C O N C R E T E   A R C H E S.

By

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C O N T E N T S.

INTRODUCTION

The masonry arch has proved itself to be the best form of bridge for many purposes. It possesses that quality which is appreciated under most conditions, that of architectural beauty.

It is by far the most permanent of structures, for example, arches built by the Romans, 2000 years B.C., notably the Ponte Rotto, are still either wholly or partially in existence. Many of these arches contained a large amount of concrete, and often steel imbedded therein, and hence give a proof of the durability of both those materials under such conditions.

Concerning the superiority of concrete or masonry, over steel bridges, the most important considerations are as enumerated below.

(1). The life of the bridge.
A steel bridge has a life of 30 to 40 years, during which time constant attention is needed to prevent undue rusting, whereas a concrete bridge has an indefinite life, and should need no repairs whatsoever.

(2). Masonry or concrete arches are generally provided with a considerable thickness of filling between the track and the arch, and hence, in comparison with the well-known noisiness of steel bridges, they have great advantages. So much does this filling improve the conditions, that no impact allowance need be made in designing heavy masonry arches for live loads.

(3). Frequently, a concrete bridge can be built more cheaply than a steel bridge of the same quality, and on the whole, be built with less skilled labour, although requiring more careful overseeing.

On the contrary, concrete bridges cannot compete with light, temporary steel trusses such as are often used, chiefly on account of the large size of the abutments necessary for an arch, in comparison with those necessary for a light truss.

Many concrete arches which have been built, have been spoilt architecturally, and from an engineering standpoint, by attempts to make them appear to be masonry arches.
Especially is this so in the case of reinforced arches, which have, indeed, been cut to imitate masonry arches as well as actually faced with masonry, with the effect, that they appeared to be neither masonry nor concrete, and had not the fine, slender appearance of an undisguised concrete arch.

Such treatment as this must, surely, have worked contrarily to theory, as, to imitate masonry, a concrete arch must be made much heavier in the arch ring than theory demands.

On the other hand, such bold designs as the reinforced arch, of 300 ft span, over the Tiber at Rome, must have been a great stimulus to further efforts in mathematical design.

APPARATUS AND TESTS.

The arch tests described here, are intended to supplement those made by Mr. V.H.Adams in 1911 - 12, and described by him in his "Strength of Masonry Arches" with a few modifications and additions.

They consist of tests of concrete arches, plain, and reinforced, under varying, uniformly distributed, dead loads.

The arches, as before, have rigid ends, and were constructed of the following dimensions.

Span 10 ft. Rise 1 ft. Radius of Soffit 13 ft

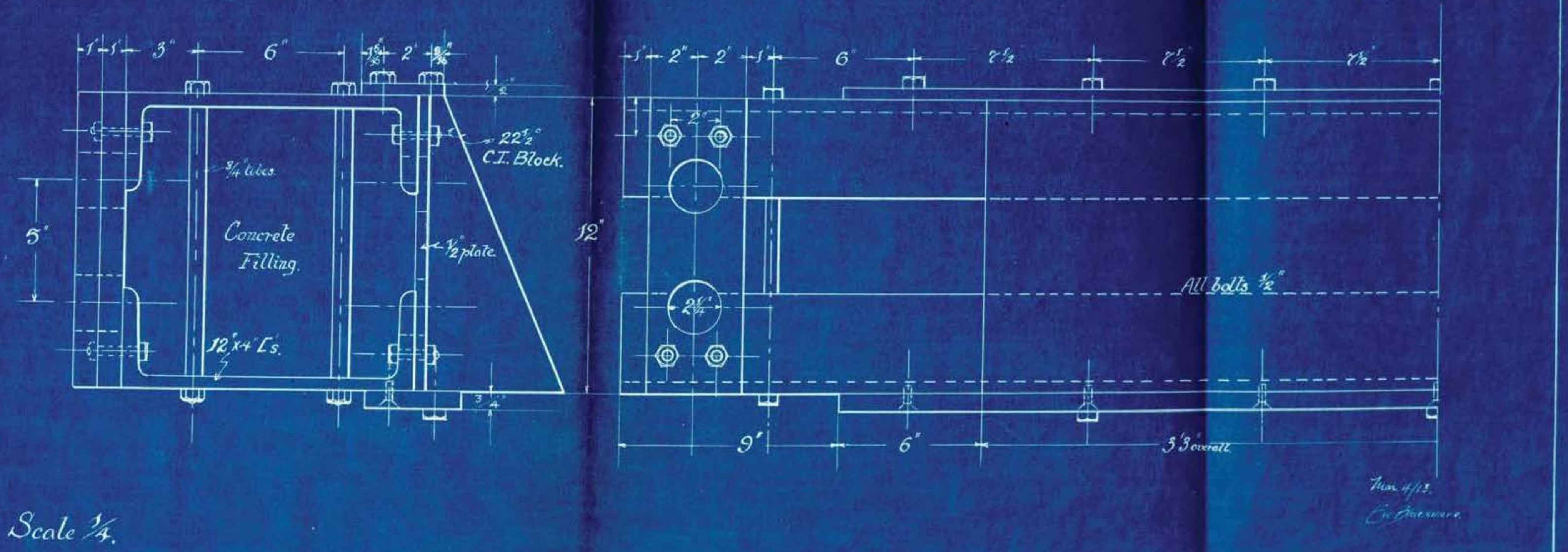
Width 3 ft overall.

In a research of this description, of necessity very little comparative work can be done in a short time, and hence it is the author's desire in carrying out these tests, to attempt to verify the elastic theory forthe particular type of arch experimented upon, especially considering the relation of horizontal thrust to loading.

The experimental arch is essentially of the form for use in large spans, being of small rise in relation to its span, (1/10th), and hence, the verification, or otherwise, of theory would be of service in designing arches of this important type.

PLATE 1.

SKEWBACK FOR 10FT. ARCH



SKEWBACKS.

As the skewbacks failed to take sufficient thrust designs were made for stronger ones, and these were constructed by the author in October 1912.

Each consists of two channels, 12" x 4" x 5'3" long, arranged withthe web horizontal, connected by four tie plates, which are drilled to allow four tie rods to pass through. The actual skewback face is that of a cast iron block 12 inches deep, roughly planed to an angle of 23^{o}, being normal to the soffit of the arch. This block is 3'3" long, and is so fixed that it may be moved sideways to accommodate skew arches. See Plate 1.

The accompanying drawing, Plate 2, shews the general arrangement of the apparatus.

It will be noticed that the skewbacks rest directly on the main joists, instead of on the 12" x 12" x 3'0" baulks which were previously used. In the tests these baulks were allowed to remain, as it was impossible to remove them, and lower the staging in the short time available for building the second arch.

It was decided that for future tests these blocks should be removed, and the skewbacks lowered into contact with the girders, so as to materially increase the stability of the apparatus.

To resist buckling, the skewbacks were raised into place, and filled with concrete.

Owing to the greatly increased weight of abutments a pair of brackets were added to the ends of the girders, and served to prevent any overturning of the 12" x 12" baulks. In the revised arrangements, these brackets are used as an extension to the main beams and the angle pieces turned round to provide a safeguard for the abutments, especially that one which rests on rollers.

The staging was also redesigned, and supported by angles, so as to take a much greater load from the derrick poles used for lifting parts of the apparatus.

It will be noticed on the drawing that the triangular cast iron beams have been replaced by 2" x 2" mild steel beams of square section, as they were deemed insufficient to transmit the necessary shear in loading.

HORIZONTAL THRUST.

The two tie rods previously used were stretched beyond their elastic limit, and hence are replaced by two pairs of 1.9/16" diameter, at 5" centres (bumped up to 2" at the ends and screwed) in order that their elasticity might

Extensometer Discs etc.

not be endangered.

These bars were calibrated for use in measuring horizontal thrust, and gave the following results:-

Bar	load for	.001" extension
1,2	600	lbs
3,4	640	lbs
5,6	640	lbs
6,7	670	lbs

Hence, an average extension of .001" on all four bars corresponds to a total thrust of 2,550 lbs.

The extensions of the tie rods were measured by means of an inside micrometer; measuring between special pegs which were fixed into clamps which carried a pair of direct reading.

Extensometers made by the author, and arranged to read in degrees. The arrangement is shewn in Plate 3, and accompanying photograph shews the clamps and extensometer discs.

These are designed to give a mean reading between a pair of rods, on a length of 100 inches, and hence are mounted at the centres of rocking clamps which are attached to the bars by means of pointed set screws thus providing hinged joints.

To one clamp is fixed the hinged end of a light tubular rod, the other end of which is pressed into contact with a steel roller or .500 in. diameter by means of a weight of 4 lbs.

The roller is mounted on ball bearings of the small-journal type, and is supplied with two diametrically opposed indicators, which are balanced in all positions, and move round a brass disc, graduated in degrees. This disc carries the ball bearings in a central boss and is mounted on the second rocking clamp, which is fixed to the bars with its pivots distant 100 inches from those of the first.

Circumference of .500 ins diameter circle

$$= 1.5708 \text{ inches}$$

$$\text{Hence } 360^{\circ} \text{ represents } 1.5708 \times 2550 \times 1000 \text{ lbs}$$

$$1^{\circ} \quad " \quad \frac{1.5708 \times 2550000}{360}$$

$$= \underline{11{,}127 \text{ lbs Horizontal Thrust}}.$$

Deflections.

The deflections of the Arch were measured by means of a finely graduated scale, fixed to a 3" x 3" angle, attached to the skewbacks, at the side of which rule moved a pointer fixed centrally into the crown of the arch.

The deflectograph used by Mr Adams, being deemed of small utility, was dispensed with in the following tests.

Theories of Arch Design.

The theoretical difficulties attending the satisfactory solution of the problem of the stability of a masonry arch are, perhaps, the greatest the engineer has to encounter in direct design.

In the first place, the elementary data, i.e. the method of loading, and the behaviour of the material under simple loading are both indeterminate.

The following seven indefinite points further illustrate the care required in any theoretical treatment.

1. The amount and distribution of external forces (effect of filling etc.)

2. The true line of resistance obtained from the above.

3. The adhesion of mortar and its effects. The elasticity of materials etc.

4. The uncertain and variable strength of masonry.

5. The quality of bedding of joints, and general workmanship.

6. Effect of striking centring.

7. Spreading of abutments, and settlement of foundations.

A voussoir arch may fail in the following ways:-

1. By direct crushing - This depends on the intensity of stress normal to any section.

2. By shear - This causes sliding of adjacent voussoirs, and is counteracted by the adhesion of the mortar, and friction of the joints. The allowable obliquity being generally 17 degrees at any joint. (17° being a conservative value of the angle of friction between adjacent joints.)

3. By opening of joints. - This is caused by bending in addition to direct compression, and only occurs when the line of stress passes without the arch ring, but this is not the allowable limit in design, as, if the line of stress passes without the middle third of the arch ring, a tension is set up in the mortar, thus breaking the bond between the voussoirs and hence much impairing the shearing strength of the joints.

Most of the above difficulties, and methods of failure apply to concrete arches. Certainly much more is known of the strength of concrete than of masonry, and although concrete construction requires very careful overseeing to obtain consistent results, it is more uniformly strong than masonry, and when reinforced, especially, is entirely free from the difficulties attending joints, and may even be designed to resist bending as in a beam, if sufficient steel is added to withstand the tensile stresses induced.

Concerning the theories to be treated here, it is proposed to deal with those referring to light, flat, arch ribs, such as are now being much constructed, more than with those referring to the more massive structures, with earth filling. These latter, it may be mentioned, very seldom fail by either of the three methods above mentioned, but fail by cracks induced by settlement of foundations, and hence may be safely designed by such methods as Alexander and Thomson's Transformed Catenary Method.

Arch ribs may now be safely designed by the elastic formulae when the

1) joints are thin

2) lateral spandrel walls are thin

3) arch is protected from horizontal earth pressure above the abutments by means of retaining walls.

For the above reasons, the elastic theory will be chiefly considered here.

The so called "Line of Stress" theories may be dismissed here, as although very ingenious in avoiding mathematical difficulties, the assumptions made (and taken as first principles), in their evolution, are obviously unwarranted, and take the most difficult part of the problem namely, the position of the line of stress, for granted.

A comparison of the strengths and efficiencies of arches of three materials, with a safety factor of 10, will shew how various are the values of the materials used.

1) Brick	2) Sandstone	3) Granite

The crushing strengths are

154,000	575,000	and 1,350,000

pounds per square foot, and the weights

112	140	and 164

pounds per cubic foot, respectively.

The efficiency depends on $\frac{\text{Strength}}{\text{weight}}$

and if brick be taken to have an efficiency of 1/6, Sandstone has one of 1/2, and granite 1. Here cost has not entered, and it is a known fact, that Brick Arches, of good bricks, are very durable and satisfactory.

A simplification of theories may be made on small arches, as whereas large spans are mostly dependent on stability, small arches, as culverts in embankments etc., are dependent on crushing strength, and may be designed by considering that the line of stress is very close to the middle ninth of the arch ring, and the stress is almost uniform over the section.

The elastic method was first used by Thacher, in America. In 1894, a highway bridge, over Rock Rapids, Iowa, of 30 feet span, was erected, being the first such concrete arch for heavy travel in America.

Present Day Practice.

The general tendency of late years, for railroad bridges, has been to use solid arch rings of concrete, reinforcement being introduced, not so much to resist tension, as to bind the materials into a solid monolith.

Railroad bridges require weight to be secure against heavy, rapidly moving loads, and hence solid spandrel filling, or a cushion of 4 or 5 feet of earth and ballast, with open spandrels, has been in favour.

For architectural effect, for highway and the lighter types of bridges, tension reinforcement has been used, some of the best examples being found in parks. For such, open spandrels, with projecting sidewalks, have been most used, and ribbed arches, sometimes with cantilever ends, have been found economical.

The most recent large arches, are constructed with two shallow, slab-like arch rings, set at 10 to 15 feet apart, and connected to form a roadway with some form of reinforced flooring.

A fine example of design, on the elastic theory, is to be found in the projected Hudson Memorial Bridge, for New York. The general arrangement of this arched bridge is as follows:-

One central arch of 703 ft span, with seven semicircular end spans of 108 feet, the central ground clearance being 183 feet. The main arch consists of twelve ribs, and will contain 8,500 tons of steel to resist both bending and compression.

Two Decks. Top. 50 ft roadway and two 15 ft sidewalks.

Lower. (to be omitted until needed) 70 ft wide to accommodate 4 lines of electric railway.

Main piers to be 80 feet wide.

Weight of falsework for construction will be 100,000 tons, giving an average pressure of 2 tons per square foot, on the whole surface under the main arch.

The cost of the whole structure is estimated at £800,000.

Such an achievement as this will be certain to establish the theory by which it was designed, and will undoubtedly serve as a fine example for future design.

The Elastic Theory.

The elasticity of a non homogeneous material such as stone or bricks and mortar, has been very little tested, and hence there is no reliable data on which to base the so called Elastic Limit of such materials.

From arch tests made at Birmingham University, a practical limit of $\frac{1}{4}$ of the crushing strength of the bricks would seem to be a conservative assumption.

Brick arches up to the present have been designed with a safety factor of 8 to 10, and seldom fail by crushing, so that, apparently, it is safe to design brick arches on such an elastic basis as is indicated above.

Of the elasticity of Plain Concrete, very much more is known, and as a Straight Line Theory is assumed for its elasticity under load in designing Beams, with a factor of safety of 4, so may we assume this theory in designing arches, where tha factor of safety is greater.

From this one would expect that in testing such an arch elastically, the results would agree with the elastic theory up to about $\frac{1}{4}$ of the crushing strength of the composite material, and then would disagree in an increasing ratio until failure, thus justifying the use of an Elastic Theory in design.

One noticeable point in the various forms of such theories is that, the greatly variable modulus of elasticity does not enter into the formulae where the slight deflections of the arch are a secondary matter.

The basis of all these theories is found in the five conditions dependent on the elasticity and method of construction of an arch.

These conditions will now be indicated, together with their applications in the form of formulae, and graphical methods of finding the all important horizontal thrust and hence the lines of stress for simple loading, and by superposition, for more complex loading.

The masonry arch rib is of the type such as are assumed wholly continuous and fixed at the abutments, i.e., are capable of sustaining a moment and a thrust there.

— The Elastic Theory.

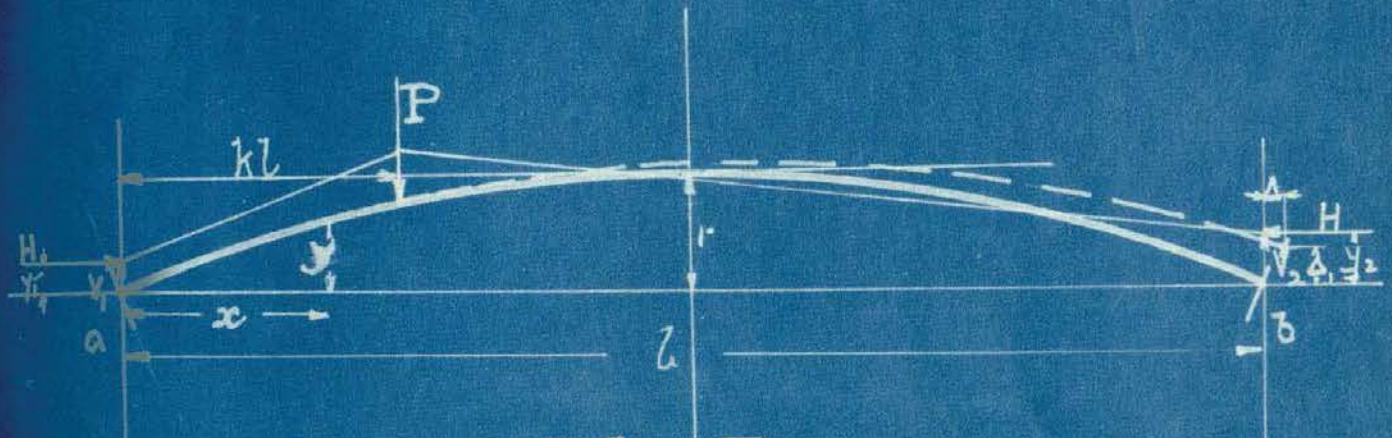


Fig. I.

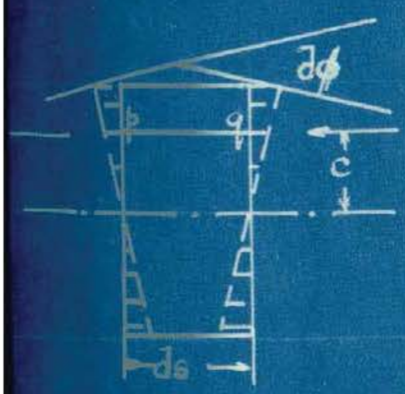


Fig. II.

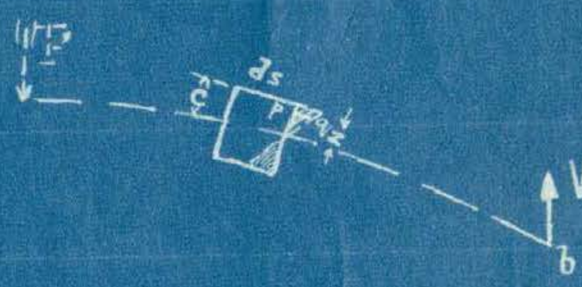


Fig. III.

Fig. I. represents the Centre Line of an Arch Rib, the distorted form under Loading, as described below, is represented by the broken Line.

P is a single load, distant kl from the left abutment.

x is the ordinate of the arch, and y the abcissa.

H is the horizontal thrust of the arch for the loading P.

V_1 & V_2 are the Vertical Components forming with H

R_1 & R_2, the reactions, which act at some distance from the Centre Line of the Arch, the abcissae y_1 & y_2 giving points in their lines of action.

First Condition. (Static) Taking moments about point a.

$$Pkl - V_2 l + H(y_1 - y_2) = 0 \quad \text{------------} \quad 1.$$

Second Condition (Static) Equilibrium of Vertical Forces.

$$V_1 + V_2 - P = 0 \quad \text{------------} \quad 2.$$

Third Condition

The tangents at a & b do not alter in direction, because arch is rigidly fixed. Consider an element of the rib, ds, Fig. II. This is deformed from having parallel sides, so that it becomes as shown, similar to a beam subjected to bending moment.

Let $d\phi$ be the inclination, and hence the change in direction, of the tangents at the bounding sections.

The stress at any section such as pq, is proportional to $c \times d\phi$, (the elongation.)

The elongation is $\frac{Mc}{I} \cdot \frac{ds}{E} = c\,d\phi$ (M is the bending moment at the section.) I the moment of Inertia & E the Modulus of Elasticity.)

Cancelling c, and integrating $\phi = \int \frac{M ds}{EI} = 0$ ------------ 3.

Fourth and Fifth Conditions.

H must be such, and y_1 & y_2 such that there are no horizontal or vertical deformations at the abutments.

Let M' be the bending moment at x, y, due to vertical forces only, and M'' that due to horizontal forces only.

Let Δ = Horizontal & Δ_1 Vertical displacements of b, if the end is assumed free to move, & then replaced by the action of H & V_2

Consider a small section of the arch as shown in Fig. III.

Horizontal Force $H = 1$ applied to the arch at b will produce a horizontal deflection Δ, and vertically Δ_1.

Work of deformation, Horizontally $= \frac{\Delta}{2}$

" " ", Vertically $= \frac{\Delta_1}{2}$

These must be equal in value to the internal work done by the flexural stresses

Fibre stress due to vertical forces $= \frac{M'z}{I}$

" " " " horizontal " $= \frac{M''z}{I}$

On a small area "a", Stress due to vertical forces & also horizontal forces

$$= \frac{M'az}{I} + \frac{M''az}{I}$$

being due to the elongations e, e' & e'', which are due to $H=1$, $H-1$, & P resp.

Work of deformation due to $H=1$, $= \frac{1}{2}\frac{M'aze}{I} + \frac{1}{2}\frac{M''aze}{I}$

The bending Moment due to force unity is $y \times 1$, i.e. is represented by the ordinate y, and hence "e" may be written $\frac{yaz}{I} \times \frac{ds}{aE}$

and work of deformation

$$\frac{1}{2}\frac{M'yaz^2ds}{EI^2} + \frac{1}{2}\frac{M''yaz^2ds}{EI^2}$$

Integrating over the section $(\int az^2 = I)$ obtain

$$\frac{1}{2}\cdot\frac{M'yds}{EI} + \frac{1}{2}\frac{M''yds}{EI}$$

and over the whole rib

$$\Delta = \sum \frac{M'yds}{EI} + \sum \frac{M''yds}{EI} \text{ - - - - - - - - - } 4.$$

similarly for vertical forces

$$\Delta_1 = \sum \frac{M'xds}{EI} + \sum \frac{M''xds}{EI} \text{ - - - - - - - - - } 5.$$

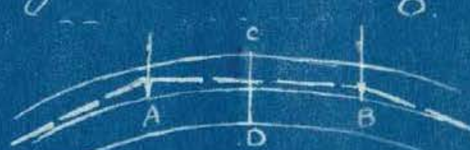
thus giving the fourth and fifth conditions.

Now, obviously M'' is always equal to $-H(y_1-y_2)$, also M' is known from the Loading conditions; being, with a simple load P, $V_1 x$ to the left of P, and $V_1 x - P(x-kl)$ to the right of P. If these values be introduced, integration furnishes three equations, giving V_1, H and y_1 & hence V_2 & y_2.

The formulae above found may be simplified & applied by dividing the arch into sections ∂s such that $\frac{\partial s}{I}$ is constant, and also by transferring the origin of co-ordinates to the crown, and considering that point.

If CD be a section at the crown, and A & B two loads, one on either side thereof, the Horizontal Motions, due to bending moments, on either side, are equal, and hence

$$\sum_c^B \frac{My\,ds}{EI} + \sum_c^A \frac{My\,ds}{EI} = 0 \quad \text{--- } 6.$$



Similarly Vertical Motions give

$$\sum_c^B \frac{Mx\,ds}{EI} - \sum_c^A \frac{Mx\,ds}{EI} = 0 \quad \text{------ } 7.$$

Changes in direction of the crown tangent are equal and opposite

hence

$$\sum_c^B \frac{M\,ds}{EI} + \sum_c^A \frac{M\,ds}{EI} = 0 \quad \text{---------- } 8.$$

Now

$$M = M_c - V_c x + H_c y - M_R \quad \text{between } A \,\&\, C \text{ ---------- } 9.$$

C referring to the crown, & M_L & M_R the moments of all the forces from the point x, y, to the crown, about that point — also

$$M = M_c + V_c x + H_c y - M_L \quad \text{between } B \,\&\, C \text{ ------ } 10$$

Hence from 6, 7 & 8 are obtained (m being the no. of divisions of the arch ÷ by 2)

$$2M_c \Sigma y + 2H_c \Sigma y^2 - \Sigma M_R y - \Sigma M_L y = 0 \quad \text{-- } 11.$$

$$2mM_c + 2H_c \Sigma y - \Sigma M_R - \Sigma M_L = 0 \quad \text{--- } 12$$

$$2V_c \Sigma x^2 - \Sigma M_L x + \Sigma M_R x = 0 \quad \text{--- } 13$$

Combining 11 & 13 $$H_c = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R \Sigma y - \Sigma M_L \Sigma y}{2[m\Sigma y^2 - (\Sigma y)^2]} \quad \text{--- 14}$$

13 gives. $$V_c = \frac{\Sigma M_L x - \Sigma M_R x}{2\Sigma x^2} \quad \text{--- 15}$$

& 12 $$M_c = \frac{\Sigma M_R + \Sigma M_L - 2H_c\Sigma y}{2m} \quad \text{--- 16}$$

The effect of rib shortening, and thrust due to changes in temperature may be found in exactly the same way, but the above treatment indicates sufficiently the method followed, and it is now proposed to analyse the arch under test for

(1) A uniform load, all over the arch

(2) A uniform load, halfway over the arch

Analysis of Loaded Arch.

Superload 1940 lbs. sq. foot.

Arch 60 " " " assumed uniform.

2000 " " "

Span 10 ft. Rise 1 ft Thickness 6" uniform.

Radius of Centre Line. 13 ft. 3 ins.

I is uniform & hence the arch divisions are equal & are 10 in number.

$$m = 5.$$

x	x^2	y	y^2	M_L (1)	M_L (2)	$M_L x$ (1)	$M_L x$ (2)	$M_L y$ (1)	$M_L y$ (2)
·54	·292	·10	·01	292	292	157	157	29	29
1·58	2·494	·18	·0324	2494	2494	3940	3940	450	450
2·63	6·909	·35	·1225	6909	6909	18190	18190	2420	2420
3·64	10·324	·59	·3471	10324	10324	37600	37600	6100	6100
4·62	21·040	·90	·8100	21040	21040	97300	97300	18936	18936
Totals:	41·059	2·12	1·3220	41059	41059	157187	157187	27935	27935

(1.) $$H_c = \frac{5 \times 27935 \times 2 - 41059 \times 2.12 \times 2}{10 \times 1.322 - 2 \times 4.492} = 24{,}920 \text{ lbs.}$$

as $M_L = M_R$ $\qquad V_c = 0$ lbs.

$$M_c = \frac{2 \times 41059 - 2 \times 24{,}920 \times 2}{2 \times 5} = -2{,}328 \text{ lbs feet.}$$

$$\text{Eccentricity} = \frac{-2{,}328 \times 12}{24{,}920} = -1.122 \text{ ins.}$$

Ratio of thrust to load = 1.22.

(2) $M_R = 0$ & all other quantities being as before,

H_c = 12,460 lbs. $\qquad M_c = -1{,}164$ lbs feet.

Eccentricity = -1.122 ins.

$$V_c = \frac{157187}{2 \times 41.059} = 1{,}915 \text{ lbs.}$$

Ratio of thrust to load = 1.22

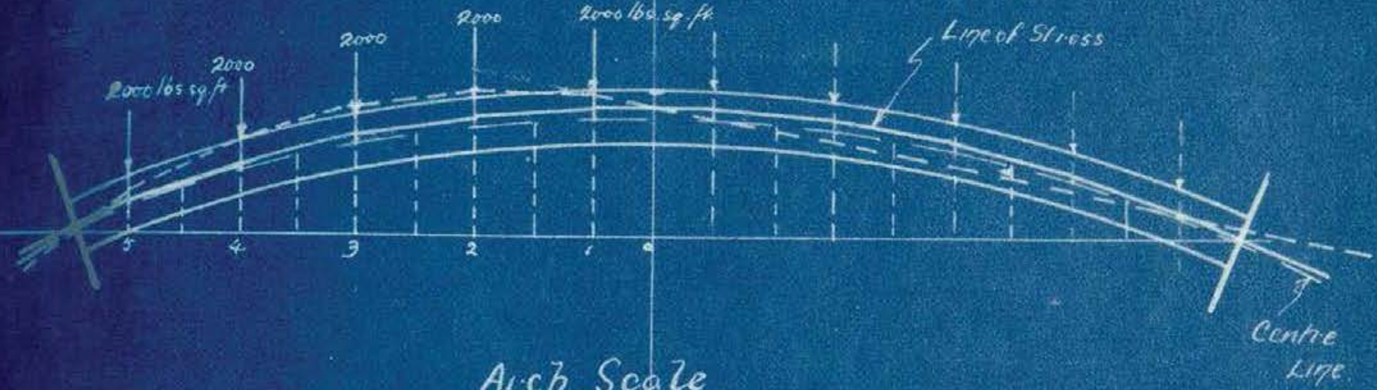
The above values are taken out for 1 ft. width of arch.

H gives the polar distance in the polar diagram for the line of stress, & the eccentricity gives a point in the line of action of the thrust at the crown, V_c gives the vertical position of the pole, & hence the line of stress may be drawn.

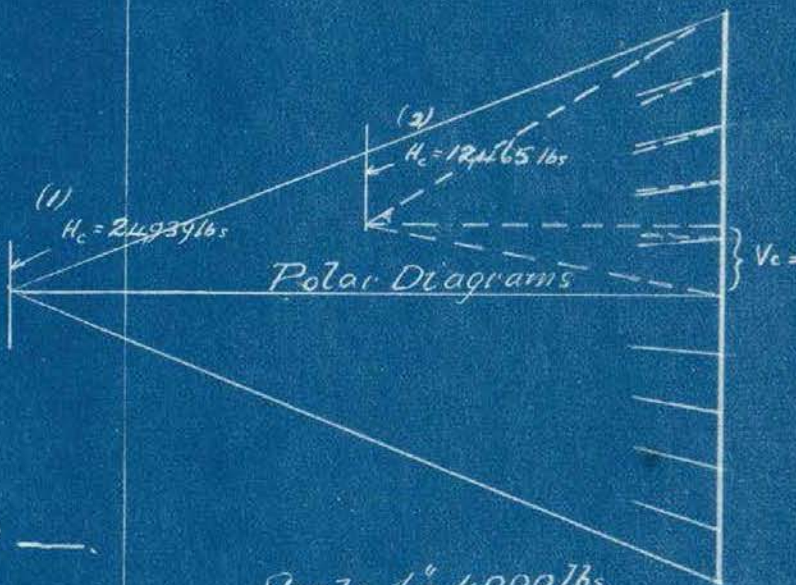
The two lines of stress are shewn in the accompanying diagram Fig.

The Bending Moment at any section is the product of the thrust at that section, & its eccentricity.

The Thrust is obtained at any point by drawing a line in the polar diagram from the pole, parallel to the line of stress at that point.



Arch Scale
$\frac{1}{2}'' = 1$ foot.



(1) Line of Stress, is full line ——.

(2) " " " , is broken line - - - -.

Scale 1" = 4000 lbs.

Weight of Arching neglected for Load half over.

Analysis of Loaded Arch.

When the foregoing methods are applied to Reinforced Concrete arches, of varying thickness, the difficulty immediately encountered, is that of determining the Moment of Inertia.

Using the ordinary, "no tension" method, obviously, the moment of inertia of an arch not wholly in compression, varies at any section according to the eccentricity of the normal thrust, and hence renders the problem indeterminate, and a solution is only possible by successive trial, first assuming the moment of inertia to be that of the whole section, and dividing the arch up into sections such that, as before, S/I is constant.

From the results this obtained, a curve for S and $\frac{1}{I}$ should be drawn, and graphically divided into equalaieas (I being calculated according to the eccentricity of stress), t thus giving a closer approximation to the true division of the arch axis.

A comparison of the results obtained with the new division of the arch axis, and the former will give an indication of the necessity, or otherwise, of proceeding to another approximation in the same manner.

With heavy rolling loads, in many cases the first approximation will be found sufficiently accurate.

ARCH TESTS.

The results obtained by Mr Adams in 1911 - 12, are given in the following tables, and indicated in the following graphs, for purposes of comparison.

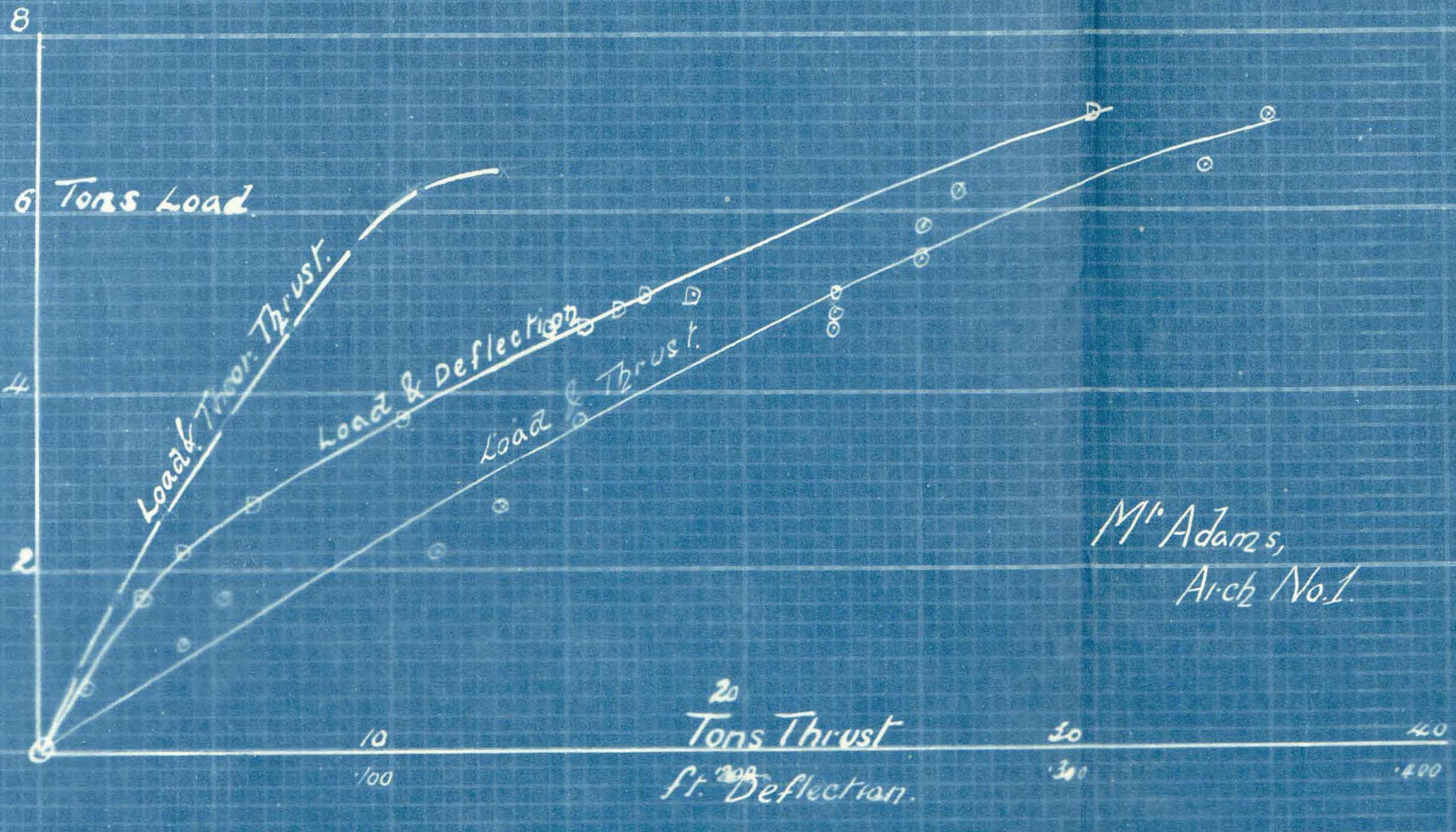
Arch No. 1. Tested by Mr Adams.

$4\frac{1}{2}$" brick arch, of wirecut bricks laid with thin sand joints, all stretchers.

Width of arch $3'1\frac{1}{2}$". Span 10' 0"
Rise 1'0" Radius of Soffit 13'0"

Centres lowered at 4 days.
Tested at 7 days.

Load tons	Thrust tons	Deflection. feet.
.7	1.31	–
1.2	4.16	–
1.7	5.38	.031
2.2	11.40	.042
2.7	13.10	.063
3.7	15.45	.104
4.7	22.6	.146
4.7	22.6	.152
4.9	22.6	.166
5.1	22.6	.177
5.1	22.6	.189
5.47	25.1	–
5.83	25.1	.199
6.23	26.2	.230
5.24	25.1	.230
5.72	27.4	.240
6.10	30.9	.260
6.49	33.4	.281
7.04	35.2	.302


8
6
4
2
Tons Load
Load & Theor: Thrust.
Load & Deflection
Load & Thrust.
Mr Adams,
Arch No.1.
10
20
30
40
Tons Thrust
·100
·200
·300
·400
ft. Deflection.

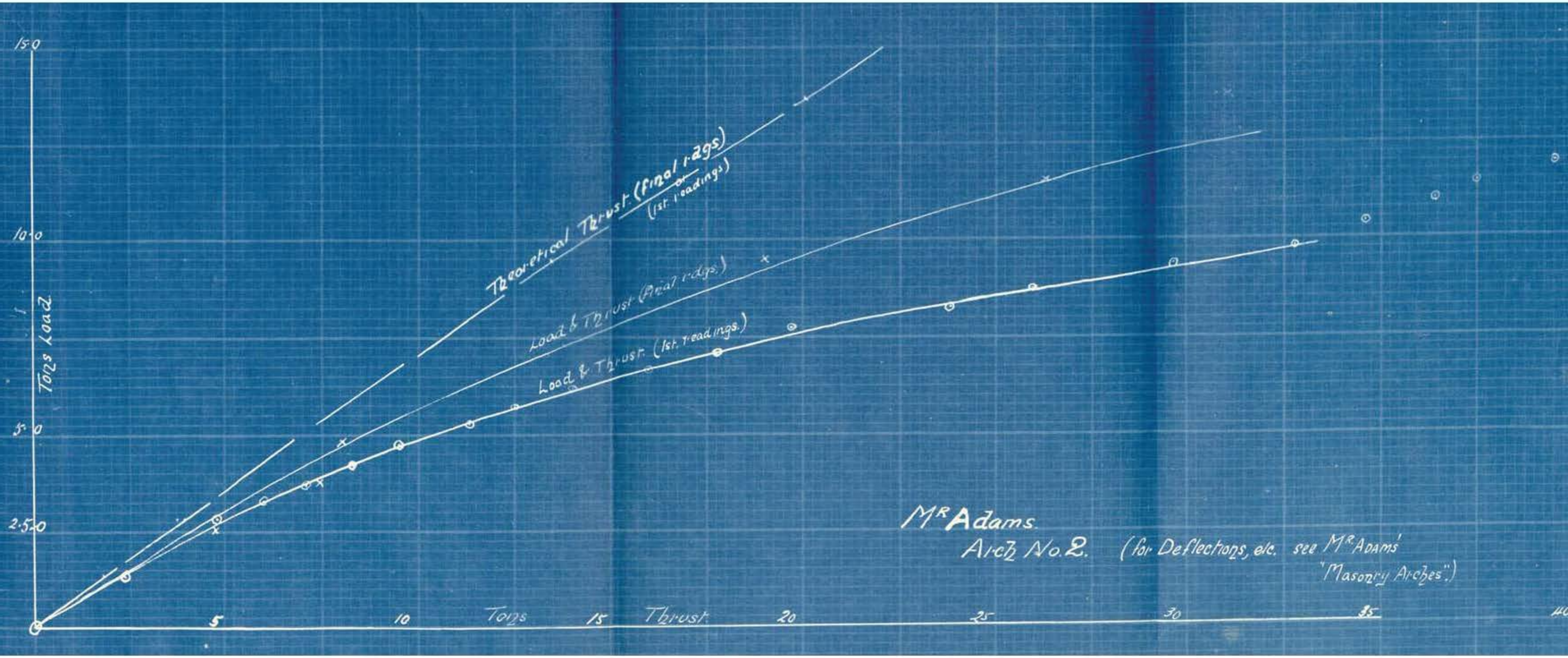
Arch No. 2 Tested by Mr Adams.

Similar to No. 1 with 1.3 P.C. Mortar joints, not greater than ½".

Centres struck at 28 days.
Tested at 109 days.

Load tons	Thrust tons	Deflection feet.
0	-	-
1.37	2.38	-
2.82	4.79	.012
2.82	4.79	.012
3.26	5.95	.018
3.71	7.16	.025
4.23	8.35	.040
4.76	9.53	.047
5.22	11.30	.053
5.70	12.5	.057
6.17	14.0	.057
6.65	16.1	.067
7.15	17.9	.067
7.75	19.7	.067
8.24	23.8	.070
9.30	29.7	.073
9.85	32.8	.073
10.4	34.6	.077
10.95	36.4	.077
11.40	37.5	.080
11.90	39.4	.087

Load tons	Thrust tons	Deflection feet.
2.82	4.79	.053
3.71	9.55	.060
4.76	12.25	.060
5.70	15.2	.063
6.65	17.9	.067
7.75	20.8	.070
8.75	22.6	.070
9.85	24.4	.070
10.95	26.8	.075
11.90	29.1	.077


Mr Adams.
Arch No. 2. (for Deflections, etc. see Mr Adams' "Masonry Arches".)
Theoretical Thrust (final or 1st. readings)
Load & Thrust (final rdgs.)
Load & Thrust (1st. readings.)
Tons Load
15.0
10.0
5.0
2.50
Tons Thrust
5
10
15
20
25
30
35
40

Arch No. 3 Tested by Mr Adams.

9½" Brick arch, of wire cut bricks laid with 1:3 P.C.

Mortar joints ½" thick.

All Stretchers Dimensions as Nos. 1 & 2

Strength of Bricks 169.5 tons sq.ft

Centres struck at 8 days.

Tested at 46 days.

Load tons	Thrust tons	Deflection. feet.
0	-	-
1.37	2.39	0
2.82	4.78	.001
2.82	4.78	.001
3.71	5.50	.001
4.76	5.50	.001
5.70	5.95	.002
6.65	7.71	.003
7.75	8.65	.004
8.75	9.31	.005
9.85	10.7	.006
10.95	11.94	.007
11.90	13.12	.009
12.84	14.16	.010
13.83	15.8	.011
14.8	16.05	.012
15.8	16.61	.013

Load tons	Thrust tons	Deflection. feet.
2.82.	4.78	.004
3.71	5.45	.004
4.76	7.71	.005
5.70	9.55	.006
6.65	11.34	.007
7.75	13.13	.008
8.75	14.16	.009
9.85	15.80	.010
10.95	16.61	.010
11.90	18.41	.011
12.84	19.20	.012
13.83	19.80	.013
14.81	21.40	.014
15.80	21.84	.014
2.82	4.77	.005
10.60	19.7	.005
11.55	20.8	.018
14.80	23.8	.021
18.70	25.6	.025
20.60	25.6	.027
23.50	28.0	.028
26.20	30.9	.030
29.30	33.5	.033
32.30	37.1	.037
35.70	41.1	.041
39.20	44.7	.056
42.40	52.2	.118

The Tests of Arch No.1

Dimensions of Arch -

Span 10'0"

Rise 1'0"

Radius of soffit 13'0"

Thickness, (constant) 6"   Width 3'0"

Concrete

1:2:5 P.C. Concrete made with unwashed Moseley Gravel, and "Ferro-crete" cement.

Strength of concrete blocks tested at same date as arch .67 tons per square inch in compression.

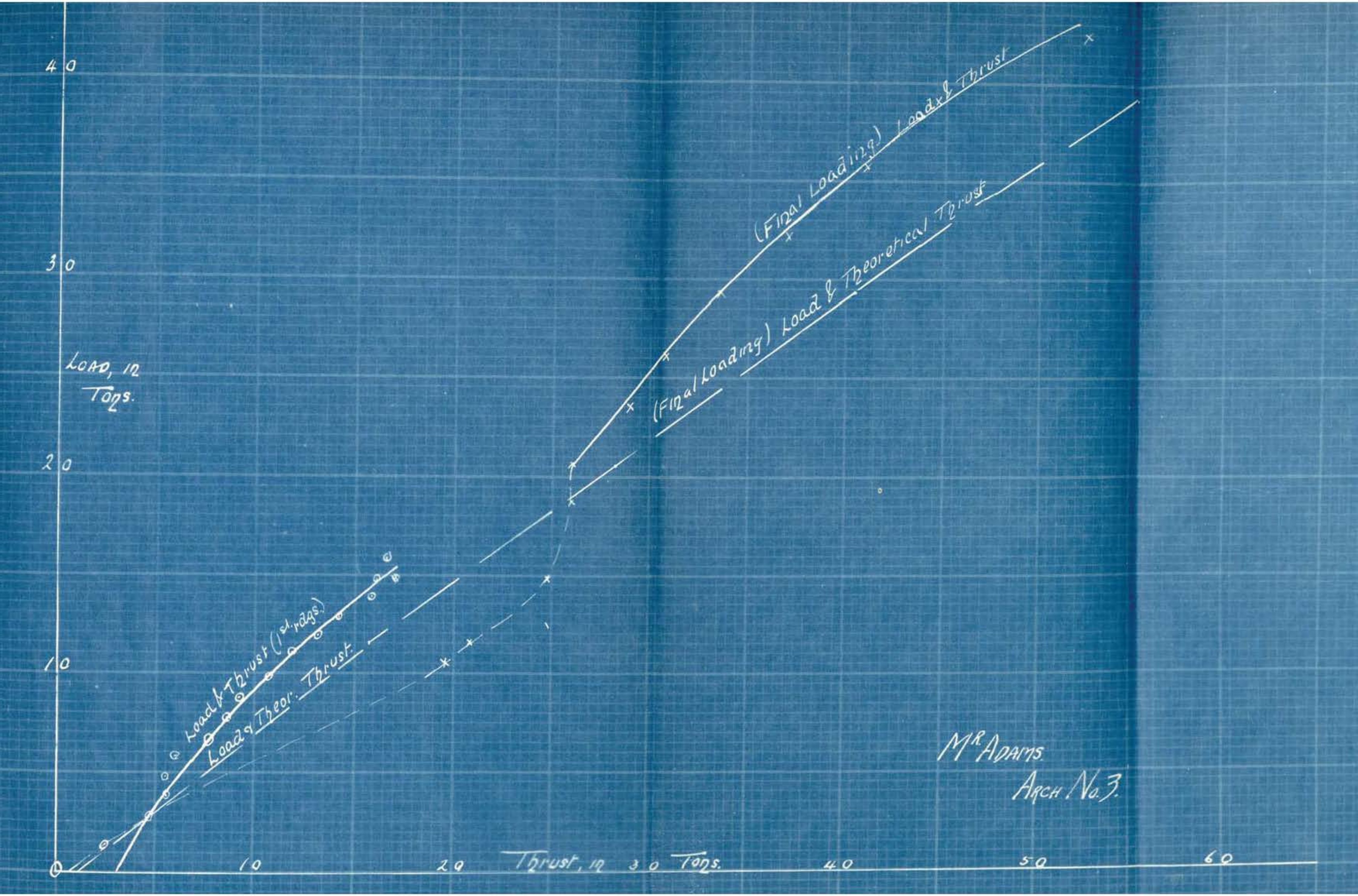
The cement complied with the British Standard Specifications, giving the following strengths.

Neat Cement at 7 days	530	lbs
" " " 28 "	730	lbs

1.3 mortar, with Standard Leighton-Buzzard

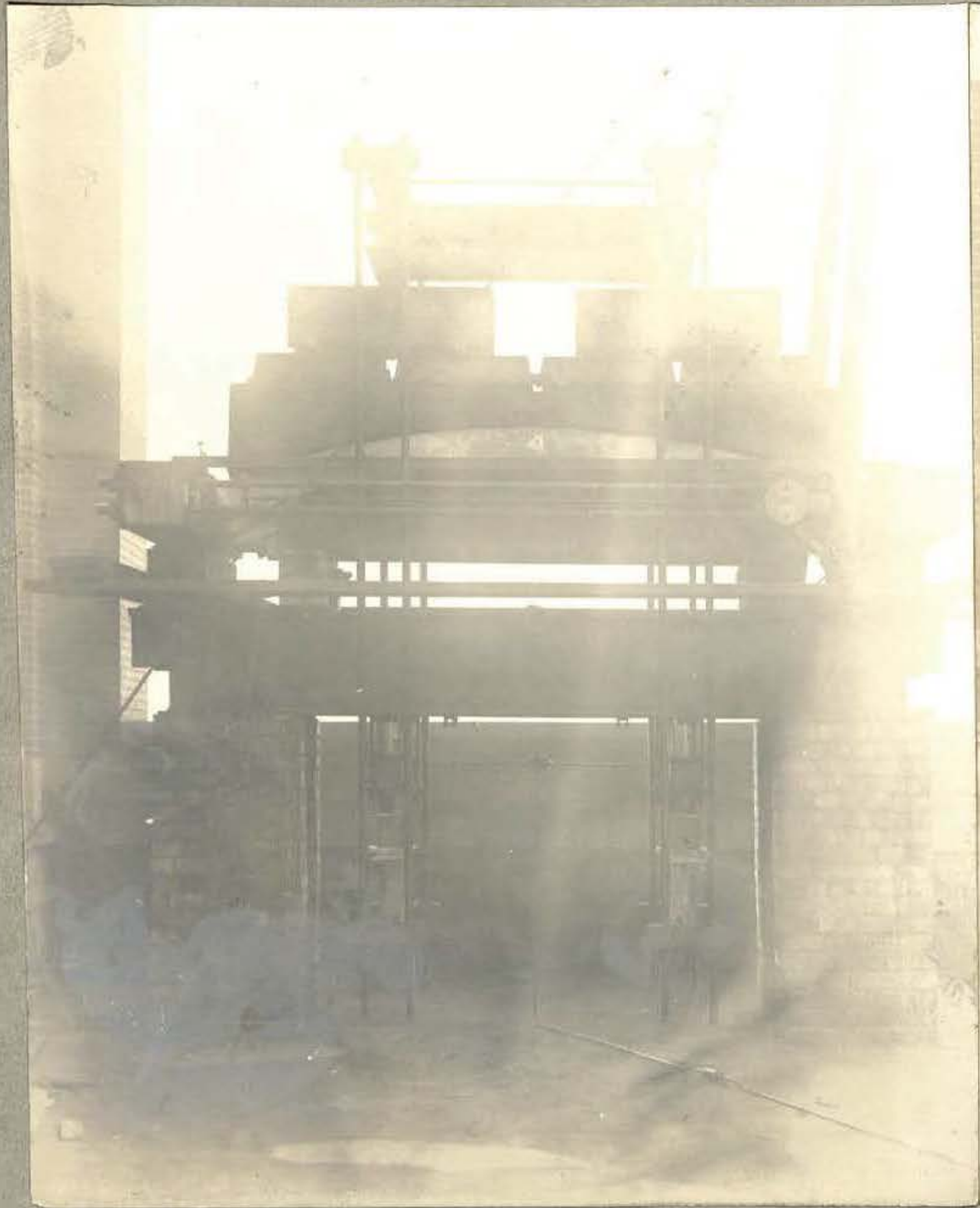
Sand, at 7 days	190	lbs
at 28 days	280	lbs

Arch constructed	Dec. 10th
Centres dropped	Dec 24th
Arch tested	February 13th and 14th
Mean interval	65 days.

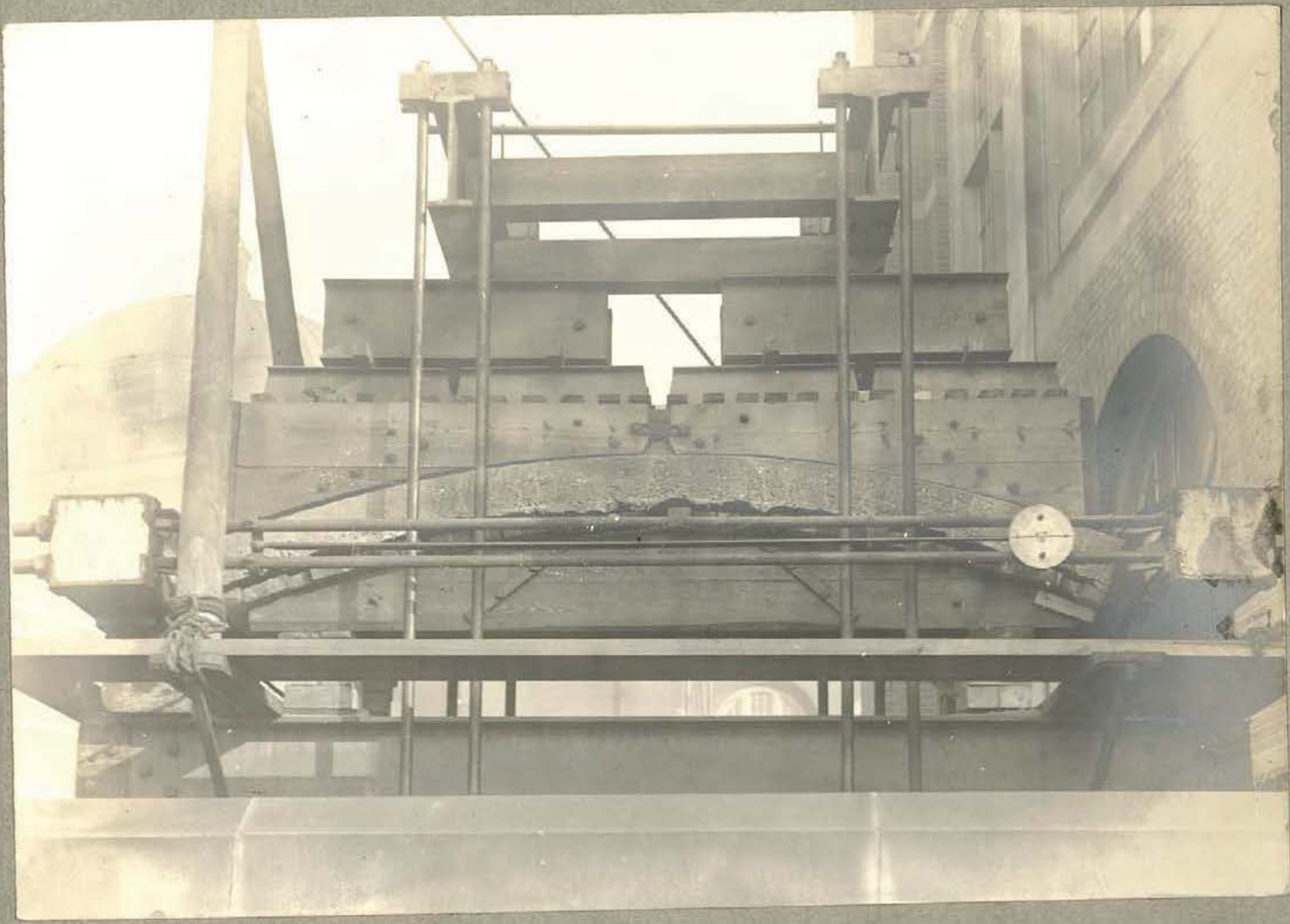


Load, in Tons.
Thrust, in Tons.
0
10
20
30
40
10
20
30
40
50
60
Load & Thrust (1st. ldgs.)
Load & Theor. Thrust.
(Final Loading) Load & Thrust
(Final Loading) Load & Theoretical Thrust
Mr. Adams.
Arch No. 3.

Arch No. I.

General Views of Apparatus.



— Arch No. 1 —

The arch was successively loaded at intervals of about 3 tons, the load being eased off after a load of 45 tons, was reached, and then put on again in somewhat larger increments than before.

Readings, of

1. Rise at crown,

2. Extension of rods on both sides, measured by (a) Inside micrometer (b) Disc extensometers

were taken for every successive load.

The value of thrust, rise, and load, thus obtained, are tabulated, and also graphs drawn shewing the relations existing between these quantities, and also load and deflection.

At a load of 18 tons, a tension crack appeared in the underside of the arch, exactly at the crown, extending for nearly $\frac{3}{4}$ of the thickness of the arch ring, indicating distinctly that the normal stress at that section was very eccentric.

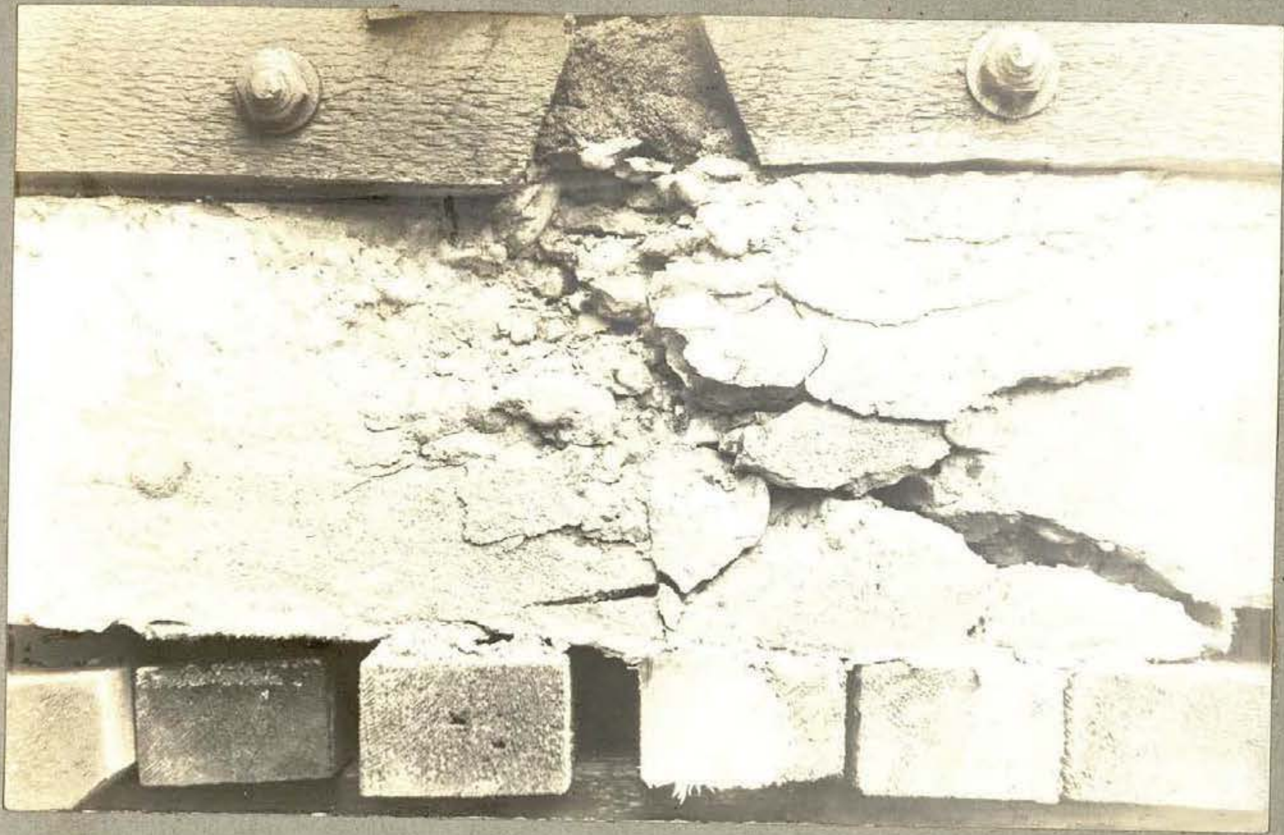
At a load of 45 tons the arch commenced to crush at the crown, and at 53.2 tons, complete failure occured, with a horizontal thrust of over 90 tons, by complete crushing at the crown, and also longitudinal splitting, induced by the wedge action of concrete when crushing as in a column.

The fractures are clearly shewn in the accompanying photographs.

Slight crushing was also noticeable at one of the haunches, near to the underside.

The maximum deflection before failure was .76 inches.

The following graphs shew the relation between Load, Thrust and Deflection etc., and also give a comparison of results with the thrust, found by assuming the arch parabolic, as it nearly is, and taking the thrust as equal to $\frac{Wl}{8R}$



FRACTURE OF ARCH No. 1.

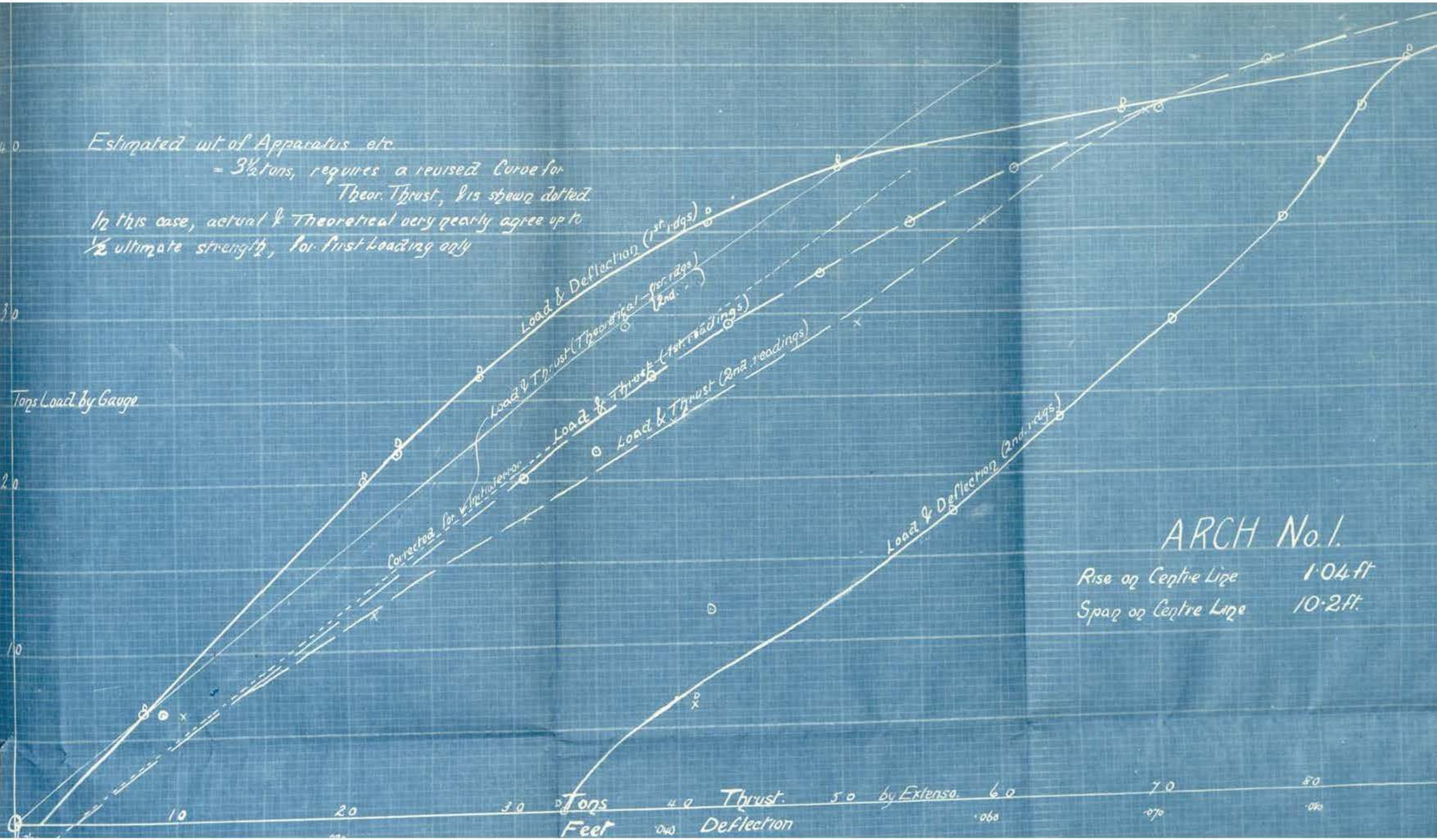
Arch No. 1

Load tons	Thrust tons	Deflection. feet
0	-	-
6.41	9.10	.008
20.16	30.7	.021
21.84	35.2	.023
26.47	38.6	.028
29.4	43.2	.037
32.5	48.9	.034
35.6	54.5	.042
38.9	61.4	.050
42.2	70.4	.068
45.1	77.2	.086
0	-	.033

Arch No. 1

Load Tons	Thrust Tons	Deflection Feet
0	–	–
6.41	10.2	.008
12.05	21.6	.009
17.80	30.8	.024
23.54	39.8	.031
29.40	51.2	.038
35.60	59.2	.045
42.20	69.5	.050
48.00	88.8	.063
0	–	.005
0	–	.043

53.20 Failure by complete crushing at the crown.


Estimated wt. of Apparatus etc.
= 3½ tons, requires a revised Curve for
Theor. Thrust, & is shewn dotted.
In this case, actual & Theoretical very nearly agree up to
½ ultimate strength, for First Loading only
Tons Load by Gauge
Load & Deflection (1st rdgs)
Load & Thrust (Theoretical - first rdgs / 2nd)
Load & Thrust (1st readings)
Load & Thrust (2nd readings)
Corrected for wt. of Apparatus
Load & Deflection (2nd rdgs)
ARCH No. I.
Rise on Centre Line 1·04 ft
Span on Centre Line 10·2 ft.
Tons Thrust by Extenso.
Feet Deflection
10
20
30
40
50
60
70
80
·040
·060
·070
·080

Arch No. 2 was loaded at intervals of about 2 tons, up to 30 tons, and again up to 47.3 tons, at which point it was about to fail, the load was then eased off, and then replaced and the arch fractured.

At 32 tons, tension cracks began to appear at the underside of the arch near to the crown.

At 34.5 tons, the concrete commenced to crush at the crown, but the arch continued to take load up to 47.3 tons.

The final failure was due to buckling of the curved steel bars, and the fracture is shewn in the accompanying photographs.

No signs of failure were visible at the haunches.

The tables following shew the relations between Load, Thrust and Deflection etc., as previously given for Arch No. 1.

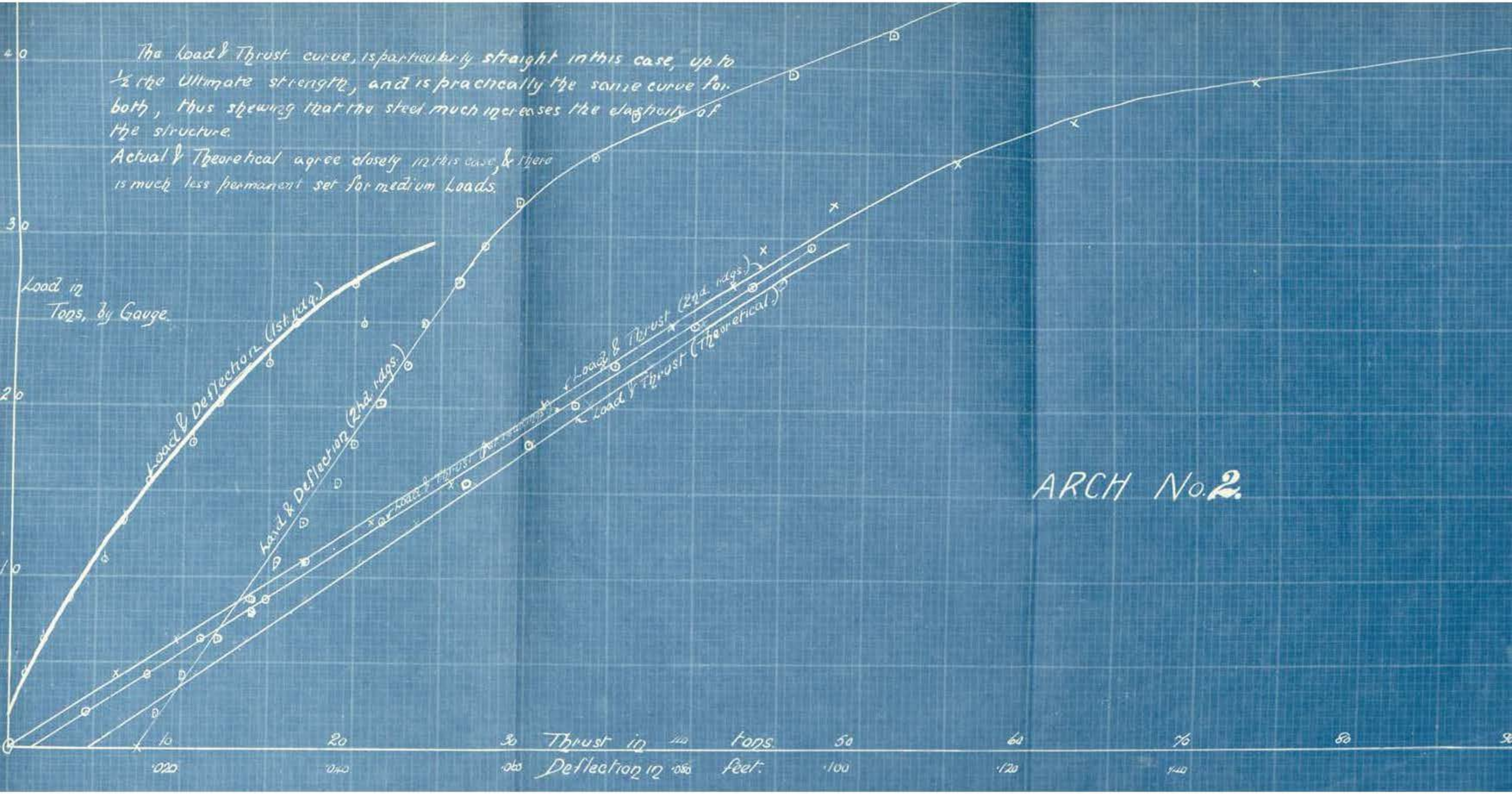
Arch No. 2.

Load tons	Thrust tons	Deflection at crown feet
0	-	-
2.14	4.55	-
4.27	8.0	.002
6.41	11.0	.004
8.69	14.8	.007
10.97	17.1	.011
13.25	21.6	.013
15.53	26.5	.016
17.8	30.2	.021
20.16	33.0	.024
22.52	35.3	.030
24.88	40.0	.033
27.24	43.3	.040
29.4	46.7	.055.

Arch No 2.

Load tons	Thrust tons	Deflections at crown. feet.
0	–	.0025
2.14	4.55	.0025
4.27	6.26	.0050
6.41	9.70	.0092
8.69	13.7	.0133
10.97	17.1	.016
13.25	21.0	.019
15.53	25.6	.023
17.8	27.9	.025
20.16	31.3	.028
22.52	34.7	.031
24.88	38.7	.033
27.24	42.2	.037
29.4	43.8	.039
31.96	48.0	.044
34.52	55.25	.053
37.08	67.8	.058
39.64	73.5	.076
42.20	91.1	.088
44.76	127.0	.128
	Permanent set	.053

47.32 Failure by crushing and after by buckling of steel bars.


The Load & Thrust curve, is particularly straight in this case, up to ½ the Ultimate strength, and is practically the same curve for both, thus shewing that the steel much increases the elasticity of the structure.
Actual & Theoretical agree closely in this case, & there is much less permanent set for medium Loads.
40
30
20
10
Load in Tons, by Gauge.
Load & Deflection (1st. ldg.)
Load & Deflection (2nd. ldgs.)
Load & Thrust (2nd. ldgs.)
Load & Thrust (Theoretical.)
ARCH No. 2.
10
20
30
Thrust in
40
tons.
50
60
70
80
90
·020
·040
·060
Deflection in
·080
feet.
·100
·120
·140

CONCLUSION.

As previously stated in this thesis, the series of tests of which this forms a part, must extend over a considerable period, and must include a great number of various arch types, but, in spite of this fact, the results obtained point out one fact, that is, that the horizontal thrust is much greater than would be expected, especially in the case of Arch No. 2

Another notable fact also, is that, after the appearance of the first crack in the arch, the horizontal thrust greatly increased in value, the crack, apparently producing the same effect as a hinge.

As regards the position of the line of stress at the crown, that suggested by the cracks produced under test, seems to agree fairly well with that found by the elastic theory, and certainly disagrees with that found by the method frequently used, in which a line of stress of least total eccentricity is used.

FUTURE TESTS.

As regards the arch testing apparatus, several modifications will be advisible.

In view of thefact that the horizontal thrust may be so great, the skewbacks should be modified to withstand a load of at least 198 tons, to be as strong as the load of

70 tons which may be applied by the jacks, requires. (arch No.2 for a load of 44.47 tons gave a thrust of 127 tons).

The tie rods for transmitting the pressure from the jacks may be shortened 9" without any detriment.

The 4" channels on top of the cross beams, having badly buckled, should be replaced, preferably being placed back to back, on either side of the vertical rods, and having a heavy washer, of $\frac{3}{4}$" plate to spread the load along their whole length. This arrangement will also considerably facilitate the erection of the apparatus.

In the test of arch No. 2 there was a considerable settlement in the sand toward the haunches of the arch, which caused the transmission of load to become eccentric, and finally caused the bursting of the pitch-pine block under the ram of one of the jacks.

To remedy this defect, it is suggested that the sand be well tamped down at the haunches, and also that a considerable quantity of inert material, such as old bricks, be put in with the sand there, as it will prevent undue sinking, and will not interfere with the distribution of the load over the back of the arch.

As regards the disc extensometers, they were found to agree very well with the readings given by micrometer, and hence, as they are much more rapid, and more trustworthy in reading, it would be possible to dispense with the micrometer readings, and devote all attention to the disc extensometers. These latter would be made much safer by the addition of a device for pressing measuring bar and roller into better contact.

Also, in this connection, it should be mentioned that cast iron, or other split bushes should be inserted in the skewbacks to completely control the position of the horizontal tie rods.

In the test of arch No.2 a permanent set was produced in the tie rods, the yield point being slightly passed, and hence the last two readings of horizontal thrust may be somewhat excessive.

Tests of reinforced arches with similar reinforcement to No.2, but much lighter, would be very useful to determine the value of light steel reinforcement for distributing purposes.

Arch No. 1 for ultimate strength, with a uniform load, would be much more economical than No. 2, but unfortunately tension cracks appeared in it at a light load, whereas theyonly appeared, and then to a much less extent, in No. 2, when stressed to very near its ultimate strength.

These tension cracks could be very easily prevented by the introduction of light reinforcement, which arrangement should produce the most economical arch of the three.

CONCRETE ARCHES.

APPENDIX

by

E. W. BLACKMORE, B. Sc.,

May. 1913.

C O N T E N T S .

Former Tests. Tests of Brick Arches.

In this appendix it was the author's original intention to include comparisons of the results previously presented, with those of contemporaneous tests in the United States, but, owing to lack of information, this important section has been omitted.

The first arch tested by Mr Adams, gave readings which did not agree within themselves, chiefly because the apparatus was in a somewhat crude form.

This fact prevents any very definite deductions from the results, or curves shewn in the first graph in the preceding thesis.

At any rate it would seem obvious that the thrust differed very much from that obtained theoretically, being much greater, although giving a curve of the same form, when plotted against load, thus indicating that it is proportional to load, and inversely proportional to the rise of the arch.
If the thrust is calculable by a formula of the form $K \frac{WL}{R}$ then K is approximately $3/8$ for this arch.

The deflection is approximately proportional to the load, for small loads, then loses its proportionality for a space, perhaps owing to some form of settlement in the arch, and then becomes again nearly proportional to the load, but in a different ratio.

The Second Arch gave much more consistent results, the apparatus having been much improved, and loading performed hydraulically.

Here again, the thrust was much greater than that expected from theoretical treatment, being somewhat less in the second set of readings than the first.

The thrust was not so great as before, neither was there any definite change in the relationship of thrust to load, the curve plotted from the readings of the two latter being smooth and continuous.

Using the same formula as suggested before, in this case is obtained $K = 1/5$ or

$$\text{THRUST} = \frac{1}{5} \times \frac{WL}{R}$$

W = total load on arch.

L = Span of arch at centre line.

R = Rise " " " " "

The Third Arch gave a still less thrust than the above, and also less than the theoretical thrust.

The curve for load and thrust readings is much more curved than before, but the results agree much more nearly with theory than the previous two tests.

In this case Thrust = $\frac{1}{10} \cdot \frac{WL}{R}$ approximately.

Tests of Concrete Arches. Arch No. 1

The curves, as given, for load and thrust for the Plain Concrete arch shew a much closer agreement up to $\frac{1}{2}$ the ultimate strength of the arch, the curves being parallel up to this point, and afterwards diverging, the actual thrust increasing in a greater ratio than the theoretical.

On reloading, after the first set of readings (a considerable permanent set having taken place), the thrust was somewhat greater than before, but it is noteworthy, that, on reaching the original maximum load, the thrust reached the same value as before, and after that the new curve obtained followed on continuously with that obtained from the first set of readings.

The possible formula for thrust in this case, is the parabolic one $T = \frac{1}{8} \cdot \frac{WL}{R}$,

the constant being the ordinary theoretical one.

It would hence seem safe, using a Factor of Safety of 4 to design a similar arch according to this rule.

Arch No. 2

A still closer agreement with theory is visible in the curves drawn to represent the readings taken from this arch. For both sets of readings, the curves are practically straight lines up to $2/3$ of the breaking load.

Very much less permanent set was caused than in the former test, owing, in all probability to the way in which the reinforcing bars hold the arch together and distribute the pressure.

The Thrust again may be written as being equal to

$$\frac{1}{8} \cdot \frac{WL}{R}$$

The curve for load and deflection is a straight line up to $2/3$ of the breaking load for the second set of readings, after which the deflections increase in proportion to the loads in much the same way as in previous tests.

The behaviour of the arches in these two tests, after the first set of readings had been taken, is a very noticeable feature of these two tests. A permanent set takes place after the first loading. On second loading, no further set takes place until the original maximum load in surpassed.

From these facts, it would seem that the concrete may be repeatedly loaded without any continually increasing permanent set, or creep, taking place, provided the load be kept within definite limits.

Although not a safe statement to make at the present stage of these tests, it would seem that the Elastic Theory is applicable to Reinforced, or Plain Concrete arches of this type, if the ordinary factor of safety of 4 be used.
(An estimate of the load on the concrete in both cases when crushing actually took place would shew the stress to be over 2000 lbs per sq. inch).

In this connection, a tentative method of finding the neutral axis of a given section under eccentric loading having been previously mentioned, it is now proposed to conclude with an illustration of that method, and that of finding the stresses involved in the component materials of a reinforced section.

Cross section of an arch under eccentric thrust.

Consider, as usual, one foot width of the arch section, and take all dimensions in pounds and inches.

Normal Thrust = T.

Its eccentricity = E. above the centre line.

Thickness of arch ring = D.

Reinforcement per foot being of steel of areas

w_1 sq ins at d_1 ins. from extrados

w_2 " " " d_2 " " intrados.

C = Compressive stress in concrete

Y = assumed depth of the neutral axis of the section.

M = the modular ratio for steel and concrete, being now generally taken as 15.

A straight line stress, strain diagram is assumed.

The compressive stress in the steel at extrados

$$f_c = \frac{c}{y} \times (m - 1) \times (y - d_1)$$

the gross area of the concrete being counted in compression.

The (tensile) stress in the lower steel

$$f_t = \frac{mc}{y} (D - y - d_2) \quad \text{-- (negative)}$$

Next finding the total forces, and their lever arms about the intrados of the arch, we have –

Compression in concrete:

$$\frac{C \times 12 \times y}{2} \qquad \text{Lever arm } (D - y/3).$$

Compression in steel:

$$f_c \times w_1 \qquad \text{Lever arm } (D - d_1).$$

Tension in steel:

$$- f_t \times w_2 \qquad \text{Lever arm: } d_2$$

Taking the algebraic sum of the moments, and dividing by the algebraic sum of the forces (i.e. by the Resultant), the point of application of the resultant force is obtained in terms of an unknown stress C, which does not affect the position of the line of action of the resultant.

If the eccentricity thus found agrees with that known from the analysis, the neutral axis assumed is correct, and the stresses may be taken out as shewn above. If there is not an agreement, a further approximation to the N.A. must be made, and the process repeated.

The above method is merely a tentative, but necessary method of avoiding a mathematical problem which is practically insolvable.

For example, an 8" arch ring sustains 20,000 lbs load per foot width, at 2½ inches from its centre line.

The Reinforcement being $\frac{3}{8}$" bars at 6" spaces top and bottom, the centres of the bars being at 1" from the outer faces of the concrete.

To find the Neutral axis, and hence the stresses on the section:-

Assume the stress on the concrete to be 600 lbs per square inch (the usual maximum for 1:2:4 concrete) and consider 12" width.

Compression in concrete (Neutral axis assumed at 6")

$\frac{600}{2} \times 12 \times 6 = 21,600$ lbs. Lever arm = 6"

Compression in steel.

$600 \times 14 \times .208 \times \frac{5}{6} = 1,456$ lbs. — Lever arm = 7"

Tension in steel, negative

$600 \times 15 \times .208 \times \frac{1}{6} = -312$ lbs. Lever arm = 1"

Resultant force = 22, 744 lbs

Total moment = 129, 600 + 10,192 - 312

= 139, 480 lbs inches.

Distance of resultant from intrados

$= \frac{139,480}{22,744} = 6.132$ inches.

Actual eccentricity from centre line = 2.132 inches which is a very close approximation to the eccentricity given.

Hence, by simple proportion:-

Stress in concrete

$$= \frac{600 \times 20,000}{22,744} = 528 \text{ lbs per square inch.}$$

Stress in compression steel = 6,160 lbs per sq. in.

" " tension " = 1,320 " " " "

The use of the tension steel (which is apparently of little value here) is manifested under a more eccentric loading, as it allows the line of stress to pass outside the section, and it is also of use in distributing the stresses, and in taking compressive stress near to the haunches of the arch, when the reinforcement is of the usual kind, following the curve of the arch at each face.