radiation being constructed to mean energy converted from the surface-wave field to some other form.*". An idealistic definition, not always achieved in reality due to interface losses.

One of the solutions of Maxwell's equations for the inhomogeneous plane wave traveling on a flat surface was proposed by Zenneck [5], we will use his solution for a wave supported by a flat surface to meet the conditions of a surface wave. If a wave is to propagate along the surface of a metal, the electric field must be polarised normal to the surface, otherwise boundary conditions will require it to be zero. Therefore, the E-mode does not exist as a tangential component across the surface. E-field is polarised normal to the surface impedance. Only the H-mode exist.

When an analysis of the full Zenneck wave is performed, similar solutions that look like plane waves are obtained, but in a homogeneous plane. It means that they look like plane waves. With phase slightly tilted, but the wave amplitude deceases exponentially about the surface.

Consequently, the solution is a trapped wave along the surface. Surface impedance is the ratio of the Electric and Magnetic field.

To achieve a high impedance surface on a high-frequency material, a periodically grid of (Perfect Electric Conductor) PEC formed by an array of square patches should be etched on one side. With $w \ll D$, where D is the periodicity and w is the width of the gap [see Fig. 1]. Where h' is the height/thickness of the substrate, hp is the height of the metallic patches, hg is the height of the ground conductor, and lp is the length of the patch. The dimensions adopted for the array are:

$$D = \frac{\lambda}{10}, \tag{1}$$

and

$$w = \frac{D}{10}, \tag{2}$$

corresponding to the results obtained in the model of C. L. Holloway [6].

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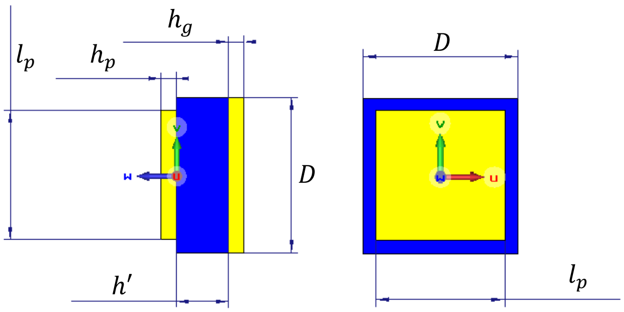


Fig. 1. Unit cell dimensions of the metallic patch on a grounded high-frequency laminate. The effective refractive index can be modulated by changing the laminate material and height (h'), as well as the patch size. Metallic parts are coloured yellow.

The patch is totally isolated from the ground plane, and the relation between the periodicity and the gap for the grid can be considered as:

$$D = l_p + w \tag{3}$$

For a grid to be considered a perfect metasurface waveguide, a confinement must be achieved. It means that the metasurface will work as a variable-impedance patch array. With a different equivalent surface impedance in each point along the lateral mesh.

The grid impedances for the TM and TE mode can be obtained from O. Luukkonen [7].

The approximation is made over the grid periodicity D, length of patch l_p, and gap w.

B. Metasurface Waveguide Design and Considerations

1) Substrate and Metal Selection

Primary attention was given to the substrate selection. The quality of the traveling wave and frequency stability will be highly determined by the dielectric used, not only for the construction, but also for the simulation. Losses in the material are always present, but the interference needs to be diminished by using highly conductive metals and low loss dielectrics.

The material used in our simulations is the high-frequency laminate Rogers RT/Duroid 5880, which has a relative permittivity $\varepsilon_r = 2.20$, and dissipation factor $\tan\delta = 0.0009$.

After running a parameter swap simulation for a single patch in CST Microwave Studio®, we obtained an optimised value of $h' = 1.54$, and $h_p = 0.07$, due to the fabrication standards of the material.

2) Excitation of the Metasurface with Waveguide Ports

For excitation purposes, not only a waveguide port was used. A physic metallic waveguide was modelled from PEC using CST Microwave Studio® with $22.86mm$ of width and $10.16mm$ of height [see Fig. 2], to create a standard $WG16$ waveguide used in $X - band$ communications with a frequency range of $8.2GHz$ to $12.4GHz$.

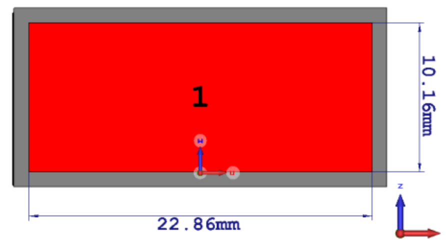


Figure. 2. WG16 Waveguide Port dimensions used as an excitation for the metasurface and leaky-wave antenna simulation in CST Microwave Studio®.

Aperture fields on a real waveguide structure with currents flowing on the outside surface, affects the actual impedance that the wave sees on that plane. The metasurface either needs uniform field or the dominant TE_{10} mode.

3) Design Equations

Transverse Magnetic TM mode, implies that the H-field is orthogonal to the axis of the waveguide. For this particular mode, $H_z = 0$. The following equations are used to define exactly what the inhomogeneous plane wave will look like on the surface.

A value for the attenuation constant α_z, is predicted according to the self-consistent answers obtained after solving the transcendental equations (4)-(10) proposed by S. Maci [8], in MATLAB®. Designing the metasurface for a required α_z, fixes the length of the patches lp, and for an invariable periodicity D, also fixes the gap w and X_s that is the surface impedance.

For the case proposed, the attenuation constant along z is $\alpha_z = 620$. Using (4), with $k = 2\pi/\lambda$, one can find the value of the equivalent refractive index n_{eq}.

$$n_{eq} = \sqrt{1 + \left(\frac{\alpha_z}{k}\right)^2} > 1 \tag{4}$$

This index must be greater than one because we look for transverse-to-z wavenumber $k_t(\boldsymbol{\rho}) > k$, with k as the wavenumber. Therefore, from (5), k_t is found and can be replaced on (6) to find k_{z1}, using an $\varepsilon_r = 2.2$.

$$k_t(\boldsymbol{\rho}) = k n_{eq}(\boldsymbol{\rho}) \tag{5}$$

$$k_{z1} = \sqrt{k^2\varepsilon_r - k_t^2}, \tag{6}$$

From here, X_s can be obtained from (7), with Z_0 as the characteristic impedance of free space, that can be calculated as the square root of the ratio of the permeability of free space u_0 in Henrys/m to the permittivity of free space ε_0 in Farads/m. This comes to 377 Ω approximately.

$$n_{eq} = \sqrt{1 + \left(\frac{X_s}{Z_0}\right)^2} \quad (7)$$

For metasurfaces conformed by patches, the equation for the surface reactance X_s (the imaginary part of the impedance seeing on the surface) can be approximate by interpolation on an array of sampled results by relating the pole-zero matching method [9]. Henceforward, the equations from [8] are solved to find the values of w, and δ that is a dielectric dependant value in function of the surface impedance along the lateral distance of the metasurface, starting from the excitation port. Solving (8) for δ, yields a value in function of the α_z conjecture.

$$X_s(w, k_t) = \frac{\zeta k_{z1} \tan(h' k_{z1})}{k\varepsilon_r - k(\varepsilon_r + 1)\delta k_{z1} \tan(h' k_{z1})} \quad (8)$$

$$\delta = \left(\frac{D}{\pi}\right) \ln\left[\frac{1}{sin\left(\frac{\pi W}{2D}\right)}\right] \quad (9)$$

The value of δ, is used in (10), from which the calculation for the gap w, in function of n_{eq} is performed.

$$w = \left(\frac{2D}{\pi}\right) \operatorname{asin}\left(\frac{1}{e^{\frac{\delta\pi}{D}}}\right), \quad (10)$$

Now that w is known, optimisation for α_z is executed using MATLAB®, inverting the equations numerically to find the quantities needed to satisfy the conditions. A value of $\alpha_z = 620.2152$ is calculated after iterative optimisation. From here, the values of δ, k_z, and X_s can be derived. These values fix the size of the patches, and the spacing between them. With X_s, n_{eq} can be found using (7), the equation that relates the refractive index with the surface impedance.

After solving the transcendental equations, the value of w is taken for various measurements, and an approximate expression for the equivalent refractive index in a 2-dimensional material can be generated using MATLAB®. A differential of the gap value w is created, and will be recalled as Δ_w in this paper. Using a variation of w in form of an average with the relation,

$$w = w_0 + \Delta_w \quad (11)$$

yields the variation of the refractive index in function of the separation w between patches. From the MATLAB® generated plot in Fig. 3, patches of different sizes can be constructed.

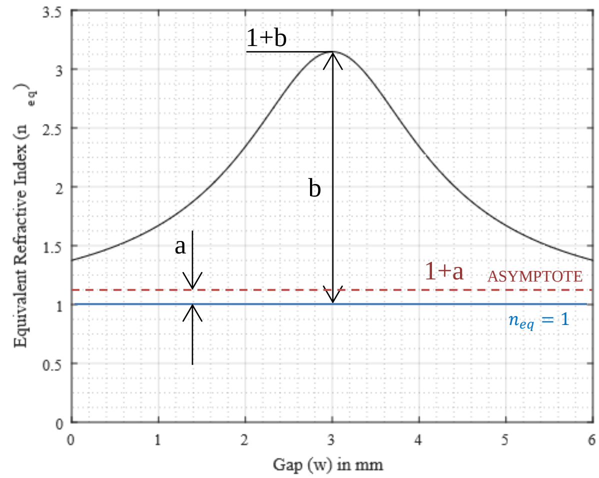


Fig. 3. Gradient-index curve representing the gap between patches w, against the equivalent refractive index n_{eq} generated with the self-consisted set of equations from *[8]*.

If we assume the curve looks roughly Gaussian, it does not have an exact analytical solution, but the nearest quadratic function (12) can be fitted at the top of the Gaussian.

$$n = h_0 - ax^2, \quad (12)$$

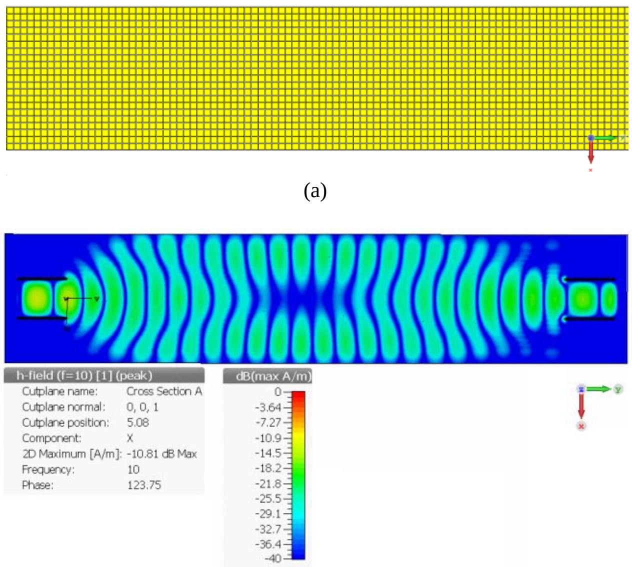
with h_0 being the maximum value in the y-axis, when $x = 0$ in the x-axis, and $b > a > 0$. The confinement width is inversely proportional to the value a, so the metasurface structure dimensions' importance arises. This issue will be analysed in the next sub-section.

An additional methodology can be taken from H. M. Barlow and J. Brown [5], the impedance is for a plane Zenneck wave with a rate of decay of the field with distance from the surface a_2, and phase change coefficient b_2, is given by,

$$Z_s = R_s + jX_s = \left[\frac{E_{x2}}{H_{z2}}\right]_{y=0} = \frac{b_2}{\omega\varepsilon_0} + j\frac{a_2}{\omega\varepsilon_0} \quad (13)$$

C. Step and Gradient Refractive Index Solutions

To effectively guide the wave on the metasurface, and avoid losses due to propagation in undesirable directions, dissimilar patch sizes representing different values of refractive-index are required. Being able to control the refractive index and create a metasurface waveguide with a constant n for the directional path, and a different refractive index for the sides, allows to guide a wave inside a linear or constrained curved path, using a rotation index tensor to guide the phase front for the case of the bent route [10]. A metasurface waveguide design with a step-index profile were designed and simulated on CST Microwave Studio®, results are shown in Fig. 4(a) and Fig. 4(b) respectively.



(a)

(b)

Fig. 4. Step-index profile metasurface waveguide. (a) Homogeneous patches structure, with $D = \lambda/10$, and $w = D/10$. (b) Magnetic field on the plane x-y, showing destructive reflections.

As it is shown in Fig. 4, a step-index profile is not very effective in guiding an electromagnetic wave over a metasurface, due to the fact that reflections on the edges of the metasurface arise destructive reflections and standing waves.

The solution proposed, is generating an inhomogeneous tapered metasurface reactance, obtained by an alteration of the gap of the patches to obtain a confined wave, according to in Fig. 3. The values of a and b define the confinement at a particular frequency of operation. The surface impedance X_s, is related to the propagation constant by,

$$X_s = \frac{a_2 \alpha}{\omega_0} \tag{14}$$

where α is a complex propagation constant in the perpendicular plane of the propagation. Moreover, we can observe that the reactance X_s depends only on a_2.

Useful engineering information about the propagation of each mode of the waveguide has been extracted by C. Constantinou [11], where a waveguide mode is given by a standing wave pattern $\varphi_{nm}(x,y)$ in the z plane, which is transverse to the direction of propagation. The field amplitude $\psi_{nm}(x,y,z)$ of a wave travelling along the waveguide axis with a propagation constant β_{nm} for the first eigenvalue is given by,

$$\varphi_{00}(x,y) = \sqrt{\frac{k_0 n_0 \sqrt{ab}}{\pi}} e^{-\frac{k(ax^2+by^2)}{2}} \tag{15}$$

with,

$$\beta_{00} = k_0 n_0 - \frac{a}{2} - \frac{b}{2} \tag{16}$$

Therefore, effectively we can confirm that the lowest of the mode for this kind of quadratic parabolic waveguide, is given by a Gaussian function. The spot size of the curve will define the beam width of the field. And the lateral confinement Δ_x can be represented as the double of the latter value as,

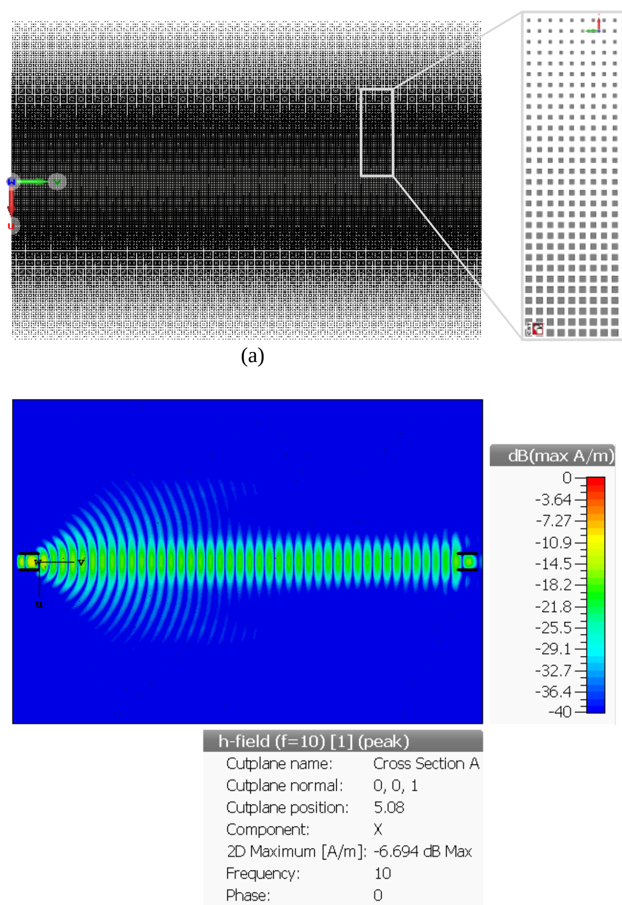
$$\Delta_x = 2\sqrt{\frac{\lambda_0}{\pi n_0 a}} \tag{17}$$

For the metasurface to remain in its instantaneous eigenstate when the homogeneity is perturbed, the gap between patches must change gradually following the adiabatic theorem [12].

It can be settled that the wavelength and the curvature of the parabola fix the lateral confinement of the field. Part of the results from the MATLAB® code generated are presented as follows,

>The lateral confinement in millimetres is:
>319.1538

Design and simulations for a metasurface waveguide with a gradient-index profile are shown in Fig. 5.



(a)

(b)

Fig. 5. Metasurface waveguide with confinement, accomplished by a tapered gradient-refractive-index. (a) Arrangement of patches over the substrate. (b) Magnetic field on the plane x-y, confirming the theory presented.

The refractive index distribution for a waveguide which tapers in the xz plane [11], is of the form,

$$n(x, y, z) = n_0 \left(1 - \frac{1}{2} a^2(z) x^2 - \frac{1}{2} b^2 y^2\right) \qquad (18)$$

Path integration technique for analysing waves propagation in graded-index optical waveguides of constant cross-sectional shape and area has been analysed in [13].

Fig. 6. Shows the perpendicular cut of the plane of propagation. Standing waves across all the 610 mm metasurface waveguide.

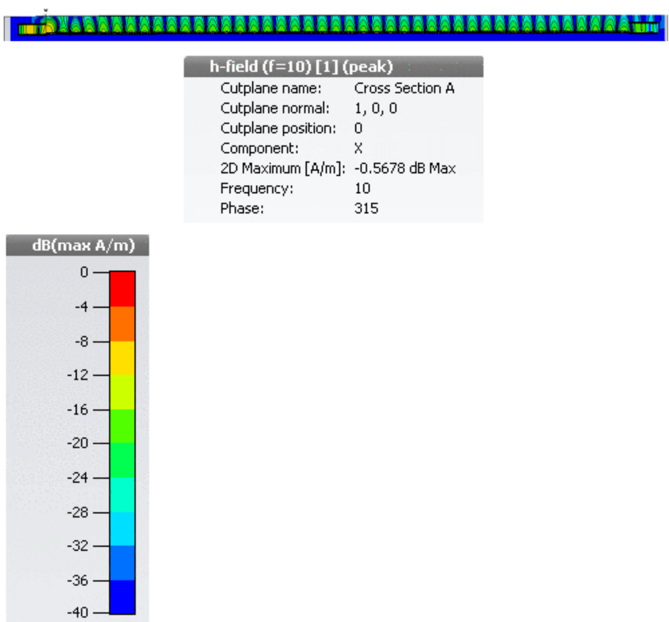


Fig. 6. Perpendicular cut of the plane of the propagation axis. Perfect standing waves across the patches are shown. The scale is measured in dB (max A/m) to obtain a clear picture of the phenomena.

To substantiate the standing wave theory, a clear picture of all the structure is shown in Fig. 7(a)., with the S-Parameters plot on Fig. 7 (b).

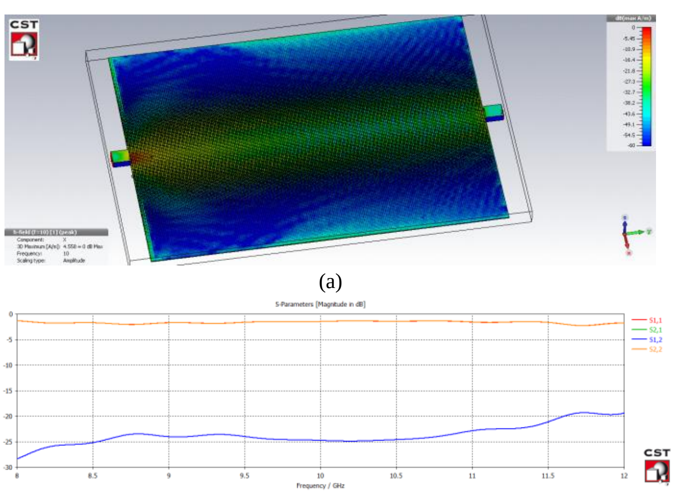


Fig. 7. Metasurface waveguide results. (a)Perspective view of the structure (457mm x 610mm) clearly showing a confined EM wave. (b)S-Parameters displays the reflections and transmissions from edge to edge.

A way to quantify the confinement in the simulations, is to check the E-field vertically, project perpendicularly to the axis of propagation, measure the decay at the maximum. If the measurement is performed logarithmically, the result is effectively an exponential decay. If I is plotted logarithmically, and convert the logarithmically decay into linear units, it yields a straight line, in which the scaled angle α can be easily extracted.

As long as you have the Pythagorean triangle expression form, the propagation constant can be defined from [5] as,

$$-(\gamma^2 + u_2^2) = k_2^2 = \omega^2 u_0 \varepsilon_0 \qquad (19)$$

In effect, the wavenumber is γ, and defines your exponential constant.

To guarantee the matching between the metasurface waveguide and the radiation structures, the alteration over distances of the order of a guided wavelength should be negligible in width, so one mode blends into the next without suffering from reflections. Unfortunately, the adiabatic theorem gives the rise to long structures.

D. *Dispersion Relation/Transition Function*

An alternative approach to analyse a metasurface waveguide is in terms of dispersion relation, which is determined by the transition equation for metasurfaces and relates wavenumber to the frequency confinement factor for a metasurface. If the DC capacitance per unit area is computed for a given metasurface, the static value for the impedance can be predicted [14].

If the estimation starting with very low into high frequencies, the transition function can be used to find the parameter X_s, which is used for the dispersion relation. Close form formulas can be smoothly extrapolated from the numerical simulation.

The dispersion characteristics of impenetrable metasurfaces can be derived, just depending on the equivalent quasi-static capacitance [14], this novel approach can be used in furtherance of this study, in favour of the design of metasurface planar lenses and transformation optics devices

The approximate formula yields to the dispersion relation, later the propagation constant along the surface and the exponential attenuation constant perpendicular to the surface can also be extracted.

III. Leaky-Wave Antennas

Design of high gain leaky-wave antennas, implies knowledge of how the wavenumber in the plane is related to the refractive index equation. The designs proposed are based on modulation of the surface impedance and controlling the wave propagation along the surface by changing the geometry. Engineering the wavenumber is essential to make the wave turn whichever way the designer want to go.

In this section, some ways to spoiling the confinement to force the surface to radiate are investigated using a metasurface.

A. *Previous Works on the Topic*

Spiral leaky-wave antennas, a particular case for obtaining a gradient-index leakage [4], had been analysed extensively, however the authors procedure to excite the spiral is adding a port in the middle of the structure. In this paper we aim to analyse methods to create leakage from a metasurface waveguide, using the Adiabatic theorem, and a gradient refractive index over the structure.

Creating this spiral in a metasurface and exciting it from a gradient refractive index implies to generate the pattern in a CAD software previously to import the design in CST Microwave Studio®. Creating scalar and tensor holographic artificial impedance surfaces [15] are excluded from this analysis due to time limit.

The intention of this project is to implement waveguide antennas and to model some candidate designs at a target operating frequency of around $10\,GHz$ in order to identify optimal configurations, and optimize it numerically using CST Microwave Studio®.

Traveling wave antennas [16], uniform leaky-wave antennas [17], and periodic leaky-wave antennas [18], exhibits the same electromagnetic properties and principles for working, phase velocity > speed of light, complex propagation wavenumber k_z, phase constant β controlling the beam angle, attenuation constant α controlling the beam width, radiation as it propagates, etc.

The parameters of the antenna will be strongly linked to the types of excitations. Discrete port and waveguide port are available in CST Microwave Studio® for simulation purposes. In the next examples, a waveguide port and the time domain solver are used.

B. *Sinusoidally Modulated Reactance Surface*

To create a sinusoidally modulated leaky-wave antenna from strips, a way to manipulate the patches to change the properties of impedance is required. The main condition here, is to vary the density of the patches until the equations get violated and they don't support the surface wave (either radiate back, or radiate into space).

Propagation of surface waves in a metasurface waveguide structure, generates fast waves. When discontinuities are introduced in the patches, the structure operates as a leaky-wave antenna. The main difference of this kind of operation in contrast to slow waves, is that $\beta < k_0$, with β as the propagation constant, and k_0 as the free space wave number.

Leaky-wave antennas are characterised by a complex propagation constant,

$$k = -j\alpha + \beta \tag{20}$$

and starting from here, an idea that has been tried and tested [3] is chosen to create a leaky wave from a strips structure, analyse the results and calculate the equivalent with patches and gradient index for the confinement.

The antenna is designed to radiate at 30° from broadside. The refractive-index vary along the waveguide axis. To create a sinusoidal modulated reactance surface [3] using strips, the change of impedance must be gradual until the radiation point. If an abrupt change in the size of the patches takes place, the wave will see a huge impedance discontinuity and will radiate the energy back.

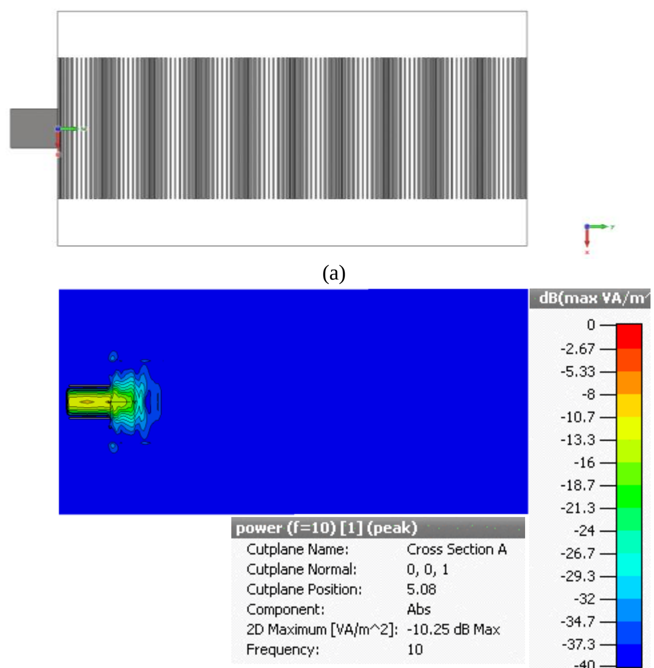
For a wave propagating in the z-axis, the refractive index can be found using the expression in [3],

$$n(z) = jX_s\left[1 + M\cos\left(\frac{2\pi z}{\lambda}\right)\right] \tag{21}$$

where, X_s is the surface impedance, and M is the modulation factor.

If the reduction of the confinement is attained by creating an aperture where the field distribution is desired, by the *Surface Equivalent Theorem: Huygens's Principle* [19] that electric field perpendicular to the aperture is not going to radiate remarkably, due to only the tangential fields that generate equivalent currents, radiates out of the antenna.

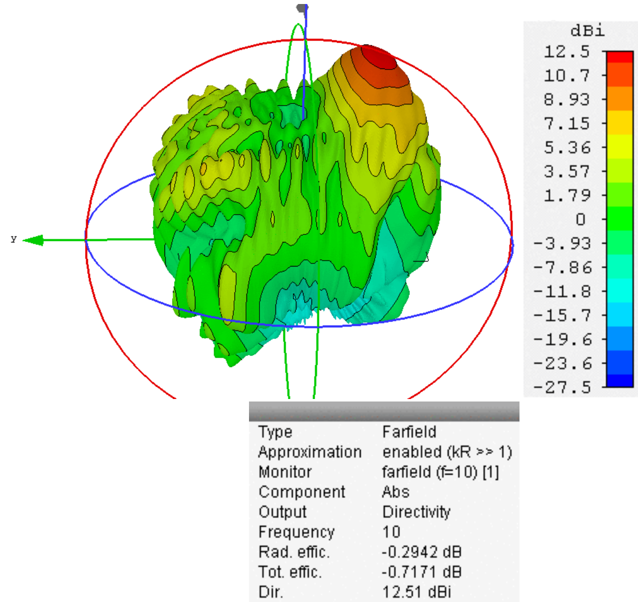
How narrow is the confinement, will determine the illumination offered to the region that radiates. The design has not been tapered to the sides. The design, as well as the magnetic field across the propagation plane is shown in Fig. 8.



(b)

Fig. 8. Sinusoidally modulated reactance surface constructed with strips. (a)strips design of the leaky-wave antenna in CST Microwave Studio®. (b)Magnetic field of the antenna shows the radiation takes place immediately.

The design resulted in a high directional antenna. The value of angle calculated, coincides with the radiation pattern simulated. The antenna reveals a gain of 12.5 dBi (17.8 linear scale). A radiation efficiency of 0.9345 in linear scale. Fig. 9. Discloses the simulation measurements of the antenna in dB, and Fig. 10. Shows the results in linear units.



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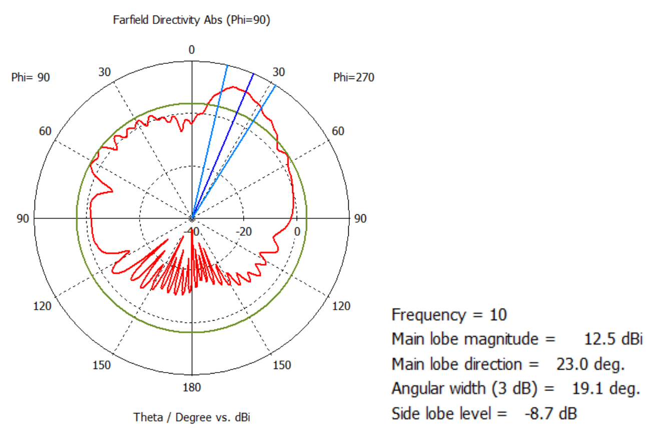

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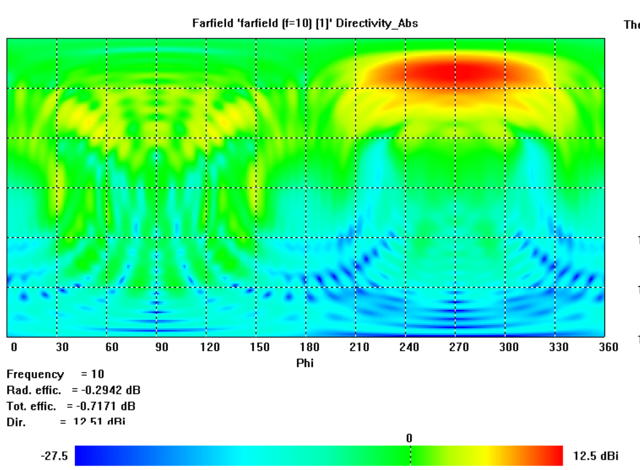
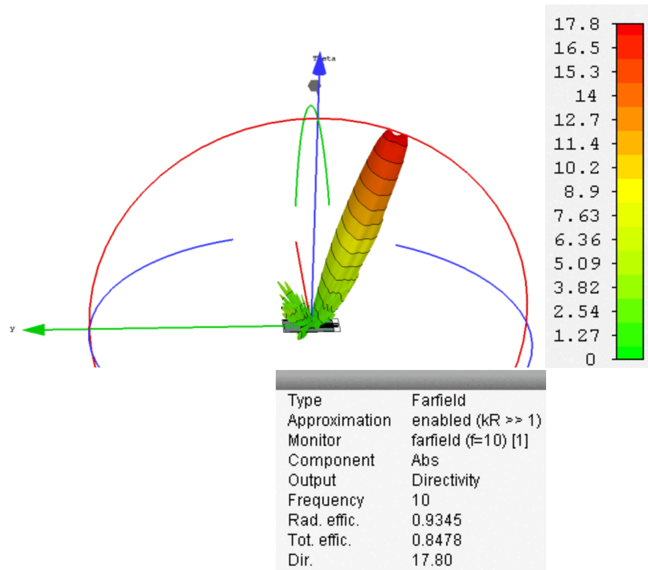


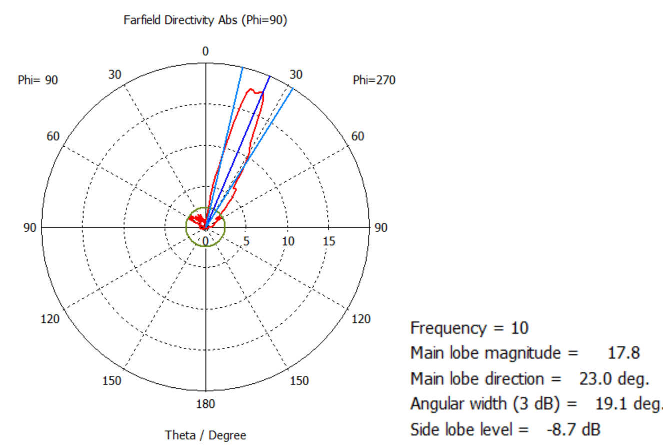
(c)

Fig. 9. Sinusoidally modulated leaky-wave antenna constructed with patches without tapering in dB units. (a)3D Radiation Pattern, (b) Polar Radiation Pattern, (c) 2D Radiation Pattern.

The same results are shown in Fig. 10. In linear units.



(a)



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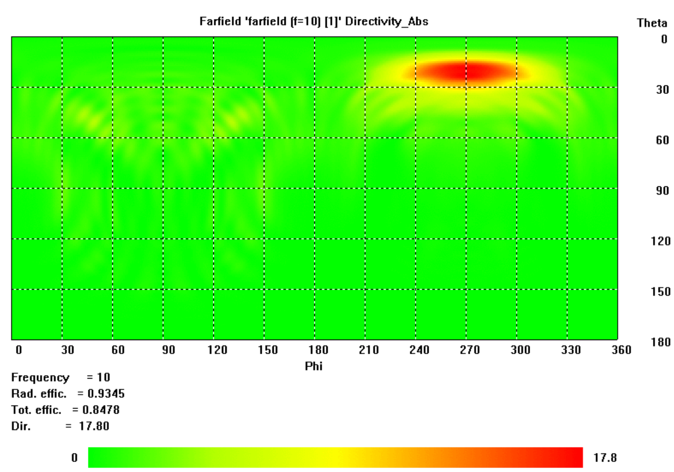
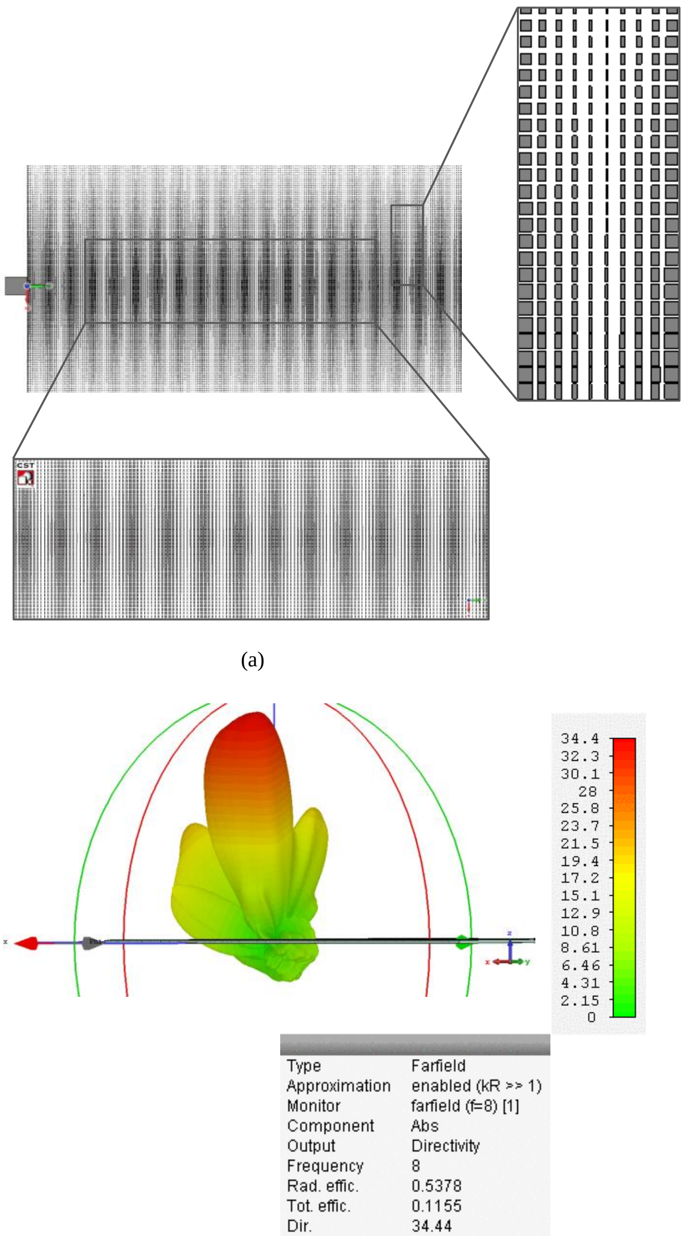


(c)

Fig. 10. Sinusoidally modulated leaky-wave antenna constructed with patches without tapering in linear units. (a)3D Radiation Pattern, (b) Polar Radiation Pattern, (c) 2D Radiation Pattern.

C. *Sinusoidally-Modulated Reactance Surface with Lateral Confinement: Patches Candidate Antenna*

The refractive-index vary along the waveguide axis, as well as the two transverse planes. To create an efficient and controlled leakage from the metasurface, the polarisation should be upset at the same time of disturbing the wave confinement. The design has to be tapered to the sides. Fig. 11. presents a proposed design based in the previous strip line modulated impedance, but with patches.



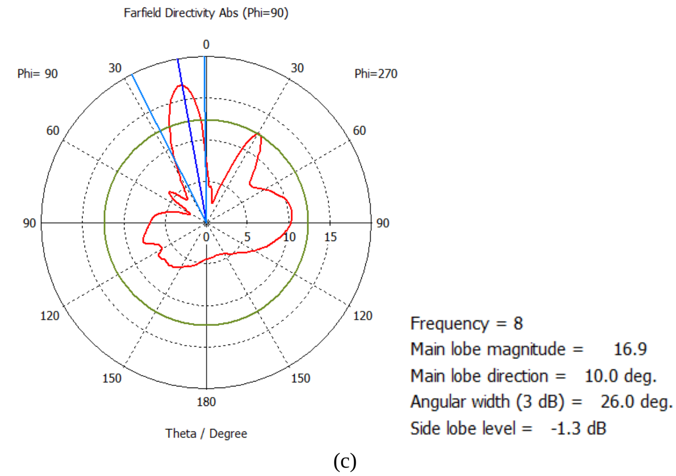


Fig. 11. Sinusoidally-Modulated Reactance Surface with Lateral Confinement. (a)Implementation of the pattern in CST Microwave Studio®. (b)3D Radiation Pattern of the antenna in linear scale. Polar Radiation Pattern of the antenna in linear units, radiating in pencil beam lobe.

D. *Luneburg Lens Antenna*

The velocity of the EM wave through a dielectric material is less than that in free space. The section of spherical EM wave that travels through the centre of the dielectric material will travel most slowly compared to both end. The velocities of the spherical wave entering the lens will be controlled and the curved wave front will become a plane wave front with constant phase in front of the dielectric antenna, that is the refraction based on Snell's law.

Where you focus the wave to a point on the lens, but at the focal point the patches are spaced in such way that they cannot support the bound-wave, and the wave becomes leaky and radiates off the focal point. The basic physics of it is that the design is forcing k_z to be equal to k_0 in the area of focus. This idea has worked in the past [8], using the Luneburg linear expansion of the electromagnetic field, where the wave amplitude is written as a series expanded in $1/k_0$ powers.

A differential equation that describes the ray trajectory is obtained. The *Luneburg Lens Solution* [20], with a radius R, and a point $r = \sqrt{y^2 + z^2}$, assuming a two-dimensional case for the yz-plane [21]

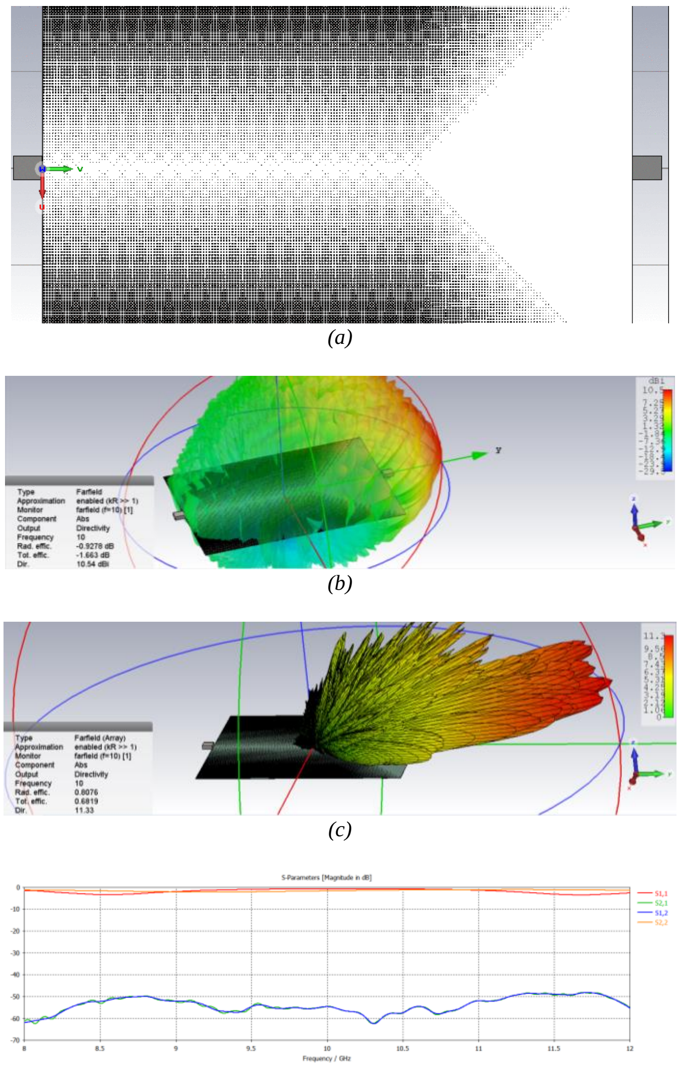
$$n = \sqrt{\varepsilon_r} = \sqrt{2 - \left(\frac{r}{R}\right)^2} \tag{22}$$

So, by manipulating ε_r, you are manipulating locally the wavenumber. The refractive index n falls from $\sqrt{2}$ to 1.

Eq. 22., contains the refractive index as a function of the distance from the centre, therefore we can infer that if n is constant, the differential equation becomes an equation of a straight line, but when n is a radial function and varies radially with the distance from the centre, reducing its value as it goes away from the centre. This shows that controlling the refractive index, we can make rays ben around it, and recalling S. Maci in [8], "*when a surface wave propagates on a planar* *inhomogeneous metasurface reactance, the curvilinear propagation path is in general accompanied by radiation*". Hence, how the Luneburg Solution can be used to create leaky-wave antennas.

E. H-Plane Sectoral Horn Antenna Radiator

The H-Plane directional radiator will act as an end-fire antenna. The shape of the horn, determines the shape of the field pattern. The ratio of the horn length, to the size of its mouth, determines the beam angle and directivity. In general, the larger the mouth of the horn, the more directive is the field pattern. Fig. 12 shows a H-plane sectorial metasurface horn design.



(a) *(b)* *(c)* *(d)*

Fig. 12. H-Plane Sectoral Metasurface Horn design in CST Microwave Studio®. (a)Patches Structure, (b)Radiation Pattern in dB units. (c)Radiation Pattern in linear units. (d)S-Parameters of the antenna.

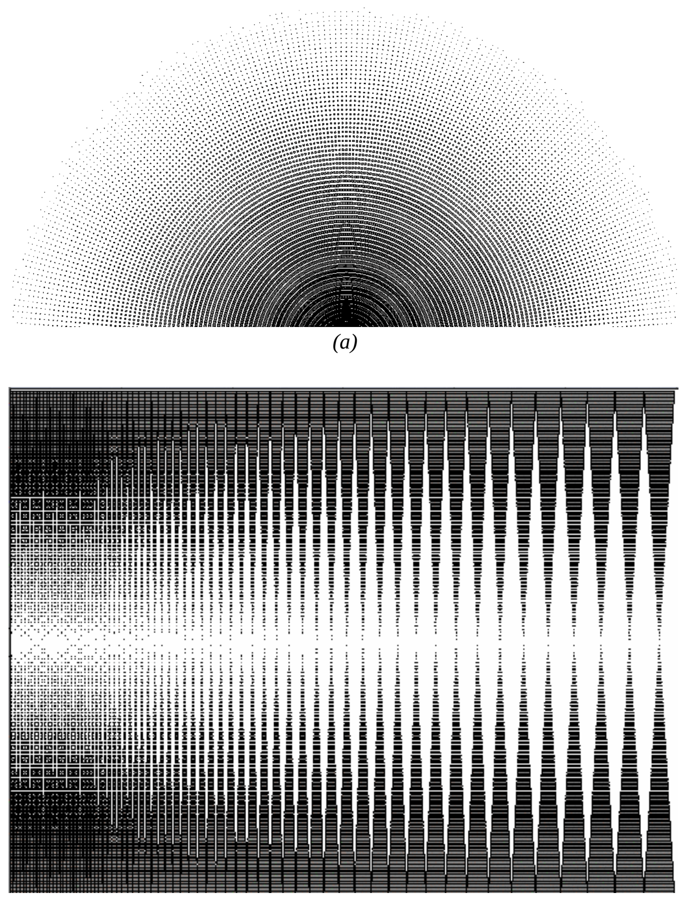
Directivity increases as the length is increased (from the mouth to the aperture). The slope of this linear trend becomes more gradual for longer lengths resulting in dwindling returns for increased length. The gain and efficiencies of the models presented in this paper are automatically calculated in CST Microwave Studio®, and shown in each graph. It can also be calculated from:

$$G_{max} = \eta_a D_{max} \tag{23}$$

In order to optimize the gain for both *pencil beam* ($10°\ HPBW$) and *wide beam* ($100°\ HPBW$), the dimensions of the implementation has to be changed.

Half-Power Beam Width is the angular separation in which the magnitude of the radiation pattern decrease by 50% (or -3dB) from the peak of the main beam, or the angular width within which the antenna is most sensitive.

Fig. 13. shows different types of leaky-wave antennas under investigation by the author.



(a) *(b)*

Fig. 13. Prototypes currently investigated by the author and supervisor. (a)Half Luneburg Lens Metasurface Antenna. (b)Refractive index variation prototype.

IV. CONCLUSIONS

In this paper, a very exhaustive overview of gradient refractive-index is presented. Design, calculations and simulations corroborate the theory proposed. The examples provided works accordingly to the objectives anticipated.

A deep comparison of step-index refractive index vs gradient-index has been presented. The use of gradient refractive index is recommended to obtain a high efficiency of transmission on the waveguide, and the antennas.

A higher gain antenna will be more directional. The dispersion characteristics of impenetrable metasurfaces can be derived, just depending on the equivalent quasi-static capacitance [14], this novel approach can be used in furtherance of this study, in favour of the design of metasurface planar lenses and transformation optics devices.

Planar optical devices and metamaterial cloaks [22]-[23], can be engineered from this analysis, meaning that the proposed theory can be extended as an answer to metamaterials transformation questions.

The confinement measured in the magnetic field plot is in concordance with the theoretical value.

CST Microwave Studio® contains optimisation techniques to enhance the performance of the designed elements in this paper to achieve a certain objective.

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VI. References

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