MULTICRITERIA OPTIMISATION

IN

DESIGN FOR RELIABILITY

By

STEPHEN BOAKYE TWUM

A thesis submitted to the

School of Mechanical Engineering

University of Birmingham

For the Degree of

DOCTOR OF PHILOSOPHY

School of Mechanical Engineering

University of Birmingham

August 2009

ABSTRACT

A novel methodology for series-parallel systems' reliability optimisation has been proposed developed and tested in this thesis. The approach has been to formulate the reliability design problem as a multi-criteria optimisation, to maximise independently but simultaneously the subsystem reliabilities while minimising the system cost modelled as a penalty function of component reliabilities, with lower bound constraints on the reliability of the subsystems. The goal was to find the Pareto optimal component reliability values that yielded or exceeded a system reliability target. This problem is common at the system design stage. The resultant continuous optimisation problem was solved using the Weighted Sum method which is efficient for it.

The methodology was applied to a number of hypothetical problems and to several applications derived from previously published work concerned with life support and electricity transmission systems' reliability. It was also tested on a gas transmission system. The results were very good and consistent with the theory of reliability and multi-criteria optimisation. For instance a comparison of the results with those for a single criterion optimisation model of the life support system indicated that higher reliability could be generated for the components/system under this new methodology; the relative levels of the component reliability values was also found to be consistent with those achieved under the single criterion formulation. The level of the reliability value allocated to a component was also consistent with their reliability importance. The cost/penalty increased with increase in component reliabilities, becoming indeterminate as component reliability approached its maximum value.

DEDICATION

To my parents: Kingsley (of blessed memory) and Susanna Twum

For investing in my education

ACKNOWLEDGMENT

My greatest gratitude goes to the Almighty God whose blessings have brought me to this achievement. My thanks and unqualified gratitude goes next to my supervisors Mrs. Elaine Aspinwall who is the Head of the Quality Research Group of which I was a member, and Professor Jorge Fliege who was formerly the Head of the Management Mathematics Group in the School of Mathematics in the University of Birmingham. Their wonderful council, encouragement and professionalism towards academic work have indeed provided me with the needed tonic and motivation for this achievement.

The support of various personalities within the School of Mechanical Engineering such as Dr. Andrew Tobias, who showed keen interest in my work, and Helen Booth the postgraduate Secretary, who offered administrative services, including the technical staff such as John Oldbury, the School's computing systems administrator, were all very helpful. The friendship from fellow postgraduates especially those I shared Office with has also been part of all the little bits that have together spurred me on, and I should like to acknowledge all of them, especially Maria Arvaiova.

The GET Fund Scholarship offer by the Government of Ghana ensured that I was able to travel to, live, and study in the U.K. for the period of my studies, and I say thank you for money well spent. I am thankful also to the Ghana High Commission in London which administered the funds.

Needless to say, my family has been supportive tolerant and sympathetic to me in all the long hours of absence from home or behind the computer, and for my sometimes seeming lack of interest in or attention for the things that mattered to them. Certainly this thesis is their benediction.

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Acronyms and Abbreviations

MCO	Multi-Criteria Optimisation
SCO	Single Criterion Optimisation
S	Feasible decision Set
M	Feasible Criterion Set
BCS	Best Compromise Solution
CP	Compromise Programming
GP	Goal Programming
GA	Genetic Algorithm
MOGA	Multi-Objective GA
VEGA	Vector Evaluation GA
EA	Evolutionary Algorithms
MOEA	Multi-Objective EA
NPGA	Niched Parameter GA
SPEA	Strength Pareto EA
NSGA	Non-dominated Sorting GA
TS	Tabu Search
MOPSO	Multi-Objective Particle Swarm Optimisation
SA	Simulated Annealing
ACO	Ant Colony Optimisation
DP	Dynamic Programming
IP	Integer Programming
FMEA	Failure Mode Effect Analysis

FTA	Fault Tree Analysis
FT	Fault Tree
ET	Event Tree
CCA	Component Criticality Analysis
RBD	Reliability Block Diagram
IT	Iteration number
FF	Feasibility Factor
Rs	Overall system reliability
Cs	Overall system cost/penalty
C’s	Actual monetary cost of system reliability
UC	The unit cost of a component
RSBi	Reliability of the i’th subsystem
Wi	Weight of the i’th criterion
LB	Lower Bound of a system’s reliability
LSS	Life Support System
BH	Breaker-and-a-Half
DGI	District Governor Installation
IR	Initial Reliability
PR	Pareto optimal reliability
FO	Failing Open
FC	Failing Closed

CHAPTER ONE

INTRODUCTION

1.1 BACK GROUND

When a tap is turned on the expectation is that water will flow, when a light is switched on there will be light etc. Any time one or more of these services or functions are interrupted or taken away various levels of disorganisation or disappointment is experienced (Murthy et al, 2008). This is only one side of the story; in more serious cases such interruptions could even endanger lives, as when a car bursts a tyre on a busy road, or a gas leakage occurs in the home etc.

At the centre of all the instances cited is the word “failure”, which at this stage can be described simply as the interruption of an expected function. Even though the factors responsible for failures in engineering products or systems are many and varied (Carter, 1997; Modarass et al 1999; Evans & Evans, 2001) the fact remains that whatever the cause, the consequence is never pleasant. Despite this, it is perhaps the most universal characteristic of all products or systems, whether man-made or naturally occurring (the focus of course in this case is on engineering products or systems). Fortunately, some products or systems (even those of the same kind) are less inclined to fail, than others, for reasons that can largely be ascribed to their design (Dhillon, 2005). In an attempt to distinguish between such products it is said that one is more reliable than the other, indicating that the one is less failure prone than the other. The word “reliability” (although vague at this stage) is derived from this perception and thus inextricably linked to the word failure and a watch word for many (if not every body) about the products (or services) they buy or use.

Indeed of the several characteristics that together describe a product's quality, such as aesthetics, performance, ease of use, reliability etc., reliability is considered by many as very, if not the most, important. For instance in their discussion of a survey report by the American Society of Quality Control, Murthy et al (2008) recounted that, of approximately one thousand individuals who were asked to assess their relative preference for ten quality characteristics in a product that they intended to buy, reliability was ranked the second highest, after performance. Even though reliability is important, the study and development of it as a subject and discipline goes back only a few decades (Pham, 2003) to world War II when it became necessary to deal with the high frequency of failure of equipment, especially complex electronic ones (Bernstein et al, 2006). Around the same time a new branch of optimisation, the specialty of which was to develop techniques for optimising a set of criteria, instead of just one, as was typical of traditional optimisation, was also growing in terms of development of theories as well as applications (Figueira et al, 2005). These two, now broadly called respectively Reliability Engineering and Multi-criteria Optimisation, have today become major fields of Operational Research, Engineering, and Mathematics. Even though traditional optimisation has been very much a part of modern Reliability Engineering, this new branch has not seen as much application.

Reliability Engineering covers all aspects of a product's life cycle from conception, to design, through manufacture, to use, and finally to death or obsolescence (Relex Software Corporation, U.S.A). A schematic diagram showing the phases of a product life cycle is given in Figure 1.1. The Conceptual Design stage is one where an idea for a new product is conceived or

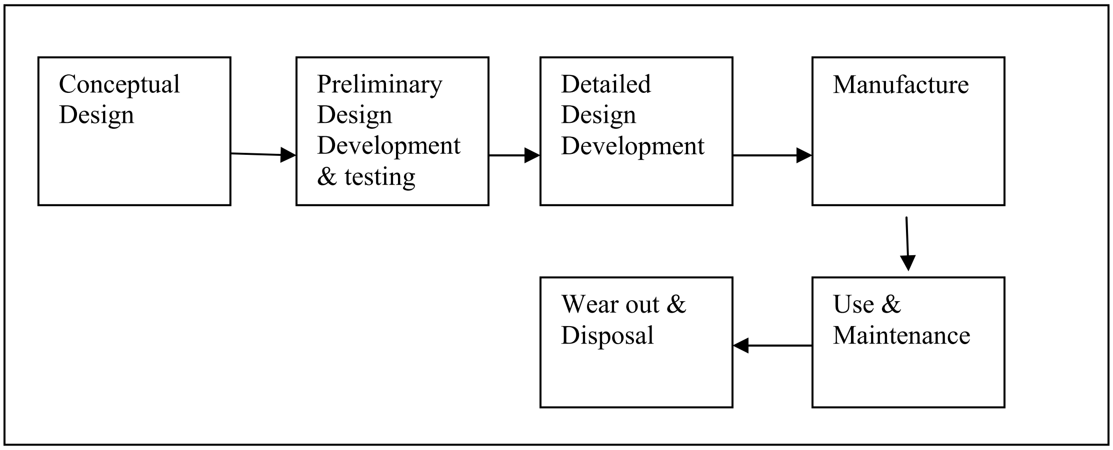

Figure 1.1: Phases of a product/system's life cycle (Source: Relex Software Corporation U.S.A.)

a request for proposal or bid documents are developed in pursuit of a grant or a contract for a product design. The assessment of the reliability metrics and the application of reliability prediction and analysis techniques at this stage can prove advantageous not only for the design but also for the bidding process. The Preliminary Design Development & Testing stage builds upon the former and usually involves the development and testing of a prototype in order to predict or evaluate the reliability metrics as well as correct mistakes or improve the design. The third stage factors all the insight and information gained from the previous stage into a final and detailed design. In the fourth to sixth stages, the final design is translated into a finished product, used and maintained (in repairable cases), and phased out.

Of the stages at which optimisation may be applied in a product's life cycle, the conceptual or preliminary design one is the most important (Amari, Relex Software Corporation, U.S.A), for two major reasons: (i) a product's reliability is an intrinsic characteristic of its design, thus optimising reliability early in the life cycle is a necessary step to ensuring reliability in later life

(ii) it is the most economically sound approach, as displayed in Figure 1.2. This is because it is easier and less costly to fix reliability problems detected in the early phases including making any design modifications, than when the product is in use. It is worthwhile therefore that more research attention to enhance reliability is directed to this area.

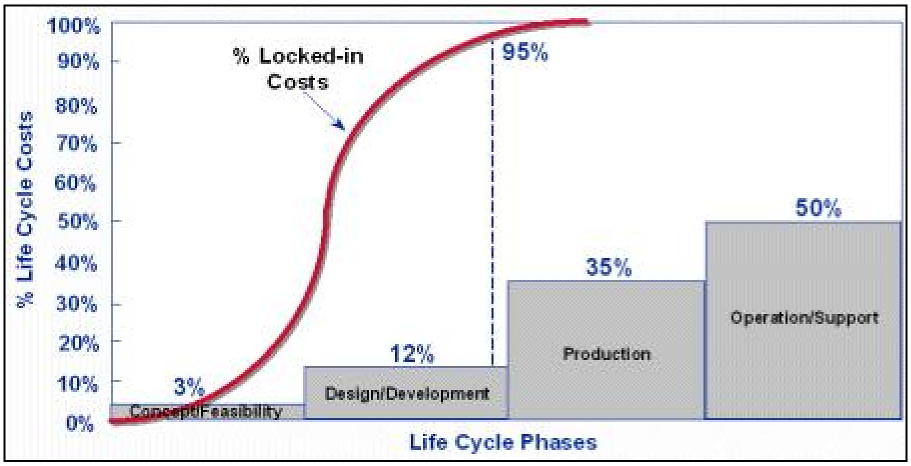

Figure 1.2: Cost of reliability at each phase of a product's life cycle
(Source: *Centre for Systems Reliability, Sandia Laboratories U.S.A.*)

1.2 MOTIVATION AND OBJECTIVES

The initial interest and incentive to research into the general area of optimal design for reliability thus stems from its importance to product reliability and the vibrancy of the area (as was determined from the literature) as a research field. The study of the literature eventually led to the following very important observations which provided the focus and direction for this work:

- While the application of traditional single criterion optimisation was very common in reliability design, it was not the case with multi-criteria optimisation. The reason for this situation could perhaps be because multi-criteria optimisation is a relatively new field and may as yet be unknown and unappreciated by many researchers and practitioners in the reliability field.

- Practical reliability design problems are generally multi-criteria by nature: for instance reliability is inextricably linked to cost, thus it is more realistic to optimise both.
- The enormous benefits that could be derived from multi-criteria optimisation had not been fully exploited in reliability design, such as: (i) the opportunity to examine a variety of potential designs and thus make informed decisions, (ii) being able to take on board and to optimise all relevant criteria simultaneously, thereby securing information on their trade-offs for effective decision-making, (iii) opportunity to factor user preferences into the analysis and so obtain acceptable solutions.
- Application of optimisation in reliability design had been inordinately concerned with just one particular type of problem - redundancy allocation at the component level. Its use in setting system and component reliability specifications at the design stage had received little or no attention.
- The applications focused mostly on the top level system reliability expression, which in the case of complex systems especially, was not easy to find.

The research work was therefore centred on the consideration of the problem of design for reliability where optimal component reliability values are sought in order to meet at least a system reliability target or specification at minimum cost. The main objectives were to:

- Develop an alternative and new approach to the problem described;
- Model the reliability optimisation problem as a multi-criteria one;
- Apply the model specifically to series-parallel and complex systems;
- Investigate the performance of the model, especially on real data if possible;
- Compare the results with those achieved using other existing models and draw conclusions.

1.3 STRUCTURE OF THE THESIS

The thesis comprises three main parts. The first provides an overview of the theoretical bases of the subjects of multi-criteria optimisation on the one hand, and reliability on the other. These are the contents of Chapters Two and Three respectively.

The second part, made up of Chapters Four and Five is concerned with the applications of optimisation in reliability design. Chapter Four discusses the state of the art in optimal design for reliability, the various formulations of the problem and the solution methods used. Perceived gaps in the literature are highlighted and the ground work is laid for a new approach to the subject. Chapter Five presents the development of a novel approach to system reliability design, and formulates it as a multi-criteria optimisation. The model considers the reliability of the subsystems of a series-parallel system and an analytical cost function as criteria which are maximised and minimised respectively. The methodology is subsequently extended to complex systems.

Part 3 presents example problems to test the performance of the model and draw conclusions. Chapter Six looks at hypothetical cases, while Chapter Seven considers three applications derived from previously published work and one case study from industry. Chapter Eight concludes the discussions by drawing attention to a number of limitations and drawbacks associated with the work carried out, which also formed the basis for a number of recommendations for future work.

CHAPTER TWO

MULTI-CRITERIA OPTIMISATION

2.1 INTRODUCTION

Decision making is more often than not, characterised by more than one criterion, and a number of constraints and decision alternatives. Consider for instance the following four scenarios: (i) a prospective purchaser of a car is considering selecting one from a number of different models. The criteria of interest may be price, size, reliability, and style. Which car would give the best value for money? (ii) The designer of a heating system wants a product that say minimises heating cost and fuel consumption. The variables to consider could be the heat pump nominal evaporator pitch, heat pump power, gas turbine pressure ratio, excess air ratio, and inlet temperature among others (Li et al 2004). What values of the variables would achieve the objectives? (iii) An oncologist is planning treatment for a malignant tumour, in order to maximise tumour dose while minimising exposure to organs at risk (Ehrgott & Burjony, 2001; Craft et al 2005). What dose levels and beam intensities would achieve these objectives? (iv) A reliability engineer seeks to design a gearbox system with high reliability, but at minimum cost and weight, using components that are available on the market. It is felt that using components in parallel will achieve high reliability (Taboada et al, 2007; Zhao et al, 2007). Which components and redundancy levels would achieve these objectives? In all the examples given, a decision alternative is sought which yields the "best" value of each criterion simultaneously while not violating the constraints. Unfortunately, as will be demonstrated later, such problems (known as multi-criteria optimisation (MCO) types) usually do not have a unique or global solution

(Petrovski & McCall, 2001), i.e., there is no decision alternative that is optimal for all the criteria simultaneously. Consequently the concept of optimality, and therefore the notion of a solution, as known in single objective optimisation are rendered untenable in this case, necessitating their reformulation or modification (Miettinen K., 1998). This has led to the development of analytical techniques and methodologies specifically tailored to the resolution of the technical difficulties (in terms of finding solutions) inherent in the problem, so that reasonable, meaningful and acceptable outcomes can be achieved. A formal statement of the general MCO problem will follow and the basic theoretical concepts and ideas that underline or underpin the subject discussed.

2.2 **FORMULATION OF THE MCO PROBLEM**

The decision making tasks described above can be formulated into a vector optimisation problem with the Pareto model as its basic structure and an ordered vector space its fundamental notion (Nemeth and Nemeth, 2006).

Consider a vector-valued criterion function $f : S \subseteq R^N \rightarrow R^K$ comprising k real-valued functions $f_i : R^N \rightarrow R$, ($i = 1,2,..,k$) defined on the vector of decision alternatives $x \in S$ where $x = [x_{1,} x_2,..., x_N]^T$ and $f(x) = [f_1(x), f_2(x),..., f_k(x)]^T$.The intention is to find a vector of decision alternatives $x = [x_{1,} x_2,..., x_N]^T$ which optimises (i.e. maximises or minimises) the vector $f(x)$ and satisfies a given set of constraints. The term "optimise" signifies the search for a solution which contains the values of all the objective functions adjudged to be acceptable to a user. This classical mathematical problem provides the context and framework within which to define and model a practical MCO problem and to analyse it for solutions. Without loss of generality the MCO problem is thus stated formally as follows:

$$
\begin{array}{rll}
\text{Find:} & x = [x_1, x_2, ..., x_N]^T & \\
\text{To minimise:} & f(x) = [f_1(x), f_2(x), ..., f_k(x)]^T & \\
\text{Subject to:} & g_i(x) \le 0\,, & i = 1,2,...,m \\
& h_j(x) = 0\,, & j = 1,2,..., p
\end{array}
\qquad (2.1)
$$

where elements of the vector $f(x)$ are functions of k identifiable criteria ($1< k \subset Z^+$); $g_i(x)$ and $h_j(x)$ are respectively m and p inequality and equality constraints; and x is a vector of decision alternatives whose elements are called decision variables. The constrained MCO problem denoted by (2.1) may be expressed compactly as:

$$
\min\{f(x) : x \in S\} \qquad (2.2)
$$

where S denotes the set of decision alternatives defined by:

$$
S = \{x : g_i(x) \le 0, i = 1,2,..., m; h_j(x) = 0, j = 1,..., p\} \qquad (2.3)
$$

2.2.1 **Decision and Criterion Set**

Two fundamental geometrical/analytical notions which arise from the problem in (2.1) and the focus of attention in the search for solutions are the decision (or variable) and the criterion (or objective) sets. While the number of decision variables generates the decision sets the number of criterion functions generates the objective one. Embedded within each of the two sets is a subset respectively referred to as the *feasible decision set* and *feasible criterion set*. For the problem in (2.1) the feasible decision set is defined by (2.3) and the feasible criterion set by $M = \{y \in R^K : y = f(x) \text{ for some } x \in S\}$. The points $y \in M$ are the images of the points $x \in S$, however, while each point in S maps onto a unique point in M the reverse may not be true. Also there may not be, in general, an explicit map of S onto M (Marler, 2005). Figure 2.1 illustrates the decision (S) and criterion (M) sets for a hypothetical bi-criteria problem with a two component decision alternative.

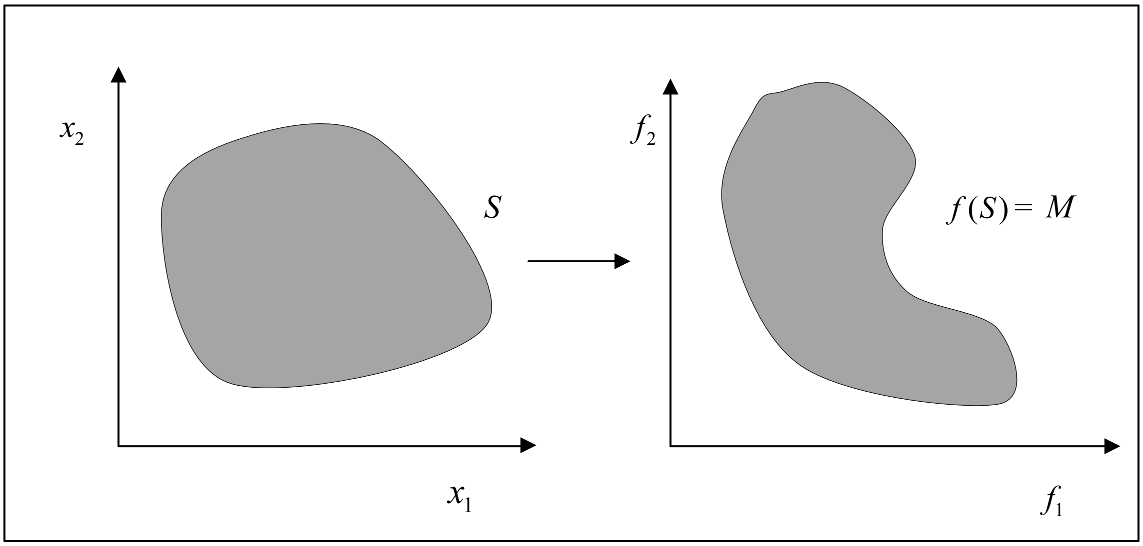

Figure 2.1: Decision and criterion sets

2.2.2 **Partially Ordered Decision and Criterion Sets**

As noted earlier an MCO problem, unlike its single criterion optimisation (SCO) counterpart, does not in most cases have a single decision alternative that is optimal for all criteria. The underlining reason for this phenomenon and a discussion of the mathematical notions employed to deal with the situation will be briefly addressed. Figure 2.2 depicts a hypothetical feasible criterion set of the functions f_1 and f_2. The figure shows that there cannot be feasible values of y_1 and y_2 which are minimum for the two functions at the same time. The point C which represents the unique minimum for the two criterion functions is infeasible. The simple reason for this absence of a unique minimum (which is characteristic of MCOs in general) is due to the presence of multiple conflicting and incommensurable criteria: what is considered a minimum for one criterion fails to be a minimum for another (Petrovski &McCall, 2001; Collette & Siarry, 2004). Instead there are many or an infinite number of points, the best (in terms of their values) of which are intuitively found along the boundary of the feasible criterion set lying between the

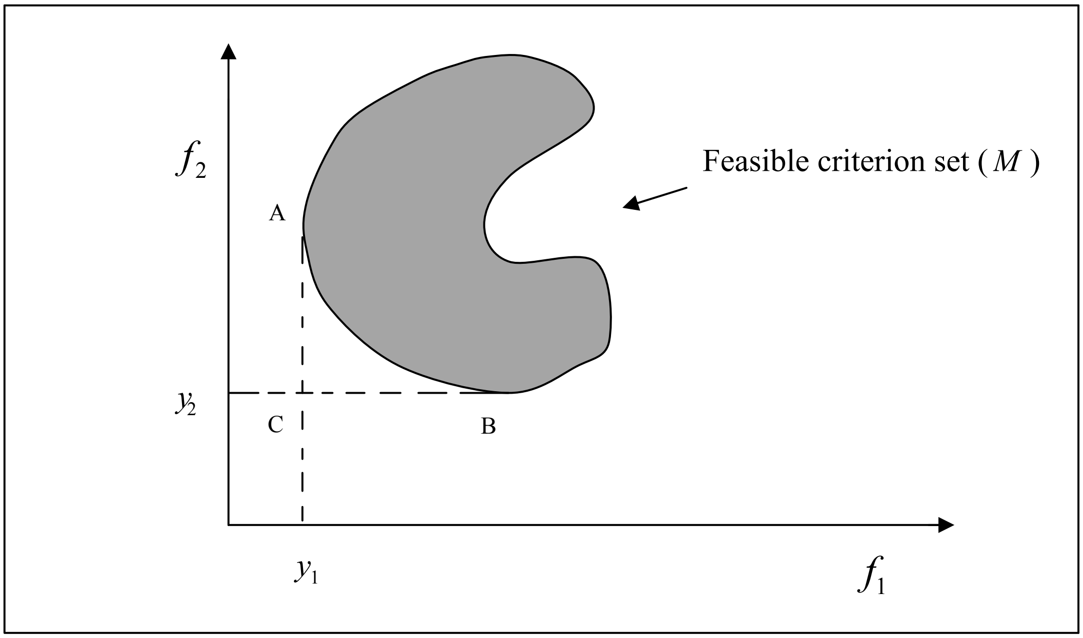

Figure 2.2: Absence of a unique minimum for f_1 and f_2

points A and B (a formal proof is given in appendix A). The criterion set is therefore characterised by the presence of vectors which are partially ordered (Jahn, 2004; Ehrgott, 2005). Consequently, one needs a basis for the comparison of the vectors, in order to decide on those which are candidates for a solution.

The subject of partially ordered vectors will not be dealt with in detail in this thesis since it is a major subject in its own right; detailed discussions can be found in Jahn (2004) and Ehrgott (2005). Suppose there are two vectors $y = [y_1, y_2, ..., y_N]^T$ and $y' = [y'_1, y'_2, ..., y'_N]^T$ to be compared, in order to determine in the context of problem (2.1) which is the better. In order to proceed, a working definition of what is meant by "better" is required. Goldberg (1989) provides this by stating that a vector y is partially less than a vector y' (denoted by $y \prec y'$) if and only if $y_i \le y'_i \ \forall i$ and there is at least i such that $y_i < y'_i, i = 1,2,...,N$. A corollary to this definition is as follows: y *dominates* y' if and only if y is partially less than y'. If y does not dominate y' and vice versa, then they are both *non-dominated* vectors (Le & Landa-silva, 2007), and according to Hawe and Sykulski (2008) both vectors are said to be equivalent. Using the concept of dominance, therefore, the vectors in a partially ordered set may be separated into two main categories: dominated and non-dominated. While it is not possible to order the latter in terms of which ones are better than the others, it is possible to do so with the former. Dominance, therefore, is the fundamental notion used for finding candidate solutions for the problem in (2.1).

2.2.3 **Notions of Optimality**

Some notions of optimality are: (i) *Pareto*, (ii) *lexicographic* (iii) *min-max,* and (iv) *lexicographic min-max* (Ehrgott, 2005). The most fundamental of these, which is derived from the concept of dominance, is, however, Pareto optimality, also called Pareto dominance. A decision alternative x^* is said to dominate a decision alternative x or be Pareto optimal if and only if the following conditions are satisfied (Louie & Strunz, 2006):

(i) $f(x^*)$ is partially less than $f(x)$, which means that $f_i(x^*) \le f_i(x)$ for all $i = 1,2,...,k$;

(ii) At least one of the inequalities is strict for some $i \in \{1,2,...,k\}$.

Where the inequality in (i) is strict for all $i = 1,2,...,k$ the result is said to be *strongly Pareto optimal*. The result is *weakly Pareto optimal* where the condition in (ii) is not satisfied. The notions of strong and weak Pareto optimality are thus special cases of Pareto optimality. If both conditions are violated, x^* cannot be said to dominate x. In this case x^* and x are non-dominated or Pareto optimal. Hence a decision alternative is Pareto optimal if it is non-dominated by any other feasible decision alternative. Consider for instance that $f(x^*) = [9,2,7]^T$

and $f(x) = [11,0,5]^T$ are the only two points in a feasible criterion set. If it is assumed that $f(x^*) \le f(x)$ then clearly condition (i) is not satisfied, neither is it satisfied in the converse relation. Thus it must be concluded that x^* and x are both Pareto optimal. The collection of all Pareto optimal solutions is called the *Pareto optimal set*. The image $y^* = f(x^*)$ of a Pareto optimal solution is called a *Pareto point*. The collection of all Pareto optimal points is called the *Pareto front*. Pareto optimal solutions are also referred to as *non-inferior or efficient solutions* (Tan et al, 2002).

Other notions of Pareto optimality (see formal definitions in appendix A) are: (i) proper, (ii) local, and (iii) global (Augugliaro et al, 2001; Le & Landa-Silva, 2007). The idea of a local Pareto optimal set is similar to that of a local optimum in classical SCO. A set in the feasible decision space is local Pareto optimal if for all the vectors it contains, there are none in a small neighbourhood of the set which dominate all of them. On the other hand such a set is global Pareto optimal if no vector exists in the decision space which dominates every vector in the set. The idea of local Pareto optimality suggests that the Pareto optimal solutions may not always be confined to a distinct set. In other words they can be distributed into a number of distinct or disjoint sets within the feasible decision set.

Pareto optimality while providing a way forward in the pursuit of solutions to MCOs, also presents some practical difficulties. These have to do with how to find them from the large number of feasible decision alternatives that may be associated with a problem, and how to manage the potentially large number, if they were found, in order to decide on the most satisfactory one. The methods and techniques used in this regard are discussed later in the chapter.

2.2.4 **Characterisation of Pareto Optimal Solutions**

There are three unresolved questions that require addressing before concluding this section. These concern the characteristics of Pareto optimal solutions and, therefore, the Pareto front: (i) what distinguishes one Pareto optimal solution from the other; (ii) is there a geometrical intuition for Pareto optimal solutions; (iii) what are the implications for choosing one Pareto optimal solution instead of the others?

Recall that the vectors in a Pareto optimal set are non-dominated with respect to each other (Taboada et al, 2007) thus one cannot be distinguished from the other on the basis of which is the better. The vectors are regarded as incomparable. Their distinguishing characteristic is that an attempt to improve upon the value of a criterion associated with a particular solution would result in the degrading or deterioration of the value of at least one other criterion (Langer et al, 2003). Therefore perhaps the only and most important distinction which may be made between any pairs of the vectors is in terms of the *tradeoffs information* they provide between the criteria. The vectors in the Pareto set thus effectively are *compromise or acceptable solutions*. The question of which one offers the best compromise or is most acceptable is incidentally a subjective one. Nevertheless there are methods designed to help the decision maker identify a best compromise solution (BCS) (see section 2.4).

Geometrically speaking, while the vectors of the Pareto optimal set could be anywhere in the feasible decision set, the Pareto front is always a subset of the boundary of the feasible criterion set. (See Appendix A for a formal statement of this geometrical property together with other topological properties of the Pareto front). From Figure 2.2 therefore, the Pareto front lies on the boundary between points A and B of the feasible criterion set. Each point between A and B provides trade-off information between f_1 and f_2. For instance while the vector corresponding to point A yields a better value of f_1 than the point B; the reverse is not the case in respect of f_2. A decision maker in this case ought to make valued judgements about which criteria to trade-off and by how much.

2.3 **CLASSIFICATION OF MCO MODELS**

The two main model classifications in MCO are deterministic and stochastic each of which can be categorised further in terms of the type and nature of their admissible solutions. The various features of the models which impact their formulation and solution are now discussed.

2.3.1 **Deterministic Models**

An MCO model is deterministic if its parameters are precisely determined, or can be assumed fixed or known as far as the problem or the optimisation is concerned. Practical problems, however, are seldom purely of this type. Variability in parameter values is all too common and in such instances they cannot be precisely determined by just a single value, but rather by a set of randomly distributed ones. Another source of uncertainty apart from heterogeneous parameter values is inadequate or inaccurate data (Ndambuki et al, 2000). In many practical situations a deterministic model may still be formulated, notwithstanding the presence of uncertainties, in order that computationally reasonable approximate solutions may be found to an otherwise intractable stochastic situation (Babayan et al, 2004; Gabriel et al, 2007; Elshafei, 2007). Deterministic models may be categorised further as continuous, discrete or mixed, depending on the type of solution output.

The deterministic continuous type is of the following general form:

$$\min\{f(x) : x \in S \subseteq R^N\} \qquad (2.4)$$

In this case the feasible decision set S which is in an N-dimensional Euclidean space is composed of decision vectors (decision alternatives) x whose components are continuous or real values. The components of the criterion vector $f(x)$ are also real-valued functions which may be linear or non-linear. In the case of the former (2.4) is described as a constrained linear MCO problem, otherwise it is said to be a constrained non-linear one; many practical MCO problems conform to the latter. The model type in (2.4) is common in practice; it is also the easiest to construct or develop and to solve. This is because the theory of continuous optimisation in particular is so well developed.

The deterministic discrete type on the other hand may take one of the following general forms:

$$\min\{f(x) : x \in S \subseteq Z^N\} \qquad (2.5)$$

$$\min\{f(x) : x \in S \subseteq R^N : 0 \leq f_i(x) \in Z, i = 1,2,...,k\} \qquad (2.6)$$

$$\min\{f(x) : x \in S \subseteq Z^N f_i(x) \in Z, i = 1,2,...,k\} \qquad (2.7)$$

$$\min\{f(x) : x_i \in [0,1], \forall x_i \in x\} \qquad (2.8)$$

In (2.5) the admissible solutions in terms of the decision variables, are required to be integer (or discrete) and may, or may not be restricted to a certain set of values, the integer restriction is limited to the decision variables only. In (2.6), however, the integer restriction is on the values of the components of the criterion vector, which may or may not be limited. In (2.7) the integer restriction is on both the decision variables and the criterion functions. In (2.8) the admissible decisions are of a binary nature; the components of the criterion vector may however assume continuous or discrete values. These types are described as constrained discrete or combinatorial MCO problems. Discrete problems are generally considered difficult to solve in view of the combinatorial nature of the solution search; in such cases the solution time grows exponentially with the size of the problem, these are called NP hard (Coit & Baheranwala, 2005; Zhao et al, 2007; Yang, 2008).

The mixed-integer types are generally of the forms:

$$\min\{f(x,y): x \in S \subseteq R^N, y \in Y \subseteq Z^N\} \tag{2.9}$$

$$\min\{f(x): x \in S \subseteq R^N, 0 \leq f_i(x) \in R, f_j(x) \in Z, j \in \{1,2,...,K\} \tag{2.10}$$

$$\min\{f(x,y): x \in S \subseteq R^N, y \in Y \subseteq Z^N, 0 \leq f_i(x,y) \in Z, i \in \{1,2,...,k\}\} \tag{2.11}$$

Y is the feasible set of integer variables. The admissible solutions in this case may be such that some specified decision variables take on only integer values while the rest are unrestricted, such as in (2.9) where the decision variables in the vector x were real valued and those in y were integer. There are cases where the integer requirement would be with respect to some identified criteria only, as in (2.10). In other cases the output of both some specified decision variables and criterion functions may be required to be integer, as in (2.11). Mixed problems like these also pose combinatorial problems rendering them NP hard. They are therefore in the class of hard MCO problems.

2.3.2 **Stochastic Models**

An MCO model is stochastic (non-deterministic) if uncertainties arising from the variations in parameters including criteria and decision variables are explicitly accounted for in its formulation. In this case stochastic modelling techniques are employed in order to estimate the values of the parameters and to construct criteria and constraints. The use of fuzzy techniques in this regard has also become popular (Pohekar & Ramachandran, 2004). Further categorisations under this type are again continuous, discrete and mixed models.

Stochastic continuous models are similar to their deterministic continuous counterparts, as far as the requirements for the admissible values of both decision variables and criteria are concerned. They are different, however, firstly in terms of the characteristics of the parameters that are used in the model and secondly in the quality of the solutions. This is on the grounds that a stochastic model is more representative of a problem where variations and uncertainties exist, and thus are expected to provide better solutions than a deterministic one. The following general forms are presented as illustrations:

$$\min\{-E[f_1(x)],\ldots,-E[f_k(x)],\mathrm{var}[f_1(x)],\ldots,\mathrm{var}[f_k(x)] : x \in S \subseteq R^N\} \quad (2.12)$$
$$\min\{f(x) : x \in S \subseteq R^N, l_i \leq x_i \leq u_i, i = 1,2,\ldots,N\} \quad (2.13)$$
$$\min\{-E[f_1(x)],-E[f_2(x)],\ldots,-E[f_k(x)] : x \in S \subseteq R^N\} \quad (2.14)$$
$$\min\{f(x) : \Pr ob(x_i \in S \forall x_i \in x, i = 1,2,\ldots,N) \geq 1-\alpha_i\} \quad (2.15)$$

All four require continuous values for their decision variables. The formulation in (2.12) illustrates the case where variability is associated with one or more criteria. This may be the case where the precise forms of the objective functions are unknown or too complicated to compute precisely, or, where their outputs are only approximations (Bao et al 2007). Thus their expected values $E[f_i(x)]$ and variances $\mathrm{var}[f_i(x)]$, $i = 1,2,\ldots,k$ are respectively maximised and minimised. In (2.13) the decision variables are subject to variation with their values expected to lie within stochastically determined upper and lower limit u_i and l_i respectively. The expression in (2.14) a variant of (2.12) on the other hand seeks to maximise only the expected values of each criterion. The expression in (2.15) illustrates the case where some or all of the constraints are subject to variation and thus to the chances of constraint violation for some decision variable values. This situation is modelled by incorporating stochastic constraints in the formulation, where $\alpha_i, i = 1,2,\ldots,N$ represents a parameter of the distribution associated with the chance of constraint violation. Various mixes of the above models can also be formulated. This type of model, even though continuous, presents both formulation and computational challenges, due to the stochastic situations involved which invariably leads to increased model complexity. Stochastic continuous models are less common in applications than deterministic continuous ones, perhaps because of these limitations.

The discrete models are similar to the continuous ones in terms of their general forms; however they differ in the nature of the solutions which are required to have only integer values. This type presents even more formidable formulation and computational challenges than the stochastic continuous ones.

The solution requirements for the mixed-integer models are the same as those for the deterministic types. However the level of difficulty in terms of modelling and finding solutions is much higher for the same reasons as given for the discrete case. Venkataraman & Hafka (2002) have indicated that the level of difficulty, also referred to as complexity (Rubenstein-Montano & Malaga, 2002), in modelling and computational terms, for the SCO versions discussed above, increases from left to right of the following list: deterministic continuous, deterministic discrete (or mixed-integer), stochastic continuous, stochastic discrete (or mixed-integer). The presence of multiple criteria further increases the complexity.

2.4 THE MCO METHODOLOGY

There are four important issues to address in any application of the MCO methodology. These are identified as: (i) knowledge of the characteristics of the problem, (ii) the model formulation, (iii) the search for solutions and, (iv) the selection of a compromise solution.

2.4.1 Problem Characteristics

An application requires specification of the various criteria, decision variables, constraints (Nocedal & Wright, 1999) and any other parameters which are related to the problem. The formulation process is an attempt to put these attributes into context and to specify their levels of interaction. Knowledge and specification of these attributes is an essential precedent to any model formulation. It is also necessary to have an appreciation of the levels of complexity which are influenced by the model size (number of parameters, decision variables, criteria and constraints) and have implications for finding solutions (Savic, 2002; Fadel et al 2005).

2.4.2 **Model Formulation and Challenges**

The formulation activity essentially centres on finding a representative model that captures the essence of the problem. It is in effect a simplification of the problem which otherwise would, according to Michalewicz &Fogel (2004) be as complex and unwieldy as the natural environment itself. Among other things this requires a precise definition of the criteria, decision variables and constraints. This crucial step (Nocedal & Wright, 1999; Savic, 2002) sets the boundaries as well as provides a structure to the problem and helps the analyst to focus on the essentials.

The criteria expressed as criterion functions are measures of effectiveness or of performance for the problem. The intention is to achieve the best values possible for these measures in the optimisation. ***The constraints*** specify the limits on the consumption of resources, or requirements which must be satisfied for the model to remain valid or feasible. A highly constrained problem, however, has the tendency to drastically reduce the feasible criterion and decision set thus rendering the problem very difficult to solve or even infeasible. Constraints may be imposed on criteria, decision variables or other attributes of the problem. ***The parameters*** are all the quantities that describe and influence the model behaviour. They generally include both the constant coefficients and the variables of a problem. They provide a means for assessing the sensitivities of the model, through the perturbations of their values. They may be known, a priori, or may be determined experimentally. In some cases assumptions have to be made about them in order to simplify the task of model development. This is the case especially where randomness exists in their values but they are assumed fixed or precisely known, in order to proceed with a deterministic formulation. There is however, the risk of over-simplification and therefore the possible misrepresentation of the essence of the nature of the problem (Igor et al 2004) which eventually could compromise the quality of the solutions. In recent years a number of techniques for dealing with randomness have evolved. These are robust and stochastic optimisation techniques which characteristically yield MCO models (Jung & Lee, 2002; Igor et al, 2004), and interval and fuzzy sets-based methods (Kuo et al, 2001; Pongthanapanich, 2003; Zang et al, 2005).

A major challenge in the formulation process is finding metrics which model the dependence of criteria or constraints on decision variables and/or parameters. Common practice that helps to address this problem has been to use already existing theoretical models (Leyland et al, 2003; Yun et al, 2004; Subbu et al, 2005). In the absence of such models, real and computational experimental approaches, for instance, have been used (Yun et al, 2004). The experiments are intended to manipulate the variables and parameters of the problem, to investigate the dependency of the criteria on decision variables and parameters, so as to formulate relations between them. Simulation models have been used (Papalambros, 2002) in cases where an explicit algebraic expression for the criteria and the constraints is absent, leaving only a formal statement of a complex procedure involving computer based calculations. Where qualitative criteria, constraints or decision variables are involved heuristic approaches such as neural net approximations and fuzzy modelling techniques have been used (Srdjevic et al, 2004; Fadel et al, 2005; Mahfouf et al, 2005). Also used are Monte Carlo simulations where stochastic criteria, constraints, or parameters are involved. The use of simulation models for both problem formulation and solution generation has also become popular (Gupta & Sivarkumar, 2002; Duvivier et al, 2003; Persson et al, 2006) especially as they do not require explicit functional forms and because powerful computer based tools for evaluating complex problems have become available.

2.4.3 **The Search for Solutions and Challenges**

The search for solutions refers to the steps taken, once a model is formulated, to find the entire Pareto optimal set in some cases, or a subset of it; or in other cases just a BCS. This aspect is an important part of the MCO methodology and constitutes a major research area by itself. It begins with a choice of a suitable algorithm which effectively incorporates the model characteristics, thus sufficient knowledge of the connection between the characteristics and a solution method or algorithm is necessary if an appropriate method is to be found (Cai & Wang, 2006). The methods and techniques used are discussed in section 2.4, while attention here is drawn to some consequences of the characteristics on the solution search.

The model type and the size of the search space have already been noted as being determinants in both model complexity and in finding solutions. The type of model for instance should provide an insight into whether or not the search space is going to be continuous, discrete, or a mixture of both. Furthermore whether or not the search space would have desirable features like convexity, connectedness and compactness may be ascertained thus serving as a guide for the selection of an appropriate algorithm. Since the size of a Pareto front is proportional to the number of criteria (Coit & Baheranwala, 2005) a large model could have a large search space.

This could pose both technical and economic difficulties in terms of finding a suitable algorithm and of the cost, especially of Central Processing Unit (CPU) time among other resources. In some cases such a model may even be unsolvable. A large search space also means that the number of possible solutions is comparable in size (Amponsah, 2003) and this could make the search, either for the Pareto front, or a compromise solution a daunting task. This difficulty is further exacerbated by a fragmented and or/sparse search space as may occur especially with discrete problems, whether deterministic or otherwise. The number of constraints also has an impact on the size of the search space. A highly constrained problem could result in a diminished search space and render many solutions infeasible or impracticable (Michalewicz & Fogel, 2004).

2.4.4 Compromise solution Selection and Challenges

Even though it is the climax to any MCO methodology, the selection of a compromise solution is not an easy or simple exercise. The reason for this is, firstly, that the Pareto front, if secured for a problem, would normally contain a large if not infinite number of possible solutions, where each one is equally good, though only one would normally be required. The question then becomes which one to select. Alternatively, if instead of the Pareto front a compromise solution is sought directly through interactive means, there still remains the need to accurately extract and model user preferences for integration into a solution algorithm. This certainly requires a lot of effort on the part of both the user and the analyst. The other reason is that since all the solutions are equally good the choice is very much a subjective one, which customarily must be exercised by the end user (decision maker). The challenge lies in the user being able to make subjective judgements about the individual solutions on the basis of the values of the criterion functions and to appreciate the trade-offs involved. In the special case of a bi-criteria problem this might be easier to do, since graphical aids could be employed in the process. In higher dimensions, as is typical with real problems, this is just not possible. While interactions between the analyst and the decision maker are a necessary way forward (Cohon, 1978), this problem remains a great challenge in the selection of a BCS.

Because of its importance, techniques for the identification of a BCS are an important research concern. A number of techniques and tools (including software packages) have been developed for this purpose. These generally are called *Decision Support Systems* (DSS) (Weistroffer & Narula, 1997; Ndambuki et al, 2000), *Multi-Criteria Decision Making* (MCDM) (Traintaphlou, 2004; Pohekar &Ramachandran, 2004; Steuer & Na, 2005) or *Multi-Criteria Decision Analysis* (MCDA) (Cheng et al, 2003). Examples are *Analytic Hierarchy Process* (AHP), *Multi-attribute Utility Theory* (MAUT), *Elimination and Choice Translating Reality* (ELECTRE), *Compromise Programming* (CP), *The Technique for Order Preference by Similar to Ideal Solutions* (TOPSIS), *Goal Programming* (GP), *Preference Ranking Organisation Method for Enrichment Evaluation* (PROMETHEE), etc. Some of these techniques, such as the AHP and the ELECTRE, are good for analysing only a few discrete solution alternatives. The ELECTRE is also able to analyse both quantitative and qualitative criteria. In the next section some of the popular solution methods, include CP and GP, are discussed in detail.

2.5 **CLASSIFICATION OF SOLUTION METHODS**

The literature reveals a large and diverse collection of methods and techniques for solving MCO problems as well as their characteristics. Marler & Arora (2004) present a comprehensive review of a number of the methods. In general, they may be classified as either *classical* (also called *traditional* or *deterministic*) methods; or as *non-deterministic* (also called *stochastic* or *heuristic/ meta-heuristic*) methods.

2.5.1 **Classical Methods**

These were the earliest to be developed, and are particularly suited to MCO models with continuous search spaces. They use various schemes to transform the vector minimisation problem defined in (2.1) into a scalar one (Fonseca & Fleming, 1995), thereby permitting the use of scalar optimisation algorithms, the outputs of which are a single solution at each run. They are thus aptly called scalar methods (Orths et al, 2001; Schmitt & Verstege, 2001). The unique optimum which results as a solution is considered Pareto optimal under certain conditions required by the particular method. An important attribute of most of the methods in this class is that they may be used interactively by incorporating the decision maker's preference directly into the solution algorithm at specific stages of the search. The expressed preferences may be incorporated into the algorithm before, during, or after the solution search (French, 1984; Marler &Arora, 2004) in order to find the BCS of the decision maker. The interactive approach, therefore, avoids the generation of the entire Pareto optimal set (Cohon, 1978). The benefit can be a great saving in time and effort to find the BCS; a drawback however is that since it depends on user expressed preferences there is a risk of obtaining a solution that is not Pareto optimal (Collette & Siarry, 2004).

Notable amongst the methods are the *weighted sum,* the *constraint method,* MAUT, GP and CP (Coit et al., 2004). These are particularly effective at dealing with deterministic (or stochastic) models characterised by continuous search spaces. The weighted sum method specifies weights for each criterion and aggregates them into a scalar function where w_i is the weight of the $i'th$ criterion function ($i = 1,2,..,k$) and is given by the following:

$$\min\left\{\sum_{i=1}^{k} w_i f_i(x):\ x \in S,\ w_i > 0,\ \sum_{i=1}^{k} w_i = 1\right\} \tag{2.16}$$

The values of the weights, which are inputs to the optimisation, are decided beforehand. This may be done interactively with the user or a set of weights may be generated independently by the analyst. The user expressed weights represent the relative preference for the individual criteria where a higher weight represents a higher preference. Where the analyst supplies the weights the solution corresponding to each set constitutes a point in the Pareto optimal set (Stadler, 1988); thus by varying the weights parametrically the analyst may generate a portion of the Pareto optimal set for consideration by the user. The major drawbacks of this method as discussed by Das & Dennis (1997) and de Weck & Kim (2004) are: (i) it fails to find Pareto optimal solutions at the non-convex areas of the Pareto front. This is because the method is implemented as a convex combination of the objectives, with a constant weights' sum and negative weights disallowed. (ii) Even where the Pareto front is convex an even spread of weights does not guarantee an even spread of points along the Pareto front. The Adaptive Weighted Sum Method by de Weck & Kim (2004) is a recent work aimed at addressing the former drawback. A challenge presented by the method is how to determine the appropriate weights especially where there is not enough information about the problem. A major advantage of the method is its computational efficiency.

The constraint method also referred to as the ε-Constraint Method or the Trade-off Method is such that a function $f_i(x)$, $1 \le i \le k$ is selected as the primary criterion for minimisation while each of the remaining functions, treated as secondary, is constrained to an upper bound u_j which is a pre-determined value for the jth criterion. The method is given by the following general form:

$$\min\{f_i(x) : x \in S,\ f_j(x) \le u_j\ \forall\ 1 \le i \le k\ ,\ j = 1,2,...i-1, i+1,..., k\ \} \qquad (2.17)$$

A positive aspect of this method is that it does not require any convexity assumption (Ehrgott, 2005) which means that it is better at locating the Pareto front in non- convex search spaces than the weighted sum approach. The constraint method systematically modifies the search space by altering the values of the upper bounds of the criteria and solving the resultant problems. In this way the entire Pareto optimal set may be generated. The user may express relative preference for each criterion through the choice of upper bounds. They must however be feasible to ensure that the solutions obtained are Pareto optimal. Its simplicity is also one of its advantages. The objective functions however can be time consuming to code, where there are too many of them. It tends also to find weak Pareto optimal solutions.

The utility function approach is premised on utility or value theory, which is based on a set of propositions of logical choice behaviour of a decision maker (Cohon, 1978) to the effect that faced with a set of decision alternatives a decision maker would make choices consistent with a utility function, one that is defined on an interval scale (cardinal function). Thus if v is a value function, defined on the criterion space, then the decision maker seeks $x \in S$ which minimises $v(f(x))$. Conditions desirable for finding x are that u is non-increasing and the feasible criterion set convex. The utility approach therefore transforms the MCO as given in (2.1) into an equivalent problem of the form:

$$\min\{v(f(x)) : x \in S\} \tag{2.18}$$

where $v(f(x)) = v(f_1(x),..., f_k(x))$ is a scalar-valued function. A well known approach for the construction of utility functions is the decomposition method which assumes that the utility of the $i'th$ criterion is mutually independent from the others and therefore the overall utility is either additive or multiplicative (Cohon, 1978). Thus the top level utility function may be expressed by:

$$v(f(x)) = \sum_{i=1}^{k} v_i f_i(x) \quad \text{or} \quad v(f(x)) = \prod_{i=1}^{k} v_i f_i(x) \tag{2.19}$$

where $v_i f_i(x)$ is the utility of the $i'th$ criterion. While the functions $v_i f_i(x)$ exist in theory, for all i, they are in practice not easy to find, due to the following reasons: (i) the functions are derived from the decision maker's expressed preference information which can be time consuming to extract, (ii) the decision maker may have difficulty making value judgements about the various criteria, (iii) modelling preferences is a difficult task and may result in making assumptions that could over-simplify the problem, and (iv) they ignore the interactions between criteria. An advantage, however, is that they allow the decision maker to concentrate on one criterion at a time while specifying utilities (Cohon, 1978). On another note, the assumptions of logical behaviour on the part of the decision maker have been questioned on the grounds that empirical evidence does not always support them (Dyer et al, 1992). Once the functions $v_i f_i(x)$ are specified the resulting scalar-valued problem can be solved by a standard scalar optimisation algorithm. The resulting Pareto optimal solution provides the BCS of the decision maker.

In GP an ideal value or goal G_i is set by the decision maker, for each criterion $f_i(x)$ $i = 1,2,..,k$ and deviations from the goals are minimised as follows:

$$\min\{ \sum_{i=1}^{k} \left| G_i - f_i(x) \right| : x \in S,\ i = 1,2,...,k \} \qquad (2.20)$$

The absolute value is used since the goals for each criterion may be less or greater than the attainable value of the criterion. An equivalent linear formulation in terms of the positive and negative deviations d_i^+ and d_i^- respectively, of the *ith* goal from the *ith* criterion is the following:

$$\min\{\sum_{i=1}^{K}(d_i^{+} + d_i^{-}) : x \in S, G_i - f_i(x) = d_i^{-} - d_i^{+}, d_i^{+}, d_i^{-} \geq 0\} \qquad (2.21)$$

It is noted that d_i^+ and d_i^- will not both be nonzero for a given goal. That is when $d_i^+ = 0$ then $d_i^- \neq 0$ and vice versa. This is because even though many combinations of d_i^+ and d_i^- would satisfy the equality constraint given by $G_i - f_i(x) = d_i^+ - d_i^-$, the ones that minimise their sum is what is required. Sometimes a decision maker may want to modify the formulation in (2.21) by a weighting scheme, to reflect his/her relative preferences for each criterion. The corresponding formulation would therefore be:

$$\min\{\sum_{i=1}^{K}(w_i^{+} d_i^{+} + w_i^{-} d_i^{-}) : x \in S, G_i - f_i(x) = d_i^{+} - d_i^{-}, d_i^{+}, d_i^{-} \geq 0, i = 1,2,...,k\} \qquad (2.22)$$

In (2.22) the weights are assumed non-negative. Special cases are when either of the weights takes on a value of zero, or when some weight is assigned a very high value relative to the others. The former results in what is called a one sided goal programming formulation, which occurs when a decision maker determines that only positive or negative deviations from the goals are of importance. The latter occurs when the relative importance of one goal is rated higher than the others (Cohon, 1978). The GP approach suffers from the risk that a set of goals may lead to a solution that is not Pareto optimal (Das, 2000). This may occur where the goal is in the feasible criterion space but not Pareto optimal. In such a situation the goals may be attained, producing a total deviation of zero. In general an ideal solution which results in a Pareto optimal solution is one that is in the Pareto optimal set, or close to it (Cohon, 1978). The formulations in (2.21) and (2.22) are scalar-valued optimisation problems which yield solutions that represent the BCS of the decision maker.

Another scalarisation approach, viewed as an extension of both the weighted sum and the utility function methods, is the normed method also called compromise programming, global criterion, or utopian point methods (Stadler, 1988; Marler &Arora, 2004). This is given by:

$$\min\left\{\left\|f(x)\right\|_P : x \in S\right\} \qquad (2.23)$$

where:

$$\left\|f(x)\right\|_P = \left\{\sum_{i=1}^{K}(f_i(x) - U_i)^P\right\}^{\frac{1}{P}} \quad 1 \le P < \infty \qquad (2.24)$$

and U_i is the unique minimum of the ith criteria. The vector $U = [U_1, U_2, ..., U_K]^T$ is therefore generally infeasible; i.e. $U \notin f(S) \forall x \in S$. Therefore U is referred to as a *utopian point*. Where some or all the independent minima of some or all the criteria cannot be determined, U may be approximated (Marler & Arora, 2004). The expression in (2.24) which is the Euclidean norm is called a distance function since it measures the distance of a solution from U. There are occasions where a weighting scheme may be introduced to model preferences. In this case the method is called weighted norm, given by:

$$\min\left\{\|wf(x)\|_P : x \in S\right\} \tag{2.25}$$

Where:

$$\|wf(x)\|_P = \left\{\sum_{i=1}^{K} w_i (f_i(x) - U_i)^P\right\}^{\frac{1}{P}} \quad 1 \leq P < \infty \tag{2.26}$$

Note that a series of scalar problems results as P is assigned specific values: the higher the value of P the closer the solutions in a feasible criterion set. The most often used values of P are 1, 2 and ∞. Where ∞ is assigned, the resulting scalar problem is called min-max optimisation as defined by Stadler (1988). A solution to (2.23) or (2.25) for fixed values of P and $w_i > 0 \ \forall i$ is Pareto optimal (Stadler, 1988). Thus by varying either P or w_i or both a set of Pareto optimal solutions may be generated (Stadler, 1988; Marler & Arora, 2004). A benefit of this method is its ability to find solutions in non-convex spaces. A major difficulty is that if the objectives functions have different units the Euclidean norm fails (mathematically speaking) to be a good representation of the distance of the solutions from the utopian point. In such situations it is better to transform the objective functions so that they are dimensionless (Marler & Arora, 2004). Table 1.1 summarises the methods discussed in terms of their major characteristics and conditions under which they generate Pareto optimal solutions. Further examples of the scalar methods are discussed by Miettinen (1998) and Ehrgott (2005). An observation that is common to all the methods is that they each have their particular strengths and weaknesses. A summary of the generic strengths and weaknesses of the methods are recounted on the next page.

Table 2.1: A summary of the main features of the Scalar methods discussed

<table>
<tr><th>Method</th><th>Characteristics</th><th>Condition for Pareto optimal solutions</th><th>Advantages</th><th>Disadvantages</th></tr>
<tr><td rowspan="3">Weighted Sum</td><td>Minimises a convex combination of weighted objective functions,</td><td>Weights are strictly positive,</td><td rowspan="2">Computationally efficient and easy to apply</td><td>Fails to find Pareto optimal solutions where feasible criterion set is non-convex</td></tr>
<tr><td rowspan="2">Uses normalised weight vectors</td><td>Weights systematically varied</td><td rowspan="2">Difficulty in determining appropriate weight</td></tr>
<tr><td>Convex feasible criterion set</td><td>Simplicity</td></tr>
<tr><td rowspan="2">ε-Constraint</td><td rowspan="2">Sets one criterion as the objective function to be minimised and constrains the others to determined limits</td><td rowspan="2">The limits on objective functions must be feasible</td><td rowspan="2">Can find Pareto optimal solutions in non convex regions of Pareto front</td><td>Could be time consuming and difficult to code for a large set of objective functions</td></tr>
<tr><td>Could yield weak Pareto optimal solutions</td></tr>
<tr><td rowspan="2">Utility Function</td><td rowspan="2">Minimises specified utility functions for all the criteria</td><td>Utility function non-increasing</td><td rowspan="2">Allows decision maker to focus on one criterion at a time</td><td>The utility function may be difficult to determine</td></tr>
<tr><td>Convex feasible criterion set</td><td>Fails under non-convex conditions</td></tr>
<tr><td>Goal Programming</td><td>Minimises deviations of the objective functions from a specified goal</td><td>The goals must be close to or in the Pareto front</td><td>The decision maker sets the goals. Efficient in bi-criterion cases</td><td>The goals can lead to dominated solutions. Setting goals for many criteria can be difficult</td></tr>
<tr><td>Compromise Programming</td><td>Minimises deviations of the objective functions from a chosen infeasible point</td><td>The weights must be strictly positive</td><td>Can find Pareto optimal solutions in non convex regions of Pareto front</td><td>Requires that the units of all criteria are the same</td></tr>
</table>

Strengths

- Generally efficient at finding Pareto optimal solutions where the search space is continuous
- Where user preference is specified or known they can find the preferred solution with relatively few computations and thus at minimal cost.
- They have well established theoretical and mathematical foundations that facilitate investigations into the convergence of the methods

Weaknesses

- Generally, they fail to find solutions in non-convex regions of the search space.
- Where the user is unable to articulate preferences, the generation of the entire Pareto set or sections of it may be required. This may constitute a heavy computational burden as well as additional cost.
- Since computational runs are independent there is no opportunity to exploit synergies between the solutions (Petrovski & McCall, 2001)
- Where the solution space is large the search for Pareto-optimal ones can be a daunting task with a huge resultant computational burden and cost
- May not provide an opportunity for post optimality analysis since the entire Pareto front may not be found.

None of the methods can thus be said to be good in all cases. Therefore the choice of any for a problem should by informed by its characteristics, time and budgetary constraints together with its strengths and weaknesses.

2.5.2 Stochastic Methods

The stochastic methods, also generically referred to as Pareto or vector minimisation methods, because of their population-based approach to finding solutions, are relatively new. They have been developed only over the last two or three decades and are still evolving; but are already proving to be the popular choice for the solution of many practical MCO problems. Some of the factors responsible for this development are on the one hand attributable to the complexity of practical problems which in many instances are non-linear, non-convex, non-differentiable, combinatorial or NP-hard (Petrovski & McCall, 2001; Venkataraman & Hafka, 2002). The classical methods are generally reputed to be deficient in adequately handling such situations compared with the stochastic ones (Marler & Arora, 2004; Taboada et al, 2007; Zhao et al, 2007; Taboada & Coit, 2007). The stochastic methods have thus been developed mainly in response to this need. Their attractiveness is also due to their relative ease of use, even though their solution output may be sub-optimal. Popular examples in this category are the class of genetic algorithms (GA) specifically devoted to MCO problems. These are generally called *Multi-objective Genetic Algorithms* (MOGA) out of which have evolved the so called *Multi-objective Evolutionary Algorithms* (MOEA) (Salazar et al., 2006). Specific instances of the latter, are, *Vector Evaluated Genetic Algorithm* (VEGA), *Niched Parameter Genetic Algorithm* (NPGA), *Strength Pareto Evolutionary Algorithm* (SPEA), and *Non-dominated Sorting Genetic Algorithms 1&2* (NSGA1&2). A detailed discussion of these methods is given by Deb (2001) and Marler & Arora (2004). Other heuristic algorithms in current use are *Simulated Annealing* (SA), *Tabu Search* (TS), *Multi-objective Particle Swarm Optimisation* (MOPSO), and *Ant Colony Optimisation* (ACO).

A common feature with these is that they are based upon and mimic some natural phenomena in their solution process; they also adopt a global search approach to the solution. For instance MOGA which is an extension of the single objective GA to multiple objectives through the introduction of a Pareto-ranking scheme emulates nature's evolution based on preferential survival. Examples are the reproduction of the fittest members of the population, maintenance of a population with diverse members, inheritance of genetic material from parents and occasional mutation of genes (Savic, 2002). MOPSO on the other hand, unlike MOGA, simulates the social behaviour of flocks and explores the search space by adjusting the trajectories of individual vectors called particles; these evaluate their positions in relation to a predetermined goal, during iteration. They are drawn stochastically towards the positions of their own previous best performance and that of their companions (Mahfouf et al. 2005). The ACO algorithm mimics the behaviour of ants in their movement from their nest to their food source and back. Ants are known to deposit a substance called *pheromone* along their paths, so by tracking the concentration of this substance they are able to find the shortest route between their nest and a food source (Shelokar et al, 2002).

The main strengths and weaknesses of these methods are noted as follows:

Strengths

- Easy to apply without modification to the problem (Coit &Baheranwala, 2005)
- Ability to search through large solution spaces
- Ability to find solutions in both convex and non-convex regions in the search space
- Ability to approximate the Pareto optimal set in a single run
- Does not require preference information a priori.
- Permits post optimality analysis and thus facilitate the search for desired solutions.

Weaknesses

- No guarantee that a Pareto-optimal solution is found, though they are reputed to consistently find very good solutions (Coit &Baheranwala, 2005; Louie & Strunz, 2006).
- Being random (stochastic) methods, the solution quality can only be evaluated empirically. Thus convergence of solutions cannot be investigated theoretically.

While the stochastic methods appear to be more frequently used in applications than the classical ones, a recent interesting development in recent times is the incorporation, occasionally, of deterministic methods in a heuristic framework in the search for solutions. For instance Celli et al (2005) used the ε-constraint method in a GA and Coit & Konak (2006) used the weighted sum method in conjunction with a heuristic in the sizing and siting of generated electricity and the redundancy allocation problems respectively. Coit & Baheranwala (2005) also used the weighted sum method in conjunction with a GA to solve a stochastic system reliability design problem. This development (a clear research area) may have good prospects in terms of the exploitation of the strengths and compensation for the weaknesses within specific methods, through such integrations.

In Chapter 3 a review of the foundational concepts in reliability engineering is undertaken to provide the platform for the discussion of applications of the MCO methodology in design for reliability (the subject of Chapter 4).

2.6 **SUMMARY**

The decision making situations that involve multi-criteria have been posed as a vector optimisation problem. Consequently, the theoretical basics of vector optimisation and especially the concept of Pareto optimality have been discussed. The fact that there is no unique solution in vector optimisation, but rather a set of equally good solutions has been emphasised. The need therefore for a decision maker to exercise subjective judgements about the criteria in order to find one that is implementable, together with the difficulties associated with the process, have been emphasised.

The mathematical models used to describe MCO problems were classified as deterministic and stochastic. Within these main models sub-categorisations were made in terms of the solution requirements of each model; these were identified as continuous, discrete, or mixed-integer versions. The characteristics of the models and their impact on finding solutions have also been given some attention.

The general methodology of MCO applied in modelling practical problems has been noted to involve among others, formulation of a model, selection of a solution algorithm and search for solutions, and identification of a user's compromise solution. Emphasis was given to the intricate and elaborate steps and processes that have to be undertaken to secure a working model, together with finding realistic solutions. The model characteristics were observed as being an important determinant in the choice of a solution algorithm. Computational complexity was especially noted as a major challenge in solving real problems.

The solution methods for MCOs were grouped under two major classes: classical and stochastic. The most popular of the former were discussed in terms of their properties and their strengths and weaknesses. Their characteristics included the fact that they allowed the incorporation of preferences before, during, or after a solution search, and then resulted in a single solution output at each optimisation. The classical methods were found to be particularly useful in the context of continuous or convex situations. They were however not capable of dealing with some notable features of MCOs occurring in real life applications; such as those which had discrete, combinatorial, non-differentiable, or non-convex search spaces. The stochastic methods use heuristic algorithms, which are characterised by a population based approach and random search techniques to find solutions. The methods were distinguished from the classical ones in terms of their ability to better handle the complexity issues and also to approximate the Pareto front at a single run of the solution algorithm. They were, however, considered to be deficient in terms of a sound mathematical basis upon which to investigate the convergence of solutions.

CHAPTER THREE

SYSTEM RELIABILITY DESIGN

3.1 INTRODUCTION

The concept of reliability as it is known today has evolved from being intuitive and non technical (Endrenyi, 1978), through a period of refinement and development during which it was considered to be vital especially in the design of electronic products for military purposes (O'Connor, 1995), to a discipline that enjoys well established theoretical and practical foundations and is considered a necessary factor for both enhanced product and process quality and safety related concerns. The fact that it is now a major engineering subject is indication not only of the tremendous growth in terms of its theory and application, but also of its importance and indispensability.

This chapter will report on the theoretical ideas upon which reliability is founded, as well as its role and relevance in the design of systems in general, and to provide a basis for the research work undertaken. The various fundamental concepts and terminologies are firstly defined and discussed. A number of models and techniques for evaluating systems' reliability will follow.

3.2 TERMINOLOGY AND DEFINITIONS

Before discussing systems reliability, the focus of this work, it is necessary to define terms such as system, subsystem/assembly, component, failure, and reliability.

- **A system** may be either a product or a process; in this thesis it will generally refer to a product or item which comprises identifiable parts or units/elements called components, subsystems and/or assemblies which are integrated to perform one or more specified functions (Kuo & Zuo, 2003). Therefore, while a computer is clearly a system since there are distinct parts such as the processor, the memory, the key board, the monitor, the mouse etc, all working together for a common purpose, the same cannot be said, for instance, of a shop shelf displaying various goods, since in this case there is no interaction between the items on the shelf. A system may also be viewed as a set whose elements are the components (or subsystems/assemblies) where all the elements must work as a unit to achieve the system goals.

- **A subsystem** may be one or more distinct components of a system. Technically speaking therefore every system is its own subsystem, which also means that a subsystem in some situations may be viewed as a system if it discharges its function(s) as an independent entity.
- **An assembly** is a subsystem. In some systems composed of subsystems, the subsystems themselves may also be composed of lower level subsystems which in this case may be called assemblies (ReliaSoft Corporation, 1992-2008). An example could be a car which has subsystems such as the engine, gear box, suspension etc. The engine itself would comprise subsystems such as the ignition, fuel, and cooling systems say.
- **A component** is a subsystem which is not decomposed further into lower level subsystems or assemblies during a given reliability analysis (Endrenyi, 1978). This definition suggests that a component could actually be made up of other components which would not be subject directly to reliability analysis. Thus while the configuration of a system may be altered the same cannot be done with a component during the analysis (Endrenyi, 1978). In order to alter their reliability, components may either be replaced

with ones that provide the required level of reliability or be redesigned (Mettas, 2000). Components may also be either repairable or non-repairable. Repairable components when failed may be returned to their original state after service (or repair) for further use. Non-repairable components, however, cannot be repaired and thus cannot be reused when failed. In this thesis components are assumed to be non-repairable.

- **Failure** is an event the occurrence of which puts a component or system into a state whereby it is no longer able to perform its specified function(s) as expected. Thus a bulb is considered to have failed when it does not light up when it is switched on, or a car, when it does not start when the ignition is fired. The former illustrates the failure of a non-repairable component or system, whilst the latter illustrates that of a repairable one. In some systems there can only be one of two states: failed or in perfect working condition (Dhillon, 1985). Such systems are called binary. In other systems a number of states are possible where they could be neither failed nor performing at their expected level. Such systems are called multi-state (Pham, 2003; Ramirez-Marquez & Coit, 2004). This thesis assumes that a system is always binary. From this definition it may be inferred in general that failure is first of all random and secondly time dependent. This observation is explored further in the next section.
- **Reliability**, according to Kuo & Zuo (2003), is “a measure of how well a system meets its design objectives”. According to Billinton & Allan (1992) it is “the probability of a device (or product) performing its purpose adequately for the period of time intended under the operating conditions encountered”. It may also be said to be the probability of a system not failing during a specified period in its operation. From these definitions it is clear that reliability is a system performance measure which is expressed as the

probability of failure not occurring in a specified time. The relationship between failure and reliability is apparent from the definitions and it is stated explicitly later. A product which is more prone to failure than another could thus be said to be less reliable than the other. It is easy to agree therefore with Hecht (2004), that the primary aim of system reliability analysis is the prevention of failure.

3.2.1 **Failure Causes and Characteristics**

Generally a system fails as a result of one or more of its components failing. It is sufficient therefore to discuss only component failures, the causes of which, especially in the case of electronic and mechanical ones, can vary. For instance a component may fail (i) when subjected to very high temperatures, (ii) when an applied load exceeds its strength, (iii) due to chemical changes such as corrosion emanating from the environment in which it is used, or (iv) as a result of age. In general failure is characterised by three main factors: design or manufacturing defects, accidents or misuse, or fatigue (Rao, 1992; Wasserman, 2003). Failures which fall into the first category occur early in the life of a component and their frequency, called failure rate or hazard rate, tends to decrease over time. Those originating from accidents occur during the useful life of a component and are characterised by a constant failure rate; and those associated with fatigue occur at the tail end of the life of a component and the number of failures increase with time. Figures 3.1 (a, b, and c) depict the *Bath-Tub Curve* (Rao, 1992; O'Connor, 1995) which provides a graphical illustration of the failure characteristics of components. The graphs are plots of the hazard rates denoted by $h(t)$ against time t.

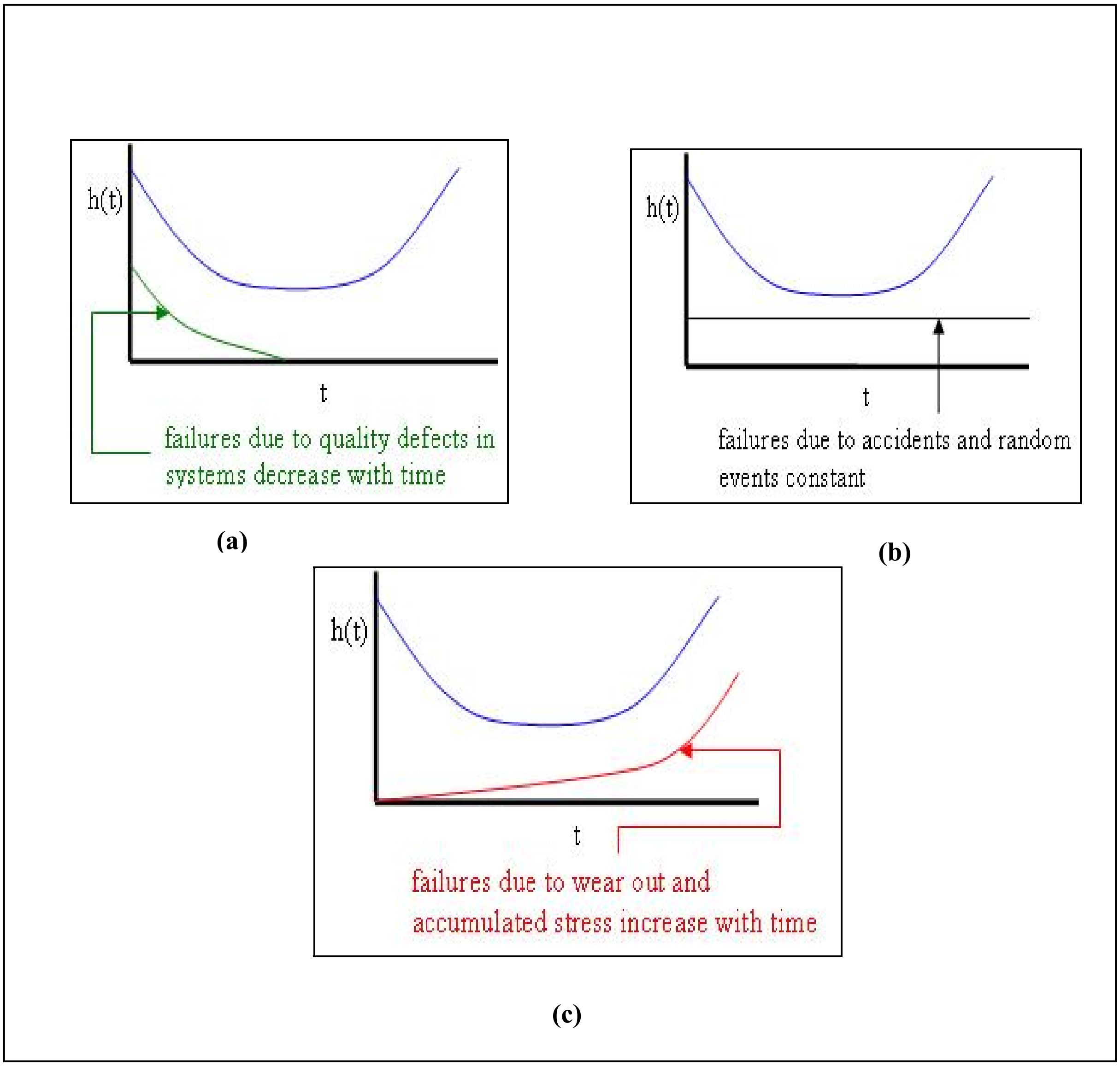

Figure 3.1: The Bath-Tub Curve (Source: O'Connor, 1995)

3.2.2 Failure Prevention Techniques

Design and reliability engineers employ a range of techniques to reduce the chance of a failure occurring in a component or system and these span across the entire life cycle of the product (i.e. from design to death or obsolescence). They encompass: (i) conservative designs, (ii) analytical tools and techniques, (iii) testing regimes and (iv) reliability enhancement approaches. The first involves the use of sufficient safety margins and high quality parts and materials while considering any environmental restrictions (Hecht, 2004). Techniques such as Failure Mode and Effect Analysis (FMEA), Fault Tree Analysis (FTA), Component Criticality Analysis (CCA) (Wang et al, 2004; Espiritu et al, 2007) etc, can be used to investigate the ways in which a system may fail and the most critical components as far as failure is concerned, so as to take remedial actions. Testing regimes comprise verification of the design margins and assessment of the product under extreme environmental conditions to determine its tolerance level. The last group involves the use of redundancy, or components with higher reliabilities, or maintenance techniques in the case of repairable products.

3.2.3 **Basic Reliability Metrics**

It was observed at the beginning of this section that failure is random and time dependent and that reliability is closely related to it. The basic time dependent measures used to quantify component or system reliability, such as the: (i) *failure and reliability functions*, (ii) *hazard function* and (iii) *mean life function*, are discussed in Appendix B.

3.2.4 **Component Failure Time Models**

Several statistical distributions can be used to model the failure times of both components and systems. Notable among them are the Exponential, Normal, Log-normal, Gamma and Weibull distributions (O'Connor, 1995). Of these the latter is the most widely applied, since it fits many life distributions (O'Connor, 1995). It actually incorporates some of the other distributions, such as the Exponential and Normal as special cases. The Weibull distribution and its properties are discussed in Appendix B.

3.3 **SYSTEM RELIABILITY MODELS**

Unfortunately the techniques for modelling and evaluating component reliabilities cannot always be readily applied directly to systems because they comprise multiple components in a variety of configurations. It is therefore impossible (or cost prohibitive) for designers to test all the components. A more cost effective approach is to model a system's reliability in terms of the reliabilities of its components (Reibman &Veeraraghavan, 1991). A basic assumption which is implicit in the models is that a given system has a *monotonic structure* (also referred to as a *coherent* system) (Endrenyi 1978; Soyer et al, 2004) characterised by the following properties:

- Each component's state is binary (i.e. either operating or failed)
- A system's state is binary
- A system is operating if all the components are operating
- A system has failed if all of its components have failed
- A failed component in a failed system cannot restore the system back to operation; neither can the replacement of a failed component in an operating system cause system failure.

3.3.1 **Types of System Configuration (Simple Systems)**

A system's reliability depends as much on its configuration as on the reliability of its components. Classical or basic system configurations (Wang, 2008) also called simple systems (or simple networks) (Billinton & Allan, 1992), comprise components in series, parallel, k out of n, (i.e. (k,n)) and mixed series-parallel (Rao, 1992, Majety et al, 1999), and defined in the following way. Suppose a system comprises n components ($n \geq 2$), and e_i denotes that the $i'th$ component is operational and e_i' that it has failed. A similar notation is used to denote the status of the system.

The components are said to be in **series configuration** if the failure of any component $i \in \{1,2,...,n\}$ results in the failure of the system. Such a system is also called a series system, and is operational if:

$$s = \bigcap_{i=1}^{n} e_i \tag{3.1}$$

and failed if:

$$s' = \bigcup_{i=1}^{n} e'_i \tag{3.2}$$

The expressions (3.1) and (3.2) respectively use the set theory language of intersection and union to characterise a series system's success and failure conditions. A series system is thus one in which operational success depends on the operational success of all its components and whose failure depends on the failure of any one of its components. A system whose components are all in series configuration is sometimes also referred to as *non-redundant* (Billinton & Allan, 1992). This is the simplest to analyse and the most commonly encountered (Heimann, 1993).

The n components of a system are said to be in **parallel configuration** if the system's failure occurs only if all components i, fail, $i = 1,2,...,n$. Such a system is also called a parallel system, and is operational if:

$$s = \bigcup_{i=1}^{n} e_i \tag{3.3}$$

and failed if:

$$s' = \bigcap_{i=1}^{n} e'_i \tag{3.4}$$

A system's operational success is thus guaranteed if one or more of its components are operational. This is characterised by the union of the operational successes of all its components as in (3.3). The system's failure occurs if all of its components fail. This is characterised by the intersection of all the failure events of its components, as in (3.4). Components in this type of configuration are also considered to be *redundant* (Reliability Edge, 2002). A parallel system is also referred to as *fully redundant* (Billinton & Allan, 1992) where all components are in active operation; otherwise it is referred to as a *standby parallel redundant* system. In this case the components in parallel are on standby until triggered into operation following the failure of an active component (Dhillon & Singh, 1981; Zhao and Liu, 2003). Redundant systems are also called *fault tolerant* (Wattanapongsakorn & Levitan, 2001; Diab & Zomaya, 2005; Mukuda et al, 2007).

The n components of a system are said to be in **(k, n) parallel configuration** when the system's operation is guaranteed if any k or more components in parallel are operational, and failed otherwise,

$(1 \le k \le n)$. In set theory terms this is expressed as:

$$s = \bigcup_{i \in \{k,..,n\}} e_i \tag{3.5}$$

and:

$$s' = \bigcap_{i \notin \{k,..,n\}} e_i \cup \bigcap_{i \in \{k,..,n\}} e'_i \tag{3.6}$$

A system with (k, n) parallel configuration is also referred to as *partially redundant* (Billinton & Allan, 1992).

Consider a system that comprises n_i distinct sets of components, ($i = 1,2,...,m$) and for each set there are k components, $k \in \{1,2,...,n\}$, then these sets are called subsystems of the system. If for each subsystem i the n_i components are in parallel and the i subsystems are in series, then the system is said to have a **series-parallel configuration**. It should be noted that this definition assumes that the components of the system are in pair wise mutually exclusive sets. When the assumption is not valid the system does not have a series-parallel configuration; however a transformation can be obtained using the concept of *cut sets* (Billinton & Allan, 1992). This will be discussed later in this chapter.

If for each subsystem i the n_i components are in series and the i subsystems are in parallel, then the system is said to have a **parallel-series configuration**. This definition also assumes that the components of the system are in pair wise mutually exclusive sets; and again if the assumption is not valid a transformation can be obtained by means of *tie sets* (Billinton & Allan, 1992) which is outside the scope of this thesis.

3.3.2 **Reliability Evaluation of Simple Systems**

Some important qualitative and quantitative reliability modelling and evaluation tools are logic diagrams, such as *Reliability Block Diagrams* (RBD), *Fault Trees* (FT) and *Event Trees* (ET) (Zafiropoulos & Dialynas, 2007). RBDs provide graphical representations of how the components of a system are connected logically, or reliability-wise (Reliability Edge, 2002), and are used to model how component and subsystem failures combine to cause system failure (Distefano & Xing, 2006). They are especially useful for modelling failures of simple systems where components are represented as blocks (or nodes) and joined together by a number of paths forming a network structure or topology, as can be seen in the following examples.

Firstly a typical RBD for a series system of n components is shown in Figure 3.2:

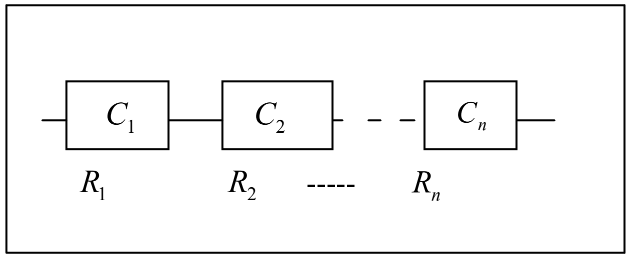

Figure 3.2: RBD of a series system of n components (Source: Hikita et al, 1992, pp 475)

where $C_1, C_2, ..., C_n$ are the individual components and $R_1, R_2, ..., R_n$ are their corresponding reliabilities.

Recall that such a system is operational if $s = \bigcap_{i=1}^{n} e_i$. Now if $P(e_i)$ is the probability that the $i'th$ component is operational then the reliability R_s of the system which is the probability of the system's successful operation $P(s)$ is given by:

$$R_s = P(s) = P(\bigcap_{i=1}^{n} e_i) = (P(e_1)P(e_2 / e_1)P(e_3 / e_1 e_2)...P(e_n / e_1 e_2 ... e_{n-1}) \quad (3.7)$$

Under the assumption of independence of component failures R_s becomes:

$$P(e_1)P(e_2)...P(e_n) = R_1 R_2 ... R_n = \prod_{i=1}^{n} R_i \quad (3.8)$$

Hence the reliability of a series system under the independence assumption is the product of the reliability of the individual components of the system (Kececioglu, 2002). A consequence of this result is that:

$$R_s \leq R_i, \quad i = 1,2,...,n \quad (3.9)$$

The equality only holds in the case where the reliability of each component is either zero or one. A series system's reliability is therefore bounded above by the least component reliability. Hence the reliability of a series system is determined by the component reliability and the number of components. To improve the reliability therefore one can either increase the reliability of the components or decrease the number of components (Heimann, 1993). Both Billinton & Allan (1992) and Heimann (1993) demonstrate graphically that the marginal gain in reliability through increased component reliability declines with an increase in the number of components. This is a major weakness of the series system configuration.

A parallel system on the other hand, has a RBD as shown in Figure 3.3:

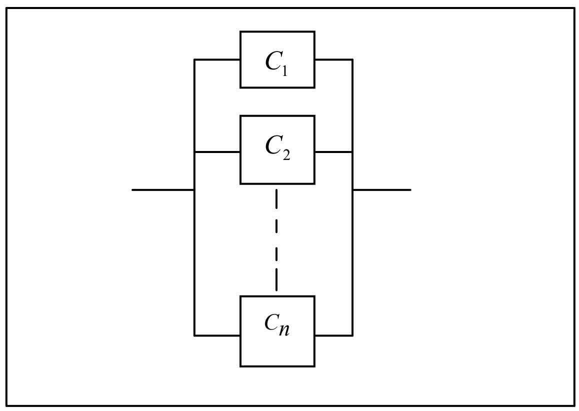

Figure 3.3: RBD of a parallel system

Again using $s = \bigcup_{i=1}^{n} e_i$ for an operational system of this type the reliability R_s of this system is:

$$R_s = P(s) = P(\bigcup_{i=1}^{n} e_i) = 1 - P(\bigcap_{i=1}^{n} e'_i) \tag{3.10}$$

Under the assumption of independence of component failures this can be written as:

$$R_s = P(s) = 1 - P(e'_1)P(e'_2)...P(e'_n) = 1 - \prod_{i=1}^{n} F_i \tag{3.11}$$

where F_i is the failure function of the $i'th$ component (also a cumulative distribution function). In terms of the component reliabilities, therefore:

$$R_s = 1 - \prod_{i=1}^{n} (1 - R_i) \qquad (3.12)$$

Hence for a parallel system:

$$R_s \geq R_i, \quad i = 1,2,...,n \qquad (3.13)$$

This shows that the reliability of a system is increased when parallel redundancy is used, and that a parallel system's reliability is bounded below by the highest component reliability. Therefore parallel redundancy is commonly used to increase a subsystem's reliability and ultimately that of an entire system (Dhingra, 1992; Prasad & Kuo, 2000; Lee et al, 2003; Zhao & Liu, 2003). Nevertheless redundancy, in general, can have adverse effects on a system and this is discussed later.

The RBD for a (k, n) redundant system configuration is similar to that of the parallel system shown in Figure 3.3. In this case, however, the successful operation of any k (or more) components (assumed identical) ensures the system's successful operation. There are $\binom{n}{i}$ mutually exclusive combinations of components' success events which would result in the system's successful operation. The probability of this event which is the required reliability is given by:

$$R_s = \sum_{i=k}^{n} \binom{n}{i} (1 - R)^{n-i} R^i \qquad (3.14)$$

In this case R is the reliability of the identical components of the parallel system and:

$$\binom{n}{i} = \frac{n!}{(n-i)!i!} \tag{3.15}$$

A standby redundant system RBD is depicted in Figure 3.4:

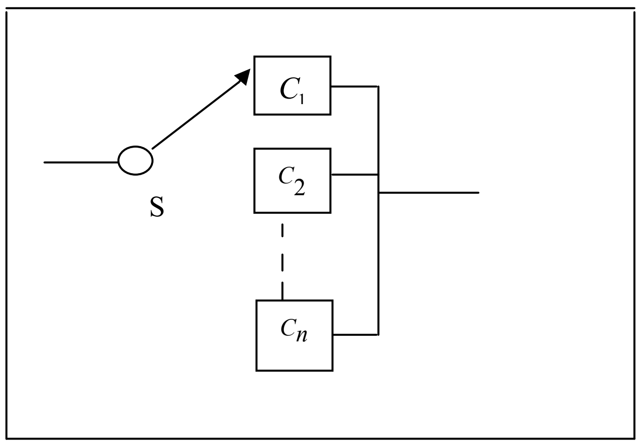

Figure 3.4: RBD of a standby redundant system (Source: Dhillon and Singh, 1981, pp 33)

It represents the situation where one component is operating and the remaining $n-1$ are on stand by until failure has occurred, then operation is switched to the next component, and so on. The system falls into either the perfect or imperfect switching type. In the former it is assumed that the switch S is failure free whilst in the latter this is not the case and the assumption is that the failure occurs in a number of different ways, for example failure of the switch S itself, or an inadvertent sensing of failure (Gordon, 1957; Kapur & Lamberson, 1977). The reliability of the system is therefore dependent on which of the categories apply and Kapur & Lamberson (1977) have discussed how the reliability expression is determined for both.

The RBD of a series-parallel system is shown in Figure 3.5:

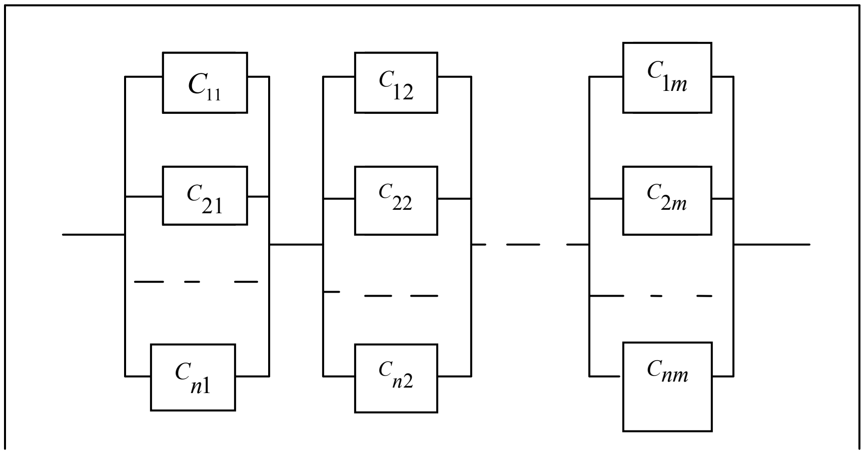

Figure 3.5: RBD of a series-parallel system (Source: Coit and Smith, 1997, pp 272)

The diagram depicts m subsystems in series configuration, with each subsystem composed of n components in parallel. Therefore, if the reliability of a subsystem is $R_{sub,i}$ $i = 1,2,...,m$, then the reliability R_s of the system is:

$$R_s = \prod_{i=1}^{m} R_{sub,i} \tag{3.16}$$

The reliability of the system in terms of the component reliabilities R_{ki} is therefore:

$$R_s = \prod_{i=1}^{m}\left(1 - \prod_{k=1}^{n}(1 - R_{ki})\right) \tag{3.17}$$

The RBD for a system with a parallel-series configuration is shown n Figure 3.6:

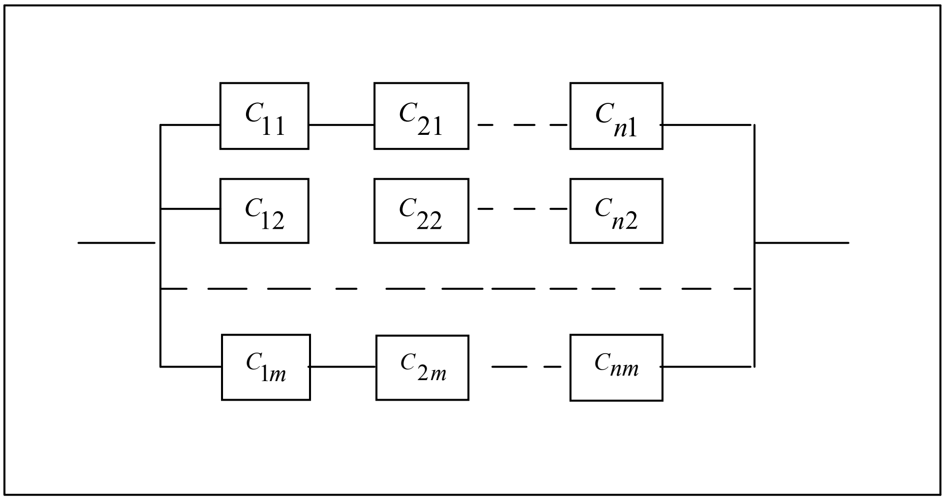

Figure 3.6: RBD of a parallel-series system

In this case there are m subsystems in parallel configuration while each subsystem is composed of n components in series configuration. The reliability R_s of the system (see Appendix B) is:

$$R_s = 1 - \prod_{i=1}^{m} \left(1 - \prod_{k=1}^{n} R_{ki} \right) \qquad (3.18)$$

The results in both (3.17) and (3.18) suggest that the reliability of mixed series-parallel systems can be obtained using series-parallel reduction techniques (Billinton & Allan, 1992), which basically reduce a subsystem of parallel components into a single entity of which the reliability is the parallel components' reliability; and a subsystem of series components into a single entity with reliability equal to the series components' reliability, and so on.

As a concluding remark for this section, apart from the series system, all the others have incorporated redundant components in one form or another. The benefit of redundancy has been noted; however it can impact negatively on a system. On the one hand it could mean increased complexity and cost, weight and volume, power consumption (especially for some electronic products), and complicated system monitoring and evaluation procedures (Hnatek, 2003). On the other hand it could actually result in a reduction in the reliability of a system (Hnatek, 2003). Billinton & Allan (1992, pp 88) have demonstrated, using a graphical illustration, that in general, the largest gain in reliability for a single component system is limited to the first redundant component; subsequent additions result in diminishing gains which asymptotically tend to zero. Zafiropoulos &Dialynas (2007) have noted also that the RBD approach while good at modelling independent component failure situations, has limited capabilities for representing operational dependencies (characteristic of complex systems), including investigating "what-if" scenarios for fault diagnosis. Under such a circumstance they suggest that the FT logic diagram was a better alternative.

3.3.3 **Reliability Evaluation of Complex Systems or Networks**

A complex system or network is one which cannot be modelled purely as a series, parallel, or mixed series-parallel system. A typical example is the bridge network (Mohan &Shanker, 1988; Kim &Yum, 1993) the RBD for which is given in Figure 3.7:

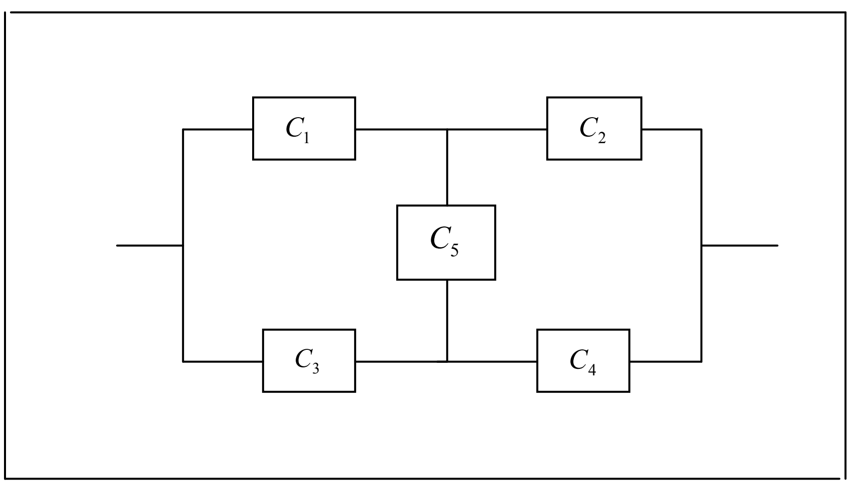

Figure 3.7: RBD of a bridge network (Example of a complex system)

Complex systems are common computer, communication, transportation, electrical and manufacturing networks (Atiqullah & Rao, 1993; Amari et al, 2005). These tend to be large spatially distributed multi-component systems with complicated configurations that pose computational challenges as far as evaluation of their reliability is concerned (Provan 1986; Thangamani, 1994; Marseguerra et al 2005). Complex system configurations are also found in manufactured products, such as a life support system in a space capsule (Zhao & Liu, 2003), an electricity transmission system (Espiritu et al, 2007) and a gas installation.

Various methods that provide exact or approximate solutions have been devised to evaluate the reliability of complex systems (Kuo &Zuo, 2003). While both provide analytical expressions for the system reliability, the exact methods which include the *cut and tie set, conditional probability* (or *pivotal decomposition*), and *enumeration methods* (Rao, 1992) are supposed to yield the precise system reliability that corresponds to the input values of the component reliabilities and the associated system configuration. (These are useful where the system size is small or moderate). The approximate methods, however only provide an estimate of the system reliability for the given input component reliability values. They seek lower or upper bounds for the system reliability and most of them involve the use of the cut and tie set methods. Notable amongst them are the *inclusion-exclusion*, *sum of disjoint products*, *Esary-Proschan*, and *linear-quadratic approximation* methods (Kuo &Zuo, 2003; Jin & Coit, 2003). The approximate methods are especially useful for large complex systems where the exact methods fail. Attention will be devoted to the cut set method, because, firstly, it is a reliability modelling tool adopted in this thesis and has wide applicability due to its relative ease of use; secondly it relates directly to the modes of system failure and furnishes the discrete set of events that characterise a failure; it is also programmable (Billinton & Allan, 1992). In addition the FT logic diagram is particularly useful in systems reliability analysis and evaluation; it provides an indirect way of identifying cut sets. A brief overview of the concepts behind it is given in Appendix B.

The Cut Set Method

The basic principle of the cut set method is the identification of sets of all combinations of components' failure events, the occurrence of which result in system failure. A cut set is thus defined as a set of system components which when failed result in the failure of the system. If a subset of components exists within a cut set, such that the subset is itself a cut set, and it contains no other cut sets, then it is called a *minimal cut set*. Suppose K_i, $i = 1,2,...,N$ is a minimal cut set for a system of N minimal cut sets. In general cut sets are not pair-wise mutually exclusive. This means that $K_i \bigcap K_j \neq \phi$ for some $i, j \in \{1,2,...,N\}$, $i \neq j$ Therefore the probability of failure F_s and the corresponding reliability R_s of the system is evaluated exactly from the following expression (Singh & Billinton, 1977):

$$
\begin{aligned}
R_s &= 1 - F_s = 1 - P(\bigcup_{i=1}^{N} K_i) \\
&= 1 - \Big(\left[P(K_1) + P(K_2) + ... + P(K_N)\right] - \left[P(K_1 \cap K_2) + ... + P(\underset{i \neq j}{K_i \cap K_j}) \right] + \\
&\left[P(K_1 \cap K_2 \cap K_3) + P(K_1 \cap K_2 \cap K_4) + ... + P(\underset{i \neq j \neq l}{K_i \cap K_j \cap K_l}) \right] ... \\
&... + (-1)^{N-1} \left[P(K_1 \cap K_2 \cap ... \cap K_N) \right] \Big)
\end{aligned}
\tag{3.19}
$$

Unless only a few minimal cut sets are involved, the expression in (3.19) can be tedious to work with. Currently there are software packages such as BlockSim of the ReliaSoft Corporation of the USA (Mettas, 2000) and PREVENT developed by ABB Netcom Ltd, Switzerland (Castelli, 1993), which reduces the effort involved in the evaluation. Even so, for very large complex systems or networks, the task of determining all the cut sets, to start with, can itself be enormous (being a combinatorial problem). The number of terms to evaluate also grows exponentially by a factor of 2^{N-1} with the number of cut sets (Singh &Billinton, 1977), which could, potentially, pose memory or storage problems, even with the use of computers. Fortunately a good approximation can be obtained for (3.19) where the component reliabilities are high (Singh &Billinton, 1977; Billinton &Allan, 1992). It has been demonstrated by Billinton & Allan (1992) that in such a case, the resultant value of the terms involving two or more cut set intersections is significantly reduced. Therefore a lower bound approximation of (3.19) is:

$$R_s \geq 1-\left[P(K_1)+P(K_2)+...+P(K_N)\right] \tag{3.20}$$

On the other hand an approximation of the system reliability can be obtained from a related series-parallel transformation, using the following characteristics of minimal cut sets: (i) the occurrence of the failure event for each minimal cut set K_i results in system failure; therefore, reliability wise, the minimal cut sets are effectively in series configuration; (ii) all components in a minimal cut set must fail for the failure event of the cut set to occur; therefore the components in a minimal cut set are effectively in redundant parallel configuration. A series-parallel transformation of the system thus follows immediately. The bridge network shown in Figure 3.7 is used to illustrate this point: The minimal cut sets are the following: $K_1=\{C_1,C_3\}$, $K_2=\{C_2,C_4\}$, $K_3=\{C_1,C_5,C_4\}$ and $K_4=\{C_3,C_5,C_2\}$. K_1 and K_2 are each called *second order cut sets* (i.e. they have two elements) and K_3 and K_4 are each *third order cut sets*. The corresponding RBD for the decomposed system is shown in Figure 3.8.

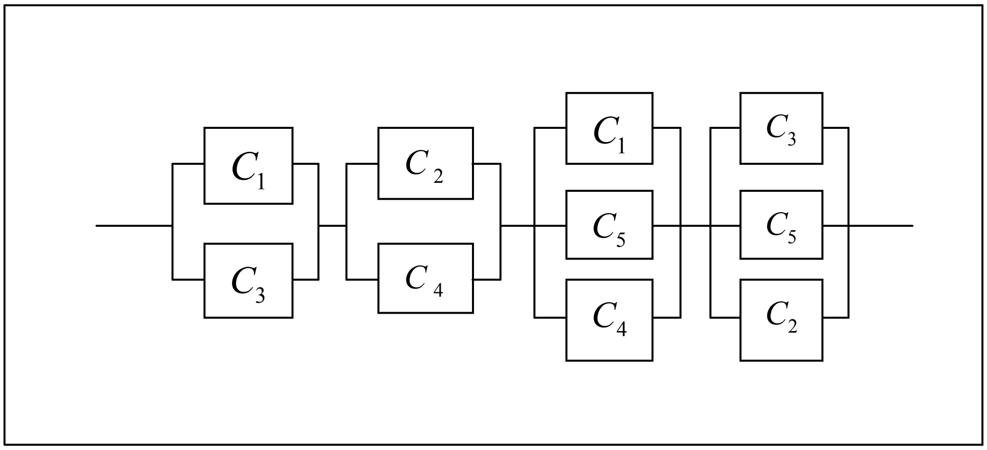

Figure 3.8: A series-parallel transformation of the bridge network

The figure is a series-parallel system with four subsystems, two of which comprise two components each, and the other two subsystems have three each. Each component is repeated in at least one other subsystem. The consequence of this is that the assumption of independence of component failures, and therefore of subsystem failures cannot be tenable. Evaluating the reliability of such systems is one of the difficult problems in reliability engineering (Coit & English, 1999). Proceeding with the independence assumption therefore means that the system reliability would be underestimated (Jin & Coit, 2001). In general, an analogous expression to (3.17) which provides instead a lower bound approximation of the precise value of the reliability of the resultant series-parallel transformation and therefore the original complex system reliability is used (Jin & Coit, 2003). This is given by:

$$R_s \geq \prod_{i=1}^{N}\left(1-\prod_{k\in K_i}(1-R_k)\right) \tag{3.21}$$

In this case, R_k is the kth component reliability in the $i'th$ subsystem (or cut set), composed of an arbitrary number of components. Once again if the component reliabilities are high, (3.21) is reputed to yield a good estimate of the required system reliability (Jin & Coit, 2003).

One advantage of a lower bound approximation of the system reliability as given in (3.20) and (3.21) is that it is better for many systems design, particularly for risk-averse ones, where operational safety is extremely critical, such as in nuclear reactors. In this case one would expect the true system reliability to be higher than the value obtained by the approximations. Therefore the better the approximations (at least to a decision maker), the better the true values are expected to be. The lower bound estimate may therefore be considered as a sort of worse case scenario for a system's reliability. Another advantage is the simplification of the computational work involved. Some draw backs of the approach, however, are the following: (i) unless the component reliabilities are known to be very high the estimate might fall short of expectation; (ii) one is also left in the dark as to how far from the exact value the approximation is, especially where high component reliabilities are not guaranteed; (iii) the lower bound estimate cannot be higher than the cut set with the least reliability, and the more cut sets there are the more the estimate could be unsatisfactory (since the cut sets are in series).

Before leaving this discussion, it is noted that while one may, by inspection, determine all the minimal cut sets of a complex system, where there are only a few components and cut sets, with real complex systems where components are literally in hundreds or more intuition becomes useless. Thus one needs to have a formal general procedure for determining the minimal cut sets. A discussion of some of the formal techniques, together with other pertinent issues, is presented in Appendix B.

3.3.4 **Reliability Importance of Components**

An important aspect of system reliability modelling and analysis is to identify the critical components or the weaknesses in a system, reliability-wise, in order to prioritise reliability improvements by directing resources and effort to the areas that have the most impact on the system's performance (Wang et al, 2004; Espiritu et al, 2007). In general reliability importance metrics are used to quantify the contribution of the individual components or minimum cut sets to the overall system performance, such as reliability, availability, risk, and safety (Espiritu et al, 2007; Chen et al, 2007). It is not the intention to discuss the subject in detail here since it is not a major theme in this work. However, it is noted that a variety of metrics have been developed to assess components or subsystems of a system for the purpose stated. The papers of Wang et al (2004) and Espiritu et al (2007) provide detailed discussions of some of the popular metrics of which the one due to Birnbaum is the first ever introduced (in 1969) and the most common. The Birnbaum importance metric is appropriate for the purposes of the work reported in this thesis and therefore discussed briefly in this section. It is applied in an example problem discussed later.

The Birnbaum importance metric for the $i'th$ component of a system, denoted by $I_i^B(t)$, is defined by:

$$I_i^B(t) = \frac{\partial R_s(t)}{\partial R_i(t)} = R_s(t, R_i(t) = 1) - R_s(t, R_i(t) = 0) \qquad (3.22)$$

where $R_s(t)$ and $R_i(t)$ are respectively the reliabilities of the system and the $i'th$ component at time t. $R_s(t, R_i(t) = 1)$ and $R_s(t, R_i(t) = 0)$ are the system's reliabilities at time t given that the $i'th$ component is operational and has failed respectively. The metric represents the maximum loss in system reliability when the $i'th$ component changes from being operational to having failed. An advantage of this metric is that it is easy to use, since it is not necessary to compute directly the partial derivative of the system reliability with respect to the $i'th$ component reliability, something that can be tedious to do, where large complex systems are concerned. A weakness of the metric is that it is determined on the basis of only the extreme values of the $i'th$ component's reliability and thus can lead to the same values being obtained for different components even when their reliabilities are not the same.

3.4 SUMMARY

In this Chapter the pertinent concepts and terminology used in the discussion of reliability in design have been defined and discussed. Reliability was described as a measure of the chance of success (or failure) of a system for a given period under specified conditions. Time was noted to be a random parameter around which a system's failure or reliability is modelled. Notable amongst the important metrics used to quantify a system's reliability were the reliability, the hazard rate, and the expected life functions.

The basic building blocks of systems reliability design, which concerned how components interacted or were configured, along with their schematic representations using reliability block diagrams, were noted to be the series, the parallel (and its variants), and the mixed series-parallel types. The associated resultant reliability of the systems, expressed as a function of their components' reliabilities, were also presented and discussed. Attention was briefly drawn to the characteristic strength and weakness of the series configuration which are respectively that they are simple to analyse and that the component with the least reliability had the greatest effect on the system's reliability, which also tended to decrease with increase in the number of components in series. The advantage and the disadvantages of the parallel configuration were also highlighted; the main advantage being that it resulted in higher system reliability than that of any of its constituent components. The disadvantages were noted to be: increased complexity, gain in weight and volume, complicated monitoring and evaluation practices, etc. A class of systems, generically called complex or networks, was mentioned with the focus on their characteristics and the ways in which their reliability could be evaluated. The computational challenges engendered by such systems were highlighted and the techniques that could be used for their reliability modelling and evaluation discussed. Particularly noted was the use of the cut set method to derive associated reliability expressions for such systems and to obtain a lower bound approximation or estimate of their reliabilities. The pros and cons of the approximation approaches were briefly discussed. Attention was also drawn briefly to the role of reliability importance metrics in reliability design and that of Birnbaum was discussed.

In the next Chapter the application of optimisation techniques in reliability design is discussed in the context of the current existing approaches, while emphasis is placed on the MCO approach. The potential gains to be derived from this are presented.

CHAPTER FOUR

OPTIMISATION IN RELIABILITY DESIGN

4.1 INTRODUCTION

A system's ability to perform its intended function, without fail, for a specified period, is an indicator of its quality, referred to as reliability. This attribute, which has far reaching consequences on the durability, availability, and life cycle cost of the system (Cranwell, 2007) and which is of great importance to the end user or the engineer, is very much a function of the design of the system (Ireson et al, 1996). There is thus a growing concern and interest among Reliability Engineers that reliability is built into a system at the earliest stages of the design process, i.e. at the conceptual or preliminary stage. The main objective of doing this is fundamentally to produce a concept or framework to predict the system's reliability (James et al, 2002). The decision process at this stage includes making choices regarding the type of components (and their associated reliabilities) to be used and the design configurations. The choices are driven by the interaction of reliability objectives with the economic costs associated with the design, manufacture and use (Marseguerra & Zio, 2000). Typically high reliability targets or specifications are set for the system, and ways to achieve them are then explored, taking into account resource constraints (Mettas, 2000). Apart from the limitations of resources, the targets set may also be in conflict. For instance a high reliability generally means a high cost, and could also mean excessive weight and volume. Also for a given system configuration its individual components may have different levels of reliability and associated costs, however the same level of system reliability may be achieved by using different component combinations. Therefore a natural optimisation problem arises (Majety et al, 1999).

Reliability considerations at the design stage provide certain advantages. For instance: (i) it allows a top-down approach (i.e. it looks at the top level design parameters), instead of a bottom-up one. This eliminates or reduces the need to make costly modifications when the design has gone into manufacture or has been commissioned for use, whereas the bottom-up one does not (Hassan & Crossley, 2002). (ii) It can also reduce warranty costs resulting from later failures during use. (iii) The cost of maintaining the system over its life time is also reduced (Cranwell, 2007; Lad et al, 2008). This research work, therefore, focuses on the optimal evaluation or prediction of a system's reliability at the design stage. The ideas and methodology resulting from the work (which is presented in chapter Five), however, are applicable to existing operational systems; Reliability and Design Engineers, in this case, would have to decide how the estimates obtained would be achieved for an existing operational system. This could mean substituting components with higher reliabilities, using redundant components, or redesigning the appropriate components or subsystems (Mettas, 2000).

The application of optimisation techniques in reliability design has been well researched and extensively reported in the literature (Mohamed et al, 1992; Kuo & Zuo, 2003) with notable benefits, such as: (i) the provision of an analytical model to represent a system. This allows testing of various scenarios to assess their impact on the system at virtually little or no cost. (ii) Estimates of the effects of individual upgrades or modifications are provided; (iii) It improves understanding of the system reliability as well as revealing any system reliability problem areas; (iv) It helps avoid wasteful resource expenditure by revealing modifications that would have an insignificant impact. The general optimal system design for reliability models are characterised by a system's performance criteria such as reliability, availability or total up-time (for repairable systems) which is expressed as an objective function(s) to be maximised, or alternatively by overall system profit or cost criteria that must be minimised. The values for the decision variables under which the objective functions are evaluated in general include the type of system configuration, the type of components, their reliability, the number of repair personnel required, preventive maintenance intervals etc. Resource restrictions which are expressed as constraints of the optimisation problem include desired reliability, desired availability, and subsystem redundancy levels, allowed downtimes, allowed weight and/or volume, and desired cost levels (Amari, 2008).

In this chapter, however, the focus for discussion is system reliability optimisation models, which have reliability as the main performance criterion and which are applicable to non-repairable systems, in particular. The main techniques used to optimise a system's reliability are discussed, to provide an overview of the field. Attention is also drawn to some perceived gaps in the literature which have essentially provided the motivation for this research.

4.2 **OPTIMAL RELIABILITY DESIGN STRATEGIES**

Three prominent strategies for systems reliability optimisation (Kuo & Prasad, 2000; Kuo et al, 2001; Zhao et al, 2007), are: (i) Redundancy Allocation, (ii) Reliability Allocation, and (iii) Redundancy-Reliability Allocation. The first, in general, seeks to find the best combination of components and levels of redundancy that together meet reliability and cost requirements and satisfy the system constraints (Coit & Smith, 1996a; Goel et al, 2003; Liang & Chen, 2007). The reliability allocation problem, on the other hand, seeks to find the best allocation of reliability to components or subsystems of a system in order to maximise the overall system reliability or minimise the system cost under specified constraints. The redundancy-reliability allocation problem combines these two strategies. Before discussing each in the context of the optimisation model types and their characteristics, a few points about the strategies should be noted.

Firstly, the diversity of system configurations, resource constraints, and options for reliability improvement have led to the construction and analysis of a number of optimisation models in respect of each of the approaches (Kuo et al, 2001; Yalaoui et al, 2005a). Secondly, the optimisation models reported in the literature have largely been single objective. Multiple objective formulations have been scarce (Kuo & Prasad, 2000; Coit et al, 2004). Thirdly, the literature shows that the use of redundancy allocation for optimal reliability design (under both single and multi-criteria situations) is more popular than the other approaches (Prasad & Kuo, 2000; Gen & Yun, 2006; Pan et al. 2007). Finally, the series-parallel system configuration has received the most attention (Coit & Smith, 1996b; Prasad & Kuo, 2000).

4.3 **OPTIMISATION UNDER A SINGLE CRITERION**

A system reliability design optimisation is described as single criterion, if and only if, one criterion is specified as the objective function to be optimised. In practice this has traditionally been the type used and a number of different formulations are reported in the literature. A generalised model representing these under each of the strategies will be discussed. Typical examples of applications are cited for illustration purposes.

4.3.1 **Existing Models**

In general, the single criterion redundancy allocation optimisation problem is of the form:

$$\begin{array}{lll} \textit{Minimise} & f(x_1, x_2, \ldots, x_N) & \\ \textit{Subject to:} & g_i(x_1, x_2, \ldots, x_N) \le 0 & i = 1,2,\ldots,m \\ & x_l \le x_j \le x_u & j = 1,2,\ldots,N \end{array} \tag{4.1}$$

This is a discrete optimisation problem since the elements of the decision vector $(x_1, x_2, ..., x_N)^T$ which specifies the redundancy levels for a set of N components or subsystems are required to be discrete in value. The objective function may be either the system's reliability expression (i.e. $-f$) or the system cost (i.e. f) which is minimised, subject to constraints on the system resources and the redundancy levels given by the functions g_i which are usually separable (Kuo & Prasad, 2000). x_l and x_u are respectively lower and upper limits on the jth component or subsystem redundancy level. The type of parallel redundancy may be total, partial, or standby (Prasad & Kuo, 2000). There are cases, albeit rare, where the decision variables concern components selection or their assignment in a system, without redundancy (Ashrafi & Berman, 1992; Atiquallah & Rao, 1993; Altiparmak et al, 1998; Wattanapongsakorn & Levitan, 2001). The model in (4.1) assumes that a component or subsystem reliability is known and remains constant throughout the optimisation process. The precise form of f depends on the criterion to be optimised; however, it is generally a non-linear function irrespective of the chosen performance measure. The constraints g_i are also generally non-linear and could be limits imposed on either the reliability of the components, subsystems, or overall system; or on cost, weight, volume or other system attributes. The type of system configuration and problem being analysed also dictate the form of both f and g_i .

Among early examples of this type of problem are the cases reported by Bala & Aggarwal (1987), Kim & Yum (1993), and Deeter & Smith (1997) which concerned redundancy allocation in complex systems or networks for their optimal reliability, and that of Coit & Smith (1997) which focused on a series-parallel system reliability optimisation. Prasad & Raghavachari (1998), considered the problem of the optimal allocation of interchangeable components, to a series-parallel system in order to maximise its reliability, with only one component allowed for each subsystem. Later Prasad & Kuo (2000) discussed the optimal allocation of redundant components to both series and complex coherent systems, to maximise their reliability, subject to constraints on the subsystems' reliability and redundancy levels. Munoz & Pierre (2004) presented a model that sought to find parallel redundancies both at the component and the system levels of a series system that minimised the cost associated with the redundancies, subject to lower bound constraints on both the system reliability and the redundancy levels. You & Chen (2005) proposed a model to maximise a series-parallel system reliability, with upper bounds on both the system cost and weight for a given redundancy level. Onishi et al (2007) considered the case of the redundancy allocation problem with, and without component mixing. Their model sought to maximise the system reliability subject to upper bounds on both the cost and the weight of the system. Further examples of the single objective redundancy allocation optimisation problem are given in Table 4.1 categorised by the criterion optimised, the constraints of the system, the decision variables, the configuration type, the specific area of application, and the source. Obviously the most studied type of redundancy allocation problem is the one that seeks to maximise systems reliability, those with cost as the objective function are very few. Another feature is the popularity of studying the series-parallel system configuration problems. The review by Mohamed et al (1992) and the book by Kuo et al (2001) are further useful sources for cases of the redundancy allocation optimisation problem.

The general reliability allocation optimisation problem is of the form:

$$\begin{aligned} &\textit{Minimise} && f(R_1, R_2, \ldots, R_N) && \\ &\textit{subject to} && g_i(R_1, R_2, \ldots, R_N) \le 0 && i = 1,2,\ldots,m \\ & && R_j^{\,l} \le R_j \le R_j^{\,u} && j = 1,2,\ldots,N \end{aligned} \tag{4.2}$$

This formulation is generally a continuous non-linear optimisation problem, since f and g_i are typically nonlinear functions expressed in terms of the reliability of the N components of a system, and the decision vector $(R_1, R_2, ..., R_N)^T$ is composed of continuous values representing the reliability of the N components. The $j'th$ component reliability is bounded below and above respectively by $R_j^{\,l}$ and $R_j^{\,u}$. This model assumes that the system configuration is determined or

Table 4.1: Further examples of the single criterion redundancy allocation optimisation problem

CRITERION	CONSTRAINTS	DECISION VARIABLES	SYSTEM TYPE	APPLICATION	SOURCE
Maximise system reliability	Linear & non-linear cost constraints, redundancy levels	Number of redundant components	Series-parallel	Unspecified	Misra & Sharma (1991a)
Maximise system reliability	Upper limit on cost & weight	Number of redundant components	Series-parallel	Hypothetical examples	Coit & Smith (1997)
Maximise system reliability	Limit on number of hardware & software components, and on cost	Choice of hardware & software components	Redundant embedded system	Computer system reliability	Wattanapongsakorn (2004)
Maximise system reliability	Upper limit on cost & weight	Number of redundant components for each subsystem	Series-parallel system with (k,n) subsystems	Extracted from literature	Coit & Liu (2000)
Maximise system reliability	Upper limit on cost & weight	Component Choices & redundancy levels	Series-parallel system with (k,n) subsystems	Extracted from literature	Coit & Smith (1996a)
Maximise system reliability	Number of hardware & software choices	Choices of hardware & software redundant components	Network	Embedded distributed speech recognition system	Wattanapongsakorn & Levitan(2001)
Maximise system reliability	Upper limit on cost & weight	Number of redundant components	Series-parallel system with (k,n) subsystems	Extracted from literature	Coit & Smith (1996b)
Maximise system reliability	Upper limit on cost & weight, and lower & upper limits on number of redundant components in a subsystem	Number of redundant components	Series-parallel, with (k,n) subsystems	Benchmark problems in the literature	Liang & Chen (2007)
Maximise system reliability	Upper limit on cost & weight, and lower limits on number of redundant components in a subsystem	Number of redundant component for each subsystem	Series-parallel system with (k,n) subsystems	Extracted from literature	Kulturel-Konak et al (2003)

known and remains unchanged (i.e. fixed) during the optimisation process. The system performance measure that is optimised in this case is reliability or cost, with constraints on the reliability of the components, the subsystems, or the system as well as on other system characteristics such as cost, weight, volume etc.

One of the earliest examples of this problem type was presented by Mohan & Shanker (1988). They discussed the reliability allocation problem in the context of a complex bridge network system. They proposed a model to allocate reliabilities to the components of the system so as to minimise the cost, subject to minimum and maximum reliability restrictions of zero and one respectively on the components. Mettas (2000) presented a formulation to minimise the cost of reliability of both series and complex systems under a lower bound constraint on the reliability of each of the systems as well as a lower and upper bound constraints on the reliability of their components. Yalaoui et al (2005a) presented a model which sought to minimise the cost associated with the reliability of a parallel-series system under a constraint on the system reliability.

The general redundancy-reliability allocation optimisation problem is modelled by the expression:

$$\begin{array}{lll} \textit{Minimise} & f(x_1, x_2, ..., x_N, R_1, R_2, ..., R_N) & \\ \textit{Subject to} & g_i(x_1, x_2, ..., x_N, R_1, R_2, ..., R_N) \le 0 & i = 1,2,...,m \\ & x_l \le x_j \le x_u, \;\; R_j^{\,l} \le R_j \le R_j^{\,u} & j = 1,2,...,N \end{array} \quad (4.3)$$

Expression (4.3) combines the formulations given by (4.1) and (4.2) into a single model which is a mixed-integer optimisation problem, with the decision variables being the number of redundant components and their reliabilities. Neither the system configuration nor the component reliabilities are assumed to be known or constant. Indeed they are the decisions that it is hoped will be made after the optimisation. The types of criterion to be optimised are similar to those discussed in the cases of (4.1) and (4.2) above.

One of the earliest examples of this model type was presented by Hikita et al in (1986). They considered the optimal selection of both component types and their reliability in order to maximise the reliability of a series parallel system, with constraints on the reliability of the components. Other cases of the model are presented by: Chi and Kuo (1990), Xu et al (1990), Hikita et al (1992), Majety et al (1999), Elegbede et al (2003), and Yalaoui et al (2005b). Chi & Kuo (1990) discussed the maximisation of the reliability of computer software by allocating both reliability and redundancy to its components (which were software programmes); the cost of a software failure was constrained to an upper limit. Xu et al (1990) discussed the reliability optimisation of a parallel-series, a complex, and a series system in which their reliabilities were maximised with constraints on cost, weight and redundancy. Hikita et al (1992) extended their studies of the series-parallel system as in Hikita et al (1986) to a complex system. Majety et al (1999) discussed their optimisation model in the context of a number of system structures, such as series, parallel, series-parallel, and, parallel-series. Their models all sought to find the allocations of both reliability and redundancy to the components which would minimise system cost and satisfy a minimum system reliability requirement. Elegbede et al (2003) after presenting theoretical results on the necessary condition for the optimal allocation of reliability to the components of a redundant subsystem, extended their result to series parallel systems and applied their ideas to a numerical example to minimise the system cost , subject to a lower bound constraint on its reliability. Yalaoui et al (2005b) presented work similar to that of Elegbede et al (2003).

4.3.2 **Methods of Solution**

A number of classical and stochastic algorithms – also categorised as approximate, exact, or heuristic/meta-heuristic, under each of the model types - have been used to find optimal solutions to the problems discussed above. Algorithms such as the surrogate worth trade-off, the Lagrange multiplier, and geometric programming methods and their variants, which are efficient for the exact solution of continuous problems of the type posed by reliability allocation optimisation, can only approximate the solution in the case of redundancy or redundancy-reliability allocation optimisation (Munoz & Pierre, 2004; You & Chen, 2005). The approximation techniques involve the use of trial and error approaches to obtain integer solutions (Xu et al, 1990; You & Chen, 2005). The approximation techniques were popular when exact solution algorithms were not well developed. The advent of the exact algorithms, such as integer programming (IP), branch-and-bound, and dynamic programming (DP) (Liang & Chen, 2007), have made the approximation techniques less popular for solving redundancy allocation problems.

The approximation and exact algorithms, though efficient with small to moderate sized problems having desirable properties such as convexity or monotonicity, are deficient with complex large scale ones, such as occurs with real life network reliability and redundancy allocation optimisation problems (Ashrafi & Berman, 1992; Atiqullah & Rao, 1993). Although the heuristic/ meta-heuristic approaches (example GA SA and TS) yield solutions which are not exact, they do have the ability to efficiently handle complexity (Altiparmak et al, 1998) and have thus become increasingly popular in the reliability optimisation field. The redundancy and the redundancy-reliability allocation optimisation problems are generally more difficult to solve than the reliability allocation ones. This is because the former belongs to the class of NP-hard problems (this phenomenon was demonstrated by Chern in 1992 (Coit et al, 2004; Coit & Konak, 2006) which involve non-convex and combinatorial search spaces and require a considerable amount of computational effort to find exact optimal solutions (Kim & Yum, 1993). The reliability allocation problems on the other hand involve continuous optimisation with a number of classical solution algorithms based on gradient and direct search methods at their disposal. They are thus relatively easier to solve. Examples of the solution algorithms which were applied in the context of the three optimisation problem types are presented in Table 4.2. Of these the heuristic methods were more popular than the exact ones. Only one example using the approximate methods was found.

Table 4.2: Summary of the Solution Algorithms used in the cited SCO cases

MODEL TYPE	SOLUTION TECHNIQUE	ALGORITHM DESCRIPTION	SOURCE
Redundancy Allocation	Approximate	Interval Arithmetic Optimisation	Munoz & Pierre (2004)
	Exact	Lagrange Relaxation algorithm in conjunction with Dynamic programming (DP)	Ashrafi &Berman (1992)
		Integer Programming (IP) Algorithm	Coit & Liu (2000)
		Lexicographic Order (P&K-Ag)	Prasad & Kuo (2000)
		Improved Surrogate Constraint (ISC) algorithm	Onishi (2007)
		IP (due to Misra)	Misra & Sharma (1991a)
	Heuristic/Meta-heuristic	Simulated Annealing (SA)	Atiqullah & Rao (1993)
		DETMAX Algorithm	Kim &Yum (1993)
		Genetic Algorithm (GA)	Deeter & Smith (1997)
		Heuristic Algorithm	Bala & Aggarwal (1987)
		GA	Coit & Smith (1997)
		SA	Wattanapongsakorn & Levitan (2001)
		Heuristic Algorithm	You & Chen (2005)
		Approximate Linear Programming Heuristic	Prasad & Raghavachari (1998)
		Tabu Search (TS)	Kulturel-Konak et al (2003)
		Variable Neighborhood Search Algorithm	Liang &Chen (2007)
		SA	Wattanapongsakorn (2004)
		GA	Coit & Smith (1996a)
		GA	Coit & Smith (1996b)
Reliability Allocation	Exact	Cutting Plane Algorithm	Majety et al (1999)
	Heuristic/Meta-heuristic	Random Search Algorithm	Mohan & Shanker (1988)
Redundancy-Reliability Allocation	Exact	Surrogate Dual Problem under DP Algorithm	Hikita et al (1986)
		Surrogate Constraint Algorithm	Hikita et al (1992)
		DP	Yaloaui et al (2005b)
		Mixed Integer Programming (MIP)Algorithm	Misra & Sharma (1991a)

4.3.3 **Some Observations and Drawbacks**

The similarities of the models formulated under single criterion optimisation (SCO) are very striking. They all virtually seek to maximise reliability as the main criterion of interest (a few were concerned with minimising cost), under similar sets of constraints. The models in effect overlook the presence of other criteria that implicitly conflict with reliability (or cost), such as weight, volume, etc. Even though such criteria feature as constraints in the models, where it is not possible to determine precise limits on the criteria (i.e. resource consumption) in order to set appropriate constraints, the single criterion approach becomes inappropriate (Kuo & Prasad, 2000). Furthermore, the fact that only a single criterion is optimised, means that decision making is limited to the consideration of just a single (unique) design, which denies decision makers the freedom of choice. The single criterion approach can thus be said to be simplistic and restrictive towards decision making. Sensitivity analysis can reduce the restrictiveness of this type of formulation by providing other design alternatives derived from a variation, within narrow ranges of selected parameters. This, however, is still limited, in comparison with the scale derivable under an MCO framework.

In all the cases where reliability was the criterion of interest, the top level (system level) expression for it was maximised as the sole objective function. The practice completely disregards any distinctive effect or influence (if there was one) that the constituent subsystems could have on the overall system reliability. In such cases, directly maximising the reliability of the relevant subsystems could be a credible, simpler alternative approach to the problem, which is likely to require a departure from a single criterion approach.

The models also virtually disregard the effects of uncertainty in the problem parameters, such as occurs with regard to component and system reliability. These parameters are based on estimates derived from data (example, failure data) which invariably are uncertain due to variability, are limited, or are in error (Nikalaidis et al, 2004). Therefore, a deterministic model runs the risk of providing only a sub-optimum of the actual optimal system reliability. Thus the variability in the system parameters could also be factored into the formulation as a constraint or a criterion (Allella et al, 2005; Azarm & Mourelatos, 2006). In the case where the variability features as criteria, however, a multi-criteria situation (bi-criteria at the least) is the result (Zhao & Liu, 2003).

The cases of reliability optimisation of other system configurations particularly that of complex systems, under the three strategies discussed earlier have been scanty. This situation is however not surprising given that complex systems' reliability expressions tend to be unwieldy, especially the large scale ones, and so formulating models and finding optimal solutions become intractable. Perhaps part of the difficulty also stems from the approach taken to solve the problem, where the top level system reliability expression is either sought as the sole criterion to be optimised, or as a constraint. Where it is possible to view such systems in terms of identifiable subsystems, with a clear understanding of their inter-connections with one another, it should be possible to proceed with simpler subsystem reliability expressions, but under multiple criteria, with the subsystems as the criteria (Li & Haimes, 1992).

4.4 **OPTIMISATION UNDER MULTIPLE CRITERIA**

The single criterion optimisation models undoubtedly result in improved system reliability, as evidenced from the many reported cases in the literature. The relatively few cases where the strategies have been approached from a multi-criteria viewpoint illustrate the additional benefits in terms of the variety of solutions that could be derived and the opportunity that is offered to decision makers to exercise discretion in the selection of the most appropriate solution. This section will cover this type of model for the three strategies, in addition to outlining their specific benefits, and pinpointing some drawbacks and gaps.

4.4.1 **The General Models**

The redundancy allocation optimisation by multi-criteria is modelled by:

$$\begin{array}{lll} \textit{Minimise} & [f_1(x), f_2(x),..., f_k(x)] & \\ \textit{Subject to} & g_i(x) \le 0\,, & i = 1,2,..., m \\ & x_l \le x_j \le x_u\,, \quad x = (x_1, x_2,...x_j..., x_N) & j = 1,2,..., N \end{array} \qquad (4.4)$$

The vector of k objective functions $[f_1(x), f_2(x),..., f_k(x)]^T$, ($k \ge 2$) represents the criteria to be optimised, which generally includes the reliability of a system, variance of the reliabilities, subsystems' reliability, system unreliability, cost, weight, volume, risk, etc. The other parameters and the assumptions of this model are the same as (or similar to) their counterparts given in equation 4.1. Among the many cases that concern the optimal allocation of redundant components, only a few were found that involved the optimal selection or assignment of components with or without redundancy, such as the cases by: Yamachi et al (2006), Zafirapoulos & Dialynas (2007), and Wattanapongsakorn & Coit (2007).

The models presented by Sakawa (1980) and those by Misra & Sharma (1991a, 1991b) were among the earliest publications found in this category. Sakawa considered the optimal allocation of both the number of standby components and their failure rates in a series-parallel system with standby redundancy, which maximised the system reliability while minimising the system cost, weight, and volume, subject to various constraints, including upper bounds on the system level reliability, cost, weight, volume, and subsystem reliability. Misra & Sharma (1991a) considered a multiple component choice redundant series-parallel system in which both the system reliability and cost were optimised subject to a set of constraints on both the system reliability and the number of redundant components. This problem was also presented by Misra and Sharma in (1991b) as one of two problems, the other being concerned with maximising a series-parallel system's reliability and minimising the system cost and weight subject to a set of expressions related to the redundancy levels of each subsystem. More recent cases of the model have been presented by Coit et al (2004), Coit & Baheranwala (2005), Marseguerra et al (2005) and Coit & Konak (2006). Coit et al (2004) presented a bi-criterion formulation in which the optimal allocation of redundant components was to be found for a series-parallel system in order to maximise its reliability and its associated variance, under system cost and weight constraints. Coit & Baheranwala (2005) presented and discussed a model which optimised reliability, cost, weight and variance of the reliabilities of a series-parallel system, with lower and upper limits set on the redundancy levels. Marseguerra et al (2005) considered a similar model formulation as Coit et al (2004) but in the context of network systems. Coit & Konak (2006) presented a model which treated each of the subsystems of a series-parallel system as a criterion to be maximised simultaneously under system cost and weight constraints. They showed that the problem could be reduced to a linear programming one, using the multi-objective weighting method. Further examples covering more recent cases are shown in Table 4.3. The table identifies the: criteria, constraints, decision variables, type of system, application, and the source.

The reliability allocation optimisation by multiple criteria is modelled by:

$$
\begin{aligned}
&\text{Minimise} && [f_1(R), f_2(R), \ldots, f_k(R)] && \\
&\text{Subject to} && g_i(R) \leq 0, && i = 1,2,\ldots,m \\
& && R_j^{\,l} \leq R_j \leq R_j^{\,u}, && j = 1,2,\ldots,N \\
& && R = (R_1, R_2, \ldots R_j \ldots, R_N) &&
\end{aligned}
\qquad (4.5)
$$

The set of k objective functions of the decision vector R whose N elements are the component reliabilities, include criteria such as the system reliability, cost, weight, volume subsystem reliability, risk etc. This type of formulation is incidentally rare in the literature.

One of the earliest examples is presented by Li & Haimes (1992) who formulated a model that decomposed a network system into a set of subsystems comprising components in both series and parallel arrangements. The network reliability expressed as a function of the subsystem reliabilities was thus maximised by simultaneously maximising the reliability of each of the subsystems. The resultant MCO was to find an optimal reliability allocation to the components of the network under the following constraints: the network cost, the reliabilities of the subsystems, and the reliabilities of the components. Further examples of recent cases are presented in Table 4.3.

The redundancy-reliability allocation by multiple criteria is modelled by:

$$\begin{array}{lll} \textit{Minimise} & [f_1(x,R), f_2(x,R), \ldots, f_k(x,R)] & \\ \textit{Subject to} & g_i(x,R) \le 0 & i = 1,2,\ldots,m \\ & x_l \le x_j \le x_u, \;\; R_j^{\,l} \le R_j \le R_j^{\,u} & j = 1,2,\ldots,N \end{array} \qquad (4.6)$$

The k criteria optimised in this case are functions of both the redundancy levels of the components or subsystems and their reliabilities. The criteria include a system or subsystem reliability, cost, weight, volume, and risk. The m constraints, g_i, describe the limits imposed on the system attributes and resources, such as cost, weight, and volume, and also on the reliabilities of the system, subsystems, and components. One of the earliest examples found in the literature of this model type was presented by Sakawa (1978) to maximise and minimise respectively the reliability and cost of a series-parallel system with upper limit constraints on the system's cost, weight, and volume. Misra & Sharma also discussed a similar model in their paper of 1991a. Further examples are presented in Table 4.3.

Table 4.3: Examples of multi-criteria redundancy, reliability & redundancy-reliability allocation optimisation problems

MODEL TYPE	CRITERIA	CONSTRAINTS	DECISION VARIABLES	SYSTEM TYPE	APPLICATION	LITERATURE
Redundancy Allocation	Minimise system cost and weight	Upper limits on system cost, weight and bounds on redundancy levels	Component redundancy levels	Series-parallel	Gearbox design, extract from the literature	Zhao et al (2007)
	Maximise System reliability, minimise associated variance	Upper limit on system cost, specified constraints on number of components selected	Number of components	Fault-tolerant system	N-Version Programming & Recovery Block	Wattanapongsakorn & Coit (2007)
	Minimise system cost & weight, maximise system reliability	Bounds on number of components selected in each subsystem	Number of components	Series-parallel	Hypothetical example	Taboada et al (2007)
Reliability Allocation	Maximise system reliability, minimise associated cost	Upper limits on system cost, & specific limits on components reliability	Reliability of components	Complex system		Shelokar et al (2002)
	Maximise reliability of subsystems	Constraint on cost of the system	Reliability of components	Complex system	Hypothetical example	Li & Haimes (1992)
	Maximise system reliability & cost	Upper bound on system level cost and lower bound on system reliability	Reliability of component	Series system	Example from literature	Kishor et al (2007)
	Maximise system reliability, minimise associated cost	Lower& upper limits on reliability of components	Reliability of components	Complex system	Life support system in a space shuttle	Salazar et al (2006)
Redundancy-Reliability Allocation	Maximise system reliability, minimise system cost & weight	Lower limit on system reliability, upper limit on system volume, weight & cost, bounds on number of components & their reliabilities	Number of components & their reliabilities	Series-parallel	Overspeed protection system of a gas turbine	Dhingra (1992)
	Minimise economic cost, maximise system reliability	technical constraints	Sizing & location of feeders & substations	Network System	A power distribution network	Ramirez-Rosado & Bernal-Agustin (2001)
	Maximise system reliability minimise cost	Upper bounds on system weight, bounds on system volume, number of components, and their reliability	Component choices & their reliability	Series-parallel	Dhingra's overspeed protection system	Huang et al (2006)

4.4.2 **Methods of Solution**

The solution methods for the MCO versions of the reliability design problems are generally of the classical or the stochastic types, and are implemented under a scalar or a Pareto approach (as discussed in Chapter Two). The methods involve a spectrum of algorithms that are approximate, exact, or heuristic. Table 4.4 lists the methods that were used in the models for the cases cited. The heuristic or meta-heuristic algorithms, based on the Pareto approach, are just as popular in the case of MCO in reliability design, whether by redundancy, or reliability allocation, or both, as they are in SCO cases. Examples of reliability allocation appear to be just as uncommon under MCO as under SCO. Again the redundancy allocation optimisation problem is still the most studied of the three types of strategies under MCO, followed by the redundancy-reliability allocation optimisation. The advent of many exact methods and algorithms, including heuristics, has resulted in a decline in the use of the approximate solution techniques.

4.4.3 **SCO versus MCO in Reliability Design**

While the single criterion formulations are the most common in the field there are clear advantages to be gained from using a multi-criteria approach. Reliability design is naturally a multi-criteria problem (Taboada et al, 2008), since one cannot just be concerned with achieving high reliability for products or systems; other factors like budget, raw materials, and technical constraints have to be considered. Since higher reliability invariably involves the consumption of additional resources in terms of additional man-hours on the job, use of higher quality and therefore more expensive materials, use of better or improved technology etc., there is clearly a conflict between reliability and these other equally important characteristics. Thus just seeking to maximise reliability alone is unrealistic.

Table 4.4: Summary of the Solution Algorithms used in the cited MCO cases of reliability design

MODEL TYPE	SOLUTION TECHNIQUE	ALGORITHM DESCRIPTION	MCO TYPE	SOURCE
Redundancy Allocation	Approximate	Surrogate Worth Trade off (SWT)Method under Dual Decomposition Algorithm	Scalar	Sakawa (1980)
		Direct Search by Min-Max Algorithm		Misra & Sharma (1991b)
	Exact	IP due to Misra		Misra & Sharma (1991a)
		The weighting Method in conjunction with a heuristic & an IP Algorithm		Coit & Konak (2006)
		Weighting Method under an IP software package		Coit et al (2004)
	Heuristic/Meta-Heuristic	GA & Monte Carlo simulation	Pareto	Marseguerra et al (2005)
		Multiobjective GA		Coit & Baheranwala (2005)
		Elitist Non-dominated Sorting GA 2 (NSGA 2)		Taboada & Coit (2007)
		GA		Wattanapongsakorn &Coit (2007)
		NSGA		Taboada et al (2008)
		Multi-objective Ant Colony		Zhao et al (2007)
		Simulated Annealing (SA)		Zafiropoulos & Dialynas (2007)
		Multio-objective GA		Yamachi et al (2006)
Reliability Allocation	Exact	Three levels Decomposition approach and the Khun Tucker multiplier method	Scalar	Li & Haimes (1992)
	Heuristic/Meta-heuristic	NSGA 2	Pareto	Salazar et al (2006)
		NSGA 2		Kishor et al (2007)
		Ant Colony (AC)		Shelokar et al (2002)
Redundancy-Reliability Allocation	Approximate	SWT	Scalar	Sakawa (1978)
		Direct Search Technique combined with the Min-Max method		Misra & Sharma (1991c)
		Goal Programming (GP) & Goal Attainment Methods (GAT)		Dhingra (1992)
	Heuristic/Meta-Heuristic	Evolutionary Algorithm (EA)	Pareto	Ramirez-Rosado & Bernal-Agustin (2001)
		GA		Huang et al (2006)

The MCO approach provides a wider range of decision alternatives and so a variety of potential designs for reliability. The opportunity to examine a multiplicity of potential designs at the conceptual stage, satisfying all design constraints, is not only desirable (Marseguerra et al. 2005; Konak et al. 2006; Taboada & Coit, 2007) but also appropriate, as this affords a careful screening of the design space for a more informed and rational decision (Limbourg & Kochs, 2007). It also offers the opportunity for engineers and decision makers to work together to arrive at a design that is acceptable and representative of the choices and compromises of all the stakeholders.

4.4.4 Drawbacks and Gaps

While the examples discussed so far point to a dynamic and exciting research field, (whether SCO or MCO) they also reveal areas where attention is necessary to advance the growth and development of the field. Apart from the distinctive advantages of the MCO approach which undoubtedly are very significant for better decision making in the reliability design environment, there seems to be very little else to choose between the two formulations (i.e. SCO and MCO). The similarity of the cases presented under each of the optimisation strategies and the relative frequencies of the examples under them, together with the solution techniques and algorithms used, are again very striking. The picture painted is one of replication of the SCO formulations under the MCO format, with hitherto constraints under SCO becoming criteria under MCO. The relative frequencies of the types of systems studied and their characteristics, in terms of the type of redundancies considered and the assumptions made especially about the components, have remained largely similar.

For example under either SCO or MCO, redundancy allocation remains the most studied of the three optimisation strategies. The series-parallel system structure is also the more popular. In almost all the cases, whether by SCO or MCO, the top level system reliability expression was the criterion maximised. The only cases found where the reliability of subsystems were the criteria for the optimisation, were those by Li & Haimes (1992), and Coit & Konak (2006). A few cases of stochastic formulations occur particularly in the context of MCO (Coit & Barheranwala, 2005; Marseguerra et al, 2005; Wattanapongsakorn & Coit, 2007), but the number of cases remains small. Other examples are the perennial assumption or consideration of redundancies involving, mostly, similar or identical components in a subsystem (i.e. components with similar failure rates that perform the same function) and also having exponential failure distributions. Redundancies involving standby components (especially cold standby) have also not received much attention (Azarm et al, 2008). These are interesting drawbacks or gaps, on account of the many practical situations which depart from such fundamentals.

As a result of the above observations, the following six areas are noted as interesting areas for further research: (i) applications of MCO in reliability design; (ii) formulations focusing on the reliability of subsystems as the criteria to be maximised (iii) examples of cases of reliability allocation optimisation; (iv) formulations which account for randomness in the design parameters in respect of all the strategies; (v) formulations which consider: component mixing in subsystems (i.e. redundant components not necessarily identical) and standby redundancies; (vi) formulations that consider other hazard rates (or failure distributions) of components other than the constant failure rate (or exponential distribution).

On the basis of the above concerns, Chapter Five presents and discusses the work aimed at addressing some of the shortfalls recounted above. Specifically, the work done covers the first three areas noted: thus a system reliability design methodology is proposed for allocating reliability to the components of a series-parallel system under an MCO format focusing on the subsystems as the criteria for the optimisation. The methodology is extendable also to complex systems. The method used to obtain Pareto optimal solutions is also discussed.

4.5 **SUMMARY**

The use of optimisation techniques especially at the conceptual or planning stage of a system design for the purposes of maximising the system's reliability and meeting such constraints as budgetary and other resource consumption limits, have been described, discussed , criticised and attention drawn to areas conceived as drawbacks and constituting a gap in the literature.

Notable among the optimisation strategies used in the pursuit of the purpose stated above are: (i) the optimal allocation of redundant components in a system, (ii) optimal allocation of reliability to the components of a system, and (iii) a combination of (i) and (ii). Among the three, the first strategy is the most common.

The most common optimisation model encountered in the reliability design literature is the one which specifies a single criterion, which invariably has been either the system reliability or its cost. The few cases where more than one criterion is specified are dominated by the bi-criterion types, which usually specify the system reliability and its cost as the criteria. The distinctive advantages of the MCO formulations over the SCO ones are in terms of the options they provide for deciding on an optimal design and the room and opportunity given to the decision maker to exercise their discretion and preferences. The optimisation models are also dominated by the deterministic types; the few stochastic formulations have been of the MCO type.

The series-parallel system structure is the most frequently studied and the types of parallel redundancies encountered are those done at the component level with the active and the partial ones occurring more often. Not much attention has been given to the study of standby redundancies especially the cold standby cases.

Heuristic or meta-heuristic algorithms are very popular solution methods in system reliability design optimisation (especially in redundancy allocation) whether by single or multi-criteria. Nevertheless the classical methods have not been completely absent and were used in cases involving reliability allocation which tend to have the required desirable characteristics.

In conclusion therefore, concerns have been raised (by the author), among others, about the low incidence of cases involving MCO formulations in reliability design and the inordinate emphasis of redundancy allocation (understandably an efficient reliability design tool) over reliability allocation (for instance), the virtues of which remain at this stage unexplored. As a response, therefore, in the next chapter a new methodology for reliability design is proposed and developed.

CHAPTER FIVE

RELIABILITY OPTIMISATION

OF

SERIES-PARALLEL SYSTEMS

5.1 INTRODUCTION

It was observed in Chapter four that the most popular decision making scenario researched in systems design for reliability concerned the determination of optimal system configuration which at least yields a specified system reliability. A less popular (in terms of research), though important, decision making scenario which is also encountered in the design of a system (or product) for reliability concerns the determination of optimal components' reliabilities which at least satisfy a given system reliability. This problem is often encountered when the reliability specification for the system is unachievable under the current component reliability specifications (Wasserman, 2003; ReliaSoft Corporation, 1992-2008). The assigned reliability is then used as the bench mark reliability specification for the components of the system.

In the above scenario it is presupposed that the system configuration is known. Thus for a simple system, such as one with two or three components in series configuration one can readily investigate the reliability specifications of the components that would achieve an overall system reliability target. Real problems are, however, usually more complex, and can involve multiple components in complicated system configurations. One needs also some measure that reflects how difficult (or expensive) it is to increase the reliability of each component. While such a measure is deemed appropriate given the reliability cost relationship - reliability is inextricably linked with cost in all phases of the design activity - it constitutes a further complication. It stands to reason, therefore, that the most acceptable component reliability assignment schemes would be those that meet the system reliability target and provide minimum values of the cost measure.

This chapter presents a novel approach to the above problem in an MCO framework. First of all, a general formulation of the problem as an MCO is presented where the system's reliability is viewed in terms of its subsystem reliabilities which are taken as the criteria to maximise and the system cost as the one to minimise. The approach is subsequently confined to a series-parallel system based on its popularity (Lobos & Momot, 2002) and its frequent occurrence in real systems. A methodology for the Pareto optimal assignment of component reliabilities in series-parallel systems which maximises system's reliability and minimises system cost is discussed. The methodology is also extended to complex systems.

5.2 **THE GENERAL MODEL**

Consider the scenario that the initial reliability of a system under design is to be improved to meet or exceed target reliability, through the use of highly reliable components. Suppose that:

(i) the system is composed of m distinct subsystems each of which comprises n_i components, $i = 1,2,...,m$;

(ii) the components may or may not be functionally equivalent i.e. although identical, they may not have the same reliability estimates, due to manufacturing practices, production costs, quality assurance provisions etc. (Coit et al, 2004);

(iii) the components may or may not be replicated in the other subsystems.

Whatever the configuration of the system being described, its overall reliability is dependent on the reliability levels of the subsystems which are in turn dependent on the reliability levels of their constituent components (Endrenyi, 1978). It follows therefore that the higher the reliability of a subsystem the higher the system reliability for a given configuration. Where a system reliability target is indicated the subsystem reliabilities must generally compete to achieve it. In other words the subsystem reliabilities cannot together increase by the same margin, some would achieve a higher increase than the others in order to realise the specified system reliability. Consequently the subsystem reliabilities may be viewed as constituting distinct and competing (or conflicting) system reliability criteria. Each subsystem may also be required to achieve a minimum (or maximum) reliability target together with the components.

The design goal has cost implications due to one of the following factors: (i) it may require use of existing components which are known to have higher reliability and quality, and are therefore more expensive to procure and install; (ii) it may require designing (or redesigning) components to have the required level of reliability, which requires expenditure in the form of better quality materials, retooling costs, etc. (Reliability Hotwire, 2001). In either case there could also be administrative, or change in vendors costs (Reliability Hotwire, 2001). If an expression can be found to describe the reliability-cost relationship of the components, then the cost associated with the entire system's reliability can be determined. The system reliability can then be maximised by maximising the subsystem reliabilities and minimising the cost simultaneously, while taking into account all constraints in terms of limits on the reliabilities of the subsystems and the components.

Let R_s , $R_{sub,i}$ and R_{ki} denote system, subsystem, and component reliabilities respectively

($i = 1,2,...,m$; $k = 1,2,...,n_i$). The vector of subsystem reliabilities to be maximised is thus of the form:

$$\left[R_{sub,1,}R_{sub,2,}...,R_{sub,m}\right]^T \tag{5.1}$$

Let C_{sy} , $c_{sub,i}$ and c_{ki} denote respectively the cost of improving the reliability of the system, the $i'th$ subsystem, and the $k'th$ component in the $i'th$ subsystem. Assuming that:

$$C_s = \sum_{i=1}^{m} c_{sub,i} \quad \text{and} \quad c_{sub,i} = \sum_{k=1}^{n_i} c_{ki} \tag{5.2}$$

then the cost of system reliability to be minimised is given by:

$$C_s = \sum_{i=1}^{m}\sum_{k=1}^{n_{i_i}} c_{ki} \tag{5.3}$$

The following constraints are imposed on the design:

$$R_{sub,i} \geq R_{\min,i} \quad \forall \quad i = 1,2,...,m \tag{5.4}$$

$$R_{ki,\min} \leq R_{ki} \leq R_{ki,\max} \text{ , } i = 1,2,...,m \text{ ; } k = 1,2,...,n_i \tag{5.5}$$

where $R_{\min,i}$ is the lower bound of the $i'th$ subsystem reliability at the specified mission time at which the optimisation is to be performed. $R_{ki,\min}$ is the initial or current reliability of the $k'th$ component in the $i'th$ subsystem and $R_{ki,\max}$ is the upper bound of the $k'th$ component reliability of the $i'th$ subsystem, which represents the maximum achievable reliability. The resultant deterministic MCO model of the reliability design problem becomes:

$$
\begin{aligned}
&\text{Maximise} \quad \left[R_{sub,1,} R_{sub,2,} \ldots, R_{sub,m}\right]^T \\
&\text{Minimise} \quad C_s = \sum_{i=1}^{m} \sum_{k=1}^{n_i} c_{ki} \\
&\text{Subject to:} \\
&\qquad R_{sub,i} \geq R_{\min,i} \ \ \forall \ \ i = 1,2,\ldots,m \\
&\qquad R_{ki,\min} \leq R_{ki} \leq R_{ki,\max} \ , \ i = 1,2,\ldots,m \ ; \ k = 1,2,\ldots,n_i
\end{aligned}
\tag{5.6}
$$

The model is general at this point and only useful where the objective functions and input parameters are known or can be determined. This requires that the configuration of the system is known, which also means that the overall system reliability can be evaluated on the basis of the output of the optimisation. The proposed model (expression 5.6) is therefore discussed in the context of a series-parallel system configuration extending to complex systems.

5.3 **A SERIES-PARALLEL SYSTEM MODEL**

Given a series-parallel system (see Figure 3.5 of Section 3.3) comprising m subsystems with n_i components in each ($n_i \geq 1$ and remain finite) enables the terms and inputs to the model given by (5.6) to be specifically determined.

5.3.1 **Subsystem Reliability**

The reliability expression for a subsystem in terms of the reliability of its components (discussed in Chapter 3) was shown (Rao, 1992; Kuo et al, 2001) to be:

$$
R_{sub,i} = 1 - \prod_{k=1}^{n_i} (1 - R_{ki}) \tag{5.7}
$$

Where all the components are identical expression 5.7 reduces to the form:

$$R_{sub,i} = 1 - (1 - R_{ki})^{n_i} \qquad (5.8)$$

The general system reliability expression was given in (3.17). Where the components in a subsystem are identical and distinct from those in the other subsystems (i.e. no mixing of components within subsystems) the result in (3.17) reduces to the form:

$$R_s = \prod_{i=1}^{m} \left[1 - (1 - R_{ki})^{n_i} \right] \qquad (5.9)$$

The expressions in (5.8) or (5.9) thus define the objective functions to be maximised in the series-parallel MCO model. The overall system reliability which is derived from the results of either (3.17) or (5.9) follows immediately from the output of the optimisation.

5.3.2 **Reliability-Cost Function and Model Parameters**

The component costs c_{ki} and the constraints need to be determined in order to proceed with the optimisation. The reliability-cost function can be derived empirically from actual cost data using past experience or that for similar components (Mettas, 2000). For instance it can be obtained from a reliability growth programme in which the stage-to-stage cost of improvement of the reliability of components or systems are tracked and quantified (Reliability Hotwire, 2001). In most cases however the necessary data is not available so a number of analytical models have been used as an alternative. Some of the more common models are discussed by Aggarwal (1994). The main features of these models are the following:

- Cost is modelled as a monotonically increasing function of reliability
- Cost is modelled as a differentiable and convex function of reliability
- Cost becomes indeterminate as reliability approaches unity

- Cost increases sharply with marginal increases in reliability where the original reliability was very high.

The analytical cost function used in this research, which exhibits the above features was developed by the ReliaSoft Corporation of the USA (Mettas, 2000). It has been chosen because unlike many of the others it incorporates a feature which accounts for and quantifies the practical difficulty or otherwise associated with increasing reliability in design, a feature considered suitable and necessary for the proposed model and consistent with the objectives of this research. The chosen cost function is defined by:

$$c_{ki} = \exp\left((1 - f_{ki}) \frac{R_{ki} - R_{ki,\min}}{R_{ki,\max} - R_{ki}} \right) \qquad (5.10)$$

where f_{ki} is a constant which measures the difficulty of increasing the reliability of the $k'th$ component in the $i'th$ subsystem relative to the other components in the subsystem. This measure, called the feasibility factor is set such that $0 < f_{ki} < 1$ (Reliability HotWire, 2001). Expression 5.10 quantifies cost as a dimensionless constant whose value is not only dependent on a component's reliability but also on its feasibility factor, which is an input parameter together with the initial and maximum achievable reliability values. The notion of cost as used in this thesis (unless otherwise stated) therefore refers to this dimensionless penalty function calibrated on a scale of one to infinity (one when no improvement in reliability is achieved and infinity when reliability approaches the maximum value), and serve as a measure or indicator of the level of resource expenditure required in order to achieve the reliability levels specified for the optimisations. A major difficulty presented by this notion is how to assess the significance of the numbers that are assigned. While a technique for converting these numbers into direct monetary terms is developed in this Chapter and illustrated in Chapters Six and Seven, it is suggested that the difficulty arises especially when the numbers are treated as absolutes. A comparative approach is better at putting them into context and facilitates a basis for assessing them for a given problem and making the appropriate cost-benefit analysis for decision making. The fact that the upper level of the scale is unbounded, however, remains a major weakness.

It is clear that the higher the value of the feasibility factor the lower the cost at a given level of component reliability and vice versa. Figure 5.1 illustrates this point with an example for selected feasibility factor values of 0.1, 0.5, and 0.9.

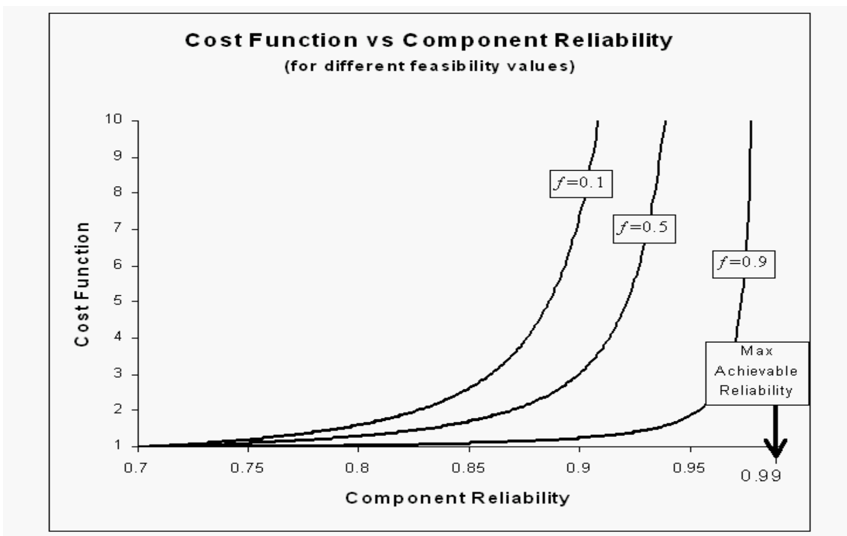

Figure 5.1: The effect of feasibility factor on component cost (Source: Mettas, 2000)

Setting appropriate values for f_{ki} for all $k = 1,2,...,n_i$ is thus necessary, even though it is not straightforward. The practice has been to use weighting factors which depend on certain influential aspects like complexity of the components, the state of the art, the operational profile, the criticality, etc (Mettas, 2000; Reliability Hotwire, 2001). Engineering judgement based on past experience, supplier quality, supplier availability, may also be used (Reliability Hotwire, 2001). There is therefore some level of subjectivity involved in the determination of the feasibility factor.

The other input parameters to the cost function model are the initial and maximum achievable reliability values, $R_{ki,\min}$ and $R_{ki,\max}$ respectively as well as the subsystem lower bound value, $R_{\min,i}$, for all i ($i = 1,2,...,m$). The initial reliability value, which may be taken as the current value of the reliability of a component, can be obtained from the component's failure data and its corresponding statistical distribution. The initial reliability values of other functionally similar components may also be used. Where a component has competing failure modes the failure data in respect of each of the failure modes would be required in order to estimate a generic reliability value for the component. In this case one ought to obtain the configuration of the failure modes (ReliaSoft Corporation, 1999-2007). Suppose, for instance, that a component has $p \geq 2$ failure modes and the occurrence of any one would result in failure of the component. If it can be established that the failures are independent then the failure modes have a series configuration. Thus if $R_1, R_2,..., R_p$ are respectively the reliability values corresponding to the p failure modes, then:

$$R = \prod_{j=1}^{p} R_j \tag{5.11}$$

where R is the generic reliability estimate of the component in question. Similar results may be determined for cases where the configuration is parallel, series-parallel, etc (as discussed in Chapter 3). The maximum achievable reliability value which is usually dictated by technological and financial constraints is a limiting value that may be approached but not necessarily attained; it is thus set very high (Reliability Hotwire, 2001). The value which eventually is a subjective estimate can be set, however, based on engineering judgement and current state of the art (Reliability Hotwire, 2001). Figure 5.2 illustrates the impact of values (i.e. 0.85, 0.9, and 0.99) of the maximum reliability set on a hypothetical cost function. The figure shows that cost rises sharply as the component reliability approaches the maximum achievable value (Mettas, 2000). A subsystem lower bound which specifies the minimum value that a subsystem can attain in the optimisation is similarly determined.

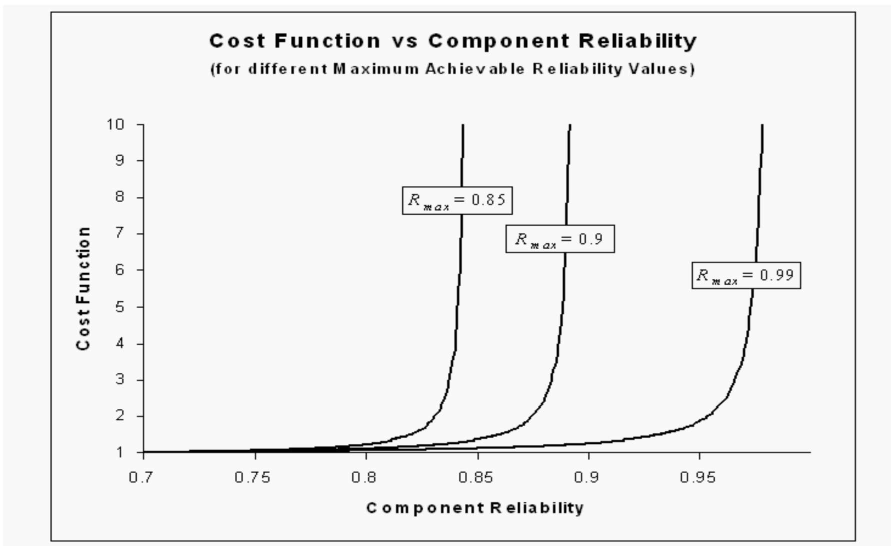

Figure 5.2: Impact of maximum reliability values on cost function (Source: Mettas, 2000)

Since one would also like to know (especially the decision maker) the direct monetary cost of reliability improvement, a methodology that converts the reliability cost/penalty value into a monetary cost estimate is discussed. It can be shown (Reliability HotWire, 2001) that by using a fault tolerant scheme (i.e. putting components in a parallel arrangement), an array of component reliability values and their associated cost units can be evolved and used to develop an analogous monetary cost function for a component. A plot of the reliability values against cost (in monetary terms) yields a curve (see Figure 5.3) similar to those given in Figure 5.1 and described by an exponential relation (Reliability HotWire, 2001). In Figure 5.3 the cost of a hypothetical component (which could well include the cost of design, manufacture, packaging etc.) is assumed to be one pound (£1) and the reliability of the component to be 0.3. The plot depicts the cost of a fault tolerant scheme involving from one through to eight identical components in a parallel arrangement with the original, and their corresponding reliabilities.

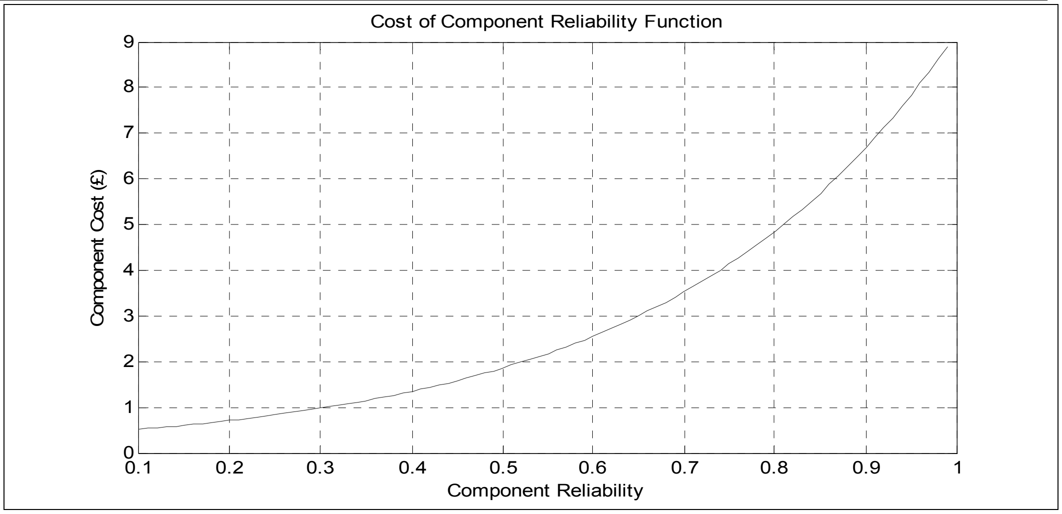

Figure 5.3: Plot of component reliability against actual cost, in a fault tolerant scheme

The cost function for the hypothetical component can be expressed by the general form given by expression 5.12 (Reliability HotWire, 2001), where c' is the monetary cost (Pounds in this instance) corresponding to a reliability value R; λ and μ are scalars to be determined.

$$c'(R) = \lambda \exp(\mu R) \qquad (5.12)$$

Note that the values of the scalars are as given in the following expressions:

$$\mu = \frac{\ln(c_1') - \ln(c_2')}{R_1 - R_2} \qquad (5.13)$$

$$\lambda = c_1' \exp\left(-R_1 \frac{\ln(c_1') - \ln(c_2')}{R_1 - R_2} \right) \qquad (5.14)$$

The ordered pairs (c_1', R_1) and (c_2', R_2) are arbitrary points on the curve defined by $c'(R)$.

For the purposes of the series-parallel system MCO model under discussion, suppose the monetary cost, c'_{ki}, of the $k'th$ component in the $i'th$ subsystem is given by

$$c'_{ki} = \lambda_{ki} \exp(\mu_{ki} R_{ki}) \tag{5.15}$$

where λ_{ki} and μ_{ki} are the associated scalars determined as given in expressions 5.13 and 5.14 respectively, for all $k = 1,2,...,n_i$ ($i = 1,2,...,m$). From the graphs of both c_{ki} and c'_{ki} one can reasonably assume that the two are proportionally related and can thus be approximated by the expression:

$$c'_{ki} = \alpha_{ki} c_{ki} \tag{5.16}$$

where α_{ki} is a scalar (cost constant) associated with c_{ki} ($k = 1,2,...,n_i$; $i = 1,2,...,m$). It is observed that when $R_{ki} = R_{ki,\min}$, $c_{ki}(R_{ki,\min}) = 1$ and $c'_{ki}(R_{ki,\min}) = c_{ki}{}^{o}$, where $c_{ki}{}^{o}$ is the original unit cost of the $k'th$ component in the $i'th$ subsystem ($k = 1,2,...,n_i$; $i = 1,2,...,m$). It follows from expression 5.16 that $\alpha_{ki} = c_{ki}{}^{o}$. It is therefore possible to obtain an estimate of the monetary cost of reliability for a component given its cost/penalty value. An estimate therefore of the monetary cost, C'_s, of reliability improvement in a series-parallel system is:

$$C'_s = \sum_{i=1}^{m} \sum_{k=1}^{n_i} c_{ki}{}^{o} c_{ki} \tag{5.17}$$

Note that although expression 5.10 is dimensionless, this is not so with the cost in expression 5.17, the dimension of which is determined by the unit of the currency in which the monetary cost is measured.

5.4 **EXTENSION TO COMPLEX SYSTEMS**

Reliability optimisation of complex systems or networks provides formidable challenges as far as modelling reliability goes. The difficulty (as discussed in Chapter 3) has resulted in the development of techniques for obtaining lower and upper bound reliability estimates for systems or networks (Jin & Coit, 2003; Espiritu et al, 2007) involving the use of network reduction techniques, such as cut and path sets. The resultant network configuration, (which is series-parallel in the case of cut sets, and parallel-series in the case of the latter), provides the setting and opportunity for applying the current MCO model. Note that each minimum cut set signifies a discrete event that describes a failure characteristic (failure mode defined by a specific combination of component failures) of an entire system. Together they constitute therefore distinct criteria or subsystems which provide measures of the probability of failure of the system. One would thus want to minimise each of the probabilities or maximise their reliabilities (or a subset of them).

Suppose therefore that a series-parallel transformation using m minimum cut sets $K_1, K_2, ..., K_m$ of any order has been achieved for a complex system. Since the minimum cut sets comprise components which reliability-wise are in parallel configuration, and which may be replicated in other cut sets, their reliability is modelled by expression 5.18 where R_k is the reliability of the $k'th$ component in the $i'th$ minimum cut set consisting of an arbitrary number of components.

$$R_{sub,i} = 1 - \prod_{k \in K_i} (1 - R_k) \qquad (5.18)$$

Even though component mixing occurs in this case, the current MCO approach allows that and it is appropriate for the following reasons: (i) the subsystems are pair wise mutually distinct, in terms of the combination of component failures that lead to system failure. Maximising each is therefore consistent with the MCO approach; (ii) each subsystem's reliability depends on the collective reliability levels of its components, whether or not the components are replicated in other subsystems; (iii) even though in general the system level reliability function (i.e. for complex systems or networks) does not increase monotonically with respect to the reliability of its components (Mohan & Shanker, 1988) it does so with respect to the reliabilities of its subsystems (Li & Haimes, 1992), thus simultaneous maximisation of their reliabilities means a search for the component reliability values which collectively increase the subsystems' and thus the system's reliability; such values, if found would thus be consistent with the dependent state of the subsystems' reliabilities.

MCO model formulation, similar to the one discussed under the series-parallel case, can therefore be derived in the form of the model given by (5.6), where the m minimum cut sets are the subsystems in which reliabilities are maximised. The cost function to be minimised and the input parameters are similarly determined as discussed in Section 5.3 and equation 5.10. The compromise solutions derived from the optimisation in this case represent the component reliability specifications which would maximise the lower bound estimate of the complex system reliability (as given by expression 3.21) at minimum cost, subject to constraints on the minimum cut sets and component reliabilities.

5.5 **CHARACTERISITCS AND ASSUMPTIONS**

The formulation given in expression 5.6 belongs to the class of constrained non-linear and continuously differentiable MCO problems: The objective functions are generally non-linear and the decision variables continuous real numbers in the domain $(0,1)$ which is a convex set. The cost function (in terms of either expression 5.10 or 5.16) is convex while the subsystem reliability functions are concave monotone (See Appendix A for formal discussions of the properties). The feasible criterion space is thus closed (Li & Haimes, 1992) and the Pareto front connected (see Appendix A). Therefore the model can be solved by classical methods employing exact algorithms.

The model does not require the system level reliability expression as input to the optimisation (although one may choose to add it as a constraint). This characteristic is considered a simplification of the problem of optimising the reliability of complex systems especially in cases where the analytical reliability expression is generally difficult to find. The system reliability for the series-parallel case can be obtained from expression 3.17 or 5.9 after the optimisation, or from a product of the subsystem reliability values obtained in the case of both series-parallel and complex systems. This is a distinguishing feature of the model.

Since all the subsystems' reliabilities are maximised, the least subsystem reliability which is crucial to the overall system reliability (see Chapter 3) is also maximised. In the case of a complex system, maximising the reliability of the minimum cut sets constitutes maximising the lower bound estimate of the system reliability.

The model assumes that the input parameters are precisely determined. Thus random variations in their values are ignored. The system may be repairable or non-repairable. The model is concerned with the time to first failure of the system during its operations. In the case of repairable systems the time to failure is assumed to be the time to the next failure after repair. The system reliability improvement cost is the aggregate cost of improvement in the reliability of all the components. Any other extraneous cost is not taken into account.

5.6 DISTINCTIVE FEATURES

Notable simplifications among other advantages of the proposed approach are the following:

- The focus on subsystems decentralises the task of finding the system reliability and greatly reduces the complexity of the problem, as far as formulation and computational efforts are concerned, which thus facilitates seeking optimal solutions (Li & Haimes, 1992; Coit & Konak, 2006).
- The reliability function of a complex system is generally non-separable with respect to its major subsystems (the components are replicated within the subsystems in this case). The MCO approach acts as a separation strategy which decomposes the problem for optimisation (Li & Haimes, 1992).
- While component mixing within a subsystem of a series-parallel system poses a major problem (especially in redundancy allocation), the current formulation allows mixing of components (Coit & Konak, 2006).
- Even though system reliability is not maximised directly, the formulation does yield high system reliability (Coit & Konak, 2006). This is on account of the fact that system reliability is an increasing function of the subsystems' reliabilities (Li & Haimes, 1992). Maximising the subsystems' reliabilities, therefore, maximises the system reliability.

5.7 THE SOLUTION METHOD

In view of the above characteristics a number of scalar methods may be appropriately applicable for generating Pareto optimal solutions (or for obtaining a compromise solution where preferences are expressed). These include such methods as Goal Attainment, Goal Programming and Weighted Sum Scalarisation (Marler & Arora, 2004). The chosen method of solution in this work was the Weighted Sum, not just because it is simple and easy to implement (Kim & de Weck, 2005) but it is also effective and efficient at providing Pareto optimal solutions under the characteristics noted. It is also particularly suitable given a characteristic of series-parallel systems (see Chapter 3), which makes it necessary to weight each subsystem reliability equally in order to avoid a situation where the Pareto optimal solutions result in a mixture of very low and very high subsystem reliabilities (Coit & Konak, 2006). In Chapters 6 and 7, it is used as a generating method (Cohon, 1978) to find Pareto optimal solutions.

In order to facilitate the application of this method the general MCO model is formatted into the following scalar form where w_i is the weight of the $i'th$ criterion, and a_j, b_j are such that $a_j > b_j$ for all $j = 1,2,..,N$. It is also assumed that each subsystem's reliability is of equal importance and thus weighted accordingly:

$$\begin{aligned} &\text{Minimise} \quad \sum_{i=1}^{m+1} w_i f_i(R_{ki}) \\ &\text{Subject to:} \quad -f_i(R_{ki}) \geq R_{i,\min}, \qquad i = 1,2,\ldots,m \\ &\qquad R_{ki,\min} \leq R_{ki} \leq R_{ki,\max}, \qquad i = 1,2,\ldots,m \quad k = 1,2,\ldots,n_i \\ &\qquad w_i > 0, \quad \sum_{i=1}^{m+1} w_i = 1, \qquad i = 1,2,\ldots,m+1 \end{aligned} \tag{5.19}$$

Where:

$0 < R_{ki,\min} \leq R_{ki} \leq R_{ki,\max} < 1$, $f_i(R_{ki}) = -R_{sub,i}$ $\forall i = 1,2,...m$, and $f_{m+1}(R_{ki}) = C_s$ (or C'_s).

And:

$$w_i = \begin{cases} \dfrac{a_j}{ma_j + b_j}, \forall i = 1,2,\ldots,m; j = 1,2,\ldots,N \\ \dfrac{b_j}{ma_j + b_j}, i = m+1; j = 1,2,\ldots,N \end{cases} \qquad a_j, b_j \in \Re^+ \ , \ N \in \mathrm{Z}^+ \tag{5.20}$$

An optimal solution to the scalar problem in expression 5.19 is Pareto optimal for the MCO problem posed in expression 5.6 so long as the weights are non-negative (Das & Dennis, 1997; Coit et al, 2004). Thus a set of Pareto optimal solutions can be secured by a number of weight generations and optimisations.

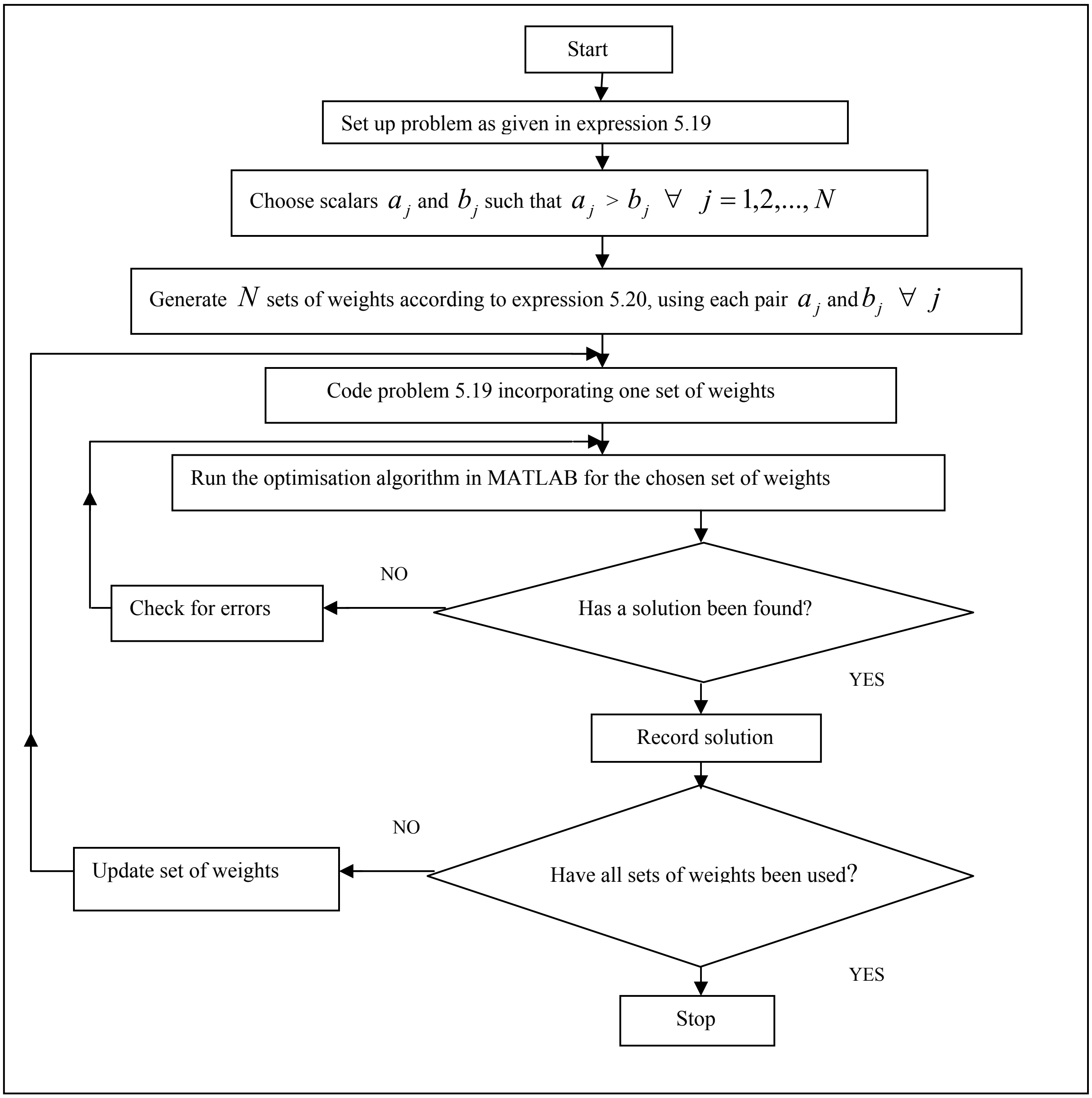

Figure 5.4: A flowchart of the solution process for the weighted sum problem

The MATLAB optimisation toolbox was used to perform the computations. The solution process is described by the flowchart in Figure 5.4. The system reliability optimisation methodology discussed in this Chapter was tested on hypothetical problems as well as some published examples as a means of verifying the model and its methodology. These will be discussed in Chapters 6 and 7.

5.8 **SUMMARY**

A new methodology for optimal design for reliability has been presented and described. It addresses the problem of assigning optimal reliabilities to the components of a system using a multi-criteria model. Unlike many of the methods found in the literature, which approach the problem from the system level, the proposed methodology identifies the subsystems as the criteria to maximise in order to maximise the system level reliability. The other criteria taken on board in the problem formulation is the cost of increasing component reliability, which is minimised in the optimisation.

A general form of the model was formulated for a system with identifiable and distinct subsystems of any configuration, and subsequently discussed in terms of series-parallel systems in particular, and also complex systems. The input parameters of the model including the expressions for the objective functions were discussed as was the type of cost function used and its characteristics. The cost function was noted to be a dimensionless increasing (exponential) function of component reliability which measured the degree of difficulty in improving reliability in terms of expenditure of resources etc. A technique to convert the value obtained into a monetary cost was presented. The mathematical features and inherent assumptions of the model, which influenced the chosen method of solution, were also briefly discussed.

The Weighted Sum Scalarisation method was identified as suitable for finding Pareto optimal solutions, given the features of the model, and its effectiveness at providing good solutions under those features. The process of implementing the optimisation from the initial scalarisation of the MCO problem, through to formatting the scalarised problem for the optimisation algorithm was depicted in a flowchart.

CHAPTER SIX

MODEL TESTING 1

6.1 INTRODUCTION

The MCO model proposed in the previous Chapter was firstly tested using seven hypothetical series-parallel examples to assess its performance in terms of the quality of its solutions and its sensitivity to the model's parameters. The tests also aimed to demonstrate the use of the model for optimal component reliability assignment in order to simultaneously maximise system reliability and minimise cost; the term cost being as defined by the expressions 5.10 and 5.3. This phase of the testing concerned relatively simple examples of the model (including a bridge system), more complicated and larger systems were used and are discussed in Chapter Seven. The examples presented in this Chapter comprise series-parallel system configurations ranging from two to four subsystems. Each subsystem was made up of two or three components which in turn were assumed functionally equivalent, but not necessarily identical in terms of their reliability characteristics.

The model associated with each specific case was evaluated under the following conditions: (i) values of w_i were varied over fixed values of f_{ki} and $R_{\min,i}$; (ii) f_{ki} was varied while holding w_i and $R_{\min,i}$ fixed; and (iii) $R_{\min,i}$ was varied for fixed values of f_{ki} and w_i. The parameters of each of the models were assigned values as follows:

- Initial reliability of all the components = 0.5
- Maximum component reliability, for all the components = 0.99 or 0.999

- The reliability lower bounds, for all the subsystems ranged from 0.999 to 0.9998 in steps of 0.0002.
- The feasibility factor of all the components was equal in each case, and assigned values ranging from 0.3 to 0.9 in steps of 0.2.

These values were selected purely on the grounds of preference; there were of course innumerable choices (within the bounds of realism) that could have been made. The maximum component reliability values represent the upper limit reliability specification set for the components and the subsystem lower bounds represent the lower limit reliability specifications for the subsystems. A system level reliability specification of 0.99 or above was expected for all the cases. The low initial components' reliability value of 0.5 was deliberate and intended not only to test the ability of both the model and the optimisation algorithm to yield very high values even from initially low ones, but also to indicate a potentially low initial system reliability.

6.2 **SCENARIO 1**

The system configurations in this design scenario involved two subsystems each with Case 1 having two components in each; Case 2 having three in one and two in the other; and Case 3 having three in both.

Case 1

The system reliability block diagram (RBD) for this configuration is shown in Figure 6.1. The first subsystem's components' reliabilities are R_{11} and R_{21} while those of the second are R_{12} and R_{22}. The MCO model corresponding to this system configuration with the parameter values as specified earlier was run using the solution algorithm described in Chapter Five.

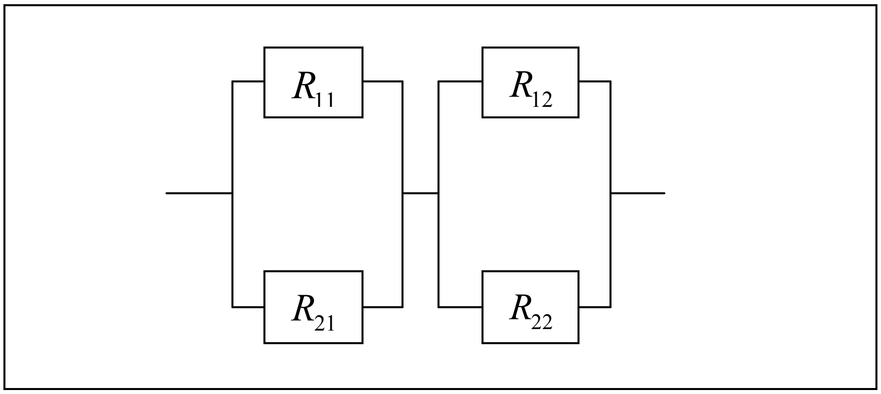

Figure 6.1: System reliability block diagram for Case 1 of Scenario 1

Results and Discussion

The values of the input parameters and the solution outputs in respect of the specified experiments are presented in Tables 6.1 to 6.4. The four were generated using ten different sets of weights but fixed feasibility factors of 0.9, 0.7, 0.5, and 0.3 respectively. The first column on the extreme left of each table shows the row titles. IT defines the iteration number, FF the feasibility factor, W1 and W2 are the weights for the reliabilities of the two subsystems and W3 that for the cost. R11, R21, R12, and R22 are the component reliabilities (Pareto optimal values) after optimisation, RSB1 and RSB2 are the subsystem reliabilities resulting from the component reliability values, Rs is the overall system reliability, and Cs is the associated cost/penalty. (The tables presented in the subsequent examples and cases are similarly formatted)**.**

Table 6.1: Results of Scenario 1 Case 1, with a feasibility factor of 0.9

IT	1	2	3	4	5	6	7	8	9	10
FF	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
W1	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W2	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W3	0.1	0.06	0.02	0.016	0.012	0.008	0.004	0.0036	0.0032	0.0028
R11	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684
R21	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684
R12	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684
R22	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684
RSB1	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014
RSB2	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014	0.9990014
Rs	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039	0.9980039
Cs	18.485993	18.485993	18.485993	18.485993	18.485993	18.485993	18.485993	18.485993	18.485993	18.485993

Table 6.2: Results of Scenario 1 Case 1, with a feasibility factor of 0.7

IT	1	2	3	4	5	6	7	8	9	10
FF	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
W1	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W2	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W3	0.1	0.06	0.02	0.016	0.012	0.008	0.004	0.0036	0.0032	0.0028
R11	0.9684	0.9683	0.9699	0.9684	0.9684	0.966	0.9675	0.9684	0.9684	0.9684
R21	0.9684	0.9684	0.9669	0.9684	0.9684	0.9787	0.9693	0.9684	0.9684	0.9684
R12	0.9684	0.9691	0.9772	0.9684	0.9684	0.9688	0.9629	0.9684	0.9684	0.9684
R22	0.9684	0.9676	0.9719	0.9684	0.9684	0.9679	0.9731	0.9684	0.9684	0.9684
RSB1	0.999001	0.9989983	0.999004	0.9990014	0.9990014	0.9992758	0.9990023	0.9990014	0.9990014	0.9990014
RSB2	0.999001	0.999	0.999359	0.9990014	0.9990014	0.9995	0.999002	0.9990014	0.9990014	0.9990014
Rs	0.998004	0.9979981	0.998364	0.9980039	0.9980039	0.998275	0.9980053	0.9980039	0.9980039	0.9980039
Cs	394.8284	393.67879	1102.44	394.82839	394.82839	1447.0276	486.9833	394.82839	394.82839	394.82839

Table 6.3: Results of Scenario 1 Case 1, with a feasibility factor of 0.5

IT	1	2	3	4	5	6	7	8	9	10
FF	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
W1	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W2	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W3	0.1	0.06	0.02	0.016	0.012	0.008	0.004	0.0036	0.0032	0.0028
R11	0.9818	0.9813	0.9684	0.9758	0.9699	0.9745	0.9802	0.9816	0.9684	0.9684
R21	0.9793	0.9808	0.9684	0.976	0.9695	0.9715	0.9784	0.9686	0.9684	0.9684
R12	0.9801	0.9814	0.9759	0.9755	0.9687	0.971	0.9511	0.9687	0.9684	0.9684
R22	0.9796	0.9668	0.973	0.9783	0.9699	0.9655	0.9795	0.9816	0.9684	0.9684
RSB1	0.999623	0.999641	0.999001	0.9994192	0.999082	0.9992733	0.9995723	0.9994222	0.9990014	0.9990014
RSB2	0.999594	0.9993825	0.999349	0.9994684	0.9990579	0.999	0.999	0.9994241	0.9990014	0.9990014
Rs	0.999217	0.9990237	0.998351	0.9988879	0.9981407	0.9982735	0.9985703	0.9988467	0.9980039	0.9980039
Cs	1963132	2219392.7	42895.71	188458.2	11561.828	26874.672	681020.75	2052209.6	8432.8421	8432.8421

Table 6.4: Results of Scenario 1 Case 1, with a feasibility factor of 0.3

IT	1	2	3	4	5	6	7	8	9	10
FF	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
W1	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W2	0.45	0.47	0.49	0.492	0.494	0.496	0.498	0.4982	0.4984	0.4986
W3	0.1	0.06	0.02	0.016	0.012	0.008	0.004	0.0036	0.0032	0.0028
R11	0.9754	0.9783	0.9815	0.9777	0.9794	0.9815	0.9702	0.9787	0.98	0.9755
R21	0.9741	0.9723	0.9693	0.9757	0.9781	0.9599	0.9665	0.9738	0.9804	0.9725
R12	0.976	0.9769	0.9692	0.9779	0.9791	0.9807	0.9681	0.9785	0.9735	0.9758
R22	0.9761	0.9781	0.9817	0.969	0.9734	0.9827	0.9692	0.9733	0.9793	0.9735
RSB1	0.999363	0.9993989	0.999432	0.9994581	0.9995489	0.9992582	0.9990017	0.9994419	0.999608	0.9993263
RSB2	0.999426	0.9994941	0.999436	0.9993149	0.9994441	0.9996661	0.9990175	0.999426	0.9994515	0.9993587
Rs	0.99879	0.9988933	0.998869	0.9987734	0.9989932	0.9989245	0.9980202	0.9988682	0.9990597	0.9986854
Cs	5992753	23451866	5.23E+08	15925721	57541662	1.334E+09	216481.86	28141108	144334254	3838459.1

It is clear that the weighting had no impact on the Pareto optimal component reliability values when the FF was set at 0.9 (see Table 6.1); this was not the case at lower FF values (see Tables 6.2 to 6.4). The unique reliability value of 0.9684 assigned to all the components, (as in Tables 6.1 & 6.2) though conspicuous, is not surprising, given the symmetry in the values of the parameters (i.e. feasibility factors, initial reliabilities, and maximum achievable reliabilities and subsystem reliability lower bounds) used with respect to both the components and the subsystems. The subsystems were treated as identical in this case. The components' reliability values represent a 46.8% improvement over their initial ones of 0.5.

The cost/penalty values shown in the tables represent the cost in terms of expenditure of resources including time, associated with the improvement in the system's reliability which is measured on a scale ranging from unity to infinity – the two limits signifying respectively 0% improvement in reliability, and reliability tending to 100% (see Chapter Five). It is the aggregate of the costs/penalty values of all the components. Thus, for instance, in Tables 6.1 & 6.2, whilst it costs 18.48 to improve the initial system reliability to the current value of 0.9980039 under a FF of 0.9, it costs 394.83 (signifying more than a twenty one fold increase to achieve the same level of improvement under a FF of 0.7. Similar observations can be made in respect of the results displayed in the other tables. As was shown in Chapter Five, estimates of the direct monetary value of the cost/penalty values of the components and the system reliabilities could be derived where the unit costs of the components were known. Using the results of the 1st iteration in Table 6.1 as an illustration, suppose the unit cost of the components associated with the reliabilities R11, R21, R12, and R22 are respectively £1k, £2k, £3k and £4k. Their cost/penalty value as computed from expression 5.10 is 4.62 (to 2 decimal places) giving the aggregate value of 18.48. The monetary cost estimates for the component reliabilities are therefore respectively £4.62k, £9.24k, £13.86k and £18.48k giving an aggregate cost for the entire system reliability of £46.2k. (In Chapter Seven the technique is illustrated further using real data from a practical example).

Observe also that the system reliability values achieved after the optimisations, in all cases satisfied the stated reliability specification or target of 0.99. Even though weighting resulted in various sets of Pareto optimal components reliability values at lower FFs (with repetitions of the solutions in several of the iterations) their impact on the system reliability did not generally show a wide variation. The cost values however show appreciable levels of variation under the iterations; they increase sharply as the FF decreases and in a number of instances become practically indeterminate. This is an indication of the fact that at very low FF reliability improvement can be cost prohibitive.

The impact of the FF values on the reliability of the components and thus that of the system, as well as the system cost, is very marked. In many of the iterations the system reliability increased (as a result of the increased reliability to some of the components) or remained the same as the FF values decreased. The cost however always increased at lower FF values. The phenomenon which can be inferred from Figure 6.2 (drawn for the fixed weight vector corresponding to the tenth iterations) seems to run counter to expectation - intuitively, one would expect that low feasibility should mean low reliability. This phenomenon is attributed to the MCO model which requires that reliability is maximised while cost is minimised and weights reliability higher above cost. Thus as feasibility decreases and cost increases, preference is given to higher subsystem reliabilities resulting in higher component reliabilities. Figure 6.3 provides an illustration to aid the intuition, for a three dimensional Pareto surface associated with Scenario 1. Since cost (Cs) is minimised and subsystems' reliabilities (RSB1 and RSB2) are concurrently maximised, the candidate Pareto points are those closest to the origin of the criterion space. These points which are ordered 3-tuples (3 being the number of criteria) would characteristically have some of the lowest values (in relative terms) in their cost co- ordinate (for a given feasibility factor value however low) and some of the largest reliability values in their subsystem reliability co-ordinates. These inevitably yield higher components' and system's reliabilities.

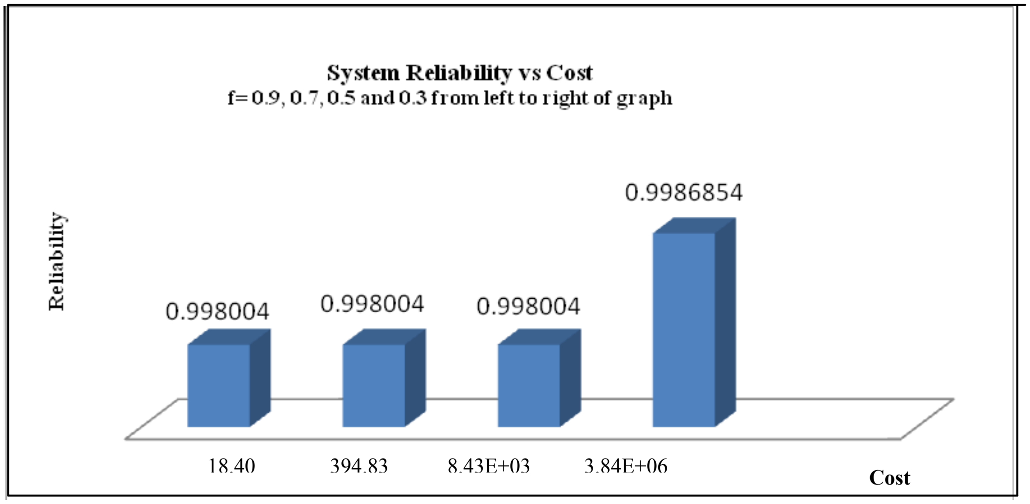

Figure 6.2: System reliability and cost for varied feasibility factor values

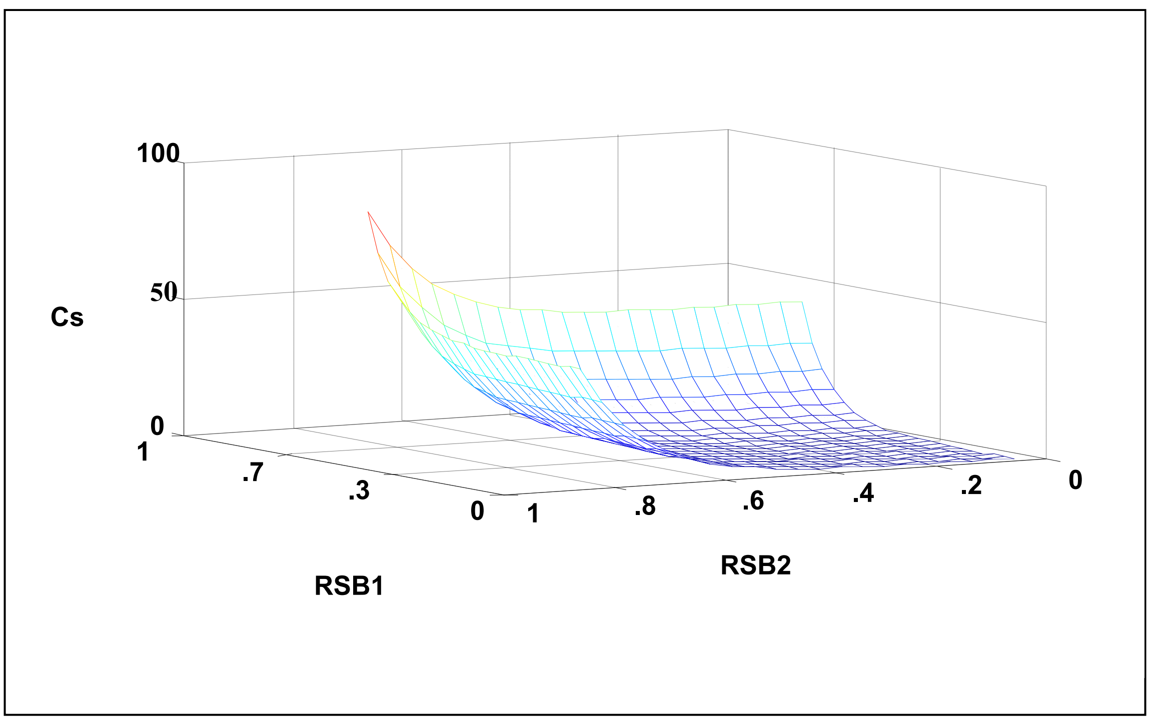

Figure 6.3: Pareto surface: Scenario 1

This observation shows that low feasibility is not necessarily a barrier to achieving higher reliability; it is the cost that invariably does this.

Tables 6.5 and 6.6 show the results obtained by varying the lower bound (LB) of the subsystems reliability iteratively from 0.9992 to 0.9998 in steps of 0.0002 (i.e. four iterations each), with the same FF values as before and a fixed weight vector corresponding to the tenth IT (in Tables 6.1 to 6.4) given by $[0.4986, 0.4986, 0.0028]^T$ which assigns the highest and lowest weights (out of the ten generated) to the subsystems and cost respectively. They show that varying the subsystem LBs results in an improvement in the components, and system reliabilities, but there is a very large increase in cost.

Table 6.5: Scenario 1Case 1results, varying the subsystem reliability lower bounds with feasibility factors of 0.9 and 0.7

IT	1	2	3	4	5	6	7	8
FF	0.9	0.9	0.9	0.9	0.7	0.7	0.7	0.7
R11	0.9717	0.9755	0.98	0.9859	0.9717	0.9755	0.98	0.9859
R21	0.9717	0.9755	0.98	0.9859	0.9717	0.9755	0.98	0.9859
R12	0.9717	0.9755	0.98	0.9859	0.9717	0.9755	0.98	0.9859
R22	0.9717	0.9755	0.98	0.9859	0.9717	0.9755	0.98	0.9859
RSB1	0.9992	0.9994	0.9996	0.9998	0.9992	0.9994	0.9996	0.9998
RSB2	0.9992	0.9994	0.9996	0.9998	0.9992	0.9994	0.9996	0.9998
Rs	0.9984	0.9988	0.9992	0.9996	0.9984	0.9988	0.9992	0.9996
Cs	18.4958	24.3087	39.14595	124.092	536.531	1299.93	5871.32	204046

Table 6.6: Scenario 1 Case 1 results, varying the subsystem reliability lower bounds with feasibility factors of 0.5 and 0.3

IT	1	2	3	4	5	6	7	8
FF	0.5	0.5	0.5	0.5	0.3	0.3	0.3	0.3
R11	0.9768	0.9762	0.9803	0.986	0.9714	0.9851	0.9834	0.9859
R21	0.9793	0.9808	0.9804	0.986	0.9828	0.985	0.9842	0.986
R12	0.9719	0.9836	0.9803	0.9661	0.9828	0.9846	0.9792	0.9861
R22	0.9715	0.9782	0.9804	0.9661	0.9716	0.9696	0.9829	0.986
RSB1	0.99952	0.99954	0.999614	0.9998	0.99951	0.99978	0.99974	0.9998
RSB2	0.9992	0.99964	0.999614	0.999	0.99951	0.99953	0.99964	0.99981
Rs	0.999	0.99919	0.999228	0.999	0.99902	0.99931	0.99938	0.99961
Cs	203277	7234928	1189744	1.3E+08	2.3E+09	5.1E+10	1E+10	7.5E+11

Case 2

In this case the system configuration's RBD is shown in Figure 6.4 and has three components in one subsystem and two in the other. The components' reliabilities in the first subsystem are: R_{11}, R_{21} and R_{31} while those in the second are R_{12}, R_{22}. The MCO model for this system configuration is optimised under the four experimental conditions outlined in section 6.1.

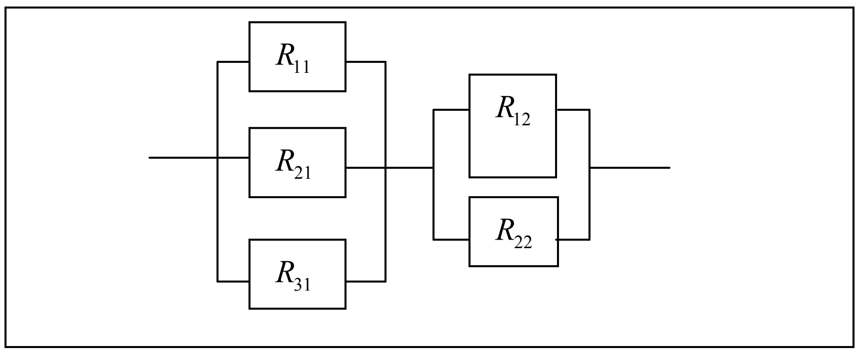

Figure 6.4: System reliability block diagram for Case 2 of Scenario 1

Results and Discussion

The results of the optimisations (showing this time only the Pareto optimal component reliabilities, the corresponding system reliability and the cost) are presented in Tables 6.7 and 6.8. The former features the results corresponding to varying the weights (the same values as for Case 1) and the feasibility factors from 0.9 to 0.3, and the latter presents those corresponding to varying the subsystem reliability LB values as before.

A similar results pattern to that of Case 1 emerges. In Table 6.7 the weighting of the criteria (especially at the highest FF value of 0.9) produced no change in the component reliabilities over the ten iterations. When the FF was changed to 0.7 a slight difference occurred in the component reliability values obtained in the second and fourth iterations only. The cost over all the iterations increased markedly from those obtained under the FF of 0.9; at the 2nd iteration it increased marginally over the 204.14 recorded for most of the other iterations, to 204.91 and at the 4th iteration it increased significantly to 833.37 representing approximately a 0.03% increase over those obtained in the other iterations. This indicates that cost even for marginal improvements in a system's reliability, can be very high. Lower FF values resulted in more heterogeneous components' and system reliabilities. They also resulted in higher cost figures in all the ITs as

Table 6.7: Results of Scenario 1 Case 2, varying weights and feasibility factor values

IT	FF	R11	R21	R31	R12	R22	Rs	Cs
1	0.9	0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
2		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
3		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
4		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
5		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
6		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
7		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
8		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
9		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
10		0.9	0.9	0.9	0.9684	0.9684	0.998	12.2387
1	0.7	0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
2		0.9264	0.8155	0.9264	0.9684	0.9684	0.998	204.913
3		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
4		0.9	0.9	0.9	0.977	0.9711	0.99834	833.367
5		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
6		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
7		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
8		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
9		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
10		0.9	0.9	0.9	0.9684	0.9684	0.998	204.135
1	0.5	0.9619	0.9183	0.9411	0.9684	0.9684	0.99882	4735.03
2		0.9319	0.9435	0.8996	0.9601	0.975	0.99862	20297
3		0.9307	0.849	0.9326	0.685	0.9687	0.99831	4477.4
4		0.8288	0.918	0.9288	0.9684	0.9684	0.998	4232.25
5		0.9387	0.9214	0.9339	0.9772	0.9708	0.99902	60940.2
6		0.9	0.9	0.9	0.9684	0.9684	0.998	4231.5
7		0.9	0.9	0.9	0.9684	0.9684	0.998	4231.5
8		0.9	0.9	0.9	0.9684	0.9684	0.998	4231.5
9		0.9606	0.9509	0.916	0.9741	0.9694	0.99905	16918.4
10		0.8995	0.8986	0.9019	0.9721	0.968	0.99811	8383.45
1	0.3	0.9	0.9	0.9	0.9684	0.9684	0.998	90089.2
2		0.8519	**0.9899**	0.9	0.9684	0.9684	0.99885	∞
3		0.9369	0.9355	0.8907	0.966	0.9745	0.99869	792201
4		0.9659	0.9281	0.9542	0.9739	0.9701	0.99911	656470
5		0.9403	0.9397	0.9602	0.9718	0.9734	0.99911	606483
6		0.9795	0.9207	0.845	0.9485	0.9806	0.99875	1.2E+08
7		0.9	0.9	0.9	0.9684	0.9684	0.998	90089.2
8		0.9	0.9	0.9	0.9684	0.9684	0.998	90089.2
9		0.9	0.9	0.9	0.9684	0.9684	0.998	90089.2
10		0.9	0.9	0.9	0.9684	0.9684	0.998	90089.2

depicted pictorially for the 10th iteration in Figure 6.5 though the system reliability does not follow the same trend. In instances where the cost was so high as to be practically indeterminate, one can see that at least one component reliability value closely approached the maximum of 0.99. (This is the case with the 2nd iteration involving the feasibility factor of 0.3 (see Table 6.7)).

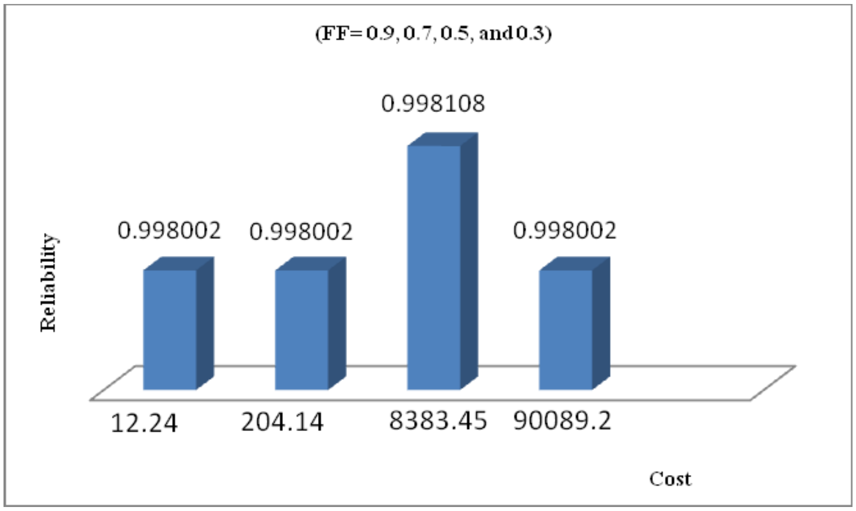

Figure 6.5: System reliability and cost for varied feasibility factor

Varying the subsystem reliability LB values from 0.9992 to 0.9998 (see Table 6.8) had an appreciable impact on the components' and system reliabilities and the cost. Note that in this case the system cost and reliability generally varied monotonically with the subsystem reliability. It should also be noted that the component reliability values obtained for the subsystem with two components were generally higher (in all the ITs) than the one which had three components, and therefore yielded higher subsystem reliability values. This is consistent with the theory (Billinton and Allan, 1992) and indicates that the higher the number of redundant components there are in a subsystem (or system) the lower their reliability levels need to be in order to achieve the subsystem reliability target. Even though this means that one can choose to design a highly reliable system using cheap components with relatively low reliabilities in an extensive redundant arrangement, the adverse consequence of redundancy (discussed in Chapters Three and Four) such as increased volume and weight, could eventually render such a system impractical.

Table 6.8: Results of Scenario 1 Case 2, varying subsystem lower bounds

IT	FF	R11	R21	R31	R12	R22	Rs	Cs
1	0.9	0.9072	0.9072	0.9072	0.9717	0.9717	0.9984	14.324
2		0.9157	0.9157	0.9157	0.9755	0.9755	0.9988	18.297
3		0.9263	0.9263	0.9263	0.98	0.98	0.9992	28.3504
4		0.9415	0.9415	0.9415	0.9859	0.9859	0.9996	85.3561
1	0.7	0.9072	0.9072	0.9072	0.9717	0.9717	0.9984	361.906
2		0.9157	0.9157	0.9157	0.9755	0.9755	0.9988	871.532
3		0.9263	0.9263	0.9263	0.98	0.98	0.9992	3920.48
4		0.9415	0.9415	0.9415	0.9859	0.9859	0.9996	136041
1	0.5	0.9072	0.9072	0.9072	0.9717	0.9717	0.9984	11308.2
2		0.9441	0.899	0.9456	0.9761	0.9769	0.99914	81224.4
3		0.9263	0.9263	0.9263	0.98	0.98	0.9992	612166
4		0.9146	0.9797	0.9408	0.9859	0.9859	0.9997	2.3E+08
1	0.3	0.8618	0.9664	0.9676	0.9759	0.9738	0.99922	2374646
2		0.9715	0.9744	0.9033	0.9754	0.9756	0.99933	3568628
3		0.9263	0.9263	0.9263	0.98	0.98	0.9992	9.6E+07
4		0.9448	0.9445	0.9449	0.9859	0.986	0.99963	4.2E+11

Case 3

The configuration and RBD for Case 3 is shown in Figure 6.6. Each subsystem comprises three components, in the first the component reliabilities are R_{11}, R_{21}, R_{31} and in the second R_{12}, R_{22}, R_{32}. The subsystems in this case have two redundant components each.

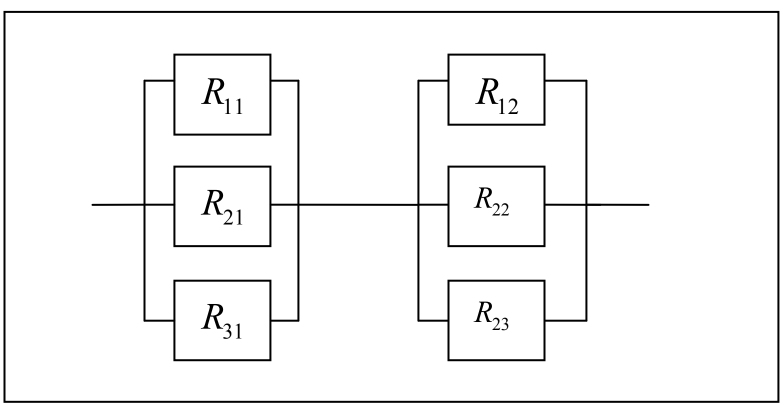

Figure 6.6: System reliability block diagram for Case 3 of scenario 1

Table 6.9: Results of Scenario 1 Case 3, varying weights and feasibility factor values

IT	FF	R11	R21	R31	R12	R22	R23	Rs	Cs
1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
2		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	7.48932
3		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	7.48932
4		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
5		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
6		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
7		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
8		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
9		0.9	0.9	0.9	0.9	0.9	0.9	0.998	7.48932
10		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	7.48932
1	0.7	0.9358	0.9454	0.927	0.927	0.9479	0.8856	0.99931	42.5767
2		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
3		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
4		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
5		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
6		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
7		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
8		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
9		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
10		0.9	0.9	0.9	0.9	0.9	0.9	0.998002	16.80303
1	0.5	0.896	0.896	0.9075	0.8781	0.955	0.9545	0.99875	359.576
2		0.9036	0.9038	0.8922	0.9402	0.9402	0.9606	0.99886	503.4751
3		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
4		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
5		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
6		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
7		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
8		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
9		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
10		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	37.69924
1	0.3	0.9001	0.9001	0.8998	0.8176	0.7825	0.9899	0.9986	2.3E+16
2		0.8603	0.8046	0.9899	0.745	0.7031	0.9899	0.99896	2.32E+16
3		0.9	0.9	0.9	0.9	0.9	0.9	0.998001	84.58194
4		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819
5		0.8865	0.9061	0.9061	0.9082	0.8853	0.9413	0.99838	277.783
6		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819
7		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819
8		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819
9		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819
10		0.9	0.9	0.9	0.9	0.9	0.9	0.998	84.5819

Results and Discussion

The results of testing the MCO model are provided in Tables 6.9 and 6.10 which are arranged in the same format as those for Case 2. The weightings over the ten iterations (for each feasibility factor value) had little or no impact on the levels of the Pareto optimal reliability values generated for each component, except in the 1st iteration, for a FF of 0.7, the 1st and 2nd iterations, for a FF of 0.5, and the 1st, 2nd, and 5th iterations for a FF of 0.3. Varying the feasibility factor values for fixed subsystem reliability lower bounds as shown in Table 6.9 only marginally affected the system reliability but had a marked effect again on the cost. Varying the subsystem reliability lower bound in this instance too had a significant impact across all the output parameters as seen in Table 6.10.

Table 6.10: Results of Scenario 1 Case 3, varying subsystem lower bounds

IT	FF	R11	R21	R31	R12	R22	R23	Rs	Cs
1	0.9	0.9072	0.9072	0.9072	0.9072	0.9072	0.9072	0.9984	9.34959
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9988	9.88283
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.9992	10.785
4		0.9415	0.9415	0.9415	0.9415	0.9415	0.9415	0.9996	12.93
1	0.7	0.9072	0.9072	0.9072	0.9072	0.9072	0.9072	0.9984	22.703
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9988	26.813
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.9992	34.845
4		0.9415	0.9415	0.9415	0.9415	0.9415	0.9415	0.9996	60.056
1	0.5	0.9072	0.9072	0.9072	0.9072	0.9072	0.9072	0.9984	47.4478
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9988	62.1437
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.9992	95.358
4		0.9415	0.9415	0.9415	0.9415	0.9415	0.9415	0.9996	233.99
1	0.3	0.9072	0.9072	0.9072	0.9072	0.9072	0.9072	0.9984	113.06
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9988	165.99
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.9992	304.66
4		0.99	0.84	0.9095	0.958	0.99	0.8808	0.9998	7E+16

6.3 **SCENARIO 2**

The series-parallel system models constructed for this scenario have three subsystems. The specific cases considered are as follows:

- Case 1: Two components in each subsystem
- Case 2: Three components in one subsystem and two in the others
- Case 3: Three components each in two subsystems and two in the other

The MCO model corresponding to each of the cases was subjected to the same tests as before. The generated weights corresponding to each of the ten iterations in each case are shown in Table 6.11. W1, W2, and W3, represent the weights for the three subsystems' reliabilities respectively, and W4 that for the cost.

Table 6.11: Weights for the criteria in the Scenario 2 Cases

	ITERATIONS									
Weight	1	2	3	4	5	6	7	8	8	10
W1	0.3098	0.3116	0.313	0.314	0.3149	0.316	0.316	0.3167	0.3171	0.3175
W2	0.3098	0.3116	0.313	0.314	0.3149	0.316	0.316	0.3167	0.3171	0.3175
W3	0.3098	0.3116	0.313	0.314	0.3149	0.316	0.316	0.3167	0.3171	0.3175
W4	0.0707	0.0651	0.061	0.058	0.0552	0.053	0.051	0.0499	0.0049	0.0475

Case 1

Results and Discussions

The results of the optimisations for this case are presented in Tables 6.12 and 6.13. The former shows the solutions when the weights are varied iteratively for fixed FF levels of 0.9, 0.7, 0.5, and 0.3 and subsystems' reliability LBs of 0.999. (The chosen weights in this instance too provide the highest and lowest weightings for the subsystems' reliabilities and the cost respectively). The latter presents the results when the subsystem reliability LBs were varied iteratively from 0.9992 to 0.9998 in steps of 0.0002, the weights were fixed as 0.3175 for each of the subsystem reliabilities and 0.0475 for the cost. The FF values were as before.

Similar trends to those for Scenario 1 are evident i.e. the effects of varying the weightings, the FF values and subsystem LBs, are similar for the components' and system reliabilities and the cost. In this case, however, the weightings had more impact on the relative levels of the reliabilities of the components for a given iteration, than was previously observed, even though the size of the improvement across the ITs could still be described as marginal. This is particularly so for feasibility factor values of 0.7 and lower. Its impact on cost was also quite marked. The FF and subsystem LB values were the parameters which again showed the greatest impact on the solutions.

A notable feature of the results in Table 6.12 especially, is the heterogeneous nature of the component reliability values obtained within and across all the ITs for the entire set of FF values except those associated with 0.9. Consequently a variety of reliability and cost values were obtained for the system. This trend is partly attributed to the relatively more sophisticated system configuration encountered in this case (i.e. increased number of subsystems and components resulting in an MCO model of higher dimensionality). Therefore, there is a multiplicity of choices for this design than has been seen in earlier cases. Since the basic system reliability specifications outlined at the beginning of this Chapter are all met, the results for each of the iterations represents a potential (Pareto) optimal design for reliability of the system under discussion.

Table 6.12: Results of Scenario 2 Case 1, varying weights and feasibility factor values

IT	FF	R11	R21	R12	R22	R13	R23	Rs	Cs
1	0.9	0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
2		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
3		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
4		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
5		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
6		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
7		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
8		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
9		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
10		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	27.729
1	0.7	0.9667	0.9699	0.9747	0.9606	0.9661	0.9705	0.997004	8024198
2		0.9684	0.9683	0.971	0.9722	0.9739	0.9617	0.997195	878.17
3		0.9684	0.9684	0.9684	0.9684	0.9684	0.9684	0.997007	592.243
4		0.9684	0.9684	0.9726	0.9731	0.9731	0.9729	0.997537	1121.47
5		0.9684	0.9685	0.9696	0.9684	0.9682	0.9698	0.997124	660.476
6		0.9684	0.9706	0.9732	0.9716	0.9663	0.9703	0.997238	840.996
7		0.9684	0.9684	0.962	0.9737	0.9715	0.9729	0.997229	914.04
8		0.9684	0.9684	0.9684	0.9683	0.9683	0.9684	0.997	589.122
9		0.9684	0.9684	0.9645	0.9781	0.9722	0.9641	0.997229	1461.66
10		0.9684	0.9684	0.968	0.9688	0.9626	0.9732	0.997	685.929
1	0.5	0.9695	0.9735	0.9684	0.9684	0.9713	0.9651	0.997194	23745.1
2		0.9542	0.9782	0.9773	0.9716	0.9788	0.9713	0.99775	308844
3		0.9779	0.9732	0.9684	0.9684	0.9785	0.9769	0.997914	262272
4		0.9726	0.9771	0.9752	0.9784	0.9743	0.969	0.998041	210796
5		0.9684	0.9684	0.9683	0.9684	0.9684	0.9683	0.997001	12538.8
6		0.9816	0.9785	0.9595	0.982	0.9822	0.9815	0.998547	5227898
7		0.9673	0.9694	0.9734	0.9643	0.9684	0.9684	0.997054	19750.4
8		0.9724	0.9704	0.9717	0.9733	0.9723	0.9639	0.99743	34218.1
9		0.9757	0.9772	0.9751	0.96	0.9768	0.9759	0.997892	180751
10		0.9678	0.9689	0.9681	0.9687	0.9735	0.9764	0.99738	57008.1
1	0.3	0.9755	0.9714	0.9792	0.974	0.9786	0.9791	0.998312	5.9E+07
2		0.9721	0.9751	0.9757	0.971	0.9728	0.9691	0.997762	3.43E+06
3		0.9781	0.9733	0.9806	0.9675	0.9792	0.9796	0.998361	1.5E+08
4		0.9672	0.978	0.9789	0.9765	0.9772	0.9755	0.998225	3.5E+07
5		0.9809	0.9709	0.9805	0.9712	0.963	0.9816	0.998203	4.6E+08
6		0.9766	0.9723	0.9791	0.9792	0.9739	0.972	0.998192	4.8E+07
7		0.9716	0.9776	0.9779	0.9723	0.9781	0.967	0.99803	2.3E+07
8		0.9772	0.9772	0.9645	0.9781	0.9751	0.9787	0.99817	3.4E+07
9		0.9721	0.9798	0.98	0.9705	0.9797	0.9731	0.998301	1.2E+08
10		0.9586	0.9809	0.9624	0.9756	0.9721	0.9777	0.99767	1.3E+08

Table 6.13: Results of Scenario 2 Case 1, varying subsystem lower bounds

IT	FF	R11	R21	R12	R22	R13	R23	Rs	Cs
1	0.9	0.9717	0.9717	0.9717	0.9717	0.9717	0.9717	0.997599	33.7709
2		0.9157	0.9157	0.9157	0.9755	0.9755	0.9755	0.9982	45.3842
3		0.98	0.98	0.98	0.98	0.98	0.98	0.9988	75.0441
4		0.9859	0.9859	0.9859	0.9859	0.9859	0.9859	0.999361	212.912
1	0.7	0.9683	0.9748	0.9648	0.9782	0.9788	0.9623	0.997636	2774.5
2		0.976	0.975	0.9676	0.9815	0.9755	0.9755	0.998201	5672.79
3		0.9806	0.9803	0.9802	0.9804	0.9801	0.9805	0.998842	13656.1
4		0.9859	0.9859	0.9859	0.9859	0.9859	0.9859	0.9994	408089
1	0.5	0.9717	0.9717	0.9707	0.9727	0.9748	0.9683	0.997608	44252.7
2		0.9755	0.9755	0.98	0.97	0.9809	0.9789	0.998398	1096113
3		0.98	0.98	0.98	0.98	0.98	0.98	0.9988	1836438
4		0.9859	0.986	0.9859	0.9859	0.986	0.9859	0.999407	7.2E+08
1	0.3	0.9714	0.9759	0.9791	0.9723	0.9788	0.9789	0.998285	5.7E+07
2		0.9542	0.9542	0.9542	0.9542	0.9184	0.99	0.994988	2.3E+16
3		0.9802	0.9798	0.9786	0.9813	0.9806	0.9815	0.998841	6.2E+08
4		0.9763	0.9763	0.9763	0.9763	0.99	0.9689	0.99856	2.3E+16

Case 2

Results and Discussion

The results for this case are presented in Tables 6.14 and 6.15. Once again the same general patterns are evident. As in Case 1 of this Scenario, the values of the components' reliabilities were generally heterogeneous. Also, the components of the subsystem with the largest number of redundant components had in general the least assigned reliabilities, a trend which has already been noted in Scenario 1 as being consistent with the theory. Another aspect worth noting is the assignment of the same reliability values to the components of a subsystem, as occurs especially where the FF value is 0.9, but is also replicated elsewhere in Table 6.14. This suggests that identical components in redundant series subsystems do yield optimal system reliability. Further, and more specific, comments on this feature are reserved until Chapter Seven, where more cases are tested and further instances of the phenomenon are observed.

While there is only a single choice design offered by the results under a FF value of 0.9, the others provide a variety. With a value of 0.7, the highest assigned system reliability value of 0.99766 occurs in the 9^{th} iteration, with a corresponding cost of 2219.55 which is also the highest in this group. The least system reliability value of 0.99701 with corresponding least cost occurred in the 1st, 2nd, 5th, 6th, and 8th iterations. Even though this reliability differs only slightly from the highest their cost values differ significantly - a further indication of the cost associated with even marginal reliability improvements. The lowest system reliability value throughout the tests are the same as those obtained under the 0.9 FF value, though the corresponding cost value of the latter was much lower. This further illustrates the observation made under Scenario 1 that low feasibility values do not necessarily mean low reliability, but the cost values tend to be very

Table 6.14: Results of Scenario 2 Case 2, varying weights and feasibility factor values

IT	FF	R11	R21	R31	R12	R22	R13	R23	Rs	Cs
1	0.9	0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
2		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
3		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
4		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
5		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
6		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
7		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
8		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
9		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
10		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	22.9796
1	0.7	0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	404.91
2		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	404.91
3		0.9704	0.6786	0.9215	0.9641	0.9765	0.9681	0.9701	0.99746	999.442
4		0.9251	0.9336	0.9189	0.9684	0.9684	0.9684	0.9684	0.9976	412.554
5		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	404.91
6		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	404.91
7		0.6391	0.9684	0.9129	0.965	0.9715	0.9764	0.9724	0.99736	1099.63
8		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	404.91
9		0.9272	0.9291	0.9108	0.9556	0.9719	0.9627	0.9734	0.99766	2219.55
10		0.8181	0.9098	0.939	0.9598	0.9751	0.9704	0.9662	0.997	647.463
1	0.5	0.9505	0.8646	0.8507	0.9681	0.9687	0.9568	0.9769	0.99701	53081.8
2		0.9417	0.9396	0.7991	0.9687	0.97	0.977	0.9737	0.99775	68382.5
3		0.9339	0.9561	0.7776	0.9684	0.9684	0.9758	0.9729	0.9977	41453.4
4		0.9	0.9	0.9	0.9684	0.9684	0.9684	0.9684	0.99701	8455.46
5		0.9137	0.9306	0.9368	0.9713	0.9708	0.9702	0.9725	0.99797	20191
6		0.9406	0.8242	0.953	0.972	0.9724	0.969	0.9678	0.99774	17904.6
7		0.8881	0.8224	0.966	0.9678	0.9704	0.9667	0.97	0.99737	11382.7
8		0.9457	0.9601	0.8397	0.9769	0.9767	0.9772	0.9778	0.99861	227805
9		0.9533	0.8219	0.9408	0.972	0.9723	0.9684	0.9684	0.99774	17594.7
10		0.9586	0.8814	0.9374	0.9751	0.9767	0.9582	0.9761	0.99811	97879.9
1	0.3									
2		0.7155	**0.9899**	**0.9899**	**0.9899**	0.9455	0.9766	**0.9899**	0.9918	9.30E+16
3		0.8765	0.9727	0.8591	0.9721	0.9718	0.973	0.9728	0.99801	1340636
4		0.8162	0.9723	0.8776	0.9733	0.9703	0.9732	0.9726	0.99785	1384681
5		0.8011	0.9675	0.9469	0.9685	0.9716	0.9663	0.9726	0.99784	548872
6		0.9541	0.6887	0.9707	0.9699	0.9736	0.9731	0.9736	0.99808	1485534
7		0.6623	0.9824	0.8842	0.9795	0.974	0.9439	0.9822	0.99778	1.2E+09
8		0.9667	0.9639	0.883	0.973	0.9704	0.9733	0.974	0.99837	1452057
9		0.7058	0.9704	0.9178	0.9728	0.9725	0.973	0.9712	0.99776	1151074
10		0.9538	0.9561	0.9363	0.9539	0.982	0.9812	0.9825	0.99871	1.4E+09

high. This fact is further supported at the 0.5 and 0.3 FF levels where some of the highest system reliability values and costs were obtained. Incidentally the first iteration (in Table 6.14) with an FF value of 0.3 failed to yield a feasible solution, thus that row is left blank and shaded. In Table 6.15 the effect of varying the subsystem reliability LBs are again evident: system reliability and cost increased with an increase in the LBs.

Table 6.15: Results of Scenario 2 Case 2, varying subsystem lower bounds

IT	FF	R11	R21	R13	R12	R22	R13	R23	Rs	Cs
1	0.9	0.9072	0.9072	0.9072	0.9717	0.9717	0.9717	0.9717	0.9976	27.1392
2		0.9157	0.9157	0.9157	0.9755	0.9755	0.9755	0.9755	0.9982	35.0722
3		0.9263	0.9263	0.9263	0.98	0.98	0.98	0.98	0.9988	55.1625
4		0.9415	0.9415	0.9415	0.9859	0.9859	0.9859	0.9859	0.994	169.15
1	0.7	0.9627	0.8983	0.8437	0.9713	0.9721	0.9717	0.9717	0.99781	721.618
2		0.94	0.9324	0.9337	0.9755	0.9755	0.9755	0.9755	0.99853	1746.99
3		0.9388	0.906	0.9421	0.9799	0.9805	0.9799	0.9805	0.99888	8612.33
4		0.9412	0.9413	0.942	0.9859	0.9859	0.9859	0.9859	0.9994	272081
1	0.5	0.9132	0.9535	0.888	0.974	0.9741	0.9717	0.9717	0.99807	38176.7
2		0.9626	0.5931	0.9654	0.9823	0.9661	0.9795	0.9707	0.99827	2092391
3		0.9567	0.919	0.9741	0.9806	0.9794	0.9806	0.9794	0.99911	1362502
4		0.9464	0.9564	0.9569	0.986	0.9859	0.9862	0.986	0.99951	5.5E+08
1	0.3	0.8283	0.6892	0.9899	0.9899	0.92	0.92	0.9899	0.99785	7E+16
2		0.8773	0.8773	0.9773	0.9542	0.9542	0.9184	0.9899	0.99674	2.3E+16
3		0.9722	0.8986	0.978	0.9816	0.9816	0.9819	0.9818	0.99927	1.2E+09
4		0.9174	0.9175	0.9175	0.969	0.9899	0.9763	0.9763	0.99856	2.3E+16

Case 3

Results and Discussion

The results presented in Tables 6.16 and 6.17 repeat the general trends already noted in earlier Cases; again the reliability assigned to the components of the subsystem with one redundant component was generally higher than those with two. Also at lower feasibility factor values some of the components reached reliability values closely approaching their maximum values (these are recorded in bold) which inevitably resulted in exorbitant cost values.

All the iterations in Table 6.16, except those under the FF of 0.9, yielded heterogeneous component reliability values, resulting in a variety of system reliabilities and costs. The highest

Table 6.16: Results of Scenario 2 Case 3, varying weights and feasibility factor values

IT	FF	R11	R21	R31	R12	R22	R32	R13	R23	Rs	Cs
1		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
2		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
3		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
4		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
5	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
6		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
7		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
8		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
9		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
10		0.9	0.9	0.9	0.9	0.9	0.9	0.9684	0.9684	0.997	18.23018
1		0.5527	0.9527	0.9527	0.8974	0.889	0.9684	0.9684	0.9466	**0.99696**	**255.0453**
2		0.9483	0.9643	0.7958	0.9296	0.9585	0.8995	0.9685	0.9682	**0.99833**	**308.0336**
3		0.8918	0.8726	0.9463	0.9	0.9	0.9	0.9684	0.9684	0.99726	225.5983
4		0.9407	0.6839	0.9463	0.9153	0.9208	0.9101	0.9683	0.9684	0.99739	232.8357
5	0.7	0.9339	0.861	0.9504	0.9294	0.7677	0.939	0.9682	0.9686	0.99755	240.0174
6		0.8784	0.9185	0.8991	0.9508	0.8952	0.8855	0.9685	0.9682	0.99741	229.0037
7		0.9458	0.9057	0.8744	0.9626	0.9456	0.5094	0.977	0.9684	0.99736	770.4695
8		0.8999	0.9001	0.9	0.8864	0.9058	0.9351	0.9684	0.9684	0.99731	221.6994
9		0.8834	0.8736	0.9497	0.9364	0.9394	0.7398	0.9761	0.9716	0.99759	725.3357
10		0.9009	0.9008	0.8983	0.9192	0.9112	0.9281	0.9684	0.9684	0.99749	222.6299
1		0.9651	0.7688	0.9632	0.9302	0.9706	0.865	0.8714	0.9661	**0.99507**	**6788.085**
2		0.9958	0.8581	0.8903	0.9616	0.9062	0.7221	0.9787	0.9755	0.99796	4.41E+31
3		0.901	0.8552	0.952	0.8475	0.9582	0.8792	0.9787	0.9755	0.99802	157188.2
4		0.9531	0.8227	0.9119	0.8864	0.917	0.917	0.9752	0.9753	0.99787	44481.59
5	0.5	0.931	0.918	0.8312	0.926	0.8715	0.9339	0.9685	0.9683	0.99742	4308.758
6		0.9378	0.9164	0.8738	0.9103	0.9079	0.9493	0.97	0.9726	0.9981	11183.56
7		0.316	0.9375	0.9506	0.9688	0.9013	0.8876	0.9714	0.9688	0.99665	9966.53
8		0.8741	0.9184	0.9171	0.8834	0.9241	0.9009	0.9728	0.9721	0.99751	14822.4
9		0.9628	0.9551	0.7929	0.9649	0.9679	0.9676	0.972	0.9723	**0.99884**	**18441.61**
10		0.9439	0.9569	0.8991	0.9273	0.8861	0.9309	0.9599	0.9751	0.99819	21428.19
1		0.562	0.9606	0.7712	0.9325	0.8994	0.956	0.9716	0.9688	0.99487	2.29E+05
2		0.7158	**0.9899**	**0.9899**	0.7158	**0.9899**	**0.9899**	0.9765	**0.9899**	0.99971	**1.16E+17**
3		0.5	**0.9899**	**0.9899**	0.5	0.829	0.8319	0.9	**0.9899**	**0.98458**	**6.97E+16**
4		0.9735	0.8453	0.9496	0.9534	0.9639	0.5697	0.9713	0.9716	0.99826	773162.1
5	0.3	0.7023	**0.9899**	**0.9899**	0.7023	**0.9899**	**0.9899**	0.9797	**0.9899**	**0.99973**	1.16E+17
6		**0.9899**	0.8222	0.8461	0.9645	0.9181	0.9181	0.9699	**0.9899**	0.99888	4.65E+16
7		0.8461	**0.9899**	0.8222	0.9645	0.9181	0.9181	0.9399	**0.9899**	0.99888	4.65E+16
8		**0.8999**	0.6544	**0.9899**	0.5	0.7602	**0.9899**	**0.9899**	**0.9899**	0.99834	9.3E+16
9		0.9773	0.689	0.898	0.9777	0.9228	0.9028	0.9712	0.9745	0.99838	12352470
10		0.6763	**0.9899**	0.5	0.8997	0.8997	0.8997	0.9	**0.9899**	0.99635	4.65E+16

and lowest system reliability values recorded for a FF value of 0.7 were 0.99833 and 0.99696 respectively with corresponding cost values of 308.03 and 225.04 respectively. These occurred in the 1st and 2nd iterations. Under the FF of 0.5 the highest and lowest system reliability values of 0.99884 and 0.99507 respectively were recorded, with resultant associated costs of 18441.61 and 6788.08 respectively. These occurred in the 9th and 1st iterations. Similar results were apparent for the FF value of 0.3. The highest and lowest system reliabilities show a narrow range of values not only from IT to IT for a given FF value, but also across FF values. For instance the differences in the values of the system reliabilities occur only in the third or fourth decimal places over all the iterations. The same cannot be said, however for the costs. These observations are replicated in the earlier Cases too.

Table 6.17 repeats the trends already noted in relation to varying the subsystem reliability LBs. It is also observed that the patterns of component reliability values as occurred in Table 6.16 are preserved under this experiment. In all the cases the highest system reliability and cost were achieved in connection with the highest subsystem reliability LBs - once more indicating the sensitivity of the MCO model to the subsystem reliability LBs.

Table 6.17: Results of Scenario 2 Case 3, varying subsystem lower bounds

IT	FF	R11	R21	R31	R12	R22	R32	R13	R23	Rs	Cs
1	0.9	0.9072	0.9072	0.9072	0.9072	0.9072	0.9072	0.9717	0.9717	0.9976	20.55708
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9755	0.9755	0.9982	24.88556
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.98	0.98	0.9988	35.54024
4		0.9415	0.9415	0.9415	0.9415	0.9415	0.9415	0.9859	0.9859	0.9994	93.9764
1	0.7	0.882	0.9481	0.8778	0.8403	0.9517	0.9348	0.9664	0.9762	0.99795	644.4917
2		0.9157	0.9157	0.9157	0.9157	0.9157	0.9157	0.9755	0.9755	0.9982	889.407
3		0.9225	0.9566	0.8996	0.9513	0.8435	0.9514	0.9803	0.9801	0.9989	4326.439
4		0.9199	0.9589	0.9502	0.9201	0.9261	0.9749	0.9859	0.9859	0.9998	136458.1
1	0.5	0.9072	0.9072	0.9072	0.8855	0.9607	0.955	0.9717	0.9717	0.9982	11908.15
2		0.9648	0.9765	0.808	0.9677	0.9478	0.9776	0.9765	0.9773	0.99927	211143.2
3		0.9263	0.9263	0.9263	0.9263	0.9263	0.9263	0.98	0.98	0.9988	612241.4
4		0.9727	0.9113	0.9596	0.9343	0.9508	0.9536	0.9859	0.9859	0.99955	2.27E+08
1	0.3	**0.9899**	0.8549	0.8649	0.9638	0.9274	0.9274	**0.9899**	0.9414	0.99902	2.32E+16
2		0.8773	0.8773	0.8773	0.8773	0.8773	0.8773	**0.9899**	0.9184	0.99549	2.32E+16
3		**0.9899**	0.7987	**0.9899**	0.5	0.9061	0.9061	**0.9899**	0.96	0.99517	4.65E+16
4		0.8816	0.9589	0.9589	0.9589	0.9589	0.9589	0.9899	0.9248	0.99924	2.32E+16

6.4 **SCENARIO 3**

In this Scenario the MCO model is applied to a relatively simple bridge network- the RBD is displayed in Figure 3.11. A series-parallel transformation of the network (Billinton & Allan, 1992), shown in Figure 6.7 yields four minimum cut sets involving two second and two third order ones, from its five components with reliabilities R_i respectively ($i = 1,2,..,5$). Thus the MCO model for the network has five criteria: the four minimum cut sets and the cost of improving the reliability of the components. Note that all the components are repeated in at least one subsystem, thus making this series-parallel system different from those considered earlier. The reliabilities of the subsystems are in this case dependent on those of the other subsystems which have identical components. The MCO model therefore amounts to finding the reliability values of the components that would jointly maximise the reliability of the subsystems (i.e. minimise the chance of failure due to the minimum cut sets) and minimise the cost. The weights for the optimisation are presented in Table 6.18, where W1 to W4 are the respective weights for the subsystem reliabilities and W5 that for the cost.

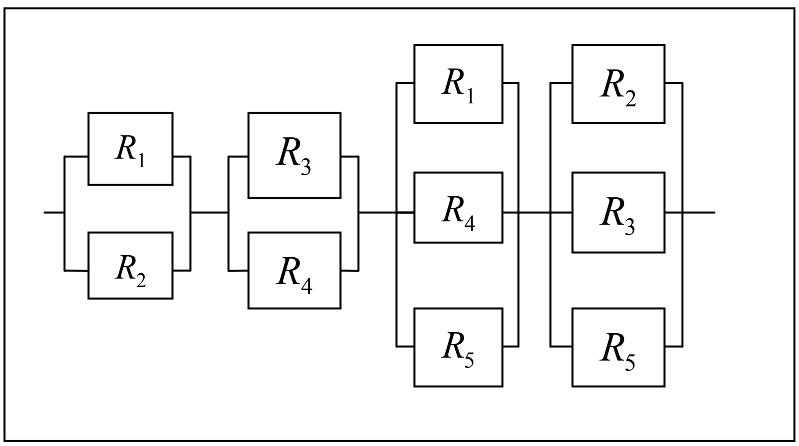

Figure 6.7: Series-parallel transformation of bridge network

Table 6.18: Weights for the criteria in Scenario 3

	ITERATION									
Weight	1	2	3	4	5	6	7	8	9	10
W1	0.2365	0.2376	0.2384	0.239	0.2395	0.2399	0.2402	0.2405	0.2408	0.241
W2	0.2365	0.2376	0.2384	0.239	0.2395	0.2399	0.2402	0.2405	0.2408	0.241
W3	0.2365	0.2376	0.2384	0.239	0.2395	0.2399	0.2402	0.2405	0.2408	0.241
W4	0.2365	0.2376	0.2384	0.239	0.2395	0.2399	0.2402	0.2405	0.2408	0.241
W5	0.0539	0.0496	0.0464	0.044	0.042	0.0404	0.039	0.0379	0.0369	0.0361

Results and Discussion

The results presented in Tables 6.19 and 6.20 show similar characteristics to those observed in the first two Scenarios (as far as varying the weights, FF, and subsystem reliability LBs are concerned). The reliabilities generated for the fifth component show little or no variation. This may at first appear odd but a reliability importance check (Mettas, 2000) of the components in the system (see the details in Appendix B) reveals that the fifth component which is also a redundant one (for this network configuration) has the least, also it occurs only in the third order minimum cut sets their failure probabilities of which are the lowest. Hence it is expected. The high system reliability values that were obtained further endorse the minimal impact of the low reliability value for the component. In this case therefore effort to find higher values for the system reliability is better focussed on upgrading the reliability of the other components. It is to be noted from Table 6.20 that the reliability of the fifth component appreciated slightly at higher subsystem reliability LBs, which again indicates the impact of the subsystem reliability LBs on the solution output. The relative impact on the overall system reliability was however negligible, although the cost increased sharply as a result.

In conclusion, the Cases discussed in this Chapter have shown promising results to the effect that the MCO model can provide the Pareto optimal levels of reliability improvements in the components of series-parallel or complex systems in order to at least achieve a system reliability target at minimum cost. The solutions associated with the individual iterations (for the assumed FF values) represent potential reliability designs for the systems discussed. Thus a variety of potential designs for reliability could be secured using the MCO model and methodology for review and decision making.

Table 6.19: Results of Scenario 3, varying weights and feasibility factor values

IT	FF	R1	R2	R3	R4	R5	Rs	Cs
1	0.9	0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
2		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
3		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
4		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
5		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
6		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
7		0.9684	0.9684	0.9684	0.9684	0.525	0.99706	19.4913
8		0.9684	0.9684	0.9684	0.9684	0.5	0.99701	19.486
9		0.9684	0.9684	0.9684	0.9684	0.5749	0.99716	19.5038
10		0.9684	0.9684	0.9684	0.9684	0.5749	0.99716	19.5038
1	0.7	0.9788	0.9527	0.9766	0.9774	0.5	0.99768	2594.65
2		0.9743	0.9792	0.979	0.9744	0.5	0.99838	3386.69
3		0.9813	0.9767	0.9729	0.9789	0.5	0.99848	5601.31
4		0.9654	0.9754	0.9622	0.9735	0.5	0.99723	791.901
5		0.9809	0.9475	0.9812	0.9688	0.5	0.99762	6342.76
6		0.9696	0.9818	0.9691	0.9817	0.5	0.99832	8938.47
7		0.9744	0.9792	0.9756	0.9739	0.5	0.99824	2482.68
8		0.9796	0.9509	0.9717	0.9766	0.5	0.99741	2450.87
9		0.9822	0.9811	0.9804	0.963	0.5	0.99842	11031.6
10		0.9769	0.9755	0.9784	0.9537	0.5	0.99764	2162.89
1	0.5	0.9395	**0.9899**	0.9342	**0.9899**	0.5	0.99809	9.8E+11
2		0.9032	0.9789	0.894	0.9881	0.5	0.99501	5.3E+09
3		**0.9899**	0.9359	0.941	**0.9899**	0.793	0.99795	9.8E+11
4		0.9541	0.97	0.9376	**0.9899**	0.5123	0.99686	4.9E+11
5		0.9764	**0.9899**	0.9673	**0.9899**	0.5	0.99915	9.8E+11
6		0.9446	**0.9899**	0.9358	**0.9899**	0.5	0.99819	9.8E+11
7		**0.9899**	0.942	**0.9899**	0.9509	0.5	0.99838	9.8E+11
8		**0.9899**	0.9387	**0.9899**	0.9475	0.5	0.99828	9.8E+11
9		0.9452	**0.9899**	0.9364	**0.9899**	0.5	0.99821	9.8E+11
10		**0.9899**	0.9394	**0.9899**	0.9483	0.5	0.9983	9.8E+11
1	0.3	0.9364	0.9753	0.9267	**0.9899**	0.5	0.99647	2.3E+16
2		**0.9899**	0.9228	0.9271	**0.9899**	0.8549	0.99765	4.60E+16
3		0.9355	0.9762	0.9235	**0.9899**	0.5	0.99646	2.3E+16
4		0.9218	**0.9899**	0.9231	**0.9899**	0.5	0.99765	4.6E+16
5		**0.9899**	0.9228	**0.9899**	0.9257	0.5	0.99771	4.6E+16
6		**0.9899**	0.923	0.9246	**0.9899**	0.8593	0.99763	4.6E+16
7		0.9353	0.9764	0.9229	**0.9899**	0.5	0.99646	2.3E+16
8		**0.9899**	0.9229	0.9242	**0.9899**	0.8603	0.99763	4.6E+16
9		**0.9899**	0.9226	0.925	**0.9899**	0.8594	0.99763	4.6E+16
10		0.9229	**0.9899**	0.9354	0.9763	0.5	0.99645	2.3E+16

Table 6.20: Results of Scenario 3, varying subsystem lower bounds

ITERAT	FFACT	R1	R2	R3	R4	R5	Rs	Cs
1	0.9	0.9717	0.9717	0.972	0.9717	0.5	0.9976	23.514
2		0.9755	0.9755	0.976	0.9755	0.5	0.9982	31.256
3		0.98	0.98	0.98	0.98	0.5	0.9988	51.029
4		0.9859	0.9859	0.986	0.9859	0.5	0.9994	164.28
1	0.7	0.9788	0.9772	0.965	0.9773	0.5	0.9981	2729.8
2		0.9758	0.9764	0.976	0.9763	**0.5503**	0.9983	2053.7
3		0.9793	0.9807	0.985	0.9743	0.5	0.9988	30512
4		0.9859	0.986	0.986	0.986	**0.5975**	0.9995	284559
1	0.5	0.9717	0.9717	0.972	0.9717	0.5	0.9976	22596
2		0.9755	0.9755	0.976	0.9755	0.5	0.9982	99045
3		0.98	0.98	0.98	0.98	0.5	0.9988	1E+06
4		0.9859	0.9859	0.986	0.9859	0.5	0.9994	5E+08
1	0.3	0.9788	0.9772	0.965	0.9773	0.5	0.9981	3E+07
2		0.9758	0.9764	0.976	0.9763	**0.5503**	0.9983	9E+06
3		0.9793	0.9807	0.985	0.9743	0.5	0.9988	2E+10
4		0.9859	0.986	0.986	0.986	**0.5975**	0.9995	8E+11

6.5 SUMMARY

The proposed MCO model was tested using seven hypothetical series-parallel systems one of which was derived from a bridge network. The following observations can be drawn:

- The MCO model and the accompanying methodology yielded very high component and therefore system reliabilities at specified minimum costs.
- The model exhibited very high stability or robustness with respect to the weighting system used (i.e. weighting all subsystem reliabilities equally but higher than the cost). This means that the selection of the BCS from those generated is relatively easy, since only a few reliability design options have to be considered.
- The parameters to which the model is most sensitive are the subsystem reliability lower bounds, followed by the feasibility factors.
- The system cost and reliability both vary monotonically with the subsystem reliability lower bounds.
- Even though the system cost varies monotonically with the feasibility factor, the system reliability does not
- The increase in the reliability levels of some components was higher at lower values of the feasibility factor, indicating that a lower feasibility factor value is not a direct limitation to improving reliability. Cost was identified as the limiting factor.

The experimental results obtained have been consistent with the theories of the subject areas of reliability and MCO. To further test the model, four problems from real engineering applications are presented and discussed in the next Chapter.

CHAPTER SEVEN

PRACTICAL APPLICATIONS

7.1 **INTRODUCTION**

In order to demonstrate the potential of the proposed model it was applied to three cases extracted from previously published work and to a practical example. The first was a problem taken from Billinton and Allan's book (1992, pp 98-99), the second from an example discussed by both Shelokar et al (2002) and Salazar et al (2006), and the third from a paper by Espiritu et al (2007). The practical application was based on a gas supply system.

The characteristics that set these problems apart from those discussed in the previous Chapter are: (i) the system configurations are more sophisticated; (ii) they are associated with specific real life systems; (iii) the latter two especially involve larger system configurations, and (iv) they provide an opportunity to compare the solutions using the model developed in this work (especially in the case of the second and third examples) with those obtained using other techniques and methodologies.

7.2 **THE BILLINTON AND ALLAN PROBLEM**

7.2.1 **Introduction**

This system design problem (Billinton and Allan, 1992, pp 98-99) has the configuration shown in Figure 7.1. The system comprises ten components and three major subsystems in series-parallel format. The latter are made up of the following:

(i) The first consists only of component 1

(ii) The second incorporates components 2 to 7; of these numbers 3, 4 and 5 are not identical but form a subsystem which remains operational when at least one of its components is operational.

(iii) Components 8, 9 and 10 are identical and form the third subsystem which remains operational when two out of the three components are operational.

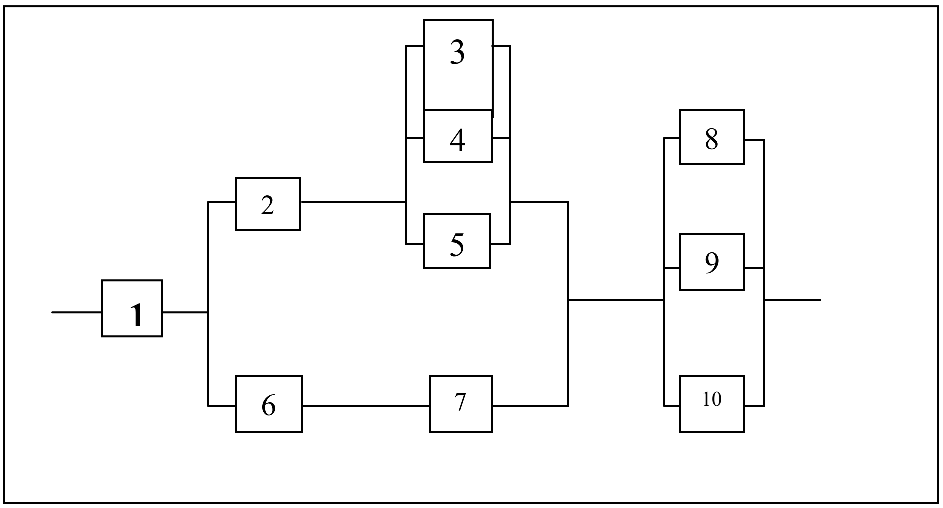

Figure 7.1: Configuration of the system (Source: Billinton & Allan, 1992)

If the reliabilities of the ten components are respectively $R_1, R_2,...R_{10}$ where $R_8 = R_9 = R_{10}$ then the reliabilities of the subsystems are respectively given by: R_1, $1-(1-R_2R^*)(1-R_6R_7)$ and $3R_8^{\ 2} - 2R_8^{\ 3}$ (since the components are identical) , where R^* is given by $1-(1-R_3)(1-R_4)(1-R_5)$. As before the objective was to maximise the subsystem reliabilities and minimise the associated cost function. The subsystem reliabilities were again weighted equally. The input parameters for the optimisation were taken to be the following:

- Initial component reliability $R_{ki,\min} = 0.8 \ \ \forall k, i \ \ k \in \{1,2,...,6\}$, $i = 1,2,3$
- Maximum component reliability $R_{ki,\max} = 0.99 \ \ \forall k, i \ \ k \in \{1,2,...,6\}$, $i = 1,2,3$

- Subsystem reliability lower bounds $R_{\min,i} = 0.99$ or $0.999 \quad \forall i = 1,2,3$
- Feasibility factor $f_{ki} = 0.9$ and 0.7. (Lower values were ignored due to the very high cost values associated with them, suggesting that reliability improvements at those levels may be practically impossible).

7.2.2 **Results and Discussion**

Tables 7.1 and 7.2 detail the output for ten iterations involving varying the weight and the feasibility factors. The tables (and subsequent ones in the Chapter) are formatted in the same way as those presented and discussed in Chapter Six.

Table 7.1 Solution output to Billinton & Allan's example for a feasibility factor of 0.9

IT	1	2	3	4	5	6	7	8	9	10
FF	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
W1	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W2	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W3	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W4	0.0168	0.0154	0.0139	0.0124	0.0108	0.0092	0.0075	0.0057	0.0039	0.002
R1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
R2	0.9813	0.9813	0.9813	0.9813	0.9813	0.9813	0.9813	0.9813	0.9813	0.9813
R3	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**
R4	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**
R5	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**	**0.8936**
R6	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746
R7	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746	0.9746
R8	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816
RSB1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
RSB2	0.999003	0.999003	0.999003	0.999003	0.999003	0.999003	0.999003	0.999003	0.999003	0.999003
RSB3	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994
Rs	0.989007	0.989007	0.989007	0.989007	0.989007	0.989007	0.989007	0.989007	0.989007	0.989007
Cs	312.755	312.755	312.755	312.755	312.755	312.755	312.755	312.755	312.755	312.755

Table 7.1 shows that varying the weighting produced no changes in the component reliabilities across the 10 iterations and hence none also in those of the subsystems and system. The components' reliabilities achieved higher levels than the initial value of 0.8 after optimisation with component 1 attaining the maximum value of 0.99 throughout. This is not unexpected since this one (also a subsystem in this case) is very crucial to the system reliability; the whole system would fail if it fails; this is recognised by the optimisation algorithm and thus it assigns it the

highest reliability value. The process also results in the same reliability value being assigned to components 3, 4, and 5 which are in redundant arrangement. This fact (replicated in the results for the examples of Chapter Six) supports the notion which is also established rigorously by Elegbede et al (2003) that for series-parallel systems a necessary condition for the optimal assignment of reliability to redundant components is that they are identical. These three also had the least improvement, indicating that the higher the number of redundant components in a system (or subsystem) the lower their reliability levels need to be to attain the specified system reliability. The overall system reliability of 0.989007 represents a 49% improvement over the initial value of 0.6635 corresponding to component reliability values of 0.8. The corresponding cost value for the improvement was 312.76. Similar results were obtained for the FF value of 0.7 as seen in Table 7.2. The various weightings this time produced some marginal differences in the values of the component reliabilities achieved. Component 1 again attained the maximum permissible value of 0.99, and the same percentage rise (49%) in system reliability was recorded. The cost rose sharply with the drop in the FF value from 0.9 to 0.7, indicating the very strong correlation between the two. Component 2 experienced a slight drop in reliability while that of the three redundant ones (i.e. 3, 4, and 5) appreciated by a relatively higher margin. The improvements in R_6, R_7 and R_8 were negligible. The same was true for the system reliability. That the net effect of the modifications achieved in the component reliability values had no measurable effect on the system reliability as previously recorded when the FF value was 0.9, and yet yielded an extremely high cost in comparison, is indication of the very strong sensitivity of the cost function to the FF value in this problem, suggesting that the pursuit of higher component reliability values- not to mention at lower FF values - is cost prohibitive.

Table 7.2: Solution output to Billinton & Allan's example for a feasibility factor of 0.7

IT	1	2	3	4	5	6	7	8	9	10
FF	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
W1	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W2	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W3	0.3277	0.3282	0.3287	0.3292	0.3297	0.3303	0.3308	0.3314	0.332	0.3327
W4	0.0168	0.0154	0.0139	0.0124	0.0108	0.0092	0.0075	0.0057	0.0039	0.002
R1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
R2	0.9796	0.9796	0.9796	0.9796	0.9795	0.9796	0.9796	0.9796	0.9796	0.9796
R3	**0.9208**	**0.9208**	**0.9208**	**0.9215**	**0.9241**	**0.9208**	**0.9208**	**0.9208**	**0.9208**	**0.9208**
R4	**0.9208**	**0.9208**	**0.9208**	**0.9215**	**0.9203**	**0.9208**	**0.9208**	**0.9208**	**0.9208**	**0.9208**
R5	**0.9208**	**0.9208**	**0.9208**	**0.9198**	**0.9224**	**0.9208**	**0.9208**	**0.9208**	**0.9208**	**0.9208**
R6	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758
R7	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758	0.9758
R8	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816	0.9816
RSB1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
RSB2	0.999001	0.999001	0.999	0.999001	0.998998	0.999001	0.999001	0.999001	0.999001	0.999001
RSB3	0.999994	0.999994	0.99999	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994	0.999994
Rs	0.989005	0.989005	0.98901	0.989005	0.989002	0.989005	0.989005	0.989005	0.989005	0.989005
Cs	12416298	12416298	1.2E+07	12416298	12416234	12416298	12416298	12416298	12416298	12416298

7.3 **A LIFE-SUPPORT SYSTEM MODEL**

7.3.1 **Introduction**

This application concerns the reliability design of a life-support system (LSS) in a space capsule (Shelokar et al, 2002; Salazar et al, 2006); its configuration is presented in Figure 7.2. The system, which requires a single path for its success, has two redundant subsystems each comprising components 1 and 4. Each of the redundant subsystems is in series with component 2 and the resultant pair of series-parallel arrangement forms two equal paths. Component 3 is inserted as a third path and backup for the pair.

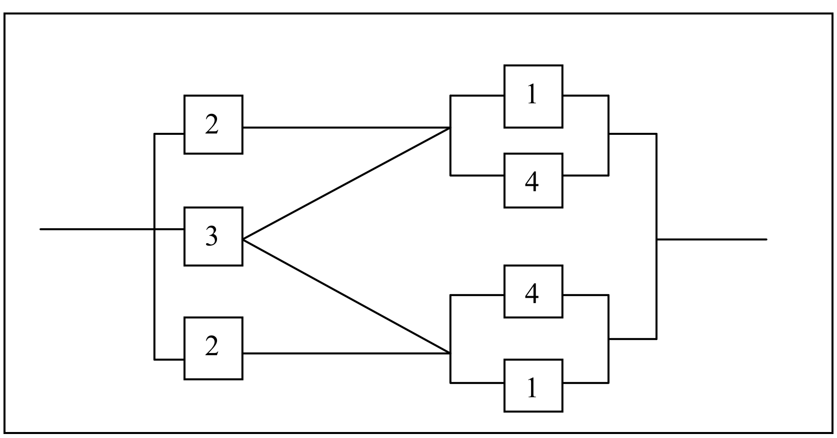

Figure 7.2: RBD of a life-support system in a space capsule (Source: Sheloker et al, 2002)

The continuous optimisation models that were originally formulated for the reliability design of this system approached the problem in two different ways: Sheloker et al (2002) adopted a single criterion methodology in which a cost function of component reliability was minimised, subject to constraints on system and components' reliabilities. On the other hand Salazar et al (2006) used a bi-criterion approach using a number of heuristic algorithms such as ACO, TS, and NSGA-2. A cost function of component reliability and the top level system reliability were respectively minimised and maximised, subject to constraints on the component reliabilities. In order to apply the MCO model the LSS configuration had to be transformed into a series-parallel format. Results of the application were then compared with those reported by Sheloker et al (2002) and Salazar et al (2006).

7.3.2 **Model Development**

The cut sets associated with the LSS are determined by inspection (see Appendix B for a discussion of techniques for determining cut sets) as follows: {3, k, k}, {2, 3, 2}, {2, 3, k}, {2, 3, k}, {2, k, k}, and {2, k, k} where k = {1, 4}. Since each subset {k, k} is itself a cut set the minimum cut sets associated with the system are: {1,4,1,4}, {2,3,2}, {2,3,1,4} and {2,3,1,4}. The minimum cut sets obtained suggest that the LSS may be extremely reliable, since all but one of its minimum cut sets are fourth order and thus could have rare failure events (Espiritu et al 2007). The results from the optimisations are therefore expected to be consistent with this observation. The series-parallel transformation of the system is shown in Figure 7.3.

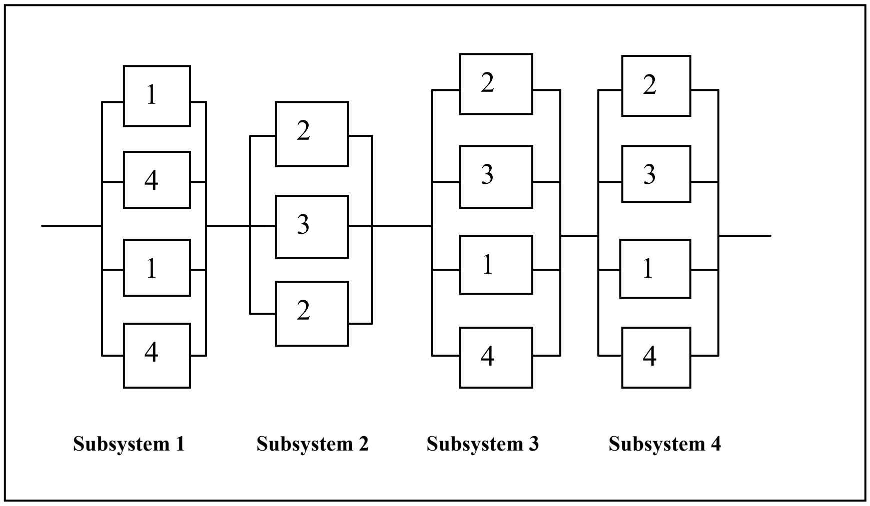

Figure 7.3: Series-parallel transformation of the configuration of LSS

The corresponding reliability expressions for each of the four derived subsystems and the associated cost function follow immediately from expressions 5.7, 5.10 and 5.3 respectively. The input parameters are the same as those used in the Scenarios of Chapter Six. The initial component reliability values of 0.5 are in accordance with those used by Sheloker et al (2002) and Salazar et al (2006).

7.3.3 **Results and Discussion**

The results of the optimisation process over ten iterations are presented in Tables 7.3 to 7.6. They include the Pareto optimal component reliability values and the corresponding subsystems and system reliabilities, as well as the cost values; the input weights are the same as those used in the previous Chapter for testing the MCO model.

As can be seen, significant improvements in reliability values were achieved for all the components over all the iterations for each of the FF values. However the values for components 1 and 4 remained at 0.8222 throughout while those for components 2 and 3 did vary a little. This resulted in a reasonably consistent system reliability value of approximately 0.998. This is a further indication of the stability of the MCO model under the weighting scheme adopted. Component 2 in almost all cases was assigned the highest reliability value while Component 3 was assigned the lowest, a situation that could be attributed to the relative importance (reliability wise) of the two components: note that the two components of type 2 occur in the third order cut set the failure of which is more likely than the others. The fact that both components 1 and 4 were assigned the same reliability value of 0.8222 is again consistent with the observation made by Elegbede et al (2003) on redundant components. Even though the improvements in the component reliability values relative to their initial ones were significant, they were not very high in absolute terms (especially those for components 1, 3, and 4). That the resultant system reliability was nevertheless high shows that the current configuration of the LSS can guarantee very high system reliability even with moderately reliable components. This fact is reflected in the cost values which are very low compared to those recorded in the last example, for instance.

Table 7.3: Results of the optimisations for the LSS MCO model for a feasibility factor value of 0.9

IT	1	2	3	4	5	6	7	8	9	10
FF	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
W1	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W2	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W3	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W4	0.0127	0.0116	0.0105	0.0093	0.0081	0.0069	0.0056	0.0043	0.0029	0.0015
R1	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
R2	0.9305	0.9305	0.9304	0.9303	0.9302	0.9301	0.9298	0.9294	0.9286	0.9264
R3	0.7927	0.793	0.7934	0.794	0.7947	0.7956	0.7971	0.9793	0.8038	0.8152
R4	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
RSB1	0.999	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001
RSB2	0.999	0.999	0.999	0.999	0.999	0.999001	0.999	0.9999	0.999	0.999
RSB3	0.99954	0.999545	0.999545	0.999546	0.999547	0.999548	0.99955	0.999954	0.999557	0.99957
RSB4	0.99954	0.999545	0.999545	0.999546	0.999547	0.999548	0.99955	0.999954	0.999557	0.99957
Rs	0.99709	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
Cs	9.70152	9.701922	9.698486	9.695342	9.692357	9.689674	9.680079	19.89911	9.644481	9.585029

Table 7.4: Results of the optimisations for the LSS MCO model for a feasibility factor value of 0.7

IT	1	2	3	4	5	6	7	8	9	10
FF	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
W1	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W2	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W3	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W4	0.0127	0.0116	0.0105	0.0093	0.0081	0.0069	0.0056	0.0043	0.0029	0.0015
R1	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
R2	0.9311	0.9311	0.9311	0.931	0.931	0.931	0.9309	0.9309	0.9307	0.9303
R3	0.7895	0.7896	0.7897	0.7898	0.7899	0.7901	0.7904	0.7908	0.7918	0.7944
R4	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
RSB1	0.999	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001
RSB2	0.999	0.999001	0.999002	0.999	0.999	0.999001	0.999	0.999001	0.999	0.999001
RSB3	0.99954	0.999542	0.999542	0.999541	0.999542	0.999542	0.999542	0.999543	0.999544	0.999547
RSB4	0.99954	0.999542	0.999542	0.999541	0.999542	0.999542	0.999542	0.999543	0.999544	0.999547
Rs	0.99709	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
Cs	21.859	21.85948	21.86	21.81703	21.81755	21.81859	21.77693	21.77902	21.69866	21.54456

Table 7.5: Results of the optimisations for the LSS MCO model for a feasibility factor value of 0.5

IT	1	2	3	4	5	6	7	8	9	10
FF	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
W1	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W2	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W3	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W4	0.0127	0.0116	0.0105	0.0093	0.0081	0.0069	0.0056	0.0043	0.0029	0.0015
R1	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
R2	0.9312	0.9312	0.9312	0.9312	0.9311	0.9311	0.9311	0.9311	0.931	0.9309
R3	0.7889	0.7889	0.789	0.789	0.789	0.7891	0.7892	0.7894	0.7897	0.77907
R4	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
RSB1	0.999	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001
RSB2	0.999	0.999001	0.999001	0.999001	0.999	0.999	0.999	0.999	0.999	0.999
RSB3	0.99954	0.999541	0.999541	0.999541	0.99954	0.999541	0.999541	0.999541	0.999541	0.999517
RSB4	0.99954	0.999541	0.999541	0.999541	0.99954	0.999541	0.999541	0.999541	0.999541	0.999517
Rs	0.99709	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
Cs	60.0287	60.02868	60.0298	60.0298	59.76987	59.771	59.77213	59.77439	59.52003	59.15255

Table 7.6: Results of the optimisations for the LSS MCO model for a feasibility factor value of 0.3

IT	1	2	3	4	5	6	7	8	9	10
FF	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
W1	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W2	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W3	0.2468	0.2471	0.2474	0.2477	0.248	0.2483	0.2486	0.2489	0.2493	0.2496
W4	0.0127	0.0116	0.0105	0.0093	0.0081	0.0069	0.0056	0.0043	0.0029	0.0015
R1	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
R2	0.9312	0.9312	0.9312	0.9312	0.9312	0.9312	0.9312	0.9312	0.9312	0.9311
R3	0.7887	0.7887	0.7887	0.7887	0.7887	0.7888	0.7888	0.7889	0.789	0.7895
R4	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222	0.8222
RSB1	0.999	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001	0.999001
RSB2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999001	0.999001	0.999001
RSB3	0.99954	0.99954	0.99954	0.99954	0.99954	0.999541	0.999541	0.999541	0.999541	0.999542
RSB4	0.99954	0.99954	0.99954	0.99954	0.99954	0.999541	0.999541	0.999541	0.999541	0.999542
Rs	0.99708	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
Cs	188.521	188.5213	188.5213	188.5213	188.5213	188.5234	188.5234	188.5254	188.5275	187.241

In a further attempt to investigate the appropriateness of the MCO model to this application it was decided to compare these results with those presented by Sheloker et al (2002) and Salazar et al (2006). While the former discuss the results of their original SCO models the latter compare the SCO results of the problem discussed earlier by Ravi et al (1997) and Rocco et al (2000) with that of their bi-criterion version which was presented graphically. Consequently a quantitative comparison with the bi-criterion version is rendered impossible. The comparison is therefore limited to the results of the former and those of Ravi et al (1997) and Rocco et al (2000) as presented by the latter. It is also limited to the reliability values of both the components and the system. The cost values are excluded because of the different cost function models involved, which do not provide a proper basis for cost comparison (the cost functions are in Appendix C). The values corresponding to the 10th IT where the highest and lowest weights' assignments occur were selected and the optimisation results are presented in Table 7.7.

Table 7.7: Reliability values of the MCO model and those from earlier published work obtained for the LSS

	MCO Model				Sheloker et al (2002)		Salazar et al (2006)	
	F = 0.9	**FF = 0.7**	**FF = 0.5**	**FF = 0.3**	**Case1**	**Case 2**	**Rocco et al**	**Ravi et al**
R1	0.8222	0.8222	0.8222	0.8222	0.500000	0.825895	0.500000009	0.50006
R2	0.9264	0.9303	0.9309	0.9311	0.83892	0.890089	0.838920148	0.83887
R3	0.8152	0.7944	0.779	0.7895	0.500000	0.627426	0.500000011	0.50001
R4	0.8222	0.8222	0.8222	0.8222	0.500000	0.728794	0.500000022	0.50002
Rs	0.998004	0.998004	0.998004	0.998004	0.900000	0.990000	0.900000619	0.90001

It can be seen that the MCO model yielded the best component reliability values throughout, except in case 2 of Sheloker et al where component 1 had the highest value. The corresponding system reliability values were the highest throughout. The similarity of the patterns, in all the cited cases with the MCO ones, is interesting. For example, in all cases component 2 was assigned the highest reliability and component 3 the least, an observation which has already drawn comment in this discussion. One cannot fail to notice also the equal (or almost equal) reliability values assigned to components 1 and 4 in all cases except one - a phenomenon which has already been noted. These observations are positive indicators of the ability of the MCO model to provide very high specifications in reliability design.

7.4 BREAKER-AND-A-HALF SYSTEM MODEL

7.4.1 Introduction

The Electricity Transmission System is a vital part of the entire network which ensures that power is transmitted from the generation sources to consumers in homes and industry. A Breaker-And-A-Half (BH) is one of many important components or subsystems of the transmission system the reliability of which ensures that there is no interruption in power to consumers. The BH which is a redundant and complex system provides an application for the MCO model. The example is extracted from a specific one of the BH system discussed by Espiritu et al (2007) in the context of component criticality measures. A schematic of the functional form of the system is reproduced in Figure 7.4.

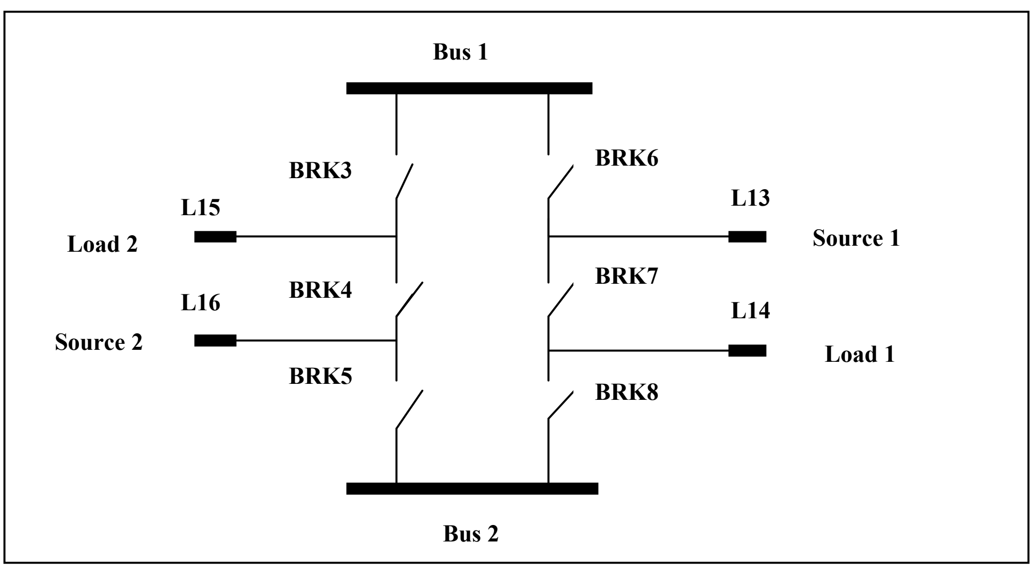

Figure 7.4: Breaker-and-a-half configuration (Source: Espiritu et al (2007))

The components of the BH are the two Buses, the six breakers (BRK 3-8) and the supply and load lines L13 and L16, and L14 and L15 respectively. Espiritu et al (2007) have analysed this system's failure characteristics in terms of cut sets, assuming failure at Load 1 as the top event. The resultant twenty one minimum cut sets (which excludes component L15 in all the instances, since it is assumed that failure occurs only at load 1 (L14)) are listed in Table 7.8. Fourth order cut sets and higher were ignored on the grounds that their failure events were rare. The cut sets form the basis for the application of the MCO model.

Table 7.8: Minimum cut sets for the BH system for failure at Load 1 (Source: Espiritu et al (2007))

{L14}	{L13, BRK 4, BRK 5}	{L13, BRK 3, BRK 8}
{BRK 7, BRK 8}	{L13, BRK 4, BUS 2}	{{L13, BUS 1, BRK 5}
{BRK7, BRK 5}	(L13, BRK 4, BRK 8}	{L13, BUS 1, BUS 2}
{BRK 7, BUS 2}	{L13, BRK3, BRK 5}	{L13, BUS 1, BRK 8}
{L13, L16}	{L13, BRK 3, BUS 2}	{L13, BRK 5, BRK 6}
{L16, BRK 6, BRK 7}	{L16, BRK 3, BRK 7}	{L13, BUS 2, BRK 6}
{L16, BUS 1, BRK 7}	{L16, BRK 4, BRK 7}	{L13, BRK 8, BRK 6}

7.4.2 **Model Development**

The model thus has twenty two criteria, twenty one of which are subsystem reliabilities to be maximised and one of which is the associated cost to be minimised.

Assumptions

- The components' outage rates as given by Espiritu et al (2007) are their failure rates
- Even though the BH system is repairable, the components are not. Thus a component's time to failure and failure frequency is assumed to be devoid of maintenance times and its effects.
- Component failures are independent
- Component failures are Exponentially distributed

- The initial component reliability is the reliability value obtained from its failure rate information

Parameters

The initial reliability of many of the components (see Table 7.9) was high so the maximum component reliability was set at an upper bound of 1. Since previous results have consistently shown the MCO model to be quite stable with respect to the weighting scheme being used, only a single weight vector was used. The subsystem reliabilities were assigned weights of 0.0476 and the cost function a weight of 0.0005. This means that a preference oriented approach was adopted where the resultant single weight vector characterised the preference. The subsystem lower bounds were fixed at 0.99. Solutions were obtained for the same set of FF values as used previously.

7.4.3 **Results and Discussion**

The results of the optimisations are presented in Table 7.9. In addition to the main components of the BH system, their failure (outage) rates, their initial reliability and their reliability values achieved under the optimisations for FF values of 0.9, 0.7, 0.5, and 0.3, the Table also shows the resultant system level reliabilities and the associated costs.

Table 7.9: Results of the optimisations for the BH system

COMPONENT TYPE	OUTAGE RATE/YR	INITIAL RELIABILITY	RELIABILITY ACHIEVED	RELIABILITY ACHIEVED	RELIABILITY ACHIEVED	RELIABILITY ACHIEVED
			FF = 0.9	FF = 0.7	FF = 0.5	FF = 0.3
L14	0.86	**0.4232**	**0.99**	**0.99**	**0.99**	**0.99**
BRK 7	0.034	0.9666	0.9855	0.977	0.9722	0.9685
BRK 8	0.056	0.9455	0.962	0.9477	0.9459	0.9455
BRK 5	0.023	0.9773	0.9773	0.9773	0.9773	0.9775
BUS 2	0.18	0.8353	0.8999	0.8875	0.872	0.8661
L13	0.93	**0.3946**	**0.9194**	**0.8996**	**0.9009**	**0.9065**
L16	0.88	**0.4148**	**0.8895**	**0.9004**	**0.8991**	**0.893**
BRK 6	0.07	0.9324	0.9417	0.9324	0.9324	0.9324
BUS 1	0.2	0.8187	0.8443	0.8353	0.8312	0.8267
BRK 4	0.076	0.9268	0.9381	0.9268	0.9268	0.9268
BRK 3	0.09	0.9139	0.929	0.9139	0.9139	0.9139
SYSTEM RELIABILITY		0.237843	0.947601	0.92706	0.916674	0.909293
COST OF RELIABILITY			301.2323	1.60E+08	4.70E+13	1.40E+19

As has already been stated, apart from components L14, L13, and L16, which had rather low initial reliabilities, all the other components had appreciably high values. The consequence of the low initial component reliabilities is reflected in the very low initial system reliability of 0.237843. The optimisation algorithm therefore assigned very high reliability values to those with initially low ones, and only barely increased those with very high ones. The system reliability improved to for instance, 0.947601 (in the case of FF = 0.9) with an associated cost of 301.23. Similar system reliability results were recorded for the other FF values, however their associated cost values were prohibitively high. This is not surprising given the drastic improvements in reliability required for components L14, L13 and L16 and also because of the characteristic difficulty associated with improving (however marginally) the reliability of components with already very high reliability values. Component L14 recorded the highest reliability improvement, which is consistent with its first order cut set status (see Table 7.8), while no improvement was obtained for BRK 5 in practically all the cases.

The results of this application have shown that the reliability of the BH system can be appreciably improved by increasing the reliability of components L14 (first and foremost), and also those of L13 and L16. Indeed, in Espiritu et al (2007), component L14 received the highest ranking in all the five metrics used to evaluate the relative importance, in reliability terms, of the components of the BH system. Component L13 had the second highest ranking in two of the metrics; component L16 had the third in two of the metrics and the fifth in the others. These observations are consistent with the results obtained in this application and serve as further confirmation that the MCO model works and can provide very useful information on the reliability levels of the components which would yield a desired system level reliability.

7.5 DISTRICT GOVERNOR INSTALLATION MODEL

7.5.1 Introduction

Having tested the MCO model on various sets of hypothetical and published data it was felt necessary and worthwhile to apply it to an existing industrial operational system. Consequently twenty companies (which were thought to be appropriate for this) were selected and contacted; only one indicated an interest. The Company (which is situated in the Midlands) operate a gas supply system.

At the outset a meeting was arranged between three engineers of the Company in the reliability field and the author, at which a formal presentation (in PowerPoint) was made of the main features of the model and its capabilities. This was followed by discussions of the specific applications available within the Company and the data requirements, etc. Subsequently the District Governor Installation (DGI) was selected and the necessary data supplied.

7.5.2 Brief Background

The DGI (shown schematically in Figure 7.5) is a gas carrying system consisting of two independent but connected streams, (i.e. the working and the standby) the function of which is to reduce the pressure of gas from 7bars at the inlet of the installation to approximately 50mbars at the outlet. The standby stream is expected to automatically supply gas upon failure of the working one. The components of both working and standby streams are the inlet, outlet and non-return valves, the relief valves, the filters, slam shuts, and monitor and active regulators. Each of the regulators exhibits two failure modes - failing open or closed - with the former being the most dangerous, as high pressure gas will pass downstream through low pressure systems, with the likelihood of leakage or rupture. On the other hand, where one or both streams fail closed, gas supply downstream is interrupted, resulting in loss of gas supply. The application and analysis were centred on this failure mode.

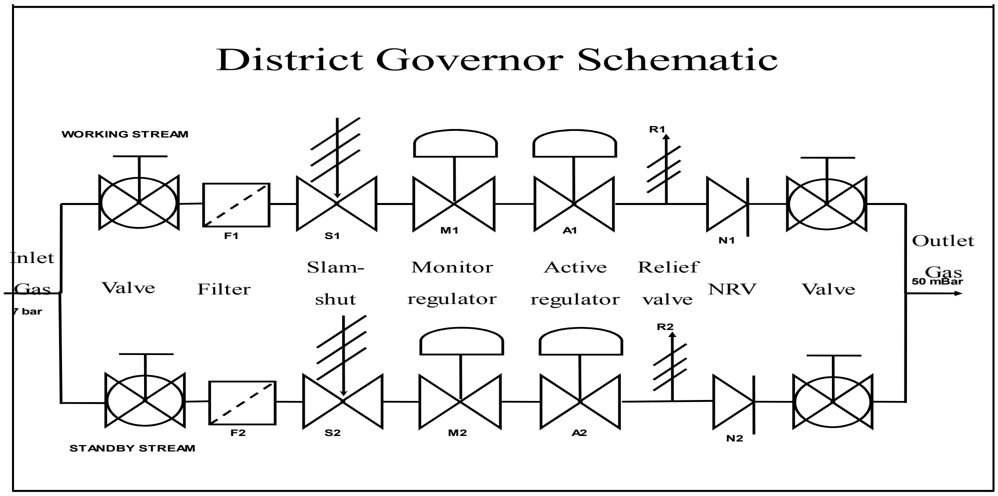

Figure 7.5: District Governor Installation (Source: Company drawing)

The DGI is a redundant system (at the system instead of component level) involving both the working and standby streams, nevertheless in reliability terms it is complexly configured (several of its components exhibit two failure modes) and not just a simple parallel-series system. Consequently the FT modelling approach was used to provide a better and easier way to analyse the system for reliability.

7.5.3 **The Data**

The full set of relevant data for the implementation was secured over approximately a one month period, during which several follow-ups were made on the initial data collected and missing ones secured. The data included: the schematic diagram shown in Figure 7.5; a FT diagram of the DGI in which the top event was gas supply failure downstream and an analysis of it in terms of cut sets (this can be seen in Appendix D); the components' failure rates and the components' costs in £s. Table 7.10 lists the latter data for each of the components for both the working and standby streams. The components with dual failure modes (i.e. failing open or closed) are denoted by letters "o" or "c" respectively in parentheses as part of their code. Apart from the Filters (F1 and F2) all the other components have at least one of the characteristics described. While the slam shut valves only fail closed all the remaining components fail either open or closed, with different failure rates (except for the Non-Return Valve in the Standby Stream). The data in the cost column gives the unit cost of each component.

Table 7.10: Part of the data supplied for the DGI

Stream	Component	Code	Failure rate	Cost (£k)
Working	Filter	F1	7.91E-04	2
	Slam shut Valve	S1(c)	1.35E-04	2
	Active Regulator	A1(o)	2.57E-03	2
	Active Regulator	A1(c)	9.88E-05	
	Monitor Regulator	M1(o)	2.99E-03	2
	Monitor Regulator	M1(c)	3.58E-04	
	Relief Valve	R1(c)	1.66E-03	0.5
	Non-Return Valve	N1(o)	3.95E-04	1
	Non-Return Valve	N1(c)	9.09E-05	
Standby	Filter	F2	2.33E-04	2
	Slam shut Valve	S2(c)	6.99E-05	2
	Active Regulator	A2(o)	9.30E-04	2
	Active Regulator	A2(c)	1.16E-04	
	Monitor Regulator	M2(o)	1.50E-03	2
	Monitor Regulator	M2(c)	4.62E-04	
	Relief Valve	R2(c)	1.94E-03	0.5
	Non-Return Valve	N2(o)	2.07E-04	1
	Non-Return Valve	N2(c)	2.07E-04	

7.5.4 **Model Development**

Forty minimum cuts sets, resulting from the FT analysis provided by the company are shown in Table 7.11. Twenty five are second order, fourteen are third and one is fourth.

Table 7.11: Minimum cut sets of the DGI failing closed

A1(c), A2(c)	S1, N2(c)	N1(c), S2	F1, F2	M1(c), A2(o), M2(o)
A1(c),S2	S1, F2	N1(c), M2(c)	A1(o), M1(o), A2(c)	N1(c), A2(o), M2(o)
A1(c),M2(c)	M1(c), A2(c)	N1(c), N2(c)	A1(o), M1(o), S2	F1, A2(o), M2(o)
A1(c), N2(c)	M1(c), S2	N1(c), F2	A1(o), M1(o), M2(c)	A1(o), M1(o), N2(o)
A1(c), F2	M1(c), M2(c)	F1, A2(c)	A1(o), M1(o), N2(c)	A1(o), M1(o), R2
S1, A2(c)	M1(c),N2(c)	F1, S2	A1(o), M1(o), F2	A2(o), M2(o), N1(o)
S1, S2	M1(c), F2	F1, M2(c)	A1(o), A2(o), M2(o)	A2(o), M2(o), R1
S1, M2(c)	N1(c), A2(c)	F1, N2(c)	S1, A2(o), M2(o)	A1(o), M1(o), A2(o), M2(o)

The reliability of the DGI can thus be approximated by a series-parallel system whose subsystems are derived from these cut sets. The MCO model thus follows immediately, where the reliability of each minimum cut set is maximised simultaneously and the corresponding cost function minimised. The decision vector in this case comprised eighteen variables (i.e. $[R_1, R_2,...R_{18}]^T$) each R_i (i = 1,2, ..., 18) corresponding respectively to the reliability of the components as listed in Table 7.10. It must be noted that the reliabilities of components with dual failure modes appear twice in this vector. The purpose of the optimisation at this stage was to find the (Pareto) optimal values of the variables which yielded maximum reliability values of the cut sets at minimum cost. Subsequently an equivalent reduced form of the decision vector, involving twelve variables instead of eighteen, which represents the reliability of each component as a single variable, was derived. Components with dual failure modes were thus assigned a generic reliability value which facilitated the evaluation of a corresponding generic system level reliability estimate.

Parameters

Once again given that the MCO model has proven to be stable under the weighting system, a single preference weight vector was generated as input to the optimisation. In this vector the minimum cut sets' reliabilities were each assigned a weight of 0.9674 and the cost function a weight of 0.0326. Only a single FF value of 0.9 was considered in this case, since the lower levels yielded similar (as far as the system reliability was concerned) or infeasible solutions; this was primarily due to the rather high initial reliability of the components obtained from their failure rate information, which ultimately imply a severely diminished search space. To ensure that solutions were nevertheless obtained in the optimisations, and noting that the MCO model was very sensitive to subsystem reliability lower bounds, these were set very high and varied between 0.99999 and 0.999999. For the same reason the maximum component reliability was set at 1.The unit cost data provided a means for quantifying the actual cost (£) of improvement in reliability using the technique developed in Chapter Five.

Assumptions

The assumptions inherent in the MCO model were applied as follows:

- The state of the DGI is binary (i.e. it is either failed or operational)
- The parameters of the model are precisely determined (i.e. parameter variations are ignored)
- The DGI is non-repairable under the current optimisations and for its mission time
- The components' failure rates are Exponentially distributed

7.5.5 **Results and Discussion**

Tables 7.12 to 7.16 show five Pareto optimal solutions (obtained for subsystem reliability LBs of 0.99999, 0.999995, 0.999997, 0.999998, and 0.999999 respectively) which were considered the most preferable, on the basis of the reliability values assigned to the components of the DGI. In each table the first column lists the components of both the working and the standby streams in terms of their specific codes. The initial or original reliability of each of the components is given in the second. The third shows their values obtained after optimisation. The fourth and fifth columns respectively show the difference between the initial reliability and that after optimisation and their percentage improvements

Table 7.12 shows that the level of reliability improvement for the components ranged from 0% to approximately 10%, corresponding to a 3.95% improvement (resulting in 0.99971) in the system reliability estimate which initially was 0.99578. Similar ranges were achieved for the component reliabilities after the optimisation in Tables 7.13 to 7.16, resulting in similar improvements in the system reliability equivalent to 0.99985, 0.99991, 0.99994, and 0.99997 respectively.

Table 7.12: Pareto optimal solution of the DGI model with RSBs of 0.99999

COMPONENT	INITIAL RELIABILITY	PARETO OPTIMAL RELIABILITY	RELIABILITY IMPROVEMENT	PERCENTAGE IMPROVEMENT
F1	0.96886	0.9962	0.02734	2.822
S1(C)	0.99461	0.9962	0.00159	0.159
A1(O)	0.90231	0.9775	0.07519	8.333
A1(C)	0.99606	0.9962	0.00014	0.014
M1(O)	0.88728	0.9741	0.08682	9.785
M1(C)	0.98578	0.9962	0.01042	1.057
R1(C)	0.93576	0.9731	0.03734	3.990
N1(O)	0.9843	0.9843	0	0
N1(C)	0.9964	0.9964	0	0
F2	0.99072	0.9974	0.00668	0.674
S2(C)	0.99721	0.9974	0.00019	0.019
A2(O)	0.96348	0.9847	0.02122	2.202
A2(C)	0.99537	0.9974	0.00203	0.204
M2(O)	0.94176	0.9756	0.03384	3.593
M2(C)	0.98169	0.9974	0.01571	1.600
R2(C)	0.92533	0.9828	0.05747	6.211
N2(O)	0.9918	0.9918	0	0
N2(C)	0.99175	0.9974	0.00565	0.569

Table 7.13: Pareto optimal solution of the DGI model with RSBs of 0.999995

COMPONENT	INITIAL RELIABILITY	PARETO OPTIMAL RELIABILITY	RELIABILITY IMPROVEMENT	PERCENTAGE IMPROVEMENT
F1	0.96886	0.9972	0.02834	2.925
S1(C)	0.99461	0.9972	0.00259	0.260
A1(O)	0.90231	0.9822	0.07989	8.854
A1(C)	0.99606	0.9972	0.00114	0.114
M1(O)	0.88728	0.9794	0.09212	10.382
M1(C)	0.98578	0.9972	0.01142	1.158
R1(C)	0.93576	0.9787	0.04294	4.589
N1(O)	0.9843	0.9843	0	0
N1(C)	0.99637	0.9972	0.00083	0.083
F2	0.99072	0.9982	0.00748	0.755
S2(C)	0.99721	0.9982	0.00099	0.099
A2(O)	0.96348	0.9879	0.02442	2.535
A2(C)	0.99537	0.9982	0.00283	0.284
M2(O)	0.94176	0.9807	0.03894	4.135
M2(C)	0.98169	0.9982	0.01651	1.682
R2(C)	0.92533	0.9864	0.06107	6.599
N2(O)	0.9918	0.9918	0	0
N2(C)	0.99175	0.9982	0.00645	0.650

Table 7.14: Pareto optimal solution of the DGI model with RSBs of 0.999997

COMPONENT	INITIAL RELIABILITY	PARETO OPTIMAL RELIABILITY	RELIABILITY IMPROVEMENT	PERCENTAGE IMPROVEMENT
F1	0.96886	0.9978	0.02894	2.987
S1(C)	0.99461	0.9978	0.00319	0.321
A1(O)	0.90231	0.985	0.08269	9.164
A1(C)	0.99606	0.9978	0.00174	0.175
M1(O)	0.88728	0.9826	0.09532	10.743
M1(C)	0.98578	0.9978	0.01202	1.219
R1(C)	0.93576	0.982	0.04624	4.941
N1(O)	0.9843	0.9843	0	0
N1(C)	0.99637	0.9978	0.00143	0.143
F2	0.99072	0.9986	0.00788	0.796
S2(C)	0.99721	0.9986	0.00139	0.139
A2(O)	0.96348	0.9898	0.02632	2.732
A2(C)	0.99537	0.9986	0.00323	0.325
M2(O)	0.94176	0.9837	0.04194	4.453
M2(C)	0.98169	0.9986	0.01691	1.722
R2(C)	0.92533	0.9885	0.06317	6.827
N2(O)	0.99175	0.9918	0	0
N2(C)	0.99175	0.9986	0.00685	0.691

Table 7.15: Pareto optimal solution of the DGI model with RSBs of 0.999998

COMPONENT	INITIAL RELIABILITY	PARETO OPTIMAL RELIABILITY	RELIABILITY IMPROVEMENT	PERCENTAGE IMPROVEMENT
F1	0.96886	0.9982	0.02934	3.028
S1(C)	0.99461	0.9982	0.00359	0.361
A1(O)	0.90231	0.9869	0.08459	9.375
A1(C)	0.99606	0.9982	0.00214	0.215
M1(O)	0.88728	0.9848	0.09752	10.991
M1(C)	0.98578	0.9982	0.01242	1.259
R1(C)	0.93576	0.9843	0.04854	5.187
N1(O)	0.9843	0.9843	0	0
N1(C)	0.99637	0.9982	0.00183	0.184
F2	0.99072	0.9989	0.00818	0.826
S2(C)	0.99721	0.9989	0.00169	0.169
A2(O)	0.96348	0.9911	0.02762	2.867
A2(C)	0.99537	0.9989	0.00353	0.355
M2(O)	0.94176	0.9858	0.04404	4.676
M2(C)	0.98169	0.9989	0.01721	1.753
R2(C)	0.92533	0.99	0.06467	6.989
N2(O)	0.9918	0.9918	0	0
N2(C)	0.99175	0.9989	0.00715	0.721

Table 7.16: Pareto optimal solution of the DGI model with RSBs of 0.999999

COMPONENT	INITIAL RELIABILITY	PARETO OPTIMAL RELIABILITY	RELIABILITY IMPROVEMENT	PERCENTAGE IMPROVEMENT
F1	0.96886	0.9987	0.02984	3.079
S1(C)	0.99461	0.9987	0.00409	0.411
A1(O)	0.90231	0.9897	0.08739	9.685
A1(C)	0.99606	0.9987	0.00264	0.265
M1(O)	0.88728	0.9881	0.10082	11.362
M1(C)	0.98578	0.9987	0.01292	1.311
R1(C)	0.93576	0.9866	0.05084	5.433
N1(O)	0.98432	0.9866	0.00228	0.232
N1(C)	0.99637	0.9987	0.00233	0.233
F2	0.99072	0.9992	0.00848	0.856
S2(C)	0.99721	0.9992	0.00199	0.199
A2(O)	0.96348	0.9932	0.02972	3.085
A2(C)	0.99537	0.9992	0.00383	0.385
M2(O)	0.94176	0.9891	0.04734	5.027
M2(C)	0.98169	0.9992	0.01751	1.784
R2(C)	0.92533	0.9919	0.06657	7.194
N2(O)	0.99175	0.9919	0.00015	0.015
N2(C)	0.99175	0.9992	0.00745	0.751

Tables 7.17 to 7.21 present the results of the conversion of both the initial and the Pareto optimal component reliability values (designated by the columns labelled IR and PR respectively) into single reliability measures for those components which exhibit dual failure modes (identified as FO and FC, for those failing open and failing closed respectively) . This conversion (the process of which was discussed in Chapter Five) is necessary, especially for reliability design purposes where a top level estimate of system reliability is required for reliability improvement planning. Columns four and seven in the tables, record the single reliability measure for each of the components in terms of the initial and the Pareto optimal values respectively. The eighth column (labelled Cs) shows the cost/penalty levels corresponding to the Pareto optimal component reliability values. Since the unit costs (UC) for each component were available (listed in column 9 in £k) the monetary cost of reliability was evaluated and shown in the last column of each table. The costs associated with the system reliability improvement are also given in the last rows of the eighth and tenth columns.

It is evident from Tables 7.17 to 7.21 that the evaluated generic initial component reliability values were also very high. That their corresponding generic Pareto optimal values show only slight improvements over the initial, in spite of the 0.9 FF assumption is thus not surprising - most of the improvements occurred after the second or third decimal places. For instance, the component coded as N1 experienced no improvement at all (see Table 7.17) when the subsystem LBs were 0.99999, consequently its cost/penalty value was unity corresponding to a zero actual (monetary) cost. As the subsystem LBs were increased slightly, as occurs in the subsequent iterations, they yielded correspondingly slight reliability improvements in the component, thereby attracting a nonzero actual cost. This trend is also seen in respect of all the other components and a further pointer to the sensitivity of the MCO model to the subsystem reliability LBs. It is observed also that even though the component labelled M1 recorded the highest reliability improvement (in relative terms) its corresponding cost/penalty value was less than that of the one labelled F1 in all the iterations. A closer scrutiny however shows that that the generic initial reliability of M1 was much lower than that of F1 which was very high. Consequently it was going to be much more difficult (i.e. expensive) to improve that of the latter than that of the former. The tables show a steady rise of both the cost/penalty and actual monetary cost values for the entire set of components as the subsystem reliability LBs are varied iteratively, resulting in a corresponding steady rise in those values for the system (see the bottom row of the tables). One should also observe that the cost/penalty values were very low (in absolute terms) for both the components and the system. This is attributed to the assumption of very high feasibility factor (0.9) for the components as well as the maximum reliability of 1 set for them. The actual cost of reliability improvement for the system for all the iterations ranged from £23217.22 to £50495.63.

Table 7.17: Results of conversion of Pareto optimal solutions for RSBs of 0.99999

CODE	IR (FO)	IR (FC)	IR	PR (FO)	PR (FC)	PR	Cs	UC (£k)	C's (£k)
F1			**0.96886**			**0.9962**	2.053352	2	4.1067044
F2			**0.99072**			**0.9974**	1.292946	2	2.5858913
S1		0.99461	**0.99461**		0.9962	**0.9962**	1.04273	2	2.0854596
S2		0.99721	**0.99721**		0.9974	**0.9974**	1.007334	2	2.0146689
A1	0.90231	0.99606	**0.8987549**	0.9775	0.9962	**0.9737855**	1.331383	2	2.6627652
A2	0.96348	0.99537	**0.9590191**	0.9847	0.9974	**0.98213978**	1.138206	2	2.2764125
R1		0.93576	**0.93576**		0.9731	**0.9731**	1.148906	0.5	0.5744531
R2		0.92533	**0.92533**		0.9828	**0.9828**	1.396722	0.5	0.6983609
N1	0.98432	0.99637	**0.9807469**	0.9843	0.9964	**0.98075652**	1	1	0
N2	0.99175	0.99175	**0.9835681**	0.9918	0.9974	**0.98922132**	1.053848	1	1.0538483
M1	0.88728	0.98578	**0.8746629**	0.9741	0.9962	**0.97039842**	1.381837	2	2.7636736
M2	0.94176	0.98169	**0.9245164**	0.9756	0.9974	**0.97306344**	1.19749	2	2.3949793
AGGREGATE							**15.04475**		**23.217217**

Table 7.18: Results of conversion of Pareto optimal solutions for RSBs of 0.999995

CODE	IR (FO)	IR (FC)	IR	PR (FO)	PR (FC)	PR	Cs	UC (£k)	C's (£k)
F1			**0.96886**			**0.9972**	2.75149075	2	5.502981508
F2			**0.99072**			**0.9982**	1.51521229	2	3.030424583
S1		0.99461	**0.99461**		0.9972	**0.9972**	1.09691314	2	2.193826283
S2		0.99721	**0.99721**		0.9982	**0.9982**	1.05654061	2	2.113081229
A1	0.9023	0.99606	**0.8987549**	0.9822	0.9972	**0.97944984**	1.48093413	2	2.961868257
A2	0.9635	0.99537	**0.95901909**	0.9879	0.9982	**0.98612178**	1.21566275	2	2.431325494
R1		0.93576	**0.93576**		0.9787	**0.9787**	1.22335397	0.5	0.611676986
R2		0.92533	**0.92533**		0.9864	**0.9864**	1.56681378	0.5	0.78340689
N1	0.9843	0.99637	**0.98074692**	0.9843	0.9972	**0.98154396**	1.00432793	1	1.004327933
N2	0.9918	0.99175	**0.98356806**	0.9918	0.9982	**0.99001476**	1.066692	1	1.066691998
M1	0.8873	0.98578	**0.87466288**	0.9794	0.9972	**0.97665768**	1.54798222	2	3.09596444
M2	0.9418	0.98169	**0.92451637**	0.9807	0.9982	**0.97893474**	1.29476897	2	2.589537936
AGGREGATE							**16.8206925**		**27.38511354**

Table 7.19: Results of conversion of Pareto optimal solutions for RSBs of 0.999997

CODE	IR (FO)	IR (FC)	IR	PR (FO)	PR (FC)	PR	Cs	UC (£k)	C's (£k)
F1			**0.96886**			**0.9978**	3.72644	2	7.45288888
F2			**0.99072**			**0.9986**	1.75568	2	3.51136315
S1		0.99461	**0.99461**		0.9978	**0.9978**	1.15604	2	2.31207914
S2		0.99721	**0.99721**		0.9986	**0.9986**	1.10438	2	2.20876358
A1	0.90231	0.99606	**0.8987549**	0.985	0.9978	**0.982833**	1.63193	2	3.26386808
A2	0.96348	0.99537	**0.95901909**	0.9898	0.9986	**0.9884143**	1.28881	2	2.57761963
R1		0.93576	**0.93576**		0.982	**0.982**	1.2929	0.5	0.64645073
R2		0.92533	**0.92533**		0.9885	**0.9885**	1.73205	0.5	0.86602385
N1	0.98432	0.99637	**0.98074692**	0.9843	0.9978	**0.9821345**	1.0078	1	1.00779731
N2	0.99175	0.99175	**0.98356806**	0.9918	0.9986	**0.9904115**	1.07398	1	1.07397954
M1	0.88728	0.98578	**0.87466288**	0.9826	0.9978	**0.9804383**	1.71725	2	3.43450795
M2	0.94176	0.98169	**0.92451637**	0.9837	0.9986	**0.9823228**	1.38682	2	2.77363539
AGGREGATE							**18.8741**		**31.1289772**

Table 7.20: Results of conversion of Pareto optimal solutions for RSBs of 0.999998

CODE	IR (FO)	IR (FC)	IR	PR (FO)	PR (FC)	PR	Cs	UC (£k)	C's (£k)
F1			**0.96886**			**0.9982**	5.10387472	2	10.2077494
F2			**0.99072**			**0.9989**	2.10357097	2	4.20714194
S1		0.99461	**0.99461**		0.9982	**0.9982**	1.22072439	2	2.44144878
S2		0.99721	**0.99721**		0.9989	**0.9989**	1.16606679	2	2.33213357
A1	0.90231	0.99606	**0.8987549**	0.9869	0.9982	**0.9851236**	1.78706457	2	3.57412914
A2	0.96348	0.99537	**0.95901909**	0.9911	0.9989	**0.9900098**	1.36371245	2	2.72742489
R1		0.93576	**0.93576**		0.9843	**0.9843**	1.36229663	0.5	0.68114832
R2		0.92533	**0.92533**		0.99	**0.99**	1.90922996	0.5	0.95461498
N1	0.98432	0.99637	**0.98074692**	0.9843	0.9982	**0.9825283**	1.01024771	1	1.01024771
N2	0.99175	0.99175	**0.98356806**	0.9918	0.9989	**0.990709**	1.07988984	1	1.07988984
M1	0.88728	0.98578	**0.87466288**	0.9848	0.9982	**0.9830274**	1.8935733	2	3.78714661
M2	0.94176	0.98169	**0.92451637**	0.9858	0.9989	**0.9847156**	1.48269477	2	2.96538953
AGGREGATE							**21.4829461**		**35.9684647**

Table 7.21: Results of conversion of Pareto optimal solutions for RSBs of 0.999999

CODE	IR (FO)	IR (FC)	IR	PR (FO)	PR (FC)	PR	Cs	UC (£k)	C's (£k)
F1			**0.96886**			**0.9987**	9.928254	2	19.8565077
F2			**0.99072**			**0.9992**	2.886371	2	5.77274198
S1		0.99461	**0.99461**		0.9987	**0.9987**	1.369732	2	2.73946478
S2		0.99721	**0.99721**		0.9992	**0.9992**	1.282421	2	2.56484278
A1	0.90231	0.99606	**0.8987549**	0.9897	0.9987	**0.98841339**	2.168013	2	4.33602673
A2	0.96348	0.99537	**0.9590191**	0.9932	0.9992	**0.99240544**	1.5521	2	3.10419977
R1		0.93576	**0.93576**		0.9866	**0.9866**	1.461412	0.5	0.73070592
R2		0.92533	**0.92533**		0.9919	**0.9919**	2.274708	0.5	1.13735418
N1	0.98432	0.99637	**0.9807469**	0.9866	0.9987	**0.98531742**	1.031618	1	1.0316183
N2	0.99175	0.99175	**0.9835681**	0.9919	0.9992	**0.99110648**	1.088459	1	1.08845911
M1	0.88728	0.98578	**0.8746629**	0.9881	0.9987	**0.98681547**	2.341139	2	4.68227896
M2	0.94176	0.98169	**0.9245164**	0.9891	0.9992	**0.98830872**	1.725713	2	3.45142631
AGGREGATE							**29.1099**		**50.495626**

Table 7.22 provides a summary of the results obtained. It shows the system reliability estimates for each of the iterations and the corresponding actual cost measures in thousands of pounds. It is apparent from the table that only marginal improvements in the system reliability values were attained in the iterations; however these translated into fairly significant increases in the cost values.

Table 7.22: Summary of the results for the DGI

ITERATION	1	2	3	4	5
SYSTEM RELIABILITY	0.99971	0.99985	0.99991	0.99994	0.99997
COST OF SYSTEM RELIABILITY (£k)	23.217	27.385	31.129	35.968	50.496

In conclusion, the methodology used to obtain the system reliability values provided a lower bound estimate of reliability; therefore the system reliability values obtained were only approximations; the exact estimates would be expected to be higher. However since the reliabilities of the components were initially high, the difference between the exact system reliabilities and their corresponding lower estimates would be quite small. Again, given the very high initial reliability of the components of the DGI, it was not surprising that the lower feasibility factors yielded infeasible solutions The very high feasibility factor value (0.9) required in order to obtain the Pareto optimal solutions suggests that unless the DGI's component's reliabilities were easy to improve, higher reliabilities than the initial ones would be impossible to attain.

Another consequence of the very high initial reliability values could mean that reliability improvement interventions were not a critical requirement for the DGI. The high levels could be sustained through (existing) planned and systematic inspection and maintenance activities. Even so in technical terms the results obtained indicate that there is room for improvement, if it were deemed necessary, except that there are on the one hand serious cost implications which have to be addressed, and on the other practical difficulties in terms of how the improvements can be achieved, considering that the DGI is an already existing operational system. The company would have to decide whether to (i) use components with comparable levels of reliability to those achieved, or (ii) use redundancy techniques to achieve the desired levels of improvement in reliability for the components, or (iii) seek to redesign the appropriate components.

The fact that higher reliability values were nevertheless achieved for the components, which could further increase the already very high system reliability, is a further demonstration of the ability of the MCO model and methodology, developed and implemented in this work, to yield very high reliability systems.

7.6 **SUMMARY**

In this Chapter, four practical examples were presented and discussed as the concluding part of the model testing process. Three which were taken from the literature involved systems with much more complicated configurations than those discussed in Chapter Six, and the fourth was an industrial application.

The examples taken from the literature did not only yield high system reliability as a result of the optimisation, they also gave results that were consistent with earlier observations made about the model on the basis of the initial testing. The examples demonstrate that the extension of the MCO model to complex systems using the series- parallel transformation derived from their minimal cut sets can provide better results than an SCO approach where the top level system reliability expression is specified as the objective function. This was particularly demonstrated by the results obtained for the LSS model. The values assigned to specific components under the MCO model were not only higher but also compared favourably with the relative levels of those assigned them in their SCO counterparts. The MCO also yielded results that were consistent with the reliability importance of the components. This was particularly noted in respect of the BH system.

The case study results endorsed the methodology as a useful tool in design for reliability. The results obtained, in spite of the size (forty one objective functions and eighteen decision variables), complexity, and very high initial reliability values all provide confidence in the practical usefulness of the MCO model proposed in this work. Another distinctive feature of this particular application which gives further confidence in the utility of the model is its applicability to situations involving components with multiple failure modes. The technique developed, as part of this research, for converting the general cost measure into real cost was also illustrated.

CHAPTER EIGHT

LIMITATIONS AND FUTURE WORK

8.1 CONCLUSIONS

A new methodology for optimising reliability in a system's design has been proposed, developed and tested in this thesis. It posed the reliability design problem as a multiple criteria one and used the path of Pareto optimal assignment of component reliabilities for the dual purpose of enhancing overall system reliability while minimising the associated cost. The novelty of the approach is in:

(i) the multi-criteria optimisation view taken of the problem (which has habitually been treated as a single criterion one);

(ii) the focus on the subsystems' reliabilities (instead of the system's) and cost as the criteria to be optimised and the fact that the top level system reliability expression was not required for the optimisation;

(iii)the view of the minimum cut sets of a complex system as criteria their reliabilities of which are to be maximised. This is a novelty that simplifies the task of maximising a complex system's reliability;

(iv) the discussion of the MCO model and methodology in the context of series-parallel and complex systems and successfully applying them to optimise the reliability of the LSS, BH, and DGI.

The approach is justified particularly for the following reasons: (i) generally, the higher the reliability of the subsystems, whether in series-parallel configuration or otherwise, the higher the overall system reliability, (ii) since the reliability of a series-parallel system is bounded above by the lowest subsystem reliability, maximising the subsystems' reliabilities in effect maximises the lower bound reliability, which in turn has a positive impact on the system reliability. (Maximising subsystem reliability was therefore an indirect attempt at maximising the system reliability).

The resultant model was a deterministic, continuous, nonlinear multi-criteria optimisation problem. All but one criterion were concave monotone functions (i.e. subsystem reliability functions) and the other a convex function (i.e. Cost function) each defined on a convex set (component reliabilities) in the domain (0, 1). Pareto optimal solutions were found using the weighted sum scalarisation method the solution algorithm of which was run using the optimisation toolbox in MATLAB. Equal weights were assigned to the subsystems to reflect their equal importance for the overall system reliability; at the same time the subsystem reliabilities were weighted higher than the cost to reflect preference for reliability against cost.

The methodology was tested, firstly on hypothetically generated examples, then on data that had been published, and finally on an industrial application. The difficulty of securing more organisations to collaborate in this research did not permit wider testing on real problems. Nevertheless the series of tests performed has provided very good results which have thus increased the confidence in both the model and the methodology. The results have led to the following conclusions about the model:

- It is very sensitive to perturbations of its primary parameters which are the feasibility factors and the subsystem reliability lower bounds, being more so with that of the latter. It was much more stable with respect to weighting (the weights being secondary parameters, derived from the solution method used).

- The very low sensitivity of the model to the weightings is an indication of its robustness under the scheme adopted. In other words the equal weighting of the subsystems along with another for the cost, greatly narrowed the spread of points tracked along the Pareto front for candidate solutions, hence the minimal variation in the sets of solutions achieved.
- Due to the model stability, weighting may be used as a preference technique, rather than as a generating one. In this case a single weight vector representing a decision maker's preference order (as was used in the case of the DGI discussed in Chapter Seven) may be specified. The resultant Pareto optimal solution thus becomes a compromise solution. Alternatively, only a few weighting vectors representing a narrow set of preferences may be considered, to provide a number of compromise solutions from which a final one may be chosen for implementation. Since the set of potential solutions in this case would be very limited, selecting one as the BCS should be relatively easy.
- The choice of weights is relatively easy, given that they principally concern the relative magnitudes by which all the subsystems on the one hand and the cost on the other are weighted; the number of subsystems thus makes very little difference to this task. Of course the relative magnitudes of the weights are very subjective, but the principle behind them ensures that whatever the choices, the Pareto points do not widely deviate from those resulting from any other potential choices.
- The extension of the methodology especially to complex systems, such as to the LSS, BH and DGI systems and the results achieved, have demonstrated that the technique introduced in this work not only simplifies an otherwise formidable complex system reliability optimisation problem, but it does yield high components' and system

reliabilities at minimum costs. The concurrence of the results with those previously achieved for the LSS under SCO, in terms of the ordering of the values of the component reliabilities and in terms of the reliability importance of some identified components as in the BH system example are reasons for having confidence in the viability of the model and the accompanying methodology. That higher component and thus system reliabilities were obtained for the DGI in spite of the high initial values suggest that the model can yield high reliabilities regardless of the starting values. This however comes at a very high cost.

8.2 **LIMITATIONS**

There were two major limitations associated with this work, one practical and one technical. In the first instance, ideally, the number of real examples used to test the model should have been higher to provide not only breadth for applications, but also further opportunity to test the concepts. For example the feasibility factors for the components had to be assumed as also were the maximum component reliabilities and the subsystems reliability lower bounds. Thus an opportunity to experience the rudiments and practical processes, as well as the pitfalls involved in setting the parameters has not been fully exploited. The fact that these parameter values were assumed means that the results, while technically acceptable, may not accurately represent the true levels of the components' and system's reliabilities and associated cost.

On the technical level, firstly it is noted that a number of assumptions were made in order to simplify the application process of the model, these included (i) the independence of component failures, (ii) the system's time to failure being the time to first failure, (iii) the system being non-repairable, and (iv) the components' failure times being modelled by the exponential distribution.

In real terms some or all of these assumptions may not be valid and a different approach would have to be adopted. This however could only be done in real situations where there is an opportunity to work with managers or engineers in charge of the systems in question to address all the pertinent issues and resolve any practical difficulties that may be encountered. Secondly, the model's deterministic formulation means that the aspect of randomness in parameter values has not been taken into account. The fact that data, even in practice, may have errors or may be inaccurate means that a single point estimate of the values derived from them could well deviate from the actual levels. Thus the failure rates data obtained from the DGI application, for instance, has to be seen in this context. The components and system reliability values which are also given by single measures are strictly only estimates; since they are also random variables for which reason an interval estimate or information on the level of variations from their expected values would have made them more precise.

Another intrinsic limitation that the model could suffer from is attributed to a weakness in series systems, and noted in Chapter Three. Since the reliability of such systems tend to reduce with the number of subsystems (or components) in series, the system reliability estimate for a large number of subsystems could be greatly compromised, especially if at least one subsystem has a relatively low reliability. Thus in some practical problems, where large scale systems are more common, the model may experience this phenomenon.

8.3 **RECOMMENDATIONS**

In order to further explore the potentials of the model, possibly refine it, and hopefully plug the gaps the following seven areas are recommended for future work:

- Further testing especially on real data taken from industry would be a necessary and worthwhile follow-up to this work and should help to assess the practical relevance and usefulness of the concepts embodied in the model, as well as shed more light on its strengths and weaknesses.
- The cost function model used in this work was analytical in nature. It would be interesting to investigate the performance of the model on the basis of an empirical cost function derived from real cost data. Furthermore, it would be a beneficial exercise to evaluate and estimate values for the parameters of the model, such as the feasibility factor and the maximum component reliability, from real data and through consultation with design engineers of components and systems for which an application is undertaken. The prospects of using other scalar (solution) methods to find Pareto optimal solutions could also be investigated.
- The MCO model was deterministic; however, for the purpose of addressing the short falls noted under section 8.2 on this subject, it would be appropriate to investigate a stochastic variant of the model. This means introducing parameters to account for the randomness in the values of the components' or system reliabilities. This should provide a way of assessing the robustness of the resultant model under random conditions.
- It is envisaged that the modelling and optimisation concept presented in this thesis can be widely applied to many design for reliability situations having the requisite background. For example it can be used to analyse and assign in an optimal fashion reliability specifications for a system's design involving subsystems which are made up of assemblies and the assemblies in turn made up of components. While traditionally this problem is approached from the top down in three major steps: assigning reliabilities

first to subsystems, then from subsystems to assemblies, then to the components, the MCO modelling approach can provide a one-stop assignment of reliability to the components directly, after which the reliability specifications for the assemblies and subsystems follow immediately from their configurations.

- A closely related example to the foregoing where the modelling approach may be useful is in designs involving systems with identifiable subsystems the components of which made up of others but which are treated as black boxes (i.e. they are not identified or directly assigned reliability), but whose reliability levels impact the top level component reliability. This may be the case where individual systems are linked up either in series parallel or some other configuration, into a huge super system. The reliability assigned to the "components" thus becomes the bench mark value for their design. An example for instance is the security system linking n airports (where n is a large positive finite integer). The entire system in this case is made up of the security systems of each airport which in turn have their own subsystems and components in that order. The MCO modelling approach could be used to assign optimal reliability values to the subsystems or "components" as the case may be.
- The model could also be useful for investigating the reliability requirements in designs where only minimal levels of redundant components are permitted in a subsystem (recall that this generally results in high component reliability assignment) due to weight and volume constraints, such as in gearbox design.
- The possibility of extending the model into other domains outside reliability design (especially in the mathematical sciences) has been envisaged, and requires investigating. In other words, suppose there was a phenomenon to be optimised, but for which there

was no explicit function describing it in terms of its variables, for which reason SCO methods could not be applied. If there were known sub functions which describe aspects of the phenomenon, they could be optimised independently but simultaneously using the general MCO model proposed, to find Pareto optimal values of the decision variables for which an acceptable value of the unknown function could be estimated. This idea is only intuitive, but more importantly a stimulus for follow-up and possible development.

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APPENDICES

A. DEFINITIONS AND OVERVIEW OF TERMINOLOGY USED IN THE THEORY OF MCO

A1 DEFINITIONS

Open Neighbourhood

Let ε > 0 be given. An open neighbourhood of $x_0 \in R^N$ denoted by $N_\varepsilon(x_0)$ is an N - dimensional hyper-sphere centred at $x_0 \in R^N$ given by the set $\{x_0 \in R^N : |x - x_0| < \varepsilon\}$, where ε is the radius of the hyper-sphere. An open neighbourhood is also referred to as an Open Ball.

Interior Point of a Set

Let S be a nonempty set. $x_0 \in S$ is said to be an interior point of S if there exist an open ball $N_\varepsilon(x_0)$ of radius ε > 0 such that $N_\varepsilon(x_0) \subset S$. The set of all interior points of S constitute the interior of the set and is denoted by $\mathrm{int}(S)$.

Boundary Point of a Set

Let S be a nonempty set in R^N. A point $x_0 \in S$ is said to be a boundary point of S if and only if $N_\varepsilon(x_0) \not\subset S$ for some ε>0. That is the open ball centred at x_0 contains points both in and outside S. The set of all boundary points of S constitute the boundary of S and are denoted by ∂S.

Open Set

Let S be a nonempty set. S is said to be open, if and only if $S = \mathrm{int}(S)$. An open set is therefore one in which members are interior points.

Closed Set

Let S be a nonempty set. S is said to be a closed set if and only if S contains all of its boundary points.

Closure of a Set

Let S be a nonempty set. The closure of S denoted by $cl(S)$ is the union of S and its boundary points; i.e. $cl(S) = S \cup \partial S$.

Another characterisation of the notion of closure of a set is the following. The closure of an arbitrary set $S \subset R^N$ is the set of all points $x_0 \in R^N$ for which there exists a sequence $\{x^k\}$ of points in S such that: $\lim_{k\to\infty} \left|x^k - x_0\right| = 0$. Therefore a closed set $S \subset R^N$ is equal to its closure; i.e. $S = cl(S)$ if S is closed, or $cl(S) = \text{int}(S) \cup \partial S$.

Limit Point of a Set

Let S be a nonempty set in R^N. A point $x \in S$ is called a limit point of S, if every neighbourhood of x contains a point $y \in S$ such that $x \neq y$.

Isolated Point of a Set

If a point $y \in S$ is such that it is not a limit point, then it is called an isolated point of S.

Bounded Set

A set $S \subset R^N$ is said to be bounded if it is contained in a ball of finite radius, i.e. if there exist $y \in R^N$ and a real number $\delta > 0$ such that $\forall\ x \in S\ |x - y| < \delta$.

Compact Set

A set that is closed and bounded is said to be compact

Connected Sets

Let S be a non empty set in R^N. S is said to be a connected set if and only if there does not exist open sets M and N such that:

i) $M \cap S$ and $N \cap S$ are nonempty disjoint sets,

ii) $S = (M \cap S) \cup (N \cap S)$

If two sets are not connected they are said to be disconnected.

While disconnected sets are referred to as **discrete sets** connected one are referred to as **continuous sets**

Convex Combination

Let $S \subset R^N$. For any points $x_1, x_2 \in S$ the line segment given by $\lambda x_1 + (1-\lambda)x_2$ such that $\lambda \in [0,1]$ is called a convex combination of x_1 and x_2. In general $\sum_{i=1}^{k} \lambda_i x_i$ such that $\sum_{i=1}^{k} \lambda_i = 1 \quad \lambda_i \geq 0 \quad i = 1,2,..,k$ is a convex combination of $x_1, x_2, ..., x_k$.

Convex Set

Let S be a set. S is said to be convex if $\forall\ x_1, x_2 \in S$ and $\lambda \in [0,1]$, $\lambda x_1 + (1-\lambda)x_2 \in S$

The definition can be restated as: A set S is convex if for any two points x_1 and x_2 belonging to S there are no points on the line between x_1 and x_2 that are not members of S. Put another way, a set S is convex if there are no points x_1 and x_2 in S such that there is a point on the line between x_1 and x_2 that does not belong to S. The restatement includes the empty set within the definition of convexity. It also includes singleton sets where x_1 and x_2 coincide and thus the line between them reduces to a point. Thus $\phi \in S$ and therefore the intersection of any two convex sets is a convex set. Examples of convex sets are the Euclidean space R^N, ($N \geq 1$), hyper planes, line segments.

Vector Space

Let $S \subseteq R^N$ be a set and $x, y, z \in S$ any arbitrary elements. S is called a vector space if it satisfies the following axioms for the scalars $\lambda, \mu \in R$.

i) $x + y = y + x$

ii) $(x + y) + z = x + (y + z)$

iii) $\exists\ 0 \in S$ such that $0 + x = x + 0 = x$, $\forall\ x \in S$
(0 is called the zero element of S)

iv) $\forall\ x \in S\ \exists\ y \in S$ such that $x + y = 0$ (y is called the additive inverse of x),

v) $\lambda(x + y) = \lambda x + \lambda y$

vi) $(\lambda + \mu)x = \lambda x + \mu x$

vii) $(\lambda\mu)x = \lambda(\mu x)$

viii) $1x = x$.

Any set which satisfy these axioms is said to have a linear structure or constitute a linear space. The Euclidean space is a well known example of which the real line is a trivial case.

Cones

Let S be a real linear space. A nonempty subset $K \subset S$ is called a cone if $\lambda x \in K$ for all scalars $\lambda \geq 0$ and $x \in K$.

Pointed Cones: A cone K is called pointed if $K \cap -K = \{0\}$. $-K$ is called the negative cone.

Reproducing Cones: A cone K is called reproducing if $K - K = S$.

Convex Cones: A cone K in a real linear space is convex if and only if $K + K \subseteq K$

Ordering Cones: If a set $K \subseteq S$ (a linear space) such that for any $x, y \in S$ the relation $x \leq_k y \Leftrightarrow y - x \in K$. Then the relation $\leq_k$ is the order induced by K. K in this case is called the ordering cone.

The following are typical examples of cones;

(a) The non-negative octant $\{ x \in R^N : x \geq 0 \}$,

(b) The space R^N

(c) The singleton set $\{0 \in R^N \}$,

Convex Function

A function $f(x)$ is convex on an interval $[a,b]$ if for any two points $x_1, x_2 \in [a,b]$ and $\lambda \in (0,1)$

$f[\lambda x_1 + (1-\lambda)x_2] \leq f(x_1) + (1-\lambda) f(x_2)$

Concave Function

If f is a convex function then $-f$ is a concave function and vice versa.

Monotone Functions

Function *f* defined on a subset of the real numbers with real values is called monotone increasing (also monotonically increasing or non-decreasing), if for all *x* and *y* such that $x \leq y$, $f(x) \leq f(y)$, (see Figure A1). Likewise, a function is called monotonically decreasing (non-increasing) if, whenever $x \leq y$, then $f(x) \geq f(y)$, (see Figure A2).

If the order ≤ in the definition of monotonicity is replaced by the strict order <, then one obtains a stronger requirement. A function with this property is called **strictly increasing**. Again, by inverting the order symbol, one finds a corresponding concept called **strictly decreasing**. Functions that are strictly increasing or decreasing are one-to-one (because for *x* not equal to *y*, either $x < y$ or $x > y$ and so, by monotonicity, either $f(x) < f(y)$ or $f(x) > f(y)$, thus $f(x)$ is not equal to $f(y)$).

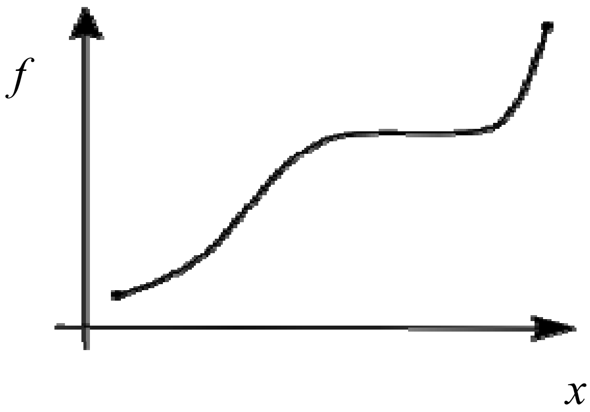

Figure A1: Monotonically increasing function

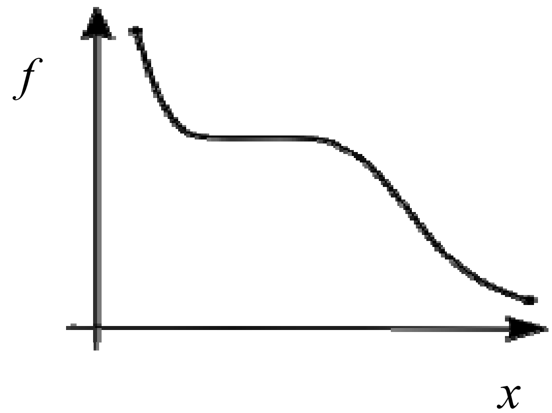

Figure A2: Monotonically decreasing function

A2 CHARACTERISATION OF PARETO POINTS

Definition

Let the set of Pareto points be denoted by $E(M,K)$ where K is the ordering cone induced on M the feasible criterion set. Then $E(M,K) = \{ y_0 \in M \mid \forall y \in M \;\; y \leq_k y_0 : y = y_0 \}$.

Theorem

If a point is in $E(M,K)$ then it is on the boundary of M .

Proof

Let $y_0 \in E(M,K)$. Suppose $y_0 \in \text{int}(M)$ then there exists an open neighbourhood $N_\delta(y_0)$, $\delta > 0$ such that $N_\delta(y_0)\delta \subseteq M$. Let there be a vector $v \in K$, then $-v \in -K$ and $\delta v / \|v\|$ is a vector of length δ such that $y_0 - \delta v / \|v\| = y \in N_\delta(y_0) \subseteq M$. It means that $y \leq_k y_0$, $y \neq y_0$ which means $y_0 \notin E(M,K)$ which is a contradiction.

Corollary: (**Necessary and sufficient condition**)

Let $y \in M$. $y \in E(M,K)$, if and only if, $y \in E(M,K) = \{y \in M : M \cap (y - K \setminus \{0\}) = \phi\} \}$.

For each $y \in E(M, K)$ there corresponds $x \in S$ such that $y = f(x)$. Therefore Pareto optimal solutions are tractable.

Theorem

If the feasible region in the criterion set is closed convex then the set of Pareto points is connected.

Remark

A connected set has desirable analytical properties and therefore a connected Pareto front provides an ideal environment for the pursuit of Pareto optimal solutions. This is guaranteed by a closed convex feasible criterion set.

Theorem

If there exist a point $y \in M$ such that $(y - K) \cap M$ is compact, then $E(M, K \setminus \{0\}) \neq \phi$.

A corollary to this theorem is the following:

If M is compact and K is a cone (without lines) then $E(M, K) \neq \phi$

Remark

Another desirable property of the feasible region in objective space is compactness. A compact (i.e. closed and bounded) feasible criterion set provides an ideal environment for the tracking of Pareto points and therefore Pareto optimal solutions.

A3 NOTIONS OF THE PARETO OPTIMAL SET

Proper Pareto Optimal Solution

x^* is properly Pareto optimal if there exists a scalar $M > 0$ for all i and $x \in S$ satisfying $f_i(x) < f_i(x^*)$ there exists a j such that $f_j(x^*) < f_j(x)$ such that:

$$\frac{f_i(x^*) - f_i(x)}{f_j(x) - f_j(x^*)} \leq M$$

Local Pareto Optimal Set

A set $s \in S$ is local Pareto optimal, if for every $x \in s$ there does not exist $x' \in N_{\varepsilon}(s)$ for some $\varepsilon > 0$ which dominates every element of s.

Global Pareto Optimal Set

A set $s \in S$ is global Pareto optimal, if there exists no $x \in S$ which dominates every element of s.

A4 PROPERTIES OF THE OBJECTIVE FUNCTIONS

Convexity of the Cost Function

It is noted that the cost function $c(R_{ki})$ is a continuously differentiable function of a single variable R_{ki} for each $k = 1,2,..,n_{i\,i}$ and $i = 1,2,..,s$; where:

$$c(R_{ki}) = \exp\left[(1 - f_{ki})\frac{R_{ki} - R_{ki,\,\min}}{R_{ki,\,\max} - R_{ki}}\right] \tag{1}$$

Therefore by the second order derivative test $c(R_{ki})$ is convex if $c''(R_{ki}) \geq 0$, $\forall R_{ki} \in [0,1]$. Since $R_{ki,\,\min}$, $R_{ki,\,\max}$ and f_{ki} are fixed for any value of R_{ki} ,equation 1 is written simply as:

$$c(R) = \exp\left[(1 - f)\frac{R - k_1}{k_2 - R}\right] \tag{2}$$

where $k_1 = R_{ki,\,\min}$, $k_2 = R_{ki,\,\max}$ and $f = f_{ki}$ are constants. For the purposes of the differentiation, equation 2 may be denoted by $c(R) =$ exp (*), where (*) represents the expression in the exponent of equation 2. The first order derivative of equation 2, by the chain and quotient rules, is the following:

$$c'(R) = \frac{(1-f)(k_2 - R) + (1-f)(R - k_1)}{(k_2 - R)^2}\exp(*)$$

$$c'(R) = \frac{(1-f)(k_2 - k_1)}{(k_2 - R)^2}\exp(*) \tag{3}$$

The second order derivative of equation 2 is the following:

$$c''(R) = \frac{(1-f)^2(k_2-k_1)^2}{(k_2-R)^4}\exp(*) + \frac{2(1-f)(k_2-k_1)(k_2-R)}{(k_2-R)^4}\exp(*)$$

$$c''(R) = \frac{(1-f)(k_2-k_1)\left[(1-f)(k_2-k_1)+2(k_2-R)\right]}{(k_2-R)^4}\exp(*) \qquad (4)$$

It is clear that $(k_2-k_1) \geq 0$, $R-k_1 \geq 0$, $(k_2-R) \geq 0$ and $(1-f) \geq 0$. Thus the right hand side of equation 4 is non-negative. It follows therefore that $c''(R_{ki}) \geq 0$.

Monotonicity & Concavity of the Subsystem Reliabilities

The subsystem reliability $R_{sub,i} = f_i(R_1, R_2, ..., R_N) = 1 - \prod_{j=1}^{N}(1-R_j)$ are monotone increasing and concave functions defined on the convex set $S \subset R^N$ where $S = [0,1]^N$.

B. OVERVIEW OF BASIC RELIABILITY METRICS, FTA AND METHODS FOR THE DETERMININATION OF CUT SETS

B1 BASIC RELIABILITY METRICS

These are the failure and reliability functions, failure rate function (also called hazard function), and the mean life function.

The Failure and Reliability Functions

Failure and therefore reliability are modelled as functions of time. The failure function $F(t)$ is defined by:

$$F(t) = \int_0^t f(s)ds \tag{B1.1}$$

where t is the *time-to-failure* or the *life time* of a component and $f(s)$ is the probability distribution or density function (i.e. PDF) of t. $F(t)$ is also called the *cumulative failure function*. Under binary assumptions the probability that a component is in operation, denoted by $R(t)$, is given as:

$$R(t) = 1 - F(t) = 1 - \int_0^t f(s)ds = \int_t^T f(s)ds = P(T \geq t) \tag{B1.2}$$

$R(t)$ is called the reliability or the *survival function* of a component, which is also the probability of the successful operation of a component for a mission time T. It follows from (B1.2) that:

$$f(t) = -\frac{d}{dt}R(t) \tag{B1.3}$$

The Failure Rate Function

The failure rate (an important metric) is the frequency with which a component or a system fails over time (ReliaSoft Corporation, 1996-2006). It is thus the rate of occurrence of failure which is instantaneous at every point in time. The instantaneous failure rate is called the failure rate function or hazard function

(Rao, 1992). The failure rate can be regarded as describing the number of components successfully performing their required functions at a given time in a population of components when others have failed (ReliaSoft Corporation, 1996-2006). The hazard function may therefore be defined in terms of the ratio of the components that have failed to those that have survived at a given time which represents a conditional probability of failure during a given time, given that there was no failure before the time. The hazard function $h(t)$ is therefore expressed as:

$$h(t) = \frac{f(t)}{R(t)} \tag{B1.4}$$

Substituting (B1.3) for $f(t)$ in (B1.4) yields the expression:

$$h(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \Leftrightarrow h(t)dt = -\frac{dR(t)}{R(t)} \tag{B1.5}$$

Integrating both sides of (B1.5) in the time interval $[0, t]$ and reliability in the interval $[1, R(t)]$ and substituting the initial conditions yields the following result:

$$\ln R(t) = -\int_0^t h(t)dt \Leftrightarrow R(t) = \exp\left(-\int_0^t h(t)dt\right) \tag{B1.6}$$

which is a general expression for the reliability of a component in terms of its hazard function. In the case where the failure rate is constant equation B1.6 reduces to:

$$\boxed{R(t) = \exp(-\lambda t) \qquad \text{(B1.7)}}$$

In this case $h(t)$ is a constant denoted by λ .Most mechanical and electronic components are known to exhibit constant failure rates (Carter, 1997) (especially during their useful life as depicted in the bath-tub curve), thus their reliability may be determined by (B1.7). The lifetime distribution $f(t)$ for a component that exhibits a constant hazard rate (from (B1.4) and (B1.7)) is given by:

$$f(t) = \lambda \exp(-\lambda t) \tag{B1.8}$$

which is the Exponential distribution.

The Mean Life Function

This provides a measure of the average time of operation before failure (i.e. expected life) of a non-repairable component or system. It is therefore defined as an expected value $E(t)$ of the time to failure and is given by:

$$E(t) = \int_0^\infty t.f(t)dt \qquad \text{(B1.9)}$$

The function is referred to as the Mean-Time-to-Failure (MTTF) (Mettas & Savva, 2001). Substituting $-\frac{d}{dt}R(t)$ for $f(t)$ and integrating the resultant expression by parts (while letting $R(t) \to 0$ as $t \to \infty$) yields the mean life function in terms of the component reliability function, i.e.

$$E(t) = \int_0^\infty R(t)dt \qquad \text{(B1.10)}$$

In the special case of constant failure rate the result becomes:

$$E(t) = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{MTTF} \qquad \text{(B1.11)}$$

Other less referred to metrics in the reliability literature are the *variance function*: which provides a measure of the variability in the failure times of components and systems, and the *median* and *modal life* functions which are also average measures of the component life time distribution function.

B2 COMPONENT FAILURE TIME MODELS

Several statistical distributions can be used to model the failure times of both components and systems. Notable among them are the Exponential, Normal, Log-normal, Gamma and Weibull distributions (O'Connor, 1995). Of these the latter is the most widely applied, since it fits many life time distributions (O'Connor, 1995). The PDF of the 2-parameter Weibull distribution is:

$$f(t) = \frac{\beta}{\eta^\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right], \quad t \geq 0 \qquad \text{(B2.1)}$$

The scalars β and η are parameters which provide important information about the distribution: β measures the variability in the failure times and describe the shape (i.e. skewness) of the distribution; η measures its spread or dispersion (O'Connor, 1995). When $\beta = 1$, the distribution is Exponential, therefore, the hazard rate is constant. β > 1 indicates increasing hazard; it also indicates wear out failures. When β = 3.44 the distribution approaches Normality. β < 1 implies decreasing hazard which is characteristic of the early life failures of components or systems. The basic reliability metrics for the Weibull distribution are:

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{B2.2}$$

$$h(t) = \frac{\beta}{\eta^{\beta}} t^{\beta - 1} \tag{B2.3}$$

$$MTTF = \eta\Gamma\left(1 + \frac{1}{\beta}\right) \tag{B2.4}$$

where Γ is a gamma function (k is a positive integer) defined by:

$$\Gamma(k) = \int_0^{\infty} x^{k-1} \exp(-x) dx \tag{B2.5}$$

Figures B1 to B4 illustrate graphically the effects of the values of β and η on the Weibull distribution and its associated reliability metrics. (ReliaSoft Corporation, 1996-2006).

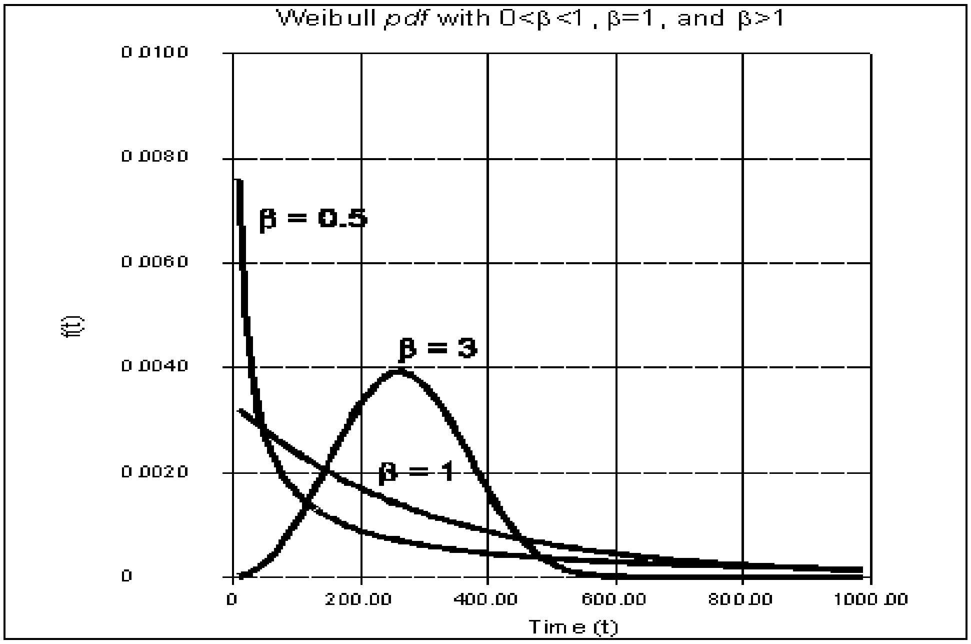

Figure B1: The effect of values of β on the Weibull distribution

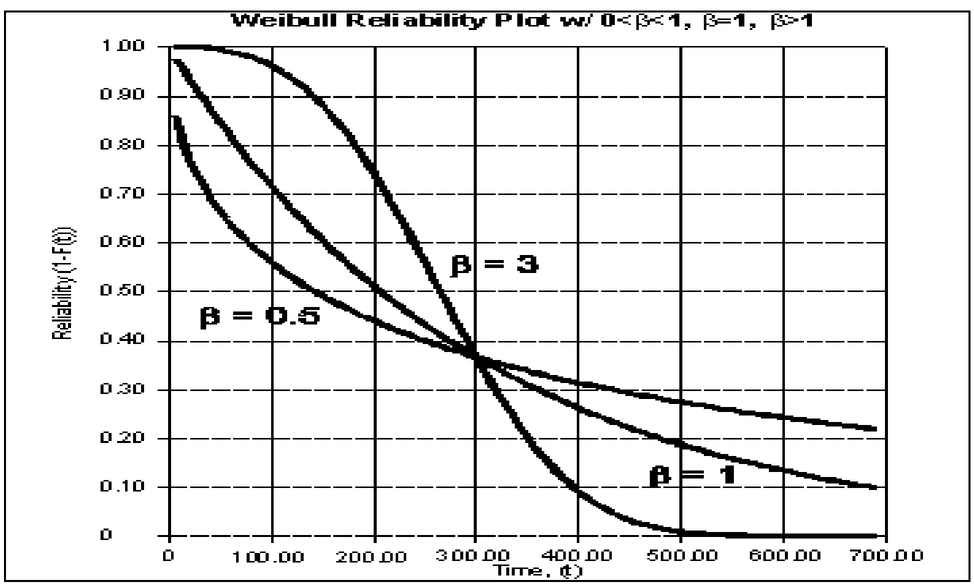

Figure B2: The effect of values of β on the reliability function

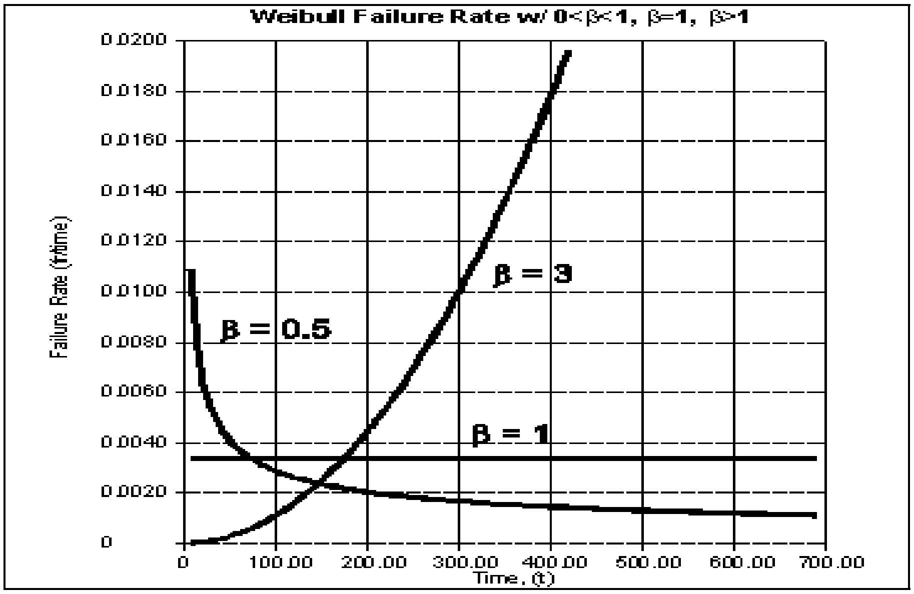

Figure B3: The effect of values of β on failure rate

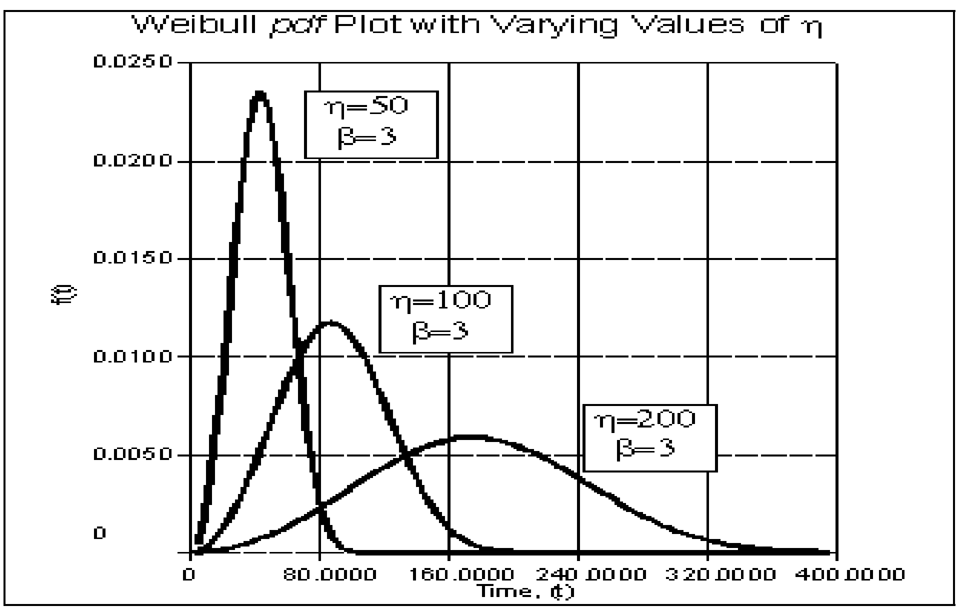

Figure B4: The effect of η on the Weibull distribution

B3 THE RELIABILITY OF A PARALLEL-SERIES SYSTEM

Refer to Figure 3.6. The parallel-series system is made up of m subsystems in parallel while each subsystem is made up of n components in series. Let $R_{sub,i}$, $F_{sub,i}$, and R_{ki} be the reliability and failure probability of the $i'th$ subsystem, and the $k'th$ component in the $i'th$ subsystem respectively $i = 1,2,...,m$, $k = 1,2,...,n$.

Note that:

$$R_{sub,i} + F_{sub,i} = 1 \tag{B3.1}$$

Since a subsystem is operational only when all of its components are operational it follows that:

$$R_{sub,i} = \prod_{k=1}^{n} R_{ki} \tag{B3.2}$$

Since the subsystems are in parallel, the system will fail only when all the subsystems fail. Therefore the system's failure probability F_s is:

$$F_s = \prod_{i=1}^{m} F_{sub,i} \tag{B3.3}$$

The system's reliability R_s therefore is:

$$R_s = 1 - \prod_{i=1}^{m} F_{sub,i} \tag{B3.4}$$

$$= 1 - \prod_{i=1}^{m} (1 - R_{sub,i})$$

$$= 1 - \prod_{i=1}^{m} (1 - \prod_{k=1}^{n} R_{ki}) \tag{B3.5}$$

B4 FAULT TREE ANALYSIS

A Fault Tree (FT) is a tool for both qualitative and quantitative analysis of systems' risks and failure characteristics. It was developed on the basis of deductive logic starting with an event of system failure known as a Top Event, and deducing which sequences of component failures could lead to the top event.

The logical interrelationship between the sequences of component failures is represented through logical connections known as logical gates, leading to a tree-like structure with the top event at the top, followed by intermediate events, and basic event at the extremes. The basic events are those for which failure rate data or failure probabilities are available and which cannot be further evolved into other branches or events.

A FT is typically a Boolean logical diagram comprising primarily AND and OR gates. The output event of an AND gate occurs only if all of the input events occur simultaneously, and the output event of an OR gate occurs if any one of the input events occurs. The symbols commonly used in the construction of FT diagrams are those for AND and OR gates, as well as intermediate, basic, and incomplete events. These are displayed in Figure B1.

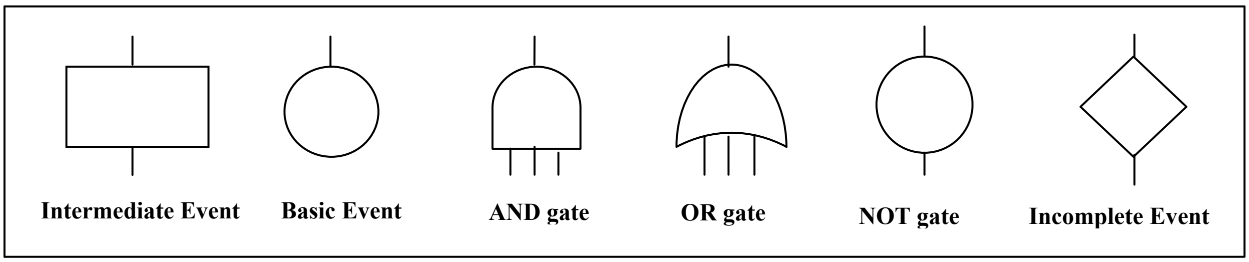

Figure B1: Common Boolean logical symbols used in FT diagrams

The symbol for the top event is similar to that for the intermediate one. The difference is that the one for the top event omits the vertical bar on top of that of the intermediate. An example of a FT diagram is shown in Figure B2. The figure denotes the top event as T, the four intermediate events as I_1 to I_4, the five logical gates four of which are labelled as G_1 to G_4, and the basic events as E_1 to E_6.

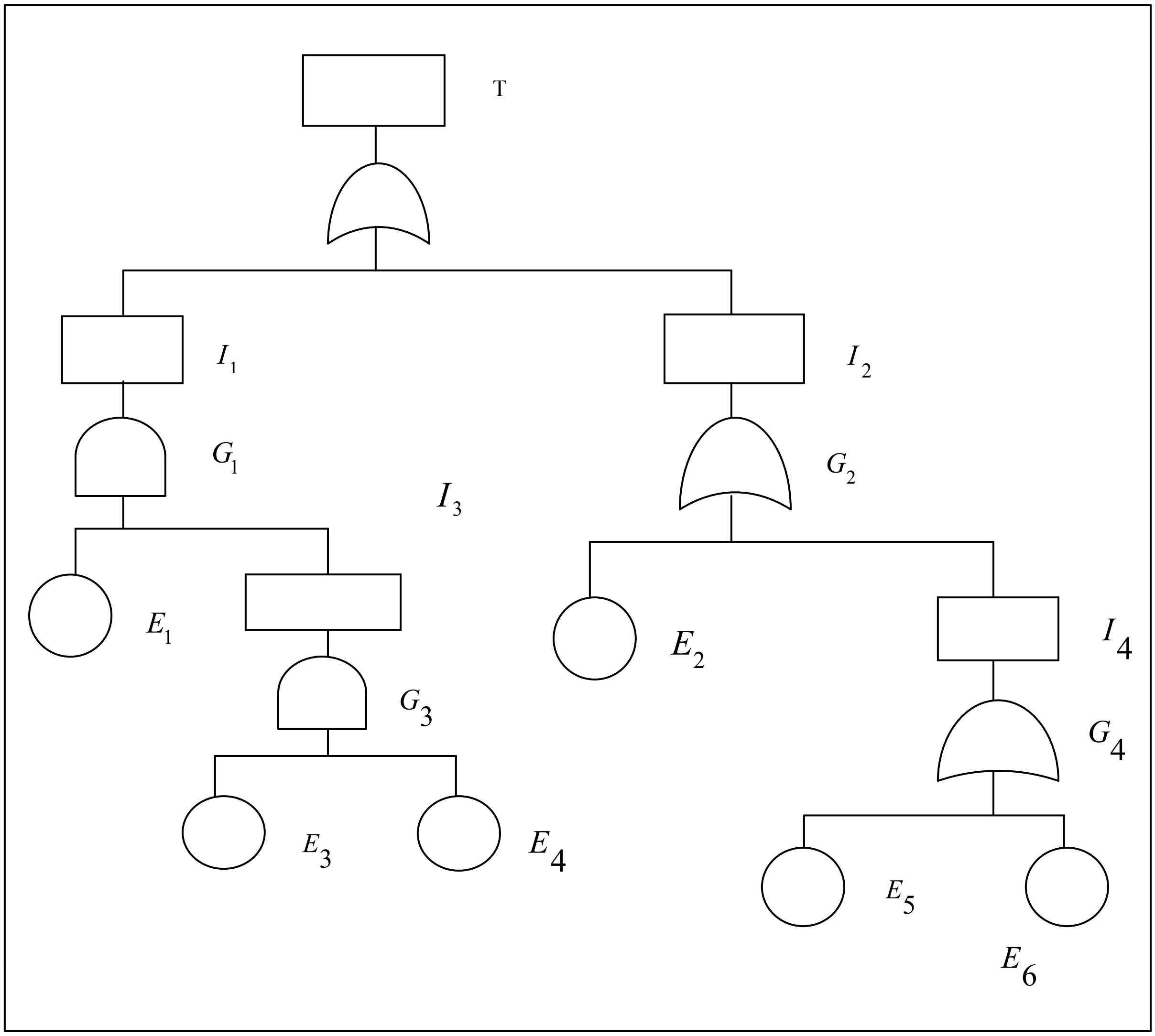

Figure B2: Example of a FT diagram (Source: Billinton and Allan, 1992)

Quantitative Analysis of FT

Two basic techniques are used to evaluate the chance of occurrence of the top event in FTs. These are using: (i) Boolean algebra and the logical structure of the FT, and (ii) a numerical approach that combines the basic event probabilities using the laws of probability and the logical structure of the FT. The minimal cut sets may also be deduced from the FT. The first technique is applied to the FT in Figure B2 to express the top event T in terms of the basic events. This is achieved by expressing the top event in terms of the events immediately succeeding it in the hierarchy and so on until all other events have been replaced leaving only the basic events. In Boolean algebra AND and OR denote product and sum respectively.

The required results are:

$$T = I_1 + I_2$$
$$= (E_1 I_3) + (E_2 + I_4)$$
$$= (E_1(E_3 E_4)) + (E_2 + (E_5 + E_6))$$
$$= E_1 E_3 E_4 + E_2 + E_5 + E_6$$

Where the probabilities $P(E_i)$ are known, $i = 1,2,..,6$ then the probability $P(T)$ of the top event is evaluated by: $P(T) = P(E_1)P(E_3)P(E_4) + P(E_2) + P(E_5) + P(E_6)$

The FTA in Five Steps

- Define the undesired or top event
- Obtain an understanding of the system under study
- Develop the fault tree
- Evaluate the fault tree
- Control the hazards identified.

B5 METHODS FOR DEDUCING MINIMAL CUT SETS.

Where a system is simple the minimal cut set can normally be identified by inspection with little or no difficulty. The task becomes more difficult in larger and more complex systems. In such cases a systematic approach is needed and there are many such approaches (Billinton and Allan, 1992). Two of the methods which are suitable for network or topological type problems and discussed by Billinton and Allan (1992) are presented here.

Method 1 (Steps)

(i) Deduce all minimal paths (A minimal path is defined as a path between input and output such that no node or intersection between branches is traversed more than once);

(ii) Construct an incidence matrix that identifies all components in each path;

(iii) If all elements of any column of the incidence matrix are non-zero, the component associated with that column forms a first order cut;

(iv) Combine two columns of the incidence matrix at a time. If all elements of the combined columns are non-zero, the components associated with three columns form a second order cut Eliminate any cut containing first order cuts to give the second order minimal cuts;

(v) Repeat step (iv) with three columns at a time to give the third order cut sets;

(vi) Continue until maximum order of cut has been reached

Method 2 (Steps)

(i) Deduce all minimal paths

(ii) Deduce all first order cuts of path 1; these being the components in the path

(iii) Deduce all first order cuts of path 2 and combine in all possible ways with all cuts from path 1. Eliminate duplicated combinations, non-minimal cut sets and cut sets of order greater than that required.

(iv) Take the next path and combine all of its first order cuts with those remaining after step (iii). Eliminate cuts as in step (iii).

(v) Repeat step (iv) until all paths have been considered.

Deducing FT Minimal Cut Sets

This method is suitable for deducing the minimal cut sets of fault trees (Billinton and Allan, 1992). It starts with the top event and proceeds as follows:

(i) If the top event gate is an OR gate, write down the inputs in terms of basic events and input gates as separate items in a list;

(ii) If the top event gate is an AND gate, write down the inputs in terms of basic events and input gates as one single item in a list;

(iii) Consider each gate in the new list and replace it as in (i) if it is an OR gate or as in (ii) if it is an AND gate;

(iv) Repeat (iii) continuously for each new list until all items in the list is in terms of basic events. Eliminate any row of this list if it is not a minimal set, i.e., if another item in the list is contained within it;

(v) The resulting list after completing all the above steps is the list of minimal cut sets.

Applying the above method to the FT in Figure B2 produces the deductions shown below:

$$G_1 \rightarrow E_1G_3 \qquad E_1G_3 \rightarrow E_1E_3E_4 \qquad E_1E_3E_4$$

$$G_2 \qquad G_2 \rightarrow \begin{cases} E_2 \\ G_4 \end{cases} \qquad \begin{matrix} E_2 \\ G_4 \rightarrow \end{matrix} \qquad \begin{matrix} E_2 \\ \begin{cases} E_5 \\ E_6 \end{cases} \end{matrix}$$

The cut sets are: $E_1E_3E_4$, E_2 , E_5 , and E_3 . Since E_3 occurs also in the first on the list the minimal cut sets are: E_2 , E_3 , and E_5 .

B6 RELIABILITY IMPORTANCE OF COMPONENT 5 OF SENARIO 3 PROBLEM

The system reliability expression for the bridge system (see Figure 3.11) discussed under Scenario 3 of Chapter Six is given by Sheloker et al (2002) as follows:

$$\begin{aligned} R_s &= R_1R_2 + R_3R_4 + R_2R_3R_5 + R_1R_4R_5 + \\ &2R_1R_2R_3R_4R_5 - R_2R_3R_4R_5 - R_1R_2R_4R_5 - R_1R_2R_3R_4 - R_1R_3R_4R_5 - R_1R_2R_3R_5 \end{aligned} \quad \text{(B1)}$$

The (static) Birnbaum component reliability importance metric for component 1 is:

$$I_1^{\ B} = R_2 + R_5R_4 + 2R_2R_3R_4R_5 - R_2R_4R_5 - R_3R_2R_4 - R_3R_5R_4 - R_3R_5R_2 \quad \text{(B2)}$$

Substituting the reliability value of 0.5 for the components yield the result:

$$I_1^{\ B} = 0.375$$

The reliability importance of component 2 is:

$$I_2^{\ B} = R_1 + R_3R_5 + 2R_1R_3R_4R_5 - R_3R_4R_5 - R_1R_4R_5 + R_1R_3R_4 - R_1R_3R_5 \quad \text{(B3)}$$

Substituting the component reliability values of 0.5 in B3 yields the same results as those for $I_3^{\ B}$ and $I_4^{\ B}$:

$$I_2^{\ B} = 0.375 = I_3^{\ B} = I_4^{\ B}$$

$$I_5^{\ B} = R_2R_3 + R_1R_4 + 2R_1R_2R_4R_5 - R_2R_3R_4 - R_1R_2R_4 - R_1R_3R_4 - R_1R_3R_2 = 0.125$$

The reliability importance of component 5 is therefore the least.

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C. THE SCO MODELS DISCUSSED BY SHELOKER ET AL AND SALAZAR ET AL

C1 THE SCO MODELS OF SHELOKER ET AL AND SALAZAR ET AL.

The SCO models presented by Sheloker et al (2002) and those by Ravi et al and Rocco et al discussed by Salazar et al (2006) are reproduced here. They seek to assign reliability to the components of the LSS at minimum cost.

The models involving two different cost function formulations were presented under two cases

The Cost functions C_S of the system are minimised subject to lower and upper bound constraints on both system and component reliabilities. K_i and α_i ($i = 1,2,3,4$) are scalars representing the system's physical characteristics. The overall reliability expression for the LSS was given as:

$$R_S = 1 - R_3[(1-R_1)(1-R_4)]^2 - (1-R_3)\{1-R_2[1-(1-R_1)(1-R_4)]\}^2$$

Case 1

$$Min\ C_S = 2K_1R_1^{\alpha_1} + 2K_2R_2^{\alpha_2} + 2K_3R_3^{\alpha_3} + 2K_4R_4^{\alpha_4}$$

subject to:

$$0.5 \le R_i \le 1, \quad i = 1,2,3,4$$

$$0.9 \le R_S \le 1$$

$$K_1 = 100,\ K_2 = 100,\ K_3 = 100,\ K_4 = 150$$

$$\alpha_i = 0.6, \qquad i = 1,2,3,4$$

Case 2

$$Min\ C_S = \sum_{i=1}^{4} K_i \left[\tan\left(\frac{\pi}{2} R_i \right) \right]^{\alpha_i}$$

subject to:

$$0.5 \le R_i \le 1, \quad i = 1,2,3,4$$

$$0.99 \le R_S \le 1$$

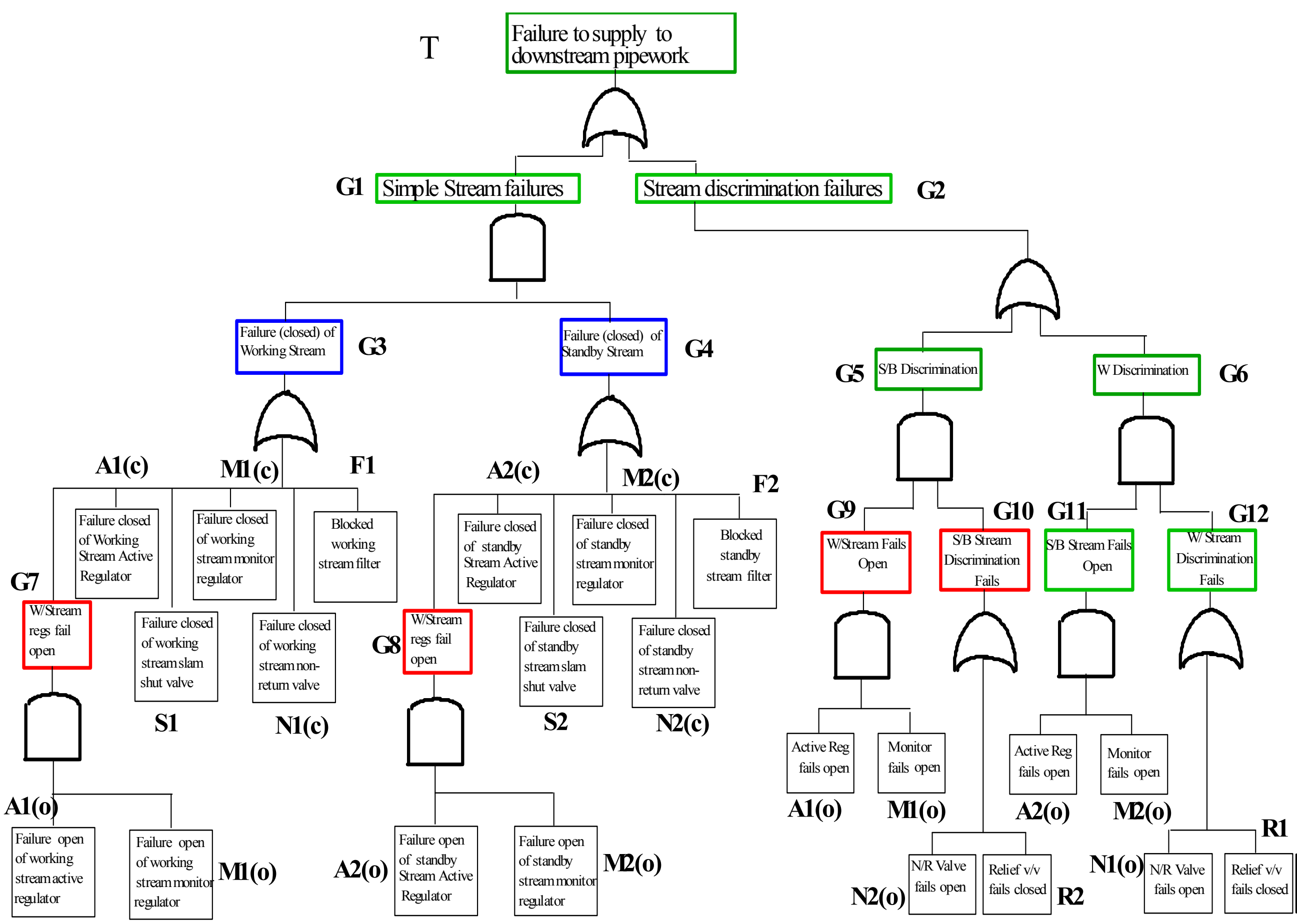
$$K_1 = 25,\ K_2 = 25,\ K_3 = 50,\ K_4 = 37.5, \qquad \alpha_i = 1.0 \quad \forall\ i$$

The SCO models discussed by Salazar in respect of Ravi et al and Rocco et al were the same as that presented under case 1 above. The difference was in the algorithms employed for their solutions.

The minimum cost was approximately 641.8 for all the cases.

D. FAULT TREE DIAGRAM AND CUT SET ANALYSIS OF THE DGI

D1 FAULT TREE DIAGRAM OF THE DGI FAILING CLOSED



D2 ANALYSIS OF FT IN TERMS OF CUT SETS

The Boolean Algebra for this system failing closed is as follows:-

T = G1 + G2

= G3.G4 + (G5 + G6)

G3 = G7 + A1(c) + S1 + M1(c) + N1(c) + F1

G4 = G8 + A2(c) + S2 + M2(c) + N2(c) + F2

substituting for G7; = [(A1(o).M1(o))]

G3 = A1(o).M1(o)) +A1(c) + S1 + M1(c) + N1(c) + F1

substituting for G8; [+(A2(o).M2(o))]

G4 = A2(o).M2(o)) + A2(c) + S2 + M2(c) + N2(c) + F2

G1 = G3.G4 = A1(o).M1(o).A2(o).M2(o) + A1(o).M1(o).A2(c) + A1(o).M1(o). S2 + A1(o).M1(o). M2(c) + A1(o).M1(o). N2(c) + A1(o).M1(o). F2 + A1(c). A2(o).M2(o) + A1(c). A2(c) + A1(c). S2 + A1(c). M2(c) + A1(c). N2(c) + A1(c). F2 + S1. A2(o).M2(o) + S1. A2(c) + S1. S2 + S1. M2(c) + S1. N2(c) + S1. F2 + M1(c). A2(o).M2(o) + M1(c). A2(c) + M1(c). S2 + M1(c). M2(c) + M1(c). N2(c) + M1(c).F2 + N1(c). A2(o).M2(o) + N1(c).A2(c) + N1(c). S2 + N1(c). M2(c) + N1(c). N2(c) + N1(c). F2 + F1. A2(o).M2(o) + F1. A2(c) + F1. S2 + F1. M2(c) + F1. N2(c) + F1. F2

G5 = G9.G10

G9 = A1(o).M1(o).

G10 = N2(o) + R2

G5 = A1(o).M1(o).(N2(o) + R2

= A1(o).M1(o).N2(o) + A1(o).M1(o). R2

G6 = G11.G12

G11 = A2(o).M2(o)

G12 = N1(o) + R1

G6 = A2(o).M2(o).(N1(o) + R1)
= A2(o).M2(o).N1(o) + A2(o).M2(o).R1

G2 = G5 + G6
= A1(o).M1(o).N2(o) + A1(o).M1(o). R2 + A2(o).M2(o).N1(o) + A2(o).M2(o).R1

T = G1 + G2

= A1(o).M1(o).A2(o).M2(o) + A1(o).M1(o).A2(c) + A1(o).M1(o). S2 + A1(o).M1(o).M2(c) + A1(o).M1(o). N2(c) + A1(o).M1(o). F2 + A1(c). A2(o).M2(o) + A1(c). A2(c) + A1(c). S2 + A1(c). M2(c) + A1(c). N2(c) + A1(c). F2 + S1. A2(o).M2(o) + S1. A2(c) + S1. S2 + S1. M2(c) + S1. N2(c) + S1. F2 + M1(c). A2(o).M2(o) + M1(c). A2(c) + M1(c). S2 + M1(c). M2(c) + M1(c). N2(c) + M1(c).F2 + N1(c). A2(o).M2(o) + N1(c).A2(c) + N1(c). S2 + N1(c). M2(c) + N1(c). N2(c) + N1(c). F2 + F1. A2(o).M2(o) + F1. A2(c) + F1. S2 + F1. M2(c) + F1. N2(c) + F1. F2 + A1(o).M1(o).N2(o) + A1(o).M1(o). R21 + A2(o).M2(o).N1(o) + A2(o).M2(o).R1

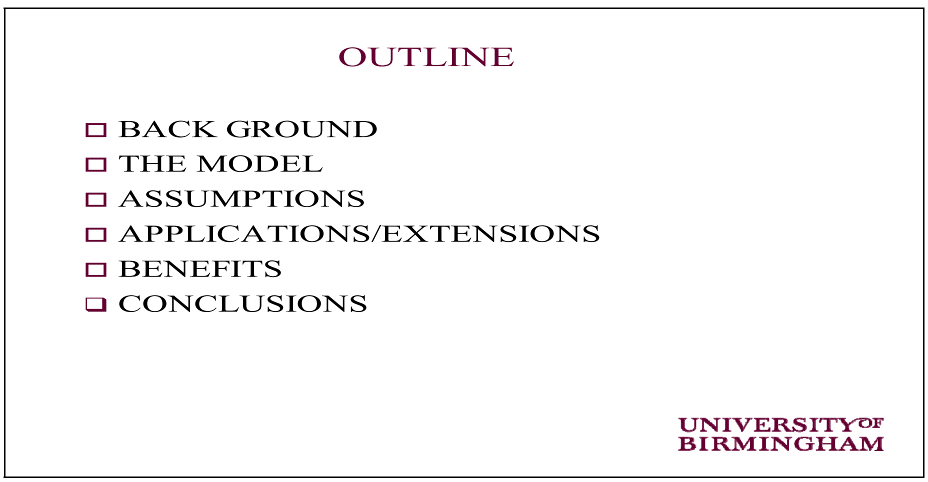
No further reduction is possible giving a very unwieldy expression which contains the following;

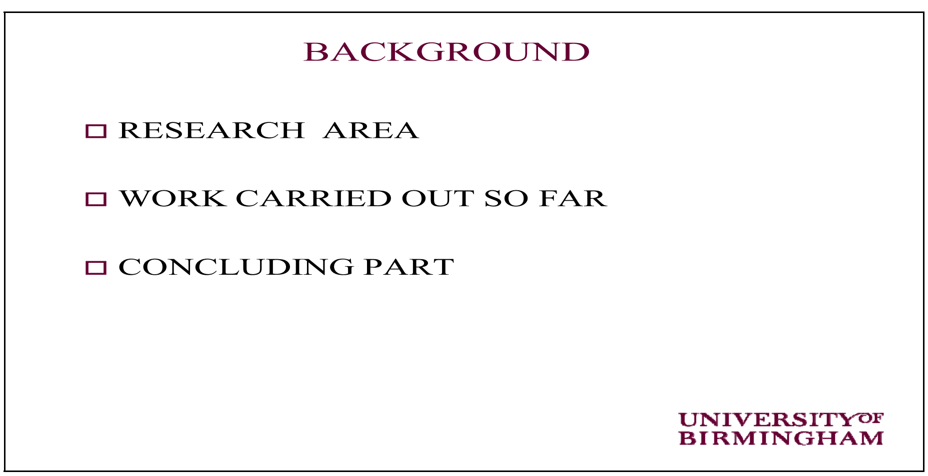
- 0 - 1st order cut sets
- **25 - 2nd order cut sets**
- 14 - 3rd order cut sets
- 1 - 4th order cut set

The most significant result is that there are 25 permutations for any two failures to cause a failure to supply. No single failures would result in failure to supply. The 3rd and 4th order cut sets (combinations) can effectively be ignored as the probabilities of these occurring are far smaller than the second order cut sets.

E. THE MCO MODEL PRESENTED IN POWER POINT TO THE GAS SUPPLY COMPANY

E1 THE SLIDES





THE MODEL

MAXIMISE: [RSUB1, RSUB2,...,RSUBK]
MININIMISE: [COST OF INCREASING RELIABILITY]

SUBJECT TO: MIN CONSTRAINT ON RSUBi, i=1,2,...,K
MIN & MAX CONSTRAINTS ON COMPONENT RELIABILITIES

CRITERIA: SUBSYSTEM RELIABILITY & COST
DECISION VARIABLES: COMPONENT RELIABILITY

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ASSUMPTIONS

- SERIES-PARALLEL SYSTEM
- NETWORKS REDUCIBLE TO SERIES-PARALLEL
- COST = SUM OF COMPONENTS' COST AT GIVEN RELIABILITY
- INDEPENDENT COMPONENTS
- BINARY SYSTEM

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PURPOSE

- RELIABILITY IMPROVEMENT OF EXISTING DESIGNS
- OPTIMAL RELIABILITY FOR NEW DESIGNS
- COST –BENEFIT OF RELIABILITY ANALYSIS

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BENEFITS

- COMPONENT RELIABILITY NECESSARY TO MAX. SYSTEM RELIABILITY AT MIN. COST
- VARIETY OF POTENTIAL DESIGNS
- TRADE-OFFS BETWEEN COST AND RELIABILITY
- SENSITIVITY OF THE SYSTEM

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BENEFITS (CONT.)

EXAMPLE 1: BREAKER-AND-HALF SYSTEM (FF=0.9)

COMPT CODE	OUTAGE RATE/YR	INITIAL RELIABILITY	ACHIEVED RELIABILITY
0014	0.8600	0.42232	0.9900
0007	0.0340	0.9666	0.9855
0008	0.0560	0.9455	0.9620
0005	0.0230	0.9773	0.9773
0002	0.1800	0.8353	0.9999
0013	0.9300	0.3946	0.9194
0016	0.8800	0.4148	0.8895
0006	0.0700	0.9324	0.9417
0001	0.2000	0.8187	0.8443
0004	0.0760	0.9268	0.9381
0003	0.0900	0.9139	0.929

INITIAL SYSTEM RELIABILITY = 0.27843
OPTIMISED SYSTEM RELIABILITY = 0.947601
COST OF OPTIMISED YSTEM RELIABILITY = 301.23

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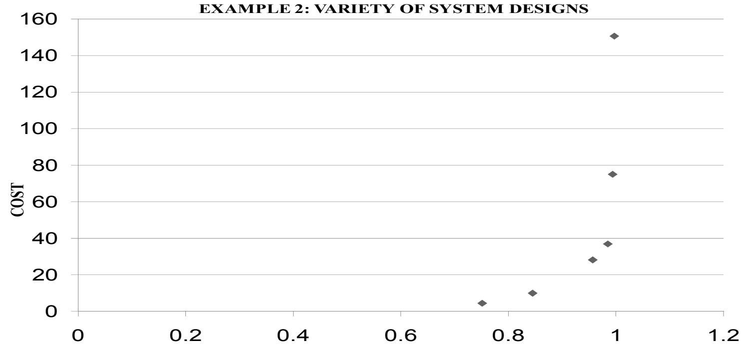
BENEFITS (CONT.)

EXAMPLE 2: VARIETY OF SYSTEM DESIGNS



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CONCLUSIONS

MODEL:

- DECISION-MAKING AID/TOOL
- RELIABILITY DESIGN OF NEW SYSTEMS
- RELIABILITY IMPROVEMENT OF EXISTING SYSTEMS
- RELIABILITY DESIGN OF SERIES-PARALLEL SYSTEMS

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THANK YOU

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F. LIST OF JOURNAL ARTICLES

F1 JOURNAL ARTICLES IN PREPARATION

Twum S.B., Aspinwall E., Fliege J. (submitted, 2009) A Multi-criteria Optimisation Model for Reliability Design of Series-Parallel Systems - Part 1, International Journal of Quality and Reliability Management.

Twum S.B., Aspinwall E., Fliege J. A Multi-criteria Optimisation Model for Reliability Design of Series-Parallel Systems - Part II.

Twum S.B., Aspinwall E., Fliege J. A review of Applications of Multi-criteria Optimisation in Reliability Design.

Twum S.B., Aspinwall E., Fliege J. Estimate of the Financial Cost of Reliability Improvement in a system, derived from an Analytical Cost/Penalty Function.