Essays on Unit Root Testing in Panel Data

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Abstract

This thesis discusses some issues on unit root testing in panel data. It first examines the intra-China price convergence by employing panel unit root tests that take cross-sectional dependence into account. Contrast to the existing literature, where tests assuming independence are employed and PPP is found in the vast majority of goods/services prices, our study finds mixed evidence in favour of PPP. Mixed panels with both $I(1)$ and $I(0)$ units are then considered, a large scale simulation study is undertaken. Size/power of panel unit root tests are examined under a variety of DGPs. A battery of procedures designed for mixed panels are employed, and their performance are examined by simulation. An application on intra-China PPP shows that, on average, only a small proportion of stationary units can be found in relative price panels. We then consider fractionally integrated processes and propose two different types of panel fractional integration test, a Fisher-type test and a multiple testing procedure that controls the false discovery rate (FDR) and classify units into null and alternative. Simulation evidence is provided. Empirical application shows that, in our intra-China PPP study, strong evidence can be found against the unit root null.

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Contents

Chapter 1

Introduction

Panel data, also called longitudinal data or cross-sectional time series data, are data where multiple units are observed at two or more time periods. There are two kinds of information in panel data: the cross-sectional information reflected in the differences between units, and the time series information reflected in the changes within units over time. Panel data testing and estimation techniques allow us to take advantage of these different types of information.

Considerable research has been undertaken on panel unit root test in the last two decades. For some literature surveys, see Banerjee and Wagner (2009) and Breitung and Pesaran (2008). Panel methods can exploit time series dimension as well as cross sectional dimension, and have been widely employed in testing the Purchasing Power Parity (PPP) and output convergence. Panel techniques have also been employed more recently to more areas of research, for example, housing price convergence, regional migration, etc.

It is well known that time series unit root tests lack power when sample size is small and auto-regressive root is close to unity. One of the main motivations of applying unit root tests to a panel of cross-sectional units is to increase the power of their univariate counterparts. In many applications, this was supported by employing the so called first generation panel unit root tests, which assume cross-sectional independence. For example, in PPP literature, time series unit root tests typically do not reject the unit root null hypothesis, result in the failure of PPP. However, Frankel and Rose (1996), Papell (1997), Parsley and Wei (1996) and Cecchetti et al. (2002) find evidence in favour of the alternative hypothesis when panel unit root tests are employed, showing that relative exchange rates are stationary and PPP holds.

However, the assumption of cross-sectional independence may be to restrictive, especially in empirical studies. Hlouskova and Wagner (2006) use Monte Carlo simulation to show that, when units in a panel are correlated, tests assuming cross-sectional independence have severe size distortion. This means that these tests reject the unit root null hypothesis too often when a panel is generated under the null, hence a rejection can be misleading. To overcome this problem, some panel unit root tests are proposed that take cross-sectional dependence into account.

Another problem of panel unit root tests is that, it is difficult to interpret the result when the unit root null hypothesis is rejected. Consider a simply dynamic panel $y_{i,t}$:

$$y_{i,t} = (1 - \phi_i)\mu_i + \phi_i y_{i,t-1} + \varepsilon_{i,t} \tag{1.1}$$

where i is cross-sectional index, $i = 1, \cdots, N$, t is time series index, $t = 1, \cdots, T$. Throughout this thesis, the term panel data refers to two-dimensional data involving time series and cross-sectional data. Re-arrange this equation we have:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \varepsilon_{i,t} \tag{1.2}$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$, $\alpha_i = (1 - \phi_i)\mu_i$, and $\rho_i = (\phi_i - 1)$. This panel is constructed in such a way that, under the null hypothesis, $\alpha_i = 0$ and $\rho_i = 0$ for all i, each series in the panel is a pure random walk. Under the alternative hypothesis, each series is a stationary autoregressive process with constant term.

Panel unit root tests are testing the null hypothesis that $\rho_i = 0$ for all i, against either the homogeneous alternative that $\rho_i = \rho < 0$ for all i, or the heterogeneous alternative that $\rho_i < 0$ for some i. For a test that assumes heterogeneous alternative, if the null is rejected, we have little information on how many unit are stationary and which units are stationary. In fact, a rejection can be caused by only a few stationary units or an overwhelming majority of stationary units, and this may have very different economic meanings. For a test that assumes homogeneous alternative, a rejection can be easily interpreted. However, panels can be heterogeneous (or mixed) in empirical studies, and the homogeneous assumption may be too restrictive. Westerlund and Breitung (2013) show that many of these tests still have power when only some of the units are stationary, therefore, a rejection of the null hypothesis does not provide convincing evidence in favour of the alternative. Recently, some procedures that account for mixed panels are proposed. These procedures aimed to provide more information about the proportion of units under null/alternative hypotheses, and which units are stationary/nonstationary.

Some researches on panel fractional integration test have been undertaking in the past few years. Time series such as real exchange rate and aggregate output may exhibit long-memory behaviour. In empirical studies with panel data, economic variables may have fractional order of integration, not necessarily $I(1)$ or $I(0)$. Therefore, it may be desirable to test the unit root null hypothesis against the fractional alternative $I(d)$, $d < 1$ in panel data. A few tests have been proposed and there are some working papers on this topic.

Above mentioned issues are the motivations of this thesis. We start our analysis from an empirical PPP study in Chapter 2. We collect 15 aggregate and disaggregated price indices from 36 major Chinese cities over 99 months, and test for the intra-China price convergence for different goods and services. Existing studies on intra-China price convergence employ first generation unit root tests, which assumes cross-sectional independence, and strong evidence is found in favour of PPP for the vast majority of goods and services. For example, see Fan and Wei (2006), Lan and Sylwester (2010). However, dependence of price indices within a country is expected to be high, as markets within one economy suffer no exchange rate volatilities and are considered have less trade barriers, and some common factors may have more influential impact on price indices within a country. Hlouskova and Wagner (2006) show that tests ignore cross-sectional dependence have severe size distortion. Thus, the existing results in favour of PPP within China may be misleading due to the size distortion of tests employed. We extend the intra-China PPP studies by employing the so called second generation panel unit root tests, which account for cross-sectional dependence. Tests that are employed are Pesaran (2007b), Moon and Perron (2004), Breitung and Das (2008), and Bai and Ng (2004). Although evidence is found in favour of PPP in aggregate CPI, little evidence is found for nonperishable consumer goods, and mixed evidence is found against PPP for service prices.

In Chapter 3, we consider the issue of mixed panel, which consist of both $I(1)$ units and $I(0)$ units. A large scale simulation study is performed in this Chapter. For the data generating process (DGP), we considered a set of dimensions $\{T, N\}$, sources of nonstationarity, proportion of units under null hypothesis (θ), number of common factors, and magnitude of cross-sectional dependence. For the purpose of illustration, in the main text, we focus on the performance of tests with respect to dimensions $\{T, N\}$, sources of nonstationarity, and proportion of units under null hypothesis (θ). Full results are provided in Appendix B.1 and B.2.

We first examine the size and power of panel unit root tests employed in Chapter 2 under different DGPs. Results show that when tests are valid under certain DGPs, some of them are very sensitive to a small proportion of units under the alternative, result in high power even if only 10% units are stationary, for example the Pesaran (2007b) test and Bai and Ng (2004) $P_{\hat{e}}$ test on idiosyncratic component. Therefore, a rejection of null hypothesis may come from either a small proportion of stationary units or the vast majority of units are stationary. This further confirms our motivation that, we should find more information to interpret the rejection. Power of other tests, on the other hand, grows gradually with greater proportion of units under alternative hypothesis.

We then simulate the performance of a battery of procedures that account for mixed panels. Tests being employed are, Chortareas and Kapetanios (2009), Romano and Wolf (2005), Moon and Perron (2012), Smeekes (2011), and Ng (2008). We examine whether the tests can estimate the stationary/nonstationary proportion correctly, and whether they can classify each cross sectional unit into stationary group and nonstationary group. Results show that satisfactory performance of tests requires $T \geq 100$. Procedures using bootstrapped critical values provide more reliable results if the underlying DGP is unknown, in which the procedure by Moon and Perron (2012) shows impressing good performance.

An application is provided by employing the procedures that account for mixed panels on the same data set in Chapter 2. We find evidence that, on average, the proportion of stationary units is small. Even for the group of perishable goods, where strong evidence can be found in favour of PPP by panel unit root tests, the stationary proportion is about only a quarter to 2 fifths.

In Chapter 4, we consider panel fractional integration test. We employ the univariate local Whittle (LW) test and exact local Whittle (ELW) tests proposed by Shimotsu and Phillips (2005, 2006) to construct two different kind of panel fractional integration tests.

The first test proposed is the Fisher-type test, which combines the p-values from univariate fractional integration tests. Suppose a cross sectional unit i , $i = 1, \cdots N$, follows a fractionally integration process $I(d_i)$, we are interested in testing the unit root null hypothesis that $d_i = 1$ for all i, against the heterogeneous long-memory alternatives that $d_i < 1$ for some i. The Fisher test has χ^2 distribution if independence is assumed. When cross-sectional units are dependent, we bootstrap the critical values of Fisher statistic, as suggested by Maddala and Wu (1999).

The second testing procedure accounts for mixed panels, where panels consist of both $I(1)$ and $I(d)$, $d < 1$ units. The idea is similar to its counterparts in panel unit root testing, the testing procedure should estimate the proportion of units under null and alternative hypothesis, and classify each unit into the null group and alternative group. Our study uses the concept of false discovery rate (FDR) in multiple testing literature, and employs the bootstrap procedure by Romano et al. (2008) to control the FDR and dependence structure. Hypothesis testing is then performed and units are classified into null group and alternative group.

Simulation experiment shows that our Fisher-type test is generally correctly sized, and is sensitive to a small proportion of units under the alternative, especially when the value of d_i under alternative is in the stationary region $(0, \frac{1}{2})$. It is shown that our multiple testing procedure successfully control the FDR at 5% in all DGPs. When $d_i = 0.3$ under the alternative, the classification mechanism works well even if $T = 50$. When $d_i = 0.7$ under the alternative, however, a satisfactory performance requires $T = 200$.

To consistent with our empirical analysis in Chapter 3, we apply our multiple testing procedure on the same intra-China price indices data set. Contrast to the results in Chapter 3, where strong evidence is found in favour of unit root null hypothesis if we classify units into $I(1)$ and $I(0)$, our finding here is that we find strong evidence against unit root null if we classify units into $I(1)$ and $I(d)$, $d < 1$. This may indicate that the relative price indices are neither $I(1)$ nor $I(0)$, but exhibit long-range dependence behaviour.

The remainder of this thesis is organised as follows. Chapter 2 examines the intra-China PPP by applying unit root tests that take cross-sectional dependence into account. In Chapter 3, a large scale simulation study is performed on a battery of panel unit root tests and a battery of procedures that account for mixed panels. In Chapter 4, we propose two kind of panel fractional integration test. A Fisher-type test and a multiple testing procedure, which takes mixed panels into account. Chapter 5 concludes.

Chapter 2

Price Indices Convergence Among Chinese Cities

2.1 Introduction

Purchasing Power Parity (PPP) states that in the long run the equilibrium exchange rates should make different currencies have the same purchasing power in different countries. The equilibrium exchange rate between two countries should be equal to the ratio of the two countries' prices of an identical basket of goods and services. PPP plays an important role in economic theories, and is often used to examine the overvaluation or undervaluation of currencies and predict the long run trend of exchange rates.

There are plenty of studies on the PPP among countries. When testing the post-1973 floating exchange rate between countries using univariate unit root tests, the random walk null hypothesis typically can not be rejected[1]. This means that shocks on real exchange rate can last permanently, and PPP fails to hold between the two countries. However, univariate unit root tests suffer from the problem of low power, i.e., they may fail to reject the random walk null while the series is actually stationary. To overcome this problem, researchers have tried to increase the power of the tests by increasing the time length of the series, see Frankel (1986), Edison (1987), Froot et al. (1995) and Lothian and Taylor (1996). Recently, panel data sets have been employed by researchers, for example, Frankel and Rose (1996), Papell (1997), Parsley and Wei (1996) and Cecchetti et al. (2002). These studies are more successful in rejecting the unit root null, implying that shocks on relative prices can not last permanently and relative prices will converge to a common mean. Estimates of the half-life of a deviation from the PPP are usually between four to six years, some studies are, Abuaf and Jorion (1990), Frankel and Rose (1996), Wu (1996), MacDonald (1996), Papell (1997), Lothian (1997) and Wei and Parsley (1995).

Some research works consider prices convergence within one country. Markets within one economy suffer no exchange rate volatilities and are considered have less trade barriers. Furthermore, factors are more integrated in markets within a country. So the prices within a country are expected to be mean-reverting and the convergence rate should be faster than cross-country convergence. Engel and Rogers (1996) show that the border between US and Canada significantly influences the variation of prices among cities. They show that the variation of price is

[1] An excellent literature survey is by Froot and Rogoff (1995)

much higher for two cities located in different countries than those located in one country with same distance. Ceglowski (2003) uses 45 consumer goods prices across 25 Canadian cities to examine the intranational PPP and finds that a majority of relative prices are stationary and the half-lives average well under a year. Parsley and Wei (1996) examine the convergence of prices among 48 US cities and estimate the half-life of tradable goods is roughly one year, and the half-life of non-tradable goods is about four years. Cecchetti et al. (2002) examine 19 US cities but conclude that the half-life of convergence is about nine years. Sonora (2005) uses CPI for 34 Mexican cities and rejects the panel unit root null hypothesis by using Levin et al. (2002) LLC test and Im et al. (2003) IPS test. Estimated half-lives fail in the 1.4 years to 2.7 years range. Morshed et al. (2006) employ impulse response function to calculate half-lives between Indian cities and find that the half-life is close to 3 months. Fan and Wei (2006) examine the price convergence within China, they collect the price data of 97 products and services in 36 Chinese cities and conclude that the law of one price holds in China for an overwhelming majority of the goods and services. They apply the ADF test and Fisher type test proposed by Maddala and Wu (1999) and get half-lives estimation from ADF regressions. The half-lives range between 1.66 months and 2.44 months for goods and services. Lan and Sylwester (2010) examine the intra-China purchasing power parity by employing the Levin et al. (2002) test, Im et al. (2003) test, and Fisher-ADF test. They find evidence in favour of PPP for the vast majority of goods and services. They use impulse response functions to obtain half-lives and the result is below 2 months for goods prices.

Some resent research uses methods that take cross-sectional dependence into account and examine the purchasing power parity. Das and Bhattacharya (2008) examine PPP among regions in India and decompose relative prices into a set of common factors and idiosyncratic components, unit root tests are then applied on the two components separately. They find that relative prices among regions are mean-reverting with a half-life for the common factor component at around 9 months and the half-life for the idiosyncratic component at around 20 months. Nagayasu and Inakura (2009) analyse the convergence of CPI among 47 Japanese municipalities and find evidence in favour of PPP, the estimated half-life is 2 years. Yazgan and Yilmazkuday (2011) applies the Pesaran (2007b) test on 48 goods prices for 52 US cities and find strong evidence in favour of PPP, the estimated half-life is 1.37 quarters for tradable goods and 2.75 quarters for non-tradable goods, which are significantly lower than the half-lives in previous studies. Basher and Carrion-i Silvestre (2009, 2011) use panel stationary tests allowing for multiple structure breaks and cross-sectional dependence to detect the price convergence over 17 US cities and find that relative prices are $I(0)$ processes and median half-lives range between 1.5 years and 2.6 years.

This study intends to investigate price index convergence in China by using aggregated CPI and 14 disaggregated price indices among 36 Chinese cities. There are basically two motivations to investigate CPI and disaggregated Chinese data. First, as mentioned earlier, there are several studies examine the price index movements in some developed economies, for example, Cecchetti et al. (2002) examine the price convergence among US cities. In relation to this, it would be an interesting comparative study to investigate price convergence among Chinese cities, because China has similar geographic expanse but its infrastructure is less developed and its market is believed to be less integrated compared to the US. Second, China is a transitional economy from a planned economy to a market economy. As price distortion is high in a planned economy, it is interesting to test price convergence in Chinese cities and see whether the liberalization of domestic trade has eased price distortion. From this point of view, this study may give some evidence on whether China has become a market economy.

Our empirical study examines price convergence among Chinese cities by using a monthly data set that consists of the aggregated consumer price index (CPI) and 14 disaggregated price indices for different commodities and services from 36 major cities over 99 months (from January 2003 to March 2011). We employ a battery of panel unit root tests that take cross-sectional dependence into account, proposed by Pesaran (2007b), Moon and Perron (2004), Breitung and Das (2008), and Bai and Ng (2004). We apply the tests to the balanced panel of monthly price indices of the 36 cities.

Our study extends previous intra-China purchasing power parity studies. Compared to existing studies, it applies panel unit root tests that account for cross-sectional dependence, which is expected to be high within a country due to single currency, high market integration, and lower trade barriers. Panel unit root tests that fail to account for cross-sectional dependence if it is present will suffer size distortions, i.e., they will favour the stationary alternative while the data is actually non-stationary, see Hlouskova and Wagner (2006).

Our econometric results show overwhelming support for purchasing power parity for aggregate CPI and prices of perishable consumer goods. The half-lives are estimated at 1.81 months for CPI, and 2.06 months or 10.95 months for perishable consumer goods, depending on the testing methods used. The half-life is surprisingly small compare to the US and Canada, see Parsley and Wei (1996), Cecchetti et al. (2002), and Ceglowski (2003). Unlike previous studies on intra-China PPP, we find little evidence to support PPP for nonperishable consumer goods, and the evidence for services is mixed. Cities that are further apart have slower convergence rates and lager half-lives.

The remainder of the chapter is divided into 4 sections. Section 2.2 describes the data set used in more detail and presents some basic descriptive statistics. Section 2.3 discusses the panel unit root tests used in the study. In section 2.4 we report the econometric test results based on the tests as well as the estimation of convergence rates and half-lives. We then examine the role of distance in explaining the convergence rate and half-lives. Section 2.5 concludes.

2.2 Data and Descriptive Statistics

There are 15 balanced panels in data set, i.e., 15 different price indices. Monthly price data are collected form 36 major cities in China[2]. Placemarks of the cities are plotted on a map and is provided in Appendix A.1. The 15 panels have the same time span, from January 2003 to March 2011. That is to say, for each panel, the time dimension is 99 and cross-sectional dimension is 36. In the following analysis, Shanghai is used as the numeraire city to calculate the relative price series[3]. The price indices[4] of January 2002 are set to be 100.

One of the 15 price indices is the aggregated consumer price index (CPI). One of the price

[2]The 36 cities are: Beijing, Tianjin, Shijiazhuang, Taiyuan, Huhehaote, Shenyang, Dalian, Changchun, Haerbin, Shanghai, Nanjing, Hangzhou, Ningbo, Hefei, Fuzhou, Xiamen, Nanchang, Jinan, Qingdao, Zhengzhou, Wuhan, Changsha, Guangzhou, Shenzhen, Nanning, Haikou, Chongqing, Chengdu, Guiyang, Kunming, Lhasa, Xian, Lanzhou, Xining, Yinchuan, Urumuqi.

[3]Though it is not shown, we also perform tests we employed in this study by using national level price indices, in order to check the robustness. In other words, we also check price convergence to the national level, which can be considered as a weighted average, in additional to price convergence to Shanghai. The results obtained from national price indices are similar to the results reported in this chapter and are available upon request.

[4]Data source: National Bureau of Statistics of China.

indices is the food index, which has 6 sub-indices: grain, meat, eggs, seafood, fresh vegetables, fresh fruits. These 6 sub-indices are subsets of the food index but not a complete basket of food. The remaining 7 indices are parallel to the food index: tobacco and wine, clothing, recreation facilities, health care, traffic and communication, cultural and educational expense, residence. For the purpose of analysis, we have re-grouped the 13 price indices of specific goods or services (except CPI and food index) into three main categories: perishables, nonperishables and services. This classification is used to facilitate the presentation of the study and is of course subjective, but this classification scheme will not change the basic results obtained. The perishable category consists of five goods, all of them are food: meat, eggs, seafood, fresh vegetables, fresh fruits. Nonperishable category consists of three goods: grain, tobacco and wine, clothing. While the remaining 5 indices belong to the service category: recreation facilities, health care, traffic and communication, cultural and educational expense, residence. The classification is summarized in Table 2.1.

Table 2.1: Price Indices Classification

Perishables:	Tobacco and Wine
Meat	Clothing
Eggs	
seafood	Services:
Fresh Vegetables	Recreation Facilities
Fresh Fruits	Health Care
	Traffic and Communication
Nonperishables:	Cultural and Educational Expense
Grain	Residence

We now turn to some basic descriptive statistics of the data set. We generate the relative prices time series for each city. The price index of Shanghai is used as the base level, so the relative prices measure the city price deviations from the price level in Shanghai. These relative prices are analogous to the real exchange rates in the cross country context, except that the nominal exchange rate is 1. Similar to Cecchetti et al. (2002) and O'Connell and Wei (2002) and Fan and Wei (2006), the relative price is defined as:

$$q_{i,t} = \ln(\frac{P_{i,t}}{P_{s,t}}) = \ln(P_{i,t}) - \ln(P_{s,t}) = p_{i,t} - p_{s,t} \quad (2.1)$$

where $q_{i,t}$ is the logarithm of relative prices of province i at time t, $P_{i,t}$ is the price index of province i at time t, $P_{s,t}$ is the price index of Shanghai at time t.

Following the procedure in Parsley and Wei (1996), we calculate the variability of price differentials and mean absolute price differentials for each of the three categories: perishables, nonperishables and services. The variability of price differentials is defined as the standard deviation of $q_{i,t}$, the relative price, over time. For example, there are 5 goods in the perishable category, and for each good there are 35 city pairs, so there are overall $5 \times 35 = 175$ time series, and hence 175 observations of variability of price differentials. Mean and standard deviation will be taken over the 175 observations. The mean absolute price differentials is defined as the mean of $|\ln(P_{i,t}/P_{s,t})|$ over time. The two measures both intuitively describe how far the prices deviate from the numeraire city over time. The results are shown in Table 2.2:

Table 2.2: Descriptive Statistics

	Variability of Price Differentials		Mean Absolute Price Differentials	
Product Group	Mean	Standard Deviation	Mean	Standard Deviation
Perishables (175)	0.121	0.078	0.134	0.083
Nonperishables (105)	0.061	0.030	0.084	0.051
Services (175)	0.041	0.018	0.064	0.036

Note: numbers in the parentheses are the number of observations in each product group.

From Table 2.2 we can see that, for both the variability of price differentials and mean absolute of price differentials, the perishables have on average, the highest value. Meanwhile, the nonperishables and services have similar but smaller values than perishables. These results are very similar to the results obtained by Parsley and Wei (1996), in which the perishables in the US has the largest variability and mean absolute of price differentials, and the values of nonperishables is only slightly greater than the value of services. The highest mean absolute differentials of perishables (meat, eggs, seafood, vegetables, fruits in our data set) indicates that the price of perishable food is sensitive to the place of production due to the fact that the arbitrage cost of perishables is high.

Although the products being investigated in Parsley and Wei (1996) is different from our data set, it is quite surprising that for both nonperishables and services, the variability of price differentials and mean absolute of price differentials are smaller in China than in the US. This may show that the Chinese domestic market is quite efficient and integrated.

One of the natural benchmarks of the price differentials between cities would be zero, which corresponds to zero arbitrage cost. However it is widely believed that the price of the same good in two different cities may differ from one to the other, but the difference is bounded by the arbitrage cost between the cities. The price difference increases with arbitrage cost, which is usually proxied by distance between cities. We therefore collect the distance between each city and the numeraire city, Shanghai. Engel and Rogers (1996) use the "greater circle distance" between the cities to measure the distance. Due to the lack of official city coordinates, however, the "greater circle distance" is not employed in our study. We collect the driving distance between the cities by using the driving distance calculator on Google Maps(`maps.google.com`). When multiple driving routes are available, the shortest route is selected.

We then examine whether the variability of price differences increases with arbitrage cost and whether the size of price differences increases with arbitrage cost. For each of the three groups, the standard deviation of $q_{i,t}$ is firstly regressed on the log of distance and product dummy variables. A squared log distance is augmented into the regression to detect the possible convexities:

$$s.d.(q_{i,j,t}) = \alpha \ln(\text{distance}_i) + dummies \tag{2.2}$$

$$s.d.(q_{i,j,t}) = \alpha \ln(\text{distance}_i) + \beta \ln^2(\text{distance}_i) + dummies \tag{2.3}$$

where $q_{i,j,t}$ is the relative price between city i and the numeraire city for commodity j at time t, $s.d.(q_{i,j,t})$ is the standard error of the time series $q_{i,j,t}$ over the whole time span. $\ln(\text{distance}_i)$ is the log distance between city i and the numeraire city. Value of jth dummy variable equals one if product is j, and zero otherwise. Similarly, the mean absolute value of $q_{i,j,t}$ is regressed on log distance and squared log distance:

$$\text{mean}(|q_{i,j,t}|) = \alpha \ln(\text{distance}_i) + dummies \tag{2.4}$$

$$\text{mean}(|q_{i,j,t}|) = \alpha \ln(\text{distance}_i) + \beta \ln^2(\text{distance}_i) + dummies \tag{2.5}$$

Results are shown in Table 2.3. The results for the variability of relative prices are reported

Table 2.3: Relative prices and distance

Panel A: Variability of relative prices	Perishables		Nonperishables		Services	
Regression number	1	2	3	4	5	6
ln(distance)	0.0263	0.0373	0.0120	0.0157	0.0057	0.0041
	(0.0010)	(0.0058)	(0.0005)	(0.0034)	(0.0004)	(0.0022)
ln(distance)-squared		-0.0015		-0.0005		0.0002
		(0.0008)		(0.0005)		(0.0003)
product dummies	yes	yes	yes	yes	yes	yes
R^2	0.6856	0.6924	0.5295	0.5352	0.1352	0.1382
σ of the regression	0.0440	0.0437	0.0205	0.0205	0.0168	0.0168
Number of observations	175	175	105	105	175	175

Panel B: Mean absolute of relative prices	Perishables		Nonperishables		Services	
Regression number	7	8	9	10	11	12
ln(distance)	0.0243	0.0268	0.0166	0.0152	0.0103	0.0086
	(0.0012)	(0.0069)	(0.0010)	(0.0073)	(0.0008)	(0.0047)
ln(distance)-squared		-0.0003		-0.0002		0.0002
		(0.0009)		(0.0010)		(0.0006)
product dummies	yes	yes	yes	yes	yes	yes
R^2	0.6171	0.6174	0.2930	0.2932	0.0772	0.0780
σ of the regression	0.0519	0.0520	0.0437	0.0439	0.0351	0.0352
Number of observations	175	175	105	105	175	175

Note: numbers in parenthesis are standard errors.

in Panel A. Distance has a positive and very significant explaining power on the variability. The coefficient is biggest for the perishables, and smallest for the services. Similar results are obtained for the mean absolute of relative prices in Panel B, where the effect of distance is positive and significant for all categories. Distance has the largest coefficient in perishable group. For both Panel A and B, the quadratic term in the regression is insignificant.

In the following section, we review the unit root tests that are employed in this chapter.

2.3 Methodology

The "traditional" approach to investigate the purchasing power parity is to apply the time series unit root tests (e.g. the ADF test) on the real exchange rate or the relative price. If the unit root null hypothesis is rejected, prices will converge in the long run and the purchasing power parity holds between the two countries/cities. If, however, the unit root null can not be rejected, any shock to the relative price will be permanent, and the PPP fails to hold.

It is well known that the time series unit root tests suffer from the problem of low power, i.e., they fail to reject the unit root null too often when the series is actually stationary, especially when the root is close to one. Levin et al. (2002) show that the use of a panel unit root test, which uses not only the time series dimension but also the cross-sectional dimension, can significantly increase the power of unit root tests. There are several panel unit root tests proposed, for example, Levin et al. (2002), Im et al. (2003), Maddala and Wu (1999). An important assumption of these tests is that the units are cross-sectionally independent. This is a restrictive assumption because countries, or cities within a country are affected by some common factors, e.g., technology, oil price, etc. When the cross-sectional dependence is present but the above tests are applied, there are severe size distortions, i.e., the tests may reject the null hypothesis too often compare to the nominal level, say 5%, see Hlouskova and Wagner (2006).

In this study, we employ a battery of panel unit root tests. The Im et al. (2003)(IPS test) is employed, which assumes cross sectional independence. A set of so called second generation panel unit root tests are employed, which account for the cross-sectional dependence. The tests are proposed by Bai and Ng (2004), Pesaran (2007b), Moon and Perron (2004), Breitung and Das (2008).

2.3.1 Im et al. (2003)

Im et al. (2003) propose a set of group-mean panel unit root tests to account for cross-sectional dependence and meanwhile allow for heterogeneity, including the heterogeneous dynamics. For the basic ADF regression:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \varepsilon_{i,t} \tag{2.6}$$

The null hypothesis of unit root is:

$$\mathrm{H}_0 : \rho_i = 0 \text{ for all } i \tag{2.7}$$

and the alternative hypothesis is:

$$\mathrm{H}_1 : \rho_i < 0,\ i = 1, 2, \cdots, N_1,\ \rho_i = 0,\ i = N_1 + 1, N_1 + 2, \cdots, N \tag{2.8}$$

where, $\dfrac{N_1}{N} = \delta,\ 0 < \delta \leq 1.$

Im et al. (2003) propose the IPS test based on the group mean of individual ADF statistics. The IPS test is considered for both serially uncorrelated and correlated cases, allowing for a constant and a constant and linear trend. They also propose that tests based on the principle of likelihood framework can be constructed, for example, the Lagrange Multiplier test. We concentrate here on the ADF-based t-bar test with constant only because of its popularity. Other test statistics can be constructed similarly and are referred to their original paper.

They consider a panel of time series $y_{i,t}$ is generated by a finite AR($p_i + 1$) processes:

$$y_{i,t} = \mu_i \phi_i(1) + \sum_{j=1}^{p_i+1} \phi_{ij} y_{i,t-j} + \varepsilon_{i,t} \tag{2.9}$$

where $i = 1, 2, \cdots, N$ and $t = 1, 2, \cdots .T$. The equation can be written as the ADF(p_i) processes:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \sum_{j=1}^{p_i} \psi_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{2.10}$$

where $i = 1, 2, \cdots, N$ and $t = 1, 2, \cdots .T$, $\phi_i(1) = 1 - \sum_{j=1}^{p_i+1} \phi_{i,j}$, $\alpha_i = \mu_i \phi_i(1)$, $\rho_i = -\phi_i(1)$, and $\psi_{i,j} = -\sum_{h=j+1}^{p_i+1} \phi_{i,h}$.

For each cross-section member, rewrite the ADF regression in the matrix form we have:

$$\Delta \boldsymbol{y}_i = \rho_i \boldsymbol{y}_{i,t-1} + \boldsymbol{Q}_i \gamma_i + \varepsilon_i \tag{2.11}$$

where $\boldsymbol{Q}_i = (\tau_T, \Delta \boldsymbol{y}_{i,t-1}, \Delta \boldsymbol{y}_{i,t-2}, \cdots, \Delta \boldsymbol{y}_{t-p_i})$, $\tau_T = (1, 1, \cdots, 1)'$ and $\gamma_i = (\alpha_i, \psi_{i,1}, \psi_{i,2}, \cdots, \psi_{i,p_i})'$.

Then the t-bar statistic, which is the cross-sectional mean of the individual t statistics for testing $\rho_i = 0$ in equation 2.10 can be obtained:

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_{i,T}(p_i, \boldsymbol{\psi}_i) \tag{2.12}$$

where individual t statistic is calculated by

$$t_{i,T}(p_i, \boldsymbol{\psi}_i) = \frac{\sqrt{T - p_i - 2}(y'_{i,t-1} M_{Q_i} \Delta y_{i,t})}{(y'_{i,t-1} M_{Q_i} \Delta y_{i,t})^{1/2} (\Delta y'_{i,t} M_{X_i} \Delta y_{i,t})^{1/2}} \tag{2.13}$$

$\psi_i = (\psi_{i,1}, \psi_{i,2}, \cdots, \psi_{i,p_i})'$, $M_{Q_i} = I_T - Q_i(Q_i'Q_i)^{-1}Q_i'$, $M_{X_i} = I_T - X_i(X_i'X_i)^{-1}X_i'$, and $X_i = (y_{i,t-1}, Q_i)$.

Im et al. (2003) show that for fixed time dimension T, the individual ADF statistics $t_{i,T}(p_i, \boldsymbol{\psi}_i)$ in equation 2.13 will depend on nuisance parameter $\psi_i = (\psi_{i,1}, \psi_{i,2}, \cdots, \psi_{i,p_i})'$ and p_i under the null hypothesis. Hence it is infeasible to standardize $t_{i,T}(p_i, \boldsymbol{\psi}_i)$ by using $E(t_{i,T}(p_i, \boldsymbol{\psi}_i))$ and $Var(t_{i,T}(p_i, \boldsymbol{\psi}_i))$. They propose two feasible standardizations. When T and N are sufficiently large, the individual statistics $t_{i,T}(p_i, \boldsymbol{\psi}_i)$ follow Dickey-Fuller distribution and are free from nuisance parameters. So the asymptotic mean and variance of Dickey-Fuller statistic can be used. In finite sample, the mean and variance of $t_{i,T}(p_i, 0)$ are evaluated under the null hypothesis. $E(t_{i,T}(p_i, 0)|\rho_i = 0)$ and $Var(t_{i,T}(p_i, 0)|\rho_i = 0)$ take the information of p_i into account, and depend on the time dimension T and ADF lag length $P_i = P$. IPS tabulate these correction terms for both cases of deterministic terms via stochastic simulations, see Table 3 in Im et al. (2003).

The IPS statistic is the cross-sectional mean of the individual t-statistics, subject to appropriate corrections:

$$IPS_{T,N} = \frac{\sqrt{N}\{\bar{t} - \frac{1}{N}\sum_{i=1}^{N} E(t_{i,T}(p_i, 0)|\rho_i = 0)\}}{\sqrt{\frac{1}{N} Var(t_{i,T}(p_i, 0)|\rho_i = 0)}} \tag{2.14}$$

The IPS statistic has a standard normal distribution under the null hypothesis and diverge to negative infinity under the alternative hypothesis.

Attention should be paid on the ADF lag length selection. First, it is not possible to simulate the correction terms for all the P_i combinations, especially when N is large. IPS impose the restriction that lag length are equal in all units to simulate the values of correction terms.

Second, IPS find that using the information criteria may lead to size distortion. They conduct simulation exercises where information criteria such as AIC and SBIC are employed to determine the ADF lag order. The tests show significant degree of size distortion. This is due to the fact that the information criteria are "not sufficiently conservative", in terms of selecting too few lags. IPS suggest to use an uniform lag length $P_i = P$, which ensures that the error term in each unit is white noise. Third, if the "step down" procedure starting from P_{max} is employed to determine the lag order, the P_{max} is often selected when N is large. However the P_{max} is not data dependent and the choice of ADF lag length ends up being driven by arbitrary choices.

2.3.2 Pesaran (2007b)

The assumption of cross-sectional dependence is believed to be too restrictive. Following the idea in Im et al. (2003) and to account for the cross-sectional dependence, Pesaran (2007b) modified the IPS test by augmenting the ADF test by the cross-sectional averages of the level and of lagged differences of the series. The idea is that when a single common factor is assumed, time effect can be used as a proxy for the common factor. The cross-sectional average is an estimate for the time effect, and heterogeneous responses to the time effect is allowed. Pesaran assumes a simple dynamic heterogeneous data generating process:

$$y_{i,t} = (1 - \phi_i)\mu_i + \phi_i y_{i,t-1} + u_{i,t},\ i = 1, \cdots, N,\ t = 1, \cdots, T \tag{2.15}$$

where the initial value $y_{i,0}$ is assumed to have a given density function with bounded expected value and variance, and the error term $u_{i,t}$ has a single factor form:

$$u_{i,t} = \lambda_i f_t + \varepsilon_{i,t} \tag{2.16}$$

where λ_i is individual-specific factor loading, and the mean of λ_i, $\bar{\lambda}$, is assumed to be non-zero as $N \rightarrow 0$, f_t is the unobserved single common factor, $\varepsilon_{i,t}$ is idiosyncratic error. The errors $\varepsilon_{i,t}$ are assumed to be independently distributed across i and t, with mean zero, variance σ_i^2, and finite fourth-order moment. Common effect f_t follows a general linear stationary process with mean zero and constant variance σ_f^2. $\varepsilon_{i,t}$, f_t, and γ_i are assumed to be independently distributed for all units. According to the assumptions, the only source of nonstationarity considered by Pesaran (2007b) is the autoregressive root $\phi_i = 1$ in equation 2.15.

The DGP can then be rewritten as:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \lambda_i f_t + \varepsilon_{i,t} \tag{2.17}$$

where $\alpha_i = (1 - \phi_i)\mu_i$, $\rho_i = -(1 - \phi_i)$ and $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$.

The unit root null hypothesis is that:

$$\mathrm{H}_0 : \rho_i = 0 \text{ for all } i \tag{2.18}$$

The heterogeneous alternative hypothesis is:

$$\mathrm{H}_1 : \rho_i < 0,\ i = 1, 2, \cdots, N_1,\ \rho_i = 0,\ i = N_1 + 1, N_1 + 2, \cdots, N \tag{2.19}$$

where $\dfrac{N_1}{N} = \delta,\ 0 < \delta \leq 1$ as $N \to \infty$.

The regression function without linear trend is:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + c_i \bar{y}_{t-1} + \sum_{j=0}^{p_i} d_{i,j} \Delta \bar{y}_{t-j} + \sum_{j=1}^{p_i} \psi_{i,j} \Delta y_{i,t-j} + e_{i,j} \tag{2.20}$$

where $\bar{y}_t = \frac{1}{N} \sum_{i=1}^{N} y_{i,t}$, $\Delta \bar{y}_t = \frac{1}{N} \sum_{i=1}^{N} \Delta y_{i,t}$, and $e_{i,t}$ regression errors.

The cross-sectionally augmented Dickey-Fuller (CADF) test statistic is defined as the t-ratio of the estimated autoregressive coefficient $\hat{\rho}_i$ in equation 2.20. The cross-sectional averages, $\bar{y}_{t-1}$ and $\Delta \bar{y}_{t-j},\ j = 1, \cdots, p_i$ are a proxy of the common factor. Pesaran (2007b) shows that the distribution of the CADF statistic is free of nuisance parameters when $N \to \infty$ with any fixed $T > 4$, as well as when N and T tend to infinity jointly with the same magnitude. To ensure that the moments of the CADF statistic exist in the finite sample, Pesaran proposes a truncated version of the CADF statistic, denoted as $CADF^*$. Pesaran (2007b) tabulates the critical values of $CADF$ and $CADF^*$, and their critical values are different only when T is very small($T = 10$ for constant and linear trend case and $T < 10$ for the other two cases).

Given that the null distribution of CADF statistic is asymptotically free from nuisance parameters, Pesaran (2007b) proposes three panel unit root tests. The first one is the cross-sectionally augmented version of IPS test, denoted as CIPS test:

$$CIPS = t\text{-bar} = \frac{1}{N}\sum_{i=1}^{N} CADF_i \tag{2.21}$$

where $CADF_i$ is cross-sectionally augmented Dickey-Fuller statistic defined by equation 2.20. Due to the existence of common factors, $CADF_i$ are dependent and central limit theorems are not applicable. However, $CIPS$ and $CIPS^*$ converge to a nonstandard distribution. Pesaran (2007b) tabulates the critical values for $CIPS$ and $CIPS^*$, the difference between which are indistinguishable when T is as large as 20.

Pesaran (2007b) considers combining the individual p-value proposed by Maddala and Wu (1999) and Choi (2001). The inverse chi-squared (Fisher) test statistic, denoted as CP test, is defined by:

$$CP = -2\sum_{i=1}^{N} \ln(p_i) \tag{2.22}$$

where p_i is the p-value from the ith individual $CADF$ test.

An inverse normal test statistic, CZ, is defined by:

$$CZ = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \Phi^{-1}(p_i) \tag{2.23}$$

In the presence of cross-sectional dependence, the CP and CZ are no longer normally distributed, hence their critical values are obtained by simulation.

2.3.3 Moon and Perron (2004)

The tests proposed by Moon and Perron (2004), denote $MP04$ test, is based on the t-ratio of a corrected pooled OLS estimator using the de-factored panel data, where the number of factors is allowed to be more than one and is taken to be known. The factors and their loadings are consistently estimated using principle component method.

Moon and Perron (2004) consider the DGP:

$$y_{i,t} = \alpha_i + y_{i,t}^0 \tag{2.24}$$

$$y_{i,t}^0 = \rho_i y_{i,t-1}^0 + u_{i,t} \tag{2.25}$$

$$u_{i,t} = \lambda_i' f_t + e_{i,t} \tag{2.26}$$

for $i = 1, \cdots, N$ and $t = 1, \cdots, T$, where ρ_i is the heterogeneous autoregressive root, f_t is a $K \times 1$ vector of common factors, λ_i is the vector of factor loadings to each cross-sectional member i, $e_{i,t}$ is the idiosyncratic error term. If the number of factors $K = 1$, the DGP is the same to the one used in Pesaran (2007b).

Some assumptions are made. The idiosyncratic error term $e_{i,t} = \sum_{j=0}^{\infty} d_{i,j} v_{i,t-j}$, where $v_{i,t}$ are i.i.d.(0,1) across i and over t, have finite eighth moment, $\inf_i \sum_{j=0}^{\infty} d_{i,j} > 0$ and $\bar{d}_j = \sup_i |d_{i,j}|$, $\sum_{j=0}^{\infty} j^m \bar{d}_j < M$ for some $m > 1$. The common factors $f_t = \sum_{j=0}^{\infty} c_j u_{t-j}$, where c_j is $K \times K$ matrix of real numbers and the K-vectors u_t are i.i.d.$(0,I_K)$, $\sum_{j=0}^{\infty} j^m \|c_j\| < M$ for some $m > 1$, $\|c_j\|$ is the norm of matrix c_j. Idiosyncratic shocks and common factors are linear stationary process and are independent of each other. The number of common factors K is bounded by a known number, $1 \leqslant K \leqslant \bar{K} < \infty$. Covariance matrix of common factors is asymptotically positive definite, $\frac{1}{T}\sum_{t=1}^{T} f_t f_t' \to_p \Sigma_f > 0$ as $T \to \infty$. The factor loading to a specific series may be zero, but the proportion of cross-sectional members which are influenced by each factor must be different from zero as N tends to infinity, $\frac{1}{N}\sum_{i=1}^{N} \lambda_i \lambda_i' \to \Sigma_\lambda > 0$ as $N \to \infty$. Define the variance of $e_{i,t}$ as $\sigma_{e,i}^2 = \sum_{j=0}^{\infty} d_{i,j}^2$, the long-run variance of $e_{i,t}$ as $\omega_{e,i}^2 = (\sum_{j=0}^{\infty} d_{i,j})^2$, and the one-sided long run variance of $e_{i,t}$ as $\nu_{e,i} = \sum_{l=1}^{\infty}\sum_{j=0}^{\infty} d_{i,j} d_{i,j+l}$. Their cross-sectional means are well defined, $\omega_e^2 = \lim_n \frac{1}{N}\sum_{i=1}^{N} \omega_{e,i}^2 > 0$, $\phi_e^4 = \lim_n \frac{1}{N}\sum_{i=1}^{N} \omega_{e,i}^4 > 0$, $\sigma_e^2 = \lim_n \frac{1}{N}\sum_{i=1}^{N} \sigma_{e,i}^2 > 0$, and $\nu_e = \lim_n \frac{1}{N}\sum_{i=1}^{N} \nu_{e,i}$.

The nonstationarity of the panel $y_{i,t}$ comes from the autoregressive root ρ_i. The null hypothesis is that $\rho_i = 1$ for all i, while the heterogeneous alternative hypothesis is that $\rho_i < 1$ for some i. Moon and Perron propose two test statistics that are based on transformed standard pooled OLS estimator using projection matrix from estimated common factor loadings.

In the first step of their procedure, by imposing the homogeneous restriction $\rho_i = 1$ for all

i, a pooled OLS regression is undertaken and the residuals are extracted:

$$\hat{u}_{i,t} = y_{i,t} - \hat{\rho}_{\text{pooled}} y_{i,t-1} \tag{2.27}$$

The principal component method is employed to estimate common factors and factor loadings. The principal component method is commonly used in the literature, for example, Bai and Ng (2002), Bai (2003). Given that the number of factors K is known beforehand, Moon and Perron (2004) use the normalization $\lambda'\lambda/N = I_K$ to estimate the unique factor loadings, which is an $N \times K$ matrix of $\sqrt{N}$ times the eigenvectors corresponding to the K largest eigenvalues of $\hat{u}'_{i,t}\hat{u}_{i,t}$. The estimate of the factor loading matrix is $\hat{\Lambda} = (\hat{\lambda}_1, \cdots, \hat{\lambda}_N)'$, a projection matrix can be constructed as $\hat{Q}_{\hat{\Lambda}} = I_N - \hat{\Lambda}(\hat{\Lambda}'\hat{\Lambda})^{-1}\hat{\Lambda}'$.

The projection matrix is used to obtain the de-factored residuals:

$$\hat{e} = \hat{u}\hat{Q}_{\hat{\Lambda}} \tag{2.28}$$

By using the de-factored residuals, long run variances can be estimated nonparametrically. Define the sample covariances by $\hat{\Gamma}_i(j) = \frac{1}{T}\sum_t \hat{e}_{i,t}\hat{e}_{i,t+j}$, where the summation is taken over $1 \leqslant t, t+j \leqslant T$. Kernel estimator for long-run variance and one-sided long-run variance for each member is obtained:

$$\hat{\nu}_{e,i} = \sum_{j=1}^{T-1} w\left(\frac{j}{h}\right)\hat{\Gamma}_i(j) \tag{2.29}$$

$$\hat{\omega}^2_{e,i} = \sum_{j=-T+1}^{T-1} w\left(\frac{j}{h}\right)\hat{\Gamma}_i(j) \tag{2.30}$$

where $\omega(\cdot)$ is a kernel function and h is a bandwidth parameter. Define their cross-sectional averages:

$$\hat{\nu}_e^N = \frac{1}{N}\sum_{i=1}^{N}\hat{\nu}_{e,i} \tag{2.31}$$

$$\hat{\omega}_e^2 = \frac{1}{N}\sum_{i=1}^{N}\hat{\omega}_{e,i}^2 \tag{2.32}$$

$$\hat{\phi}_e^4 = \frac{1}{N}\sum_{i=1}^{N}\hat{\omega}_{e,i}^4 \tag{2.33}$$

Two test statistics, denote t_a^* and t_b^*, can now be constructed as:

$$t_a^* = \frac{\sqrt{N}T\,(\rho^*_{\text{Pooled}} - 1)}{\sqrt{2\hat{\phi}_e^4/\hat{\omega}_e^4}} \tag{2.34}$$

$$t_b* = \frac{\sqrt{N}T\,(\rho^*_{\text{Pooled}} - 1)}{\hat{\phi}_e^2}\sqrt{\frac{1}{NT^2}tr(Y_{-1}Q_{\hat{\Lambda}}Y'_{-1})\hat{\omega}_e} \tag{2.35}$$

where

$$\rho^*_{Pooled} = \frac{tr(Y_{-1}Q_{\hat{\Lambda}}Y') - NT\hat{\nu}_e^N}{tr(Y_{-1}Q_{\hat{\Lambda}}Y'_{-1})}$$

Moon and Perron (2004) show shat t_a^* and t_b^* have standard normal distribution under the null hypothesis, as $N, T \to \infty$ with $N/T \to 0$

2.3.4 Breitung and Das (2008)

Breitung and Das (2008) propose various tests of panel unit root with cross-sectional dependence due to the presence of dynamic common factors. The data generated process they employ is similar to the one in Bai and Ng (2004), which is more general than the ones in Moon and Perron (2004) and Pesaran (2007b), etc. The consider a panel of time series generated by:

$$y_{i,t} = \lambda_i' f_t + u_{i,t} \tag{2.36}$$

$$f_t = \Phi f_{t-1} + v_t \tag{2.37}$$

$$u_{i,t} = \theta u_{i,t-1} + \varepsilon_{i,t} \tag{2.38}$$

where f_t is an $K \times 1$ vector of common factors, λ_i $(K \times 1)$ is the corresponding non-random factor loads for member i, $\Lambda = (\lambda_i, \cdots, \lambda_N)'$ is an $N \times K$ matrix of the factor loadings. Authors focus on the special case of Φ when $\Phi = \rho I_K$ and $|\rho| \leqslant 1$, i.e., the common factors have the homogeneous autoregressive root and the lag of factor f_m only enters into the function of f_m, $m = 1, \cdots, K$. Breitung and Das (2008) illustrate their analysis assuming no constant and linear trend in the DGP. When a model is believed to have individual specific constants, Breitung and Das (2008) suggest to remove the constant by subtracting the first observation, i.e., use $y^*_{i,t} = y_{i,t} - y_{i,0}$ instead of $y_{i,t}$.

Notice the homogeneous restrictions of ρ_i and θ_i that $\rho_i = \rho$ and $\theta_i = \theta$ are assumed. A special case of this DGP is that when $\rho = \theta$, the common component and idiosyncratic component have same autoregressive process, this is similar to the DGP considered in Phillips and Sul (2003), Moon and Perron (2004) and Pesaran (2007b).

Assumptions for the error terms v_t and $\varepsilon_{i,t}$ are made. The vector $\varepsilon_t = (\varepsilon_{1,t}, \cdots, \varepsilon_{N,t})'$ is independently and identically distributed with mean zero and heteroscedasticity, $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_t') = \Sigma = diag(\sigma_1^2, \cdots, \sigma_N^2)$. The matrix $\lim_{N\to\infty} \frac{1}{N}\Lambda'\Lambda > 0$. The vector v_t is independently and identically distributed with $E(v_t) = 0$ and $E(v_t v_t') = I_K$, and $E(\varepsilon_t v_t') = 0$. Additionally assuming that $E(\varepsilon_{i,t}^4) < \infty$ for all i, t, and $E(v_{j,t}^2 v_{k,t}^2) < \infty$ for all j, k, t.

The panel unit root tests are based on the "reduced form" autoregression function:

$$\Delta y_t = \phi y_{t-1} + e_t \tag{2.39}$$

where $\Delta y_t = (\Delta y_{1,t}, \cdots, \Delta y_{N,t})'$, $y_{t-1} = (y_{1,t-1}, \cdots, y_{N,t-1})'$, and $e_t = (e_{1,t}, \cdots, e_{N,t})'$.

Following Breitung (2005) and Jonsson (2005), the t_{rob} statistic is considered, which assumes "Panel correlated standard errors"(PCSE). A consistent estimator of the panel correlated variance of the OLS estimator is suggested by Beck and Katz (1995) and Jonsson (2005):

$$var(\hat{\phi}) = \frac{\sum_{t=1}^{T} y_{t-1}'\hat{\Omega} y_{t-1}}{(\sum_{t=1}^{T} y_{t-1}' y_{t-1})^2} \tag{2.40}$$

The robust version of the OLS test statistic is defined as:

$$t_{rob} = \frac{\sum_{t=1}^{T} y'_{t-1}\Delta y_t}{\sqrt{\sum_{t=1}^{T} y'_{t-1}\hat{\Omega} y_{t-1}}} \tag{2.41}$$

where $\hat{\phi}$ is the pooled OLS estimator of the autoregressive root, $\hat{\Omega}$ is the estimated variance-covariance matrix:

$$\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t \hat{e}'_t \tag{2.42}$$

If weak dependence is assumed in the data[5], Breitung and Das (2005) show that the t_{rob} statistic is asymptotically normally distributed given that $T \to \infty$ followed by $N \to \infty$.

The GLS estimator for the autoregressive root in equation 2.39 is more efficient than the OLS estimator hence is more powerful than the OLS estimator. The GLS $t-statistic$ is defined by:

$$t_{gls} = \frac{\sum_{t=1}^{T} y'_{t-1}\hat{\Omega}^{-1}\Delta y_t}{\sqrt{\sum_{t=1}^{T} y'_{t-1}\hat{\Omega}^{-1} y_{t-1}}} \tag{2.43}$$

Note that the GLS statistic is defined only when $T > N$, when it is ensured that $\hat{\Omega}$ is non-singular and its inverse exists.

Breitung and Das (2008) consider three different cases of data generating process. In the first case, the common factors and the idiosyncratic components are both nonstationary. Breitung and Das (2008) show that for a single-factor model, the null distribution of t_{rob} statistic is dominated by the common factor and is equivalent to the asymptotic distribution of the Dickey-Full test. The t_{gls} statistic is standard normally distributed as $N \to \infty$, $T \to \infty$, and $N^2/T \to 0$. In the second case, common factors are $I(1)$ processes and idiosyncratic components are stationary. This case is referred to *cross-unit cointegration*, following Banerjee et al. (2005). Both OLS based test statistics and GLS based test statistics are invalid. In the third case, the common factors are $I(0)$ and the idiosyncratic components are $I(1)$. As $N \to \infty$, $T \to \infty$, and $N/T \to 0$, t_{rob} diverges to $-\infty$ and t_{gls} has standard normal distribution as $N \to \infty$, $T \to \infty$, and $N^2/T \to 0$.

[5] Pesaran and Tosetti (2011) provide a formal discussion on weak and strong dependence in panels.

2.3.5 Bai and Ng (2004)

In contrast to Moon and Perron (2004), Pesaran (2007b), where the common factors are assumed to be stationary, the Panel Analysis of Nonstationarity in Idiosyncratic and Common components(PANIC) proposed by Bai and Ng (2004) allows the nonstationarity of a panel $Y_{i,t}$ comes from either common factor component or idiosyncratic component, or both. The key point of PANIC analysis is to get consistent estimates of common factors and idiosyncratic components. Unit root tests are then proposed and applied to both components separately to determine whether the the source of nonstationarity is pervasive of idiosyncratic.

Bai and Ng (2004) consider the data generating process of panel $Y_{i,t}$ to be:

$$Y_{i,t} = c_i + \beta_i t + \lambda_i' F_t + e_{i,t} \tag{2.44}$$

$$(1-L)F_t = \Phi(L)u_t \tag{2.45}$$

$$(1-\rho_i L)e_{i,t} = \Psi_i(L)\varepsilon_{i,t} \tag{2.46}$$

where $\Phi(L) = \sum_{j=0}^{\infty} \phi_j L^j$ and $\Psi_i(L) = \sum_{j=0}^{\infty} \psi_{i,j} L^j$. Suppose the number of common factors is K, then k_0 stationary common factors and k_1 nonstationary common factors are allowed, with the relationship that $K = k_0 + k_1$. The aim of the test is to determine the number of common trends $k_1 (0 \leqslant k_1 \leqslant K)$, and test whether $\rho_i = 1$ for each member i.

Some assumptions are made. For nonrandom λ_i, $\|\lambda_i\| \leqslant M < \infty$, for random λ_i, $E\|\lambda_i\|^4 < M$. $\frac{1}{N}\sum_{i=1}^{N} \lambda_i \lambda_i' \xrightarrow{p} \Sigma_\Lambda > 0$, where Σ_Λ is a $K \times K$ matrix. This assumption ensures the factor structure are identifiable and redundant factors are excluded from the common factors. $u_{i,t} \sim$ i.i.d.$(0, \Sigma_u)$, $E\|u_t\|^4 < M$, $var(\Delta F_t) = \sum_{j=0}^{\infty} \Phi_j \Sigma_u \Phi_j' > 0$, $\sum_{j=0}^{\infty} j \|\Phi_j\| < M$ and $\Phi(1)$ has rank k_1, $0 \leqslant k_1 \leqslant K$. The long-run covariance matrix of ΔF_t may have reduced rank to allow for combinations of $I(1)$ factors to be stationary. $\varepsilon_{i,t} \sim$ i.i.d.$(0, \sigma_{\varepsilon_i^2})$ for each member i, $E|\varepsilon_{i,t}|^8 \leqslant M$, $\omega_{\varepsilon_i}^2 = \Psi_i(1)^2 \sigma_{\varepsilon_i^2} > 0$, $E(\varepsilon_{i,t}, \varepsilon_{j,t}) = \tau_{i,j}$ with $\sum_{i=1}^{N} |\tau_{i,j}| \leqslant M$ for all j, $E|N^{-1/2}\sum_{i=1}^{N}(\varepsilon_{i,t}, \varepsilon_{i,s} - E(\varepsilon_{i,t}, \varepsilon_{i,s}))|^4 \leqslant M$ for all t, s. This permits serial correlation and cross-sectional correlation, as long as the column summation of the covariance matrix of the error term is bounded. Finally, the errors $\varepsilon_{i,t}$, u_t and factor loadings λ_i are mutually independent.

Bai and Ng (2004) apply the principle component method on the first-differenced data for the constant only case and differenced and demeaned data for the linear trend case. It is to ensure that consistent estimator of common factor F_t and idiosyncratic component $e_{i,t}$ can be obtained, regardless of whether $e_{i,t}$ is stationary or integrated of order 1. For the constant only case, denote $y_{i,t} = \Delta Y_{i,t}$, $f_t = \Delta F_t$, and $z_{i,t} = \Delta e_{i,t}$. For the linear trend case, denote $y_{i,t} = \Delta Y_{i,t} - \overline{\Delta Y_i}$, $f_t = \Delta F_t - \overline{\Delta F}$ and $z_{i,t} = \Delta e_{i,t} - \overline{\Delta e_i}$, where $\overline{\Delta Y_i} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta Y_{i,t}$, $\overline{\Delta F} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta F_t$, and $\overline{\Delta e_i} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta e_{i,t}$. Then the factor model for the transformed data is:

$$y_{i,t} = \lambda_i' f_t + z_{i,t} \tag{2.47}$$

The principal component method is employed to get consistent estimator of factors and their associated loadings, $\hat{f}_t$ and $\hat{\lambda}_i$ respectively. The transformed idiosyncratic component $z_{i,t}$ can also be estimated consistently.

Bai and Ng (2004) re-accumulate the estimated values to get the common component and idiosyncratic component. This procedure allows the common factor to be nonstationary and remove the effect of possible over-differencing. Define:

$$\hat{F}_t = \sum_{s=2}^{t} f_s \tag{2.48}$$

$$\hat{e}_{i,t} = \sum_{s=2}^{t} z_{i,s},\ i = 1, \cdots, N \tag{2.49}$$

Two univariate tests are proposed, namely $ADF^c_{\hat{e}_i}$ ($ADF^\tau_{\hat{e}_i}$ for linear trend case) for idiosyncratic component and $ADF_{\hat{F}}$ for common factor component. The ADF unit root test is applied to each estimated idiosyncratic component $\hat{e}_i$ individually without deterministic terms. When the idiosyncratic components are being tested individually, weak cross-sectional dependence is allowed. However, the assumption of cross-sectional independence is imposed later when pooled tests of idiosyncratic components are performed. If it is believed that there is only one common factor in the data generating process, the ADF test is also employed to test the unit root null of the estimated factor $\hat{F}_t$, where an intercept is included in the ADF regression. Bai and Ng (2004) show that the limiting distribution of $ADF^c_{\hat{e}_i}$ and $ADF^\tau_{\hat{e}_i}$ coincides with the DF distribution with no deterministic terms. The asymptotic critical value of the test is hence -1.95 for 5%

level. The asymptotic distribution of $ADF_{\hat{F}}$ is the same to the DF distribution with constant only, for which the 5% asymptotic critical value is -2.86. The autoregressive order p_i is chosen in a way that $p \to \infty$ and $p^3/\min(T,N) \to 0$, to ensure the convergence of $ADF^c_{\hat{e}_i}(ADF^\tau_{\hat{e}_i})$ and $ADF_{\hat{F}}$.

In the case when the number of common factors is greater than one, Bai and Ng (2004) propose modified Q_f and Q_c tests, which are proposed by Stock and Watson (1988). This is an iterative procedure. For the constant only case, denote the demeaned factors as $\hat{F}^c_t = \hat{F}_t - \overline{\hat{F}}$, where $\overline{\hat{F}} = \frac{1}{T-1}\sum_{t=2}^T \hat{F}_t$. For the linear trend case, denote the demeaned and detrended common factors as $\hat{F}^\tau_t$, which is the residual from a regression of $\hat{F}_t$ on a constant and linear trend. The testing procedures are then identical, hence only $\hat{F}^c_t$ is used for the purpose of illustration. Starts from $m = K$, when all factors are nonstationary.

1. Let $\hat{\beta}_\perp$ be the m eigenvectors associated with the m largest eigenvalues of $T^{-2}\sum_{t=2}^T \hat{F}^c_t \hat{F}^{c\prime}_t$. Define $\hat{X}^c_t = \hat{\beta}'_\perp \hat{F}^c_t$. Two statistics are considered:

2. Construct the test statistics.

   (a) Let $\hat{\xi}^c_t$ be the residuals from estimating a VAR(1) in $\hat{Y}^c_t$ and let

   $$\hat{\Sigma}^c_1 = \sum_{j=1}^J K(j)\left(T^{-1}\sum_{t=2}^T \hat{\xi}^c_t \hat{\xi}^{c\prime}_t\right)$$

   Let $\hat{v}^c_c(m)$ be the smallest eigenvalue of

   $$\hat{\Phi}^c_c(m) = \frac{1}{2}\left[\sum_{t=2}^T (\hat{X}^c_t \hat{X}^{c\prime}_{t-1} + \hat{X}^c_{t-1}\hat{X}^{c\prime}_t) - T(\hat{\Sigma}^c_1 \hat{\Sigma}^{c\prime}_1)\right]\left(\sum_{t=2}^T \hat{X}^c_{t-1}\hat{X}^{c\prime}_{t-1}\right)^{-1}$$

   Define test statistic $MQ^c_c(m) = T(\hat{v}^c_c(m) - 1)$ for constant only case.

   (b) For p fixed and does not depend on N or T.

   Estimate a VAR(p) in $\Delta\hat{X}^c_t$ to obtain $\hat{\Pi}(L) = I_m - \sum_{i=1}^p \hat{\Pi}_i L^i$. Filter $\hat{X}^c_t$ by $\hat{\Pi}(L)$ to get $\hat{x}^c_t = \hat{\Pi}(L)\hat{X}^c_t$. Let $\hat{v}^c_f(m)$ be the smallest eigenvalue of

   $$\hat{\Phi}^c_f(m) = \frac{1}{2}\left[\sum_{t=2}^T (\hat{x}^c_t \hat{x}^{c\prime}_{t-1} + \hat{x}^c_{t-1}\hat{x}^{c\prime}_t)\right]\left(\sum_{t=2}^T \hat{x}^c_{t-1}\hat{x}^{c\prime}_{t-1}\right)^{-1}$$

Define $MQ_f^c(m) = T(\hat{v}_f^c(m) - 1)$ as the second test statistic.

3. If the null: $k_1 = m$ is rejected, let $m = m - 1$ and go back to step 1. If the null is not rejected, $\hat{k}_1 = m$ and stop.

When a constant and linear trend is in the data generating process, corresponding statistics $MQ_c^\tau(m)$ and $MQ_f^\tau(m)$ can be calculated similarly.

Bai and Ng (2004) point out that the $MQ_c(m)$ test statistic is more general than the $MQ_f(m)$ test statistic. $MQ_f(m)$ requires the nonstationary common factors to be have finite order VAR(p) processes, while the $MQ_c(m)$ is more general and allows the the unit root processes to have a more general dynamics, e.g., moving average error, as long as the errors satisfy the weak dependence assumptions of the PANIC test. Authors obtain the critical values of $MQ_c(m)$ and $MQ_f(m)$ by simulation and tabulate these value in Table I in their article.

Pooled tests are then proposed for the idiosyncratic components in order to gain more power. Cross-sectional independence is now imposed on the idiosyncratic component to validate the tests pooling on independent p-values. Consider testing the null hypothesis that $\rho_i = 1$ for all i against the heterogeneous alternative $\rho_i < 1$ for some i. Suppose $p_{\hat{e}_i}^c$is the p-value associated with the ADF regression on idiosyncratic component $\hat{e}_i$ with constant only, then the test statistic is defined as:

$$P_{\hat{e}}^c = \frac{-2\sum_{i=1}^N \ln(p_{\hat{e}_i}^c - 2N)}{\sqrt{4N}}$$

For constant and linear trend case, simply replace the $p_{\hat{e}_i}^c$ with $p_{\hat{e}_i}^\tau$, where $p_{\hat{e}_i}^\tau$ is the p-value from the ADF regression on $\hat{e}_i$ with constant and a linear trend. The test statistics $p_{\hat{e}_i}^c$ and $p_{\hat{e}_i}^\tau$ are asymptotically standard normally distributed.

The PANIC approach has some attractive features. The tests performed on common factors is independent of the stationarity of the idiosyncratic component, i.e., tests on $\hat{F}_t$ are valid no mater whether $e_{i,t} \sim I(0)$ or $e_{i,t} \sim I(1)$. The test on the idiosyncratic component for unit i is independent of the stationarity of the idiosyncratic component for unit j, $i \neq j$. And the tests on the idiosyncratic components do not depend on whether $F_t \sim I(0)$ or $F_t \sim I(1)$.

2.4 Econometric Analysis

2.4.1 Testing Purchasing Power Parity

Following section 2.3, we report the econometric test and estimation results in this section. The main purpose is to test whether the unit root null hypothesis can be rejected for each relative price panel. If the unit root null is rejected for a relative price panel, the PPP holds in that specific index. Then the convergence rate is estimated and the half-life is calculated accordingly.

We start our analysis from the IPS test of Im et al. (2003). The test assumes cross-sectional independence in panels and it may not be a suitable test in our case. It is known that if the dependence is ignored, the IPS test suffers from size distortion, that is, the probability that a true null hypothesis is rejected is greater the significance level. We use IPS as a benchmark to show how misleading it could be by ignoring cross-sectional dependence, therefore, we do not pay much attention on the results of IPS. The test statistic for each panel with lag $p = 1, \cdots, 4$ are provided in Table 2.4. A general conclusion is that, as we expected, IPS rejects the unit root null hypothesis for the vast majority of price indices, regardless of the lag length.

Table 2.4: IPS panel unit root test results

panel/p	1	2	3	4
CPI	-15.871***	-12.906***	-9.567***	-7.800***
Food	-19.102***	-16.713***	-11.716***	-10.068***
Grain	-10.326***	-7.911***	-5.802***	-4.266***
Meat	-13.605***	-7.939***	-7.226***	-5.585***
Eggs	-19.414***	-13.809***	-13.057***	-11.656***
Seafood	-16.331***	-7.736***	-7.983***	-6.302***
Fresh Vegetables	-32.367***	-28.371***	-18.413***	-12.138***
Fresh Fruits	-25.121***	-22.935***	-18.602***	-16.835***
Tobacco and Wine	-4.358***	-3.072***	-0.980	0.183
Clothing	-8.654***	-6.923***	-4.962***	-2.695***
Recreation Facilities	7.170	6.832	7.282	7.983
Health Care	-12.993***	-13.284***	-12.905***	-11.502***
Traf. and Comm.	-4.126***	-2.113**	-2.178**	-1.481*
Cul. and Edu. Exp.	-10.980***	-9.843***	-10.941***	-7.190***
Residence	-6.191***	-5.726***	-4.443***	-3.321***

Note: *,**,*** denote significance at 10 percent, 5 percent, and 1 percent levels, respectively.

Testing results of the so called second generation tests are reported in Table 2.5. The first column is the 15 price indices. For each of the 15 price panels, we first estimate the CADF equation 2.20 individually, i.e., unit by unit. The CADF statistics are then calculated from the estimations and are compared to the critical values provided in Table I(b) of Pesaran (2007b). The relative price between i and the numeraire city is deemed as stationary if CADF statistic rejects the null. The number of stationary cities pairs (out of 35) is reported in the second column in Table 2.5[6]. The CIPS test statistics of Pesaran (2007b) [7] are reported in the third column, which is the cross-sectional average of the individual CADF statistics. The fourth column reports the t_a^* test statistic of Moon and Perron (2004) [8]. The t_{rob} statistic of Breitung and Das (2008) is reported in the fifth column. The sixth column reports the number of nonstationary common factors selected by the MQ_c test of Bai and Ng (2004)[9]. The seventh column reports the $P_{\hat{e}}$ test of Bai and Ng (2004) for the idiosyncratic components. If the $CIPS$ test rejects the unit root null hypothesis for a specific panel, we calculate the half-life of that panel and report it in column eight. The half-life is calculated from the cross-sectional average of $\hat{\rho}_i$ in equation 2.20: $\ln(0.5)/\ln(1 + \frac{1}{N}\sum_{i=1}^{N}\hat{\rho}_i)$. In the last column, we report the half-life calculated from t_{rob} test. If a panel is stationary, we use the homogeneous autoregressive coefficient $\hat{\phi}$ in equation 2.39 to calculate the half-life: $\ln(0.5)/\ln(1 + \hat{\phi})$. If a panel is nonstationary according to the corresponding test, we mark the half-life as N/A. For Pesaran (2007b) tests, $p = 4$ is used, the significance level of CADF is 5%. For Breitung and Das (2008) tests and $P_{\hat{e}}$ test of Bai and Ng (2004), lag order is selected by BIC with $p_{max} = 4$. For Moon and Perron (2004) tests and MQ_c test of Bai and Ng (2004) , the number of common factors is set at two.

The test results show that, for the two aggregate price indices, CPI and food, the Bai and Ng (2004) MQ_c test selects 0 nonstationary common factor and the idiosyncratic component is stationary by $P_{\hat{e}}$ test, all other tests reject the unit root null hypothesis, expect that the t_{rob} test fails to reject the null in the CPI panel. These evidences show that the relative prices of CPI and food are stationary, the purchasing power parity holds in the general consumer basket and in the food basket, among the 36 Chinese cities. The half-life is around 2 months for both indices by $CIPS$ test and 11 months for food price index by t_{rob} test. The vast majority of

[6]Details of the stationary/nonstationary classification by CADF is provided in Table A.1 (on page 138) in Appendix A.2.

[7]The CP and CZ tests of Pesaran (2007b) are also employed and similar results are obtained.

[8]The t_b^* of Moon and Perron (2004) is calculated and results are similar to t_a^*.

[9]The MQ_f test of Bai and Ng (2004) shows some conflict results to the MQ_c. We employ the MQ_c as it allows a more general underlying data generating process, as pointed out by Bai and Ng (2004).

Table 2.5: Panel Unit Root Test Results

Product Name	Number of Conv. Cities	$CIPS$	t_a^*	t_{rob}	MQ_c	$P_{\hat{e}}$	HL $CIPS$	HL t_{rob}
CPI	16	-3.094***	-11.826***	-0.751	0	14.720***	1.811	N/A
Food	12	-2.693***	-38.043***	-3.546***	0	12.946***	2.173	11.380
Perishables								
Meat	6	-2.489***	-36.675***	-2.535***	0	10.673***	3.069	16.358
Eggs	8	-2.555***	-29.155***	-3.826***	0	9.852***	2.573	13.816
seafood	7	-2.403***	-24.005***	-1.191	0	10.115***	2.900	N/A
Fresh Vegetables	27	-4.084***	-110.922***	-3.998***	0	23.444***	0.783	7.716
Fresh Fruits	16	-3.471***	-111.024***	-5.326***	0	29.031***	0.992	5.916
Nonperishables								
Grain	3	-1.975	-13.773***	-1.245	0	4.435***	N/A	N/A
Tobacco and Wine	7	-2.317***	-3.235***	0.948	1	4.273***	3.078	N/A
Clothing	4	-1.576	-1.193	-1.044	2	0.719	N/A	N/A
Services								
Recreation Facility	6	-2.274**	-1.127	1.704	2	0.264	3.577	N/A
Health Care	8	-2.635***	-4.840***	-2.839***	0	4.888***	3.214	54.368
Traf. and Comm.	8	-2.546***	0.000	0.222	1	1.529*	3.239	N/A
Cul. and Edu. Exp.	18	-3.353***	-4.855***	-1.014	1	5.280***	1.485	N/A
Residence	3	-2.161**	-5.763***	-0.567	1	1.148	4.005	N/A

Note: *,**,*** denote significance at 10 percent, 5 percent, and 1 percent levels, respectively. First column is product name, second column is the number of convergence cities out of 35, third column reports test statistic of Pesaran (2007b) CIPS, forth column reports t_a^* test statistic of Moon and Perron (2004), fifth column shows t_{rob} statistic of Breitung and Das (2008), columns six and seven report MQ_c and $P_{\hat{e}}$ tests of Bai and Ng (2004), last two columns report half-livies calculated from CIPS test and t_{rob} tests.

tests strongly reject the unit root null hypothesis for the 5 products in the perishable group. The PANIC test selects 0 nonstationary common factor for all products, and the idiosyncratic components are stationary as well. All tests reject the null for the 5 panels except that t_{rob} test fails to reject the null for the seafood price panel. This is a strong evidence to prove that, PPP holds in all goods in the perishable group. The half-life ranges between less than 1 months and 3 months by $CIPS$ test, and ranges between 6 months and 16 months by t_{rob} test. The results of nonperishables show that, for all the three goods in this group, at least two of the tests fail to reject the unit root null hypothesis: $CIPS$ and t_{rob} tests fail to reject the null for grain price, t_{rob} test can not reject the null for tobacco and wine price and the PANIC test detects 1 nonstationary common factor and stationary idiosyncratic component, all tests fail to reject the null for clothing price panel and PANIC test shows that the panel consists of 2 nonstationary common factors and nonstationary idiosyncratic component. The only half-life that can be defined is 3 months for the tobacco and wine price by $CIPS$ test. Finally, evidence for the price indices in the services group is mixed. All tests reject the unit root null for the health care panel, but the tests provide conflict results for the other 4 prices. PANIC test shows that recreation facility price consists of two $I(1)$ common factors and $I(1)$ idiosyncratic component, traffic and communication price and cultural and educational expense consist of one $I(1)$ common factor and $I(0)$ idiosyncratic component, residence price consists of one $I(1)$ common factors and $I(1)$ idiosyncratic component. $CIPS$ rejects the null for all of the 4 prices, while t_{rob} dose not reject the null for all of the 4 prices. t_a^* fails to reject the null for recreation facility price and traffic and communication price, but rejects the null for cultural and educational expense and residence price. Half price by $CIPS$ test ranges between 1.5 months and 4 month, but the only half-life defined by t_{rob} test for health care is as large as 54 month.

Some general results can be observed in Table 2.5. First, for the 2 aggregate price indices and 5 products in the perishable group, there are strong evidence in favour of purchasing power parity. For the 3 products in the nonperishable group, more evidence show that the panels are nonstationary and purchasing power parity fails to hold. Test statistics for the price indices in the services group provide mixed evidence. Second, the PANIC test shows that, if a panel is deemed as nonstationary, the nonstationarity may come from common factor component alone

Table 2.6: Results Summary

Product Group	% of Conver. Cities	HL $CIPS$	HL t_{rob}
Perishables	36.57	2.06	10.95
Nonperishables	13.33	3.08	N/A
Services	24.57	3.10	54.37

or common factor component along with idiosyncratic component, and there may be more than one $I(1)$ common factors. Third, for panels that are deemed as stationary by PANIC test, the test results from other tests are generally consistent. However, for the 6 panels that are deemed as nonstationary by PANIC test, the results from other tests are conflicting: $CIPS$ rejects 5 panels out of 6, t_a^* rejects 3 panels, and t_{rob} rejects none of the 6 panels. Fourth, the $CIPS$ rejects unit root in 13 panels out of 15, which is the largest number of rejections in the tests. t_{rob} rejects unit root in only 6 panels, which is the smallest number of rejections in the tests. Fifth, the difference of half-lives calculated from different tests can be large.

To directly compare the three product groups, we calculate the percentage of stationary relative price series for each product group. The percentage is calculated as the total number of time series for which the unit root null is rejected in the CADF test, for example, there are $3+4+7 = 14$ stationary series in nonperishable group, out of 35×3 time series, so the percentage is $14/105 = 13\%$. From the results we find that, the proportion of cities with stationary relative price is highest for perishables (37%), followed by services (25%), and followed by nonperishables (13%), these results are summarized in column two in Table 2.6. The average half-lives for the three groups are calculated as the simple average of half-lives within groups, as long as the unit root null hypothesis is rejected in a specific panel and the half-life can be defined. The half-life estimation by $CIPS$ test is reported in the third column and half-life by t_{rob} test is reported in the last column.

There are two noteworthy observations here. First, comparing to the descriptive statistics shown in Table 2.2, the perishables have the biggest mean absolute price differentials but also have the biggest percentage of stationary series. This illustrates that although the prices of the perishables deviate from the numeraire city relatively far, the relative prices in the group is more stationary than the relative prices in other groups. This may due to the fact that, although the arbitrage cost of perishables is high, the products in this group are highly competitive and the prices of which are closely monitored by the government and some price protection or price control may be present. Second, the nonperishables have the lowest percentage of cities that reject the unit root null. This is counter-intuitive because the nonperishables have low storage and transportation cost and we would expect that the purchasing power parity holds in these products. Interestingly, Fan and Wei (2006) examine a large number of prices in Chinese cities and they find very similar result in their study. They employ univariate ADF test on each unit in panels and find that the percentage of cities that rejected the unit root null is highest for perishable consumer goods(71%), followed by nonperishable consumer goods (27%), and services(22%). Parsley and Wei (1996) examine the price convergence within US and find that, on average, convergence speed in perishable group is faster than the nonperishable group. Neither of the studies provide an explanation. A possibility to this puzzle is that there are market segmentation in these products and the arbitrage cost is high, so a shock to the relative price does not diminish quickly. But we are lack of more information to find out an explanation to the nonstationarity of relative prices in the nonperishable group.

2.4.2 The Role of Distance

Distance between cities or countries is often used in the literature as a proxy of arbitrage cost, and it is believed that a shock to the relative price should diminish more rapidly for cities that are closer to each other. In this section we examine the role of distance in explaining the magnitude of convergence rate and half-life between city i and the numeraire city.

We run regression of autoregressive coefficient of relative price on the log of distance. For each price index in each of the three groups, we define the autoregressive coefficient of relative price between city i and numeraire city as $\hat{\phi}_i = (1 + \hat{\rho}_i)$, where $\hat{\rho}_i$ is estimated from the CADF equation 2.20. For example, in the perishable group, there are 5 price indices, and for each price index there are 35 city pairs and 35 relative price autoregressive coefficients, therefore the number of autoregressive coefficients in the perishable group is $5 \times 35 = 175$. Similarly, there are 105 observations in the nonperishable group and 175 observations in the service group. In order to get a general result, we also combine all 13 price indices together into the overall group, in which we have 455 observations. Note that, we use the autoregressive coefficients for all price indices and cities, regardless of whether the unit root null is rejected by CADF test for each city. The autoregressive coefficients are then regressed on the log of distance and product dummy variables. To detect the possible convexity between distance and autoregressive coefficient, the squared log distance is also augmented into the regression:

$$\hat{\phi}_{i,j} = \alpha \ln(\text{distance}_i) + dummies \tag{2.50}$$

$$\hat{\phi}_{i,j} = \alpha \ln(\text{distance}_i) + \beta \ln^2(\text{distance}_i) + dummies \tag{2.51}$$

where $\hat{\phi}_{i,j}$ is the autoregressive coefficient of relative price between city i and numeraire city for price index j, $\ln(\text{distance}_i)$ is the log of distance between city i and the numeraire city. Value of jth dummy variable equals one if product is j, and zero otherwise. The estimation results are summarized in Table 2.7 Panel A.

Similarly, we examine the relationship between half-life and distance. The half-life is calculated from the CADF autoregressive coefficient: $\hat{\text{HL}} = \ln(0.5)/\ln(\hat{\phi}_i) = \ln(0.5)/\ln(1+\hat{\rho}_i)$, where $\hat{\rho}_i$ is estimated from the CADF equation 2.20. Note that in the half-life regressions, we only use the relative prices for which the unit root null hypothesis are rejected by CADF test. There are 62 observations in the perishable group, 14 in the nonperishable group, 43 in the services group, and 119 observations in the overall group. The estimated half-lives are regressed on log distance, product dummies and squared log distance:

$$\hat{\text{HL}}_{i,j} = \alpha \ln(\text{distance}_i) + dummies \tag{2.52}$$

$$\hat{\text{HL}}_{i,j} = \alpha \ln(\text{distance}_i) + \beta \ln^2(\text{distance}_i) + dummies \tag{2.53}$$

where $\hat{\text{HL}}_{i,j}$ is the half-life of relative price between city i and numeraire city for price index j. Value of jth dummy variable equals one if product is j, and zero otherwise. Results for these estimations are reported in Table 2.7 Panel B.

Results from Panel A show that, when the quadratic term is not augmented, there is a positive and significant relationship between distance and the magnitude of autoregressive coefficient of relative price for all commodity/service groups, which indicates that the further the cities

Table 2.7: Convergence Rate and Distance

Panel A: Autoregressive Coefficient	Perishables		Nonperishables		Services		Overall	
Regression number	1	2	3	4	5	6	7	8
ln(distance)	0.067	0.146	0.121	0.256	0.112	0.277	0.105	0.258
	(0.004)	(0.024)	(0.004)	(0.023)	(0.004)	(0.020)	(0.004)	(0.014)
ln(distance)-squared		-0.011		-0.018		-0.022		-0.019
		(0.003)		(0.003)		(0.003)		(0.002)
product dummis	yes	yes	yes	yes	yes	yes	yes	yes
R^2	0.432	0.468	-0.230	0.086	-0.100	0.231	0.295	0.446
σ of the regression	0.183	0.178	0.160	0.139	0.178	0.149	0.178	0.158
Number of observations	175	175	105	105	175	175	455	455

Panel B: half-life	Perishables		Nonperishables		Services		Overall	
Regression number	9	10	11	12	13	14	15	16
ln(distance)	0.108	0.145	0.241	0.429	0.195	0.918	0.180	0.639
	(0.018)	(0.112)	(0.087)	(0.656)	(0.079)	(0.338)	(0.056)	(0.196)
ln(distance)-squared		-0.005		-0.028		-0.089		-0.051
		(0.015)		(0.096)		(0.040)		(0.021)
product dummis	yes	yes	yes	yes	yes	yes	yes	yes
R^2	0.317	0.318	0.054	0.062	0.056	0.164	0.220	0.262
σ of the regression	0.511	0.515	1.134	1.185	1.086	1.036	0.836	0.817
Number of observations	62	62	14	14	43	43	119	119

Note: numbers in parenthesis are standard errors.

are separated apart, the slower the convergence will be. The effect of distance on autoregressive coefficient is much higher for the nonperishable and service groups than for the perishable group. When quadratic term of log distance is added into the regression, the quadratic terms are negative and significant for all groups, showing a concave relationship between distance and autoregressive coefficient and a diminishing effect of distance. The turning point of the log distance ranges between 6.30(545km) to 7.11(1224km). Results from Panel B show that, without the quadratic term, the log distance is positive and significant in the half-life regressions for all groups. Similar to Panel A, the distance effect on nonperishable and service groups is much higher than the effect on the perishable group, where the effect on nonperishable group is the largest. This shows that, for prices in all groups, the half-life is longer for cities that are further apart. The quadratic term is insignificant in perishable and nonperishable groups, but negative and significant in service and overall groups. This shows a concave relationship and diminishing effect of distance on half-life. The turning point of the log distance is 5.16(174km) for service group and 6.26(523km) for overall group.

2.5 Conclusion and Remarks

In this study, we collect 15 aggregate and disaggregated monthly price indices for 36 Chinese cities and examine whether the purchasing power parity holds in these price indices. To overcome the low power problem of time series unit root tests and size distortion problem of the panel unit root tests that assume cross-sectional dependence, we employ a battery of panel unit root tests that take the cross-sectional dependence into account. Overwhelming evidence show that the purchasing power parity holds in the aggregate price index and the commodities in the perishable group. The half-life for the CPI is is 1.81 months and the half-life for the perishable group is, on average, 2.06 months or 10.95 months, depending on the methods employed. Previous intra-China PPP studies by Fan and Wei (2006) and Lan and Sylwester (2010), which employ the panel unit root tests that assume cross-sectional independence, show that PPP holds in the vast majority of goods/services prices and the half-lives range between 1.66 months to 2.44 months. In contrast, our research finds little evidence to support the purchasing power parity for the nonperishable goods and mixed evidence to support the PPP for services. Although the stationarity of service prices is supported by the Pesaran (2007b) test and the average half-life is 3.10 base on the CADF test , which is very close to the previous studies, Bai and Ng (2004) and Breitung and Das (2008) tests fail to reject the nonstationary null for most prices within this group. For the only price for which PPP holds, the estimated half-life is 54.37 months by the Breitung and Das (2008) t_{rob} test. Therefore, we suggest that caution should be exercised when interpreting the results from the previous studies and PPP may fail to hold in nonperishable and service groups if the dependence is taken into account.

It remains unsolved that why tests may provide conflict results some times: the Pesaran (2007b) test rejects the unit root null for 13 panels (out of 15), but the Breitung and Das (2008) test rejects the null for only 6 panels. The conflict results are much more obvious when the Bai and Ng (2004) test suggests that a panel is nonstationary: there are 6 nonstationary panels detected by the Bai and Ng (2004) PANIC test, and Pesaran (2007b) test rejects the unit root null in 5 of them, Moon and Perron (2004) test rejects the null in 3 of them, and Breitung and Das (2008) test rejects the null for none of them. These results may confuse us when drawing a conclusion regarding to purchasing power parity. We will try to examine why this may happen in the empirical study and make a suggestion on how to interpret this in Chapter 3.

Chapter 3

On Properties of Panel Unit Root Tests in Mixed Panels and Performances of Mixed Panel Tests: A Large Scale Simulation Study

3.1 Introduction

Considerable research has been undertaken on panel unit root test over the past two decades. For a recent literature survey, see Breitung and Pesaran (2008). The null hypothesis for most panel unit root tests is that all cross-section units in the panel have unit roots. Different from time series unit root tests, there are different ways that the alternative hypotheses are treated in panel unit root tests. Basically, panel unit root tests assume either homogeneous alternative or heterogeneous alternatives, depending on what assumptions have been made on the the panel.

Suppose a dynamic panel $y_{i,t}$ with time series dimension T and cross-sectional dimension N is generated as:

$$y_{i,t} = (1 - \phi_i)\mu_i + \phi_i y_{i,t-1} + \varepsilon_{i,t} \tag{3.1}$$

where $i = 1, \cdots, N$, $t = 1, \cdots, T$, initial values $y_{i,0}$ are given. Re-arrange this equation we have:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \varepsilon_{i,t} \tag{3.2}$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$, $\alpha_i = (1 - \phi_i)\mu_i$, and $\rho_i = (\phi_i - 1)$. For homogeneous panel unit root tests, the null and alternative hypotheses are usually designed as:

$$\begin{aligned} H_0 &: \rho_i = 0 \qquad \forall i \\ H_1 &: \rho_i = \rho < 0 \, \forall i \end{aligned} \tag{3.3}$$

For heterogeneous panel unit root tests, the null and alternative hypothesis are:

$$\begin{aligned} H_0 &: \rho_i = 0 \, \forall i \\ H_1 &: \rho_i < 0 \, for\ some\ i \end{aligned} \tag{3.4}$$

A problem of heterogeneous hypothesis testing is that, once the null hypothesis is rejected, the alternative is hard to be interpreted in empirical researches: it could be only a few units in the panel that are stationary, or it could be that all units are stationary. The two extreme situations may have very different economic meanings in empirical studies, but they are hardly to be distinguished. Homogeneous panel unit root tests, on the other hand, do not have this problem. The homogeneous alternative that all units in the panel are stationary helps researchers to draw a conclusion in empirical study, however, Westerlund and Breitung (2013) show that many of these tests still have power when only some of the units are stationary. Therefore, it is argued that the homogeneous restriction of these kind of tests may be too restrictive, especially in empirical studies such as PPP testing, and a rejection of the null hypothesis does not provide convincing evidence in favour of the alternative.

Some resent research works focus on mixed panels, in which fully heterogeneity is assumed and stationary and nonstationary units are mixed together. Pesaran (2007a) proposes a pair-wise approach to test the output convergence and suggests to apply individual unit root test on each of the $N(N-1)/2$ possible log real per capita output gaps, in which a null is rejected at level of significance, α. Pesaran shows that although the time series unit root tests are not independent, under the null hypotheses that all time series are nonstationary, the rejection fraction among the $N(N-1)/2$ decisions converges to α, if N and T jointly tend to infinity.

Some procedures are proposed in order to not only estimate the stationary/nonstationary proportion, but also identify the stationary and nonstationary units in the panel. Kapetanios (2003) and Chortareas and Kapetanios (2009) propose a sequential selection method. The Im et al. (2003) IPS panel unit root test is applied to a panel, if the unit root null hypothesis is rejected, the unit with smallest t-statistic in ADF regression is deemed to be stationary and is removed from the panel. The IPS test is then applied on the reduced panel, and this process carries on until the IPS test fails to reject the null hypothesis that all units are nonstationary. Smeekes (2011) also proposes a sequential testing process, in which order statistics are obtained for user-specified stationary proportions and are compared with critical values by block bootstrap. Some procedures use the concepts of family-wise error rate and false discovery rate from multiple testing literature to identify the stationary and nonstationary units. The family-wise error rate is defined as the possibility that at least one rejection is false, that is, the possibility that at least 1 unit is nonstationary in the classified stationary units. Controlling FWE is less desirable when the cross-sectional dimension is large. The false discovery rate is defined as the possibility of false rejection, that is, the expected fraction of classified stationary units that are actually nonstationary. Hanck (2006) applies the Romano and Wolf (2005) bootstrap approach to control the family-wise error rate (FWE) and examine a PPP data set in order to identify the countries for which PPP hold. Moon and Perron (2012) employs the Romano et al. (2008) bootstrap method to control the false discovery rate (FDR). Another approach is to estimate the nonstationary proportion and then identify the units accordingly. Ng (2008) shows that the if the nonstationary proportion in a panel is θ, then the cross-sectional variance of the mixed panel is dominated by a linear trend which grows at rate θ. The nonstationary proportion, θ, is then estimated as the average increase of cross-sectional variance over time. She suggests to order the units by the size of the autoregressive roots, from the largest to the smallest, and the first $[\theta \cdot N]$ units are deemed to be nonstationary units.

The motivation and contribution of our simulation study are the following. We first apply panel unit root tests on mixed panels. Hlouskova and Wagner (2006) run Monte Carlo simulation on a set of first-generation panel unit root tests and analyse the effect of T, N, serial correlation, autoregressive coefficient, etc., on different tests. Karlsson and Lothgren (2000) perform simulation study on Levin and Lin (1992) and Pasaran et al. (1995) tests in mixed panels and examine the power of tests for different proportion of stationary units. Gengenbach et al. (2010) run simulation on a set of second-generation panel unit root tests under different data generating processes. The DGPs are differ by different sources of unit root, number of common factors, and variance of errors in common factor component. The size and power properties of the tests are compared and analysed. We extend the research work by Karlsson and Lothgren (2000) and Gengenbach et al. (2010) by looking at the performance of second-generation panel unit root tests on mixed panels. Tests employed are Pesaran (2007b) $CIPS$ test, Moon and Perron (2004) t_a^* and t_b^* tests, Breitung and Das (2008) t_{rob} and t_{gls} tests, and Bai and Ng (2004) PANIC test. The nonstationarity in our experiments may come from common factor component, idiosyncratic component, or both, as described in Bai and Ng (2004), Breitung and Das (2008), and the nonstationarity may also from autoregressive coefficient, where both common factor component and idiosyncratic component are stationary, as in Moon and Perron (2004, 2012), Pesaran (2007b). We consider different panel dimensions, one or two common factors, different magnitude of cross-sectional dependence, and most importantly, different stationary/nonstationary proportion in panels. We then compare the rejection frequency of the tests under different DGPs.

We then run Monte Carlo experiments to examine the performance of mixed panel tests under the same DGPs. We employ the sequential test by Kapetanios (2003) and Chortareas and Kapetanios (2009), but replace the IPS test with $CIPS$ test to account for cross-sectional dependence. Sequential test on order statistic with bootstrap critical value by Smeekes (2011) is employed. The multiple testing procedures by Romano and Wolf (2005) and Moon and Perron (2012) are employed, where the former controls FWE and the later controls FDR. The Ng (2008) estimator is also employed. Similar to the panel unit root test analysis, the effect of panel dimensions, sources of unit roots, number of common factors, magnitude of cross-sectional dependence and proportion of nonstationary units are analysed.

An empirical application is performed by using the data set of Chinese city price indices. We provide an explanation of the conflicting results observed in Chapter 2. We find evidence that, on average, only a few units in the nonperishable group and services group are stationary, which conflicts the findings in the literature that almost all goods and services prices among Chinese cities are stationary and the convergence rate is very fast (2 to 3 months). Even for the perishable group, where strong evidence can be found in favour of PPP by panel unit root tests, the stationary proportion is about only a quarter to 2 fifths.

The rest of this Chapter is organised as follows: In Section 3.2, we describe the procedures employed in some detail. In Section 3.3, we present the design of the data generating processes. In Section 3.4 we specify the parameters used in the DGP and tests. Results from simulations are reported and analysed. The empirical application is given in Section 3.5. Section 3.6 concludes.

3.2 Methodology

3.2.1 Panel Unit Root Tests

We employ a battery of second-generation panel unit root tests that take cross-sectional dependence into account. The tests employed here are same to those used in Chapter 2: Pesaran (2007b) $CIPS$ test, Moon and Perron (2004) t_a^* and t_b^* tests, Breitung and Das (2008) t_{rob} and

t_{gls} tests, and Bai and Ng (2004) PANIC test. Details of these tests are described in section 2.3. To save space, we do not repeat the construction of these tests in this Chapter.

3.2.2 Chortareas and Kapetanios (2009)

Chortareas and Kapetanios (2009) introduce a procedure that enables the researchers to re-group the series in a panel and identify the stationary series and nonstationary series, which is referred to as sequential panel selection method (SPSM). They propose to use a sequence of panel unit root tests to identify the series in the panel that are stationary and those are nonstationary. The procedure starts from the whole panel and test the null hypothesis that all units in the panel are nonstationary. If the null is not rejected, the procedure stops and draw the conclusion that all units are nonstationary processes. If the null is rejected, we remove the series which has most evidence in favour of stationarity. This series is deemed as an $I(0)$ series. We redo the panel unit root test on the remaining panel. This procedure is carried on sequentially until the null hypothesis is no longer rejected. As the result, we pick up the series that are deemed to be stationary based on the panel unit root test, and the remaining series are nonstationary series.

Chortareas and Kapetanios (2009) illustrate their method by using the Im et al. (2003) panel unit root tests. Consider a panel of data with time series dimension T and cross-sectional dimension N, $y_{i,t}$ is generated as:

$$y_{i,t} = (1-\delta_i)\mu_i + \delta_i y_{i,t-1} + \varepsilon_{i,t} \tag{3.5}$$

where $i = 1, \cdots, N$, $t = 1, \cdots, T$. Rearrange the equation to have:

$$\Delta y_{i,t} = \alpha_i + \beta_i y_{i,t-1} + \varepsilon_{i,t} \tag{3.6}$$

where $\alpha_i = (1-\delta_i)\mu_i$, $\beta_i = \delta_i - 1$. The null hypothesis is then $H_0 : \beta_i = 0$ for all i (pure random walk). The error term $\varepsilon_{i,t}$ is assumed to be i.i.d. random variables for all i, t and with heterogeneous variance σ_i^2. The zero serial correlation assumption in the errors is then released by the authors, a more general ADF(p) data generating process for each unit is allowed:

$$\Delta y_{i,t} = \alpha_i + \beta_i y_{i,t-1} + \sum_{j=1}^{p_i} \gamma_j \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{3.7}$$

with $i = 1, \cdots, N$, $t = 1, \cdots, T$.

The Im et al. (2003) panel unit root test is then employed. The test is reviewed in section 2.3.1 and we do not repeat it here to save space. Chortareas and Kapetanios (2009) show that under certain conditions, their procedure to identify the stationary and nonstationary series is consistent. They further claim that any heterogeneous panel unit root tests under similar conditions can be applied in their procedure, for example, the Pesaran (2007b) $CIPS$ test, which takes account for cross-sectional dependence.

3.2.3 Romano and Wolf (2005)

Romano and Wolf (2005) propose a stepwise multiple testing procedure that asymptotically controls the Familywise Error Rate (FWE). The method is designed to improve the power of single-step testing methods, e.g., White (2000), and meanwhile control the FWE. They claim that a single-step multiple testing method stops in the first round of a stepwise multiple testing method, hence more false null hypothesis should be rejected in a stepwise testing procedure and more power is gained.

The testing method is proposed in a general form, which can be applied to any multiple testing problem with N individual decisions to be made. Authors call the N individual tests to be N "strategies", and critical values to be "benchmarks". In our panel unit root test scenario, comparing test statistics to their critical values is equivalent to comparing strategies to benchmarks. Define a data matrix $y_{i,t}$, where $1 \leq t \leq T$ and $1 \leq i \leq N+1$, t and i are the row and column index of the element in the matrix, respectively. The data is generated from an unknown underlying probability mechanism P. By assumption, T tends to infinity and the number of "strategies" N is fixed. The last column $N+1$ is the benchmark, which can be the critical value of test statistics. For each strategy i, define a test statistic $w_{i,t}$, which tests the parameter θ_i. The parameter θ_i is defined in a way that under the null hypothesis, the strategy

i does not beat the benchmark, i.e., $\theta_i \le 0$.

The parameter θ_i is a function of the underlying probability mechanism P, i.e., $\theta_i = \theta_i(P)$. We are interested in testing whether the strategy i beats the benchmark and meanwhile control the FWE at a pre-specified level α. For a specific strategy i, the null hypothesis is $H_0^i : \theta_i \le 0$ against the alternative hypothesis $H_1^i : \theta_i > 0$. Romano and Wolf (2005) propose the StepM testing procedure:

1. Re-label the strategies in descending order of individual test statistics $w_{i,t}$. Label r_1 to the strategy with the largest test statistic, r_N to the strategy with the smallest statistic, so that $w_{r_1,T} > w_{r_2,T} > \cdots > w_{r_N,T}$.

2. Set $j = 1$ and $R_0 = 0$

3. For $R_{j-1} + 1 \le i \le N$, if zero is out of the confidence interval, reject the null. That is, reject $H_0^{r_i} \le 0$ if $0 \notin [w_{r_i,T} - \hat{c}_j, \infty)$, where $\hat{c}_j$ is a common value to all strategies, chosen in such a way to ensure the proper joint asymptotic coverage probability.

4. If no null hypothesis is rejected, stop. Otherwise, suppose the number of rejected null hypothesis so far is R_j, let $j = j + 1$, go to step 3.

In order to ensure the proper joint coverage probability, the common value c_j is chosen as the $1 - \alpha$ quantile of the sampling distribution of $\max_{R_{j-1}+1 \le i \le N}(w_{r_s,T} - \theta_{i,T})$, where R_{j-1} is the number of rejected null hypothesis in the first $j - 1$ steps. Formally, define:

$$\begin{aligned} \hat{c}_j &= c_j(1-\alpha, \hat{P}_T) && (3.8) \\ &= \inf\left\{x : \Pr_{\hat{P}_t}\left\{\max_{R_{j-1}+1 \le i \le N}(w^*_{r_i,T} - \theta^*_{r_i,T}) \le x\right\} \le 1-\alpha\right\} && (3.9) \end{aligned}$$

where $\hat{P}_T$ is the feasible estimation of the unknown probability mechanism P, $w^*_{r_i,T}$ is the test statistic under sampling distribution $\hat{P}_T$, and $\theta^*_{r_i,T}$ is the parameter under $\hat{P}_T$.

To account for the dependence of the statistics and avoid parametric assumptions, Romano and Wolf (2005) propose to compute $\hat{c}_j$ via bootstrap, the algorithm is as follows:

1. The order $r_1, \cdots, r_N$ and numerical values of $R_0, R_1, \cdots$ are given in the previous StepM testing algorithm.

2. Generate B bootstrap data matrix $y_T^{*,1}, \cdots, y_T^{*,B}$.

3. From each bootstrap data matrix $y_Y^{*,b}$, $b = 1, \cdots, B$, calculate the individual test statistics $w_{1,T}^{*,b}, \cdots, w_{N,T}^{*,b}$

4. For $b = 1, \cdots, B$, compute $\max_{j,T}^{*,b} = \max_{R_{j-1}+1 \leq i \leq N}(w_{T,r_i}^{*,b} - \theta_{r_i,T}^{*})$.

5. Set $\hat{c}_j$ as the $1 - \alpha$ quantile of the B replications of $\max_{j,T}^{*,1}, \cdots, \max_{j,T}^{*,B}$.

Romano and Wolf (2005) then propose a studentize version of the testing procedure, details of which can be found in the original paper. They prove that the stepwise procedure proposed can asymptotically control the FWE at level α.

3.2.4 Moon and Perron (2012)

The alternative hypothesis for heterogeneous panel unit root tests is that at least one of the units in the panel is stationary. Researchers are sometimes interested in identifying the stationary units, and this is often carried out by using individual time series tests. Moon and Perron (2012) use the concept of false discover rate (FDR) proposed by Benjamini and Hochberg (1995) to perform an multiple testing in order to gain an aggregate decision and meanwhile control the FDR.

They propose two ways of using the FDR in practice, one is to adjust the significance level for individual time series tests, given a selected FDR to be controlled, say 5%. The second one is to fix the significance level for individual tests and estimate the FDR. Note that for other mixed panel classification methods, for example, Ng (2008), Chortareas and Kapetanios (2009), the quantity of FDR cannot be estimated. In order to use the FDR as a basis to compare the different methods, we only review the first way, which controls the FDR in practice. Details for the second way can be found in Moon and Perron (2012).

They use the structure model to generate the panel data $y_{i,t}$:

$$y_{i,t} = d_{i,t} + y_{i,t}^0 \tag{3.10}$$

$$y_{i,t}^0 = \rho_i y_{i,t-1}^0 + \varepsilon_{i,t} \tag{3.11}$$

where $i = 1, \cdots, N$ is the cross-sectional dimension, $t = 1, \cdots, T$ is the time series dimension, $d_{i,t}$ is the deterministic component, and $y^0_{i,t}$ is the idiosyncratic component. The error term $\varepsilon_{i,t}$ is assumed to be stationary, so that the nonstationarity of the idiosyncratic component comes from $\rho_i = 1$. The nonstationary series have $\rho_i = 1$ and stationary series have $\rho_i < 1$.

Moon and Perron (2012) use bootstrap method to account for cross-sectional dependence. The bootstrap is used to mimic the joint distribution of time series test statistics under dependence, and calculate a proper set of critical values. They use the moving block bootstrap on the first differenced data to account for serial dependence, cross-sectional correlation and nonstationarity. The algorithm is the following:

1. Calculate the first difference of $y_{i,t}$, $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$, and collect these as a vector in each time period $\Delta Y_t = (\Delta y_{i,t}, \Delta y_{2,t}, \cdots, \Delta y_{N,t})'$.

2. Choose a block size b, which is the same to the bandwidth selection when estimating long-variance in Andrews (1991), draw $[T/b]$ blocks of b consecutive observations of ΔY_t with replacement. Draw the last block with length $T - [T/b]b$, denote the new bootstrap sample ΔY^*.

3. Cumulating the bootstrap sample ΔY^* to get the sample of level variable, $Y^*_t = \sum_{j=1}^{t} \Delta Y^*_j$.

4. Perform ADF test on each of the bootstrapped units.

5. Repeat steps 2-4 B times.

6. Compute the N critical values by solving equation (7) in Romano et al. (2008) for $n_0 = 1, \cdots, N$.

7. Denote the critical values as $\{\hat{c}_1, \hat{c}_2, \cdots, \hat{c}_n\}$. The null hypothesis can now be tested sequentially. Rank the ADF statistics from the most stationary (most negative) to the most nonstationary. Compare the first ADF statistic with $\hat{c}_1$, if the statistic is greater than $\hat{c}_1$ and the unit root null is not rejected, stop the testing procedure and all units are nonstationary. If the null is rejected, compare the second ADF statistic with $\hat{c}_2$, reject the null if ADF statistic is less than $\hat{c}_2$, so on and so forth. Stop the procedure at the first

non-rejection, call it j^*, then the stationary units are $i = 1, \cdots, j^* - 1$, and nonstationary units are $i = j^*, \cdots, N$.

Moon and Perron (2012) point out that the panel to be tested must be a balance panel, otherwise there will be holes in the bootstrap sample. The method computes the joint distribution of N ADF statistics, and is therefore subject to the curse of dimensionality.

3.2.5 Smeekes (2011)

Smeekes (2011) proposes an sequential approach to estimate the proportion of stationary units in a mixed panel and identify the stationary and nonstationary units. The sequential method uses quantile test statistics based on an increasing pre-defined proportions of stationary units, where critical values are obtained by bootstrap. For example, suppose the pre-defined proportion of stationary units to be tested is $q_j = 0.5$, the procedure bootstraps the critical value as the 50% quantile of N univariate unit root test statistics, and compares this critical value with the order statistic corresponding to $q_j = 0.5$. He claims that the sequential method has an advantage over multiple testing methods that it can exploit the information cross-sectionally, especially when T is small and N is relatively large.

Denote $k_0 = 0, 1, \cdots, N$ as the number of stationary units in the panel and $q_0 = k_0/N$ the proportion of stationary units. Define $q_1, \cdots, q_r$ as a set of user-defined numbers with $0 = q_1 < q_2 < \cdots < q_r < 1$. These numbers are the proportion of stationary units to be tested sequentially. Define $k_j = [q_j N]$ as the corresponding number of stationary units in the panel. The null hypothesis $H_0(q_j)$ is that the proportion of stationary units is q_j, i.e., the number of stationary units is k_j. The alternative hypothesis $H_1(q_{j+1})$ is that the proportion of stationary units is at least q_{j+1}, or at least k_{j+1} units are stationary in the panel. Denote $\tau(q_j, q_{j+1})$ be a test statistic to test the null against the alternative, and we reject the null hypothesis if $\tau(q_j, q_{j+1}) < c_\alpha(q_j, q_{j+1})$, where $c_\alpha(q_j, q_{j+1})$ is an appropriate critical value corresponding to the significance level α.

Smeekes (2011) proposes the following Sequential Quantile Test algorithm:

1. Test the null hypothesis $H_0(q_1)$ against the alternative $H_1(q_2)$. Reject the null if $\tau(q_1, q_2) < c_\alpha(q_1, q_2)$ and go to step 2. If the null is not rejected, stop and set $\hat{q} = q_1 = 0$.

2. Test $H_0(q_2)$ against $H_1(q_3)$, stop if a non-rejection is found and set $\hat{q} = q_2$. Go to step 3 if null is not rejected.

3. Keep testing until the null $H_0(q_j)$ is no longer rejected and set $\hat{q} = q_j$. If all null hypothesis are rejected, set $\hat{q} = 1$.

Denote θ_i to be any individual unit root test statistic applied to cross-sectional unit i, some assumptions about the individual test statistic are made. We should reject the unit root null for small values of θ_i. Under the null hypothesis, θ_i is asymptotically distributed as ξ_i, i.e. $\theta_i \xrightarrow{p} \xi_i$ as $T \to \infty$. θ_i is consistent, in particular, under the alternative, we have $\theta_i \xrightarrow{p} -\infty$ as $T \to \infty$. the Dickey-Fuller test is one of the unit root tests that satisfy the assumptions and is thereafter used in the sequential quantile test procedures. Note the individual test statistics are correlated if cross-sectional dependence is present.

Define $\theta_{(1)}, \cdots, \theta_{(N)}$ as the order statistics of $\theta_1, \cdots, \theta_N$ such that $\theta_{(1)} \leq \cdots \leq \theta_{(N)}$. The panel test statistic is then taken as the order statistic corresponding to the null hypothesis to be tested, i.e., $\tau(q_j, q_{j+1}) = \theta_{(k_{j+1})} = \theta_{([q_{j+1}N])}$. Due to the cross-sectional dependence, Smeekes (2011) proposes to use bootstrap to obtain the critical values.

To test the null $H_0(q_j)$ against the alternative $H_1(q_{j+1})$, the algorithm of Bootstrap Sequential Quantile Test ($BSQT$) is proposed as the following. Let

$$S_{k_j} = \{i : \theta_i \leq \theta_{(k_j)}\} \tag{3.12}$$

1. For each unit, detrend the data:

$$\tilde{y}_{i,t} = y_{i,t} - \hat{\beta}_i' z_t \tag{3.13}$$

where $\hat{\beta}_i$ is the OLS or GLS estimator of β_i. z_t is deterministic term(s).

2. For each unit, get the estimation of autoregressive root ρ_i:

$$\hat{\rho}_i = \frac{\sum_{t=1}^{T} \tilde{y}_{i,t}\tilde{y}_{i,t-1}}{\sum_{t=1}^{T} \tilde{y}_{i,t-1}^2} \tag{3.14}$$

and calculate

$$\hat{u}_{i,t} = \tilde{y}_{i,t} - \hat{\rho}_i \tilde{y}_{i,t-1} - \frac{1}{T-1} \sum_{t=2}^{T} (\tilde{y}_{i,t} - \hat{\rho}_i \tilde{y}_{i,t-1}) \tag{3.15}$$

where $t = 2, \cdots, T$. Let $\hat{u}_t = (\hat{u}_{1,t}, \cdots, \hat{u}_{N,t})'$.

3. Choose block length b, draw $i_0, \cdots, i_{k-1} \sim iidU[1, T-b]$, where $k = \lfloor (T-2)/b \rfloor + 1$ is the number of blocks. The "floor" operator $\lfloor\rfloor$ returns the nearest integer rounded towards $-\infty$. Smeekes (2011) suggests to use $b = 1.75T^{\frac{1}{3}}$ as in Palm et al. (2011).

4. Generate bootstrap error terms as

$$u_t^* = \hat{u}_{i_m+s} \tag{3.16}$$

where $m = \lfloor (t-2)/b \rfloor$ and $s = t - mb - 1$.

5. Generate the pseudo-observations:

$$y_{i,t}^* = \rho_i^* y_{i,t-1}^* + u_{i,t}^* \tag{3.17}$$

where $\rho_i^* = 1$ for all $i = 1, \cdots, N$.

6. Obtain θ_i^* for all $i \in S_{k_j}^c$ and the bootstrap order statistic:

$$\tau^*(q_j, q_{j+1}) = \theta^*_{(k_{j+1}-k_j : S_{k_j}^c)} \tag{3.18}$$

7. Repeat steps 3 to 6 B times, collect the B bootstrap statistics $\tau^{*,s}(q_j, q_{j+1})$, $s = 1, \cdots, B$. The bootstrap critical value $c_\alpha^*(q_j, q_{j+1})$ is selected as the α-quantile of the bootstrap statistics $\tau^{*,s}(q_j, q_{j+1})$, $s = 1, \cdots, B$.

The set of units that are deemed as stationary is then $S_{\hat{k}} = S_{[\hat{q}N]}$.

Smeekes (2011) then points out that the selection of the quantiles to be tested should be as spread out as possible, and meanwhile still being able to draw meaningful economic conclusions. The test combines the information of cross-sectional units that are within two neighboured quantiles, which increase the power of the test. So the more spread out of the quantiles, the more power the test has. On the other hand, since we can only draw the conclusion that the true proportion of stationary units is within the interval (q_{j-1}, q_{j+1}), the conclusion is more interpretable if the estimated quantile is closer to its neighbours.

Some extensions are made in additional to the $BSQT$ test. Although the individual test statistic for a stationary unit diverges to negative infinity asymptotically, it may overlap the distribution of test statistic for a nonstationary unit in finite samples. This may influence the ordered statistic in finite samples. The $BSQT$ method ignores this by removing the stationary units in the sample in next round of testing. A modification, denoted $BSQT_2$, is proposed in a way that it keeps the stationary units to obtain the bootstrap critical values for the order statistic. Bootstrap errors u_t^* is obtained in the same way as in the previous algorithm for $BSQT$, from step 5, the modified algorithm is as the following:

5. Generate

$$y_{i,t}^* = \rho_i^* y_{i,t-1}^* + u_{i,t}^* \tag{3.19}$$

   where

$$\rho_i^* = \begin{cases} \hat{\rho}_i & if\ i \in S_{k_j} \\ 1 & if\ i \in S_{k_j}^c \end{cases} \tag{3.20}$$

6. Obtain θ_i^* for all units and define the bootstrap order statistic

$$\tau_2^*(q_j, q_{j+1}) = \theta_{(k_{j+1})}^* \tag{3.21}$$

7. Repeat step 3 to 6 B times, collect the B bootstrap order statistics $\tau_2^{*,s}(q_j, q_{j+1}), s = 1, \cdots, B$. The bootstrap critical value $c_\alpha^*(q_j, q_{j+1})$ is selected as the α-quantile of the bootstrap statistics $\tau_2^{*,s}(q_j, q_{j+1})$, $s = 1, \cdots, B$.

Another modification to the $BSQT$ and $BSQT_2$ is proposed. Smeekes (2011) shows that after an estimation of $\hat{q}$ by $BSQT$, the true proportion of stationary units is between $\hat{q}_{-1}$ and $\hat{q}_{+1}$, where $\hat{q}_{-1}$ and $\hat{q}_{+1}$ are the nearest quantiles smaller and larger than $\hat{q}$, respectively. That is, $\lim_{T\to\infty} P(q_0 \in [\hat{q}_{-1}, \hat{q}_{+1}] \geq 1-\alpha)$. The $BSQT$ and $BSQT_2$ stop here, to obtain a more precise estimation of q_0, a further sequential quantile test ($BSQT$ or $BSQT_2$) can be applied on the interval $[\hat{q}_{-1}, \hat{q}_{+1}]$. Similarly, the estimated interval can be further shortened, and this procedure continues until an user specific level of precision. This procedure is labelled Iterative Bootstrap Sequential Quantile Test ($IBSQT$), or $IBSQT_2$ corresponding to $BSQT_2$. It is pointed out that the probability of making an error is accumulated with the number of iterations, that is, $\lim_{T\to\infty} P\{\hat{q}^{(I)} = q_0\} \geq (1-\alpha)^I$. So Smeekes suggests that the number of iterations should be selected fairly small.

3.2.6 Ng (2008)

Ng (2008) proposes an estimator to consistently estimate the proportion of a panel that has a unit root. Suppose that the fraction of the unit root processes in a panel is θ, Ng shows that the cross-sectional variance of the mixed panel is dominated by a linear trend which grows at rate θ. Once the proportion θ is estimated, hypothesis testing can be performed to test the null that $\theta = \theta^0$ for any $\theta^0 \in (0, 1]$. The test statistic is asymptotically standard normal, and the critical values are independent of whether cross-sectional dependence is present in the data, assuming that the cross-sectional variation is stationary.

Without cross-sectional dependence, Ng (2008) considers the following data generating process:

$$y_{i,t} = \lambda_i + u_{i,t} \tag{3.22}$$

$$u_{i,t} = \alpha_i u_{i,t-1} + e_{i,t} \tag{3.23}$$

where $i = 1, \cdots, N$ and $t = 1, \cdots, T$, $y_{i,t}$ is observed, $u_{i,t}$ and $e_{i,t}$ are not observed. $\alpha_i \in [0, 1]$ and is independent of $e_{j,t}$ for all i, j, t. $0 \leqslant \lambda_i < \infty$ for all i and λ_i is independent of $e_{i,t}$ for all i, t. Denote the cross-sectional mean and variance of λ_i as $\Lambda_N = \frac{1}{N}\sum_{i=1}^{N}\lambda_i$ and $var_{i,N}(\lambda_i) = \frac{1}{N}\sum_{i=1}^{N}(\lambda_i - \Lambda_N)^2$, then finite variance is assumed, such that $0 \leqslant var_{i,N}(\lambda_i) < \infty$. For all i, $e_{i,t}$ is i.i.d. over i and t with $u_{i,0} = O_p(1)$. Denote the cross-sectional mean and variance of $y_{i,t}$ as $Y_{t,N} = \frac{1}{N}\sum_{i=1}^{N} y_{i,t}$ and $V_{t,N} = \frac{1}{N}\sum_{i=1}^{N}(y_{i,t} - Y_{t,N})^2$, define average increase in cross-sectional variance as $\hat{\theta} = \frac{1}{T}\sum_{t=1}^{T}\Delta V_{t,N}$. Ng (2008) shows that if the DGP and assumptions hold, as $N \to \infty$ and then $T \to \infty$,

$$\sqrt{N}(\hat{\theta} - \theta) \xrightarrow{d} N(0, 2\theta) \tag{3.24}$$

$\hat{\theta}$ is $\sqrt{N}$ consistent for θ.

With higher serial correlation and heteroscedastic error terms, Ng (2008) consider the DGP

$$y_{i,t} = \lambda_i + u_{i,t} \tag{3.25}$$

$$\alpha_i(L)u_{i,t} = e_{i,t} \tag{3.26}$$

where, $e_{i,t} \sim iid(0, \sigma_i^2)$, $\alpha_i(L) = 1 - \alpha_{i,1}L - \cdots - \alpha_{i,p}L^p$, p is finite. Ng (2008) proposes to construct correcting terms to control the variance of all cross-sectional units raising from higher order serial correlation and unit specific variance. The θ can be estimated as the following:

1. For each i, estimate the $AR(p_i)$ model

$$y_{i,t} = \alpha_0 + \alpha_1 y_{i,t-1} + \cdots + \alpha_p y_{i,t-p} + e_{i,t} \tag{3.27}$$

   Denote the estimated coefficients as $(\hat{\alpha}_0, \cdots, \hat{\alpha}_p)$ and extract the residuals $\hat{e}_{i,t}$.

2. Estimate the unit specific variance $\hat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_{i,t}$.

3. Denote $\hat{\phi}_{i,1} > \hat{\phi}_{i,2} > \cdots > \hat{\phi}_{i,p}$ be the reciprocal of the roots of $1 - \hat{\alpha}_{i,1}L - \cdots - \hat{\alpha}_{i,p}L^p = 0$.

4. Construct the serial correlation adjustment term D_i, Ng recommends the approximation that $\hat{D}_i = \hat{\phi}_{i,1}^{p-1} - \hat{\phi}_{i,1}^{p-2} \sum_{j=2}^{p} \hat{\phi}_{i,j}$.

5. Rescale the data as $\hat{y}_{i,t} = \frac{\hat{D}_i y_{i,t}}{\hat{\sigma}_i}$, calculate the cross-sectional variance of $\hat{y}_{i,t}$, $\hat{V}_{t,N} = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_{i,t} - \hat{Y}_{t,N})^2$, where $\hat{Y}_{t,N} = \frac{1}{N}\sum_{i=1}^{N} \hat{y}_{i,t}$.

6. Let $\hat{\theta} = \frac{1}{T}\sum_{t=1}^{T} \Delta\hat{V}_{t,N}$ and $\hat{\eta}_{t,N} = \Delta\hat{V}_{t,N} - \hat{\theta}$. Let $\hat{\gamma}_\eta(k) = \frac{1}{T}\sum_{t=1}^{T-1} \hat{\eta}_{t,N}\hat{\eta}_{t-k,N}$ and $\omega_{\hat{\theta}}^2 = \hat{\gamma}_\eta(0) + 2\sum_{s=1}^{M} K(s,M)\hat{\gamma}_\eta(s)$, where $K(s,M)$ is a kernel with a truncation point of M. Ng (2008) uses the Newey-West kernel $K(s,M) = 1 - \frac{s}{M+1}$ with $M = 2$.

7. The estimated fraction is then $\hat{\theta}$ and the studentized statistic

$$t = \frac{\hat{\theta} - \theta}{\omega_{\hat{\theta}}} \sim N(0,1) \tag{3.28}$$

   can be used to test the null that $\theta = \theta^0$ for any value of $\theta^0 \in (0, 1]$.

Ng (2008) then considers the case that cross-sectional dependence is present in the panel. Following the tests of Moon and Perron (2004) and Bai and Ng (2004), a factor structure is assumed in the data generating process:

$$y_{i,t} = \lambda_i F_t + u_{i,t} \tag{3.29}$$

$$\alpha_i(L) u_{i,t} = e_{i,t} \tag{3.30}$$

where $\alpha_i(L)$ is a finite pth order polynomial in the lag operator L, F_t is the single common factor that may or may not be observed. If F_t is not observed, Ng (2008) suggests to use the principal component of $y_{i,t}$ or the average of $y_{i,t}$ to estimate the common factor. Additional to the previous assumptions in the case of cross-sectional independence, Ng (2008) further assumes that $\beta(L)F_t = v_t$, where the roots of $\beta(L) = 0$ are outside the unit circle, $E(u_{it}v_s) = 0$ for all i, t, s, $E(\alpha_i v_s) = 0$ for all i, s, and $E(\lambda_i \alpha_j) = 0$ for all i, j. According to the assumption, the common factor F_t must be stationary. Ng (2008) shows that the previous estimator remains valid, however, accounting for F_t when constructing D_i may yield the estimates of the largest eigenvalues more precisely. Another estimator that accounts for cross-sectional dependence is proposed and can be calculated in the following:

1. For each unit i, estimate the least square regression of $y_{i,t}$:

$$y_{i,t} = \alpha_0 + \alpha_1 y_{i,t-1} + \cdots + \alpha_p y_{i,t-p} + \lambda_{i,0}\hat{F}_t + \cdots + \lambda_{i,q}\hat{F}_{t-q} + e_{i,t} \tag{3.31}$$

   where $\hat{F}_t$ is the first principal component of $y_{i,t}$ or an aggregate variable. Denote the estimated coefficients $(\hat{\alpha}_0, \hat{\alpha}_{i,1}, \cdots, \hat{\alpha}_{i,p}, \hat{\lambda}_{i,0}, \hat{\lambda}_{i,1}, \cdots \hat{\lambda}_{i,q})$ and estimated residuals $\hat{e}_{i,t}$.

Then follow steps 2-7 in the estimator without cross-sectional dependence, we have $\hat{\theta} = \frac{1}{T}\sum_{t=1}^{T} \Delta \hat{V}_{t,N} \xrightarrow{p} \theta$ and $(\hat{\theta} - \theta)/\omega_{\hat{\theta}} \sim N(0,1)$, asymptotically.

3.3 Data Generating Process

Similar to Gengenbach et al. (2010), we are going to consider four different data generating processes. The first three DGPs are similar to those used by Bai and Ng (2004) :

$$
\begin{aligned}
Y_{i,t} &= \alpha_i + \lambda'_{i,m} F_{m,t} + E_{i,t} && (3.32)\\
F_{m,t} &= \varphi_m F_{m,t-1} + f_{m,t} && (3.33)\\
E_{i,t} &= \delta_i E_{i,t-1} + e_{i,t} && (3.34)
\end{aligned}
$$

where $i = 1, \cdots, N$, $t = 1, \cdots, T$, and $m = 1, \ldots, K$, and K is the number of factors. Each unit is the sum of a fixed effect α_i, common factors with different factor loadings $\lambda'_{i,m}F_{m,t}$, and idiosyncratic components $E_{i,t}$. The cross-sectional dependency is induced by the common factors. The error terms in the common factors and idiosyncratic components are generated as $MA(1)$ processes:

$$
\begin{aligned}
f_{m,t} &= \eta_{m,t} + \zeta_m \eta_{m,t-1} && (3.35)\\
e_{i,t} &= \varepsilon_{i,t} + \rho_i \varepsilon_{i,t-1} && (3.36)
\end{aligned}
$$

where shocks are drawn from independent standard normal distributions, i.e., $\eta_{m,t} \sim i.i.d.N(0,1)$ and $\varepsilon_{i,t} \sim i.i.d.N(0,1)$.

Furthermore, out of the N cross-sectional units, we define the proportion of nonstationary units as θ and the number of nonstationary units as N_1 (the subscript 1 is used to denote I(1) units, and N_1 is $\theta \cdot N$ rounded to the nearest integer). The N_1 nonstationary units are set to be the first N_1 units in the panel in the sense that the first to the N_1^{th} columns in the $T \times N$ panel matrix are nonstationary. We use the subscript 0 to denote the I(0) units, so that there are $N_0 = N - N_1$ units that are stationary, and they correspond to the last N_0 columns in the panel matrix.

The nonstationarity arises from two sources, the common factor component and the idiosyncratic component. A cross-sectional unit is nonstationary if either the common factors have a unit root or its idiosyncratic component has a unit root, or both. A cross-sectional unit is stationary if both its common factor and idiosyncratic component are stationary. Another possibility is that, the cross-sectional unit is stationary if it had stationary idiosyncratic component and zero factor loadings when the factors are nonstationary. So we need to consider all three different cases for the mixed panel.

1. *Nonstationary common factor components and nonstationary idiosyncratic components.* In this first case, the common factors are nonstationary, i.e., $\varphi_m = 1$ for all m. The nonstationary cross-sectional units have non-zero factor loadings, $\lambda_i \neq 0$ for $i = 1, \cdots, N_1$, hence the common factor components in these units are nonstationary. The idiosyncratic components in the N_1 nonstationary units are also nonstationary, such that, $\delta_i = 1$ for $i = 1, \cdots, N_1$. For the stationary units, within this framework, we assume that the factor loadings are zeros, $\lambda_i = 0$ for $i = N_1 + 1, \cdots, N$. The idiosyncratic components in these units are stationary, that is, $\delta_i < 1$ for $i = N_1 + 1, \cdots, N$.

2. *Nonstationary common factor components and stationary idiosyncratic components.* In this second case, the common factors are nonstationary, $\varphi_m = 1$ for all m. The nonstationary cross-sectional units have non-zero factor loadings, $\lambda_i \neq 0$ for $i = 1, \cdots, N_1$. The factor loadings for the remaining N_0 units are set to be zero, $\lambda_i = 0$ for $i = N_1 + 1, \cdots, N$. The idiosyncratic components for all the units are stationary, that is, $\delta_i < 1$ for all i.

3. *Stationary common factor components and nonstationary idiosyncratic components.* In this case, the common factors are all stationary, $\varphi_m < 1$ for all m. All cross-sectional units have non-zero factor loadings, $\lambda_i \neq 0$ for all i. The idiosyncratic component for the first N_1 units is nonstationary, such that $\delta_i = 1$ for $i = 1, \cdots, N_1$. The rest N_0 units have stationary idiosyncratic components, in the way that $\delta_i < 1$ for $i = N_1 + 1, \cdots, N$.

In addition to the three data generating process above, we also consider a fourth DGP which is similar to Moon and Perron (2004, 2012) and Pesaran (2007b). The DGP has a single stationary common factor f_t:

$$Y_{i,t} = \alpha_i + \delta_i Y_{i,t-1} + u_{i,t} \tag{3.37}$$

$$u_{i,t} = \lambda_i f_t + e_{i,t} \tag{3.38}$$

where the common factor is generated as an $AR(1)$ process, and the idiosyncratic component is generated as an $ARMA(1,1)$ process:

$$f_t = \eta_t + \zeta f_{t-1} \tag{3.39}$$

$$e_{i,t} = \rho_i e_{i,t-1} + \varepsilon_{i,t} + \psi_i \varepsilon_{i,t-1} \tag{3.40}$$

To summarise, the first three DGPs considered are those under the Bai and Ng (2004) framework and the last DGP is similar to Moon and Perron (2004, 2012) and Pesaran (2007b):

1. Unit roots are in both common and idiosyncratic components for nonstationary units. Zero factor loadings and I(0) idiosyncratic components for stationary units.

2. I(1) common factors and I(0) idiosyncratic components for nonstationary units. Zero factor loadings and I(0) idiosyncratic components for stationary units.

3. Stationary common factors with non-zero loadings for all the units. I(1) idiosyncratic components for the nonstationary units and I(0) idiosyncratic components for the stationary units.

4. One single stationary common factor. The autoregressive roots for the nonstationary units are equal to one, and the autoregressive roots for the stationary units are less than one.

3.4 Monte Carlo Simulation

3.4.1 Simulation Design

We consider three different values for T and N respectively. T is taken from the set $\{50, 100, 200\}$, and N is taken from $\{10,\ 30,\ 50\}$. The number of time series and cross-sectional dimension is chosen so that it is consistent with most macro-panels in the empirical studies. To extract the common factors, we use principle components as in Moon and Perron (2004), Bai and Ng (2004). Note that however, Pesaran (2007b) uses the cross-sectional average to proxy the common factor. Both methods require the cross-sectional dimension to be relatively large to ensure that the common factors can be estimated correctly, so we are not going to use too small values of N. We consider two cases of the number of common factors, K. First, one single common factor is considered, i.e., $K = 1$. The single factor is generated as an $AR(1)$ process with the autoregressive coefficient that equals to unity or less than unity, depending on the nature of the factor. The case where two common factors are present, i.e., $K = 2$ is also considered. For this case, the two factors have the same autoregressive parameters and homogeneous variances of their innovations, but are taken from different drawings.

We choose to take the fraction of nonstationary units, θ, from the set $\{0, 0.1, 0.3, 0.5, 0.7, 0.9, 1\}$. The fraction 0 implies that all the series in the panel are stationary, while $\theta = 1$ implies all cross-sectional units are I(1) processes. When θ is strictly between 0 and 1, there are N_1 nonstationary units and $N_0 = N - N_1$ stationary units, where N_1 is $\theta \cdot N$ rounded to the nearest integer.

The fixed effects in DGP equation 3.32 and equation 3.37 are generated as independent standard normals for each unit, such that $\alpha_i \sim i.i.d.N(0, 1)$. The error terms in the common factors and idiosyncratic components are generated from independent standard normals too, such that, in equation 3.35, $\eta_{m,t} \sim i.i.d.N(0, 1)$, in equation 3.36, $\varepsilon_{i,t} \sim i.i.d.N(0, 1)$. The η_t and $\varepsilon_{i,t}$ in equation 3.39 and equation 3.40 are $i.i.d.N(0, 1)$, respectively.

For the first three DGPs, the MA roots in equation 3.35 and equation 3.36 are drawn independently, $\zeta_m \sim U[0.2, 0.5]$ and $\rho_i \sim U[0.2, 0.5]$. For the last DGP, two cases for the $ARMA$ roots in equation 3.40 are considered. In the first case, the ARMA roots are set to be zero, $\rho_i = 0$ and $\psi_i = 0$ for all i_s, so that the idiosyncratic component is actually independent draw from the standard normal distribution. In the second case, the roots have uniform distribution, $\rho_i \sim iidU[-0.5, 0.5]$ and $\psi_i \sim iidU[-0.5, 0.5]$, so that the idiosyncratic component is an $ARMA(1, 1)$ process.

In the first and second DGPs, where the common factors are nonstationary, $\varphi_m = 1$ for all m in equation 3.33. The stationarity/nonstationarity for the common component in any individual unit is controlled by the factor loadings. For the nonstationary units $i = 1, \cdots, N_1$, two cases of factor loadings are considered, $\lambda_i \sim iidU[-1, 2]$ for low dependence and $\lambda_i \sim iidU[-1, 4]$ for high dependence[1]. For the stationary units $i = N_1 + 1, \cdots, N$, the factor loadings are set to be

[1]Consistency of the Pesaran (2007b) test requires the mean of factor loadings to be nonzero.

zero, $\lambda_i = 0$.

In the first DGP, the idiosyncratic component for the first N_1 units has unit root, such that $\delta_i = 1$. The idiosyncratic component is stationary for the remaining N_0 units, and the root is taken from independent uniform distributions, such that $\delta_i \sim i.i.d.U[0, 1 - \frac{c}{T}]$ and $c = 10$. In the second DGP, the idiosyncratic component is stationary for all units and $\delta_i \sim iidU[0, 1 - \frac{10}{T}]$ for all i.

In the third DGP, the common factors are stationary with $\varphi_m = 0.5$ for all m. The factor loadings are nonzero for all units and as above, $\lambda_i \sim iidU[-1, 2]$ for low dependence and $\lambda_i \sim iidU[-1, 4]$ for high dependence. The idiosyncratic root $\delta_i = 1$ for $i = 1, \cdots, N_1$ and $\delta_i \sim iidU[0, 1 - \frac{10}{T}]$ for $i = N_1 + 1, \cdots, N$.

In the last DGP, one stationary common factor is present, the AR root ζ in equation 3.39 is 0.5. Three cases of factor loadings are considered. $\lambda_i = 0$ for no dependence, $\lambda_i \sim iidU[-1, 2]$ for low dependence and $\lambda_i \sim iidU[-1, 4]$ for high dependence. The δ_i in equation 3.37 equals to one for nonstationary units, and $\delta_i \sim iidU[0, 1 - \frac{10}{T}]$ for stationary units.

The main differences for the four data generating processes can be summarised in Table 3.1.

For panel unit root tests, the level of significance is set at 5% for all cases, the constant only case for the deterministic terms is considered. For the Pesaran (2007b) $CIPS$ test, following Im et al. (2003) and Pesaran (2007b), we use an arbitrary homogeneous lag length in the $ADF/CADF$ regression for all units. The homogeneous lag length is set at one to ensure that error terms are white noises.[2] Cross-sectional mean $\bar{y}_{t-1}$ and $\Delta\bar{y}_t$ are used to proxy and filter out the unobserved common factor. The critical values for $CIPS$ test statistic are obtained from Table II(b) in Pesaran (2007b). For the Moon and Perron (2004) t_a^* and t_b^* test statistics, the number of common factors should be know beforehand. Due to the fact that the information criteria by Bai and Ng (2002) tends to select the largest number of common factors imposed, we use two methods to select the number of factors. The first one is to use the correct number of factors (in our framework, the number of factors is either one or two) and is therefore not feasible, the second one is to use the information criteria by Bai and Ng (2002) with largest number of common factors set at two. The two methods generate similar results and only the results

[2]We have also checked the lag length for $p_i = p = 2, 3, 4$ and results are similar.

Table 3.1: Data Generating Process

	Sources of Nonstationarity	Units	Common Factor	Factor Loading	Idiosyncratic Component
DGP 1	both common and idiosyncratic component	$i = 1, \cdots, N_1$	$\varphi_m = 1 \ \forall \ m$	$\lambda_i \sim iidU[-1,2]$ or $\lambda_i \sim iidU[-1,4]$	$\delta_i = 1$
		$i = N_1 + 1, \cdots, N$	N/A	$\lambda_i = 0$	$\delta_i \sim iidU[0, 1 - \frac{10}{T}]$
DGP 2	common component only	$i = 1, \cdots, N_1$	$\varphi_m = 1 \ \forall \ m$	$\lambda_i \sim iidU[-1,2]$ or $\lambda_i \sim iidU[-1,4]$	$\delta_i \sim iidU[0, 1 - \frac{10}{T}]$
		$i = N_1 + 1, \cdots, N$	N/A	$\lambda_i = 0$	same as $i = 1, \cdots, \mathrm{N}_1$
DGP 3	idiosyncratic component only	$i = 1, \cdots, N_1$	$\varphi_m = 0.5 \ \forall \ m$	$\lambda_i \sim iidU[-1,2]$ or $\lambda_i \sim iidU[-1,4]$	$\delta_i = 1$
		$i = N_1 + 1, \cdots, N$	$\varphi_m = 0.5 \ \forall \ m$	same as $i = 1, \cdots, \mathrm{N}_1$	$\delta_i \sim iidU[0, 1 - \frac{10}{T}]$
DGP 4	autoregressive root $\delta_i = 1$ for $i = 1, \cdots, N_1$ $\delta_i \sim iidU[0, 1 - \frac{10}{T}]$ for $i = N_1 + 1, \cdots, N$	$i = 1, \cdots, N_1$	$\zeta = 0.5$	$\lambda_i = 0$ or $\lambda_i \sim iidU[-1,2]$ or $\lambda_i \sim iidU[-1,4]$	$e_{i,t} \sim iidN(0,1)$ or ARMA(1,1) with roots drawn from $iidU[-0.5, 0.5]$
		$i = N_1 + 1, \cdots, N$	same as $i = 1, \cdots, N_1$	same as $i = 1, \cdots, N_1$	same as $i = 1, \cdots, N_1$

Note: Summary of Data Generating Processes in this Chapter.

by the first method are reported. Similar to Moon and Perron (2004), the long-run variances is estimated by quadratic spectral kernel with prewhitening and the bandwidth is selected by Andrews and Monahan (1992) estimator. The t_a^* and t_b^* have standard normal distribution, hence we reject the null hypothesis if the statistic is less than -1.64 for a one-sided test. For the Breitung and Das (2008) t_{rob} and t_{gls} test statistics, the lag length is obtained by AIC with maximum lag length $p_{max} = 4$. The first observation is subtracted from the data, following the advise by Breitung and Das (2008). The t_{rob} and t_{gls} are standard normally distributed and we reject the null hypothesis if the statistic is less than -1.64. For Bai and Ng (2004) ADF$_{\hat{F}}$ test, the lag length is chosen by AIC with $p_{max} = 4$. Following the suggestions by the authors, bartlett kernel is employed and the bandwidth is selected by Andrews (1991), the lag length is selected by the ASIC, which is proposed by Aznar and Salvador (2002). The ADF$_{\hat{F}}$ statistics has Dickey-Fuller distribution with constant only, hence we reject the null hypothesis if ADF$_{\hat{F}}$ is less than -2.86, which is the asymptotic critical value of DF distribution. The $P_{\hat{e}}$ test statistic has standard normal distribution and the null is rejected if $P_{\hat{e}} > 1.64$. For MQ_c and MQ_f tests, the maximum number of possible common stochastic trends is set as two [3] The critical values for MQ_c and MQ_f are obtained from Table I in Bai and Ng (2004).

The parameters for mixed panel tests are set as the following. The significance level for all tests is set at 5%, the level of FWE or FDR to be controlled at is set to be 5%, the constant only case is considered for the deterministic terms when appropriate. For Chortareas and Kapetanios (2009) $SPSM$ test, the settings are similar to the $CIPS$ unit root tests. The lag length for $CADF$ regression is set at one. Cross-sectional mean $\bar{y}_{t-1}$ and $\Delta\bar{y}_t$ are used to proxy and filter out the unobserved common factor. The critical value for the $CIPS$ in each round of the procedure is obtained from Table II(b) in Pesaran (2007b). For Romano and Wolf (2005) stepwise multiple testing method, the lag length of ADF regression is chosen by BIC with maximum number of lag length $p_{max} = 4$. The block bootstrap method is employed and the block length is data dependent and taken to be $b = \lfloor 1.75T^{\frac{1}{3}} \rfloor$ as in Palm et al. (2011), where $\lfloor\rfloor$ returns the nearest integer rounded towards $-\infty$. The number of replications for

[3] Gengenbach et al. (2010) use simulation to show that MQ_c and MQ_f tests tend to select the maximum number of possible common stochastic trends and hence tend to over-estimate the number of I(1) common factors. They show that the impact of over-estimating the number of I(1) common factor is less severe than the impact of under-estimating the number of I(1) common factors. We, however, do not focus on this issue here.

each bootstrap is 500 due to the computational burden. For Moon and Perron (2012) multiple testing procedure, the ADF lag length is selected by BIC with $p_{max} = 4$. Block bootstrap is used and the block size $b = \lfloor 3.21T^{\frac{1}{3}} \rfloor$ is the same to the bandwidth of long-run variance estimation in Andrews (1991). For Smeekes (2011) $BSQT$ tests, the lag length is chosen by BIC with maximum length $p_{max} = 4$. Block bootstrap is used with block size to be set at $b = \lfloor 1.75T^{\frac{1}{3}} \rfloor$. The number of bootstrap replication is 500. All individual units are demeaned by OLS as suggested by the author. The user-specified quantiles are chosen as {0, 0.1, 0.3, 0.5, 0.7, 0.9}. [4] For $IBSQT$ and $IBSQT2$ tests, the number of iterations is set at 3. For Ng (2008) test, the lag length is selected by BIC with maximum length $p_{max} = 4$. The common factor is estimated in a way similar to Bai and Ng (2004), which applies the principal component method on the first differenced data and accumulates common and idiosyncratic components back to their original level for allowing nonstationary common factors. We use the true number of factors in the DGP[5]. The kernel employed is the Newey-West kernel $K(s, M) = 1 - \frac{s}{M+1}$, with a data dependant truncation bandwidth $M = \lfloor 4(\frac{T}{100})^{2/9} \rfloor$. Similar to Ng (2008), three interesting hypothesis testing are performed: $H_0^A : \theta = 0.01$ against $H_1^A : \theta > 0.01$, $H_0^B : \theta = \theta_0$ against $H_1^B : \theta \neq \theta_0$, $H_0^C : \theta = 1$ against $H_1^C : \theta < 1$, where θ_0 is the (infeasible) true value of θ. The first hypothesis testing is used to examine whether all units are stationary, the second one gives us the size of the test, the third hypothesis testing checks whether all units are nonstationary against some units are stationary, which is often the null and alternative hypothesis for panel unit root tests. The test is asymptotically standard normally distributed. The decision rule is that we reject H_0^A if $t < -1.64$, reject H_0^B if $|t| > 1.96$, and reject H_0^C if $t > 1.64$.

All calculations are performed with Gauss 10. The number of replications is taken to be 1000 for all cases.

[4] We also use {0, 0.2, 0.4, 0.6, 0.8} as the quantiles and the results are similar.

[5] We also use number of factors that is different from the true number. Results show that the performance of Ng08 test relies on the correct number of factors. We do not discuss this issue here as this is not the key point.

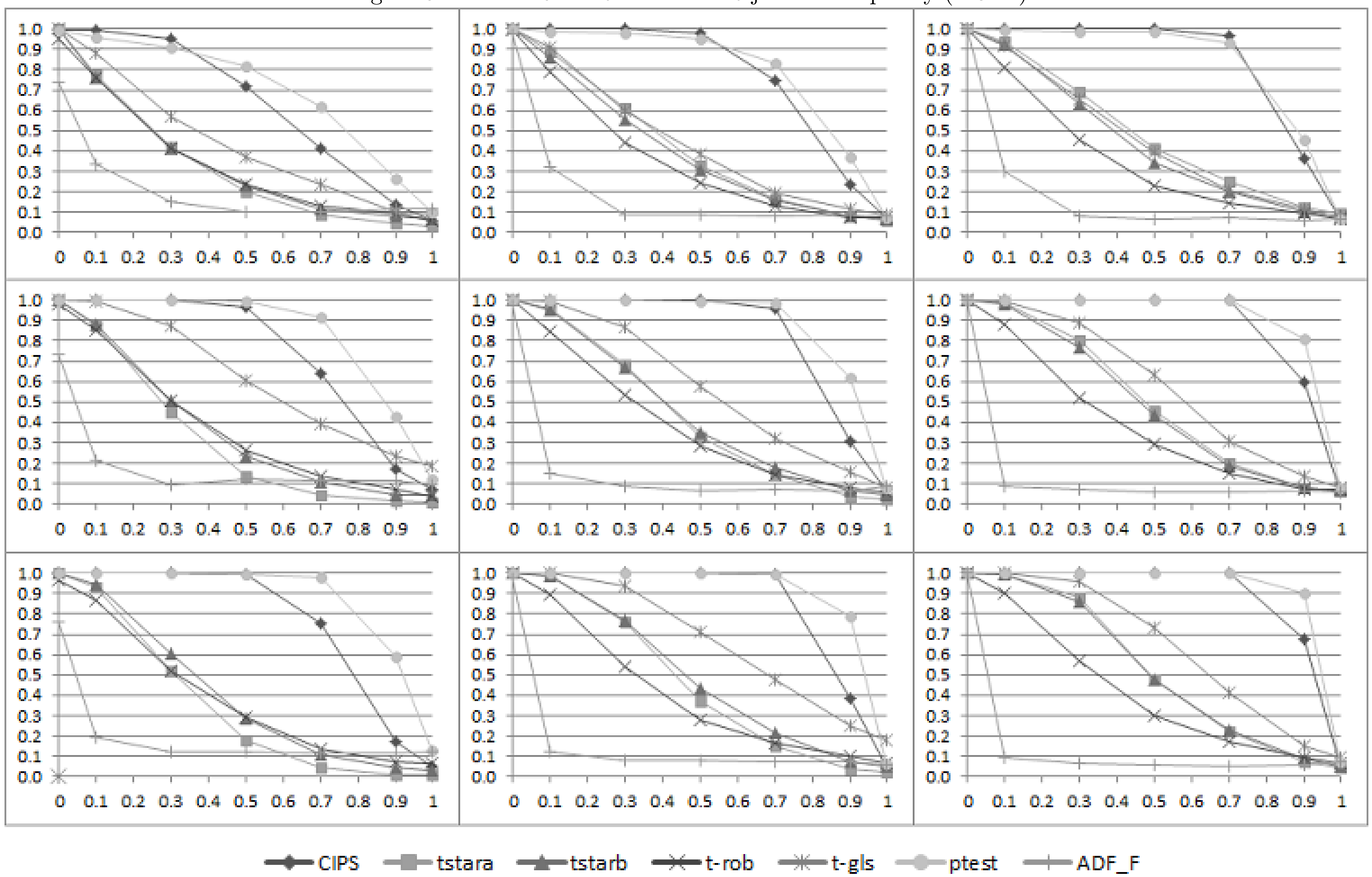
3.4.2 Simulation Results for Panel Unit Root Tests

Full Monte Carlo simulation results for panel unit root tests for DGP1 to DGP4 are reported and analysed in Appendix B.1. In order to simplify the illustration, we only focus on the effect of panel dimensions $\{T,\ N\}$, and proportion of unit root processes θ in the main text. For DGP1 to DGP3, we report the results when the number of common factors is 1, and the cross-sectional dependence is low (with factor loadings $\lambda_i \sim iidU[-1,2]$). For DGP4, we report the results when cross-sectional dependence is low, and idiosyncratic components are $ARMA(1,1)$ processes. For each combination of $\{T,\ N,\ \theta\}$, we report the rejection frequency of Pesaran (2007b) $CIPS$ test statistic, Moon and Perron (2004) t_a^* and t_b^* test statistics, Breitung and Das (2008) t_{rob} and t_{gls} test statistics and Bai and Ng (2004) $P_{\hat{e}}$ and $ADF_{\hat{F}}$ statistics. We use a 3×3 graph for each data generating process, where three columns corresponding to $T = 50$, $T = 100$, $T = 200$, and three rows corresponding to $N = 10$, $N = 30$, $N = 50$. The horizontal axis is nonstationary proportion θ, and vertical axis is rejection frequency of tests.

Figure 3.1 plots the results obtained for DGP1, where a unit root is present in both common factor and idiosyncratic components. For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls}, $P_{\hat{e}}$ test statistics, rejection frequencies when $\theta = 1$(all units are nonstationary) report the sizes of tests, and rejection frequencies when $\theta =\{0,\ 0.1,\ \cdots,\ 0.9\}$ report the power of tests. For the t_{rob} and t_{gls} homogeneous tests with the alternative that all units are stationary, however, a mixed panel with $\theta =\{0.1,\ \cdots,\ 0.9\}$ is somewhere between the null and alternative hypothesis. For $ADF_{\hat{F}}$ statistic, rejection frequencies with $\theta =\{0.1,\ \cdots,\ 0.9,\ 1\}$ report the size of test. When $\theta = 0$, there is no common factor in the panel, and $ADF_{\hat{F}}$ is not an appropriate test here, but we still extract one common factor and perform the $ADF_{\hat{F}}$ test and deem this as the "power" of it.

For the Pesaran (2007b) $CIPS$ test, it shows nearly no size distortion, even when T and N are small. The power of test increases with both T and N, and decreases with nonstationary proportion θ. The test is very sensitive to the stationary units in panel. For a moderate panel with $T = 100$ and $N = 30$, the power of $CIPS$ is close to 1 if 30% of units are stationary. For a large panel with $T = 200$ and $N = 50$, the rejection frequency of $CIPS$ is close to 0.7 even if there are only 10% stationary units in panel. The performance of Moon and Perron (2004) t_a^* and t_b^* tests are similar. When T is small, t_a^* is under-sized, causing relatively low power. When

Figure 3.1: Panel Unit Root Tests: Rejection Frequency (DGP1)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively.
Horizontal axis is nonstationary proportion θ, vertical axis is rejection frequency.

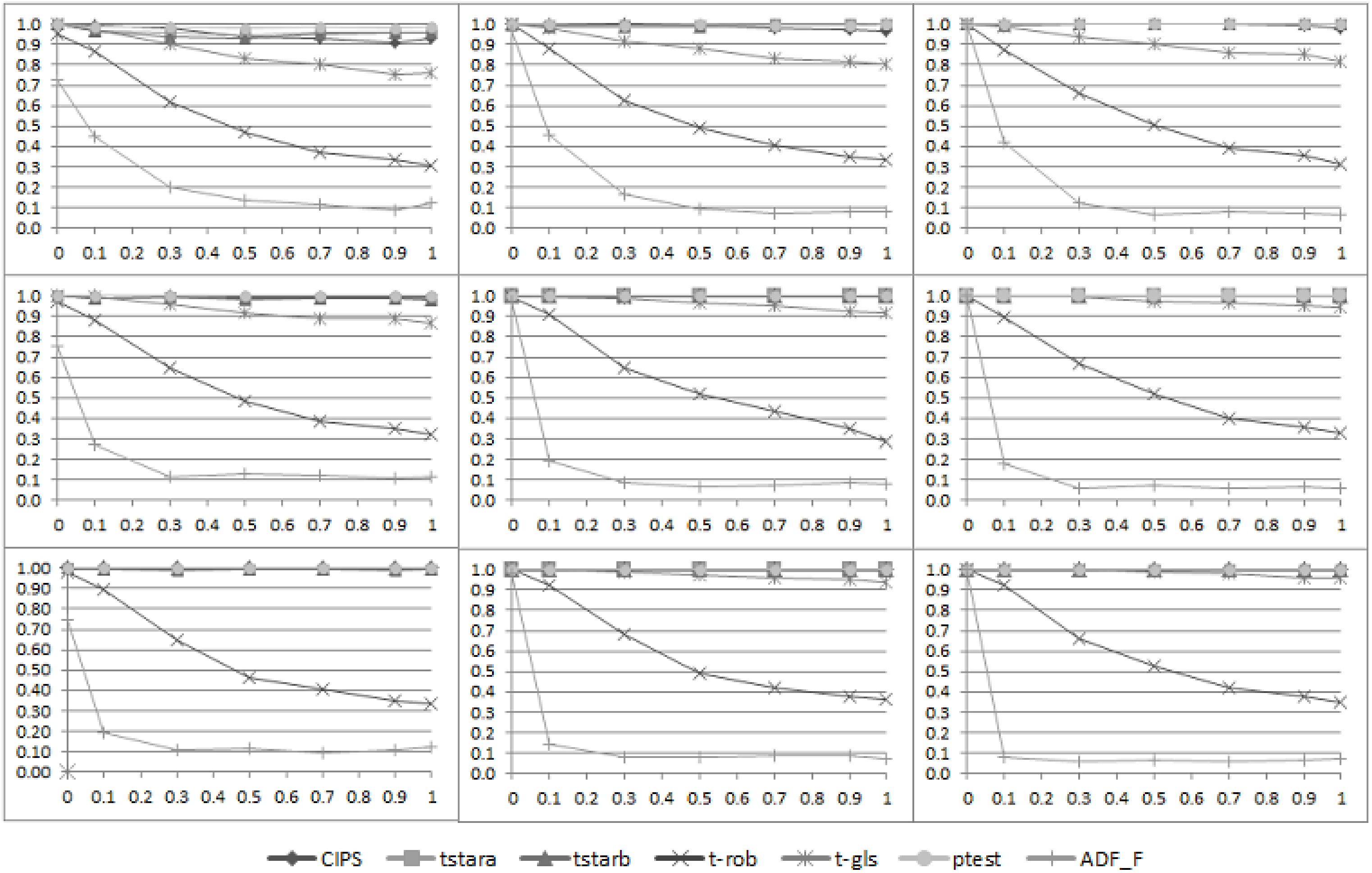
Nonstationary units by DGP1 have $I(1)$ common factors and $I(1)$ idiosyncratic components.

$T \geq 100$, tests are correctly sized. Power of the two tests are similar, which grows gradually with smaller nonstationary proportion, and increases with both T and N. The t_{rob} of Breitung and Das (2008) is slightly over-sized, t_{gls} is also over-sized and the size-distortion is more severe with small T and greater N. Similar to t_a^* and t_b^*, power of t_{rob} and t_{gls} grows gradually with smaller nonstationary proportion. The power of t_{rob} test does not increase with T and N, and for most cases, it has the smallest power among the tests we employed. Power of t_{gls} test increases with N but does not increase with T, and it is greater than the power of t_a^* and t_b^* in most cases. The $P_{\hat{e}}$ test of Bai and Ng (2004) is slightly over-sized when T is small and correctly-sized with greater value of T. Similar to $CIPS$ test, power of $P_{\hat{e}}$ increases with both T and N, and is very sensitive to a small proportion of stationary units. In a moderate panel with $T = 100$ and $N = 30$, the power of $P_{\hat{e}}$ is over 0.6 when there are only 10% stationary units, and power approaches 1 when there are 30% stationary units. The $ADF_{\hat{F}}$ is slightly over-sized when T and N are relatively small, especially when $\theta = 0.1$, but the size distortion is very small when $T \geq 100$ and $N \geq 30$. The rejection frequency of $ADF_{\hat{F}}$ jumps to 1 when $\theta = 0$ if $T \geq 100$, indicating that the test is very powerful.

Figure 3.2 plots the rejection frequencies in DGP2, where unit root is present only in the common factor component (that is, $I(1)$ common factor component and $I(0)$ idiosyncratic components). For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls} test statistics, rejection frequencies when $\theta = 1$ report the sizes of tests, and rejection frequencies when $\theta =\{0, 0.1, \cdots, 0.9\}$ report the powers of tests. For $P_{\hat{e}}$ test, all numbers are power of test. For $ADF_{\hat{F}}$ test, rejection frequency when $\theta = 0$ is deemed as the power of test, and rejection frequencies when $\theta = \{0.1, \cdots, 0.9, 1\}$ are the size of test.

For Pesaran (2007b) $CIPS$ test, Moon and Perron (2004) t_a^* and t_b^* tests, Breitung and Das (2008) t_{rob} and t_{gls} tests, severe size distortion is found. $CIPS$, t_a^*, and t_b^* tests have size close to 1, regardless of the value of T and N. Size of t_{gls} is around 0.8 when $N = 10$ and approaches to 1 when $N = 50$. t_{rob} has relatively less severe size distortion, which is around 0.3 to 0.35, regardless of the dimensions T and N. The high power of these tests are misleading due to the severe size distortion. The $P_{\hat{e}}$ test of Bai and Ng (2004) is very powerful, which is 1 in most cases. The performance of $ADF_{\hat{F}}$ test is similar to DGP 1.

Figure 3.2: Panel Unit Root Tests: Rejection Frequency (DGP2)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is rejection frequency.

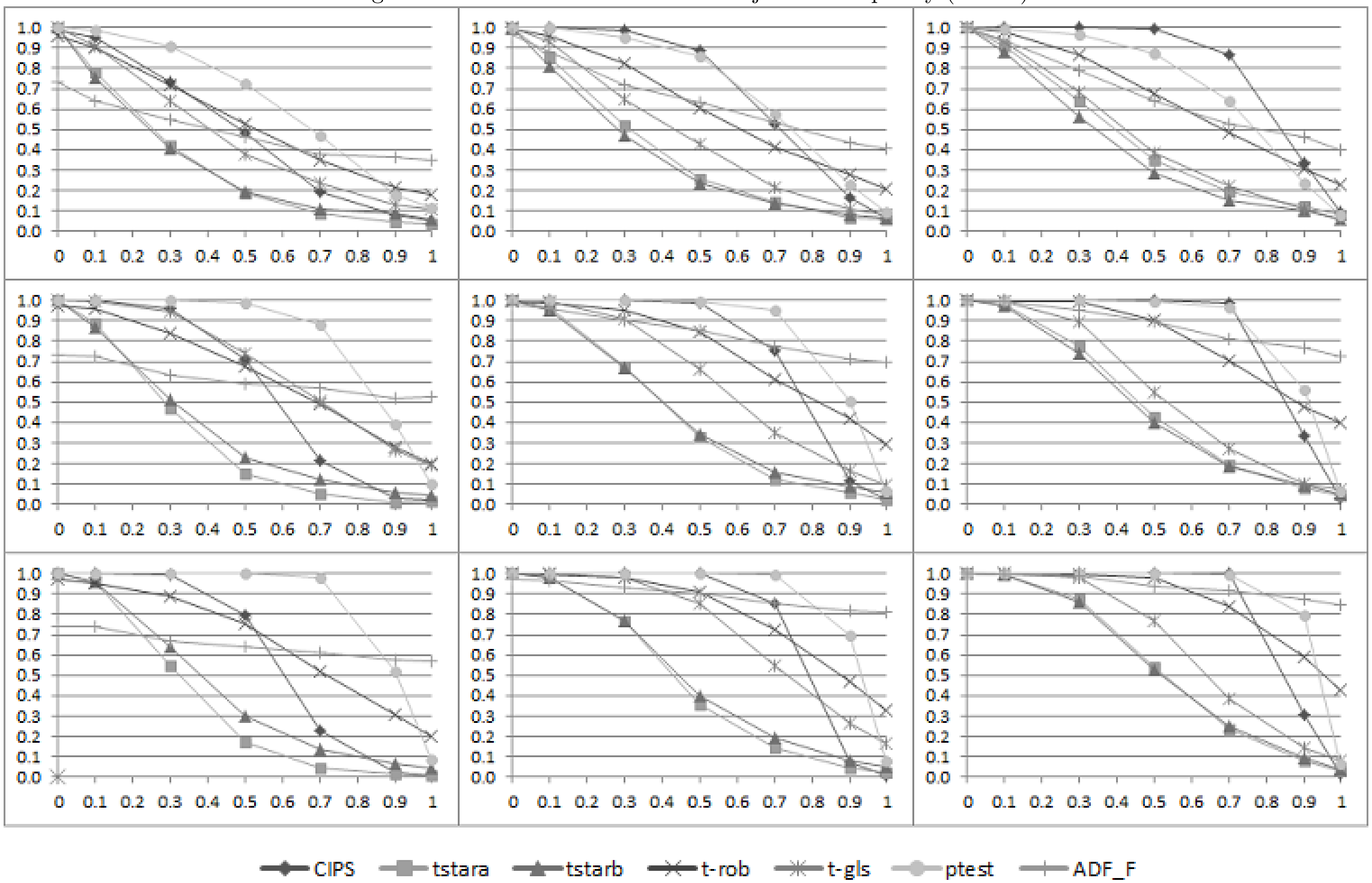
Nonstationary units by DGP2 have $I(1)$ common factors and $I(0)$ idiosyncratic components.

Figure 3.3 plots the rejection frequency of tests in DGP3, where panels are generated with $I(0)$ common factor component and $I(1)$ idiosyncratic component. For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls}, $P_{\hat{e}}$ tests, the rejection frequencies when $\theta = 1$ are sizes of tests, rejection frequencies with $\theta = \{0, 0.1, \cdots, 0.9\}$ are powers. Results for $ADF_{\hat{F}}$ reveal the power of the test.

The Pesaran (2007b) $CIPS$ test is slightly under-sized when N is large. Power of $CIPS$ increases with both T and N, but the gain is more significant with greater value of T. The power is sensitive to a small proportion of stationary units, especially when the sample size {T, N} is fairly large. But powers are less than their counterparts in DGP1, where nonstationary units have both $I(1)$ common factor component and $I(1)$ idiosyncratic component. The Moon and Perron (2004) t_a^* test is slightly over-sized when $N = 10$ and slightly under-sized when $N > 10$. t_b^* is generally correctly sized. Power of the two tests are similar, which increases with both T and N, and grows slowly with smaller θ. Moreover, powers of t_a^* and t_b^* are very close to those in DGP1. The Breitung and Das (2008) t_{rob} test is severely over-sized, which increases with both T and N. In a moderate panel with $T = 100$ and $N = 30$, the size is around 0.3, and this is increased to over 0.4 with $T = 200$ and $N = 50$. The size of t_{gls} decreases with T and increases with N. t_{gls} is generally over-sized, but the size distortion is very small when $T = 200$. When the size distortion is small, the power of t_{gls} is similar to their counterparts in DGP1, and greater than the power of t_a^* and t_b^* tests. The $P_{\hat{e}}$ test of Bai and Ng (2004) is slightly over-sized when T is small and correctly sized when $T = 200$. The test is sensitive to stationary units and is the most powerful test in most cases. The power of $ADF_{\hat{F}}$ increases with T, N, and has a negative relationship with the value of θ. Gain of power is significant when T is increased from 50 to 100, and N is increased from 10 to 30. In a panel with $T = 100$ and $N = 30$, the power of $ADF_{\hat{F}}$ ranges between 0.7 to 1.

Figure 3.4 shows the rejection frequencies of tests in DGP4, where panels are generated by equations 3.37 and 3.38 with single $I(0)$ common factor and $I(0)$ idiosyncratic component and nonstatonarity comes from autoregressive root $\delta_i = 1$. For $CIPS$, t_a^*, t_b^*, t_{rob} and t_{gls} tests, rejection frequencies when $\theta = 1$ report size of tests, rejection frequencies when $\theta = \{0, 0.1, \cdots, 0.9\}$ report power of tests. For $P_{\hat{e}}$ and $ADF_{\hat{F}}$ tests, all rejection frequencies reveal the power of tests.

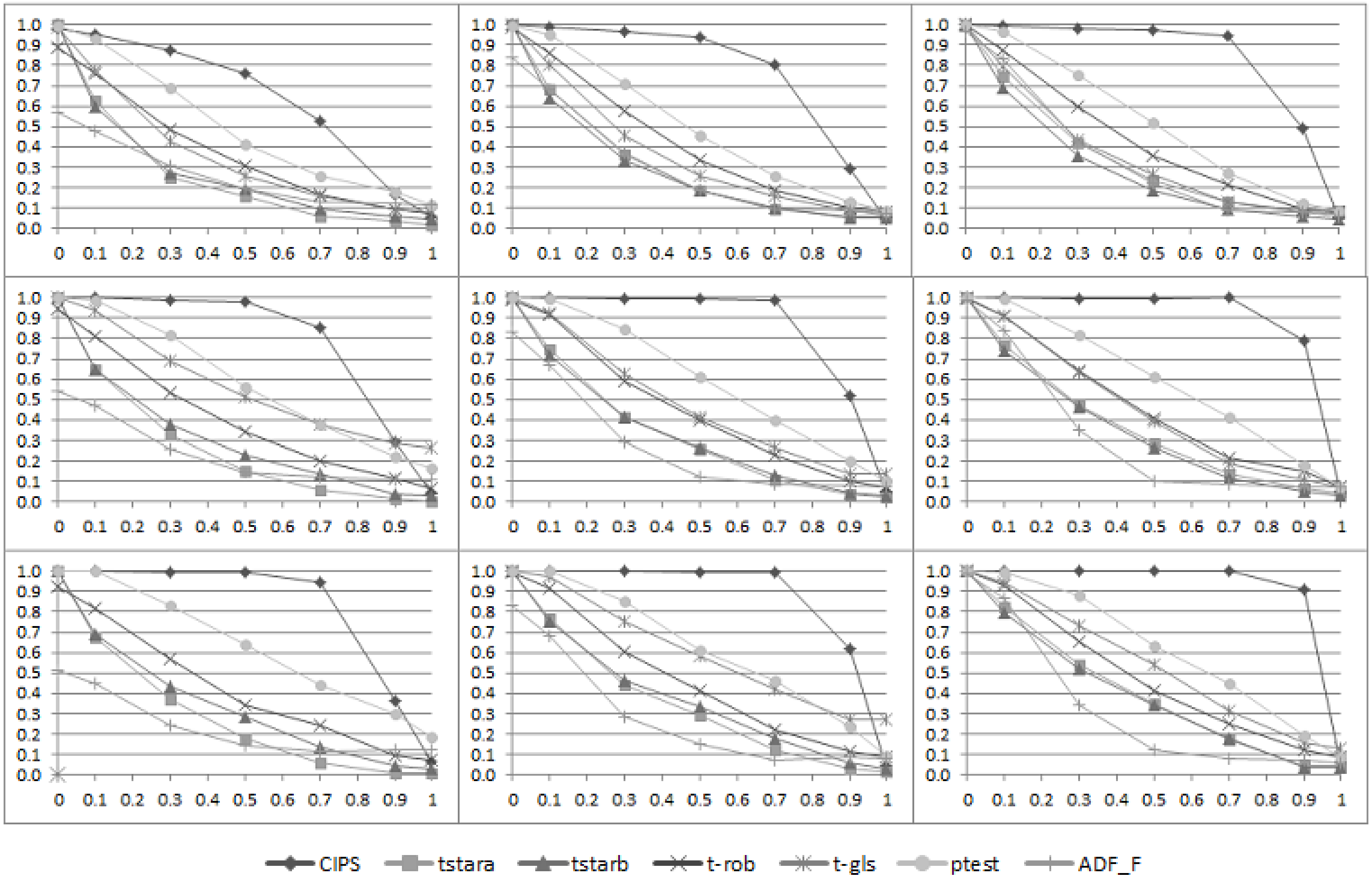
Figure 3.3: Panel Unit Root Tests: Rejection Frequency (DGP3)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is rejection frequency.

Nonstationary units by DGP3 have $I(0)$ common factors and $I(1)$ idiosyncratic components.

Figure 3.4: Panel Unit Root Tests: Rejection Frequency (DGP4)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is rejection frequency.

Nonstationary units by DGP4 have unit autoregressive roots, $I(0)$ common factors and $I(0)$ idiosyncratic components.

The Pesaran (2007b) $CIPS$ test is correctly sized and extremely powerful. In a panel with $T = 200$ and $N = 50$, the power is over 0.9 when there are only 10% stationary units, and the power is increased to 1 when there are 30% stationary units. Moon and Perron (2004) t_a^* and t_b^* tests show small size-distortion, which can be under-sized or over-sized, depending on the value of T and N. Power of the tests are similar, which grows slowly with greater T, N, and smaller θ. t_{rob} test of Breitung and Das (2008) is slightly over-sized in all cases. t_{gls} is over-sized, the size distortion increases with N and decreases with T. Power of the tests are similar, which is slightly greater that the power of t_a^* and t_b^*. The Bai and Ng (2004) $P_{\hat{e}}$ is over-sized, the size distortion increases with N and decreases with T. The power of $P_{\hat{e}}$ increases with N, with smaller θ, but does not increase with greater value of T. The power does not jump to 1.00 for $\theta \leq 0.7$ when sample is large, instead, it grows gradually with θ getting smaller. $ADF_{\hat{F}}$ test shows little power, especially when $\theta \geq 0.5$. Power of $ADF_{\hat{F}}$ decreases with N, increases with T when θ is close to zero, but decreases with T when θ is close to one.

Some general conclusions can be summarized. First, in DGP 2 where the idiosyncratic component is stationary and the unit root is present in the common factor alone, $CIPS$ of Pesaran (2007b), t_a^* and t_b^* of Moon and Perron (2004), t_{rob} and t_{gls} of Breitung and Das (2008) are invalid. t_{rob} is the one has less size distortion, the size of which ranges between 0.28 to 0.36. Similar result is also found in Gengenbach et al. (2010). These tests indeed test for a unit root in the idiosyncratic component and reject the unit root null hypothesis if the unit root is present in common factors alone. Second, the $CIPS$ test performs reasonably well in DGP 1 and in DGP 4. The test is correctly sized and very sensitive to a small proportion of stationary units. The high power property of $CIPS$ makes it a natural candidate to the Sequential Panel Selection Method proposed in Chortareas and Kapetanios (2009). Third, satisfactory size property of Moon and Perron (2004) t_a^* test appears when $T \geq 100$ and $N \geq 30$. t_b^* is generally correctly sized when $N \geq 30$. The powers of t_a^* and t_b^* are similar, increase with both $\{T, N\}$ and grow gradually as θ getting smaller. Fourth, as pointed out by Breitung and Das (2008), t_{rob} is invalid in DGP 2 and DGP 3, t_{gls} is invalid in DGP 2. t_{gls} is over-sized in most cases, the size distortion is worsened with greater N but is reduced with greater T. Therefore, t_{gls} is more suitable for panels with large T and small N. Fifth, the $P_{\hat{e}}$ and $ADF_{\hat{F}}$ test of Bai and Ng (2004) perform very well in DGP 1 to DGP 3. $P_{\hat{e}}$ is only slightly over-sized when $T \geq 100$. $P_{\hat{e}}$ test is sensitive to the stationary units and therefore the power of $P_{\hat{e}}$ is very high even when θ is close to one. This property is similar to $CIPS$ test, but the power is even higher. The $ADF_{\hat{F}}$ is only slightly over-sized if $T \geq 100$. When N is small and θ is close to 0, the size of $ADF_{\hat{F}}$ is upward distorted, this is mainly due to the fact that the estimation of common factor requires N to be large, especially when the factor only affects a small proportion of units. The power of $ADF_{\hat{F}}$ is satisfactory. In DGP 4, the $P_{\hat{e}}$ is slightly over-sized, but the power behaves differently, it increases gradually with smaller θ, which is similar to t_a^*, t_b^*, t_{rob} and t_{gls}. $ADF_{\hat{F}}$ has little power to reject the null hypothesis, unless θ is very close to 0.

3.4.3 Simulation Results for Mixed Panel Procedures

The tests designed for mixed panels aim to estimate the proportion of stationary and nonstationary units. These tests can further identify which units are stationary and which are nonstationary in the panel. We are, therefore, mainly interested in two aspects of the tests: can they estimate the nonstationary proportion efficiently and consistently, and can they classify the stationary and nonstationary units correctly. We report the average estimated θ, ave($\hat{\theta}$), and the standard deviation of estimated θ, se($\hat{\theta}$), for each combination of $\{T,N,\theta\}$, where ave($\hat{\theta}$) is the simple average and se($\hat{\theta}$) is the standard deviation of $\hat{\theta}$'s over the replication. As a standard in the literature, we report the simple average of False Discovery Rate (FDR) of tests for each $\{T,N,\theta\}$ over replication, where the FDR is calculated as the proportion of false rejection in all rejections, that is, the proportion of $I(1)$ units in the units that are deemed as $I(0)$. The FDR alone is not always a good description of classification accuracy, for example, when θ is upward biased, tests reject less null hypothesis than they suppose to, which tends to reduce the possibility of wrong rejection. However, when θ is upward biased, the proportion of wrong non-rejection should increase. We therefore consider this measurement and report the false non-rejection rate (labelled as FNR), which is the proportion of $I(0)$ units in the units that are deemed as $I(1)$. The FDR can be deemed as an aggregate size measurement, and FNR can be deemed as an aggregate power measurement. A test that can identify the stationary and nonstationary units precisely should keep both FDR and FNR low. The multiple testing framework by Romano and

Wolf (2005) is designed to control the Familywise Error Rate (FWE), which is the possibility of incorrectly rejecting at least one null hypothesis. The FWE is not a meaningful measurement when N is large. The multiple testing procedure by Moon and Perron (2012) is designed to control the FDR. For a comparison, we report FWE for both RW05 and MP12 tests, where FWE is calculated as the frequency that tests reject at least one null incorrectly over the replication. The Ng (2008) estimator of nonstationary proportion θ enables us to perform hypothesis testing. Therefore, for Ng (2008) test, we report the rejection frequency for three hypothesis testing: $H_0^A : \theta = 0.01$ against $H_1^A : \theta > 0.01$, $H_0^B : \theta = \theta_0$ against $H_1^B : \theta \neq \theta_0$, $H_0^C : \theta = 1$ against $H_1^C : \theta < 1$, where θ_0 is the (infeasible) true value of θ.

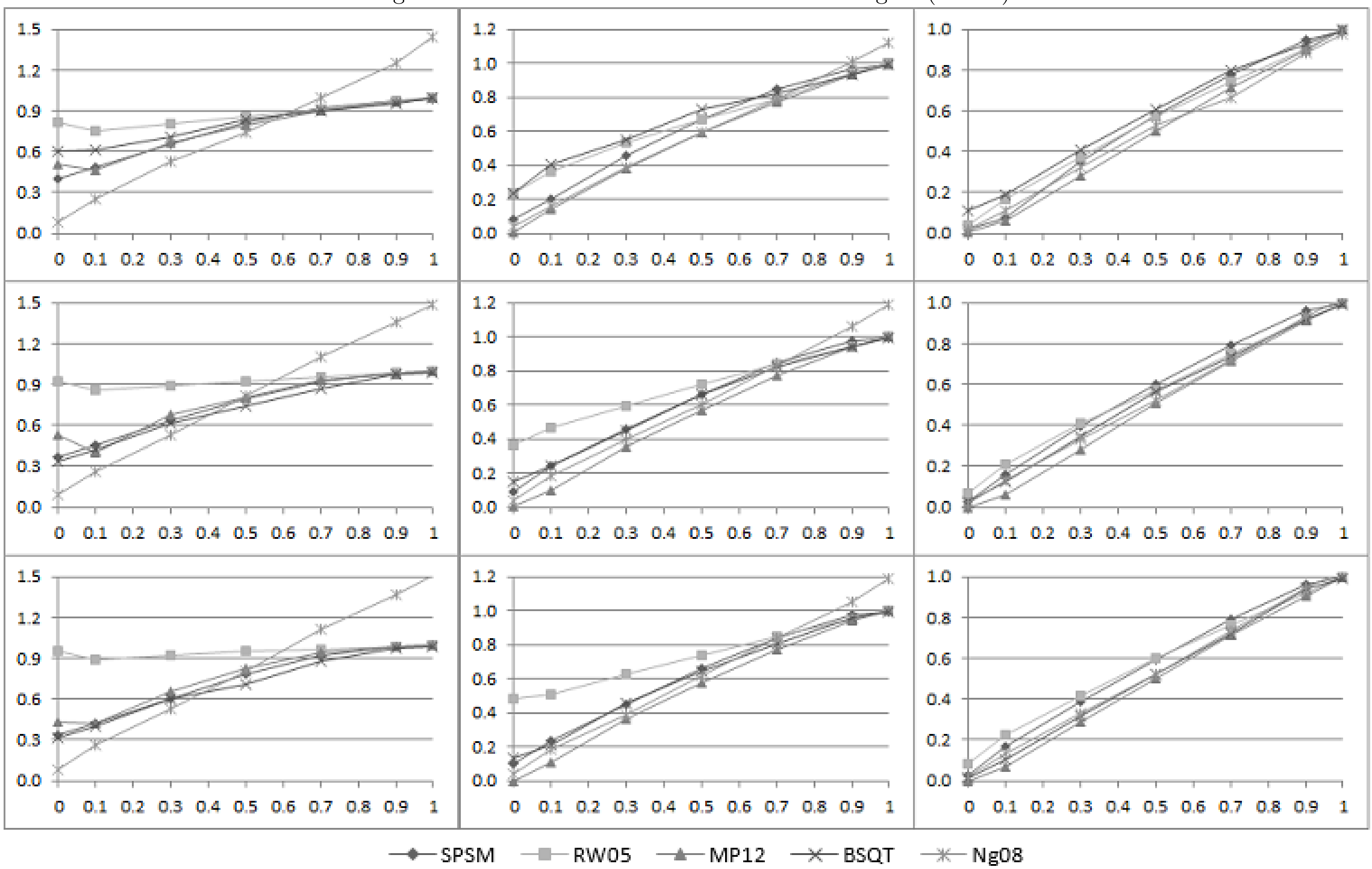
The total set of results and analysis, which comprises over 140 tables, are provided in Appendix 3.4.3. In the main text, we discuss the main observations and display some representative results graphically. Similar to the previous section, we focus on the effect of panel dimensions $\{T, N\}$, and proportion of unit root processes θ. For DGP1 to DGP3, we report the results when the number of common factors is 1, and the cross-sectional dependence is low (with factor loadings $\lambda_i \sim iidU[-1, 2]$). For DGP4, we report the results when cross-sectional dependence is low, and idiosyncratic components are $ARMA(1, 1)$ processes. For each combination of $\{T, N, \theta\}$, we report the average estimated value of θ, the false discovery rate (FDR), and false non-rejection rate (FNR). For Smeekes (2011) tests, we only report the results of BSQT test in the main text. We use a 3×3 graph for each data generating process, where three columns corresponding to $T = 50$, $T = 100$, $T = 200$, and three rows corresponding to $N = 10$, $N = 30$, $N = 50$.

When panels are generated by DGP 1, where unit roots are present in both common factor and idiosyncratic components, results are shown in Figure 3.5 to Figure 3.7. Figure 3.5 plots the average $\hat{\theta}$ of tests. The $SPSM$ estimates of θ is generally upward biased. The bias is reduced with greater T, but not sensitive to greater N. The RW05 test is upward biased and the bias decreases with T and increases with N. $RW05$ is more sensitive to N than $SPSM$ method, due to the fact that it aims to control the FWE at a fixed level. It shows the largest bias when $N \geq 30$. Results reveal that the RW05 test is suitable when T is large and N is small. For MP12 test, the ave$(\hat{\theta})$ is upward biased when T is small and the bias is reduced with T, little bias can be seen when $T = 200$. The $BSQT$ test is upward biased when sample size is small, but the bias is reduced with both T and N. Unless the multiple testing procedures $RW05$ and $MP12$, the tests of Smeekes (2011) can get use of the cross-sectional information and improve the performance with larger N. The Ng08 test is upward biased when $T \leq 100$ and close to the true value when $T = 200$, and the bias is not affected by N. Figure 3.6 plots the false discovery rate of tests (familywise error rate for RW05 test). The FDR of $SPSM$ test is close to 0.1 when $T = 50$, but is reduced with greater T. RW05 and MP12 tests successfully control the FWE and FDR at 5%, respectively, as they supposed to. The FDR of $BSQT$ test is slightly increased with greater N, and is around 0.1 when N is large. The FDR of Ng08 test shows a pattern which is similar to the pattern of FDR if a set of individual unit root tests are applied to each unit in the panel[6]. This shows that the classification mechanism of Ng08 test can not control the FDR properly, comparing to other tests employed. Figure 3.7 plots the false non-rejection rate of tests. FNR of $SPSM$ is reduced with T but increased with N. FNR of RW05 test decreases with T and increases with N, which meets our expectation that the power of RW05 is reduced with greater N. When $T \geq 100$, power of MP12 test increases dramatically, which is close to 0 when $T = 200$. This shows that the MP12 test successfully controls the FDR at 5% and meanwhile has very high power. FNR of $BSQT$ is reduced with both T and N, which satisfies our intuition that $BSQT$ exploits the cross-sectional dimension and becomes more powerful with greater N. The FNR decreases with T and increases with N, but is large when θ is close to 0.

Figure 3.8 to Figure 3.10 show the average $\hat{\theta}$, FDR(FWE), and FNR, respectively, when panels are generated by DGP 2, where unit root is present in the common factor alone. Similar to $CIPS$, the $SPSM$ does not work under this data generating process, the ave($\hat{\theta}$) is severely downward biased, and ave($\hat{\theta}$) is approaching 0 for all values of θ when T and N are large. The results are consistent with the $CIPS$ simulation in section 3.4.2, where the size distortion is approaching 1.00 for large T and N. The performance of RW05 generally improves with greater T, and smaller N. The ave($\hat{\theta}$) of RW05 is upward biased for small θ and downward biased for large θ, but becomes satisfactory when $T = 200$. It does not control the FWE at 5%, and shows severe size distortion. MP12 test under estimates the nonstationary proportion θ, and does not

[6] An example of the FDR if we ignore the multiple testing problem and apply a set of individual tests is given in Chapter 4.

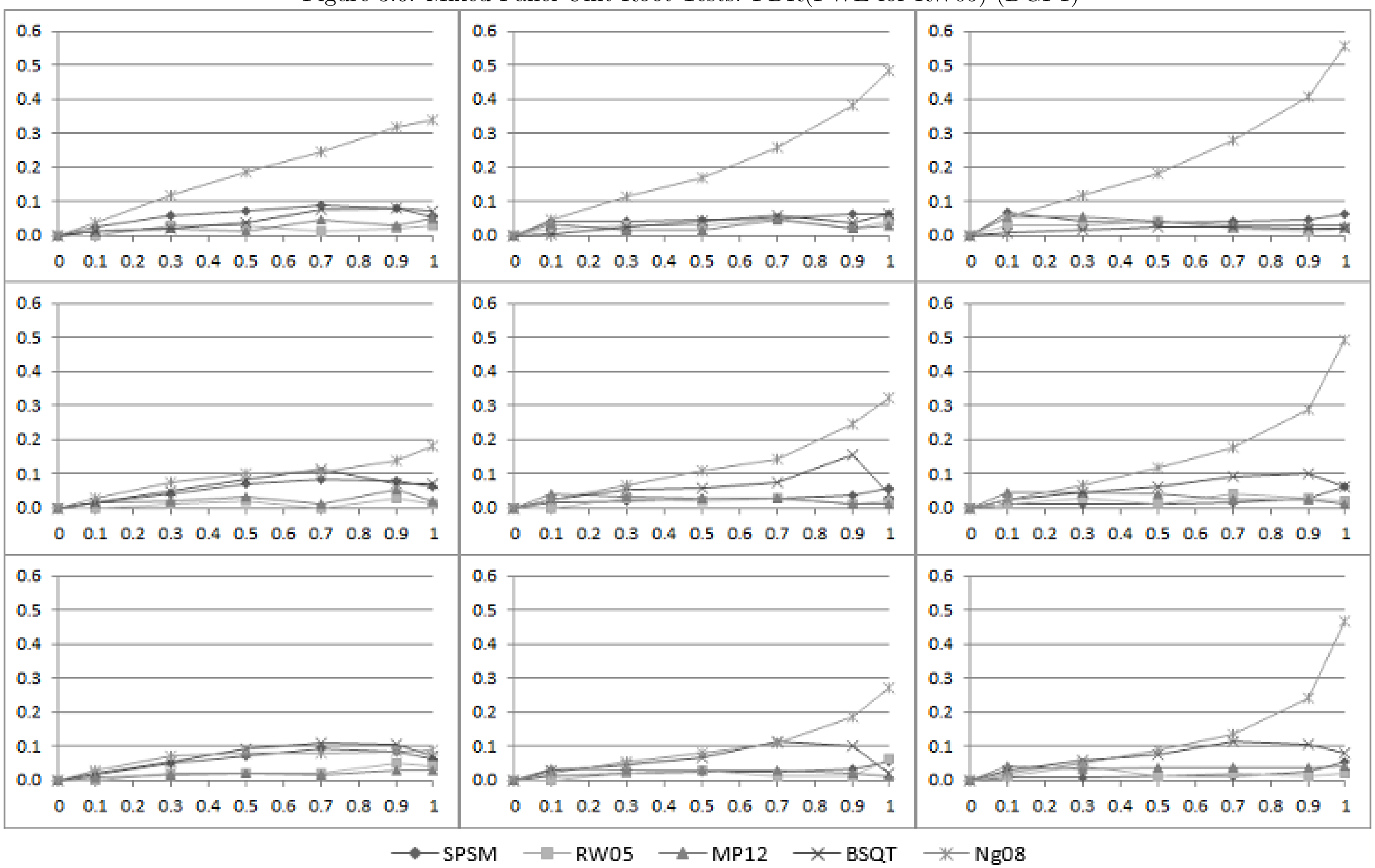
Figure 3.5: Mixed Panel Unit Root Tests: average $\hat{\theta}$ (DGP1)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is average $\hat{\theta}$.

Nonstationary units by DGP1 have $I(1)$ common factors and $I(1)$ idiosyncratic components.

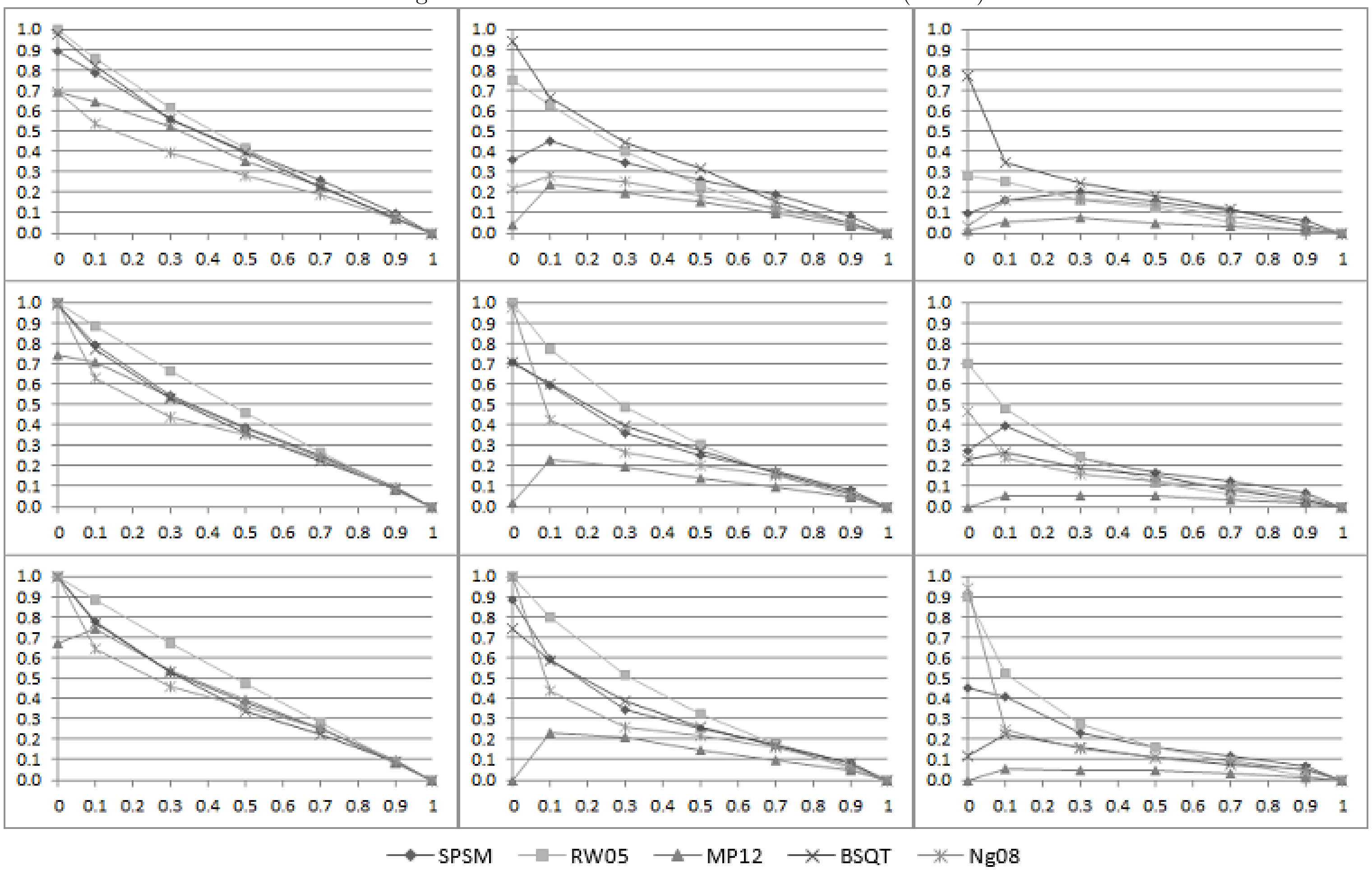
Figure 3.6: Mixed Panel Unit Root Tests: FDR(FWE for RW05) (DGP1)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FDR (FWE for RW05).

Nonstationary units by DGP1 have $I(1)$ common factors and $I(1)$ idiosyncratic components.

Figure 3.7: Mixed Panel Unit Root Tests: FNR (DGP1)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FNR.

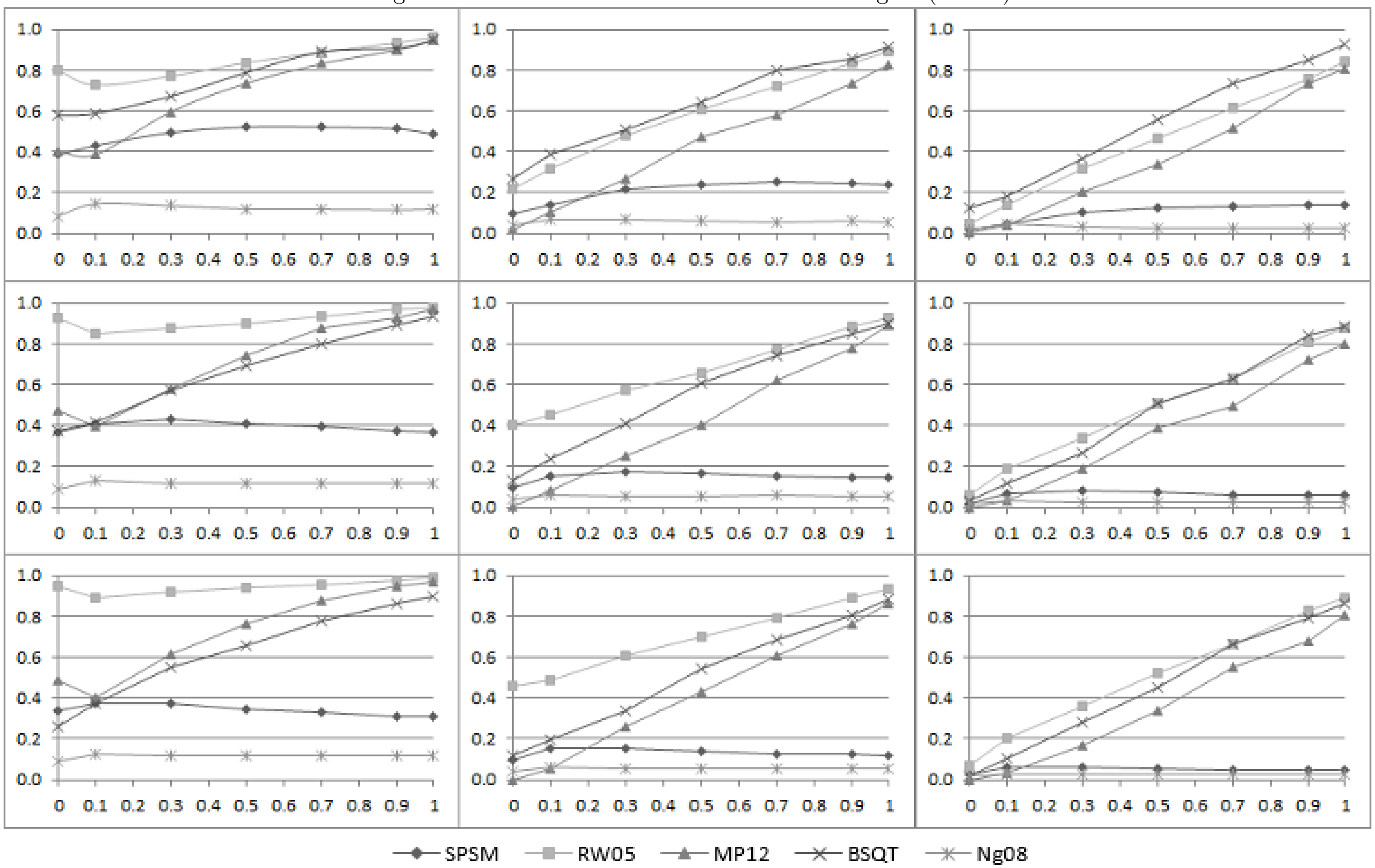
Nonstationary units by DGP1 have $I(1)$ common factors and $I(1)$ idiosyncratic components.

control the FDR at 5%. $BSQT$ test performs relatively well under this DGP, its estimator is downward biased, but it shows the smallest FDR in most cases. The Ng08 estimator has no ability to estimate the correct nonstationary proportion θ, ave($\hat{\theta}$) is close to 0 for all cases. The FNR of tests are similar to DGP 1, but this power measurement may not be meaningful for $SPSM$ and Ng08 tests due to the size distortion.

In DGP 3, unit roots are present in idiosyncratic components alone. Results are reported in Figure 3.11 to Figure 3.13. Average $\hat{\theta}$ by $SPSM$ test is upward biased, and the bias is reduced with T. FDR of $SPSM$ is small, but FNR is high and the test is lack of power, especially when T is small. RW05 estimates θ well when T is large and N is small, MP12 test shows little bias of θ when $T \geq 100$, $BSQT$ has little bias when T and N are both large. Ng08 estimator is upward biased and the bias is reduced with T. Although not shown in the main text, RW05, MP12, and $BSQT$ tests tend to under-estimate the nonstationary proportion θ with higher cross-sectional dependence and multi common factors. FDR (or FWE for RW05 test) diverges to 1 correspondingly. The FNR is therefore not meaningful for these tests. The FNR of Ng08 test decreases with T and increases with N, but is large when θ is close to 0.

Figure 3.14 to Figure 3.16 show the results of tests in DGP4, where panels are generated by equations 3.37 and 3.38 with single $I(0)$ common factor and $I(0)$ idiosyncratic component and nonstatonarity comes from autoregressive root $\delta_i = 1$. For $SPSM$ test, average $\hat{\theta}$ is upward biased, and the bias is reduced with T. The FDR is low due to the upward bias, and is below 5% for a moderate panel with $T = 100$ and $N = 30$. The FNR decreases with T but increases with N, is high when θ is close to 0, which means caution needs to be paid on the $I(1)$ units when $\hat{\theta}$ is close to zero. RW05 test performs well with large T and small N. It controls FWE successfully at 5%. However, FNR of RW05 procedure is high, especially when N is large, indicating that the procedure is lack of power. MP12 test estimates θ correctly when $T \geq 100$, successfully controls the FDR below 5%, and keeps the FNR low. $BSQT$ over-estimate θ when sample size is small, but the bias is reduced with greater T and N. FDR of $BSQT$ is generally below 10%, but is slightly increased with greater N. The FNR is high when T, N, and θ are small, but is reduced when sample size is large. Average $\hat{\theta}$ by Ng08 test is severely upward biased. This result is due to the way we estimate the common factor. We use the Bai and

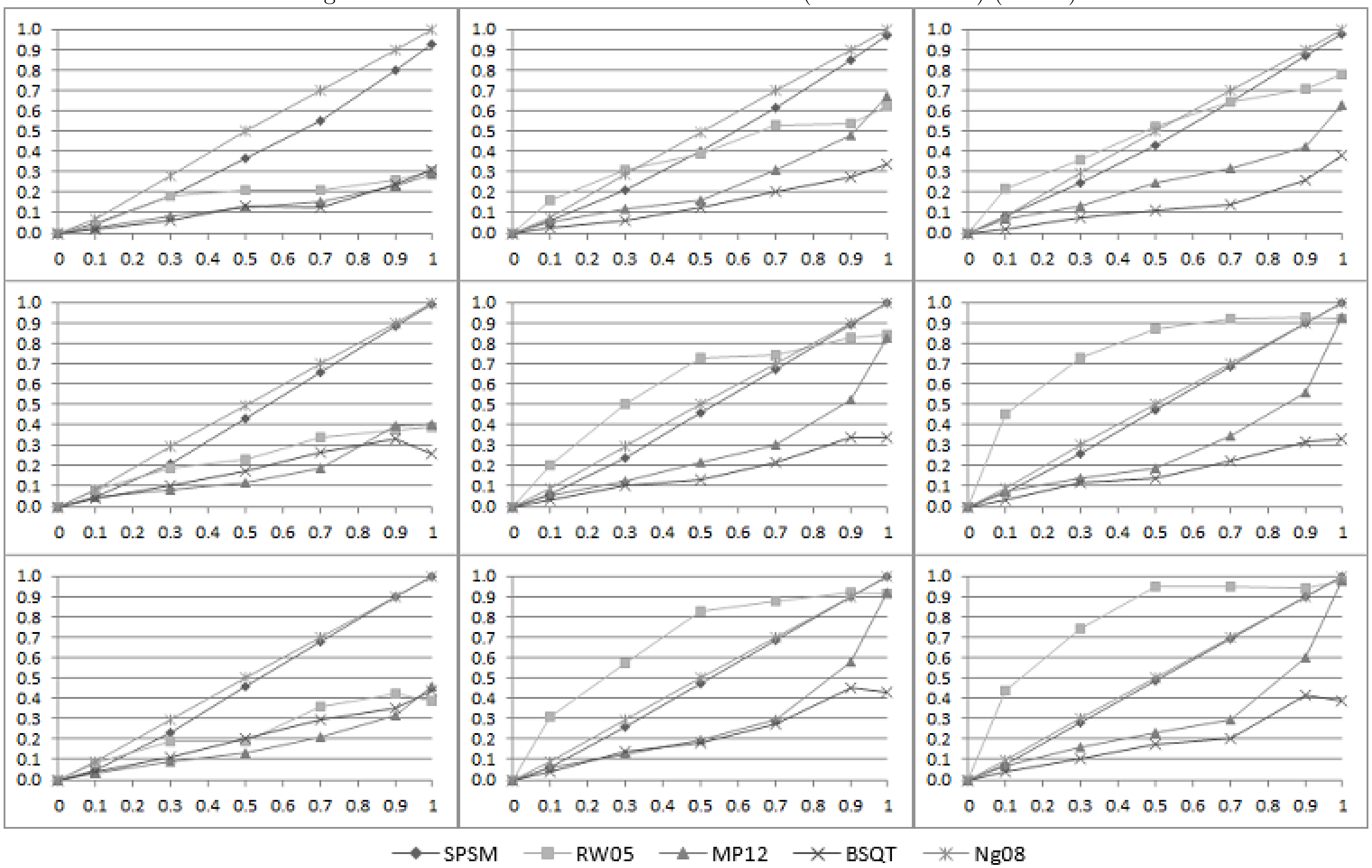
Figure 3.8: Mixed Panel Unit Root Tests: average $\hat{\theta}$ (DGP2)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is average $\hat{\theta}$.

Nonstationary units by DGP2 $I(1)$ common factors and $I(0)$ idiosyncratic components.

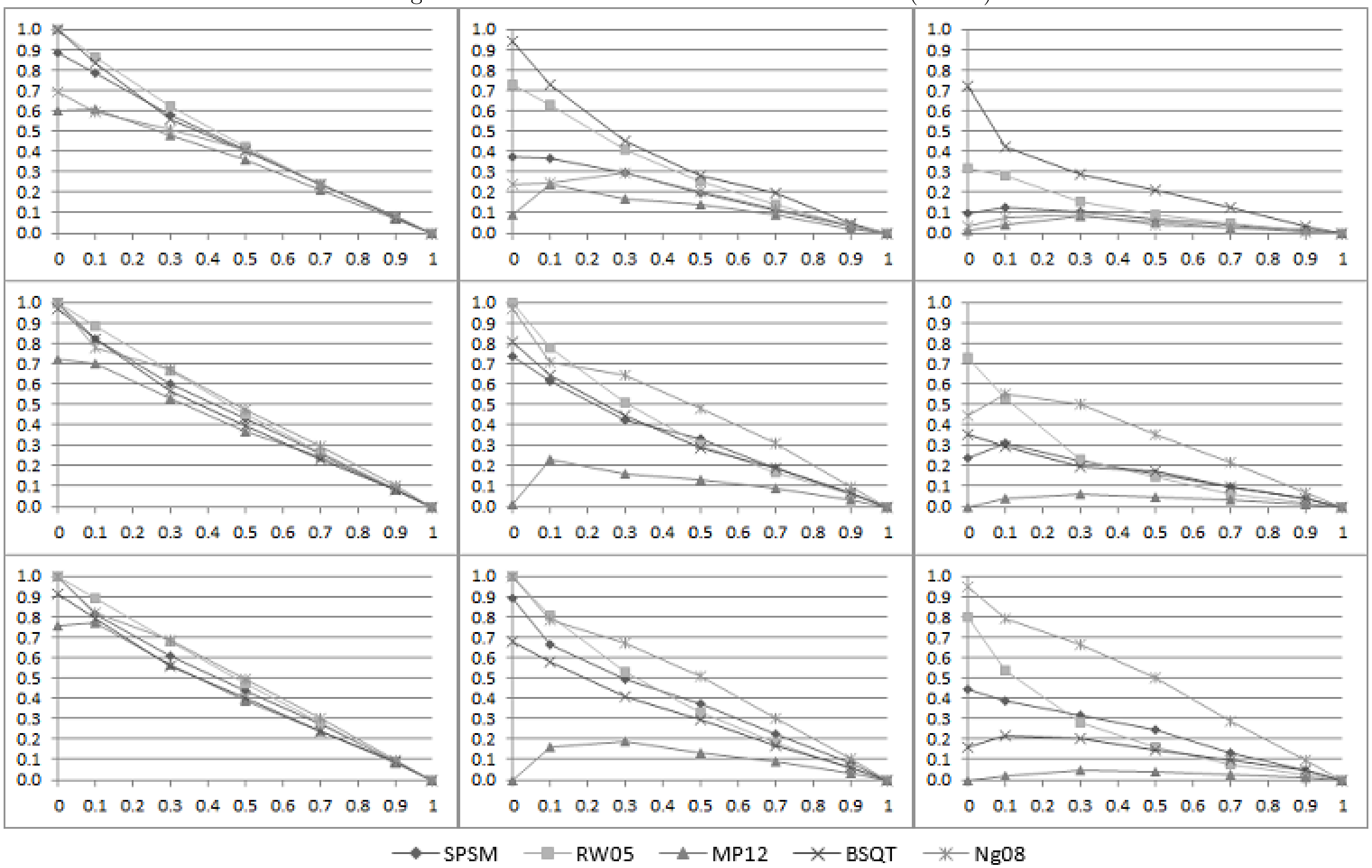
Figure 3.9: Mixed Panel Unit Root Tests: FDR(FWE for RW05) (DGP2)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FDR (FWE for RW05).

Nonstationary units by DGP2 $I(1)$ common factors and $I(0)$ idiosyncratic components.

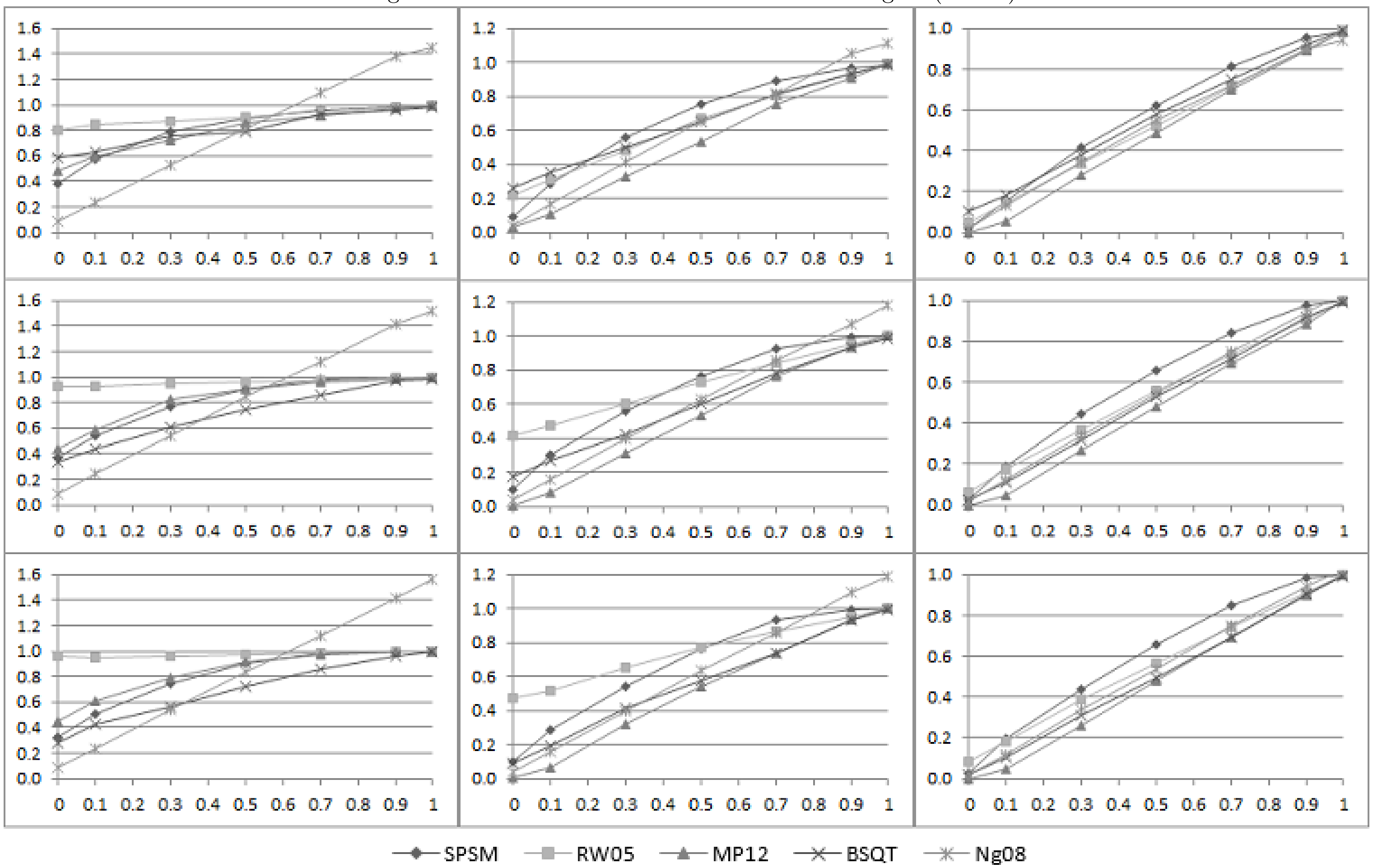
Figure 3.10: Mixed Panel Unit Root Tests: FNR (DGP2)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FNR.

Nonstationary units by DGP2 $I(1)$ common factors and $I(0)$ idiosyncratic components.

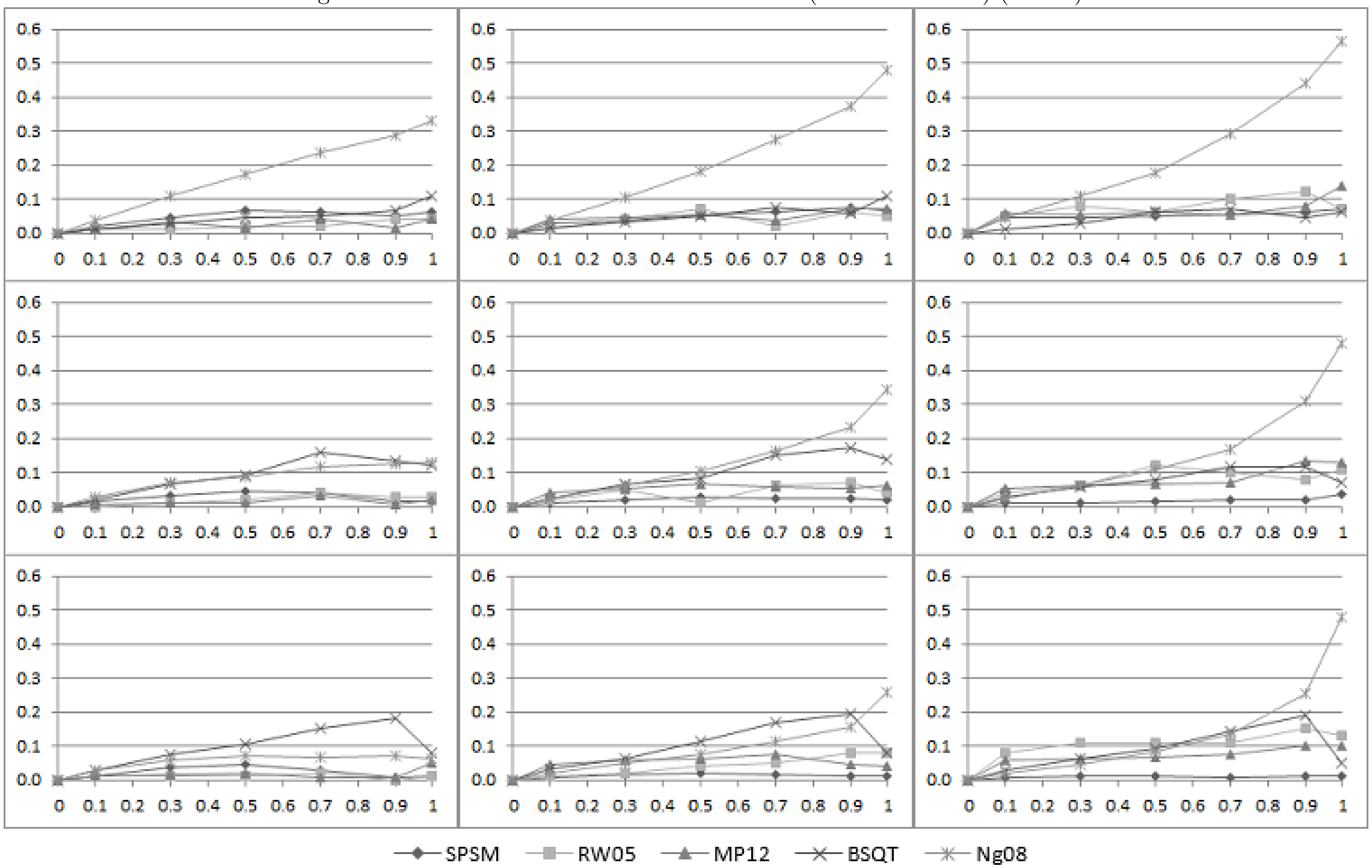
Figure 3.11: Mixed Panel Unit Root Tests: average $\hat{\theta}$ (DGP3)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is average $\hat{\theta}$.

Nonstationary units by DGP3 have $I(0)$ common factors and $I(1)$ idiosyncratic components.

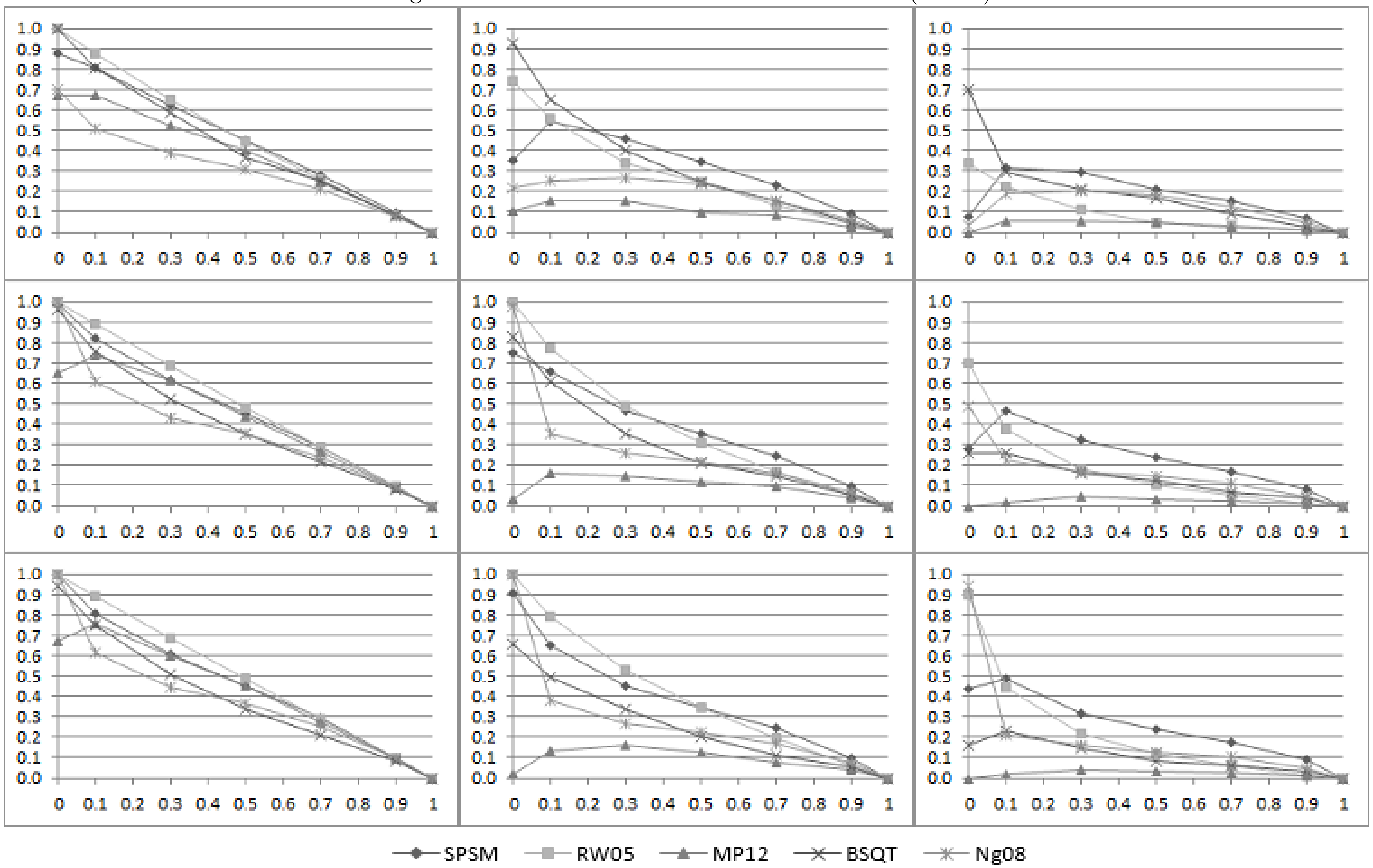
Figure 3.12: Mixed Panel Unit Root Tests: FDR(FWE for RW05) (DGP3)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FDR (FWE for RW05).

Nonstationary units by DGP3 have $I(0)$ common factors and $I(1)$ idiosyncratic components.

Figure 3.13: Mixed Panel Unit Root Tests: FNR (DGP3)



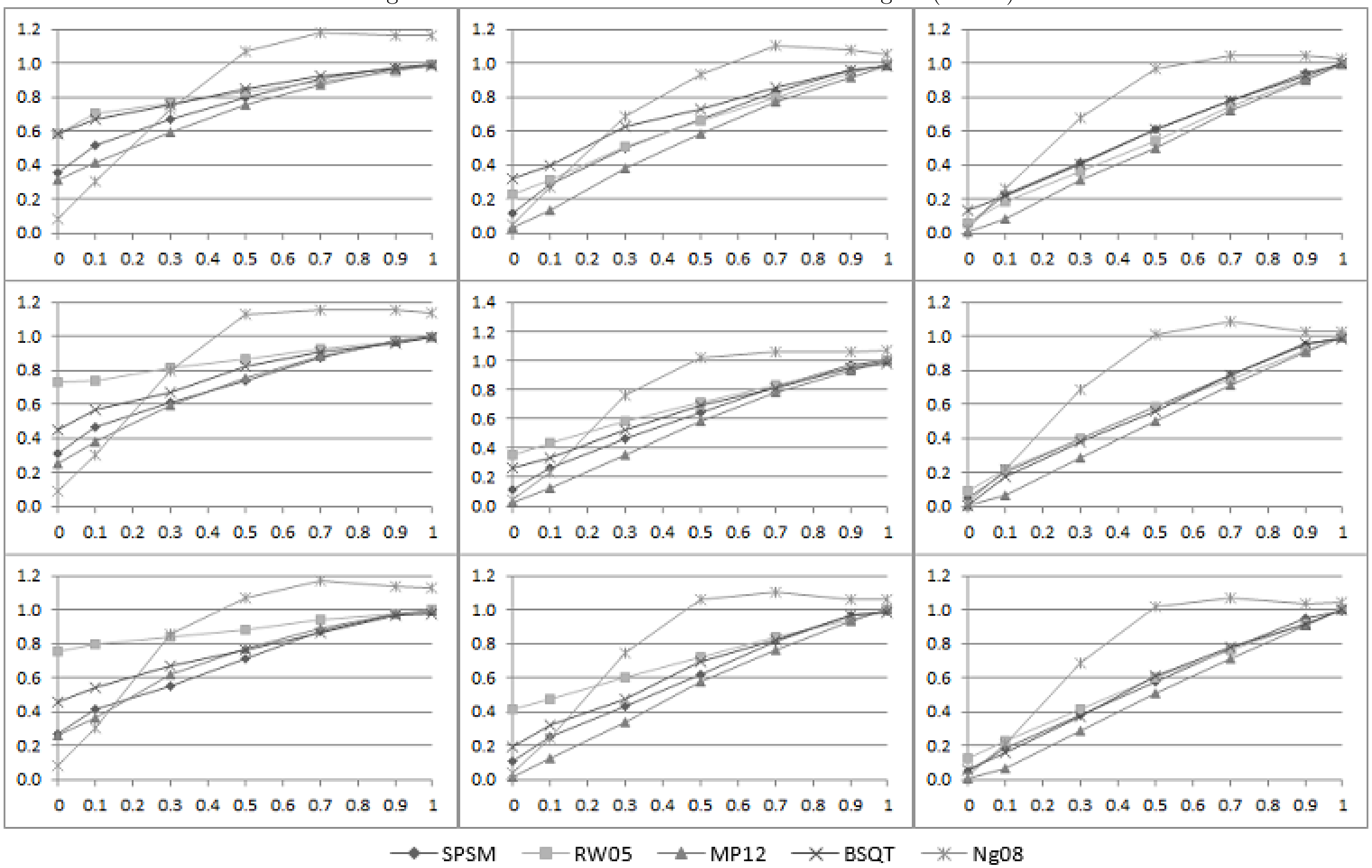
Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FNR.

Nonstationary units by DGP3 have $I(0)$ common factors and $I(1)$ idiosyncratic components.

Ng (2004) common factor estimator, which applies the principal component method to the differenced data and accumulates the common factor back in order to account for possible $I(1)$ common factor. The results of Ng (2008) is consistent with our simulations of $ADF_{\hat{F}}$ test of Bai and Ng (2004) reported in section 3.4.2. In DGP 4 with cross-sectional dependence, the $ADF_{\hat{F}}$ has low power when $\theta \geq 0.3$, therefore an $I(1)$ common factor is incorrectly estimated and a nonstationary idiosyncratic component is left. The Ng (2008) test is then applied on the idiosyncratic component, and the results are misleading. The FDR and FNR of Ng08 test show similar pattern to previous DGPs.

In sum, some general conclusions can be drawn. First, the property of Chortareas and Kapetanios (2009) $SPSM$ test and Ng (2008) test depends on the underlying panel unit root test and the way we deal with common factors, respectively. Both tests detect the nonstationary proportion of idiosyncratic component. The $SPSM$ and Ng08 tests are not valid when panel is generated with $I(1)$ common factor and $I(0)$ idiosyncratic component, the estimated nonstationary proportion is severely downward biased for all values of θ. The Ng08 test is not valid in DGP 4 with cross-sectional dependence, because the common factor can not be extracted correctly. Second, in DGP 2 and DGP3 where unit root is present in common factor component or idiosyncratic component alone, the multiple testing methods by Romano and Wolf (2005) and Moon and Perron (2012) can not control the FWE and FDR, respectively, as they suppose to. In DGP 1 and DGP 4, the tests can successfully control the FWE and FDR at 5%, respectively. Third, time series dimension T is crucial for all tests and a satisfactory performance requires T to be no less than 100. The multiple testing methods RW05 and MP12 do not explore the cross-sectional information, but the Smeekes (2011) quantile tests can get use of cross-sectional information and therefore benefit from greater N. The RW05 test is more suitable for small N because the FWE is harder to be controlled with large N. Generally speaking, all tests require T to be no less than 100. RW05 test is more suitable for panels with small N, while $BSQT$ and Ng08 benefit from greater N. Fourth, in DGP 1 and DGP 4, the Moon and Perron (2012) has impressing good property when T is large, in terms of being unbiased, and keeping both FDR and FNR low. In DGP 2 and DGP 3, all tests have size distortion and the distortion is more severe in DGP 3. An exception that the Ng08 estimator has little size distortion in DGP

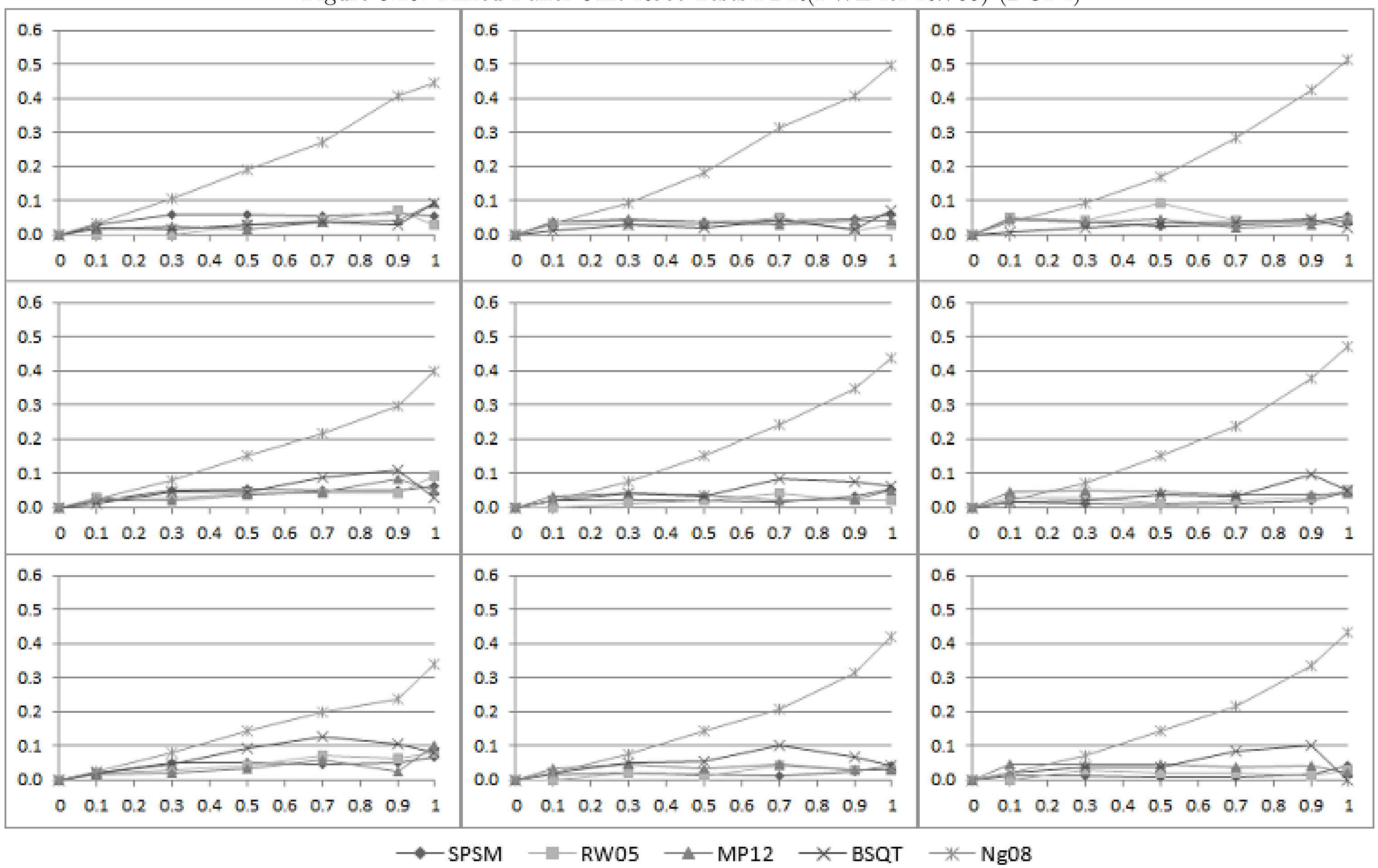
Figure 3.14: Mixed Panel Unit Root Tests: average $\hat{\theta}$ (DGP4)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is average $\hat{\theta}$.

Nonstationary units by DGP4 have unit autoregressive roots, $I(0)$ common factors and $I(0)$ idiosyncratic components.

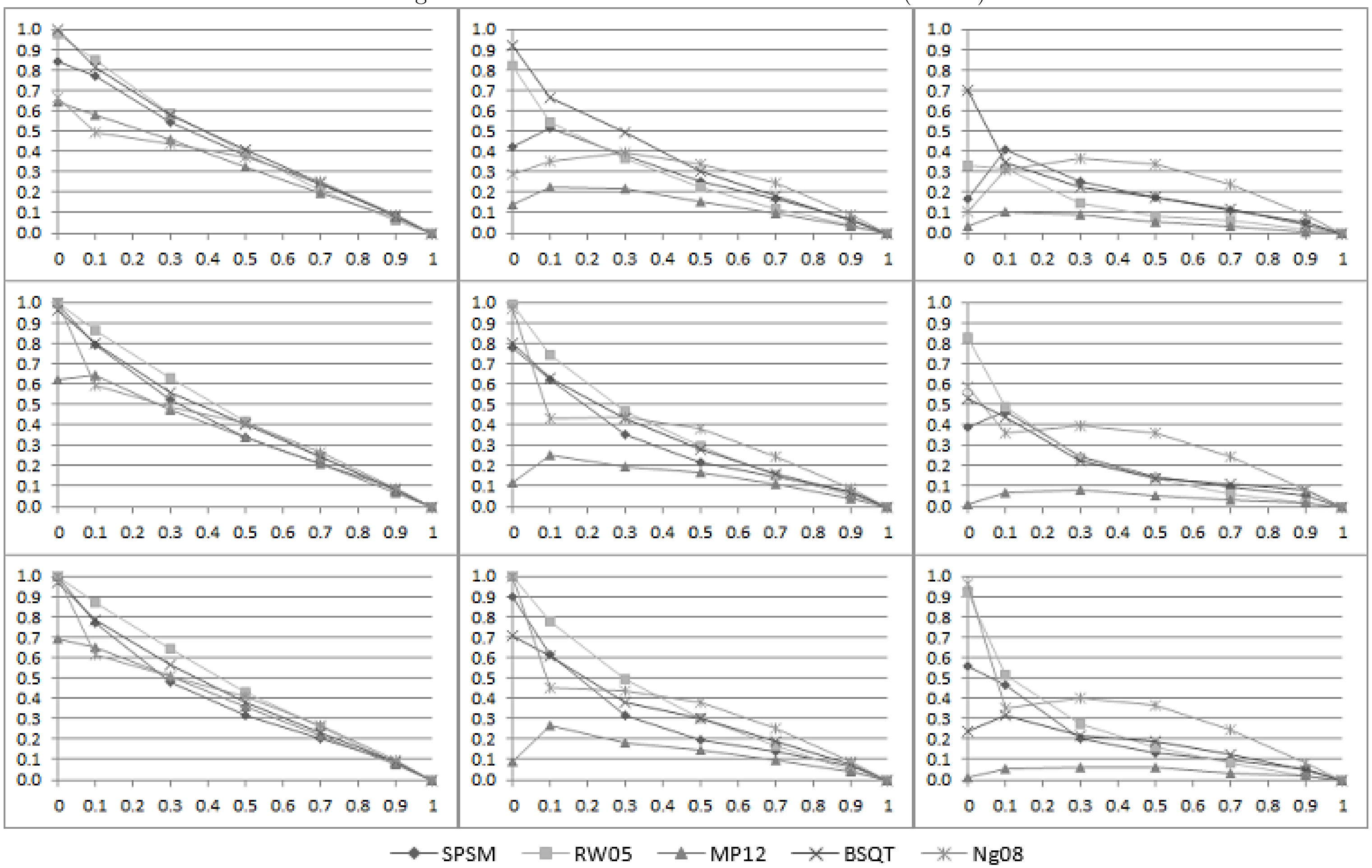
Figure 3.15: Mixed Panel Unit Root Tests:FDR(FWE for RW05) (DGP4)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FDR (FWE for RW05).

Nonstationary units by DGP4 have unit autoregressive roots, $I(0)$ common factors and $I(0)$ idiosyncratic components.

Figure 3.16: Mixed Panel Unit Root Tests:FNR (DGP4)



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is FNR.

Nonstationary units by DGP4 have unit autoregressive roots, $I(0)$ common factors and $I(0)$ idiosyncratic components.

3 if sample size is large and true number of factors is specified. If, however, the true DGP is not correctly specified, the tests base on bootstrap perform much better than others. So in an empirical application where the true DGP is unknown, the tests with bootstrapped critical values should be more reliable. Fifth, the Ng (2008) procedure can provide accurate estimator of the stationary/nonstationary proportion in DGP1 and DGP 3, if $T = 200$. However, simulation evidence shows that, even when the underlying DGP is correctly specified and sample size is large, it can not keep both FDR and FNR low, indicating that its classification mechanism does poorly. This is fair as the Ng08 estimator is primarily proposed to estimate the proportion of nonstationary units. Ng08 test provides us a tool to perform hypothesis testing, especially for two extreme hypotheses: $H_0^A : \theta = 0.01$ and $H_0^C : \theta = 1$, although the high power of test depends on the correct specification of underlying DGP and large sample size.

3.5 Empirical Application: a PPP study

In this section we apply the tests for mixed panels to real data and extend our purchasing power parity study in Chapter 2.

We use the same data set as in Chapter 2. Monthly price indices are collected for 15 different goods and services from 36 Chinese cities. We regroup the 15 price indices and there are 2 aggregate price indices, 5 prices in perishable group, 3 prices in nonperishable group, and 5 prices in services group. The 15 price indices have same time span, from January 2003 to March 2011. Shanghai is used as the numeraire city and for each price index, relative price $q_{i,t}$ is calculated between city i and Shanghai at time t as: $q_{i,t} = p_{i,t} - p_{s,t}$, where $p_{i,t}$ and $p_{s,t}$ are the log price index in city i and in Shanghai at time t, respectively. Therefore, 15 balanced panels of relative price $q_{i,t}$ are constructed with $T = 99$ and $N = 35$. Details of the data set are given in section 2.2.

We apply the tests we employed in the simulation study that are designed for mixed panels onto our relative prices panels. For Chortareas and Kapetanios (2009) $SPSM$ test, the Pesaran $CIPS$ test is employed in each round. In order to consistent with the $CIPS$ settings in Chapter 2, we use lag $p = 4$ in the CADF regressions, Cross-sectional mean $\bar{y}_{t-1}$ and $\Delta \bar{y}_t$ are used to proxy the unobserved common factor. Critical value of $CIPS$ test is obtained from Table II(b) in Pesaran (2007b). For Romano and Wolf (2005) test, the lag length of ADF regression is chosen by BIC with $p_{max} = 4$. The block bootstrap method is employed and the block length taken to be $b = \lfloor 1.75T^{1/3} \rfloor$, where floor operator $\lfloor \rfloor$ returns the nearest integer rounded towards $-\infty$ as in Palm et al. (2011). Number of replications for each bootstrap is 500. For Moon and Perron (2012) multiple testing procedure, the ADF lag length is selected by BIC with $p_{max} = 4$. Block bootstrap is used and the block size $b = \lfloor 3.21T^{1/3} \rfloor$ is the same to the bandwidth of long-run variance estimation in Andrews (1991), number of bootstrap replication is 500. For Smeekes (2011) BSQT tests, the lag length selection, block length selection, and number of replications for bootstrap are same to those in RW05 test. Units are demeaned by OLS, quantiles to be tested are chosen to be $\{0, 0.1, 0.3, 0.5, 0.7, 0.9\}$. For IBSQT and IBSQT2 tests, the number of iterations is set at 3. For Ng (2008) test, the lag length is selected by BIC $p_{max} = 4$. We employ the Bai and Ng (2004) method to extract common factors, and the number of common factors is set as two. The kernel employed is the Newey-West kernel $K(s, M) = 1 - \frac{s}{M+1}$, with a data dependant truncation bandwidth $M = \lfloor 4(\frac{T}{100})^{2/9} \rfloor$. For all tests, the level of significance is 5% and the level of FWE and FDR to be controlled at is 5%.

In Table 3.2, we report the proportion of stationary relative prices $(1 - \hat{\theta})$ in each of the 15 panels[7]. The first column is the names of the price indices. Stationary proportion by Chortareas and Kapetanios (2009) $SPSM$ test is reported in the second column. Third column reports the Romano and Wolf (2005) test results and fourth column reports the Moon and Perron (2012) test results. Results of Smeekes (2011) $BSQT$ and $BSQT2$ tests are similar, and results of $IBSQT$ and $IBSQT2$ tests are similar, hence we report the results from $BSQT$ and $IBSQT$ tests in column 5 and 6, respectively. Column 7 reports the Ng (2008) estimates of $I(0)$ proportion, as well as their standard errors. In order to compare the results here to our PPP testing results in Chapter 2, we copy some of the panel unit root test results from Table 2.5 into column 8 to column 12: column 8 to 9 is the Bai and Ng (2004) PANIC test, where column 8 reports the number of $I(1)$ common factors detected by MQ_c test and column 9 reports the $P_{\hat{e}}$ test on idiosyncratic component, column 10 is Pesaran (2007b) $CIPS$ test, column 11 is the Moon and Perron (2004) t_a^* test, and last column is Breitung and Das (2008) t_{rob} test.

[7] Details of the $I(0)$ and $I(1)$ classification of units in each of the 15 panels by different methods are reported in Appendix B.3, from Table B.163 on page 322 to Table B.168 on page 327.

Table 3.2: Stationary Proportion in Relative Price Panels

Product Name	$SPSM$	RW05	MP12	$BSQT$	$IBSQT$	Ng08	MQ_c	$P_{\hat{e}}$	$CIPS$	t_a^*	t_{rob}
CPI	0.66	0.14	0.29	0.50	0.30	0.75(0.08)	0	14.72***	-3.09***	-11.83***	-0.75
Food	0.23	0.23	0.54	0.50	0.43	0.88(0.07)	0	12.95***	-2.69***	-38.04***	-3.55***
Perishables											
Meat	0.29	0.06	0.26	0.30	0.19	0.82(0.10)	0	10.67***	-2.49***	-36.68***	-2.54***
Eggs	0.23	0.26	0.40	0.50	0.30	0.85(0.08)	0	9.85***	-2.56***	-29.16***	-3.83***
seafood	0.29	0.17	0.31	0.30	0.23	0.88(0.10)	0	10.12***	-2.40***	-24.01***	-1.19
Fresh Vegetables	0.77	0.40	0.49	0.30	0.38	0.96(0.04)	0	23.44***	-4.08***	-110.92***	-4.00***
Fresh Fruits	0.97	0.43	0.57	0.50	0.43	0.98(0.05)	0	29.03***	-3.47***	-111.02***	-5.33***
Nonperishables											
Grain	0.00	0.00	0.00	0.30	0.23	0.56(0.23)	0	4.44***	-1.98	-13.77***	-1.25
Tobacco and Wine	0.11	0.00	0.00	0.10	0.00	0.46(0.19)	1	4.27***	-2.32***	-3.24***	0.95
Clothing	0.00	0.03	0.03	0.10	0.00	0.16(0.22)	2	0.72	-1.58	-1.19	-1.04
Services											
Recreation Facility	0.09	0.00	0.00	0.00	0.00	0.42(0.18)	2	0.26	-2.27**	-1.13	1.70
Health Care	0.20	0.14	0.23	0.10	0.12	0.43(0.17)	0	4.89***	-2.64***	-4.84***	-2.84***
Traf. and Comm.	0.29	0.03	0.03	0.10	0.03	0.03(0.18)	1	1.53*	-2.55***	0.00	0.22
Cul. and Edu. Exp.	0.69	0.03	0.06	0.10	0.10	0.69(0.14)	1	5.28***	-3.35***	-4.86***	-1.01
Residence	0.03	0.03	0.06	0.10	0.03	0.54(0.19)	1	1.15	-2.16**	-5.76***	-0.57

Note: Number is parentheses are standard errors. *,**,*** denote significance at 10 percent, 5 percent, and 1 percent levels, respectively. The first column is the names of the price indices. Stationary proportion by Chortareas and Kapetanios (2009) $SPSM$ test is reported in the second column. Third column reports the Romano and Wolf (2005) test results and fourth column reports the Moon and Perron (2012) test results. Results of Smeekes (2011) $BSQT$ and $BSQT2$ tests are similar, and results of $IBSQT$ and $IBSQT2$ tests are similar, hence we report the results from $BSQT$ and $IBSQT$ tests in column 5 and 6, respectively. Column 7 reports the Ng (2008) estimates of $I(0)$ proportion, as well as their standard errors. In order to compare the results here to our PPP testing results in Chapter 2, we copy some of the panel unit root test results from Table 2.5 into column 8 to column 12: column 8 to 9 is the Bai and Ng (2004) PANIC test, where column 8 reports the number of $I(1)$ common factors detected by MQ_c test and column 9 reports the $P_{\hat{e}}$ test on idiosyncratic component, column 10 is Pesaran (2007b) $CIPS$ test, column 11 is the Moon and Perron (2004) t_a^* test, and last column is Breitung and Das (2008) t_{rob} test.

We first revisit the panel unit root testing results obtained in Chapter 2, section 2.4.1, these results are repeated in the last 5 columns in Table 3.2. As we mentioned in Chapter 2, the Pesaran (2007b) test rejects the unit root null for 13 panels (out of 15), but the Breitung and Das (2008) test rejects the null for only 6 panels. The conflict results are much more obvious when the Bai and Ng (2004) test suggests that a panel is nonstationary: there are 6 nonstationary panels detected by the PANIC test, and Pesaran (2007b) test rejects the unit root null in 5 of them, Moon and Perron (2004) test rejects the null in 3 of them, and Breitung and Das (2008) test rejects the null in none of them. This conflict of testing results may confuse a researcher from drawing conclusion in the empirical study. Our simulation study in section 3.4.2 provides us a possible explanation: if the panels in hand are mixed panels but we use panel unit root tests to get an overall conclusion, that is, we want to decide whether the panel is stationary or nonstationary, then mixed conclusions can be obtained.

According to the Bai and Ng (2004) PANIC test, three panels are constructed by $I(1)$ common factor component and $I(1)$ idiosyncratic component: clothing price index and residence price index have two $I(1)$ common factors and recreation facility price index has one $I(1)$ common factor. Our calculation shows that under such a DGP, for example in Figure 3.1, the $CIPS$ test is very sensitive to a small proportion of stationary units, and the power is close to 1 when $\theta = 0.7$. The rejection frequency of t_{rob} test, on the other hand, grows slowly as θ getting smaller, and the rejection frequency is only 0.23 when $\theta = 0.3$. The rejection frequency of t_a^* grows faster than t_{rob} test, but still much lower than the $CIPS$ test. In our example, $CIPS$ rejects the null in 2 panels (recreation facility and residence) out of 3, t_a^* rejects the null in 1 panel (residence), and t_{rob} rejects none of them.

The PANIC test detects three panels that are constructed by $I(1)$ common factor component and $I(0)$ idiosyncratic component: tobacco and wine price index, traffic and communication price index, and cultural and educational expense. Our simulation shows that under this DGP, for example in Figure 3.2, $CIPS$ and t_a^* have severe size distortion and the rejection frequency is close to 1 even under the null hypothesis. The size distortion of t_{rob} test is less severe, and the power of t_{rob} still grows slowly as θ getting smaller, for example, the rejection frequency is 0.65 when $\theta = 0.3$. In the three panels with $I(1)$ common factor component and $I(0)$ idiosyncratic component, $CIPS$ rejects all of them, t_a^* rejects 2 of them, and t_{rob} rejects none of them.

We then look at the estimation of stationary proportion $(1 - \hat{\theta})$ in our empirical data set. For the aggregate CPI and food price index, the Romano and Wolf (2005) test has the smallest proportion, and Ng (2008) estimator shows the largest proportion. The tests use bootstrap critical values estimate a smaller stationary proportion, comparing to $SPSM$ and Ng08 methods, ranging between 0.14 to 0.54. For the price indices in the perishable group, the Ng08 test always estimate the largest stationary proportion, and null hypothesis that all units are stationary can not be rejected in all panels. Fresh vegetables and fresh fruits price indices have greater stationary proportion than others, according to most of the tests. The difference of estimates from different methods can be large, for example, in the meat price index, the estimated stationary proportion by RW05 is 0.06, and the proportion by Ng08 is 0.82. Estimates by Moon and Perron (2012) test and Smeekes (2011) tests are generally close to each other, ranging from 0.19 to 0.57 in this group of consumer goods prices. For price indices in the nonperishable group, the Ng08 estimator has large standard deviation and wide confidence interval (0 is in the confidence interval for clothing price index). The stationary proportions are generally close to 0 by other estimation methods, expect the Smeekes (2011) tests on grain price index. For panels in the services group, the Ng08 estimator is generally greater than others, but with a wide confidence interval. Bootstrap methods show that the stationary proportion is less than 0.10 in this group, expect the health care price. The $SPSM$ stationary proportion is consistent with others in 3 panels but significantly larger than other estimates in the traffic and communication price panel and cultural and educational expense panel.

Some noteworthy observations here. First, our simulation study in section 3.4.3 shows that a satisfactory performance of Ng08 estimator requires T to be large and correct DGP can be specified. We therefore doubt the reliability of the Ng08 estimator. The confidence interval of the estimator can be large, for example, the 95% confidence interval for stationary proportion in grain price panel is $(0.10, 0.92)$, therefore, reasonable conclusion can hardly be obtained. Second, our simulation can also explain the abnormal large stationary proportion by $SPSM$ in panels with $I(1)$ common factor component and $I(0)$ idiosyncratic component (tobacco and wine, traffic and communication, and cultural and educational expense panels). Figure 3.2 shows

Table 3.3: Average Stationary Proportion for Each Group

Product Group	$SPSM$	RW05	MP12	$BSQT$	$IBSQT$	Ng08
Perishables	0.51	0.26	0.41	0.38	0.31	0.90
Nonperishables	0.04	0.01	0.01	0.17	0.08	0.40
Services	0.26	0.05	0.07	0.08	0.06	0.42

that, under this DGP, $SPSM$ under estimates θ and over estimate the stationary proportion $1-\theta$. Therefore, the stationary proportion of $SPSM$ in the nonperishable group and services group are biased upwards and not reliable. Third, simulation shows that the performance of RW05 test is satisfactory when N is as small as 10, when N is large, RW05 over estimates θ and under estimates the stationary proportion. The cross-sectional dimension $N = 35$ in our data set, which explains that RW05 stationary proportion is the smallest in most panels and indicates that this proportion might be downward biased. Fourth, in our simulation we see that the $IBSQT$ and $IBSQT2$ tests do not improve the $BSQT$ and $BSQT2$ tests, respectively. This result is based on a large number of replications, where the issue of user specified quantile selection is not important. In the empirical study, however, the selection of quantile remains ambiguous, and the $IBSQT$ may still provide a more reasonable result. Fifty, results from bootstrap methods are generally close to each other, especially the tests by Moon and Perron (2012) and Smeekes (2011), but results by $SPSM$ and Ng08 tests can deviate from bootstrap estimates quite far. Our simulation suggests that the satisfactory performance of $SPSM$ and Ng08 tests relies more on the correct specification of underlying DGP, while the bootstrap methods avoid this problem. We therefore argue that the bootstrap methods may provide more reliable results in empirical studies.

We summarise the stationary proportions into Table 3.3. Simple average is taken for the proportions within each group for each testing methods. Considering the bootstrap methods, the stationary proportion ranges between 0.26 to 0.41 in perishable group, between 0.01 to 0.17 in nonstationary proportion, and between 0.05 to 0.08 in services group. The $SPSM$ proportion in nonperishable and services groups are upward biased due to the existence of panels with $I(1)$ common factor and $I(0)$ idiosyncratic component. The large values estimated by Ng08 test may due to the unknown underlying DGP.

The results shed some light on the traditional purchasing power parity studies, which employ the panel unit root tests and PPP is said to be held in the panel as a whole if the unit root null hypothesis is rejected. Fan and Wei (2006), Lan and Sylwester (2010) apply first-generation panel unit root tests on Chinese city price indices and find evidence in favour of PPP for the vast majority of goods and services. We employ the second-generation panel unit root tests on Chinese city price indices in Chapter 2 and find strong evidence for PPP in perishable group. We, however, find little evidence in favour of PPP in nonperishable group and find mixed evidence in services group. Our mixed panel tests show some surprising results, even for the perishable group, for which strong evidence can be found for PPP, the proportion of stationary relative price is just about a quarter to 2 fifths. For nonperishable and services groups, the proportion of stationary relative price is generally less than 10%, that is to say, only a few units out of 35 are stationary in the panel. The mixed evidence we found in the services group comes from the fact that, even if the stationary proportion is very small, panel unit root test may still reject the unit root null hypothesis and gain a conclusion that the panel is stationary as a whole. Some tests are very sensitive to a small proportion of stationary units, for example, the Pesaran (2007b) $CIPS$ test and Bai and Ng (2004) $P_{\hat{e}}$ test. We suggest that caution should be taken when a unit root null is rejected, as this may be a result of only a small proportion of stationary units.

3.6 Conclusion

In this study we perform a large scale Monte Carlo simulation on a set of second-generation panel unit root tests and tests designed for mixed panels. For the data generating processes, we generate a set of mixed panels, considering different dimensions, sources of unit roots, number of common factors, magnitude of cross-sectional dependence, and stationary/nonstationary proportions. We first apply the panel unit root tests on mixed panels and analyse the size/power of the tests. Tests employed are Pesaran (2007b) $CIPS$ test, Moon and Perron (2004) t_a^* and t_b^* tests, Breitung and Das (2008) t_{rob} and t_{gls} tests, and Bai and Ng (2004) PANIC test. Results show that tests of Pesaran (2007b), Moon and Perron (2004), Breitung and Das (2008) test for a unit in the idiosyncratic component and reject the null hypothesis if the unit root is present in common factors alone. Power of Pesaran (2007b) $CIPS$ test and Bai and Ng (2004) $P_{\hat{e}}$ test are very sensitive to a small proportion of units under alternative hypothesis, while power of other tests grows gradually with greater proportion of units under the alternative.

We then apply the tests designed for mixed panels and compare their performances, in terms of estimating the proportion and identifying the $I(0)/I(1)$ units correctly. Tests employed are Chortareas and Kapetanios (2009), Smeekes (2011), Romano and Wolf (2005), Moon and Perron (2012) and Ng (2008). Our experiment shows that, satisfactory performance of these tests requires relatively large sample size ($T \geq 100$). Tests using bootstrapped critical values outperform others when the underlying DGP is not correctly specified. Moon and Perron (2012) procedure show impressing good property in terms of its high power and meanwhile being correctly sized.

An empirical study on Chinese city price indices is provided. Different form existing literature, in which strong evidence is found in favour of PPP within China, our results show that for vast majority of goods and services, only a small proportion of stationary relative prices is present in panel. This result suggests that caution should be exercised when interpreting a rejection of panel unit root tests.

Chapter 4

Testing Unit Root against Long-Memory in a Mixed Panel

4.1 Introduction

Literature in panel unit root tests has been developing fast over the past two decades, for example, see Breitung and Pesaran (2008). The panel unit root tests are testing the null hypothesis that all units are $I(1)$ in a panel against either the homogeneous alternative that all units are $I(0)$, or the heterogeneous alternative that at least one unit is $I(0)$ in a panel. Recently, some panel unit root testing procedures are proposed for mixed panels, where panels consist of both $I(1)$ and $I(0)$ units. The goal of these procedures is to estimate the proportion of stationary and nonstationary units, and classify units in to $I(1)$ group and $I(0)$ group. A short review of the procedures is provided in Chapter 3.

However, time series such as real exchange rate and aggregate output may exhibit long-memory behaviour. In empirical studies with panel data, economic variables may have fractional order of integration, not necessarily $I(1)$ or $I(0)$. Therefore, it may be desirable to test the unit root null hypothesis against the fractional alternative $I(d)$, $d < 1$ in panel data. Furthermore, in order to interpret the null and alternative hypothesis in panel data, it is helpful to account for mixed panel and classify units into $I(1)$ group and $I(d)$ group, where $d < 1$.

To our knowledge, there are only a few papers on the panel fractional integration test. Chen (2008) considers local Whittle estimator for fixed effect panel data but cross-sectional dependence is not allowed. Hassler et al. (2011) adapt regression-based LM integration test for panels with correlated units, and use feasible generalised least squares (GLS) to construct a panel test statistic. Robinson and Velasco (2013) propose four ways to cope with individual fixed effects to estimate the fractional order of integration, the cross-sectional units are however assumed to be independent and the order of integration is homogeneous. There are a few working papers at preliminary stage. For example, Ergemen and Velasco (2013) use a conditional-sum-of-squares type estimation on first-differences to estimate the fractional integration parameter, nonstationarity behaviour is allowed in both common factor and idiosyncratic component, cross-sectional dependence and heterogeneous memory are allowed.

We start our analysis by constructing a Fisher-type fractional integration test in panel data. The Fisher test combines p-values from individual fractional integration tests. We are interested in testing the null that $d_i = 1$ for all i, against the heterogeneous alternative that $d_i < 1$ for some

i. The asymptotic distribution of Fisher test statistic assumes cross-sectional independence, which is always not the case in empirical studies. To take the dependence into account, we bootstrap the critical values of our Fisher-type test statistic, as suggest by Maddala and Wu (1999).

We then carry on our analysis and aim to propose a testing procedure that takes mixed panel in to account. In empirical studies, panels can be mixed, that is, panels may consist of both $I(1)$ and $I(d)$, $d < 1$ units. When a panel is mixed and our Fisher test rejects the null hypothesis that all units are $I(1)$, we have little information to tell how many units are under the alternative and which units are under the alternative.

Therefore, motivation of constructing a test that takes mixed panel into account is the following. First, researchers often carry out uncorrelated individual long memory test on each unit in a panel to obtain detailed information for each unit. In order to test the unit root null in a panel context and evaluate the aggregation of individual testing results, we use concept from multiple testing literature to construct a procedure that controls the aggregate performance of individual tests. Second, the procedure should account for dependency among units. Third, in order to get interpretable results in empirical study, the procedure should allow for heterogeneous order of integration. Fourth, we want to find the proportion of true null hypothesis ($I(1)$) and true alternative hypothesis ($I(d)$, $d < 1$), and classify each unit into the null group and alternative group.

Our study uses the concept of *false discovery rate* (FDR) in multiple testing to construct a panel fractional integration test for mixed panels of unit root processes and long-memory processes. FDR is the limit proportion of false rejection in all rejections, that is, the proportion of $I(1)$ units in the classified $I(d)$, $d < 1$ units. We employ the Romano et al. (2008) method to bootstrap a set of critical values to control the FDR in N individual tests, where N is the cross-sectional dimension. Test statistics in units are calculated individually, but the critical values are bootstrapped (rather than taken from individual asymptotic distribution), in order to account for cross-sectional dependence and control the FDR.

In both Fisher-type test and multiple testing procedure, we employ the local Whittle (LW) and exact local Whittle (ELW) tests by Shimotsu and Phillips (2005, 2006) as the individual long-memory tests. Shimotsu and Phillips (2005), among others, show that LW estimator is discontinuous at $d = \frac{3}{4}$ and $d = 1$, non normally distributed when $d \geq \frac{3}{4}$, and inconsistent when $d > 1$. It is shown that ELW tests are consistent and normally distributed when the optimization covers an interval of width less that $\frac{9}{2}$. Due to the fact that we use a set of bootstrapped critical values, the non-normal distribution of LW when $d \in [\frac{3}{4}, 1]$ does not matter, as long as d can be consistently estimated. Therefore, our bootstrap procedure is valid by using LW test if d is in stationary region $(-\frac{1}{2}, \frac{1}{2})$ and nonstationary region $[\frac{1}{2}, 1]$, and is valid by using ELW tests when the optimization covers an interval of width less that $\frac{9}{2}$.

Simulation experiments are performed to examine the performance of our tests. The Fisher test shows correct size and high power even if the proportion of units under the alternative is small, especially when d is in the stationary region. It is also shown that our multiple testing procedure successfully controls the FDR at around 5%. When d is in the stationary region, the test is very powerful even if $T = 50$, providing accurate estimate of the unit root and long-memory proportion and classifying units into null group and alternative group correctly. When d is in the nonstationary region, however, a powerful test requires $T > 100$.

We provide an empirical application on the data set of Chinese city relative price indices. Contrast to the results in Chapter 3, where strong evidence is found in favour of unit root null hypothesis if we classify units into $I(1)$ and $I(0)$, our finding here is that we find strong evidence against unit root null if we classify units into $I(1)$ and $I(d)$, $d < 1$. This may indicate that the relative price indices are neither $I(1)$ nor $I(0)$, but exhibit long-range dependence behaviour.

The remainder of this Chapter is organised as follows. Section 4.2 describes the LW and ELW univariate long-memory tests. In section 4.3, we discuss the construction of our Fisher-type panel fractional integration test. In section 4.4 we describe the concept of FDR and ways to control FDR in multiple testing literature. Section 4.5 discusses our multiple testing bootstrap algorithm. Section 4.6 provides simulation evidence and section 4.7 provides an application with Chinese city relative price indices. Section 4.8 concludes.

4.2 Time Series Long Memory Testing Approaches

A stationary time series $\{y_t\}$ has long memory (see Granger and Joyeux (1980)) if there is a non-zero $d \in (-0.5,\ 0.5)$ such that its spectral density obeys a power law $f(\lambda) \sim k\lambda^{-2d}$ as $\lambda \to 0^+$. Clearly, as $\lambda \to 0$, $f(\lambda) \to \infty$ if $d > 0$, or $f(\lambda) \to 0$ if $d < 0$. In the case where $d = 0$, $\{y_t\}$ will have short memory and $(0 < f(0) < \infty)$ such as in the case of all the stationary and invertible ARMA processes. Hosking (1981) establishes the relationship between long memory models and fractional differencing, proposing a class of models denominated fractional ARIMA, where the degree of differencing can be any real number. The simplest of these is the fractionally integrated noise or $ARIMA(0, d, 0)$ defined for $d > -1$, that is:

$$\Delta^d y_t = \varepsilon_t \tag{4.1}$$

where $\varepsilon \sim iid(0, \sigma^2)$, and $\Delta^d = (1 - L)^d$. To obtain an ARIMA(0,d,0) for d that lies outside the range $(-0.5, 0.5)$, it is sufficient to start with an ARIMA$(0, \bar{d}, 0)$, with $\bar{d} \in (-0.5, 0.5)$, such that $d - \bar{d}$ is an integer. If $d > 0.5$, we need to integrate $d - \bar{d}$ times, if $d < 0.5$, we difference $d - \bar{d}$ times. The result will be an ARIMA(0,d,0) which however will not be stationary if $d \geq 0.5$.

The more general fractional ARIMA(p, d, q) can be written as:

$$\phi(L)\Delta^d y_t = \theta(L)\varepsilon_t \tag{4.2}$$

where ε_t is as before a zero mean white noise process, $d \in (-0.5,\ 0.5)$, and $\phi(L)$ and $\theta(L)$ are polynomial in the lag operator or order p and q, respectively. The ARIMA(p,d,q) can also be written as:

$$\phi(L)y_t = \theta(L)(\Delta^{-d}\varepsilon_t) \tag{4.3}$$

showing that the ARIMA(p,d,q) can be thought of as an ARMA(p,q) driven by a fractionally integrated ARIMA (0,d,0) noise, $\Delta^{-d}\varepsilon_t$. The spectral density for the ARIMA (p,d,q) is:

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi}\left|\theta(e^{-i\lambda})/\phi(e^{-i\lambda})\right|^2 \left|1 - e^{-i\lambda}\right|^{-2d} \tag{4.4}$$

and as $\lambda \to 0^+$

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} \left|\theta(1)/\phi(1)\right|^2 |\lambda|^{-2d} \tag{4.5}$$

Again, if $d \neq 0$, then y_t is a long memory process and it can be shown that its auto-covariances obey the power law $c_r \sim kr^{2d-1}$ as $r \to \infty$.

Alternatively we could write the spectral density of the fractional ARIMA (p,d,q) as:

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} \left|1 - e^{-i\lambda}\right|^{-2d} g(\lambda) \tag{4.6}$$

where $g(\lambda)$ is the spectral density of an ARMA(p,q).

The ARMA parameters as well as d are unknown and must be estimated. Assuming that p and q are known and that $\{\varepsilon_t\}$ are Gaussian, we can use maximum likelihood estimation (MLE) to estimate the ARMA parameters as well as d. However, the MLE of this type of model requires $O(n^3)$ operations to evaluate the likelihood function. To avoid such intense computation, Fox and Taqqu (1986) suggest using the Whittle approximation to the likelihood function, that is:

$$-2\log l(\Theta) \approx \sum_{j=1}^{T/2} [\log f_\Theta(\lambda_j) + I(\lambda)/f_\Theta(\lambda_j)] \tag{4.7}$$

where Θ is the vector of ARMA parameters plus d, $I(\lambda) = \frac{1}{2\pi T} \left|\sum_{t=1}^{T} y_t e^{it\lambda}\right|^2$ is the periodogram of y_t, $f_\Theta(\lambda)$ is the spectral density of the ARIMA (p,d,q) process, and $\lambda_j = 2\pi j/T$. The Whittle estimator (WE) is obtained by minimisation with respect to Θ. Fox and Taqqu (1986) show that the Whittle estimator of Θ, say $\hat{\Theta}_W$, is asymptotically equivalent to the exact MLE assuming that $d > 0$, the model is correct, and the order of p and q is known.

However, p and q are unknown, and as a result both the ML and W estimators of d will be asymptotically biased. The same problem will arise if $g(\lambda)$ is not ARMA but it is assumed that it is.

To overcome the problem, Robinson (1995) proposes a semiparametric Gaussian estimator for d known as the Local Whittle (LW) estimator, which is developed under the assumption that y_t is stationary and its spectral density behaves like $G\lambda^{-2d}$ as $\lambda \to 0^+$. The frequency domain

Gaussian likelihood in the vicinity of the origin is:

$$Q_m(G,d) = m^{-1} \sum_{j=1}^{m} \left[\log(G\lambda_j^{-2d}) + \frac{\lambda_j^{2d}}{G} I(\lambda_j) \right] \tag{4.8}$$

where as before $m < T$ is an integer controlling the number of frequencies included in the local likelihood. Estimates of G and d are then obtained through the minimisation of $Q_m(G,d)$ such that:

$$(\hat{G}, \hat{d}) = \arg \min_{G \in (0,\infty), d \in [\Delta_1, \Delta_2]} Q_m(G,d) \tag{4.9}$$

with Δ_1 and Δ_2 are numbers such that $-\infty < \Delta_1 < \Delta_2 < \frac{1}{2}$. Concentrating the likelihood with respect to G, it is found that $\hat{d}$ satisfies

$$\arg \min_{d \in [\Delta_1, \Delta_2]} \log \hat{G}(d) - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \tag{4.10}$$

where $\hat{G}(d) = m^{-1} \sum_{j=1}^{m} \lambda_j^{2d} I(\lambda_j)$.

Under appropriate assumptions and conditions, and for $d_0 \in (-\frac{1}{2}, \frac{1}{2})$, Robinson (1995) and Shimotsu and Phillips (2006) show that $\sqrt{m}(\hat{d} - d_0) \to N(0, \frac{1}{4})$.

Shimotsu and Phillips (2005), among others, show that when $d > \frac{1}{2}$, the LW estimator exhibits non standard behaviour. Although it is consistent for $d \in (\frac{1}{2}, 1]$ and asymptotically normal for $d \in (\frac{1}{2}, \frac{3}{4})$, the LW has non normal asymptotic distribution for $d \in [\frac{3}{4}, 1]$ and $d > 1$, but also converges to 1 in probability and is inconsistent.

The solution to the problem is provided by Shimotsu and Phillips (2005, 2006), who developed the Exact Local Whittle (ELW) estimator and its variants, whose asymptotics are based on the exact frequency domain (or its estimate which will give rise to FELW estimator) of the data generating process which is obtained from the minimisation of the objective function:

$$Q_m^*(G,d) = m^{-1} \sum_{j=1}^{m} \left[\log(G\lambda_j^{-2d}) + \frac{\lambda_j^{2d}}{G} I_{(1-L)^d y}(\lambda_j) \right] \tag{4.11}$$

The ELW is computationally more demanding than LW but is shown to be consistent and asymptotically normally distributed for any value of d and therefore is valid under a wider range of cases.

Shimotsu and Phillips (2006) discuss the importance of the choice of m, explaining that as in the case of LW and ELW estimation, m has to grow fast for $\hat{d}$ to be consistent, but also that a too large value of m may induce a bias to the estimator from the short run dynamics. A rule of thumb suggests that within $m = T^{\alpha}$, we should choose a value of α in the interval $[0.5, 0.65]$. We use LW, ELW and FELW to estimate $\hat{d}$ and perform the test H_0: $d = 1$ vs H_1: $d < 1$, notice that we shall only be concerned with the hypothesis that $d = 1$ (the series tested has a unit root) against the one-sided alternative that $d < 1$ (the series has long memory but is mean reverting and possibly covariance stationary).

4.3 Fisher-Type Panel Test of Unit Root Against Long-Memory Alternative

Under the unit root null hypothesis, exact local Whittle tests by Shimotsu and Phillips (2005, 2006) are normally distributed, therefore inference on the individual order of integration is straightforward. We first assume that N cross-sectional units are independent. Based on the exact local Whittle tests, we can construct a Fisher-type panel test, which combines the individual p-values from independent time series tests. We are interested in testing the unit root null hypothesis against long-memory alternative, such that[1]:

$$H_0: \quad d_i = 1, \quad for\ all\ i \tag{4.12}$$

$$H_1: \quad d_i < 1, \quad for\ some\ i \tag{4.13}$$

To calculate the Fisher-type statistic, we perform N individual hypothesis testing of the null that $d = 1$ against the alternative that $d < 1$, and collect N individual p-values. Under the null hypothesis that each cross-sectional unit is a unit root process, given that the test statistics are continuous, the p-values are independently and uniformly distributed, such that

[1]Note that Fisher type procedure can be used in other types of null/alternative hypothesis.

$p_i \sim iidU(0,1)$. Therefore, $-2\log(p_i)$ has a χ^2 distribution with two degrees of freedom, and $\lambda = -2\sum_{i=1}^{N}\log(p_i)$ has a χ^2 distribution with $2N$ degrees of freedom.

A problem of the Fisher test is that it assumes that the N individual p-values are independent. While the p-values are correlated, the distribution of Fisher statistic is unknown. As suggested by Maddala and Wu (1999), we block bootstrap the critical values of Fisher statistic to allow for cross-sectional dependence. Our algorithm is the following.

1. For each cross-sectional unit i, apply the Shimotsu and Phillips (2005, 2006) tests and obtain the estimator $\hat{d}_i$ and p-value while testing $d_i = 1$ against $d_i < 1$. Calculate the Fisher statistic, $\lambda = -2\sum_{i=1}^{N}\log(p_i)$.

2. For each unit i, calculate the residuals as:

$$\hat{u}_{i,t}I\{t \geq 1\} = (1-L)^{\hat{d}_i} y_{i,t} \tag{4.14}$$

$$= \sum_{k=0}^{t} \frac{(-\hat{d}_i)_k}{k!} y_{i,t-k} \tag{4.15}$$

   where $t = 0, \pm 1, \cdots$, $I\{t \geq 1\}$ is the indicator function, and

$$(\hat{d}_i)_k = \frac{\Gamma(\hat{d}_i + k)}{\Gamma(\hat{d}_i)} = (\hat{d}_i)(\hat{d}_i + 1)\cdots(\hat{d}_i + k - 1) \tag{4.16}$$

   is the Pochhammer's symbol for the forward fractional function and $\Gamma(\cdot)$ is the gamma function. Denote $\hat{u}_t = (\hat{u}_{1,t}, \cdots, \hat{u}_{N,t})'$.

3. Choose block length b, generate $i_0, \cdots, i_{k-1}$ i.i.d. random variables from uniform distribution on a set of integers $\{1,\cdots,T-b\}$, where k is the number of blocks and $k = \lfloor (T-2)/b \rfloor + 1$.

4. Generate bootstrap error terms $u_t^* = (u_{1,t}^*, \cdots, u_{N,t}^*)'$, $t = 2, \cdots, T$, as:

$$u_t^* = \hat{u}_{i_m + s} \tag{4.17}$$

   where $m = \lfloor (T-2)/b \rfloor$, and $s = t - mb - 1$.

5. Generate bootstrap panels $y^*_{i,t}$ under the unit root null hypothesis by accumulating the bootstrap errors:
$$y^*_{i,t} = \sum_{j=1}^{t} u^*_{i,j} \tag{4.18}$$
6. Apply the Shimotsu and Phillips (2005, 2006) tests unit by unit in the bootstrapped panel and calculate the Fisher test statistic, $\lambda^* = -2\sum_{i=1}^{N}\log(p^*_i)$.
7. Repeat step 3 to step 6 B times.
8. Obtain the 95% quantile of Fisher statistic, label as λ^*_c.
9. Compare the Fisher statistic in step 1 to the bootstrapped critical value, reject the null hypothesis that $d_i = 1 \;\forall\; i$ if $\lambda > \lambda^*_c$.

We provide a simulation study of the property of our Fisher type test in Section 4.6.

4.4 Multiple Testing Framework

Applying individual tests simultaneously has some advantage over an overall conclusion, for example, Breuer et al. (2001) suggest to use a seemingly unrelated ADF test to examine how many series are stationary in a panel. However, this kind of methods ignore the issue of multiple testing, and may suffer some statistical problems.

Consider the following simple example given by Hanck (2006). Suppose there are 20 individual tests to be performed, and the significance level for each test is set to be 0.05. Suppose that the null hypothesis is true for all tests, then the probability of a rejection (size) is 0.05 for each test. Consider the event of rejection as a Bernoulli random variable with success probability equal to 0.05. The probability of at least one successful rejection is given by:

$$P = \sum_{i=1}^{20} \begin{pmatrix} 20 \\ i \end{pmatrix} 0.05^i (1-0.05)^{20-i} = 0.64 \tag{4.19}$$

That is to say, even if the null hypothesis holds in all individual tests, the probability of making at least one mistake in a bunch of tests is high, and this probability is higher with greater number of tests to be performed. Therefore, there are incentives to discipline and evaluate the aggregation of individual tests.

4.4.1 FWE and FDR

Suppose we have N individual tests $i = 1, \cdots, N$. In each of the tests, we reject the null hypothesis $H_{0,i}$ if the p-value is less than a pre-specified significance level α, that is, reject $H_{0,i}$ if $p_i < \alpha$. In order to evaluate the aggregation of individual tests, we use the concepts of familywise error rate and false discovery rate in the multiple testing literature.

The *Familywise Error Rate* (FWE) is defined as the probability of making at least one false rejection:

$$\text{FWE} = \Pr\{\text{at least one of } \{p_1, \cdots, p_N\} < \alpha | H_{0,1}, \cdots, H_{0,N}\} \tag{4.20}$$

where p_i, $i = 1, \cdots, N$ is the p-value of the test i, α is the pre-specified significance level, $H_{0,i}$ indicates the null hypothesis of test i.

Some procedures are proposed to control the FWE in multiple testing. The well known Bonferroni method adjusts the level of significance from α to $\frac{\alpha}{N}$ for each test, and reject $H_{0,i}$ if $p_i < \frac{\alpha}{N}$. Holm (1979) proposes a step-wise procedure to control the FWE. The p-values of tests are ordered from most significant to least significant ($p_{(1)} < \cdots < p_{(N)}$), where the corresponding null hypothesis are labelled accordingly. The i-th smallest p-value is then compared with $\frac{\alpha}{N-i+1}$. For example, the first p-value is compared with $\frac{\alpha}{N}$, the second p-value is compared with $\frac{\alpha}{N-1}$. It follows that procedure until the first non-rejection is found. The Holm (1979) step-wise procedure controls the FWE and meanwhile increase the power of Bonferroni method, since it rejects more null hypothesis. White (2000) and Romano and Wolf (2005) propose methods that use bootstrap to account for the dependence of test statistics.

It is well known that the methods that control the FWE are conservative, in terms of having low power, and the power decreases as the number of hypotheses increases. In practice, the concept of FWE may be too restrictive, and it may not be desirable to sacrifice the power of tests to control the FWE. In fact, a researcher would like to reject as much null hypotheses as possible, while allowing a small proportion of false discoveries. Benjamini and Hochberg (1995) introduce the concept of *False Discovery Rate* (FDR), which is the expected value of the proportion of true null hypotheses among the rejections (*False Discovery Proportion*, FDP):

$$\text{FDR}_p = E\left(\frac{R_{1|0}}{\max(1,R)}\right) \tag{4.21}$$

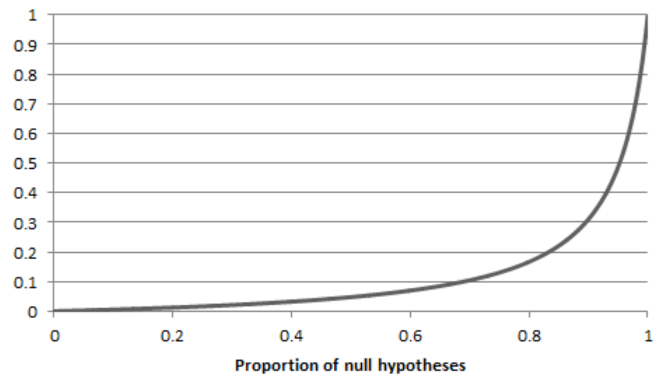
where $R_{1|0}$ is the number of rejection when the null hypotheses are true, R is the total number of rejections. When there is no rejection ($R = R_{1|0} = 0$), the FDR is set at 0.

Consider an asymptotic example to calculate the FDR. Suppose we have N independent individual tests $i = 1, \cdots, N$, and the proportion of true null hypothesis is θ among the N tests. Therefore, the number of true null hypotheses is $\theta \cdot N$, the number of true alternative hypotheses is $(1-\theta)\cdot N$. Asymptotically, the total number of rejection should be the number of true alternative hypotheses plus the size of individual test (say 5%) multiplied by the number of true null hypotheses: $R = (1-\theta)\cdot N + 0.05\cdot\theta\cdot N$. Number of false rejection is $R_{1|0} = 0.05\cdot\theta\cdot N$. Therefore, the FDR can be calculated as:

$$\begin{aligned} \text{FDR}_p &= \frac{0.05\cdot\theta\cdot N}{(1-\theta)\cdot N + 0.05\cdot\theta\cdot N} \\ &= \frac{0.05\theta}{1-0.95\theta} \end{aligned} \tag{4.22}$$

In our asymptotic example, the FDR is an increasing function with the proportion of null hypothesis. A plotting of the function is given in Figure 4.1. However, in finite sample of empirical data, the value of FDR is affected by size and power of individual test, dependence structure, proportion of null hypotheses, etc. Therefore, it is sometimes desirable to control the FDR in a multiple testing problem at a fixed level to discipline and evaluate the aggregation of individual tests.

Figure 4.1: FDR



4.4.2 Controlling FDR in Multiple Testing

In this subsection, we briefly review some previous methods that are designed to control FDR. We discuss the method by Romano et al. (2008) in some detail, which is employed in this study.

Similar to Holm (1979), Benjamini and Hochberg (1995) propose a stepwise procedure based on the p-value of each null hypothesis. Order the p-values from smallest to largest as $p_{(1)} < \cdots < p_{(N)}$, and label the corresponding null hypothesis as $H_{0,(1)}, \cdots, H_{0,(N)}$. The i-th smallest p-value is compared with the adjusted level of significance

$$\alpha_i = \frac{i}{N}\alpha \tag{4.23}$$

For example, the first p-value is compared with $\alpha_1 = \frac{1}{N}\alpha$, the second p-value is compared with $\alpha_2 = \frac{2}{N}\alpha$, and so on. The null hypothesis $H_{0,(i)}$ is rejected if $p_i < \alpha_i$. The procedure stops after the first non-rejection. Benjamini and Hochberg (1995) method is designed for independent tests, Benjamini and Yekutieli (2001) show that the independence can be replaced by some weak form of dependence.

It can be showed that replacing N by N_0, the number of true null hypothesis, can also control the FDR. Since $N_0 \leq N$, the i-th level of significance is greater when N_0 is substituted in, and more null hypothesis will be rejected. Storey et al. (2004) propose an estimator of the unknown value N_0:

$$\hat{N}_0 = \frac{\sum_{i=1}^{N} 1(p_i > \lambda) + 1}{1 - \lambda} \tag{4.24}$$

where $\lambda \in (0, 1)$ is a user specified parameter. Storey et al. (2004) show that by replacing the N in equation 4.23 with $\hat{N}_0$, the procedure controls FDR when a weak dependence holds.

Benjamini et al. (2006) propose an alternative 2-step procedure. In the first step, apply the Benjamini and Hochberg (1995) procedure at nominal level $\alpha' = \dfrac{\alpha}{1+\alpha}$, and let R be the number of rejections. If $R = 0$ or $R = N$, the procedure stops and we reject none or all of the null hypotheses, respectively. If $0 < R < N$, we set the number of true null hypothesis to be $\hat{N}_0 = N - R$. In the second step, perform the Benjamini and Hochberg (1995) procedure and replace the N in equation 4.23 with $\hat{N}_0$. Authors prove that the procedure controls FDR under independency. They also use simulation to show that the methods is still valid under positive dependency.

The above mentioned methods do not consider the dependence structure of test statistics. Yekutieli and Benjamini (1999) suggest a procedure of controlling FDR that incorporate information about the dependence structure, but the procedure requires the test statistics under the null hypotheses are independent of those under the alternative. Troendle (2000) also proposes a procedure to control FDR and exploit the dependence structure. The procedure assumes that the joint distribution of statistics has a symmetric multivariate t-distribution.

In this study, we employ the Romano et al. (2008) bootstrap procedure to control the FDR, which exploits the dependence structure of test statistics. The approach relaxes the assumptions in Yekutieli and Benjamini (1999), and Troendle (2000), but relies on the exchangeability assumption.

The idea of Romano et al. (2008) procedure is the following. Suppose we have test statistics T_i, $i = 1, \cdots, N$, for the hypothesis testing H_i, $i = 1, \cdots, N$, the critical value for H_i is c_i, and a greater value of T_i indicates a rejection. Note that we may take $T_i = -p_i$, where p_i is the p-value of H_i. For any step-down procedure based on a set of critical values $\{c_i, \cdots, c_N\}$, we have:

$$\begin{aligned}
\mathrm{FDR}_p &= E\left(\frac{R_{1|0}}{\max(1,R)}\right) &(4.25)\\
&= \sum_{r=1}^{N}\frac{1}{r}E(R_{1|0}|R=r)\Pr\{R=r\} &(4.26)\\
&= \sum_{r=1}^{N}\frac{1}{r}E(R_{1|0}|R=r) &(4.27)\\
&\quad \times\Pr\{T_{(N)}\geq c_N,\cdots,T_{(N-r+1)}\geq c_{N-r+1},T_{(N-r)}<c_{N-r}\}
\end{aligned}$$

where $T_{(N-r)} < c_{N-r}$ is understood to be true when $r = N$. Suppose N_0 is the number of true null hypotheses. Denote $T_{r:n}$ be the r-th largest test statistic of the statistics $T_1, \cdots, T_n$. When $n = N_0$, $T_{r:N_0}$ is the r-th largest statistic among the true null hypotheses. Romano et al. (2008) prove that all false hypothesis will be rejected with a probability tending to 1 under weak assumptions. Then, we have:

$$\begin{aligned}
\mathrm{FDR}_p &= \sum_{r=N-N_0+1}^{N}\frac{r-(N-N_0)}{r} &(4.28)\\
&\quad \times\Pr\{T_{N_0:N_0}\geq c_{N_0},\cdots,T_{N-r+1:N_0}\geq c_{N-r+1},T_{N-r:N_0}\leq c_{N-r}\}
\end{aligned}$$

where the event $T_{N-r:N_0} \leq c_{N-r}$ is understood to be true when $r = N$.

In order to control the FDR, equation 4.28 should be bounded above by α asymptotically.

1. Consider $N_0 = 1$, equation 4.28 is simplified to

$$\mathrm{FDR}_p = \frac{1}{N}\Pr\{T_{1:1}\geq c_1\} \tag{4.29}$$

   c_1 should be chosen as the smallest value that ensures the FDR is bounded above by α.

2. After determining $c_1, \cdots, c_{j-1}$, consider $N_0 = j$, equation 4.28 is simplified to

$$\begin{aligned} \text{FDR}_p &= \sum_{r=N-j+1}^{N} \frac{r-(N-j)}{r} \\ &\quad \times \Pr\{T_{j:j} \geq c_j, \cdots, T_{N-r+1:j} \geq c_{N-r+1}, T_{N-r:j} \leq c_{N-r}\} \end{aligned} \tag{4.30}$$

c_j should be chosen as the smallest value for which FDR is bounded above by α.

3. When $N_0 = N$, equation 4.28 simplifies to

$$\text{FDR}_p = \Pr\{T_{N:N} \geq c_N\} \tag{4.31}$$

and c_N is the smallest value to ensure FDR is bounded above by α.

Romano et al. (2008) propose to use bootstrap test statistics $T_1, \cdots, T_n$ to find the appropriate critical values $c_1, \cdots, c_N$.

4.5 Long Memory Testing in Mixed Panels: Bootstrap Algorithm

We now outline the bootstrap algorithm of applying the Shimotsu and Phillips (2005, 2006) fractional integration tests under the Romano et al. (2008) multiple testing framework. Suppose we have a panel $y_{i,t}$, $i = 1, \cdots, N$, $t = 1, \cdots, T$. We want to test the unit root null hypothesis $H_{0,i} : d_i = 1$ against the fractional integration alternative $H_{1,i} : d_i < 1$, and meanwhile control the FDR at $\alpha = 0.05$.

1 For each cross-sectional unit i, apply the Shimotsu and Phillips (2005, 2006) tests and obtain the estimator $\hat{d}_i$. Collect the test statistics T_i, $i = 1, \cdots, N$. Order the test statistics from most significant to least significant, label the corresponding unit being tested.

2-5 Same to steps 2-5 in the previous algorithm in Section 4.3.

6 Apply the Shimotsu and Phillips (2005, 2006) tests unit by unit in the bootstrap panel and collect the test statistics T_i^*, $i = 1, \cdots, N$.

7 Repeat step 3 to step 6 B times.

8 Compute the critical values $\{c_1, \cdots, c_N\}$ recursively by solving equation 4.28 on page 113.

9 Compare the ordered test statistics in step 1 to the ordered critical values in step 8 sequentially. Reject the most significant null hypothesis if $T_{(1)} < c_1$. If the first statistic is rejected, compare the second statistic to the second critical value, reject the null hypothesis if $T_{(2)} < c_2$. Carry on this sequentially testing process until the first non-rejection is found. The units for which the statistics are rejected are deemed as fractionally integrated series and others are deemed as unit root series. The proportion of unit root processes and long memory processes is calculated accordingly.

We employ the data dependent block size $b = \lfloor 1.75T^{1/3} \rfloor$, as in Palm et al. (2011). As pointed out by Romano et al. (2008), Moon and Perron (2012), the bootstrap is employed to examine the dependence structure of the test statistics. The procedure is subject to the curse of dimensionality, as it requires N computations of up to N dimensional distribution. When N is large, the N dimensional distribution is dubious and the computational burden can be large.

4.6 Monte Carlo Simulations

In this section, we perform a simulation study on a number of mixed panels consist of unit root processes and fractionally integrated processes. We first apply our Fisher-type test described in Section 4.3 on a set of mixed panels, size and power of the test are examined. We then apply our multiple testing procedure on mixed panels. We want to check whether the FDR is controlled successfully, and analyse the estimator of the fraction of true null hypothesis (unit root processes), and see whether the procedure can classify the true null and true alternative correctly.

4.6.1 Data Generating Process

We consider the following data generating process of a panel $y_{i,t}$:

$$y_{i,t} = (1-L)^{-d_i} u_{i,t} \tag{4.32}$$

$$= \sum_{k=0}^{t-1} \frac{(d_i)_k}{k!} u_{i,t-k} \tag{4.33}$$

where $u_{i,t}$ is a zero mean white noise process, $(d_i)_k = \dfrac{\Gamma(d_i+k)}{\Gamma(d_i)} = (d_i)(d_i+1)\cdots(d_i+k-1)$, $\Gamma(\cdot)$ is the gamma function. $d_i = 1$ for the cross-sectional units under the null hypothesis, and $d_i < 1$ for the units under the alternative hypothesis. The cross-sectional dependence of the test statistics is introduced by the covariance matrix of error terms $\Omega = E(u'_{i,t}u_{i,t})$.

In order to be consistent with our simulation study in Chapter 3, we choose the time series dimension T from the set {50, 100, 200}, and choose the cross-sectional dimension from set {10, 30, 50}. We consider the fraction of true null hypotheses, which is the proportion of unit root processes, to be $\theta = \{0,\ 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9,\ 1\}$. When $\theta = 0$, all cross-sectional units are fractionally integrated will $d_i < 1$, $i = 1, \cdots, N$. When $\theta = 1$, all units are unit root processes with $d_i = 1$, $i = 1, \cdots, N$. When $0 < \theta < 1$, we set the first $\theta \cdot N$ units to be unit root processes with $d_i = 1$, $i = 1, \cdots, \theta \cdot N$, and the last $(1-\theta)\cdot N$ units to be fractionally integrated processes with $d_i < 1$, $i = \theta \cdot N + 1, \cdots, N$.

We consider two different values of d_i for the fractionally integrated units. We first set $d_i = 0.3$, $i = \theta \cdot N + 1, \cdots, N$, which is in the stationary region. We then consider $d_i = 0.7$, $i = \theta \cdot N + 1, \cdots, N$, which is in the nonstationary region. Similar to Hlouskova and Wagner (2006), we consider three different cases of the covariance matrix of errors. The first one is independent case, $\Omega_0 = I(N)$, where I is identity matrix. The second case is cross-sectional dependent case with constant covariance Ω_{CC}, and the third case is cross-sectional dependent case with a covariance matrix in Toeplitz form Ω_{TP}, where

$$\Omega_{CC} = \begin{bmatrix} 1 & \omega & \cdots & \omega & \omega \\ \omega & 1 & \cdots & \omega & \omega \\ \vdots & \vdots & & \vdots & \vdots \\ \omega & \omega & \cdots & 1 & \omega \\ \omega & \omega & \cdots & \omega & 1 \end{bmatrix}$$

and

$$\Omega_{TP} = \begin{bmatrix} 1 & \omega & \omega^2 & \cdots & \omega^{N-1} \\ \omega & 1 & \omega & \cdots & \omega^{N-2} \\ \omega^2 & \omega & 1 & \cdots & \omega^{N-3} \\ \vdots & \vdots & \vdots & & \vdots \\ \omega^{N-1} & \omega^{N-2} & \omega^{N-3} & \cdots & 1 \end{bmatrix}$$

The value of ω is set to be 0.5. Note for the independent case and constant covariance case, the position of unit root processes does not matter, but for the Toeplitz covariance matrix, the position of unit root processes affects the magnitude of covariance of statistics under the null and alternative. When the true null statistics are next to each other (like in our DGP), the covariance is high among the null statistics. We denote the the DGPs corresponding to the three cases of covariance matrices to be DGP1, DGP2, and DGP3.

4.6.2 Simulation Results for Fisher Test

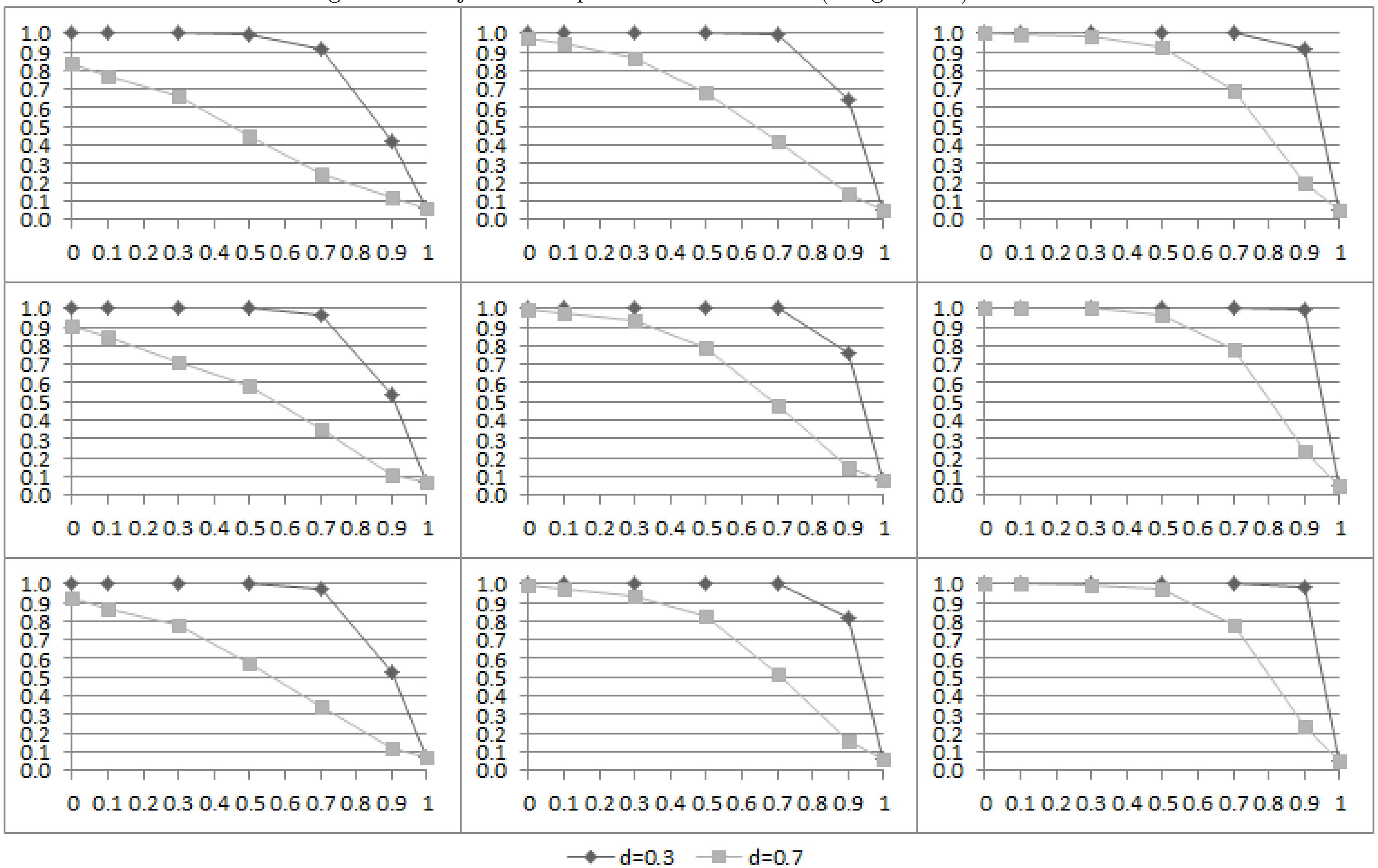
In our simulation experiment, we choose $\alpha = 0.65$ in Shimotsu and Phillips (2005, 2006) tests, where α is a parameter decides m ($m = T^{\alpha}$), which controls the number of frequencies included in the local likelihood. In each individual test, p-value is calculated from standard normal distribution. Number of bootstrap is set to be 500. Level of significance of Fisher statistic is 0.05. Number of replications of Monte Carlo simulation is set to be 1000. All calculations are performed in Gauss 10.

For each combination of T, N, d, θ, and covariance matrix, we apply our Fisher testing procedure using LW, ELW, FELW, and FELWd tests. We calculate the rejection frequencies over the replications. When $\theta = 1$, all units are unit root processes, rejection frequencies are sizes of our test. When $\theta < 1$, panels are mixed and some units are long-memory processes, rejection frequencies are powers of our test. Results are provided in Appendix C.1, Table C.1 on page 330 to Table C.3 on page 332, corresponding to DGP1, DGP2, and DGP3, respectively. In order to provide some representative results in main text, we plot the rejection frequencies of Fisher test when feasible exact local Whittle (FELW) estimator is employed, and panels are generated with DGP2 (constant covariance). In Figure 4.2, we use a 3×3 graph to show the results. Three columns are T=50, T=100, T=200, and three rows are N=10, N=30, N=50. Horizontal axis is the proportion of units under the unit root null hypothesis, vertical axis is the rejection frequency.

We start from looking at the size property of our Fisher test. The magnitude of cross-sectional dependence has little impact on size property. Size is slightly reduced with greater value of T. When FELW estimator is employed, our test is slightly over-sized when $T = 50$, ranging between 0.06 to 0.08, and is correctly sized when $T = 200$. When other estimators (LW, ELW, FELW with detrend data) are employed, our test is under-sized when $T \geq 100$, ranging between 0.01 to 0.03 in most cases. The impact of this size distortion is not severe, as when T is large, the power of test is high, so the size distortion does not affact the power of test too much.

We now look at the power property of our Fisher test. We use Figure 4.2 as an example. Generally speaking, the test is powerful and sensitive to a small proportion of units under the alternative hypothesis, especially when the value of d_i under alternative hypothesis is not close the value under the null. For example, in a panel with $T = 100$, $N = 30$, and $d_i = 0.3$ under alternative, power of test is over 0.7 when only 10% units are under alternative ($\theta = 0.9$), and power is 1 when 30% units are under alternative ($\theta = 0.7$). Power of test is increased with both T and N, and the gain from greater T is more significant. Some points are noteworthy. First, power of test is reduced with greater magnitude of cross-sectional dependence. In our DGP's, power is highest when units are independent (DGP1), and is lowest when units have constant covariance (DGP2). Second, patterns of relationship between powers and null proportion are similar when different estimators are employed. Third, Fisher test using LW and FELW estimator is slightly

Figure 4.2: Rejection Frequencies of Fisher Test (using FELW) in DGP2



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is rejection frequency.

Panels are generated by DGP2 with constant covariance, $d_i = 0.3$ and $d_i = 0.7$ are values of d_i under the alternative hypothesis.

more powerful than others, and the power is lowest when FELWd is employed.

4.6.3 Simulation Results for Multiple Testing Procedure

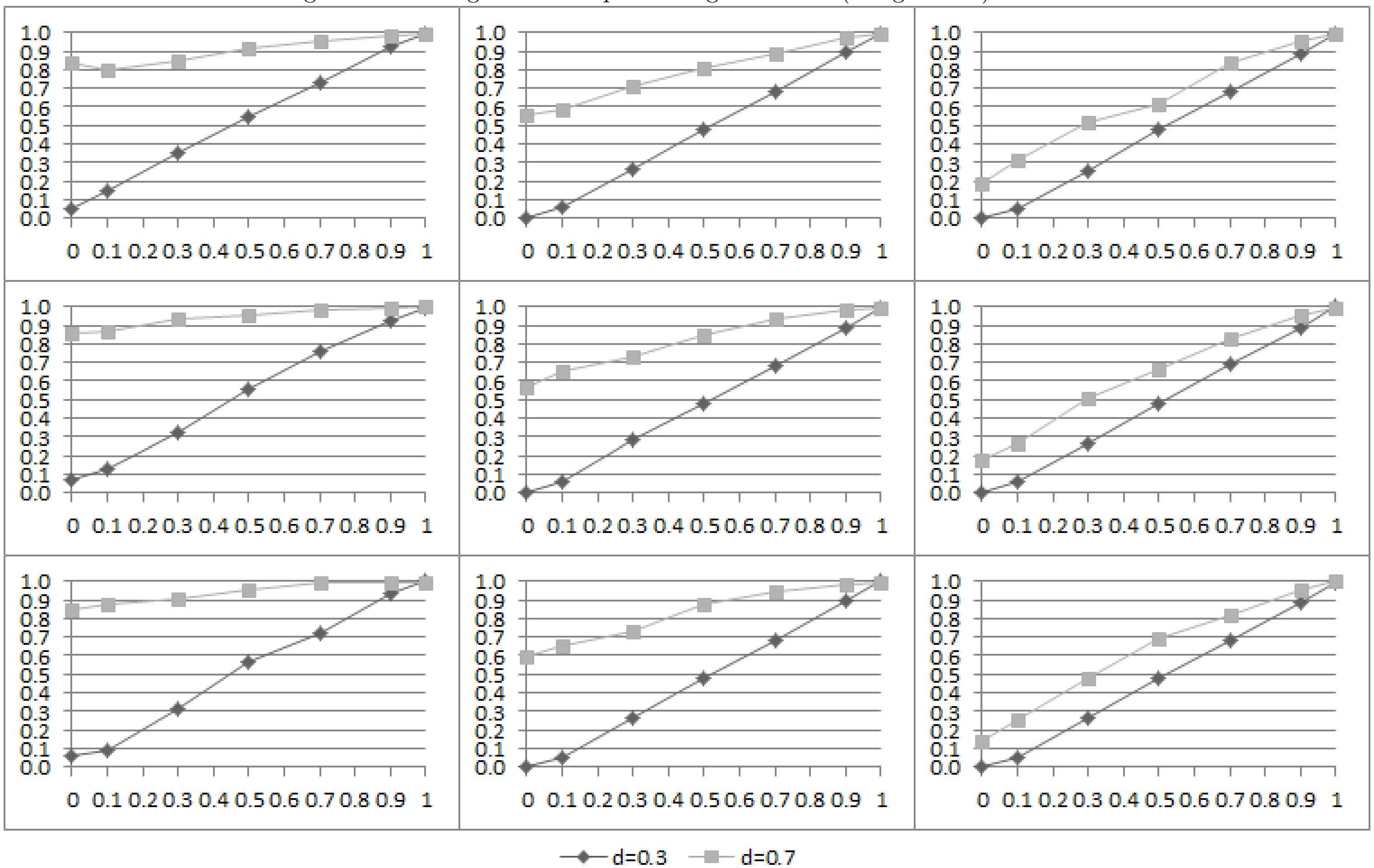
In our simulation experiment, we choose $\alpha = 0.65$ in Shimotsu and Phillips (2005, 2006) tests, where α is a parameter decides m ($m = T^{\alpha}$). The number of replications of Romano et al. (2008) bootstrap is set to be 500. The FDR is to be controlled at 0.05. The multiple testing procedure is computationally demanding, since each replication of bootstrap involves optimisation calculation for N individual tests and the Romano et al. (2008) bootstrap procedure requires N computations of up to N dimensional distribution. Number of replications of Monte Carlo simulation is set to be 100. All calculations are performed in Gauss 10.

For each combination of T, N, d, θ, and covariance matrix, we apply our multiple testing procedure using LW, ELW, FELW, and FELWd tests. We calculate the simple average of the estimated proportion of unit root processes, $\hat{\theta}$, over the replications, the standard deviation of $\hat{\theta}$ over the replications, the false discovery rate (FDR), and the proportion of false non-rejection (denote FNR). Multiple testing procedures that control the FWE or FDR usually suffer from the problem of low power, that is, they are too conservative to reject the null hypothesis, in order to avoid false rejections. The proportion of false non-rejection can be seen as a measurement of power: a more powerful procedure should have lower proportion of false non-rejection.

The results of our multiple testing procedure using LW test are reported in Appendix C.2, Table C.4 on page 334 to Table C.6 on page 336, corresponding to DGP1, DGP2, and DGP3. Similarly, Table C.7 on page 337 to Table C.15 on page 345 report the results of our multiple testing procedure using ELW, FELW, and FELWd tests. In order to provide some representative results in main text, we plot the average $\hat{\theta}$ of our procedure using FELW estimator under DGP2 (constant covariance). Again, we use a 3×3 graph to show the possible power gain from T and N. Results are provided in Figure 4.3. Three columns are $T = 50$, $T = 100$, $T = 200$, and three rows are $N = 10$, $N = 30$, and $N = 50$. Horizontal axis is the proportion of units under the unit root null hypothesis (θ), vertical axis is average $\hat{\theta}$.

The first important result from our simulation experiment is that, in general, the bootstrap multiple testing procedure successfully control the FDR at 5% in all data generating processes.

Figure 4.3: Average $\hat{\theta}$ of Multiple Testing Procedure (using FELW) in DGP2



Note: Three columns corresponding to $T = 50$, $T = 100$, and $T = 200$, respectively. Three rows corresponding to $N = 10$, $N = 30$, and $N = 50$, respectively. Horizontal axis is nonstationary proportion θ, vertical axis is average $\hat{\theta}$.

Panels are generated by DGP2 with constant covariance, $d_i = 0.3$ and $d_i = 0.7$ are values of d_i under the alternative hypothesis.

Although the FDR is slightly above 5% in some cases, this may come from the fact that the number of replications in our simulation is small. Second, the FDR is robust to different cross-sectional covariance matrices, which shows that the dependence structure can be taken into account correctly by bootstrap. However, power of our testing procedure is reduced with greater cross-sectional dependence. Third, the time series dimension T plays a crucial role in the performance of the procedure, an increase in T would increase the power of tests and hence increase the accuracy of the estimation of $I(1)$ and $I(d)$, $d < 1$ proportions. The value of N has little impact on the power of tests, but a greater value of N slightly reduces the standard deviation of $\hat{\theta}$. Fourth, when $d = 0.3$, the procedure is very powerful. Most tests only slightly over estimate the $I(1)$ proportion when $T = 50$, indicating the high power of tests. When $T \geq 100$, tests even slightly under estimate the $I(1)$ proportion, and the FNR is very close to 0. In other words, the procedure successfully controls the FDR at 5% and meanwhile has power close to 1, every true alternative hypothesis is rejected. However, when $d = 0.7$, the power is less encouraging. For example, as shown in Figure 4.3, the procedure has little power when $T \leq 100$. When $T = 100$, $N = 30$, and true proportion of $I(1)$ processes is 0 ($\theta = 0$), the procedure tells us that, on average, the proportion of $I(1)$ processes is 0.66. When $T = 200$, the power is significantly increased, and the upward bias of $\hat{\theta}$ is relatively small. Using the same example, when $T = 200$, $N = 50$, and $\theta = 0$, the procedure results in an average $\hat{\theta}$ to be 0.14. Fifth, the simulation results show that, under our data generating process, procedures using LW and FELW are slightly more powerful than procedures using ELW and FELWd. And the procedure using FELWd has the lowest power among the four univariate tests.

4.7 Empirical Application

Most PPP empirical studies employ unit root tests or cointegration tests. When univairate unit root tests are employed, the unit root null hypothesis usually can not be rejected, indicating the failure of PPP, see Froot and Rogoff (1995) for a review. However, it is pointed out that the failure of supporting the PPP may come from the low power of univariate unit root tests or cointegration tests. To overcome this problem, time series with longer time span are used, see Frankel (1986), Edison (1987), Froot et al. (1995) and Lothian and Taylor (1996).

An alternative approach of testing PPP employs panel unit root tests, in order to increase the power of univariate unit root tests. To name a few, Frankel and Rose (1996), Papell (1997), Parsley and Wei (1996). The first generation of panel unit root tests assume cross-sectional independence and may have size distortion when dependence is present, hence evidence in favour of PPP may be misleading. Some resent empirical studies employ the second generation panel unit root tests, which take dependence into account, see for example, Wagner (2008).

Another alternative approach that has been considered is the fractional integration tests, which extends the unit root testing framework. For example, Cheung and Lai (2001), Achy (2003) find long-memory but mean reversion in real exchange rates in the floating period. Okimoto and Shimotsu (2010) find that the persistence of real exchange rates of 17 industrial counties decline in their two sub-samples, but the unit root null can not be rejected in both time periods. Aloy et al. (2011) use real exchange rate from 78 countries and find that the null of unit root can not be rejected in the vast majority of the real exchange rates. In order to take the cross-sectional dependence into account, Caporale et al. (2013) apply fractionally integrated vector autoregressive model (FIVAR) on four real exchange rates. The null hypothesis that $d = 1$ is rejected for all real exchange rates.

In this section, we use the same data set in Chapter 2 and Chapter 3, which is the disaggregated price indices among Chinese cities. We briefly repeat the data description here. Monthly price indices are collected for 15 different goods and services from 36 Chinese cities. We regroup the 15 price indices and there are 2 aggregated price indices, 5 prices in perishable group, 3 prices in nonperishable group, and 5 prices in services group. The 15 price indices have same time span, from January 2003 to March 2011. Shanghai is used as the numeraire city and for each price index, relative price $q_{i,t}$ is calculated between city i and Shanghai at time t as: $q_{i,t} = p_{i,t} - p_{s,t}$, where $p_{i,t}$ and $p_{s,t}$ are the log price index in city i and in Shanghai at time t, respectively. Therefore, 15 balanced panels of relative price $q_{i,t}$ are constructed with $T = 99$ and $N = 35$. Details of the dataset are given in section 2.2.

We apply the multiple testing method proposed in this Chapter, which controls the FDR and meanwhile takes the cross-sectional dependence into account. The multiple testing procedure is applied on each of the 15 panels: in each panel, the value of d for each unit (city) is firstly

Table 4.1: Cross-sectional Average of $\hat{d}$ in 15 Panels

Product Name	LW	ELW	FELW	FELWd
CPI	0.47	0.42	0.53	0.30
Food	0.40	0.37	0.47	0.29
Perishables				
Meat	0.61	0.60	0.66	0.56
Eggs	0.50	0.48	0.53	0.45
seafood	0.56	0.54	0.61	0.52
Fresh Vegetables	0.12	0.11	0.14	0.00
Fresh Fruits	0.28	0.27	0.30	0.15
Nonperishables				
Grain	0.70	0.71	0.75	0.71
Tobacco and Wine	0.77	0.62	0.71	0.53
Clothing	0.68	0.63	0.68	0.61
Services				
Recreation Facility	0.85	0.90	0.92	0.86
Health Care	0.66	0.62	0.67	0.59
Traf. and Comm.	0.83	0.65	0.78	0.61
Cul. and Edu. Exp.	0.68	0.57	0.64	0.49
Residence	0.73	0.63	0.75	0.66

estimated, and then units are classified into $I(1)$ and $I(d)$, $d < 1$. The value of m in Shimotsu and Phillips (2005, 2006) tests is taken to be $m = T^{0.65}$. Number of bootstrap is set to be 2000.

First, we estimate the order of fractional integration using local Whittle, exact local Whittle, feasible exact local Whittle and feasible exact local Whittle on demean, detrend data. For each of the 15 panel, the estimation of d is obtained for each cross-sectional unit, results are shown in Appendix C.3. Table C.16 on page 347 to Table C.19 on page 350 report the estimation results for LW, ELW, FELW, and FELWd, respectively. In order to have a general picture of the results, we calculate the simple average of $\hat{d}$ across units within each panel, ave$(\hat{d}) = \frac{1}{N}\sum_{i=1}^{N}\hat{d}_i$. Results are shown in Table 4.1, first column is the price indices, second column to fifth column report the results by LW, ELW, FELW, and FELWd, respectively.

As shown in Table 4.1, for the two aggregate price indices, CPI and Food, strong evidence shows that the order of fractional integration is less than 0.5 (FELW estimator on CPI is the only exception), indicating stationary behaviour of the relative prices. Both stationary and

Table 4.2: Simple Average of $\hat{d}$ within Each Group

Product Group	LW	ELW	FELW	FELWd
Perishables	0.41	0.40	0.45	0.34
Nonperishables	0.72	0.65	0.72	0.61
Services	0.75	0.67	0.75	0.64

nonstationary behaviour can be found in the tradable goods within the perishables group. The order of fractional integration is highest for meat and lowest for fresh vegetables. In both nonperishables group and services group, strong evidence is found in favour of nonstationary behaviour. The $\hat{d}$ is smallest for cultural and educational expense and largest for recreation facility.

A general view can be given by calculating the simple average of $\hat{d}$ for all relative prices within each group of goods and services. As shown in Table 4.2, the average $\hat{d}$ of the perishables group ranges between 0.34 and 0.45, indicating the stationarity of relative prices on average. The average $\hat{d}$ of the nonperishables group and services group range from 0.61 to 0.72, and from 0.64 to 0.75, respectively. The order of fractional integration is slightly higher in services group than in the nonperishables group.

Next, we apply the Romano et al. (2008) bootstrap procedure to perform hypothesis testing in order to take cross-sectional dependence into account and meanwhile control the FDR. We test the null hypothesis that a cross-sectional unit is a unit root process against the alternative that it is a long memory process, $H_{0,i}: \ d_i = 1$ vs $H_{1,i}: \ d_i < 1$. Detailed classifications for each city in each panel are reported in Appendix C.3. Table C.20 on page 351 to Table C.23 on page 354 report the results for LW, ELW, FELW, and FELWd, respectively.

We calculate the proportion of units that the null are rejected in each panel, that is, the proportion of $I(d)$, $d < 1$ units in each panel. The results are reported in Table 4.3. For the two aggregate price indices, all units in the panel are long memory processes, indicating a strong rejection of unit root null hypothesis. For goods in the perishables group, the long memory proportion is estimated to be 100% by almost all tests, similar to the results for aggregate price indices, indicating a strong rejection of unit root null. Results for goods in nonperishables group are mixed, the long memory proportion for Grain is relatively small, but the proportion

Table 4.3: Proportion of $I(d)$, $d < 1$, Units in 15 Panels

Product Name	LW	ELW	FELW	FELWd
CPI	1.00	1.00	1.00	1.00
Food	1.00	1.00	1.00	1.00
Perishables				
Meat	1.00	1.00	0.66	0.66
Eggs	1.00	1.00	1.00	1.00
seafood	1.00	1.00	1.00	1.00
Fresh Vegetables	1.00	1.00	1.00	1.00
Fresh Fruits	1.00	1.00	1.00	1.00
Nonperishables				
Grain	0.43	0.34	0.20	0.20
Tobacco and Wine	0.11	1.00	1.00	1.00
Clothing	0.54	1.00	0.51	0.51
Services				
Recreation Facility	0.00	0.00	0.00	0.00
Health Care	0.51	0.71	0.54	0.54
Traf. and Comm.	0.14	0.69	0.40	0.40
Cul. and Edu. Exp.	0.43	0.86	0.91	0.91
Residence	0.46	0.80	0.46	0.46

Table 4.4: Simple Average of $I(d)$, $d < 1$, Proportion within Each Group

Product Group	LW	ELW	FELW	FELWd
Perishables	1.00	1.00	0.93	0.93
Nonperishables	0.36	0.78	0.57	0.57
Services	0.31	0.61	0.46	0.46

for Tobacco and Wine is large. The long memory proportion is smallest for services. The proportion is 0 for Recreation Facility, indicating a random walk processes for all units in the panel.

In order to gain a general view of the long memory proportions, we calculate the simple average of the proportions for panels in each group. Results are reported in Table 4.4. The average long memory proportion is highest in the perishables group, with a value very close to 1. The average proportion ranges between 0.36 and 0.78 in nonperishables group, and between 0.31 and 0.61 in services group.

Table 4.5: Average $I(0)$ Proportion for Each Group

Product Group	RW05	MP12	$BSQT$	$IBSQT$
Perishables	0.26	0.41	0.38	0.31
Nonperishables	0.01	0.01	0.17	0.08
Services	0.05	0.07	0.08	0.06

Note that for the goods and services in nonperishables and services groups, the average $\hat{d}$ is around 0.7 (see Table 4.2) , and the dimension of the panels is that $T = 99$ and $N = 35$. Our simulation results show that, when the value of d is close to unity, for example $d = 0.7$, the procedure has low power if $T \leq 100$. That is to say, our procedure fails to reject the null hypothesis for some units while d is less than but close to unity. In this circumstance, the estimator of long memory proportion is downward biased. Therefore, for the nonperishables and services group, we would expect that the actual long memory proportion is higher than the estimated proportions, indicating more evidence against the unit root null hypothesis.

It is interesting to compare the results obtained here with the results in Chapter 3, where a set of procedures for mixed panels are employed to estimate the proportion of $I(1)$ and $I(0)$ series, and classify the units into $I(1)$ and $I(0)$. We repeat some of the results in Chapter 3 here: the results by procedure with bootstrap critical values (Romano and Wolf (2005), Moon and Perron (2012), and Smeekes (2011)), the average proportion of $I(0)$ units within each group is given in Table 4.5. The reason to choose these procedures is that they share the same null hypothesis with our panel long memory procedure. All these tests use bootstrap method to mimic the same cross-sectional dependence(same data set) and calculate the corresponding critical values. Our panel long memory procedure results are especially comparable to the results obtained by Moon and Perron (2012) (Table 4.5, column 3), as both procedures apply the Romano et al. (2008) bootstrap method to control the FDR: Moon and Perron (2012) employ ADF unit root test and we employ Shimotsu and Phillips (2005, 2006) long memory tests.

A comparison of results in Table 4.4 and Table 4.5 immediately shows that, for both sets of tests, the proportion of alternative hypothesis ($I(0)$ and $I(d)$, $d < 1$) is highest in the perishables group, and lowest in the services group, and the proportion in nonperishables group is very close to services group. A significant difference is noted, for all three groups, the alternative proportion is much higher in our panel long memory testing procedure. That is to say, by employing the fractional integration tests into panel data, we find stronger evidence against the unit root null hypothesis. For example, by using the Romano and Wolf (2005), Moon and Perron (2012), and Smeekes (2011) procedures, over 50% of units are classified as $I(1)$ in perishables group, and around 90% of units are classified as $I(1)$ in nonperishables and services groups. By employing the panel fractional integration tests, less than 10% of units are classified as $I(1)$ in perishables group, and less than 50% of units are classified as $I(1)$ in nonperishables and services groups. Therefore, we shed some light on our intra-China PPP study, which employs panel unit root tests and recently developed procedures that classify the units into $I(1)$ and $I(0)$ groups. Even though more evidence shows that PPP fails to hold among vast majority of cities when we classify units into $I(1)$ and $I(0)$, the unit root null hypothesis can be rejected in favour of long memory processes.

4.8 Conclusion

In this Chapter, we propose two different fractional integration tests in panel data, which extend time series fractional integration tests.

A Fisher-type fractional integration test is first proposed, which combines the p-values from individual long-memory tests. It tests the null hypothesis that $d_i = 1$ for all i, against the heterogeneous alternative that $d_i < 1$ for some i. To account for cross-sectional dependence, we bootstrap the critical values of Fisher test, as suggested by Maddala and Wu (1999).

We then extend our analysis into mixed panel data by getting use of the concept of false discovery rate (FDR) in multiple testing literature. The aim of our panel long memory testing procedure is to classify each of the cross-sectional unit into null ($I(1)$) and alternative ($I(d)$, $d < 1$) hypothesis, and meanwhile control the FDR. We apply the Romano et al. (2008) bootstrap procedure, which takes cross-sectional dependence into account and controls the FDR at a pre-specified level, say 5%.

For both Fisher-type test and multiple testing procedure, we employ the local Whittle fractional integration test and exact local Whittle tests proposed by Shimotsu and Phillips (2005, 2006) as the individual long memory tests.

We perform simulation experiments to look at the performance of our testing procedures. The Fisher-type test is correctly sized and has high power, especially when d is in the stationary region. It is shown that the FDR is successfully controlled by using our multiple testing procedure. When the value of d under alternative is relatively small ($d = 0.3$), the procedure is very powerful, even if $T = 50$. However, when the value of d under alternative is close to the unity null ($d = 0.7$), a powerful procedure requires T to be 200.

We then apply our panel long memory multiple testing procedure in an empirical study. Following Chapter 2 and Chapter 3, we use the data set of 15 balanced panels of relative price indices among 35 Chinese cities. Contrast to the results in Chapter 3, where strong evidence is found in favour of unit root processes if we classify units into $I(1)$ and $I(0)$, results in this Chapter show evidence against the unit root null hypothesis when we classify units into $I(1)$ and $I(d)$, $d < 1$.

Chapter 5

Conclusion

This thesis discusses some issues on unit root testing in panel data. It contributes to the literature in several aspects. First, it extends the existing intra-China PPP studies by employing panel unit root tests that take cross-sectional dependence into account. Second, it considers the case of mixed panel, where both stationary units and non-stationary units are present. It simulates the performance of a battery of panel unit root tests in mixed panels. Third, it employs a battery of recently proposed procedures designed for mixed panels and simulates their performances under different DGPs. Fourth, it extends panel unit root tests into panel long-memory tests. Two different panel fractional integration tests are proposed, with and without taking mixed panel into account. Fifth, it further deepens the intra-China PPP study by considering mixed panel and possible fractional integration under the alternative.

This thesis starts from an empirical study on the convergence of Chinese city price indices. Previous studies on intra-China price convergence employ panel unit root tests that assume cross-sectional independence and find that PPP holds in the vast majority of goods/services prices and half-life ranges between 1.66 and 2.44 months. It is shown that tests assuming cross-sectional independence suffer from size-distortion when units are dependent. We therefore employ the so called second generation panel unit root tests, which account for the cross-sectional dependence. Tests employed are Pesaran (2007b) $CIPS$ test, Moon and Perron (2004) t_a^* and t_b^* tests, Breitung and Das (2008) t_{rob} and t_{gls} tests, and Bai and Ng (2004) PANIC test. Our research finds little evidence to support the purchasing power parity for the nonperishable goods and mixed evidence to support the PPP for services. Therefore, we suggest that caution should be exercised when interpreting the results from the previous studies and PPP may fail to hold in nonperishable and service groups if the dependence is taken into account.

Next issue discussed in this thesis is the presence of mixed panel (or heterogeneous panel). We perform a large scale simulation study to examine the performance of a battery of panel unit root tests without or with taking mixed panels into account. For data generating processes, we consider different sources of unit root, number of common factors, and variance of errors in common factor component, and most importantly, proportion of units under null and alternative hypothesis. We first look at the size and power of panel unit root test employed in the previous empirical study. Simulation evidence shows that, for some tests, a rejection of the null hypothesis can be caused by either a small proportion of units under the alternative hypothesis, or that all units are under the alternative. In empirical studies, this may have very different economic meanings. Hence, a rejection of unit root null can hardly be interpreted in empirical studies. We then show the performance of procedures designed for mixed panels: Chortareas and Kapetanios (2009), Smeekes (2011), Romano and Wolf (2005), Moon and Perron (2012) and Ng (2008). We focus on whether these procedures can correctly estimate the proportion of stationary/nonstationary units, and classify each unit into null group and alternative group. Experiment results show that the satisfactory performance of these procedures require large sample size. When the underlying DGP can not be correctly specified, which is always the case in empirical studies, procedures using bootstrapped critical values show much better performance. An empirical study on intra-China PPP is performed, and results sheds some light on existing literature of PPP within China, in which PPP is found for vast majority of goods and services. Our results show that, on average, only a small proportion of units are stationary in panels, and the rejection of panel unit root tests is the result of a small proportion of stationary units. Therefore, when a rejection is found in panel unit root tests, we should be careful drawing the conclusion that purchasing power parity is suggested in such panel.

The third issue discussed is the fractional integration test in panel data. We propose two different types of panel fractional integration test. A Fisher-type test is first proposed, which combines p-value from individual long memory test statistics and tests the null hypothesis that $d_i = 1$ for all i, against the heterogeneous alternative that $d_i < 1$ for some i. A multiple testing procedure is then proposed, which controls the false discovery rate (FDR) and cross-sectional dependence, and classify units into $I(1)$ group and $I(d)$, $d < 1$ group. Simulation study shows that our tests perform well, especially when d_i under alternative is in stationary region. The multiple testing procedure is applied on our intra-China PPP data set, results show strong evidence against the unit root null hypothesis when we classify units into $I(1)$ and $I(d)$, $d < 1$. These result conflict with testing results we obtained in previous Chapter, where strong evidence is found against the unit root null when time series are classified into $I(1)$ and $I(0)$.

There are still limitations in this thesis. In the empirical study of Purchasing Power Parity, the choice of numeraire city may introduce a factor, which could introduce additional dependence by construction. Pesaran et al. (2009) suggest to avoid this problem by using a pairwise procedure. The impact of the choice of numeraire city and testing results by employing the pairwise procedure is left for further research.

The selection of ADF lag length in panel unit root tests in empirical studies remains a problem. Basically speaking, size problem should gain more attention than power in a panel framework, as panel tests tend to have high power but being incorrectly sized. Im et al. (2003) find that using the information criteria may lead to size distortion. They conduct simulation exercises where information criteria such as AIC and SBIC are employed to determine the ADF lag order. The tests show significant degree of size distortion. This is due to the fact that the information criteria are "not sufficiently conservative", in terms of selecting too few lags. However, if the "step down" procedure starting from P_{max} is employed to determine the lag order, the P_{max} is often selected when N is large. However the P_{max} is not data dependent and the choice of ADF lag length ends up being driven by arbitrary choices. In this study, the choices of ADF lag length in panel unit root tests are consistent with original papers, hence no uniform rule of lag selection is applied. Further research of lag selection within panel framework can be undertaken to improve the properties of panel unit root tests.

The selection of the number of common factors is also a limitation. In this thesis, the number of common factors in empirical studies are set as an arbitrary number. Due to the fact that the information criteria by Bai and Ng (2002) tends to select the largest number of common factors imposed, the selection of this number is still a problem in empirical studies, as the testing result is not always robust to the largest number of common factors. Selection of the number of common factors is left for further research.

Appendix A

Price Indices Convergence Among Chinese Cities

A.1 Cities Placemarks

A.2 Econometric Analysis

Table A.1: I(0) and I(1) Classification by CADF test

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	1
2	0	1	1	0	1	0	1	1	0	0	1	1	1	0	1
3	0	0	1	0	1	0	0	1	0	1	0	1	0	1	1
4	1	1	0	1	1	0	0	1	1	1	1	1	1	1	1
5	0	0	0	1	1	0	0	0	0	1	1	0	1	0	1
6	1	0	1	1	1	0	1	1	1	1	1	0	1	0	1
7	0	1	1	1	1	0	0	1	1	1	0	0	0	0	1
8	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1
9	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1
10	1	0	0	1	1	0	0	1	1	1	0	1	1	1	1
11	1	1	1	1	0	0	1	1	1	0	1	1	1	0	1
12	0	0	1	1	1	1	0	1	1	1	1	1	1	0	1
13	0	0	1	0	1	0	1	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1	1	1	1	1	1	0	1
15	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1
16	0	1	1	0	0	1	1	1	1	1	1	1	1	0	1
17	0	0	0	1	1	0	1	1	1	1	1	0	1	1	1
18	1	1	1	1	1	1	0	1	1	1	1	0	1	0	1
19	0	1	1	1	0	0	1	1	1	1	1	1	1	0	1
20	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1
21	0	0	1	1	1	0	1	1	1	1	1	0	1	1	1
22	1	1	1	1	1	0	1	1	0	1	1	1	1	0	1
23	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0
24	0	1	1	1	1	1	0	1	0	0	1	1	1	0	1
25	1	1	1	0	1	0	0	1	1	1	1	1	1	0	0
26	1	0	1	1	1	0	1	1	1	1	0	1	1	1	1
27	1	0	1	1	1	0	0	1	1	1	1	1	0	1	0
28	1	1	1	1	0	0	1	1	0	1	1	0	0	1	1
29	0	1	1	1	1	0	1	1	1	1	0	1	1	0	1
30	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1
31	1	1	1	1	1	0	1	1	1	1	1	0	1	1	1
32	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
33	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1
34	0	1	1	1	1	1	1	0	0	1	1	1	1	0	1
35	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Appendix B

On Properties of Panel Unit Root Tests in Mixed Panels and Performances of Mixed Panel Tests: A Large Scale Simulation Study

B.1 Simulation Results for Panel Unit Root Tests

In this section of Appendix, we report the rejection frequency of the panel unit root tests, corresponding to section 3.4.2 on page 66.

In order to facilitate the illustration, we label DGP 1 with low cross-sectional dependence and one common factor as DGP1a, DGP 1 with low cross-sectional dependence and two common factors as DGP1b, DGP 1 with high cross-sectional dependence and one common factor as DGP1c, DGP 1 with high cross-sectional dependence and two common factors as DGP1d. DGP2a to

DGP2d, DGP3a to DGP3d are used similarly. DGP4a is DGP 4 with no cross-sectional dependence and iidN idiosyncratic component, DGP4b is DGP 4 with no cross-sectional dependence and $ARMA(1,1)$ idiosyncratic component. DGP4c is DGP 4 with low dependence and iidN idiosyncratic component, DGP4d is DGP 4 with low dependence and $ARMA(1,1)$ idiosyncratic component. DGP4e and DGP4f are 4 with high dependence and iidN and $ARMA(1,1)$ idiosyncratic component, respectively.

Monte Carlo simulation results for panel unit root tests for DGP1a to DGP4f are reported in Table B.1 on page 150 to Table B.18 on page 167. When the number of common factor is one in the data generating process, we report the results for Pesaran (2007b) $CIPS$ test statistic, Moon and Perron (2004) t_a^* and t_b^* test statistics, Breitung and Das (2008) t_{rob} and t_{gls} test statistics and Bai and Ng (2004) $P_{\hat{e}}$ and $ADF_{\hat{F}}$ statistics. When two common factors are present in the data generating process, we report MQ_c and MQ_f statistics instead of $ADF_{\hat{F}}$. The rejection frequency for each combination of $\{T,\ N,\ \theta\}$ is reported for each statistic. For MQ_c and MQ_f statistics, we report the frequency that the tests choose the correct number of common stochastic trends. The construction of t_{gls} statistic requires $T > N$, so when $T = N = 50$, the results of t_{gls} are labelled with N/A. Note that, if $I(1)$ common factor(s) is assumed (DGP 1 and DGP 2), when $N = 10$ and $\theta = 0.1$, the nonstationary common factor(s) affects only one unit through nonzero factor loading(s) and therefore it is actually an idiosyncratic component.

The results in Table B.1 on page 150 to B.4 on page 153 are obtained for DGP 1, where a unit root is present in both common factors and idiosyncratic components. For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls}, $P_{\hat{e}}$ test statistics, the columns with $\theta = 1$(all units are nonstationary) report the size of the tests, and columns with $\theta = \{0,\ 0.1,\ \cdots,\ 0.9\}$ report the power of the tests. For the t_{rob} and t_{gls} homogeneous tests with the alternative that all units are stationary, however, a mixed panel with $\theta = \{0.1,\ \cdots,\ 0.9\}$ is somewhere between the null and alternative hypothesis. For $ADF_{\hat{F}}$ statistic, columns with $\theta = \{0.1,\ \cdots,\ 0.9,\ 1\}$ report the size of the test. When $\theta = 0$, there is no common factor in the panel, and $ADF_{\hat{F}}$ is not an appropriate test here, but we still extract one common factor and perform the $ADF_{\hat{F}}$ test and deem this as the "power" of it. For MQ_c and MQ_f statistics, the numbers reveal the frequency that the tests can choose the correct number of common stochastic trends. When $\theta = 0$, or $N = 10$ and $\theta = 0.1$, the number of $I(1)$ common factors is zero, for other cases, the number of $I(1)$ common factors is two.

For Pesaran (2007b) $CIPS$ test, when a single factor is present, the magnitude of factor loadings has little impact on the size and power of tests. $CIPS$ test shows nearly no size distortion, even when T and N are small. The power of test increases with both T and N, and decreases with nonstationary proportion θ. The test is very sensitive to the stationary units in panel. For $\theta = 0.7$, the rejection frequency is 0.97 and 0.96 for {T, N}={200, 10} or {100, 30} respectively. For $T = 200$ and $N = 50$, the rejection is 0.67 when $\theta = 0.9$ and 1.00 for θ smaller than 0.9. When two common factors are present in the data generating process, the test is over-sized and higher cross-sectional dependence leads to more severe size distortion. The size distortion increases with N and is stable with T. For low dependence panels, the size is around 0.08 when $N = 10$ and increases to 0.14 when $N = 50$. For high dependence panels, the size distortions increase to 0.12 and 0.21 when $N = 10$ and $N = 50$, respectively. The power of $CIPS$ is similar when two $I(1)$ common factors are present. For Moon and Perron (2004) t_a^* and t_b^* tests, when $N = 10$, the tests are more powerful with high cross-sectional dependence or two common factors, when $N = 30$ or 50, the powers are robust for all data generating processes. Both tests show slightly size distortions. When $N = 10$, t_a^* is under-sized with small T and over-sized with large T, ranging from 0.03 when $T = 50$ to 0.10 when T=200, $t_b^*$is slightly over-sized. When $N = 30$ or 50, the size of t_a^* increases from 0.01 for $T = 50$, to 0.02 for $T = 100$ and to 0.05 for $T = 200$, t_b^* is generally correctly sized. The power of the tests are similar and gradually increases with the value of θ, for example, when $T = 200$ and $N = 50$, the power is less than 0.10 for $\theta = 0.9$, around 0.50 for $\theta = 0.5$, and slightly below 0.90 for $\theta = 0.3$. For Breitung and Das (2008), both t_{rob} and t_{gls} tests are over-sized. The t_{rob} test is slightly over-sized for most cases, ranging from 0.04 to 0.10, and there is no clear patten of the size changing with T and N. The size distortion for t_{gls} test increases with N and decreases with T, for example, for high dependence and one common factor in DGP, the sizes are 0.07, 0.18, 0.34 if $T = 100$ and $N = 10$, 30, 50, respectively. When $N = 50$, if we increase T from 100 to 200, the size decreases from 0.34 to 0.18. The t_{gls} is more powerful than t_{rob}, partly due to its severe size distortion. The power of t_{rob} tests grows very gradually when θ gets smaller, and for most cases, t_{rob} test has the lowest power among the tests we employed. Higher cross-sectional dependence slightly increases the size distortion of t_{rob} and reduce the power of t_{rob}. High dependence also increases the size distortion of t_{gls} and increases the power if it when θ is close to 1, this power gain should be a consequence of more severe size distortion. When two common factors are present, it has little impact on the size but reduces the power of of t_{rob}, it increases the size distortion and the power of t_{gls} when θ is close to 1, which is similar to the case of high dependence. The $P_{\hat{e}}$ test of Bai and Ng (2004) is slightly over-sized and the distortion is reduced with greater value of T, when $T = 50$, it size ranges from 0.10 to 0.14, and the size is reduced to around 0.08 if $T = 200$. $P_{\hat{e}}$ test is very powerful against the alternatives, for $T = 100$ and $N = 30$, the power is around 0.60 when $\theta = 0.9$ and close to 1.00 when $\theta = 0.7$, for $T = 200$ and $N = 50$, the power is around 0.90 when $\theta = 0.9$ and equals 1.00 if θ is smaller than 0.9. The magnitude of dependence has little impact on the size/power of $P_{\hat{e}}$. When two common factors are present, the size of $P_{\hat{e}}$ remains unchanged, and the power of $P_{\hat{e}}$ is slightly reduced, but still remains high, compare to other tests. The $ADF_{\hat{F}}$ in tables reveals the size of the test, except the cases with $\theta = 0$, where no common factor is present in panels and one common factor is extracted and tested for unit root. The $ADF_{\hat{F}}$ is over-sized when T is small and θ is close to 0, ranging from 0.09 to 0.19 for $T = 50$. When T is increased to 200, $ADF_{\hat{F}}$ is only slightly over-sized for most cases. N has little effect on the size of $ADF_{\hat{F}}$. Higher cross-sectional dependence reduce the size distortion of $ADF_{\hat{F}}$, for example, when $T = 200$, the test is correctly sized even for $\theta = 0.1$, that is, the method can extract and test the common factor correctly even when the factor affects only 10% of the units. The MQ_c and MQ_f tests select the correct number of $I(1)$ common factors for most cases. This is, however, because that the maximum number of possible $I(1)$ factors is set at two, and the tests tend to select the maximum number possible, see Gengenbach et al. (2010). When no common factor is present, the tests have no ability to choose the correct number when $T = 50$. When T is increased to 100, MQ_c can choose zero $I(1)$ factors with 75% probability, and MQ_f can choose the correct number with 65% probability. When $T = 200$, both tests can choose zero $I(1)$ common factors with 100% probability. Higher dependence can slightly increase the ability of detecting the correct number of common stochastic trends.

For DGP 2, where unit root(s) is present only in the common factor component (that is, $I(1)$ common factor component and $I(0)$ idiosyncratic component), the simulation results are reported in Table B.5 on page 154 to Table B.8 on page 157. For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls}, tests, the column with $\theta = 1$ is the size of tests, columns with $\theta = \{0,\ 0.1, \cdots,\ 0.9\}$ are the power of tests. For $P_{\hat{e}}$ test, all numbers are the power of test. For $ADF_{\hat{F}}$ test, column with $\theta = 0$ is deemed as the power of the test, columns with $\theta = \{0.1, \cdots,\ 0.9,\ 1\}$ are the size of the test. Results for MQ_c and MQ_f is the frequency to select the correct number of $I(1)$ factors.

The $CIPS$ test of Pesaran (2007b) shows significant size distortion. When a single factor is present, the size is one for most cases, with the only exception that when low dependence is present and $N = 10$, the size is between 0.90 and 1.00. When two common factors are present, the size distortion is reduced to around 0.50 if $T = 50$ and $N = 10$. The size distortion increases with both T and N, and when $T = 200$ and $N = 50$, the size is increased to 0.87 and 0.69 with low or high dependence, respectively. The power of $CIPS$ is one almost every due to the size distortion. Similar results can be seen for t_a^* and t_b^* of Moon and Perron (2004). The size is very close to 1 when $T = 50$, and for greater value of T, the size is one for both tests in call cases. Misleading high power is also gained for t_a^* and t_b^*. The t_{rob} test of Breitung and Das (2008) shows smaller size distortion. The size is distortion is robust to both T and N, but gets smaller when higher dependence is present or two common factors are used in DGP. The size is around 0.33 for low dependence, one factor panel, 0.18 for low dependence, two factor panel, 0.15 for high dependence, one factor panel, and 0.10 for high dependence, two factor model. Interestingly, the power of t_{rob} seems to be robust of T and N and grows very slowly when θ is getting smaller, for example, for panels with high dependence and two factors, the power is only about 0.40 when $\theta = 0.1$. When $\theta = 0$, the power then jump to over 0.90 when $T = 50$ and 1.00 when $T > 50$. The size distortion of t_{gls} increases with both T and N, but is reduced with high cross-sectional dependence and two common factors. For panels with high dependence and two factors, the size is still 0.47 when $T = 50$ and $N = 10$, and is increased to 0.82 when $T = 200$ and $N = 50$. Similar to t_{rob}, the power of t_{gls} grows slowly when θ is getting smaller, but the power is still high due to the large size distortion. The $P_{\hat{e}}$ test of Bai and Ng (2004) is very powerful, with power close to unity when $T = 50$ and $N = 10$, and power is 1.00 for all other cases. The performance of $ADF_{\hat{F}}$, MQ_c and MQ_f tests is similar to those in DGP 1. When $T = 50$, the size ranges from 0.08 to 0.19 for low dependence panel and from 0.10 to 0.13 for high dependence panel. When $T = 100$ or $T = 200$, $ADF_{\hat{F}}$ is only slightly over-sized. MQ_c and MQ_f can always choose two common factors with moderate $\{T, N\}$ and $\theta \geq 0.3$. when $\theta = 0$, the frequency that the tests can successfully choose zero common factor increases with T. Higher dependence can improve the performance of the tests, especially when $\theta = 0.1$.

Results in Table B.9 on page 158 to Table B.12 on page 161 are obtained for DGP 3, where panels are generated with $I(0)$ common factor component and $I(1)$ idiosyncratic component. For $CIPS$, t_a^*, t_b^*, t_{rob}, t_{gls}, $P_{\hat{e}}$ tests, the column with $\theta = 1$ is the size of tests, columns with $\theta = \{0, 0.1, \cdots, 0.9\}$ are the powers. Results for $ADF_{\hat{F}}$ reveal the power of the test. Results for MQ_c and MQ_f show the frequency that the tests choose zero as the number of $I(1)$ common factors.

The $CIPS$ test of Pesaran (2007b) has size distortion under the data generating process. If one common factor is present in the panel and cross-sectional dependence is low, the test is slightly over-sized when $N = 10$. When $N = 30$, the size of the test is 0.02 to 0.03. When N is increased to 50, the size is further reduced to 0.01. With high dependence, the size is reduced to 0.02 when $N = 10$ and 0.00 when $N = 30$ and 50. For panels with two common factors, the test is generally over-sized and the size increases with T. When cross-sectional dependence is low, the size decreases with N, ranging from 0.03 to 0.17. When the dependence is high, the size distortion is higher, ranging from 0.08 to 0.31. The power of $CIPS$ is still high, but smaller than the power in DGP 1. For Moon and Perron (2004) t_a^* test, if one common factor is present, the size increases with T and decreases with N and the test is generally under-sized. The magnitude of dependence has little effect on the size. t_a^* is only over-sized when $N = 10$, and under-sized when $N = 30$ and 50. t_b^* is slightly over-sized when $N = 10$ and correctly sized when $N = 30$ and 50. If two common factors are present in the panel and cross-sectional dependence is low, similar results are obtained for t_a^* and t_b^*. However, when the dependence is high, t_a^* is over-sized for most cases, ranging from 0.04 to 0.31, and t_b^* is also over-sized, ranging from 0.08 to 0.04. The power of the two tests are similar and getting higher with both T and N, the power grows gradually as θ gets smaller. The t_{rob} test of Breitung and Das (2008) is severely over-sized. The size distortion increases with both $\{T, N\}$, and when two common factors or high dependence is present. The size of t_{rob} ranges from 0.18 to 0.43 for DGP3a and ranges from 0.70 to 1.00 for

DGP3d. Power of t_{rob} is high due to the size distortion. The size of t_{gls} decreases with T and increases with N, regardless of the magnitude of dependence and number of common factors. When $T = 200$, t_{gls} is only slightly oversize for DGP 3a, at around 0.07. When T is small and N is large, however, the test can be severely over-sized, up to 0.82 for DGP 3d. The power of t_{gls} is somewhere between Moon and Perron (2004) tests and t_{rob} test. The $P_{\hat{e}}$ test of Bai and Ng (2004) is slightly over-sized. The size distortion is robust to N, magnitude of dependence, number of factors, and is reduced with greater T. The size is around 0.10 for $T = 50$, 0.08 for $T = 100$ and 0.07 for $T = 200$. The power of $P_{\hat{e}}$ is the highest among the tests employed and increases with both $\{T, N\}$, and the magnitude of dependence. The number of factors has no effect on the power. For a moderate panel with $T = 100$ and $N = 30$, the power is 0.95 when $\theta = 0.7$, and for a large panel with $T = 200$ and $N = 50$, the power is 0.80 when $\theta = 0.9$ and 1.00 when $\theta \leq 0.7$. Similar to the $CIPS$ test, the powers of $P_{\hat{e}}$ is smaller in DGP 3 than those in DGP 1. The power of $ADF_{\hat{F}}$ increases with T, N, and the magnitude of dependence. The power has a negative relationship with the value of θ. With low dependence, the power is greater than 0.70 when N is as large as 30 and T is as large as 100. With high dependence, the power is greater than 0.90 for a moderate panel with same dimensions. For MQ_c and MQ_f tests, the ability of selecting the correct number of $I(1)$ factors is generally poor. The frequency increases with T and N, but is much more sensitive to the time series dimension T. Similar to $ADF_{\hat{F}}$ test, the frequency has a negative relationship with θ and a positive relationship with the magnitude of dependence. The MQ_c test has better performance than MQ_f test. With low dependence, the tests have satisfactory performance only when $T = 200$ and $N \geq 30$. The probability that the MQ_c test selects the correct number of $I(1)$ factors is over 0.65 when $T = 200$ and $N = 30$, and increases to over 0.81 when T=200 and $N = 50$. With high dependence, results are acceptable when $T = 100$ and $N \geq 30$, or $T = 200$ and $N \geq 10$, where the probability is generally greater than 0.60. The probability is approaching 1.00 when $T = 200$ and $N \geq 30$.

Results for DGP 4 are reported in Table B.13 on page 162 to Table B.18 on page 167, where panels are generated by equations 3.37 and 3.38 with single $I(0)$ common factor and $I(0)$ idiosyncratic component and nonstatonarity comes from autoregressive root $\delta_i = 1$. For $CIPS$, t_a^*, t_b^*, t_{rob} and t_{gls} tests, column with $\theta = 1$ reports size of tests, columns with $\theta = \{0, 0.1, \cdots,$

0.9} report the power of tests. For $P_{\hat{e}}$ and $ADF_{\hat{F}}$ tests, all columns reveal the power of tests.

For Pesaran (2007b) $CIPS$ test, when cross-sectional units are independent, $CIPS$ is correctly sized. The power increases with T and N. When cross-sectional dependence is present in panels, $CIPS$ is slightly over-sized when T, N is small, ranging from 0.08 to 0.06. High cross-sectional dependence and $ARMA(1,1)$ idiosyncratic component reduce the size distortion, and the test is correctly sized with $ARMA(1,1)$ idiosyncratic component. The presence of dependence increases the power of test, while the presence of $ARMA(1,1)$ idiosyncratic component slightly reduce the power. $CIPS$ test is still very powerful, with high dependence and iidN idiosyncratic component, the power is 0.81 when $T = 100$, $N = 30$ and $\theta = 0.9$, for a large panel with $T = 200$, $N = 50$, the power is 0.99 when $\theta = 0.9$. The power dominates other tests everywhere under the DGP. For Moon and Perron (2004) t_a^* test, the size is more sensitive to N, and the correct size requires N to be fairly large. When $N = 10$, the size of t_a^* grows with T, t_a^* is under-sized when $T = 50$, ranging from 0.02 to 0.04, and it becomes over-sized when $T = 200$, ranging 0.06 to 0.10. When $N = 30$ or 50, t_a^* is under-sized with small T and correctly sized with $T = 200$. The size of t_b^* is sensitive to N but has no pattern with T. When $N = 10$, t_b^* is slightly over-sized with iidN idiosyncratic component, ranging from 0.05 to 0.09, and slightly under-sized with $ARMA(1,1)$ idiosyncratic component, ranging from 0.03 to 0.05. The magnitude of dependence has little impact on the size of t_b^*. The powers of t_a^* and t_b^* are similar, increase gradually with T, N and cross-sectional dependence, irrelevance with the idiosyncratic component. For a moderate panel ($T = 100$ and $N = 30$) and high dependence, the power is around 0.40 when $\theta = 0.5$ and 0.60 when $\theta = 0.1$ and 1.00 when $\theta = 0$. For t_{rob} test of Breitung and Das (2008), when no dependence is present, the size decreases with N and increases with T, and the test is slightly under-sized if T is small and N is large, and correctly sized for other cases. With cross-sectional dependence, the size of t_{rob} is not sensitive to T, N, magnitude of dependence. The size ranges from 0.07 to 0.12 with iidN idiosyncratic component and ranges from 0.07 to 0.10 with $ARMA(1,1)$ idiosyncratic component. t_{gls} test is generally over-sized and the size distortion increases with N, magnitude of dependence, decreases with T. The presence of $ARMA(1,1)$ slightly reduces the size distortion. For panels with high dependence and $ARMA(1,1)$ idiosyncratic component, the size of t_{gls} is 0.35 when $T = 50$ and $N = 30$, and is reduced to 0.10 when $T = 200$ and $N = 30$. The power of t_{rob} and t_{gls} is lower than the power of $CIPS$ but greater than the power of t_a^* and t_b^*. With cross-sectional dependence, t_{rob} is more powerful than t_{gls} when $N = 10$, and t_{gls} is more powerful than t_{rob} when $N = 30$ or 50, but the power is partly due to the severe size distortion of t_{gls} when N is large. For a moderate panel ($T = 100$ and $N = 30$) and high dependence, the power of t_{rob} is 0.33 and the power of t_{gls} is 0.41 when $\theta = 0.5$, the powers of t_{rob} and t_{gls} are 0.80 and 0.86, respectively, when $\theta = 0.1$. Without cross-sectional dependence, the performance of Bai and Ng (2004) $P_{\hat{e}}$ is satisfactory. The size decreases with T and increases with N, when $T = 50$ and $N = 10$, the size is 0.18, when T is increased to 200, the size is 0.07, which is only slightly over-sized. The power is very high, only smaller than the power of $CIPS$ test. For $T = 100$ and $N = 30$, the power of $P_{\hat{e}}$ is 0.42 when $\theta = 0.9$ and 0.96 when $\theta = 0.7$. The presence of $ARMA(1,1)$ idiosyncratic component slightly increase the power of $P_{\hat{e}}$. When cross-sectional dependence is present in panels, the size property remains similar, but the power shrinks dramatically. The power of $P_{\hat{e}}$ increases with N, but does not increase with greater value of T. The power is worsened with higher dependence. The power does not jump to 1.00 for $\theta \leq 0.7$ when sample is large, instead, it grows gradually with θ getting smaller, and the power is very similar to those of t_{rob} and t_{gls} tests. For our moderate panel ($T = 100$ and $N = 30$) and high dependence, the power is 0.45 when $\theta = 0.5$, and 0.97 when $\theta = 0.1$. For $ADF_{\hat{F}}$ test, the power is very low for all cases and the presence of cross-sectional dependence worsens the power. The power of $ADF_{\hat{F}}$ decreases with N, increases with T when θ is close to zero, but decreases with T when θ is close to one. The power of $ADF_{\hat{F}}$ when $T = 100$ and $N = 30$ is below 0.20 for $\theta \geq 0.5$ and can hardly reject the unit root null hypothesis.

Some general conclusions can be summarized. First, in DGP 2 where the idiosyncratic component is stationary and the unit root is present in the common factor alone, $CIPS$ of Pesaran (2007b), t_a^* and t_b^* of Moon and Perron (2004), t_{rob} and t_{gls} of Breitung and Das (2008) are invalid. t_{rob} is moderately over-sized, ranging between 0.08 to 0.16 with high dependence and 0.17 to 0.36 with low dependence. Size of t_{gls} ranges between 0.47 to 0.96. Sizes of $CIPS$, t_a^* and t_b^* are distorted to 1.00 for most cases. Therefore, the high power of these tests is misleading. Second, the $CIPS$ test performs reasonably well in DGP 1 when one common factor is present, and in DGP 4. The test is correctly sized and very sensitive to a small proportion of stationary units. For a moderate panel ($T = 100$ and $N = 30$), the power is close to 1.00 when $\theta = 0.7$. For a large panel ($T = 200$ and $N = 50$) with high dependence in DGP 4, the rejection frequency jump from 0.05 when $\theta = 1$ to 0.99 when $\theta = 0.9$. The high power property of $CIPS$ makes it a natural candidate to the Sequential Panel Selection Method proposed in Chortareas and Kapetanios (2009). In DGP 3 with unit root in idiosyncratic component alone, the size is downward distorted. When two common factors are present in the panel, the size is upward distorted. Third, satisfactory size property of t_a^* test appears when $T \geq 100$ and $N \geq 30$. When $N = 10$, t_a^* is under-sized with small T and over-sized with large T. When $N = 30$ or 50, t_a^* is under-sized with small T and correctly sized with large T. t_b^* is generally correctly sized when $N \geq 30$, and slightly over-sized when $N = 10$. The powers of t_a^* and t_b^* are similar, increase with both $\{T, N\}$ and grow gradually as θ getting smaller. Fourth, as pointed out by Breitung and Das (2008), t_{rob} is invalid in DGP 2 and DGP 3, t_{gls} is invalid in DGP 2. The size of t_{rob} is approaching 1.00 with high dependence and more factors in DGP 3. t_{gls} is over-sized in most cases, the size distortion is worsened with greater N, cross-sectional dependence, number of common factors, but is reduced with greater T. For example, in DGP 1 with high dependence and one factor, the size is 0.32 when $T = 50$ and $N = 30$, and is reduced to 0.07 when $T = 200$ and $N = 10$. Therefore, t_{gls} is more suitable for panels with large T and small N. The powers of t_{rob} and t_{gls} behave similarly and grow slowly with smaller value of θ, which is similar to the case of t_a^* and t_b^*. Fifth, the $P_{\hat{e}}$ and $ADF_{\hat{F}}$ test of Bai and Ng (2004) perform very well in DGP 1 to DGP 3. The size of $P_{\hat{e}}$ is slightly above 0.12 when $T = 50$ and is reduced to a value between 0.06 and 0.08 when $T \geq 100$. $P_{\hat{e}}$ test is sensitive to the stationary units and therefore the power of $P_{\hat{e}}$ is very high even when θ is close to one, for example, in DGP 1 with low dependence and one factor, for a large panel ($T = 200$ and $N = 50$), the rejection frequency of $P_{\hat{e}}$ jump from 0.07 when $\theta = 1$ to 0.90 when $\theta = 0.90$. This property is similar to $CIPS$ test, but the power is even higher. The $ADF_{\hat{F}}$ is only slightly over-sized if $T \geq 100$. When N is small and θ is close to 0, the size of $ADF_{\hat{F}}$ is upward distorted, this is mainly due to the fact that the estimation of common factor requires N to be large, especially when the factor only affects a small proportion of units. The power of $ADF_{\hat{F}}$ is satisfactory, for example, for a moderate panel ($T = 100$ and $N = 30$), the power is over 0.70 with low dependence and over 0.90 with high dependence. In DGP 4 with cross-sectional dependence, the $P_{\hat{e}}$ is slightly over-sized, but the power behaves differently, it increases gradually with smaller θ, which is similar to t_a^*, t_b^*, t_{rob} and t_{gls}. $ADF_{\hat{F}}$ has little power to reject the null hypothesis, unless θ is very close to 0. The MQ_c and MQ_f tend to choose the maximum possible $I(1)$ factors, as pointed out by Gengenbach et al. (2010). In DGP 3 with two common factors, MQ_c is more powerful than MQ_f and tends to select the correct number of $I(1)$ factors, which is zero, more frequently than MQ_f. A satisfactory performance, however, requires the panel to be relatively large, for example, $T = 200$ and $N \geq 30$.

A very general but important conclusion for this section is that, if we apply panel unit root tests on mixed panels in order to draw an extreme conclusion: the panel is either stationary or nonstationary, conflicting results may be obtained, although the performance depends on the underlying DGP.

Table B.1: DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
					50							100							200			
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.99	0.99	0.96	0.72	0.42	0.14	0.06	1.00	1.00	1.00	0.98	0.75	0.23	0.07	1.00	1.00	1.00	1.00	0.97	0.36	0.07
	$t_a^\star$	1.00	0.77	0.42	0.20	0.09	0.04	0.03	1.00	0.89	0.61	0.33	0.16	0.08	0.06	1.00	0.94	0.69	0.41	0.25	0.12	0.09
	$t_b^\star$	1.00	0.76	0.41	0.23	0.11	0.08	0.07	1.00	0.86	0.56	0.31	0.16	0.08	0.06	1.00	0.93	0.63	0.34	0.20	0.10	0.08
10	t_{rob}	0.95	0.76	0.41	0.24	0.13	0.09	0.06	1.00	0.79	0.44	0.24	0.13	0.07	0.07	1.00	0.81	0.45	0.23	0.14	0.09	0.06
	t_{gls}	1.00	0.88	0.57	0.37	0.24	0.10	0.09	1.00	0.91	0.60	0.39	0.19	0.11	0.08	1.00	0.92	0.65	0.39	0.21	0.11	0.07
	$P_{\hat{e}}$	1.00	0.96	0.91	0.82	0.62	0.26	0.10	1.00	0.99	0.98	0.95	0.83	0.37	0.07	1.00	1.00	0.99	0.99	0.93	0.46	0.07
	$\text{ADF}_{\hat{F}}$	0.74	0.34	0.15	0.10	0.10	0.12	0.11	0.97	0.32	0.09	0.08	0.08	0.08	0.08	1.00	0.30	0.08	0.06	0.07	0.05	0.06
	CIPS	1.00	1.00	1.00	0.97	0.64	0.17	0.07	1.00	1.00	1.00	1.00	0.96	0.31	0.07	1.00	1.00	1.00	1.00	1.00	0.60	0.06
	$t_a^\star$	1.00	0.87	0.45	0.13	0.05	0.01	0.01	1.00	0.96	0.68	0.32	0.14	0.04	0.02	1.00	0.99	0.80	0.46	0.20	0.08	0.06
	$t_b^\star$	1.00	0.88	0.51	0.23	0.11	0.05	0.05	1.00	0.95	0.67	0.35	0.18	0.07	0.04	1.00	0.98	0.77	0.44	0.19	0.08	0.07
30	t_{rob}	0.98	0.85	0.50	0.27	0.14	0.07	0.04	1.00	0.85	0.53	0.28	0.14	0.08	0.06	1.00	0.88	0.52	0.29	0.15	0.07	0.07
	t_{gls}	1.00	0.99	0.87	0.60	0.39	0.24	0.19	1.00	0.99	0.86	0.58	0.32	0.16	0.08	1.00	1.00	0.89	0.63	0.31	0.14	0.08
	$P_{\hat{e}}$	1.00	1.00	1.00	0.99	0.92	0.43	0.12	1.00	1.00	1.00	1.00	0.99	0.62	0.08	1.00	1.00	1.00	1.00	1.00	0.81	0.08
	$\text{ADF}_{\hat{F}}$	0.73	0.22	0.09	0.12	0.11	0.12	0.09	0.98	0.15	0.09	0.07	0.07	0.07	0.08	1.00	0.09	0.07	0.06	0.06	0.06	0.06
	CIPS	1.00	1.00	1.00	1.00	0.75	0.17	0.05	1.00	1.00	1.00	1.00	0.99	0.38	0.05	1.00	1.00	1.00	1.00	1.00	0.67	0.05
	$t_a^\star$	1.00	0.93	0.52	0.17	0.05	0.01	0.01	1.00	0.99	0.76	0.37	0.15	0.04	0.02	1.00	1.00	0.88	0.48	0.22	0.07	0.05
	$t_b^\star$	1.00	0.95	0.61	0.29	0.11	0.04	0.03	1.00	0.99	0.77	0.43	0.21	0.07	0.05	1.00	1.00	0.86	0.47	0.23	0.09	0.05
50	t_{rob}	0.97	0.86	0.52	0.29	0.13	0.07	0.07	1.00	0.89	0.54	0.28	0.16	0.10	0.06	1.00	0.90	0.57	0.30	0.17	0.09	0.06
	t_{gls}				N/A				1.00	1.00	0.94	0.71	0.47	0.25	0.18	1.00	1.00	0.96	0.73	0.41	0.15	0.10
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.98	0.59	0.13	1.00	1.00	1.00	1.00	1.00	0.79	0.06	1.00	1.00	1.00	1.00	1.00	0.90	0.07
	$\text{ADF}_{\hat{F}}$	0.76	0.19	0.12	0.12	0.12	0.11	0.12	0.97	0.12	0.08	0.08	0.07	0.07	0.06	1.00	0.10	0.06	0.06	0.05	0.06	0.07

Note:

Table B.2: DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	0.99	0.99	0.94	0.74	0.40	0.14	0.09	1.00	1.00	1.00	0.98	0.75	0.25	0.07	1.00	1.00	1.00	1.00	0.95	0.43	0.08
	$t_a^\star$	1.00	0.87	0.46	0.21	0.10	0.04	0.03	1.00	0.95	0.65	0.37	0.16	0.08	0.05	1.00	0.98	0.77	0.51	0.25	0.14	0.10
	$t_b^\star$	1.00	0.86	0.46	0.24	0.15	0.09	0.07	1.00	0.94	0.60	0.35	0.15	0.09	0.06	1.00	0.98	0.72	0.45	0.20	0.11	0.07
	t_{rob}	0.96	0.65	0.32	0.16	0.11	0.08	0.07	1.00	0.68	0.32	0.17	0.10	0.09	0.06	1.00	0.70	0.36	0.17	0.12	0.08	0.07
	t_{gls}	1.00	0.89	0.60	0.35	0.18	0.10	0.09	1.00	0.91	0.63	0.38	0.20	0.09	0.07	1.00	0.91	0.64	0.39	0.17	0.11	0.07
	$P_{\hat{e}}$	1.00	0.94	0.81	0.67	0.49	0.22	0.10	1.00	0.99	0.93	0.86	0.70	0.29	0.10	1.00	1.00	0.97	0.96	0.84	0.42	0.09
	MQ_c	0.04	0.02	0.94	1.00	1.00	1.00	1.00	0.73	0.04	0.90	0.99	1.00	1.00	1.00	1.00	0.08	0.88	1.00	1.00	1.00	1.00
	MQ_f	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.58	0.02	0.95	1.00	1.00	1.00	1.00	1.00	0.05	0.93	1.00	1.00	1.00	1.00
30	CIPS	1.00	1.00	1.00	0.96	0.63	0.22	0.10	1.00	1.00	1.00	1.00	0.93	0.35	0.12	1.00	1.00	1.00	1.00	1.00	0.60	0.11
	$t_a^\star$	1.00	0.85	0.41	0.14	0.03	0.01	0.00	1.00	0.97	0.70	0.33	0.11	0.04	0.02	1.00	0.99	0.84	0.45	0.20	0.07	0.05
	$t_b^\star$	1.00	0.86	0.49	0.24	0.11	0.06	0.03	1.00	0.97	0.70	0.36	0.15	0.07	0.04	1.00	0.98	0.81	0.41	0.19	0.08	0.06
	t_{rob}	0.98	0.71	0.31	0.16	0.11	0.07	0.05	1.00	0.77	0.36	0.20	0.13	0.07	0.06	1.00	0.76	0.40	0.21	0.12	0.10	0.07
	t_{gls}	1.00	1.00	0.84	0.58	0.44	0.29	0.23	1.00	0.99	0.86	0.60	0.33	0.18	0.12	1.00	1.00	0.86	0.58	0.29	0.12	0.08
	$P_{\hat{e}}$	1.00	1.00	0.99	0.98	0.86	0.37	0.13	1.00	1.00	1.00	1.00	0.97	0.51	0.08	1.00	1.00	1.00	1.00	1.00	0.71	0.09
	MQ_c	0.05	0.93	1.00	1.00	1.00	1.00	1.00	0.78	0.75	1.00	1.00	1.00	1.00	1.00	1.00	0.69	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	0.99	1.00	1.00	1.00	1.00	1.00	0.66	0.86	1.00	1.00	1.00	1.00	1.00	1.00	0.77	1.00	1.00	1.00	1.00	1.00
50	CIPS	1.00	1.00	1.00	0.97	0.71	0.27	0.14	1.00	1.00	1.00	1.00	0.96	0.43	0.14	1.00	1.00	1.00	1.00	1.00	0.67	0.13
	$t_a^\star$	1.00	0.89	0.46	0.14	0.03	0.01	0.00	1.00	0.99	0.72	0.33	0.13	0.03	0.01	1.00	1.00	0.86	0.56	0.22	0.07	0.04
	$t_b^\star$	1.00	0.91	0.56	0.27	0.12	0.04	0.05	1.00	0.99	0.75	0.39	0.17	0.05	0.04	1.00	1.00	0.86	0.55	0.23	0.08	0.05
	t_{rob}	0.99	0.75	0.35	0.19	0.12	0.07	0.06	1.00	0.78	0.37	0.21	0.12	0.08	0.05	1.00	0.77	0.35	0.20	0.13	0.08	0.08
	t_{gls}				N/A				1.00	1.00	0.92	0.73	0.50	0.31	0.23	1.00	1.00	0.95	0.71	0.44	0.20	0.11
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.96	0.47	0.12	1.00	1.00	1.00	1.00	1.00	0.68	0.08	1.00	1.00	1.00	1.00	1.00	0.86	0.08
	MQ_c	0.05	0.97	1.00	1.00	1.00	1.00	1.00	0.76	0.92	1.00	1.00	1.00	1.00	1.00	1.00	0.93	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.66	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	1.00	1.00

Table B.3: DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.99	1.00	0.96	0.81	0.43	0.12	0.06	1.00	1.00	1.00	0.98	0.76	0.21	0.06	1.00	1.00	1.00	1.00	0.98	0.42	0.06
	$t_a^\star$	1.00	0.83	0.47	0.24	0.09	0.04	0.03	1.00	0.92	0.62	0.33	0.18	0.09	0.06	1.00	0.97	0.72	0.41	0.23	0.12	0.09
	$t_b^\star$	1.00	0.82	0.46	0.26	0.11	0.07	0.06	1.00	0.91	0.58	0.30	0.16	0.10	0.06	1.00	0.95	0.65	0.36	0.18	0.09	0.07
10	t_{rob}	0.84	0.58	0.28	0.13	0.11	0.07	0.07	0.97	0.60	0.27	0.17	0.11	0.08	0.10	1.00	0.62	0.27	0.16	0.11	0.10	0.08
	t_{gls}	1.00	0.88	0.58	0.36	0.23	0.12	0.08	1.00	0.91	0.63	0.37	0.21	0.10	0.07	1.00	0.91	0.66	0.40	0.20	0.10	0.07
	$P_{\hat{e}}$	1.00	0.97	0.93	0.86	0.63	0.27	0.11	1.00	0.99	0.99	0.96	0.82	0.36	0.09	1.00	1.00	1.00	1.00	0.94	0.48	0.08
	$\text{ADF}_{\hat{F}}$	0.73	0.25	0.12	0.12	0.11	0.11	0.11	0.98	0.22	0.08	0.08	0.07	0.08	0.07	1.00	0.20	0.07	0.06	0.06	0.06	0.07
	CIPS	1.00	1.00	1.00	0.97	0.67	0.16	0.06	1.00	1.00	1.00	1.00	0.98	0.33	0.06	1.00	1.00	1.00	1.00	1.00	0.61	0.04
	$t_a^\star$	1.00	0.91	0.49	0.13	0.05	0.02	0.01	1.00	0.97	0.67	0.31	0.13	0.06	0.03	1.00	0.98	0.81	0.46	0.23	0.06	0.04
	$t_b^\star$	1.00	0.91	0.55	0.23	0.14	0.06	0.04	1.00	0.96	0.67	0.33	0.16	0.08	0.06	1.00	0.98	0.78	0.42	0.23	0.07	0.05
30	t_{rob}	0.83	0.55	0.21	0.12	0.09	0.08	0.08	0.97	0.57	0.23	0.15	0.11	0.08	0.06	1.00	0.60	0.24	0.12	0.12	0.09	0.08
	t_{gls}	1.00	0.99	0.87	0.68	0.46	0.38	0.32	1.00	0.99	0.86	0.63	0.38	0.24	0.18	1.00	0.99	0.88	0.62	0.35	0.16	0.11
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.93	0.46	0.11	1.00	1.00	1.00	1.00	1.00	0.61	0.08	1.00	1.00	1.00	1.00	1.00	0.77	0.07
	$\text{ADF}_{\hat{F}}$	0.77	0.13	0.12	0.11	0.10	0.11	0.10	0.98	0.10	0.07	0.10	0.07	0.06	0.08	1.00	0.07	0.07	0.06	0.07	0.07	0.06
	CIPS	1.00	1.00	1.00	1.00	0.73	0.18	0.05	1.00	1.00	1.00	1.00	1.00	0.37	0.05	1.00	1.00	1.00	1.00	1.00	0.68	0.04
	$t_a^\star$	1.00	0.93	0.53	0.15	0.04	0.02	0.01	1.00	0.99	0.74	0.36	0.13	0.03	0.02	1.00	1.00	0.88	0.53	0.19	0.07	0.04
	$t_b^\star$	1.00	0.94	0.61	0.28	0.13	0.07	0.06	1.00	0.99	0.76	0.41	0.19	0.07	0.05	1.00	1.00	0.87	0.52	0.20	0.09	0.04
50	t_{rob}	0.84	0.54	0.20	0.14	0.11	0.08	0.06	0.97	0.53	0.23	0.15	0.09	0.10	0.08	1.00	0.56	0.24	0.15	0.11	0.10	0.08
	t_{gls}				N/A				1.00	1.00	0.93	0.75	0.55	0.39	0.34	1.00	1.00	0.95	0.75	0.48	0.24	0.18
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.99	0.55	0.11	1.00	1.00	1.00	1.00	1.00	0.77	0.08	1.00	1.00	1.00	1.00	1.00	0.92	0.09
	$\text{ADF}_{\hat{F}}$	0.74	0.12	0.10	0.09	0.11	0.11	0.11	0.98	0.09	0.08	0.07	0.07	0.07	0.08	1.00	0.05	0.05	0.06	0.06	0.06	0.04

Table B.4: DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	0.98	0.99	0.96	0.77	0.39	0.20	0.12	1.00	1.00	1.00	0.98	0.75	0.28	0.11	1.00	1.00	1.00	1.00	0.94	0.45	0.12
	$t_a^\star$	1.00	0.91	0.53	0.24	0.09	0.05	0.03	1.00	0.97	0.73	0.39	0.20	0.09	0.07	1.00	1.00	0.80	0.54	0.27	0.14	0.10
	$t_b^\star$	1.00	0.90	0.53	0.26	0.13	0.09	0.07	1.00	0.96	0.68	0.37	0.19	0.10	0.08	1.00	0.99	0.77	0.48	0.21	0.11	0.08
	t_{rob}	0.89	0.41	0.14	0.11	0.09	0.06	0.08	0.98	0.41	0.16	0.11	0.09	0.06	0.06	1.00	0.43	0.17	0.12	0.10	0.07	0.07
	t_{gls}	1.00	0.88	0.61	0.37	0.21	0.13	0.11	1.00	0.92	0.63	0.40	0.23	0.12	0.07	1.00	0.90	0.65	0.41	0.21	0.09	0.08
	$P_{\hat{e}}$	1.00	0.96	0.86	0.75	0.53	0.22	0.12	1.00	0.99	0.95	0.91	0.75	0.33	0.09	1.00	1.00	0.99	0.98	0.88	0.43	0.08
	MQ_c	0.05	0.01	0.99	1.00	1.00	1.00	1.00	0.75	0.02	0.96	1.00	1.00	1.00	1.00	1.00	0.03	0.95	1.00	1.00	1.00	1.00
	MQ_f	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.67	0.01	0.98	1.00	1.00	1.00	1.00	1.00	0.01	0.97	1.00	1.00	1.00	1.00
30	CIPS	0.99	1.00	1.00	0.93	0.61	0.28	0.19	1.00	1.00	1.00	1.00	0.89	0.42	0.20	1.00	1.00	1.00	1.00	0.98	0.59	0.19
	$t_a^\star$	1.00	0.86	0.46	0.16	0.05	0.02	0.01	1.00	0.97	0.71	0.32	0.13	0.04	0.03	1.00	0.99	0.82	0.47	0.22	0.09	0.05
	$t_b^\star$	1.00	0.87	0.52	0.25	0.13	0.07	0.04	1.00	0.97	0.70	0.34	0.18	0.06	0.06	1.00	0.88	0.79	0.45	0.22	0.09	0.06
	t_{rob}	0.90	0.35	0.15	0.10	0.09	0.08	0.07	0.99	0.35	0.14	0.12	0.08	0.09	0.08	1.00	0.38	0.15	0.12	0.09	0.10	0.10
	t_{gls}	1.00	0.99	0.83	0.63	0.48	0.40	0.35	1.00	0.99	0.86	0.63	0.39	0.24	0.22	1.00	0.99	0.89	0.63	0.38	0.19	0.13
	$P_{\hat{e}}$	1.00	1.00	1.00	0.98	0.88	0.37	0.12	1.00	1.00	1.00	1.00	0.97	0.56	0.08	1.00	1.00	1.00	1.00	1.00	0.69	0.09
	MQ_c	0.03	0.97	1.00	1.00	1.00	1.00	1.00	0.73	0.91	1.00	1.00	1.00	1.00	1.00	1.00	0.90	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.65	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.93	1.00	1.00	1.00	1.00	1.00
50	CIPS	1.00	1.00	1.00	0.94	0.69	0.33	0.20	1.00	1.00	1.00	1.00	0.92	0.44	0.21	1.00	1.00	1.00	1.00	1.00	0.61	0.22
	$t_a^\star$	1.00	0.90	0.51	0.15	0.05	0.02	0.01	1.00	0.99	0.75	0.36	0.11	0.05	0.02	1.00	1.00	0.88	0.53	0.21	0.07	0.04
	$t_b^\star$	1.00	0.91	0.60	0.27	0.15	0.07	0.06	1.00	0.98	0.76	0.41	0.16	0.08	0.05	1.00	1.00	0.87	0.52	0.23	0.09	0.05
	t_{rob}	0.90	0.32	0.15	0.10	0.08	0.07	0.07	0.98	0.37	0.15	0.10	0.10	0.08	0.08	1.00	0.37	0.15	0.10	0.09	0.08	0.09
	t_{gls}				N/A				1.00	1.00	0.91	0.71	0.55	0.41	0.36	1.00	1.00	0.94	0.74	0.50	0.30	0.25
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.96	0.49	0.14	1.00	1.00	1.00	1.00	1.00	0.72	0.09	1.00	1.00	1.00	1.00	1.00	0.87	0.08
	MQ_c	0.04	0.99	1.00	1.00	1.00	1.00	1.00	0.75	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00

Table B.5: DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	1.00	1.00	0.98	0.94	0.93	0.91	0.93	1.00	1.00	1.00	1.00	0.98	0.98	0.97	1.00	1.00	1.00	1.00	1.00	1.00	0.98
	$t_a^\star$	1.00	0.97	0.96	0.95	0.96	0.96	0.96	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.96	0.94	0.93	0.95	0.96	0.96	1.00	0.99	0.99	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
10	t_{rob}	0.95	0.86	0.62	0.47	0.37	0.33	0.31	1.00	0.88	0.63	0.49	0.40	0.35	0.34	1.00	0.87	0.66	0.51	0.39	0.35	0.31
	t_{gls}	1.00	0.97	0.90	0.83	0.80	0.75	0.76	1.00	0.98	0.92	0.88	0.83	0.82	0.81	1.00	0.99	0.94	0.91	0.86	0.85	0.82
	$P_{\hat{e}}$	1.00	0.99	0.99	0.98	0.99	0.98	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ADF$_{\hat{F}}$	0.72	0.45	0.20	0.14	0.12	0.08	0.12	0.97	0.46	0.17	0.10	0.08	0.08	0.08	1.00	0.42	0.12	0.06	0.08	0.07	0.06
	CIPS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_a^\star$	1.00	0.99	0.99	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.99	0.99	0.98	0.98	0.99	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	t_{rob}	0.97	0.88	0.65	0.48	0.39	0.35	0.32	1.00	0.91	0.65	0.52	0.43	0.35	0.28	1.00	0.90	0.67	0.52	0.40	0.35	0.33
	t_{gls}	1.00	1.00	0.96	0.92	0.89	0.86	0.87	1.00	1.00	0.99	0.97	0.95	0.93	0.91	1.00	1.00	0.99	0.97	0.96	0.95	0.95
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ADF$_{\hat{F}}$	0.75	0.27	0.12	0.13	0.12	0.11	0.12	0.98	0.19	0.09	0.07	0.07	0.08	0.08	1.00	0.18	0.06	0.07	0.06	0.06	0.06
	CIPS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_a^\star$	1.00	0.99	0.99	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.99	0.99	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	t_{rob}	0.98	0.89	0.65	0.46	0.40	0.35	0.34	1.00	0.92	0.68	0.49	0.42	0.38	0.36	1.00	0.92	0.66	0.52	0.42	0.38	0.35
	t_{gls}				N/A				1.00	1.00	0.99	0.97	0.96	0.95	0.94	1.00	1.00	1.00	0.99	0.98	0.96	0.96
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ADF$_{\hat{F}}$	0.74	0.19	0.11	0.12	0.10	0.11	0.12	0.98	0.14	0.08	0.08	0.09	0.09	0.07	1.00	0.08	0.05	0.06	0.06	0.07	0.07

Table B.6: DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.98	1.00	0.96	0.87	0.73	0.58	0.53	1.00	1.00	1.00	0.99	0.92	0.77	0.66	1.00	1.00	1.00	1.00	0.98	0.87	0.74
	$t_a^\star$	1.00	0.95	0.89	0.86	0.89	0.87	0.89	1.00	1.00	0.97	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.93	0.86	0.83	0.86	0.86	0.88	1.00	0.99	0.96	0.98	0.98	0.99	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.96	0.69	0.43	0.28	0.23	0.20	0.18	1.00	0.76	0.45	0.30	0.22	0.20	0.19	1.00	0.73	0.45	0.34	0.22	0.22	0.17
10	t_{gls}	1.00	0.94	0.80	0.69	0.64	0.58	0.58	1.00	0.96	0.85	0.76	0.70	0.65	0.63	1.00	0.96	0.87	0.79	0.72	0.70	0.66
	$P_{\hat{e}}$	1.00	0.97	0.94	0.95	0.94	0.96	0.96	1.00	1.00	0.98	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.04	0.04	0.92	0.98	1.00	1.00	1.00	0.70	0.13	0.80	0.97	0.99	1.00	1.00	1.00	0.14	0.75	0.96	1.00	1.00	1.00
	MQ_f	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.56	0.07	0.89	0.98	1.00	1.00	1.00	1.00	0.09	0.82	0.97	1.00	1.00	1.00
	CIPS	1.00	1.00	1.00	0.97	0.87	0.75	0.63	1.00	1.00	1.00	1.00	0.97	0.86	0.76	1.00	1.00	1.00	1.00	1.00	0.93	0.84
	$t_a^\star$	1.00	0.96	0.96	0.96	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.96	0.96	0.97	0.96	0.96	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.98	0.75	0.41	0.29	0.22	0.17	0.18	1.00	0.77	0.44	0.32	0.25	0.22	0.19	1.00	0.81	0.50	0.31	0.27	0.21	0.20
30	t_{gls}	1.00	1.00	0.94	0.86	0.78	0.76	0.76	1.00	1.00	0.97	0.92	0.90	0.86	0.86	1.00	1.00	0.98	0.95	0.90	0.88	0.86
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.04	0.89	1.00	1.00	1.00	1.00	1.00	0.76	0.64	1.00	1.00	1.00	1.00	1.00	1.00	0.61	1.00	0.99	1.00	1.00	1.00
	MQ_f	0.00	0.99	1.00	1.00	1.00	1.00	1.00	0.65	0.77	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	1.00	1.00	1.00	1.00
	CIPS	1.00	1.00	1.00	0.98	0.88	0.77	0.68	1.00	1.00	1.00	1.00	0.98	0.88	0.80	1.00	1.00	1.00	1.00	1.00	0.96	0.87
	$t_a^\star$	1.00	0.98	0.97	0.98	0.98	0.97	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.98	0.98	0.98	0.98	0.98	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.98	0.78	0.43	0.30	0.22	0.20	0.17	1.00	0.81	0.49	0.31	0.26	0.23	0.19	1.00	0.82	0.49	0.32	0.25	0.19	0.19
50	t_{gls}				N/A				1.00	1.00	0.97	0.95	0.90	0.90	0.88	1.00	1.00	0.99	0.97	0.94	0.91	0.90
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.05	0.96	1.00	1.00	1.00	1.00	1.00	0.75	0.86	1.00	1.00	1.00	1.00	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	0.99	1.00	1.00	1.00	1.00	1.00	0.68	0.91	1.00	1.00	1.00	1.00	1.00	1.00	0.90	1.00	1.00	1.00	1.00	1.00

Table B.7: DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_a^\star$	1.00	0.96	0.98	0.96	0.97	0.97	0.97	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.95	0.97	0.95	0.95	0.96	0.96	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.85	0.62	0.35	0.22	0.18	0.18	0.14	0.97	0.64	0.37	0.23	0.17	0.15	0.16	1.00	0.67	0.35	0.26	0.19	0.16	0.16
	t_{gls}	1.00	0.95	0.84	0.79	0.74	0.70	0.70	1.00	0.96	0.89	0.83	0.77	0.75	0.74	1.00	0.97	0.90	0.84	0.79	0.78	0.75
	$P_{\hat{e}}$	1.00	0.98	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$ADF_{\hat{F}}$	0.71	0.34	0.11	0.11	0.10	0.11	0.10	0.98	0.29	0.10	0.08	0.09	0.07	0.08	1.00	0.30	0.08	0.07	0.07	0.06	0.06
30	CIPS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_a^\star$	1.00	0.98	0.98	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.98	0.98	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.85	0.60	0.26	0.20	0.18	0.14	0.13	0.97	0.60	0.29	0.24	0.18	0.17	0.14	1.00	0.61	0.28	0.20	0.17	0.16	0.14
	t_{gls}	1.00	0.99	0.93	0.85	0.81	0.81	0.81	1.00	1.00	0.96	0.93	0.91	0.87	0.85	1.00	1.00	0.97	0.94	0.91	0.88	0.88
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$ADF_{\hat{F}}$	0.74	0.13	0.10	0.11	0.12	0.12	0.11	0.98	0.10	0.08	0.08	0.08	0.08	0.07	1.00	0.08	0.06	0.05	0.06	0.06	0.08
50	CIPS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_a^\star$	1.00	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.83	0.56	0.30	0.18	0.17	0.17	0.16	0.97	0.58	0.28	0.21	0.18	0.15	0.16	1.00	0.58	0.27	0.22	0.18	0.15	0.14
	t_{gls}				N/A				1.00	1.00	0.96	0.94	0.90	0.91	0.90	1.00	1.00	0.98	0.95	0.93	0.90	0.89
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$ADF_{\hat{F}}$	0.72	0.13	0.11	0.12	0.10	0.10	0.12	0.98	0.09	0.07	0.08	0.08	0.08	0.10	1.00	0.07	0.06	0.07	0.06	0.06	0.06

Table B.8: DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	0.99	0.99	0.97	0.85	0.67	0.50	0.44	1.00	1.00	1.00	1.00	0.88	0.64	0.55	1.00	1.00	1.00	1.00	0.96	0.75	0.63
	$t_a^\star$	1.00	0.96	0.87	0.89	0.91	0.91	0.90	1.00	1.00	0.99	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.96	0.85	0.87	0.89	0.88	0.89	1.00	0.99	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.88	0.43	0.21	0.15	0.12	0.11	0.13	0.98	0.46	0.21	0.16	0.11	0.11	0.08	1.00	0.46	0.19	0.13	0.12	0.10	0.11
	t_{gls}	1.00	0.92	0.74	0.62	0.57	0.53	0.47	1.00	0.93	0.80	0.67	0.59	0.58	0.56	1.00	0.94	0.81	0.73	0.66	0.59	0.56
	$P_{\hat{e}}$	1.00	0.97	0.94	0.96	0.95	0.97	0.97	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.04	0.02	0.98	1.00	1.00	1.00	1.00	0.75	0.06	0.90	1.00	1.00	1.00	1.00	1.00	0.06	0.91	0.99	1.00	1.00	1.00
	MQ_f	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.64	0.02	0.96	1.00	1.00	1.00	1.00	1.00	0.03	0.94	0.99	1.00	1.00	1.00
30	CIPS	1.00	1.00	1.00	0.93	0.77	0.62	0.56	1.00	1.00	1.00	1.00	0.92	0.73	0.60	1.00	1.00	1.00	1.00	0.99	0.83	0.70
	$t_a^\star$	1.00	0.96	0.96	0.97	0.97	0.96	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.96	0.96	0.97	0.97	0.96	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.91	0.38	0.15	0.13	0.11	0.10	0.09	0.99	0.37	0.16	0.14	0.11	0.12	0.11	1.00	0.41	0.19	0.14	0.13	0.11	0.10
	t_{gls}	1.00	0.98	0.87	0.77	0.72	0.70	0.65	1.00	0.99	0.93	0.87	0.81	0.75	0.72	1.00	0.99	0.95	0.88	0.83	0.78	0.78
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.05	0.95	1.00	1.00	1.00	1.00	1.00	0.77	0.86	1.00	1.00	1.00	1.00	1.00	1.00	0.83	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	0.91	1.00	1.00	1.00	1.00	1.00	1.00	0.87	1.00	1.00	1.00	1.00	1.00
50	CIPS	1.00	1.00	1.00	0.96	0.80	0.64	0.55	1.00	1.00	1.00	1.00	0.95	0.76	0.64	1.00	1.00	1.00	1.00	1.00	0.87	0.69
	$t_a^\star$	1.00	0.97	0.98	0.98	0.97	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$t_b^\star$	1.00	0.97	0.99	0.98	0.97	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t_{rob}	0.90	0.37	0.17	0.14	0.12	0.10	0.09	0.98	0.37	0.20	0.14	0.13	0.12	0.11	1.00	0.39	0.16	0.14	0.13	0.11	0.12
	t_{gls}				N/A				1.00	1.00	0.95	0.88	0.84	0.80	0.77	1.00	1.00	0.96	0.92	0.88	0.83	0.82
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MQ_c	0.33	0.99	1.00	1.00	1.00	1.00	1.00	0.75	0.98	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00
	MQ_f	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	0.98	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00

Table B.9: DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.99	0.95	0.73	0.48	0.19	0.08	0.05	1.00	1.00	0.99	0.89	0.53	0.17	0.06	1.00	1.00	1.00	0.99	0.87	0.33	0.09
	$t_a^\star$	1.00	0.78	0.42	0.18	0.08	0.04	0.03	1.00	0.85	0.52	0.26	0.14	0.07	0.06	1.00	0.91	0.64	0.35	0.19	0.12	0.08
	$t_b^\star$	1.00	0.76	0.41	0.20	0.11	0.08	0.06	1.00	0.81	0.47	0.24	0.13	0.08	0.07	1.00	0.88	0.56	0.28	0.15	0.10	0.06
10	t_{rob}	0.96	0.91	0.72	0.52	0.35	0.22	0.18	1.00	0.96	0.82	0.60	0.41	0.28	0.20	1.00	0.98	0.87	0.68	0.48	0.31	0.23
	t_{gls}	1.00	0.91	0.64	0.38	0.23	0.13	0.11	1.00	0.93	0.65	0.43	0.22	0.11	0.08	1.00	0.93	0.68	0.39	0.22	0.10	0.06
	$P_{\hat{e}}$	1.00	0.99	0.91	0.72	0.47	0.18	0.11	1.00	1.00	0.95	0.86	0.57	0.23	0.09	1.00	1.00	0.97	0.88	0.64	0.24	0.08
	ADF$_{\hat{F}}$	0.74	0.64	0.55	0.46	0.38	0.37	0.35	0.97	0.88	0.72	0.64	0.53	0.44	0.41	1.00	0.94	0.79	0.64	0.53	0.46	0.40
	CIPS	1.00	1.00	0.96	0.71	0.21	0.03	0.02	1.00	1.00	1.00	0.99	0.75	0.11	0.02	1.00	1.00	1.00	1.00	0.99	0.34	0.03
	$t_a^\star$	1.00	0.89	0.47	0.15	0.05	0.01	0.01	1.00	0.97	0.68	0.33	0.12	0.06	0.03	1.00	0.98	0.78	0.43	0.20	0.08	0.04
	$t_b^\star$	1.00	0.87	0.51	0.23	0.12	0.06	0.04	1.00	0.95	0.67	0.34	0.16	0.09	0.06	1.00	0.97	0.74	0.40	0.19	0.09	0.05
30	t_{rob}	0.97	0.96	0.84	0.68	0.49	0.28	0.20	1.00	0.99	0.95	0.84	0.61	0.42	0.29	1.00	1.00	0.99	0.90	0.70	0.48	0.40
	t_{gls}	1.00	1.00	0.94	0.74	0.51	0.26	0.20	1.00	1.00	0.91	0.66	0.35	0.16	0.09	1.00	0.99	0.90	0.55	0.27	0.10	0.07
	$P_{\hat{e}}$	1.00	1.00	1.00	0.99	0.88	0.39	0.10	1.00	1.00	1.00	1.00	0.95	0.50	0.06	1.00	1.00	1.00	1.00	0.97	0.56	0.06
	ADF$_{\hat{F}}$	0.74	0.72	0.63	0.59	0.57	0.52	0.53	0.98	0.96	0.90	0.85	0.78	0.71	0.70	1.00	0.99	0.96	0.90	0.81	0.77	0.73
	CIPS	1.00	1.00	0.99	0.79	0.23	0.02	0.01	1.00	1.00	1.00	1.00	0.85	0.07	0.01	1.00	1.00	1.00	1.00	1.00	0.31	0.01
	$t_a^\star$	1.00	0.96	0.55	0.17	0.04	0.02	0.01	1.00	0.98	0.77	0.36	0.14	0.04	0.02	1.00	1.00	0.87	0.54	0.24	0.08	0.03
	$t_b^\star$	1.00	0.96	0.64	0.30	0.13	0.07	0.04	1.00	0.98	0.77	0.40	0.20	0.08	0.05	1.00	1.00	0.86	0.52	0.25	0.09	0.04
50	t_{rob}	0.97	0.95	0.89	0.75	0.52	0.30	0.20	1.00	0.99	0.98	0.91	0.73	0.47	0.33	1.00	1.00	1.00	0.98	0.84	0.59	0.43
	t_{gls}				N/A				1.00	1.00	0.98	0.85	0.55	0.26	0.16	1.00	1.00	0.98	0.77	0.39	0.14	0.08
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.98	0.52	0.09	1.00	1.00	1.00	1.00	1.00	0.70	0.08	1.00	1.00	1.00	1.00	1.00	0.80	0.07
	ADF$_{\hat{F}}$	0.74	0.74	0.67	0.64	0.62	0.58	0.57	0.98	0.97	0.93	0.90	0.85	0.81	0.81	1.00	1.00	0.99	0.94	0.92	0.88	0.84

Table B.10: DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.99	0.94	0.77	0.51	0.26	0.12	0.06	1.00	1.00	1.00	0.93	0.62	0.26	0.11	1.00	1.00	1.00	0.99	0.92	0.45	0.17
	$t_a^\star$	1.00	0.82	0.42	0.18	0.10	0.05	0.04	1.00	0.89	0.54	0.33	0.14	0.09	0.06	1.00	0.88	0.62	0.38	0.21	0.12	0.10
	$t_b^\star$	1.00	0.78	0.40	0.21	0.15	0.07	0.07	1.00	0.86	0.48	0.30	0.14	0.09	0.07	1.00	0.85	0.54	0.32	0.18	0.09	0.08
	t_{rob}	0.98	0.91	0.80	0.67	0.49	0.36	0.29	1.00	0.98	0.89	0.78	0.62	0.44	0.38	1.00	0.99	0.94	0.85	0.69	0.49	0.45
10	t_{gls}	1.00	0.94	0.71	0.49	0.27	0.17	0.14	1.00	0.94	0.72	0.44	0.25	0.13	0.09	1.00	0.94	0.68	0.43	0.25	0.14	0.09
	$P_{\hat{e}}$	1.00	0.96	0.81	0.55	0.28	0.14	0.09	1.00	0.99	0.91	0.68	0.39	0.13	0.09	1.00	1.00	0.90	0.71	0.39	0.15	0.09
	MQ_c	0.04	0.03	0.02	0.01	0.01	0.00	0.00	0.75	0.59	0.35	0.20	0.13	0.07	0.05	1.00	0.91	0.68	0.43	0.29	0.21	0.12
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.35	0.14	0.07	0.02	0.02	0.01	1.00	0.86	0.54	0.29	0.17	0.11	0.05
	CIPS	1.00	1.00	0.97	0.77	0.35	0.09	0.04	1.00	1.00	1.00	1.00	0.90	0.26	0.08	1.00	1.00	1.00	1.00	1.00	0.57	0.14
	$t_a^\star$	1.00	0.90	0.47	0.17	0.05	0.02	0.01	1.00	0.96	0.63	0.32	0.11	0.04	0.02	1.00	0.98	0.78	0.44	0.21	0.08	0.04
	$t_b^\star$	1.00	0.87	0.52	0.25	0.10	0.06	0.05	1.00	0.94	0.62	0.32	0.15	0.06	0.04	1.00	0.97	0.74	0.41	0.20	0.08	0.05
	t_{rob}	0.98	0.98	0.92	0.83	0.67	0.49	0.41	1.00	1.00	0.99	0.94	0.84	0.69	0.59	1.00	1.00	1.00	0.98	0.91	0.77	0.70
30	t_{gls}	1.00	1.00	0.97	0.90	0.68	0.45	0.39	1.00	1.00	0.97	0.78	0.52	0.27	0.18	1.00	1.00	0.93	0.66	0.35	0.13	0.08
	$P_{\hat{e}}$	1.00	1.00	1.00	0.96	0.77	0.30	0.09	1.00	1.00	1.00	0.99	0.88	0.36	0.09	1.00	1.00	1.00	0.99	0.91	0.39	0.07
	MQ_c	0.04	0.04	0.02	0.02	0.02	0.01	0.01	0.78	0.70	0.61	0.48	0.41	0.31	0.29	1.00	1.00	0.96	0.87	0.75	0.65	0.66
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.53	0.33	0.22	0.16	0.11	0.10	1.00	0.99	0.92	0.77	0.62	0.48	0.47
	CIPS	1.00	1.00	0.99	0.86	0.42	0.06	0.03	1.00	1.00	1.00	1.00	0.96	0.24	0.05	1.00	1.00	1.00	1.00	1.00	0.63	0.10
	$t_a^\star$	1.00	0.95	0.57	0.15	0.04	0.02	0.00	1.00	0.98	0.77	0.40	0.14	0.04	0.02	1.00	1.00	0.89	0.50	0.24	0.07	0.04
	$t_b^\star$	1.00	0.94	0.64	0.30	0.14	0.07	0.04	1.00	0.98	0.78	0.44	0.18	0.08	0.04	1.00	1.00	0.86	0.49	0.25	0.09	0.06
	t_{rob}	0.98	0.98	0.95	0.86	0.75	0.56	0.46	1.00	1.00	0.99	0.97	0.91	0.77	0.69	1.00	1.00	1.00	0.99	0.97	0.89	0.81
50	t_{gls}				N/A				1.00	1.00	1.00	0.95	0.80	0.53	0.41	1.00	1.00	0.99	0.90	0.57	0.29	0.17
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.93	0.40	0.09	1.00	1.00	1.00	1.00	0.98	0.55	0.07	1.00	1.00	1.00	1.00	0.99	0.61	0.07
	MQ_c	0.05	0.05	0.02	0.02	0.03	0.02	0.02	0.76	0.73	0.68	0.58	0.52	0.46	0.45	1.00	1.00	0.99	0.97	0.92	0.84	0.81
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.55	0.45	0.36	0.25	0.20	0.18	1.00	1.00	0.97	0.91	0.84	0.74	0.70

Table B.11: DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

	Test	T																				
					50							100							200			
N\ θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	0.99	0.93	0.63	0.24	0.08	0.03	0.01	1.00	1.00	0.93	0.69	0.26	0.05	0.02	1.00	1.00	0.99	0.91	0.59	0.11	0.03
	$t_a^\star$	1.00	0.82	0.51	0.25	0.15	0.06	0.04	1.00	0.88	0.60	0.36	0.19	0.12	0.07	1.00	0.93	0.68	0.41	0.21	0.13	0.09
	$t_b^\star$	1.00	0.75	0.43	0.23	0.14	0.07	0.06	1.00	0.82	0.52	0.30	0.16	0.10	0.08	1.00	0.88	0.59	0.33	0.16	0.10	0.07
	t_{rob}	0.85	0.81	0.74	0.69	0.56	0.49	0.46	0.98	0.96	0.89	0.82	0.76	0.68	0.65	1.00	0.98	0.97	0.91	0.83	0.77	0.69
	t_{gls}	1.00	0.93	0.71	0.49	0.30	0.16	0.12	1.00	0.93	0.73	0.45	0.25	0.11	0.09	1.00	0.95	0.72	0.42	0.23	0.11	0.07
	$P_{\hat{e}}$	1.00	0.99	0.92	0.73	0.49	0.21	0.10	1.00	1.00	0.97	0.85	0.61	0.24	0.09	1.00	1.00	0.97	0.91	0.68	0.28	0.07
	$\text{ADF}_{\hat{F}}$	0.74	0.73	0.66	0.63	0.61	0.59	0.55	0.97	0.96	0.93	0.87	0.81	0.78	0.79	1.00	0.99	0.95	0.93	0.87	0.83	0.80
30	CIPS	1.00	1.00	0.94	0.59	0.11	0.01	0.00	1.00	1.00	1.00	0.96	0.51	0.02	0.00	1.00	1.00	1.00	1.00	0.93	0.08	0.00
	$t_a^\star$	1.00	0.94	0.60	0.26	0.07	0.03	0.01	1.00	0.97	0.74	0.41	0.17	0.08	0.03	1.00	0.98	0.83	0.53	0.22	0.11	0.06
	$t_b^\star$	1.00	0.90	0.56	0.29	0.14	0.08	0.05	1.00	0.94	0.69	0.40	0.20	0.11	0.05	1.00	0.97	0.78	0.48	0.21	0.11	0.06
	t_{rob}	0.85	0.80	0.75	0.69	0.66	0.56	0.51	0.98	0.96	0.94	0.90	0.85	0.80	0.77	1.00	1.00	0.98	0.98	0.95	0.92	0.90
	t_{gls}	1.00	1.00	0.97	0.86	0.71	0.49	0.39	1.00	1.00	0.95	0.77	0.51	0.28	0.20	1.00	1.00	0.93	0.67	0.36	0.13	0.10
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.90	0.39	0.11	1.00	1.00	1.00	1.00	0.96	0.53	0.08	1.00	1.00	1.00	1.00	0.98	0.60	0.05
	$\text{ADF}_{\hat{F}}$	0.70	0.72	0.72	0.67	0.67	0.64	0.66	0.97	0.97	0.96	0.95	0.95	0.92	0.91	1.00	1.00	0.99	0.99	0.99	0.98	0.98
50	CIPS	1.00	1.00	1.00	0.80	0.14	0.00	0.00	1.00	1.00	1.00	1.00	0.73	0.01	0.00	1.00	1.00	1.00	1.00	0.98	0.07	0.00
	$t_a^\star$	1.00	0.99	0.66	0.30	0.09	0.03	0.01	1.00	0.99	0.86	0.52	0.18	0.06	0.03	1.00	1.00	0.91	0.63	0.27	0.11	0.05
	$t_b^\star$	1.00	0.95	0.65	0.40	0.18	0.09	0.06	1.00	0.98	0.83	0.53	0.23	0.11	0.06	1.00	0.99	0.89	0.60	0.26	0.12	0.06
	t_{rob}	0.85	0.81	0.79	0.70	0.66	0.58	0.56	0.97	0.96	0.95	0.94	0.89	0.83	0.81	1.00	1.00	0.99	0.98	0.97	0.95	0.93
	t_{gls}				N/A				1.00	1.00	1.00	0.94	0.79	0.55	0.42	1.00	1.00	0.99	0.88	0.58	0.30	0.17
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.99	0.52	0.11	1.00	1.00	1.00	1.00	1.00	0.70	0.08	1.00	1.00	1.00	1.00	1.00	0.80	0.07
	$\text{ADF}_{\hat{F}}$	0.75	0.71	0.71	0.73	0.72	0.68	0.68	0.98	0.98	0.97	0.96	0.96	0.95	0.94	1.00	1.00	1.00	1.00	1.00	0.99	0.99

Table B.12: DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.98	0.94	0.77	0.51	0.30	0.14	0.09	1.00	1.00	0.99	0.90	0.62	0.31	0.16	1.00	1.00	1.00	0.99	0.89	0.49	0.24
	$t_a^\star$	1.00	0.90	0.63	0.39	0.20	0.12	0.11	1.00	0.93	0.71	0.45	0.30	0.18	0.12	1.00	0.95	0.73	0.51	0.33	0.21	0.18
	$t_b^\star$	1.00	0.83	0.51	0.30	0.17	0.12	0.13	1.00	0.88	0.61	0.35	0.24	0.15	0.11	1.00	0.89	0.61	0.39	0.22	0.16	0.14
	t_{rob}	0.88	0.87	0.83	0.76	0.74	0.70	0.66	0.98	0.97	0.95	0.93	0.91	0.87	0.85	1.00	1.00	1.00	0.98	0.97	0.94	0.91
10	t_{gls}	1.00	0.96	0.83	0.65	0.47	0.65	0.27	1.00	0.97	0.81	0.61	0.41	0.26	0.20	1.00	0.96	0.80	0.55	0.34	0.20	0.14
	$P_{\hat{e}}$	1.00	0.97	0.83	0.57	0.32	0.13	0.09	1.00	0.99	0.89	0.67	0.39	0.16	0.08	1.00	1.00	0.90	0.75	0.43	0.17	0.08
	MQ_c	0.05	0.04	0.02	0.02	0.02	0.02	0.02	0.74	0.70	0.60	0.49	0.43	0.36	0.31	1.00	0.99	0.92	0.84	0.77	0.67	0.62
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.53	0.38	0.24	0.18	0.13	0.11	1.00	0.98	0.86	0.74	0.66	0.52	0.45
	CIPS	1.00	0.99	0.95	0.81	0.52	0.20	0.08	1.00	1.00	1.00	1.00	0.93	0.42	0.17	1.00	1.00	1.00	1.00	1.00	0.76	0.28
	$t_a^\star$	1.00	0.98	0.77	0.44	0.18	0.10	0.04	1.00	0.99	0.87	0.61	0.32	0.14	0.09	1.00	0.99	0.91	0.65	0.36	0.19	0.11
	$t_b^\star$	1.00	0.94	0.66	0.39	0.21	0.13	0.08	1.00	0.96	0.77	0.54	0.31	0.15	0.11	1.00	0.98	0.83	0.58	0.31	0.18	0.10
	t_{rob}	0.88	0.92	0.85	0.84	0.82	0.78	0.74	0.98	0.98	0.97	0.96	0.96	0.94	0.93	1.00	1.00	1.00	1.00	0.99	0.99	0.98
30	t_{gls}	1.00	1.00	1.00	0.97	0.94	0.87	0.82	1.00	1.00	0.99	0.95	0.84	0.69	0.63	1.00	1.00	0.98	0.91	0.69	0.47	0.36
	$P_{\hat{e}}$	1.00	1.00	1.00	0.97	0.79	0.27	0.10	1.00	1.00	1.00	0.98	0.88	0.33	0.07	1.00	1.00	1.00	0.99	0.91	0.42	0.07
	MQ_c	0.03	0.04	0.04	0.03	0.03	0.03	0.02	0.76	0.74	0.72	0.69	0.65	0.64	0.57	1.00	1.00	1.00	0.99	0.98	0.97	0.96
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.60	0.55	0.48	0.42	0.37	0.33	1.00	1.00	1.00	0.98	0.95	0.90	0.90
	CIPS	1.00	0.99	0.97	0.87	0.62	0.20	0.11	1.00	1.00	1.00	1.00	0.98	0.49	0.16	1.00	1.00	1.00	1.00	1.00	0.87	0.31
	$t_a^\star$	1.00	1.00	0.89	0.54	0.22	0.07	0.04	1.00	1.00	0.96	0.73	0.39	0.14	0.09	1.00	1.00	0.97	0.77	0.45	0.20	0.14
	$t_b^\star$	1.00	0.99	0.82	0.54	0.29	0.17	0.09	1.00	0.99	0.91	0.70	0.40	0.18	0.13	1.00	1.00	0.94	0.73	0.42	0.19	0.14
	t_{rob}	0.92	0.88	0.87	0.85	0.83	0.80	0.79	0.99	0.99	0.98	0.97	0.97	0.96	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	t_{gls}				N/A				1.00	1.00	1.00	1.00	0.98	0.94	0.91	1.00	1.00	1.00	0.98	0.94	0.79	0.68
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.95	0.42	0.09	1.00	1.00	1.00	1.00	0.98	0.54	0.08	1.00	1.00	1.00	1.00	0.98	0.64	0.06
	MQ_c	0.05	0.05	0.05	0.04	0.03	0.04	0.03	0.77	0.75	0.72	0.71	0.70	0.68	0.68	1.00	1.00	1.00	1.00	1.00	0.99	0.99
	MQ_f	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.68	0.59	0.53	0.49	0.45	0.45	1.00	1.00	1.00	0.99	0.99	0.97	0.98

Table B.13: DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	1.00	1.00	0.98	0.83	0.46	0.13	0.06	1.00	1.00	1.00	1.00	0.85	0.23	0.05	1.00	1.00	1.00	1.00	0.98	0.46	0.04
	$t_a^\star$	1.00	0.84	0.45	0.19	0.09	0.04	0.04	1.00	0.90	0.61	0.30	0.17	0.08	0.06	1.00	0.94	0.67	0.42	0.20	0.12	0.07
	$t_b^\star$	1.00	0.81	0.46	0.21	0.13	0.08	0.08	1.00	0.87	0.57	0.28	0.17	0.09	0.07	1.00	0.91	0.60	0.34	0.16	0.10	0.07
10	t_{rob}	1.00	0.95	0.77	0.45	0.22	0.07	0.05	1.00	0.97	0.79	0.49	0.26	0.11	0.06	1.00	0.98	0.81	0.54	0.29	0.12	0.05
	t_{gls}	1.00	0.92	0.69	0.37	0.22	0.09	0.07	1.00	0.94	0.71	0.41	0.22	0.12	0.07	1.00	0.95	0.72	0.45	0.24	0.12	0.05
	$P_{\hat{e}}$	1.00	1.00	0.97	0.87	0.54	0.17	0.12	1.00	1.00	1.00	0.95	0.66	0.18	0.09	1.00	1.00	1.00	0.99	0.77	0.14	0.08
	$\text{ADF}_{\hat{F}}$	0.94	0.85	0.70	0.54	0.38	0.19	0.11	0.98	0.87	0.71	0.51	0.37	0.18	0.08	1.00	0.93	0.73	0.54	0.40	0.17	0.06
	CIPS	1.00	1.00	1.00	1.00	0.77	0.20	0.04	1.00	1.00	1.00	1.00	1.00	0.42	0.04	1.00	1.00	1.00	1.00	1.00	0.72	0.05
	$t_a^\star$	1.00	0.97	0.62	0.25	0.07	0.01	0.01	1.00	0.99	0.79	0.46	0.15	0.05	0.04	1.00	1.00	0.87	0.61	0.23	0.08	0.05
	$t_b^\star$	1.00	0.97	0.69	0.38	0.16	0.07	0.08	1.00	0.99	0.78	0.50	0.19	0.07	0.06	1.00	0.99	0.85	0.58	0.23	0.08	0.06
30	t_{rob}	1.00	1.00	0.97	0.71	0.28	0.07	0.02	1.00	1.00	0.97	0.79	0.38	0.10	0.04	1.00	1.00	0.98	0.80	0.40	0.08	0.05
	t_{gls}	1.00	1.00	0.92	0.67	0.35	0.14	0.08	1.00	1.00	0.93	0.68	0.33	0.14	0.07	1.00	1.00	0.94	0.71	0.35	0.09	0.07
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.92	0.40	0.14	1.00	1.00	1.00	1.00	0.96	0.42	0.09	1.00	1.00	1.00	1.00	1.00	0.48	0.08
	$\text{ADF}_{\hat{F}}$	0.93	0.85	0.69	0.50	0.37	0.17	0.11	0.98	0.87	0.60	0.40	0.28	0.16	0.08	1.00	0.90	0.61	0.40	0.24	0.13	0.06
	CIPS	1.00	1.00	1.00	1.00	0.91	0.24	0.05	1.00	1.00	1.00	1.00	1.00	0.46	0.04	1.00	1.00	1.00	1.00	1.00	0.80	0.05
	$t_a^\star$	1.00	1.00	0.77	0.33	0.10	0.01	0.01	1.00	1.00	0.92	0.56	0.20	0.05	0.02	1.00	1.00	0.96	0.70	0.31	0.08	0.05
	$t_b^\star$	1.00	1.00	0.84	0.50	0.23	0.09	0.04	1.00	1.00	0.93	0.62	0.27	0.10	0.05	1.00	1.00	0.96	0.69	0.32	0.10	0.07
50	t_{rob}	1.00	1.00	1.00	0.82	0.33	0.05	0.01	1.00	1.00	1.00	0.90	0.46	0.09	0.02	1.00	1.00	1.00	0.91	0.53	0.10	0.04
	t_{gls}				N/A				1.00	1.00	0.99	0.81	0.45	0.15	0.07	1.00	1.00	0.99	0.81	0.46	0.14	0.07
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.99	0.56	0.18	1.00	1.00	1.00	1.00	1.00	0.63	0.10	1.00	1.00	1.00	1.00	1.00	0.68	0.07
	$\text{ADF}_{\hat{F}}$	0.94	0.87	0.66	0.50	0.33	0.20	0.11	0.99	0.86	0.54	0.40	0.25	0.16	0.06	1.00	0.88	0.53	0.35	0.21	0.13	0.06

Table B.14: DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	1.00	1.00	0.97	0.81	0.45	0.12	0.06	1.00	1.00	1.00	1.00	0.80	0.23	0.04	1.00	1.00	1.00	1.00	0.96	0.43	0.04
	$t_a^\star$	1.00	0.81	0.44	0.18	0.09	0.02	0.03	1.00	0.87	0.57	0.28	0.15	0.06	0.07	1.00	0.90	0.64	0.38	0.18	0.11	0.06
	$t_b^\star$	1.00	0.79	0.41	0.19	0.11	0.05	0.07	1.00	0.82	0.51	0.23	0.14	0.07	0.07	1.00	0.86	0.54	0.28	0.11	0.07	0.04
10	t_{rob}	1.00	0.94	0.67	0.42	0.20	0.09	0.05	1.00	0.95	0.75	0.45	0.23	0.10	0.07	1.00	0.95	0.75	0.48	0.23	0.11	0.06
	t_{gls}	1.00	0.92	0.60	0.39	0.22	0.12	0.09	1.00	0.93	0.67	0.41	0.22	0.12	0.08	1.00	0.93	0.67	0.42	0.21	0.10	0.07
	$P_{\hat{e}}$	1.00	0.99	0.96	0.81	0.53	0.21	0.12	1.00	1.00	0.98	0.91	0.63	0.21	0.10	1.00	1.00	1.00	0.98	0.77	0.25	0.08
	ADF$_{\hat{F}}$	0.86	0.74	0.51	0.37	0.23	0.15	0.11	0.97	0.84	0.57	0.38	0.24	0.12	0.07	0.99	0.88	0.65	0.42	0.23	0.13	0.06
	CIPS	1.00	1.00	1.00	1.00	0.77	0.21	0.06	1.00	1.00	1.00	1.00	0.99	0.41	0.06	1.00	1.00	1.00	1.00	1.00	0.67	0.04
	$t_a^\star$	1.00	0.87	0.46	0.17	0.04	0.01	0.01	1.00	0.93	0.61	0.29	0.12	0.03	0.03	1.00	0.95	0.73	0.41	0.17	0.07	0.06
	$t_b^\star$	1.00	0.87	0.47	0.23	0.09	0.05	0.04	1.00	0.91	0.57	0.29	0.13	0.06	0.04	1.00	0.93	0.64	0.33	0.14	0.06	0.05
30	t_{rob}	1.00	1.00	0.92	0.63	0.27	0.08	0.05	1.00	1.00	0.93	0.67	0.35	0.10	0.04	1.00	1.00	0.94	0.71	0.35	0.12	0.06
	t_{gls}	1.00	0.99	0.88	0.64	0.40	0.23	0.15	1.00	0.99	0.86	0.61	0.34	0.15	0.08	1.00	0.99	0.90	0.59	0.32	0.13	0.09
	$P_{\hat{e}}$	1.00	1.00	1.00	0.99	0.87	0.36	0.15	1.00	1.00	1.00	1.00	0.97	0.44	0.09	1.00	1.00	1.00	1.00	0.99	0.58	0.07
	ADF$_{\hat{F}}$	0.87	0.74	0.45	0.30	0.17	0.13	0.12	0.97	0.78	0.47	0.29	0.18	0.11	0.08	0.99	0.88	0.53	0.33	0.19	0.12	0.07
	CIPS	1.00	1.00	1.00	1.00	0.87	0.25	0.07	1.00	1.00	1.00	1.00	1.00	0.46	0.06	1.00	1.00	1.00	1.00	1.00	0.79	0.06
	$t_a^\star$	1.00	0.91	0.51	0.15	0.04	0.01	0.01	1.00	0.96	0.71	0.34	0.11	0.02	0.01	1.00	0.97	0.80	0.48	0.23	0.08	0.04
	$t_b^\star$	1.00	0.89	0.54	0.23	0.11	0.06	0.04	1.00	0.95	0.67	0.34	0.13	0.05	0.04	1.00	0.96	0.73	0.42	0.21	0.07	0.05
50	t_{rob}	1.00	1.00	0.98	0.74	0.33	0.07	0.02	1.00	1.00	0.99	0.80	0.37	0.09	0.04	1.00	1.00	0.98	0.83	0.46	0.14	0.06
	t_{gls}				N/A				1.00	1.00	0.95	0.76	0.43	0.18	0.12	1.00	1.00	0.96	0.74	0.42	0.18	0.10
	$P_{\hat{e}}$	1.00	1.00	1.00	1.00	0.98	0.55	0.18	1.00	1.00	1.00	1.00	1.00	0.59	0.11	1.00	1.00	1.00	1.00	1.00	0.76	0.07
	ADF$_{\hat{F}}$	0.88	0.73	0.46	0.25	0.18	0.12	0.11	0.97	0.76	0.43	0.25	0.14	0.08	0.08	1.00	0.83	0.47	0.28	0.14	0.09	0.06

Table B.15: DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	CIPS	0.99	0.97	0.90	0.83	0.62	0.20	0.08	1.00	1.00	0.97	0.95	0.86	0.37	0.07	1.00	1.00	0.99	0.97	0.95	0.59	0.07
	$t_a^\star$	1.00	0.64	0.30	0.19	0.08	0.04	0.03	1.00	0.74	0.39	0.23	0.13	0.06	0.04	1.00	0.76	0.43	0.25	0.17	0.11	0.09
	$t_b^\star$	1.00	0.62	0.33	0.25	0.14	0.09	0.06	1.00	0.70	0.37	0.24	0.14	0.07	0.05	1.00	0.72	0.40	0.22	0.16	0.10	0.07
	t_{rob}	0.89	0.76	0.51	0.31	0.18	0.12	0.07	0.98	0.85	0.59	0.38	0.22	0.12	0.09	1.00	0.89	0.62	0.40	0.24	0.12	0.09
	t_{gls}	0.99	0.79	0.45	0.26	0.16	0.10	0.07	1.00	0.81	0.49	0.28	0.17	0.09	0.06	1.00	0.82	0.47	0.26	0.16	0.10	0.07
	$P_{\hat{e}}$	1.00	0.94	0.72	0.48	0.25	0.17	0.12	1.00	0.95	0.74	0.46	0.26	0.16	0.10	1.00	0.95	0.76	0.46	0.26	0.13	0.09
	$ADF_{\hat{F}}$	0.55	0.48	0.32	0.19	0.13	0.12	0.11	0.82	0.70	0.40	0.17	0.10	0.08	0.08	0.97	0.79	0.46	0.21	0.07	0.07	0.07
30	CIPS	1.00	1.00	1.00	0.98	0.92	0.32	0.08	1.00	1.00	1.00	1.00	0.99	0.61	0.06	1.00	1.00	1.00	1.00	1.00	0.88	0.06
	$t_a^\star$	1.00	0.69	0.39	0.23	0.10	0.02	0.01	1.00	0.74	0.45	0.31	0.17	0.06	0.02	1.00	0.78	0.54	0.39	0.24	0.07	0.04
	$t_b^\star$	1.00	0.69	0.46	0.33	0.20	0.10	0.05	1.00	0.73	0.47	0.36	0.23	0.09	0.06	1.00	0.77	0.54	0.40	0.24	0.08	0.05
	t_{rob}	0.93	0.82	0.56	0.36	0.24	0.13	0.09	0.99	0.90	0.60	0.42	0.26	0.15	0.10	1.00	0.93	0.68	0.42	0.26	0.14	0.11
	t_{gls}	1.00	0.94	0.70	0.53	0.39	0.29	0.28	1.00	0.92	0.66	0.45	0.31	0.20	0.17	1.00	0.92	0.69	0.43	0.27	0.13	0.11
	$P_{\hat{e}}$	1.00	0.99	0.82	0.58	0.43	0.25	0.18	1.00	1.00	0.82	0.58	0.39	0.21	0.11	1.00	0.99	0.84	0.55	0.41	0.18	0.08
	$ADF_{\hat{F}}$	0.52	0.43	0.26	0.13	0.13	0.14	0.12	0.86	0.69	0.30	0.13	0.12	0.08	0.09	0.98	0.85	0.38	0.14	0.09	0.07	0.07
50	CIPS	1.00	1.00	1.00	1.00	0.98	0.42	0.07	1.00	1.00	1.00	1.00	1.00	0.72	0.08	1.00	1.00	1.00	1.00	1.00	0.96	0.06
	$t_a^\star$	1.00	0.71	0.45	0.35	0.14	0.03	0.01	1.00	0.76	0.56	0.42	0.22	0.04	0.02	1.00	0.81	0.56	0.45	0.27	0.08	0.04
	$t_b^\star$	1.00	0.73	0.51	0.45	0.26	0.10	0.05	1.00	0.76	0.57	0.47	0.27	0.08	0.04	1.00	0.81	0.56	0.47	0.28	0.10	0.05
	t_{rob}	0.93	0.82	0.56	0.39	0.24	0.12	0.09	0.99	0.91	0.64	0.41	0.24	0.14	0.11	1.00	0.94	0.64	0.42	0.26	0.14	0.09
	t_{gls}				N/A				1.00	0.96	0.76	0.60	0.44	0.31	0.31	1.00	0.96	0.74	0.55	0.38	0.22	0.16
	$P_{\hat{e}}$	1.00	1.00	0.86	0.63	0.48	0.29	0.18	1.00	1.00	0.84	0.60	0.47	0.24	0.11	1.00	1.00	0.83	0.63	0.46	0.20	0.07
	$ADF_{\hat{F}}$	0.50	0.42	0.26	0.16	0.10	0.10	0.13	0.87	0.68	0.27	0.13	0.08	0.07	0.08	0.99	0.88	0.35	0.14	0.08	0.07	0.06

Table B.16: DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	0.98	0.95	0.87	0.76	0.53	0.16	0.06	1.00	0.99	0.97	0.94	0.81	0.29	0.04	1.00	1.00	0.98	0.98	0.94	0.49	0.05
	$t_a^\star$	1.00	0.63	0.25	0.16	0.06	0.04	0.02	1.00	0.68	0.36	0.18	0.09	0.06	0.06	1.00	0.75	0.42	0.23	0.13	0.09	0.09
	$t_b^\star$	1.00	0.60	0.27	0.19	0.09	0.06	0.04	1.00	0.64	0.33	0.19	0.10	0.05	0.05	1.00	0.69	0.36	0.19	0.09	0.06	0.04
10	t_{rob}	0.89	0.76	0.48	0.30	0.16	0.10	0.07	0.99	0.86	0.57	0.33	0.19	0.10	0.08	1.00	0.87	0.59	0.35	0.21	0.09	0.08
	t_{gls}	0.99	0.78	0.42	0.25	0.16	0.10	0.08	1.00	0.81	0.45	0.25	0.16	0.08	0.06	1.00	0.79	0.44	0.26	0.13	0.08	0.07
	$P_{\hat{e}}$	1.00	0.93	0.69	0.42	0.26	0.18	0.12	1.00	0.95	0.71	0.46	0.26	0.13	0.08	1.00	0.97	0.75	0.52	0.27	0.12	0.09
	$\text{ADF}_{\hat{F}}$	0.57	0.48	0.31	0.19	0.13	0.12	0.12	0.84	0.68	0.37	0.18	0.09	0.09	0.07	0.97	0.83	0.43	0.22	0.09	0.08	0.06
	CIPS	1.00	1.00	0.99	0.98	0.85	0.28	0.05	1.00	1.00	1.00	1.00	0.99	0.52	0.05	1.00	1.00	1.00	1.00	1.00	0.79	0.05
	$t_a^\star$	1.00	0.65	0.33	0.15	0.06	0.01	0.00	1.00	0.74	0.41	0.26	0.11	0.03	0.02	1.00	0.77	0.47	0.29	0.13	0.07	0.04
	$t_b^\star$	1.00	0.65	0.38	0.23	0.14	0.04	0.03	1.00	0.72	0.41	0.27	0.13	0.04	0.03	1.00	0.74	0.47	0.27	0.12	0.05	0.03
30	t_{rob}	0.94	0.81	0.53	0.34	0.20	0.11	0.07	0.99	0.92	0.59	0.40	0.23	0.10	0.07	1.00	0.91	0.64	0.41	0.21	0.15	0.07
	t_{gls}	1.00	0.94	0.69	0.51	0.38	0.28	0.26	1.00	0.93	0.63	0.41	0.26	0.13	0.14	1.00	0.91	0.64	0.39	0.19	0.11	0.07
	$P_{\hat{e}}$	1.00	0.99	0.82	0.56	0.38	0.22	0.17	1.00	1.00	0.84	0.61	0.40	0.20	0.10	1.00	1.00	0.82	0.61	0.42	0.18	0.07
	$\text{ADF}_{\hat{F}}$	0.54	0.47	0.25	0.14	0.12	0.11	0.11	0.83	0.67	0.29	0.12	0.09	0.08	0.07	0.99	0.84	0.35	0.10	0.09	0.07	0.07
	CIPS	1.00	1.00	1.00	1.00	0.94	0.37	0.06	1.00	1.00	1.00	1.00	1.00	0.62	0.04	1.00	1.00	1.00	1.00	1.00	0.91	0.06
	$t_a^\star$	1.00	0.68	0.37	0.18	0.05	0.01	0.01	1.00	0.77	0.44	0.29	0.12	0.03	0.02	1.00	0.82	0.54	0.35	0.17	0.04	0.04
	$t_b^\star$	1.00	0.69	0.43	0.28	0.14	0.04	0.03	1.00	0.75	0.46	0.33	0.18	0.06	0.03	1.00	0.79	0.52	0.34	0.18	0.04	0.04
50	t_{rob}	0.93	0.82	0.57	0.34	0.24	0.09	0.07	0.99	0.92	0.60	0.41	0.22	0.12	0.08	1.00	0.93	0.65	0.42	0.25	0.12	0.08
	t_{gls}				N/A				1.00	0.97	0.75	0.59	0.42	0.27	0.27	1.00	0.95	0.73	0.54	0.31	0.16	0.13
	$P_{\hat{e}}$	1.00	1.00	0.83	0.64	0.44	0.30	0.18	1.00	1.00	0.86	0.61	0.46	0.23	0.09	1.00	1.00	0.88	0.63	0.45	0.19	0.09
	$\text{ADF}_{\hat{F}}$	0.51	0.45	0.24	0.15	0.11	0.12	0.12	0.83	0.68	0.29	0.15	0.08	0.09	0.08	0.99	0.87	0.34	0.12	0.08	0.07	0.06

Table B.17: DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	1.00	0.99	0.98	0.97	0.90	0.33	0.06	1.00	1.00	1.00	0.99	0.98	0.57	0.07	1.00	1.00	1.00	0.99	1.00	0.79	0.06
	$t_a^\star$	1.00	0.55	0.29	0.20	0.11	0.04	0.03	1.00	0.66	0.36	0.26	0.15	0.07	0.05	1.00	0.67	0.37	0.27	0.18	0.08	0.10
	$t_b^\star$	1.00	0.49	0.29	0.25	0.16	0.07	0.05	1.00	0.59	0.35	0.25	0.16	0.08	0.07	1.00	0.60	0.31	0.24	0.14	0.06	0.09
10	t_{rob}	0.75	0.61	0.42	0.28	0.19	0.13	0.09	0.92	0.80	0.52	0.34	0.20	0.12	0.10	0.99	0.85	0.58	0.34	0.22	0.12	0.09
	t_{gls}	0.98	0.67	0.36	0.21	0.17	0.12	0.11	1.00	0.70	0.36	0.20	0.13	0.09	0.08	1.00	0.68	0.34	0.19	0.12	0.07	0.07
	$P_{\hat{e}}$	0.99	0.90	0.57	0.34	0.19	0.13	0.12	1.00	0.88	0.60	0.37	0.23	0.12	0.11	1.00	0.91	0.61	0.36	0.23	0.09	0.10
	$\text{ADF}_{\hat{F}}$	0.53	0.47	0.27	0.20	0.13	0.12	0.11	0.82	0.67	0.38	0.18	0.11	0.08	0.09	0.97	0.81	0.42	0.19	0.11	0.08	0.06
	CIPS	1.00	1.00	1.00	1.00	0.99	0.49	0.07	1.00	1.00	1.00	1.00	1.00	0.81	0.06	1.00	1.00	1.00	1.00	1.00	0.96	0.06
	$t_a^\star$	1.00	0.53	0.40	0.32	0.19	0.04	0.02	1.00	0.61	0.43	0.37	0.25	0.05	0.03	1.00	0.65	0.47	0.38	0.26	0.07	0.05
	$t_b^\star$	1.00	0.50	0.43	0.40	0.28	0.10	0.05	1.00	0.56	0.45	0.41	0.29	0.08	0.05	1.00	0.59	0.46	0.39	0.25	0.07	0.06
30	t_{rob}	0.76	0.65	0.42	0.29	0.16	0.11	0.08	0.94	0.79	0.54	0.32	0.20	0.11	0.12	0.99	0.88	0.56	0.34	0.19	0.15	0.10
	t_{gls}	1.00	0.90	0.64	0.51	0.43	0.38	0.36	1.00	0.85	0.60	0.44	0.33	0.25	0.24	1.00	0.83	0.55	0.39	0.25	0.18	0.15
	$P_{\hat{e}}$	1.00	0.97	0.65	0.44	0.30	0.17	0.16	1.00	0.97	0.64	0.45	0.30	0.15	0.12	1.00	0.97	0.66	0.44	0.30	0.14	0.10
	$\text{ADF}_{\hat{F}}$	0.48	0.44	0.25	0.16	0.12	0.11	0.12	0.86	0.69	0.31	0.14	0.08	0.08	0.08	0.99	0.84	0.38	0.15	0.07	0.07	0.06
	CIPS	1.00	1.00	1.00	1.00	1.00	0.60	0.07	1.00	1.00	1.00	1.00	1.00	0.86	0.05	1.00	1.00	1.00	1.00	1.00	0.99	0.05
	$t_a^\star$	1.00	0.55	0.42	0.35	0.24	0.07	0.01	1.00	0.64	0.47	0.43	0.30	0.08	0.02	1.00	0.66	0.51	0.43	0.34	0.10	0.05
	$t_b^\star$	1.00	0.54	0.46	0.42	0.34	0.16	0.06	1.00	0.61	0.50	0.46	0.34	0.11	0.05	1.00	0.63	0.52	0.45	0.35	0.11	0.06
50	t_{rob}	0.76	0.64	0.42	0.27	0.16	0.11	0.10	0.93	0.80	0.52	0.33	0.19	0.12	0.09	1.00	0.87	0.57	0.35	0.22	0.12	0.10
	t_{gls}				N/A				1.00	0.91	0.71	0.57	0.45	0.38	0.37	1.00	0.86	0.64	0.50	0.39	0.29	0.26
	$P_{\hat{e}}$	1.00	0.98	0.68	0.47	0.37	0.23	0.17	1.00	0.99	0.68	0.45	0.34	0.19	0.10	1.00	0.98	0.67	0.45	0.35	0.15	0.09
	$\text{ADF}_{\hat{F}}$	0.49	0.43	0.23	0.15	0.13	0.11	0.12	0.82	0.71	0.29	0.13	0.09	0.09	0.08	1.00	0.87	0.35	0.12	0.08	0.07	0.07

Table B.18: DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\θ		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	CIPS	1.00	0.98	0.95	0.95	0.83	0.30	0.06	1.00	1.00	0.99	0.99	0.97	0.49	0.04	1.00	1.00	1.00	1.00	0.99	0.71	0.03
	$t_a^\star$	1.00	0.56	0.29	0.18	0.10	0.02	0.03	1.00	0.64	0.30	0.22	0.12	0.05	0.05	1.00	0.66	0.37	0.29	0.14	0.09	0.08
	$t_b^\star$	1.00	0.51	0.29	0.21	0.15	0.05	0.04	1.00	0.58	0.27	0.23	0.13	0.04	0.04	1.00	0.59	0.32	0.23	0.11	0.06	0.04
10	t_{rob}	0.77	0.61	0.42	0.27	0.18	0.10	0.09	0.92	0.79	0.53	0.31	0.19	0.12	0.09	0.99	0.86	0.57	0.35	0.19	0.11	0.09
	t_{gls}	0.97	0.68	0.36	0.23	0.14	0.11	0.11	0.99	0.71	0.38	0.18	0.10	0.08	0.07	1.00	0.69	0.35	0.17	0.08	0.06	0.06
	$P_{\hat{e}}$	0.97	0.85	0.52	0.32	0.19	0.14	0.13	1.00	0.89	0.62	0.38	0.22	0.13	0.09	1.00	0.92	0.63	0.39	0.21	0.11	0.07
	$\text{ADF}_{\hat{F}}$	0.53	0.46	0.30	0.18	0.12	0.11	0.13	0.84	0.68	0.33	0.19	0.09	0.08	0.05	0.97	0.83	0.43	0.19	0.08	0.07	0.08
	CIPS	1.00	1.00	1.00	1.00	0.99	0.46	0.04	1.00	1.00	1.00	1.00	1.00	0.71	0.05	1.00	1.00	1.00	1.00	1.00	0.92	0.03
	$t_a^\star$	1.00	0.53	0.34	0.28	0.14	0.04	0.01	1.00	0.64	0.41	0.35	0.23	0.04	0.03	1.00	0.68	0.46	0.34	0.22	0.08	0.04
	$t_b^\star$	1.00	0.49	0.39	0.36	0.20	0.09	0.03	1.00	0.59	0.42	0.38	0.26	0.05	0.03	1.00	0.60	0.44	0.36	0.22	0.06	0.03
30	t_{rob}	0.77	0.63	0.42	0.27	0.20	0.12	0.08	0.93	0.80	0.54	0.33	0.23	0.11	0.10	1.00	0.86	0.56	0.33	0.20	0.12	0.10
	t_{gls}	1.00	0.88	0.64	0.51	0.43	0.37	0.35	1.00	0.86	0.60	0.41	0.29	0.20	0.19	1.00	0.82	0.54	0.35	0.18	0.13	0.10
	$P_{\hat{e}}$	1.00	0.97	0.63	0.44	0.30	0.19	0.16	1.00	0.97	0.66	0.43	0.29	0.17	0.10	1.00	0.98	0.68	0.45	0.29	0.15	0.09
	$\text{ADF}_{\hat{F}}$	0.50	0.44	0.22	0.16	0.13	0.11	0.10	0.82	0.68	0.29	0.13	0.08	0.08	0.08	0.98	0.87	0.35	0.14	0.09	0.06	0.07
	CIPS	1.00	1.00	1.00	1.00	1.00	0.56	0.06	1.00	1.00	1.00	1.00	1.00	0.84	0.04	1.00	1.00	1.00	1.00	1.00	0.98	0.04
	$t_a^\star$	1.00	0.54	0.41	0.31	0.19	0.04	0.01	1.00	0.61	0.45	0.40	0.24	0.08	0.01	1.00	0.69	0.48	0.43	0.28	0.10	0.05
	$t_b^\star$	1.00	0.53	0.45	0.38	0.30	0.09	0.03	1.00	0.58	0.46	0.43	0.28	0.13	0.03	1.00	0.66	0.48	0.44	0.29	0.10	0.04
50	t_{rob}	0.77	0.65	0.44	0.25	0.18	0.12	0.08	0.94	0.80	0.51	0.32	0.18	0.11	0.10	0.99	0.87	0.56	0.36	0.19	0.13	0.08
	t_{gls}				N/A				1.00	0.90	0.69	0.56	0.43	0.35	0.37	1.00	0.87	0.64	0.49	0.31	0.22	0.20
	$P_{\hat{e}}$	1.00	0.98	0.68	0.46	0.34	0.23	0.20	1.00	0.99	0.69	0.52	0.35	0.18	0.11	1.00	0.99	0.73	0.50	0.38	0.18	0.07
	$\text{ADF}_{\hat{F}}$	0.49	0.40	0.24	0.14	0.11	0.11	0.13	0.86	0.69	0.29	0.13	0.08	0.09	0.07	0.99	0.86	0.34	0.13	0.07	0.07	0.07

B.2 Simulation Results for Mixed Panel Tests

This section of Appendix reports the test statistics in section 3.4.3 on page 74.

The tests designed for mixed panels aim to estimate the proportion of stationary and nonstationary units. These tests can further identify which units are stationary and which are nonstationary in the panel. We are, therefore, mainly interested in two aspects of the tests: can they estimate the nonstationary proportion efficiently and consistently, and can they classify the stationary and nonstationary units correctly. We report the average estimated θ, ave($\hat{\theta}$), and the standard deviation of estimated θ, se($\hat{\theta}$), for each combination of $\{T,N,\theta\}$, where ave($\hat{\theta}$) is the simple average and se($\hat{\theta}$) is the standard deviation of $\hat{\theta}$'s over the replication. As a standard in the literature, we report the simple average of False Discovery Rate (FDR) of tests for each $\{T,N,\theta\}$ over replication, where the FDR is calculated as the proportion of false rejection in all rejections, that is, the proportion of $I(1)$ units in the units that are deemed as $I(0)$. The FDR alone is not always a good description of classification accuracy, for example, when θ is upward biased, tests reject less null hypothesis than they suppose to, which tends to reduce the possibility of wrong rejection. However, when θ is upward biased, the proportion of wrong non-rejection should increase. We therefore consider this measurement and report the false non-rejection rate (labelled as FNR), which is the proportion of $I(0)$ units in the units that are deemed as $I(1)$. The FDR can be deemed as an aggregate size measurement, and FNR can be deemed as an aggregate power measurement. A test that can identify the stationary and nonstationary units precisely should keep both FDR and FNR low. The multiple testing framework by Romano and Wolf (2005) is designed to control the Familywise Error Rate (FWE), which is the possibility of incorrectly rejecting at least one null hypothesis. The FWE is not a meaningful measurement when N is large. The multiple testing procedure by Moon and Perron (2012) is designed to control the FDR. For a comparison, we report FWE for both RW05 and MP12 tests, where FWE is calculated as the frequency that tests reject at least one null incorrectly over the replication. The Ng (2008) estimator of nonstationary proportion θ enables us to perform hypothesis testing. Therefore, for Ng (2008) test, we report the rejection frequency for three hypothesis testing: $H_0^A : \theta = 0.01$ against $H_1^A : \theta > 0.01$, $H_0^B : \theta = \theta_0$ against $H_1^B : \theta \neq \theta_0$, $H_0^C : \theta = 1$ against $H_1^C : \theta < 1$, where θ_0 is the (infeasible) true value of θ.

Simulation results for $SPSM$ test of Chortareas and Kapetanios (2009) are reported in Table B.19 on page 177 to Table B.36 on page 194. Results for Romano and Wolf (2005), Moon and Perron (2012) are reported in Table B.37 on page 195 to Table B.54 on page 212 and Table B.55 on page 213 to Table B.72 on page 230, respectively. The results for $BSQT$, $BSQT2$, $IBSQT$ and $IBSQT2$ tests of Smeekes (2011) are reported in Table B.73 on page 231 to Table B.144 on page 302. Ng (2008) test results are reported in Table B.145 on page 303 to Table B.162 on page 320.In order to compare the methods, we analyse the results from all tests for each data generating processes.

For DGP 1, unit roots are present in both common factor component and idiosyncratic component. The $SPSM$ is not affected by the magnitude of dependence and number of common factors. The $SPSM$ estimates of θ is generally upward biased. The bias is sensitive to time series dimension T, but not sensitive to N. In DGP 1a, when $N = 30$ and $\theta = 0.5$, ave$(\hat{\theta}) = 0.80$ if $T = 50$, ave$(\hat{\theta}) = 0.66$ if $T = 100$, and ave$(\hat{\theta}) = 0.60$ if $T = 200$. Both T and N reduce the standard deviation of estimates, as well as the FDR. The FDR of $SPSM$ is low, for example, in a moderate panel ($T = 100$ and $N = 30$), the FDR is 0.06 when $\theta = 1$, and is approaching 0.00 when θ is close to 0. The good FDR is, however, partly due to the fact that the test rejects less null hypothesis than it supposes to, which reduces the probability of wrong rejection. The FNR shows the proportion of $I(1)$ units that are actually $I(0)$. The results show that the FNR decreases with T but increases with N, especially when θ is close to 0. For our moderate panel, the FNR is 0.59 when $\theta = 0.1$ and 0.71 when $\theta = 0$, so the units that are classified as $I(1)$ units are not reliable when $\hat{\theta}$ is close to 0. The RW05 test is robust to the cross-sectional dependence and number of common factors. The results show that $\hat{\theta}$ is upward biased and the bias decreases with T and increases with N. $RW05$ is more sensitive to N than $SPSM$ method, due to the fact that it aims to control the FWE at a fixed level. For example, in DGP 1a, when $T = 100$ and $\theta = 0.5$, ave$(\hat{\theta}) = 0.67$ if $N = 10$, ave$(\hat{\theta}) = 0.72$ if $N = 30$, and ave$(\hat{\theta}) = 0.74$ if $N = 50$. The standard deviation is reduced with both T and N. The FWE is successfully controlled at 5% level. The FDR is lower than FWE and close to 0.00 for most cases, due to the upward bias. The FNR decreases with T but increases with N, for example, in DGP 1a with $T = 200$ and $\theta = 0.1$, FNR=0.26 if $N = 10$, FNR=0.48 if $N = 30$ and FNR=0.53 if $N = 50$. Similar to

$SPSM$, the classified $I(1)$ units are not reliable when $\hat{\theta}$ is close to 0. Results reveal that the RW05 test is suitable when T is large and N is small. The MP12 test is not affected by the cross-sectional dependence and number of common factors. The ave$(\hat{\theta})$ is upward biased when T is small and the bias is reduced with T, little bias can be seen when $T = 200$. In DGP 1a with $N = 30$ and $\theta = 0.5$, ave$(\hat{\theta}) = 0.81$ if $T = 50$, ave$(\hat{\theta}) = 0.57$ if $T = 100$, and ave$(\hat{\theta}) = 0.51$ if $T = 200$. The cross-sectional dimension N has no effect on ave$(\hat{\theta})$. The standard deviation of $\hat{\theta}$ is reduced with both T and N. The FDR is successfully controlled at 5% level, and the FWE increases with both T and N. The FNR is also low, especially when $T = 200$, both FDR and FNR are below approximately 5%. Results show that when T is as large as 100, the MP12 test can estimate θ and classify the units into $I(0)$ and $I(1)$ successfully. The $BSQT$ test is upward biased when sample size is small, but the bias is reduced with both T and N. Unless the multiple testing procedures $RW05$ and $MP12$, the tests of Smeekes (2011) can get use of the cross-sectional information and improve the performance with larger N. In DGP 1a, the ave$(\hat{\theta}) = 0.66$ for our moderate panel, and ave$(\hat{\theta}) = 0.53$ for large panel. The standard deviation of $\hat{\theta}$ decreases with both T and N. There is no clear pattern of FDR with T and N: larger T and N should improve the FDR, but the upward bias may reduce the FDR when T and N are small. However, it is clear that larger T and N can reduce the FNR, for example, when $T = 200$ and $\theta = 0.5$, the FNR is 0.18 if $N = 10$, 0.15 if $N = 30$, and 0.11 if $N = 50$. The presence of high cross-sectional dependence and two common factors improve the performance of tests for most combination of T and N, but slightly deteriorate the performance when $T = 200$ and $N \geq 30$. The $BSQT2$ test, as it is designed for, improves the finite sample performance of $BSQT$ test, for example, in our moderate panel with $\theta = 0.5$, ave$(\hat{\theta}) = 0.66$ for $BSQT$, and ave$(\hat{\theta}) = 0.59$ for $BSQT2$. Smaller FDR and FNR are also gained in $BSQT2$. However, there is no evidence that $IBSQT$ and $IBSQT2$ can improve the performance of $BSQT$ and $BSQT2$, respectively. The performance of Ng08 test is not affected by high dependence and two common factors. The Ng08 test is upward biased when $T \leq 100$ and close to the true value when $T = 200$, and the bias is not affected by N. In DGP 1a with $N = 50$ and $\theta = 0.5$, ave$(\hat{\theta}) = 0.80$ if $T = 50$, ave$(\hat{\theta}) = 0.62$ if $T = 100$, and ave$(\hat{\theta}) = 0.52$ if $T = 200$. The standard deviation is larger when θ is close to 1, and is reduced with both T and N. Note that, the standard deviation of Ng08 estimator is much higher than the standard deviation from other tests for most cases, which shows that the confidence interval of Ng08 estimator is wide and the estimator is not very reliable, especially when sample size is small. The power of Ng08 test for the null hypothesis $H_0^A : \theta = 0.01$ is reduced with greater value of T, this is due to the fact that when T is small, $\hat{\theta}$ is upward biased and $H_0^A : \theta = 0.01$ is easier to be rejected. The power of the test for the null hypothesis $H_0^C : \theta = 1$ increases with T. Both powers increases with N. In DGP 1a with $\theta = 0.5$, rejection frequency for H_0^A is 0.95 and rejection frequency of H_0^C is 0.67 for moderate panel, and rejection frequency for H_0^A is 1.00 and rejection frequency of H_0^C is 0.91 for large panel. The rejection frequency for $H_0^B : \theta = \theta_0$ is the size of test. The test is generally oversized, and the size distortion is reduced with T, and for $T = 200$, the size distortion is reduced with N. The size ranges between 0.08 to 0.20 for moderate panel and ranges between 0.06 to 0.13 for large panel. The FDR increases with T and decreases with N, but is large when θ is close to 1. The FNR decreases with T and increases with N, but is large when θ is close to 0. In short, for a satisfactory performance of Ng08 test, $T = 200$ and $N \geq 30$ is required, and the classified $I(0)$ units when $\hat{\theta}$ is close to 1 and classified $I(1)$ units when $\hat{\theta}$ is close to 0 are not reliable.

In DGP 2, panel is generated with $I(1)$ common factor component and $I(0)$ idiosyncratic component. Similar to $CIPS$, the $SPSM$ does not work under this data generating process. When one common factor is present, the ave($\hat{\theta}$) is severely downward biased, and ave($\hat{\theta}$) is approaching 0 for all values of θ when T and N are large. The presence of high dependence causes an even lower ave($\hat{\theta}$). The results are consistent with the $CIPS$ simulation in section 3.4.2, where the size distortion is approaching 1.00 for large T and N. When two common factors are present, the downward bias of ave($\hat{\theta}$) is reduced, but still shifts toward 0 with T. This is due to the fact that the $CIPS$ test is designed for single common factor and the effect of multifactors can not be proxied efficiently, and size distortion of $CIPS$ is less severe with two common factors. The performance of RW05 generally improves with greater T, smaller N, high cross-sectional dependence, and the presence of two common factors. The ave($\hat{\theta}$) of RW05 is upward biased for small θ and downward biased for large θ, but becomes precise when $T = 200$. The standard deviation decreases with both T and N, and the gain from greater N is more significant. The FWE is not controlled under this DGP and is increased with both T and N, and a greater value of θ. Similar pattern of FDR can be observed, and FDR can be high when θ is close to 1. Standard deviation of $\hat{\theta}$, FWE, and FDR are all reduced with higher dependence and two common factors. The FNR decreases with T and increases with N. The best results can be obtained in Table B.44 on page 202, where high dependence and two common factors are present. For $T = 200$ and $N = 10$, ave$(\hat{\theta}) = 0.16$ when $\theta = 0.1$, ave$(\hat{\theta}) = 0.55$ when $\theta = 0.5$, and ave$(\hat{\theta}) = 0.90$ when $\theta = 0.9$. The FWE is below 10%, the FDR is below 5% expect when $\theta = 1$, FNR is 0.28 for $\theta = 0.1$, 0.15 for $\theta = 0.3$, and below 10% for other values of θ. FWE, FDR, and FNR are deteriorated for greater N. For MP12 test, the presence of two factor improves the performance significantly, and high dependence has little effect on the test. In Table B.60 on page 218, ave$(\hat{\theta})$ is upward biased when T is small and slightly downward biased when $T = 200$. The standard deviation of $\hat{\theta}$ is reduced with T and N. The FDR is not controlled at 5%, but is below 10% for most cases. The FDR grows slowly with T and greater θ, FDR can be high when θ is close to or equal to 1. The FNR shrinks with T. For our moderate panel, ave$(\hat{\theta}) = 0.10$ when $\theta = 0.1$, ave$(\hat{\theta}) = 0.53$ when $\theta = 0.5$, and ave$(\hat{\theta}) = 0.92$ when $\theta = 0.9$. The FDR is above 0.10 when $\theta = \{0.7, 0.9\}$, FWE is 0.22 when $\theta = 0.1$, and getting smaller with greater θ. The $BSQT$ test of Smeekes (2011) is upward biased when θ is close to 0 and downward biased when θ is close to 1. The bias is reduced with both T and N, but there is no clear relationship between the bias and dependence, as well as between bias and number of factors. The standard deviation of $\hat{\theta}$ and FDR are reduced with higher dependence and two common factors, but no clear relationship between standard deviation and $\{T, N\}$, FDR and $\{T, N\}$ can be observed. FNR is reduced for greater T and N. In Table B.80 on page 238, for a moderate panel, ave$(\hat{\theta}) = 0.15$ when $\theta = 0.1$, ave$(\hat{\theta}) = 0.58$ when $\theta = 0.5$, and ave$(\hat{\theta}) = 0.962$ when $\theta = 0.9$. The FDR is below 8%, and the FNR is 0.38 when $\theta = 0.1$ and 0.25 when $\theta = 0.3$. By construction, $BSQT2$ always estimate a $\hat{\theta}$ that is no more than the $\hat{\theta}$ by $BSQT$. In this DGP, when $BSQT$ is downward biased, $BSQT2$ provides a worse estimate of θ. The $BSQT2$ does not dominate $BSQT$ under this DGP. No evidence shows that $IBSQT$ and $IBSQT2$ can improve the performance of $BSQT$ and $BSQT2$, respectively. The Ng08 estimator has no ability to estimate the correct nonstationary proportion θ, ave$(\hat{\theta})$ is close to 0 for all cases.

In DGP 3, panels are generated with $I(0)$ common factor component and $I(1)$ idiosyncratic component. The $SPSM$ test of Chortareas and Kapetanios (2009) is upward biased and the bias is reduced with T and high cross-sectional dependence. The standard deviation of $\hat{\theta}$ is reduced with both T and N. The FDR is below 5% for most cases due to the bias of $\hat{\theta}$. The FNR decreases with T, but increases with N when θ is close to 0. The FNR is high when θ is close than one, therefore, the classified $I(1)$ are not reliable with small $\hat{\theta}$. In Table B.27 on page 185, for the moderate panel, ave($\hat{\theta}$) $= 0.30$ when $\theta = 0.1$, ave($\hat{\theta}$) $= 0.77$ when $\theta = 0.5$, and ave($\hat{\theta}$) $= 0.99$ when $\theta = 0.9$. The FDR is below 3%, but FNR is 0.65 when $\theta = 0.1$ and is 0.45 when $\theta = 0.3$. The Romano and Wolf (2005) test performs well with great T and small N. In the DGP with low dependence and one common factor (Table B.45 on page 203), RW05 is slightly over biased when $T = 200$ and $N = 10$, for example, ave($\hat{\theta}$) $= 0.52$ when $\theta = 0.5$. The FWR is not controlled at 5% and ranges between 5% to 12%. The FDR is below 8% and FNR is 0.22 when $\theta = 0.1$ and is 0.11 when $\theta = 0.3$. The presence of high dependence and multi-factors, however, shifts the ave($\hat{\theta}$) down, and the test is downward biased with high dependence and two common factors. The FWE and FDR are also increased significantly due to the bias, while the FNR is reduced. FWE and FDR increase with T but FNR decreases with T. In Table B.48 on page 206, with $T = 200$ and $N = 10$, the FWE is increased and ranges between 0.34 to 0.87, FDR is 0.52 when $\theta = 0.9$, and FNR is 0.09 when $\theta = 0.1$. Similar to RW05 test, the Moon and Perron (2012) test has good performance when $T \geq 100$ with low dependence and one common factor, but is downward biased when $T \geq 100$ with high dependence and two common factors. The ave($\hat{\theta}$) is reduced with T and the bias is more severe with greater T. The FDR is not controlled at 5% and is increased with T, N, high dependence, and multi-factors. FNR is reduced with T, high dependence, and multi-factors. In Table B.48 on page 206, for a moderate panel, ave($\hat{\theta}$) $= 0.31$ when $\theta = 0.5$, FDR ranges from 0.09 to 0.51, FNR is below 5%. Similar results are also obtained in Smeekes (2011) tests: high dependence and multi-factors reduce ave($\hat{\theta}$) and hence increase the FDR. The $BSQT$ is downward biased when θ is close to 1. Unlike RW05 and MP12 tests, the ave($\hat{\theta}$) of $BSQT$ is reduced not only with T, but also with N. Therefore, the FDR increases with both T and N, and FNR decreases with both T and N. In Table B.84 on page 242, for our moderate panel, ave($\hat{\theta}$) $= 0.29$ when $\theta = 0.5$, FDR ranges from 0.06 to 0.92, FNR is 0.21 when $\theta = 0.1$ and 0.07 when $\theta = 0.3$. The $BSQT2$ test selects smaller ave($\hat{\theta}$) and worsens the bias in most cases. $IBSQT$ and $IBSQT2$ do not improve the performance of $BSQT$ and $BSQT2$, respectively. The Ng (2008) is not affected by the magnitude of dependence and number of factors. Ng08 estimator is upward biased and the bias is reduced with T. When $T = 200$ and $N = 30$, ave($\hat{\theta}$) $= 0.13$ if $\theta = 0.1$, ave($\hat{\theta}$) $= 0.54$ if $\theta = 0.5$, ave($\hat{\theta}$) $= 0.94$ if $\theta = 0.9$. The standard deviation is reduced with both T and N, but is much higher than the standard deviation of other tests employed. The power of Ng08 test for $H_0^A : \theta = 0.01$ decreases slightly with T due to the correction of bias and increases significantly with N. The test is over-sized and size distortion is reduced with both T and N, the size is between 0.06 and 0.13 with $T = 200$ and $N = 50$. The power for $H_0^C : \theta = 1$ increases with both T and N. The FDR increases with T due to the bias correction and decreases with N. The FNR decreases with T and slightly increases with N. In Table B.145 on page 303, for a moderate panel, the power for $H_0^A : \theta = 0.01$ is 0.83 when $\theta = 0.3$ and is 0.97 when $\theta = 0.5$, power of $H_0^C : \theta = 1$ is 0.33 when $\theta = 0.7$ and is 0.62 when $\theta = 0.5$. The size ranges between 0.07 to 0.19. FDR is 0.23 when $\theta = 0.9$ and FNR is 0.35 when $\theta = 0.1$.

In DGP4, panels are generated with single $I(0)$ common factor component and $I(0)$ idiosyncratic component, nonstationarity comes from the autoregressive root of $y_{i,t}$. The $SPSM$ of Chortareas and Kapetanios (2009) is not sensitive to the magnitude of cross-sectional dependence and whether the idiosyncratic component is auto correlated. The ave($\hat{\theta}$) is upward biased and the bias is reduce with T. In Table B.34 with low dependence and $ARMA(1,1)$ idiosyncratic component, for a moderate panel, ave($\hat{\theta}$) $= 0.27$ when $\theta = 0.1$, ave($\hat{\theta}$) $= 0.64$ when $\theta = 0.5$, ave($\hat{\theta}$) $= 0.97$ when $\theta = 0.9$. The standard deviation of $\hat{\theta}$ decreases with both T and N. The FDR is low due to the upward bias, and is below 5% for the moderate panel. The FNR decreases with T but increases with N when θ is close to 0, which means caution needs to be paid on the $I(1)$ units when $\hat{\theta}$ is close to zero. In the moderate panel, FNR is 0.62 when $\theta = 0.1$ and is 0.35 when $\theta = 0.3$. The Romano and Wolf (2005) test is not sensitive to the autocorrelation of idiosyncratic component. Higher cross-sectional dependence tends to shift ave($\hat{\theta}$) upwards, but the difference is small when T is large. The ave($\hat{\theta}$) is upward biased, the bias decreases with T but increases with N. The best estimates can be obtained when T is large and N is small. In Table B.52, the best estimate of $\theta = 0.5$ is obtained when $T = 200$ and $N = 10$, and ave$(\hat{\theta}) = 0.54$. The standard deviation is reduced with both T and N. The FWE is generally controlled at the pre-specified level 5%. The FDR is close to 0 for most cases due to the bias. The FNR decreases with T and increases with N. When $T = 200$ and $N = 10$, the FWE is below 5% with an exception when $\theta = 0.5$, which should be a computational error due to the small number of replications. FDR ranges between 0.01 to 0.04, FNR is 0.32 when $\theta = 0.1$ and 0.15 when $\theta = 0.3$. Similar to RW05 test, the ave$(\hat{\theta})$ of Moon and Perron (2012) test is greater when cross-sectional dependence is high, which leads higher bias when T is small but the difference becomes small when T is large. In Table B.70, when $\theta = 0.5$, the ave$(\hat{\theta}) = 0.59$ for moderate panel, and ave$(\hat{\theta}) = 0.51$ for large panel. The standard deviation decreases with both T and N. The FDR is successfully controlled at 5%, especially when $T \geq 100$. The FNR decreases with T and is below 10% when $T = 200$. For Smeekes tests ave$(\hat{\theta})$ is generally upward biased and the bias is reduced with both T and N. Both higher dependence and autocorrelation of idiosyncratic component increase ave$(\hat{\theta})$, deteriorate the bias, but the difference is small when T is large. The $BSQT2$ improves the $BSQT$ test under the DGP, but $IBSQT$ and $IBSQT2$ do not improve the tests. In Table B.106, when $\theta = 0.5$, ave$(\hat{\theta}) = 0.66$ for moderate panel, and ave$(\hat{\theta}) = 0.59$ for large panel. Standard deviation is reduced with T. FDR increases with N and ranges between 0.02 to 0.10 when $N = 30$ and between 0.03 to 0.14 when $N = 50$. FNR is reduced with T, ranges between 0.06 to 0.59 for moderate panel, and between 0.04 to 0.19 for large panel. The Ng (2008) test is not valid when cross-sectional dependence is present under this DGP. The ave$(\hat{\theta})$ is severely upward biased when $\theta = 0.3$ and even above 1.00 when $\theta \geq 0.5$. This result is due to the way we estimate the common factor. We use the Bai and Ng (2004) common factor estimator, which applies the principal component method to the differenced data and accumulates the common factor back in order to account for possible $I(1)$ common factor. The results of Ng (2008) is consistent with the simulations of $ADF_{\hat{F}}$ test of Bai and Ng (2004) reported in section 3.4.2. In DGP 4 with cross-sectional dependence, the $ADF_{\hat{F}}$ has low power when $\theta \geq 0.3$, therefore an $I(1)$ common factor is incorrectly estimated and a nonstationary idiosyncratic component is left. The Ng (2008) test is then applied on the idiosyncratic component, and the results are misleading.

In sum, some general conclusions can be drawn. First, the property of Chortareas and Kapetanios (2009) $SPSM$ test and Ng (2008) test depends on the underlying panel unit root test and the way we deal with common factor, respectively. Both tests detect the nonstationary proportion of idiosyncratic component. The $SPSM$ and Ng08 tests are not valid when panel is generated with $I(1)$ common factor and $I(0)$ idiosyncratic component, especially when the true number of common factors is used, the estimated nonstationary proportion is close to 0 for all values of θ. The Ng08 test is not valid in DGP 4 with cross-sectional dependence, because the common factor can not be extracted correctly. Second, in DGP 2 and DGP3 where unit root is present in common factor component or idiosyncratic component alone, the multiple testing methods by Romano and Wolf (2005) and Moon and Perron (2012) can not control the FWE and FDR, respectively, as they suppose to. In DGP 1 and DGP 4, the tests can successfully control the FWE and FDR at 5%, respectively. Third, time series dimension T is more crucial for all tests and a satisfactory performance requires T to be no less than 100. The multiple testing methods RW05 and MP12 do not explore the cross-sectional information, but the Smeekes (2011) quantile tests can get use of cross-sectional information and therefore benefit from greater N. The RW05 test is more suitable for small N because the FWE is harder to be controlled with large N. Generally speaking, all tests require T to be no less than 100. RW05 test is more suitable for panels with small N, while $BSQT$ and Ng08 benefit from greater N. Fourth, in DGP 1 and DGP 4, the Moon and Perron (2012) has impressing good property when T is large, in terms of being unbiased, and keeping both FDR and FNR low. In DGP 2 and DGP 3, all tests have size distortion and the distortion is more severe in DGP 3. An exception that the Ng08 estimator has little size distortion in DGP 3 if sample size is large and true number of factors is specified. If, however, the true DGP is not correctly specified, the tests base on bootstrap perform much better than others. So in an empirical application where the true DGP is unknown, the tests with bootstrapped critical values should be more reliable. Fifth, the Ng (2008) estimator generally performs poorly compared to others, but it provides us a tool to perform hypothesis testing, especially for two extreme hypothesis: $H_0^A : \theta = 0.01$ and $H_0^C : \theta = 1$, although the high power of test depends on the correct specification of underlying DGP and large sample size.

Table B.19: SPSM test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.40	0.48	0.66	0.82	0.92	0.98	0.99	0.09	0.20	0.45	0.67	0.85	0.97	0.99	0.02	0.07	0.35	0.58	0.78	0.95	0.99
	sd($\hat{\theta}$)	0.23	0.20	0.20	0.17	0.13	0.07	0.04	0.13	0.17	0.16	0.15	0.12	0.08	0.05	0.06	0.11	0.13	0.12	0.11	0.08	0.05
	FDR	0.00	0.02	0.06	0.07	0.09	0.08	0.05	0.00	0.04	0.04	0.04	0.05	0.06	0.06	0.00	0.07	0.04	0.04	0.04	0.04	0.06
	FNR	0.89	0.79	0.56	0.41	0.26	0.09	0.00	0.36	0.45	0.35	0.26	0.19	0.08	0.00	0.09	0.16	0.20	0.15	0.11	0.06	0.00
30	ave($\hat{\theta}$)	0.36	0.46	0.64	0.80	0.93	0.98	0.99	0.09	0.24	0.46	0.66	0.85	0.98	1.00	0.02	0.16	0.39	0.60	0.80	0.96	1.00
	sd($\hat{\theta}$)	0.12	0.10	0.11	0.11	0.09	0.05	0.03	0.08	0.08	0.08	0.10	0.08	0.05	0.02	0.04	0.06	0.06	0.06	0.06	0.05	0.02
	FDR	0.00	0.02	0.04	0.07	0.09	0.08	0.06	0.00	0.01	0.02	0.03	0.03	0.04	0.06	0.00	0.01	0.01	0.01	0.01	0.03	0.06
	FNR	1.00	0.79	0.55	0.39	0.25	0.09	0.00	0.71	0.59	0.36	0.25	0.18	0.08	0.00	0.27	0.40	0.24	0.17	0.12	0.07	0.00
50	ave($\hat{\theta}$)	0.33	0.42	0.61	0.78	0.92	0.99	1.00	0.10	0.24	0.45	0.66	0.84	0.98	1.00	0.03	0.17	0.39	0.60	0.79	0.96	1.00
	sd($\hat{\theta}$)	0.09	0.09	0.10	0.09	0.08	0.04	0.02	0.06	0.06	0.07	0.08	0.07	0.04	0.02	0.03	0.05	0.05	0.05	0.05	0.04	0.01
	FDR	0.00	0.02	0.05	0.07	0.09	0.08	0.06	0.00	0.01	0.02	0.02	0.03	0.03	0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.05
	FNR	1.00	0.78	0.53	0.38	0.25	0.09	0.00	0.88	0.59	0.34	0.25	0.17	0.09	0.00	0.45	0.41	0.23	0.16	0.12	0.07	0.00

Table B.20: SPSM test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.38	0.47	0.66	0.81	0.91	0.97	0.98	0.07	0.18	0.45	0.65	0.84	0.96	0.98	0.01	0.07	0.36	0.57	0.78	0.94	0.98
10	sd($\hat{\theta}$)	0.25	0.22	0.21	0.18	0.13	0.09	0.06	0.12	0.16	0.16	0.16	0.14	0.09	0.07	0.05	0.11	0.13	0.13	0.13	0.10	0.06
	FDR	0.00	0.03	0.06	0.07	0.09	0.10	0.08	0.00	0.05	0.04	0.04	0.05	0.08	0.08	0.00	0.07	0.03	0.04	0.05	0.07	0.08
	FNR	0.85	0.77	0.56	0.40	0.25	0.09	0.00	0.30	0.41	0.34	0.25	0.18	0.08	0.00	0.05	0.15	0.20	0.15	0.12	0.06	0.00
	ave($\hat{\theta}$)	0.35	0.45	0.63	0.78	0.90	0.97	0.99	0.08	0.23	0.46	0.66	0.84	0.96	0.99	0.02	0.16	0.39	0.59	0.79	0.95	0.99
30	sd($\hat{\theta}$)	0.15	0.10	0.12	0.13	0.11	0.06	0.05	0.07	0.08	0.09	0.10	0.11	0.07	0.05	0.04	0.06	0.07	0.08	0.08	0.07	0.05
	FDR	0.00	0.01	0.04	0.08	0.11	0.11	0.12	0.00	0.01	0.02	0.03	0.04	0.08	0.11	0.00	0.01	0.01	0.02	0.03	0.06	0.12
	FNR	0.99	0.79	0.54	0.38	0.25	0.09	0.00	0.65	0.58	0.36	0.25	0.17	0.08	0.00	0.21	0.37	0.24	0.16	0.12	0.06	0.00
	ave($\hat{\theta}$)	0.32	0.41	0.60	0.77	0.91	0.97	0.98	0.08	0.23	0.45	0.65	0.83	0.97	0.99	0.02	0.16	0.39	0.59	0.78	0.95	0.99
50	sd($\hat{\theta}$)	0.13	0.08	0.11	0.12	0.10	0.06	0.05	0.05	0.06	0.08	0.10	0.10	0.06	0.05	0.03	0.05	0.06	0.07	0.08	0.06	0.04
	FDR	0.00	0.02	0.05	0.08	0.10	0.12	0.16	0.00	0.01	0.02	0.03	0.04	0.07	0.13	0.00	0.01	0.01	0.02	0.03	0.06	0.12
	FNR	1.00	0.78	0.53	0.38	0.25	0.09	0.00	0.83	0.58	0.34	0.24	0.17	0.08	0.00	0.32	0.40	0.23	0.16	0.12	0.06	0.00

Table B.21: SPSM test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.40	0.44	0.63	0.81	0.93	0.98	0.99	0.09	0.17	0.42	0.65	0.84	0.97	0.99	0.02	0.06	0.34	0.58	0.78	0.95	0.99
10	sd($\hat{\theta}$)	0.25	0.22	0.20	0.17	0.11	0.07	0.06	0.14	0.16	0.17	0.15	0.13	0.08	0.05	0.07	0.10	0.13	0.12	0.11	0.09	0.04
	FDR	0.00	0.04	0.06	0.07	0.07	0.07	0.07	0.00	0.05	0.04	0.05	0.04	0.05	0.06	0.00	0.07	0.04	0.03	0.03	0.04	0.04
	FNR	0.88	0.77	0.55	0.40	0.26	0.09	0.00	0.35	0.40	0.31	0.25	0.17	0.08	0.00	0.10	0.14	0.17	0.15	0.11	0.06	0.00
	ave($\hat{\theta}$)	0.38	0.43	0.61	0.79	0.92	0.99	1.00	0.11	0.22	0.46	0.66	0.85	0.98	1.00	0.03	0.15	0.39	0.60	0.79	0.96	1.00
30	sd($\hat{\theta}$)	0.12	0.11	0.11	0.11	0.08	0.04	0.02	0.08	0.08	0.08	0.08	0.08	0.04	0.02	0.05	0.06	0.06	0.06	0.06	0.04	0.02
	FDR	0.00	0.02	0.05	0.07	0.08	0.07	0.04	0.00	0.02	0.02	0.02	0.02	0.03	0.06	0.00	0.02	0.01	0.01	0.01	0.02	0.05
	FNR	1.00	0.79	0.54	0.38	0.25	0.09	0.00	0.77	0.56	0.35	0.25	0.18	0.08	0.00	0.33	0.36	0.24	0.17	0.12	0.07	0.00
	ave($\hat{\theta}$)	0.34	0.40	0.59	0.78	0.92	0.99	1.00	0.11	0.22	0.45	0.65	0.84	0.98	1.00	0.03	0.16	0.39	0.60	0.79	0.96	1.00
50	sd($\hat{\theta}$)	0.09	0.08	0.10	0.09	0.08	0.03	0.02	0.06	0.06	0.07	0.07	0.07	0.04	0.02	0.04	0.04	0.05	0.05	0.05	0.04	0.01
	FDR	0.00	0.02	0.05	0.06	0.09	0.07	0.06	0.00	0.01	0.02	0.02	0.02	0.04	0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.05
	FNR	1.00	0.78	0.53	0.38	0.25	0.09	0.00	0.92	0.58	0.34	0.24	0.17	0.08	0.00	0.49	0.39	0.23	0.16	0.12	0.07	0.00

Table B.22: SPSM test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.33	0.42	0.63	0.79	0.91	0.96	0.97	0.06	0.13	0.42	0.64	0.83	0.95	0.97	0.01	0.04	0.34	0.57	0.77	0.93	0.97
10	sd($\hat{\theta}$)	0.26	0.23	0.21	0.18	0.14	0.11	0.10	0.11	0.15	0.17	0.18	0.16	0.12	0.09	0.05	0.09	0.14	0.13	0.14	0.11	0.09
	FDR	0.00	0.04	0.06	0.08	0.09	0.12	0.13	0.00	0.06	0.05	0.06	0.06	0.09	0.12	0.00	0.08	0.04	0.04	0.06	0.08	0.12
	FNR	0.76	0.76	0.53	0.39	0.25	0.09	0.00	0.25	0.35	0.31	0.24	0.18	0.08	0.00	0.05	0.09	0.17	0.15	0.11	0.06	0.00
	ave($\hat{\theta}$)	0.32	0.42	0.61	0.78	0.90	0.95	0.97	0.06	0.21	0.45	0.65	0.82	0.95	0.97	0.01	0.15	0.39	0.59	0.77	0.93	0.97
30	sd($\hat{\theta}$)	0.19	0.11	0.13	0.15	0.13	0.10	0.09	0.06	0.08	0.11	0.13	0.14	0.09	0.08	0.03	0.06	0.08	0.09	0.12	0.10	0.08
	FDR	0.00	0.02	0.04	0.07	0.10	0.17	0.21	0.00	0.02	0.03	0.04	0.07	0.11	0.19	0.00	0.01	0.02	0.02	0.06	0.11	0.19
	FNR	0.98	0.78	0.53	0.38	0.24	0.09	0.00	0.60	0.55	0.35	0.25	0.17	0.07	0.00	0.17	0.34	0.24	0.17	0.12	0.06	0.00
	ave($\hat{\theta}$)	0.30	0.39	0.59	0.77	0.88	0.95	0.96	0.06	0.22	0.43	0.65	0.82	0.95	0.97	0.01	0.16	0.38	0.58	0.77	0.93	0.97
50	sd($\hat{\theta}$)	0.15	0.09	0.14	0.14	0.14	0.11	0.10	0.05	0.06	0.10	0.12	0.13	0.10	0.09	0.02	0.05	0.07	0.10	0.11	0.10	0.08
	FDR	0.00	0.02	0.05	0.07	0.13	0.17	0.24	0.00	0.01	0.03	0.04	0.06	0.12	0.21	0.00	0.01	0.02	0.03	0.06	0.11	0.22
	FNR	1.00	0.77	0.53	0.38	0.24	0.09	0.00	0.77	0.57	0.33	0.25	0.17	0.07	0.00	0.29	0.39	0.23	0.16	0.12	0.06	0.00

Table B.23: SPSM test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.39	0.43	0.49	0.52	0.52	0.51	0.49	0.09	0.14	0.22	0.24	0.25	0.24	0.24	0.02	0.05	0.10	0.12	0.13	0.14	0.14
10	sd($\hat{\theta}$)	0.24	0.21	0.23	0.25	0.27	0.27	0.27	0.14	0.15	0.19	0.22	0.24	0.25	0.24	0.07	0.09	0.14	0.17	0.19	0.21	0.22
	FDR	0.00	0.05	0.18	0.36	0.55	0.80	0.93	0.00	0.06	0.21	0.40	0.61	0.85	0.97	0.00	0.08	0.24	0.43	0.65	0.87	0.98
	FNR	0.88	0.78	0.58	0.41	0.24	0.08	0.00	0.37	0.36	0.30	0.20	0.11	0.04	0.00	0.09	0.12	0.11	0.07	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.37	0.41	0.43	0.41	0.39	0.37	0.37	0.10	0.16	0.18	0.17	0.16	0.14	0.14	0.02	0.07	0.08	0.07	0.06	0.06	0.06
30	sd($\hat{\theta}$)	0.13	0.11	0.15	0.15	0.15	0.14	0.14	0.08	0.09	0.12	0.13	0.12	0.10	0.11	0.04	0.07	0.09	0.09	0.08	0.07	0.07
	FDR	0.00	0.04	0.21	0.43	0.66	0.89	1.00	0.00	0.06	0.24	0.46	0.68	0.89	1.00	0.00	0.07	0.26	0.47	0.69	0.90	1.00
	FNR	1.00	0.82	0.60	0.43	0.26	0.09	0.00	0.74	0.61	0.43	0.33	0.19	0.06	0.00	0.24	0.31	0.23	0.16	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.34	0.37	0.37	0.35	0.33	0.31	0.31	0.09	0.15	0.15	0.14	0.13	0.12	0.12	0.02	0.06	0.06	0.05	0.05	0.05	0.05
50	sd($\hat{\theta}$)	0.10	0.09	0.11	0.11	0.11	0.10	0.10	0.06	0.07	0.09	0.08	0.07	0.06	0.06	0.03	0.06	0.06	0.06	0.05	0.04	0.04
	FDR	0.00	0.05	0.23	0.46	0.68	0.90	1.00	0.00	0.06	0.26	0.48	0.69	0.90	1.00	0.00	0.07	0.28	0.49	0.69	0.90	1.00
	FNR	1.00	0.82	0.61	0.44	0.27	0.09	0.00	0.90	0.67	0.50	0.38	0.22	0.08	0.00	0.44	0.39	0.31	0.25	0.13	0.05	0.00

Table B.24: SPSM test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.37	0.44	0.57	0.64	0.72	0.78	0.79	0.07	0.15	0.31	0.44	0.54	0.63	0.68	0.01	0.05	0.22	0.33	0.44	0.55	0.62
10	sd($\hat{\theta}$)	0.25	0.21	0.24	0.25	0.26	0.26	0.26	0.12	0.15	0.20	0.25	0.30	0.32	0.33	0.05	0.10	0.16	0.23	0.29	0.33	0.35
	FDR	0.00	0.04	0.11	0.22	0.34	0.46	0.54	0.00	0.05	0.13	0.23	0.38	0.57	0.67	0.00	0.08	0.15	0.26	0.41	0.61	0.74
	FNR	0.85	0.78	0.54	0.37	0.23	0.08	0.00	0.28	0.38	0.29	0.19	0.12	0.04	0.00	0.06	0.12	0.14	0.09	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.36	0.41	0.51	0.59	0.68	0.72	0.76	0.07	0.18	0.31	0.43	0.52	0.62	0.65	0.01	0.10	0.24	0.34	0.45	0.55	0.59
30	sd($\hat{\theta}$)	0.15	0.11	0.15	0.20	0.24	0.26	0.26	0.07	0.09	0.15	0.21	0.27	0.31	0.32	0.03	0.07	0.14	0.20	0.27	0.32	0.34
	FDR	0.00	0.03	0.12	0.26	0.38	0.57	0.65	0.00	0.04	0.13	0.24	0.40	0.59	0.75	0.00	0.04	0.13	0.26	0.42	0.63	0.83
	FNR	1.00	0.80	0.54	0.37	0.22	0.08	0.00	0.64	0.57	0.34	0.21	0.12	0.04	0.00	0.19	0.31	0.20	0.11	0.07	0.03	0.00
	ave($\hat{\theta}$)	0.32	0.38	0.48	0.56	0.65	0.71	0.73	0.08	0.19	0.29	0.41	0.51	0.58	0.63	0.02	0.11	0.22	0.32	0.45	0.52	0.58
50	sd($\hat{\theta}$)	0.12	0.09	0.15	0.21	0.25	0.28	0.28	0.05	0.07	0.14	0.21	0.27	0.32	0.33	0.03	0.06	0.13	0.20	0.27	0.32	0.34
	FDR	0.00	0.04	0.14	0.26	0.40	0.57	0.68	0.00	0.03	0.14	0.26	0.41	0.64	0.77	0.00	0.04	0.15	0.28	0.42	0.67	0.84
	FNR	1.00	0.79	0.54	0.36	0.21	0.07	0.00	0.83	0.60	0.34	0.21	0.12	0.04	0.00	0.32	0.38	0.20	0.12	0.07	0.03	0.00

Table B.25: SPSM test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.38	0.42	0.42	0.40	0.37	0.36	0.34	0.10	0.14	0.15	0.13	0.11	0.12	0.12	0.03	0.04	0.05	0.04	0.04	0.04	0.05
	sd($\hat{\theta}$)	0.24	0.21	0.23	0.24	0.22	0.23	0.21	0.14	0.15	0.17	0.16	0.14	0.14	0.14	0.07	0.09	0.10	0.09	0.09	0.08	0.09
	FDR	0.00	0.04	0.22	0.43	0.66	0.89	0.99	0.00	0.06	0.25	0.47	0.69	0.90	1.00	0.00	0.08	0.27	0.48	0.69	0.90	1.00
	FNR	0.86	0.78	0.56	0.41	0.24	0.08	0.00	0.36	0.36	0.26	0.17	0.10	0.03	0.00	0.11	0.10	0.06	0.04	0.03	0.01	0.00
30	ave($\hat{\theta}$)	0.37	0.38	0.35	0.33	0.31	0.31	0.31	0.11	0.13	0.11	0.11	0.10	0.10	0.10	0.03	0.05	0.04	0.04	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.12	0.11	0.12	0.12	0.11	0.11	0.12	0.08	0.09	0.08	0.08	0.07	0.08	0.07	0.05	0.06	0.05	0.05	0.05	0.04	0.04
	FDR	0.00	0.05	0.26	0.48	0.69	0.90	1.00	0.00	0.07	0.28	0.49	0.69	0.90	1.00	0.00	0.08	0.29	0.49	0.70	0.90	1.00
	FNR	1.00	0.82	0.64	0.47	0.29	0.09	0.00	0.76	0.61	0.43	0.33	0.18	0.06	0.00	0.34	0.26	0.18	0.13	0.08	0.03	0.00
50	ave($\hat{\theta}$)	0.34	0.33	0.31	0.29	0.28	0.28	0.28	0.11	0.12	0.11	0.10	0.09	0.09	0.09	0.03	0.04	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.08	0.09	0.08	0.08	0.09	0.08	0.08	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.03
	FDR	0.00	0.06	0.28	0.49	0.70	0.90	1.00	0.00	0.08	0.29	0.49	0.70	0.90	1.00	0.00	0.09	0.29	0.50	0.70	0.90	1.00
	FNR	1.00	0.83	0.66	0.49	0.30	0.10	0.00	0.90	0.69	0.56	0.40	0.25	0.08	0.00	0.51	0.38	0.28	0.21	0.12	0.04	0.00

Table B.26: SPSM test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.36	0.41	0.53	0.65	0.73	0.77	0.80	0.06	0.13	0.31	0.46	0.60	0.68	0.71	0.01	0.05	0.22	0.37	0.51	0.64	0.68
10	sd($\hat{\theta}$)	0.28	0.23	0.24	0.27	0.29	0.31	0.30	0.12	0.14	0.21	0.27	0.32	0.36	0.36	0.04	0.09	0.17	0.24	0.31	0.36	0.37
	FDR	0.00	0.04	0.11	0.20	0.28	0.39	0.43	0.00	0.07	0.12	0.20	0.31	0.45	0.54	0.00	0.08	0.14	0.22	0.33	0.48	0.61
	FNR	0.79	0.74	0.52	0.36	0.22	0.08	0.00	0.23	0.34	0.26	0.19	0.13	0.05	0.00	0.05	0.12	0.12	0.10	0.07	0.03	0.00
	ave($\hat{\theta}$)	0.33	0.39	0.52	0.63	0.72	0.76	0.77	0.06	0.18	0.34	0.47	0.60	0.65	0.70	0.01	0.10	0.26	0.41	0.50	0.61	0.66
30	sd($\hat{\theta}$)	0.18	0.11	0.18	0.24	0.27	0.29	0.29	0.06	0.09	0.17	0.24	0.30	0.35	0.35	0.03	0.07	0.15	0.24	0.30	0.36	0.37
	FDR	0.00	0.03	0.11	0.21	0.31	0.45	0.55	0.00	0.03	0.11	0.21	0.32	0.52	0.62	0.00	0.04	0.12	0.21	0.36	0.54	0.68
	FNR	0.98	0.79	0.55	0.38	0.22	0.08	0.00	0.58	0.55	0.34	0.22	0.14	0.05	0.00	0.18	0.29	0.19	0.13	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.31	0.36	0.49	0.60	0.68	0.74	0.75	0.06	0.18	0.33	0.46	0.57	0.67	0.68	0.02	0.11	0.26	0.39	0.50	0.60	0.66
50	sd($\hat{\theta}$)	0.17	0.09	0.17	0.24	0.28	0.29	0.31	0.05	0.08	0.16	0.24	0.31	0.34	0.36	0.03	0.07	0.15	0.23	0.30	0.36	0.36
	FDR	0.00	0.03	0.12	0.22	0.34	0.49	0.56	0.00	0.03	0.11	0.22	0.34	0.52	0.64	0.00	0.04	0.12	0.22	0.36	0.55	0.72
	FNR	1.00	0.78	0.54	0.37	0.23	0.00	0.08	0.75	0.58	0.34	0.23	0.15	0.05	0.00	0.31	0.35	0.20	0.13	0.08	0.04	0.00

Table B.27: SPSM test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.38	0.58	0.79	0.89	0.96	0.99	0.99	0.09	0.28	0.56	0.76	0.90	0.97	0.99	0.01	0.15	0.41	0.62	0.82	0.96	0.99
10	sd($\hat{\theta}$)	0.23	0.24	0.19	0.15	0.10	0.06	0.04	0.13	0.21	0.19	0.17	0.13	0.08	0.05	0.06	0.16	0.17	0.16	0.14	0.08	0.06
	FDR	0.00	0.02	0.04	0.07	0.06	0.05	0.06	0.00	0.03	0.04	0.05	0.06	0.07	0.07	0.00	0.05	0.04	0.05	0.05	0.06	0.07
	FNR	0.88	0.81	0.62	0.45	0.28	0.10	0.00	0.35	0.54	0.46	0.35	0.23	0.09	0.00	0.08	0.32	0.30	0.21	0.16	0.07	0.00
	ave($\hat{\theta}$)	0.37	0.54	0.77	0.91	0.98	1.00	1.00	0.10	0.30	0.56	0.77	0.93	0.99	1.00	0.02	0.19	0.44	0.66	0.84	0.98	1.00
30	sd($\hat{\theta}$)	0.13	0.14	0.13	0.09	0.04	0.02	0.01	0.08	0.11	0.11	0.10	0.07	0.03	0.01	0.04	0.08	0.08	0.08	0.07	0.04	0.02
	FDR	0.00	0.01	0.03	0.05	0.04	0.01	0.01	0.00	0.01	0.02	0.03	0.02	0.02	0.02	0.00	0.01	0.01	0.01	0.02	0.02	0.04
	FNR	1.00	0.82	0.62	0.45	0.29	0.10	0.00	0.75	0.66	0.47	0.35	0.25	0.09	0.00	0.28	0.46	0.32	0.24	0.17	0.08	0.00
	ave($\hat{\theta}$)	0.33	0.51	0.75	0.91	0.99	1.00	1.00	0.10	0.28	0.54	0.76	0.93	1.00	1.00	0.03	0.20	0.44	0.66	0.85	0.99	1.00
50	sd($\hat{\theta}$)	0.09	0.11	0.10	0.08	0.03	0.01	0.01	0.06	0.07	0.08	0.08	0.06	0.02	0.00	0.03	0.06	0.06	0.07	0.05	0.03	0.01
	FDR	0.00	0.01	0.03	0.04	0.03	0.01	0.01	0.00	0.01	0.02	0.02	0.02	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01
	FNR	1.00	0.81	0.61	0.45	0.29	0.10	0.00	0.90	0.65	0.45	0.35	0.25	0.10	0.00	0.44	0.49	0.31	0.24	0.18	0.09	0.00

Table B.28: SPSM test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.37	0.55	0.77	0.88	0.95	0.98	0.99	0.06	0.27	0.52	0.72	0.87	0.96	0.98	0.01	0.13	0.37	0.57	0.77	0.93	0.97
10	sd($\hat{\theta}$)	0.24	0.25	0.20	0.15	0.10	0.07	0.04	0.11	0.19	0.20	0.18	0.14	0.09	0.07	0.05	0.15	0.16	0.16	0.16	0.11	0.08
	FDR	0.00	0.02	0.06	0.08	0.10	0.09	0.07	0.00	0.03	0.05	0.07	0.10	0.11	0.10	0.00	0.05	0.06	0.08	0.09	0.12	0.17
	FNR	0.85	0.78	0.62	0.45	0.28	0.10	0.00	0.26	0.52	0.44	0.32	0.22	0.08	0.00	0.05	0.29	0.26	0.19	0.13	0.06	0.00
	ave($\hat{\theta}$)	0.36	0.51	0.72	0.88	0.97	0.99	1.00	0.07	0.27	0.52	0.71	0.88	0.98	0.99	0.02	0.17	0.40	0.60	0.80	0.95	0.99
30	sd($\hat{\theta}$)	0.15	0.14	0.14	0.10	0.06	0.03	0.02	0.07	0.10	0.10	0.10	0.09	0.04	0.03	0.03	0.08	0.08	0.08	0.08	0.06	0.04
	FDR	0.00	0.02	0.05	0.07	0.08	0.05	0.04	0.00	0.01	0.03	0.04	0.05	0.06	0.06	0.00	0.02	0.02	0.04	0.05	0.08	0.13
	FNR	0.99	0.81	0.60	0.44	0.28	0.10	0.00	0.64	0.63	0.43	0.31	0.21	0.09	0.00	0.19	0.43	0.28	0.19	0.13	0.07	0.00
	ave($\hat{\theta}$)	0.32	0.48	0.70	0.86	0.97	1.00	1.00	0.07	0.26	0.49	0.71	0.88	0.99	1.00	0.02	0.17	0.40	0.60	0.80	0.96	0.99
50	sd($\hat{\theta}$)	0.12	0.13	0.12	0.09	0.05	0.02	0.01	0.05	0.07	0.08	0.08	0.07	0.03	0.01	0.03	0.06	0.06	0.07	0.06	0.05	0.03
	FDR	0.00	0.02	0.05	0.07	0.07	0.04	0.03	0.00	0.01	0.02	0.04	0.05	0.05	0.03	0.00	0.01	0.02	0.03	0.03	0.05	0.10
	FNR	1.00	0.80	0.58	0.43	0.28	0.00	0.10	0.81	0.62	0.41	0.30	0.21	0.09	0.00	0.34	0.43	0.27	0.18	0.13	0.07	0.00

Table B.29: SPSM test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.40	0.61	0.85	0.95	0.98	1.00	1.00	0.10	0.37	0.67	0.85	0.96	0.99	1.00	0.03	0.25	0.55	0.75	0.91	0.99	1.00
10	sd($\hat{\theta}$)	0.25	0.24	0.17	0.10	0.06	0.02	0.03	0.14	0.23	0.19	0.15	0.08	0.04	0.03	0.08	0.19	0.18	0.16	0.11	0.05	0.03
	FDR	0.00	0.01	0.03	0.03	0.03	0.01	0.01	0.00	0.02	0.03	0.03	0.03	0.02	0.02	0.00	0.03	0.03	0.03	0.02	0.02	0.03
	FNR	0.88	0.81	0.64	0.48	0.29	0.10	0.00	0.37	0.62	0.54	0.41	0.27	0.10	0.00	0.11	0.48	0.44	0.33	0.23	0.09	0.00
	ave($\hat{\theta}$)	0.37	0.55	0.79	0.94	0.99	1.00	1.00	0.11	0.35	0.62	0.82	0.96	1.00	1.00	0.03	0.25	0.52	0.73	0.91	1.00	1.00
30	sd($\hat{\theta}$)	0.12	0.12	0.11	0.07	0.02	0.00	0.00	0.08	0.10	0.10	0.09	0.05	0.01	0.00	0.05	0.09	0.08	0.08	0.06	0.01	0.00
	FDR	0.00	0.01	0.03	0.04	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00
	FNR	1.00	0.82	0.62	0.47	0.30	0.10	0.00	0.75	0.71	0.51	0.39	0.27	0.10	0.00	0.32	0.58	0.42	0.31	0.23	0.10	0.00
	ave($\hat{\theta}$)	0.34	0.51	0.76	0.92	0.99	1.00	1.00	0.11	0.31	0.58	0.80	0.96	1.00	1.00	0.03	0.23	0.49	0.72	0.90	1.00	1.00
50	sd($\hat{\theta}$)	0.09	0.10	0.09	0.07	0.02	0.00	0.00	0.06	0.07	0.08	0.07	0.04	0.00	0.00	0.03	0.06	0.06	0.06	0.04	0.01	0.00
	FDR	0.00	0.01	0.03	0.03	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	FNR	1.00	0.81	0.61	0.46	0.30	0.10	0.00	0.92	0.68	0.48	0.37	0.27	0.10	0.00	0.51	0.56	0.39	0.30	0.22	0.10	0.00

Table B.30: SPSM test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.33	0.53	0.75	0.87	0.94	0.98	0.99	0.06	0.27	0.52	0.72	0.86	0.95	0.97	0.01	0.18	0.42	0.60	0.77	0.91	0.95
	sd($\hat{\theta}$)	0.27	0.27	0.22	0.16	0.13	0.07	0.05	0.12	0.20	0.20	0.19	0.15	0.10	0.08	0.05	0.16	0.18	0.18	0.15	0.12	0.10
	FDR	0.00	0.02	0.07	0.10	0.11	0.10	0.08	0.00	0.03	0.07	0.10	0.15	0.16	0.17	0.00	0.04	0.08	0.11	0.16	0.21	0.29
	FNR	0.76	0.76	0.61	0.45	0.28	0.10	0.00	0.24	0.51	0.45	0.34	0.22	0.08	0.00	0.07	0.37	0.35	0.24	0.15	0.06	0.00
30	ave($\hat{\theta}$)	0.33	0.47	0.68	0.83	0.93	0.98	0.99	0.06	0.26	0.48	0.68	0.85	0.96	0.99	0.01	0.18	0.41	0.59	0.77	0.93	0.97
	sd($\hat{\theta}$)	0.19	0.18	0.17	0.13	0.09	0.05	0.04	0.06	0.10	0.10	0.11	0.10	0.06	0.04	0.03	0.08	0.08	0.09	0.08	0.06	0.05
	FDR	0.00	0.02	0.07	0.11	0.14	0.12	0.10	0.00	0.02	0.04	0.08	0.10	0.14	0.15	0.00	0.02	0.04	0.07	0.09	0.17	0.29
	FNR	0.98	0.80	0.58	0.42	0.27	0.10	0.00	0.57	0.62	0.41	0.29	0.20	0.08	0.00	0.18	0.48	0.31	0.20	0.13	0.06	0.00
50	ave($\hat{\theta}$)	0.30	0.44	0.65	0.81	0.92	0.98	0.99	0.06	0.23	0.46	0.65	0.83	0.96	0.99	0.02	0.17	0.38	0.58	0.76	0.93	0.98
	sd($\hat{\theta}$)	0.17	0.16	0.15	0.13	0.09	0.05	0.03	0.05	0.06	0.08	0.09	0.08	0.05	0.03	0.02	0.06	0.06	0.07	0.06	0.06	0.04
	FDR	0.00	0.02	0.07	0.11	0.16	0.15	0.11	0.00	0.02	0.04	0.07	0.10	0.14	0.13	0.00	0.02	0.04	0.06	0.08	0.16	0.25
	FNR	1.00	0.78	0.56	0.41	0.26	0.09	0.00	0.76	0.60	0.38	0.26	0.18	0.08	0.00	0.32	0.46	0.27	0.17	0.11	0.05	0.00

Table B.31: SPSM test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.25	0.39	0.61	0.78	0.92	0.98	0.99	0.08	0.18	0.43	0.65	0.84	0.97	0.99	0.02	0.09	0.36	0.57	0.78	0.95	0.99
10	sd($\hat{\theta}$)	0.17	0.18	0.17	0.16	0.11	0.06	0.05	0.10	0.15	0.14	0.14	0.12	0.06	0.04	0.05	0.11	0.12	0.12	0.10	0.07	0.03
	FDR	0.00	0.02	0.04	0.05	0.06	0.06	0.06	0.00	0.04	0.04	0.04	0.04	0.04	0.05	0.00	0.06	0.04	0.03	0.03	0.03	0.04
	FNR	0.87	0.73	0.52	0.37	0.24	0.09	0.00	0.47	0.46	0.32	0.24	0.17	0.07	0.00	0.16	0.24	0.21	0.14	0.11	0.06	0.00
	ave($\hat{\theta}$)	0.26	0.38	0.59	0.76	0.91	0.99	1.00	0.07	0.22	0.46	0.65	0.83	0.97	1.00	0.02	0.16	0.40	0.59	0.79	0.95	1.00
30	sd($\hat{\theta}$)	0.10	0.09	0.10	0.09	0.08	0.03	0.02	0.06	0.07	0.07	0.08	0.07	0.04	0.02	0.04	0.06	0.05	0.06	0.06	0.04	0.02
	FDR	0.00	0.01	0.03	0.04	0.05	0.05	0.04	0.00	0.01	0.01	0.02	0.02	0.03	0.04	0.00	0.01	0.01	0.01	0.01	0.02	0.04
	FNR	1.00	0.75	0.50	0.35	0.24	0.09	0.00	0.75	0.56	0.35	0.24	0.16	0.08	0.00	0.27	0.38	0.24	0.16	0.11	0.06	0.00
	ave($\hat{\theta}$)	0.24	0.35	0.57	0.75	0.91	0.99	1.00	0.07	0.22	0.44	0.65	0.83	0.98	1.00	0.02	0.17	0.39	0.59	0.79	0.96	1.00
50	sd($\hat{\theta}$)	0.06	0.07	0.08	0.08	0.07	0.03	0.01	0.05	0.05	0.06	0.07	0.06	0.04	0.02	0.03	0.04	0.04	0.05	0.04	0.04	0.01
	FDR	0.00	0.01	0.03	0.04	0.05	0.04	0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.04
	FNR	1.00	0.73	0.49	0.35	0.23	0.09	0.00	0.89	0.56	0.33	0.23	0.16	0.08	0.00	0.43	0.41	0.23	0.16	0.11	0.06	0.00

Table B.32: SPSM test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.32	0.44	0.65	0.82	0.93	0.98	0.99	0.10	0.22	0.47	0.67	0.85	0.97	0.99	0.03	0.11	0.37	0.60	0.78	0.95	0.99
	sd($\hat{\theta}$)	0.19	0.19	0.19	0.15	0.11	0.06	0.04	0.11	0.16	0.15	0.14	0.11	0.06	0.04	0.06	0.12	0.13	0.12	0.11	0.07	0.04
	FDR	0.00	0.02	0.04	0.05	0.05	0.05	0.05	0.00	0.03	0.04	0.04	0.04	0.05	0.04	0.00	0.05	0.04	0.03	0.03	0.03	0.05
	FNR	0.91	0.76	0.54	0.40	0.26	0.09	0.00	0.51	0.51	0.38	0.27	0.18	0.08	0.00	0.19	0.26	0.22	0.17	0.11	0.06	0.00
30	ave($\hat{\theta}$)	0.31	0.41	0.61	0.78	0.91	0.99	1.00	0.10	0.24	0.47	0.67	0.84	0.97	1.00	0.03	0.17	0.40	0.60	0.79	0.96	1.00
	sd($\hat{\theta}$)	0.10	0.10	0.10	0.09	0.08	0.04	0.02	0.07	0.08	0.07	0.08	0.07	0.04	0.02	0.04	0.06	0.06	0.06	0.06	0.04	0.02
	FDR	0.00	0.01	0.03	0.05	0.06	0.06	0.05	0.00	0.01	0.01	0.02	0.02	0.03	0.05	0.00	0.01	0.01	0.01	0.01	0.02	0.06
	FNR	1.00	0.77	0.52	0.37	0.24	0.09	0.00	0.85	0.59	0.36	0.25	0.17	0.08	0.00	0.37	0.41	0.25	0.17	0.12	0.06	0.00
50	ave($\hat{\theta}$)	0.28	0.39	0.59	0.77	0.90	0.98	1.00	0.09	0.23	0.45	0.65	0.84	0.98	1.00	0.03	0.18	0.39	0.59	0.79	0.96	1.00
	sd($\hat{\theta}$)	0.07	0.07	0.09	0.08	0.07	0.04	0.02	0.05	0.05	0.06	0.07	0.06	0.04	0.02	0.03	0.04	0.05	0.05	0.05	0.04	0.01
	FDR	0.00	0.01	0.03	0.05	0.07	0.07	0.06	0.00	0.01	0.01	0.02	0.02	0.03	0.07	0.00	0.01	0.01	0.01	0.01	0.02	0.05
	FNR	1.00	0.76	0.51	0.36	0.23	0.09	0.00	0.94	0.59	0.35	0.24	0.17	0.08	0.00	0.56	0.43	0.24	0.16	0.12	0.06	0.00

Table B.33: SPSM test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

	Test	T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	$ave(\hat{\theta})$	0.28	0.46	0.62	0.75	0.87	0.97	0.99	0.11	0.27	0.48	0.64	0.81	0.94	0.99	0.05	0.19	0.40	0.57	0.75	0.93	0.99
10	$sd(\hat{\theta})$	0.22	0.26	0.22	0.19	0.14	0.07	0.05	0.17	0.25	0.21	0.17	0.13	0.09	0.05	0.14	0.23	0.18	0.14	0.11	0.08	0.05
	FDR	0.00	0.03	0.06	0.07	0.08	0.06	0.07	0.00	0.04	0.04	0.04	0.04	0.06	0.07	0.00	0.05	0.04	0.04	0.04	0.05	0.07
	FNR	0.79	0.73	0.52	0.35	0.22	0.08	0.00	0.39	0.50	0.36	0.23	0.14	0.06	0.00	0.17	0.36	0.23	0.14	0.09	0.04	0.00
	$ave(\hat{\theta})$	0.25	0.41	0.56	0.71	0.86	0.97	0.99	0.09	0.26	0.44	0.61	0.79	0.95	0.99	0.04	0.20	0.39	0.57	0.76	0.94	0.99
30	$sd(\hat{\theta})$	0.13	0.14	0.13	0.12	0.09	0.05	0.04	0.09	0.13	0.11	0.09	0.08	0.05	0.03	0.07	0.12	0.09	0.07	0.06	0.05	0.03
	FDR	0.00	0.02	0.04	0.05	0.06	0.07	0.08	0.00	0.02	0.02	0.02	0.02	0.04	0.07	0.00	0.02	0.01	0.01	0.02	0.03	0.07
	FNR	0.98	0.76	0.48	0.31	0.19	0.08	0.00	0.70	0.61	0.32	0.19	0.12	0.06	0.00	0.33	0.44	0.22	0.12	0.08	0.04	0.00
	$ave(\hat{\theta})$	0.22	0.36	0.51	0.69	0.84	0.97	0.99	0.08	0.24	0.41	0.60	0.79	0.96	1.00	0.04	0.18	0.36	0.56	0.76	0.94	1.00
50	$sd(\hat{\theta})$	0.08	0.10	0.11	0.10	0.08	0.05	0.02	0.06	0.10	0.08	0.06	0.06	0.04	0.01	0.05	0.08	0.06	0.05	0.05	0.04	0.02
	FDR	0.00	0.02	0.04	0.04	0.05	0.06	0.08	0.00	0.02	0.02	0.02	0.02	0.02	0.06	0.00	0.01	0.01	0.01	0.01	0.02	0.06
	FNR	1.00	0.74	0.44	0.29	0.18	0.08	0.00	0.85	0.59	0.28	0.17	0.11	0.07	0.00	0.48	0.43	0.18	0.11	0.08	0.05	0.00

Table B.34: SPSM test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.35	0.51	0.67	0.80	0.91	0.98	0.99	0.12	0.29	0.50	0.67	0.83	0.96	0.99	0.05	0.22	0.41	0.61	0.78	0.94	0.99
10	sd($\hat{\theta}$)	0.24	0.26	0.22	0.18	0.12	0.07	0.05	0.17	0.25	0.22	0.17	0.13	0.07	0.05	0.12	0.23	0.19	0.14	0.11	0.08	0.04
	FDR	0.00	0.03	0.06	0.06	0.05	0.06	0.05	0.00	0.04	0.04	0.04	0.04	0.04	0.06	0.00	0.04	0.04	0.02	0.03	0.04	0.05
	FNR	0.84	0.77	0.55	0.38	0.24	0.09	0.00	0.42	0.51	0.38	0.25	0.16	0.07	0.00	0.17	0.41	0.25	0.17	0.11	0.05	0.00
	ave($\hat{\theta}$)	0.31	0.46	0.61	0.74	0.88	0.98	1.00	0.12	0.27	0.46	0.64	0.82	0.97	1.00	0.05	0.20	0.40	0.59	0.77	0.95	1.00
30	sd($\hat{\theta}$)	0.13	0.15	0.14	0.12	0.09	0.04	0.02	0.10	0.13	0.11	0.09	0.07	0.05	0.03	0.08	0.12	0.09	0.07	0.06	0.04	0.02
	FDR	0.00	0.02	0.05	0.05	0.05	0.05	0.06	0.00	0.02	0.02	0.02	0.02	0.03	0.05	0.00	0.02	0.01	0.01	0.01	0.02	0.05
	FNR	0.99	0.79	0.52	0.34	0.21	0.09	0.00	0.78	0.62	0.35	0.22	0.15	0.07	0.00	0.39	0.47	0.24	0.15	0.10	0.06	0.00
	ave($\hat{\theta}$)	0.27	0.41	0.55	0.71	0.87	0.98	1.00	0.10	0.25	0.43	0.62	0.81	0.97	1.00	0.04	0.19	0.38	0.58	0.77	0.95	1.00
50	sd($\hat{\theta}$)	0.09	0.11	0.11	0.10	0.08	0.04	0.02	0.07	0.08	0.08	0.07	0.06	0.04	0.02	0.05	0.08	0.06	0.05	0.05	0.04	0.02
	FDR	0.00	0.02	0.05	0.05	0.05	0.05	0.07	0.00	0.01	0.02	0.02	0.01	0.02	0.04	0.00	0.01	0.01	0.01	0.01	0.01	0.04
	FNR	1.00	0.77	0.48	0.32	0.20	0.09	0.00	0.90	0.61	0.31	0.20	0.14	0.07	0.00	0.56	0.47	0.21	0.14	0.10	0.06	0.00

Table B.35: SPSM test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.20	0.34	0.48	0.64	0.81	0.95	0.99	0.07	0.17	0.37	0.55	0.75	0.93	0.99	0.02	0.12	0.33	0.52	0.72	0.90	0.99
	sd($\hat{\theta}$)	0.18	0.25	0.21	0.18	0.13	0.09	0.05	0.13	0.21	0.18	0.12	0.11	0.08	0.05	0.08	0.19	0.14	0.10	0.09	0.10	0.05
	FDR	0.00	0.05	0.10	0.08	0.06	0.07	0.06	0.00	0.06	0.08	0.05	0.04	0.05	0.07	0.00	0.07	0.06	0.04	0.03	0.05	0.08
	FNR	0.67	0.64	0.42	0.25	0.15	0.07	0.00	0.27	0.37	0.24	0.13	0.08	0.04	0.00	0.10	0.25	0.15	0.07	0.04	0.02	0.00
30	ave($\hat{\theta}$)	0.18	0.35	0.46	0.63	0.81	0.96	0.99	0.05	0.18	0.37	0.57	0.77	0.94	0.99	0.02	0.13	0.34	0.55	0.75	0.93	1.00
	sd($\hat{\theta}$)	0.09	0.12	0.10	0.09	0.08	0.05	0.03	0.06	0.09	0.06	0.06	0.05	0.05	0.03	0.04	0.08	0.05	0.04	0.05	0.04	0.02
	FDR	0.00	0.03	0.06	0.04	0.04	0.05	0.07	0.00	0.04	0.03	0.01	0.01	0.04	0.07	0.00	0.04	0.01	0.01	0.01	0.02	0.05
	FNR	0.95	0.74	0.40	0.22	0.15	0.07	0.00	0.53	0.54	0.23	0.13	0.09	0.05	0.00	0.22	0.37	0.14	0.09	0.06	0.04	0.00
50	ave($\hat{\theta}$)	0.18	0.32	0.44	0.63	0.81	0.96	1.00	0.06	0.18	0.36	0.57	0.77	0.95	1.00	0.02	0.13	0.34	0.54	0.75	0.93	1.00
	sd($\hat{\theta}$)	0.07	0.09	0.08	0.08	0.07	0.04	0.02	0.04	0.07	0.05	0.05	0.05	0.04	0.02	0.03	0.05	0.03	0.04	0.04	0.03	0.02
	FDR	0.00	0.03	0.05	0.03	0.03	0.04	0.07	0.00	0.03	0.02	0.01	0.01	0.02	0.07	0.00	0.03	0.01	0.01	0.01	0.02	0.07
	FNR	0.98	0.74	0.37	0.22	0.14	0.07	0.00	0.78	0.54	0.20	0.13	0.09	0.05	0.00	0.34	0.39	0.13	0.08	0.07	0.04	0.00

Table B.36: SPSM test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

	Test	T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.25	0.38	0.54	0.68	0.83	0.97	0.99	0.07	0.20	0.38	0.58	0.77	0.94	0.99	0.03	0.12	0.33	0.54	0.73	0.92	1.00
10	sd($\hat{\theta}$)	0.20	0.26	0.23	0.17	0.13	0.07	0.04	0.12	0.23	0.18	0.13	0.11	0.07	0.04	0.09	0.19	0.16	0.11	0.09	0.08	0.03
	FDR	0.00	0.05	0.10	0.09	0.06	0.04	0.04	0.00	0.06	0.08	0.05	0.03	0.04	0.04	0.00	0.07	0.06	0.03	0.03	0.03	0.03
	FNR	0.73	0.68	0.48	0.29	0.17	0.08	0.00	0.29	0.40	0.28	0.16	0.11	0.05	0.00	0.12	0.25	0.16	0.09	0.06	0.03	0.00
	ave($\hat{\theta}$)	0.23	0.38	0.49	0.66	0.84	0.97	1.00	0.07	0.21	0.38	0.59	0.79	0.95	1.00	0.02	0.13	0.34	0.56	0.76	0.94	1.00
30	sd($\hat{\theta}$)	0.11	0.13	0.12	0.10	0.07	0.05	0.02	0.07	0.11	0.08	0.07	0.06	0.04	0.02	0.05	0.08	0.06	0.04	0.04	0.03	0.02
	FDR	0.00	0.03	0.09	0.07	0.04	0.05	0.05	0.00	0.04	0.05	0.02	0.01	0.02	0.05	0.00	0.04	0.03	0.01	0.01	0.01	0.04
	FNR	0.97	0.77	0.46	0.27	0.17	0.08	0.00	0.60	0.58	0.28	0.16	0.11	0.06	0.00	0.26	0.38	0.18	0.11	0.08	0.04	0.00
	ave($\hat{\theta}$)	0.22	0.36	0.46	0.64	0.83	0.97	0.99	0.07	0.19	0.36	0.59	0.79	0.96	1.00	0.02	0.13	0.34	0.56	0.76	0.94	1.00
50	sd($\hat{\theta}$)	0.07	0.10	0.10	0.08	0.07	0.04	0.02	0.05	0.08	0.06	0.05	0.05	0.04	0.02	0.03	0.06	0.04	0.04	0.04	0.03	0.01
	FDR	0.00	0.03	0.08	0.06	0.04	0.04	0.09	0.00	0.03	0.05	0.02	0.01	0.02	0.05	0.00	0.03	0.02	0.01	0.01	0.01	0.05
	FNR	1.00	0.77	0.44	0.26	0.16	0.08	0.00	0.80	0.59	0.25	0.16	0.11	0.06	0.00	0.38	0.41	0.16	0.10	0.08	0.05	0.00

Table B.37: RW05 test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.82	0.75	0.80	0.86	0.91	0.97	1.00	0.23	0.36	0.53	0.67	0.79	0.94	1.00	0.04	0.17	0.37	0.58	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.16	0.16	0.14	0.11	0.09	0.05	0.02	0.23	0.17	0.13	0.12	0.09	0.05	0.02	0.07	0.10	0.10	0.08	0.07	0.03	0.02
10	FWE	0.00	0.00	0.03	0.03	0.01	0.02	0.03	0.00	0.02	0.03	0.03	0.05	0.02	0.04	0.00	0.03	0.03	0.04	0.02	0.01	0.02
	FDR	0.00	0.00	0.01	0.01	0.01	0.02	0.03	0.00	0.00	0.01	0.01	0.02	0.02	0.04	0.00	0.00	0.00	0.01	0.01	0.01	0.02
	FNR	1.00	0.86	0.62	0.41	0.22	0.07	0.00	0.75	0.63	0.40	0.23	0.11	0.05	0.00	0.28	0.26	0.16	0.12	0.06	0.01	0.00
	ave($\hat{\theta}$)	0.92	0.86	0.89	0.93	0.96	0.99	1.00	0.37	0.46	0.60	0.72	0.84	0.95	1.00	0.07	0.21	0.41	0.57	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.06	0.08	0.06	0.05	0.04	0.02	0.00	0.18	0.11	0.08	0.07	0.05	0.03	0.00	0.06	0.06	0.06	0.05	0.04	0.02	0.01
30	FWE	0.00	0.00	0.01	0.02	0.00	0.03	0.01	0.00	0.00	0.03	0.02	0.03	0.01	0.02	0.00	0.01	0.03	0.01	0.04	0.03	0.02
	FDR	0.00	0.00	0.00	0.00	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.02
	FNR	1.00	0.88	0.66	0.46	0.27	0.09	0.00	1.00	0.77	0.49	0.30	0.16	0.05	0.00	0.70	0.48	0.25	0.12	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.96	0.90	0.92	0.95	0.97	0.99	1.00	0.48	0.51	0.63	0.74	0.85	0.95	1.00	0.08	0.22	0.42	0.60	0.77	0.92	1.00
	sd($\hat{\theta}$)	0.04	0.05	0.04	0.03	0.03	0.02	0.00	0.16	0.08	0.07	0.05	0.04	0.02	0.00	0.06	0.05	0.05	0.04	0.03	0.02	0.00
50	FWE	0.00	0.00	0.02	0.02	0.02	0.05	0.04	0.00	0.00	0.02	0.03	0.01	0.01	0.06	0.00	0.01	0.04	0.01	0.02	0.01	0.02
	FDR	0.00	0.00	0.00	0.01	0.02	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.02
	FNR	1.00	0.89	0.67	0.47	0.28	0.09	0.00	1.00	0.80	0.52	0.32	0.17	0.05	0.00	0.90	0.53	0.27	0.16	0.09	0.02	0.00

Table B.38: RW05 test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.87	0.75	0.83	0.88	0.90	0.96	1.00	0.18	0.37	0.53	0.68	0.81	0.94	1.00	0.03	0.16	0.38	0.56	0.74	0.92	0.99
	sd($\hat{\theta}$)	0.14	0.15	0.12	0.09	0.09	0.05	0.02	0.20	0.17	0.13	0.11	0.09	0.05	0.01	0.06	0.09	0.09	0.07	0.06	0.05	0.02
10	FWE	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.00	0.00	0.02	0.02	0.03	0.01	0.02	0.00	0.04	0.02	0.00	0.03	0.04	0.06
	FDR	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.00	0.00	0.01	0.00	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.02	0.06
	FNR	1.00	0.86	0.63	0.43	0.22	0.07	0.00	0.60	0.63	0.39	0.24	0.12	0.04	0.00	0.19	0.26	0.17	0.09	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.96	0.85	0.89	0.94	0.95	0.98	1.00	0.39	0.45	0.58	0.73	0.84	0.94	1.00	0.03	0.21	0.40	0.58	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.05	0.08	0.06	0.05	0.04	0.02	0.00	0.21	0.10	0.09	0.07	0.05	0.03	0.00	0.04	0.06	0.06	0.05	0.04	0.02	0.01
30	FWE	0.00	0.00	0.03	0.00	0.03	0.03	0.02	0.00	0.00	0.02	0.02	0.03	0.01	0.02	0.00	0.03	0.04	0.01	0.02	0.03	0.04
	FDR	0.00	0.00	0.01	0.00	0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.04
	FNR	1.00	0.88	0.66	0.46	0.26	0.08	0.00	0.98	0.77	0.47	0.31	0.16	0.04	0.00	0.49	0.48	0.24	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.97	0.90	0.92	0.94	0.96	0.99	1.00	0.49	0.51	0.64	0.73	0.85	0.95	1.00	0.04	0.22	0.41	0.59	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.04	0.05	0.04	0.04	0.03	0.02	0.01	0.19	0.08	0.06	0.05	0.04	0.02	0.00	0.04	0.06	0.05	0.04	0.03	0.02	0.00
50	FWE	0.00	0.00	0.02	0.03	0.03	0.08	0.05	0.00	0.00	0.01	0.01	0.03	0.02	0.02	0.00	0.02	0.02	0.01	0.02	0.02	0.01
	FDR	0.00	0.00	0.01	0.01	0.01	0.07	0.05	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	FNR	1.00	0.89	0.67	0.47	0.27	0.09	0.00	1.00	0.80	0.52	0.31	0.17	0.05	0.00	0.74	0.52	0.26	0.15	0.07	0.02	0.00

Table B.39: RW05 test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.86	0.72	0.79	0.88	0.91	0.97	1.00	0.20	0.33	0.50	0.66	0.80	0.93	1.00	0.03	0.17	0.38	0.55	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.19	0.15	0.14	0.09	0.08	0.05	0.01	0.25	0.16	0.14	0.10	0.09	0.05	0.00	0.05	0.09	0.10	0.07	0.06	0.04	0.01
10	FWE	0.00	0.00	0.00	0.03	0.06	0.03	0.01	0.00	0.01	0.02	0.03	0.01	0.01	0.00	0.00	0.02	0.03	0.02	0.03	0.03	0.02
	FDR	0.00	0.00	0.00	0.01	0.06	0.03	0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02
	FNR	1.00	0.85	0.61	0.43	0.23	0.08	0.00	0.61	0.58	0.35	0.22	0.11	0.03	0.00	0.23	0.25	0.17	0.08	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.93	0.85	0.89	0.93	0.96	0.98	1.00	0.31	0.47	0.59	0.72	0.83	0.94	1.00	0.03	0.22	0.40	0.57	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.08	0.07	0.07	0.05	0.04	0.02	0.01	0.24	0.10	0.09	0.07	0.05	0.03	0.00	0.04	0.07	0.06	0.05	0.04	0.02	0.01
30	FWE	0.00	0.00	0.00	0.01	0.02	0.02	0.04	0.00	0.01	0.00	0.04	0.03	0.03	0.01	0.00	0.01	0.03	0.01	0.01	0.01	0.06
	FDR	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
	FNR	1.00	0.88	0.66	0.46	0.27	0.09	0.00	0.97	0.77	0.48	0.30	0.15	0.04	0.00	0.48	0.49	0.24	0.12	0.07	0.02	0.00
	ave($\hat{\theta}$)	0.95	0.89	0.91	0.94	0.97	0.99	1.00	0.46	0.50	0.63	0.73	0.84	0.95	1.00	0.04	0.23	0.41	0.59	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.06	0.05	0.04	0.04	0.02	0.02	0.00	0.24	0.07	0.06	0.06	0.04	0.02	0.00	0.04	0.05	0.04	0.05	0.03	0.02	0.02
50	FWE	0.00	0.01	0.00	0.04	0.02	0.06	0.03	0.00	0.01	0.00	0.02	0.00	0.02	0.03	0.00	0.02	0.02	0.02	0.07	0.00	0.04
	FDR	0.00	0.00	0.00	0.02	0.01	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.01	0.00	0.04
	FNR	1.00	0.89	0.67	0.47	0.28	0.09	0.00	1.00	0.79	0.52	0.31	0.17	0.05	0.00	0.75	0.54	0.27	0.15	0.07	0.02	0.00

Table B.40: RW05 test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.84	0.75	0.83	0.87	0.93	0.97	1.00	0.14	0.34	0.52	0.68	0.77	0.94	1.00	0.02	0.17	0.36	0.55	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.23	0.16	0.12	0.11	0.07	0.04	0.02	0.22	0.17	0.14	0.13	0.09	0.05	0.02	0.04	0.09	0.08	0.08	0.06	0.04	0.02
10	FWE	0.00	0.00	0.00	0.02	0.02	0.01	0.03	0.00	0.01	0.00	0.02	0.03	0.03	0.04	0.00	0.03	0.06	0.04	0.02	0.03	0.05
	FDR	0.00	0.00	0.00	0.01	0.02	0.01	0.03	0.00	0.00	0.00	0.00	0.02	0.02	0.04	0.00	0.00	0.01	0.01	0.01	0.02	0.05
	FNR	0.97	0.86	0.63	0.42	0.24	0.07	0.00	0.41	0.61	0.38	0.24	0.09	0.04	0.00	0.13	0.26	0.16	0.09	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.96	0.85	0.88	0.92	0.95	0.98	1.00	0.31	0.47	0.59	0.71	0.83	0.94	1.00	0.01	0.21	0.40	0.58	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.07	0.07	0.06	0.05	0.04	0.02	0.01	0.29	0.10	0.09	0.07	0.05	0.03	0.00	0.03	0.06	0.05	0.05	0.04	0.02	0.01
30	FWE	0.00	0.02	0.01	0.05	0.02	0.04	0.04	0.00	0.01	0.00	0.00	0.00	0.04	0.02	0.00	0.00	0.02	0.00	0.02	0.01	0.03
	FDR	0.00	0.00	0.00	0.03	0.01	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.03
	FNR	1.00	0.88	0.66	0.46	0.26	0.09	0.00	0.83	0.78	0.48	0.29	0.16	0.05	0.00	0.23	0.47	0.24	0.14	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.98	0.90	0.92	0.93	0.96	0.99	1.00	0.39	0.51	0.62	0.73	0.84	0.95	1.00	0.01	0.22	0.41	0.59	0.76	0.92	1.00
	sd($\hat{\theta}$)	0.04	0.05	0.04	0.04	0.02	0.02	0.00	0.28	0.08	0.08	0.05	0.04	0.02	0.01	0.03	0.06	0.05	0.03	0.03	0.02	0.00
50	FWE	0.00	0.00	0.01	0.01	0.04	0.05	0.04	0.00	0.00	0.01	0.00	0.02	0.02	0.01	0.00	0.01	0.01	0.00	0.00	0.01	0.00
	FDR	0.00	0.00	0.00	0.00	0.02	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	FNR	1.00	0.89	0.67	0.46	0.27	0.09	0.00	0.96	0.80	0.51	0.31	0.16	0.05	0.00	0.32	0.52	0.25	0.14	0.07	0.02	0.00

Table B.41: RW05 test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.80	0.73	0.77	0.84	0.89	0.94	0.96	0.22	0.32	0.48	0.61	0.72	0.84	0.90	0.05	0.14	0.32	0.47	0.62	0.76	0.84
	sd($\hat{\theta}$)	0.18	0.15	0.15	0.12	0.10	0.09	0.07	0.20	0.18	0.18	0.15	0.13	0.11	0.13	0.08	0.10	0.11	0.14	0.16	0.17	0.17
10	FWE	0.00	0.04	0.18	0.21	0.21	0.26	0.30	0.00	0.16	0.31	0.39	0.53	0.54	0.62	0.00	0.22	0.36	0.52	0.64	0.71	0.78
	FDR	0.00	0.01	0.06	0.10	0.16	0.23	0.30	0.00	0.02	0.07	0.11	0.25	0.39	0.62	0.00	0.02	0.06	0.13	0.24	0.48	0.78
	FNR	1.00	0.86	0.62	0.42	0.24	0.08	0.00	0.73	0.63	0.41	0.25	0.14	0.03	0.00	0.32	0.28	0.16	0.09	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.93	0.85	0.88	0.90	0.93	0.97	0.98	0.41	0.45	0.57	0.66	0.77	0.89	0.93	0.06	0.19	0.34	0.51	0.63	0.81	0.88
	sd($\hat{\theta}$)	0.06	0.07	0.07	0.05	0.04	0.04	0.03	0.17	0.11	0.09	0.09	0.08	0.07	0.06	0.05	0.08	0.08	0.06	0.10	0.08	0.10
30	FWE	0.00	0.08	0.19	0.23	0.34	0.37	0.39	0.00	0.20	0.50	0.73	0.74	0.83	0.84	0.00	0.45	0.73	0.87	0.92	0.93	0.92
	FDR	0.00	0.01	0.07	0.09	0.17	0.31	0.39	0.00	0.01	0.05	0.14	0.22	0.53	0.84	0.00	0.02	0.07	0.13	0.27	0.51	0.92
	FNR	1.00	0.88	0.67	0.45	0.27	0.09	0.00	1.00	0.78	0.51	0.31	0.16	0.06	0.00	0.73	0.53	0.23	0.14	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.95	0.89	0.92	0.94	0.96	0.98	0.99	0.46	0.48	0.61	0.70	0.79	0.89	0.94	0.07	0.20	0.36	0.52	0.67	0.83	0.89
	sd($\hat{\theta}$)	0.04	0.04	0.04	0.03	0.03	0.02	0.02	0.16	0.09	0.07	0.07	0.06	0.05	0.05	0.06	0.05	0.07	0.07	0.08	0.07	0.06
50	FWE	0.00	0.08	0.19	0.19	0.36	0.42	0.39	0.00	0.31	0.57	0.83	0.88	0.92	0.91	0.00	0.44	0.74	0.95	0.95	0.94	0.98
	FDR	0.00	0.02	0.04	0.07	0.19	0.34	0.39	0.00	0.01	0.04	0.11	0.23	0.50	0.91	0.00	0.01	0.06	0.13	0.24	0.47	0.98
	FNR	1.00	0.89	0.68	0.47	0.28	0.09	0.00	1.00	0.80	0.53	0.33	0.18	0.05	0.00	0.80	0.54	0.28	0.16	0.08	0.02	0.00

Table B.42: RW05 test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.82	0.74	0.78	0.88	0.93	0.97	0.98	0.16	0.39	0.51	0.66	0.78	0.91	0.97	0.03	0.15	0.35	0.51	0.72	0.89	0.97
	sd($\hat{\theta}$)	0.20	0.18	0.15	0.11	0.07	0.06	0.07	0.20	0.18	0.14	0.14	0.10	0.07	0.05	0.06	0.09	0.09	0.08	0.07	0.09	0.07
10	FWE	0.00	0.01	0.03	0.03	0.03	0.08	0.14	0.00	0.03	0.12	0.16	0.18	0.15	0.23	0.00	0.11	0.19	0.22	0.19	0.16	0.25
	FDR	0.00	0.01	0.02	0.03	0.02	0.07	0.14	0.00	0.00	0.02	0.04	0.06	0.09	0.23	0.00	0.01	0.03	0.05	0.06	0.09	0.25
	FNR	1.00	0.85	0.60	0.43	0.24	0.08	0.00	0.62	0.67	0.39	0.24	0.12	0.03	0.00	0.23	0.26	0.17	0.07	0.06	0.01	0.00
	ave($\hat{\theta}$)	0.95	0.84	0.89	0.93	0.95	0.98	1.00	0.37	0.46	0.59	0.70	0.82	0.94	0.99	0.04	0.19	0.40	0.57	0.73	0.89	0.98
	sd($\hat{\theta}$)	0.06	0.08	0.06	0.05	0.04	0.02	0.01	0.19	0.11	0.08	0.07	0.06	0.04	0.02	0.05	0.06	0.07	0.05	0.05	0.05	0.02
30	FWE	0.00	0.02	0.03	0.06	0.00	0.13	0.12	0.00	0.03	0.12	0.12	0.22	0.24	0.33	0.00	0.13	0.19	0.24	0.29	0.44	0.34
	FDR	0.00	0.01	0.01	0.02	0.00	0.11	0.12	0.00	0.00	0.01	0.02	0.04	0.09	0.33	0.00	0.01	0.01	0.02	0.04	0.17	0.34
	FNR	1.00	0.88	0.66	0.46	0.26	0.09	0.00	0.99	0.77	0.49	0.29	0.16	0.05	0.00	0.58	0.44	0.25	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.98	0.90	0.92	0.94	0.97	0.99	1.00	0.49	0.49	0.62	0.74	0.84	0.94	0.99	0.04	0.22	0.40	0.57	0.75	0.91	0.98
	sd($\hat{\theta}$)	0.03	0.05	0.04	0.03	0.02	0.02	0.01	0.19	0.08	0.07	0.06	0.04	0.03	0.01	0.04	0.05	0.05	0.04	0.04	0.03	0.02
50	FWE	0.00	0.00	0.04	0.05	0.09	0.09	0.14	0.00	0.07	0.14	0.19	0.23	0.41	0.31	0.00	0.14	0.23	0.36	0.33	0.43	0.50
	FDR	0.00	0.00	0.01	0.01	0.04	0.06	0.14	0.00	0.00	0.01	0.02	0.03	0.14	0.31	0.00	0.00	0.01	0.02	0.04	0.12	0.50
	FNR	1.00	0.89	0.67	0.47	0.28	0.09	0.00	1.00	0.79	0.52	0.32	0.17	0.05	0.00	0.73	0.53	0.26	0.14	0.08	0.03	0.00

Table B.43: RW05 test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.82	0.72	0.76	0.85	0.89	0.95	0.98	0.16	0.35	0.48	0.65	0.75	0.87	0.95	0.02	0.15	0.34	0.50	0.68	0.84	0.89
	sd($\hat{\theta}$)	0.25	0.16	0.17	0.13	0.10	0.08	0.05	0.19	0.17	0.17	0.14	0.12	0.10	0.08	0.05	0.10	0.12	0.11	0.10	0.10	0.14
10	FWE	0.00	0.02	0.15	0.10	0.24	0.18	0.22	0.00	0.07	0.24	0.30	0.34	0.46	0.41	0.00	0.19	0.33	0.41	0.51	0.48	0.62
	FDR	0.00	0.00	0.05	0.04	0.15	0.15	0.22	0.00	0.01	0.04	0.08	0.14	0.33	0.41	0.00	0.02	0.05	0.10	0.17	0.30	0.62
	FNR	0.98	0.86	0.60	0.41	0.23	0.08	0.00	0.61	0.64	0.38	0.26	0.13	0.04	0.00	0.15	0.28	0.18	0.10	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.93	0.84	0.90	0.91	0.94	0.98	0.99	0.34	0.42	0.57	0.68	0.80	0.90	0.95	0.04	0.19	0.35	0.53	0.69	0.85	0.92
	sd($\hat{\theta}$)	0.08	0.07	0.05	0.05	0.04	0.03	0.02	0.26	0.10	0.09	0.08	0.07	0.05	0.05	0.05	0.06	0.08	0.07	0.08	0.08	0.07
30	FWE	0.00	0.06	0.10	0.13	0.25	0.22	0.24	0.00	0.16	0.33	0.57	0.58	0.66	0.65	0.00	0.30	0.47	0.68	0.73	0.82	0.84
	FDR	0.00	0.01	0.03	0.04	0.16	0.17	0.24	0.00	0.01	0.03	0.09	0.13	0.34	0.65	0.00	0.01	0.04	0.08	0.16	0.38	0.84
	FNR	1.00	0.88	0.67	0.46	0.26	0.09	0.00	0.94	0.77	0.48	0.31	0.15	0.04	0.00	0.58	0.50	0.22	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.96	0.90	0.91	0.93	0.96	0.98	0.99	0.40	0.49	0.59	0.71	0.81	0.91	0.96	0.04	0.22	0.39	0.55	0.70	0.85	0.93
	sd($\hat{\theta}$)	0.05	0.05	0.04	0.04	0.03	0.02	0.01	0.24	0.09	0.06	0.06	0.06	0.04	0.03	0.03	0.07	0.07	0.05	0.06	0.05	0.05
50	FWE	0.00	0.10	0.11	0.19	0.28	0.33	0.32	0.00	0.21	0.54	0.59	0.63	0.74	0.86	0.00	0.37	0.62	0.75	0.89	0.89	0.92
	FDR	0.00	0.02	0.03	0.07	0.14	0.24	0.32	0.00	0.01	0.04	0.06	0.13	0.33	0.86	0.00	0.01	0.04	0.07	0.17	0.39	0.92
	FNR	1.00	0.89	0.67	0.47	0.28	0.09	0.00	0.99	0.80	0.51	0.32	0.17	0.05	0.00	0.70	0.57	0.28	0.15	0.07	0.02	0.00

Table B.44: RW05 test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.86	0.73	0.80	0.86	0.92	0.98	0.99	0.14	0.37	0.52	0.65	0.78	0.92	1.00	0.01	0.16	0.35	0.55	0.73	0.90	0.99
	sd($\hat{\theta}$)	0.20	0.15	0.13	0.11	0.08	0.04	0.04	0.21	0.17	0.14	0.11	0.09	0.05	0.02	0.03	0.10	0.09	0.09	0.08	0.05	0.04
10	FWE	0.00	0.00	0.04	0.05	0.02	0.01	0.10	0.00	0.00	0.03	0.04	0.10	0.07	0.04	0.00	0.09	0.08	0.06	0.08	0.07	0.10
	FDR	0.00	0.00	0.02	0.02	0.01	0.01	0.10	0.00	0.00	0.01	0.01	0.03	0.05	0.04	0.00	0.01	0.01	0.01	0.03	0.04	0.10
	FNR	0.99	0.86	0.62	0.41	0.24	0.08	0.00	0.45	0.65	0.39	0.22	0.11	0.03	0.00	0.07	0.28	0.15	0.09	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.96	0.85	0.89	0.92	0.94	0.98	1.00	0.30	0.46	0.60	0.70	0.83	0.94	1.00	0.01	0.20	0.38	0.57	0.74	0.91	0.99
	sd($\hat{\theta}$)	0.07	0.07	0.07	0.05	0.04	0.02	0.01	0.29	0.12	0.09	0.08	0.05	0.04	0.01	0.02	0.06	0.05	0.05	0.04	0.03	0.02
30	FWE	0.00	0.02	0.01	0.05	0.06	0.07	0.07	0.00	0.02	0.07	0.11	0.08	0.09	0.07	0.00	0.07	0.02	0.12	0.17	0.22	0.23
	FDR	0.00	0.00	0.00	0.02	0.02	0.06	0.07	0.00	0.00	0.00	0.01	0.02	0.04	0.07	0.00	0.00	0.00	0.01	0.02	0.08	0.23
	FNR	1.00	0.88	0.66	0.46	0.25	0.09	0.00	0.82	0.77	0.49	0.28	0.16	0.05	0.00	0.25	0.45	0.20	0.12	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.97	0.89	0.92	0.94	0.96	0.99	1.00	0.39	0.49	0.62	0.73	0.84	0.95	1.00	0.02	0.21	0.40	0.59	0.75	0.91	0.99
	sd($\hat{\theta}$)	0.04	0.05	0.04	0.04	0.03	0.01	0.01	0.30	0.08	0.07	0.05	0.04	0.02	0.01	0.02	0.05	0.04	0.04	0.04	0.02	0.01
50	FWE	0.00	0.00	0.01	0.04	0.07	0.04	0.09	0.00	0.06	0.08	0.09	0.17	0.12	0.19	0.00	0.02	0.06	0.17	0.17	0.18	0.28
	FDR	0.00	0.00	0.00	0.01	0.02	0.03	0.09	0.00	0.00	0.00	0.01	0.02	0.03	0.19	0.00	0.00	0.00	0.01	0.02	0.04	0.28
	FNR	1.00	0.89	0.67	0.47	0.27	0.09	0.00	0.96	0.79	0.51	0.32	0.17	0.05	0.00	0.43	0.51	0.25	0.15	0.08	0.02	0.00

Table B.45: RW05 test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.80	0.85	0.88	0.91	0.95	0.98	1.00	0.22	0.31	0.49	0.67	0.81	0.93	1.00	0.05	0.15	0.34	0.52	0.72	0.90	0.99
	sd($\hat{\theta}$)	0.19	0.14	0.13	0.10	0.08	0.04	0.02	0.19	0.16	0.14	0.13	0.09	0.06	0.02	0.08	0.08	0.08	0.06	0.06	0.04	0.03
10	FWE	0.00	0.01	0.01	0.02	0.02	0.04	0.04	0.00	0.01	0.04	0.07	0.02	0.06	0.05	0.00	0.05	0.08	0.06	0.10	0.12	0.07
	FDR	0.00	0.00	0.01	0.01	0.01	0.04	0.04	0.00	0.00	0.01	0.02	0.01	0.04	0.05	0.00	0.01	0.01	0.01	0.03	0.08	0.07
	FNR	1.00	0.88	0.65	0.45	0.26	0.09	0.00	0.74	0.56	0.34	0.24	0.13	0.04	0.00	0.34	0.22	0.11	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.93	0.93	0.95	0.96	0.98	0.99	1.00	0.41	0.48	0.60	0.73	0.84	0.95	1.00	0.06	0.18	0.37	0.56	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.06	0.06	0.06	0.05	0.03	0.02	0.01	0.19	0.14	0.11	0.09	0.06	0.03	0.01	0.06	0.06	0.05	0.04	0.04	0.02	0.01
30	FWE	0.00	0.00	0.01	0.02	0.04	0.03	0.03	0.00	0.02	0.05	0.01	0.06	0.07	0.04	0.00	0.04	0.06	0.12	0.10	0.08	0.11
	FDR	0.00	0.00	0.01	0.01	0.03	0.03	0.03	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.00	0.00	0.00	0.01	0.01	0.02	0.11
	FNR	1.00	0.89	0.68	0.48	0.28	0.09	0.00	1.00	0.77	0.49	0.31	0.16	0.06	0.00	0.70	0.38	0.18	0.10	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.96	0.95	0.96	0.97	0.99	0.99	1.00	0.47	0.52	0.66	0.77	0.87	0.96	1.00	0.08	0.19	0.39	0.57	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.04	0.03	0.03	0.03	0.02	0.01	0.00	0.17	0.13	0.11	0.08	0.05	0.03	0.01	0.06	0.04	0.04	0.04	0.02	0.02	0.01
50	FWE	0.00	0.01	0.01	0.01	0.02	0.00	0.01	0.00	0.01	0.02	0.04	0.05	0.08	0.08	0.00	0.08	0.11	0.11	0.11	0.15	0.13
	FDR	0.00	0.00	0.01	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.00	0.00	0.00	0.01	0.01	0.03	0.13
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	1.00	0.79	0.53	0.35	0.19	0.06	0.00	0.90	0.44	0.22	0.12	0.06	0.01	0.00

Table B.46: RW05 test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.87	0.88	0.90	0.95	0.95	0.97	1.00	0.21	0.31	0.46	0.64	0.78	0.93	0.99	0.04	0.13	0.32	0.51	0.71	0.89	0.97
	sd($\hat{\theta}$)	0.16	0.13	0.12	0.08	0.07	0.05	0.02	0.21	0.17	0.15	0.13	0.09	0.07	0.03	0.08	0.08	0.07	0.07	0.06	0.05	0.05
10	FWE	0.00	0.00	0.02	0.05	0.02	0.11	0.05	0.00	0.03	0.09	0.13	0.10	0.10	0.07	0.00	0.06	0.13	0.12	0.14	0.20	0.27
	FDR	0.00	0.00	0.02	0.04	0.02	0.09	0.05	0.00	0.00	0.01	0.03	0.03	0.06	0.07	0.00	0.01	0.02	0.03	0.04	0.11	0.27
	FNR	1.00	0.88	0.66	0.47	0.26	0.08	0.00	0.68	0.53	0.32	0.21	0.11	0.04	0.00	0.25	0.16	0.08	0.04	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.95	0.96	0.97	0.98	0.98	0.99	1.00	0.38	0.46	0.62	0.73	0.85	0.95	1.00	0.04	0.15	0.35	0.54	0.72	0.90	0.99
	sd($\hat{\theta}$)	0.06	0.04	0.04	0.03	0.03	0.01	0.01	0.20	0.18	0.12	0.09	0.06	0.03	0.01	0.05	0.05	0.05	0.05	0.03	0.02	0.02
30	FWE	0.00	0.00	0.01	0.02	0.03	0.03	0.05	0.00	0.02	0.07	0.07	0.12	0.14	0.10	0.00	0.14	0.13	0.23	0.27	0.28	0.25
	FDR	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.00	0.00	0.01	0.01	0.03	0.07	0.10	0.00	0.01	0.01	0.02	0.04	0.08	0.25
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	0.97	0.74	0.50	0.31	0.18	0.06	0.00	0.52	0.29	0.15	0.08	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.97	0.97	0.98	0.99	0.99	1.00	1.00	0.48	0.58	0.69	0.78	0.87	0.96	1.00	0.05	0.16	0.37	0.55	0.72	0.90	0.99
	sd($\hat{\theta}$)	0.03	0.03	0.03	0.02	0.01	0.01	0.01	0.21	0.16	0.12	0.08	0.05	0.02	0.00	0.04	0.04	0.04	0.03	0.03	0.02	0.01
50	FWE	0.00	0.02	0.02	0.01	0.03	0.05	0.06	0.00	0.02	0.09	0.09	0.13	0.12	0.06	0.00	0.16	0.23	0.26	0.31	0.34	0.31
	FDR	0.00	0.01	0.01	0.01	0.02	0.04	0.06	0.00	0.00	0.00	0.01	0.02	0.05	0.06	0.00	0.00	0.01	0.01	0.03	0.07	0.31
	FNR	1.00	0.90	0.69	0.49	0.30	0.10	0.00	1.00	0.81	0.55	0.36	0.19	0.06	0.00	0.77	0.38	0.18	0.10	0.04	0.01	0.00

Table B.47: RW05 test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.80	0.84	0.90	0.94	0.93	0.98	0.99	0.21	0.29	0.44	0.61	0.76	0.89	0.96	0.02	0.11	0.30	0.48	0.67	0.82	0.92
	sd($\hat{\theta}$)	0.21	0.20	0.15	0.09	0.11	0.05	0.04	0.26	0.21	0.20	0.17	0.12	0.09	0.07	0.04	0.07	0.08	0.09	0.09	0.10	0.09
10	FWE	0.00	0.01	0.04	0.03	0.13	0.05	0.11	0.00	0.08	0.34	0.29	0.28	0.32	0.33	0.00	0.18	0.31	0.38	0.44	0.55	0.51
	FDR	0.00	0.00	0.01	0.01	0.08	0.04	0.11	0.00	0.01	0.06	0.06	0.10	0.22	0.33	0.00	0.02	0.04	0.08	0.14	0.34	0.51
	FNR	0.99	0.86	0.66	0.46	0.26	0.09	0.00	0.60	0.48	0.33	0.19	0.11	0.03	0.00	0.18	0.13	0.09	0.05	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.93	0.94	0.96	0.97	0.98	0.99	1.00	0.32	0.41	0.59	0.72	0.85	0.93	0.99	0.03	0.13	0.32	0.51	0.68	0.85	0.94
	sd($\hat{\theta}$)	0.10	0.08	0.06	0.04	0.03	0.02	0.01	0.26	0.22	0.17	0.13	0.08	0.05	0.03	0.04	0.05	0.05	0.04	0.05	0.05	0.05
30	FWE	0.00	0.02	0.03	0.06	0.06	0.05	0.14	0.00	0.13	0.23	0.24	0.36	0.32	0.31	0.00	0.27	0.47	0.58	0.79	0.80	0.77
	FDR	0.00	0.01	0.01	0.02	0.04	0.05	0.14	0.00	0.01	0.02	0.03	0.09	0.13	0.31	0.00	0.01	0.03	0.06	0.14	0.34	0.77
	FNR	1.00	0.89	0.69	0.48	0.29	0.10	0.00	0.94	0.70	0.46	0.30	0.19	0.05	0.00	0.44	0.26	0.11	0.06	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.96	0.96	0.97	0.98	0.99	0.99	1.00	0.41	0.56	0.65	0.77	0.86	0.95	0.99	0.03	0.14	0.33	0.51	0.69	0.87	0.96
	sd($\hat{\theta}$)	0.03	0.04	0.04	0.02	0.01	0.02	0.01	0.25	0.21	0.15	0.12	0.08	0.03	0.02	0.04	0.04	0.05	0.04	0.04	0.04	0.03
50	FWE	0.00	0.01	0.01	0.01	0.03	0.07	0.05	0.00	0.13	0.20	0.23	0.31	0.25	0.28	0.00	0.38	0.63	0.77	0.79	0.82	0.85
	FDR	0.00	0.00	0.00	0.00	0.03	0.05	0.05	0.00	0.00	0.01	0.02	0.05	0.09	0.28	0.00	0.01	0.03	0.06	0.12	0.30	0.85
	FNR	1.00	0.90	0.69	0.49	0.30	0.09	0.00	1.00	0.80	0.52	0.34	0.19	0.06	0.00	0.67	0.34	0.14	0.08	0.04	0.01	0.00

Table B.48: RW05 test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.84	0.89	0.91	0.94	0.94	0.97	0.99	0.09	0.22	0.39	0.58	0.71	0.85	0.92	0.01	0.08	0.24	0.41	0.57	0.74	0.82
	sd($\hat{\theta}$)	0.21	0.16	0.16	0.09	0.10	0.06	0.04	0.16	0.22	0.19	0.17	0.17	0.10	0.12	0.03	0.07	0.09	0.11	0.12	0.13	0.12
10	FWE	0.00	0.02	0.07	0.11	0.12	0.10	0.08	0.00	0.23	0.38	0.40	0.52	0.57	0.40	0.00	0.34	0.53	0.63	0.73	0.81	0.87
	FDR	0.00	0.01	0.03	0.08	0.08	0.09	0.08	0.00	0.02	0.07	0.10	0.22	0.38	0.40	0.00	0.03	0.09	0.15	0.27	0.52	0.87
	FNR	0.99	0.88	0.66	0.48	0.27	0.08	0.00	0.39	0.36	0.26	0.19	0.12	0.03	0.00	0.04	0.09	0.04	0.02	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.31	0.39	0.56	0.71	0.80	0.93	0.96	0.01	0.10	0.26	0.44	0.62	0.79	0.88
	sd($\hat{\theta}$)	0.04	0.06	0.03	0.03	0.03	0.01	0.02	0.28	0.24	0.20	0.15	0.12	0.05	0.04	0.02	0.05	0.05	0.06	0.06	0.07	0.06
30	FWE	0.00	0.03	0.00	0.09	0.08	0.08	0.19	0.00	0.17	0.46	0.47	0.61	0.52	0.61	0.00	0.49	0.84	0.85	0.94	0.96	0.95
	FDR	0.00	0.02	0.00	0.05	0.06	0.08	0.19	0.00	0.01	0.04	0.07	0.16	0.29	0.61	0.00	0.03	0.08	0.12	0.23	0.50	0.95
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	0.83	0.64	0.45	0.31	0.16	0.06	0.00	0.27	0.17	0.06	0.03	0.01	0.01	0.00
	ave($\hat{\theta}$)	0.98	0.97	0.98	0.99	0.99	0.99	1.00	0.47	0.50	0.63	0.76	0.85	0.94	0.97	0.01	0.10	0.28	0.46	0.63	0.81	0.90
	sd($\hat{\theta}$)	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.29	0.25	0.20	0.14	0.09	0.06	0.03	0.02	0.03	0.04	0.05	0.05	0.05	0.05
50	FWE	0.00	0.00	0.04	0.04	0.07	0.12	0.10	0.00	0.25	0.51	0.48	0.51	0.41	0.59	0.00	0.61	0.92	0.95	0.97	0.98	1.00
	FDR	0.00	0.00	0.02	0.03	0.06	0.10	0.10	0.00	0.01	0.04	0.06	0.10	0.22	0.59	0.00	0.02	0.07	0.11	0.22	0.46	1.00
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	0.95	0.75	0.50	0.34	0.19	0.06	0.00	0.30	0.20	0.08	0.05	0.03	0.01	0.00

Table B.49: RW05 test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	$ave(\hat{\theta})$	0.30	0.40	0.55	0.70	0.82	0.93	0.99	0.09	0.20	0.39	0.57	0.74	0.90	1.00	0.04	0.14	0.34	0.52	0.72	0.90	1.00
	$sd(\hat{\theta})$	0.18	0.18	0.13	0.11	0.09	0.05	0.03	0.10	0.10	0.11	0.09	0.07	0.05	0.01	0.07	0.08	0.07	0.06	0.05	0.03	0.01
10	FWE	0.00	0.01	0.04	0.03	0.05	0.04	0.08	0.00	0.05	0.07	0.07	0.05	0.10	0.02	0.00	0.06	0.04	0.06	0.04	0.04	0.03
	FDR	0.00	0.00	0.01	0.01	0.02	0.03	0.08	0.00	0.01	0.01	0.02	0.01	0.06	0.02	0.00	0.01	0.01	0.01	0.01	0.02	0.03
	FNR	0.89	0.67	0.44	0.27	0.14	0.03	0.00	0.54	0.38	0.20	0.12	0.06	0.02	0.00	0.25	0.21	0.09	0.04	0.03	0.01	0.00
	$ave(\hat{\theta})$	0.43	0.52	0.64	0.76	0.86	0.95	1.00	0.14	0.25	0.42	0.60	0.76	0.92	1.00	0.05	0.15	0.35	0.55	0.72	0.91	1.00
	$sd(\hat{\theta})$	0.13	0.10	0.09	0.07	0.05	0.03	0.00	0.09	0.08	0.06	0.04	0.04	0.03	0.00	0.05	0.04	0.04	0.04	0.03	0.02	0.01
30	FWE	0.00	0.00	0.00	0.00	0.01	0.02	0.01	0.00	0.02	0.07	0.01	0.02	0.04	0.03	0.00	0.04	0.06	0.01	0.08	0.04	0.03
	FDR	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.01	0.01	0.03
	FNR	1.00	0.80	0.52	0.34	0.18	0.06	0.00	0.92	0.56	0.28	0.16	0.08	0.02	0.00	0.65	0.29	0.14	0.08	0.03	0.01	0.00
	$ave(\hat{\theta})$	0.52	0.57	0.69	0.79	0.88	0.96	1.00	0.16	0.27	0.45	0.62	0.77	0.92	1.00	0.06	0.17	0.36	0.55	0.73	0.91	1.00
	$sd(\hat{\theta})$	0.10	0.08	0.06	0.06	0.03	0.02	0.00	0.07	0.06	0.06	0.05	0.03	0.02	0.00	0.05	0.04	0.03	0.03	0.03	0.01	0.01
50	FWE	0.00	0.00	0.02	0.03	0.03	0.03	0.05	0.00	0.01	0.04	0.05	0.02	0.03	0.06	0.00	0.03	0.05	0.02	0.05	0.03	0.08
	FDR	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.00	0.00	0.00	0.00	0.00	0.01	0.08
	FNR	1.00	0.82	0.56	0.37	0.21	0.06	0.00	0.99	0.61	0.32	0.19	0.09	0.03	0.00	0.87	0.39	0.16	0.09	0.04	0.01	0.00

Table B.50: RW05 test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.45	0.53	0.62	0.72	0.83	0.94	0.99	0.17	0.24	0.44	0.60	0.76	0.92	1.00	0.05	0.15	0.35	0.55	0.72	0.91	0.99
	sd($\hat{\theta}$)	0.19	0.17	0.17	0.13	0.10	0.07	0.04	0.16	0.15	0.12	0.10	0.08	0.04	0.02	0.09	0.09	0.07	0.06	0.06	0.04	0.03
10	FWE	0.00	0.02	0.07	0.12	0.15	0.09	0.10	0.00	0.03	0.08	0.06	0.03	0.03	0.05	0.00	0.11	0.04	0.04	0.07	0.04	0.05
	FDR	0.00	0.01	0.02	0.03	0.09	0.07	0.10	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.00	0.01	0.01	0.01	0.02	0.02	0.05
	FNR	0.97	0.79	0.49	0.31	0.17	0.05	0.00	0.74	0.44	0.29	0.15	0.08	0.02	0.00	0.33	0.26	0.12	0.08	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.55	0.59	0.69	0.77	0.87	0.95	0.99	0.23	0.33	0.51	0.66	0.79	0.93	1.00	0.07	0.19	0.38	0.56	0.73	0.91	1.00
	sd($\hat{\theta}$)	0.10	0.10	0.08	0.07	0.05	0.03	0.01	0.10	0.08	0.08	0.07	0.05	0.02	0.01	0.05	0.05	0.05	0.04	0.04	0.02	0.01
30	FWE	0.00	0.03	0.10	0.14	0.12	0.20	0.18	0.00	0.01	0.02	0.06	0.06	0.05	0.10	0.00	0.03	0.04	0.03	0.04	0.04	0.08
	FDR	0.00	0.00	0.01	0.02	0.03	0.10	0.18	0.00	0.00	0.00	0.01	0.01	0.02	0.10	0.00	0.00	0.00	0.00	0.00	0.01	0.08
	FNR	1.00	0.83	0.56	0.36	0.19	0.06	0.00	0.99	0.68	0.40	0.23	0.12	0.03	0.00	0.78	0.43	0.19	0.10	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.58	0.63	0.72	0.80	0.88	0.96	1.00	0.28	0.38	0.53	0.68	0.81	0.93	1.00	0.09	0.19	0.39	0.57	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.08	0.06	0.07	0.05	0.04	0.02	0.01	0.07	0.08	0.06	0.05	0.04	0.02	0.01	0.06	0.05	0.04	0.04	0.03	0.02	0.00
50	FWE	0.00	0.01	0.08	0.16	0.18	0.19	0.23	0.00	0.01	0.07	0.08	0.11	0.06	0.10	0.00	0.05	0.05	0.06	0.10	0.07	0.03
	FDR	0.00	0.00	0.01	0.02	0.03	0.09	0.23	0.00	0.00	0.00	0.01	0.01	0.02	0.10	0.00	0.00	0.00	0.00	0.01	0.01	0.03
	FNR	1.00	0.84	0.58	0.38	0.21	0.06	0.00	1.00	0.72	0.43	0.26	0.13	0.04	0.00	0.91	0.45	0.24	0.13	0.06	0.02	0.00

Table B.51: RW05 test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.54	0.59	0.69	0.81	0.88	0.96	1.00	0.24	0.29	0.45	0.65	0.79	0.93	1.00	0.06	0.18	0.35	0.56	0.74	0.91	0.99
	sd($\hat{\theta}$)	0.22	0.20	0.13	0.12	0.08	0.06	0.02	0.17	0.17	0.11	0.11	0.09	0.05	0.02	0.09	0.10	0.09	0.08	0.06	0.05	0.03
10	FWE	0.00	0.01	0.01	0.05	0.04	0.04	0.04	0.00	0.04	0.04	0.02	0.03	0.03	0.02	0.00	0.04	0.05	0.01	0.01	0.02	0.06
	FDR	0.00	0.00	0.01	0.02	0.02	0.03	0.04	0.00	0.00	0.01	0.00	0.01	0.02	0.02	0.00	0.00	0.01	0.00	0.01	0.01	0.06
	FNR	0.98	0.79	0.54	0.37	0.20	0.06	0.00	0.84	0.52	0.31	0.22	0.10	0.03	0.00	0.40	0.32	0.12	0.09	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.68	0.74	0.79	0.85	0.92	0.97	1.00	0.30	0.38	0.53	0.66	0.80	0.94	1.00	0.10	0.21	0.39	0.58	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.12	0.10	0.10	0.07	0.05	0.03	0.01	0.13	0.11	0.08	0.07	0.04	0.03	0.01	0.08	0.08	0.06	0.05	0.04	0.03	0.00
30	FWE	0.00	0.00	0.00	0.02	0.02	0.03	0.05	0.00	0.02	0.03	0.00	0.01	0.02	0.04	0.00	0.04	0.02	0.01	0.00	0.03	0.01
	FDR	0.00	0.00	0.00	0.00	0.01	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.01	0.01
	FNR	1.00	0.86	0.61	0.41	0.23	0.07	0.00	0.99	0.71	0.42	0.24	0.13	0.04	0.00	0.85	0.46	0.22	0.12	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.75	0.76	0.84	0.88	0.93	0.98	1.00	0.36	0.42	0.57	0.69	0.82	0.94	1.00	0.13	0.23	0.40	0.59	0.75	0.92	1.00
	sd($\hat{\theta}$)	0.10	0.09	0.07	0.06	0.03	0.02	0.00	0.10	0.09	0.07	0.07	0.04	0.02	0.00	0.08	0.07	0.05	0.04	0.03	0.02	0.00
50	FWE	0.00	0.01	0.01	0.01	0.04	0.02	0.00	0.00	0.01	0.02	0.02	0.03	0.00	0.04	0.00	0.01	0.05	0.01	0.03	0.00	0.06
	FDR	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.06
	FNR	1.00	0.87	0.64	0.43	0.25	0.08	0.00	1.00	0.75	0.47	0.27	0.14	0.04	0.00	0.94	0.53	0.24	0.15	0.07	0.02	0.00

Table B.52: RW05 test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.59	0.70	0.77	0.84	0.89	0.95	1.00	0.23	0.31	0.51	0.66	0.80	0.94	1.00	0.05	0.18	0.36	0.54	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.22	0.16	0.15	0.12	0.09	0.06	0.02	0.18	0.17	0.14	0.13	0.09	0.05	0.02	0.09	0.11	0.09	0.08	0.08	0.04	0.03
10	FWE	0.00	0.00	0.00	0.03	0.04	0.07	0.03	0.00	0.03	0.03	0.03	0.05	0.01	0.03	0.00	0.05	0.04	0.09	0.04	0.04	0.04
	FDR	0.00	0.00	0.00	0.01	0.02	0.06	0.03	0.00	0.00	0.01	0.01	0.01	0.01	0.03	0.00	0.01	0.01	0.02	0.01	0.02	0.04
	FNR	0.98	0.85	0.59	0.39	0.21	0.06	0.00	0.82	0.55	0.37	0.22	0.12	0.04	0.00	0.33	0.32	0.15	0.08	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.73	0.74	0.81	0.86	0.92	0.97	1.00	0.36	0.44	0.58	0.72	0.83	0.95	1.00	0.09	0.22	0.40	0.58	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.11	0.10	0.09	0.07	0.05	0.03	0.01	0.15	0.12	0.09	0.07	0.05	0.03	0.00	0.07	0.08	0.06	0.05	0.04	0.02	0.01
30	FWE	0.00	0.03	0.03	0.04	0.04	0.04	0.09	0.00	0.00	0.01	0.02	0.04	0.02	0.02	0.00	0.03	0.03	0.01	0.02	0.03	0.05
	FDR	0.00	0.00	0.01	0.02	0.02	0.03	0.09	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.05
	FNR	1.00	0.86	0.63	0.42	0.24	0.07	0.00	0.99	0.75	0.47	0.30	0.15	0.05	0.00	0.83	0.49	0.24	0.14	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.76	0.80	0.84	0.88	0.94	0.98	1.00	0.41	0.48	0.60	0.72	0.84	0.95	1.00	0.12	0.23	0.42	0.60	0.76	0.92	1.00
	sd($\hat{\theta}$)	0.09	0.08	0.06	0.06	0.04	0.02	0.01	0.13	0.10	0.08	0.06	0.04	0.02	0.00	0.08	0.07	0.05	0.05	0.03	0.02	0.01
50	FWE	0.00	0.02	0.03	0.04	0.07	0.06	0.08	0.00	0.00	0.02	0.01	0.04	0.03	0.03	0.00	0.00	0.03	0.02	0.02	0.01	0.03
	FDR	0.00	0.00	0.01	0.01	0.03	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.03
	FNR	1.00	0.87	0.64	0.43	0.26	0.08	0.00	1.00	0.78	0.49	0.30	0.17	0.05	0.00	0.92	0.51	0.27	0.16	0.08	0.02	0.00

Table B.53: RW05 test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.70	0.78	0.81	0.91	0.96	1.00	0.27	0.35	0.50	0.66	0.79	0.94	1.00	0.07	0.18	0.37	0.55	0.73	0.91	0.99
	sd($\hat{\theta}$)	0.23	0.19	0.17	0.14	0.08	0.05	0.03	0.21	0.18	0.14	0.12	0.08	0.05	0.01	0.11	0.11	0.09	0.07	0.06	0.04	0.04
10	FWE	0.00	0.00	0.01	0.04	0.03	0.01	0.03	0.00	0.00	0.01	0.01	0.00	0.01	0.02	0.00	0.03	0.02	0.05	0.03	0.02	0.04
	FDR	0.00	0.00	0.00	0.02	0.01	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.01	0.01	0.01	0.04
	FNR	0.98	0.84	0.59	0.37	0.22	0.06	0.00	0.86	0.59	0.34	0.22	0.11	0.04	0.00	0.41	0.29	0.16	0.09	0.04	0.02	0.00
	ave($\hat{\theta}$)	0.78	0.79	0.86	0.90	0.93	0.98	1.00	0.33	0.44	0.56	0.71	0.83	0.94	1.00	0.09	0.21	0.39	0.58	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.14	0.09	0.08	0.07	0.05	0.02	0.00	0.16	0.14	0.11	0.09	0.06	0.03	0.00	0.10	0.08	0.07	0.06	0.03	0.03	0.00
30	FWE	0.00	0.01	0.01	0.01	0.01	0.03	0.02	0.00	0.01	0.01	0.01	0.03	0.03	0.00	0.00	0.04	0.02	0.03	0.03	0.02	0.02
	FDR	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02
	FNR	1.00	0.87	0.65	0.44	0.25	0.08	0.00	0.95	0.73	0.44	0.28	0.15	0.04	0.00	0.74	0.45	0.22	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.82	0.83	0.88	0.90	0.96	0.98	1.00	0.41	0.46	0.60	0.72	0.84	0.95	1.00	0.12	0.24	0.42	0.59	0.76	0.92	1.00
	sd($\hat{\theta}$)	0.10	0.09	0.07	0.05	0.04	0.02	0.00	0.14	0.14	0.08	0.07	0.05	0.03	0.01	0.09	0.07	0.06	0.05	0.04	0.02	0.00
50	FWE	0.00	0.01	0.01	0.02	0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.00	0.02	0.03	0.02	0.02	0.03	0.03
	FDR	0.00	0.00	0.00	0.01	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.00	0.00	0.00	0.00	0.00	0.01	0.03
	FNR	1.00	0.88	0.66	0.45	0.27	0.08	0.00	1.00	0.76	0.49	0.30	0.17	0.05	0.00	0.89	0.53	0.27	0.15	0.08	0.02	0.00

Table B.54: RW05 test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.70	0.71	0.80	0.85	0.90	0.96	0.99	0.27	0.40	0.53	0.68	0.79	0.94	1.00	0.09	0.18	0.35	0.56	0.74	0.91	1.00
	sd($\hat{\theta}$)	0.21	0.19	0.16	0.12	0.09	0.05	0.03	0.21	0.17	0.16	0.12	0.10	0.05	0.02	0.12	0.10	0.09	0.09	0.06	0.04	0.02
10	FWE	0.00	0.00	0.01	0.03	0.02	0.04	0.03	0.00	0.01	0.03	0.04	0.06	0.01	0.03	0.00	0.02	0.04	0.05	0.03	0.01	0.05
	FDR	0.00	0.00	0.01	0.01	0.02	0.04	0.03	0.00	0.00	0.00	0.01	0.02	0.01	0.03	0.00	0.00	0.01	0.01	0.01	0.01	0.05
	FNR	0.98	0.84	0.60	0.40	0.22	0.07	0.00	0.81	0.67	0.38	0.24	0.12	0.04	0.00	0.49	0.33	0.13	0.10	0.05	0.01	0.00
	ave($\hat{\theta}$)	0.79	0.80	0.84	0.90	0.94	0.98	1.00	0.36	0.47	0.60	0.72	0.83	0.95	1.00	0.12	0.21	0.41	0.58	0.76	0.92	1.00
	sd($\hat{\theta}$)	0.13	0.12	0.09	0.08	0.05	0.03	0.00	0.15	0.14	0.11	0.09	0.06	0.03	0.00	0.10	0.07	0.07	0.06	0.04	0.03	0.01
30	FWE	0.00	0.01	0.02	0.02	0.04	0.05	0.02	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.00	0.02	0.04	0.01	0.02	0.04	0.05
	FDR	0.00	0.00	0.00	0.01	0.02	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.05
	FNR	1.00	0.87	0.64	0.44	0.26	0.08	0.00	0.99	0.76	0.48	0.30	0.16	0.05	0.00	0.82	0.45	0.25	0.13	0.07	0.02	0.00
	ave($\hat{\theta}$)	0.81	0.85	0.89	0.93	0.94	0.98	1.00	0.41	0.50	0.65	0.75	0.84	0.95	1.00	0.12	0.23	0.43	0.59	0.76	0.92	1.00
	sd($\hat{\theta}$)	0.09	0.09	0.06	0.05	0.03	0.02	0.01	0.15	0.12	0.09	0.07	0.05	0.02	0.00	0.09	0.07	0.06	0.05	0.04	0.03	0.00
50	FWE	0.00	0.01	0.01	0.02	0.04	0.06	0.07	0.00	0.01	0.01	0.04	0.03	0.03	0.01	0.00	0.05	0.00	0.01	0.03	0.03	0.00
	FDR	0.00	0.00	0.00	0.00	0.02	0.03	0.07	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	FNR	1.00	0.88	0.66	0.46	0.26	0.09	0.00	1.00	0.79	0.53	0.33	0.17	0.05	0.00	0.89	0.54	0.29	0.15	0.08	0.02	0.00

Table B.55: MP12 test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.51	0.46	0.67	0.79	0.90	0.97	0.99	0.01	0.14	0.38	0.60	0.77	0.93	1.00	0.00	0.06	0.28	0.50	0.71	0.90	1.00
	sd($\hat{\theta}$)	0.40	0.30	0.21	0.16	0.12	0.05	0.03	0.03	0.14	0.11	0.12	0.10	0.05	0.02	0.01	0.07	0.10	0.07	0.07	0.04	0.02
10	FWE	0.00	0.10	0.08	0.05	0.09	0.03	0.05	0.00	0.34	0.12	0.08	0.14	0.03	0.03	0.00	0.53	0.31	0.22	0.10	0.05	0.03
	FDR	0.00	0.01	0.02	0.01	0.04	0.03	0.05	0.00	0.03	0.02	0.02	0.04	0.02	0.03	0.00	0.05	0.05	0.04	0.03	0.03	0.03
	FNR	0.69	0.64	0.52	0.35	0.23	0.07	0.00	0.04	0.24	0.19	0.15	0.10	0.03	0.00	0.01	0.05	0.07	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.53	0.40	0.67	0.81	0.93	0.98	1.00	0.00	0.10	0.35	0.57	0.77	0.94	1.00	0.00	0.06	0.28	0.51	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.41	0.20	0.17	0.12	0.07	0.03	0.01	0.01	0.09	0.08	0.07	0.05	0.03	0.00	0.00	0.05	0.06	0.04	0.03	0.03	0.00
30	FWE	0.00	0.21	0.19	0.19	0.05	0.09	0.02	0.00	0.55	0.41	0.27	0.21	0.03	0.01	0.00	0.61	0.53	0.44	0.18	0.08	0.01
	FDR	0.00	0.01	0.02	0.03	0.01	0.06	0.02	0.00	0.04	0.03	0.03	0.03	0.01	0.01	0.00	0.05	0.05	0.04	0.02	0.03	0.01
	FNR	0.74	0.70	0.54	0.38	0.25	0.08	0.00	0.02	0.23	0.20	0.14	0.10	0.05	0.00	0.00	0.05	0.05	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.44	0.43	0.66	0.83	0.94	0.98	1.00	0.00	0.10	0.36	0.57	0.77	0.94	1.00	0.00	0.07	0.29	0.50	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.43	0.17	0.12	0.11	0.05	0.02	0.00	0.00	0.06	0.06	0.05	0.05	0.03	0.00	0.00	0.05	0.04	0.03	0.03	0.02	0.01
50	FWE	0.00	0.20	0.23	0.20	0.08	0.05	0.03	0.00	0.64	0.52	0.41	0.23	0.08	0.01	0.00	0.72	0.59	0.53	0.38	0.15	0.04
	FDR	0.00	0.01	0.01	0.02	0.01	0.03	0.03	0.00	0.03	0.03	0.03	0.03	0.02	0.01	0.00	0.04	0.03	0.04	0.04	0.03	0.04
	FNR	0.67	0.74	0.54	0.39	0.26	0.08	0.00	0.00	0.23	0.21	0.14	0.10	0.04	0.00	0.00	0.06	0.05	0.04	0.03	0.01	0.00

Table B.56: MP12 test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.57	0.46	0.69	0.80	0.89	0.96	1.00	0.01	0.13	0.37	0.58	0.76	0.93	1.00	0.00	0.07	0.30	0.51	0.71	0.90	1.00
	sd($\hat{\theta}$)	0.43	0.28	0.23	0.17	0.10	0.05	0.02	0.03	0.13	0.15	0.13	0.10	0.05	0.01	0.01	0.08	0.08	0.08	0.07	0.04	0.02
10	FWE	0.00	0.12	0.09	0.07	0.04	0.07	0.04	0.00	0.36	0.26	0.13	0.12	0.03	0.01	0.00	0.42	0.23	0.17	0.13	0.08	0.04
	FDR	0.00	0.01	0.02	0.02	0.02	0.06	0.04	0.00	0.04	0.04	0.03	0.04	0.02	0.01	0.00	0.04	0.04	0.03	0.04	0.05	0.04
	FNR	0.71	0.66	0.52	0.35	0.21	0.07	0.00	0.06	0.22	0.19	0.15	0.08	0.03	0.00	0.01	0.06	0.06	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.68	0.45	0.69	0.83	0.93	0.98	1.00	0.00	0.10	0.36	0.60	0.77	0.93	1.00	0.00	0.05	0.29	0.51	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.42	0.21	0.15	0.12	0.07	0.03	0.01	0.00	0.07	0.09	0.08	0.07	0.03	0.00	0.00	0.05	0.05	0.05	0.05	0.03	0.00
30	FWE	0.00	0.19	0.16	0.08	0.03	0.06	0.07	0.00	0.44	0.43	0.20	0.19	0.04	0.03	0.00	0.66	0.48	0.34	0.25	0.08	0.02
	FDR	0.00	0.01	0.02	0.01	0.01	0.02	0.07	0.00	0.03	0.03	0.02	0.03	0.01	0.03	0.00	0.05	0.03	0.04	0.03	0.02	0.02
	FNR	0.79	0.73	0.55	0.39	0.24	0.08	0.00	0.00	0.18	0.21	0.16	0.10	0.04	0.00	0.00	0.03	0.06	0.05	0.04	0.02	0.00
	ave($\hat{\theta}$)	0.63	0.41	0.67	0.83	0.93	0.99	1.00	0.00	0.11	0.37	0.57	0.77	0.94	1.00	0.00	0.06	0.29	0.51	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.43	0.16	0.13	0.10	0.06	0.02	0.00	0.00	0.07	0.07	0.06	0.05	0.03	0.00	0.00	0.05	0.04	0.04	0.04	0.02	0.00
50	FWE	0.00	0.27	0.26	0.15	0.13	0.05	0.04	0.00	0.61	0.53	0.41	0.21	0.08	0.00	0.00	0.79	0.70	0.54	0.37	0.09	0.02
	FDR	0.00	0.01	0.02	0.01	0.02	0.03	0.04	0.00	0.03	0.03	0.03	0.03	0.02	0.00	0.00	0.05	0.04	0.04	0.04	0.02	0.02
	FNR	0.75	0.73	0.55	0.40	0.25	0.09	0.00	0.00	0.24	0.23	0.15	0.10	0.04	0.00	0.00	0.07	0.06	0.05	0.03	0.02	0.00

Table B.57: MP12 test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.49	0.49	0.67	0.80	0.88	0.97	0.99	0.01	0.12	0.36	0.58	0.77	0.92	1.00	0.00	0.07	0.31	0.51	0.71	0.90	1.00
	sd($\hat{\theta}$)	0.43	0.26	0.21	0.16	0.13	0.05	0.04	0.05	0.14	0.14	0.12	0.09	0.09	0.01	0.00	0.07	0.09	0.09	0.07	0.06	0.01
10	FWE	0.00	0.09	0.07	0.10	0.04	0.05	0.02	0.00	0.41	0.15	0.15	0.07	0.05	0.01	0.00	0.44	0.13	0.14	0.10	0.06	0.01
	FDR	0.00	0.01	0.01	0.04	0.02	0.05	0.02	0.00	0.04	0.03	0.03	0.02	0.04	0.01	0.00	0.04	0.02	0.03	0.03	0.04	0.01
	FNR	0.65	0.71	0.51	0.37	0.20	0.07	0.00	0.07	0.22	0.17	0.14	0.09	0.04	0.00	0.00	0.05	0.07	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.59	0.45	0.69	0.83	0.94	0.98	1.00	0.01	0.10	0.36	0.57	0.78	0.93	1.00	0.00	0.07	0.30	0.51	0.71	0.92	0.99
	sd($\hat{\theta}$)	0.44	0.23	0.16	0.10	0.06	0.03	0.00	0.05	0.07	0.09	0.07	0.05	0.04	0.00	0.00	0.06	0.06	0.07	0.05	0.03	0.04
30	FWE	0.00	0.15	0.13	0.07	0.02	0.03	0.02	0.00	0.50	0.36	0.35	0.13	0.06	0.02	0.00	0.52	0.29	0.36	0.18	0.06	0.03
	FDR	0.00	0.01	0.02	0.01	0.00	0.02	0.02	0.00	0.04	0.03	0.04	0.02	0.02	0.02	0.00	0.04	0.03	0.04	0.03	0.02	0.03
	FNR	0.70	0.73	0.56	0.39	0.25	0.08	0.00	0.03	0.23	0.20	0.15	0.10	0.04	0.00	0.00	0.06	0.06	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.63	0.43	0.63	0.82	0.92	0.98	1.00	0.01	0.10	0.36	0.58	0.77	0.93	1.00	0.00	0.06	0.30	0.51	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.44	0.18	0.14	0.10	0.06	0.03	0.00	0.05	0.07	0.08	0.08	0.05	0.05	0.00	0.00	0.05	0.06	0.05	0.04	0.02	0.00
50	FWE	0.00	0.19	0.22	0.14	0.10	0.04	0.00	0.00	0.58	0.48	0.27	0.19	0.09	0.03	0.00	0.75	0.49	0.44	0.22	0.12	0.02
	FDR	0.00	0.01	0.02	0.02	0.02	0.03	0.00	0.00	0.04	0.03	0.02	0.02	0.03	0.03	0.00	0.05	0.03	0.04	0.02	0.04	0.02
	FNR	0.73	0.74	0.52	0.39	0.24	0.09	0.00	0.02	0.23	0.21	0.15	0.10	0.04	0.00	0.00	0.05	0.08	0.05	0.03	0.01	0.00

Table B.58: MP12 test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.45	0.66	0.81	0.90	0.97	1.00	0.02	0.15	0.38	0.59	0.74	0.94	1.00	0.00	0.07	0.30	0.50	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.46	0.28	0.22	0.18	0.10	0.05	0.01	0.10	0.15	0.13	0.10	0.09	0.05	0.00	0.00	0.07	0.08	0.08	0.07	0.04	0.00
10	FWE	0.00	0.09	0.07	0.05	0.04	0.04	0.02	0.00	0.29	0.11	0.05	0.09	0.02	0.00	0.00	0.42	0.13	0.14	0.12	0.05	0.00
	FDR	0.00	0.01	0.01	0.02	0.02	0.03	0.02	0.00	0.03	0.02	0.01	0.03	0.01	0.00	0.00	0.04	0.02	0.03	0.03	0.03	0.00
	FNR	0.66	0.64	0.51	0.37	0.22	0.07	0.00	0.03	0.27	0.19	0.15	0.07	0.04	0.00	0.00	0.06	0.05	0.04	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.69	0.42	0.66	0.82	0.91	0.97	1.00	0.00	0.12	0.37	0.59	0.77	0.93	1.00	0.00	0.06	0.30	0.50	0.70	0.90	1.00
	sd($\hat{\theta}$)	0.43	0.18	0.17	0.11	0.07	0.05	0.00	0.00	0.08	0.08	0.08	0.06	0.04	0.02	0.00	0.05	0.06	0.08	0.09	0.05	0.00
30	FWE	0.00	0.13	0.12	0.06	0.09	0.04	0.01	0.00	0.40	0.22	0.21	0.14	0.08	0.01	0.00	0.64	0.38	0.26	0.22	0.12	0.00
	FDR	0.00	0.01	0.01	0.01	0.02	0.02	0.01	0.00	0.03	0.02	0.02	0.02	0.03	0.01	0.00	0.05	0.03	0.04	0.05	0.05	0.00
	FNR	0.75	0.73	0.53	0.39	0.23	0.08	0.00	0.01	0.25	0.21	0.16	0.10	0.04	0.00	0.00	0.06	0.07	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.79	0.41	0.67	0.82	0.93	0.98	1.00	0.02	0.10	0.36	0.57	0.77	0.93	1.00	0.00	0.07	0.29	0.51	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.38	0.17	0.14	0.10	0.06	0.02	0.00	0.09	0.06	0.06	0.06	0.05	0.03	0.00	0.00	0.05	0.05	0.07	0.03	0.02	0.00
50	FWE	0.00	0.24	0.20	0.13	0.03	0.09	0.04	0.00	0.55	0.41	0.30	0.19	0.07	0.04	0.00	0.62	0.47	0.43	0.22	0.09	0.01
	FDR	0.00	0.01	0.02	0.02	0.01	0.03	0.04	0.00	0.03	0.02	0.02	0.02	0.02	0.04	0.00	0.04	0.03	0.04	0.02	0.02	0.01
	FNR	0.82	0.73	0.54	0.39	0.25	0.09	0.00	0.02	0.23	0.21	0.14	0.10	0.04	0.00	0.00	0.06	0.06	0.06	0.03	0.01	0.00

Table B.59: MP12 test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.40	0.39	0.60	0.74	0.83	0.90	0.95	0.02	0.11	0.26	0.47	0.58	0.74	0.83	0.00	0.04	0.21	0.34	0.51	0.73	0.81
	sd($\hat{\theta}$)	0.40	0.28	0.29	0.21	0.18	0.18	0.11	0.05	0.13	0.17	0.21	0.28	0.24	0.22	0.01	0.07	0.15	0.18	0.23	0.27	0.27
10	FWE	0.00	0.23	0.32	0.33	0.30	0.30	0.29	0.00	0.54	0.52	0.54	0.67	0.66	0.67	0.00	0.67	0.59	0.82	0.71	0.59	0.63
	FDR	0.00	0.03	0.08	0.13	0.15	0.23	0.29	0.00	0.05	0.12	0.16	0.31	0.48	0.67	0.00	0.07	0.13	0.25	0.32	0.42	0.63
	FNR	0.60	0.61	0.48	0.36	0.21	0.07	0.00	0.09	0.24	0.17	0.14	0.09	0.02	0.00	0.01	0.04	0.08	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.47	0.40	0.58	0.75	0.87	0.93	0.97	0.00	0.08	0.25	0.40	0.62	0.78	0.89	0.00	0.03	0.19	0.39	0.50	0.72	0.80
	sd($\hat{\theta}$)	0.42	0.23	0.19	0.16	0.14	0.11	0.06	0.01	0.08	0.14	0.21	0.17	0.20	0.13	0.00	0.05	0.12	0.14	0.24	0.20	0.24
30	FWE	0.00	0.50	0.60	0.50	0.49	0.53	0.40	0.00	0.68	0.80	0.89	0.93	0.85	0.83	0.00	0.76	0.89	0.89	0.86	0.95	0.93
	FDR	0.00	0.04	0.08	0.12	0.19	0.40	0.40	0.00	0.05	0.12	0.22	0.30	0.52	0.83	0.00	0.07	0.14	0.19	0.35	0.56	0.93
	FNR	0.72	0.70	0.53	0.37	0.25	0.08	0.00	0.01	0.23	0.16	0.13	0.09	0.04	0.00	0.00	0.04	0.06	0.04	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.49	0.40	0.62	0.76	0.87	0.95	0.97	0.00	0.06	0.26	0.43	0.61	0.77	0.86	0.00	0.03	0.17	0.34	0.55	0.68	0.81
	sd($\hat{\theta}$)	0.41	0.17	0.16	0.16	0.11	0.09	0.08	0.00	0.06	0.14	0.19	0.21	0.19	0.18	0.00	0.05	0.12	0.17	0.20	0.25	0.22
50	FWE	0.00	0.60	0.72	0.63	0.57	0.44	0.46	0.00	0.87	0.88	0.93	0.95	0.99	0.92	0.00	0.85	0.97	0.98	0.97	0.99	0.98
	FDR	0.00	0.03	0.09	0.14	0.21	0.32	0.46	0.00	0.06	0.12	0.19	0.30	0.58	0.92	0.00	0.07	0.16	0.23	0.29	0.60	0.98
	FNR	0.76	0.77	0.56	0.39	0.24	0.08	0.00	0.00	0.16	0.19	0.13	0.09	0.03	0.00	0.00	0.02	0.05	0.04	0.02	0.01	0.00

Table B.60: MP12 test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.60	0.43	0.66	0.76	0.86	0.96	0.98	0.01	0.12	0.35	0.55	0.75	0.92	0.96	0.00	0.05	0.25	0.44	0.66	0.88	0.96
	sd($\hat{\theta}$)	0.41	0.28	0.26	0.18	0.13	0.06	0.08	0.03	0.14	0.16	0.15	0.14	0.07	0.09	0.00	0.06	0.12	0.17	0.14	0.09	0.08
10	FWE	0.00	0.19	0.16	0.12	0.13	0.13	0.07	0.00	0.48	0.29	0.27	0.27	0.16	0.22	0.00	0.59	0.36	0.39	0.29	0.23	0.24
	FDR	0.00	0.02	0.04	0.04	0.06	0.11	0.07	0.00	0.05	0.06	0.07	0.10	0.09	0.22	0.00	0.06	0.07	0.11	0.10	0.13	0.24
	FNR	0.75	0.65	0.51	0.34	0.19	0.07	0.00	0.05	0.21	0.20	0.15	0.11	0.04	0.00	0.00	0.04	0.05	0.05	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.69	0.44	0.66	0.79	0.91	0.97	1.00	0.00	0.10	0.31	0.53	0.73	0.92	0.98	0.00	0.05	0.25	0.48	0.68	0.87	0.96
	sd($\hat{\theta}$)	0.42	0.22	0.18	0.12	0.08	0.04	0.01	0.01	0.09	0.12	0.12	0.15	0.06	0.04	0.00	0.05	0.10	0.08	0.06	0.10	0.11
30	FWE	0.00	0.25	0.32	0.25	0.26	0.13	0.09	0.00	0.56	0.57	0.52	0.42	0.33	0.31	0.00	0.63	0.65	0.65	0.60	0.42	0.43
	FDR	0.00	0.02	0.03	0.05	0.08	0.08	0.09	0.00	0.04	0.07	0.08	0.11	0.14	0.31	0.00	0.05	0.08	0.09	0.10	0.17	0.43
	FNR	0.77	0.72	0.54	0.38	0.24	0.08	0.00	0.03	0.22	0.17	0.14	0.10	0.04	0.00	0.00	0.04	0.05	0.06	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.66	0.44	0.64	0.83	0.92	0.98	1.00	0.00	0.09	0.34	0.55	0.75	0.93	0.99	0.00	0.06	0.25	0.49	0.67	0.88	0.96
	sd($\hat{\theta}$)	0.44	0.18	0.16	0.10	0.07	0.02	0.01	0.00	0.07	0.10	0.09	0.09	0.03	0.02	0.00	0.05	0.09	0.06	0.09	0.05	0.12
50	FWE	0.00	0.29	0.45	0.27	0.22	0.16	0.13	0.00	0.72	0.63	0.61	0.47	0.38	0.32	0.00	0.75	0.82	0.76	0.70	0.63	0.56
	FDR	0.00	0.02	0.04	0.04	0.05	0.08	0.13	0.00	0.04	0.05	0.07	0.07	0.14	0.32	0.00	0.05	0.09	0.08	0.11	0.19	0.56
	FNR	0.75	0.76	0.54	0.40	0.25	0.09	0.00	0.01	0.23	0.22	0.16	0.10	0.04	0.00	0.00	0.07	0.07	0.06	0.03	0.01	0.00

Table B.61: MP12 test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.53	0.43	0.61	0.75	0.85	0.93	0.96	0.00	0.13	0.33	0.47	0.67	0.81	0.91	0.00	0.06	0.24	0.42	0.62	0.81	0.84
	sd($\hat{\theta}$)	0.43	0.28	0.25	0.19	0.19	0.13	0.13	0.02	0.14	0.15	0.21	0.23	0.20	0.16	0.00	0.07	0.13	0.17	0.19	0.17	0.23
10	FWE	0.00	0.15	0.22	0.23	0.24	0.25	0.21	0.00	0.42	0.31	0.47	0.48	0.54	0.50	0.00	0.55	0.47	0.51	0.52	0.52	0.63
	FDR	0.00	0.02	0.04	0.07	0.14	0.21	0.21	0.00	0.04	0.06	0.14	0.22	0.36	0.50	0.00	0.06	0.09	0.14	0.20	0.33	0.63
	FNR	0.67	0.63	0.49	0.34	0.22	0.07	0.00	0.03	0.24	0.17	0.13	0.10	0.03	0.00	0.00	0.05	0.05	0.04	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.68	0.39	0.64	0.77	0.87	0.92	0.98	0.01	0.09	0.28	0.52	0.67	0.85	0.94	0.00	0.06	0.24	0.42	0.59	0.80	0.89
	sd($\hat{\theta}$)	0.41	0.21	0.21	0.15	0.13	0.15	0.04	0.07	0.09	0.14	0.13	0.20	0.13	0.06	0.00	0.06	0.11	0.14	0.18	0.16	0.17
30	FWE	0.00	0.33	0.44	0.42	0.41	0.41	0.27	0.00	0.57	0.68	0.71	0.75	0.78	0.80	0.00	0.56	0.69	0.85	0.83	0.85	0.81
	FDR	0.00	0.02	0.07	0.09	0.13	0.28	0.27	0.00	0.05	0.10	0.11	0.22	0.41	0.80	0.00	0.05	0.09	0.15	0.24	0.42	0.81
	FNR	0.76	0.69	0.54	0.38	0.22	0.07	0.00	0.02	0.22	0.18	0.14	0.10	0.04	0.00	0.00	0.06	0.04	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.73	0.40	0.62	0.78	0.87	0.96	0.99	0.01	0.08	0.27	0.49	0.68	0.84	0.93	0.00	0.05	0.24	0.43	0.60	0.79	0.88
	sd($\hat{\theta}$)	0.39	0.16	0.17	0.14	0.16	0.09	0.02	0.05	0.07	0.13	0.17	0.12	0.20	0.10	0.00	0.05	0.10	0.11	0.16	0.18	0.16
50	FWE	0.00	0.37	0.55	0.56	0.50	0.29	0.29	0.00	0.74	0.87	0.84	0.91	0.86	0.80	0.00	0.74	0.86	0.90	0.97	0.92	0.91
	FDR	0.00	0.02	0.06	0.09	0.16	0.21	0.29	0.00	0.05	0.11	0.14	0.20	0.40	0.80	0.00	0.06	0.09	0.13	0.23	0.41	0.91
	FNR	0.84	0.73	0.55	0.39	0.24	0.08	0.00	0.02	0.19	0.18	0.14	0.09	0.04	0.00	0.00	0.04	0.06	0.04	0.03	0.01	0.00

Table B.62: MP12 test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.62	0.50	0.62	0.80	0.90	0.97	0.99	0.00	0.13	0.36	0.55	0.78	0.92	0.98	0.00	0.06	0.26	0.48	0.69	0.88	0.99
	sd($\hat{\theta}$)	0.46	0.30	0.26	0.15	0.13	0.05	0.08	0.01	0.14	0.15	0.18	0.10	0.07	0.10	0.00	0.07	0.12	0.12	0.11	0.10	0.05
10	FWE	0.00	0.14	0.13	0.07	0.05	0.01	0.05	0.00	0.39	0.20	0.24	0.12	0.11	0.06	0.00	0.51	0.30	0.18	0.22	0.13	0.08
	FDR	0.00	0.02	0.03	0.02	0.03	0.01	0.05	0.00	0.04	0.04	0.07	0.04	0.08	0.06	0.00	0.05	0.06	0.05	0.07	0.08	0.08
	FNR	0.67	0.69	0.45	0.36	0.22	0.07	0.00	0.01	0.24	0.20	0.15	0.11	0.03	0.00	0.00	0.05	0.04	0.04	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.71	0.43	0.71	0.81	0.90	0.97	1.00	0.01	0.10	0.34	0.56	0.75	0.93	1.00	0.00	0.07	0.29	0.50	0.69	0.87	0.98
	sd($\hat{\theta}$)	0.42	0.21	0.19	0.12	0.08	0.05	0.00	0.07	0.09	0.10	0.11	0.11	0.04	0.01	0.00	0.05	0.07	0.06	0.08	0.18	0.06
30	FWE	0.00	0.20	0.16	0.13	0.15	0.06	0.01	0.00	0.53	0.44	0.31	0.30	0.16	0.07	0.00	0.49	0.46	0.42	0.32	0.27	0.19
	FDR	0.00	0.01	0.02	0.03	0.04	0.04	0.01	0.00	0.04	0.04	0.05	0.06	0.07	0.07	0.00	0.04	0.04	0.05	0.06	0.11	0.19
	FNR	0.76	0.73	0.56	0.38	0.22	0.08	0.00	0.01	0.23	0.18	0.15	0.10	0.04	0.00	0.00	0.05	0.07	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.77	0.40	0.67	0.80	0.92	0.98	1.00	0.00	0.11	0.33	0.58	0.76	0.92	1.00	0.00	0.05	0.26	0.48	0.70	0.88	0.98
	sd($\hat{\theta}$)	0.40	0.16	0.14	0.10	0.07	0.04	0.01	0.00	0.07	0.09	0.05	0.07	0.08	0.01	0.00	0.05	0.09	0.10	0.07	0.13	0.09
50	FWE	0.00	0.29	0.20	0.27	0.08	0.06	0.06	0.00	0.51	0.53	0.29	0.37	0.27	0.11	0.00	0.73	0.56	0.45	0.47	0.36	0.28
	FDR	0.00	0.02	0.02	0.03	0.02	0.03	0.06	0.00	0.03	0.05	0.03	0.06	0.10	0.11	0.00	0.05	0.06	0.06	0.07	0.11	0.28
	FNR	0.79	0.75	0.55	0.38	0.25	0.08	0.00	0.00	0.26	0.19	0.16	0.10	0.04	0.00	0.00	0.05	0.06	0.05	0.03	0.01	0.00

Table B.63: MP12 test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.48	0.59	0.73	0.86	0.92	0.98	1.00	0.03	0.11	0.33	0.53	0.76	0.91	0.99	0.00	0.06	0.28	0.49	0.70	0.89	0.99
	sd($\hat{\theta}$)	0.40	0.38	0.27	0.17	0.10	0.05	0.02	0.10	0.12	0.13	0.14	0.10	0.07	0.03	0.00	0.07	0.10	0.10	0.07	0.06	0.04
10	FWE	0.00	0.11	0.13	0.06	0.09	0.03	0.04	0.00	0.39	0.26	0.25	0.13	0.13	0.07	0.00	0.56	0.30	0.29	0.20	0.15	0.14
	FDR	0.00	0.01	0.03	0.02	0.04	0.02	0.04	0.00	0.04	0.05	0.05	0.04	0.07	0.07	0.00	0.06	0.05	0.06	0.06	0.08	0.14
	FNR	0.67	0.67	0.53	0.40	0.24	0.08	0.00	0.10	0.15	0.15	0.10	0.08	0.03	0.00	0.00	0.05	0.05	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.44	0.58	0.83	0.90	0.96	0.99	1.00	0.00	0.08	0.31	0.53	0.76	0.93	1.00	0.00	0.05	0.26	0.48	0.70	0.89	1.00
	sd($\hat{\theta}$)	0.42	0.33	0.19	0.13	0.06	0.03	0.00	0.02	0.07	0.08	0.07	0.08	0.04	0.01	0.00	0.05	0.06	0.04	0.04	0.03	0.01
30	FWE	0.00	0.13	0.10	0.08	0.08	0.01	0.02	0.00	0.60	0.61	0.65	0.37	0.16	0.06	0.00	0.67	0.69	0.67	0.47	0.41	0.13
	FDR	0.00	0.01	0.01	0.01	0.03	0.01	0.02	0.00	0.04	0.05	0.07	0.06	0.05	0.06	0.00	0.05	0.06	0.07	0.07	0.13	0.13
	FNR	0.65	0.73	0.62	0.44	0.27	0.09	0.00	0.03	0.16	0.15	0.12	0.10	0.04	0.00	0.00	0.02	0.04	0.03	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.44	0.61	0.79	0.92	0.97	1.00	1.00	0.00	0.07	0.32	0.54	0.74	0.93	1.00	0.00	0.05	0.26	0.48	0.69	0.90	1.00
	sd($\hat{\theta}$)	0.41	0.35	0.22	0.11	0.05	0.01	0.00	0.01	0.05	0.09	0.07	0.06	0.03	0.00	0.00	0.04	0.04	0.03	0.03	0.02	0.01
50	FWE	0.00	0.23	0.22	0.13	0.05	0.01	0.05	0.00	0.81	0.80	0.70	0.61	0.19	0.04	0.00	0.94	0.87	0.83	0.63	0.42	0.10
	FDR	0.00	0.01	0.02	0.02	0.01	0.01	0.05	0.00	0.05	0.06	0.06	0.07	0.04	0.04	0.00	0.06	0.06	0.07	0.07	0.10	0.10
	FNR	0.67	0.75	0.60	0.45	0.28	0.10	0.00	0.02	0.13	0.16	0.12	0.08	0.04	0.00	0.00	0.02	0.04	0.03	0.02	0.01	0.00

Table B.64: MP12 test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.60	0.64	0.74	0.88	0.97	0.98	1.00	0.01	0.06	0.28	0.51	0.73	0.91	0.98	0.00	0.05	0.22	0.44	0.66	0.86	0.98
	sd($\hat{\theta}$)	0.41	0.37	0.28	0.18	0.05	0.07	0.02	0.03	0.08	0.11	0.16	0.14	0.09	0.05	0.01	0.06	0.11	0.10	0.09	0.07	0.06
10	FWE	0.00	0.15	0.17	0.08	0.04	0.09	0.04	0.00	0.59	0.46	0.40	0.26	0.19	0.14	0.00	0.58	0.48	0.44	0.36	0.39	0.16
	FDR	0.00	0.02	0.03	0.02	0.04	0.06	0.04	0.00	0.06	0.07	0.10	0.08	0.12	0.14	0.00	0.06	0.09	0.10	0.11	0.21	0.16
	FNR	0.75	0.72	0.55	0.42	0.28	0.09	0.00	0.05	0.08	0.10	0.11	0.08	0.04	0.00	0.01	0.02	0.02	0.01	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.62	0.69	0.85	0.94	0.98	0.99	1.00	0.00	0.05	0.28	0.50	0.73	0.94	1.00	0.00	0.03	0.23	0.44	0.66	0.88	0.99
	sd($\hat{\theta}$)	0.44	0.38	0.22	0.11	0.04	0.02	0.01	0.00	0.06	0.08	0.08	0.08	0.05	0.01	0.00	0.04	0.05	0.06	0.06	0.04	0.03
30	FWE	0.00	0.19	0.11	0.08	0.05	0.04	0.06	0.00	0.79	0.76	0.68	0.52	0.21	0.10	0.00	0.84	0.91	0.85	0.80	0.53	0.28
	FDR	0.00	0.01	0.01	0.02	0.03	0.02	0.06	0.00	0.06	0.08	0.09	0.09	0.09	0.10	0.00	0.07	0.09	0.11	0.16	0.19	0.28
	FNR	0.71	0.72	0.62	0.46	0.28	0.09	0.00	0.00	0.11	0.11	0.09	0.07	0.05	0.00	0.00	0.02	0.03	0.02	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.67	0.72	0.90	0.96	0.98	1.00	1.00	0.00	0.05	0.28	0.50	0.72	0.93	1.00	0.00	0.04	0.23	0.44	0.66	0.88	0.99
	sd($\hat{\theta}$)	0.43	0.37	0.17	0.08	0.03	0.01	0.00	0.01	0.05	0.07	0.06	0.07	0.04	0.01	0.00	0.04	0.04	0.05	0.04	0.04	0.02
50	FWE	0.00	0.19	0.15	0.08	0.03	0.04	0.03	0.00	0.94	0.93	0.85	0.73	0.36	0.08	0.00	0.93	0.98	0.97	0.90	0.65	0.35
	FDR	0.00	0.01	0.01	0.02	0.01	0.04	0.03	0.00	0.06	0.08	0.09	0.11	0.11	0.08	0.00	0.07	0.09	0.13	0.15	0.18	0.35
	FNR	0.79	0.75	0.65	0.48	0.29	0.10	0.00	0.01	0.09	0.12	0.10	0.08	0.05	0.00	0.00	0.03	0.02	0.02	0.02	0.01	0.00

Table B.65: MP12 test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.57	0.66	0.80	0.88	0.91	0.98	0.99	0.02	0.06	0.26	0.43	0.71	0.85	0.94	0.00	0.04	0.18	0.40	0.57	0.78	0.90
	sd($\hat{\theta}$)	0.44	0.38	0.28	0.22	0.14	0.06	0.03	0.08	0.10	0.17	0.16	0.16	0.15	0.11	0.00	0.06	0.11	0.12	0.14	0.14	0.14
10	FWE	0.00	0.16	0.15	0.12	0.16	0.09	0.07	0.00	0.67	0.56	0.67	0.40	0.42	0.33	0.00	0.69	0.71	0.66	0.68	0.66	0.53
	FDR	0.00	0.02	0.05	0.04	0.09	0.08	0.07	0.00	0.07	0.10	0.18	0.15	0.28	0.33	0.00	0.07	0.14	0.16	0.27	0.41	0.53
	FNR	0.68	0.69	0.58	0.43	0.24	0.09	0.00	0.10	0.07	0.10	0.08	0.09	0.03	0.00	0.00	0.03	0.02	0.02	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.63	0.62	0.78	0.90	0.94	0.99	1.00	0.01	0.05	0.24	0.45	0.67	0.86	0.97	0.00	0.03	0.19	0.38	0.58	0.79	0.93
	sd($\hat{\theta}$)	0.43	0.41	0.31	0.18	0.11	0.03	0.01	0.06	0.06	0.14	0.12	0.11	0.11	0.07	0.00	0.04	0.08	0.08	0.07	0.08	0.07
30	FWE	0.00	0.30	0.25	0.13	0.19	0.10	0.06	0.00	0.80	0.87	0.88	0.75	0.65	0.33	0.00	0.86	0.93	0.95	0.98	0.95	0.76
	FDR	0.00	0.02	0.04	0.03	0.06	0.07	0.06	0.00	0.07	0.12	0.17	0.21	0.33	0.33	0.00	0.08	0.14	0.20	0.29	0.49	0.76
	FNR	0.75	0.66	0.57	0.44	0.26	0.09	0.00	0.01	0.07	0.10	0.08	0.07	0.03	0.00	0.00	0.01	0.02	0.02	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.61	0.68	0.84	0.92	0.96	0.99	1.00	0.00	0.05	0.23	0.45	0.66	0.88	0.98	0.00	0.03	0.20	0.38	0.57	0.79	0.92
	sd($\hat{\theta}$)	0.43	0.39	0.25	0.16	0.08	0.02	0.02	0.00	0.06	0.07	0.12	0.12	0.09	0.05	0.00	0.04	0.05	0.06	0.07	0.07	0.07
50	FWE	0.00	0.24	0.23	0.19	0.18	0.04	0.08	0.00	0.89	0.98	0.93	0.89	0.68	0.36	0.00	0.98	1.00	1.00	1.00	0.96	0.83
	FDR	0.00	0.02	0.03	0.04	0.08	0.04	0.08	0.00	0.07	0.11	0.17	0.24	0.30	0.36	0.00	0.08	0.13	0.20	0.30	0.50	0.83
	FNR	0.68	0.72	0.62	0.45	0.28	0.10	0.00	0.00	0.10	0.09	0.09	0.07	0.04	0.00	0.00	0.02	0.02	0.02	0.01	0.01	0.00

Table B.66: MP12 test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.58	0.74	0.84	0.93	0.95	0.98	0.00	0.03	0.18	0.34	0.55	0.74	0.85	0.00	0.02	0.13	0.26	0.43	0.61	0.68
	sd($\hat{\theta}$)	0.44	0.44	0.33	0.27	0.16	0.11	0.07	0.01	0.06	0.19	0.22	0.23	0.21	0.19	0.00	0.04	0.11	0.16	0.17	0.23	0.26
10	FWE	0.00	0.30	0.23	0.20	0.15	0.13	0.12	0.00	0.76	0.75	0.80	0.69	0.71	0.55	0.00	0.84	0.83	0.89	0.94	0.85	0.82
	FDR	0.00	0.03	0.06	0.07	0.12	0.11	0.12	0.00	0.08	0.16	0.26	0.29	0.49	0.55	0.00	0.08	0.18	0.29	0.44	0.61	0.82
	FNR	0.67	0.58	0.55	0.42	0.27	0.09	0.00	0.01	0.04	0.06	0.06	0.04	0.02	0.00	0.00	0.00	0.02	0.01	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.72	0.77	0.84	0.91	0.97	0.98	0.99	0.00	0.01	0.16	0.31	0.55	0.77	0.91	0.00	0.01	0.11	0.27	0.43	0.62	0.74
	sd($\hat{\theta}$)	0.42	0.38	0.31	0.19	0.09	0.05	0.04	0.00	0.03	0.17	0.15	0.18	0.18	0.13	0.00	0.03	0.07	0.09	0.11	0.13	0.14
30	FWE	0.00	0.20	0.20	0.19	0.14	0.11	0.12	0.00	0.94	0.97	0.97	0.90	0.77	0.51	0.00	0.94	1.00	1.00	1.00	1.00	0.95
	FDR	0.00	0.02	0.05	0.07	0.09	0.08	0.12	0.00	0.09	0.19	0.29	0.37	0.50	0.51	0.00	0.09	0.21	0.31	0.46	0.70	0.95
	FNR	0.76	0.73	0.60	0.46	0.29	0.09	0.00	0.00	0.02	0.05	0.05	0.05	0.03	0.00	0.00	0.01	0.00	0.01	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.78	0.82	0.89	0.94	0.98	0.99	0.99	0.00	0.01	0.15	0.32	0.55	0.80	0.93	0.00	0.01	0.11	0.25	0.43	0.63	0.74
	sd($\hat{\theta}$)	0.39	0.35	0.24	0.18	0.08	0.03	0.02	0.00	0.03	0.07	0.14	0.17	0.15	0.11	0.00	0.02	0.06	0.08	0.10	0.11	0.12
50	FWE	0.00	0.18	0.16	0.13	0.14	0.09	0.11	0.00	0.99	1.00	0.98	0.92	0.76	0.56	0.00	0.98	1.00	1.00	1.00	1.00	0.98
	FDR	0.00	0.03	0.03	0.05	0.09	0.06	0.11	0.00	0.09	0.19	0.29	0.37	0.46	0.56	0.00	0.09	0.22	0.33	0.46	0.70	0.98
	FNR	0.80	0.77	0.64	0.47	0.29	0.10	0.00	0.00	0.01	0.04	0.06	0.06	0.03	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00

Table B.67: MP12 test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.06	0.20	0.38	0.57	0.79	0.94	0.99	0.01	0.08	0.31	0.52	0.71	0.91	1.00	0.00	0.06	0.27	0.49	0.70	0.89	1.00
	sd($\hat{\theta}$)	0.12	0.19	0.16	0.14	0.11	0.05	0.04	0.04	0.08	0.09	0.10	0.08	0.05	0.02	0.00	0.06	0.08	0.06	0.06	0.04	0.02
10	FWE	0.00	0.24	0.28	0.17	0.16	0.03	0.10	0.00	0.44	0.23	0.20	0.21	0.05	0.05	0.00	0.46	0.33	0.23	0.11	0.13	0.03
	FDR	0.00	0.02	0.05	0.04	0.05	0.02	0.10	0.00	0.04	0.03	0.04	0.06	0.03	0.05	0.00	0.05	0.05	0.04	0.03	0.07	0.03
	FNR	0.25	0.35	0.24	0.14	0.12	0.04	0.00	0.03	0.08	0.10	0.07	0.04	0.02	0.00	0.00	0.03	0.03	0.03	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.01	0.14	0.37	0.59	0.78	0.94	1.00	0.00	0.08	0.30	0.52	0.72	0.91	1.00	0.00	0.07	0.27	0.49	0.70	0.90	1.00
	sd($\hat{\theta}$)	0.04	0.10	0.08	0.09	0.07	0.03	0.01	0.01	0.06	0.06	0.05	0.05	0.03	0.00	0.00	0.04	0.05	0.04	0.04	0.02	0.01
30	FWE	0.00	0.51	0.60	0.40	0.23	0.13	0.05	0.00	0.54	0.60	0.43	0.41	0.15	0.02	0.00	0.58	0.65	0.56	0.42	0.16	0.06
	FDR	0.00	0.03	0.05	0.04	0.04	0.06	0.05	0.00	0.04	0.05	0.04	0.05	0.05	0.02	0.00	0.04	0.05	0.05	0.06	0.05	0.06
	FNR	0.12	0.33	0.25	0.18	0.11	0.05	0.00	0.02	0.09	0.10	0.07	0.05	0.02	0.00	0.00	0.02	0.03	0.03	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.01	0.13	0.39	0.59	0.79	0.95	1.00	0.00	0.07	0.30	0.51	0.72	0.91	1.00	0.00	0.06	0.28	0.49	0.70	0.90	1.00
	sd($\hat{\theta}$)	0.03	0.08	0.06	0.06	0.05	0.03	0.00	0.00	0.04	0.05	0.05	0.03	0.02	0.00	0.00	0.04	0.03	0.03	0.03	0.02	0.01
50	FWE	0.00	0.66	0.63	0.55	0.33	0.12	0.02	0.00	0.86	0.75	0.70	0.45	0.32	0.03	0.00	0.87	0.75	0.66	0.57	0.23	0.05
	FDR	0.00	0.03	0.03	0.04	0.04	0.03	0.02	0.00	0.04	0.04	0.05	0.04	0.07	0.03	0.00	0.04	0.04	0.04	0.06	0.05	0.05
	FNR	0.13	0.32	0.26	0.18	0.12	0.06	0.00	0.00	0.09	0.11	0.06	0.04	0.02	0.00	0.00	0.03	0.04	0.03	0.02	0.01	0.00

Table B.68: MP12 test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.11	0.20	0.50	0.70	0.79	0.94	0.99	0.01	0.08	0.34	0.52	0.73	0.92	0.99	0.00	0.06	0.27	0.48	0.69	0.89	0.99
	sd($\hat{\theta}$)	0.17	0.18	0.18	0.14	0.11	0.07	0.04	0.05	0.09	0.12	0.12	0.09	0.06	0.04	0.01	0.06	0.08	0.08	0.07	0.06	0.03
10	FWE	0.00	0.38	0.23	0.18	0.18	0.13	0.12	0.00	0.44	0.22	0.31	0.20	0.09	0.13	0.00	0.51	0.31	0.28	0.19	0.20	0.10
	FDR	0.00	0.04	0.04	0.04	0.07	0.09	0.12	0.00	0.04	0.04	0.06	0.06	0.05	0.13	0.00	0.05	0.05	0.05	0.05	0.11	0.10
	FNR	0.41	0.43	0.39	0.28	0.13	0.06	0.00	0.07	0.10	0.17	0.10	0.07	0.03	0.00	0.01	0.03	0.02	0.03	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.07	0.22	0.45	0.66	0.81	0.95	0.99	0.01	0.07	0.31	0.54	0.72	0.92	1.00	0.00	0.06	0.27	0.49	0.70	0.90	1.00
	sd($\hat{\theta}$)	0.11	0.11	0.12	0.10	0.07	0.04	0.01	0.02	0.07	0.08	0.07	0.05	0.04	0.01	0.00	0.05	0.05	0.05	0.04	0.02	0.01
30	FWE	0.00	0.40	0.44	0.43	0.43	0.23	0.17	0.00	0.67	0.62	0.50	0.49	0.23	0.09	0.00	0.65	0.67	0.55	0.32	0.22	0.08
	FDR	0.00	0.02	0.03	0.06	0.07	0.12	0.17	0.00	0.05	0.05	0.06	0.07	0.08	0.09	0.00	0.05	0.06	0.05	0.05	0.06	0.08
	FNR	0.37	0.53	0.35	0.26	0.15	0.06	0.00	0.05	0.12	0.13	0.12	0.06	0.03	0.00	0.00	0.03	0.05	0.04	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.05	0.21	0.47	0.65	0.80	0.94	0.99	0.00	0.07	0.32	0.53	0.75	0.92	1.00	0.00	0.06	0.27	0.49	0.70	0.90	1.00
	sd($\hat{\theta}$)	0.09	0.11	0.08	0.08	0.06	0.03	0.01	0.00	0.05	0.04	0.05	0.04	0.03	0.01	0.00	0.04	0.04	0.04	0.03	0.02	0.01
50	FWE	0.00	0.59	0.68	0.60	0.63	0.44	0.31	0.00	0.83	0.80	0.68	0.39	0.19	0.10	0.00	0.89	0.85	0.70	0.48	0.30	0.06
	FDR	0.00	0.03	0.04	0.06	0.09	0.16	0.31	0.00	0.05	0.04	0.05	0.04	0.04	0.10	0.00	0.05	0.05	0.05	0.05	0.06	0.06
	FNR	0.32	0.53	0.39	0.26	0.15	0.06	0.00	0.00	0.12	0.14	0.11	0.07	0.03	0.00	0.00	0.04	0.04	0.04	0.02	0.01	0.00

Table B.69: MP12 test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.27	0.35	0.59	0.73	0.85	0.96	1.00	0.02	0.12	0.35	0.57	0.75	0.92	1.00	0.01	0.07	0.29	0.52	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.30	0.25	0.21	0.15	0.11	0.05	0.01	0.06	0.14	0.13	0.12	0.10	0.05	0.02	0.03	0.08	0.10	0.07	0.07	0.05	0.02
10	FWE	0.00	0.13	0.10	0.08	0.05	0.03	0.02	0.00	0.39	0.17	0.14	0.11	0.02	0.04	0.00	0.44	0.23	0.10	0.13	0.04	0.03
	FDR	0.00	0.01	0.02	0.02	0.02	0.03	0.02	0.00	0.04	0.03	0.03	0.04	0.01	0.04	0.00	0.04	0.04	0.02	0.04	0.02	0.03
	FNR	0.56	0.55	0.45	0.30	0.17	0.06	0.00	0.10	0.19	0.16	0.13	0.08	0.03	0.00	0.02	0.08	0.07	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.22	0.31	0.55	0.73	0.87	0.96	1.00	0.03	0.12	0.37	0.55	0.77	0.93	0.99	0.00	0.07	0.29	0.50	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.23	0.18	0.15	0.12	0.08	0.03	0.01	0.08	0.09	0.09	0.08	0.05	0.04	0.03	0.00	0.05	0.07	0.07	0.05	0.03	0.01
30	FWE	0.00	0.21	0.16	0.15	0.09	0.04	0.03	0.00	0.39	0.20	0.23	0.12	0.10	0.08	0.00	0.53	0.41	0.33	0.11	0.09	0.05
	FDR	0.00	0.01	0.01	0.03	0.01	0.02	0.03	0.00	0.03	0.02	0.03	0.02	0.03	0.08	0.00	0.04	0.04	0.05	0.02	0.03	0.05
	FNR	0.58	0.56	0.43	0.31	0.19	0.06	0.00	0.13	0.25	0.21	0.12	0.09	0.04	0.00	0.00	0.05	0.07	0.06	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.21	0.33	0.55	0.71	0.86	0.97	1.00	0.01	0.13	0.35	0.57	0.76	0.93	1.00	0.00	0.08	0.30	0.51	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.22	0.20	0.13	0.10	0.07	0.03	0.00	0.05	0.09	0.08	0.06	0.06	0.03	0.00	0.01	0.05	0.06	0.05	0.06	0.03	0.00
50	FWE	0.00	0.39	0.33	0.28	0.15	0.04	0.01	0.00	0.54	0.45	0.34	0.28	0.07	0.03	0.00	0.58	0.56	0.45	0.34	0.09	0.01
	FDR	0.00	0.02	0.02	0.03	0.03	0.02	0.01	0.00	0.03	0.03	0.03	0.03	0.02	0.03	0.00	0.03	0.04	0.03	0.04	0.03	0.01
	FNR	0.61	0.61	0.45	0.30	0.19	0.07	0.00	0.08	0.26	0.19	0.15	0.09	0.04	0.00	0.02	0.08	0.08	0.06	0.04	0.01	0.00

Table B.70: MP12 test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.31	0.41	0.60	0.76	0.87	0.96	0.99	0.03	0.13	0.38	0.58	0.77	0.92	0.99	0.01	0.08	0.31	0.50	0.72	0.90	0.99
	sd($\hat{\theta}$)	0.31	0.30	0.22	0.15	0.11	0.06	0.03	0.08	0.14	0.16	0.15	0.09	0.10	0.07	0.04	0.09	0.10	0.08	0.06	0.04	0.03
10	FWE	0.00	0.16	0.13	0.06	0.10	0.06	0.09	0.00	0.36	0.25	0.17	0.07	0.07	0.04	0.00	0.47	0.25	0.18	0.07	0.05	0.04
	FDR	0.00	0.02	0.03	0.01	0.04	0.04	0.09	0.00	0.04	0.05	0.04	0.03	0.04	0.04	0.00	0.05	0.04	0.05	0.02	0.03	0.04
	FNR	0.64	0.58	0.46	0.32	0.20	0.07	0.00	0.14	0.22	0.22	0.15	0.10	0.03	0.00	0.03	0.11	0.09	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.25	0.38	0.59	0.75	0.88	0.97	1.00	0.03	0.12	0.35	0.59	0.79	0.93	1.00	0.00	0.07	0.29	0.50	0.71	0.91	0.99
	sd($\hat{\theta}$)	0.27	0.23	0.17	0.13	0.08	0.03	0.01	0.07	0.10	0.10	0.09	0.05	0.07	0.01	0.01	0.06	0.08	0.07	0.04	0.03	0.05
30	FWE	0.00	0.28	0.19	0.25	0.16	0.12	0.05	0.00	0.43	0.40	0.34	0.13	0.07	0.05	0.00	0.51	0.50	0.37	0.24	0.11	0.04
	FDR	0.00	0.02	0.02	0.04	0.05	0.09	0.05	0.00	0.03	0.04	0.04	0.02	0.03	0.05	0.00	0.04	0.05	0.05	0.04	0.04	0.04
	FNR	0.62	0.64	0.48	0.33	0.21	0.07	0.00	0.12	0.25	0.20	0.17	0.11	0.04	0.00	0.01	0.07	0.08	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.26	0.36	0.62	0.78	0.89	0.97	1.00	0.01	0.12	0.34	0.58	0.76	0.94	1.00	0.00	0.06	0.28	0.51	0.71	0.91	1.00
	sd($\hat{\theta}$)	0.24	0.20	0.13	0.10	0.06	0.02	0.01	0.05	0.08	0.10	0.07	0.06	0.03	0.00	0.01	0.05	0.06	0.06	0.04	0.04	0.00
50	FWE	0.00	0.29	0.30	0.34	0.24	0.06	0.10	0.00	0.59	0.52	0.40	0.32	0.10	0.03	0.00	0.70	0.64	0.52	0.39	0.14	0.02
	FDR	0.00	0.01	0.02	0.03	0.06	0.03	0.10	0.00	0.03	0.04	0.03	0.05	0.03	0.03	0.00	0.04	0.05	0.04	0.04	0.04	0.02
	FNR	0.69	0.65	0.51	0.36	0.22	0.08	0.00	0.09	0.27	0.18	0.15	0.10	0.04	0.00	0.01	0.06	0.06	0.06	0.04	0.02	0.00

Table B.71: MP12 test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.34	0.48	0.60	0.79	0.87	0.96	0.99	0.03	0.14	0.41	0.58	0.77	0.94	1.00	0.00	0.06	0.33	0.52	0.70	0.91	1.00
	sd($\hat{\theta}$)	0.34	0.32	0.26	0.16	0.13	0.06	0.07	0.09	0.15	0.16	0.14	0.11	0.05	0.02	0.03	0.06	0.11	0.11	0.08	0.04	0.01
10	FWE	0.00	0.11	0.15	0.03	0.07	0.05	0.03	0.00	0.34	0.08	0.09	0.06	0.01	0.04	0.00	0.48	0.15	0.14	0.06	0.03	0.01
	FDR	0.00	0.01	0.03	0.01	0.02	0.05	0.03	0.00	0.03	0.02	0.03	0.02	0.01	0.04	0.00	0.05	0.03	0.03	0.02	0.02	0.01
	FNR	0.60	0.60	0.45	0.34	0.20	0.06	0.00	0.14	0.22	0.23	0.15	0.09	0.04	0.00	0.02	0.03	0.11	0.06	0.01	0.01	0.00
	ave($\hat{\theta}$)	0.40	0.46	0.60	0.79	0.90	0.97	1.00	0.04	0.15	0.38	0.58	0.77	0.93	1.00	0.00	0.06	0.29	0.50	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.32	0.28	0.22	0.14	0.08	0.03	0.01	0.11	0.13	0.11	0.11	0.09	0.08	0.00	0.00	0.07	0.09	0.08	0.05	0.04	0.01
30	FWE	0.00	0.17	0.22	0.15	0.09	0.04	0.03	0.00	0.32	0.27	0.20	0.18	0.07	0.02	0.00	0.59	0.38	0.21	0.11	0.09	0.03
	FDR	0.00	0.01	0.03	0.03	0.03	0.02	0.03	0.00	0.03	0.03	0.03	0.03	0.03	0.02	0.00	0.05	0.04	0.03	0.02	0.03	0.03
	FNR	0.74	0.67	0.46	0.36	0.22	0.08	0.00	0.16	0.27	0.23	0.15	0.10	0.04	0.00	0.00	0.07	0.07	0.05	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.26	0.43	0.66	0.81	0.91	0.97	1.00	0.03	0.12	0.37	0.57	0.79	0.93	1.00	0.00	0.06	0.29	0.51	0.70	0.91	1.00
	sd($\hat{\theta}$)	0.27	0.26	0.20	0.12	0.06	0.04	0.00	0.09	0.12	0.14	0.13	0.06	0.04	0.00	0.00	0.05	0.09	0.08	0.09	0.02	0.00
50	FWE	0.00	0.23	0.24	0.18	0.10	0.07	0.05	0.00	0.47	0.33	0.27	0.11	0.06	0.00	0.00	0.57	0.45	0.25	0.27	0.10	0.00
	FDR	0.00	0.01	0.02	0.02	0.02	0.04	0.05	0.00	0.04	0.04	0.04	0.01	0.02	0.00	0.00	0.04	0.04	0.03	0.05	0.02	0.00
	FNR	0.59	0.64	0.51	0.37	0.23	0.08	0.00	0.11	0.23	0.22	0.15	0.12	0.04	0.00	0.00	0.04	0.08	0.05	0.03	0.02	0.00

Table B.72: MP12 test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.44	0.50	0.66	0.80	0.88	0.97	0.99	0.05	0.17	0.37	0.58	0.77	0.93	1.00	0.00	0.06	0.31	0.49	0.69	0.90	0.99
	sd($\hat{\theta}$)	0.35	0.30	0.22	0.17	0.12	0.05	0.06	0.13	0.18	0.18	0.15	0.13	0.05	0.02	0.03	0.06	0.09	0.13	0.12	0.08	0.08
10	FWE	0.00	0.05	0.06	0.09	0.06	0.01	0.07	0.00	0.35	0.17	0.12	0.05	0.05	0.05	0.00	0.52	0.21	0.17	0.13	0.11	0.02
	FDR	0.00	0.01	0.01	0.03	0.04	0.01	0.07	0.00	0.04	0.04	0.03	0.02	0.03	0.05	0.00	0.05	0.03	0.05	0.05	0.07	0.02
	FNR	0.70	0.66	0.49	0.37	0.20	0.07	0.00	0.18	0.29	0.19	0.14	0.10	0.03	0.00	0.02	0.03	0.08	0.05	0.03	0.02	0.00
	ave($\hat{\theta}$)	0.36	0.49	0.67	0.80	0.89	0.98	1.00	0.04	0.13	0.35	0.58	0.77	0.93	1.00	0.00	0.06	0.30	0.52	0.71	0.90	1.00
	sd($\hat{\theta}$)	0.33	0.28	0.22	0.14	0.08	0.03	0.00	0.10	0.11	0.13	0.10	0.07	0.09	0.00	0.01	0.06	0.07	0.07	0.07	0.08	0.00
30	FWE	0.00	0.17	0.14	0.11	0.12	0.06	0.02	0.00	0.36	0.27	0.22	0.17	0.07	0.03	0.00	0.53	0.30	0.17	0.23	0.05	0.01
	FDR	0.00	0.01	0.01	0.02	0.04	0.03	0.02	0.00	0.03	0.04	0.03	0.03	0.04	0.03	0.00	0.05	0.03	0.02	0.04	0.02	0.01
	FNR	0.66	0.69	0.51	0.37	0.21	0.08	0.00	0.14	0.24	0.19	0.15	0.10	0.04	0.00	0.01	0.05	0.06	0.06	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.36	0.49	0.68	0.81	0.92	0.97	1.00	0.04	0.15	0.35	0.58	0.77	0.94	1.00	0.00	0.07	0.28	0.52	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.31	0.28	0.18	0.12	0.06	0.05	0.01	0.10	0.13	0.12	0.11	0.06	0.03	0.00	0.01	0.06	0.09	0.05	0.06	0.03	0.00
50	FWE	0.00	0.24	0.18	0.22	0.09	0.12	0.06	0.00	0.43	0.41	0.29	0.24	0.09	0.00	0.00	0.58	0.52	0.38	0.26	0.09	0.02
	FDR	0.00	0.02	0.02	0.03	0.03	0.08	0.06	0.00	0.03	0.04	0.04	0.03	0.02	0.00	0.00	0.04	0.04	0.03	0.03	0.02	0.02
	FNR	0.65	0.69	0.53	0.38	0.24	0.08	0.00	0.15	0.27	0.21	0.17	0.10	0.04	0.00	0.01	0.08	0.06	0.05	0.04	0.02	0.00

Table B.73: BSQT test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.62	0.71	0.83	0.90	0.96	0.99	0.24	0.41	0.55	0.73	0.82	0.94	0.99	0.11	0.19	0.41	0.61	0.80	0.93	1.00
10	sd($\hat{\theta}$)	0.31	0.21	0.20	0.15	0.11	0.07	0.03	0.18	0.20	0.15	0.14	0.12	0.07	0.04	0.10	0.11	0.12	0.12	0.11	0.07	0.01
	FDR	0.00	0.01	0.02	0.04	0.07	0.08	0.07	0.00	0.00	0.02	0.04	0.06	0.03	0.06	0.00	0.01	0.02	0.02	0.02	0.02	0.02
	FNR	0.98	0.82	0.56	0.40	0.23	0.07	0.00	0.94	0.67	0.45	0.32	0.16	0.05	0.00	0.77	0.35	0.25	0.18	0.12	0.03	0.00
	ave($\hat{\theta}$)	0.34	0.43	0.61	0.74	0.87	0.98	0.99	0.15	0.24	0.45	0.66	0.83	0.94	1.00	0.03	0.13	0.34	0.56	0.74	0.92	0.99
30	sd($\hat{\theta}$)	0.24	0.16	0.12	0.11	0.11	0.05	0.04	0.17	0.13	0.13	0.11	0.10	0.07	0.02	0.05	0.07	0.10	0.11	0.09	0.05	0.02
	FDR	0.00	0.02	0.05	0.09	0.11	0.07	0.07	0.00	0.02	0.06	0.06	0.08	0.15	0.04	0.00	0.02	0.04	0.06	0.09	0.10	0.06
	FNR	0.99	0.77	0.53	0.36	0.22	0.09	0.00	0.71	0.60	0.39	0.27	0.17	0.07	0.00	0.23	0.27	0.19	0.15	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.32	0.40	0.60	0.71	0.88	0.98	0.99	0.13	0.21	0.45	0.65	0.81	0.96	1.00	0.01	0.10	0.32	0.53	0.72	0.94	0.99
50	sd($\hat{\theta}$)	0.21	0.13	0.10	0.10	0.08	0.05	0.04	0.13	0.11	0.10	0.09	0.10	0.09	0.03	0.03	0.05	0.08	0.09	0.09	0.06	0.04
	FDR	0.00	0.02	0.05	0.09	0.11	0.11	0.07	0.00	0.03	0.05	0.07	0.11	0.10	0.02	0.00	0.03	0.06	0.07	0.11	0.11	0.08
	FNR	1.00	0.77	0.53	0.34	0.22	0.09	0.00	0.74	0.59	0.38	0.26	0.17	0.08	0.00	0.12	0.22	0.16	0.11	0.08	0.06	0.00

Table B.74: BSQT test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.35	0.49	0.64	0.73	0.87	0.98	1.00	0.04	0.22	0.43	0.60	0.80	0.97	0.99	0.01	0.18	0.37	0.56	0.76	0.98	0.98
10	sd($\hat{\theta}$)	0.32	0.19	0.16	0.15	0.11	0.06	0.03	0.08	0.12	0.12	0.09	0.10	0.07	0.03	0.03	0.07	0.09	0.10	0.09	0.06	0.09
	FDR	0.00	0.01	0.01	0.05	0.03	0.06	0.02	0.00	0.01	0.01	0.03	0.04	0.07	0.03	0.00	0.01	0.02	0.04	0.05	0.07	0.07
	FNR	0.68	0.75	0.51	0.32	0.19	0.09	0.00	0.18	0.50	0.30	0.19	0.13	0.09	0.00	0.03	0.44	0.22	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.19	0.25	0.47	0.69	0.81	0.98	1.00	0.01	0.18	0.37	0.59	0.78	0.96	1.00	0.00	0.15	0.35	0.57	0.76	0.96	1.00
30	sd($\hat{\theta}$)	0.26	0.13	0.14	0.11	0.08	0.06	0.03	0.03	0.08	0.09	0.06	0.06	0.10	0.03	0.02	0.09	0.09	0.07	0.08	0.08	0.00
	FDR	0.00	0.03	0.05	0.05	0.10	0.07	0.02	0.00	0.02	0.03	0.03	0.04	0.10	0.02	0.00	0.03	0.04	0.03	0.05	0.11	0.00
	FNR	0.44	0.60	0.39	0.29	0.16	0.09	0.00	0.03	0.46	0.24	0.17	0.12	0.09	0.00	0.01	0.38	0.20	0.15	0.10	0.08	0.00
	ave($\hat{\theta}$)	0.11	0.22	0.47	0.66	0.82	0.98	1.00	0.01	0.17	0.37	0.57	0.77	0.95	1.00	0.00	0.17	0.34	0.56	0.76	0.96	0.99
50	sd($\hat{\theta}$)	0.20	0.10	0.12	0.11	0.08	0.06	0.03	0.04	0.08	0.08	0.07	0.07	0.10	0.03	0.00	0.08	0.10	0.08	0.08	0.08	0.03
	FDR	0.00	0.03	0.05	0.06	0.09	0.07	0.02	0.00	0.02	0.03	0.04	0.05	0.12	0.02	0.00	0.02	0.04	0.03	0.06	0.10	0.03
	FNR	0.33	0.57	0.41	0.27	0.16	0.09	0.00	0.05	0.43	0.24	0.17	0.12	0.08	0.00	0.00	0.42	0.19	0.14	0.10	0.08	0.00

Table B.75: BSQT test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.53	0.43	0.61	0.77	0.87	0.98	1.00	0.05	0.21	0.45	0.63	0.82	0.97	0.98	0.01	0.16	0.36	0.58	0.78	0.99	0.99
10	sd($\hat{\theta}$)	0.36	0.22	0.19	0.15	0.12	0.05	0.02	0.13	0.12	0.11	0.12	0.09	0.09	0.08	0.04	0.10	0.09	0.09	0.06	0.05	0.04
	FDR	0.00	0.01	0.03	0.04	0.03	0.05	0.01	0.00	0.01	0.01	0.02	0.03	0.08	0.07	0.00	0.02	0.03	0.02	0.03	0.04	0.05
	FNR	0.77	0.70	0.49	0.35	0.19	0.09	0.00	0.18	0.48	0.32	0.21	0.15	0.09	0.00	0.04	0.39	0.21	0.16	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.26	0.25	0.53	0.70	0.83	0.98	1.00	0.01	0.17	0.41	0.60	0.78	0.99	0.98	0.00	0.16	0.37	0.56	0.76	0.98	0.99
30	sd($\hat{\theta}$)	0.36	0.14	0.12	0.12	0.13	0.08	0.03	0.05	0.08	0.07	0.08	0.10	0.07	0.08	0.02	0.08	0.08	0.08	0.11	0.06	0.04
	FDR	0.00	0.03	0.04	0.05	0.08	0.07	0.02	0.00	0.02	0.02	0.03	0.04	0.03	0.05	0.00	0.02	0.02	0.03	0.05	0.04	0.04
	FNR	0.46	0.57	0.46	0.30	0.18	0.10	0.00	0.07	0.44	0.28	0.18	0.12	0.10	0.00	0.01	0.39	0.23	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.28	0.22	0.49	0.68	0.83	0.98	0.99	0.01	0.16	0.37	0.59	0.78	0.99	0.99	0.00	0.15	0.36	0.57	0.78	0.98	0.99
50	sd($\hat{\theta}$)	0.34	0.12	0.12	0.12	0.12	0.05	0.05	0.07	0.08	0.11	0.08	0.11	0.04	0.07	0.02	0.09	0.08	0.08	0.07	0.06	0.04
	FDR	0.00	0.03	0.04	0.07	0.08	0.06	0.03	0.00	0.03	0.04	0.04	0.05	0.02	0.04	0.00	0.02	0.03	0.03	0.04	0.04	0.02
	FNR	0.53	0.55	0.43	0.30	0.18	0.10	0.00	0.05	0.41	0.24	0.18	0.13	0.10	0.00	0.01	0.38	0.21	0.15	0.11	0.09	0.00

Table B.76: BSQT test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.46	0.45	0.62	0.76	0.87	0.99	1.00	0.02	0.20	0.43	0.61	0.80	0.98	1.00	0.00	0.17	0.37	0.58	0.77	0.97	0.99
10	sd($\hat{\theta}$)	0.37	0.22	0.20	0.14	0.11	0.04	0.03	0.07	0.11	0.11	0.11	0.09	0.08	0.00	0.02	0.08	0.10	0.09	0.09	0.08	0.08
	FDR	0.00	0.01	0.02	0.03	0.05	0.04	0.02	0.00	0.01	0.02	0.02	0.04	0.07	0.00	0.00	0.02	0.03	0.02	0.04	0.08	0.02
	FNR	0.71	0.71	0.49	0.34	0.20	0.10	0.00	0.09	0.47	0.30	0.19	0.14	0.09	0.00	0.01	0.42	0.23	0.16	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.38	0.27	0.51	0.71	0.84	0.99	0.99	0.01	0.16	0.38	0.60	0.79	0.97	0.99	0.00	0.15	0.37	0.56	0.77	0.98	0.99
30	sd($\hat{\theta}$)	0.40	0.14	0.11	0.10	0.10	0.05	0.04	0.03	0.09	0.10	0.06	0.04	0.07	0.05	0.00	0.09	0.07	0.09	0.07	0.06	0.03
	FDR	0.00	0.02	0.03	0.05	0.08	0.04	0.02	0.00	0.03	0.03	0.02	0.02	0.08	0.04	0.00	0.02	0.02	0.04	0.05	0.06	0.03
	FNR	0.56	0.62	0.43	0.31	0.18	0.10	0.00	0.03	0.41	0.26	0.18	0.13	0.09	0.00	0.00	0.38	0.22	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.24	0.22	0.51	0.68	0.83	1.00	0.99	0.01	0.16	0.39	0.59	0.77	0.98	0.99	0.00	0.15	0.36	0.57	0.78	0.98	0.99
50	sd($\hat{\theta}$)	0.35	0.12	0.12	0.10	0.10	0.03	0.06	0.07	0.08	0.06	0.07	0.08	0.07	0.09	0.00	0.09	0.09	0.09	0.07	0.06	0.04
	FDR	0.00	0.03	0.04	0.07	0.09	0.01	0.04	0.00	0.03	0.02	0.03	0.04	0.06	0.03	0.00	0.03	0.03	0.03	0.03	0.05	0.04
	FNR	0.39	0.55	0.44	0.30	0.18	0.10	0.00	0.03	0.42	0.26	0.18	0.12	0.09	0.00	0.00	0.37	0.22	0.15	0.11	0.09	0.00

Table B.77: BSQT test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.58	0.59	0.67	0.79	0.89	0.91	0.95	0.27	0.39	0.51	0.64	0.80	0.86	0.91	0.13	0.18	0.37	0.56	0.74	0.85	0.93
	sd($\hat{\theta}$)	0.29	0.22	0.21	0.18	0.15	0.16	0.11	0.22	0.21	0.18	0.17	0.20	0.19	0.16	0.15	0.12	0.17	0.18	0.16	0.18	0.14
	FDR	0.00	0.02	0.06	0.13	0.13	0.24	0.31	0.00	0.02	0.06	0.13	0.20	0.27	0.34	0.00	0.02	0.07	0.11	0.14	0.26	0.38
	FNR	1.00	0.84	0.56	0.40	0.24	0.08	0.00	0.94	0.73	0.45	0.28	0.20	0.05	0.00	0.72	0.42	0.29	0.21	0.12	0.04	0.00
30	ave($\hat{\theta}$)	0.37	0.42	0.57	0.70	0.80	0.89	0.94	0.13	0.24	0.41	0.61	0.75	0.85	0.90	0.04	0.12	0.27	0.51	0.63	0.84	0.89
	sd($\hat{\theta}$)	0.23	0.17	0.15	0.20	0.18	0.18	0.14	0.13	0.12	0.18	0.18	0.22	0.25	0.23	0.05	0.08	0.16	0.17	0.26	0.22	0.26
	FDR	0.00	0.04	0.10	0.17	0.27	0.33	0.26	0.00	0.03	0.10	0.13	0.22	0.34	0.34	0.00	0.03	0.11	0.14	0.23	0.32	0.33
	FNR	0.97	0.82	0.56	0.39	0.23	0.08	0.00	0.81	0.64	0.44	0.29	0.19	0.07	0.00	0.35	0.29	0.20	0.17	0.10	0.04	0.00
50	ave($\hat{\theta}$)	0.26	0.38	0.55	0.66	0.78	0.86	0.90	0.12	0.19	0.34	0.55	0.69	0.81	0.88	0.02	0.10	0.28	0.46	0.67	0.79	0.87
	sd($\hat{\theta}$)	0.19	0.14	0.16	0.21	0.21	0.23	0.19	0.16	0.13	0.19	0.23	0.26	0.25	0.25	0.05	0.08	0.17	0.23	0.23	0.28	0.29
	FDR	0.00	0.04	0.11	0.20	0.29	0.35	0.44	0.00	0.04	0.14	0.18	0.27	0.45	0.43	0.00	0.04	0.11	0.17	0.20	0.41	0.39
	FNR	0.91	0.79	0.56	0.40	0.24	0.09	0.00	0.68	0.58	0.41	0.30	0.17	0.06	0.00	0.16	0.22	0.20	0.15	0.10	0.04	0.00

Table B.78: BSQT test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.45	0.45	0.60	0.72	0.86	0.95	0.98	0.04	0.22	0.38	0.58	0.72	0.93	0.96	0.01	0.16	0.31	0.54	0.71	0.90	0.95
10	sd($\hat{\theta}$)	0.34	0.22	0.18	0.17	0.11	0.09	0.07	0.11	0.13	0.15	0.13	0.16	0.12	0.13	0.04	0.09	0.15	0.12	0.13	0.16	0.13
	FDR	0.00	0.01	0.03	0.07	0.08	0.14	0.09	0.00	0.02	0.05	0.06	0.12	0.18	0.13	0.00	0.02	0.07	0.06	0.13	0.23	0.15
	FNR	0.77	0.72	0.49	0.33	0.20	0.09	0.00	0.15	0.50	0.27	0.19	0.10	0.07	0.00	0.04	0.39	0.18	0.13	0.08	0.07	0.00
	ave($\hat{\theta}$)	0.13	0.25	0.50	0.68	0.78	0.96	0.98	0.01	0.13	0.33	0.55	0.73	0.93	0.97	0.00	0.12	0.29	0.52	0.73	0.93	0.96
30	sd($\hat{\theta}$)	0.23	0.13	0.12	0.13	0.13	0.09	0.07	0.05	0.09	0.13	0.13	0.15	0.11	0.08	0.00	0.10	0.15	0.16	0.16	0.12	0.11
	FDR	0.00	0.03	0.05	0.09	0.20	0.14	0.11	0.00	0.04	0.08	0.09	0.13	0.19	0.14	0.00	0.04	0.08	0.08	0.09	0.17	0.15
	FNR	0.34	0.61	0.44	0.30	0.18	0.09	0.00	0.07	0.36	0.23	0.18	0.11	0.07	0.00	0.00	0.29	0.17	0.13	0.10	0.07	0.00
	ave($\hat{\theta}$)	0.12	0.19	0.43	0.61	0.76	0.95	0.99	0.01	0.12	0.32	0.55	0.76	0.89	0.96	0.00	0.11	0.27	0.51	0.72	0.93	0.95
50	sd($\hat{\theta}$)	0.20	0.12	0.15	0.20	0.19	0.14	0.05	0.04	0.10	0.15	0.12	0.11	0.20	0.14	0.00	0.10	0.16	0.17	0.13	0.12	0.12
	FDR	0.00	0.04	0.08	0.14	0.18	0.13	0.06	0.00	0.04	0.07	0.09	0.11	0.21	0.13	0.00	0.04	0.10	0.09	0.12	0.16	0.18
	FNR	0.35	0.50	0.39	0.28	0.16	0.09	0.00	0.05	0.33	0.23	0.17	0.13	0.07	0.00	0.00	0.29	0.16	0.14	0.09	0.07	0.00

Table B.79: BSQT test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.49	0.47	0.59	0.71	0.83	0.94	0.96	0.05	0.18	0.41	0.58	0.73	0.90	0.95	0.01	0.14	0.32	0.50	0.66	0.85	0.89
	sd($\hat{\theta}$)	0.38	0.20	0.21	0.18	0.15	0.14	0.14	0.10	0.14	0.14	0.16	0.17	0.12	0.09	0.03	0.10	0.14	0.17	0.21	0.20	0.24
	FDR	0.00	0.02	0.06	0.13	0.18	0.13	0.12	0.00	0.03	0.06	0.09	0.16	0.28	0.22	0.00	0.04	0.07	0.10	0.20	0.34	0.28
	FNR	0.76	0.75	0.51	0.34	0.21	0.08	0.00	0.20	0.43	0.33	0.20	0.12	0.07	0.00	0.03	0.37	0.20	0.12	0.08	0.05	0.00
30	ave($\hat{\theta}$)	0.27	0.24	0.48	0.65	0.81	0.92	0.94	0.02	0.12	0.33	0.54	0.68	0.88	0.94	0.00	0.11	0.33	0.53	0.69	0.84	0.90
	sd($\hat{\theta}$)	0.34	0.14	0.18	0.20	0.14	0.19	0.18	0.06	0.11	0.16	0.18	0.23	0.21	0.16	0.03	0.10	0.14	0.15	0.16	0.19	0.25
	FDR	0.00	0.04	0.09	0.14	0.19	0.20	0.15	0.00	0.05	0.09	0.12	0.23	0.28	0.19	0.00	0.05	0.07	0.11	0.18	0.36	0.22
	FNR	0.55	0.58	0.46	0.32	0.21	0.08	0.00	0.10	0.33	0.26	0.20	0.13	0.07	0.00	0.02	0.30	0.23	0.16	0.09	0.05	0.00
50	ave($\hat{\theta}$)	0.20	0.19	0.43	0.65	0.80	0.93	0.96	0.02	0.08	0.33	0.54	0.73	0.88	0.92	0.00	0.09	0.30	0.50	0.68	0.88	0.90
	sd($\hat{\theta}$)	0.31	0.13	0.19	0.20	0.12	0.20	0.12	0.05	0.10	0.15	0.16	0.17	0.22	0.23	0.00	0.10	0.17	0.18	0.22	0.14	0.25
	FDR	0.00	0.04	0.11	0.13	0.22	0.14	0.15	0.00	0.06	0.09	0.13	0.20	0.24	0.18	0.00	0.06	0.09	0.12	0.20	0.30	0.25
	FNR	0.41	0.50	0.43	0.30	0.19	0.09	0.00	0.08	0.23	0.26	0.20	0.13	0.07	0.00	0.00	0.22	0.22	0.15	0.11	0.05	0.00

Table B.80: BSQT test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.50	0.45	0.61	0.75	0.84	0.98	0.99	0.03	0.22	0.44	0.60	0.79	0.97	0.98	0.00	0.15	0.36	0.58	0.76	0.95	0.98
10	sd($\hat{\theta}$)	0.35	0.18	0.15	0.14	0.14	0.06	0.05	0.11	0.13	0.14	0.13	0.10	0.08	0.08	0.00	0.09	0.11	0.07	0.10	0.11	0.11
	FDR	0.00	0.02	0.02	0.02	0.07	0.05	0.06	0.00	0.02	0.03	0.04	0.05	0.08	0.07	0.00	0.03	0.03	0.02	0.07	0.12	0.05
	FNR	0.76	0.75	0.50	0.32	0.20	0.09	0.00	0.13	0.52	0.31	0.19	0.13	0.09	0.00	0.00	0.38	0.21	0.15	0.11	0.08	0.00
	ave($\hat{\theta}$)	0.32	0.26	0.51	0.70	0.86	0.99	0.98	0.01	0.15	0.37	0.58	0.76	0.96	0.99	0.00	0.14	0.36	0.56	0.78	0.97	0.98
30	sd($\hat{\theta}$)	0.36	0.14	0.18	0.13	0.15	0.05	0.05	0.07	0.09	0.12	0.15	0.14	0.13	0.09	0.00	0.09	0.10	0.12	0.08	0.12	0.08
	FDR	0.00	0.02	0.05	0.07	0.07	0.03	0.08	0.00	0.03	0.04	0.06	0.08	0.08	0.04	0.00	0.03	0.03	0.04	0.03	0.06	0.07
	FNR	0.50	0.57	0.43	0.32	0.20	0.10	0.00	0.04	0.38	0.25	0.19	0.13	0.09	0.00	0.00	0.35	0.23	0.15	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.31	0.20	0.48	0.69	0.85	0.98	0.99	0.00	0.14	0.36	0.58	0.78	0.98	0.98	0.00	0.14	0.36	0.57	0.75	0.98	0.98
50	sd($\hat{\theta}$)	0.37	0.13	0.17	0.16	0.16	0.08	0.06	0.00	0.09	0.11	0.11	0.11	0.07	0.11	0.02	0.09	0.11	0.08	0.14	0.08	0.09
	FDR	0.00	0.03	0.05	0.07	0.09	0.07	0.05	0.00	0.03	0.04	0.06	0.06	0.05	0.05	0.00	0.03	0.03	0.03	0.07	0.04	0.07
	FNR	0.52	0.49	0.42	0.32	0.20	0.09	0.00	0.00	0.37	0.24	0.19	0.13	0.09	0.00	0.01	0.36	0.22	0.15	0.11	0.09	0.00

Table B.81: BSQT test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.59	0.64	0.76	0.80	0.93	0.97	0.99	0.26	0.36	0.50	0.66	0.82	0.94	0.99	0.10	0.18	0.38	0.58	0.75	0.92	0.99
10	sd($\hat{\theta}$)	0.31	0.25	0.22	0.18	0.10	0.08	0.04	0.21	0.18	0.15	0.16	0.13	0.07	0.03	0.11	0.15	0.14	0.15	0.10	0.07	0.02
	FDR	0.00	0.01	0.03	0.04	0.05	0.07	0.11	0.00	0.01	0.03	0.05	0.08	0.06	0.11	0.00	0.01	0.03	0.06	0.07	0.04	0.06
	FNR	1.00	0.81	0.59	0.37	0.25	0.08	0.00	0.93	0.65	0.40	0.25	0.15	0.05	0.00	0.70	0.30	0.21	0.17	0.09	0.03	0.00
	ave($\hat{\theta}$)	0.34	0.43	0.61	0.74	0.86	0.97	0.99	0.17	0.27	0.42	0.60	0.78	0.93	0.98	0.03	0.11	0.32	0.53	0.71	0.92	0.99
30	sd($\hat{\theta}$)	0.25	0.21	0.19	0.14	0.12	0.05	0.03	0.17	0.17	0.15	0.11	0.11	0.07	0.04	0.04	0.07	0.08	0.09	0.07	0.06	0.03
	FDR	0.00	0.02	0.07	0.09	0.16	0.14	0.12	0.00	0.02	0.07	0.09	0.15	0.17	0.14	0.00	0.03	0.06	0.08	0.12	0.12	0.07
	FNR	0.96	0.76	0.52	0.35	0.21	0.08	0.00	0.83	0.60	0.35	0.21	0.15	0.06	0.00	0.26	0.26	0.16	0.12	0.07	0.04	0.00
	ave($\hat{\theta}$)	0.28	0.43	0.56	0.72	0.86	0.96	0.99	0.09	0.19	0.42	0.57	0.74	0.93	0.99	0.02	0.11	0.31	0.49	0.69	0.91	1.00
50	sd($\hat{\theta}$)	0.22	0.23	0.16	0.13	0.12	0.05	0.03	0.11	0.15	0.14	0.10	0.08	0.06	0.03	0.06	0.06	0.07	0.06	0.06	0.05	0.02
	FDR	0.00	0.03	0.08	0.10	0.15	0.18	0.08	0.00	0.03	0.06	0.11	0.17	0.19	0.08	0.00	0.03	0.06	0.09	0.14	0.19	0.05
	FNR	0.94	0.75	0.51	0.34	0.21	0.08	0.00	0.66	0.49	0.34	0.20	0.11	0.06	0.00	0.16	0.23	0.15	0.08	0.06	0.03	0.00

Table B.82: BSQT test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.48	0.52	0.67	0.78	0.87	0.96	0.99	0.05	0.15	0.35	0.52	0.72	0.89	0.95	0.01	0.14	0.31	0.50	0.70	0.89	0.94
10	sd($\hat{\theta}$)	0.34	0.27	0.21	0.17	0.13	0.08	0.05	0.09	0.10	0.13	0.12	0.12	0.13	0.08	0.04	0.10	0.12	0.13	0.11	0.13	0.11
	FDR	0.00	0.01	0.03	0.06	0.11	0.13	0.06	0.00	0.03	0.05	0.09	0.15	0.27	0.23	0.00	0.03	0.06	0.08	0.14	0.27	0.26
	FNR	0.79	0.75	0.53	0.36	0.21	0.09	0.00	0.23	0.39	0.20	0.13	0.09	0.05	0.00	0.04	0.35	0.15	0.10	0.07	0.05	0.00
	ave($\hat{\theta}$)	0.14	0.28	0.50	0.65	0.79	0.92	0.98	0.01	0.12	0.30	0.47	0.67	0.85	0.94	0.00	0.14	0.29	0.45	0.62	0.82	0.91
30	sd($\hat{\theta}$)	0.20	0.24	0.20	0.15	0.11	0.10	0.06	0.04	0.10	0.11	0.10	0.10	0.11	0.10	0.00	0.09	0.11	0.10	0.08	0.09	0.11
	FDR	0.00	0.03	0.07	0.13	0.19	0.24	0.10	0.00	0.04	0.08	0.13	0.20	0.40	0.29	0.00	0.03	0.07	0.14	0.23	0.45	0.40
	FNR	0.40	0.54	0.43	0.29	0.17	0.07	0.00	0.04	0.32	0.16	0.09	0.06	0.04	0.00	0.00	0.35	0.12	0.06	0.02	0.02	0.00
	ave($\hat{\theta}$)	0.11	0.24	0.43	0.62	0.76	0.90	0.97	0.01	0.14	0.28	0.47	0.64	0.84	0.95	0.00	0.14	0.23	0.44	0.61	0.81	0.91
50	sd($\hat{\theta}$)	0.19	0.20	0.19	0.14	0.11	0.10	0.07	0.05	0.09	0.10	0.09	0.10	0.08	0.09	0.02	0.09	0.10	0.08	0.07	0.06	0.10
	FDR	0.00	0.04	0.08	0.14	0.24	0.31	0.13	0.00	0.03	0.09	0.13	0.24	0.43	0.26	0.00	0.03	0.11	0.14	0.24	0.48	0.43
	FNR	0.33	0.52	0.36	0.28	0.16	0.07	0.00	0.06	0.38	0.14	0.09	0.05	0.03	0.00	0.01	0.35	0.05	0.05	0.02	0.01	0.00

Table B.83: BSQT test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.43	0.48	0.65	0.77	0.86	0.92	0.94	0.04	0.15	0.29	0.50	0.66	0.82	0.91	0.01	0.11	0.27	0.43	0.63	0.76	0.84
10	sd($\hat{\theta}$)	0.35	0.32	0.30	0.20	0.17	0.12	0.12	0.12	0.12	0.16	0.14	0.16	0.15	0.14	0.04	0.10	0.12	0.14	0.15	0.15	0.13
	FDR	0.00	0.03	0.06	0.09	0.14	0.24	0.20	0.00	0.03	0.10	0.12	0.22	0.42	0.36	0.00	0.05	0.09	0.16	0.22	0.50	0.66
	FNR	0.75	0.69	0.50	0.36	0.22	0.08	0.00	0.14	0.35	0.17	0.12	0.07	0.04	0.00	0.04	0.29	0.12	0.06	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.32	0.41	0.53	0.63	0.77	0.89	0.93	0.02	0.13	0.26	0.45	0.58	0.78	0.86	0.00	0.14	0.24	0.38	0.55	0.71	0.81
30	sd($\hat{\theta}$)	0.38	0.32	0.27	0.21	0.17	0.13	0.11	0.08	0.09	0.12	0.12	0.10	0.12	0.12	0.00	0.09	0.10	0.10	0.11	0.12	0.13
	FDR	0.00	0.02	0.08	0.16	0.22	0.31	0.34	0.00	0.04	0.11	0.17	0.33	0.53	0.64	0.00	0.03	0.11	0.20	0.33	0.62	0.75
	FNR	0.53	0.62	0.42	0.28	0.18	0.07	0.00	0.05	0.37	0.15	0.09	0.04	0.02	0.00	0.00	0.35	0.09	0.03	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.20	0.39	0.53	0.64	0.78	0.87	0.93	0.00	0.14	0.24	0.42	0.58	0.77	0.86	0.00	0.14	0.22	0.38	0.56	0.72	0.81
50	sd($\hat{\theta}$)	0.31	0.35	0.29	0.22	0.17	0.12	0.12	0.03	0.09	0.10	0.08	0.10	0.12	0.09	0.00	0.09	0.08	0.08	0.09	0.10	0.09
	FDR	0.00	0.04	0.10	0.15	0.24	0.37	0.29	0.00	0.04	0.12	0.19	0.33	0.55	0.68	0.00	0.03	0.12	0.20	0.31	0.62	0.89
	FNR	0.41	0.60	0.44	0.29	0.18	0.06	0.00	0.02	0.37	0.12	0.07	0.05	0.02	0.00	0.00	0.35	0.07	0.03	0.01	0.00	0.00

Table B.84: BSQT test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.49	0.53	0.67	0.69	0.83	0.91	0.89	0.02	0.10	0.25	0.39	0.55	0.69	0.77	0.00	0.08	0.19	0.30	0.46	0.65	0.69
10	sd($\hat{\theta}$)	0.41	0.37	0.30	0.25	0.20	0.15	0.17	0.05	0.12	0.16	0.18	0.17	0.19	0.18	0.03	0.10	0.13	0.18	0.18	0.18	0.19
	FDR	0.00	0.03	0.09	0.16	0.18	0.24	0.36	0.00	0.05	0.12	0.22	0.35	0.60	0.75	0.00	0.06	0.14	0.26	0.40	0.64	0.91
	FNR	0.68	0.65	0.54	0.34	0.22	0.08	0.00	0.08	0.24	0.15	0.08	0.05	0.02	0.00	0.02	0.21	0.06	0.03	0.01	0.01	0.00
	ave($\hat{\theta}$)	0.29	0.35	0.49	0.56	0.70	0.81	0.85	0.00	0.10	0.18	0.29	0.45	0.62	0.71	0.00	0.09	0.18	0.27	0.40	0.56	0.62
30	sd($\hat{\theta}$)	0.37	0.34	0.29	0.23	0.19	0.17	0.16	0.02	0.12	0.10	0.11	0.14	0.15	0.14	0.00	0.10	0.09	0.11	0.12	0.13	0.14
	FDR	0.00	0.05	0.12	0.24	0.35	0.52	0.53	0.00	0.06	0.17	0.30	0.45	0.71	0.92	0.00	0.06	0.15	0.30	0.48	0.75	0.99
	FNR	0.47	0.54	0.43	0.28	0.17	0.06	0.00	0.01	0.26	0.07	0.03	0.03	0.01	0.00	0.00	0.25	0.04	0.01	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.33	0.37	0.37	0.59	0.67	0.78	0.83	0.01	0.10	0.18	0.30	0.44	0.59	0.64	0.00	0.11	0.18	0.26	0.38	0.52	0.57
50	sd($\hat{\theta}$)	0.38	0.34	0.28	0.27	0.21	0.17	0.16	0.03	0.10	0.08	0.11	0.13	0.15	0.15	0.00	0.10	0.07	0.11	0.10	0.12	0.13
	FDR	0.00	0.05	0.16	0.22	0.39	0.53	0.63	0.00	0.06	0.16	0.30	0.45	0.74	0.99	0.00	0.05	0.15	0.32	0.50	0.78	1.00
	FNR	0.50	0.58	0.34	0.28	0.16	0.05	0.00	0.03	0.28	0.09	0.04	0.02	0.01	0.00	0.00	0.29	0.03	0.02	0.01	0.00	0.00

Table B.85: BSQT test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.37	0.50	0.63	0.74	0.87	0.95	0.99	0.04	0.21	0.38	0.57	0.76	0.95	0.99	0.01	0.17	0.37	0.52	0.74	0.94	0.99
10	sd($\hat{\theta}$)	0.22	0.23	0.17	0.17	0.11	0.06	0.03	0.09	0.10	0.09	0.09	0.09	0.10	0.04	0.04	0.09	0.08	0.11	0.10	0.10	0.04
	FDR	0.00	0.01	0.04	0.06	0.05	0.05	0.09	0.00	0.01	0.03	0.04	0.07	0.13	0.04	0.00	0.02	0.02	0.07	0.08	0.14	0.05
	FNR	0.98	0.79	0.53	0.33	0.20	0.06	0.00	0.19	0.48	0.24	0.15	0.10	0.07	0.00	0.05	0.42	0.23	0.11	0.09	0.07	0.00
	ave($\hat{\theta}$)	0.04	0.19	0.35	0.58	0.74	0.94	0.99	0.01	0.16	0.34	0.56	0.72	0.92	0.99	0.00	0.15	0.34	0.52	0.73	0.92	1.00
30	sd($\hat{\theta}$)	0.08	0.08	0.10	0.08	0.10	0.09	0.04	0.05	0.08	0.09	0.08	0.10	0.10	0.05	0.03	0.09	0.10	0.10	0.09	0.10	0.03
	FDR	0.00	0.02	0.06	0.07	0.12	0.15	0.04	0.00	0.02	0.05	0.04	0.10	0.21	0.06	0.00	0.03	0.04	0.07	0.09	0.20	0.02
	FNR	0.19	0.49	0.25	0.18	0.11	0.07	0.00	0.07	0.41	0.20	0.15	0.08	0.06	0.00	0.02	0.38	0.18	0.11	0.08	0.06	0.00
	ave($\hat{\theta}$)	0.02	0.17	0.36	0.56	0.73	0.92	1.00	0.01	0.17	0.31	0.53	0.73	0.90	0.99	0.00	0.16	0.31	0.52	0.71	0.88	0.99
50	sd($\hat{\theta}$)	0.06	0.07	0.08	0.08	0.10	0.10	0.03	0.03	0.07	0.10	0.10	0.10	0.10	0.04	0.00	0.08	0.10	0.10	0.10	0.10	0.04
	FDR	0.00	0.02	0.05	0.07	0.14	0.21	0.02	0.00	0.02	0.06	0.07	0.10	0.24	0.04	0.00	0.02	0.06	0.07	0.11	0.31	0.04
	FNR	0.12	0.47	0.24	0.17	0.10	0.07	0.00	0.03	0.43	0.16	0.12	0.09	0.05	0.00	0.00	0.41	0.14	0.10	0.07	0.04	0.00

Table B.86: BSQT test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.50	0.57	0.68	0.77	0.89	0.97	0.99	0.29	0.36	0.54	0.69	0.84	0.94	1.00	0.10	0.17	0.39	0.59	0.76	0.93	1.00
10	sd($\hat{\theta}$)	0.24	0.21	0.18	0.15	0.10	0.06	0.04	0.19	0.19	0.17	0.15	0.13	0.08	0.02	0.09	0.12	0.13	0.11	0.11	0.05	0.03
	FDR	0.00	0.02	0.02	0.03	0.04	0.08	0.07	0.00	0.01	0.02	0.05	0.07	0.05	0.05	0.00	0.01	0.02	0.04	0.05	0.04	0.03
	FNR	1.00	0.81	0.54	0.35	0.22	0.08	0.00	0.96	0.63	0.41	0.28	0.18	0.05	0.00	0.77	0.28	0.22	0.16	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.32	0.45	0.57	0.70	0.84	0.95	0.99	0.12	0.23	0.45	0.59	0.78	0.95	1.00	0.02	0.11	0.32	0.52	0.72	0.93	1.00
30	sd($\hat{\theta}$)	0.16	0.16	0.11	0.10	0.11	0.07	0.03	0.09	0.12	0.13	0.11	0.10	0.06	0.02	0.04	0.06	0.08	0.08	0.08	0.04	0.02
	FDR	0.00	0.03	0.07	0.11	0.13	0.19	0.07	0.00	0.02	0.05	0.11	0.12	0.15	0.03	0.00	0.02	0.05	0.07	0.11	0.09	0.03
	FNR	0.99	0.80	0.51	0.33	0.20	0.08	0.00	0.86	0.58	0.37	0.23	0.14	0.06	0.00	0.22	0.20	0.17	0.10	0.07	0.04	0.00
	ave($\hat{\theta}$)	0.28	0.40	0.54	0.67	0.81	0.96	1.00	0.07	0.19	0.42	0.60	0.75	0.94	1.00	0.01	0.11	0.30	0.50	0.70	0.91	1.00
50	sd($\hat{\theta}$)	0.14	0.12	0.09	0.08	0.10	0.05	0.01	0.06	0.10	0.10	0.10	0.09	0.06	0.02	0.03	0.06	0.04	0.07	0.05	0.04	0.02
	FDR	0.00	0.03	0.07	0.12	0.18	0.19	0.02	0.00	0.03	0.07	0.10	0.17	0.19	0.03	0.00	0.03	0.06	0.09	0.13	0.18	0.05
	FNR	0.98	0.78	0.50	0.31	0.19	0.08	0.00	0.67	0.52	0.36	0.23	0.13	0.06	0.00	0.12	0.25	0.14	0.09	0.06	0.03	0.00

Table B.87: BSQT test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.24	0.46	0.58	0.71	0.83	0.97	0.99	0.11	0.24	0.43	0.62	0.79	0.96	0.99	0.04	0.19	0.37	0.57	0.77	0.97	0.99
	sd($\hat{\theta}$)	0.24	0.23	0.17	0.14	0.12	0.07	0.03	0.15	0.14	0.15	0.14	0.11	0.12	0.07	0.09	0.09	0.09	0.10	0.08	0.07	0.04
	FDR	0.00	0.00	0.02	0.03	0.05	0.07	0.03	0.00	0.02	0.03	0.03	0.05	0.09	0.04	0.00	0.01	0.02	0.04	0.05	0.08	0.05
	FNR	0.59	0.71	0.46	0.30	0.16	0.09	0.00	0.41	0.52	0.29	0.20	0.13	0.09	0.00	0.16	0.46	0.22	0.16	0.10	0.08	0.00
30	ave($\hat{\theta}$)	0.22	0.35	0.49	0.69	0.83	0.97	0.98	0.08	0.21	0.43	0.59	0.79	0.97	1.00	0.02	0.18	0.38	0.57	0.78	0.97	0.98
	sd($\hat{\theta}$)	0.21	0.17	0.15	0.14	0.09	0.08	0.07	0.12	0.09	0.09	0.08	0.07	0.08	0.03	0.06	0.07	0.07	0.07	0.07	0.08	0.10
	FDR	0.00	0.01	0.03	0.03	0.05	0.07	0.06	0.00	0.01	0.01	0.02	0.04	0.07	0.02	0.00	0.01	0.02	0.02	0.03	0.06	0.05
	FNR	0.60	0.66	0.40	0.28	0.17	0.09	0.00	0.32	0.51	0.30	0.17	0.12	0.09	0.00	0.10	0.44	0.23	0.15	0.12	0.09	0.00
50	ave($\hat{\theta}$)	0.21	0.31	0.51	0.68	0.84	0.98	0.99	0.06	0.19	0.39	0.59	0.79	0.96	0.97	0.01	0.17	0.36	0.57	0.76	0.98	1.00
	sd($\hat{\theta}$)	0.20	0.16	0.14	0.14	0.12	0.05	0.04	0.11	0.09	0.12	0.10	0.07	0.10	0.12	0.05	0.07	0.09	0.09	0.10	0.07	0.02
	FDR	0.00	0.02	0.03	0.06	0.07	0.05	0.05	0.00	0.02	0.04	0.04	0.04	0.09	0.07	0.00	0.01	0.03	0.03	0.05	0.05	0.01
	FNR	0.61	0.63	0.42	0.29	0.18	0.09	0.00	0.24	0.49	0.27	0.18	0.13	0.09	0.00	0.06	0.44	0.22	0.15	0.11	0.09	0.00

Table B.88: BSQT test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.58	0.67	0.76	0.85	0.92	0.97	0.99	0.32	0.40	0.63	0.73	0.86	0.96	0.99	0.14	0.21	0.40	0.61	0.78	0.92	1.00
	sd($\hat{\theta}$)	0.25	0.27	0.20	0.15	0.11	0.08	0.05	0.27	0.23	0.24	0.16	0.12	0.06	0.05	0.17	0.16	0.14	0.15	0.14	0.08	0.01
	FDR	0.00	0.02	0.01	0.03	0.04	0.03	0.09	0.00	0.01	0.03	0.02	0.04	0.02	0.07	0.00	0.01	0.02	0.03	0.04	0.05	0.02
	FNR	1.00	0.81	0.58	0.41	0.24	0.08	0.00	0.92	0.66	0.49	0.30	0.18	0.06	0.00	0.70	0.35	0.23	0.18	0.12	0.04	0.00
30	ave($\hat{\theta}$)	0.45	0.57	0.67	0.83	0.91	0.96	1.00	0.26	0.34	0.53	0.69	0.81	0.95	0.98	0.01	0.17	0.38	0.56	0.78	0.96	0.99
	sd($\hat{\theta}$)	0.28	0.24	0.20	0.14	0.11	0.09	0.03	0.28	0.20	0.19	0.15	0.13	0.09	0.10	0.03	0.07	0.08	0.11	0.06	0.11	0.06
	FDR	0.00	0.01	0.04	0.05	0.09	0.11	0.03	0.00	0.02	0.04	0.03	0.08	0.08	0.06	0.00	0.02	0.02	0.04	0.03	0.09	0.05
	FNR	0.96	0.80	0.56	0.40	0.24	0.08	0.00	0.80	0.63	0.43	0.28	0.16	0.07	0.00	0.53	0.44	0.22	0.14	0.11	0.09	0.00
50	ave($\hat{\theta}$)	0.46	0.54	0.67	0.76	0.87	0.97	0.97	0.19	0.32	0.47	0.70	0.83	0.97	0.98	0.06	0.16	0.37	0.61	0.78	0.92	1.00
	sd($\hat{\theta}$)	0.30	0.25	0.18	0.14	0.13	0.08	0.10	0.22	0.21	0.16	0.12	0.14	0.07	0.09	0.14	0.12	0.12	0.12	0.13	0.11	0.00
	FDR	0.00	0.02	0.05	0.09	0.12	0.10	0.08	0.00	0.02	0.05	0.05	0.10	0.07	0.04	0.00	0.02	0.04	0.04	0.08	0.10	0.00
	FNR	0.97	0.79	0.56	0.38	0.23	0.09	0.00	0.71	0.61	0.38	0.30	0.19	0.08	0.00	0.24	0.32	0.21	0.19	0.13	0.05	0.00

Table B.89: BSQT test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.43	0.54	0.65	0.80	0.89	0.98	0.99	0.13	0.29	0.47	0.64	0.81	0.98	0.98	0.06	0.19	0.41	0.59	0.77	0.97	1.00
10	sd($\hat{\theta}$)	0.26	0.23	0.18	0.14	0.11	0.06	0.09	0.18	0.16	0.16	0.10	0.09	0.10	0.10	0.10	0.09	0.10	0.10	0.08	0.10	0.02
	FDR	0.00	0.01	0.02	0.03	0.03	0.06	0.02	0.00	0.01	0.02	0.02	0.03	0.05	0.04	0.00	0.01	0.01	0.02	0.04	0.05	0.01
	FNR	0.89	0.78	0.52	0.37	0.21	0.09	0.00	0.43	0.58	0.34	0.21	0.14	0.09	0.00	0.25	0.46	0.26	0.17	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.34	0.50	0.59	0.76	0.86	0.98	0.99	0.10	0.24	0.44	0.62	0.82	0.99	0.99	0.02	0.18	0.39	0.59	0.78	0.97	1.00
30	sd($\hat{\theta}$)	0.27	0.23	0.19	0.16	0.13	0.06	0.05	0.15	0.13	0.13	0.13	0.08	0.06	0.05	0.06	0.08	0.06	0.06	0.08	0.11	0.00
	FDR	0.00	0.01	0.04	0.03	0.06	0.05	0.06	0.00	0.01	0.02	0.03	0.02	0.04	0.03	0.00	0.01	0.01	0.01	0.02	0.05	0.00
	FNR	0.71	0.74	0.48	0.34	0.20	0.10	0.00	0.37	0.54	0.31	0.22	0.15	0.09	0.00	0.10	0.44	0.24	0.17	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.34	0.43	0.62	0.76	0.88	1.00	0.99	0.07	0.25	0.45	0.62	0.80	0.98	0.99	0.02	0.18	0.38	0.57	0.78	0.99	0.97
50	sd($\hat{\theta}$)	0.25	0.20	0.20	0.15	0.11	0.03	0.04	0.13	0.13	0.12	0.11	0.08	0.11	0.06	0.06	0.06	0.06	0.10	0.10	0.03	0.11
	FDR	0.00	0.01	0.02	0.03	0.05	0.01	0.02	0.00	0.01	0.02	0.03	0.02	0.04	0.01	0.00	0.01	0.01	0.03	0.03	0.02	0.08
	FNR	0.74	0.71	0.50	0.34	0.21	0.10	0.00	0.27	0.55	0.33	0.21	0.13	0.10	0.00	0.09	0.45	0.24	0.15	0.12	0.10	0.00

Table B.90: BSQT test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.43	0.56	0.66	0.80	0.89	0.99	0.99	0.15	0.28	0.45	0.66	0.82	0.97	0.99	0.04	0.18	0.38	0.58	0.76	0.97	0.99
10	sd($\hat{\theta}$)	0.28	0.24	0.20	0.18	0.12	0.07	0.05	0.17	0.17	0.14	0.11	0.09	0.10	0.08	0.09	0.09	0.10	0.09	0.12	0.10	0.07
	FDR	0.00	0.01	0.02	0.03	0.06	0.02	0.06	0.00	0.01	0.02	0.01	0.04	0.06	0.03	0.00	0.01	0.02	0.02	0.05	0.06	0.05
	FNR	0.84	0.79	0.51	0.36	0.22	0.10	0.00	0.49	0.55	0.32	0.23	0.15	0.09	0.00	0.20	0.45	0.24	0.16	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.40	0.44	0.64	0.77	0.87	0.97	0.98	0.10	0.26	0.45	0.63	0.80	0.97	1.00	0.02	0.17	0.37	0.59	0.79	0.98	0.99
30	sd($\hat{\theta}$)	0.29	0.23	0.18	0.13	0.14	0.10	0.13	0.15	0.13	0.14	0.11	0.08	0.13	0.03	0.05	0.09	0.11	0.07	0.06	0.08	0.09
	FDR	0.00	0.01	0.03	0.03	0.07	0.05	0.04	0.00	0.01	0.02	0.03	0.03	0.05	0.02	0.00	0.02	0.03	0.02	0.02	0.05	0.02
	FNR	0.76	0.72	0.51	0.35	0.21	0.10	0.00	0.35	0.56	0.33	0.22	0.13	0.09	0.00	0.08	0.41	0.23	0.16	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.37	0.43	0.61	0.77	0.89	0.98	0.98	0.09	0.23	0.43	0.64	0.79	0.98	0.99	0.01	0.17	0.36	0.59	0.78	0.98	1.00
50	sd($\hat{\theta}$)	0.29	0.24	0.16	0.12	0.12	0.10	0.09	0.14	0.13	0.12	0.10	0.10	0.08	0.06	0.06	0.08	0.11	0.07	0.09	0.06	0.02
	FDR	0.00	0.02	0.03	0.04	0.04	0.05	0.05	0.00	0.02	0.02	0.01	0.04	0.04	0.02	0.00	0.02	0.04	0.01	0.02	0.04	0.01
	FNR	0.71	0.69	0.50	0.36	0.22	0.10	0.00	0.32	0.50	0.32	0.22	0.13	0.09	0.00	0.05	0.43	0.22	0.17	0.12	0.09	0.00

Table B.91: BSQT2 test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.53	0.54	0.68	0.80	0.90	0.96	0.99	0.17	0.29	0.48	0.70	0.80	0.93	0.99	0.08	0.16	0.37	0.60	0.78	0.92	1.00
10	sd($\hat{\theta}$)	0.36	0.25	0.24	0.18	0.12	0.08	0.03	0.20	0.19	0.17	0.16	0.14	0.08	0.05	0.10	0.10	0.13	0.12	0.12	0.07	0.01
	FDR	0.00	0.01	0.03	0.05	0.07	0.09	0.07	0.00	0.01	0.05	0.05	0.08	0.04	0.07	0.00	0.01	0.03	0.03	0.03	0.03	0.02
	FNR	0.89	0.76	0.54	0.38	0.22	0.07	0.00	0.66	0.50	0.41	0.30	0.15	0.05	0.00	0.52	0.24	0.20	0.17	0.11	0.03	0.00
	ave($\hat{\theta}$)	0.23	0.32	0.55	0.71	0.85	0.98	0.99	0.09	0.19	0.38	0.59	0.79	0.93	0.99	0.02	0.12	0.32	0.54	0.73	0.92	0.99
30	sd($\hat{\theta}$)	0.23	0.17	0.16	0.15	0.13	0.06	0.05	0.12	0.13	0.15	0.16	0.13	0.11	0.05	0.05	0.08	0.09	0.12	0.09	0.06	0.04
	FDR	0.00	0.03	0.06	0.11	0.12	0.08	0.07	0.00	0.03	0.08	0.10	0.10	0.16	0.04	0.00	0.03	0.05	0.07	0.10	0.10	0.07
	FNR	0.85	0.70	0.49	0.34	0.21	0.09	0.00	0.50	0.49	0.32	0.23	0.15	0.07	0.00	0.21	0.25	0.16	0.14	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.21	0.25	0.51	0.66	0.86	0.97	0.98	0.09	0.14	0.37	0.59	0.77	0.95	0.99	0.01	0.10	0.30	0.51	0.70	0.93	0.99
50	sd($\hat{\theta}$)	0.21	0.12	0.13	0.15	0.10	0.08	0.08	0.11	0.10	0.12	0.12	0.12	0.10	0.07	0.03	0.05	0.08	0.09	0.10	0.08	0.05
	FDR	0.00	0.04	0.08	0.12	0.13	0.09	0.08	0.00	0.04	0.08	0.11	0.15	0.13	0.02	0.00	0.03	0.06	0.08	0.13	0.13	0.07
	FNR	0.88	0.68	0.48	0.32	0.21	0.09	0.00	0.60	0.43	0.31	0.23	0.15	0.08	0.00	0.08	0.21	0.14	0.10	0.07	0.05	0.00

Table B.92: BSQT2 test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.27	0.44	0.61	0.71	0.86	0.98	1.00	0.03	0.20	0.43	0.59	0.78	0.98	0.99	0.01	0.17	0.37	0.55	0.76	0.97	0.98
10	sd($\hat{\theta}$)	0.31	0.21	0.20	0.17	0.12	0.06	0.03	0.08	0.11	0.12	0.10	0.11	0.07	0.04	0.03	0.07	0.09	0.10	0.10	0.10	0.10
	FDR	0.00	0.01	0.02	0.05	0.04	0.05	0.02	0.00	0.02	0.02	0.04	0.05	0.06	0.04	0.00	0.02	0.02	0.05	0.05	0.07	0.06
	FNR	0.58	0.73	0.48	0.31	0.19	0.09	0.00	0.17	0.46	0.29	0.18	0.12	0.09	0.00	0.03	0.43	0.22	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.10	0.21	0.44	0.66	0.81	0.99	1.00	0.00	0.17	0.35	0.58	0.77	0.96	1.00	0.00	0.15	0.36	0.57	0.76	0.95	1.00
30	sd($\hat{\theta}$)	0.20	0.12	0.14	0.13	0.09	0.05	0.03	0.03	0.08	0.08	0.08	0.09	0.10	0.03	0.02	0.09	0.08	0.08	0.09	0.09	0.02
	FDR	0.00	0.03	0.05	0.06	0.10	0.05	0.02	0.00	0.02	0.04	0.04	0.05	0.09	0.02	0.00	0.03	0.03	0.03	0.05	0.11	0.01
	FNR	0.27	0.55	0.36	0.27	0.16	0.10	0.00	0.02	0.45	0.23	0.16	0.11	0.09	0.00	0.01	0.37	0.21	0.14	0.10	0.08	0.00
	ave($\hat{\theta}$)	0.06	0.18	0.42	0.63	0.81	0.98	1.00	0.01	0.16	0.36	0.57	0.77	0.95	0.99	0.00	0.16	0.33	0.55	0.75	0.96	0.99
50	sd($\hat{\theta}$)	0.13	0.11	0.13	0.12	0.10	0.05	0.02	0.04	0.08	0.09	0.09	0.08	0.11	0.05	0.00	0.08	0.10	0.09	0.09	0.09	0.05
	FDR	0.00	0.04	0.07	0.07	0.10	0.05	0.01	0.00	0.02	0.04	0.05	0.06	0.11	0.04	0.00	0.02	0.04	0.04	0.06	0.10	0.03
	FNR	0.23	0.49	0.36	0.25	0.16	0.09	0.00	0.05	0.42	0.23	0.16	0.11	0.08	0.00	0.00	0.41	0.18	0.13	0.10	0.08	0.00

Table B.93: BSQT2 test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.43	0.39	0.59	0.76	0.87	0.98	1.00	0.03	0.19	0.43	0.62	0.81	0.96	0.97	0.01	0.15	0.36	0.58	0.78	0.98	0.98
	sd($\hat{\theta}$)	0.39	0.23	0.22	0.15	0.11	0.06	0.00	0.09	0.11	0.12	0.12	0.12	0.12	0.11	0.03	0.10	0.10	0.08	0.06	0.06	0.09
	FDR	0.00	0.01	0.03	0.03	0.02	0.05	0.00	0.00	0.02	0.02	0.02	0.04	0.07	0.07	0.00	0.03	0.03	0.02	0.03	0.05	0.05
	FNR	0.68	0.66	0.47	0.34	0.19	0.10	0.00	0.13	0.45	0.30	0.20	0.14	0.09	0.00	0.03	0.38	0.21	0.16	0.11	0.09	0.00
30	ave($\hat{\theta}$)	0.21	0.21	0.48	0.69	0.82	0.96	0.99	0.02	0.16	0.40	0.58	0.77	0.98	0.99	0.00	0.16	0.37	0.56	0.75	0.99	0.99
	sd($\hat{\theta}$)	0.33	0.14	0.15	0.14	0.14	0.12	0.06	0.05	0.08	0.07	0.11	0.11	0.10	0.08	0.02	0.08	0.09	0.10	0.14	0.06	0.06
	FDR	0.00	0.03	0.05	0.06	0.08	0.08	0.02	0.00	0.02	0.02	0.04	0.05	0.05	0.04	0.00	0.02	0.02	0.04	0.06	0.04	0.05
	FNR	0.38	0.50	0.41	0.30	0.18	0.10	0.00	0.08	0.43	0.27	0.17	0.12	0.09	0.00	0.01	0.39	0.23	0.14	0.11	0.09	0.00
50	ave($\hat{\theta}$)	0.19	0.19	0.45	0.65	0.82	0.98	0.99	0.01	0.16	0.35	0.56	0.77	0.99	0.98	0.00	0.15	0.36	0.57	0.77	0.97	0.99
	sd($\hat{\theta}$)	0.31	0.11	0.13	0.14	0.14	0.07	0.04	0.07	0.08	0.11	0.11	0.10	0.04	0.10	0.02	0.09	0.09	0.10	0.09	0.11	0.04
	FDR	0.00	0.03	0.06	0.08	0.08	0.05	0.02	0.00	0.03	0.04	0.05	0.05	0.03	0.05	0.00	0.03	0.03	0.03	0.04	0.05	0.02
	FNR	0.41	0.51	0.39	0.28	0.17	0.10	0.00	0.04	0.41	0.23	0.16	0.13	0.10	0.00	0.01	0.38	0.21	0.15	0.11	0.09	0.00

Table B.94: BSQT2 test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.39	0.39	0.59	0.75	0.87	0.99	0.99	0.02	0.19	0.41	0.59	0.79	0.96	1.00	0.00	0.16	0.37	0.57	0.76	0.97	0.99
10	sd($\hat{\theta}$)	0.40	0.23	0.21	0.16	0.12	0.05	0.07	0.09	0.11	0.12	0.13	0.09	0.15	0.02	0.02	0.08	0.11	0.11	0.12	0.11	0.08
	FDR	0.00	0.02	0.02	0.03	0.04	0.04	0.02	0.00	0.02	0.02	0.04	0.04	0.08	0.01	0.00	0.02	0.03	0.03	0.06	0.07	0.02
	FNR	0.60	0.66	0.47	0.33	0.20	0.10	0.00	0.09	0.46	0.29	0.18	0.13	0.09	0.00	0.01	0.41	0.22	0.15	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.28	0.23	0.48	0.68	0.83	0.99	0.99	0.01	0.15	0.36	0.59	0.78	0.96	0.99	0.00	0.15	0.36	0.56	0.76	0.97	0.99
30	sd($\hat{\theta}$)	0.39	0.14	0.13	0.13	0.15	0.04	0.08	0.04	0.09	0.12	0.08	0.09	0.10	0.06	0.00	0.09	0.09	0.10	0.13	0.07	0.08
	FDR	0.00	0.03	0.04	0.06	0.08	0.03	0.02	0.00	0.03	0.04	0.03	0.03	0.09	0.05	0.00	0.02	0.03	0.04	0.05	0.07	0.02
	FNR	0.43	0.56	0.40	0.30	0.18	0.10	0.00	0.04	0.39	0.24	0.18	0.12	0.09	0.00	0.00	0.38	0.22	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.19	0.18	0.48	0.67	0.82	0.99	0.98	0.01	0.15	0.38	0.59	0.76	0.96	0.98	0.00	0.15	0.36	0.56	0.77	0.98	0.99
50	sd($\hat{\theta}$)	0.33	0.12	0.13	0.13	0.14	0.05	0.09	0.07	0.08	0.07	0.09	0.12	0.11	0.12	0.00	0.09	0.10	0.10	0.09	0.11	0.10
	FDR	0.00	0.04	0.04	0.07	0.10	0.02	0.04	0.00	0.03	0.03	0.03	0.05	0.08	0.05	0.00	0.03	0.03	0.03	0.04	0.04	0.03
	FNR	0.33	0.49	0.40	0.29	0.18	0.10	0.00	0.03	0.41	0.25	0.18	0.12	0.09	0.00	0.00	0.38	0.22	0.15	0.11	0.09	0.00

Table B.95: BSQT2 test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.45	0.50	0.62	0.75	0.87	0.89	0.93	0.20	0.29	0.43	0.59	0.77	0.81	0.88	0.10	0.14	0.33	0.48	0.71	0.81	0.91
10	sd($\hat{\theta}$)	0.37	0.28	0.26	0.22	0.17	0.19	0.18	0.21	0.20	0.23	0.21	0.26	0.27	0.24	0.13	0.11	0.18	0.23	0.22	0.26	0.22
	FDR	0.00	0.03	0.08	0.14	0.15	0.24	0.29	0.00	0.03	0.09	0.13	0.21	0.29	0.36	0.00	0.03	0.08	0.15	0.14	0.28	0.39
	FNR	0.84	0.76	0.53	0.39	0.23	0.07	0.00	0.78	0.61	0.44	0.26	0.20	0.04	0.00	0.59	0.28	0.21	0.17	0.11	0.04	0.00
	ave($\hat{\theta}$)	0.25	0.29	0.45	0.63	0.74	0.84	0.93	0.09	0.17	0.30	0.54	0.71	0.83	0.86	0.03	0.10	0.24	0.49	0.59	0.84	0.85
30	sd($\hat{\theta}$)	0.22	0.18	0.21	0.25	0.26	0.31	0.18	0.12	0.13	0.21	0.22	0.27	0.28	0.31	0.04	0.08	0.17	0.20	0.30	0.23	0.32
	FDR	0.00	0.05	0.14	0.20	0.30	0.30	0.25	0.00	0.05	0.14	0.16	0.24	0.36	0.36	0.00	0.04	0.13	0.15	0.25	0.33	0.33
	FNR	0.87	0.74	0.52	0.39	0.23	0.08	0.00	0.57	0.48	0.35	0.26	0.17	0.07	0.00	0.25	0.22	0.14	0.16	0.09	0.04	0.00
	ave($\hat{\theta}$)	0.17	0.23	0.43	0.58	0.72	0.83	0.84	0.08	0.13	0.23	0.47	0.60	0.73	0.83	0.02	0.09	0.25	0.41	0.64	0.74	0.84
50	sd($\hat{\theta}$)	0.15	0.14	0.23	0.28	0.30	0.30	0.30	0.15	0.11	0.21	0.28	0.33	0.35	0.34	0.05	0.08	0.17	0.25	0.26	0.34	0.34
	FDR	0.00	0.06	0.14	0.23	0.31	0.35	0.38	0.00	0.05	0.18	0.21	0.32	0.48	0.42	0.00	0.04	0.12	0.20	0.22	0.43	0.39
	FNR	0.81	0.73	0.51	0.38	0.22	0.09	0.00	0.49	0.40	0.32	0.26	0.15	0.05	0.00	0.12	0.17	0.15	0.13	0.09	0.04	0.00

Table B.96: BSQT2 test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.36	0.38	0.57	0.71	0.86	0.94	0.98	0.04	0.20	0.36	0.55	0.70	0.92	0.95	0.01	0.16	0.30	0.52	0.70	0.88	0.95
10	sd($\hat{\theta}$)	0.34	0.22	0.22	0.18	0.13	0.15	0.07	0.10	0.12	0.16	0.15	0.18	0.15	0.16	0.04	0.09	0.15	0.15	0.14	0.19	0.14
	FDR	0.00	0.02	0.04	0.07	0.09	0.13	0.06	0.00	0.02	0.06	0.09	0.14	0.19	0.13	0.00	0.02	0.08	0.07	0.13	0.23	0.14
	FNR	0.68	0.66	0.46	0.32	0.20	0.08	0.00	0.13	0.47	0.25	0.17	0.09	0.07	0.00	0.04	0.39	0.17	0.12	0.08	0.07	0.00
	ave($\hat{\theta}$)	0.10	0.20	0.45	0.64	0.74	0.95	0.96	0.01	0.14	0.31	0.53	0.72	0.92	0.97	0.00	0.12	0.29	0.51	0.72	0.91	0.94
30	sd($\hat{\theta}$)	0.20	0.11	0.15	0.16	0.21	0.14	0.15	0.05	0.09	0.14	0.16	0.17	0.16	0.09	0.00	0.10	0.15	0.16	0.19	0.19	0.18
	FDR	0.00	0.04	0.07	0.09	0.22	0.14	0.12	0.00	0.04	0.09	0.11	0.13	0.20	0.14	0.00	0.04	0.09	0.08	0.09	0.17	0.15
	FNR	0.30	0.57	0.40	0.27	0.17	0.09	0.00	0.07	0.37	0.21	0.17	0.11	0.07	0.00	0.00	0.29	0.16	0.12	0.10	0.07	0.00
	ave($\hat{\theta}$)	0.09	0.18	0.37	0.57	0.72	0.93	0.98	0.01	0.12	0.30	0.53	0.73	0.88	0.94	0.00	0.13	0.27	0.51	0.70	0.91	0.93
50	sd($\hat{\theta}$)	0.18	0.12	0.16	0.23	0.23	0.19	0.11	0.04	0.10	0.15	0.16	0.17	0.22	0.19	0.00	0.10	0.17	0.17	0.18	0.17	0.20
	FDR	0.00	0.04	0.10	0.15	0.20	0.13	0.06	0.00	0.04	0.09	0.10	0.13	0.23	0.13	0.00	0.04	0.10	0.09	0.13	0.17	0.19
	FNR	0.28	0.47	0.36	0.25	0.16	0.09	0.00	0.05	0.33	0.21	0.17	0.12	0.07	0.00	0.00	0.32	0.15	0.14	0.09	0.07	0.00

Table B.97: BSQT2 test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.44	0.41	0.56	0.68	0.79	0.93	0.95	0.05	0.18	0.39	0.55	0.73	0.89	0.94	0.00	0.14	0.32	0.49	0.65	0.85	0.88
10	sd($\hat{\theta}$)	0.39	0.23	0.22	0.20	0.24	0.16	0.18	0.10	0.14	0.15	0.18	0.17	0.16	0.14	0.03	0.11	0.14	0.17	0.21	0.21	0.26
	FDR	0.00	0.02	0.06	0.13	0.19	0.17	0.11	0.00	0.03	0.07	0.12	0.16	0.30	0.22	0.00	0.04	0.07	0.11	0.21	0.32	0.30
	FNR	0.69	0.67	0.48	0.32	0.20	0.08	0.00	0.20	0.42	0.30	0.19	0.13	0.07	0.00	0.02	0.35	0.19	0.12	0.08	0.05	0.00
	ave($\hat{\theta}$)	0.23	0.20	0.41	0.60	0.81	0.92	0.92	0.02	0.12	0.32	0.52	0.64	0.87	0.93	0.00	0.11	0.32	0.52	0.69	0.84	0.90
30	sd($\hat{\theta}$)	0.33	0.14	0.22	0.24	0.16	0.19	0.24	0.06	0.10	0.17	0.20	0.28	0.21	0.18	0.02	0.10	0.15	0.16	0.17	0.20	0.26
	FDR	0.00	0.04	0.11	0.14	0.18	0.19	0.13	0.00	0.05	0.09	0.13	0.25	0.29	0.21	0.00	0.05	0.08	0.11	0.18	0.37	0.22
	FNR	0.50	0.51	0.39	0.30	0.21	0.08	0.00	0.09	0.32	0.24	0.19	0.12	0.07	0.00	0.01	0.29	0.22	0.16	0.09	0.04	0.00
	ave($\hat{\theta}$)	0.15	0.16	0.36	0.60	0.78	0.92	0.93	0.02	0.08	0.31	0.54	0.72	0.86	0.92	0.00	0.08	0.29	0.50	0.68	0.86	0.89
50	sd($\hat{\theta}$)	0.28	0.13	0.22	0.24	0.19	0.22	0.21	0.05	0.10	0.17	0.16	0.19	0.26	0.24	0.00	0.10	0.17	0.18	0.22	0.20	0.25
	FDR	0.00	0.05	0.13	0.15	0.22	0.14	0.16	0.00	0.06	0.10	0.13	0.20	0.25	0.18	0.00	0.06	0.10	0.13	0.20	0.31	0.27
	FNR	0.34	0.44	0.37	0.28	0.19	0.09	0.00	0.08	0.22	0.24	0.20	0.13	0.07	0.00	0.00	0.21	0.21	0.15	0.10	0.05	0.00

Table B.98: BSQT2 test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.38	0.39	0.54	0.73	0.84	0.97	0.99	0.03	0.21	0.42	0.58	0.77	0.97	0.98	0.00	0.15	0.36	0.58	0.75	0.93	0.98
	sd($\hat{\theta}$)	0.37	0.20	0.19	0.17	0.15	0.10	0.04	0.10	0.13	0.14	0.15	0.14	0.08	0.09	0.00	0.09	0.11	0.07	0.12	0.17	0.11
	FDR	0.00	0.02	0.03	0.02	0.07	0.05	0.05	0.00	0.02	0.03	0.05	0.07	0.09	0.07	0.00	0.02	0.03	0.02	0.07	0.13	0.04
	FNR	0.60	0.70	0.44	0.32	0.20	0.09	0.00	0.10	0.49	0.29	0.18	0.13	0.09	0.00	0.00	0.38	0.21	0.16	0.10	0.08	0.00
30	ave($\hat{\theta}$)	0.23	0.24	0.46	0.67	0.83	0.98	0.97	0.01	0.14	0.36	0.58	0.73	0.96	0.98	0.00	0.14	0.36	0.55	0.77	0.96	0.98
	sd($\hat{\theta}$)	0.35	0.13	0.20	0.17	0.19	0.11	0.09	0.07	0.10	0.13	0.14	0.19	0.13	0.11	0.00	0.09	0.10	0.13	0.08	0.12	0.10
	FDR	0.00	0.02	0.07	0.09	0.08	0.04	0.09	0.00	0.03	0.05	0.06	0.10	0.07	0.04	0.00	0.03	0.03	0.05	0.04	0.07	0.06
	FNR	0.39	0.54	0.39	0.31	0.19	0.10	0.00	0.04	0.37	0.25	0.19	0.12	0.09	0.00	0.00	0.35	0.22	0.15	0.12	0.09	0.00
50	ave($\hat{\theta}$)	0.23	0.16	0.43	0.67	0.84	0.97	0.96	0.00	0.13	0.34	0.56	0.76	0.97	0.96	0.00	0.14	0.36	0.56	0.74	0.97	0.97
	sd($\hat{\theta}$)	0.34	0.12	0.19	0.18	0.20	0.13	0.18	0.00	0.09	0.13	0.13	0.16	0.09	0.18	0.02	0.09	0.11	0.10	0.17	0.13	0.14
	FDR	0.00	0.04	0.07	0.08	0.10	0.07	0.07	0.00	0.04	0.06	0.07	0.07	0.07	0.06	0.00	0.03	0.04	0.04	0.07	0.06	0.08
	FNR	0.42	0.43	0.38	0.30	0.20	0.09	0.00	0.00	0.35	0.23	0.18	0.13	0.09	0.00	0.01	0.36	0.22	0.15	0.11	0.09	0.00

Table B.99: BSQT2 test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.52	0.54	0.71	0.77	0.92	0.98	0.99	0.19	0.26	0.45	0.62	0.79	0.93	0.99	0.06	0.16	0.37	0.55	0.74	0.91	0.99
10	sd($\hat{\theta}$)	0.35	0.33	0.28	0.21	0.12	0.06	0.03	0.22	0.19	0.17	0.18	0.15	0.08	0.04	0.09	0.14	0.14	0.17	0.10	0.07	0.04
	FDR	0.00	0.02	0.04	0.06	0.05	0.04	0.10	0.00	0.02	0.05	0.07	0.10	0.08	0.10	0.00	0.01	0.03	0.09	0.08	0.05	0.06
	FNR	0.93	0.71	0.54	0.36	0.25	0.08	0.00	0.71	0.50	0.36	0.22	0.14	0.05	0.00	0.46	0.23	0.19	0.15	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.22	0.34	0.53	0.70	0.84	0.97	0.99	0.13	0.21	0.36	0.56	0.75	0.93	0.98	0.02	0.11	0.30	0.52	0.70	0.91	0.99
30	sd($\hat{\theta}$)	0.23	0.22	0.23	0.17	0.14	0.05	0.03	0.16	0.16	0.16	0.12	0.13	0.07	0.06	0.04	0.07	0.07	0.09	0.08	0.08	0.03
	FDR	0.00	0.03	0.09	0.11	0.17	0.14	0.10	0.00	0.03	0.09	0.11	0.18	0.18	0.16	0.00	0.03	0.06	0.09	0.14	0.14	0.09
	FNR	0.84	0.68	0.46	0.33	0.21	0.08	0.00	0.64	0.53	0.30	0.19	0.13	0.06	0.00	0.20	0.24	0.14	0.11	0.06	0.04	0.00
	ave($\hat{\theta}$)	0.20	0.31	0.48	0.66	0.84	0.96	0.99	0.06	0.16	0.36	0.53	0.71	0.92	0.99	0.02	0.10	0.30	0.48	0.68	0.90	1.00
50	sd($\hat{\theta}$)	0.20	0.22	0.16	0.16	0.13	0.07	0.02	0.10	0.13	0.13	0.11	0.10	0.07	0.05	0.06	0.05	0.07	0.08	0.07	0.06	0.02
	FDR	0.00	0.04	0.10	0.14	0.17	0.19	0.06	0.00	0.04	0.08	0.14	0.19	0.22	0.10	0.00	0.03	0.06	0.11	0.16	0.21	0.05
	FNR	0.87	0.67	0.46	0.31	0.20	0.08	0.00	0.45	0.43	0.29	0.17	0.10	0.05	0.00	0.14	0.23	0.14	0.08	0.05	0.03	0.00

Table B.100: BSQT2 test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.37	0.45	0.62	0.75	0.84	0.96	0.98	0.04	0.15	0.34	0.52	0.71	0.88	0.94	0.01	0.14	0.30	0.51	0.70	0.88	0.93
	sd($\hat{\theta}$)	0.37	0.27	0.25	0.19	0.17	0.09	0.06	0.08	0.10	0.13	0.13	0.12	0.14	0.09	0.03	0.10	0.12	0.13	0.11	0.14	0.12
	FDR	0.00	0.01	0.04	0.07	0.13	0.14	0.07	0.00	0.03	0.06	0.09	0.16	0.28	0.27	0.00	0.03	0.06	0.08	0.12	0.27	0.28
	FNR	0.65	0.70	0.49	0.34	0.20	0.09	0.00	0.18	0.39	0.20	0.13	0.09	0.05	0.00	0.03	0.35	0.15	0.10	0.07	0.05	0.00
30	ave($\hat{\theta}$)	0.07	0.23	0.43	0.60	0.78	0.91	0.98	0.01	0.12	0.30	0.46	0.66	0.84	0.93	0.00	0.14	0.29	0.45	0.63	0.82	0.92
	sd($\hat{\theta}$)	0.13	0.20	0.19	0.18	0.13	0.11	0.07	0.04	0.10	0.11	0.11	0.11	0.11	0.10	0.00	0.09	0.11	0.10	0.08	0.09	0.10
	FDR	0.00	0.04	0.09	0.15	0.21	0.26	0.11	0.00	0.04	0.08	0.14	0.21	0.41	0.32	0.00	0.03	0.07	0.13	0.23	0.45	0.36
	FNR	0.29	0.50	0.38	0.25	0.17	0.07	0.00	0.04	0.32	0.16	0.08	0.06	0.03	0.00	0.00	0.36	0.12	0.06	0.03	0.02	0.00
50	ave($\hat{\theta}$)	0.08	0.19	0.37	0.57	0.72	0.90	0.97	0.01	0.14	0.27	0.45	0.62	0.83	0.93	0.00	0.14	0.22	0.43	0.61	0.80	0.91
	sd($\hat{\theta}$)	0.14	0.17	0.19	0.17	0.14	0.11	0.08	0.04	0.09	0.10	0.10	0.10	0.09	0.10	0.02	0.09	0.10	0.08	0.08	0.07	0.10
	FDR	0.00	0.04	0.10	0.16	0.27	0.30	0.13	0.00	0.04	0.09	0.15	0.26	0.44	0.31	0.00	0.03	0.11	0.15	0.24	0.49	0.43
	FNR	0.30	0.47	0.31	0.24	0.14	0.07	0.00	0.05	0.37	0.13	0.08	0.05	0.03	0.00	0.01	0.36	0.05	0.04	0.02	0.01	0.00

Table B.101: BSQT2 test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.36	0.42	0.59	0.76	0.85	0.91	0.93	0.03	0.15	0.27	0.48	0.65	0.81	0.90	0.01	0.11	0.27	0.42	0.63	0.75	0.84
	sd($\hat{\theta}$)	0.37	0.34	0.32	0.22	0.18	0.14	0.16	0.09	0.13	0.15	0.15	0.16	0.17	0.15	0.03	0.10	0.12	0.14	0.15	0.17	0.14
	FDR	0.00	0.03	0.06	0.08	0.13	0.23	0.21	0.00	0.04	0.10	0.13	0.23	0.42	0.37	0.00	0.05	0.09	0.16	0.22	0.50	0.64
	FNR	0.62	0.60	0.44	0.35	0.21	0.08	0.00	0.12	0.34	0.15	0.11	0.07	0.04	0.00	0.03	0.29	0.12	0.06	0.05	0.02	0.00
30	ave($\hat{\theta}$)	0.24	0.36	0.46	0.57	0.75	0.89	0.92	0.01	0.13	0.26	0.43	0.57	0.77	0.84	0.00	0.14	0.25	0.38	0.55	0.71	0.81
	sd($\hat{\theta}$)	0.36	0.32	0.28	0.22	0.19	0.15	0.12	0.07	0.10	0.12	0.12	0.11	0.13	0.13	0.00	0.09	0.10	0.10	0.11	0.12	0.13
	FDR	0.00	0.03	0.09	0.18	0.23	0.30	0.33	0.00	0.04	0.11	0.18	0.33	0.54	0.65	0.00	0.03	0.10	0.20	0.33	0.62	0.76
	FNR	0.45	0.58	0.36	0.25	0.16	0.07	0.00	0.04	0.35	0.14	0.08	0.04	0.02	0.00	0.00	0.36	0.09	0.03	0.02	0.01	0.00
50	ave($\hat{\theta}$)	0.13	0.32	0.45	0.57	0.74	0.85	0.93	0.00	0.14	0.23	0.40	0.56	0.75	0.85	0.00	0.13	0.21	0.38	0.56	0.72	0.79
	sd($\hat{\theta}$)	0.26	0.33	0.30	0.25	0.20	0.15	0.13	0.03	0.09	0.09	0.07	0.12	0.12	0.10	0.00	0.10	0.08	0.08	0.09	0.11	0.10
	FDR	0.00	0.04	0.12	0.18	0.25	0.37	0.27	0.00	0.04	0.12	0.20	0.35	0.57	0.72	0.00	0.04	0.13	0.21	0.32	0.62	0.92
	FNR	0.31	0.55	0.37	0.24	0.17	0.06	0.00	0.02	0.38	0.12	0.06	0.04	0.02	0.00	0.00	0.33	0.05	0.03	0.01	0.01	0.00

Table B.102: BSQT2 test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.38	0.45	0.60	0.62	0.80	0.91	0.88	0.01	0.09	0.25	0.37	0.52	0.65	0.76	0.01	0.09	0.19	0.30	0.46	0.63	0.68
10	sd($\hat{\theta}$)	0.43	0.39	0.34	0.29	0.22	0.15	0.18	0.05	0.11	0.15	0.16	0.17	0.22	0.19	0.03	0.10	0.14	0.17	0.18	0.19	0.19
	FDR	0.00	0.04	0.09	0.17	0.19	0.22	0.37	0.00	0.06	0.12	0.23	0.38	0.62	0.74	0.00	0.06	0.15	0.27	0.40	0.65	0.92
	FNR	0.50	0.56	0.49	0.28	0.21	0.08	0.00	0.07	0.23	0.14	0.06	0.05	0.01	0.00	0.03	0.22	0.06	0.03	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.20	0.29	0.39	0.49	0.65	0.76	0.82	0.00	0.09	0.17	0.28	0.43	0.60	0.66	0.00	0.09	0.18	0.27	0.41	0.55	0.61
30	sd($\hat{\theta}$)	0.34	0.33	0.30	0.25	0.22	0.21	0.21	0.02	0.12	0.10	0.11	0.14	0.16	0.16	0.00	0.10	0.08	0.11	0.12	0.13	0.15
	FDR	0.00	0.05	0.15	0.25	0.38	0.54	0.52	0.00	0.06	0.17	0.31	0.47	0.73	0.96	0.00	0.06	0.15	0.30	0.48	0.75	1.00
	FNR	0.37	0.49	0.35	0.23	0.16	0.05	0.00	0.01	0.24	0.07	0.03	0.02	0.01	0.00	0.00	0.25	0.04	0.01	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.23	0.28	0.31	0.52	0.62	0.74	0.79	0.00	0.09	0.18	0.29	0.43	0.58	0.62	0.00	0.12	0.18	0.25	0.38	0.53	0.57
50	sd($\hat{\theta}$)	0.35	0.32	0.26	0.29	0.24	0.22	0.21	0.03	0.10	0.08	0.11	0.15	0.15	0.16	0.00	0.10	0.07	0.11	0.10	0.12	0.13
	FDR	0.00	0.05	0.17	0.24	0.40	0.53	0.61	0.00	0.06	0.16	0.30	0.47	0.75	0.99	0.00	0.04	0.15	0.32	0.50	0.78	1.00
	FNR	0.39	0.48	0.30	0.24	0.15	0.05	0.00	0.02	0.26	0.09	0.03	0.02	0.01	0.00	0.00	0.30	0.03	0.02	0.01	0.00	0.00

Table B.103: BSQT2 test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.29	0.41	0.57	0.72	0.86	0.95	0.99	0.04	0.20	0.37	0.57	0.76	0.94	0.99	0.01	0.17	0.37	0.51	0.74	0.95	0.99
10	sd($\hat{\theta}$)	0.22	0.26	0.20	0.18	0.12	0.07	0.03	0.09	0.10	0.10	0.09	0.09	0.10	0.04	0.04	0.08	0.08	0.12	0.09	0.09	0.05
	FDR	0.00	0.02	0.06	0.07	0.06	0.07	0.08	0.00	0.01	0.03	0.04	0.07	0.14	0.04	0.00	0.02	0.03	0.08	0.08	0.12	0.06
	FNR	0.92	0.69	0.50	0.32	0.19	0.06	0.00	0.19	0.48	0.23	0.15	0.10	0.07	0.00	0.05	0.44	0.22	0.10	0.09	0.08	0.00
	ave($\hat{\theta}$)	0.03	0.19	0.35	0.58	0.74	0.94	0.99	0.01	0.16	0.33	0.55	0.71	0.91	0.99	0.00	0.15	0.34	0.52	0.73	0.93	0.99
30	sd($\hat{\theta}$)	0.08	0.08	0.10	0.08	0.10	0.09	0.04	0.05	0.08	0.09	0.09	0.10	0.10	0.05	0.03	0.09	0.10	0.10	0.10	0.10	0.03
	FDR	0.00	0.02	0.06	0.07	0.12	0.15	0.04	0.00	0.02	0.05	0.05	0.11	0.23	0.07	0.00	0.03	0.04	0.07	0.09	0.19	0.03
	FNR	0.17	0.48	0.24	0.18	0.11	0.07	0.00	0.07	0.41	0.19	0.14	0.08	0.06	0.00	0.02	0.37	0.18	0.11	0.08	0.06	0.00
	ave($\hat{\theta}$)	0.02	0.17	0.35	0.56	0.74	0.92	1.00	0.01	0.17	0.31	0.51	0.72	0.89	0.99	0.00	0.16	0.31	0.52	0.71	0.88	0.99
50	sd($\hat{\theta}$)	0.06	0.07	0.09	0.08	0.09	0.10	0.03	0.03	0.07	0.10	0.10	0.10	0.10	0.05	0.00	0.08	0.10	0.10	0.10	0.10	0.04
	FDR	0.00	0.02	0.05	0.07	0.13	0.21	0.02	0.00	0.02	0.06	0.08	0.11	0.27	0.07	0.00	0.02	0.06	0.07	0.12	0.32	0.04
	FNR	0.12	0.48	0.23	0.17	0.11	0.07	0.00	0.03	0.43	0.16	0.11	0.08	0.05	0.00	0.00	0.41	0.14	0.10	0.07	0.04	0.00

Table B.104: BSQT2 test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.41	0.48	0.62	0.75	0.89	0.97	0.99	0.19	0.28	0.47	0.67	0.81	0.94	1.00	0.07	0.15	0.37	0.57	0.74	0.92	0.99
10	sd($\hat{\theta}$)	0.27	0.25	0.22	0.16	0.11	0.06	0.04	0.19	0.21	0.18	0.17	0.14	0.08	0.02	0.08	0.11	0.14	0.11	0.12	0.05	0.05
	FDR	0.00	0.02	0.04	0.05	0.05	0.07	0.06	0.00	0.01	0.04	0.07	0.10	0.04	0.05	0.00	0.01	0.03	0.04	0.07	0.04	0.02
	FNR	0.94	0.73	0.51	0.34	0.22	0.08	0.00	0.74	0.49	0.36	0.28	0.17	0.05	0.00	0.62	0.24	0.19	0.15	0.08	0.03	0.00
	ave($\hat{\theta}$)	0.20	0.34	0.47	0.64	0.83	0.95	1.00	0.07	0.17	0.37	0.53	0.75	0.93	0.99	0.02	0.10	0.32	0.51	0.71	0.92	1.00
30	sd($\hat{\theta}$)	0.14	0.19	0.15	0.14	0.13	0.08	0.02	0.07	0.12	0.13	0.12	0.11	0.08	0.04	0.04	0.06	0.08	0.07	0.09	0.06	0.02
	FDR	0.00	0.04	0.09	0.14	0.14	0.19	0.05	0.00	0.03	0.08	0.15	0.16	0.17	0.06	0.00	0.03	0.06	0.08	0.12	0.11	0.04
	FNR	0.89	0.73	0.46	0.30	0.20	0.08	0.00	0.64	0.48	0.31	0.19	0.13	0.06	0.00	0.20	0.19	0.16	0.10	0.07	0.04	0.00
	ave($\hat{\theta}$)	0.18	0.28	0.45	0.61	0.80	0.96	1.00	0.05	0.14	0.35	0.54	0.72	0.92	0.99	0.01	0.11	0.30	0.49	0.69	0.91	0.99
50	sd($\hat{\theta}$)	0.14	0.13	0.11	0.11	0.11	0.06	0.01	0.05	0.09	0.10	0.11	0.09	0.07	0.03	0.03	0.06	0.05	0.08	0.06	0.04	0.02
	FDR	0.00	0.04	0.10	0.15	0.19	0.19	0.02	0.00	0.04	0.09	0.14	0.20	0.21	0.04	0.00	0.03	0.06	0.10	0.14	0.18	0.06
	FNR	0.87	0.70	0.45	0.29	0.19	0.08	0.00	0.47	0.44	0.30	0.19	0.11	0.05	0.00	0.09	0.24	0.13	0.09	0.05	0.03	0.00

Table B.105: BSQT2 test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.22	0.43	0.56	0.71	0.82	0.96	0.98	0.10	0.24	0.42	0.62	0.78	0.96	0.98	0.03	0.18	0.37	0.56	0.76	0.97	0.98
10	sd($\hat{\theta}$)	0.24	0.24	0.19	0.15	0.14	0.11	0.10	0.15	0.14	0.17	0.14	0.12	0.13	0.08	0.09	0.09	0.10	0.14	0.10	0.07	0.07
	FDR	0.00	0.01	0.03	0.03	0.06	0.07	0.04	0.00	0.02	0.03	0.03	0.05	0.09	0.05	0.00	0.02	0.02	0.05	0.05	0.07	0.06
	FNR	0.57	0.67	0.43	0.29	0.16	0.09	0.00	0.37	0.53	0.28	0.20	0.13	0.08	0.00	0.15	0.43	0.22	0.16	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.19	0.34	0.47	0.69	0.82	0.97	0.98	0.07	0.20	0.41	0.59	0.78	0.97	0.99	0.02	0.18	0.38	0.57	0.78	0.97	0.98
30	sd($\hat{\theta}$)	0.21	0.18	0.16	0.15	0.11	0.11	0.10	0.11	0.10	0.11	0.08	0.08	0.08	0.07	0.05	0.07	0.07	0.08	0.09	0.09	0.11
	FDR	0.00	0.02	0.04	0.04	0.06	0.07	0.05	0.00	0.02	0.03	0.02	0.04	0.07	0.03	0.00	0.01	0.02	0.03	0.03	0.06	0.04
	FNR	0.55	0.64	0.38	0.28	0.17	0.09	0.00	0.30	0.48	0.29	0.17	0.12	0.09	0.00	0.08	0.45	0.23	0.15	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.18	0.29	0.51	0.66	0.83	0.98	0.99	0.05	0.19	0.38	0.58	0.78	0.96	0.97	0.01	0.17	0.36	0.56	0.76	0.98	1.00
50	sd($\hat{\theta}$)	0.19	0.16	0.14	0.15	0.13	0.08	0.07	0.10	0.08	0.12	0.11	0.07	0.13	0.13	0.04	0.07	0.09	0.10	0.11	0.10	0.04
	FDR	0.00	0.02	0.03	0.07	0.07	0.05	0.04	0.00	0.02	0.04	0.05	0.04	0.09	0.07	0.00	0.01	0.03	0.04	0.05	0.05	0.01
	FNR	0.55	0.61	0.42	0.28	0.18	0.09	0.00	0.20	0.48	0.26	0.18	0.12	0.09	0.00	0.04	0.44	0.22	0.15	0.11	0.09	0.00

Table B.106: BSQT2 test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.44	0.59	0.72	0.84	0.91	0.97	0.99	0.20	0.32	0.56	0.70	0.85	0.95	0.99	0.10	0.19	0.39	0.59	0.76	0.92	1.00
10	sd($\hat{\theta}$)	0.32	0.31	0.23	0.15	0.12	0.08	0.06	0.27	0.24	0.27	0.18	0.13	0.09	0.05	0.17	0.16	0.15	0.17	0.16	0.09	0.01
	FDR	0.00	0.02	0.02	0.03	0.05	0.03	0.07	0.00	0.02	0.05	0.04	0.04	0.04	0.07	0.00	0.01	0.02	0.05	0.05	0.05	0.02
	FNR	0.86	0.74	0.55	0.40	0.24	0.08	0.00	0.65	0.57	0.42	0.27	0.18	0.06	0.00	0.49	0.26	0.21	0.17	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.32	0.44	0.57	0.80	0.88	0.95	0.99	0.20	0.25	0.46	0.66	0.79	0.93	0.98	0.01	0.17	0.37	0.55	0.77	0.96	0.98
30	sd($\hat{\theta}$)	0.32	0.29	0.28	0.18	0.16	0.11	0.08	0.27	0.19	0.21	0.18	0.16	0.14	0.10	0.03	0.07	0.08	0.11	0.07	0.11	0.08
	FDR	0.00	0.02	0.08	0.06	0.09	0.10	0.03	0.00	0.03	0.06	0.05	0.10	0.09	0.05	0.00	0.02	0.02	0.04	0.04	0.09	0.05
	FNR	0.78	0.69	0.49	0.39	0.24	0.08	0.00	0.59	0.50	0.37	0.26	0.15	0.06	0.00	0.03	0.43	0.22	0.14	0.11	0.09	0.00
	ave($\hat{\theta}$)	0.37	0.43	0.59	0.72	0.84	0.96	0.96	0.13	0.24	0.40	0.63	0.80	0.95	0.98	0.04	0.14	0.35	0.59	0.74	0.91	1.00
50	sd($\hat{\theta}$)	0.34	0.28	0.23	0.18	0.18	0.10	0.13	0.20	0.20	0.19	0.19	0.17	0.10	0.11	0.11	0.11	0.13	0.15	0.18	0.14	0.00
	FDR	0.00	0.03	0.06	0.11	0.14	0.11	0.08	0.00	0.02	0.08	0.09	0.12	0.08	0.04	0.00	0.02	0.05	0.05	0.11	0.10	0.00
	FNR	0.80	0.73	0.51	0.36	0.23	0.09	0.00	0.55	0.51	0.32	0.26	0.17	0.07	0.00	0.19	0.29	0.19	0.17	0.11	0.04	0.00

Table B.107: BSQT2 test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.37	0.48	0.62	0.78	0.89	0.98	0.99	0.12	0.27	0.46	0.63	0.80	0.97	0.98	0.06	0.19	0.40	0.59	0.77	0.97	1.00
10	sd($\hat{\theta}$)	0.29	0.26	0.21	0.16	0.12	0.08	0.09	0.17	0.16	0.16	0.11	0.13	0.13	0.13	0.10	0.09	0.11	0.10	0.12	0.11	0.04
	FDR	0.00	0.01	0.02	0.03	0.03	0.04	0.03	0.00	0.01	0.02	0.02	0.03	0.05	0.04	0.00	0.01	0.02	0.02	0.04	0.06	0.01
	FNR	0.78	0.70	0.49	0.36	0.21	0.09	0.00	0.37	0.56	0.33	0.20	0.14	0.09	0.00	0.28	0.46	0.26	0.16	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.29	0.47	0.59	0.76	0.86	0.97	0.97	0.09	0.24	0.44	0.62	0.82	0.97	0.98	0.02	0.18	0.38	0.59	0.78	0.97	0.99
30	sd($\hat{\theta}$)	0.27	0.25	0.21	0.15	0.14	0.14	0.13	0.13	0.14	0.14	0.15	0.09	0.11	0.11	0.05	0.08	0.06	0.09	0.10	0.14	0.06
	FDR	0.00	0.01	0.04	0.03	0.06	0.06	0.06	0.00	0.01	0.03	0.03	0.02	0.05	0.05	0.00	0.02	0.01	0.02	0.03	0.04	0.01
	FNR	0.63	0.71	0.47	0.34	0.20	0.09	0.00	0.35	0.53	0.31	0.21	0.15	0.09	0.00	0.08	0.43	0.24	0.16	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.30	0.40	0.60	0.75	0.87	0.99	0.99	0.06	0.25	0.43	0.61	0.79	0.96	0.99	0.02	0.18	0.38	0.57	0.77	0.98	0.97
50	sd($\hat{\theta}$)	0.26	0.21	0.21	0.16	0.13	0.07	0.09	0.12	0.14	0.12	0.14	0.09	0.16	0.06	0.05	0.06	0.07	0.10	0.13	0.11	0.11
	FDR	0.00	0.01	0.03	0.04	0.06	0.02	0.02	0.00	0.02	0.02	0.04	0.03	0.05	0.01	0.00	0.01	0.02	0.03	0.04	0.03	0.07
	FNR	0.65	0.68	0.48	0.34	0.21	0.10	0.00	0.24	0.54	0.31	0.20	0.12	0.09	0.00	0.08	0.45	0.23	0.16	0.12	0.10	0.00

Table B.108: BSQT2 test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.37	0.51	0.64	0.79	0.89	0.98	0.98	0.12	0.27	0.43	0.64	0.81	0.97	0.98	0.04	0.19	0.38	0.57	0.76	0.97	0.97
10	sd($\hat{\theta}$)	0.27	0.26	0.24	0.20	0.13	0.09	0.08	0.17	0.17	0.14	0.12	0.11	0.11	0.11	0.09	0.08	0.11	0.10	0.12	0.10	0.13
	FDR	0.00	0.01	0.03	0.03	0.06	0.03	0.07	0.00	0.01	0.02	0.02	0.04	0.07	0.03	0.00	0.01	0.02	0.03	0.05	0.06	0.06
	FNR	0.77	0.74	0.50	0.36	0.23	0.09	0.00	0.40	0.53	0.29	0.22	0.15	0.09	0.00	0.19	0.45	0.24	0.15	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.33	0.42	0.61	0.74	0.87	0.97	0.98	0.07	0.25	0.45	0.62	0.79	0.96	0.99	0.02	0.17	0.36	0.59	0.78	0.97	0.98
30	sd($\hat{\theta}$)	0.30	0.25	0.21	0.18	0.14	0.11	0.14	0.13	0.13	0.15	0.14	0.09	0.15	0.06	0.05	0.08	0.11	0.07	0.09	0.13	0.10
	FDR	0.00	0.02	0.04	0.04	0.06	0.05	0.03	0.00	0.01	0.02	0.04	0.03	0.06	0.02	0.00	0.02	0.03	0.02	0.03	0.05	0.04
	FNR	0.66	0.68	0.49	0.33	0.21	0.10	0.00	0.28	0.54	0.33	0.22	0.13	0.09	0.00	0.08	0.41	0.23	0.16	0.12	0.09	0.00
	ave($\hat{\theta}$)	0.33	0.39	0.57	0.74	0.89	0.97	0.99	0.08	0.22	0.42	0.63	0.78	0.98	0.99	0.01	0.17	0.36	0.57	0.78	0.98	1.00
50	sd($\hat{\theta}$)	0.29	0.25	0.20	0.18	0.14	0.13	0.08	0.13	0.13	0.13	0.11	0.12	0.10	0.07	0.06	0.08	0.11	0.11	0.10	0.10	0.02
	FDR	0.00	0.02	0.04	0.05	0.04	0.06	0.03	0.00	0.02	0.03	0.02	0.05	0.04	0.03	0.00	0.02	0.04	0.03	0.03	0.04	0.01
	FNR	0.65	0.65	0.47	0.34	0.22	0.10	0.00	0.32	0.49	0.30	0.21	0.13	0.10	0.00	0.05	0.42	0.22	0.16	0.12	0.09	0.00

Table B.109: IBSQT test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.72	0.73	0.79	0.86	0.92	0.97	0.99	0.33	0.50	0.64	0.77	0.84	0.94	0.99	0.18	0.28	0.47	0.65	0.80	0.93	1.00
	sd($\hat{\theta}$)	0.24	0.17	0.15	0.11	0.08	0.05	0.03	0.19	0.17	0.13	0.12	0.08	0.06	0.02	0.14	0.12	0.12	0.10	0.08	0.06	0.02
	FDR	0.00	0.01	0.01	0.03	0.06	0.08	0.09	0.00	0.00	0.01	0.01	0.02	0.05	0.06	0.00	0.00	0.01	0.01	0.00	0.02	0.02
	FNR	0.99	0.85	0.61	0.42	0.24	0.07	0.00	0.96	0.78	0.52	0.34	0.17	0.05	0.00	0.80	0.56	0.34	0.22	0.11	0.03	0.00
30	ave($\hat{\theta}$)	0.51	0.60	0.77	0.87	0.93	0.99	1.00	0.27	0.41	0.59	0.75	0.88	0.95	1.00	0.12	0.27	0.48	0.65	0.80	0.94	1.00
	sd($\hat{\theta}$)	0.21	0.14	0.10	0.07	0.06	0.02	0.01	0.17	0.10	0.10	0.07	0.05	0.04	0.01	0.06	0.06	0.07	0.07	0.05	0.03	0.01
	FDR	0.00	0.01	0.02	0.03	0.06	0.05	0.10	0.00	0.01	0.02	0.02	0.01	0.05	0.06	0.00	0.00	0.01	0.02	0.02	0.01	0.04
	FNR	1.00	0.84	0.61	0.43	0.25	0.09	0.00	1.00	0.76	0.50	0.34	0.20	0.06	0.00	1.00	0.62	0.37	0.23	0.13	0.04	0.00
50	ave($\hat{\theta}$)	0.50	0.58	0.74	0.84	0.93	0.99	1.00	0.25	0.36	0.58	0.75	0.87	0.95	1.00	0.11	0.23	0.45	0.64	0.80	0.94	1.00
	sd($\hat{\theta}$)	0.21	0.12	0.07	0.08	0.04	0.02	0.03	0.16	0.10	0.09	0.05	0.04	0.05	0.01	0.03	0.06	0.06	0.05	0.04	0.02	0.02
	FDR	0.00	0.01	0.02	0.04	0.05	0.02	0.04	0.00	0.01	0.02	0.02	0.01	0.04	0.02	0.00	0.01	0.02	0.02	0.02	0.03	0.04
	FNR	1.00	0.83	0.60	0.41	0.25	0.09	0.00	1.00	0.74	0.49	0.34	0.20	0.07	0.00	0.99	0.58	0.35	0.23	0.13	0.05	0.00

Table B.110: IBSQT test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.47	0.65	0.75	0.85	0.93	0.99	1.00	0.04	0.28	0.50	0.66	0.81	0.99	1.00	0.00	0.14	0.36	0.55	0.73	0.98	0.99
10	sd($\hat{\theta}$)	0.37	0.17	0.13	0.11	0.08	0.02	0.02	0.11	0.17	0.14	0.11	0.10	0.03	0.01	0.03	0.10	0.09	0.07	0.07	0.04	0.06
	FDR	0.00	0.00	0.00	0.03	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.01	0.00	0.02	0.01	0.01	0.01	0.02	0.02
	FNR	0.69	0.82	0.59	0.41	0.24	0.09	0.00	0.19	0.51	0.37	0.24	0.13	0.09	0.00	0.03	0.24	0.16	0.09	0.05	0.09	0.00
	ave($\hat{\theta}$)	0.37	0.42	0.63	0.83	0.93	0.99	1.00	0.07	0.28	0.49	0.67	0.82	0.95	1.00	0.00	0.18	0.39	0.57	0.75	0.92	1.00
30	sd($\hat{\theta}$)	0.25	0.13	0.11	0.08	0.05	0.02	0.00	0.08	0.07	0.07	0.06	0.05	0.05	0.00	0.02	0.05	0.05	0.04	0.04	0.03	0.00
	FDR	0.00	0.01	0.02	0.02	0.03	0.01	0.00	0.00	0.00	0.01	0.00	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.01	0.02	0.00
	FNR	0.99	0.77	0.54	0.40	0.25	0.09	0.00	0.41	0.63	0.38	0.25	0.14	0.06	0.00	0.07	0.40	0.22	0.11	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.28	0.39	0.62	0.80	0.92	0.99	1.00	0.08	0.23	0.47	0.66	0.82	0.94	1.00	0.04	0.18	0.39	0.57	0.75	0.92	1.00
50	sd($\hat{\theta}$)	0.18	0.11	0.09	0.08	0.04	0.02	0.00	0.03	0.05	0.06	0.05	0.03	0.04	0.00	0.03	0.04	0.04	0.03	0.04	0.02	0.01
	FDR	0.00	0.01	0.02	0.02	0.02	0.02	0.01	0.00	0.01	0.01	0.01	0.01	0.03	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.03
	FNR	1.00	0.75	0.53	0.38	0.24	0.09	0.00	0.99	0.57	0.37	0.24	0.15	0.05	0.00	0.69	0.43	0.22	0.13	0.07	0.02	0.00

Table B.111: IBSQT test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.65	0.58	0.74	0.86	0.93	1.00	1.00	0.04	0.26	0.50	0.65	0.81	0.99	0.99	0.00	0.13	0.35	0.55	0.72	0.99	1.00
	sd($\hat{\theta}$)	0.35	0.21	0.16	0.12	0.08	0.03	0.00	0.12	0.17	0.12	0.13	0.11	0.05	0.05	0.02	0.12	0.09	0.08	0.05	0.03	0.03
	FDR	0.00	0.00	0.02	0.02	0.00	0.02	0.00	0.00	0.02	0.00	0.01	0.01	0.02	0.05	0.00	0.03	0.01	0.01	0.01	0.01	0.03
	FNR	0.82	0.79	0.59	0.41	0.24	0.10	0.00	0.14	0.50	0.37	0.22	0.13	0.09	0.00	0.04	0.24	0.15	0.09	0.04	0.09	0.00
30	ave($\hat{\theta}$)	0.43	0.43	0.69	0.84	0.92	0.98	1.00	0.07	0.27	0.51	0.67	0.82	0.95	0.99	0.01	0.18	0.38	0.57	0.73	0.91	1.00
	sd($\hat{\theta}$)	0.33	0.15	0.10	0.08	0.08	0.03	0.00	0.08	0.06	0.07	0.07	0.07	0.05	0.04	0.02	0.05	0.06	0.05	0.07	0.03	0.02
	FDR	0.00	0.01	0.02	0.02	0.02	0.03	0.02	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.00	0.00	0.00	0.01	0.01	0.02	0.03
	FNR	0.98	0.76	0.57	0.40	0.24	0.09	0.00	0.45	0.63	0.41	0.25	0.15	0.05	0.00	0.09	0.41	0.21	0.12	0.05	0.02	0.00
50	ave($\hat{\theta}$)	0.43	0.39	0.65	0.81	0.92	0.98	1.00	0.09	0.24	0.47	0.66	0.82	0.95	1.00	0.03	0.18	0.39	0.57	0.75	0.91	1.00
	sd($\hat{\theta}$)	0.33	0.10	0.09	0.08	0.08	0.03	0.02	0.07	0.05	0.07	0.06	0.07	0.02	0.04	0.03	0.03	0.04	0.06	0.04	0.03	0.02
	FDR	0.00	0.01	0.02	0.04	0.03	0.03	0.05	0.00	0.01	0.01	0.01	0.02	0.01	0.02	0.00	0.00	0.00	0.01	0.01	0.01	0.02
	FNR	1.00	0.75	0.54	0.39	0.25	0.09	0.00	0.98	0.60	0.37	0.25	0.15	0.05	0.00	0.64	0.44	0.23	0.13	0.06	0.02	0.00

Table B.112: IBSQT test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.57	0.60	0.75	0.85	0.92	1.00	1.00	0.04	0.28	0.49	0.65	0.81	0.99	1.00	0.00	0.13	0.36	0.55	0.72	0.98	0.99
10	sd($\hat{\theta}$)	0.40	0.21	0.17	0.11	0.09	0.02	0.02	0.14	0.18	0.13	0.10	0.09	0.05	0.00	0.01	0.11	0.10	0.08	0.08	0.04	0.06
	FDR	0.00	0.01	0.01	0.02	0.02	0.02	0.02	0.00	0.01	0.01	0.00	0.01	0.02	0.00	0.00	0.02	0.01	0.01	0.01	0.03	0.02
	FNR	0.72	0.79	0.59	0.41	0.24	0.10	0.00	0.10	0.53	0.36	0.22	0.13	0.09	0.00	0.01	0.24	0.16	0.10	0.04	0.09	0.00
	ave($\hat{\theta}$)	0.53	0.45	0.66	0.85	0.94	0.99	1.00	0.04	0.27	0.49	0.68	0.83	0.95	1.00	0.00	0.18	0.39	0.56	0.75	0.92	1.00
30	sd($\hat{\theta}$)	0.37	0.14	0.10	0.06	0.06	0.03	0.01	0.07	0.08	0.08	0.06	0.05	0.03	0.02	0.01	0.05	0.06	0.05	0.03	0.03	0.00
	FDR	0.00	0.01	0.01	0.01	0.04	0.01	0.02	0.00	0.01	0.01	0.01	0.00	0.01	0.02	0.00	0.00	0.00	0.01	0.01	0.00	0.02
	FNR	0.96	0.78	0.54	0.41	0.26	0.09	0.00	0.29	0.62	0.39	0.26	0.15	0.05	0.00	0.06	0.40	0.21	0.11	0.07	0.02	0.00
	ave($\hat{\theta}$)	0.42	0.39	0.67	0.81	0.92	0.99	0.99	0.08	0.24	0.49	0.67	0.82	0.94	0.99	0.01	0.18	0.39	0.58	0.75	0.92	0.99
50	sd($\hat{\theta}$)	0.35	0.11	0.09	0.05	0.06	0.02	0.03	0.07	0.05	0.06	0.06	0.06	0.02	0.06	0.02	0.03	0.05	0.04	0.04	0.02	0.08
	FDR	0.00	0.02	0.02	0.03	0.05	0.03	0.07	0.00	0.01	0.01	0.01	0.01	0.00	0.06	0.00	0.00	0.01	0.00	0.01	0.01	0.05
	FNR	1.00	0.77	0.55	0.39	0.25	0.09	0.00	0.92	0.59	0.40	0.26	0.15	0.05	0.00	0.37	0.44	0.24	0.14	0.07	0.02	0.00

Table B.113: IBSQT test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.70	0.71	0.77	0.85	0.91	0.93	0.96	0.38	0.48	0.60	0.69	0.82	0.87	0.92	0.20	0.28	0.44	0.60	0.75	0.86	0.94
10	sd($\hat{\theta}$)	0.22	0.17	0.17	0.12	0.11	0.11	0.07	0.21	0.18	0.15	0.15	0.14	0.16	0.15	0.18	0.13	0.16	0.15	0.12	0.14	0.09
	FDR	0.00	0.02	0.06	0.10	0.13	0.22	0.31	0.00	0.02	0.03	0.10	0.17	0.25	0.36	0.00	0.01	0.06	0.11	0.12	0.28	0.39
	FNR	1.00	0.86	0.62	0.43	0.25	0.08	0.00	0.96	0.78	0.50	0.32	0.20	0.04	0.00	0.71	0.58	0.35	0.24	0.12	0.03	0.00
	ave($\hat{\theta}$)	0.53	0.59	0.73	0.82	0.88	0.93	0.96	0.27	0.39	0.55	0.71	0.78	0.87	0.92	0.12	0.25	0.40	0.60	0.68	0.85	0.89
30	sd($\hat{\theta}$)	0.21	0.16	0.12	0.15	0.11	0.13	0.08	0.13	0.12	0.16	0.15	0.17	0.19	0.18	0.06	0.07	0.14	0.15	0.22	0.19	0.23
	FDR	0.00	0.03	0.08	0.11	0.23	0.36	0.46	0.00	0.02	0.08	0.10	0.21	0.37	0.59	0.00	0.02	0.07	0.10	0.18	0.33	0.60
	FNR	1.00	0.85	0.62	0.43	0.26	0.09	0.00	1.00	0.76	0.55	0.36	0.21	0.06	0.00	0.98	0.64	0.38	0.26	0.14	0.04	0.00
	ave($\hat{\theta}$)	0.41	0.54	0.71	0.79	0.87	0.91	0.93	0.23	0.35	0.48	0.65	0.75	0.84	0.90	0.11	0.22	0.39	0.53	0.71	0.82	0.89
50	sd($\hat{\theta}$)	0.18	0.13	0.14	0.17	0.15	0.17	0.16	0.16	0.12	0.17	0.20	0.21	0.22	0.22	0.05	0.08	0.15	0.21	0.20	0.24	0.25
	FDR	0.00	0.03	0.08	0.16	0.23	0.37	0.46	0.00	0.03	0.10	0.14	0.23	0.46	0.59	0.00	0.02	0.07	0.14	0.17	0.38	0.61
	FNR	1.00	0.84	0.62	0.44	0.26	0.09	0.00	1.00	0.76	0.54	0.37	0.21	0.06	0.00	1.00	0.61	0.39	0.25	0.15	0.05	0.00

Table B.114: IBSQT test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.59	0.59	0.73	0.83	0.91	0.98	1.00	0.05	0.28	0.45	0.62	0.77	0.95	0.97	0.00	0.12	0.32	0.53	0.69	0.93	0.97
10	sd($\hat{\theta}$)	0.37	0.20	0.15	0.11	0.09	0.05	0.02	0.15	0.15	0.15	0.11	0.15	0.09	0.12	0.02	0.10	0.14	0.11	0.10	0.11	0.09
	FDR	0.00	0.00	0.00	0.04	0.07	0.05	0.02	0.00	0.02	0.03	0.04	0.07	0.12	0.11	0.00	0.02	0.05	0.03	0.07	0.14	0.11
	FNR	0.77	0.79	0.58	0.40	0.24	0.09	0.00	0.15	0.58	0.34	0.21	0.12	0.08	0.00	0.04	0.20	0.16	0.09	0.03	0.07	0.00
	ave($\hat{\theta}$)	0.33	0.42	0.66	0.81	0.91	0.99	0.99	0.08	0.25	0.46	0.65	0.79	0.93	0.98	0.01	0.17	0.35	0.54	0.72	0.90	0.98
30	sd($\hat{\theta}$)	0.23	0.13	0.11	0.11	0.09	0.03	0.03	0.09	0.08	0.11	0.11	0.11	0.04	0.04	0.04	0.05	0.08	0.09	0.09	0.04	0.07
	FDR	0.00	0.02	0.03	0.04	0.10	0.06	0.07	0.00	0.01	0.03	0.05	0.08	0.14	0.23	0.00	0.01	0.04	0.04	0.06	0.16	0.22
	FNR	1.00	0.78	0.56	0.40	0.25	0.09	0.00	0.49	0.61	0.38	0.26	0.15	0.05	0.00	0.14	0.41	0.20	0.11	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.29	0.37	0.59	0.75	0.87	0.96	1.00	0.09	0.23	0.43	0.65	0.81	0.89	0.97	0.04	0.17	0.35	0.54	0.72	0.90	0.97
50	sd($\hat{\theta}$)	0.19	0.12	0.13	0.16	0.17	0.10	0.01	0.04	0.05	0.11	0.09	0.08	0.16	0.09	0.03	0.04	0.09	0.10	0.09	0.04	0.05
	FDR	0.00	0.02	0.04	0.09	0.12	0.12	0.08	0.00	0.01	0.04	0.04	0.07	0.15	0.36	0.00	0.01	0.04	0.05	0.07	0.15	0.56
	FNR	1.00	0.76	0.52	0.38	0.25	0.09	0.00	0.99	0.59	0.36	0.26	0.15	0.05	0.00	0.76	0.44	0.20	0.13	0.07	0.02	0.00

Table B.115: IBSQT test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.60	0.73	0.81	0.90	0.97	0.97	0.06	0.23	0.47	0.61	0.77	0.92	0.96	0.00	0.13	0.31	0.48	0.64	0.87	0.90
10	sd($\hat{\theta}$)	0.38	0.23	0.17	0.15	0.12	0.11	0.13	0.13	0.18	0.16	0.16	0.14	0.11	0.09	0.02	0.12	0.13	0.14	0.18	0.19	0.23
	FDR	0.00	0.01	0.04	0.07	0.14	0.06	0.12	0.00	0.03	0.05	0.08	0.11	0.22	0.18	0.00	0.04	0.06	0.09	0.18	0.27	0.29
	FNR	0.78	0.79	0.60	0.40	0.25	0.09	0.00	0.22	0.46	0.39	0.23	0.14	0.07	0.00	0.03	0.27	0.16	0.08	0.04	0.05	0.00
	ave($\hat{\theta}$)	0.42	0.42	0.67	0.79	0.90	0.96	0.96	0.06	0.26	0.45	0.62	0.74	0.86	0.94	0.01	0.17	0.35	0.51	0.67	0.82	0.88
30	sd($\hat{\theta}$)	0.33	0.15	0.15	0.17	0.11	0.11	0.14	0.08	0.09	0.14	0.13	0.18	0.15	0.12	0.03	0.06	0.09	0.11	0.10	0.16	0.22
	FDR	0.00	0.03	0.06	0.10	0.16	0.22	0.24	0.00	0.02	0.06	0.10	0.19	0.37	0.56	0.00	0.02	0.05	0.10	0.15	0.39	0.70
	FNR	0.96	0.80	0.58	0.42	0.26	0.09	0.00	0.42	0.64	0.42	0.27	0.15	0.05	0.00	0.11	0.44	0.22	0.12	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.38	0.36	0.60	0.76	0.90	0.94	0.97	0.09	0.22	0.44	0.61	0.75	0.87	0.91	0.03	0.17	0.32	0.51	0.65	0.84	0.88
50	sd($\hat{\theta}$)	0.31	0.11	0.18	0.16	0.06	0.15	0.08	0.04	0.06	0.11	0.13	0.15	0.15	0.19	0.03	0.05	0.10	0.12	0.17	0.09	0.19
	FDR	0.00	0.03	0.08	0.11	0.15	0.30	0.43	0.00	0.02	0.07	0.11	0.18	0.40	0.82	0.00	0.02	0.07	0.11	0.20	0.39	0.92
	FNR	1.00	0.77	0.57	0.41	0.25	0.09	0.00	0.99	0.62	0.41	0.28	0.17	0.05	0.00	0.68	0.50	0.22	0.14	0.08	0.02	0.00

Table B.116: IBSQT test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.59	0.59	0.73	0.84	0.90	0.99	1.00	0.03	0.28	0.49	0.63	0.78	0.98	0.99	0.00	0.12	0.35	0.55	0.71	0.95	0.99
10	sd($\hat{\theta}$)	0.41	0.19	0.17	0.11	0.10	0.03	0.00	0.12	0.17	0.14	0.12	0.10	0.05	0.06	0.00	0.11	0.12	0.08	0.08	0.12	0.10
	FDR	0.00	0.01	0.02	0.01	0.03	0.03	0.00	0.00	0.01	0.02	0.03	0.04	0.03	0.05	0.00	0.03	0.02	0.01	0.04	0.09	0.02
	FNR	0.73	0.81	0.59	0.40	0.22	0.10	0.00	0.12	0.52	0.38	0.21	0.11	0.09	0.00	0.00	0.22	0.16	0.09	0.04	0.08	0.00
	ave($\hat{\theta}$)	0.47	0.44	0.67	0.82	0.93	0.98	1.00	0.03	0.26	0.49	0.66	0.80	0.93	0.99	0.00	0.18	0.38	0.56	0.74	0.90	0.99
30	sd($\hat{\theta}$)	0.34	0.14	0.16	0.10	0.10	0.03	0.01	0.09	0.07	0.10	0.11	0.10	0.08	0.08	0.01	0.05	0.08	0.06	0.05	0.08	0.05
	FDR	0.00	0.01	0.03	0.05	0.05	0.03	0.03	0.00	0.01	0.01	0.03	0.06	0.08	0.06	0.00	0.01	0.02	0.02	0.02	0.06	0.11
	FNR	0.97	0.77	0.58	0.40	0.25	0.09	0.00	0.22	0.61	0.38	0.26	0.15	0.05	0.00	0.04	0.42	0.22	0.12	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.45	0.37	0.63	0.81	0.92	0.98	1.00	0.07	0.24	0.47	0.65	0.82	0.94	0.98	0.01	0.18	0.37	0.57	0.74	0.91	0.98
50	sd($\hat{\theta}$)	0.34	0.12	0.14	0.12	0.11	0.04	0.02	0.07	0.05	0.10	0.07	0.08	0.02	0.08	0.02	0.04	0.06	0.05	0.09	0.04	0.10
	FDR	0.00	0.01	0.04	0.05	0.06	0.03	0.09	0.00	0.01	0.02	0.03	0.03	0.05	0.20	0.00	0.00	0.01	0.01	0.05	0.06	0.27
	FNR	1.00	0.74	0.55	0.40	0.25	0.09	0.00	0.94	0.59	0.38	0.26	0.15	0.05	0.00	0.35	0.44	0.21	0.13	0.07	0.02	0.00

Table B.117: IBSQT test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.69	0.74	0.83	0.86	0.94	0.97	0.99	0.37	0.43	0.58	0.71	0.84	0.94	0.99	0.18	0.27	0.45	0.62	0.78	0.92	0.99
10	sd($\hat{\theta}$)	0.26	0.20	0.16	0.13	0.08	0.05	0.03	0.20	0.18	0.15	0.12	0.10	0.06	0.03	0.15	0.16	0.12	0.12	0.08	0.06	0.03
	FDR	0.00	0.00	0.03	0.03	0.04	0.06	0.09	0.00	0.01	0.01	0.03	0.05	0.05	0.10	0.00	0.00	0.01	0.03	0.04	0.06	0.07
	FNR	0.98	0.85	0.64	0.41	0.26	0.08	0.00	0.94	0.71	0.46	0.29	0.17	0.05	0.00	0.72	0.48	0.30	0.20	0.10	0.03	0.00
	ave($\hat{\theta}$)	0.51	0.60	0.76	0.86	0.93	0.99	0.99	0.32	0.40	0.56	0.73	0.86	0.95	1.00	0.11	0.23	0.45	0.62	0.79	0.93	1.00
30	sd($\hat{\theta}$)	0.23	0.19	0.15	0.09	0.06	0.02	0.01	0.18	0.15	0.13	0.09	0.07	0.04	0.01	0.05	0.08	0.07	0.07	0.06	0.03	0.01
	FDR	0.00	0.01	0.02	0.04	0.05	0.02	0.15	0.00	0.01	0.02	0.02	0.04	0.05	0.08	0.00	0.01	0.02	0.03	0.02	0.05	0.07
	FNR	1.00	0.83	0.60	0.42	0.25	0.09	0.00	0.99	0.74	0.46	0.31	0.19	0.06	0.00	0.98	0.54	0.35	0.21	0.12	0.04	0.00
	ave($\hat{\theta}$)	0.45	0.60	0.73	0.85	0.93	0.99	1.00	0.21	0.33	0.55	0.71	0.84	0.96	1.00	0.11	0.22	0.43	0.61	0.78	0.93	1.00
50	sd($\hat{\theta}$)	0.22	0.21	0.13	0.10	0.06	0.02	0.00	0.12	0.13	0.10	0.07	0.05	0.03	0.01	0.06	0.07	0.08	0.06	0.05	0.02	0.01
	FDR	0.00	0.01	0.04	0.04	0.09	0.08	0.03	0.00	0.01	0.02	0.03	0.04	0.08	0.03	0.00	0.01	0.02	0.02	0.03	0.05	0.09
	FNR	1.00	0.82	0.60	0.42	0.26	0.09	0.00	1.00	0.68	0.45	0.30	0.18	0.07	0.00	0.98	0.56	0.32	0.19	0.12	0.04	0.00

Table B.118: IBSQT test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.60	0.69	0.82	0.90	0.95	0.99	1.00	0.05	0.18	0.45	0.60	0.78	0.94	0.99	0.01	0.09	0.29	0.49	0.69	0.93	0.98
10	sd($\hat{\theta}$)	0.36	0.25	0.17	0.12	0.08	0.02	0.02	0.12	0.15	0.12	0.11	0.10	0.08	0.02	0.03	0.09	0.10	0.08	0.07	0.09	0.06
	FDR	0.00	0.01	0.02	0.03	0.06	0.02	0.02	0.00	0.03	0.02	0.03	0.06	0.10	0.05	0.00	0.03	0.04	0.04	0.05	0.12	0.16
	FNR	0.80	0.82	0.63	0.44	0.27	0.10	0.00	0.21	0.35	0.33	0.19	0.12	0.06	0.00	0.04	0.12	0.07	0.04	0.03	0.05	0.00
	ave($\hat{\theta}$)	0.32	0.45	0.66	0.80	0.93	0.99	1.00	0.08	0.22	0.42	0.60	0.78	0.94	0.99	0.01	0.13	0.33	0.52	0.71	0.89	0.98
30	sd($\hat{\theta}$)	0.21	0.24	0.18	0.12	0.08	0.01	0.00	0.08	0.06	0.07	0.06	0.06	0.05	0.03	0.03	0.05	0.05	0.04	0.04	0.04	0.04
	FDR	0.00	0.02	0.05	0.07	0.06	0.03	0.01	0.00	0.01	0.02	0.03	0.07	0.10	0.09	0.00	0.01	0.01	0.03	0.05	0.13	0.22
	FNR	0.98	0.76	0.55	0.39	0.25	0.10	0.00	0.49	0.56	0.30	0.19	0.13	0.05	0.00	0.18	0.25	0.12	0.06	0.04	0.01	0.00
	ave($\hat{\theta}$)	0.30	0.40	0.61	0.80	0.91	0.99	1.00	0.08	0.19	0.39	0.59	0.77	0.93	1.00	0.03	0.14	0.33	0.52	0.71	0.89	0.99
50	sd($\hat{\theta}$)	0.19	0.22	0.17	0.11	0.08	0.03	0.01	0.03	0.05	0.05	0.06	0.06	0.03	0.02	0.03	0.03	0.04	0.03	0.03	0.03	0.02
	FDR	0.00	0.02	0.04	0.07	0.07	0.07	0.05	0.00	0.01	0.03	0.04	0.08	0.08	0.08	0.00	0.01	0.02	0.03	0.06	0.12	0.36
	FNR	1.00	0.73	0.51	0.38	0.24	0.10	0.00	0.99	0.51	0.27	0.18	0.11	0.05	0.00	0.69	0.31	0.12	0.08	0.04	0.01	0.00

Table B.119: IBSQT test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.55	0.62	0.77	0.87	0.93	0.98	0.99	0.05	0.17	0.36	0.57	0.73	0.89	0.95	0.00	0.07	0.28	0.45	0.65	0.82	0.91
	sd($\hat{\theta}$)	0.38	0.33	0.25	0.14	0.11	0.05	0.05	0.15	0.17	0.18	0.13	0.12	0.12	0.09	0.02	0.08	0.10	0.12	0.10	0.12	0.11
	FDR	0.00	0.02	0.03	0.07	0.05	0.09	0.08	0.00	0.03	0.07	0.06	0.11	0.25	0.26	0.00	0.04	0.06	0.11	0.14	0.36	0.48
	FNR	0.76	0.73	0.59	0.44	0.26	0.09	0.00	0.13	0.31	0.24	0.16	0.09	0.05	0.00	0.04	0.10	0.09	0.04	0.02	0.02	0.00
30	ave($\hat{\theta}$)	0.47	0.54	0.67	0.77	0.91	0.98	0.99	0.07	0.20	0.38	0.58	0.73	0.89	0.95	0.00	0.12	0.30	0.48	0.66	0.83	0.91
	sd($\hat{\theta}$)	0.36	0.30	0.23	0.19	0.12	0.06	0.04	0.12	0.06	0.08	0.10	0.09	0.08	0.07	0.01	0.05	0.05	0.06	0.08	0.07	0.07
	FDR	0.00	0.01	0.04	0.10	0.11	0.10	0.11	0.00	0.02	0.05	0.08	0.15	0.29	0.40	0.00	0.02	0.05	0.10	0.17	0.40	0.78
	FNR	0.98	0.78	0.53	0.37	0.24	0.09	0.00	0.38	0.54	0.27	0.18	0.10	0.04	0.00	0.06	0.23	0.11	0.06	0.03	0.01	0.00
50	ave($\hat{\theta}$)	0.34	0.52	0.67	0.78	0.89	0.96	0.98	0.08	0.18	0.35	0.53	0.71	0.87	0.96	0.03	0.12	0.30	0.46	0.65	0.83	0.90
	sd($\hat{\theta}$)	0.28	0.31	0.26	0.17	0.12	0.07	0.06	0.03	0.05	0.08	0.07	0.08	0.08	0.05	0.03	0.03	0.05	0.06	0.05	0.05	0.05
	FDR	0.00	0.02	0.06	0.09	0.13	0.16	0.15	0.00	0.02	0.06	0.10	0.19	0.35	0.50	0.00	0.02	0.04	0.11	0.18	0.43	0.92
	FNR	1.00	0.76	0.55	0.38	0.24	0.09	0.00	0.98	0.50	0.24	0.15	0.09	0.04	0.00	0.68	0.27	0.12	0.06	0.03	0.01	0.00

Table B.120: IBSQT test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.64	0.65	0.79	0.82	0.92	0.97	0.96	0.02	0.09	0.32	0.48	0.66	0.80	0.90	0.00	0.04	0.19	0.33	0.51	0.72	0.77
10	sd($\hat{\theta}$)	0.40	0.38	0.25	0.19	0.12	0.07	0.10	0.06	0.14	0.19	0.17	0.14	0.16	0.12	0.02	0.05	0.13	0.16	0.14	0.16	0.17
	FDR	0.00	0.02	0.04	0.10	0.14	0.10	0.21	0.00	0.06	0.09	0.16	0.25	0.47	0.55	0.00	0.06	0.12	0.23	0.34	0.53	0.85
	FNR	0.77	0.69	0.61	0.42	0.27	0.09	0.00	0.08	0.17	0.21	0.14	0.09	0.04	0.00	0.03	0.02	0.02	0.02	0.01	0.01	0.00
	ave($\hat{\theta}$)	0.43	0.52	0.62	0.72	0.85	0.93	0.95	0.05	0.16	0.29	0.43	0.61	0.77	0.86	0.00	0.08	0.24	0.38	0.53	0.70	0.77
30	sd($\hat{\theta}$)	0.35	0.33	0.27	0.21	0.15	0.13	0.10	0.08	0.11	0.09	0.10	0.12	0.13	0.11	0.00	0.05	0.06	0.07	0.09	0.10	0.12
	FDR	0.00	0.03	0.08	0.16	0.23	0.29	0.24	0.00	0.03	0.10	0.20	0.30	0.56	0.75	0.00	0.04	0.10	0.21	0.36	0.64	0.96
	FNR	0.94	0.79	0.53	0.37	0.23	0.08	0.00	0.35	0.42	0.20	0.11	0.07	0.02	0.00	0.02	0.12	0.06	0.03	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.46	0.48	0.50	0.74	0.85	0.92	0.96	0.07	0.13	0.27	0.42	0.59	0.75	0.80	0.01	0.09	0.24	0.36	0.51	0.66	0.73
50	sd($\hat{\theta}$)	0.34	0.32	0.26	0.24	0.17	0.12	0.10	0.03	0.05	0.07	0.09	0.12	0.13	0.12	0.02	0.03	0.05	0.07	0.08	0.09	0.11
	FDR	0.00	0.03	0.13	0.14	0.22	0.26	0.24	0.00	0.03	0.11	0.20	0.32	0.60	0.90	0.00	0.03	0.09	0.22	0.38	0.69	1.00
	FNR	1.00	0.76	0.46	0.37	0.24	0.08	0.00	0.95	0.43	0.18	0.09	0.05	0.02	0.00	0.36	0.18	0.05	0.03	0.01	0.00	0.00

Table B.121: IBSQT test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.48	0.58	0.70	0.79	0.87	0.95	0.99	0.03	0.18	0.38	0.54	0.74	0.97	1.00	0.00	0.11	0.32	0.52	0.71	0.97	1.00
	sd($\hat{\theta}$)	0.18	0.19	0.14	0.12	0.09	0.06	0.03	0.08	0.12	0.10	0.07	0.07	0.06	0.00	0.03	0.09	0.08	0.06	0.05	0.06	0.00
	FDR	0.00	0.00	0.02	0.02	0.04	0.06	0.10	0.00	0.01	0.01	0.01	0.02	0.03	0.00	0.00	0.02	0.01	0.02	0.02	0.04	0.00
	FNR	0.99	0.81	0.57	0.36	0.20	0.06	0.00	0.18	0.37	0.19	0.09	0.06	0.07	0.00	0.03	0.15	0.09	0.06	0.02	0.07	0.00
30	ave($\hat{\theta}$)	0.18	0.29	0.50	0.69	0.83	0.96	1.00	0.06	0.20	0.40	0.59	0.76	0.91	1.00	0.02	0.14	0.36	0.54	0.72	0.91	1.00
	sd($\hat{\theta}$)	0.08	0.08	0.08	0.07	0.05	0.03	0.00	0.08	0.06	0.05	0.05	0.05	0.03	0.01	0.03	0.05	0.05	0.04	0.03	0.02	0.00
	FDR	0.00	0.01	0.02	0.01	0.03	0.02	0.01	0.00	0.01	0.01	0.01	0.02	0.02	0.03	0.00	0.00	0.00	0.01	0.01	0.01	0.01
	FNR	0.89	0.65	0.40	0.28	0.16	0.06	0.00	0.45	0.49	0.25	0.15	0.08	0.02	0.00	0.20	0.26	0.15	0.08	0.04	0.01	0.00
50	ave($\hat{\theta}$)	0.15	0.27	0.48	0.66	0.83	0.95	1.00	0.08	0.19	0.39	0.59	0.76	0.92	1.00	0.04	0.15	0.35	0.54	0.73	0.91	1.00
	sd($\hat{\theta}$)	0.06	0.06	0.06	0.06	0.04	0.02	0.00	0.02	0.04	0.05	0.04	0.03	0.02	0.00	0.03	0.03	0.03	0.03	0.02	0.01	0.00
	FDR	0.00	0.01	0.02	0.02	0.03	0.03	0.02	0.00	0.00	0.01	0.01	0.01	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.01	0.04
	FNR	1.00	0.63	0.38	0.25	0.16	0.06	0.00	1.00	0.47	0.25	0.16	0.09	0.03	0.00	0.77	0.33	0.14	0.08	0.05	0.01	0.00

Table B.122: IBSQT test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.61	0.67	0.76	0.82	0.91	0.97	0.99	0.40	0.46	0.61	0.72	0.85	0.94	0.99	0.18	0.28	0.48	0.63	0.78	0.92	1.00
10	sd($\hat{\theta}$)	0.19	0.15	0.14	0.12	0.08	0.05	0.03	0.18	0.19	0.15	0.13	0.10	0.05	0.02	0.13	0.12	0.12	0.09	0.09	0.06	0.02
	FDR	0.00	0.01	0.01	0.02	0.04	0.06	0.07	0.00	0.00	0.01	0.02	0.05	0.03	0.06	0.00	0.00	0.01	0.02	0.02	0.06	0.03
	FNR	1.00	0.85	0.59	0.38	0.23	0.08	0.00	0.95	0.74	0.49	0.30	0.18	0.05	0.00	0.78	0.56	0.35	0.21	0.10	0.03	0.00
	ave($\hat{\theta}$)	0.49	0.62	0.72	0.84	0.91	0.97	1.00	0.28	0.40	0.58	0.72	0.86	0.96	1.00	0.12	0.24	0.46	0.64	0.80	0.93	1.00
30	sd($\hat{\theta}$)	0.15	0.12	0.10	0.07	0.05	0.03	0.01	0.10	0.11	0.09	0.08	0.05	0.03	0.01	0.04	0.06	0.07	0.06	0.05	0.03	0.01
	FDR	0.00	0.01	0.03	0.03	0.03	0.07	0.10	0.00	0.01	0.03	0.03	0.03	0.04	0.08	0.00	0.01	0.01	0.02	0.02	0.03	0.06
	FNR	1.00	0.84	0.59	0.41	0.24	0.08	0.00	1.00	0.75	0.49	0.31	0.19	0.06	0.00	1.00	0.58	0.36	0.22	0.13	0.04	0.00
	ave($\hat{\theta}$)	0.46	0.57	0.70	0.81	0.91	0.98	1.00	0.22	0.35	0.56	0.72	0.86	0.96	1.00	0.10	0.21	0.42	0.63	0.79	0.94	1.00
50	sd($\hat{\theta}$)	0.13	0.11	0.08	0.07	0.04	0.02	0.01	0.10	0.10	0.08	0.05	0.04	0.03	0.00	0.02	0.05	0.05	0.06	0.04	0.02	0.00
	FDR	0.00	0.01	0.03	0.04	0.04	0.05	0.02	0.00	0.01	0.02	0.03	0.04	0.05	0.01	0.00	0.01	0.02	0.02	0.02	0.03	0.01
	FNR	1.00	0.83	0.58	0.39	0.24	0.09	0.00	1.00	0.72	0.48	0.32	0.19	0.06	0.00	1.00	0.53	0.31	0.21	0.12	0.05	0.00

Table B.123: IBSQT test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.30	0.55	0.68	0.80	0.88	0.99	0.99	0.11	0.27	0.47	0.64	0.79	0.97	0.99	0.03	0.13	0.36	0.56	0.72	0.98	1.00
10	sd($\hat{\theta}$)	0.28	0.23	0.15	0.13	0.11	0.03	0.04	0.17	0.20	0.16	0.13	0.11	0.11	0.05	0.07	0.10	0.10	0.09	0.06	0.05	0.02
	FDR	0.00	0.00	0.01	0.01	0.01	0.02	0.03	0.00	0.02	0.02	0.01	0.03	0.03	0.03	0.00	0.02	0.01	0.01	0.01	0.03	0.02
	FNR	0.60	0.76	0.54	0.36	0.19	0.09	0.00	0.37	0.49	0.33	0.21	0.11	0.09	0.00	0.15	0.19	0.14	0.10	0.04	0.09	0.00
	ave($\hat{\theta}$)	0.39	0.50	0.62	0.80	0.89	0.97	1.00	0.19	0.30	0.49	0.66	0.80	0.94	1.00	0.05	0.19	0.38	0.58	0.74	0.91	0.99
30	sd($\hat{\theta}$)	0.20	0.17	0.13	0.09	0.06	0.06	0.02	0.13	0.09	0.09	0.07	0.06	0.04	0.00	0.07	0.06	0.06	0.05	0.05	0.03	0.06
	FDR	0.00	0.01	0.02	0.01	0.02	0.03	0.03	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03
	FNR	0.99	0.78	0.52	0.37	0.22	0.08	0.00	0.78	0.64	0.37	0.24	0.13	0.05	0.00	0.40	0.40	0.19	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.37	0.45	0.64	0.78	0.90	0.97	1.00	0.15	0.28	0.47	0.65	0.81	0.93	0.99	0.07	0.19	0.39	0.57	0.75	0.91	1.00
50	sd($\hat{\theta}$)	0.19	0.16	0.12	0.09	0.06	0.02	0.00	0.09	0.09	0.09	0.07	0.05	0.04	0.08	0.04	0.05	0.06	0.05	0.06	0.03	0.00
	FDR	0.00	0.01	0.01	0.03	0.03	0.03	0.04	0.00	0.00	0.01	0.01	0.01	0.04	0.06	0.00	0.00	0.01	0.01	0.02	0.02	0.02
	FNR	1.00	0.76	0.53	0.36	0.23	0.08	0.00	1.00	0.62	0.36	0.24	0.13	0.04	0.00	0.91	0.43	0.23	0.13	0.07	0.02	0.00

Table B.124: IBSQT test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.69	0.73	0.80	0.89	0.94	0.97	0.99	0.40	0.51	0.69	0.76	0.86	0.96	0.99	0.20	0.29	0.47	0.66	0.79	0.93	1.00
10	sd($\hat{\theta}$)	0.20	0.21	0.16	0.11	0.08	0.06	0.03	0.26	0.22	0.19	0.13	0.10	0.05	0.04	0.20	0.16	0.13	0.13	0.11	0.06	0.01
	FDR	0.00	0.01	0.01	0.01	0.01	0.03	0.07	0.00	0.00	0.01	0.00	0.03	0.03	0.06	0.00	0.00	0.01	0.01	0.03	0.03	0.02
	FNR	1.00	0.85	0.61	0.43	0.25	0.08	0.00	0.91	0.75	0.54	0.32	0.18	0.06	0.00	0.66	0.52	0.32	0.22	0.11	0.03	0.00
	ave($\hat{\theta}$)	0.61	0.70	0.79	0.90	0.94	0.97	1.00	0.38	0.48	0.64	0.77	0.87	0.94	0.99	0.04	0.19	0.38	0.57	0.75	0.91	1.00
30	sd($\hat{\theta}$)	0.25	0.19	0.15	0.09	0.06	0.05	0.01	0.26	0.20	0.16	0.11	0.08	0.06	0.07	0.06	0.05	0.06	0.07	0.04	0.05	0.02
	FDR	0.00	0.02	0.03	0.02	0.03	0.05	0.06	0.00	0.01	0.02	0.01	0.03	0.04	0.07	0.00	0.00	0.00	0.01	0.00	0.03	0.02
	FNR	1.00	0.85	0.62	0.44	0.26	0.08	0.00	1.00	0.77	0.52	0.34	0.19	0.06	0.00	0.33	0.43	0.21	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.61	0.69	0.79	0.86	0.92	0.98	0.99	0.30	0.45	0.60	0.76	0.87	0.96	0.99	0.15	0.27	0.46	0.67	0.81	0.93	1.00
50	sd($\hat{\theta}$)	0.25	0.21	0.14	0.10	0.08	0.03	0.06	0.23	0.20	0.14	0.10	0.09	0.05	0.07	0.14	0.13	0.10	0.10	0.07	0.06	0.00
	FDR	0.00	0.01	0.02	0.05	0.08	0.05	0.11	0.00	0.01	0.02	0.02	0.04	0.05	0.04	0.00	0.01	0.01	0.01	0.03	0.04	0.01
	FNR	1.00	0.84	0.62	0.43	0.26	0.09	0.00	1.00	0.75	0.49	0.34	0.21	0.07	0.00	0.98	0.59	0.35	0.25	0.14	0.04	0.00

Table B.125: IBSQT test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.53	0.64	0.72	0.86	0.92	0.99	0.99	0.14	0.30	0.49	0.65	0.81	0.98	0.98	0.04	0.15	0.37	0.55	0.71	0.98	1.00
10	sd($\hat{\theta}$)	0.27	0.19	0.17	0.13	0.10	0.03	0.06	0.19	0.19	0.15	0.13	0.11	0.08	0.11	0.09	0.12	0.11	0.11	0.10	0.06	0.00
	FDR	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.00	0.01	0.01	0.01	0.01	0.03	0.00
	FNR	0.90	0.83	0.56	0.41	0.23	0.10	0.00	0.44	0.50	0.36	0.22	0.13	0.09	0.00	0.25	0.23	0.19	0.09	0.04	0.09	0.00
	ave($\hat{\theta}$)	0.47	0.63	0.73	0.85	0.92	0.98	1.00	0.20	0.32	0.52	0.67	0.82	0.94	1.00	0.05	0.18	0.38	0.58	0.74	0.91	1.00
30	sd($\hat{\theta}$)	0.25	0.19	0.16	0.11	0.07	0.05	0.02	0.15	0.13	0.11	0.10	0.06	0.04	0.01	0.07	0.07	0.07	0.06	0.05	0.06	0.00
	FDR	0.00	0.00	0.02	0.02	0.02	0.05	0.05	0.00	0.00	0.01	0.02	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.01	0.03	0.01
	FNR	0.97	0.82	0.57	0.41	0.24	0.09	0.00	0.77	0.63	0.40	0.26	0.15	0.05	0.00	0.40	0.39	0.20	0.14	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.48	0.56	0.74	0.84	0.92	0.98	1.00	0.19	0.33	0.52	0.68	0.82	0.93	1.00	0.08	0.20	0.40	0.57	0.75	0.92	0.99
50	sd($\hat{\theta}$)	0.23	0.19	0.15	0.10	0.06	0.02	0.02	0.12	0.12	0.10	0.08	0.06	0.08	0.04	0.05	0.06	0.05	0.07	0.06	0.02	0.05
	FDR	0.00	0.01	0.01	0.02	0.02	0.05	0.02	0.00	0.00	0.01	0.01	0.01	0.03	0.04	0.00	0.00	0.00	0.02	0.01	0.01	0.06
	FNR	1.00	0.80	0.59	0.40	0.24	0.08	0.00	1.00	0.65	0.42	0.27	0.15	0.05	0.00	0.89	0.46	0.24	0.14	0.07	0.02	0.00

Table B.126: IBSQT test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.51	0.66	0.74	0.86	0.94	0.99	0.99	0.17	0.33	0.50	0.66	0.83	0.98	1.00	0.03	0.14	0.37	0.55	0.72	0.98	0.99
10	sd($\hat{\theta}$)	0.29	0.21	0.20	0.15	0.08	0.04	0.04	0.20	0.21	0.15	0.11	0.11	0.08	0.03	0.08	0.11	0.12	0.11	0.10	0.08	0.04
	FDR	0.00	0.01	0.02	0.02	0.02	0.01	0.06	0.00	0.01	0.01	0.01	0.00	0.02	0.01	0.00	0.02	0.01	0.01	0.02	0.04	0.02
	FNR	0.84	0.83	0.57	0.42	0.25	0.10	0.00	0.49	0.58	0.37	0.23	0.14	0.09	0.00	0.16	0.22	0.16	0.10	0.05	0.09	0.00
	ave($\hat{\theta}$)	0.52	0.59	0.77	0.86	0.92	0.97	0.99	0.19	0.34	0.54	0.69	0.83	0.93	1.00	0.05	0.20	0.38	0.58	0.74	0.91	0.99
30	sd($\hat{\theta}$)	0.26	0.20	0.15	0.10	0.06	0.07	0.09	0.16	0.13	0.12	0.08	0.06	0.10	0.02	0.07	0.07	0.08	0.06	0.05	0.05	0.07
	FDR	0.00	0.00	0.01	0.02	0.02	0.07	0.02	0.00	0.00	0.01	0.01	0.01	0.04	0.02	0.00	0.00	0.01	0.00	0.01	0.03	0.03
	FNR	0.98	0.80	0.60	0.42	0.24	0.09	0.00	0.74	0.67	0.43	0.27	0.15	0.05	0.00	0.35	0.45	0.21	0.13	0.06	0.02	0.00
	ave($\hat{\theta}$)	0.52	0.57	0.75	0.85	0.92	0.97	0.99	0.21	0.31	0.52	0.69	0.83	0.95	0.99	0.08	0.20	0.39	0.58	0.75	0.92	1.00
50	sd($\hat{\theta}$)	0.28	0.22	0.14	0.09	0.06	0.07	0.06	0.13	0.11	0.11	0.08	0.07	0.03	0.06	0.06	0.06	0.08	0.05	0.06	0.02	0.00
	FDR	0.00	0.01	0.01	0.02	0.02	0.06	0.10	0.00	0.00	0.01	0.01	0.02	0.03	0.07	0.00	0.00	0.01	0.01	0.02	0.01	0.01
	FNR	1.00	0.80	0.59	0.41	0.24	0.09	0.00	1.00	0.65	0.42	0.28	0.16	0.06	0.00	0.94	0.46	0.23	0.14	0.07	0.02	0.00

Table B.127: IBSQT2 test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.54	0.60	0.71	0.82	0.91	0.96	0.99	0.19	0.36	0.54	0.73	0.82	0.93	0.99	0.10	0.19	0.41	0.61	0.80	0.92	1.00
10	sd($\hat{\theta}$)	0.36	0.23	0.22	0.16	0.10	0.06	0.03	0.20	0.20	0.17	0.15	0.11	0.08	0.04	0.12	0.11	0.12	0.11	0.09	0.08	0.02
	FDR	0.00	0.01	0.02	0.05	0.07	0.07	0.08	0.00	0.01	0.03	0.02	0.05	0.06	0.07	0.00	0.01	0.02	0.02	0.01	0.03	0.02
	FNR	0.89	0.79	0.55	0.40	0.24	0.08	0.00	0.68	0.62	0.43	0.31	0.16	0.04	0.00	0.54	0.37	0.25	0.17	0.11	0.03	0.00
	ave($\hat{\theta}$)	0.27	0.38	0.62	0.76	0.88	0.98	0.99	0.14	0.23	0.44	0.62	0.83	0.93	0.99	0.06	0.16	0.35	0.56	0.76	0.93	0.99
30	sd($\hat{\theta}$)	0.21	0.17	0.17	0.13	0.11	0.03	0.04	0.15	0.12	0.14	0.13	0.10	0.10	0.03	0.06	0.07	0.08	0.10	0.07	0.04	0.03
	FDR	0.00	0.02	0.05	0.07	0.08	0.05	0.07	0.00	0.02	0.06	0.07	0.06	0.09	0.07	0.00	0.01	0.04	0.05	0.05	0.02	0.05
	FNR	0.98	0.75	0.53	0.37	0.23	0.09	0.00	0.79	0.58	0.38	0.24	0.17	0.06	0.00	0.77	0.37	0.20	0.15	0.09	0.04	0.00
	ave($\hat{\theta}$)	0.25	0.34	0.56	0.71	0.88	0.97	0.98	0.12	0.19	0.41	0.63	0.80	0.94	0.99	0.04	0.14	0.34	0.54	0.74	0.93	0.99
50	sd($\hat{\theta}$)	0.20	0.13	0.12	0.13	0.08	0.06	0.08	0.12	0.09	0.11	0.10	0.07	0.08	0.05	0.05	0.05	0.07	0.06	0.07	0.03	0.03
	FDR	0.00	0.03	0.06	0.10	0.10	0.06	0.07	0.00	0.03	0.06	0.07	0.08	0.08	0.02	0.00	0.02	0.04	0.06	0.08	0.05	0.04
	FNR	0.98	0.75	0.52	0.34	0.22	0.09	0.00	0.84	0.55	0.34	0.25	0.15	0.06	0.00	0.56	0.34	0.20	0.12	0.09	0.04	0.00

Table B.128: IBSQT2 test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
$N\backslash\theta_0$		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.33	0.54	0.69	0.81	0.92	0.99	1.00	0.03	0.22	0.47	0.62	0.80	0.99	1.00	0.00	0.12	0.35	0.53	0.72	0.98	0.99
10	sd($\hat{\theta}$)	0.36	0.23	0.19	0.15	0.12	0.03	0.03	0.08	0.15	0.14	0.10	0.11	0.04	0.01	0.03	0.09	0.09	0.08	0.08	0.06	0.08
	FDR	0.00	0.01	0.01	0.03	0.01	0.01	0.02	0.00	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.02	0.01	0.02	0.03	0.03	0.03
	FNR	0.56	0.76	0.55	0.38	0.24	0.09	0.00	0.18	0.40	0.33	0.19	0.12	0.09	0.00	0.03	0.21	0.14	0.07	0.05	0.09	0.00
	ave($\hat{\theta}$)	0.21	0.30	0.53	0.76	0.90	0.99	1.00	0.03	0.19	0.41	0.61	0.78	0.94	1.00	0.00	0.14	0.34	0.55	0.73	0.91	1.00
30	sd($\hat{\theta}$)	0.22	0.14	0.13	0.12	0.08	0.03	0.00	0.06	0.08	0.09	0.08	0.08	0.07	0.00	0.01	0.04	0.06	0.04	0.05	0.04	0.00
	FDR	0.00	0.02	0.03	0.04	0.04	0.03	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.01	0.00	0.01	0.01	0.01	0.02	0.03	0.00
	FNR	0.81	0.66	0.44	0.35	0.23	0.09	0.00	0.34	0.43	0.28	0.19	0.12	0.05	0.00	0.06	0.27	0.14	0.09	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.16	0.27	0.52	0.73	0.88	0.98	1.00	0.04	0.17	0.39	0.59	0.78	0.93	1.00	0.02	0.13	0.34	0.53	0.73	0.92	0.99
50	sd($\hat{\theta}$)	0.16	0.12	0.14	0.12	0.08	0.02	0.02	0.03	0.05	0.08	0.09	0.07	0.06	0.02	0.02	0.04	0.04	0.05	0.04	0.02	0.04
	FDR	0.00	0.02	0.05	0.04	0.04	0.03	0.03	0.00	0.01	0.03	0.04	0.04	0.05	0.02	0.00	0.01	0.01	0.02	0.02	0.02	0.06
	FNR	0.98	0.64	0.46	0.33	0.22	0.09	0.00	0.77	0.43	0.26	0.19	0.12	0.05	0.00	0.50	0.25	0.13	0.07	0.05	0.02	0.00

Table B.129: IBSQT2 test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.49	0.46	0.67	0.83	0.92	0.99	1.00	0.02	0.20	0.47	0.62	0.80	0.98	0.99	0.00	0.12	0.35	0.54	0.72	0.99	0.99
10	sd($\hat{\theta}$)	0.40	0.26	0.21	0.16	0.12	0.04	0.00	0.08	0.15	0.13	0.14	0.12	0.10	0.06	0.02	0.11	0.09	0.08	0.06	0.03	0.06
	FDR	0.00	0.01	0.03	0.04	0.01	0.02	0.00	0.00	0.02	0.01	0.02	0.02	0.03	0.05	0.00	0.03	0.01	0.01	0.01	0.01	0.03
	FNR	0.68	0.67	0.53	0.39	0.24	0.10	0.00	0.13	0.40	0.33	0.20	0.12	0.09	0.00	0.04	0.19	0.14	0.08	0.03	0.09	0.00
	ave($\hat{\theta}$)	0.30	0.32	0.57	0.78	0.88	0.97	0.99	0.04	0.18	0.43	0.61	0.80	0.94	0.99	0.01	0.14	0.34	0.53	0.72	0.91	0.99
30	sd($\hat{\theta}$)	0.36	0.15	0.16	0.12	0.13	0.09	0.04	0.06	0.07	0.08	0.10	0.09	0.07	0.06	0.02	0.05	0.06	0.06	0.08	0.03	0.05
	FDR	0.00	0.02	0.04	0.03	0.04	0.04	0.03	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.00	0.01	0.01	0.03	0.03	0.03	0.04
	FNR	0.81	0.67	0.50	0.36	0.22	0.09	0.00	0.39	0.45	0.31	0.20	0.13	0.05	0.00	0.10	0.28	0.13	0.08	0.04	0.02	0.00
	ave($\hat{\theta}$)	0.26	0.27	0.53	0.74	0.87	0.98	0.99	0.04	0.16	0.39	0.59	0.78	0.94	0.99	0.02	0.13	0.34	0.54	0.73	0.90	0.99
50	sd($\hat{\theta}$)	0.31	0.10	0.14	0.13	0.13	0.05	0.05	0.07	0.06	0.09	0.10	0.10	0.02	0.06	0.02	0.04	0.04	0.05	0.05	0.06	0.05
	FDR	0.00	0.02	0.04	0.05	0.05	0.05	0.07	0.00	0.02	0.03	0.03	0.04	0.01	0.04	0.00	0.01	0.01	0.01	0.02	0.03	0.03
	FNR	0.96	0.66	0.47	0.34	0.23	0.09	0.00	0.72	0.43	0.27	0.18	0.12	0.05	0.00	0.45	0.24	0.14	0.09	0.05	0.02	0.00

Table B.130: IBSQT2 test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.41	0.49	0.67	0.80	0.91	0.99	0.99	0.03	0.22	0.43	0.62	0.80	0.97	1.00	0.00	0.11	0.35	0.53	0.72	0.98	0.99
10	sd($\hat{\theta}$)	0.40	0.23	0.20	0.16	0.11	0.05	0.05	0.12	0.16	0.13	0.12	0.10	0.11	0.00	0.01	0.10	0.09	0.11	0.09	0.06	0.09
	FDR	0.00	0.01	0.02	0.02	0.02	0.02	0.03	0.00	0.02	0.01	0.02	0.01	0.04	0.00	0.00	0.02	0.01	0.02	0.02	0.03	0.02
	FNR	0.58	0.72	0.53	0.37	0.23	0.10	0.00	0.10	0.40	0.29	0.19	0.12	0.09	0.00	0.01	0.17	0.15	0.08	0.04	0.09	0.00
	ave($\hat{\theta}$)	0.34	0.32	0.56	0.78	0.90	0.98	0.99	0.02	0.19	0.39	0.62	0.79	0.94	0.99	0.00	0.13	0.35	0.53	0.73	0.91	0.99
30	sd($\hat{\theta}$)	0.38	0.13	0.14	0.12	0.14	0.03	0.05	0.05	0.08	0.10	0.08	0.11	0.06	0.05	0.01	0.05	0.06	0.07	0.06	0.02	0.06
	FDR	0.00	0.02	0.03	0.03	0.04	0.02	0.04	0.00	0.01	0.03	0.02	0.02	0.04	0.04	0.00	0.01	0.01	0.03	0.02	0.01	0.02
	FNR	0.77	0.70	0.47	0.36	0.24	0.09	0.00	0.18	0.47	0.27	0.20	0.13	0.05	0.00	0.04	0.21	0.14	0.08	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.25	0.27	0.56	0.72	0.88	0.98	0.99	0.03	0.16	0.41	0.62	0.78	0.93	0.98	0.01	0.13	0.34	0.54	0.73	0.91	0.99
50	sd($\hat{\theta}$)	0.34	0.11	0.13	0.13	0.10	0.06	0.08	0.04	0.06	0.09	0.08	0.09	0.09	0.10	0.02	0.03	0.07	0.06	0.07	0.07	0.08
	FDR	0.00	0.03	0.03	0.06	0.05	0.04	0.07	0.00	0.02	0.02	0.02	0.04	0.02	0.06	0.00	0.01	0.02	0.02	0.02	0.03	0.05
	FNR	0.89	0.69	0.48	0.34	0.23	0.09	0.00	0.62	0.41	0.29	0.20	0.12	0.05	0.00	0.32	0.25	0.14	0.09	0.05	0.02	0.00

Table B.131: IBSQT2 test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.51	0.55	0.67	0.79	0.88	0.90	0.93	0.25	0.34	0.47	0.62	0.78	0.83	0.90	0.13	0.17	0.35	0.51	0.71	0.82	0.90
10	sd($\hat{\theta}$)	0.36	0.26	0.25	0.19	0.15	0.18	0.17	0.22	0.21	0.21	0.20	0.22	0.24	0.21	0.16	0.12	0.18	0.21	0.19	0.23	0.22
	FDR	0.00	0.02	0.06	0.10	0.13	0.24	0.29	0.00	0.03	0.07	0.13	0.19	0.27	0.36	0.00	0.02	0.07	0.14	0.14	0.30	0.40
	FNR	0.84	0.80	0.55	0.41	0.24	0.08	0.00	0.79	0.66	0.43	0.29	0.19	0.04	0.00	0.60	0.36	0.24	0.17	0.11	0.04	0.00
	ave($\hat{\theta}$)	0.30	0.35	0.51	0.68	0.77	0.85	0.92	0.13	0.22	0.36	0.57	0.69	0.83	0.85	0.05	0.15	0.27	0.49	0.60	0.83	0.86
30	sd($\hat{\theta}$)	0.22	0.19	0.21	0.24	0.25	0.28	0.18	0.12	0.12	0.20	0.20	0.24	0.26	0.30	0.05	0.07	0.15	0.17	0.27	0.22	0.30
	FDR	0.00	0.05	0.12	0.16	0.28	0.38	0.44	0.00	0.03	0.13	0.15	0.24	0.40	0.59	0.00	0.03	0.11	0.14	0.24	0.35	0.60
	FNR	0.96	0.76	0.56	0.40	0.25	0.08	0.00	0.88	0.62	0.46	0.29	0.18	0.07	0.00	0.70	0.43	0.26	0.16	0.12	0.04	0.00
	ave($\hat{\theta}$)	0.21	0.30	0.47	0.62	0.74	0.85	0.88	0.12	0.18	0.27	0.49	0.62	0.72	0.82	0.05	0.12	0.27	0.42	0.63	0.75	0.85
50	sd($\hat{\theta}$)	0.16	0.14	0.23	0.26	0.29	0.28	0.26	0.15	0.11	0.20	0.26	0.30	0.34	0.32	0.05	0.06	0.14	0.22	0.23	0.31	0.30
	FDR	0.00	0.05	0.13	0.22	0.30	0.41	0.53	0.00	0.04	0.17	0.20	0.30	0.52	0.61	0.00	0.03	0.11	0.18	0.22	0.43	0.60
	FNR	0.92	0.78	0.56	0.41	0.24	0.09	0.00	0.87	0.59	0.42	0.32	0.15	0.06	0.00	0.59	0.36	0.25	0.13	0.08	0.04	0.00

Table B.132: IBSQT2 test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.41	0.47	0.65	0.77	0.91	0.97	0.99	0.04	0.22	0.40	0.59	0.74	0.94	0.96	0.00	0.11	0.29	0.52	0.68	0.90	0.97
	sd($\hat{\theta}$)	0.37	0.24	0.22	0.17	0.12	0.08	0.05	0.11	0.15	0.17	0.12	0.17	0.14	0.15	0.02	0.10	0.14	0.14	0.10	0.18	0.12
	FDR	0.00	0.01	0.03	0.06	0.07	0.07	0.02	0.00	0.02	0.04	0.05	0.09	0.12	0.09	0.00	0.02	0.06	0.05	0.09	0.17	0.10
	FNR	0.66	0.71	0.52	0.36	0.24	0.09	0.00	0.13	0.46	0.27	0.19	0.10	0.08	0.00	0.04	0.16	0.12	0.09	0.03	0.07	0.00
30	ave($\hat{\theta}$)	0.20	0.30	0.56	0.74	0.86	0.97	0.97	0.04	0.18	0.38	0.57	0.75	0.91	0.97	0.01	0.13	0.30	0.50	0.69	0.86	0.96
	sd($\hat{\theta}$)	0.22	0.12	0.14	0.16	0.17	0.11	0.14	0.06	0.08	0.13	0.15	0.16	0.11	0.10	0.02	0.05	0.09	0.12	0.15	0.14	0.15
	FDR	0.00	0.03	0.05	0.07	0.13	0.11	0.11	0.00	0.02	0.06	0.08	0.10	0.19	0.21	0.00	0.02	0.05	0.07	0.08	0.18	0.22
	FNR	0.78	0.70	0.50	0.36	0.22	0.09	0.00	0.40	0.45	0.29	0.19	0.12	0.04	0.00	0.10	0.25	0.12	0.08	0.05	0.02	0.00
50	ave($\hat{\theta}$)	0.17	0.26	0.48	0.67	0.82	0.95	0.99	0.05	0.15	0.36	0.57	0.75	0.88	0.95	0.02	0.12	0.30	0.50	0.69	0.88	0.94
	sd($\hat{\theta}$)	0.19	0.11	0.15	0.21	0.21	0.14	0.08	0.05	0.07	0.11	0.14	0.15	0.19	0.17	0.03	0.04	0.09	0.12	0.11	0.09	0.14
	FDR	0.00	0.03	0.07	0.12	0.14	0.15	0.11	0.00	0.02	0.06	0.08	0.10	0.17	0.40	0.00	0.01	0.06	0.07	0.10	0.17	0.59
	FNR	0.91	0.67	0.45	0.34	0.23	0.09	0.00	0.77	0.43	0.28	0.20	0.13	0.04	0.00	0.56	0.26	0.13	0.09	0.05	0.02	0.00

Table B.133: IBSQT2 test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.48	0.49	0.63	0.76	0.84	0.96	0.96	0.04	0.18	0.43	0.58	0.74	0.92	0.95	0.00	0.11	0.30	0.47	0.63	0.87	0.90
10	sd($\hat{\theta}$)	0.42	0.25	0.25	0.21	0.25	0.15	0.17	0.09	0.16	0.17	0.18	0.17	0.13	0.13	0.02	0.10	0.13	0.16	0.19	0.20	0.23
	FDR	0.00	0.01	0.05	0.10	0.15	0.07	0.12	0.00	0.03	0.06	0.09	0.15	0.23	0.20	0.00	0.04	0.06	0.09	0.18	0.28	0.27
	FNR	0.66	0.71	0.53	0.38	0.23	0.09	0.00	0.19	0.38	0.34	0.21	0.12	0.07	0.00	0.03	0.21	0.14	0.07	0.04	0.05	0.00
	ave($\hat{\theta}$)	0.29	0.31	0.54	0.67	0.86	0.92	0.93	0.03	0.17	0.35	0.53	0.65	0.84	0.93	0.01	0.13	0.31	0.48	0.65	0.81	0.86
30	sd($\hat{\theta}$)	0.33	0.15	0.21	0.27	0.16	0.18	0.23	0.06	0.10	0.16	0.19	0.26	0.18	0.14	0.02	0.06	0.10	0.14	0.14	0.16	0.25
	FDR	0.00	0.03	0.09	0.14	0.18	0.25	0.26	0.00	0.03	0.08	0.13	0.24	0.40	0.56	0.00	0.02	0.06	0.11	0.17	0.41	0.70
	FNR	0.85	0.69	0.54	0.38	0.24	0.08	0.00	0.36	0.45	0.31	0.21	0.13	0.04	0.00	0.11	0.29	0.16	0.10	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.24	0.25	0.44	0.68	0.84	0.91	0.95	0.04	0.14	0.34	0.55	0.71	0.83	0.90	0.02	0.12	0.28	0.46	0.62	0.83	0.86
50	sd($\hat{\theta}$)	0.30	0.11	0.21	0.21	0.16	0.20	0.18	0.03	0.07	0.14	0.16	0.17	0.22	0.22	0.02	0.05	0.11	0.14	0.19	0.13	0.23
	FDR	0.00	0.04	0.12	0.13	0.19	0.31	0.43	0.00	0.03	0.09	0.13	0.20	0.42	0.86	0.00	0.02	0.08	0.13	0.22	0.41	0.91
	FNR	0.95	0.72	0.55	0.36	0.23	0.08	0.00	0.81	0.49	0.31	0.25	0.13	0.05	0.00	0.49	0.27	0.14	0.09	0.06	0.02	0.00

Table B.134: IBSQT2 test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.46	0.47	0.64	0.81	0.89	0.99	0.99	0.03	0.22	0.46	0.59	0.77	0.98	0.99	0.00	0.11	0.33	0.54	0.70	0.94	0.99
10	sd($\hat{\theta}$)	0.42	0.22	0.20	0.16	0.14	0.05	0.06	0.10	0.16	0.16	0.15	0.13	0.05	0.06	0.00	0.10	0.12	0.08	0.10	0.16	0.10
	FDR	0.00	0.02	0.02	0.01	0.04	0.04	0.01	0.00	0.02	0.02	0.04	0.06	0.04	0.05	0.00	0.03	0.02	0.01	0.05	0.11	0.02
	FNR	0.62	0.76	0.52	0.37	0.22	0.10	0.00	0.11	0.44	0.33	0.18	0.11	0.09	0.00	0.00	0.18	0.13	0.08	0.04	0.08	0.00
	ave($\hat{\theta}$)	0.31	0.33	0.56	0.75	0.90	0.97	0.99	0.02	0.18	0.41	0.60	0.76	0.93	0.98	0.00	0.14	0.33	0.52	0.72	0.90	0.98
30	sd($\hat{\theta}$)	0.36	0.14	0.20	0.15	0.16	0.08	0.05	0.07	0.08	0.13	0.14	0.16	0.09	0.09	0.01	0.06	0.08	0.13	0.10	0.10	0.08
	FDR	0.00	0.01	0.05	0.06	0.06	0.03	0.06	0.00	0.01	0.03	0.04	0.07	0.09	0.09	0.00	0.01	0.02	0.04	0.04	0.07	0.10
	FNR	0.69	0.68	0.50	0.36	0.25	0.09	0.00	0.18	0.44	0.29	0.20	0.13	0.05	0.00	0.05	0.28	0.14	0.09	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.29	0.26	0.52	0.74	0.88	0.96	0.97	0.03	0.16	0.39	0.59	0.77	0.93	0.97	0.01	0.13	0.33	0.53	0.71	0.90	0.97
50	sd($\hat{\theta}$)	0.35	0.11	0.18	0.17	0.17	0.11	0.14	0.04	0.06	0.10	0.12	0.14	0.04	0.15	0.02	0.04	0.07	0.08	0.14	0.08	0.12
	FDR	0.00	0.02	0.06	0.07	0.08	0.05	0.11	0.00	0.01	0.03	0.05	0.05	0.07	0.23	0.00	0.01	0.02	0.03	0.06	0.07	0.29
	FNR	0.89	0.66	0.49	0.37	0.25	0.09	0.00	0.66	0.40	0.29	0.21	0.13	0.04	0.00	0.28	0.25	0.14	0.09	0.06	0.02	0.00

Table B.135: IBSQT2 test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.53	0.58	0.74	0.80	0.92	0.97	0.99	0.23	0.31	0.49	0.64	0.81	0.93	0.99	0.09	0.20	0.40	0.57	0.77	0.92	0.99
10	sd($\hat{\theta}$)	0.33	0.32	0.26	0.20	0.10	0.08	0.03	0.23	0.20	0.18	0.16	0.12	0.06	0.04	0.11	0.15	0.13	0.14	0.09	0.06	0.04
	FDR	0.00	0.01	0.02	0.04	0.04	0.06	0.11	0.00	0.02	0.03	0.04	0.08	0.06	0.10	0.00	0.01	0.02	0.06	0.04	0.06	0.08
	FNR	0.93	0.74	0.56	0.37	0.24	0.08	0.00	0.71	0.56	0.38	0.23	0.15	0.04	0.00	0.51	0.36	0.24	0.16	0.09	0.03	0.00
	ave($\hat{\theta}$)	0.29	0.38	0.58	0.76	0.90	0.98	0.99	0.18	0.26	0.42	0.62	0.79	0.94	0.99	0.06	0.15	0.37	0.56	0.75	0.93	1.00
30	sd($\hat{\theta}$)	0.22	0.22	0.22	0.15	0.09	0.03	0.02	0.17	0.15	0.13	0.12	0.09	0.06	0.03	0.06	0.07	0.08	0.08	0.07	0.04	0.02
	FDR	0.00	0.02	0.07	0.07	0.09	0.05	0.13	0.00	0.02	0.06	0.07	0.10	0.08	0.10	0.00	0.02	0.04	0.06	0.06	0.06	0.08
	FNR	0.95	0.71	0.51	0.36	0.23	0.09	0.00	0.91	0.59	0.34	0.22	0.15	0.06	0.00	0.68	0.36	0.23	0.15	0.09	0.04	0.00
	ave($\hat{\theta}$)	0.23	0.37	0.54	0.74	0.88	0.98	1.00	0.11	0.21	0.42	0.60	0.77	0.93	1.00	0.04	0.15	0.36	0.54	0.74	0.92	1.00
50	sd($\hat{\theta}$)	0.20	0.23	0.17	0.14	0.10	0.04	0.01	0.11	0.12	0.12	0.10	0.07	0.04	0.02	0.05	0.07	0.08	0.07	0.06	0.02	0.01
	FDR	0.00	0.03	0.08	0.09	0.14	0.12	0.03	0.00	0.03	0.05	0.09	0.10	0.11	0.05	0.00	0.02	0.04	0.06	0.08	0.06	0.07
	FNR	0.94	0.71	0.49	0.35	0.23	0.09	0.00	0.86	0.54	0.33	0.22	0.13	0.05	0.00	0.59	0.37	0.22	0.13	0.09	0.03	0.00

Table B.136: IBSQT2 test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.46	0.54	0.73	0.86	0.92	0.99	0.99	0.05	0.14	0.39	0.56	0.76	0.93	0.99	0.00	0.09	0.28	0.49	0.69	0.93	0.98
10	sd($\hat{\theta}$)	0.40	0.30	0.24	0.16	0.14	0.03	0.04	0.11	0.13	0.12	0.13	0.11	0.09	0.03	0.02	0.09	0.11	0.08	0.07	0.10	0.07
	FDR	0.00	0.01	0.04	0.03	0.05	0.03	0.03	0.00	0.03	0.03	0.06	0.07	0.12	0.06	0.00	0.03	0.04	0.04	0.06	0.12	0.16
	FNR	0.68	0.72	0.57	0.41	0.25	0.10	0.00	0.21	0.26	0.23	0.15	0.10	0.06	0.00	0.04	0.13	0.06	0.04	0.03	0.06	0.00
	ave($\hat{\theta}$)	0.19	0.33	0.55	0.71	0.88	0.98	1.00	0.04	0.16	0.36	0.54	0.74	0.92	0.99	0.01	0.12	0.32	0.51	0.70	0.89	0.98
30	sd($\hat{\theta}$)	0.15	0.23	0.20	0.17	0.09	0.05	0.02	0.06	0.06	0.08	0.08	0.07	0.07	0.04	0.03	0.05	0.05	0.04	0.04	0.04	0.04
	FDR	0.00	0.02	0.07	0.10	0.10	0.07	0.01	0.00	0.01	0.04	0.06	0.10	0.16	0.12	0.00	0.01	0.02	0.04	0.06	0.13	0.23
	FNR	0.87	0.66	0.47	0.33	0.22	0.09	0.00	0.39	0.40	0.21	0.12	0.09	0.04	0.00	0.17	0.19	0.09	0.04	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.16	0.27	0.47	0.70	0.85	0.97	0.99	0.04	0.15	0.34	0.54	0.72	0.91	0.99	0.02	0.12	0.31	0.50	0.69	0.88	0.98
50	sd($\hat{\theta}$)	0.14	0.18	0.17	0.15	0.12	0.06	0.04	0.03	0.05	0.06	0.06	0.07	0.04	0.03	0.02	0.03	0.04	0.04	0.04	0.03	0.03
	FDR	0.00	0.03	0.07	0.11	0.16	0.12	0.06	0.00	0.02	0.05	0.06	0.12	0.13	0.19	0.00	0.01	0.03	0.06	0.08	0.16	0.38
	FNR	0.98	0.64	0.41	0.33	0.21	0.09	0.00	0.72	0.38	0.19	0.13	0.08	0.03	0.00	0.56	0.22	0.09	0.06	0.03	0.01	0.00

Table B.137: IBSQT2 test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.38	0.47	0.68	0.84	0.91	0.95	0.97	0.03	0.13	0.30	0.54	0.70	0.87	0.95	0.00	0.07	0.28	0.44	0.64	0.81	0.90
10	sd($\hat{\theta}$)	0.40	0.37	0.31	0.19	0.14	0.11	0.09	0.10	0.15	0.15	0.13	0.15	0.15	0.09	0.02	0.07	0.10	0.12	0.10	0.14	0.12
	FDR	0.00	0.03	0.05	0.08	0.08	0.13	0.12	0.00	0.04	0.08	0.08	0.15	0.27	0.23	0.00	0.05	0.06	0.12	0.16	0.38	0.51
	FNR	0.61	0.60	0.50	0.41	0.25	0.09	0.00	0.13	0.24	0.17	0.13	0.08	0.05	0.00	0.04	0.09	0.09	0.04	0.02	0.02	0.00
	ave($\hat{\theta}$)	0.34	0.47	0.58	0.67	0.85	0.95	0.97	0.04	0.14	0.32	0.52	0.68	0.86	0.93	0.00	0.10	0.28	0.46	0.64	0.81	0.90
30	sd($\hat{\theta}$)	0.37	0.31	0.27	0.20	0.16	0.11	0.08	0.08	0.07	0.09	0.11	0.10	0.09	0.08	0.01	0.05	0.05	0.06	0.08	0.08	0.08
	FDR	0.00	0.01	0.06	0.14	0.16	0.15	0.21	0.00	0.03	0.07	0.10	0.20	0.32	0.51	0.00	0.02	0.06	0.11	0.20	0.42	0.79
	FNR	0.81	0.71	0.47	0.31	0.22	0.09	0.00	0.33	0.37	0.19	0.13	0.07	0.03	0.00	0.05	0.18	0.09	0.05	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.21	0.38	0.54	0.67	0.84	0.93	0.96	0.04	0.13	0.31	0.48	0.66	0.85	0.94	0.02	0.11	0.29	0.45	0.64	0.81	0.89
50	sd($\hat{\theta}$)	0.27	0.33	0.27	0.21	0.16	0.10	0.10	0.03	0.05	0.07	0.07	0.10	0.09	0.06	0.02	0.04	0.05	0.06	0.05	0.06	0.07
	FDR	0.00	0.03	0.09	0.14	0.17	0.23	0.21	0.00	0.03	0.07	0.13	0.23	0.40	0.58	0.00	0.02	0.05	0.12	0.20	0.45	0.92
	FNR	0.95	0.67	0.47	0.31	0.22	0.08	0.00	0.79	0.36	0.19	0.10	0.07	0.03	0.00	0.52	0.22	0.10	0.05	0.02	0.01	0.00

Table B.138: IBSQT2 test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.44	0.50	0.67	0.69	0.88	0.95	0.93	0.01	0.07	0.27	0.42	0.60	0.74	0.85	0.00	0.04	0.18	0.33	0.51	0.71	0.75
10	sd($\hat{\theta}$)	0.45	0.42	0.32	0.28	0.17	0.12	0.15	0.04	0.12	0.17	0.16	0.15	0.21	0.18	0.02	0.05	0.13	0.16	0.14	0.16	0.17
	FDR	0.00	0.03	0.05	0.15	0.16	0.12	0.25	0.00	0.06	0.10	0.19	0.28	0.49	0.56	0.00	0.06	0.13	0.23	0.35	0.53	0.85
	FNR	0.55	0.55	0.52	0.34	0.25	0.09	0.00	0.06	0.11	0.13	0.09	0.06	0.03	0.00	0.03	0.03	0.03	0.02	0.01	0.01	0.00
	ave($\hat{\theta}$)	0.30	0.36	0.50	0.60	0.76	0.86	0.90	0.02	0.12	0.24	0.39	0.56	0.72	0.82	0.00	0.07	0.23	0.37	0.51	0.68	0.76
30	sd($\hat{\theta}$)	0.36	0.31	0.28	0.23	0.20	0.17	0.16	0.05	0.11	0.08	0.10	0.13	0.14	0.14	0.00	0.05	0.06	0.08	0.09	0.11	0.12
	FDR	0.00	0.04	0.11	0.20	0.31	0.40	0.38	0.00	0.03	0.12	0.23	0.35	0.62	0.77	0.00	0.04	0.10	0.22	0.37	0.67	0.96
	FNR	0.74	0.66	0.45	0.30	0.20	0.08	0.00	0.26	0.28	0.13	0.08	0.05	0.02	0.00	0.02	0.10	0.05	0.02	0.01	0.00	0.00
	ave($\hat{\theta}$)	0.29	0.36	0.40	0.63	0.75	0.85	0.89	0.03	0.09	0.24	0.39	0.54	0.70	0.76	0.01	0.08	0.23	0.35	0.50	0.65	0.71
50	sd($\hat{\theta}$)	0.35	0.33	0.26	0.27	0.21	0.16	0.16	0.04	0.04	0.07	0.08	0.12	0.14	0.13	0.02	0.03	0.05	0.07	0.08	0.09	0.11
	FDR	0.00	0.04	0.15	0.19	0.29	0.39	0.47	0.00	0.04	0.13	0.22	0.37	0.64	0.94	0.00	0.03	0.10	0.23	0.39	0.70	1.00
	FNR	0.87	0.67	0.38	0.31	0.20	0.07	0.00	0.61	0.30	0.14	0.07	0.04	0.01	0.00	0.28	0.13	0.04	0.02	0.01	0.00	0.00

Table B.139: IBSQT2 test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.33	0.45	0.62	0.75	0.86	0.95	0.99	0.03	0.17	0.38	0.54	0.74	0.97	1.00	0.01	0.12	0.32	0.52	0.71	0.97	1.00
10	sd($\hat{\theta}$)	0.23	0.25	0.17	0.16	0.11	0.06	0.03	0.08	0.12	0.11	0.07	0.07	0.06	0.00	0.03	0.09	0.07	0.06	0.05	0.06	0.00
	FDR	0.00	0.02	0.03	0.04	0.04	0.06	0.09	0.00	0.01	0.01	0.01	0.02	0.04	0.00	0.00	0.02	0.01	0.03	0.02	0.04	0.00
	FNR	0.90	0.73	0.52	0.33	0.19	0.06	0.00	0.18	0.34	0.20	0.08	0.06	0.07	0.00	0.04	0.17	0.08	0.05	0.02	0.08	0.00
	ave($\hat{\theta}$)	0.14	0.25	0.46	0.67	0.81	0.96	1.00	0.05	0.19	0.39	0.58	0.75	0.91	1.00	0.01	0.14	0.35	0.54	0.72	0.91	1.00
30	sd($\hat{\theta}$)	0.09	0.08	0.09	0.08	0.06	0.03	0.01	0.07	0.06	0.05	0.05	0.05	0.03	0.01	0.03	0.05	0.05	0.04	0.03	0.02	0.01
	FDR	0.00	0.01	0.02	0.02	0.04	0.02	0.01	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.00	0.01	0.00	0.01	0.01	0.01	0.01
	FNR	0.81	0.59	0.36	0.25	0.14	0.06	0.00	0.42	0.45	0.23	0.14	0.07	0.02	0.00	0.18	0.26	0.14	0.07	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.12	0.23	0.45	0.64	0.81	0.94	1.00	0.06	0.18	0.38	0.58	0.76	0.92	1.00	0.03	0.15	0.34	0.54	0.73	0.91	1.00
50	sd($\hat{\theta}$)	0.07	0.06	0.06	0.06	0.05	0.02	0.01	0.03	0.04	0.05	0.04	0.03	0.02	0.01	0.03	0.03	0.03	0.03	0.02	0.01	0.00
	FDR	0.00	0.01	0.02	0.03	0.04	0.04	0.04	0.00	0.01	0.02	0.01	0.01	0.01	0.05	0.00	0.00	0.00	0.01	0.00	0.01	0.03
	FNR	0.95	0.59	0.35	0.23	0.14	0.05	0.00	0.91	0.44	0.23	0.14	0.08	0.03	0.00	0.72	0.30	0.13	0.08	0.04	0.01	0.00

Table B.140: IBSQT2 test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.42	0.52	0.68	0.79	0.89	0.97	0.99	0.25	0.35	0.51	0.69	0.83	0.94	0.99	0.11	0.20	0.41	0.60	0.77	0.92	0.99
10	sd($\hat{\theta}$)	0.26	0.25	0.19	0.16	0.10	0.05	0.04	0.21	0.22	0.17	0.15	0.12	0.07	0.03	0.12	0.12	0.14	0.11	0.09	0.06	0.05
	FDR	0.00	0.02	0.03	0.03	0.04	0.07	0.07	0.00	0.01	0.04	0.04	0.07	0.03	0.06	0.00	0.01	0.02	0.02	0.03	0.06	0.04
	FNR	0.93	0.73	0.54	0.36	0.22	0.08	0.00	0.77	0.61	0.41	0.27	0.17	0.05	0.00	0.60	0.39	0.26	0.17	0.09	0.03	0.00
	ave($\hat{\theta}$)	0.28	0.41	0.56	0.72	0.87	0.95	1.00	0.13	0.25	0.45	0.63	0.80	0.95	1.00	0.06	0.17	0.38	0.57	0.77	0.93	1.00
30	sd($\hat{\theta}$)	0.15	0.18	0.14	0.11	0.09	0.05	0.01	0.09	0.11	0.11	0.09	0.09	0.04	0.01	0.05	0.06	0.08	0.07	0.06	0.04	0.01
	FDR	0.00	0.03	0.07	0.09	0.08	0.12	0.10	0.00	0.02	0.05	0.07	0.08	0.07	0.10	0.00	0.01	0.03	0.04	0.04	0.05	0.04
	FNR	0.98	0.79	0.50	0.34	0.21	0.07	0.00	0.94	0.60	0.38	0.24	0.15	0.06	0.00	0.70	0.42	0.25	0.15	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.25	0.36	0.53	0.70	0.85	0.97	1.00	0.11	0.22	0.42	0.62	0.80	0.94	1.00	0.03	0.14	0.35	0.56	0.74	0.93	1.00
50	sd($\hat{\theta}$)	0.13	0.12	0.11	0.10	0.07	0.03	0.01	0.07	0.09	0.09	0.07	0.07	0.04	0.01	0.04	0.05	0.06	0.06	0.05	0.03	0.00
	FDR	0.00	0.03	0.07	0.10	0.13	0.09	0.05	0.00	0.02	0.06	0.08	0.10	0.10	0.02	0.00	0.02	0.03	0.05	0.06	0.06	0.01
	FNR	0.97	0.76	0.49	0.33	0.21	0.08	0.00	0.91	0.58	0.36	0.24	0.16	0.06	0.00	0.53	0.35	0.20	0.15	0.08	0.04	0.00

Table B.141: IBSQT2 test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.25	0.49	0.63	0.78	0.86	0.98	0.99	0.10	0.25	0.45	0.63	0.78	0.97	0.99	0.02	0.13	0.35	0.53	0.72	0.97	0.99
10	sd($\hat{\theta}$)	0.27	0.25	0.19	0.15	0.13	0.09	0.10	0.16	0.19	0.17	0.13	0.12	0.11	0.06	0.07	0.10	0.10	0.12	0.06	0.09	0.04
	FDR	0.00	0.01	0.02	0.02	0.02	0.03	0.02	0.00	0.02	0.02	0.02	0.03	0.03	0.04	0.00	0.01	0.01	0.03	0.02	0.03	0.03
	FNR	0.56	0.69	0.49	0.34	0.19	0.09	0.00	0.36	0.46	0.29	0.20	0.11	0.09	0.00	0.15	0.20	0.14	0.09	0.03	0.08	0.00
	ave($\hat{\theta}$)	0.32	0.44	0.55	0.76	0.87	0.96	0.99	0.14	0.26	0.46	0.63	0.79	0.94	1.00	0.03	0.17	0.36	0.56	0.73	0.91	0.98
30	sd($\hat{\theta}$)	0.22	0.19	0.17	0.12	0.08	0.08	0.07	0.13	0.10	0.10	0.08	0.07	0.05	0.04	0.06	0.06	0.06	0.06	0.07	0.05	0.11
	FDR	0.00	0.01	0.03	0.02	0.02	0.05	0.02	0.00	0.01	0.01	0.01	0.01	0.03	0.01	0.00	0.00	0.00	0.01	0.02	0.03	0.05
	FNR	0.90	0.73	0.45	0.34	0.20	0.08	0.00	0.66	0.57	0.33	0.21	0.11	0.04	0.00	0.36	0.34	0.16	0.11	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.29	0.38	0.59	0.73	0.87	0.97	1.00	0.12	0.23	0.43	0.62	0.79	0.92	0.98	0.04	0.16	0.37	0.55	0.73	0.91	1.00
50	sd($\hat{\theta}$)	0.20	0.18	0.14	0.12	0.10	0.04	0.02	0.10	0.09	0.11	0.10	0.07	0.10	0.10	0.05	0.05	0.06	0.08	0.09	0.06	0.02
	FDR	0.00	0.01	0.02	0.04	0.04	0.03	0.06	0.00	0.01	0.02	0.03	0.03	0.05	0.05	0.00	0.00	0.01	0.02	0.03	0.03	0.03
	FNR	0.95	0.68	0.49	0.33	0.21	0.08	0.00	0.92	0.54	0.32	0.21	0.12	0.04	0.00	0.66	0.34	0.19	0.10	0.06	0.02	0.00

Table B.142: IBSQT2 test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.49	0.62	0.76	0.85	0.93	0.97	0.99	0.25	0.38	0.60	0.72	0.85	0.96	0.99	0.14	0.22	0.41	0.63	0.77	0.93	1.00
10	sd($\hat{\theta}$)	0.31	0.31	0.21	0.15	0.10	0.07	0.04	0.28	0.25	0.26	0.16	0.12	0.06	0.06	0.19	0.18	0.14	0.15	0.15	0.07	0.01
	FDR	0.00	0.02	0.00	0.02	0.02	0.03	0.07	0.00	0.02	0.03	0.02	0.03	0.02	0.07	0.00	0.01	0.02	0.03	0.03	0.04	0.02
	FNR	0.87	0.76	0.57	0.41	0.24	0.08	0.00	0.65	0.63	0.45	0.28	0.17	0.06	0.00	0.51	0.36	0.23	0.19	0.09	0.03	0.00
	ave($\hat{\theta}$)	0.39	0.50	0.63	0.82	0.89	0.96	0.99	0.24	0.30	0.50	0.68	0.81	0.93	0.98	0.02	0.15	0.36	0.54	0.73	0.90	0.99
30	sd($\hat{\theta}$)	0.31	0.27	0.25	0.16	0.15	0.10	0.07	0.27	0.20	0.21	0.17	0.13	0.12	0.09	0.04	0.07	0.06	0.09	0.04	0.08	0.05
	FDR	0.00	0.02	0.06	0.04	0.08	0.08	0.08	0.00	0.02	0.05	0.04	0.06	0.06	0.09	0.00	0.01	0.01	0.02	0.02	0.04	0.03
	FNR	0.91	0.74	0.52	0.40	0.24	0.08	0.00	0.87	0.58	0.41	0.27	0.16	0.05	0.00	0.24	0.29	0.15	0.10	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.40	0.49	0.64	0.77	0.86	0.97	0.97	0.17	0.31	0.46	0.66	0.81	0.95	0.97	0.08	0.19	0.36	0.60	0.75	0.91	1.00
50	sd($\hat{\theta}$)	0.32	0.26	0.22	0.16	0.15	0.05	0.12	0.20	0.21	0.18	0.15	0.14	0.07	0.13	0.15	0.12	0.11	0.12	0.12	0.10	0.00
	FDR	0.00	0.02	0.05	0.09	0.11	0.08	0.11	0.00	0.02	0.05	0.06	0.08	0.08	0.04	0.00	0.01	0.03	0.03	0.07	0.07	0.01
	FNR	0.94	0.75	0.54	0.38	0.23	0.09	0.00	0.84	0.60	0.37	0.27	0.17	0.07	0.00	0.55	0.39	0.21	0.18	0.11	0.03	0.00

Table B.143: IBSQT2 test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.42	0.54	0.68	0.83	0.91	0.99	0.99	0.12	0.28	0.46	0.64	0.80	0.98	0.97	0.04	0.14	0.36	0.54	0.70	0.98	1.00
	sd($\hat{\theta}$)	0.31	0.29	0.21	0.15	0.11	0.07	0.10	0.18	0.19	0.17	0.14	0.13	0.10	0.14	0.09	0.11	0.12	0.11	0.13	0.10	0.04
	FDR	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.00	0.01	0.02	0.01	0.02	0.03	0.04	0.00	0.01	0.02	0.01	0.03	0.04	0.01
	FNR	0.77	0.69	0.53	0.39	0.23	0.09	0.00	0.39	0.46	0.31	0.21	0.12	0.09	0.00	0.26	0.21	0.17	0.08	0.03	0.09	0.00
30	ave($\hat{\theta}$)	0.37	0.56	0.68	0.81	0.90	0.97	0.98	0.14	0.25	0.45	0.63	0.81	0.93	0.99	0.03	0.15	0.35	0.56	0.73	0.90	1.00
	sd($\hat{\theta}$)	0.28	0.26	0.20	0.16	0.12	0.11	0.10	0.15	0.15	0.15	0.14	0.08	0.08	0.08	0.05	0.07	0.07	0.08	0.09	0.08	0.01
	FDR	0.00	0.01	0.02	0.02	0.02	0.05	0.06	0.00	0.01	0.02	0.03	0.02	0.03	0.02	0.00	0.01	0.01	0.01	0.03	0.03	0.02
	FNR	0.81	0.75	0.53	0.37	0.23	0.08	0.00	0.62	0.50	0.32	0.22	0.13	0.05	0.00	0.33	0.28	0.14	0.11	0.06	0.02	0.00
50	ave($\hat{\theta}$)	0.38	0.47	0.66	0.81	0.90	0.97	0.99	0.12	0.28	0.47	0.63	0.79	0.91	0.99	0.05	0.16	0.37	0.54	0.73	0.92	0.98
	sd($\hat{\theta}$)	0.26	0.22	0.22	0.13	0.12	0.05	0.07	0.13	0.14	0.12	0.13	0.09	0.13	0.05	0.06	0.06	0.07	0.09	0.11	0.01	0.10
	FDR	0.00	0.01	0.02	0.02	0.03	0.05	0.03	0.00	0.01	0.01	0.03	0.02	0.05	0.03	0.00	0.00	0.01	0.03	0.04	0.01	0.07
	FNR	0.96	0.71	0.53	0.38	0.23	0.08	0.00	0.79	0.55	0.35	0.22	0.12	0.04	0.00	0.64	0.32	0.19	0.11	0.06	0.02	0.00

Table B.144: IBSQT2 test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.42	0.58	0.69	0.83	0.92	0.99	0.99	0.12	0.27	0.45	0.64	0.80	0.98	0.99	0.03	0.13	0.35	0.54	0.71	0.97	0.98
10	sd($\hat{\theta}$)	0.30	0.27	0.24	0.18	0.13	0.07	0.06	0.18	0.21	0.16	0.13	0.13	0.10	0.09	0.07	0.10	0.12	0.11	0.11	0.10	0.12
	FDR	0.00	0.01	0.03	0.03	0.03	0.01	0.05	0.00	0.01	0.02	0.01	0.02	0.04	0.02	0.00	0.02	0.01	0.01	0.02	0.04	0.04
	FNR	0.77	0.76	0.53	0.38	0.25	0.10	0.00	0.38	0.44	0.31	0.20	0.13	0.09	0.00	0.17	0.21	0.13	0.09	0.05	0.09	0.00
	ave($\hat{\theta}$)	0.41	0.48	0.69	0.82	0.90	0.95	0.98	0.11	0.27	0.47	0.63	0.80	0.92	0.99	0.03	0.14	0.34	0.55	0.73	0.90	0.98
30	sd($\hat{\theta}$)	0.31	0.25	0.21	0.15	0.12	0.15	0.14	0.15	0.15	0.14	0.13	0.09	0.14	0.05	0.06	0.06	0.10	0.07	0.05	0.08	0.11
	FDR	0.00	0.01	0.02	0.03	0.04	0.09	0.04	0.00	0.01	0.01	0.03	0.01	0.06	0.01	0.00	0.01	0.03	0.01	0.02	0.03	0.03
	FNR	0.86	0.71	0.54	0.39	0.23	0.08	0.00	0.55	0.53	0.35	0.23	0.13	0.05	0.00	0.23	0.27	0.15	0.10	0.05	0.02	0.00
	ave($\hat{\theta}$)	0.42	0.45	0.65	0.80	0.89	0.96	0.99	0.14	0.23	0.45	0.65	0.80	0.94	0.98	0.05	0.15	0.35	0.54	0.74	0.90	1.00
50	sd($\hat{\theta}$)	0.31	0.25	0.21	0.17	0.10	0.11	0.08	0.14	0.12	0.13	0.11	0.09	0.06	0.09	0.06	0.06	0.09	0.10	0.07	0.09	0.01
	FDR	0.00	0.01	0.03	0.03	0.03	0.08	0.10	0.00	0.01	0.02	0.01	0.03	0.04	0.07	0.00	0.01	0.02	0.02	0.02	0.04	0.02
	FNR	0.93	0.70	0.52	0.38	0.23	0.09	0.00	0.82	0.50	0.32	0.23	0.14	0.06	0.00	0.60	0.30	0.16	0.10	0.06	0.02	0.00

Table B.145: Ng(2008) test for DGP 1a. I(1) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.08	0.25	0.52	0.74	1.00	1.25	1.44	0.04	0.16	0.39	0.59	0.79	1.01	1.13	0.02	0.11	0.32	0.53	0.66	0.88	0.98
	sd($\hat{\theta}$)	0.22	0.33	0.30	0.17	0.39	0.44	0.39	0.06	0.20	0.30	0.19	0.20	0.24	0.21	0.10	0.15	0.45	0.41	0.50	0.22	0.29
	$\theta = 0.01$	0.00	0.12	0.35	0.53	0.68	0.79	0.84	0.00	0.08	0.27	0.47	0.63	0.75	0.79	0.00	0.06	0.20	0.41	0.60	0.73	0.79
10	$\theta = \theta_0$	0.00	0.04	0.09	0.15	0.16	0.21	0.22	0.00	0.02	0.09	0.13	0.15	0.17	0.20	0.00	0.03	0.12	0.15	0.13	0.12	0.15
	$\theta = 1$	1.00	0.85	0.61	0.42	0.26	0.14	0.09	1.00	0.93	0.73	0.53	0.36	0.20	0.16	1.00	0.96	0.78	0.59	0.41	0.21	0.17
	FDR	0.00	0.04	0.12	0.19	0.24	0.32	0.34	0.00	0.04	0.11	0.17	0.26	0.38	0.48	0.00	0.05	0.12	0.18	0.28	0.41	0.56
	FNR	0.69	0.54	0.39	0.28	0.19	0.07	0.00	0.22	0.28	0.25	0.18	0.12	0.05	0.00	0.03	0.16	0.16	0.14	0.08	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.26	0.53	0.82	1.10	1.36	1.48	0.04	0.18	0.40	0.60	0.84	1.07	1.19	0.02	0.13	0.33	0.52	0.72	0.93	1.02
	sd($\hat{\theta}$)	0.11	0.12	0.15	0.17	0.36	0.25	0.39	0.04	0.09	0.11	0.21	0.24	0.23	0.22	0.04	0.10	0.16	0.20	0.19	0.19	0.22
	$\theta = 0.01$	0.02	0.45	0.86	0.97	0.99	1.00	1.00	0.00	0.34	0.81	0.95	0.99	1.00	1.00	0.00	0.24	0.78	0.94	0.99	1.00	1.00
30	$\theta = \theta_0$	0.01	0.14	0.22	0.27	0.31	0.33	0.34	0.00	0.08	0.12	0.16	0.16	0.20	0.17	0.00	0.03	0.10	0.14	0.13	0.12	0.12
	$\theta = 1$	1.00	0.95	0.73	0.38	0.13	0.04	0.03	1.00	0.99	0.90	0.67	0.35	0.14	0.06	1.00	1.00	0.96	0.81	0.48	0.19	0.11
	FDR	0.00	0.03	0.07	0.10	0.11	0.14	0.18	0.00	0.02	0.07	0.11	0.14	0.24	0.32	0.00	0.03	0.07	0.12	0.18	0.29	0.49
	FNR	1.00	0.63	0.44	0.35	0.24	0.08	0.00	0.98	0.42	0.27	0.20	0.15	0.06	0.00	0.47	0.24	0.16	0.12	0.10	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.27	0.53	0.80	1.12	1.37	1.51	0.04	0.18	0.39	0.62	0.84	1.05	1.19	0.02	0.13	0.33	0.52	0.74	0.95	1.03
	sd($\hat{\theta}$)	0.07	0.09	0.12	0.18	0.28	0.16	0.21	0.04	0.06	0.10	0.17	0.25	0.17	0.25	0.03	0.06	0.10	0.09	0.14	0.17	0.16
	$\theta = 0.01$	0.02	0.68	0.98	1.00	1.00	1.00	1.00	0.00	0.58	0.96	1.00	1.00	1.00	1.00	0.00	0.45	0.97	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.26	0.31	0.37	0.47	0.49	0.47	0.00	0.12	0.14	0.19	0.19	0.19	0.22	0.00	0.06	0.10	0.09	0.12	0.13	0.11
	$\theta = 1$	1.00	0.99	0.83	0.42	0.14	0.03	0.01	1.00	1.00	0.97	0.75	0.38	0.11	0.05	1.00	1.00	0.99	0.91	0.56	0.20	0.08
	FDR	0.00	0.03	0.07	0.08	0.08	0.09	0.09	0.00	0.02	0.05	0.08	0.11	0.19	0.27	0.00	0.02	0.05	0.09	0.13	0.24	0.47
	FNR	1.00	0.65	0.46	0.36	0.25	0.09	0.00	1.00	0.44	0.26	0.22	0.16	0.07	0.00	0.94	0.24	0.16	0.11	0.10	0.05	0.00

Table B.146: Ng(2008) test for DGP 1b. I(1) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.21	0.55	0.79	1.02	1.30	1.44	0.04	0.14	0.41	0.60	0.81	1.01	1.11	0.02	0.07	0.35	0.51	0.71	0.91	0.98
	sd($\hat{\theta}$)	0.15	0.23	0.18	0.34	0.65	0.59	0.51	0.10	0.15	0.17	0.29	0.30	0.59	0.30	0.07	0.23	0.11	0.21	0.60	0.31	0.46
	$\theta = 0.01$	0.01	0.08	0.28	0.46	0.61	0.77	0.80	0.00	0.04	0.20	0.37	0.57	0.71	0.75	0.00	0.01	0.15	0.34	0.50	0.67	0.72
10	$\theta = \theta_0$	0.00	0.02	0.07	0.12	0.15	0.19	0.19	0.00	0.01	0.07	0.12	0.14	0.18	0.15	0.00	0.02	0.10	0.12	0.16	0.15	0.16
	$\theta = 1$	1.00	0.89	0.54	0.37	0.22	0.12	0.09	1.00	0.95	0.67	0.52	0.31	0.19	0.14	1.00	0.98	0.72	0.56	0.37	0.20	0.17
	FDR	0.00	0.05	0.12	0.19	0.25	0.32	0.36	0.00	0.05	0.13	0.20	0.26	0.40	0.47	0.00	0.07	0.13	0.20	0.28	0.41	0.57
	FNR	0.71	0.60	0.43	0.30	0.19	0.07	0.00	0.26	0.33	0.30	0.20	0.13	0.05	0.00	0.07	0.13	0.20	0.15	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.30	0.58	0.82	1.08	1.38	1.48	0.04	0.20	0.43	0.64	0.86	1.10	1.17	0.02	0.15	0.34	0.53	0.73	0.92	1.03
	sd($\hat{\theta}$)	0.10	0.14	0.26	0.33	0.23	0.49	0.42	0.06	0.12	0.14	0.25	0.25	0.26	0.25	0.04	0.07	0.12	0.21	0.22	0.27	0.23
	$\theta = 0.01$	0.02	0.50	0.89	0.97	0.99	1.00	1.00	0.00	0.31	0.79	0.95	0.99	1.00	1.00	0.00	0.21	0.73	0.94	0.99	1.00	1.00
30	$\theta = \theta_0$	0.01	0.17	0.25	0.27	0.29	0.33	0.33	0.00	0.06	0.12	0.14	0.18	0.19	0.16	0.00	0.05	0.09	0.10	0.12	0.11	0.11
	$\theta = 1$	1.00	0.93	0.65	0.37	0.15	0.05	0.03	1.00	0.97	0.84	0.59	0.32	0.11	0.07	1.00	0.98	0.95	0.78	0.46	0.21	0.11
	FDR	0.00	0.03	0.08	0.12	0.14	0.15	0.18	0.00	0.02	0.07	0.10	0.16	0.22	0.33	0.00	0.03	0.07	0.11	0.17	0.31	0.47
	FNR	1.00	0.66	0.49	0.36	0.24	0.09	0.00	0.98	0.46	0.31	0.23	0.16	0.07	0.00	0.52	0.28	0.17	0.13	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.30	0.56	0.83	1.11	1.37	1.50	0.04	0.20	0.41	0.64	0.85	1.08	1.20	0.02	0.14	0.34	0.54	0.75	0.93	1.06
	sd($\hat{\theta}$)	0.07	0.11	0.08	0.14	0.13	0.16	0.39	0.04	0.05	0.14	0.15	0.18	0.27	0.20	0.03	0.08	0.10	0.16	0.15	0.19	0.18
	$\theta = 0.01$	0.03	0.76	0.99	1.00	1.00	1.00	1.00	0.00	0.58	0.98	1.00	1.00	1.00	1.00	0.00	0.43	0.95	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.30	0.34	0.37	0.41	0.41	0.46	0.00	0.14	0.15	0.20	0.21	0.19	0.22	0.00	0.04	0.10	0.08	0.13	0.12	0.14
	$\theta = 1$	1.00	0.98	0.78	0.37	0.10	0.03	0.02	1.00	1.00	0.95	0.70	0.37	0.10	0.04	1.00	1.00	0.99	0.89	0.55	0.20	0.09
	FDR	0.00	0.02	0.08	0.09	0.09	0.10	0.11	0.00	0.02	0.06	0.09	0.12	0.17	0.26	0.00	0.02	0.05	0.08	0.13	0.27	0.43
	FNR	1.00	0.68	0.49	0.38	0.26	0.09	0.00	1.00	0.48	0.29	0.24	0.16	0.07	0.00	0.96	0.29	0.17	0.12	0.10	0.04	0.00

Table B.147: Ng(2008) test for DGP 1c. I(1) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.08	0.21	0.49	0.82	1.09	1.31	1.46	0.04	0.13	0.35	0.61	0.81	1.06	1.11	0.02	0.08	0.31	0.50	0.71	0.92	1.02
	sd($\hat{\theta}$)	0.09	0.28	0.76	0.74	0.23	0.62	0.41	0.09	0.09	0.15	0.20	0.35	0.51	0.35	0.05	0.07	0.20	0.29	0.47	0.33	0.34
	$\theta = 0.01$	0.00	0.10	0.31	0.56	0.67	0.78	0.85	0.00	0.06	0.24	0.48	0.64	0.77	0.79	0.00	0.03	0.19	0.40	0.61	0.74	0.80
10	$\theta = \theta_0$	0.00	0.03	0.12	0.16	0.20	0.23	0.22	0.00	0.02	0.11	0.16	0.17	0.17	0.18	0.00	0.04	0.12	0.14	0.13	0.16	0.15
	$\theta = 1$	1.00	0.90	0.63	0.38	0.24	0.16	0.10	1.00	0.94	0.76	0.54	0.35	0.19	0.15	1.00	0.98	0.80	0.61	0.36	0.20	0.15
	FDR	0.00	0.05	0.12	0.16	0.21	0.30	0.32	0.00	0.06	0.12	0.17	0.25	0.34	0.48	0.00	0.07	0.12	0.18	0.25	0.38	0.52
	FNR	0.68	0.53	0.37	0.29	0.19	0.07	0.00	0.25	0.27	0.20	0.18	0.13	0.05	0.00	0.04	0.14	0.15	0.13	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.27	0.53	0.80	1.08	1.34	1.51	0.04	0.17	0.39	0.62	0.86	1.05	1.18	0.02	0.13	0.33	0.53	0.74	0.93	1.02
	sd($\hat{\theta}$)	0.05	0.17	0.14	0.30	0.16	0.19	0.34	0.10	0.21	0.16	0.15	0.16	0.24	0.30	0.04	0.05	0.10	0.16	0.19	0.16	0.25
	$\theta = 0.01$	0.02	0.44	0.86	0.96	0.99	1.00	1.00	0.00	0.30	0.81	0.96	0.99	1.00	1.00	0.00	0.23	0.74	0.95	0.99	1.00	1.00
30	$\theta = \theta_0$	0.01	0.15	0.21	0.27	0.31	0.32	0.36	0.00	0.06	0.11	0.16	0.15	0.16	0.18	0.00	0.06	0.11	0.11	0.13	0.13	0.14
	$\theta = 1$	1.00	0.94	0.74	0.42	0.16	0.05	0.03	1.00	0.98	0.90	0.64	0.32	0.13	0.07	1.00	0.99	0.96	0.79	0.46	0.21	0.12
	FDR	0.00	0.03	0.08	0.11	0.12	0.14	0.14	0.00	0.03	0.06	0.10	0.14	0.24	0.34	0.00	0.03	0.07	0.11	0.17	0.31	0.49
	FNR	1.00	0.62	0.44	0.34	0.23	0.08	0.00	0.98	0.39	0.26	0.21	0.16	0.06	0.00	0.46	0.23	0.15	0.13	0.10	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.27	0.52	0.81	1.10	1.36	1.51	0.04	0.16	0.40	0.62	0.84	1.07	1.18	0.02	0.13	0.33	0.53	0.73	0.95	1.02
	sd($\hat{\theta}$)	0.07	0.11	0.12	0.14	0.18	0.23	0.26	0.04	0.08	0.11	0.16	0.21	0.19	0.21	0.02	0.10	0.11	0.13	0.15	0.22	0.18
	$\theta = 0.01$	0.03	0.72	0.98	1.00	1.00	1.00	1.00	0.00	0.52	0.97	1.00	1.00	1.00	1.00	0.00	0.41	0.95	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.24	0.31	0.39	0.45	0.44	0.49	0.00	0.09	0.16	0.17	0.19	0.21	0.21	0.00	0.06	0.10	0.12	0.10	0.12	0.12
	$\theta = 1$	1.00	0.99	0.84	0.41	0.12	0.04	0.01	1.00	1.00	0.96	0.75	0.38	0.11	0.04	1.00	1.00	0.99	0.91	0.60	0.17	0.10
	FDR	0.00	0.03	0.07	0.08	0.08	0.10	0.10	0.00	0.02	0.05	0.08	0.12	0.18	0.28	0.00	0.02	0.05	0.08	0.14	0.24	0.48
	FNR	1.00	0.66	0.45	0.36	0.25	0.09	0.00	1.00	0.40	0.27	0.21	0.16	0.07	0.00	0.93	0.23	0.15	0.12	0.09	0.05	0.00

Table B.148: Ng(2008) test for DGP 1d. I(1) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.18	0.53	0.86	1.07	1.35	1.48	0.04	0.09	0.41	0.61	0.85	1.06	1.13	0.02	0.05	0.34	0.54	0.71	0.92	1.03
	sd($\hat{\theta}$)	0.24	0.23	0.36	0.32	0.64	0.60	0.42	0.08	0.16	0.24	0.53	0.37	0.70	0.59	0.06	0.12	0.35	0.26	0.62	0.41	0.31
	$\theta = 0.01$	0.00	0.04	0.25	0.48	0.61	0.74	0.78	0.00	0.01	0.17	0.37	0.54	0.73	0.78	0.00	0.01	0.16	0.32	0.48	0.69	0.73
10	$\theta = \theta_0$	0.00	0.01	0.08	0.14	0.19	0.21	0.21	0.00	0.01	0.09	0.13	0.16	0.16	0.14	0.00	0.04	0.11	0.13	0.13	0.15	0.15
	$\theta = 1$	1.00	0.94	0.59	0.38	0.25	0.13	0.08	1.00	0.98	0.68	0.50	0.32	0.17	0.11	1.00	1.00	0.73	0.54	0.38	0.21	0.15
	FDR	0.00	0.06	0.13	0.18	0.25	0.30	0.34	0.00	0.07	0.14	0.20	0.25	0.37	0.48	0.00	0.08	0.14	0.19	0.28	0.39	0.52
	FNR	0.75	0.60	0.42	0.31	0.19	0.07	0.00	0.30	0.30	0.28	0.20	0.13	0.05	0.00	0.06	0.11	0.19	0.15	0.09	0.04	0.00
	ave($\hat{\theta}$)	0.09	0.29	0.58	0.82	1.11	1.35	1.51	0.04	0.19	0.42	0.64	0.86	1.10	1.20	0.02	0.13	0.34	0.53	0.74	0.95	1.03
	sd($\hat{\theta}$)	0.11	0.12	0.24	0.17	0.18	0.45	0.33	0.08	0.10	0.17	0.16	0.17	0.26	0.24	0.04	0.25	0.08	0.15	0.25	0.19	0.23
	$\theta = 0.01$	0.02	0.44	0.86	0.97	0.99	0.99	1.00	0.00	0.27	0.80	0.96	0.99	1.00	1.00	0.00	0.15	0.70	0.93	0.99	1.00	1.00
30	$\theta = \theta_0$	0.01	0.13	0.23	0.24	0.32	0.30	0.35	0.00	0.05	0.12	0.16	0.16	0.19	0.18	0.00	0.03	0.10	0.12	0.13	0.14	0.11
	$\theta = 1$	1.00	0.92	0.68	0.37	0.13	0.05	0.03	1.00	0.96	0.85	0.59	0.30	0.11	0.06	1.00	0.98	0.94	0.75	0.46	0.17	0.11
	FDR	0.00	0.03	0.08	0.11	0.12	0.15	0.19	0.00	0.03	0.06	0.11	0.15	0.22	0.30	0.00	0.03	0.07	0.11	0.17	0.29	0.46
	FNR	1.00	0.67	0.48	0.36	0.24	0.09	0.00	0.98	0.44	0.30	0.23	0.16	0.07	0.00	0.52	0.27	0.18	0.14	0.10	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.29	0.55	0.82	1.11	1.37	1.53	0.04	0.19	0.41	0.64	0.87	1.08	1.19	0.02	0.14	0.34	0.54	0.75	0.95	1.05
	sd($\hat{\theta}$)	0.08	0.11	0.09	0.18	0.16	0.20	0.30	0.06	0.10	0.12	0.16	0.16	0.25	0.15	0.03	0.06	0.10	0.13	0.17	0.17	0.18
	$\theta = 0.01$	0.04	0.75	0.99	1.00	1.00	1.00	1.00	0.00	0.55	0.96	1.00	1.00	1.00	1.00	0.00	0.41	0.96	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.27	0.31	0.37	0.41	0.44	0.46	0.00	0.10	0.18	0.20	0.21	0.23	0.20	0.00	0.05	0.10	0.11	0.12	0.12	0.13
	$\theta = 1$	1.00	0.98	0.80	0.40	0.12	0.03	0.01	1.00	0.99	0.95	0.72	0.32	0.10	0.04	1.00	1.00	0.99	0.89	0.53	0.18	0.08
	FDR	0.00	0.03	0.08	0.10	0.10	0.10	0.10	0.00	0.02	0.05	0.08	0.12	0.19	0.26	0.00	0.02	0.05	0.08	0.13	0.24	0.43
	FNR	1.00	0.69	0.49	0.37	0.25	0.09	0.00	1.00	0.47	0.30	0.23	0.17	0.07	0.00	0.97	0.29	0.17	0.13	0.10	0.05	0.00

Table B.149: Ng(2008) test for DGP 2a. I(1) common factor component and I(0) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.14	0.14	0.12	0.12	0.12	0.12	0.04	0.07	0.07	0.06	0.06	0.06	0.06	0.02	0.04	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.18	0.11	0.35	0.13	0.13	0.23	0.13	0.06	0.12	0.12	0.12	0.13	0.10	0.17	0.05	0.26	0.03	0.07	0.04	0.03	0.05
	$\theta = 0.01$	0.01	0.05	0.03	0.02	0.03	0.02	0.03	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	$\theta = \theta_0$	0.00	0.01	0.18	0.71	0.92	0.98	0.99	0.00	0.01	0.63	0.95	1.00	1.00	1.00	0.00	0.08	0.91	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	0.96	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.28	0.50	0.70	0.90	1.00	0.00	0.07	0.29	0.50	0.70	0.90	1.00	0.00	0.08	0.30	0.50	0.70	0.90	1.00
	FNR	0.69	0.59	0.51	0.41	0.24	0.08	0.00	0.24	0.24	0.29	0.20	0.12	0.04	0.00	0.03	0.08	0.09	0.04	0.02	0.01	0.00
	ave($\hat{\theta}$)	0.09	0.13	0.12	0.12	0.12	0.12	0.12	0.04	0.06	0.06	0.06	0.06	0.06	0.06	0.02	0.03	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.09	0.10	0.11	0.09	0.11	0.09	0.10	0.06	0.08	0.06	0.07	0.05	0.06	0.07	0.03	0.04	0.04	0.03	0.04	0.06	0.05
	$\theta = 0.01$	0.02	0.15	0.12	0.12	0.10	0.11	0.13	0.00	0.04	0.01	0.03	0.04	0.02	0.03	0.00	0.01	0.01	0.00	0.00	0.00	0.00
30	$\theta = \theta_0$	0.00	0.01	0.48	0.94	1.00	1.00	1.00	0.00	0.02	0.96	1.00	1.00	1.00	1.00	0.00	0.36	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.08	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.78	0.67	0.48	0.30	0.10	0.00	0.97	0.71	0.64	0.48	0.31	0.10	0.00	0.45	0.55	0.50	0.35	0.21	0.07	0.00
	ave($\hat{\theta}$)	0.09	0.12	0.12	0.12	0.12	0.12	0.12	0.04	0.06	0.06	0.06	0.06	0.06	0.05	0.02	0.03	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.08	0.09	0.08	0.07	0.11	0.06	0.08	0.04	0.04	0.04	0.06	0.06	0.05	0.06	0.03	0.04	0.04	0.03	0.03	0.03	0.03
	$\theta = 0.01$	0.03	0.20	0.21	0.22	0.18	0.19	0.19	0.00	0.09	0.06	0.06	0.06	0.07	0.05	0.00	0.01	0.01	0.01	0.01	0.00	0.01
50	$\theta = \theta_0$	0.01	0.02	0.66	0.99	1.00	1.00	1.00	0.00	0.02	0.99	1.00	1.00	1.00	1.00	0.00	0.68	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.10	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.82	0.68	0.50	0.30	0.10	0.00	1.00	0.79	0.67	0.51	0.30	0.10	0.00	0.95	0.80	0.67	0.50	0.29	0.09	0.00

Table B.150: Ng(2008) test for DGP 2b. I(1) common factor component and I(0) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.16	0.20	0.17	0.17	0.16	0.16	0.04	0.09	0.11	0.09	0.08	0.08	0.08	0.02	0.04	0.06	0.05	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.16	0.13	0.22	0.15	0.26	0.19	0.16	0.07	0.09	0.10	0.23	0.12	0.10	0.16	0.04	0.08	0.09	0.11	0.08	0.06	0.10
	$\theta = 0.01$	0.00	0.03	0.08	0.05	0.05	0.05	0.04	0.00	0.01	0.02	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	$\theta = \theta_0$	0.00	0.00	0.07	0.46	0.77	0.93	0.96	0.00	0.00	0.34	0.81	0.97	1.00	1.00	0.00	0.04	0.66	0.96	1.00	1.00	1.00
	$\theta = 1$	1.00	0.95	0.92	0.96	0.97	0.97	0.97	1.00	0.99	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.25	0.48	0.69	0.90	1.00	0.00	0.07	0.27	0.49	0.70	0.90	1.00	0.00	0.08	0.28	0.50	0.70	0.90	1.00
	FNR	0.72	0.62	0.49	0.41	0.25	0.10	0.00	0.30	0.30	0.32	0.29	0.18	0.06	0.00	0.05	0.09	0.17	0.12	0.06	0.03	0.00
	ave($\hat{\theta}$)	0.09	0.18	0.17	0.15	0.15	0.15	0.15	0.04	0.10	0.08	0.08	0.08	0.07	0.07	0.02	0.05	0.04	0.04	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.12	0.21	0.12	0.12	0.12	0.13	0.12	0.05	0.06	0.07	0.06	0.06	0.06	0.06	0.03	0.05	0.04	0.04	0.05	0.05	0.04
	$\theta = 0.01$	0.01	0.24	0.25	0.23	0.22	0.21	0.21	0.00	0.09	0.08	0.07	0.07	0.05	0.07	0.00	0.03	0.01	0.01	0.01	0.01	0.01
30	$\theta = \theta_0$	0.01	0.03	0.21	0.88	0.99	1.00	1.00	0.00	0.01	0.83	0.99	1.00	1.00	1.00	0.00	0.11	0.99	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.06	0.29	0.50	0.70	0.90	1.00	0.00	0.07	0.29	0.50	0.70	0.90	1.00	0.00	0.08	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.71	0.65	0.50	0.29	0.10	0.00	0.99	0.56	0.64	0.47	0.29	0.10	0.00	0.51	0.41	0.57	0.43	0.26	0.09	0.00
	ave($\hat{\theta}$)	0.09	0.17	0.16	0.16	0.15	0.15	0.15	0.04	0.09	0.08	0.07	0.07	0.07	0.08	0.02	0.04	0.04	0.04	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.08	0.15	0.10	0.09	0.07	0.08	0.10	0.05	0.07	0.09	0.05	0.05	0.05	0.05	0.03	0.06	0.03	0.03	0.03	0.03	0.03
	$\theta = 0.01$	0.04	0.37	0.39	0.39	0.36	0.39	0.38	0.00	0.16	0.15	0.11	0.15	0.14	0.16	0.00	0.04	0.03	0.03	0.03	0.03	0.02
50	$\theta = \theta_0$	0.01	0.07	0.39	0.94	1.00	1.00	1.00	0.00	0.01	0.95	1.00	1.00	1.00	1.00	0.00	0.31	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.29	0.50	0.70	0.90	1.00	0.00	0.08	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.74	0.65	0.49	0.29	0.10	0.00	1.00	0.68	0.68	0.48	0.28	0.09	0.00	0.95	0.65	0.68	0.48	0.29	0.11	0.00

Table B.151: Ng(2008) test for DGP 2c. I(1) common factor component and I(0) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.13	0.12	0.12	0.12	0.12	0.11	0.04	0.08	0.06	0.06	0.06	0.05	0.05	0.02	0.04	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.18	0.18	0.22	0.16	0.17	0.42	0.17	0.05	0.04	0.09	0.11	0.06	0.18	0.09	0.07	0.08	0.06	0.10	0.09	0.09	0.11
	$\theta = 0.01$	0.01	0.03	0.02	0.02	0.01	0.02	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	$\theta = \theta_0$	0.00	0.00	0.22	0.73	0.92	0.98	0.99	0.00	0.01	0.70	0.96	0.99	1.00	1.00	0.00	0.08	0.94	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	0.97	0.99	0.99	1.00	0.99	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.29	0.50	0.70	0.90	1.00	0.00	0.07	0.30	0.50	0.70	0.90	1.00	0.00	0.08	0.30	0.50	0.70	0.90	1.00
	FNR	0.74	0.55	0.54	0.40	0.23	0.09	0.00	0.24	0.28	0.28	0.21	0.13	0.03	0.00	0.03	0.06	0.07	0.06	0.03	0.01	0.00
	ave($\hat{\theta}$)	0.09	0.12	0.12	0.12	0.12	0.12	0.12	0.04	0.06	0.05	0.06	0.06	0.06	0.06	0.02	0.03	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.12	0.09	0.09	0.07	0.12	0.10	0.12	0.08	0.05	0.05	0.07	0.05	0.06	0.06	0.03	0.03	0.05	0.03	0.04	0.03	0.06
	$\theta = 0.01$	0.02	0.13	0.11	0.10	0.12	0.12	0.11	0.00	0.04	0.02	0.03	0.03	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	$\theta = \theta_0$	0.00	0.01	0.48	0.98	1.00	1.00	1.00	0.00	0.02	0.97	1.00	1.00	1.00	1.00	0.00	0.46	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.10	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.84	0.70	0.49	0.32	0.10	0.00	0.98	0.82	0.68	0.48	0.31	0.09	0.00	0.48	0.62	0.50	0.36	0.21	0.08	0.00
	ave($\hat{\theta}$)	0.09	0.12	0.11	0.12	0.12	0.12	0.12	0.04	0.06	0.05	0.06	0.06	0.06	0.05	0.02	0.03	0.03	0.03	0.03	0.03	0.03
	sd($\hat{\theta}$)	0.07	0.08	0.08	0.07	0.08	0.07	0.09	0.06	0.05	0.07	0.06	0.05	0.05	0.04	0.03	0.04	0.04	0.03	0.03	0.03	0.04
	$\theta = 0.01$	0.03	0.21	0.19	0.22	0.19	0.20	0.19	0.00	0.07	0.06	0.06	0.06	0.06	0.05	0.00	0.01	0.01	0.01	0.01	0.01	0.01
50	$\theta = \theta_0$	0.01	0.01	0.70	0.99	1.00	1.00	1.00	0.00	0.03	1.00	1.00	1.00	1.00	1.00	0.00	0.74	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.10	0.30	0.50	0.70	0.90	1.00	0.00	0.10	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.87	0.69	0.50	0.30	0.10	0.00	1.00	0.86	0.70	0.50	0.30	0.10	0.00	0.94	0.86	0.69	0.49	0.32	0.09	0.00

Table B.152: Ng(2008) test for DGP 2d. I(1) common factor component and I(0) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.15	0.19	0.16	0.15	0.15	0.16	0.04	0.08	0.10	0.08	0.08	0.08	0.08	0.02	0.04	0.05	0.04	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.13	0.19	0.32	0.18	0.17	0.26	0.21	0.08	0.08	0.14	0.14	0.12	0.11	0.07	0.06	0.10	0.06	0.07	0.08	0.09	0.10
	$\theta = 0.01$	0.00	0.03	0.05	0.04	0.03	0.03	0.03	0.00	0.01	0.02	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	$\theta = \theta_0$	0.00	0.00	0.09	0.52	0.82	0.93	0.96	0.00	0.00	0.39	0.85	0.98	1.00	1.00	0.00	0.05	0.73	0.98	1.00	1.00	1.00
	$\theta = 1$	1.00	0.96	0.93	0.96	0.98	0.98	0.98	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.26	0.50	0.70	0.90	1.00	0.00	0.07	0.28	0.50	0.70	0.90	1.00	0.00	0.08	0.29	0.50	0.70	0.90	1.00
	FNR	0.74	0.61	0.56	0.46	0.29	0.09	0.00	0.30	0.29	0.36	0.31	0.19	0.05	0.00	0.05	0.09	0.17	0.10	0.07	0.02	0.00
	ave($\hat{\theta}$)	0.09	0.18	0.16	0.15	0.15	0.15	0.15	0.04	0.09	0.07	0.07	0.07	0.07	0.07	0.02	0.05	0.04	0.03	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.09	0.20	0.11	0.08	0.10	0.09	0.11	0.05	0.06	0.05	0.10	0.07	0.09	0.06	0.08	0.06	0.05	0.05	0.05	0.03	0.05
	$\theta = 0.01$	0.02	0.25	0.24	0.21	0.22	0.23	0.22	0.00	0.06	0.05	0.06	0.05	0.07	0.05	0.00	0.02	0.01	0.00	0.00	0.01	0.01
30	$\theta = \theta_0$	0.01	0.03	0.25	0.87	1.00	1.00	1.00	0.00	0.00	0.88	1.00	1.00	1.00	1.00	0.00	0.13	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.07	0.30	0.50	0.70	0.90	1.00	0.00	0.07	0.30	0.50	0.70	0.90	1.00	0.00	0.08	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.75	0.68	0.50	0.29	0.10	0.00	0.98	0.66	0.68	0.48	0.29	0.10	0.00	0.51	0.52	0.59	0.42	0.27	0.09	0.00
	ave($\hat{\theta}$)	0.09	0.16	0.15	0.15	0.15	0.15	0.15	0.04	0.08	0.07	0.07	0.07	0.07	0.08	0.02	0.04	0.04	0.03	0.04	0.04	0.04
	sd($\hat{\theta}$)	0.08	0.14	0.08	0.11	0.11	0.08	0.08	0.04	0.04	0.06	0.07	0.06	0.06	0.06	0.03	0.05	0.04	0.03	0.04	0.03	0.03
	$\theta = 0.01$	0.03	0.39	0.37	0.40	0.38	0.38	0.40	0.00	0.16	0.13	0.15	0.16	0.13	0.13	0.00	0.04	0.02	0.02	0.03	0.03	0.03
50	$\theta = \theta_0$	0.01	0.04	0.40	0.95	1.00	1.00	1.00	0.00	0.01	0.96	1.00	1.00	1.00	1.00	0.00	0.42	1.00	1.00	1.00	1.00	1.00
	$\theta = 1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	FDR	0.00	0.08	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00	0.00	0.09	0.30	0.50	0.70	0.90	1.00
	FNR	1.00	0.82	0.68	0.48	0.30	0.10	0.00	1.00	0.79	0.67	0.50	0.30	0.09	0.00	0.95	0.79	0.69	0.47	0.30	0.11	0.00

Table B.153: Ng(2008) test for DGP 3a. I(0) common factor component and I(1) idiosyncratic component. Low dependence, one common factor.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.08	0.23	0.53	0.81	1.09	1.38	1.45	0.04	0.16	0.42	0.65	0.81	1.05	1.12	0.02	0.13	0.35	0.55	0.72	0.90	0.94
	sd($\hat{\theta}$)	0.17	0.19	0.18	0.67	0.41	0.43	0.35	0.07	0.14	0.44	0.20	0.32	0.81	0.60	0.06	0.12	0.23	0.31	0.47	0.32	0.41
	$\theta=0.01$	0.00	0.12	0.37	0.59	0.72	0.82	0.83	0.00	0.10	0.32	0.52	0.66	0.79	0.80	0.00	0.09	0.26	0.46	0.60	0.74	0.76
10	$\theta=\theta_0$	0.00	0.04	0.11	0.16	0.19	0.21	0.22	0.00	0.03	0.10	0.13	0.14	0.15	0.18	0.00	0.03	0.10	0.11	0.13	0.14	0.14
	$\theta=1$	1.00	0.89	0.59	0.37	0.20	0.10	0.10	1.00	0.94	0.72	0.48	0.30	0.19	0.15	1.00	0.95	0.77	0.55	0.38	0.21	0.17
	FDR	0.00	0.04	0.11	0.17	0.24	0.29	0.33	0.00	0.04	0.10	0.18	0.28	0.37	0.48	0.00	0.04	0.11	0.18	0.29	0.44	0.57
	FNR	0.70	0.51	0.39	0.31	0.21	0.08	0.00	0.22	0.25	0.26	0.24	0.15	0.06	0.00	0.03	0.19	0.20	0.18	0.13	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.24	0.54	0.85	1.12	1.42	1.52	0.04	0.16	0.40	0.63	0.86	1.07	1.18	0.02	0.13	0.34	0.54	0.75	0.94	1.03
	sd($\hat{\theta}$)	0.09	0.13	0.16	0.33	0.30	0.26	0.49	0.05	0.09	0.18	0.16	0.22	0.23	0.44	0.04	0.07	0.18	0.15	0.19	0.22	0.27
	$\theta=0.01$	0.02	0.44	0.87	0.98	0.99	1.00	1.00	0.00	0.32	0.83	0.97	0.99	1.00	1.00	0.00	0.29	0.79	0.96	0.99	1.00	1.00
30	$\theta=\theta_0$	0.01	0.14	0.22	0.31	0.34	0.38	0.38	0.00	0.07	0.14	0.14	0.18	0.17	0.19	0.00	0.04	0.10	0.10	0.11	0.13	0.13
	$\theta=1$	1.00	0.97	0.72	0.36	0.15	0.04	0.02	1.00	0.99	0.89	0.62	0.33	0.10	0.07	1.00	1.00	0.95	0.78	0.43	0.18	0.11
	FDR	0.00	0.03	0.07	0.09	0.12	0.13	0.13	0.00	0.02	0.06	0.10	0.16	0.23	0.34	0.00	0.02	0.06	0.11	0.17	0.31	0.48
	FNR	1.00	0.61	0.43	0.35	0.24	0.09	0.00	0.97	0.35	0.26	0.22	0.16	0.07	0.00	0.49	0.22	0.17	0.14	0.11	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.23	0.54	0.84	1.12	1.42	1.56	0.04	0.16	0.40	0.64	0.85	1.10	1.19	0.02	0.12	0.34	0.53	0.75	0.94	1.03
	sd($\hat{\theta}$)	0.08	0.11	0.08	0.19	0.17	0.20	0.34	0.06	0.05	0.12	0.10	0.14	0.16	0.22	0.03	0.05	0.08	0.11	0.15	0.17	0.23
	$\theta=0.01$	0.03	0.65	0.99	1.00	1.00	1.00	1.00	0.00	0.58	0.97	1.00	1.00	1.00	1.00	0.00	0.47	0.96	1.00	1.00	1.00	1.00
50	$\theta=\theta_0$	0.02	0.19	0.33	0.44	0.46	0.52	0.55	0.00	0.09	0.15	0.19	0.21	0.23	0.22	0.00	0.06	0.10	0.11	0.13	0.13	0.13
	$\theta=1$	1.00	0.99	0.82	0.39	0.12	0.02	0.01	1.00	1.00	0.96	0.73	0.35	0.09	0.04	1.00	1.00	0.99	0.90	0.53	0.20	0.10
	FDR	0.00	0.03	0.06	0.07	0.07	0.07	0.06	0.00	0.02	0.05	0.07	0.11	0.16	0.26	0.00	0.02	0.05	0.08	0.13	0.26	0.48
	FNR	1.00	0.61	0.45	0.37	0.25	0.09	0.00	1.00	0.38	0.27	0.22	0.17	0.07	0.00	0.94	0.21	0.16	0.12	0.10	0.05	0.00

Table B.154: Ng(2008) test for DGP 3b. I(0) common factor component and I(1) idiosyncratic component. Low dependence, two common factors.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.27	0.59	0.88	1.18	1.41	1.52	0.04	0.19	0.45	0.73	0.93	1.06	1.16	0.02	0.16	0.39	0.63	0.79	0.93	0.99
	sd($\hat{\theta}$)	0.18	0.21	0.28	0.22	0.28	0.75	0.42	0.09	0.14	0.18	0.19	0.58	0.39	0.51	0.05	0.22	0.25	0.23	0.32	0.35	0.38
	$\theta = 0.01$	0.00	0.13	0.37	0.54	0.68	0.80	0.80	0.00	0.09	0.29	0.52	0.65	0.71	0.79	0.00	0.09	0.27	0.46	0.60	0.69	0.73
10	$\theta = \theta_0$	0.00	0.04	0.10	0.15	0.21	0.22	0.23	0.00	0.03	0.07	0.11	0.13	0.16	0.16	0.00	0.03	0.06	0.07	0.11	0.12	0.15
	$\theta = 1$	1.00	0.84	0.52	0.31	0.18	0.10	0.09	1.00	0.91	0.65	0.39	0.22	0.17	0.12	1.00	0.94	0.72	0.42	0.29	0.18	0.17
	FDR	0.00	0.03	0.11	0.19	0.26	0.31	0.31	0.00	0.03	0.11	0.19	0.28	0.41	0.47	0.00	0.04	0.11	0.19	0.30	0.47	0.55
	FNR	0.73	0.54	0.43	0.36	0.24	0.09	0.00	0.29	0.32	0.33	0.30	0.20	0.07	0.00	0.06	0.22	0.25	0.26	0.17	0.06	0.00
	ave($\hat{\theta}$)	0.09	0.25	0.57	0.89	1.18	1.48	1.60	0.04	0.17	0.42	0.64	0.89	1.13	1.22	0.02	0.13	0.35	0.57	0.76	0.96	1.04
	sd($\hat{\theta}$)	0.10	0.18	0.12	0.34	0.20	0.44	0.33	0.07	0.07	0.31	0.15	0.18	0.22	0.27	0.03	0.08	0.11	0.10	0.14	0.22	0.28
	$\theta = 0.01$	0.02	0.42	0.88	0.97	0.99	1.00	1.00	0.00	0.32	0.81	0.95	0.99	1.00	1.00	0.00	0.31	0.80	0.96	0.99	1.00	1.00
30	$\theta = \theta_0$	0.00	0.14	0.27	0.34	0.39	0.39	0.42	0.00	0.06	0.14	0.15	0.21	0.20	0.18	0.00	0.04	0.08	0.11	0.12	0.10	0.13
	$\theta = 1$	1.00	0.96	0.68	0.30	0.12	0.03	0.01	1.00	0.99	0.86	0.62	0.28	0.09	0.05	1.00	1.00	0.96	0.72	0.41	0.14	0.10
	FDR	0.00	0.03	0.07	0.09	0.12	0.11	0.11	0.00	0.02	0.06	0.11	0.17	0.23	0.29	0.00	0.02	0.06	0.11	0.18	0.32	0.47
	FNR	1.00	0.61	0.47	0.38	0.25	0.09	0.00	0.98	0.38	0.29	0.24	0.18	0.07	0.00	0.54	0.25	0.20	0.18	0.13	0.06	0.00
	ave($\hat{\theta}$)	0.09	0.25	0.56	0.88	1.18	1.49	1.64	0.04	0.16	0.42	0.64	0.89	1.11	1.25	0.02	0.13	0.34	0.56	0.77	0.95	1.08
	sd($\hat{\theta}$)	0.06	0.14	0.10	0.26	0.23	0.30	0.30	0.06	0.07	0.10	0.13	0.16	0.20	0.15	0.03	0.06	0.10	0.11	0.18	0.18	0.13
	$\theta = 0.01$	0.04	0.69	0.97	1.00	1.00	1.00	1.00	0.00	0.57	0.97	1.00	1.00	1.00	1.00	0.00	0.48	0.97	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.22	0.38	0.45	0.52	0.58	0.61	0.00	0.09	0.21	0.19	0.23	0.23	0.27	0.00	0.06	0.09	0.11	0.13	0.11	0.13
	$\theta = 1$	1.00	0.99	0.77	0.34	0.09	0.01	0.00	1.00	1.00	0.93	0.70	0.30	0.07	0.03	1.00	1.00	1.00	0.88	0.50	0.16	0.06
	FDR	0.00	0.03	0.06	0.08	0.07	0.05	0.05	0.00	0.02	0.05	0.08	0.11	0.17	0.21	0.00	0.02	0.05	0.08	0.14	0.26	0.38
	FNR	1.00	0.63	0.47	0.39	0.27	0.10	0.00	1.00	0.38	0.30	0.24	0.18	0.08	0.00	0.95	0.24	0.16	0.15	0.12	0.05	0.00

Table B.155: Ng(2008) test for DGP 3c. I(0) common factor component and I(1) idiosyncratic component. High dependence, one common factor.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.24	0.52	0.83	1.11	1.41	1.52	0.04	0.17	0.41	0.63	0.85	1.05	1.17	0.02	0.14	0.35	0.56	0.77	0.94	0.99
	sd($\hat{\theta}$)	0.13	0.27	0.25	0.40	0.37	0.41	0.61	0.08	0.14	0.23	0.27	0.43	0.25	0.40	0.06	0.13	0.21	0.30	0.39	0.44	0.31
	$\theta = 0.01$	0.00	0.13	0.38	0.60	0.71	0.82	0.87	0.00	0.11	0.33	0.51	0.66	0.78	0.81	0.00	0.08	0.29	0.48	0.66	0.77	0.78
10	$\theta = \theta_0$	0.00	0.05	0.11	0.15	0.21	0.23	0.23	0.00	0.03	0.09	0.13	0.15	0.17	0.16	0.00	0.03	0.08	0.11	0.11	0.13	0.13
	$\theta = 1$	1.00	0.86	0.60	0.35	0.22	0.11	0.09	1.00	0.93	0.72	0.49	0.30	0.17	0.11	1.00	0.96	0.78	0.54	0.32	0.20	0.15
	FDR	0.00	0.04	0.11	0.16	0.25	0.29	0.31	0.00	0.04	0.11	0.18	0.26	0.39	0.46	0.00	0.04	0.10	0.18	0.27	0.42	0.53
	FNR	0.70	0.49	0.38	0.31	0.21	0.08	0.00	0.24	0.27	0.27	0.23	0.16	0.06	0.00	0.03	0.19	0.20	0.19	0.14	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.24	0.54	0.85	1.15	1.45	1.56	0.04	0.16	0.40	0.64	0.86	1.08	1.19	0.02	0.13	0.34	0.54	0.74	0.94	1.05
	sd($\hat{\theta}$)	0.08	0.08	0.12	0.34	0.33	0.16	0.21	0.05	0.15	0.16	0.16	0.23	0.16	0.26	0.04	0.11	0.12	0.16	0.30	0.34	0.27
	$\theta = 0.01$	0.01	0.44	0.86	0.97	0.99	1.00	1.00	0.00	0.33	0.83	0.97	0.99	1.00	1.00	0.00	0.29	0.80	0.95	0.99	1.00	1.00
30	$\theta = \theta_0$	0.00	0.12	0.26	0.30	0.36	0.41	0.40	0.00	0.07	0.13	0.15	0.16	0.20	0.18	0.00	0.06	0.10	0.10	0.12	0.12	0.11
	$\theta = 1$	1.00	0.97	0.72	0.34	0.13	0.03	0.01	1.00	0.99	0.90	0.62	0.29	0.13	0.06	1.00	1.00	0.95	0.78	0.45	0.18	0.09
	FDR	0.00	0.03	0.07	0.09	0.11	0.10	0.13	0.00	0.02	0.06	0.10	0.14	0.25	0.32	0.00	0.02	0.06	0.11	0.17	0.32	0.44
	FNR	1.00	0.60	0.44	0.36	0.24	0.09	0.00	0.98	0.35	0.26	0.23	0.16	0.07	0.00	0.45	0.24	0.17	0.14	0.11	0.05	0.00
	ave($\hat{\theta}$)	0.09	0.24	0.56	0.84	1.15	1.40	1.59	0.04	0.16	0.39	0.64	0.87	1.09	1.20	0.02	0.12	0.33	0.55	0.75	0.94	1.05
	sd($\hat{\theta}$)	0.08	0.07	0.18	0.18	0.15	0.15	0.30	0.04	0.07	0.10	0.19	0.11	0.22	0.14	0.03	0.07	0.09	0.11	0.20	0.14	0.19
	$\theta = 0.01$	0.03	0.64	0.98	1.00	1.00	1.00	1.00	0.00	0.56	0.96	1.00	1.00	1.00	1.00	0.00	0.51	0.96	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.20	0.36	0.43	0.50	0.51	0.57	0.00	0.09	0.14	0.20	0.22	0.23	0.23	0.00	0.05	0.11	0.12	0.12	0.11	0.12
	$\theta = 1$	1.00	0.99	0.79	0.40	0.09	0.02	0.00	1.00	1.00	0.97	0.72	0.32	0.10	0.03	1.00	1.00	0.99	0.88	0.54	0.17	0.08
	FDR	0.00	0.03	0.05	0.07	0.06	0.07	0.04	0.00	0.02	0.05	0.07	0.11	0.17	0.23	0.00	0.02	0.05	0.08	0.13	0.25	0.43
	FNR	1.00	0.61	0.46	0.37	0.26	0.09	0.00	1.00	0.37	0.25	0.23	0.17	0.07	0.00	0.94	0.22	0.15	0.13	0.11	0.05	0.00

Table B.156: Ng(2008) test for DGP 3d. I(0) common factor component and I(1) idiosyncratic component. High dependence, two common factors.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.28	0.61	0.91	1.20	1.51	1.61	0.04	0.18	0.45	0.74	0.94	1.12	1.18	0.02	0.14	0.39	0.63	0.78	0.98	1.00
	sd($\hat{\theta}$)	0.19	0.13	0.17	0.39	0.24	1.04	0.75	0.11	0.09	0.47	0.21	0.33	0.21	0.31	0.08	0.53	0.26	0.25	0.18	0.27	0.29
	$\theta = 0.01$	0.00	0.15	0.39	0.58	0.70	0.81	0.85	0.00	0.09	0.30	0.52	0.64	0.75	0.80	0.00	0.07	0.27	0.45	0.56	0.73	0.75
10	$\theta = \theta_0$	0.00	0.06	0.11	0.16	0.19	0.24	0.25	0.00	0.02	0.07	0.12	0.14	0.17	0.16	0.00	0.02	0.05	0.08	0.08	0.12	0.13
	$\theta = 1$	1.00	0.82	0.52	0.30	0.16	0.08	0.08	1.00	0.91	0.64	0.39	0.23	0.16	0.13	1.00	0.94	0.69	0.43	0.29	0.16	0.14
	FDR	0.00	0.03	0.11	0.18	0.24	0.26	0.28	0.00	0.04	0.11	0.18	0.28	0.38	0.45	0.00	0.04	0.11	0.20	0.32	0.42	0.55
	FNR	0.74	0.55	0.45	0.37	0.24	0.09	0.00	0.32	0.31	0.33	0.30	0.20	0.08	0.00	0.04	0.21	0.27	0.25	0.17	0.07	0.00
	ave($\hat{\theta}$)	0.09	0.25	0.58	0.88	1.19	1.51	1.61	0.04	0.17	0.44	0.67	0.89	1.12	1.23	0.02	0.13	0.36	0.58	0.76	0.95	1.06
	sd($\hat{\theta}$)	0.07	0.14	0.32	0.22	0.23	0.39	0.18	0.07	0.14	0.10	0.19	0.25	0.24	0.36	0.05	0.13	0.11	0.15	0.16	0.21	0.26
	$\theta = 0.01$	0.02	0.42	0.88	0.97	0.99	1.00	1.00	0.00	0.33	0.83	0.97	1.00	1.00	1.00	0.00	0.29	0.78	0.96	0.99	1.00	1.00
30	$\theta = \theta_0$	0.01	0.13	0.26	0.32	0.39	0.45	0.42	0.00	0.06	0.14	0.16	0.18	0.19	0.21	0.00	0.06	0.09	0.11	0.11	0.11	0.11
	$\theta = 1$	1.00	0.95	0.67	0.33	0.10	0.02	0.02	1.00	1.00	0.87	0.58	0.26	0.09	0.05	1.00	1.00	0.94	0.72	0.41	0.16	0.07
	FDR	0.00	0.03	0.07	0.10	0.11	0.09	0.12	0.00	0.02	0.06	0.11	0.16	0.23	0.30	0.00	0.02	0.06	0.11	0.19	0.32	0.45
	FNR	1.00	0.62	0.47	0.37	0.26	0.09	0.00	0.98	0.38	0.31	0.26	0.18	0.07	0.00	0.54	0.24	0.20	0.18	0.13	0.06	0.00
	ave($\hat{\theta}$)	0.09	0.25	0.56	0.89	1.19	1.49	1.67	0.04	0.16	0.41	0.66	0.89	1.14	1.23	0.02	0.13	0.34	0.56	0.76	0.97	1.06
	sd($\hat{\theta}$)	0.09	0.09	0.19	0.17	0.24	0.33	0.21	0.06	0.08	0.15	0.12	0.16	0.19	0.22	0.04	0.08	0.09	0.14	0.15	0.15	0.16
	$\theta = 0.01$	0.02	0.66	0.97	1.00	1.00	1.00	1.00	0.00	0.57	0.96	1.00	1.00	1.00	1.00	0.00	0.48	0.95	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.01	0.22	0.37	0.48	0.54	0.60	0.63	0.00	0.09	0.18	0.22	0.24	0.26	0.24	0.00	0.07	0.10	0.11	0.11	0.13	0.11
	$\theta = 1$	1.00	0.98	0.79	0.33	0.09	0.01	0.00	1.00	1.00	0.95	0.68	0.30	0.07	0.03	1.00	1.00	0.99	0.87	0.51	0.16	0.06
	FDR	0.00	0.03	0.06	0.07	0.07	0.05	0.04	0.00	0.02	0.05	0.08	0.12	0.16	0.23	0.00	0.02	0.05	0.08	0.14	0.25	0.42
	FNR	1.00	0.63	0.47	0.39	0.26	0.10	0.00	1.00	0.39	0.28	0.25	0.19	0.08	0.00	0.96	0.23	0.17	0.15	0.12	0.06	0.00

Table B.157: Ng(2008) test for DGP 4a. I(0) common factor component and I(0) idiosyncratic component. No dependence(zero factor loading), iidN idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.07	0.15	0.36	0.55	0.75	0.87	0.98	0.03	0.12	0.31	0.49	0.69	0.87	0.92	0.02	0.11	0.31	0.49	0.67	0.89	0.91
	sd($\hat{\theta}$)	0.16	0.11	0.10	0.48	0.30	0.76	0.29	0.05	0.21	0.18	0.33	0.31	0.33	0.37	0.06	0.21	0.49	0.39	0.45	0.58	0.33
	$\theta = 0.01$	0.00	0.08	0.30	0.48	0.60	0.70	0.75	0.00	0.08	0.25	0.41	0.55	0.69	0.68	0.00	0.07	0.24	0.39	0.53	0.67	0.66
10	$\theta = \theta_0$	0.00	0.02	0.05	0.08	0.09	0.10	0.12	0.00	0.01	0.05	0.07	0.08	0.09	0.10	0.00	0.01	0.06	0.06	0.05	0.08	0.10
	$\theta = 1$	1.00	0.97	0.79	0.52	0.31	0.17	0.14	1.00	0.99	0.85	0.59	0.34	0.16	0.13	1.00	0.98	0.84	0.60	0.33	0.16	0.13
	FDR	0.00	0.03	0.09	0.18	0.29	0.51	0.55	0.00	0.04	0.10	0.18	0.30	0.50	0.59	0.00	0.04	0.10	0.18	0.29	0.51	0.60
	FNR	0.60	0.32	0.22	0.19	0.15	0.07	0.00	0.17	0.19	0.16	0.14	0.12	0.07	0.00	0.03	0.15	0.15	0.14	0.12	0.08	0.00
	ave($\hat{\theta}$)	0.07	0.18	0.37	0.58	0.77	0.96	1.05	0.04	0.13	0.33	0.52	0.72	0.94	1.00	0.02	0.11	0.32	0.51	0.70	0.89	0.99
	sd($\hat{\theta}$)	0.07	0.08	0.13	0.26	0.16	0.15	0.26	0.05	0.07	0.16	0.15	0.20	0.23	0.26	0.04	0.05	0.13	0.24	0.17	0.21	0.25
	$\theta = 0.01$	0.00	0.36	0.81	0.95	0.99	1.00	1.00	0.00	0.26	0.74	0.96	0.99	1.00	1.00	0.00	0.21	0.72	0.93	0.99	1.00	1.00
30	$\theta = \theta_0$	0.00	0.06	0.07	0.09	0.10	0.09	0.10	0.00	0.03	0.04	0.06	0.06	0.08	0.07	0.00	0.01	0.06	0.06	0.05	0.05	0.06
	$\theta = 1$	1.00	1.00	0.95	0.72	0.38	0.11	0.07	1.00	1.00	0.98	0.80	0.45	0.13	0.07	1.00	1.00	0.98	0.83	0.47	0.16	0.07
	FDR	0.00	0.02	0.06	0.10	0.18	0.34	0.47	0.00	0.02	0.06	0.10	0.18	0.34	0.52	0.00	0.02	0.06	0.11	0.19	0.39	0.51
	FNR	1.00	0.44	0.24	0.19	0.14	0.06	0.00	0.97	0.26	0.16	0.13	0.10	0.06	0.00	0.59	0.19	0.14	0.11	0.09	0.05	0.00
	ave($\hat{\theta}$)	0.07	0.17	0.38	0.58	0.79	0.98	1.06	0.04	0.14	0.34	0.53	0.74	0.92	1.03	0.02	0.12	0.32	0.52	0.72	0.92	1.00
	sd($\hat{\theta}$)	0.06	0.11	0.10	0.14	0.23	0.15	0.24	0.05	0.07	0.08	0.13	0.13	0.15	0.24	0.03	0.07	0.09	0.13	0.22	0.18	0.20
	$\theta = 0.01$	0.00	0.54	0.95	1.00	1.00	1.00	1.00	0.00	0.48	0.95	1.00	1.00	1.00	1.00	0.00	0.38	0.93	1.00	1.00	1.00	1.00
50	$\theta = \theta_0$	0.00	0.07	0.09	0.10	0.11	0.11	0.08	0.00	0.04	0.05	0.07	0.06	0.07	0.08	0.00	0.02	0.05	0.05	0.06	0.06	0.07
	$\theta = 1$	1.00	1.00	0.99	0.84	0.41	0.12	0.04	1.00	1.00	1.00	0.89	0.51	0.15	0.07	1.00	1.00	1.00	0.93	0.54	0.15	0.08
	FDR	0.00	0.02	0.05	0.09	0.13	0.27	0.39	0.00	0.02	0.05	0.08	0.14	0.30	0.45	0.00	0.02	0.05	0.08	0.14	0.30	0.51
	FNR	1.00	0.44	0.25	0.19	0.14	0.07	0.00	1.00	0.28	0.16	0.12	0.10	0.05	0.00	0.99	0.19	0.13	0.10	0.09	0.05	0.00

Table B.158: Ng(2008) test for DGP 4b. I(0) common factor component and I(0) idiosyncratic component. No dependence (zero factor loading), ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.07	0.18	0.36	0.61	0.82	1.06	1.10	0.03	0.13	0.35	0.57	0.76	0.95	1.07	0.02	0.12	0.32	0.53	0.74	0.90	1.01
	sd($\hat{\theta}$)	0.13	0.12	0.15	0.40	0.48	0.55	0.27	0.09	0.14	0.29	0.32	0.55	0.39	0.39	0.07	0.11	0.27	0.29	0.33	0.69	0.72
	$\theta = 0.01$	0.00	0.11	0.29	0.49	0.63	0.74	0.73	0.00	0.07	0.26	0.44	0.58	0.67	0.72	0.00	0.08	0.23	0.39	0.53	0.60	0.67
10	$\theta = \theta_0$	0.00	0.03	0.09	0.12	0.14	0.17	0.16	0.00	0.02	0.09	0.10	0.11	0.13	0.12	0.00	0.03	0.08	0.08	0.09	0.10	0.11
	$\theta = 1$	1.00	0.95	0.79	0.53	0.31	0.19	0.14	1.00	0.97	0.79	0.53	0.34	0.19	0.12	1.00	0.96	0.81	0.56	0.30	0.16	0.11
	FDR	0.00	0.04	0.13	0.21	0.31	0.43	0.49	0.00	0.04	0.11	0.19	0.31	0.45	0.48	0.00	0.05	0.11	0.19	0.29	0.48	0.55
	FNR	0.54	0.39	0.26	0.24	0.18	0.08	0.00	0.16	0.21	0.21	0.20	0.15	0.06	0.00	0.03	0.17	0.18	0.18	0.14	0.06	0.00
	ave($\hat{\theta}$)	0.07	0.17	0.39	0.62	0.85	1.05	1.18	0.03	0.14	0.33	0.55	0.78	0.98	1.06	0.02	0.11	0.32	0.52	0.75	0.94	1.06
	sd($\hat{\theta}$)	0.06	0.11	0.19	0.12	0.25	0.18	0.23	0.04	0.09	0.14	0.16	0.18	0.21	0.33	0.03	0.09	0.17	0.18	0.20	0.16	0.29
	$\theta = 0.01$	0.01	0.32	0.79	0.95	0.99	1.00	1.00	0.00	0.25	0.76	0.92	0.99	1.00	1.00	0.00	0.21	0.72	0.93	0.99	1.00	1.00
30	$\theta = \theta_0$	0.00	0.05	0.12	0.16	0.18	0.17	0.19	0.00	0.04	0.08	0.11	0.11	0.12	0.12	0.00	0.02	0.07	0.07	0.09	0.09	0.08
	$\theta = 1$	1.00	0.99	0.91	0.63	0.33	0.14	0.08	1.00	1.00	0.96	0.73	0.38	0.15	0.09	1.00	1.00	0.97	0.79	0.41	0.13	0.06
	FDR	0.00	0.03	0.08	0.14	0.21	0.33	0.36	0.00	0.03	0.07	0.12	0.18	0.34	0.45	0.00	0.03	0.07	0.12	0.19	0.34	0.44
	FNR	1.00	0.48	0.30	0.24	0.18	0.07	0.00	0.96	0.28	0.19	0.17	0.14	0.06	0.00	0.42	0.20	0.15	0.13	0.12	0.05	0.00
	ave($\hat{\theta}$)	0.07	0.18	0.39	0.62	0.85	1.07	1.15	0.03	0.14	0.33	0.55	0.76	0.98	1.08	0.02	0.12	0.32	0.52	0.73	0.95	1.05
	sd($\hat{\theta}$)	0.07	0.08	0.15	0.14	0.15	0.30	0.24	0.04	0.09	0.12	0.14	0.17	0.20	0.21	0.04	0.07	0.13	0.13	0.17	0.24	0.21
	$\theta = 0.01$	0.01	0.55	0.95	1.00	1.00	1.00	1.00	0.00	0.47	0.94	1.00	1.00	1.00	1.00	0.00	0.40	0.94	0.99	1.00	1.00	1.00
50	$\theta = \theta_0$	0.00	0.09	0.14	0.19	0.20	0.19	0.19	0.00	0.04	0.09	0.10	0.12	0.13	0.11	0.00	0.04	0.07	0.07	0.07	0.09	0.08
	$\theta = 1$	1.00	1.00	0.95	0.71	0.36	0.09	0.07	1.00	1.00	0.99	0.86	0.49	0.16	0.07	1.00	1.00	1.00	0.89	0.52	0.15	0.05
	FDR	0.00	0.03	0.07	0.12	0.17	0.26	0.32	0.00	0.02	0.06	0.10	0.17	0.27	0.40	0.00	0.02	0.05	0.10	0.15	0.27	0.43
	FNR	1.00	0.51	0.30	0.24	0.18	0.08	0.00	1.00	0.31	0.19	0.16	0.13	0.06	0.00	0.93	0.21	0.14	0.12	0.10	0.06	0.00

Table B.159: Ng(2008) test for DGP 4c. I(0) common factor component and I(0) idiosyncratic component. Low dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.08	0.35	0.80	1.01	1.08	1.00	0.94	0.04	0.27	0.70	1.01	1.01	0.98	0.92	0.03	0.24	0.65	0.95	1.01	0.93	0.93
	sd($\hat{\theta}$)	0.14	0.14	0.32	1.07	0.48	0.32	0.50	0.16	0.12	0.25	0.22	0.29	0.55	0.34	0.08	0.26	0.20	0.35	1.02	0.30	0.31
	$\theta = 0.01$	0.00	0.21	0.38	0.49	0.65	0.73	0.73	0.00	0.13	0.32	0.41	0.58	0.71	0.70	0.00	0.12	0.27	0.37	0.53	0.67	0.70
10	$\theta = \theta_0$	0.00	0.09	0.12	0.13	0.10	0.11	0.13	0.00	0.05	0.10	0.11	0.08	0.09	0.09	0.00	0.04	0.08	0.09	0.07	0.06	0.09
	$\theta = 1$	0.99	0.79	0.43	0.25	0.16	0.13	0.16	1.00	0.83	0.46	0.23	0.16	0.12	0.13	1.00	0.86	0.47	0.22	0.13	0.12	0.12
	FDR	0.00	0.03	0.08	0.17	0.29	0.46	0.58	0.00	0.03	0.08	0.16	0.30	0.46	0.60	0.00	0.03	0.09	0.17	0.30	0.48	0.59
	FNR	0.65	0.48	0.43	0.36	0.25	0.08	0.00	0.29	0.37	0.37	0.34	0.23	0.08	0.00	0.12	0.29	0.36	0.33	0.24	0.08	0.00
	ave($\hat{\theta}$)	0.08	0.30	0.75	1.04	1.06	1.02	1.01	0.05	0.24	0.74	1.02	1.01	0.98	0.98	0.03	0.21	0.63	0.98	1.01	0.98	0.98
	sd($\hat{\theta}$)	0.09	0.15	0.60	0.57	0.29	0.18	0.27	0.05	0.19	0.31	0.13	0.17	0.22	0.20	0.08	0.25	0.38	0.19	0.84	0.24	0.23
	$\theta = 0.01$	0.01	0.36	0.59	0.76	0.93	1.00	0.99	0.00	0.27	0.51	0.67	0.91	1.00	1.00	0.00	0.21	0.41	0.62	0.86	1.00	1.00
30	$\theta = \theta_0$	0.00	0.13	0.16	0.17	0.16	0.10	0.09	0.00	0.09	0.14	0.15	0.11	0.07	0.10	0.00	0.06	0.08	0.13	0.11	0.08	0.05
	$\theta = 1$	1.00	0.88	0.51	0.24	0.14	0.10	0.10	1.00	0.91	0.51	0.24	0.16	0.10	0.10	1.00	0.93	0.53	0.26	0.14	0.10	0.06
	FDR	0.00	0.02	0.07	0.13	0.23	0.37	0.49	0.00	0.02	0.06	0.13	0.26	0.40	0.56	0.00	0.02	0.06	0.14	0.26	0.40	0.54
	FNR	1.00	0.58	0.46	0.38	0.25	0.09	0.00	0.98	0.42	0.41	0.36	0.24	0.08	0.00	0.67	0.33	0.37	0.35	0.24	0.08	0.00
	ave($\hat{\theta}$)	0.08	0.30	0.79	1.03	1.05	1.01	1.03	0.04	0.23	0.70	1.00	1.04	1.00	1.00	0.02	0.21	0.67	1.03	1.01	1.00	1.00
	sd($\hat{\theta}$)	0.10	0.26	0.49	0.23	0.19	0.12	0.19	0.06	0.36	0.17	0.26	0.55	0.17	0.21	0.09	0.80	0.22	0.63	0.43	0.19	0.19
	$\theta = 0.01$	0.02	0.45	0.69	0.79	0.98	1.00	1.00	0.00	0.32	0.51	0.72	0.97	1.00	1.00	0.00	0.28	0.46	0.65	0.95	1.00	1.00
50	$\theta = \theta_0$	0.01	0.15	0.19	0.21	0.19	0.11	0.11	0.00	0.09	0.13	0.14	0.15	0.07	0.08	0.00	0.06	0.11	0.15	0.13	0.09	0.06
	$\theta = 1$	1.00	0.90	0.53	0.27	0.15	0.09	0.09	1.00	0.94	0.52	0.26	0.14	0.07	0.07	1.00	0.94	0.54	0.25	0.15	0.08	0.07
	FDR	0.00	0.02	0.06	0.13	0.21	0.36	0.48	0.00	0.02	0.06	0.13	0.21	0.34	0.51	0.00	0.01	0.06	0.13	0.22	0.38	0.52
	FNR	1.00	0.59	0.46	0.38	0.25	0.09	0.00	1.00	0.44	0.40	0.37	0.24	0.09	0.00	0.98	0.35	0.38	0.36	0.24	0.08	0.00

Table B.160: Ng(2008) test for DGP 4d. I(0) common factor component and I(0) idiosyncratic component. Low dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.08	0.30	0.73	1.07	1.18	1.16	1.16	0.04	0.27	0.69	0.94	1.10	1.08	1.05	0.02	0.26	0.68	0.97	1.04	1.05	1.03
	sd($\hat{\theta}$)	0.24	0.10	0.20	0.31	0.52	0.54	0.37	0.15	0.27	0.17	1.10	0.99	0.25	0.29	0.08	0.12	0.39	0.63	0.47	0.53	0.40
	$\theta = 0.01$	0.01	0.17	0.35	0.53	0.67	0.76	0.76	0.00	0.12	0.29	0.41	0.55	0.71	0.74	0.00	0.13	0.26	0.42	0.54	0.67	0.70
10	$\theta = \theta_0$	0.00	0.07	0.12	0.17	0.16	0.14	0.15	0.00	0.05	0.09	0.11	0.12	0.11	0.09	0.00	0.07	0.09	0.09	0.10	0.08	0.09
	$\theta = 1$	0.99	0.83	0.48	0.27	0.15	0.14	0.14	1.00	0.84	0.45	0.25	0.15	0.11	0.10	1.00	0.84	0.48	0.21	0.15	0.11	0.10
	FDR	0.00	0.03	0.10	0.19	0.27	0.41	0.45	0.00	0.03	0.09	0.18	0.31	0.41	0.50	0.00	0.04	0.09	0.17	0.29	0.42	0.51
	FNR	0.66	0.49	0.44	0.38	0.26	0.09	0.00	0.29	0.35	0.40	0.34	0.24	0.09	0.00	0.10	0.31	0.37	0.34	0.24	0.09	0.00
	ave($\hat{\theta}$)	0.09	0.30	0.80	1.13	1.16	1.15	1.14	0.04	0.23	0.76	1.02	1.06	1.06	1.07	0.02	0.22	0.69	1.01	1.09	1.02	1.03
	sd($\hat{\theta}$)	0.12	0.22	0.16	0.21	0.33	0.22	0.28	0.09	0.10	0.37	0.37	0.19	0.26	0.25	0.08	0.16	0.74	0.60	0.33	0.31	0.31
	$\theta = 0.01$	0.01	0.37	0.61	0.78	0.95	1.00	1.00	0.00	0.25	0.50	0.67	0.91	1.00	1.00	0.00	0.21	0.44	0.61	0.89	1.00	1.00
30	$\theta = \theta_0$	0.00	0.13	0.20	0.23	0.25	0.20	0.15	0.00	0.08	0.14	0.14	0.16	0.11	0.10	0.00	0.06	0.13	0.11	0.13	0.11	0.07
	$\theta = 1$	1.00	0.89	0.47	0.23	0.11	0.08	0.08	1.00	0.91	0.48	0.24	0.15	0.09	0.07	1.00	0.91	0.50	0.23	0.14	0.09	0.07
	FDR	0.00	0.02	0.08	0.15	0.21	0.30	0.40	0.00	0.02	0.07	0.15	0.24	0.35	0.44	0.00	0.02	0.07	0.15	0.24	0.38	0.47
	FNR	1.00	0.60	0.49	0.41	0.27	0.09	0.00	0.97	0.43	0.44	0.38	0.25	0.09	0.00	0.59	0.36	0.40	0.36	0.25	0.08	0.00
	ave($\hat{\theta}$)	0.08	0.30	0.85	1.07	1.17	1.14	1.13	0.04	0.24	0.75	1.06	1.10	1.06	1.06	0.02	0.21	0.69	1.02	1.07	1.04	1.04
	sd($\hat{\theta}$)	0.11	0.14	0.24	0.21	0.24	0.22	0.16	0.07	0.19	0.27	0.26	0.25	0.17	0.14	0.06	0.23	0.12	0.56	0.63	0.16	0.23
	$\theta = 0.01$	0.02	0.47	0.70	0.84	0.98	1.00	1.00	0.00	0.33	0.54	0.75	0.96	1.00	1.00	0.00	0.25	0.45	0.65	0.95	1.00	1.00
50	$\theta = \theta_0$	0.01	0.15	0.23	0.24	0.29	0.23	0.18	0.00	0.11	0.14	0.16	0.20	0.15	0.11	0.00	0.07	0.11	0.14	0.15	0.10	0.09
	$\theta = 1$	1.00	0.90	0.50	0.23	0.12	0.06	0.08	1.00	0.91	0.52	0.27	0.14	0.07	0.06	1.00	0.93	0.50	0.25	0.14	0.08	0.06
	FDR	0.00	0.02	0.08	0.14	0.20	0.24	0.34	0.00	0.02	0.07	0.14	0.21	0.31	0.42	0.00	0.02	0.07	0.14	0.21	0.34	0.43
	FNR	1.00	0.61	0.51	0.41	0.27	0.09	0.00	1.00	0.45	0.44	0.38	0.25	0.09	0.00	0.97	0.35	0.40	0.36	0.25	0.09	0.00

Table B.161: Ng(2008) test for DGP 4e. I(0) common factor component and I(0) idiosyncratic component. High dependence, iidN idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.10	0.41	1.06	1.41	1.39	1.12	0.98	0.05	0.32	0.96	1.26	1.26	1.06	0.98	0.03	0.27	0.86	1.28	1.26	1.05	0.98
	sd($\hat{\theta}$)	0.10	0.21	0.75	1.06	0.99	0.39	0.44	0.07	0.20	0.70	1.83	0.42	0.32	0.59	0.10	0.57	0.68	0.55	0.47	0.58	0.43
	$\theta = 0.01$	0.01	0.20	0.34	0.43	0.60	0.77	0.73	0.00	0.15	0.28	0.39	0.51	0.74	0.74	0.00	0.12	0.26	0.32	0.47	0.70	0.73
10	$\theta = \theta_0$	0.00	0.10	0.15	0.16	0.13	0.09	0.12	0.00	0.07	0.12	0.11	0.10	0.10	0.09	0.00	0.05	0.11	0.11	0.10	0.09	0.08
	$\theta = 1$	0.96	0.73	0.42	0.19	0.12	0.12	0.13	1.00	0.81	0.39	0.18	0.11	0.11	0.11	1.00	0.84	0.40	0.17	0.11	0.12	0.09
	FDR	0.00	0.04	0.13	0.21	0.28	0.42	0.55	0.00	0.04	0.12	0.20	0.30	0.44	0.54	0.00	0.04	0.11	0.20	0.32	0.46	0.56
	FNR	0.66	0.59	0.55	0.41	0.26	0.09	0.00	0.32	0.45	0.48	0.39	0.25	0.09	0.00	0.12	0.35	0.43	0.37	0.25	0.09	0.00
	ave($\hat{\theta}$)	0.10	0.34	1.09	1.61	1.43	1.08	1.03	0.05	0.29	0.96	1.46	1.35	1.07	1.01	0.03	0.25	0.87	1.39	1.28	1.07	0.99
	sd($\hat{\theta}$)	0.09	0.25	0.57	1.97	0.52	0.20	0.21	0.08	0.26	1.00	1.03	0.33	0.21	0.24	0.06	0.42	0.30	1.03	0.33	0.28	0.24
	$\theta = 0.01$	0.00	0.26	0.44	0.56	0.80	1.00	0.99	0.00	0.20	0.34	0.46	0.74	1.00	1.00	0.00	0.16	0.30	0.40	0.69	1.00	1.00
30	$\theta = \theta_0$	0.00	0.10	0.16	0.20	0.18	0.12	0.10	0.00	0.07	0.12	0.15	0.15	0.11	0.09	0.00	0.05	0.12	0.15	0.12	0.09	0.06
	$\theta = 1$	0.99	0.82	0.40	0.18	0.10	0.08	0.09	1.00	0.84	0.39	0.15	0.10	0.07	0.08	1.00	0.86	0.39	0.16	0.09	0.07	0.08
	FDR	0.00	0.03	0.12	0.17	0.24	0.37	0.48	0.00	0.03	0.11	0.18	0.25	0.38	0.53	0.00	0.02	0.10	0.18	0.28	0.37	0.52
	FNR	1.00	0.65	0.58	0.43	0.27	0.09	0.00	0.95	0.51	0.51	0.41	0.25	0.09	0.00	0.64	0.39	0.46	0.38	0.25	0.09	0.00
	ave($\hat{\theta}$)	0.10	0.33	1.13	1.54	1.43	1.10	1.02	0.05	0.27	0.91	1.49	1.42	1.06	1.01	0.03	0.25	0.81	1.38	1.35	1.05	1.00
	sd($\hat{\theta}$)	0.16	0.12	0.67	0.31	0.30	0.17	0.17	0.08	0.11	0.41	0.56	0.22	0.34	0.23	0.06	0.18	1.49	0.38	1.49	0.15	0.18
	$\theta = 0.01$	0.01	0.30	0.49	0.60	0.86	1.00	1.00	0.00	0.20	0.35	0.49	0.79	1.00	1.00	0.00	0.18	0.28	0.41	0.77	1.00	1.00
50	$\theta = \theta_0$	0.00	0.11	0.19	0.18	0.20	0.15	0.10	0.00	0.07	0.11	0.16	0.16	0.11	0.10	0.00	0.06	0.11	0.14	0.14	0.10	0.07
	$\theta = 1$	0.98	0.84	0.40	0.17	0.09	0.07	0.09	1.00	0.87	0.41	0.15	0.08	0.07	0.08	1.00	0.87	0.39	0.14	0.08	0.08	0.07
	FDR	0.00	0.03	0.11	0.16	0.23	0.32	0.48	0.00	0.03	0.11	0.17	0.23	0.36	0.50	0.00	0.02	0.10	0.17	0.24	0.38	0.51
	FNR	1.00	0.65	0.58	0.42	0.27	0.09	0.00	1.00	0.51	0.52	0.42	0.26	0.09	0.00	0.95	0.39	0.46	0.39	0.25	0.09	0.00

Table B.162: Ng(2008) test for DGP 4f. I(0) common factor component and I(0) idiosyncratic component. High dependence, ARMA(1,1) idiosyncratic component.

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.10	0.42	1.19	1.47	1.41	1.29	1.16	0.05	0.34	0.93	1.47	1.37	1.16	1.09	0.03	0.31	0.93	1.27	1.27	1.14	1.05
	sd($\hat{\theta}$)	0.20	0.18	1.16	1.02	0.64	0.43	0.51	0.05	0.45	0.29	0.34	0.27	0.49	0.38	0.06	0.04	1.56	0.61	1.00	0.48	0.39
	$\theta = 0.01$	0.01	0.19	0.36	0.46	0.61	0.77	0.75	0.00	0.16	0.28	0.40	0.53	0.74	0.74	0.00	0.14	0.26	0.35	0.47	0.70	0.72
10	$\theta = \theta_0$	0.00	0.09	0.15	0.17	0.15	0.16	0.18	0.00	0.07	0.12	0.14	0.14	0.11	0.10	0.00	0.06	0.12	0.11	0.09	0.10	0.08
	$\theta = 1$	0.97	0.73	0.38	0.20	0.13	0.11	0.13	0.99	0.79	0.41	0.17	0.12	0.09	0.09	1.00	0.80	0.38	0.16	0.08	0.09	0.09
	FDR	0.00	0.04	0.12	0.20	0.28	0.35	0.48	0.00	0.04	0.12	0.20	0.30	0.40	0.50	0.00	0.04	0.11	0.19	0.31	0.41	0.51
	FNR	0.66	0.61	0.55	0.41	0.27	0.09	0.00	0.35	0.46	0.48	0.41	0.26	0.09	0.00	0.13	0.39	0.45	0.38	0.25	0.09	0.00
	ave($\hat{\theta}$)	0.10	0.39	1.10	1.51	1.52	1.23	1.14	0.05	0.29	0.94	1.44	1.36	1.10	1.09	0.03	0.25	0.90	1.27	1.37	1.06	1.05
	sd($\hat{\theta}$)	0.10	0.10	1.50	0.71	0.74	0.19	0.20	0.17	0.39	0.87	0.36	0.99	0.56	0.24	0.19	0.15	0.27	0.66	0.43	0.69	0.33
	$\theta = 0.01$	0.01	0.32	0.47	0.57	0.86	0.99	1.00	0.00	0.21	0.34	0.46	0.76	0.99	1.00	0.00	0.17	0.28	0.39	0.71	0.99	1.00
30	$\theta = \theta_0$	0.00	0.14	0.18	0.18	0.25	0.23	0.17	0.00	0.07	0.14	0.16	0.15	0.12	0.12	0.00	0.05	0.11	0.13	0.15	0.09	0.09
	$\theta = 1$	0.98	0.79	0.39	0.18	0.09	0.06	0.10	1.00	0.83	0.41	0.15	0.08	0.07	0.07	1.00	0.86	0.38	0.14	0.08	0.08	0.08
	FDR	0.00	0.04	0.12	0.19	0.21	0.29	0.40	0.00	0.03	0.12	0.18	0.25	0.34	0.42	0.00	0.03	0.11	0.19	0.27	0.40	0.43
	FNR	1.00	0.69	0.58	0.44	0.28	0.10	0.00	0.93	0.54	0.51	0.42	0.27	0.09	0.00	0.58	0.42	0.47	0.39	0.26	0.09	0.00
	ave($\hat{\theta}$)	0.10	0.35	1.09	1.53	1.40	1.22	1.16	0.05	0.27	0.86	1.37	1.30	1.13	1.07	0.03	0.24	0.87	1.39	1.30	1.12	1.05
	sd($\hat{\theta}$)	0.17	0.11	0.85	0.45	0.28	0.24	0.18	0.17	0.13	0.45	0.52	0.61	0.23	0.20	0.09	0.11	0.62	0.34	0.85	0.28	0.27
	$\theta = 0.01$	0.02	0.31	0.51	0.62	0.87	1.00	1.00	0.00	0.22	0.33	0.47	0.81	1.00	1.00	0.00	0.19	0.30	0.39	0.73	1.00	1.00
50	$\theta = \theta_0$	0.01	0.14	0.18	0.18	0.22	0.26	0.19	0.00	0.08	0.12	0.14	0.15	0.18	0.11	0.00	0.05	0.11	0.15	0.13	0.13	0.08
	$\theta = 1$	0.98	0.82	0.39	0.17	0.08	0.05	0.07	1.00	0.87	0.42	0.16	0.10	0.06	0.07	1.00	0.88	0.36	0.15	0.08	0.05	0.05
	FDR	0.00	0.04	0.12	0.18	0.23	0.23	0.34	0.00	0.03	0.12	0.19	0.26	0.32	0.40	0.00	0.03	0.10	0.17	0.27	0.29	0.45
	FNR	1.00	0.68	0.57	0.44	0.28	0.10	0.00	1.00	0.52	0.51	0.41	0.26	0.09	0.00	0.92	0.41	0.48	0.39	0.26	0.09	0.00

B.3 Empirical Application: a PPP Study

Table B.163: I(0) and I(1) Classification by $SPSM$ Test

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	1	0	1	1	1	0	0	1	1	1	1	1	1	0	1
2	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1
3	0	1	0	1	1	0	0	1	0	1	1	1	0	0	1
4	0	1	0	1	1	1	0	1	1	1	1	1	1	0	1
5	0	0	0	1	1	0	0	1	1	1	1	1	1	0	1
6	1	0	1	0	0	0	0	1	1	1	1	1	1	0	1
7	0	1	0	0	1	0	0	1	1	1	1	0	0	1	0
8	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1
10	0	1	0	1	0	1	0	1	1	1	1	1	0	1	1
11	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1
12	0	1	0	1	1	1	0	1	1	1	1	1	1	0	1
13	0	0	1	0	0	0	0	1	1	1	1	1	1	1	1
14	0	0	1	0	1	0	0	1	1	1	1	1	1	0	1
15	1	1	1	1	0	0	1	1	1	1	1	1	0	0	1
16	0	1	0	1	0	1	0	1	1	1	1	1	1	0	1
17	0	0	0	1	0	0	0	1	1	1	0	0	1	1	1
18	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1
19	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1
20	0	1	1	0	1	0	0	1	1	1	1	0	0	1	1
21	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
22	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1
23	0	1	0	1	0	0	0	1	1	1	1	1	1	0	1
24	0	1	0	1	1	1	0	1	0	1	1	1	1	0	1
25	1	1	1	0	1	0	0	1	1	1	1	1	1	0	1
26	1	0	1	1	1	0	0	1	1	1	0	0	0	0	1
27	1	1	1	1	0	0	0	1	1	1	1	1	0	1	1
28	1	1	1	1	0	0	0	1	0	1	1	0	0	1	1
29	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1
30	0	0	1	0	0	0	0	1	0	1	1	0	0	0	1
31	0	1	1	1	1	1	0	1	1	1	1	0	1	1	1
32	1	1	1	1	1	0	0	1	1	1	1	1	0	1	1
33	1	1	1	1	1	0	0	1	1	1	0	1	1	0	1
34	0	1	1	0	1	0	0	1	1	1	1	1	1	0	1
35	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table B.164: I(0) and I(1) Classification by RW05 Test

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1
2	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1
3	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1
6	0	1	1	0	1	1	1	1	1	1	1	0	1	1	1
7	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1
11	0	1	1	1	0	1	0	1	1	1	1	1	1	1	1
12	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1
13	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1
14	1	1	1	1	0	0	0	1	1	1	1	0	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1
17	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
19	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1
25	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
26	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1
32	1	1	1	0	1	1	0	1	1	1	1	1	0	1	1
33	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1
34	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1
35	1	1	1	0	0	0	0	1	1	1	1	0	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table B.165: I(0) and I(1) Classification by MP12 Test

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1
2	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1
3	1	0	1	1	1	1	0	1	1	1	1	0	1	0	1
4	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1
6	0	1	1	0	1	0	0	1	1	1	1	0	1	1	0
7	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	0	1	1	1	0	1	1	1	1	1	0	1	1	1
11	0	1	1	1	0	1	0	1	1	1	1	1	1	1	1
12	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1
13	1	1	0	0	0	1	0	1	1	1	1	0	1	1	1
14	1	1	1	0	0	0	0	1	1	1	1	0	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	0	1	1	0	1	0	1	1	1	1	1	1	1	1
17	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1
18	1	0	1	0	1	1	0	1	1	1	1	1	1	1	1
19	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1
20	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1
22	0	0	1	1	0	1	0	1	1	1	1	1	1	1	0
23	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1
24	1	0	0	1	1	0	1	1	1	0	1	1	1	1	1
25	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
26	1	0	1	0	1	0	1	1	1	1	1	0	1	1	1
27	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1
28	0	1	1	1	1	1	0	1	1	1	1	0	1	1	1
29	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	0	0	1	0	0	0	0	1	1	1	1	1	1	1	1
32	1	1	1	0	1	0	0	1	1	1	1	1	0	1	1
33	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
34	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1
35	0	1	1	0	0	0	0	1	1	1	1	0	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table B.166: I(0) and I(1) Classification by *BSQT* Test

City/Price Index	1	2	Perishables 4	5	6	7	8	Nonperi. 3	9	10	Services 11	12	13	14	15
1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1
2	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1
3	1	0	1	0	1	1	0	0	0	0	1	0	1	0	1
4	0	1	0	0	1	1	1	1	1	1	1	1	1	1	0
5	1	0	1	1	1	0	0	0	1	1	1	1	1	1	1
6	0	1	1	0	1	1	0	1	1	1	1	0	1	1	0
7	0	0	1	0	1	1	0	0	1	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
9	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
10	1	0	1	0	1	1	1	1	1	1	1	1	0	1	1
11	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1
12	0	0	0	1	0	0	1	0	0	1	1	1	1	1	1
13	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1
14	1	1	1	0	0	0	0	0	1	1	1	0	1	0	1
15	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	0	0	0	1	0	1	0	0	1	1	1	1	1	1	1
17	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1
18	1	0	1	0	1	1	0	1	1	1	1	1	1	1	1
19	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1
20	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1
22	0	1	1	1	0	1	0	0	1	1	1	1	1	1	0
23	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
24	1	0	0	1	1	0	1	1	1	0	1	1	1	1	1
25	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1
26	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1
27	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1
28	0	1	1	1	1	1	0	1	0	1	1	0	1	1	1
29	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1
30	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
31	0	0	1	0	0	1	0	1	1	1	1	1	0	1	1
32	1	1	1	0	1	1	0	1	1	1	1	1	0	1	1
33	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
34	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1
35	0	1	1	0	0	0	0	0	1	1	1	1	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table B.167: I(0) and I(1) Classification by $IBSQT$ Test

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	1
2	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1
3	1	0	1	1	1	1	1	0	1	1	1	0	1	0	1
4	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	1	1	1	0	0	0	1	1	1	1	1	1	1
6	0	1	1	0	1	1	1	1	1	1	1	0	1	1	1
7	0	0	1	0	1	1	0	0	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1
11	0	1	1	1	0	1	0	1	1	1	1	1	1	1	1
12	1	0	0	1	0	0	1	0	1	1	1	1	1	1	1
13	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1
14	1	1	1	1	0	0	0	0	1	1	1	0	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1
17	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
18	1	0	1	0	1	1	0	1	1	1	1	1	1	1	1
19	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
20	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
22	0	1	1	1	0	1	0	1	1	1	1	1	1	1	0
23	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
24	1	0	0	1	1	0	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1
26	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1
27	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
28	0	1	1	1	1	1	0	1	1	1	1	0	1	1	1
29	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	0	0	1	0	0	1	0	1	1	1	1	1	1	1	1
32	1	1	1	0	1	1	0	1	1	1	1	1	0	1	1
33	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
34	1	1	1	0	1	0	1	0	1	1	1	1	1	1	1
35	0	1	1	0	0	0	0	1	1	1	1	1	1	0	1

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table B.168: I(0) and I(1) Classification by Ng08 Test

City/Price Index	1	2	Perishables 4	5	6	7	8	Nonperi. 3	9	10	Services 11	12	13	14	15
1	1	0	0	0	0	0	0	0	1	0	1	1	1	0	1
2	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0
3	0	0	0	0	1	0	0	1	0	1	1	1	1	0	1
4	0	0	0	1	1	0	0	1	1	0	0	1	1	1	0
5	0	0	0	1	0	0	0	0	0	0	1	0	1	0	1
6	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	1	0	0	0	1	0	1	0	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1
10	0	0	0	0	0	0	0	0	1	1	1	0	1	1	0
11	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0
12	0	0	0	0	0	0	0	0	1	1	0	1	1	0	1
13	0	0	0	0	0	0	0	0	1	1	1	0	1	0	1
14	1	0	0	0	0	0	0	0	0	1	1	0	1	1	1
15	1	0	1	0	0	0	1	1	0	1	1	0	1	0	0
16	0	0	0	0	0	0	0	1	1	0	1	1	1	0	1
17	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1
18	0	1	1	0	0	0	0	1	1	1	1	1	1	0	0
19	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0
20	0	0	0	0	0	1	0	0	1	1	1	0	1	0	0
21	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0
22	0	0	1	0	0	0	0	1	0	1	0	1	1	0	0
23	0	0	0	0	0	0	0	1	0	1	1	1	1	1	0
24	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1
25	0	0	0	0	0	0	0	1	1	1	0	0	1	1	0
26	1	0	0	0	1	0	0	1	0	1	1	1	1	1	0
27	1	0	0	0	0	0	0	0	1	1	0	1	1	0	0
28	0	1	0	1	0	0	0	0	0	1	0	0	1	1	0
29	1	0	0	0	0	0	0	0	1	1	0	1	1	1	1
30	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0
31	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1
32	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1
33	1	1	1	0	0	0	0	1	1	1	1	1	1	0	1
34	1	0	1	0	1	0	0	0	1	1	0	1	1	0	1
35	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0

Note: Zero indicates that the relative price of the city is stationary, one indicates that it is nonstationary. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Appendix C

Testing Unit Root against Long Memory in a Mixed Panel

C.1 Simulation Results for Fisher Test

Table C.1: Rejection Frequencies of Fisher Test for DGP1

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	1.00	1.00	1.00	1.00	0.99	0.50	0.02	1.00	1.00	1.00	1.00	1.00	0.89	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.02
10	ELW	1.00	0.99	1.00	1.00	0.99	0.46	0.05	1.00	1.00	1.00	1.00	1.00	0.84	0.03	1.00	1.00	1.00	1.00	1.00	0.99	0.02
	FELW	1.00	1.00	1.00	1.00	1.00	0.66	0.07	1.00	1.00	1.00	1.00	1.00	0.91	0.05	1.00	1.00	1.00	1.00	1.00	0.99	0.05
	FELWd	1.00	1.00	1.00	1.00	0.97	0.42	0.02	1.00	1.00	1.00	1.00	1.00	0.82	0.02	1.00	1.00	1.00	1.00	1.00	0.99	0.02
	LW	1.00	1.00	1.00	1.00	1.00	0.89	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.01
30	ELW	0.98	0.99	0.99	0.99	1.00	0.84	0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.01
	FELW	1.00	1.00	1.00	1.00	1.00	0.94	0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.04
	FELWd	1.00	1.00	1.00	1.00	1.00	0.81	0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.02
	LW	1.00	1.00	1.00	1.00	1.00	0.99	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.00
50	ELW	0.98	0.98	0.98	0.98	0.99	0.96	0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.02
	FELW	1.00	1.00	1.00	1.00	1.00	0.99	0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.04
	FELWd	1.00	1.00	1.00	1.00	1.00	0.95	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	0.92	0.95	0.87	0.63	0.37	0.13	0.02	1.00	1.00	0.99	0.93	0.65	0.20	0.01	1.00	1.00	1.00	1.00	0.92	0.35	0.02
10	ELW	0.95	0.93	0.82	0.58	0.33	0.10	0.05	1.00	1.00	0.99	0.92	0.62	0.20	0.03	1.00	1.00	1.00	1.00	0.93	0.38	0.02
	FELW	0.99	0.97	0.92	0.74	0.43	0.15	0.07	1.00	1.00	1.00	0.95	0.69	0.21	0.05	1.00	1.00	1.00	1.00	0.94	0.37	0.05
	FELWd	0.94	0.90	0.77	0.56	0.31	0.11	0.02	1.00	1.00	0.99	0.91	0.59	0.18	0.02	1.00	1.00	1.00	1.00	0.91	0.33	0.02
	LW	1.00	1.00	1.00	0.96	0.74	0.21	0.00	1.00	1.00	1.00	1.00	0.96	0.38	0.01	1.00	1.00	1.00	1.00	1.00	0.75	0.01
30	ELW	1.00	1.00	1.00	0.94	0.67	0.18	0.05	1.00	1.00	1.00	1.00	0.96	0.35	0.01	1.00	1.00	1.00	1.00	1.00	0.70	0.01
	FELW	1.00	1.00	1.00	0.98	0.80	0.23	0.08	1.00	1.00	1.00	1.00	0.98	0.44	0.05	1.00	1.00	1.00	1.00	1.00	0.71	0.04
	FELWd	1.00	1.00	1.00	0.92	0.66	0.18	0.02	1.00	1.00	1.00	1.00	0.97	0.39	0.00	1.00	1.00	1.00	1.00	1.00	0.72	0.02
	LW	1.00	1.00	1.00	1.00	0.90	0.27	0.00	1.00	1.00	1.00	1.00	1.00	0.54	0.01	1.00	1.00	1.00	1.00	1.00	0.87	0.00
50	ELW	1.00	1.00	1.00	1.00	0.87	0.24	0.04	1.00	1.00	1.00	1.00	1.00	0.52	0.02	1.00	1.00	1.00	1.00	1.00	0.87	0.02
	FELW	1.00	1.00	1.00	1.00	0.94	0.32	0.08	1.00	1.00	1.00	1.00	1.00	0.58	0.06	1.00	1.00	1.00	1.00	1.00	0.85	0.04
	FELWd	1.00	1.00	1.00	0.99	0.85	0.23	0.01	1.00	1.00	1.00	1.00	1.00	0.50	0.00	1.00	1.00	1.00	1.00	1.00	0.87	0.01

Table C.2: Rejection Frequencies of Fisher Test for DGP2

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	1.00	1.00	1.00	1.00	0.89	0.32	0.02	1.00	1.00	1.00	1.00	1.00	0.62	0.02	1.00	1.00	1.00	1.00	1.00	0.94	0.03
10	ELW	1.00	0.99	1.00	0.99	0.84	0.32	0.06	1.00	1.00	1.00	1.00	1.00	0.56	0.03	1.00	1.00	1.00	1.00	1.00	0.91	0.04
	FELW	1.00	1.00	1.00	1.00	0.91	0.42	0.07	1.00	1.00	1.00	1.00	1.00	0.64	0.06	1.00	1.00	1.00	1.00	1.00	0.92	0.06
	FELWd	1.00	1.00	1.00	0.98	0.81	0.28	0.03	1.00	1.00	1.00	1.00	0.99	0.56	0.02	1.00	1.00	1.00	1.00	1.00	0.92	0.03
	LW	1.00	1.00	1.00	1.00	0.95	0.44	0.02	1.00	1.00	1.00	1.00	1.00	0.74	0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.01
30	ELW	0.98	0.99	0.99	0.99	0.92	0.40	0.05	1.00	1.00	1.00	1.00	1.00	0.67	0.04	1.00	1.00	1.00	1.00	1.00	0.99	0.03
	FELW	1.00	1.00	1.00	1.00	0.97	0.54	0.07	1.00	1.00	1.00	1.00	1.00	0.76	0.08	1.00	1.00	1.00	1.00	1.00	0.99	0.05
	FELWd	1.00	1.00	1.00	1.00	0.91	0.37	0.02	1.00	1.00	1.00	1.00	1.00	0.68	0.03	1.00	1.00	1.00	1.00	1.00	0.99	0.02
	LW	1.00	1.00	1.00	1.00	0.97	0.44	0.02	1.00	1.00	1.00	1.00	1.00	0.80	0.01	1.00	1.00	1.00	1.00	1.00	0.99	0.02
50	ELW	0.98	0.99	0.98	0.98	0.95	0.38	0.05	1.00	1.00	1.00	1.00	1.00	0.74	0.03	1.00	1.00	1.00	1.00	1.00	0.98	0.03
	FELW	1.00	1.00	1.00	1.00	0.98	0.53	0.07	1.00	1.00	1.00	1.00	1.00	0.81	0.06	1.00	1.00	1.00	1.00	1.00	0.98	0.05
	FELWd	1.00	1.00	1.00	0.99	0.94	0.36	0.03	1.00	1.00	1.00	1.00	1.00	0.75	0.02	1.00	1.00	1.00	1.00	1.00	0.98	0.03

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	0.79	0.74	0.60	0.42	0.24	0.10	0.02	0.94	0.93	0.85	0.67	0.44	0.15	0.02	1.00	0.99	0.98	0.92	0.71	0.22	0.03
10	ELW	0.72	0.70	0.58	0.41	0.23	0.09	0.06	0.92	0.92	0.83	0.66	0.41	0.13	0.03	1.00	1.00	0.98	0.92	0.67	0.21	0.04
	FELW	0.84	0.77	0.66	0.45	0.25	0.12	0.07	0.97	0.94	0.87	0.68	0.42	0.14	0.06	1.00	1.00	0.99	0.93	0.69	0.19	0.06
	FELWd	0.69	0.64	0.55	0.38	0.21	0.10	0.03	0.92	0.91	0.82	0.65	0.40	0.13	0.02	1.00	1.00	0.98	0.92	0.67	0.21	0.03
	LW	0.87	0.82	0.67	0.54	0.33	0.12	0.02	0.98	0.97	0.91	0.76	0.48	0.14	0.02	1.00	1.00	0.99	0.96	0.80	0.27	0.01
30	ELW	0.81	0.78	0.62	0.49	0.31	0.11	0.05	0.96	0.97	0.90	0.76	0.46	0.15	0.04	1.00	1.00	1.00	0.96	0.77	0.25	0.03
	FELW	0.91	0.85	0.71	0.58	0.35	0.11	0.07	0.99	0.98	0.93	0.79	0.48	0.15	0.08	1.00	1.00	1.00	0.96	0.78	0.23	0.05
	FELWd	0.83	0.76	0.58	0.47	0.30	0.10	0.02	0.96	0.96	0.90	0.74	0.47	0.14	0.03	1.00	1.00	1.00	0.96	0.78	0.24	0.02
	LW	0.93	0.84	0.72	0.52	0.31	0.11	0.02	0.98	0.97	0.92	0.81	0.54	0.17	0.01	1.00	1.00	1.00	0.96	0.81	0.27	0.02
50	ELW	0.85	0.81	0.70	0.51	0.30	0.10	0.05	0.97	0.96	0.91	0.79	0.52	0.18	0.03	1.00	1.00	1.00	0.97	0.80	0.25	0.03
	FELW	0.93	0.87	0.78	0.57	0.34	0.12	0.07	0.99	0.98	0.94	0.83	0.52	0.16	0.06	1.00	1.00	1.00	0.97	0.78	0.24	0.05
	FELWd	0.88	0.79	0.69	0.50	0.27	0.10	0.03	0.96	0.96	0.90	0.77	0.52	0.16	0.02	1.00	1.00	0.99	0.97	0.79	0.25	0.03

Table C.3: Rejection Frequencies of Fisher Test for DGP3

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	1.00	1.00	1.00	1.00	0.97	0.45	0.01	1.00	1.00	1.00	1.00	1.00	0.80	0.01	1.00	1.00	1.00	1.00	1.00	0.99	0.02
10	ELW	1.00	1.00	1.00	1.00	0.95	0.41	0.04	1.00	1.00	1.00	1.00	1.00	0.74	0.02	1.00	1.00	1.00	1.00	1.00	0.98	0.03
	FELW	1.00	1.00	1.00	1.00	0.97	0.59	0.06	1.00	1.00	1.00	1.00	1.00	0.84	0.06	1.00	1.00	1.00	1.00	1.00	0.98	0.05
	FELWd	1.00	1.00	1.00	1.00	0.93	0.39	0.03	1.00	1.00	1.00	1.00	1.00	0.75	0.01	1.00	1.00	1.00	1.00	1.00	0.98	0.02
	LW	1.00	1.00	1.00	1.00	1.00	0.82	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.01
30	ELW	0.98	0.98	0.99	0.99	1.00	0.76	0.05	1.00	1.00	1.00	1.00	1.00	0.99	0.03	1.00	1.00	1.00	1.00	1.00	1.00	0.02
	FELW	1.00	1.00	1.00	1.00	1.00	0.89	0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.03
	FELWd	1.00	1.00	1.00	1.00	1.00	0.74	0.01	1.00	1.00	1.00	1.00	1.00	0.99	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.01
	LW	1.00	1.00	1.00	1.00	1.00	0.94	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01
50	ELW	0.98	0.97	0.99	0.98	0.99	0.91	0.04	1.00	1.00	0.99	1.00	1.00	1.00	0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.01
	FELW	1.00	1.00	1.00	1.00	1.00	0.98	0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.07	1.00	1.00	1.00	1.00	1.00	1.00	0.04
	FELWd	1.00	1.00	1.00	1.00	1.00	0.89	0.01	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.01

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	LW	0.92	0.88	0.79	0.58	0.31	0.10	0.01	0.99	0.99	0.96	0.87	0.53	0.18	0.01	1.00	1.00	1.00	0.99	0.87	0.31	0.02
10	ELW	0.91	0.84	0.74	0.52	0.28	0.10	0.04	1.00	1.00	0.96	0.84	0.49	0.17	0.02	1.00	1.00	1.00	0.99	0.86	0.30	0.03
	FELW	0.96	0.92	0.82	0.62	0.36	0.14	0.06	1.00	1.00	0.98	0.88	0.56	0.20	0.06	1.00	1.00	1.00	0.99	0.87	0.30	0.05
	FELWd	0.87	0.81	0.69	0.49	0.27	0.09	0.03	1.00	1.00	0.95	0.82	0.47	0.16	0.01	1.00	1.00	1.00	0.99	0.86	0.30	0.02
	LW	1.00	1.00	0.98	0.92	0.65	0.16	0.01	1.00	1.00	1.00	1.00	0.92	0.36	0.01	1.00	1.00	1.00	1.00	1.00	0.55	0.01
30	ELW	1.00	1.00	0.98	0.88	0.58	0.15	0.05	1.00	1.00	1.00	1.00	0.91	0.33	0.03	1.00	1.00	1.00	1.00	1.00	0.56	0.02
	FELW	1.00	1.00	1.00	0.94	0.70	0.19	0.08	1.00	1.00	1.00	1.00	0.95	0.37	0.06	1.00	1.00	1.00	1.00	1.00	0.56	0.03
	FELWd	1.00	1.00	0.97	0.88	0.57	0.17	0.01	1.00	1.00	1.00	1.00	0.90	0.32	0.01	1.00	1.00	1.00	1.00	1.00	0.58	0.01
	LW	1.00	1.00	1.00	0.99	0.83	0.25	0.00	1.00	1.00	1.00	1.00	0.99	0.46	0.00	1.00	1.00	1.00	1.00	1.00	0.79	0.01
50	ELW	1.00	1.00	1.00	0.98	0.79	0.23	0.04	1.00	1.00	1.00	1.00	0.98	0.39	0.02	1.00	1.00	1.00	1.00	1.00	0.78	0.01
	FELW	1.00	1.00	1.00	1.00	0.88	0.28	0.08	1.00	1.00	1.00	1.00	0.99	0.45	0.07	1.00	1.00	1.00	1.00	1.00	0.77	0.04
	FELWd	1.00	1.00	1.00	0.97	0.78	0.25	0.01	1.00	1.00	1.00	1.00	0.98	0.40	0.00	1.00	1.00	1.00	1.00	1.00	0.77	0.01

C.2 Simulation Results for Multiple Testing Procedure

Table C.4: LW Test for DGP1. Cross-Sectionally Independent

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.01	0.06	0.26	0.50	0.71	0.92	0.99	0.00	0.06	0.23	0.46	0.68	0.90	0.99	0.00	0.05	0.25	0.45	0.67	0.89	1.00
	sd($\hat{\theta}$)	0.03	0.07	0.14	0.11	0.09	0.07	0.04	0.00	0.05	0.10	0.07	0.06	0.04	0.03	0.00	0.05	0.08	0.07	0.06	0.03	0.02
	FDR	0.00	0.05	0.07	0.06	0.08	0.07	0.12	0.00	0.04	0.08	0.06	0.05	0.05	0.08	0.00	0.05	0.05	0.07	0.08	0.03	0.04
	FNR	0.04	0.07	0.06	0.05	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	ave($\hat{\theta}$)	0.00	0.06	0.27	0.49	0.72	0.94	1.00	0.00	0.06	0.23	0.45	0.68	0.89	1.00	0.00	0.05	0.25	0.45	0.67	0.89	1.00
	sd($\hat{\theta}$)	0.01	0.04	0.05	0.06	0.07	0.04	0.01	0.00	0.04	0.09	0.08	0.03	0.02	0.01	0.00	0.04	0.08	0.09	0.04	0.02	0.01
	FDR	0.00	0.05	0.06	0.07	0.07	0.07	0.08	0.00	0.04	0.08	0.08	0.06	0.07	0.08	0.00	0.05	0.06	0.08	0.07	0.05	0.04
	FNR	0.01	0.02	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	ave($\hat{\theta}$)	0.00	0.06	0.27	0.50	0.73	0.94	1.00	0.00	0.05	0.26	0.47	0.68	0.90	1.00	0.00	0.05	0.26	0.47	0.68	0.89	1.00
	sd($\hat{\theta}$)	0.00	0.04	0.05	0.05	0.05	0.03	0.00	0.00	0.04	0.03	0.03	0.02	0.02	0.01	0.00	0.04	0.03	0.03	0.02	0.01	0.00
	FDR	0.00	0.05	0.06	0.06	0.05	0.04	0.04	0.00	0.05	0.06	0.06	0.05	0.06	0.07	0.00	0.06	0.05	0.05	0.07	0.05	0.05
	FNR	0.00	0.04	0.04	0.06	0.06	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.80	0.89	0.88	0.92	0.96	0.99	1.00	0.47	0.59	0.74	0.85	0.93	0.98	0.98	0.11	0.18	0.41	0.62	0.82	0.95	0.99
	sd($\hat{\theta}$)	0.23	0.18	0.19	0.13	0.08	0.04	0.02	0.35	0.33	0.26	0.18	0.11	0.05	0.05	0.22	0.22	0.22	0.20	0.13	0.06	0.02
	FDR	0.00	0.00	0.02	0.03	0.03	0.06	0.02	0.00	0.02	0.03	0.02	0.04	0.05	0.13	0.00	0.04	0.05	0.05	0.04	0.05	0.06
	FNR	1.00	0.88	0.64	0.45	0.27	0.09	0.00	0.78	0.73	0.55	0.40	0.25	0.09	0.00	0.30	0.28	0.25	0.19	0.15	0.06	0.00
30	ave($\hat{\theta}$)	0.93	0.94	0.96	0.98	0.99	0.99	1.00	0.56	0.66	0.83	0.91	0.95	0.99	1.00	0.06	0.14	0.39	0.65	0.83	0.97	1.00
	sd($\hat{\theta}$)	0.11	0.10	0.07	0.04	0.03	0.02	0.01	0.32	0.27	0.18	0.10	0.07	0.02	0.01	0.14	0.11	0.14	0.14	0.10	0.04	0.01
	FDR	0.00	0.00	0.02	0.01	0.08	0.02	0.07	0.00	0.01	0.01	0.01	0.02	0.06	0.06	0.00	0.03	0.04	0.04	0.04	0.05	0.03
	FNR	1.00	0.89	0.69	0.49	0.29	0.10	0.00	0.93	0.80	0.62	0.45	0.26	0.09	0.00	0.27	0.30	0.24	0.23	0.16	0.07	0.00
50	ave($\hat{\theta}$)	0.95	0.96	0.98	0.98	0.99	1.00	1.00	0.63	0.65	0.82	0.92	0.96	0.99	1.00	0.01	0.12	0.41	0.64	0.84	0.98	1.00
	sd($\hat{\theta}$)	0.07	0.06	0.04	0.03	0.02	0.01	0.01	0.26	0.23	0.14	0.08	0.05	0.02	0.00	0.04	0.09	0.11	0.10	0.08	0.03	0.01
	FDR	0.00	0.01	0.00	0.01	0.03	0.03	0.07	0.00	0.00	0.01	0.02	0.06	0.01	0.03	0.00	0.04	0.04	0.04	0.04	0.05	0.07
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	0.99	0.82	0.63	0.45	0.27	0.09	0.00	0.10	0.30	0.30	0.23	0.17	0.08	0.00

Table C.5: LW Test for DGP2. Constant Covariance

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.02	0.13	0.31	0.52	0.70	0.91	0.99	0.00	0.04	0.25	0.46	0.66	0.88	1.00	0.00	0.06	0.22	0.46	0.66	0.89	0.99
	sd($\hat{\theta}$)	0.12	0.24	0.20	0.22	0.16	0.11	0.04	0.00	0.05	0.08	0.08	0.10	0.11	0.02	0.00	0.05	0.11	0.10	0.09	0.07	0.03
	FDR	0.00	0.05	0.06	0.08	0.09	0.09	0.10	0.00	0.06	0.06	0.06	0.07	0.05	0.04	0.00	0.04	0.08	0.06	0.08	0.04	0.07
	FNR	0.03	0.11	0.10	0.10	0.06	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	ave($\hat{\theta}$)	0.02	0.06	0.27	0.53	0.74	0.91	1.00	0.00	0.05	0.25	0.45	0.66	0.89	1.00	0.00	0.05	0.25	0.45	0.67	0.89	1.00
	sd($\hat{\theta}$)	0.13	0.10	0.16	0.15	0.12	0.09	0.02	0.00	0.05	0.07	0.09	0.10	0.04	0.02	0.00	0.05	0.08	0.09	0.05	0.02	0.01
	FDR	0.00	0.05	0.08	0.06	0.06	0.08	0.07	0.00	0.05	0.06	0.08	0.08	0.09	0.10	0.00	0.05	0.06	0.07	0.07	0.06	0.05
	FNR	0.05	0.04	0.06	0.08	0.08	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	ave($\hat{\theta}$)	0.02	0.05	0.27	0.51	0.69	0.94	1.00	0.00	0.04	0.25	0.46	0.67	0.89	1.00	0.00	0.05	0.24	0.46	0.67	0.89	1.00
	sd($\hat{\theta}$)	0.14	0.05	0.12	0.13	0.13	0.05	0.00	0.00	0.04	0.05	0.07	0.06	0.03	0.01	0.00	0.04	0.07	0.08	0.04	0.04	0.01
	FDR	0.00	0.06	0.07	0.07	0.10	0.06	0.04	0.00	0.06	0.06	0.07	0.07	0.08	0.06	0.00	0.06	0.07	0.06	0.07	0.07	0.06
	FNR	0.04	0.03	0.06	0.08	0.04	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.81	0.74	0.85	0.90	0.94	0.98	0.99	0.53	0.56	0.66	0.77	0.87	0.96	0.99	0.19	0.28	0.46	0.57	0.82	0.95	0.99
	sd($\hat{\theta}$)	0.29	0.34	0.25	0.23	0.11	0.05	0.03	0.42	0.38	0.33	0.28	0.20	0.14	0.06	0.35	0.33	0.31	0.24	0.17	0.06	0.05
	FDR	0.00	0.02	0.04	0.05	0.05	0.06	0.05	0.00	0.01	0.04	0.07	0.06	0.07	0.06	0.00	0.03	0.06	0.07	0.07	0.03	0.07
	FNR	0.92	0.77	0.62	0.45	0.26	0.09	0.00	0.70	0.63	0.47	0.35	0.22	0.08	0.00	0.26	0.34	0.29	0.14	0.16	0.06	0.00
30	ave($\hat{\theta}$)	0.83	0.83	0.94	0.94	0.97	0.99	1.00	0.55	0.64	0.73	0.83	0.92	0.98	1.00	0.17	0.20	0.45	0.62	0.82	0.94	0.99
	sd($\hat{\theta}$)	0.32	0.30	0.14	0.12	0.11	0.02	0.01	0.45	0.40	0.31	0.23	0.13	0.05	0.01	0.34	0.28	0.28	0.23	0.12	0.08	0.06
	FDR	0.00	0.01	0.01	0.02	0.07	0.05	0.05	0.00	0.02	0.03	0.07	0.04	0.03	0.07	0.00	0.04	0.05	0.07	0.04	0.09	0.08
	FNR	0.94	0.83	0.67	0.47	0.29	0.09	0.00	0.64	0.67	0.53	0.39	0.25	0.09	0.00	0.26	0.27	0.27	0.21	0.15	0.06	0.00
50	ave($\hat{\theta}$)	0.80	0.86	0.90	0.96	0.97	0.99	0.99	0.57	0.63	0.71	0.86	0.94	0.99	1.00	0.13	0.23	0.45	0.65	0.81	0.96	1.00
	sd($\hat{\theta}$)	0.34	0.29	0.20	0.09	0.11	0.02	0.04	0.45	0.38	0.30	0.21	0.13	0.03	0.01	0.31	0.32	0.28	0.23	0.15	0.04	0.01
	FDR	0.00	0.01	0.03	0.02	0.04	0.07	0.08	0.00	0.02	0.04	0.04	0.04	0.06	0.11	0.00	0.04	0.05	0.06	0.07	0.05	0.05
	FNR	0.89	0.82	0.66	0.48	0.29	0.10	0.00	0.68	0.70	0.52	0.41	0.26	0.09	0.00	0.19	0.29	0.27	0.22	0.15	0.07	0.00

Table C.6: LW Test for DGP3. Covariance Matrix in Toeplitz Form

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.00	0.07	0.28	0.50	0.73	0.92	0.99	0.00	0.04	0.25	0.45	0.66	0.89	0.99	0.00	0.05	0.23	0.46	0.66	0.89	0.99
10	sd($\hat{\theta}$)	0.03	0.13	0.12	0.16	0.12	0.07	0.05	0.00	0.05	0.08	0.09	0.08	0.04	0.02	0.00	0.05	0.10	0.08	0.07	0.03	0.03
	FDR	0.00	0.05	0.06	0.07	0.05	0.06	0.06	0.00	0.06	0.06	0.07	0.08	0.07	0.06	0.00	0.05	0.08	0.06	0.08	0.04	0.05
	FNR	0.02	0.07	0.05	0.06	0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.00	0.05	0.27	0.49	0.73	0.94	1.00	0.00	0.05	0.24	0.47	0.68	0.89	1.00	0.00	0.06	0.25	0.45	0.68	0.89	1.00
30	sd($\hat{\theta}$)	0.01	0.05	0.05	0.07	0.07	0.05	0.01	0.00	0.04	0.06	0.03	0.03	0.02	0.01	0.00	0.04	0.08	0.08	0.03	0.02	0.01
	FDR	0.00	0.06	0.05	0.07	0.05	0.05	0.04	0.00	0.05	0.07	0.06	0.05	0.07	0.05	0.00	0.04	0.06	0.08	0.06	0.08	0.04
	FNR	0.02	0.04	0.03	0.05	0.06	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.00	0.06	0.27	0.50	0.73	0.95	1.00	0.00	0.04	0.25	0.46	0.68	0.90	1.00	0.00	0.05	0.25	0.47	0.68	0.89	1.00
50	sd($\hat{\theta}$)	0.00	0.04	0.06	0.05	0.05	0.04	0.00	0.00	0.04	0.04	0.04	0.02	0.02	0.01	0.00	0.04	0.05	0.04	0.02	0.01	0.00
	FDR	0.00	0.05	0.06	0.06	0.05	0.04	0.04	0.00	0.06	0.06	0.07	0.07	0.06	0.07	0.00	0.05	0.06	0.06	0.05	0.07	0.02
	FNR	0.00	0.05	0.06	0.05	0.06	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.79	0.84	0.93	0.93	0.96	0.98	0.99	0.61	0.60	0.73	0.81	0.91	0.97	1.00	0.11	0.25	0.46	0.63	0.79	0.93	0.99
10	sd($\hat{\theta}$)	0.29	0.27	0.11	0.12	0.08	0.05	0.04	0.36	0.35	0.27	0.22	0.14	0.06	0.03	0.21	0.27	0.24	0.21	0.17	0.11	0.03
	FDR	0.00	0.01	0.01	0.01	0.01	0.06	0.08	0.00	0.01	0.05	0.04	0.08	0.03	0.03	0.00	0.03	0.03	0.07	0.06	0.08	0.10
	FNR	0.94	0.83	0.67	0.46	0.27	0.09	0.00	0.84	0.69	0.54	0.36	0.23	0.08	0.00	0.29	0.34	0.29	0.21	0.13	0.05	0.00
	ave($\hat{\theta}$)	0.90	0.91	0.94	0.97	0.98	1.00	1.00	0.54	0.67	0.81	0.90	0.95	0.98	1.00	0.05	0.13	0.42	0.66	0.83	0.96	1.00
30	sd($\hat{\theta}$)	0.15	0.16	0.12	0.05	0.05	0.01	0.01	0.33	0.30	0.17	0.11	0.07	0.03	0.01	0.11	0.11	0.17	0.14	0.11	0.04	0.02
	FDR	0.00	0.00	0.02	0.04	0.03	0.07	0.04	0.00	0.01	0.02	0.02	0.05	0.02	0.06	0.00	0.03	0.04	0.04	0.04	0.04	0.07
	FNR	1.00	0.88	0.68	0.49	0.29	0.10	0.00	0.89	0.79	0.62	0.44	0.26	0.09	0.00	0.21	0.28	0.28	0.24	0.16	0.07	0.00
	ave($\hat{\theta}$)	0.93	0.95	0.97	0.98	0.99	1.00	1.00	0.57	0.59	0.84	0.92	0.97	0.99	1.00	0.02	0.12	0.41	0.64	0.85	0.97	1.00
50	sd($\hat{\theta}$)	0.13	0.08	0.06	0.03	0.02	0.01	0.00	0.32	0.28	0.18	0.09	0.05	0.02	0.01	0.06	0.11	0.12	0.12	0.09	0.03	0.01
	FDR	0.00	0.01	0.01	0.01	0.02	0.04	0.04	0.00	0.01	0.01	0.03	0.03	0.01	0.09	0.00	0.04	0.03	0.04	0.05	0.02	0.08
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	0.92	0.78	0.62	0.45	0.28	0.09	0.00	0.15	0.26	0.28	0.22	0.18	0.08	0.00

Table C.7: ELW Test for DGP1. Cross-Sectionally Independent

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.04	0.13	0.40	0.57	0.79	0.94	0.99	0.00	0.06	0.27	0.48	0.70	0.90	0.99	0.00	0.06	0.26	0.47	0.68	0.89	1.00
	sd($\hat{\theta}$)	0.11	0.20	0.20	0.15	0.11	0.07	0.03	0.00	0.05	0.10	0.05	0.08	0.04	0.03	0.00	0.05	0.07	0.06	0.04	0.03	0.01
	FDR	0.00	0.04	0.03	0.03	0.03	0.05	0.06	0.00	0.04	0.05	0.04	0.04	0.04	0.07	0.00	0.04	0.04	0.05	0.04	0.03	0.02
	FNR	0.13	0.17	0.20	0.12	0.11	0.05	0.00	0.00	0.00	0.01	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	ave($\hat{\theta}$)	0.01	0.10	0.35	0.60	0.81	0.96	1.00	0.00	0.08	0.26	0.47	0.69	0.90	1.00	0.00	0.05	0.26	0.47	0.69	0.90	1.00
	sd($\hat{\theta}$)	0.03	0.11	0.12	0.12	0.09	0.04	0.01	0.00	0.14	0.08	0.06	0.03	0.03	0.01	0.00	0.05	0.07	0.05	0.02	0.01	0.01
	FDR	0.00	0.03	0.03	0.03	0.03	0.04	0.06	0.00	0.04	0.05	0.05	0.04	0.05	0.03	0.00	0.05	0.04	0.04	0.04	0.03	0.07
	FNR	0.05	0.14	0.17	0.17	0.14	0.07	0.00	0.00	0.02	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	ave($\hat{\theta}$)	0.02	0.11	0.36	0.60	0.82	0.97	1.00	0.00	0.06	0.27	0.48	0.70	0.90	1.00	0.00	0.06	0.27	0.48	0.69	0.90	1.00
	sd($\hat{\theta}$)	0.14	0.14	0.14	0.11	0.08	0.03	0.00	0.00	0.03	0.03	0.02	0.04	0.02	0.00	0.00	0.04	0.02	0.02	0.01	0.01	0.00
	FDR	0.00	0.03	0.03	0.03	0.03	0.02	0.03	0.00	0.04	0.04	0.04	0.03	0.04	0.04	0.00	0.04	0.04	0.03	0.04	0.04	0.03
	FNR	0.04	0.14	0.17	0.17	0.15	0.07	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.91	0.94	0.93	0.96	0.98	0.99	1.00	0.67	0.76	0.85	0.92	0.96	0.99	0.99	0.12	0.25	0.48	0.69	0.88	0.97	1.00
	sd($\hat{\theta}$)	0.14	0.12	0.15	0.08	0.06	0.03	0.02	0.34	0.28	0.19	0.12	0.07	0.04	0.03	0.25	0.30	0.23	0.19	0.12	0.05	0.01
	FDR	0.00	0.00	0.03	0.03	0.04	0.04	0.03	0.00	0.01	0.02	0.04	0.03	0.03	0.09	0.00	0.04	0.02	0.03	0.02	0.03	0.01
	FNR	1.00	0.89	0.67	0.48	0.29	0.10	0.00	0.92	0.82	0.64	0.45	0.27	0.09	0.00	0.27	0.36	0.32	0.25	0.20	0.07	0.00
30	ave($\hat{\theta}$)	0.98	0.97	0.98	0.99	0.99	1.00	1.00	0.76	0.83	0.92	0.97	0.98	0.99	1.00	0.06	0.14	0.49	0.72	0.89	0.98	1.00
	sd($\hat{\theta}$)	0.04	0.05	0.04	0.02	0.02	0.01	0.01	0.30	0.21	0.11	0.05	0.04	0.02	0.01	0.18	0.12	0.20	0.13	0.09	0.03	0.01
	FDR	0.00	0.00	0.01	0.01	0.07	0.04	0.04	0.00	0.01	0.01	0.00	0.04	0.01	0.05	0.00	0.03	0.02	0.02	0.03	0.02	0.03
	FNR	1.00	0.90	0.69	0.49	0.30	0.10	0.00	0.95	0.87	0.67	0.48	0.28	0.10	0.00	0.20	0.28	0.34	0.29	0.21	0.08	0.00
50	ave($\hat{\theta}$)	0.98	0.99	0.99	0.99	0.99	1.00	1.00	0.84	0.87	0.93	0.97	0.99	1.00	1.00	0.03	0.15	0.47	0.72	0.89	0.98	1.00
	sd($\hat{\theta}$)	0.03	0.03	0.02	0.02	0.01	0.01	0.01	0.18	0.15	0.09	0.04	0.02	0.01	0.00	0.12	0.11	0.15	0.12	0.08	0.02	0.00
	FDR	0.00	0.00	0.01	0.02	0.01	0.04	0.06	0.00	0.01	0.01	0.02	0.01	0.02	0.03	0.00	0.03	0.02	0.02	0.02	0.05	0.03
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	1.00	0.88	0.68	0.49	0.29	0.10	0.00	0.12	0.34	0.35	0.30	0.21	0.08	0.00

Table C.8: ELW Test for DGP2. Constant Covariance

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.11	0.22	0.46	0.60	0.77	0.95	0.99	0.00	0.06	0.27	0.48	0.69	0.90	0.99	0.00	0.05	0.26	0.48	0.69	0.89	0.99
	sd($\hat{\theta}$)	0.28	0.30	0.30	0.23	0.14	0.06	0.06	0.00	0.11	0.07	0.07	0.07	0.05	0.03	0.00	0.05	0.08	0.05	0.04	0.04	0.04
	FDR	0.00	0.03	0.04	0.06	0.05	0.03	0.07	0.00	0.05	0.04	0.03	0.04	0.04	0.07	0.00	0.05	0.04	0.03	0.03	0.05	0.05
	FNR	0.18	0.24	0.24	0.17	0.10	0.06	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	ave($\hat{\theta}$)	0.15	0.20	0.39	0.63	0.82	0.95	1.00	0.00	0.06	0.28	0.48	0.70	0.90	1.00	0.00	0.06	0.26	0.48	0.69	0.89	1.00
	sd($\hat{\theta}$)	0.34	0.31	0.24	0.19	0.13	0.04	0.01	0.00	0.05	0.06	0.04	0.05	0.05	0.01	0.00	0.05	0.05	0.03	0.03	0.02	0.00
	FDR	0.00	0.04	0.04	0.04	0.03	0.03	0.07	0.00	0.04	0.03	0.04	0.04	0.07	0.03	0.00	0.04	0.05	0.04	0.03	0.04	0.02
	FNR	0.19	0.19	0.17	0.19	0.14	0.05	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	ave($\hat{\theta}$)	0.11	0.14	0.42	0.67	0.79	0.96	1.00	0.00	0.05	0.28	0.48	0.69	0.90	1.00	0.00	0.05	0.27	0.48	0.68	0.89	1.00
	sd($\hat{\theta}$)	0.29	0.22	0.26	0.22	0.12	0.05	0.00	0.00	0.05	0.08	0.05	0.04	0.03	0.00	0.00	0.04	0.05	0.03	0.03	0.01	0.00
	FDR	0.00	0.04	0.04	0.03	0.05	0.04	0.02	0.00	0.05	0.04	0.05	0.03	0.05	0.04	0.00	0.05	0.04	0.03	0.05	0.04	0.04
	FNR	0.17	0.19	0.20	0.21	0.11	0.06	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.92	0.86	0.91	0.94	0.97	0.99	0.99	0.60	0.66	0.75	0.83	0.91	0.98	1.00	0.22	0.33	0.52	0.63	0.84	0.96	1.00
	sd($\hat{\theta}$)	0.19	0.22	0.19	0.16	0.06	0.06	0.06	0.43	0.39	0.34	0.23	0.18	0.07	0.03	0.38	0.36	0.31	0.22	0.16	0.06	0.01
	FDR	0.00	0.01	0.03	0.05	0.02	0.03	0.05	0.00	0.01	0.04	0.06	0.05	0.04	0.04	0.00	0.03	0.04	0.05	0.04	0.04	0.02
	FNR	0.98	0.86	0.66	0.46	0.28	0.09	0.00	0.70	0.68	0.52	0.39	0.24	0.09	0.00	0.31	0.38	0.32	0.19	0.17	0.07	0.00
30	ave($\hat{\theta}$)	0.90	0.93	0.96	0.98	0.99	0.99	1.00	0.68	0.75	0.82	0.92	0.98	0.99	1.00	0.22	0.31	0.54	0.68	0.86	0.97	1.00
	sd($\hat{\theta}$)	0.24	0.17	0.14	0.05	0.03	0.02	0.01	0.42	0.35	0.26	0.17	0.05	0.02	0.01	0.37	0.37	0.30	0.21	0.11	0.05	0.01
	FDR	0.00	0.01	0.02	0.02	0.05	0.05	0.06	0.00	0.01	0.02	0.03	0.01	0.03	0.04	0.00	0.03	0.03	0.04	0.02	0.04	0.03
	FNR	0.96	0.88	0.68	0.49	0.30	0.10	0.00	0.77	0.76	0.59	0.45	0.28	0.09	0.00	0.31	0.33	0.34	0.24	0.18	0.08	0.00
50	ave($\hat{\theta}$)	0.91	0.93	0.96	0.97	1.00	1.00	1.00	0.71	0.77	0.87	0.92	0.97	0.99	1.00	0.17	0.29	0.52	0.71	0.87	0.98	1.00
	sd($\hat{\theta}$)	0.21	0.20	0.09	0.06	0.01	0.01	0.01	0.40	0.34	0.22	0.14	0.07	0.02	0.01	0.36	0.37	0.31	0.23	0.13	0.04	0.00
	FDR	0.00	0.00	0.02	0.04	0.01	0.05	0.04	0.00	0.01	0.02	0.02	0.03	0.05	0.09	0.00	0.04	0.04	0.04	0.03	0.03	0.02
	FNR	0.98	0.87	0.69	0.49	0.30	0.10	0.00	0.81	0.79	0.63	0.45	0.28	0.10	0.00	0.20	0.33	0.32	0.27	0.19	0.08	0.00

Table C.9: ELW Test for DGP3. Covariance Matrix in Toeplitz Form

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.05	0.19	0.40	0.61	0.80	0.95	1.00	0.00	0.06	0.27	0.48	0.68	0.90	1.00	0.00	0.05	0.25	0.48	0.69	0.89	1.00
10	sd($\hat{\theta}$)	0.15	0.27	0.24	0.19	0.13	0.06	0.02	0.00	0.05	0.07	0.07	0.09	0.04	0.02	0.00	0.05	0.09	0.05	0.05	0.03	0.02
	FDR	0.00	0.03	0.03	0.03	0.03	0.06	0.01	0.00	0.04	0.03	0.04	0.06	0.03	0.04	0.00	0.05	0.06	0.03	0.03	0.03	0.03
	FNR	0.16	0.21	0.18	0.15	0.12	0.06	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.02	0.11	0.37	0.59	0.81	0.96	1.00	0.01	0.06	0.27	0.48	0.69	0.91	1.00	0.00	0.06	0.26	0.47	0.69	0.89	1.00
30	sd($\hat{\theta}$)	0.14	0.17	0.19	0.14	0.10	0.04	0.01	0.10	0.04	0.04	0.03	0.03	0.03	0.00	0.00	0.05	0.08	0.07	0.02	0.02	0.01
	FDR	0.00	0.04	0.03	0.03	0.03	0.04	0.03	0.00	0.04	0.05	0.03	0.03	0.04	0.01	0.00	0.04	0.05	0.05	0.03	0.05	0.07
	FNR	0.06	0.15	0.15	0.15	0.13	0.06	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.05	0.13	0.35	0.60	0.83	0.97	1.00	0.00	0.07	0.27	0.48	0.69	0.91	1.00	0.00	0.06	0.27	0.48	0.69	0.89	1.00
50	sd($\hat{\theta}$)	0.20	0.18	0.11	0.14	0.09	0.03	0.00	0.00	0.10	0.03	0.03	0.02	0.02	0.00	0.00	0.04	0.03	0.02	0.02	0.01	0.00
	FDR	0.00	0.03	0.03	0.03	0.03	0.03	0.03	0.00	0.04	0.04	0.04	0.03	0.04	0.05	0.00	0.05	0.04	0.03	0.04	0.05	0.03
	FNR	0.09	0.17	0.16	0.16	0.15	0.07	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.88	0.94	0.95	0.96	0.98	0.99	0.99	0.74	0.77	0.82	0.90	0.94	0.99	0.99	0.14	0.30	0.51	0.69	0.86	0.95	1.00
10	sd($\hat{\theta}$)	0.23	0.13	0.10	0.08	0.06	0.03	0.03	0.34	0.28	0.25	0.16	0.11	0.03	0.03	0.28	0.31	0.26	0.21	0.11	0.06	0.02
	FDR	0.00	0.01	0.01	0.00	0.01	0.02	0.06	0.00	0.00	0.02	0.01	0.05	0.02	0.05	0.00	0.02	0.03	0.04	0.01	0.07	0.04
	FNR	0.97	0.89	0.69	0.48	0.28	0.10	0.00	0.89	0.82	0.59	0.43	0.25	0.09	0.00	0.26	0.39	0.32	0.26	0.18	0.06	0.00
	ave($\hat{\theta}$)	0.96	0.96	0.97	0.99	0.99	0.99	1.00	0.79	0.84	0.92	0.96	0.98	0.99	1.00	0.07	0.18	0.50	0.75	0.88	0.98	1.00
30	sd($\hat{\theta}$)	0.07	0.08	0.05	0.03	0.03	0.02	0.01	0.29	0.21	0.12	0.06	0.04	0.02	0.01	0.18	0.23	0.20	0.15	0.10	0.03	0.00
	FDR	0.00	0.00	0.02	0.02	0.05	0.06	0.03	0.00	0.00	0.00	0.03	0.05	0.03	0.05	0.00	0.03	0.02	0.02	0.03	0.03	0.00
	FNR	1.00	0.89	0.69	0.49	0.29	0.10	0.00	0.96	0.86	0.67	0.48	0.29	0.09	0.00	0.20	0.30	0.35	0.31	0.20	0.08	0.00
	ave($\hat{\theta}$)	0.98	0.98	0.98	0.99	1.00	1.00	1.00	0.80	0.85	0.94	0.97	0.99	0.99	1.00	0.02	0.17	0.49	0.72	0.90	0.98	1.00
50	sd($\hat{\theta}$)	0.04	0.03	0.03	0.02	0.01	0.01	0.00	0.26	0.20	0.11	0.05	0.02	0.01	0.01	0.07	0.17	0.17	0.13	0.08	0.03	0.01
	FDR	0.00	0.01	0.00	0.00	0.01	0.04	0.04	0.00	0.01	0.01	0.02	0.03	0.04	0.07	0.00	0.02	0.02	0.02	0.01	0.04	0.04
	FNR	1.00	0.90	0.69	0.50	0.30	0.10	0.00	0.99	0.87	0.67	0.48	0.29	0.10	0.00	0.12	0.33	0.36	0.30	0.22	0.08	0.00

Table C.10: FELW Test for DGP1. Cross-Sectionally Independent

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.03	0.09	0.33	0.53	0.73	0.92	0.99	0.00	0.06	0.26	0.48	0.68	0.89	1.00	0.00	0.06	0.26	0.47	0.68	0.89	1.00
10	sd($\hat{\theta}$)	0.07	0.10	0.14	0.11	0.08	0.06	0.03	0.01	0.05	0.08	0.06	0.06	0.04	0.01	0.00	0.05	0.07	0.06	0.05	0.03	0.02
	FDR	0.00	0.04	0.04	0.04	0.03	0.04	0.07	0.00	0.04	0.05	0.04	0.05	0.06	0.02	0.00	0.04	0.04	0.05	0.04	0.04	0.03
	FNR	0.13	0.13	0.12	0.08	0.05	0.03	0.00	0.02	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.01	0.09	0.31	0.53	0.73	0.94	1.00	0.00	0.06	0.26	0.47	0.69	0.90	1.00	0.00	0.05	0.27	0.47	0.69	0.90	1.00
30	sd($\hat{\theta}$)	0.03	0.06	0.06	0.06	0.07	0.04	0.01	0.00	0.04	0.08	0.06	0.02	0.02	0.01	0.00	0.04	0.06	0.05	0.02	0.01	0.01
	FDR	0.00	0.03	0.04	0.04	0.05	0.05	0.05	0.00	0.04	0.05	0.05	0.04	0.04	0.06	0.00	0.05	0.04	0.04	0.04	0.04	0.03
	FNR	0.06	0.16	0.11	0.08	0.06	0.04	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.01	0.09	0.31	0.53	0.74	0.93	1.00	0.00	0.06	0.27	0.48	0.69	0.90	1.00	0.00	0.06	0.27	0.48	0.69	0.89	1.00
50	sd($\hat{\theta}$)	0.03	0.05	0.05	0.05	0.04	0.03	0.00	0.00	0.03	0.03	0.02	0.02	0.01	0.01	0.00	0.03	0.02	0.03	0.02	0.01	0.00
	FDR	0.00	0.03	0.03	0.04	0.03	0.03	0.02	0.00	0.04	0.04	0.04	0.04	0.04	0.05	0.00	0.04	0.04	0.03	0.04	0.04	0.05
	FNR	0.05	0.15	0.10	0.09	0.06	0.04	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.80	0.90	0.89	0.93	0.97	0.99	1.00	0.49	0.62	0.77	0.84	0.93	0.97	0.99	0.15	0.26	0.46	0.69	0.85	0.96	0.99
10	sd($\hat{\theta}$)	0.20	0.17	0.16	0.11	0.06	0.03	0.02	0.35	0.30	0.22	0.13	0.10	0.05	0.03	0.28	0.30	0.20	0.18	0.12	0.06	0.02
	FDR	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.00	0.01	0.03	0.03	0.04	0.03	0.07	0.00	0.04	0.03	0.03	0.02	0.03	0.06
	FNR	1.00	0.88	0.65	0.46	0.28	0.09	0.00	0.85	0.74	0.59	0.40	0.24	0.08	0.00	0.31	0.37	0.30	0.26	0.17	0.06	0.00
	ave($\hat{\theta}$)	0.94	0.94	0.95	0.98	0.99	0.99	1.00	0.55	0.67	0.82	0.90	0.94	0.99	1.00	0.06	0.17	0.45	0.69	0.85	0.97	1.00
30	sd($\hat{\theta}$)	0.09	0.10	0.07	0.04	0.02	0.02	0.01	0.30	0.23	0.14	0.10	0.06	0.02	0.01	0.14	0.15	0.13	0.12	0.09	0.03	0.01
	FDR	0.00	0.00	0.01	0.00	0.07	0.02	0.06	0.00	0.00	0.01	0.01	0.04	0.03	0.04	0.00	0.02	0.03	0.02	0.04	0.04	0.05
	FNR	1.00	0.89	0.69	0.49	0.30	0.10	0.00	0.93	0.83	0.63	0.44	0.26	0.09	0.00	0.24	0.34	0.33	0.26	0.18	0.07	0.00
	ave($\hat{\theta}$)	0.94	0.96	0.97	0.98	0.99	1.00	1.00	0.62	0.66	0.82	0.90	0.95	0.99	1.00	0.02	0.16	0.46	0.68	0.84	0.97	1.00
50	sd($\hat{\theta}$)	0.09	0.07	0.05	0.03	0.02	0.01	0.01	0.18	0.17	0.13	0.08	0.05	0.01	0.00	0.07	0.11	0.11	0.08	0.06	0.03	0.00
	FDR	0.00	0.01	0.01	0.02	0.01	0.02	0.07	0.00	0.00	0.01	0.02	0.03	0.06	0.02	0.00	0.02	0.02	0.03	0.02	0.04	0.06
	FNR	1.00	0.90	0.69	0.49	0.29	0.10	0.00	1.00	0.84	0.63	0.44	0.27	0.09	0.00	0.13	0.36	0.35	0.27	0.17	0.08	0.00

Table C.11: FELW Test for DGP2. Constant Covariance

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.05	0.14	0.35	0.55	0.73	0.92	0.99	0.01	0.06	0.27	0.48	0.68	0.90	0.99	0.00	0.05	0.26	0.48	0.69	0.88	0.99
10	sd($\hat{\theta}$)	0.15	0.20	0.21	0.19	0.11	0.07	0.03	0.05	0.05	0.07	0.06	0.07	0.04	0.03	0.00	0.05	0.09	0.06	0.04	0.06	0.04
	FDR	0.00	0.03	0.04	0.06	0.05	0.06	0.09	0.00	0.04	0.04	0.03	0.05	0.05	0.07	0.00	0.05	0.05	0.04	0.03	0.05	0.06
	FNR	0.16	0.18	0.14	0.11	0.05	0.04	0.00	0.02	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.07	0.13	0.32	0.55	0.76	0.92	1.00	0.00	0.06	0.28	0.48	0.68	0.89	1.00	0.00	0.06	0.26	0.48	0.69	0.89	1.00
30	sd($\hat{\theta}$)	0.23	0.18	0.17	0.14	0.11	0.05	0.01	0.00	0.05	0.05	0.04	0.04	0.05	0.01	0.00	0.05	0.06	0.03	0.02	0.02	0.01
	FDR	0.00	0.04	0.05	0.04	0.03	0.06	0.04	0.00	0.04	0.03	0.04	0.05	0.07	0.07	0.00	0.04	0.05	0.04	0.03	0.06	0.03
	FNR	0.13	0.18	0.10	0.11	0.08	0.03	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.06	0.09	0.31	0.57	0.72	0.94	1.00	0.00	0.05	0.27	0.48	0.69	0.89	1.00	0.00	0.05	0.27	0.48	0.68	0.89	1.00
50	sd($\hat{\theta}$)	0.18	0.10	0.12	0.15	0.09	0.05	0.00	0.00	0.05	0.05	0.04	0.04	0.02	0.01	0.00	0.04	0.05	0.03	0.03	0.02	0.01
	FDR	0.00	0.04	0.05	0.04	0.06	0.05	0.06	0.00	0.05	0.04	0.05	0.04	0.07	0.07	0.00	0.05	0.04	0.03	0.05	0.06	0.08
	FNR	0.17	0.15	0.11	0.12	0.06	0.04	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.84	0.80	0.85	0.92	0.95	0.98	0.99	0.56	0.59	0.71	0.81	0.88	0.97	0.99	0.19	0.31	0.52	0.61	0.83	0.95	1.00
10	sd($\hat{\theta}$)	0.27	0.28	0.23	0.18	0.10	0.07	0.03	0.41	0.36	0.31	0.22	0.19	0.06	0.02	0.33	0.35	0.29	0.22	0.15	0.06	0.02
	FDR	0.00	0.02	0.02	0.04	0.03	0.01	0.05	0.00	0.01	0.03	0.04	0.05	0.05	0.06	0.00	0.03	0.04	0.05	0.05	0.03	0.04
	FNR	0.95	0.83	0.62	0.45	0.27	0.09	0.00	0.74	0.67	0.51	0.37	0.22	0.08	0.00	0.28	0.38	0.32	0.18	0.17	0.06	0.00
	ave($\hat{\theta}$)	0.86	0.87	0.94	0.95	0.98	0.99	1.00	0.57	0.66	0.73	0.84	0.93	0.99	1.00	0.17	0.26	0.51	0.66	0.83	0.96	1.00
30	sd($\hat{\theta}$)	0.26	0.24	0.15	0.10	0.05	0.02	0.01	0.41	0.34	0.26	0.20	0.09	0.03	0.01	0.32	0.31	0.26	0.19	0.11	0.06	0.01
	FDR	0.00	0.01	0.01	0.02	0.04	0.03	0.03	0.00	0.01	0.02	0.03	0.03	0.02	0.06	0.00	0.03	0.03	0.04	0.03	0.05	0.05
	FNR	0.96	0.87	0.67	0.47	0.29	0.10	0.00	0.74	0.74	0.55	0.39	0.25	0.09	0.00	0.28	0.32	0.34	0.23	0.15	0.07	0.00
	ave($\hat{\theta}$)	0.85	0.88	0.91	0.95	0.99	0.99	1.00	0.59	0.65	0.74	0.87	0.94	0.99	1.00	0.14	0.25	0.47	0.70	0.82	0.96	1.00
50	sd($\hat{\theta}$)	0.26	0.23	0.19	0.11	0.04	0.03	0.01	0.38	0.35	0.24	0.16	0.08	0.03	0.01	0.29	0.31	0.27	0.21	0.12	0.04	0.01
	FDR	0.00	0.01	0.02	0.03	0.01	0.05	0.07	0.00	0.01	0.02	0.03	0.02	0.03	0.12	0.00	0.03	0.04	0.04	0.05	0.05	0.05
	FNR	0.98	0.87	0.66	0.47	0.29	0.10	0.00	0.82	0.75	0.55	0.42	0.25	0.09	0.00	0.24	0.34	0.30	0.26	0.15	0.06	0.00

Table C.12: FELW Test for DGP3. Covariance Matrix in Toeplitz Form

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.03	0.12	0.34	0.54	0.73	0.91	1.00	0.00	0.06	0.27	0.47	0.68	0.89	1.00	0.00	0.05	0.25	0.48	0.68	0.90	1.00
10	sd($\hat{\theta}$)	0.08	0.14	0.16	0.15	0.10	0.06	0.01	0.02	0.05	0.08	0.07	0.07	0.04	0.02	0.00	0.05	0.09	0.05	0.05	0.02	0.01
	FDR	0.00	0.04	0.04	0.04	0.04	0.04	0.01	0.00	0.04	0.04	0.05	0.06	0.06	0.04	0.00	0.05	0.05	0.03	0.04	0.02	0.02
	FNR	0.12	0.17	0.13	0.09	0.05	0.02	0.00	0.01	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.01	0.09	0.31	0.52	0.73	0.93	1.00	0.00	0.06	0.27	0.48	0.69	0.90	1.00	0.00	0.06	0.26	0.47	0.69	0.89	1.00
30	sd($\hat{\theta}$)	0.03	0.06	0.07	0.05	0.06	0.04	0.01	0.00	0.05	0.04	0.04	0.03	0.02	0.01	0.00	0.04	0.07	0.07	0.02	0.02	0.00
	FDR	0.00	0.04	0.04	0.04	0.04	0.03	0.04	0.00	0.04	0.05	0.04	0.04	0.05	0.03	0.00	0.04	0.05	0.05	0.04	0.05	0.01
	FNR	0.09	0.17	0.10	0.07	0.06	0.04	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.00	0.10	0.32	0.53	0.74	0.94	1.00	0.00	0.06	0.28	0.48	0.69	0.90	1.00	0.00	0.06	0.27	0.48	0.69	0.89	1.00
50	sd($\hat{\theta}$)	0.02	0.05	0.05	0.05	0.06	0.04	0.00	0.00	0.04	0.03	0.02	0.02	0.02	0.01	0.00	0.04	0.03	0.02	0.02	0.01	0.00
	FDR	0.00	0.02	0.03	0.03	0.03	0.03	0.04	0.00	0.04	0.04	0.04	0.04	0.06	0.05	0.00	0.05	0.04	0.03	0.04	0.05	0.03
	FNR	0.04	0.17	0.11	0.08	0.06	0.05	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.83	0.86	0.91	0.94	0.97	0.98	1.00	0.68	0.64	0.72	0.83	0.91	0.97	1.00	0.12	0.27	0.49	0.66	0.84	0.95	0.99
10	sd($\hat{\theta}$)	0.27	0.21	0.15	0.11	0.06	0.05	0.02	0.31	0.31	0.23	0.17	0.11	0.05	0.02	0.25	0.29	0.24	0.20	0.11	0.06	0.03
	FDR	0.00	0.00	0.02	0.00	0.02	0.04	0.04	0.00	0.01	0.03	0.02	0.03	0.05	0.02	0.00	0.02	0.03	0.05	0.01	0.06	0.09
	FNR	0.96	0.87	0.67	0.46	0.28	0.09	0.00	0.92	0.77	0.55	0.37	0.23	0.08	0.00	0.24	0.37	0.32	0.23	0.16	0.06	0.00
	ave($\hat{\theta}$)	0.90	0.92	0.95	0.97	0.98	0.99	1.00	0.54	0.69	0.80	0.89	0.95	0.98	1.00	0.06	0.15	0.45	0.69	0.85	0.96	1.00
30	sd($\hat{\theta}$)	0.14	0.11	0.09	0.04	0.05	0.01	0.00	0.28	0.24	0.16	0.10	0.07	0.03	0.01	0.13	0.14	0.14	0.11	0.09	0.04	0.02
	FDR	0.00	0.00	0.01	0.03	0.03	0.05	0.02	0.00	0.00	0.01	0.01	0.04	0.05	0.03	0.00	0.03	0.02	0.02	0.03	0.04	0.08
	FNR	1.00	0.89	0.68	0.49	0.29	0.10	0.00	0.91	0.83	0.61	0.43	0.26	0.08	0.00	0.19	0.28	0.33	0.27	0.17	0.06	0.00
	ave($\hat{\theta}$)	0.94	0.95	0.97	0.98	0.99	1.00	1.00	0.58	0.62	0.81	0.91	0.96	0.99	1.00	0.03	0.17	0.46	0.68	0.86	0.97	1.00
50	sd($\hat{\theta}$)	0.11	0.08	0.05	0.03	0.02	0.01	0.00	0.25	0.20	0.16	0.09	0.05	0.02	0.01	0.08	0.14	0.12	0.10	0.09	0.03	0.01
	FDR	0.00	0.01	0.01	0.00	0.01	0.03	0.03	0.00	0.00	0.01	0.02	0.02	0.04	0.07	0.00	0.02	0.02	0.03	0.03	0.03	0.07
	FNR	1.00	0.90	0.69	0.49	0.30	0.10	0.00	0.98	0.82	0.61	0.45	0.27	0.09	0.00	0.17	0.36	0.34	0.26	0.18	0.08	0.00

Table C.13: FELWd Test for DGP1. Cross-Sectionally Independent

d=0.3

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.09	0.24	0.47	0.69	0.85	0.96	0.99	0.00	0.07	0.27	0.49	0.70	0.91	1.00	0.00	0.06	0.27	0.47	0.68	0.89	0.99
10	sd($\hat{\theta}$)	0.21	0.29	0.23	0.19	0.13	0.05	0.03	0.01	0.06	0.08	0.05	0.07	0.04	0.01	0.00	0.05	0.07	0.06	0.04	0.03	0.02
	FDR	0.00	0.03	0.02	0.02	0.03	0.03	0.04	0.00	0.04	0.04	0.03	0.04	0.01	0.01	0.00	0.04	0.04	0.05	0.04	0.03	0.06
	FNR	0.22	0.31	0.28	0.24	0.17	0.07	0.00	0.02	0.03	0.02	0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.03	0.15	0.42	0.71	0.91	0.98	1.00	0.00	0.06	0.27	0.47	0.70	0.91	1.00	0.00	0.06	0.27	0.48	0.69	0.90	1.00
30	sd($\hat{\theta}$)	0.07	0.14	0.17	0.17	0.09	0.03	0.01	0.00	0.04	0.08	0.07	0.03	0.03	0.01	0.00	0.04	0.06	0.06	0.02	0.01	0.00
	FDR	0.00	0.03	0.02	0.02	0.01	0.02	0.04	0.00	0.04	0.05	0.05	0.03	0.04	0.03	0.00	0.04	0.04	0.04	0.03	0.02	0.02
	FNR	0.22	0.29	0.26	0.28	0.23	0.08	0.00	0.00	0.01	0.02	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.03	0.14	0.43	0.74	0.92	0.98	1.00	0.00	0.07	0.28	0.49	0.70	0.91	1.00	0.00	0.06	0.27	0.48	0.68	0.89	1.00
50	sd($\hat{\theta}$)	0.11	0.09	0.16	0.14	0.07	0.02	0.00	0.00	0.03	0.03	0.03	0.02	0.02	0.00	0.00	0.03	0.02	0.02	0.02	0.01	0.00
	FDR	0.00	0.02	0.02	0.02	0.01	0.03	0.02	0.00	0.04	0.03	0.03	0.03	0.02	0.03	0.00	0.04	0.04	0.03	0.05	0.05	0.03
	FNR	0.14	0.30	0.27	0.31	0.24	0.09	0.00	0.00	0.02	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\\θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.95	0.95	0.96	0.98	0.99	0.99	1.00	0.72	0.83	0.87	0.92	0.96	0.98	1.00	0.18	0.34	0.52	0.72	0.88	0.97	0.99
10	sd($\hat{\theta}$)	0.11	0.11	0.11	0.04	0.03	0.02	0.02	0.31	0.24	0.16	0.11	0.07	0.04	0.01	0.32	0.35	0.23	0.20	0.12	0.05	0.03
	FDR	0.00	0.00	0.02	0.00	0.01	0.02	0.03	0.00	0.00	0.01	0.03	0.04	0.04	0.02	0.00	0.03	0.02	0.03	0.04	0.01	0.05
	FNR	1.00	0.89	0.68	0.49	0.29	0.10	0.00	0.96	0.84	0.65	0.45	0.27	0.09	0.00	0.31	0.42	0.35	0.28	0.20	0.07	0.00
	ave($\hat{\theta}$)	0.99	0.99	0.99	0.99	1.00	1.00	1.00	0.86	0.87	0.94	0.98	0.98	1.00	1.00	0.11	0.22	0.53	0.78	0.91	0.98	1.00
30	sd($\hat{\theta}$)	0.02	0.03	0.02	0.02	0.01	0.01	0.00	0.18	0.16	0.09	0.04	0.04	0.01	0.01	0.25	0.24	0.22	0.15	0.08	0.03	0.01
	FDR	0.00	0.00	0.01	0.03	0.04	0.01	0.02	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.00	0.02	0.02	0.01	0.03	0.05	0.03
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	0.99	0.88	0.68	0.49	0.29	0.10	0.00	0.29	0.38	0.38	0.34	0.23	0.08	0.00
	ave($\hat{\theta}$)	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.90	0.91	0.96	0.98	0.99	1.00	1.00	0.07	0.18	0.51	0.77	0.90	0.98	1.00
50	sd($\hat{\theta}$)	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.15	0.11	0.05	0.04	0.02	0.01	0.00	0.19	0.15	0.17	0.13	0.07	0.02	0.00
	FDR	0.00	0.01	0.02	0.01	0.04	0.02	0.02	0.00	0.00	0.00	0.03	0.02	0.04	0.03	0.00	0.02	0.02	0.03	0.02	0.03	0.03
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	1.00	0.89	0.69	0.49	0.29	0.10	0.00	0.19	0.38	0.39	0.34	0.22	0.09	0.00

Table C.14: FELWd Test for DGP2. Constant Covariance

d=0.3

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.21	0.32	0.53	0.69	0.85	0.97	0.99	0.01	0.06	0.27	0.50	0.70	0.91	1.00	0.00	0.06	0.26	0.48	0.69	0.90	0.99
10	sd($\hat{\theta}$)	0.37	0.39	0.33	0.23	0.13	0.05	0.03	0.04	0.07	0.09	0.09	0.07	0.05	0.02	0.00	0.05	0.07	0.05	0.03	0.03	0.02
	FDR	0.00	0.03	0.03	0.03	0.04	0.02	0.04	0.00	0.04	0.04	0.03	0.03	0.03	0.04	0.00	0.04	0.04	0.03	0.03	0.03	0.06
	FNR	0.30	0.34	0.31	0.24	0.17	0.07	0.00	0.02	0.02	0.02	0.03	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.17	0.27	0.45	0.73	0.89	0.97	1.00	0.01	0.06	0.29	0.50	0.70	0.91	1.00	0.00	0.06	0.26	0.48	0.69	0.89	1.00
30	sd($\hat{\theta}$)	0.35	0.35	0.28	0.21	0.12	0.03	0.00	0.05	0.05	0.06	0.07	0.04	0.04	0.00	0.00	0.04	0.06	0.03	0.02	0.02	0.01
	FDR	0.00	0.03	0.03	0.02	0.02	0.02	0.01	0.00	0.04	0.03	0.03	0.04	0.03	0.01	0.00	0.04	0.05	0.03	0.02	0.04	0.04
	FNR	0.24	0.30	0.24	0.28	0.20	0.08	0.00	0.01	0.01	0.03	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ave($\hat{\theta}$)	0.22	0.23	0.45	0.77	0.88	0.98	1.00	0.00	0.06	0.27	0.49	0.70	0.91	1.00	0.00	0.05	0.26	0.48	0.68	0.90	1.00
50	sd($\hat{\theta}$)	0.39	0.32	0.25	0.22	0.12	0.03	0.00	0.00	0.06	0.04	0.05	0.03	0.03	0.00	0.00	0.04	0.05	0.03	0.03	0.01	0.00
	FDR	0.00	0.03	0.03	0.02	0.02	0.02	0.02	0.00	0.05	0.04	0.04	0.03	0.03	0.01	0.00	0.05	0.04	0.03	0.05	0.04	0.04
	FNR	0.30	0.28	0.26	0.31	0.19	0.08	0.00	0.00	0.02	0.01	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

		T																				
		50							100							200						
N\ θ_0		0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1	0	0.1	0.3	0.5	0.7	0.9	1
	ave($\hat{\theta}$)	0.93	0.92	0.94	0.97	0.98	0.99	1.00	0.69	0.74	0.85	0.89	0.93	0.99	0.99	0.32	0.36	0.61	0.68	0.87	0.96	1.00
10	sd($\hat{\theta}$)	0.18	0.19	0.15	0.10	0.05	0.03	0.01	0.40	0.31	0.23	0.19	0.13	0.04	0.03	0.43	0.39	0.33	0.25	0.15	0.05	0.01
	FDR	0.00	0.00	0.03	0.02	0.03	0.03	0.02	0.00	0.00	0.01	0.03	0.04	0.03	0.06	0.00	0.03	0.03	0.04	0.04	0.02	0.01
	FNR	0.99	0.87	0.67	0.48	0.29	0.09	0.00	0.80	0.79	0.61	0.43	0.25	0.09	0.00	0.39	0.40	0.40	0.23	0.19	0.07	0.00
	ave($\hat{\theta}$)	0.96	0.97	0.99	0.99	1.00	1.00	1.00	0.80	0.80	0.87	0.92	0.98	1.00	1.00	0.27	0.38	0.61	0.72	0.88	0.98	1.00
30	sd($\hat{\theta}$)	0.14	0.11	0.03	0.04	0.02	0.01	0.00	0.33	0.31	0.22	0.17	0.04	0.02	0.00	0.40	0.41	0.30	0.21	0.12	0.04	0.01
	FDR	0.00	0.00	0.01	0.00	0.01	0.01	0.02	0.00	0.01	0.02	0.02	0.03	0.01	0.01	0.00	0.03	0.03	0.04	0.02	0.04	0.03
	FNR	0.99	0.89	0.70	0.49	0.30	0.10	0.00	0.91	0.79	0.63	0.45	0.29	0.10	0.00	0.36	0.39	0.40	0.28	0.20	0.08	0.00
	ave($\hat{\theta}$)	0.97	0.98	0.98	0.99	1.00	1.00	1.00	0.78	0.83	0.91	0.96	0.97	1.00	1.00	0.20	0.35	0.58	0.75	0.89	0.98	1.00
50	sd($\hat{\theta}$)	0.12	0.09	0.08	0.02	0.01	0.02	0.01	0.34	0.28	0.16	0.09	0.06	0.02	0.00	0.38	0.39	0.32	0.22	0.12	0.03	0.00
	FDR	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.00	0.01	0.01	0.01	0.02	0.02	0.06	0.00	0.03	0.03	0.04	0.03	0.02	0.02
	FNR	0.99	0.90	0.69	0.50	0.30	0.10	0.00	0.89	0.82	0.66	0.48	0.28	0.10	0.00	0.27	0.40	0.37	0.30	0.21	0.08	0.00

Table C.15: FELWd Test for DGP3. Covariance Matrix in Toeplitz Form

d=0.3

N\ θ_0		T=50: 0	0.1	0.3	0.5	0.7	0.9	1	T=100: 0	0.1	0.3	0.5	0.7	0.9	1	T=200: 0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.10	0.26	0.52	0.67	0.86	0.95	1.00	0.00	0.07	0.28	0.49	0.70	0.90	1.00	0.00	0.05	0.26	0.48	0.68	0.90	0.99
	sd($\hat{\theta}$)	0.23	0.33	0.28	0.21	0.13	0.06	0.01	0.02	0.07	0.06	0.07	0.06	0.03	0.02	0.00	0.05	0.08	0.05	0.06	0.03	0.04
	FDR	0.00	0.03	0.02	0.03	0.02	0.04	0.02	0.00	0.04	0.03	0.04	0.03	0.02	0.03	0.00	0.05	0.05	0.03	0.04	0.03	0.08
	FNR	0.27	0.31	0.32	0.22	0.17	0.06	0.00	0.01	0.03	0.02	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	ave($\hat{\theta}$)	0.06	0.16	0.44	0.71	0.88	0.98	1.00	0.00	0.06	0.28	0.49	0.70	0.91	1.00	0.00	0.06	0.26	0.47	0.69	0.89	1.00
	sd($\hat{\theta}$)	0.17	0.17	0.21	0.18	0.10	0.04	0.01	0.00	0.04	0.04	0.04	0.03	0.03	0.00	0.00	0.05	0.07	0.06	0.02	0.01	0.00
	FDR	0.00	0.02	0.02	0.02	0.02	0.02	0.04	0.00	0.04	0.04	0.03	0.03	0.03	0.00	0.00	0.04	0.05	0.04	0.03	0.04	0.02
	FNR	0.24	0.29	0.25	0.27	0.20	0.08	0.00	0.00	0.01	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	ave($\hat{\theta}$)	0.03	0.15	0.47	0.69	0.91	0.98	1.00	0.00	0.07	0.28	0.49	0.70	0.91	1.00	0.00	0.05	0.27	0.48	0.69	0.90	1.00
	sd($\hat{\theta}$)	0.08	0.09	0.20	0.16	0.08	0.02	0.00	0.00	0.04	0.03	0.03	0.02	0.03	0.00	0.00	0.04	0.03	0.02	0.02	0.01	0.00
	FDR	0.00	0.02	0.02	0.02	0.02	0.01	0.03	0.00	0.04	0.03	0.03	0.02	0.03	0.05	0.00	0.05	0.04	0.04	0.03	0.04	0.01
	FNR	0.21	0.32	0.30	0.26	0.23	0.09	0.00	0.00	0.02	0.01	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d=0.7

N\ θ_0		T=50: 0	0.1	0.3	0.5	0.7	0.9	1	T=100: 0	0.1	0.3	0.5	0.7	0.9	1	T=200: 0	0.1	0.3	0.5	0.7	0.9	1
10	ave($\hat{\theta}$)	0.94	0.97	0.97	0.97	1.00	1.00	1.00	0.80	0.80	0.89	0.92	0.96	0.99	1.00	0.17	0.36	0.58	0.70	0.87	0.96	1.00
	sd($\hat{\theta}$)	0.13	0.08	0.07	0.06	0.02	0.02	0.01	0.29	0.26	0.19	0.12	0.07	0.03	0.02	0.32	0.37	0.30	0.22	0.12	0.06	0.02
	FDR	0.00	0.01	0.02	0.01	0.00	0.02	0.02	0.00	0.00	0.00	0.01	0.03	0.01	0.04	0.00	0.03	0.02	0.04	0.02	0.03	0.04
	FNR	1.00	0.90	0.69	0.49	0.30	0.10	0.00	0.95	0.83	0.63	0.45	0.27	0.09	0.00	0.28	0.44	0.37	0.26	0.19	0.07	0.00
30	ave($\hat{\theta}$)	0.98	0.99	0.99	0.99	1.00	1.00	1.00	0.86	0.89	0.93	0.96	0.98	0.99	1.00	0.09	0.22	0.50	0.78	0.90	0.98	1.00
	sd($\hat{\theta}$)	0.05	0.03	0.03	0.02	0.02	0.01	0.01	0.20	0.15	0.10	0.06	0.05	0.02	0.01	0.23	0.25	0.20	0.16	0.08	0.03	0.01
	FDR	0.00	0.00	0.01	0.00	0.00	0.03	0.04	0.00	0.00	0.00	0.01	0.04	0.04	0.03	0.00	0.02	0.02	0.02	0.04	0.03	0.06
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	0.99	0.88	0.67	0.48	0.29	0.09	0.00	0.20	0.33	0.35	0.34	0.22	0.08	0.00
50	ave($\hat{\theta}$)	0.99	0.99	0.99	1.00	1.00	1.00	1.00	0.88	0.89	0.95	0.98	0.99	1.00	1.00	0.04	0.22	0.51	0.76	0.91	0.98	1.00
	sd($\hat{\theta}$)	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.14	0.16	0.09	0.04	0.02	0.01	0.00	0.11	0.22	0.17	0.14	0.07	0.02	0.00
	FDR	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.00	0.01	0.00	0.02	0.01	0.03	0.04	0.00	0.02	0.02	0.02	0.02	0.05	0.06
	FNR	1.00	0.90	0.70	0.50	0.30	0.10	0.00	1.00	0.88	0.68	0.49	0.29	0.10	0.00	0.15	0.40	0.37	0.33	0.23	0.09	0.00

C.3 Empirical Application

Table C.16: Estimated order of fractional integration using LW

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0.76	0.21	0.79	0.39	0.50	0.09	0.00	0.66	1.03	0.48	0.97	0.96	0.62	0.46	0.87
2	0.05	0.01	0.54	0.32	0.83	0.00	0.33	0.85	1.01	0.85	0.89	0.61	0.78	0.02	0.90
3	0.61	0.40	0.51	0.61	0.64	0.06	0.27	0.42	0.59	0.38	0.86	0.10	0.84	0.40	0.93
4	0.46	0.33	0.35	0.62	0.89	-0.23	0.50	0.24	0.84	0.82	0.96	0.06	0.79	0.92	0.83
5	0.21	0.41	0.52	0.51	0.54	0.34	0.27	0.41	0.79	0.44	0.89	0.50	0.87	-0.05	0.96
6	-0.06	-0.07	0.72	0.49	0.64	0.08	0.25	0.53	0.80	0.71	0.84	0.57	1.19	0.82	0.39
7	-0.03	0.49	0.65	0.37	0.71	0.21	0.19	0.45	0.74	0.43	0.61	0.80	0.49	0.71	0.52
8	0.18	0.23	0.70	0.47	0.56	0.26	0.61	0.95	0.73	0.62	0.90	0.54	0.48	1.09	0.78
9	0.10	0.51	0.76	0.24	0.73	0.20	0.33	0.93	0.91	0.75	0.69	1.11	0.97	0.81	0.70
10	0.55	0.22	0.25	0.70	0.56	0.22	0.01	0.46	0.78	0.55	0.54	0.52	0.39	0.85	0.54
11	0.40	0.45	0.54	-0.03	0.52	0.31	0.51	0.79	0.65	0.46	0.89	1.13	0.83	0.57	0.74
12	0.50	0.21	0.25	0.79	0.71	0.40	0.08	0.55	0.45	0.46	1.02	1.11	0.99	0.43	0.80
13	0.57	0.29	0.45	0.59	0.38	-0.03	0.21	0.65	0.93	0.67	0.94	0.78	1.02	0.57	1.06
14	0.45	0.38	0.44	0.12	0.44	0.23	0.35	0.61	0.78	0.98	0.87	0.32	0.96	0.36	0.66
15	0.56	0.53	0.73	0.34	0.30	0.31	0.78	0.81	0.83	1.13	0.92	0.78	0.95	0.52	0.89
16	0.60	0.46	0.71	0.85	0.47	0.32	0.21	0.44	0.63	0.70	0.69	0.56	0.81	0.95	1.04
17	0.16	0.04	0.42	0.77	0.51	-0.15	0.16	0.78	1.05	0.83	0.58	0.72	0.71	1.03	0.50
18	0.29	0.58	0.62	0.56	0.95	0.14	0.28	0.89	0.90	0.72	0.81	0.78	0.99	1.04	0.52
19	0.75	0.42	0.74	0.43	0.58	0.03	0.23	0.79	0.91	0.42	0.92	0.58	0.90	0.93	0.65
20	0.49	0.29	0.64	0.86	0.63	0.13	-0.23	0.90	0.90	0.34	0.95	0.30	0.93	0.83	0.86
21	0.49	0.37	0.75	0.55	0.61	0.31	0.66	0.40	0.60	0.65	0.88	0.17	1.21	0.99	0.77
22	0.54	0.59	0.56	0.62	0.47	0.18	0.32	0.78	0.94	0.87	1.03	0.62	0.96	0.56	0.40
23	0.51	0.49	0.46	0.64	0.41	0.24	0.19	0.99	0.76	0.51	0.81	0.98	1.01	0.74	0.54
24	0.70	0.54	0.48	0.73	0.41	0.28	0.05	0.63	0.19	0.62	0.67	0.80	0.91	0.33	1.05
25	0.56	0.70	0.72	0.32	0.54	0.19	0.29	0.77	0.66	0.77	0.85	0.45	0.98	1.07	0.70
26	0.59	0.34	0.84	0.47	0.61	0.01	0.43	1.01	0.87	1.01	0.98	0.32	1.02	1.04	0.64
27	0.73	0.60	0.85	0.41	0.35	0.27	0.52	0.79	0.83	0.71	1.00	1.13	0.81	0.92	0.47
28	0.61	0.46	0.67	0.56	0.29	0.01	0.37	0.71	0.52	1.01	0.75	1.04	0.90	1.03	0.69
29	0.69	0.44	0.62	0.51	0.51	-0.08	0.10	0.35	0.83	0.81	0.95	0.88	0.64	0.49	0.81
30	0.39	0.56	0.27	0.86	0.73	0.01	0.29	0.59	0.37	0.34	0.94	0.29	0.62	0.80	0.32
31	0.35	0.43	0.72	0.39	0.47	0.13	0.37	0.73	0.71	0.59	0.51	0.50	0.52	0.57	0.45
32	0.60	0.43	1.04	0.49	0.49	-0.15	0.23	1.22	0.76	0.97	0.76	1.15	0.37	0.78	0.91
33	0.78	0.48	0.77	0.39	0.66	-0.31	0.55	0.99	1.00	0.69	1.01	0.84	0.87	0.85	0.86
34	0.45	0.31	0.70	0.25	0.68	-0.18	-0.25	0.71	0.86	0.55	0.74	0.80	0.95	-0.04	0.94
35	0.71	0.80	0.63	0.26	0.30	0.22	0.30	0.70	0.82	0.86	0.98	0.32	0.89	0.45	0.88

Note: Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.17: Estimated order of fractional integration using ELW

City/Price Index	1	2	Perishables					Nonperishables			Services				
			4	5	6	7	8	3	9	10	11	12	13	14	15
1	0.57	0.24	0.77	0.35	0.53	0.06	-0.13	0.73	0.75	0.53	0.89	0.99	0.64	0.41	0.84
2	0.10	-0.03	0.48	0.36	0.87	-0.05	0.35	0.81	0.80	0.72	0.66	0.68	0.58	0.02	0.79
3	0.45	0.46	0.55	0.55	0.63	0.05	0.24	0.45	0.43	0.40	1.05	0.11	0.56	0.41	0.89
4	0.36	0.32	0.38	0.47	0.84	-0.10	0.66	0.29	0.71	0.74	0.90	-0.19	0.62	0.64	1.11
5	0.16	0.43	0.50	0.51	0.43	0.38	0.30	0.50	0.59	0.50	1.13	0.70	0.72	-0.15	0.71
6	0.01	-0.25	0.76	0.51	0.51	-0.03	0.25	0.54	0.69	0.64	0.85	0.51	1.21	0.60	0.47
7	0.08	0.43	0.64	0.33	0.53	0.23	0.21	0.46	0.68	0.47	0.85	0.84	0.36	0.75	0.48
8	0.19	0.22	0.72	0.45	0.35	0.27	0.37	0.83	0.46	0.71	1.10	0.45	0.35	1.02	0.99
9	0.19	0.41	0.56	0.24	0.54	0.21	0.30	0.90	0.85	0.87	0.95	0.97	0.59	0.64	0.67
10	0.41	0.24	0.28	0.63	0.46	0.25	0.06	0.47	0.82	0.50	0.66	0.70	0.42	0.67	0.43
11	0.43	0.40	0.56	0.05	0.35	0.35	0.43	0.83	0.47	0.53	1.19	0.94	1.00	0.43	0.72
12	0.34	0.09	0.14	0.71	0.72	0.41	0.10	0.57	0.54	0.41	0.99	1.03	1.01	0.37	0.57
13	0.54	0.35	0.46	0.63	0.42	-0.11	0.25	0.55	0.85	0.66	0.99	0.78	0.63	0.62	1.05
14	0.47	0.36	0.42	0.11	0.44	0.25	0.33	0.64	0.70	0.87	0.92	0.38	0.90	0.27	0.67
15	0.61	0.50	0.80	0.43	0.27	0.32	0.67	0.83	0.59	0.82	0.90	0.87	0.70	0.48	0.65
16	0.45	0.50	0.75	0.90	0.51	0.33	0.22	0.48	0.49	0.78	0.83	0.67	0.44	0.73	0.76
17	0.19	0.07	0.46	0.71	0.51	-0.15	0.19	0.79	0.60	0.79	0.64	0.62	0.52	0.89	0.45
18	0.22	0.47	0.57	0.53	0.78	0.11	0.30	0.88	0.85	0.69	0.89	0.89	0.63	0.71	0.40
19	0.47	0.31	0.76	0.39	0.59	0.12	0.27	0.66	0.63	0.43	1.06	0.49	0.54	0.67	0.54
20	0.30	0.31	0.69	0.87	0.69	0.09	-0.22	0.96	0.68	0.36	1.04	0.32	0.75	0.88	0.45
21	0.38	0.35	0.69	0.55	0.68	0.26	0.69	0.44	0.49	0.69	1.01	0.11	0.92	0.76	0.44
22	0.57	0.46	0.49	0.66	0.54	0.20	0.30	0.87	0.55	0.89	0.98	0.56	0.76	0.55	0.26
23	0.46	0.44	0.36	0.57	0.45	0.27	0.21	1.01	0.55	0.44	0.88	0.69	0.86	0.67	0.23
24	0.57	0.51	0.46	0.66	0.45	0.28	0.12	0.63	0.23	0.71	0.74	0.72	0.73	0.36	0.77
25	0.52	0.58	0.56	0.27	0.60	0.22	0.31	0.76	0.45	0.69	0.91	0.44	0.91	0.91	0.56
26	0.52	0.36	0.92	0.39	0.66	0.04	0.47	1.00	0.66	0.67	1.17	0.38	0.58	0.83	0.51
27	0.60	0.76	0.87	0.29	0.41	0.22	0.44	0.82	0.60	0.61	1.00	0.72	0.41	0.74	0.33
28	0.60	0.47	0.69	0.48	0.32	-0.04	0.37	0.78	0.49	0.67	0.71	0.96	0.60	0.89	0.68
29	0.44	0.35	0.64	0.44	0.53	-0.01	0.10	0.38	0.62	0.72	0.91	0.66	0.40	0.30	0.64
30	0.42	0.38	0.35	0.74	0.64	0.07	0.31	0.43	0.26	0.33	0.98	0.41	0.44	0.71	0.33
31	0.34	0.46	0.73	0.37	0.53	-0.25	0.33	0.75	0.53	0.47	0.65	0.51	0.37	0.59	0.41
32	0.61	0.48	1.02	0.52	0.52	-0.02	0.23	1.16	0.77	0.76	0.75	0.93	0.37	0.64	0.88
33	0.82	0.54	0.79	0.48	0.67	-0.10	0.50	1.01	0.88	0.70	0.92	0.68	0.81	0.65	1.02
34	0.37	0.30	0.64	0.28	0.68	-0.13	-0.17	0.76	0.80	0.43	0.58	1.02	0.76	-0.02	0.74
35	0.76	0.85	0.67	0.22	0.27	-0.17	0.28	0.81	0.76	0.78	0.84	0.07	0.51	0.33	0.61

Note: Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.18: Estimated order of fractional integration using FELW

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0.67	0.23	0.77	0.42	0.52	0.12	0.03	0.77	0.86	0.51	0.92	0.98	0.79	0.64	0.84
2	0.09	0.03	0.65	0.34	0.88	0.03	0.38	0.86	0.77	0.78	0.73	0.70	0.76	0.03	0.86
3	0.70	0.66	0.55	0.58	0.66	0.11	0.26	0.62	0.63	0.49	1.07	0.11	0.75	0.42	0.93
4	0.65	0.40	0.39	0.66	0.86	-0.23	0.56	0.27	0.83	0.69	0.89	0.04	0.77	0.72	0.99
5	0.27	0.70	0.65	0.50	0.67	0.37	0.30	0.46	0.74	0.48	1.13	0.58	0.82	0.05	0.73
6	-0.04	-0.09	0.77	0.51	0.72	0.11	0.26	0.51	0.83	0.66	0.93	0.68	1.21	0.73	0.48
7	0.03	0.65	0.68	0.37	0.72	0.23	0.21	0.66	0.69	0.46	0.88	0.83	0.64	0.78	0.65
8	0.28	0.22	0.74	0.47	0.68	0.27	0.67	0.86	0.70	0.74	1.11	0.65	0.47	1.02	1.00
9	0.19	0.64	0.70	0.28	0.70	0.22	0.33	0.94	0.85	0.88	0.94	1.01	0.76	0.70	0.72
10	0.66	0.27	0.28	0.67	0.69	0.25	-0.03	0.51	0.87	0.70	0.63	0.55	0.41	0.71	0.70
11	0.43	0.62	0.58	0.01	0.60	0.34	0.66	0.87	0.66	0.62	1.19	0.88	1.00	0.65	0.79
12	0.64	0.31	0.23	0.73	0.74	0.44	0.07	0.56	0.50	0.64	1.01	1.03	1.02	0.64	0.70
13	0.65	0.35	0.46	0.60	0.41	0.00	0.25	0.67	0.89	0.64	0.98	0.84	0.74	0.61	1.08
14	0.49	0.38	0.62	0.15	0.65	0.27	0.33	0.70	0.76	0.90	0.99	0.36	0.88	0.41	0.73
15	0.59	0.60	0.80	0.40	0.39	0.32	0.72	0.90	0.70	0.83	0.96	0.83	0.77	0.66	0.73
16	0.67	0.49	0.74	0.90	0.62	0.33	0.22	0.66	0.66	0.78	0.85	0.70	0.73	0.74	0.94
17	0.19	0.06	0.44	0.74	0.60	-0.16	0.19	0.84	0.73	0.81	0.74	0.69	0.69	0.88	0.60
18	0.34	0.67	0.68	0.67	0.82	0.17	0.31	0.89	0.86	0.74	0.89	0.92	0.81	0.74	0.53
19	0.71	0.60	0.76	0.43	0.59	0.09	0.28	0.70	0.72	0.48	1.06	0.67	0.74	0.73	0.69
20	0.66	0.34	0.63	0.87	0.65	0.16	-0.24	1.02	0.74	0.43	1.09	0.33	0.89	0.87	0.82
21	0.65	0.35	0.71	0.58	0.67	0.39	0.72	0.41	0.67	0.68	1.11	0.16	0.93	0.73	0.84
22	0.55	0.63	0.67	0.69	0.53	0.20	0.31	0.89	0.72	0.89	0.98	0.65	0.88	0.63	0.42
23	0.56	0.49	0.68	0.69	0.46	0.26	0.21	1.01	0.67	0.67	0.90	0.72	1.01	0.71	0.60
24	0.73	0.57	0.63	0.72	0.45	0.36	0.12	0.65	0.24	0.71	0.78	0.75	0.75	0.36	0.97
25	0.62	0.68	0.66	0.28	0.67	0.21	0.30	0.76	0.68	0.69	0.90	0.63	0.93	0.94	0.80
26	0.67	0.41	0.92	0.49	0.64	0.01	0.47	1.02	0.71	0.74	1.26	0.40	0.78	0.84	0.59
27	0.80	0.80	0.86	0.43	0.41	0.30	0.63	0.89	0.71	0.65	0.93	0.76	0.73	0.74	0.64
28	0.70	0.50	0.67	0.63	0.33	0.03	0.39	0.81	0.58	0.79	0.76	1.00	0.85	0.89	0.69
29	0.68	0.63	0.67	0.62	0.59	-0.06	0.10	0.36	0.70	0.72	0.90	0.70	0.70	0.54	0.75
30	0.44	0.51	0.40	0.86	0.66	0.05	0.31	0.68	0.38	0.46	0.97	0.39	0.72	0.77	0.33
31	0.40	0.46	0.73	0.39	0.54	0.16	0.35	0.81	0.68	0.70	0.71	0.61	0.59	0.56	0.53
32	0.67	0.50	1.02	0.70	0.53	-0.13	0.23	1.19	0.76	0.76	0.72	0.96	0.42	0.70	0.88
33	0.87	0.57	0.78	0.47	0.66	-0.29	0.67	1.02	0.94	0.77	0.86	0.69	0.79	0.70	1.15
34	0.50	0.37	0.71	0.28	0.75	-0.16	-0.31	0.80	0.82	0.69	0.77	1.14	0.84	-0.04	0.85
35	0.76	0.86	0.69	0.28	0.33	0.25	0.30	0.80	0.78	0.69	0.77	0.41	0.75	0.57	0.80

Note: Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.19: Estimated order of fractional integration using FELWd

			Perishables					Nonperi.			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0.35	0.20	0.77	0.41	0.52	0.12	-0.05	0.74	0.77	0.51	0.71	0.96	0.78	0.34	0.84
2	-0.11	-0.05	0.35	0.33	0.86	-0.11	0.24	0.81	0.66	0.81	0.50	0.70	0.64	0.02	0.87
3	0.10	0.43	0.55	0.63	0.59	-0.26	0.14	0.18	0.47	0.36	1.07	0.11	0.29	0.40	0.92
4	-0.04	-0.05	0.39	0.69	0.80	-0.28	0.55	0.24	0.62	0.71	0.85	0.03	0.67	0.26	1.00
5	0.19	0.70	0.62	0.32	0.46	0.28	0.03	0.65	0.51	0.47	1.13	0.56	0.77	-0.21	0.49
6	-0.09	-0.09	0.75	0.51	0.49	0.07	0.17	0.34	0.60	0.48	0.89	0.78	1.21	0.52	0.44
7	-0.05	0.27	0.67	0.30	0.30	0.04	-0.07	0.65	0.65	0.44	0.83	0.81	0.05	0.79	0.44
8	0.19	0.05	0.72	0.18	0.27	0.07	0.05	0.77	0.64	0.73	1.11	0.32	0.44	0.98	1.01
9	0.06	0.05	0.27	0.27	0.40	-0.04	-0.09	0.97	0.72	0.86	0.94	0.98	0.60	0.53	0.37
10	-0.17	0.20	0.27	0.69	0.22	-0.14	-0.06	0.36	0.78	0.28	0.50	0.57	0.40	0.65	0.66
11	0.43	0.00	0.37	0.00	0.41	0.20	0.36	0.88	0.11	0.63	1.20	0.76	1.00	0.10	0.79
12	0.33	0.31	0.20	0.69	0.74	0.44	0.07	0.59	0.48	0.47	1.00	1.03	1.02	0.05	0.30
13	0.51	0.35	0.39	0.63	0.40	-0.15	0.23	0.60	0.77	0.64	0.94	0.84	0.59	0.65	1.07
14	0.49	0.22	0.34	0.15	0.24	0.26	0.32	0.70	0.62	0.86	0.92	0.32	0.87	0.30	0.72
15	0.64	0.33	0.78	0.37	0.31	0.03	0.52	0.88	0.50	0.75	0.84	0.83	0.71	0.64	0.77
16	0.13	0.43	0.74	0.90	0.65	0.12	0.16	0.65	0.48	0.80	0.82	0.71	0.12	0.62	0.88
17	0.04	0.03	0.44	0.64	0.49	-0.17	-0.27	0.82	0.41	0.77	0.61	0.32	0.66	0.80	0.31
18	0.17	0.26	0.39	0.66	0.82	-0.11	0.28	0.91	0.89	0.74	0.82	0.91	0.65	0.73	0.31
19	0.63	0.17	0.77	0.13	0.61	0.04	0.13	0.65	0.50	0.46	1.04	0.24	0.64	0.67	0.64
20	0.36	0.32	0.62	0.87	0.65	0.14	-0.24	1.02	0.43	0.40	1.06	0.33	0.81	0.92	0.76
21	0.41	0.19	0.63	0.53	0.64	0.38	0.67	0.37	0.45	0.72	1.10	0.14	0.80	0.65	0.84
22	0.55	0.10	0.17	0.70	0.53	0.05	0.00	0.91	0.37	0.89	0.97	0.46	0.79	0.37	0.31
23	0.40	0.32	0.37	0.58	0.44	0.16	-0.16	1.01	0.32	0.48	0.78	0.53	0.92	0.63	0.64
24	0.53	0.41	0.46	0.65	0.44	-0.27	0.01	0.67	-0.02	0.70	0.70	0.77	0.38	0.36	0.98
25	-0.10	0.52	0.42	0.19	0.67	0.12	0.18	0.80	0.56	0.68	0.90	0.41	0.65	0.82	0.67
26	0.56	0.39	0.91	0.28	0.67	0.00	0.47	0.99	0.14	0.46	1.26	0.30	0.50	0.79	0.64
27	0.76	0.79	0.86	0.19	0.41	0.27	0.32	0.89	0.47	0.29	0.90	0.83	0.39	0.77	-0.05
28	0.67	0.46	0.65	0.25	0.33	-0.07	0.37	0.78	0.18	0.73	0.68	1.01	0.67	0.87	0.68
29	0.31	0.18	0.64	0.31	0.49	-0.08	0.01	0.15	0.64	0.67	0.85	0.42	0.35	0.45	0.61
30	0.31	0.28	0.35	0.92	0.62	-0.24	0.30	0.33	-0.01	0.33	0.89	0.30	0.24	0.57	-0.05
31	0.13	0.46	0.73	0.28	0.55	0.02	0.34	0.79	0.61	0.55	0.61	0.60	0.34	0.27	0.52
32	0.45	0.35	1.01	0.41	0.51	-0.12	0.10	1.19	0.69	0.74	0.59	1.00	0.31	0.29	0.87
33	0.77	0.48	0.78	0.44	0.65	-0.32	0.48	1.01	0.88	0.61	0.77	0.36	0.81	0.16	1.12
34	-0.15	0.09	0.59	0.26	0.62	-0.34	-0.31	0.78	0.85	0.43	0.67	1.11	0.65	-0.08	0.76
35	0.76	0.86	0.70	0.27	0.32	-0.19	0.07	0.81	0.69	0.72	0.76	0.41	0.63	0.43	0.84

Note: Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.20: Classification into $I(1)$ and $I(d)$, $d < 1$, using LW

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1
2	0	0	0	0	0	0	0	1	1	1	1	0	1	0	1
3	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
4	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1
5	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
6	0	0	0	0	0	0	0	0	1	1	1	0	1	1	0
7	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0
8	0	0	0	0	0	0	0	1	1	0	1	0	0	1	1
9	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
10	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0
11	0	0	0	0	0	0	0	1	1	0	1	1	1	0	1
12	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1
13	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1
14	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0
15	0	0	0	0	0	0	0	1	1	1	1	1	1	0	1
16	0	0	0	0	0	0	0	0	1	0	1	0	1	1	1
17	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
18	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
19	0	0	0	0	0	0	0	1	1	0	1	0	1	1	0
20	0	0	0	0	0	0	0	1	1	0	1	0	1	1	1
21	0	0	0	0	0	0	0	0	1	0	1	0	1	1	1
22	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0
23	0	0	0	0	0	0	0	1	1	0	1	1	1	1	0
24	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1
25	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0
26	0	0	0	0	0	0	0	1	1	1	1	0	1	1	0
27	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
28	0	0	0	0	0	0	0	1	0	1	1	1	1	1	0
29	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1
30	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0
31	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0
32	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1
33	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1
34	0	0	0	0	0	0	0	1	1	0	1	1	1	0	1
35	0	0	0	0	0	0	0	1	1	1	1	0	1	0	1

Note: number 1 indicates $I(1)$ series, number 0 indicates $I(d)$, $d < 1$, series. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.21: Classification into $I(1)$ and $I(d)$, $d < 1$, using ELW

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1
2	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1
9	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0
12	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
13	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
14	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
15	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
17	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0
18	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
19	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0
21	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
22	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
23	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
24	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
25	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0
26	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
27	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
28	0	0	0	0	0	0	0	1	0	0	1	1	0	1	0
29	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
31	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
32	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1
33	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1
34	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0
35	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0

Note: number 1 indicates $I(1)$ series, number 0 indicates $I(d)$, $d < 1$, series. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.22: Classification into $I(1)$ and $I(d)$, $d < 1$, using FELW

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	1	1	0	1	1	1	0	1
2	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1
3	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
4	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
5	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
6	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1
9	0	0	0	0	0	0	0	1	1	0	1	1	1	0	0
10	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	1	1	1	0	1
12	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
13	0	0	0	0	0	0	0	0	1	0	1	1	1	0	1
14	0	0	0	0	0	0	0	1	1	0	1	0	1	0	0
15	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
17	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0
18	0	0	0	0	0	0	0	1	1	0	1	1	1	0	0
19	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
20	0	0	0	0	0	0	0	1	0	0	1	0	1	1	1
21	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
22	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
23	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
24	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
25	0	0	0	0	0	0	0	1	0	0	1	0	1	1	1
26	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0
27	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
28	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0
29	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
31	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
32	0	0	0	0	0	0	0	1	1	0	1	1	0	0	1
33	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1
34	0	0	0	0	0	0	0	1	1	0	1	1	1	0	1
35	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1

Note: number 1 indicates $I(1)$ series, number 0 indicates $I(d)$, $d < 1$, series. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

Table C.23: Classification into $I(1)$ and $I(d)$, $d < 1$, using FELWd

			Perishables					Nonperishables			Services				
City/Price Index	1	2	4	5	6	7	8	3	9	10	11	12	13	14	15
1	0	0	1	0	0	0	0	1	0	0	1	1	1	0	1
2	0	0	0	0	0	0	0	1	0	1	1	1	1	0	1
3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
4	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1
5	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
6	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0
7	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0
8	0	0	1	0	0	0	0	1	0	1	1	0	0	1	1
9	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	1	1	1	0	1
12	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0
13	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1
14	0	0	0	0	0	0	0	1	0	1	1	0	1	0	1
15	0	0	1	0	0	0	0	1	0	1	1	1	1	0	1
16	0	0	1	0	0	0	0	1	0	1	1	1	0	0	1
17	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0
18	0	0	0	0	0	0	0	1	0	1	1	1	1	0	0
19	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0
20	0	0	0	0	0	0	0	1	0	0	1	0	1	1	1
21	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1
22	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0
23	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0
24	0	0	0	0	0	0	0	1	0	1	1	1	0	0	1
25	0	0	0	0	0	0	0	1	0	1	1	0	1	0	1
26	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
27	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0
28	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1
29	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
31	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
32	0	0	1	0	0	0	0	1	0	1	1	1	0	0	1
33	0	0	1	0	0	0	0	1	0	0	1	0	1	0	1
34	0	0	0	0	0	0	0	1	0	0	1	1	1	0	1
35	0	0	1	0	0	0	0	1	0	1	1	0	1	0	1

Note: number 1 indicates $I(1)$ series, number 0 indicates $I(d)$, $d < 1$, series. Categories of price indices are: 1. CPI, 2. Food, 3. Grain, 4. Meat, 5. Eggs, 6. Seafood, 7. Fresh Vegetables, 8. Fresh Fruits, 9. Tobacco and Wine, 10. Clothing, 11. Recreation Facilities, 12. Health Care, 13. Traffic and Communication, 14. Cultural and Educational Expense, 15. Residence

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