



AUTONOMOUS RADIATION MAPPING WITH A CSI(TL)-EQUIPPED MOBILE ROBOT: LOCALISATION AND ISOTOPE IDENTIFICATION

By

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ABSTRACT

In recent years, mobile robotic technologies have assumed a critical role in detecting and controlling radiological hazards, especially for nuclear facilities and emergency response scenarios. This thesis presents a comprehensive study on the location estimation and identification of gamma radiation sources using a mobile robot integrated with a CsI(Tl) module detector. In this system, Maximum Likelihood Estimation (MLE), Bayesian inference and Triangulation methods were optimised and applied in detail. The methods were tested in single-source conditions and then in a multi-source environment. The comparative evaluation showed that the Bayesian method provides the highest accuracy with an average error of approximately 2 cm, especially in environments with a low signal-to-noise ratio. In multi-source scenarios, the margin of error was approximately 6 cm. Subsequently, spectral analysis algorithms were developed to identify the sources and estimate their activity. While the results obtained for strong sources agreed with the reference measurements, for weak sources, there were limits such as low counting efficiency and measurement limits that reduced the accuracy. Experimental evaluations of the developed system have demonstrated the ability of the mobile robot-assisted radiation source detection system to successfully locate and identify the source. The combination of Bayesian probabilistic grounded approach

and spectral analysis algorithms has resulted in highly accurate radiation source detection. This work provides a solid foundation and contribution to radiation detection in nuclear safety and radiological emergency response areas.

DEDICATION

Dedicated to my parents, the two most important people in my life, who have always supported me with their unconditional love, patience, and encouragement.

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Acronyms

GEB Gaussian energy broadening. 45

L-BFGS-B Limited-memory Broyden-Fletcher-Goldfarb-Shanno with Box constraints Optimisation Algorithm. 58–60, 64, 73, 77, 139–142, 166

MCMC Markov Chain Monte Carlo. 151

MLE Maximum likelihood estimation. 2, 3, 57, 61, 67, 73, 130–132, 134, 139, 140, 142–145, 166

MPPC multi-pixel photon counter. 21

PMT Photomultiplier Tube. 15, 16

ROS Robot Operating System. 51, 52, 93

Chapter One

Introduction

1.1 Overview

Nuclear energy plays a critical role in meeting global energy demands. However, the risks associated with radioactive environments—such as high radiation exposure and hazardous operational conditions—necessitate innovative safety solutions [1]. Mobile robotics have been widely used in nuclear facilities to improve safety and reduce human exposure. Robots can perform demanding and complex tasks such as inspection, maintenance, search and rescue and decontamination with high precision and safety without human intervention [2],[3]. Robotic technologies have been used effectively, especially after the Three Mile Island and Chernobyl accidents. In the Three Mile Island accident, the remotely controllable ROVER robot used to inspect the damage in the reactor core and detect melted fuel residues was the pioneer of the use of robots in nuclear accidents [4][5],[6]. For example, the Little Sunfish robot in Fukushima in 2011 [7] significantly contributed to assessing

the accident in locating melted nuclear fuel on the reactor floor due to its small size and resistance to high radiation levels [1],[8]. More recently, in 2024, the Telesco robot developed by Tokyo Electric Power Company Holdings (TEPCO) was designed to sample melted fuel debris under the primary containment vessel of the Fukushima No. 2 reactor. With a telescopic arm and a fishing rod-like structure, it successfully cut and picked up pieces of melted fuel weighing 5 mm and 3 grams [9][10]. This finding is crucial for ensuring the safe decommissioning and effective clean-up of the nuclear plant. These examples show the importance of robotics in reducing the effects of nuclear accidents and improving the safety of nuclear facilities [2],[11]. With the development of robotic systems and their increasing use in nuclear environments, robots began to be designed according to this need. To meet the challenges of the nuclear environment, robots must have several important technical features, such as high radiation resistance, the ability to move in confined spaces, and the ability to be controlled remotely. In addition, advanced sensor systems allow robots to detect radiation levels accurately and perform detailed radiation mapping [2],[12],[13]. Mobile robots also play an important role in the location of radiation sources. Robotic systems combine information from sensors with geometric and algorithmic methods to accurately determine the location of radiation sources. In particular, Maximum likelihood estimation (MLE), Bayesian and triangulation methods are powerful methods used to estimate the location of radiation sources [14],[15]. Maximum likelihood estimation (MLE) is a method that can find the source location with high accuracy, even in complex scenarios supported by optimisation techniques [16]. The Bayesian method gives good results when environmental uncertainties are high, while the triangulation method estimates quickly using a geometric approach [17],[18]. As a result, thanks

to these methods, the location of radiation sources in nuclear environments with mobile robots has been determined with high accuracy and has significantly contributed to raising safety standards in operational processes [15]. This study aims to accurately determine the location of radiation sources using mobile robots and sensors by optimising various methods (MLE, Bayesian and triangulation) with algorithms and to identify these sources.

1.2 Thesis Objectives

This thesis aims to develop a low-cost, fast, and highly accurate system supported by a mobile robot for the location and identification of radiation sources. The originality of the study lies in the integration of algorithms, optimisation methods, comparative analyses, and gamma spectrum analysis within a single platform. The CsI(Tl) crystal scintillator detector integrated into the mobile robot performs spatial information extraction and isotope identification with the data collected in real-time from the field. Thus, a system that does not only detect the location of the source but also identifies the source and estimates its activity has emerged. To compare the Maximum Likelihood Estimation (MLE), Bayesian Approach and Triangulation algorithms experimentally, the accuracy and stability of the algorithms in different signal-to-noise scenarios have been analyzed. The performances for single and multiple-source scenarios have been evaluated. By evaluating the effects of the geometrical location errors of the robot on spectral analysis and activity estimation, the limitations that the system may encounter in field applications have been revealed in line with these findings.

Chapter Two

Radiation Detection and Transport Theory

2.1 Interaction of Gamma Rays with Matter

Photons can interact with the material in many ways. The most observed interactions in gamma sources are the photoelectric effect, Compton scattering and pair production. Rayleigh and Mie processes can also be observed, but these interactions are rare. [Figure 2.1](#) shows these interaction processes.

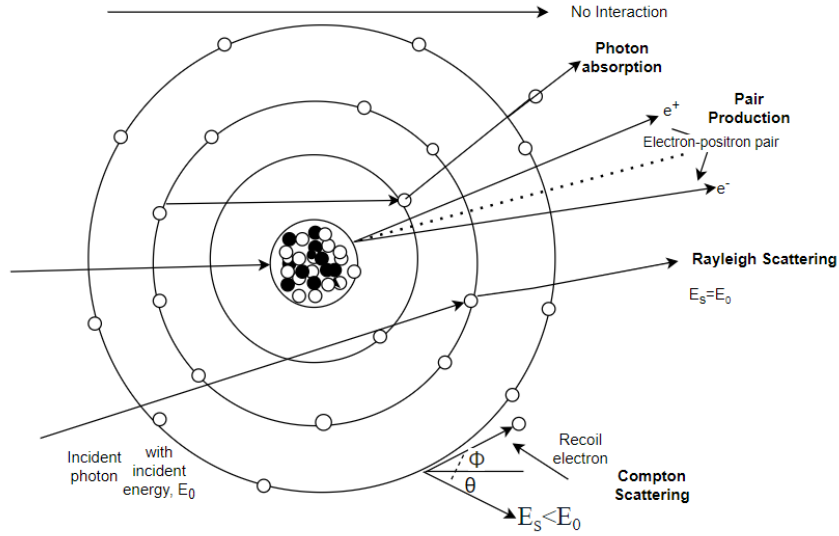


Figure 2.1: Schematic of the main mechanisms through which photons interact with matter during detection [19].

Photoelectric absorption: When a photon interacts with an atom, its energy can be completely absorbed and lost by the atom. Under these conditions, the absorbed energy is transferred to one of the bound electrons of the atom, increasing the energy level of the electron, and this electron is ejected, forming a charge carriers. During this interaction, part of the energy of the gamma ray is transferred as binding energy, while the remaining energy is converted into kinetic energy for the ejected electron:

$$T = E_{\gamma} - B_e, \quad (2.1)$$

⁰Coherent scattering is the re-emission of gamma radiation in a different direction where the energy remains constant but the direction changes [20]. It usually occurs at very low energies and can be disregarded in most gamma-ray measurements.

where T , E_γ , and B_e represent kinetic energy, gamma energy, and binding energy, respectively (2.1).

When a photon ejects a K-shell electron, the vacancy is filled by an outer-shell electron, releasing energy. This energy either produces a characteristic X-ray (radiative) or ejects an Auger electron (non-radiative). The Auger process competes with X-ray emission, with low-Z materials favoring Auger electrons and high-Z materials favoring X-rays. The energy of the Auger electron depends on the binding energy differences between the involved shells, rather than on the energy of the original photon. While there is no single estimate of the probability of photoelectric absorption for each atomic number and E range, higher Z materials increase the probability of absorption. This tendency is represented by $\tau \equiv \text{constant} \times \frac{Z^n}{(E_\gamma)^{3.5}}$. The n-value can vary between 4 and 5 depending on the photon energy in the gamma ray region of interest. In this equation, Z is the charge of the nucleus, and E represents energy. Due to the excellent detection efficiency, high Z components are often preferred in NaI detectors and often used in gamma ray spectroscopy [19],[21].

In the process of Compton scattering, the gamma ray absorber communicates directly with the electron in the material, transferring only a fraction of the energy of the gamma ray to the electron. The probability of Compton scattering is related to the density of the material. Only a fraction of the energy is transferred when the electron is not bound tightly enough by the atom to absorb the recoil energy. This is a direct result of the fundamental conservation laws of energy and momentum, which govern the process. The photon deviates from its original direction by an angle θ . Some energy is transferred to the recoiling electron. Since all

scattering angles can occur, the energy of the gamma ray can be transferred from zero to a significant amount. The following Equation 2.2 illustrates the Compton scattering process:

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos\theta)}, \quad (2.2)$$

where $h\nu$ and $h\nu'$ are the energies of the photon before and after scattering, respectively, and m is the rest mass of the electron and is 0.511 MeV. The variable divergence angle θ can take a value between 0 and 180. As the photon energy increases, a double production event begins to be observed. In this process, unlike the photoelectric and Compton processes, gamma rays affect the whole atom. This event takes place in the Coulomb field of the nucleus and the complete annihilation of the incident gamma ray produces an electron-positron pair. This event is seen in the gamma ray spectrum when the energy is at least 1.022 MeV. However, when the energy of the incident gamma ray is greater than this value, the excess energy is shared as kinetic energy between the electron and positron [20],[22],[23].

2.2 Gamma Spectroscopy

Analysing the physical properties of detectors and their interaction mechanisms can help us understand energy spectra characteristics. The peaks and interactions of such spectra are shown in the Figure 2.2. One of the most essential features of gamma ray spectra is the total energy peak. This peak ideally appears as a thin peak at a given energy level. Although this peak appears to be a thin peak at a certain energy level under ideal conditions, in practice, this peak widens due

to electronic noise and changes in the operating conditions of the detector. This results in a widened energy distribution instead of an ideal peak.

Detectors do not always capture the full energy during energy accumulation. They usually detect only a fraction of the full energy. The interaction mechanisms discussed in Section 2.1 often lead to the deposition of only a portion of the full energy. Lower energy structures arise in the spectrum due to these energy losses, which impede full energy deposition. As a result, the spectrum has considerable energy losses due to the physical constraints and detector limits, which sometimes prevent full energy retention. The finite size of the detector and the physical processes involved may result in the accumulation of only a fraction of the full energy stored. For example, Compton scattering occurs when photons interact with electrons, deposit some of their energy and are scattered. This process produces a Compton continuum spectrum in the detector, spread over a wide energy range. The maximum point of the scattered photons is defined as the Compton edge, which is observed at a scattering angle of 180 degrees. The formation of pairs characterises another critical structure in the spectrum. In this interaction, an electron-positron pair is formed, and the annihilation of this pair emits two photons with an energy of 511 keV. If one or both of these photons escape from the detector, peaks in the spectrum known as single escape $E_{\text{full}} - 511 \text{ keV}$ or double escape $E_{\text{full}} - 1022 \text{ keV}$ are seen. Figure 2.2 presents the key features that may be present in an energy spectrum in a schematic form.

In addition, the characteristic X-rays and Auger electrons seen in the spectrum are produced after the photoelectric effect. The characteristic X-rays and Auger electrons emitted by filling a vacancy in an inner shell with an electron from

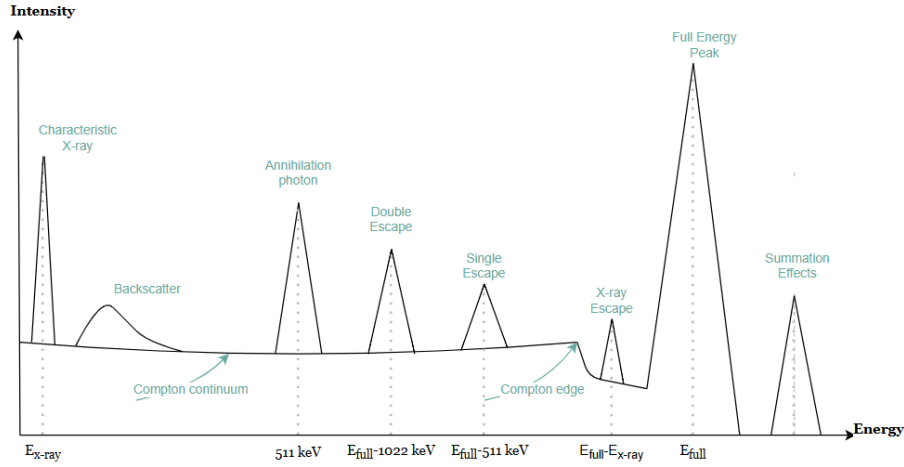


Figure 2.2: Schematic representation of the main features that can be included in the energy spectrum.

a higher energy level can also be observed. If these X-rays escape from the detector, there will be another peak in the spectrum as $E_{full} - E_{X-ray}$, whereas Auger electrons, because they follow a short free path, leave their energy inside the detector and cannot escape. Summation effects occur when a radioactive source emits successive gamma rays. These photons reach the detector simultaneously, producing an actual coincidence sum at the detector. For example, the ^{60}Co isotope emits gamma spectra with energies of 1.17 MeV and 1.33 MeV. When detected together, the ^{60}Co shows a gamma-ray peak at around 2.5 MeV. Random coincidences can also be observed at high activity levels because photons from different sources reach the detector simultaneously by chance [20],[22]. In addition to internal detector interactions, environmental conditions must be considered when analysing the spectrum. For example, photons reflected from materials near the detector may return to the detector to form backscatter peaks, usually seen around 200

keV and originate from a wide range of scattering angles. Backscattering reflects the environment and environmental conditions in which the detector is placed and shapes the spectrum. Consequently, understanding the structure of gamma spectra is only possible by considering the structural properties of the detectors, their interaction mechanisms, and environmental conditions. Accurate analyses of these relationships are of critical importance in areas such as nuclear physics, safety and radiation measurements.

2.3 Detection Efficiency

Radiation detectors produce an output pulse due to each quantum interaction of radiation in their active volume. While charged particles, such as alpha and beta particles, interact immediately in the active region of the detector by direct ionisation and excitation, uncharged radiation, such as gamma and neutron radiation, can travel long distances without interacting with the detector. This results in a detector efficiency of less than 100% for detecting uncharged radiation; therefore, the detector efficiency needs to be precisely characterised [19],[24]. Efficiency types can be categorised as follows:

Absolute Efficiency(ε_{abs})

It is the ratio of the counts produced by the detector relative to the gamma photons emitted in all directions from the source, expressed as in [Formula 2.3](#).

$$\varepsilon_{\text{abs}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta emitted by source}} \quad (2.3)$$

Absolute efficiency depends not only on the physical characteristics of the detector but also on the geometry between the source and the detector.

Intrinsic Efficiency(ε_{int})

Intrinsic efficiency is the rate at which gamma photons reaching the detector can be detected according to the physical properties of the detector itself, formulated as follows:

$$\varepsilon_{\text{int}} = \frac{\text{number of pulses recorded}}{\text{number of radiation quanta incident on detector}} \quad (2.4)$$

Intrinsic efficiency reflects the effectiveness of the detector material and design and indicates how much of the photons incident on the detector can be absorbed and detected. The relationship between absolute and intrinsic efficiency depends on the solid angle (Ω) subtended by the detector at the source.

$$\varepsilon_{\text{int}} = \varepsilon_{\text{abs}} \cdot \left(\frac{4\pi}{\Omega} \right) \quad (2.5)$$

Here, (Ω) is the solid angle (in steradians) subtended by the detector from the source position [20].

Geometric Efficiency(ε_{geo})

Geometric efficiency refers to the ratio of the total number of gamma photons emitted by a radiation source to the number of photons that successfully reach the detector. It is expressed mathematically as follows:

$$\varepsilon_{\text{geo}} = \frac{\text{Number of gamma photons reaching the detector}}{\text{Total number of gamma photons emitted by the source}} \quad (2.6)$$

The geometrical efficiency depends on both the source-detector distance and the effective surface area of the detector. If the source is a point source and emits isotropically (uniformly in all directions), the gamma photons are scattered in a 4π steradian sphere. The detector sees only a small part of this sphere. In particular, the geometric efficiency for a point source and a system of uniform detectors can be expressed approximately as in [Formula 2.7](#):

$$\varepsilon_{\text{geo}} \approx \frac{A_{\text{det}}}{4\pi r^2} \quad (2.7)$$

Where: - A_{det} : Effective surface area of the detector, - r : The distance between the source and the centre of the detector Geometric efficiency decreases with the inverse square of the distance while it increases proportionally with the detector area. The product of the geometric efficiency (ε_{geo}) and the intrinsic efficiency (ε_{int}) gives the total efficiency [20],[24].

2.4 Scintillation for Radiation Detection

Insulators and semiconductor materials have certain bandgap levels to which electrons have access, where the lower band, the valence band, represents the electrons that are bound in the crystal organisation. The conduction band is in which electrons have sufficient energy to move freely within the crystal. Between these two bands is a forbidden band, which does not contain any electron in the pure crystal state. When energy is absorbed, the primary electron gains energy from the incoming radiation and transfers part of this energy to secondary electrons through a series of inelastic collisions. These secondary electrons, in turn, may acquire sufficient energy to be excited from the valence band to the conduction band by crossing the forbidden energy region, resulting in the formation of electron-hole pairs. This is represented in [Figure 2.3](#) for conductors, insulators and semiconductors.

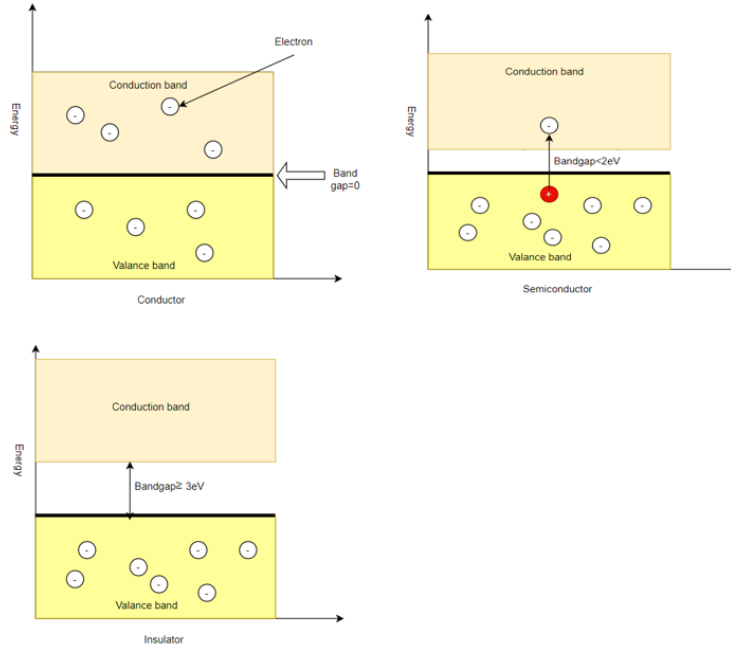


Figure 2.3: Energy band diagram of conductor, semiconductor and insulator.

In pure crystals, the efficiency of the return of electrons to the valence band is generally low, and typical band gaps prevent the emitted photons from entering the visible light range. Therefore, impurities are introduced into inorganic luminescent materials to enhance this process and increase the probability of visible light absorption. These additives are called activators. Activators add another energy level to the bandgap. In this case, the emission of photons at frequencies around the visible spectrum is facilitated by a series of de-excitations because when these levels are de-excited, the energy of the emitted photon will be lower, which means that the electromagnetic radiation can be at longer wavelengths and even in the visible range. In addition, since the wavelength of the emitted beam will no longer match the absorption characteristics of the scintillator, the loss of

light before the photomultiplier can measure it will be significantly reduced, which means more efficient light collection and detection.

The scintillation light emitted by the crystal is converted into a measurable signal by the Photomultiplier Tube (PMT). The PMT converts the scintillation-emitted light into photoelectrons, which contribute to the electronic signal and provide meaningful information about the energy deposited by the incident radiation. PMT is a complex process that starts with the scintillation light reaching the photocathode and proceeds through several amplification stages. The diagram of PMT is provided in [Figure 2.4](#). An incident photon causes the release of an electron on the photocathode. This process has very low efficiency, i.e., on average, five photons are required to produce one photoelectron since the rate of occurrence is about 20%.

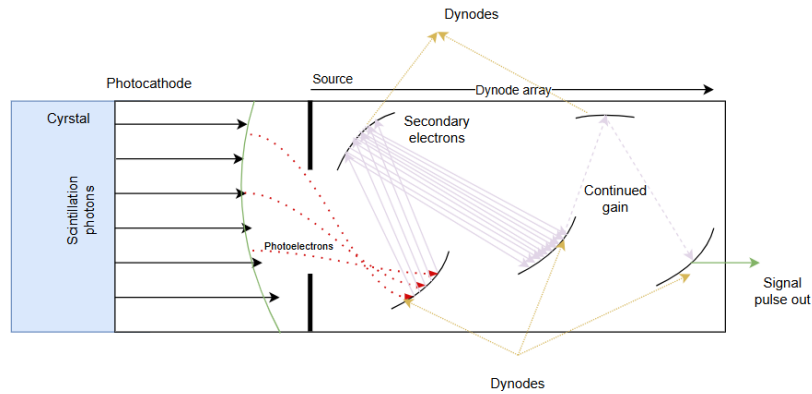


Figure 2.4: Diagram of Photomultiplier Tube.

The high electric potential difference accelerates the release of the photoelectron, which hits the first dynode. As a result of this collision, several secondary electrons are released, and these new electrons accelerate towards the following

dynodes. In this phase, each colliding electron releases more electrons. This results in a high gain amplification. When the last dynode is reached, a sufficient amount of electrons is collected, which generates a measurable electrical pulse. This pulse provides information about the energy deposited by the incident radiation. At the same time, the number of pulses generated per unit time indicates the radiation intensity. The surface barrier in PMT requires overcoming an energy threshold before electrons can diffuse from the photocathode surface into the vacuum. Therefore, the photocathode layer must be thick enough to absorb light sufficiently and thin enough to allow photoelectrons to escape from the surface. For an electron to propagate into the vacuum, it must overcome the vacuum level barrier, that is, the work function of the material. When a photon hits the photocathode, the electrons in the material are excited by receiving energy from these photons. They can spread through the material and reach the surface with sufficient energy. The photocathode layer is usually 25 nm thick [20],[25]. When the energy of the electron exceeds the work function, it can escape into the vacuum and thus overcome the vacuum level barrier. The process of emission of electrons can be treated as a series of events, each of which occurs with a certain probability, which allows the quantum efficiency, which determines the ratio of photoelectrons to incident photons, to be expressed as a series of probability multipliers.

The ideal scintillation materials for the detection of radiation should have specific properties 1-High efficiency in converting the kinetic energy of charged particles into light. 2-Light output should respond linearly to varying energy levels. 3-Using a material that can effectively stop gamma radiation (i.e. with high density and atomic number). 4-Scintillator must be transparent so the generated

light reaches the sensors without loss. 5-Short decay time is required for fast signal pulses. 6-High optical quality and convenient detector dimensions are required. 7-To ensure an effective connection with photomultiplier tubes, the refractive index of the material should be 1.5, which is the same as that of glass. It is not easy to find these properties in only one material. Inorganics are generally selected for their best light output performance and linear response, while their response time is slow. Although the response time of organic scintillators is fast, they produce less light. Therefore, the choice of material should be made according to the purpose [20],[23].

2.4.1 Inorganic Scintillators

Inorganic scintillators are mainly composed of ionic crystals, and their luminescence is related to the band pattern of solids. Luminous centres, which form localised energy levels in the crystal lattice, make light emission possible. Inorganic scintillators are often used to detect gamma rays. Ideally, a linear relationship between gamma rays and light output is desired, but many scintillators deviate from this linear relationship. A series of de-excitations is thus facilitated by the emission of photons at frequencies around the visible spectrum. A small amount of phosphorescence can also be present, as it is kept at energy levels that prevent the de-excitation from being interrupted, where additional excitation transitions can separate electrons. When gamma-ray interactions produce primary and secondary electrons, the secondary electrons are excited to the conduction band, leaving corresponding holes in the valence band. These electron-hole pairs act as the pri-

mary information carriers in the detection process. Their movement in the applied electric field enables the detector to generate a primary electrical signal. Electromagnetic radiation is emitted when the electrons are excited by falling back into the valence band. When this radiation is close to or within optical wavelengths, a photomultiplier or other light detection device can detect it and provide a detector signal [20],[26],[27].

When radiation particles interact with matter, they transfer some of their energy to these atoms, putting them into a briefly excited state. The excited atoms return to their ground state and emit photons at energy levels that are visible or close to the visible wavelength. This process is called radioluminance, but it is also called glow.

Each material can only fulfil some of the criteria simultaneously. The most used scintillators are inorganic alkali halide crystals; sodium iodide is generally preferred. Inorganics are preferred because they have the best output performance and linearity, but most have slowed response times. Organic scintillators are faster but give less light. In gamma spectroscopy, inorganics is preferred because the components have a high Z value, and inorganic crystals have a high density.

CsI(Tl) Detector CsI-based crystals are resistant to gamma radiation due to their high density and adequate atomic numbers, which was mentioned in the section on inorganic scintillators. These crystals can be scintillators without doping or with active dopants such as Na or Tl. Compared to CsI and NaI, CsI has a higher density and absorption coefficient. It is less brittle and highly resistant

to intense shocks and vibrations. Its higher plasticity makes it easier to process mechanically. When electron-hole pairs are produced in CsI(Tl), the decay times vary and are determined by multiple components, with decay times ranging from 0.6 μs to 3.5 μs . One of the most essential properties of CsI(Tl) is that the speed of light emission has a limited increase depending on the thallium concentration and ionisation intensity. This characteristic allows for distinguishing between different types of radiation and is particularly advantageous in distinguishing between protons, alpha particles, and electrons. Because of its non-hygroscopicity, it must be protected from high humidity and water [28]. When cesium iodide is activated with thallium, the light emission peaks at a wavelength of 565 nm; however, this wavelength is outside the highest sensitivity of standard photomultiplier tubes, so the light efficiency is relatively low. To improve this situation, a photodiode detector can often be used. CsI(Tl)-equipped photodiode-based detectors are now being produced industrially, and due to their small size and robust construction, they are used in many space-constrained applications such as medical devices. At the same time, when thallium and sodium additives are added to the CsI crystal, it has higher radiation stability. That is, it can resist radiation damage up to a certain dose level, and after a certain period, the properties of the crystals can be largely restored [20],[29],[30]. Research has shown that CsI crystal is not subjected to severe radiation damage at doses up to 1000 Gy. This is one of the most critical features that support the selection of CsI crystal as a detector material. In CsI(Tl) crystals, radiation damage can be observed at dose levels above 10 Gy, but some damage can be self-healing. Radiation-induced damage usually forms optical absorption bands that appear at low wavelengths. These bands can reduce the light transmission and signal quality of the crystal. Again, the use of

photodiodes provides a significant advantage here. Photodiodes protect the detector performance by minimising the adverse effects of radiation while recording the light signals emitted during scintillation.

Radiation detection module C12137 The radiation detection module developed by Hamamatsu and used in this study is designed to detect low-energy gamma rays with high sensitivity. These modules can detect and convert incoming gamma rays into light utilizing a CsI(Tl) scintillator, which converts photons into visible light [31],[32]. A multi-pixel photon counter detects this light and measures gamma rays from 30 keV to 2 MeV with high precision even at low light levels. The module also contains signal processing circuitry and an analogue-to-digital converter so that the incoming data can be processed and analysed on a computer system. These modules, with their modern interface options such as USB or RS-232C, offer users the flexibility to integrate them into a wide range of systems. Figure 2.5 provides a detailed view of the internal structure of the detector, showcasing the careful design that maximises its detection capacity.

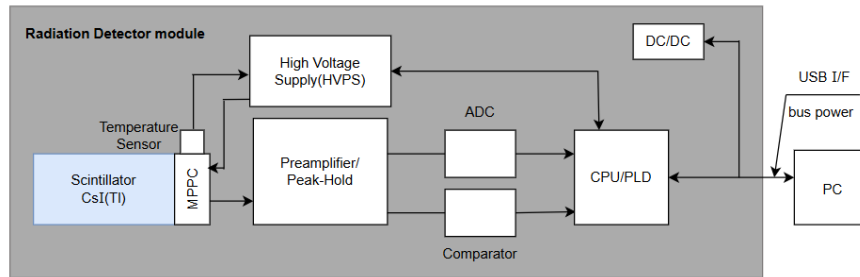


Figure 2.5: Radiation Detector Module C12137.

By increasing the scintillator capacity in the C12137 series, both detection

efficiency and measurement times have been reduced. However, by increasing the capacity of the scintillator, light particles (photons) are prevented from moving freely within the scintillator material. The amount of light reaching the photosensor decreases as the light encounters more reflection and absorption in the material. This reduces the lower detection time limit and makes it difficult to detect low-energy gamma rays. However, due to the high gain provided by the multi-pixel photon counter (MPPC), detection can be made even at lower light levels than PIN photodetector and APD. The MPPC is compact and thin, occupying minimal space in the module so that the scintillator occupies most of the module. This design allows the module to offer high photon detection efficiency despite its small size and is ideal for applications with low radiation emission. The C12137 series gamma ray detection module is designed for reliability, with a temperature compensation circuit that plays a crucial role in minimising errors. This circuit, combined with the temperature stability of the module, exhibits superior performance against temperature variations. As a result, it effectively minimises drifts in energy detection and reduces measurement errors. The amount of light emitted from the scintillator and the sensitivity of the photosensor to ambient temperature can cause errors in the measurement of gamma-ray energy. However, the temperature compensation circuit in the C12137 series gamma-ray detection module ensures that such errors are minimised, thereby enhancing the reliability and performance of the module. The sensor, a critical part of the project, takes the information obtained directly and converts it into user-friendly data. The steps for this sensor function are detailed below and designed for ease of use. 1-Firstly, the measurement data and information about the ambient temperature are received. 2-The gamma rays detected by the sensor are converted to voltage by the analogue

processor depending on their energy level. 3-This voltage is then converted into digital format, i.e., 16-bit data, so it is suitable for processing. 4-The module stores the gamma ray energy data sequence for 100 milliseconds (ms). This process is repeated every 100 ms by updating the previous data with new incoming data. 5-This stored data array contains the individual data values detected and allows the user to create an energy spectrum that can be visualised [31],[32]. These steps have been designed to ensure that the radiation detector module can accurately detect gamma rays and that the obtained data can be analysed and interpreted effectively. The capabilities of the detector module constitute the theoretical basis of the radiation detection system presented in this thesis.

2.5 The Monte Carlo Method

Monte Carlo simulations are a robust and adaptable numerical approach for solving neutron and gamma ray transport problems, which has increased demand for this computational method in nuclear technology applications. It is an ideal method for studying radiation transport in particle transport. This modelling method simulates in detail the journeys of particles, their random transitions up to their collision points, and the consequences of these interactions - whether in the form of energy direction changes or the complete dissipation of the particle. The history of each particle, shaped by these interactions, forms the history of the particle, which is crucial for understanding how the system dynamics react. The Monte Carlo method, which can model each particle in detail, is an ideal tool for studying radiation transport in particle transport. This efficiency is particularly beneficial

for engineers in nuclear engineering, reactor design, and radiation protection, as it allows them to quickly and effectively compare different designs. The method simplifies the design process by providing a detailed understanding of particle behaviour, making it a significant time-saving advantage.

The particle transport process can be defined as a process in which particles emitted from radiation sources undergo random events. The life of the particle begins when the source determines the initial position, direction of motion, and energy of each particle. The particle moves randomly through the system unless it encounters a collision, or a system leave event [33],[34],[35] . As a result, the particle is either absorbed or scattered. In scattering, the direction and energy of the particle change, while in absorption, either new particles are formed, or the particle is destroyed. These events continue until the particle leaves the system or is absorbed. This process is called "particle history". The physical basis of this process is described by the steady-state Boltzmann transport (2.8).

$$\begin{aligned} & \underline{\Omega} \cdot \underline{\nabla} \psi(\underline{x}, \underline{\Omega}, E) + \Sigma_t(\underline{x}, E) \psi(\underline{x}, \underline{\Omega}, E) \\ &= \int_0^\infty \int_{4\pi} \Sigma_s(\underline{x}, \underline{\Omega}' \cdot \underline{\Omega}, E' \rightarrow E) \psi(\underline{x}, \underline{\Omega}', E') d\Omega' dE' + \frac{Q(\underline{x}, E)}{4\pi}, \end{aligned} \quad (2.8)$$

$$\underline{x} \in V, \quad 0 < E < \infty, \quad \underline{\Omega} \in 4\pi,$$

$$\psi(\underline{x}, \underline{\Omega}, E) = \psi^b(\underline{x}, \underline{\Omega}, E), \quad \underline{x} \in \partial V, \quad 0 < E < \infty, \quad \underline{\Omega} \cdot \underline{n} < 0,$$

$\underline{x}$ is the coordinates describing the position of the particles, $\underline{\Omega}$ is the unit direction vector, and E is the energy of the particles,

where V represents the volume of the system, while $\underline{n}$ is the outward-directed unit surface vector.

Furthermore, Ψ is the angular flux, Σ_t is the total macroscopic cross-sectional area, Σ_s is the differential macroscopic cross-sectional area, which determines the scattering events, and Q is the isotopic external source. ψ^b represents the boundary condition, which can indicate either a reflective vacuum boundary or a predetermined flow of events.

The Monte Carlo method randomly selects particles from a given source distribution. First, it calculates the position of the particle in space, the initial direction of motion and the initial energy. This step is based on the initial source estimation, especially in criticality problems. In subsequent steps, the distance a particle travels before the next collision is sampled, and the probability of collision at this distance is calculated using a defined equation. In this process, Monte Carlo provides a single realisation of the problem. The following [Equation 2.9](#) expresses the probability of a particle colliding at a point s around a distance ds .

$$f(s)ds = e^{-\Sigma_t s} \Sigma_t ds \quad (2.9)$$

The inverse coherent distribution approach is used to sample the distance s over which the particle travels until the subsequent collision. At the same time, the total cross-sectional area is presumed to remain constant along the travelling path to simplify the calculations.

$$s = -\frac{1}{\Sigma_t} \ln(1 - \xi) \quad (2.10)$$

A key component of our particle physics simulations is ξ , a random number that is equally distributed between 0 and 1. The practical use of this equation, in which $1 - \xi$ is likewise evenly spread over the range from 0 to 1, offers valuable insights for our research. Suppose the collision distance exceeds the distance to the closest boundary in the direction of travel of the particle. The particle is relocated to that boundary, and the subsequent collision is resampled using the specified Equation 2.10. The value of Σ_t for the region beyond the boundary is then updated.

When a collision occurs, it is analyzed based on a discrete probability density function (PDF) that includes macroscopic cross-sectional areas for each type of interaction. If the collision results in absorption, the history of the particle ends. A new flight path and energy are drawn from the corresponding PDFs for scattering collisions, and the particle continues toward the subsequent collision and boundary. The particles in the system must be monitored stochastically, and a Monte Carlo simulation needs to obtain the necessary information. Monte Carlo sums obtain this information by performing counting experiments. A total score is accumulated for each occurrence or history using specific events, such as collisions or boundary crossings. At the end of the simulation, these scores are divided by the particle count to obtain the probability of a desired event. Monte Carlo sums serve as statistical estimates of particle motions in simulations of physical problems, and their total variance can be calculated as the inverse of the particle tally in the simulation. In theory, as the number of particles approaches infinity, Monte

Carlo results converge to the exact solution of the Boltzmann equation. However, simulating many particles incurs high computational costs, and insufficient sampling in certain regions may introduce bias in the results. Variance reduction techniques address this by lowering the variance of the solution while using fewer particles.

In Monte Carlo simulations, each particle is treated as a historically independent Markov chain. In a Markov process, the forward motion of a particle is related only to the instantaneous state, simplifying the calculation of random walks. The Monte Carlo method has many advantages over deterministic approaches. One of the most important advantages is the ability to completely model complex geometries and physical phenomena. Studies on integrating CAD (computer-aided design) systems and the geometry input methods of Monte Carlo allow simulations to be run directly from design drawings without approximation. Professionals in the nuclear industry highly prefer the method due to its flexibility and high accuracy [33],[36],[37].

Chapter Three

Experimental work and Methodology

The first stage focuses on understanding the working principles of the CsI(Tl) detector, testing its performance, and examining its applicability in radiation environments. The primary purpose of these experiments and subsequent simulation studies is to examine the interaction of the detector with different radiation sources, evaluate how it will perform when integrated into the robot, and determine the interface design between the robot and the detector. The CsI(Tl) detector used in the experiment is a device that can accurately detect the energy and dose rates of nuclear radiation. To examine the working principles of this device in detail, some tests were carried out using radiation sources at different energy levels and distances. Eleven radioactive sources were used in the nuclear laboratory during the experiments, and the activity data of these sources are given below. The experiments started on 12 October 2021, and these activity values are for that date.

- 1 – ^{137}Cs (A0697) - Activity: $124300 \text{ Bq} \pm 4600$
- 2 – ^{137}Cs (Ans32[2549]) - Activity: $30700 \text{ Bq} \pm 1210$
- 3 – ^{60}Co (C3N19/18[DC617]) - Activity: $114190 \text{ Bq} \pm 4820$
- 4 – ^{22}Na (C3N19/91[EW618]) - Activity: $\pm 79100 \text{ Bq} \pm 410$
- 5 – ^{44}Ti (U0B08123) - Activity: $21170 \text{ Bq} \pm 1110$
- 6 – ^{68}Ge (U0B22111) - Activity: $115800 \text{ Bq} \pm 6940$
- 7 – ^{133}Ba (133.278.17) - Activity: $70770 \text{ Bq} \pm 3680$
- 8 – ^{152}Eu (C3N19/92(EW617)) - Activity: $110200 \text{ Bq} \pm 5510$
- 9 – ^{207}Bi (C3N19/89) - Activity: $177900 \text{ Bq} \pm 900$
- 10 – ^{241}Am (1Q599) - Activity: $397800 \text{ Bq} \pm 19900$
- 11 – ^{108}Ag (U0B940115) - Activity: $7860 \text{ Bq} \pm 480$

Firstly, a single source spectrum was used in the experiments, as shown in [Figure 3.1](#). Dose measurements were carried out using ^{60}Co .

During these measurements, the distance between the source and the detector was set to 1cm, 5cm, 10 and 30 cm. At distances of 1cm and 5cm, it was observed that the characteristic energy peaks of ^{60}Co , 1173.28 keV and 1332.492 keV were detected more clearly and accurately; however, at greater distances, such as 10 cm and 30 cm, these peaks were not so visible. [Figure 3.1](#), [Figure 3.2](#), and [Figure 3.3](#) show the spectra obtained for the distances, respectively.

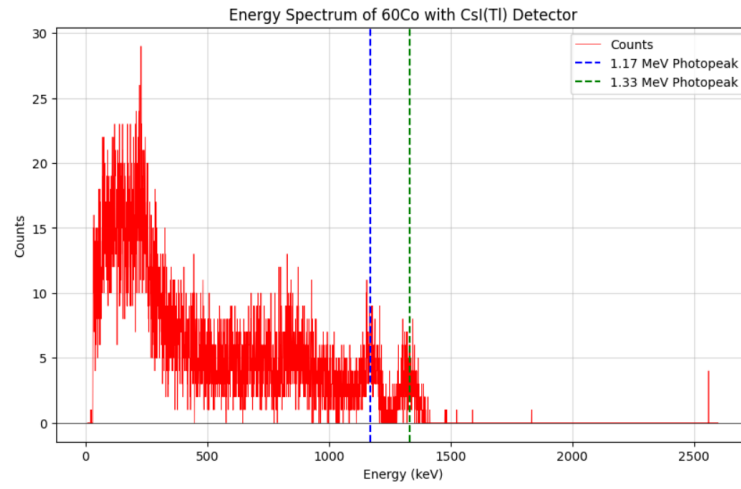


Figure 3.1: Energy Spectrum of ^{60}Co with CsI(Tl) detector-5cm away from the detector.

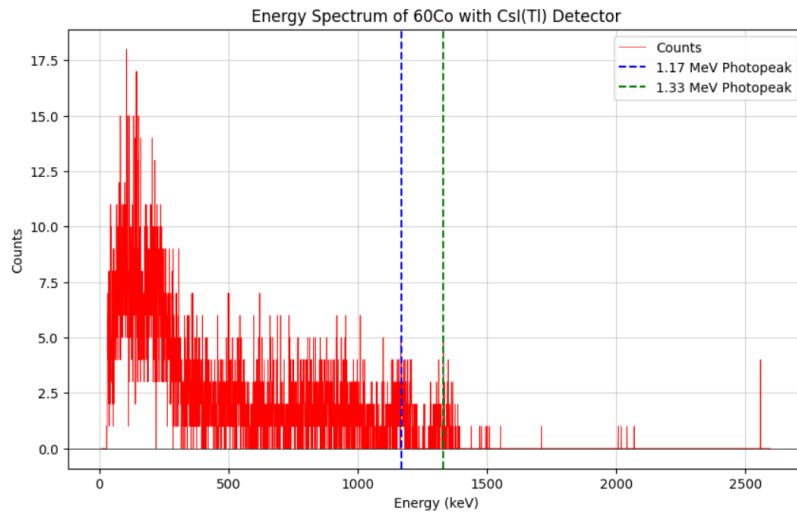


Figure 3.2: Energy Spectrum of ^{60}Co with CsI(Tl) detector-10cm away from the detector.

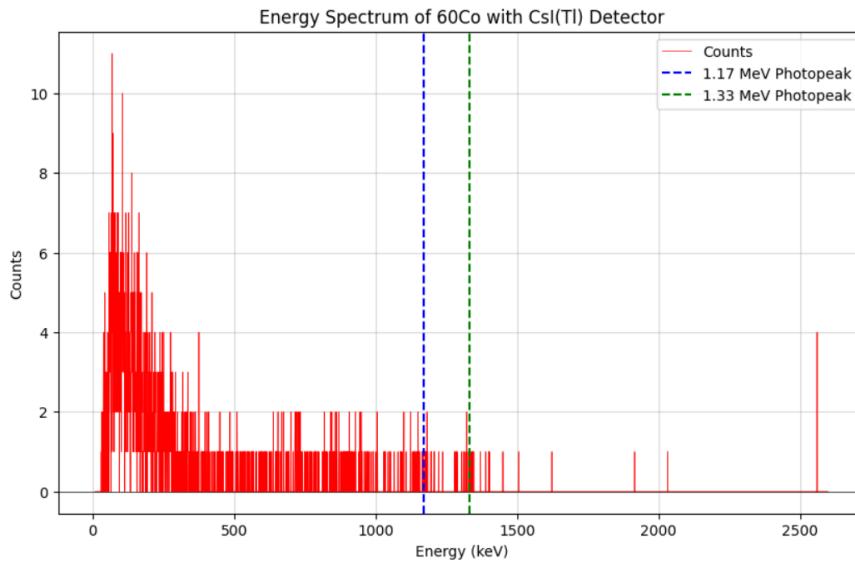


Figure 3.3: Energy Spectrum of ^{60}Co with CsI(Tl) detector-30cm away from the detector.

All gamma-ray spectra presented in Figures 3.1 - 3.3 were acquired with a counting time of 60 seconds. These results show that as the distance between the detector and the radioactive source increases, the sensitivity in detecting energy peaks decreases. This decrease is related to the working principle and physical limits of the detector and can be explained by the fact that the energy emitted from the radiation source is more dispersed with distance, and the signal collected by the detector is weakened.

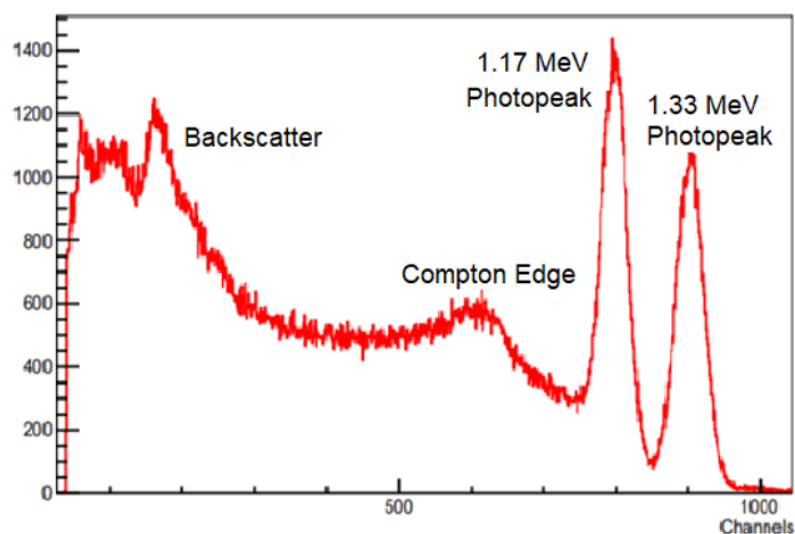


Figure 3.4: Energy spectrum of ^{60}Co obtained using a $2'' \times 2''$ NaI(Tl) detector placed 5 cm away from the source. The measurement was conducted for 180 seconds.

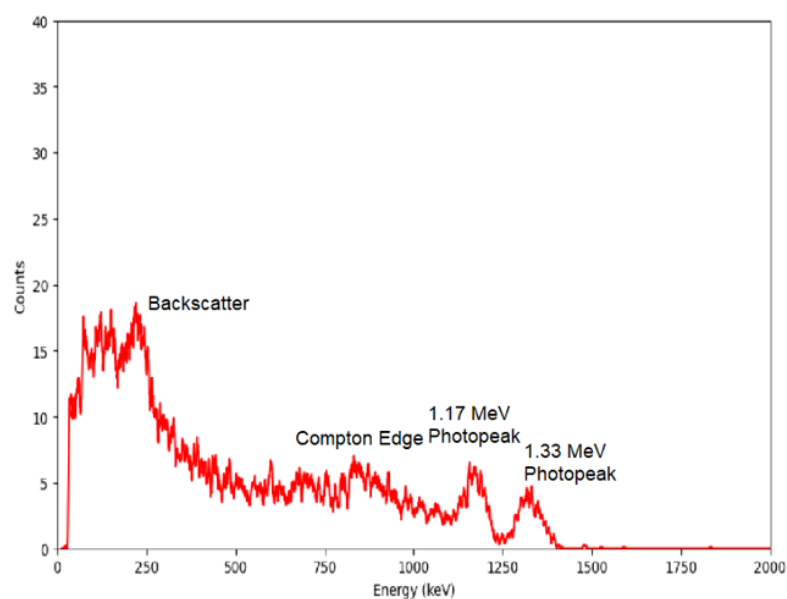


Figure 3.5: Energy spectrum of ^{60}Co obtained using a CsI(Tl) detector with a crystal size of $13 \times 13 \times 20 \text{ mm}^3$, placed 5 cm away from the source. The measurement was conducted for a duration of 180 seconds.

The graphs are a typical ^{60}Co measurement performed with an inorganic scintillator. A NaI(Tl) detector was used to obtain the spectrum shown in the [Figure 3.4](#). Two peaks are seen at energy levels of 1.17 MeV and 1.33 MeV. Photopeaks are produced when the entire energy of an incoming gamma photon is absorbed within the detector crystal, most often through the photoelectric effect, either directly or following a number of Compton interactions. Compton scattering effects are also seen in the spectrum. As the photons travel through the detector, they interact with the electron and change direction before losing their energy. As a result of this interaction, the energy of the scattered electrons reaches a maximum at a scattering angle of 180 degrees. This causes a constant counting rate in the detector until it reaches E_{max} , known as the Compton edge. In addition, the scattering of gamma photons in various directions by interaction with electrons before they lose entirely their energy during their movement through the detector creates a region in the spectrum called Compton continuum. The larger the scintillation crystal, the fewer photons escape from the detector due to Compton scattering, so more giant detectors provide a more apparent energy spectrum [38]. This feature is essential for the characterisation of radiation sources. The NaI(Tl) detector ($2'' \times 2''$) is larger than the CsI(Tl) detector ($13 \times 13 \times 20 \text{ mm}^3$) used in this study. . When we compare [Figure 3.4](#) and [Figure 3.5](#), we see that the spectrum peaks are more pronounced in the larger detector. These ratios are essential for analysing the spectra and identifying the peaks, especially in robotic tests. Determining such ratios is useful in radiation source identification and improves detector efficiency and sensitivity. In the continuation of the study, some shielding materials (such as iron and aluminium) were used to examine the operation of the detector in detail and to investigate its ability to identify radiation sources. In

these tests, the shielding materials were positioned between the detector and the source, and different thicknesses were used.

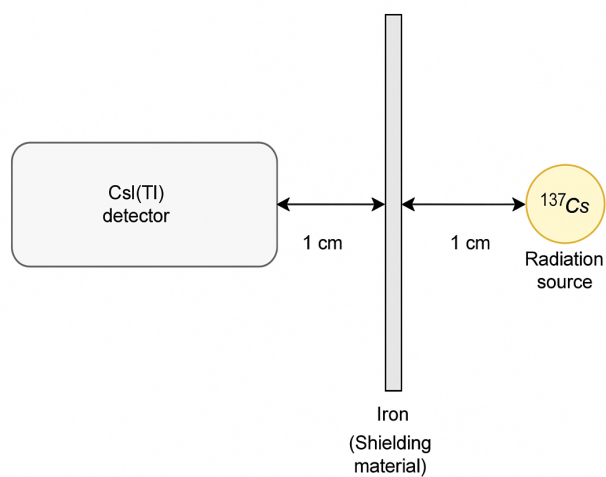


Figure 3.6: ^{137}Cs source placed 1 cm away from the CsI(Tl) detector, with an iron shielding material of 1.9 cm thickness positioned between them.

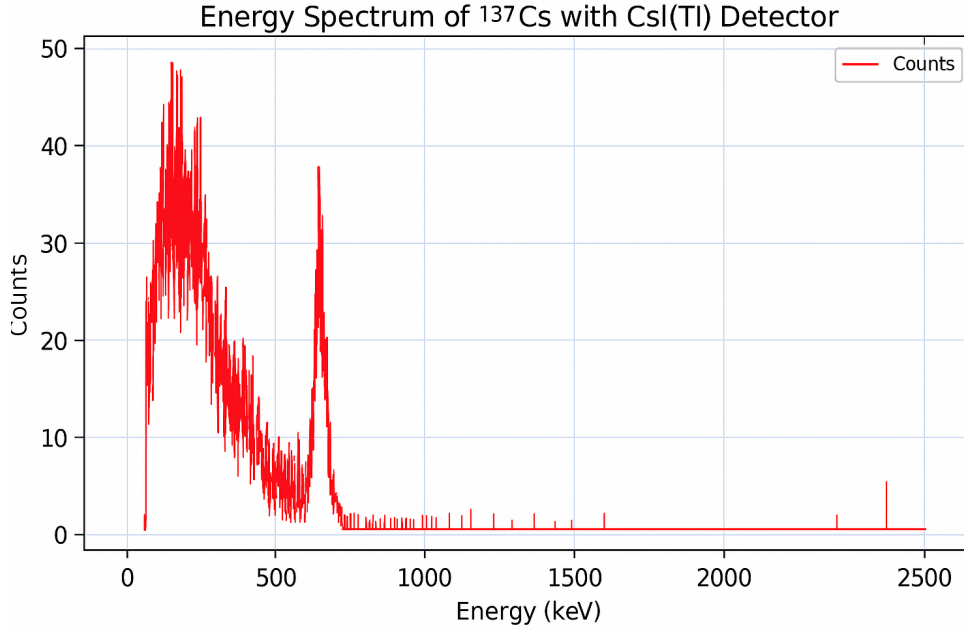


Figure 3.7: Energy spectrum for the ^{137}Cs source placed 1 cm away from the detector with an iron shielding material. The measurement was performed for 60 s.

This is shown representatively in Figure 3.6. When the distance between the detector and the shielding material was 1 cm and between the shield and the source was 1 cm, the ^{137}Cs peak was observed, as shown in Figure 3.7. However, as the thickness of the shielding material and the distance increased, the energy peak gradually weakened and could not be detected. In addition, because of comparing the measurements made with and without shielding, it was observed that there was a specific decrease in the dose rate. This decrease is related to the thickness and chemical structure of the shielding material. As a result of these tests, the effect of the shielding material on the performance of the detector was examined in detail. These various tests were carried out to evaluate the usability of the detector in real-world conditions.

When the detector is integrated into the robot, it is desired to detect sources not only in areas with high radiation levels but also in areas with low radiation levels. Additional experiments were carried out by placing two different sources of ^{60}Co and ^{137}Cs in front of the detector. Figure 3.8 illustrates the ability of the detector to distinguish between two different sources at this distance. The peaks of both ^{60}Co and ^{137}Cs are clearly visible.

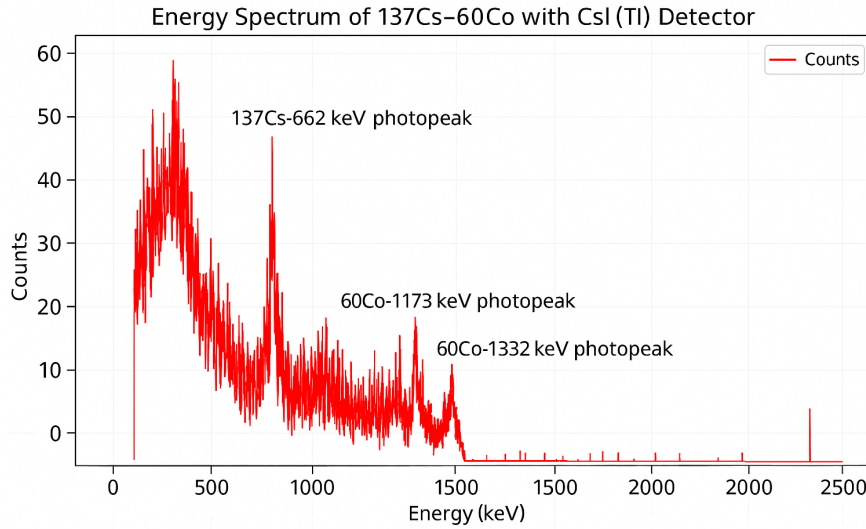


Figure 3.8: Two different radiation sources ^{137}Cs and ^{60}Co energy spectrum. The sources were placed 5 cm away from the detector, and the measurement was performed for 60 s.

In addition, the scintillator is located on the side of the detector, so it was investigated how the source would be detected with different angles of the detector. It was observed that when the source is placed on the side, the source is better detected because the scintillator is here. However, when the source is positioned at the back or side of the detector, it is observed that the sources can still be detected even though the dose rate decreases. The primary purpose of all these

studies is to provide data on the methodology to determine the source location when the detector is integrated into the robot and to understand the performance of the detector in different environmental scenarios. The data obtained provided information on the spatial position of the source locations relative to the detector and the ability of the detector to detect these sources at different angles, which provided an important basis for optimising the robot for radiation source detection and environmental mapping functions.

Chapter Four

Monte Carlo Simulations for Radiation Detection

In simulation programmes based on the Monte Carlo method, the probability distributions governing the phenomena are statistically sampled to indicate the amount of physical phenomena being simulated. This method offers high accuracy in modelling particle transport and radiation interactions, ensuring that the results match the experimental results with high accuracy. Accurate measurement and analysis of gamma radiation are essential to obtaining reliable results in experiments. Determining energy resolution with scintillation detectors allows precise interpretation of energy spectra. This study analysed gamma spectra obtained from radioactive sources (^{137}Cs , ^{60}Co , ^{133}Ba , etc.) in the nuclear laboratory using Monte Carlo simulations (MCBEND and OPENMC) to determine their agreement with experimental results. The method was used to achieve this agreement.

4.1 MCBEND

MCBEND is a commercial simulation program developed by ANSWERS software service in the United Kingdom [39]. It is designed to solve radiation transport problems using the Monte Carlo method. The program simulates the transport and interaction of particles, such as gamma rays, neutrons, and charged particles. MCBEND has been under development for over 30 years, mainly focusing on radiation transport problems in subcritical systems. MCBEND can simulate complex systems in detail using simple geometric shapes. It uses fractal geometry, in which three-dimensional geometric shapes such as boxes and cylinders are combined to form 'regions', i.e., larger structures of the system. When these regions are joined together, the overall geometry of the system is formed. In addition, creating strings of parts for repetitive geometric structures efficiently uses memory and reduces modelling time. For each geometric shape, the user can select the required material from a list prepared by MCBEND, or if a unique mixture is required, the user can select the required material from a list prepared in advance by MCBEND. This way, simulations can be made closer to reality, and the effects of different materials on the transport calculation can be examined. MCBEND uses cross-sectional data containing 13,193 ultra-fine energy groups and a DICE module to process neutron collisions. Information is taken from various evaluated nuclear data files such as JEF2.2, ENDF/B-VI and JENDL3.2 for these calculations. For gamma rays, the GAMBLE module is used to calculate an equivalent point energy library. The condensed history method is used to transport charged particles. Sources with different energy spectra can be modelled in the simulation, such as points, lines, etc. The user can define these manually, or the built-in spec-

tra provided by MCBEND can be used. MCBEND has many applications in many fields, such as reactor design, particle accelerators and fusion devices. This wide range of applications has made MCBEND a reference point in nuclear engineering applications and radiation transport and safety analyses. The programme provides a detailed understanding of the simulation results and error calculations for each case. In addition, it has a 'dump and restart' feature that allows the results to be obtained with higher accuracy when a particular variance is reached by running more significant sample numbers in the later stages of the simulation, and this feature makes the program useful for complex and large-scale simulations. As a result, MCBEND offers the user high accuracy and detailed modelling of radiation transport calculations. The flexibility of geometric modelling and the advanced physical calculation capabilities make MCBEND highly desirable software for nuclear engineering and radiation transport simulations [39],[40],[41].

The MCBEND code simulated the response function of the detector to gamma radiation emitted from eleven different radioactive sources used in the laboratory. The detector model used in the Monte Carlo simulations is the C12137 Radiation Detection module manufactured by Hamamatsu. The sources used for the calculations were placed at different distances from the detector. [Figure 4.1](#) shows the 2D and 3D representations of the simulated detector and source.

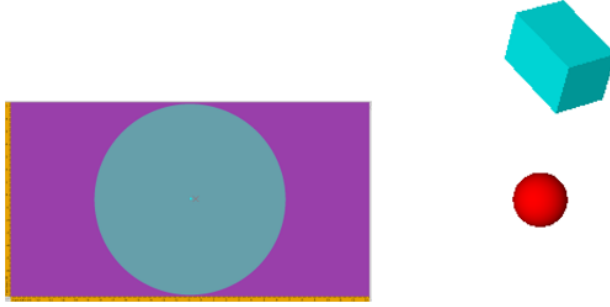


Figure 4.1: Two-dimensional (left) and three-dimensional (right) geometrical models simulated in MCBEND. The 2D model shows a circular source region within a rectangular boundary representing the surrounding medium, where radiation transport is analyzed on a planar section. The 3D model includes a red spherical source and a cyan cubic detector, representing a full spatial simulation of source–detector geometry..

4.2 OPENMC

OpenMC is an open-source Monte Carlo particle transport code developed by the Massachusetts Institute of Technology (MIT) [42]. OpenMC was designed from the ground up using state-of-the-art development methods and high-performance algorithms. The designers used continuous energy cross sections and constructive solid geometry (CSG) models to model nuclear reactors and other systems with high accuracy while using XML file format for input data and HDF5 file format for output data to provide flexibility for the user. The CSG method facilitates the design of complex structures by the user by using Boolean operators over simpler regions. The intersection of several half-spaces represents closed volumes and cells. The specified positive or negative face of a plane or quadratic surface for a half-space allows models with curved surfaces, such as spheres, to be designed accurately

without mesh discretisation. OpenMC uses the `openmc.Material` class to specify materials. Nuclides and elements of a given density can then be added with the `Material.add_nuclide()` and `Material.add_element()` commands. OpenMC was presented to the public for the first time in 2012. Compared to other popular Monte Carlo codes, it is a relatively new coding programme. Initially focused on neutron transport, the code is still being developed for other areas [42],[43]. The system modelled in MCBEND is modelled in the same way in OPENMC as in [Figure 4.2](#).

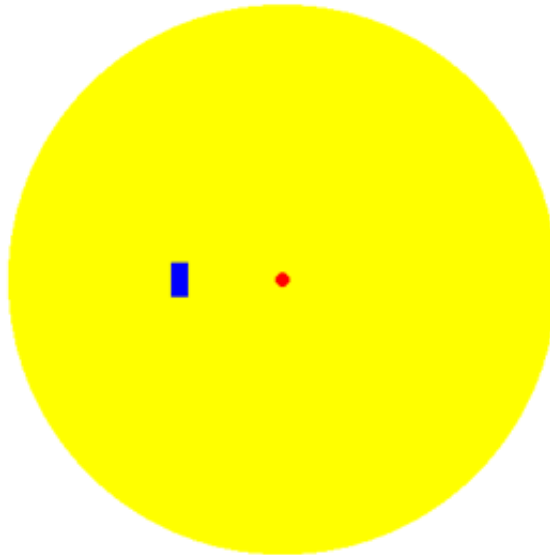


Figure 4.2: Simulated model with OpenMC. The yellow circle represents the surrounding medium, the red point indicates the radiation source, and the blue rectangle shows the detector position.

In the next stage, the OPENMC and MCBEND results were compared to determine which program more accurately matched the lab results, and the pre-

liminary studies we conducted for our experiments with the robot were continued with that program.

4.3 Comparison of Dose Rate Results

In this section, the accuracy and compatibility of both MCBEND and OpenMC codes are analysed by comparing the dose rates obtained from experimental data. The simple geometry of the detector is assumed to be comparable to the simulations, which indicates that the results are sufficient for a meaningful analysis. As described in Chapter 3, Experimental Study and Methodology, various radiation sources were used as radiation sources in the nuclear laboratory; Table 4.1 and Table 4.2 below show a comparison of the measured values and the dose rates found by the simulation program for ^{137}Cs (A0697) and ^{60}Co (C3N1918), respectively. Both codes were run with a batch size of 100 million samples to approximate the actual results.

Table 4.1: Dose rate comparison and errors for ^{137}Cs (A0697).

Dist. (cm)	Experiment ($\mu\text{Sv h}^{-1}$)	OpenMC ($\mu\text{Sv h}^{-1}$)	Δ OpenMC	% Δ OpenMC	MCBEND ($\mu\text{Sv h}^{-1}$)	Δ MCBEND	% Δ MCBEND
1	11.55	12.35	0.80	6.93	12.25	0.70	6.06
5	2.62	2.65	0.03	1.15	2.68	0.06	2.29
10	0.91	1.08	0.17	12.04	0.97	0.06	6.59
30	0.20	0.15	-0.05	-25.00	0.17	-0.03	-15.00

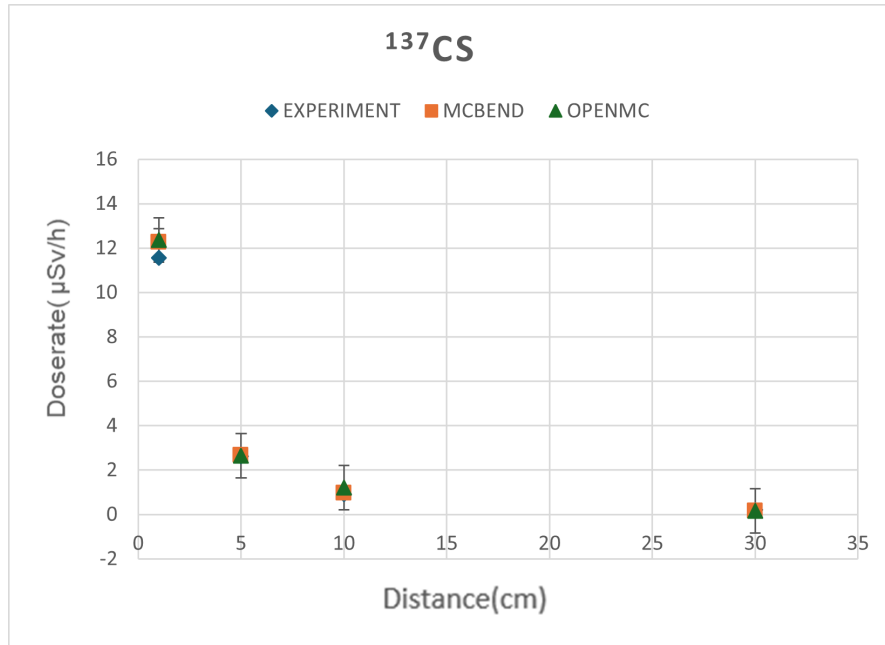
Note. Errors are defined as $\Delta = D_{\text{sim}} - D_{\text{exp}}$ (positive means simulation overestimates). Percent error is $\% \Delta = 100 \times \Delta / D_{\text{exp}}$ (experiment in the denominator).

Table 4.2: Dose rate comparison and errors for ^{60}Co (C3N1918).

Dist. (cm)	Experiment ($\mu\text{Sv h}^{-1}$)	OpenMC ($\mu\text{Sv h}^{-1}$)	Δ OpenMC	% Δ OpenMC	MCBEND ($\mu\text{Sv h}^{-1}$)	Δ MCBEND	% Δ MCBEND
1	19.02	20.50	1.48	7.78	18.68	-0.34	-1.79
5	4.80	4.34	-0.46	-9.58	4.49	-0.31	-6.46
10	1.18	1.21	0.03	2.54	1.20	0.02	1.69
30	0.34	0.26	-0.08	-23.53	0.28	-0.06	-17.65

Note. Errors are defined as $\Delta = D_{\text{sim}} - D_{\text{exp}}$ (positive means simulation overestimates). Percent error is $\% \Delta = 100 \times \Delta / D_{\text{exp}}$ (experiment in the denominator).

As a first step of the study, the radiation sources were placed 1,5,10 and 30 cm away from the detector, respectively, and the dose rates were measured and compared with the simulation results. Figure 4.3 and Figure 4.4 below show the measured error rates for two sources.


 Figure 4.3: Dose rate & Distance for ^{137}Cs (Experiment-MCBEND-OpenMC.)

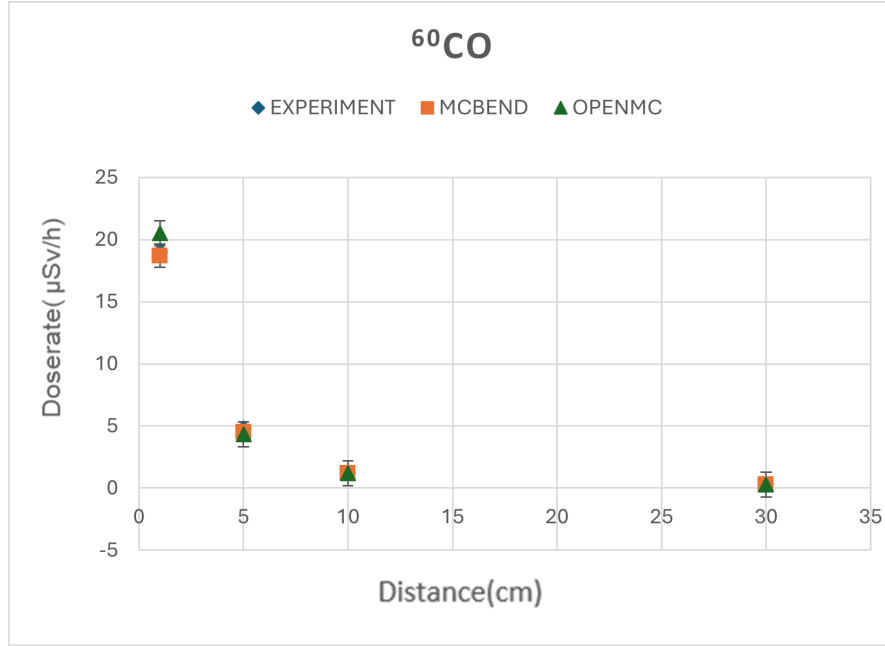


Figure 4.4: Dose rate & Distance for ^{60}Co (Experiment-MCBEND-OpenMC.)

The accuracy of the methods used to determine the position of the radiation source was first determined by using the detector before the detector was integrated into the robot to determine the position of the radiation source in the later stages of the study. When the results were analysed, the discrepancy between the error rates did not exceed 10% (Table 4.1 and Table 4.2). Although some properties of the detector (crystal homogeneity, irregularities in Tl dopant distribution) cannot be fully modelled in the simulation, MCBEND has been validated as a powerful tool for scintillation detector simulations due to its ability to integrate dead time, crystal efficiency, and geometry parameters into the model, its ability to simulate high energy resolution ($\pm 2\%$ accuracy), and its fast computational capacity [44].

However, real-world environmental (temperature, humidity, electronic noise) and material-based complexities (such as crystal inhomogeneity) cannot be fully

reflected in the model, which is among the main reasons for the discrepancies between simulation and experimental data.

Monte Carlo simulations allow accurate modelling of the response functions of radiation detectors. In this study, the distance between the radiation source and the detector is taken as 5 cm, per the actual experimental conditions. The response function is calculated using a pulse height tally. To ensure the accuracy and reliability of the simulation results, it is preferred to sample 10^8 photons in each simulation. This high sampling number ensures that the energy spectra are free from statistical fluctuations and that the spectrum is consistent with the actual experiment, so the simulation results can accurately represent the response of the detector to gamma rays. A Gaussian energy broadening (GEB) method models the energy resolution of the detector to ensure that the simulations agree with the experimental results. This method includes the statistical and physical expansion effects of the model, allowing for more accurate spectral outputs. A Gaussian distribution is sampled from the energy values to simulate the realistic response of the detector with GEB [45],[46]. The function of this distribution is as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-E)^2}{2\sigma^2}} \quad (4.1)$$

E represents the energy value, and the distribution width, σ , is determined according to 4.1.

$$\sigma = \frac{\text{FWHM}}{2\sqrt{\ln(2)}} \quad (4.2)$$

The full-width maximum (FWHM) is energy-dependent and is expressed as in 4.3.

$$\text{FWHM} = a + b\sqrt{E + cE^2} \quad (4.3)$$

The parameters a , b , and c were found based on the experimental results and are in [Table 4.3](#).

Table 4.3: Fitted Parameters.

a	b	c
5.8114(3)	0.9203(2)	$1.1776(4) \times 10^{-5}$

[Equation 4.3](#) calculates the energy resolution of the detector as a function of the energy level, allowing the simulation data to correlate more accurately with the experimental data. Another critical point addressed here is the ability to accurately model gamma rays in the detector in partial energy loss states, such as Compton scattering and full-energy peaks. Accurate simulation of Compton scattering is essential to ensure that all features in the energy spectra of the detector are consistent with the values obtained from experimental measurements. Firstly, the spectra obtained from experimental and MCBEND results for ^{137}Cs are compared in [Figure 4.5](#).

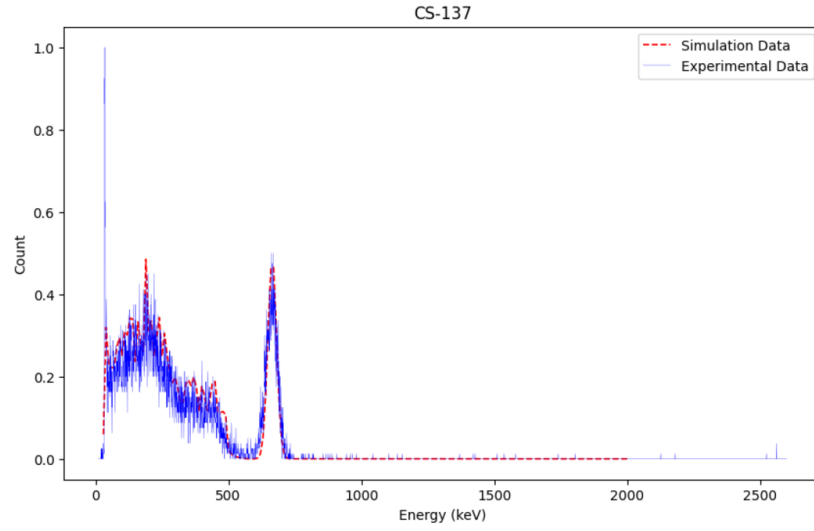


Figure 4.5: Comparison of experimental and MCBEND simulation spectra for ^{137}Cs (measurement time: 60 s, source–detector distance: 5 cm).

The simulated and experimental results agree well with the observed ^{137}Cs radiation source with a photopeak of 662 keV. However, there are slight differences for the low energy range, possibly due to the low background of the simulated spectra and electronic noise.

Figure 4.6 shows a good correspondence between the peak values of 1173 keV and 1332 keV.

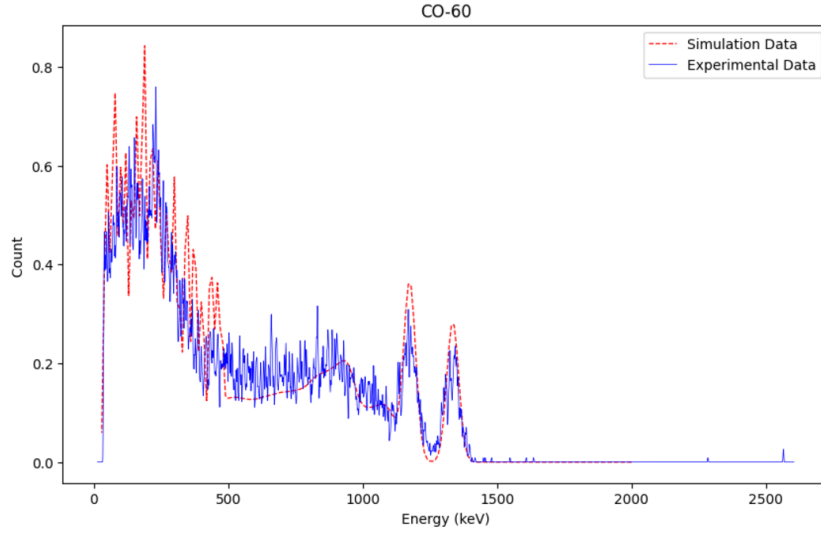


Figure 4.6: Comparison of experimental and MCBEND simulation spectra for ^{60}Co (measurement time: 60 s, source–detector distance: 5 cm).

This correspondence demonstrates the ability of the detector to determine the energy spectra and confirms that the simulation models operate correctly. In the later stages of the study, the robot went to that location after the radiation source location was found. It took measurements and tried to identify the radiation source. In the first stage, the detector independent of the robot was used to minimise the potential problems that may occur when the detector is integrated into the robot. This study also compares the ability of OpenMC and MCBEND simulation programs to simulate gamma sources. OpenMC is a new and powerful program developed for neutron physics; however, it requires further optimisation to simulate gamma sources. The error rate in simulations is higher than that of MCBEND. MCBEND produced more accurate and precise results in gamma spectrum analyses and demonstrated a lower error rate. Therefore, it was decided that the next steps of the study would be continued using MCBEND.

Chapter Five

Radiation Source Localisation and Identification with Mobile Robot

5.1 Introduction to the Mobile Robot System

Mobile robots are autonomous robots with a wide range of applications. They can move independently in a specific direction and location, move in a defined area without human intervention, collect data, and perform tasks. Mobile robots are preferred in areas such as military, security, and environmental monitoring that may be dangerous for human safety. Mobile robots have operational flexibility and security advantages, especially in radiation detection and mapping applications. This study prefers a mobile robot with a compact structure controlled by a joystick [47],[48]. The mobile robot platform used is a Husky UGV manufactured by Clearpath [49]. [Figure 5.1](#) below shows pictures of the robot. The robot is integrated with a LIDAR sensor and a CsI(Tl) detector, details of which are described

in Section 2.3.1.



(a) Husky mobile robot-Velodyne LIDAR sensor-CsI(Tl) detector.



(b) Joystick.

Figure 5.1: Husky mobile robot with LIDAR sensor, CsI(Tl) detector and joystick.

Table 5.1 below presents the characteristics of the robot.

Table 5.1: Husky UGV Specifications [49].

Feature	Description
Dimensions (mm)	990 x 670 x 390
Weight (kg)	50 kg
Maximum Speed (m/s)	1.0 m/s (2.2 mph)
Payload Capacity (kg)	75 kg
Operating Time (hours)	3 hours (typical), 8 hours (maximum)
Battery	24 V, 20 Ah
Drivers/APIs	ROS, C++, Mathworks
Control Models	Direct voltage, Velocity, Wheel Speed, Torque
Communication	RS-232, 115200 baud
Feedback	Battery, Motor, Odometry, Control System Output

This robot is equipped with the Puck (VLP-16) model produced by Velodyne. Due to its compact and efficient features, this LIDAR sensor is highly preferred in environmental detection and mapping applications [50]. This technology optimises navigation and mapping processes in robotic systems. The combination of data from the LIDAR sensor and the radiation detector plays a vital role in mapping the radiation intensity and determining the location of the radiation source. The specifications of the LIDAR sensor used are listed in [Table 5.2](#) below.

Robot Operating System (ROS) is the most widely used open-source robot software platform. It is used as the programming environment of the robot and aims to accelerate and simplify the software development process. The working logic of ROS is based on nodes, which allow tasks to be divided into independent modules (nodes). As seen in [Figure 5.2](#), the infrastructure of ROS consists of a mechanism based on subscription and publishing.

Table 5.2: Velodyne Puck (VLP-16) LIDAR Sensor Specifications [50].

Feature	Description
Measurement Range	100 m
Dimensions	103 mm - 72 mm
Field of View (Horizontal)	360°
Field of View (Vertical)	+15° to -15° (30° total)
Rotation Rate	5 Hz - 20 Hz
Angular Resolution	0.1° - 0.4°
Data Output	300,000 - 600,000 points per second
Operating Temperature	-10°C to +60°C
Protection Rating	IP67 (dust and water-resistant)
Power Consumption	8 W
Connection	100 Mbps Ethernet Connection

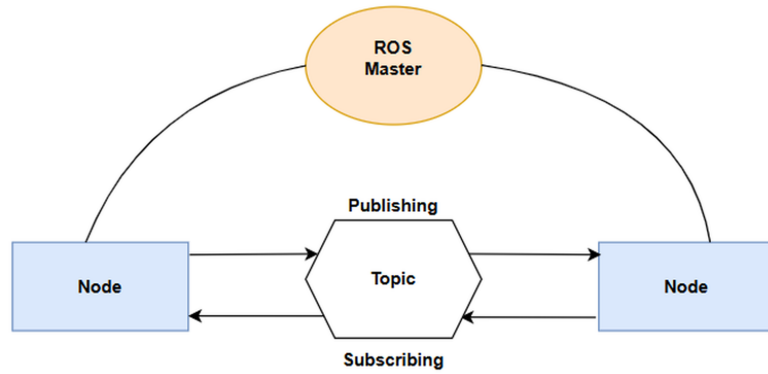


Figure 5.2: ROS Workflow.

These nodes enable the creation of ROS packages using Python or C++. Each node contains information such as LIDAR data, camera images, etc. The ROS Master is necessary for the nodes to communicate with each other. In addition, the movements of the robot and the environmental information are visualised in this study through RViz [51],[52]. The RViz interface is shown in [Figure 5.3](#), where the robot and the environment are visualised.

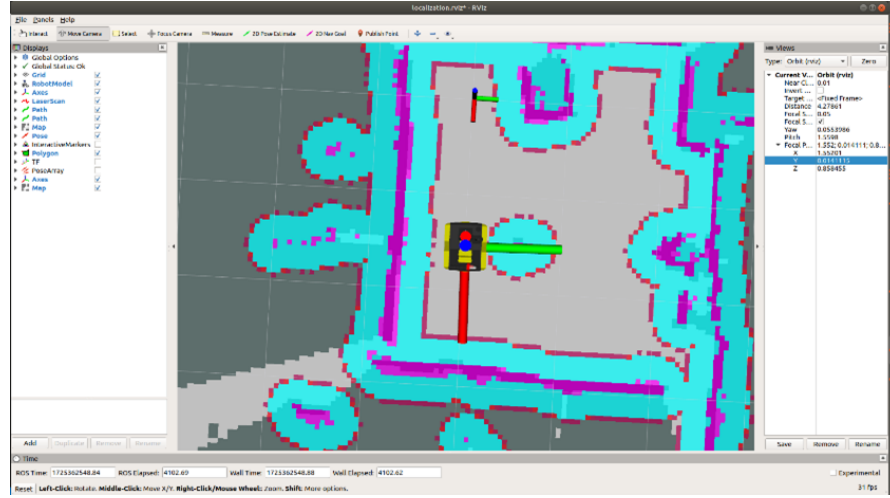
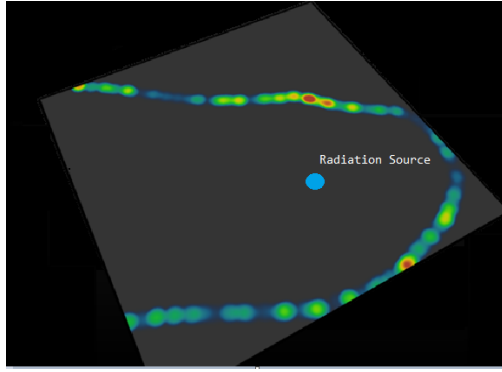
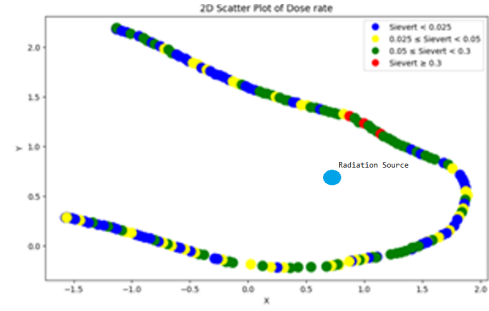


Figure 5.3: RViz interface illustrating the localization and navigation process of the mobile robot within the mapped environment. The cyan regions represent the detected obstacles and environmental boundaries, the red line indicates the planned navigation path, and the green circle marks the designated target position within the workspace.

The first map, [Figure 5.4a](#), shows the radiation intensity in a 2D plane with colour gradations, where colours such as red and yellow highlight higher radiation levels. The other, [Figure 5.4b](#), expresses specific radiation intensity ranges with specific colours. These visualisations were generated using the spatial and dose-rate data obtained from the LIDAR sensor and the CsI(Tl) detector mounted on the mobile robot. The maps collectively illustrate how radiation levels vary along the trajectory of the robot and confirm that higher dose rates occur near the source position. Thanks to these maps, it was observed that the robot accurately collects and processes spatial radiation data, and that the ROS-based framework visualises and analyses it in real time.



(a) Radiation map.



(b) Radiation map-Python.

Figure 5.4: (a) This is a 2D visualisation of the dose rates detected by the robot along the path. The red colour represents higher radiation intensities. (b) A 2-D scatter plot showing the dose rates recorded at each robot position. The colour coding shows the intensity of the detected radiation.

This study used 4 ^{137}Cs and 3 ^{60}Co sources of different activities with low dose rates in the nuclear laboratory. The scanning process took about 5 minutes. For sources with low activity, slightly longer scanning times are needed. The system was used to visualise and analyse the distribution of environmental radiation.

5.2 Radiation Source Localisation Methods

In past research, basic methods such as Maximum Likelihood Estimation (MLE), Bayesian inference, and triangulation have been widely used for radioactive source location. This work continues to develop these methods for radioactive source detection by combining them with different algorithms. These methods have different advantages; this chapter will describe them in detail, and theoretical and experimental comparisons will evaluate their performance.

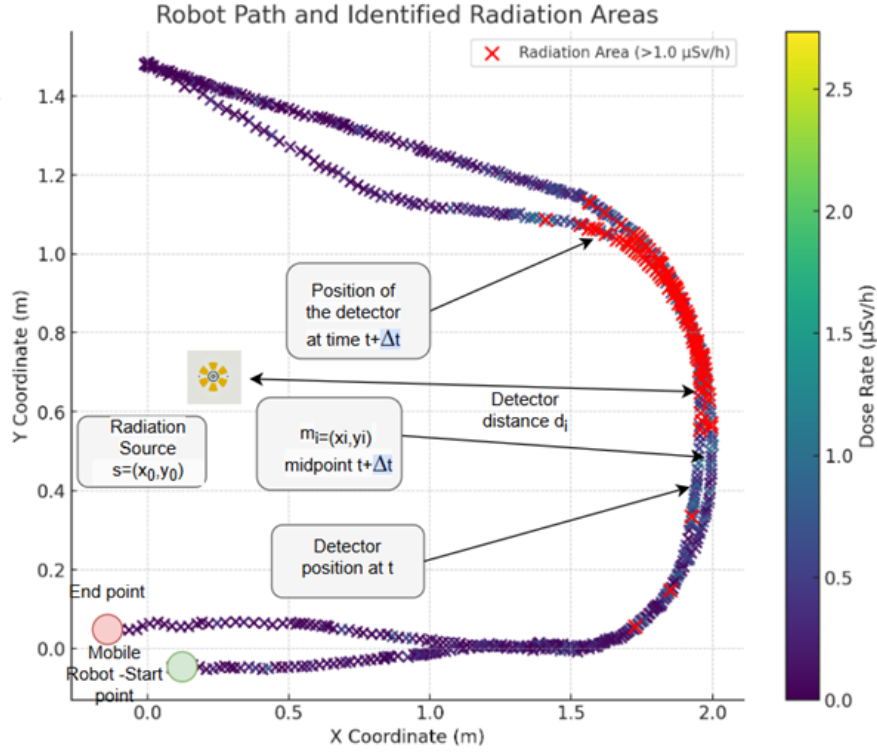


Figure 5.5: Mobile robotic system for exploring and pinpointing radioactive sources.

The unknown position of the radiation source $s, s=(x_0, y_0)$, is searched in the area scanned by the mobile robot, as shown in the Figure 5.5. The area is scanned by a scanning model with a search focus. When the radiation level in the area scanned by the robot exceeds $1.0 \mu\text{Sv/h}$, the region is considered above the threshold and shown in red. In this process, the detector records the distance $\Delta m=(\Delta x, \Delta y)$ between time t and $t + \Delta t$. The midpoint $m=(x,y)$ is related to the distance d_i of the detector to the source and the counted values. Firstly, these data are modelled with a Poisson distribution to improve the accuracy of the source position estimation [15]. The gamma counts recorded by the detector

with a Poisson distribution are expressed 5.1 as follows:

$$P(c(m_i)) = \frac{\lambda_i^{*c(m_i)} e^{-\lambda_i^*}}{c(m_i)!}, \quad i = 1, 2, 3, \dots, n \quad (5.1)$$

$c(m_i)$ is the gamma count value detected by the detector at m_i point and λ_i^* is the theoretically expected count value. The value of λ_i^* is inversely proportional to the distance to the source location 5.2.

$$\lambda_i^* = \frac{\lambda_{ss}^* Q \mu}{4\pi d_i^{*2}} e^{-\rho d_i^*} + \lambda_b^*, \quad i = 1, 2, 3, \dots, n \quad (5.2)$$

The scattering effect of radiation is neglected, so $\rho \approx 0$ is assumed. μ represents the detection efficiency of the detector, λ_{ss}^* characterizes the emitting intensity of the source, and Q denotes the surface area [53],[54],[55]. The distance between the detector and the source is calculated as follows 5.3:

$$d_i = \sqrt{(x - x^*)^2 + (y - y^*)^2} \quad (5.3)$$

where:

- (x, y) : Position of the detector.
- (x^*, y^*) : Position of the source.
- d_i : Euclidean distance between the source and the detector.

Following this study, the estimated source locations found by optimizing these estimates with the Maximum Likelihood method and two different algorithms

will be explained in detail.

5.2.1 Maximum Likelihood Estimation (MLE)

It is an effective statistical estimator that maximises the probability of observing the data and estimates parameters accordingly. This model was first developed by R.A. Fisher and is widely used in data analysis, especially in the estimation of model parameters [56]. The most appropriate values are selected by analysing the most probable model and parameter combinations of the observed data. The flexible structure of the MLE allows it to be used in different probability distributions, which increases the theoretical estimation capacity. While high accuracy can be achieved with large data sets, more computational resources are required as the models become more complex. Small data sets, on the other hand, have a disadvantage in that the results may need to be more accurate. The importance of the MLE in this study lies in its ability to increase the reliability of the results by providing high accuracy in data analysis during the determination of the radiation source location. It works effectively at different distances and with factors such as background noise [57],[58]. Equation 5.1 shows how the Poisson distribution models the gamma counts detected by the detector. The logarithmic likelihood function is obtained by taking the logarithm of the likelihood function (5.4).

$$L(\theta) = \sum_{i=1}^n [c(m_i) \ln \lambda_i - \lambda_i - \ln(c(m_i)!)] \quad (5.4)$$

In this formula, θ denotes the model parameters. Negative log-likelihood is

minimised since it is based on maximising the log-likelihood function [53],[59]. The negative likelihood method was chosen for the study. Many optimisation methods look for the smallest value of a function. Therefore, the negative likelihood function is minimised to make it suitable for the algorithm. It also ensures numerical stability by working with sums instead of multiplication. Mathematically, since it involves sums of logarithmic functions, differentiation is easier, which increases optimisation. The negative log-likelihood is expressed as:

$$-L(\theta) = - \sum_{i=1}^n [c(m_i) \ln \lambda_i - \lambda_i - \ln(c(m_i)!)] \quad (5.5)$$

The expected count values λ_i , computed from the optimised model parameters, show the highest agreement with the observed gamma counts.

L-BFGS-B (Limited-memory Broyden-Fletcher-Goldfarb-Shanno with Box constraints Optimisation Algorithm

Two different algorithms have been used to optimise the maximum likelihood method. The first algorithm chosen is Limited-memory Broyden-Fletcher-Goldfarb-Shanno with Box constraints Optimisation Algorithm (L-BFGS-B) . The algorithm is a derivative-based method for solving large-scale optimisation problems. Derived from the classical BFGS method, it successfully handles large data sets with limited memory usage [60]. One of the most important reasons for choosing this algorithm is that the study is structured as an optimisation problem, and the parameters must be within certain limits. Four different features of the algorithm were used to find the estimated location of the radiation source. 1-The ability to

control the boundaries: Given that the radiation sources are physically located within a limited area, the ability to control the parameter boundaries of the algorithm has been highlighted. 2-Memory efficiency: Since the data received from the radiation detector is large, it was important to optimise memory use. This algorithm facilitates the processing of these large data sets and gives fast and efficient results. 3-Deterministic approach: A deterministic approach was needed when trying to find the estimated radiation source location using the log likelihood function. L-BFGS-B optimises this function with gradient information and produces fast and accurate results. 4-Experimental agreement: Using dose rates in both experimental and simulation data and in combination with the L-FGBS algorithm helped to successfully find the estimated position of the design [61],[62]. The deterministic approach of the algorithm showed good agreement with the physical modelling process, which supported the results of the study. The L-BFGS-B algorithm is a quasi-Newton method often used in derivative-based optimisation problems. The objective of this algorithm is to minimise the objective function, as shown in (5.6).

$$\min_x f(x) \quad (5.6)$$

$f(x)$ is a twice derivable function with a continuous structure. The algorithm also optimises the parameters within certain limits ($l \leq x \leq u$). l lower limit, u is predetermined to be the upper limit. These constraints ensure that in practical problems, the parameters retain their current mathematical meaning and are optimised by physical constraints. Similar to the method of Newton, the L-BFGS-B algorithm proceeds with a quadratic approximation using the Hessian matrix. However, unlike the approach of Newton, it does not compute the Hessian matrix

exactly. However, it makes an approximate calculation based on the information from previous iterations, significantly reducing computational cost and memory usage. The approximate update of the Hessian matrix is equal to 5.7:

$$H_{k+1} = V_k^T H_k V_k + \rho_k s_k s_k^T, \quad (5.7)$$

where

- $s_k = x_{k+1} - x_k$, represents the parameter change during a step,
- $y_k = \nabla f(x_{k+1}) - \nabla f(x_k)$, represents the gradient difference,
- $\rho_k = \frac{1}{y_k^T s_k}$, is a normalization term,
- $V_k = I - \rho_k y_k s_k^T$, is a term used to update the Hessian matrix.

L-BFGS-B does not store the entire Hessian matrix but only the vector pairs (s_k, y_k) from the last m iterations. This strategy reduces the memory requirement to $O(mn)$ ¹ while increasing computational speed of the algorithm. n is the number of parameters being optimised. The algorithm uses projection to keep the parameters within lower and upper bounds. If the parameters updated for each iteration exceed these bounds, the algorithm destroys them within the bounds [61],[63].

The operation of the projection is defined as follows:

$$x_k^{\text{proj}} = \max(l, \min(x_k, u)), \quad (5.8)$$

¹The $O(mn)$ complexity arises from storing only the last m iterations' vector pairs (s_k, y_k) instead of the full Hessian matrix. Here, n is the number of parameters being optimised. This approach reduces memory requirements and computational costs compared to methods requiring $O(n^2)$.

where:

- l : The lower bound,
- u : The upper bound,
- x_k^{proj} : The value of the parameter after the projection process, constrained within the bounds.

Algorithm 1 Maximum Likelihood Estimation-L-BFGS-B

```

1: Input: Detector positions  $\{(X_i, Y_i)\}_{i=1}^n$ , Measured dose rates  $\{D_i\}_{i=1}^n$ , Small
   constant  $\epsilon$ 
2: Output: Estimated source position  $(x_0, y_0)$ , Errors  $(\Delta X, \Delta Y, \Delta d)$ 
3:
4: Initialize:  $x_0 \leftarrow \text{mean}(X_i)$ ,  $y_0 \leftarrow \text{mean}(Y_i)$  ▷ Initial guess
5: Define Negative Log-Likelihood Function:
6:    $L(x_0, y_0) = -\sum_{i=1}^n [\log(D_i) - \log((X_i - x_0)^2 + (Y_i - y_0)^2 + \epsilon)]$ 
7:
8: Minimise the Negative Log-Likelihood Function:
9:   result  $\leftarrow$  minimise( $L, (x_0, y_0)$ , method = L-BFGS-B)
10:  estimated_position  $\leftarrow$  result.x
11:
12: Calculate Errors:
13:    $\Delta X \leftarrow (\text{True}_x - \text{estimated\_position}[0]) \times 100$ 
14:    $\Delta Y \leftarrow (\text{True}_y - \text{estimated\_position}[1]) \times 100$ 
15:    $\Delta d \leftarrow \sqrt{\Delta X^2 + \Delta Y^2}$ 
16:
17: Output:
18:   Estimated source position:  $(x_0, y_0) = \text{estimated\_position}$ 
19:   X Error:  $\Delta X$ 
20:   Y Error:  $\Delta Y$ 
21:   Total Distance Error:  $\Delta d$ 

```

In this algorithm, the radiation source location is estimated using the MLE method based on the data obtained from the detector integrated into the robot

and moved throughout the environment. The estimation process relies solely on the dose-rate data (D_i) measured by the detector; the energy information from the gamma spectrum is not utilised in this implementation, as the localisation depends on spatial variations in the measured radiation intensity. Four different ^{137}Cs and three different ^{60}Co sources were used for these experiments. All sources were tested in the same experimental setup and placed at the same location to ensure consistent measurement conditions.

The activity values of these radiation sources for 03.09.24 are as follows:

- 1 – ^{137}Cs (1S684) - Activity: $149210 \text{ Bq} \pm 4450$
- 2 – ^{137}Cs (A0468) - Activity: $74815 \text{ Bq} \pm 1170$
- 3 – ^{137}Cs (A0697) - Activity: $111064 \text{ Bq} \pm 4110$
- 4 – ^{137}Cs (Cs137.280.17) - Activity: $66092 \text{ Bq} \pm 2450$
- 5 – ^{60}Co (C3N.19/18[DC617]) - Activity: $59983 \text{ Bq} \pm 2530$
- 6 – ^{60}Co (Co60.284.17) - Activity: $33543 \text{ Bq} \pm 3980$
- 7 – ^{60}Co (C3N.19/62[EE908]) - Activity: $8505 \text{ Bq} \pm 362$

The total activity of the radiation sources is approximately 0.545 MBq, which is a relatively low activity level that may lead to some errors in estimating the source location.

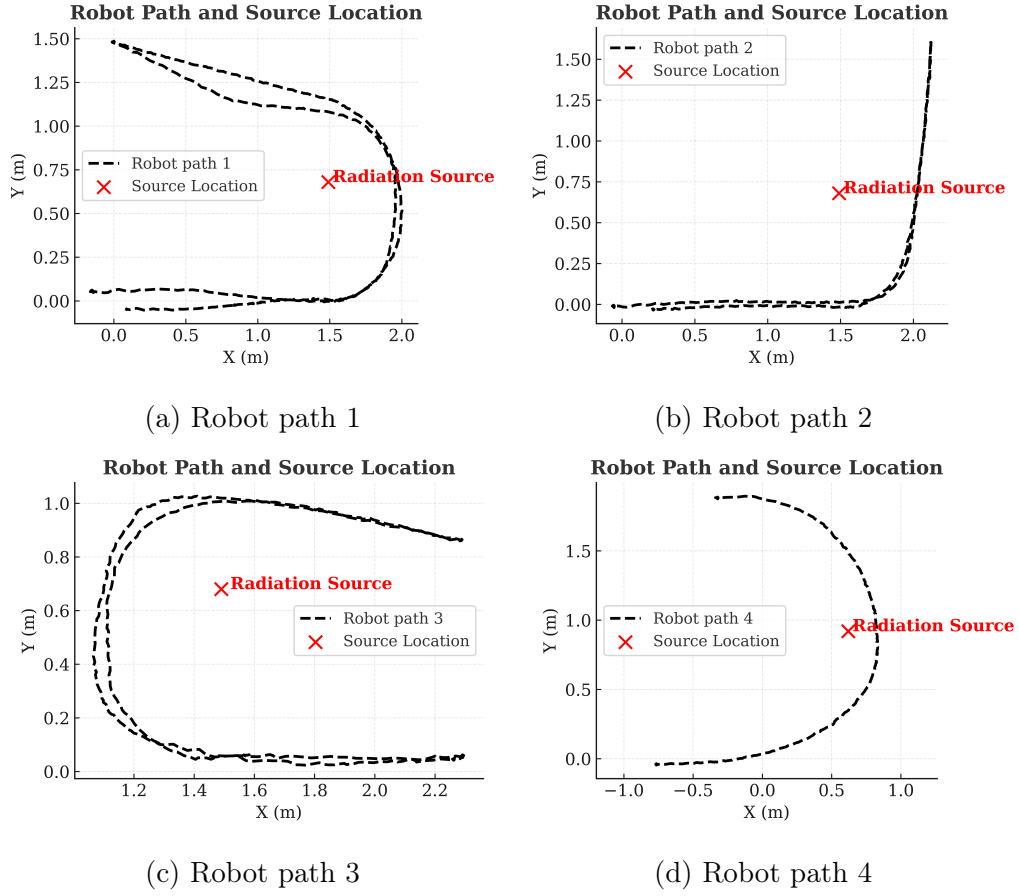


Figure 5.6: Different paths the robot follows: (a) Robot path 1, (b) Robot path 2, (c) Robot path 3 ve (d) Robot path 4.

In Figure 5.6, four random paths followed by the robot are selected to compare the performance of the algorithm under different paths. The position of the detector (X_i, Y_i) and the measured dose rate are expressed as D_i . The algorithm uses the negative log-likelihood function to estimate the optimal source location (x_0, y_0) based on the measured dose rate.

$$L(x_0, y_0) = - \sum_{i=1}^n [\log(D_i) - \log((X_i - x_0)^2 + (Y_i - y_0)^2 + \epsilon)] . \quad (5.9)$$

In this formula, ϵ is a small constant that prevents division by zero. The algorithm minimises the negative log-likelihood function using the L-BFGS-B optimisation method. Thus, in the robot-assisted scanning system, measurements are taken at different locations with a single detector, and these data are processed to locate the radiation source quickly and accurately.

Figure 5.7 shows the estimated source position given by this algorithm for the first path followed by the robot. The actual position of the source is $(x,y)=(1.49, 0.68)$, and the estimated source position is $(x,y) = (1.63, 0.75)$, where all coordinates are expressed in metres.

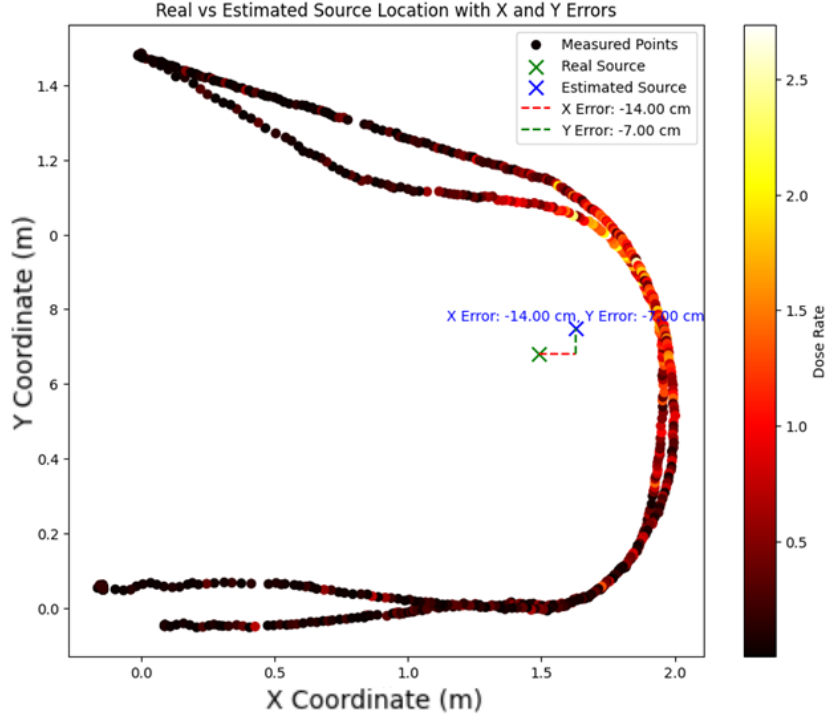


Figure 5.7: Radiation source location estimation with MLE algorithm-Robot Path 1. Comparison of real and estimated source locations with X and Y coordinate error. The real source position is marked in green, the estimated source position is in blue, and the X and Y coordinate error is annotated in red and blue. The colour bar indicates the measured dose rates, with higher intensities in yellow and lower in dark red.

In Figure 5.8, the RViz environment shows the actual source location in purple, while the yellow dot indicates the estimated source location.



Figure 5.9: The position of the robot in the estimated position found by the MLE method.

The estimated source positions obtained using the MLE method for other paths are shown in [Figure 5.10](#) below.

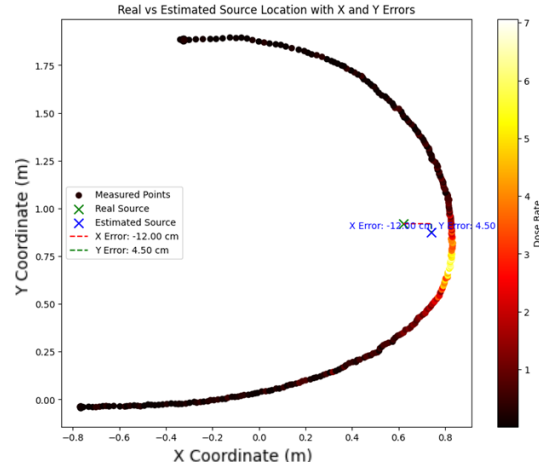
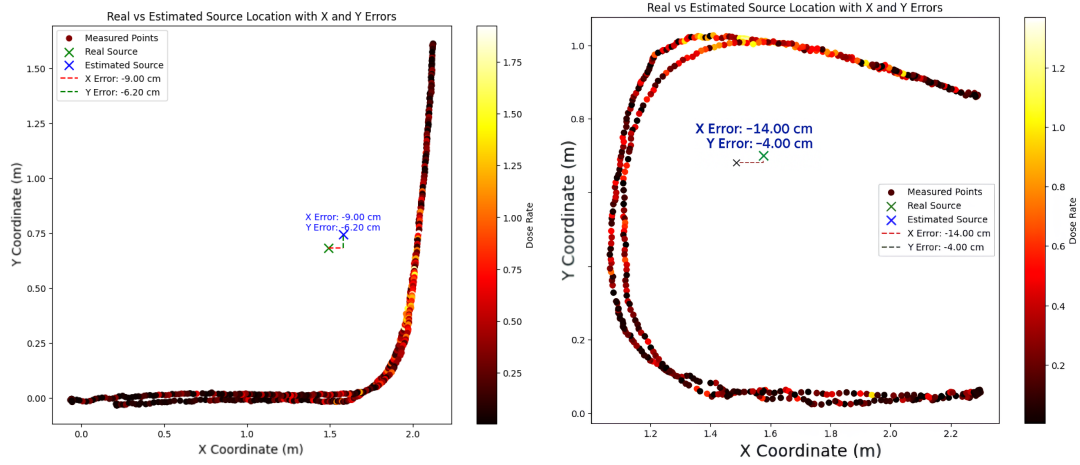


Figure 5.10: Comparison of real and estimated source locations for different paths obtained using the MLE method. The paths were generated and visualized using Python.

Table 5.3 presents the absolute position errors (in meters) for the estimated

source locations along the paths followed by the robot. The X and Y coordinate errors, as well as the total errors, are provided for four different paths.

Table 5.3: Error Analysis Table with Absolute and Total Errors for L-BFGS-B Method (in meters)

Path	Real X (m)	Real Y (m)	Est. X (m)	Est. Y (m)	X Error (m)	Y Error (m)	Total Error (m)
Path 1	1.49	0.68	1.63	0.75	0.14	0.07	0.16
Path 2	1.49	0.68	1.70	0.87	0.21	0.19	0.28
Path 3	1.49	0.68	1.63	0.72	0.14	0.04	0.15
Path 4	0.62	0.92	0.74	0.875	0.12	0.045	0.13

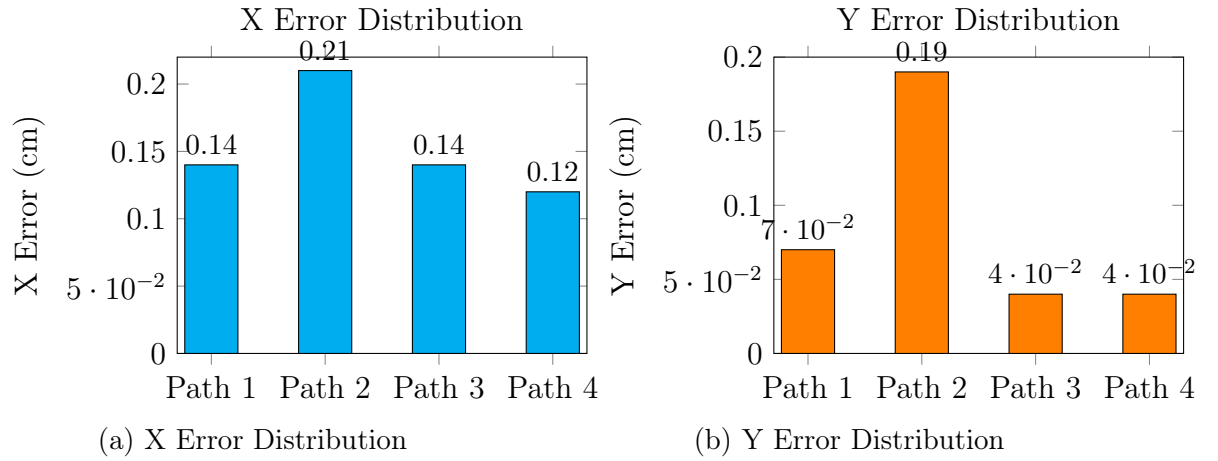


Figure 5.11: Comparison of X and Y error distributions in MLE localisation. (a) X error distribution, (b) Y error distribution.

Path 4 shows the lowest total position error, with an error of **13 cm**. This demonstrates that the Bayesian localisation algorithm provides the most accurate estimation for this trajectory. The second-lowest total error (15 cm) is observed for Path 3, indicating similarly good performance. For Path 1, the total error slightly increases to **16 cm**, yet the accuracy still remains within acceptable limits. In

contrast, Path 2 exhibits the largest total error (28 cm), with **X-axis and Y-axis errors of 21 cm and 19 cm**, respectively, showing that the algorithm performs less reliably for this path.

Although Path 3 yields a better result than Path 2, with a total position error of **15 cm**, it is not as successful as Path 1. It was observed that the error level could not be reduced to the desired range because the X and Y errors remained around **14 cm** and **4 cm**, respectively, and the algorithm tended to become trapped in local minima despite being well modelled.

In Path 4, a total position error of **13 cm** was achieved, yielding the most accurate result among all paths. However, further optimisation and noise suppression are still required to enhance localisation precision under varying trajectory conditions.

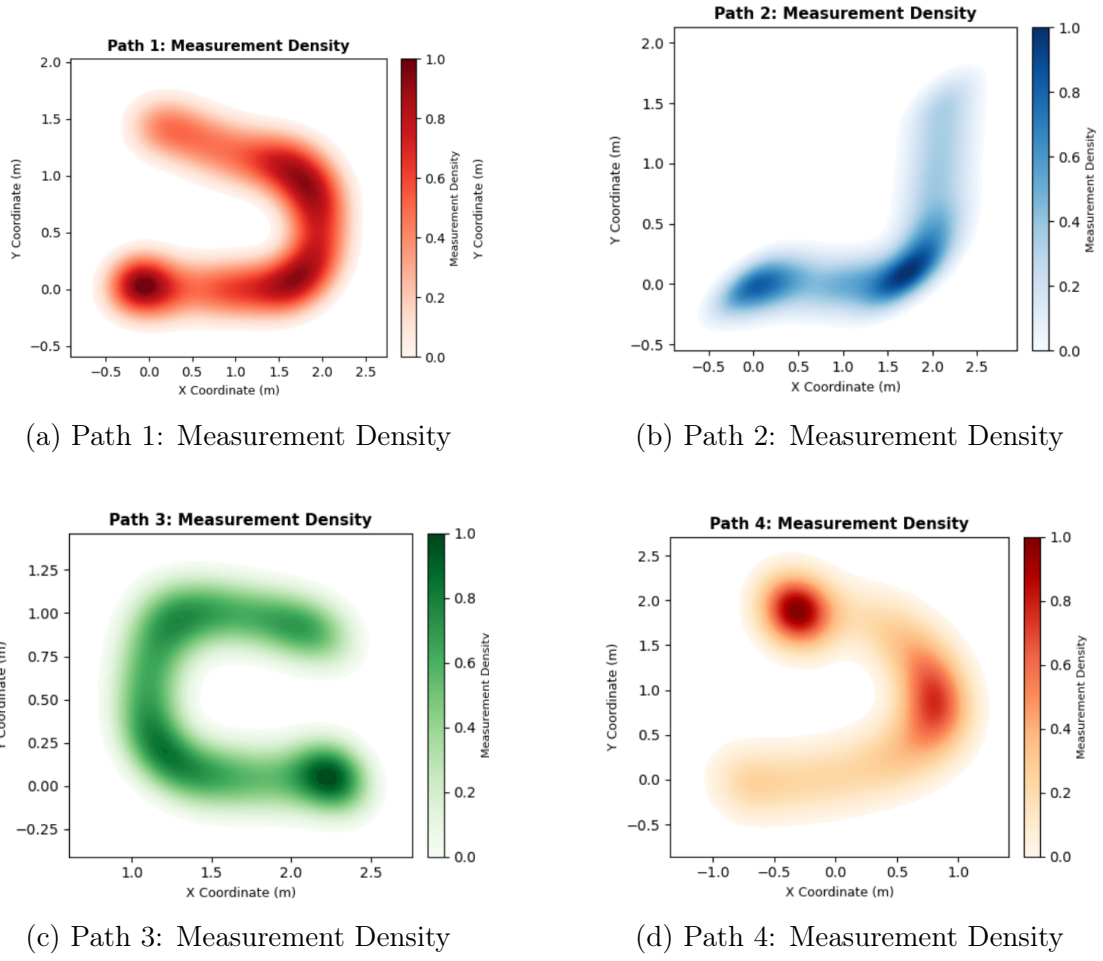


Figure 5.12: Measurement density distributions for four paths. The colour intensity represents the measurement density (number of readings per spatial unit). Darker regions indicate higher measurement frequency. (a) Path 1 shows two dense regions. (b) Path 2 displays a broad spread with lower density. (c) Path 3 reveals a C-shaped density distribution. (d) Path 4 exhibits two localized dense regions.

The density distribution presented in Figure 5.12a for Path 1 indicates that the measurement points are concentrated in specific regions. In this case, the derivative-based optimisation of the **L-BFGS-B** algorithm works more effectively. Thanks to these findings, the method reaches the global minimum more decisively and provides a high-accuracy result.

When the density map for Path 2 is examined, the vast and irregular distribution of the measurement points causes the algorithm to get stuck in local minima, increasing the error rate.

When Path 3 is analyzed, the densities are gathered in certain regions, although not as much as in Path 1.

It has been observed that the measurement points are locally concentrated in Path 4, and increasing the number of measurement points is necessary to reduce the error rate.



Figure 5.13: Correlation between Dose Rate and Total Error for Path 2. A negative correlation is observed: as the dose rate ($\mu\text{Sv/h}$) decreases, the total error (m) increases.

The x error correlation -0.43 and y error correlation -0.39 values obtained in the analysis show that the error rate increases with decreasing dose rate. It is seen that the total error values vary between 0.5–2.5 m in the regions where the dose rate is less than 0.5 $\mu\text{Sv/h}$. This is due to the weak measurements made by the detector in the region far from the source. Inadequate measurements have biased

the algorithm towards the local minimum when finding the result. To solve this problem, the detector measurements should be more homogeneously distributed, and more measurements should be performed. In general, the reasons for errors are examined in detail. Firstly, in the MLE optimisation process, the algorithm may sometimes fail to find a global minimum while searching for a solution and stay at a local minimum point. When the recorded positions of the robot are irregular, it is difficult for the algorithm to reach the global minimum. Also, the noise in the data received from the detector can cause the negative log-likelihood function to get stuck in the local minimum. The initial guess is significant since the L-BFGS-B algorithm is a derivative-based method. If the initial guess is incorrect, the algorithm may not reach the global minimum and stop at the local minimum. Therefore, other algorithms have been searched for better results, and Monte Carlo optimisation with the MLE method has been added to this work to reduce the dependence on the initial guess.

Monte-Carlo Optimisation Algorithm

Monte Carlo optimisation is a numerical technique for solving complex mathematical problems by random sampling. Optimisation methods generally use the knowledge of the derivative and gradient to determine the minimum and maximum values. However, in some cases, it is difficult or impossible to calculate the derivative of functions, and in such cases, the Monte Carlo method is used. In this method, the optimal result is determined by taking random samples. Since it performs strongly in noisy and incomplete data environments, it is frequently used in nuclear applications, machine learning, etc. One of the advantages of Monte

Carlo is that it is easy to apply and flexible. It is successful in the search for global minima by scanning multiple starting points and provides effective results in complex and high-dimensional parameter spaces. Thanks to these features, it becomes an important method in multi-parameter problems such as finding the radiation source location. The basic approach is to randomly select the possible values of the parameters and evaluate these values with negative log-likelihood to find the parameter that minimises the error.

The negative log-likelihood function minimises the difference between the measured dose-rate data and the dose rates predicted by the model. As shown below, Monte Carlo optimisation is integrated into the negative log-likelihood function.

$$\text{NLL}(x_0, y_0, S, B, \mu) = - \sum_{i=1}^N \left[D_i \cdot \ln(\hat{D}_i + \epsilon) - \hat{D}_i \right]. \quad (5.10)$$

$\hat{D}_i$ is the dose rate predicted by the model.

$$\hat{D}_i = \frac{S}{d_i^2 \cdot e^{\mu \cdot \sqrt{d_i^2}} + B}. \quad (5.11)$$

$$d_i = \sqrt{(X_i - x_0)^2 + (Y_i - y_0)^2}. \quad (5.12)$$

d_i is the distance between the source and the measurement. The previously described function 5.9. has been optimised to estimate the strength(S), the background rate (B), and the attenuation coefficient (μ). Since these parameters need to be optimised over a complex objective function with multiple local minima, Monte Carlo optimisation offers an efficient solution to reach the global optimum. The properties of robustness and variance reduction describe the reliability of

Monte Carlo results.

$$\mathbb{E} [I_f^{MC}[h]] = I_f[h], \quad (5.13)$$

$$\mathbb{V} [I_f^{MC}[h]] = \frac{1}{N} \mathbb{V}_{X \sim f} [h(X)]. \quad (5.14)$$

$I_f[h]$: The actual value of the integral. N : The number of samples. $I_f^{MC}[h]$: Monte Carlo integral estimator. Based on these statements: 1-The value obtained by Monte Carlo estimation is the same as the actual value, so this method is bias-free. 2-As can be seen in 5.14 ; the error rate decreases inversely with the square of the number of samples (N), which shows that the method provides more accurate results when more samples are used. At the same time, confidence intervals measure the statistical reliability of the values found by the Monte Carlo method. The Monte Carlo theorem is standardised according to the Central Limit Theorem (CLT).

$$\sqrt{N} \cdot \frac{I_f^{MC}[h] - I_f[h]}{\sqrt{\text{Var}_X[h(X)]}} \xrightarrow{d} \mathcal{N}(0, 1). \quad (5.15)$$

According to this expression, the $(1 - \alpha)\%$ confidence interval is calculated as follows:

$$\left(I_f^{MC}[h] - z_{\alpha/2} \sqrt{\frac{\text{Var}_X[h(X)]}{N}}, I_f^{MC}[h] + z_{\alpha/2} \sqrt{\frac{\text{Var}_X[h(X)]}{N}} \right). \quad (5.16)$$

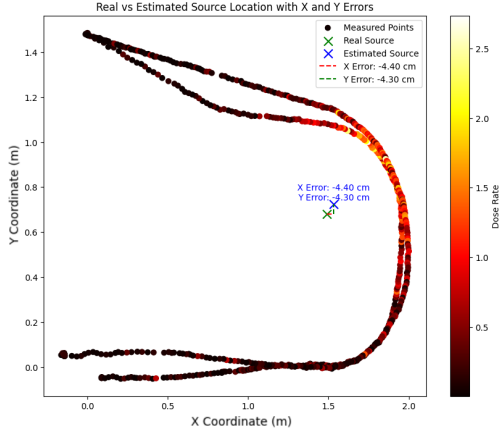
These confidence intervals are critical for assessing the reliability of the predicted results. The larger the N values, the narrower the confidence intervals and the more sensitive the results become [64],[61],[65],[66].

Algorithm 2 Monte Carlo Optimisation for Radiation Source Localisation

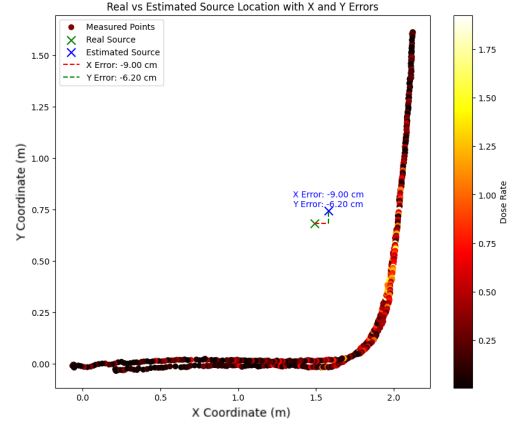
-
- 1: **Input:** Detector positions $\{(X_i, Y_i)\}_{i=1}^N$, Measured dose rates $\{D_i\}_{i=1}^N$, Small constant ϵ , Number of trials T
 - 2: **Output:** Estimated source parameters (x_0, y_0, S, B, μ) , Errors $(\Delta X, \Delta Y, \Delta d)$
 - 3: **Step 1: Filter Data**
 - 4: Filter data points with dose rate D_i :
 - 5: $D_i^{\text{filtered}} = \{D_i : D_i > \text{threshold}\}$
 - 6: **Step 2: Define Negative Log-Likelihood (NLL) Function**
 - 7: $L(x_0, y_0, S, B, \mu) = -\sum_{i=1}^N \left[D_i \cdot \ln(\hat{D}_i + \epsilon) - \hat{D}_i \right]$
 - 8: where $\hat{D}_i = \frac{S}{d_i^2 \cdot e^{\mu \cdot d_i}} + B$ and $d_i = \sqrt{(X_i - x_0)^2 + (Y_i - y_0)^2}$
 - 9: **Step 3: Monte Carlo Optimisation Loop**
 - 10: Initialize: $\text{best_NLL} \leftarrow \infty$, $\text{best_params} \leftarrow \emptyset$
 - 11: **for** $t = 1$ to T **do**
 - 12: Sample initial parameters:
 - 13: $x_0 \sim U(\min(X_i), \max(X_i))$
 - 14: $y_0 \sim U(\min(Y_i), \max(Y_i))$
 - 15: $S \sim U(0.01, 10)$
 - 16: $B \sim U(0.01, 1)$
 - 17: $\mu \sim U(0.001, 1)$
 - 18: Minimise NLL using L-BFGS-B:
 - 19: $\text{result} \leftarrow \text{minimise}(L, \text{initial_params}, \text{method} = \text{L-BFGS-B})$
 - 20: **if** $\text{result.fun} < \text{best_NLL}$ **then**
 - 21: Update best parameters:
 - 22: $\text{best_NLL} \leftarrow \text{result.fun}$
 - 23: $\text{best_params} \leftarrow \text{result.x}$
 - 24: **end if**
 - 25: **end for**
 - 26: **Step 4: Extract Results**
 - 27: Extract estimated parameters: $(x_0, y_0, S, B, \mu) \leftarrow \text{best_params}$
 - 28: **Step 5: Calculate Errors**
 - 29: $\Delta X \leftarrow (\text{True}_x - x_0) \cdot 100$
 - 30: $\Delta Y \leftarrow (\text{True}_y - y_0) \cdot 100$
 - 31: $\Delta d \leftarrow \sqrt{\Delta X^2 + \Delta Y^2}$
 - 32: **Output:**
 - 33: Estimated source parameters: (x_0, y_0, S, B, μ)
 - 34: Errors: $\Delta X, \Delta Y, \Delta d$
-

Random selection of initial parameters prevents getting stuck in local min-

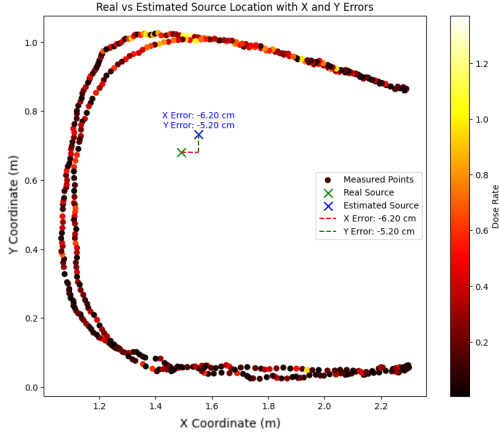
ima by reaching the global optimum. Thanks to the Monte Carlo method, a search is made at multiple starting points. This method is combined with the L-BFGS-B method and minimises the NLL function. The precision of the measured dose rates is directly related to the quality of the predictions. Increasing noisy data may adversely affect the accuracy of the estimates. At the same time, better results can be obtained as the number of samples (T) increases, but choosing an optimum T value is important as the processing time increases. Figure 5.15 shows the estimated source locations of the radiation source based on the results of this algorithm.



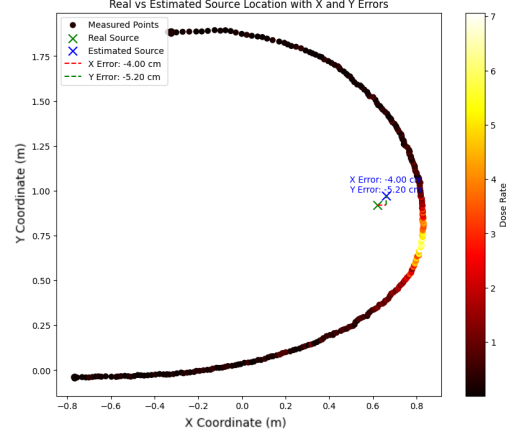
(a) Estimated source location with X and Y errors for Path 1.



(b) Estimated source location with X and Y errors for Path 2.



(c) Estimated source location with X and Y errors for Path 3.



(d) Estimated source location with X and Y errors for Path 4.

Figure 5.14: Comparison of actual and predicted source locations for different paths determined by Monte Carlo optimisation. The paths were generated and visualised in Python, demonstrating the ability of the algorithm to predict source locations with varying levels of accuracy depending on the data distribution and path characteristics.

Table 5.4 shows that better results are obtained compared to the other algorithm when the errors are analysed.

Table 5.4: Error Analysis Table with Absolute and Total Errors for Monte Carlo Method (m)

Path	Real X	Real Y	Est. X	Est. Y	X Er- ror (m)	Y Er- ror (m)	Total Error (m)
Path 1	1.49	0.68	1.534	0.723	0.044	0.043	0.061
Path 2	1.49	0.68	1.580	0.742	0.090	0.062	0.110
Path 3	1.49	0.68	1.552	0.732	0.062	0.052	0.081
Path 4	0.62	0.92	0.660	0.972	0.040	0.052	0.065

Path 1 has the lowest total error of **6.1 cm** compared to the other paths. As can be seen in [Figure 5.14a](#), the agreement between the predicted and actual position is excellent. According to this result, the Monte Carlo optimisation for Path 1 works efficiently and provides high accuracy.

The highest total error (**11.0 cm**) is observed for Path 2. The error distribution of X and Y has a broader range compared to the other paths, and it is observed that the algorithm performs less well than expected for this path ([Figure 5.15](#)).

The total error in Path 3 is moderate at **8.1 cm**. The error distribution of X and Y is slightly higher than Path 1, but it provides better accuracy than Path 2. This path is better modelled by the algorithm.

Path 4 gives a better result than Path 2, with a total error of **6.5 cm**. The algorithm shows a more stable performance for this path.

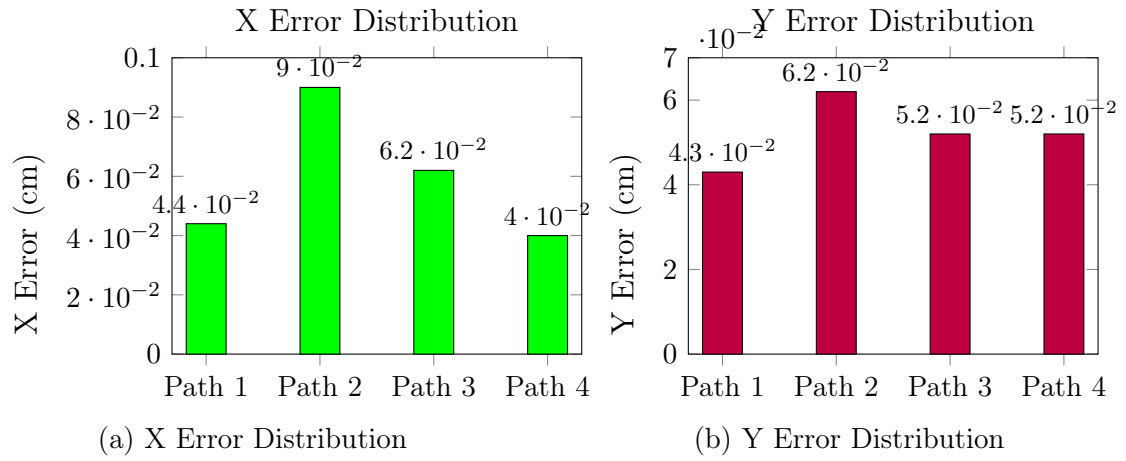


Figure 5.15: Comparison of X and Y error distributions in Monte Carlo localisation. (a) X error distribution, (b) Y error distribution.



Figure 5.16: The position of the robot in the estimated position found by the Monte Carlo optimisation for Path 3.

In Figure 5.16, the robot is sent to the estimated source position for Path 3, and as can be seen, the robot and the source are very close to each other. In Path 3, Table 5.4 shows that the Y-axis error is **5.2 cm**, while the X-axis error is **6.2 cm**. In this case, the Rviz map (5.17) indicates a more noticeable deviation between the actual position and the estimated position along the Y-axis.

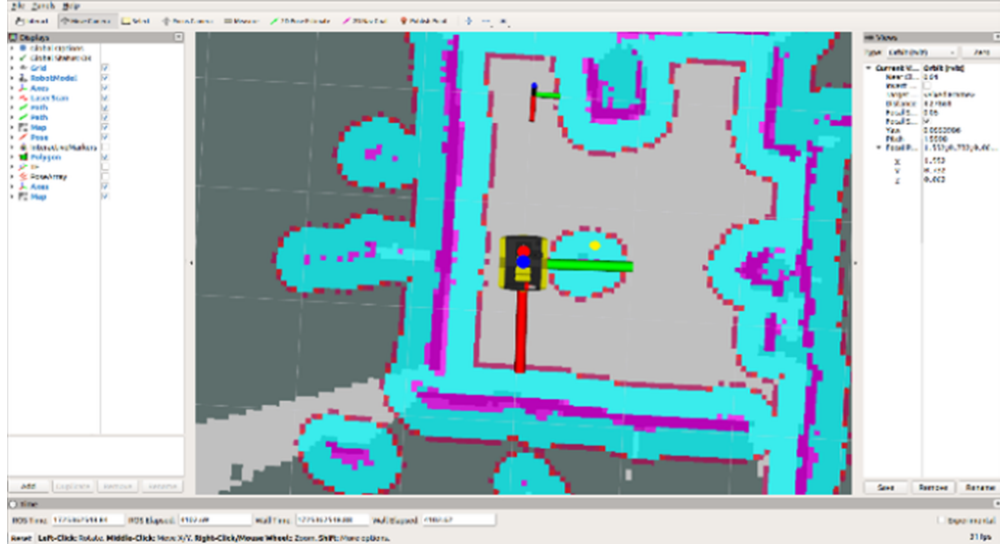


Figure 5.17: Actual and estimated source locations in the RViz environment. The robot follows Path 3 based on the Monte Carlo optimisation.

This study shows that Monte Carlo optimisation produces variable results depending on the geometry of the path and the data distribution. While the high accuracy obtained for Path 1 demonstrates the effectiveness of the method, some aspects need to be improved to address the difficulties encountered in Path 2. The flexibility and noise resistance of the algorithm increase reliability in real-world applications.

5.2.2 Bayesian Method

The Bayesian method is a modelling approach often used in statistical analysis and machine learning, especially in uncertain situations. This method updates the probability distribution of a parameter based on Bayes' theorem according to available information and new data. This method uses incomplete data and uncertainty to make more flexible and robust estimates. The basis of Bayesian inference is expressed mathematically as Bayes' theorem:

$$P(\theta|D) = \frac{P(D|\theta) \cdot P(\theta)}{P(D)} \quad (5.17)$$

where

$P(\theta)$: Prior Distribution - Parameter-related information and assumptions

$P(D|\theta)$: Likelihood - The probability of the data occurring at a given value of the parameter

$P(\theta|D)$: Posterior Distribution - Probability distribution of the parameter in the light of new data

$P(D)$: Normalization Factor - The coefficient normalizes the sum of probabilities

To find this value, the values of the maximum likelihood $P(\theta)$ and the likelihood function $P(D|\theta)$ are multiplied and summed or integrated over all values of θ .

When calculating the integral when θ is a continuous variable,

$$P(D) = \int P(D|\theta)P(\theta) d\theta \quad (5.18)$$

When it is a discrete variable, the sum is used:

$$P(D) = \sum_{\theta} P(D|\theta)P(\theta) \quad (5.19)$$

The marginal likelihood $P(D)$ does not depend on the parameter θ . Therefore, Bayes' theorem is formulated as follows:

$$P(\theta|D) \propto P(D|\theta)P(\theta) \quad (5.20)$$

This expression (5.20) shows that the posterior distribution is directly proportional to the product of the likelihood function and the prior distribution [67],[68]. There are three essential components in Bayesian modelling:

1-Prior Distribution It expresses the information obtained about the parameter before any observation is made. In the absence of prior information, non-informative priors are typically used, often in a uniform distribution. However, when certain prior information is available, using informative priors based on standard probability distributions such as Gaussian (Normal), Beta, Gamma, or Dirichlet distributions is more appropriate [69]. For example, since the initial location of the radiation source is not exactly known for this study, a wide prior distribution about its possible location is preferred.

2-Likelihood Function The likelihood function expresses the probability of ob-

serving data depending on a given parameter. This function is expressed as follows:

$$P(D|\theta) = \prod_{i=1}^N P(d_i|\theta) \quad (5.21)$$

where

d_i : The data collected for each measurement point.

N : The total number of points measured.

$P(d_i|\theta)$: Represents the probability of occurrence of the data d_i for a given θ .

In this study, if the data recorded by the detector follow a Poisson distribution, the function is expressed as:

$$P(D|\theta) = \prod_{i=1}^N \frac{e^{-\lambda(\theta)} \lambda(\theta)^{d_i}}{d_i!} \quad (5.22)$$

where

$\lambda(\theta)$: Represents the expected count rate, which varies depending on the source location.

d_i : Denotes the observed count (number of detected events) at the i^{th} measurement point.

$d_i!$: Is the factorial of the observed count, used for normalisation.

3-Posterior Distribution

The posterior distribution $P(\theta|D)$ is the parameter distribution updated with the data, calculated by Bayes' Theorem. From Bayes' Theorem:

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

The posterior distribution, which is a combination of a priori information and data from observations, can best model the uncertainties. The mean value or maximum a posteriori point (MAP) of this distribution helps to estimate the most probable value of the parameter:

$$\hat{\theta}_{MAP} = \arg \max_{\theta} P(\theta|D) = \arg \max_{\theta} P(D|\theta)P(\theta) \quad (5.23)$$

The posterior distribution can remain within the same type when the prior distribution and the likelihood function belong to the same distribution. However, in most cases, an analytical solution is not possible and requires computation with stochastic sampling methods such as Markov Chain Monte Carlo (MCMC) [67],[70],[71].

Based on the Bayesian inference technique, [Algorithm 3](#) is written to determine the most probable location of a radiation source using radiation data obtained from different points.

Algorithm 3 Bayesian Source Localisation with Dose Rate Interpolation and Gaussian Smoothing

- 1: Load data file (CSV) containing positions (X_i, Y_i) and dose rates (D_i) .
- 2: Define actual source coordinates (X_{real}, Y_{real}) and initial estimated coordinates (X_{est}, Y_{est}) .
- 3: Apply Gaussian smoothing to dose rate data:

$$D'_i = \frac{1}{\sqrt{2\pi\sigma^2}} \sum_j D_j e^{-\frac{(i-j)^2}{2\sigma^2}}$$

- 4: Create interpolation grid (X_{grid}, Y_{grid}) using linear interpolation method.
- 5: Define Bayesian model parameters and priors:

$$P(S) \sim \text{Uniform}, \quad \sigma \sim \text{HalfNormal}(0, 1)$$

State **Bayesian Estimation Procedure:**

- 6: Define likelihood function:

$$P(D|S, \sigma) = \prod_i \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(D_i - f(S, X_i, Y_i))^2}{2\sigma^2}\right)$$

- 7: Calculate posterior distribution:

$$P(S, \sigma|D) \propto P(D|S, \sigma)P(S)P(\sigma)$$

- 8: Compute Bayesian estimate by maximizing the posterior:

$$S_{Bayes} = \arg \max_S P(S|D)$$

9: **Visualization Procedure:**

- 10: Generate dose rate heatmap and contour lines from interpolated grid.
- 11: Plot robot path based on loaded positions.
- 12: Mark actual (X_{real}, Y_{real}) and estimated (X_{est}, Y_{est}) source locations.
- 13: Calculate positional differences:

$$\Delta X = |X_{real} - X_{est}| \times 100 \quad \text{cm}, \quad \Delta Y = |Y_{real} - Y_{est}| \times 100 \quad \text{cm}$$

- 14: Save and display the visualization.
 - 15: **Main Workflow:**
 - 16: Sequentially execute data loading, preprocessing, Bayesian estimation, and visualization procedures.
-

The radiation measurement data (D) collected by the CsI(Tl) detector installed on the robot, the coordinates of the measurement points (X, Y) and the actual source coordinates (S_{real}) are taken as input. A grid structure is determined to estimate the possible source location. The coordinates most likely to be the source are estimated using the radiation dose rates measured at different locations. The noise observed in sensor measurements is considered statistically independent and uniformly distributed zero-mean Gaussian noise. This assumption is widely used in similar studies in the literature and is supported by the Central Limit Theorem [55],[72],[73]. Gaussian smoothing is applied to increase the spatial consistency of the measurement data and to make the data more reliable by reducing the effect of sensor noise. This process reduces measurement errors, and an accurate estimation process is created and calculated using the following [Formula 5.24](#):

$$d'_i = \frac{1}{2\pi\sigma^2} \sum_j d_j e^{-\frac{(i-j)^2}{2\sigma^2}} \quad (5.24)$$

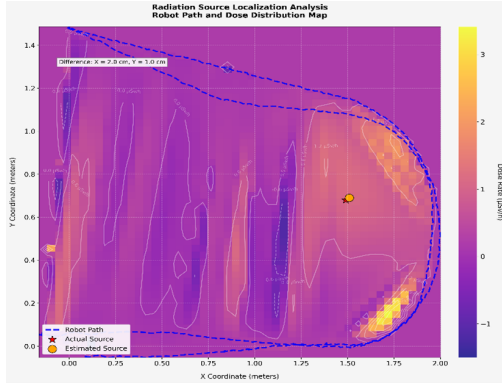
This filtering method ensures the stability of the algorithm by suppressing sudden changes in the sensor data. The sigma parameter is determined using experimental data to express the smoothing intensity. The signal-to-noise ratio (SNR) and localisation errors determine the optimal sigma. Using Bayesian inference theory, a two-dimensional grid is constructed to determine the radiation source location. The computational load was controlled when selecting the grid size, and localisation accuracy was considered. The purpose of this grid is to make the computation process manageable by providing a finite representation of the continuous space with equally spaced points over the area covered by the measurement data. The probability of the source location being at these grid points is

calculated for the data observed at these grid points. The likelihood function and the Gaussian noise model are expressed in [Algorithm 3](#) as follows:

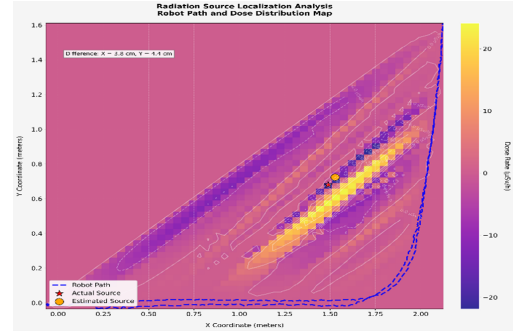
$$P(D | S) = \prod_{i=1}^N \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(d_i - f(S, x_i, y_i))^2}{2\sigma^2}\right) \quad (5.25)$$

Here $f(S, x_i, y_i)$ represents the expected radiation dose rate from the source to the point (x_i, y_i) . This rate is modelled according to the inverse square law. Then, if there are previous study measurements, the uniform distribution is accepted and used as additional information. If there is no additional information to be used, it is accepted as a uniform distribution. Finally, the most optimal source location is determined using the Maximum posterior prediction (MAP) method:

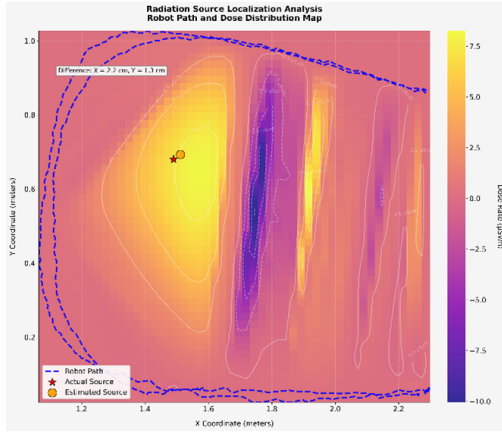
$$S_{\text{Bayes}} = \arg \max_{S \in G} P(S | D) \quad (5.26)$$



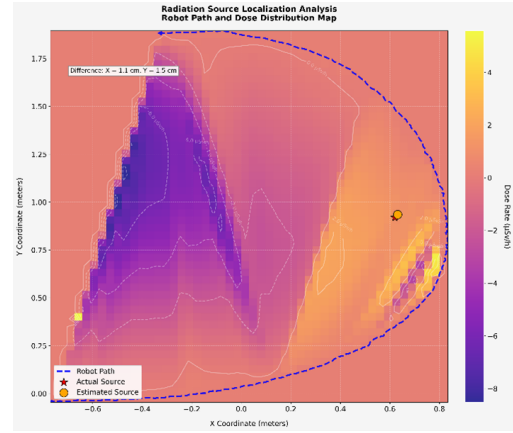
(a) Bayesian localisation result showing the robot path, dose rate distribution, and estimated source position with an error of $X = 2$ cm, $Y = 1$ cm. (Path 1).



(b) Bayesian localisation result showing the robot path, dose rate distribution, and estimated source position with an error of $X = 3.8$ cm, $Y = 4.4$ cm. (Path 2).



(c) Bayesian localisation result showing the robot path, dose rate distribution, and estimated source position with an error of $X = 2.2$ cm, $Y = 1.1$ cm. (Path 3).



(d) Bayesian localisation result showing the robot path, dose rate distribution, and estimated source position with an error of $X = 1.1$ cm, $Y = 1.5$ cm. (Path 4).

Figure 5.18: Bayesian-based radiation source localisation analysis, illustrating the estimated source location, actual source position, and robot path within the dose rate distribution map.

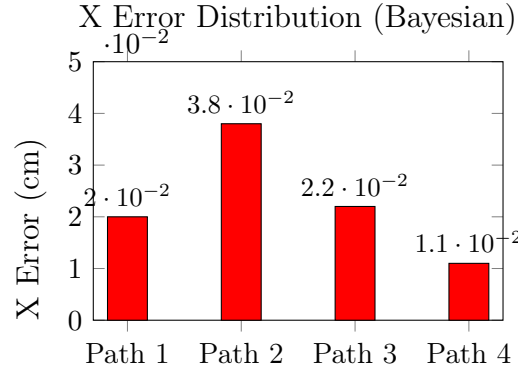
[Algorithm 3](#) was evaluated for various path scenarios to assess its reliability and performance. Four representative paths were selected as examples for the

thesis study. The blue dashed line in the images indicates the trajectory followed by the robot. A colour map was used to represent the radiation dose rate, where yellow regions correspond to higher dose rates and blue regions to lower ones. The yellow areas appear more intense near the radiation source location, indicating that the radiation dose rate increases around the source. The orange dot represents the estimated source location, while the red star marks the actual source position.

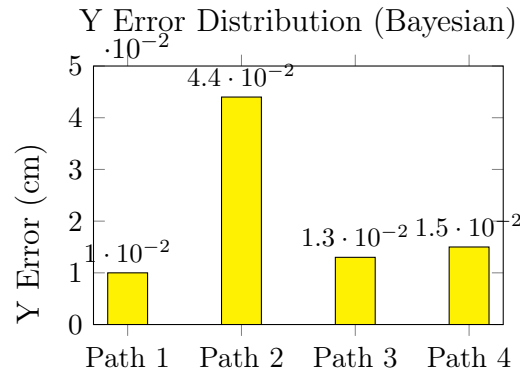
Table 5.5: Error Analysis Table with Absolute and Total Errors for Bayesian Method (m)

Path	Real X	Real Y	Est. X	Est. Y	X Er- ror (m)	Y Er- ror (m)	Total Error (m)
Path 1	1.49	0.68	1.510	0.690	0.020	0.010	0.022
Path 2	1.49	0.68	1.528	0.724	0.038	0.044	0.058
Path 3	1.49	0.68	1.512	0.693	0.022	0.013	0.026
Path 4	0.62	0.92	0.631	0.935	0.011	0.015	0.019

Table 5.5 shows the differences between the actual and estimated source positions for four different paths. Path 1 and Path 4 have the lowest total position errors, while Path 2 shows the highest value, similar to the MLE method. For Path 1, the total error is **2.2 cm**, indicating that the Bayesian algorithm provides a highly accurate estimation. Path 2, on the other hand, has a total error of **5.8 cm**, which means the predicted position deviates more from the true source location. This difference can be attributed to the smaller number of measurements collected by the robot along Path 2, which reduces the estimation reliability. As shown in the table, the measurement intensity directly influences the accuracy of the predicted source position.



(a) X Error Distribution (Bayesian, in meters)



(b) Y Error Distribution (Bayesian, in meters)

Figure 5.19: Comparison of X and Y error distributions in Bayesian localisation. (a) X error distribution in red, (b) Y error distribution in yellow. All errors are expressed in santimeters (cm).

Figure 5.19 illustrates the error distributions in X and Y coordinates. The errors vary in a smaller range for the X coordinates. In Path 1, the error in the X coordinate is 2 cm, while it is 1 cm for the Y coordinate. In Path 2, the error in the X coordinate is 3.8 cm, and the error in the Y coordinate is 4.4 cm. These results indicate that the Bayesian approach is more stable for the X-axis.



Figure 5.20: Real-world experiment showing the robot reaching the estimated source location

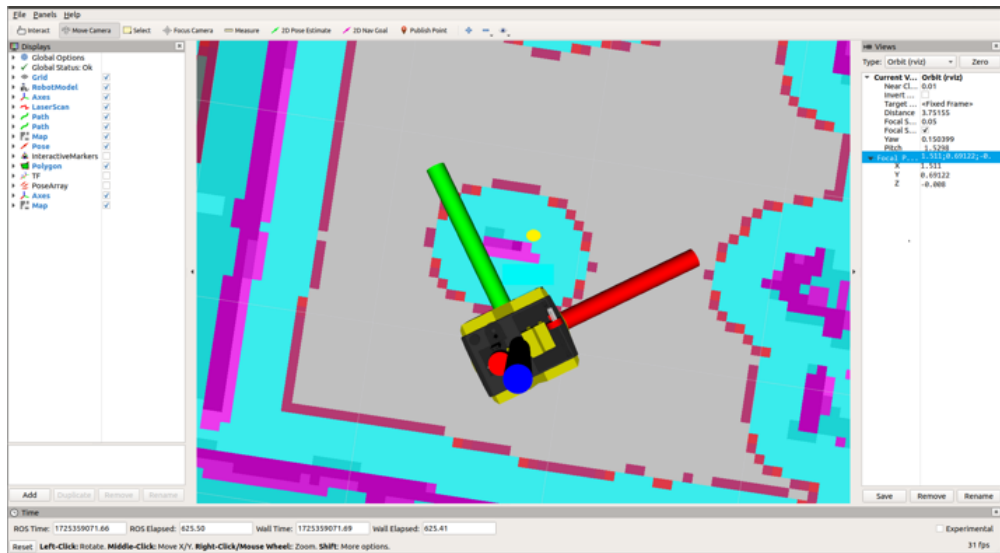


Figure 5.21: Visualization of the estimated source location using the Bayesian approach in the ROS - Rviz environment.

[Figure 5.20](#) shows the real-world view of the region where the robot has reached the estimated source location, while [Figure 5.21](#) presents the environmental map, the actual source position, and a simulation conducted in the ROS environment using Rviz. When the robot follows Path 1 and collects the data, the estimated source location determined by the robot is the region indicated by the yellow dot. The actual location of the source is in the pink region, and it can be seen that the actual location is very close to the estimated source location. The difference between the estimated source position and the actual source position was only 1.41 cm in the measurements made on the slam-based map. This extremely small, almost negligible margin of error indicates that the positioning and mapping process is highly accurate.

The Bayesian approach is a model that generates posterior distributions by considering uncertainties and using prior information. This model generates posterior distributions based on prior information that are updated in response to new incoming data. This results in a better fit to the available data and more reliable forecasts with lower error rates over time. One of the most important advantages of this model is that it minimises measurement noise and reduces errors by reducing the effect of sensor errors. Since measurement errors are handled directly within the probabilistic framework, a single measurement error cannot significantly affect the result of the model. Including prior information in the modelling process and its refinement with initial estimates allows us to achieve very low error rates. The effectiveness of this approach is compared with other estimated methods in [Section 5.4](#) and is examined in detail.

5.2.3 Triangulation Method

Triangulation is a classical method for determining the position of an unknown point based on the principles of geometric intersection. In the two-dimensional plane, the position of a source is expressed in (x,y) coordinates, while the distance of the source to any sensor i is described by d_i and is represented by the Euclidean distance formula.

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (5.27)$$

Where these distances can be measured directly, circles with a radius of d_i centred on each sensor can be drawn, and the intersection of the circles gives the true position of the source. This method is known as triangulation. However, direct distance measurement is usually impossible in physical systems. Since gamma rays are composed of high-energy photons, they travel freely through the air and follow an almost linear path unless they encounter an obstacle. The attenuation during propagation is due to the geometric decrease with distance. For rays emitted from a point source, the intensity per unit area decreases inversely with the square of the distance according to the inverse square law $D \propto \frac{1}{r^2}$. This means that the measured radiation rate drops to one-fourth at twice the distance from the source. In open-air conditions, the effect of air on gamma rays can be negligible for short distances, but absorption and scattering must be taken into account for long distances. In addition, the existing background radiation may also affect the gamma-ray measurements and should be modelled or subtracted. The dose rate received by the detector should increase as the mobile robot approaches the radioactive source and decrease as it moves away. This results in one or more pairs

of independent points. When the number of gamma photons or dose rates detected by the detector at different locations are equal, these points are called Equal Count Points (ECPs). Geometrically, these equal or nearly equal points are used to form a double-edged triangle. The location of the radioactive source should be along an axis perpendicular to the base of this triangle. When more than one pair of ECPs is detected, the intersection point of the perpendicular lines formed by these pairs indicates the location of the source. This method provides a practical and logical solution for determining the location of radioactive sources through geometric and statistical analysis. The radioactive decay process is random, so the measurements made with the detector are not exact values but show fluctuations over time in accordance with Poisson statistics. When the mobile robot is in motion, small random fluctuations can be observed in the dose values recorded consecutively by the detector instead of changing smoothly. In the same way, background radiation can also cause these fluctuations. These fluctuations create uncertainty in the total measurement and make it difficult to determine the source location. It is necessary to filter the raw data to remove noise and uncertainties using statistical methods to correct this situation. The purpose of the triangulation method is to determine the intersection of the location of the source with the information obtained from the measurement points. If at least two ECP pairs are available, their intersection can be utilised to determine the position of the source. In cases of inaccurate estimation, the data can be analysed and the ECP pair responsible for the error can be excluded from the evaluation. In summary, the principle of twin-edge triangulation is the basic rule of geometric extraction from ECP pairs [15],[74],[75]. When measuring with the mobile robot to determine the location of the radioactive source, more than one pair of equal dose rates can be identified.

As shown in [Figure 5.22](#), the robot follows a symmetrical elliptical path, and some points with an equal dose ratio are detected. The measurement data are equally distributed according to the symmetry axis. This ensures that the perpendicular lines of the measurement points intersect at a single point, and the determination of the source position becomes solvable. However, if the robot follows a symmetric path, the matrix conditionals may increase, leading to numerical instability. The hill-climbing and simulated annealing optimisation algorithms achieve faster convergence when symmetric data is used.

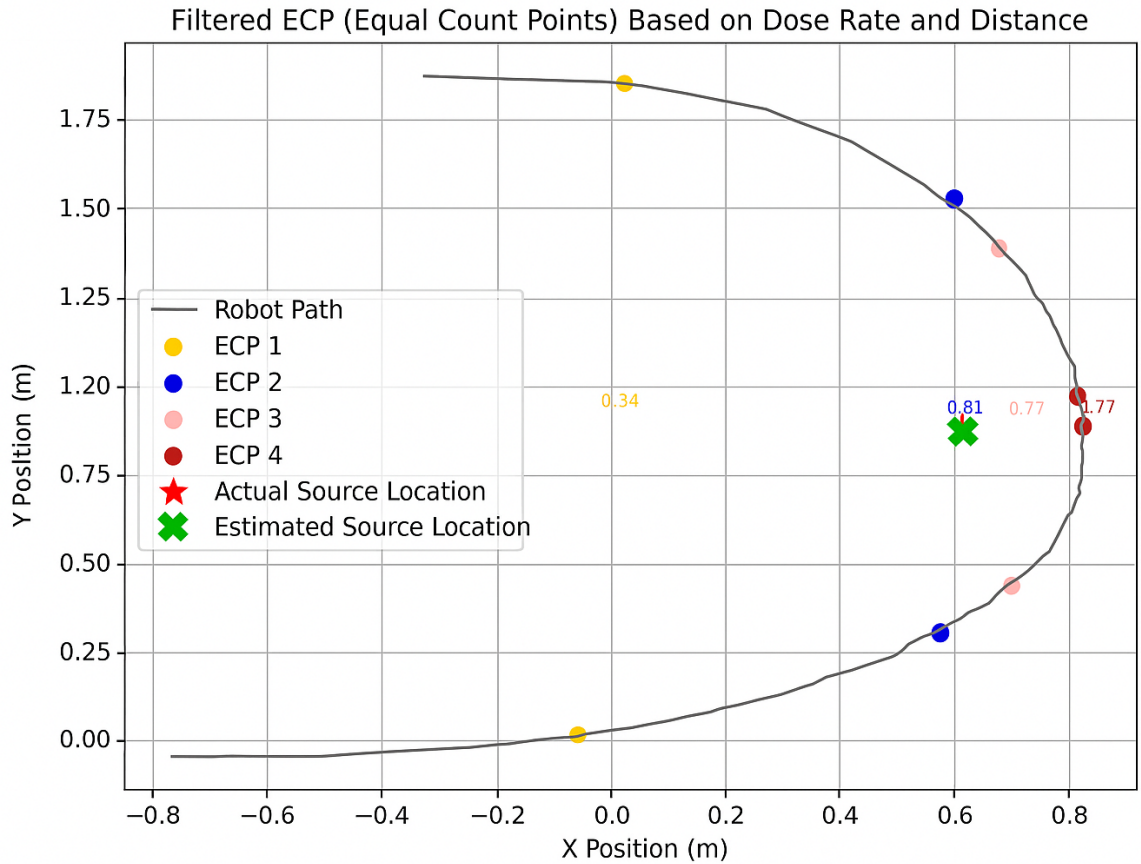


Figure 5.22: Filtered Equal Count Points (ECP) based on dose rate and distance. The robot follows a nearly symmetric path (grey line), representing different ECP groups with distinct colours. The source location is marked with a red star, while a green 'X' indicates the estimated source location'. The symmetry of the robot path allows for a more balanced distribution of measurement points, aiding in source localisation. Numerical values next to some ECP points represent dose rate ($\mu\text{Sv/h}$) measurements.

When the mobile robot follows an asymmetric path, as shown in [Figure 5.23](#), the measurement points may show a linear relationship on the vertical axis, making it challenging to locate the radiation source with a single solution.

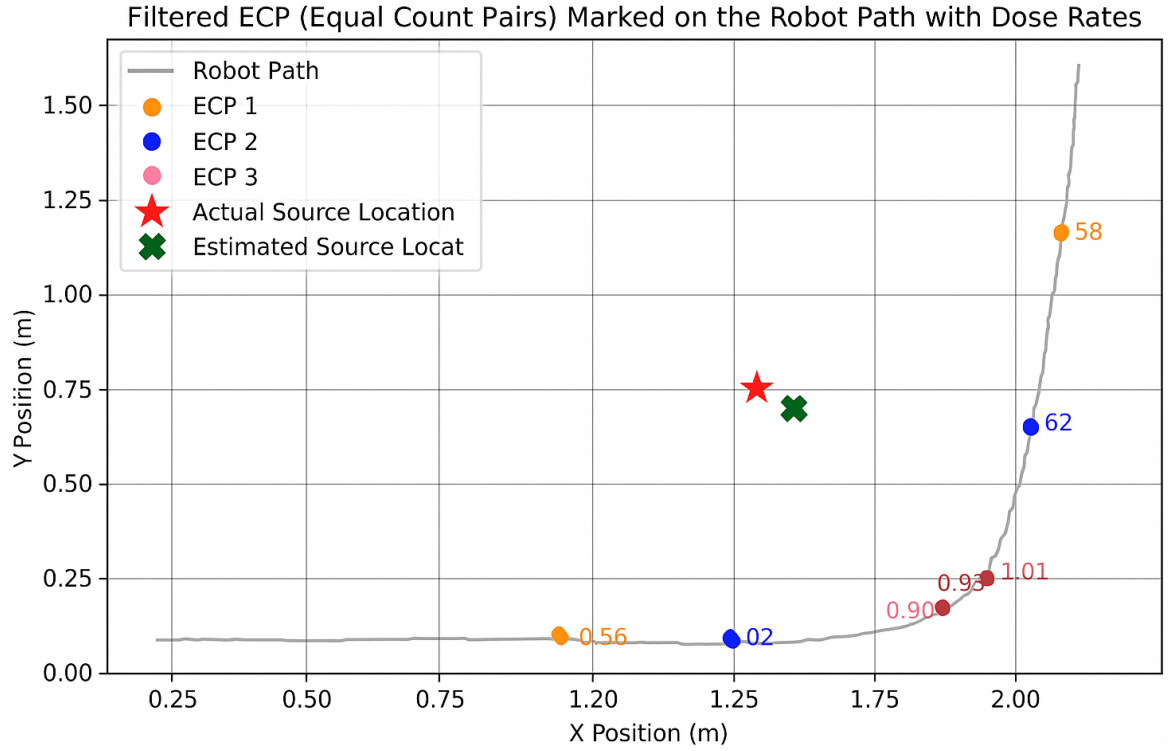


Figure 5.23: : Equal Counting Points (ECPs) are shown along the path the robot moves, determined and filtered according to the measured dose rates. In contrast to the previously used symmetric route, the path followed by the robot is asymmetric (grey line), leading to an irregular positioning of the measurement points. The different clusters of ECPs are shown in various colours to distinguish them. A red star indicates the actual location of the radiation source and the estimated source location by a green “X”. The numbers next to the ECP points represent dose rates ($\mu\text{Sv/h}$), and the differences in the data distribution from the symmetric distribution are shown.

The resulting ECP pairs help to find the radiation source position (x_0, y_0) . The dose rate of the source at point i is modelled using the inverse square law and the background radiation (B).

$$D(p_i) = \frac{A}{(x_i - x_0)^2 + (y_i - y_0)^2} + B + \epsilon_i \quad (5.28)$$

Where A is the source strength, ϵ_i represents the measurement noise obeying Poisson statistics.

Here, the equal count points (ECP) are pairs of points satisfying the condition $D(p_i) = D(p_j)$, and the twin edge on an axis perpendicular to this source position forms the base of the triangle.

However, since the dose rate $D(p_i)$ measurements are subject to a Poisson statistical distribution, i.e., the measurement points are not uniformly distributed, determining the correct ECP points can be challenging. To solve this problem, the $D(p_i)$ values are assumed to follow a specific trend during the measurement process. For example, the condition $D(p_i) = D(p_j)$ is satisfied with an error margin of $\pm\sqrt{D(p_i)}$ due to noise.

$$|D(p_i) - D(p_j)| \leq \sqrt{D(p_i) + D(p_j)} \quad (5.29)$$

With this assumption, the errors caused by statistical fluctuations are minimised, and the accuracy of the dose rate measurements is improved. In this context, the $D(p_i)$ measurements are smoothed by applying an N-point circular mean filter to control the variability of the data in the energy spectrum.

$$D'_{\text{filter}}(p_i) = \frac{1}{2N+1} \sum_{k=i-N}^{i+N} D(p_k) \quad (5.30)$$

The main objective here is to maximise the accuracy of the estimated radioactive source location by finding ECP pairs and reducing the Poisson variance by $\frac{1}{2N+1}$.

The main objective here is to maximise the accuracy of the estimated ra-

radioactive source location by finding ECP pairs and reducing the Poisson variance by $\frac{1}{2N+1}$. The radiation detector is limited to detecting low dose rates beyond a certain distance. Most dose rates are measured at points outside the detection zone of the detector ($r_i; \max$) approach the background radiation level (B). Since these points do not contain meaningful data, they are not used in ECP analysis [20],[24]. The following criteria determine the determination of points outside the detection region:

$$D'_{filter}(p_i) \approx B \quad (5.31)$$

The radiation zone threshold (D_{th}) is defined as follows:

$$D_{th} = D_{min} + \frac{D_{max} - D_{min}}{3} \quad (5.32)$$

Where

D_{max} : Maximum value of the filtered dose rate. D_{min} : Minimum value of the filtered dose rate. In this formula, the dose rate distribution is divided into three different regions and divided into 3 in order to select an intermediate threshold value. It is possible to base this threshold value on a statistical approach and use a standard deviation method.

$$D_{th} = B + k\sigma_B, \quad (k = 2, 3, 4) \quad (5.33)$$

Here, σ_B is the standard deviation of the background radiation level. This method optimises the false positive and negative rates for more accurate results.

The parameter k is determined experimentally. The threshold value determines whether the source is within the field.

$D_{max} > 2D_{min} \Rightarrow$ The robot has entered the area where the source is detected.

However, this condition is valid when the radiation is homogeneously distributed. Normalisation may be required when the conditions are different:

$$D_{norm} = \frac{D - D_{min}}{D_{max} - D_{min}} \quad (5.34)$$

Secondly, by using points of equal dose rate, a more precise analysis can be performed to find the position of the radioactive source. Two points, $p_{a_j} = (x_{a_j}, y_{a_j})^T$ and $p_{b_j} = (x_{b_j}, y_{b_j})^T$, are considered ECP pairs if they satisfy the condition in [Formula 5.34](#).

μ_0 = ECP acceptance threshold

m = number of couples detected

If μ_0 is calculated by taking Poisson noise into account, it is expressed as follows:

$$\mu_0 = \sqrt{D'_{filter}(p_i)} \quad (5.35)$$

This reduces the statistical fluctuations caused by low dose rates.

In other words, there are two important steps to be followed in determining the ECP points. The first step is to exclude points outside the detected region from the analysis.

Points within the

$$D'_{filter}(p_i) < D_{th} \quad (5.36)$$

condition are considered background radiation and are ignored for ECP analysis.

Secondly, all points are ordered according to the decreasing value, considering the $D'_{filter}(p_i)$ value. The absolute difference (μ_j) between consecutive pairs of points is calculated as:

$$\mu_j = |D'_{filter}(p_j) - D'_{filter}(p_{j+1})| \quad (5.37)$$

,and then $\mu_j < \mu_0$ is determined as ECP with the pair that fulfils this condition [20],[76].

Two optimisation algorithms have been used for the triangulation method. The first is the **Hill-Climbing algorithm**, one of the preferred algorithms for deterministic local optimisation to find a radiation source location [15],[77]. The basic principle of the algorithm is to find a better solution by iteratively evaluating neighbouring points from a given starting position. The starting point is

determined by the ECP method. Then, new candidate solutions are generated at points close to the selected region and compared with the existing solution. The algorithm is evaluated in a particular iteration and halted when there is no improvement between successive iterations. The fast convergence of the algorithm is an advantage, while the tendency to diverge to local optimum solutions instead of global optimum is a disadvantage. The stability of the algorithm is enhanced by employing a filter with threshold values to reduce the impact of Poisson noise, especially in areas with a low dose rate.

The study has two necessary steps to determine the position of the radiation source using this method. Firstly, when the dose rate difference between the measurements recorded by the robot exceeds a particular threshold value, the average of the measurement points is considered the estimated initial position of the radiation source. After this step, the hill-climbing algorithm improves the initial estimate. The algorithm determines the new point that minimises the dose rate difference by creating neighbouring points around the initial position. This process is continued iteratively until the position estimate is constant. The steps of the method are described in [Algorithm 4](#) below.

Algorithm 4 Radiation Source Localisation using Triangulation and Hill-Climbing

1: **Input:** Measurement data (X, Y, D) , actual source S_{real} , threshold range T_{range}
 2: **Output:** Estimated source location S_{final} and coordinate errors $(\Delta x, \Delta y)$ (in cm)
 3: **procedure** FINDECPPAIRS(X, Y, D, T)
 4: Initialize empty list for ECP pairs
 5: **for** $i = 1$ to $N - 1$ **do**
 6: **if** $|D_i - D_{i+1}| > T$ **then**
 7: Compute midpoint:

$$(X_m, Y_m) \leftarrow \left(\frac{X_i + X_{i+1}}{2}, \frac{Y_i + Y_{i+1}}{2} \right)$$

8: Add (X_m, Y_m) to ECP pairs
 9: **end if**
 10: **end for**
 11: **return** ECP pairs
 12: **end procedure**
 13: **procedure** TRIANGULATE(ECP pairs)
 14: Compute initial estimate:

$$S_{\text{init}} \leftarrow \frac{1}{M} \sum_{j=1}^M (X_{m,j}, Y_{m,j})$$

15: **return** S_{init}
 16: **end procedure**
 17: **procedure** HILLCLIMBING(S_{init}, X, Y, D)
 18: Set current best: $S_{\text{best}} \leftarrow S_{\text{init}}$
 19: **for** iteration = 1 to MaxIterations **do**
 20: Generate neighbor points:

$$S_{\text{neighbor}} \leftarrow S_{\text{best}} + (\Delta x, \Delta y), \quad \Delta x, \Delta y \in \{-s, 0, s\}$$

21: Interpolate dose rate at neighbors to obtain D_{interp}
 22: Compute total error:

$$\text{Error} = \sum_{i=1}^N |D_i - D_{\text{interp},i}|$$

23: **if** no improvement is observed **then**
 24: **break** the loop
 25: **else**
 26: Update $S_{\text{best}} \leftarrow S_{\text{neighbor}}$
 27: **end if**
 28: **end for**
 29: **return** S_{best}
 30: **end procedure**

(continued)

```

1: procedure OPTIMISETHRESHOLD( $X, Y, D, S_{\text{real}}, T_{\text{range}}$ )
2:   Initialize best threshold  $T_{\text{best}}$  and lowest error  $E_{\text{lowest}} \leftarrow \infty$ 
3:   for each  $T$  in  $T_{\text{range}}$  do
4:      $\mathcal{E} \leftarrow \text{FINDECPPAIRS}(X, Y, D, T)$ 
5:     if  $\mathcal{E}$  is empty then
6:       continue
7:     end if
8:      $S_{\text{init}} \leftarrow \text{TRIANGULATE}(\mathcal{E})$ 
9:      $S_{\text{est}} \leftarrow \text{HILLCLIMBING}(S_{\text{init}}, X, Y, D)$ 
10:    Compute error:

```

$$E = \sqrt{(S_{\text{est},x} - S_{\text{real},x})^2 + (S_{\text{est},y} - S_{\text{real},y})^2}$$

```

11:    if  $E < E_{\text{lowest}}$  then
12:      Update  $E_{\text{lowest}} \leftarrow E$ , and  $T_{\text{best}} \leftarrow T$ 
13:    end if
14:  end for
15:  return ( $T_{\text{best}}, E_{\text{lowest}}$ )
16: end procedure
17: procedure MAIN
18:   Load data ( $X, Y, D$ )
19:   Set actual source  $S_{\text{real}}$  and define threshold range  $T_{\text{range}}$ 
20:   ( $T_{\text{best}}, E_{\text{lowest}}$ )  $\leftarrow \text{OPTIMISETHRESHOLD}(X, Y, D, S_{\text{real}}, T_{\text{range}})$ 
21:   Obtain ECP pairs:

```

$$\text{ECP pairs} \leftarrow \text{FINDECPPAIRS}(X, Y, D, T_{\text{best}})$$

```

22:   Compute initial estimate:

```

$$S_{\text{init}} \leftarrow \text{TRIANGULATE}(\text{ECP pairs})$$

```

23:   Determine final source location:

```

$$S_{\text{final}} \leftarrow \text{HILLCLIMBING}(S_{\text{init}}, X, Y, D)$$

```

24:   Calculate coordinate errors (in cm):

```

$$(\Delta x, \Delta y) \leftarrow 100 \times (S_{\text{final}} - S_{\text{real}})$$

```

25:   Visualize results (scatter plots, interpolation grids, etc.)
26: end procedure
27: Run: MAIN

```

The graph in Figure 5.24 shows the source location estimation optimised by triangulation method and hill-climbing algorithm at different threshold values.

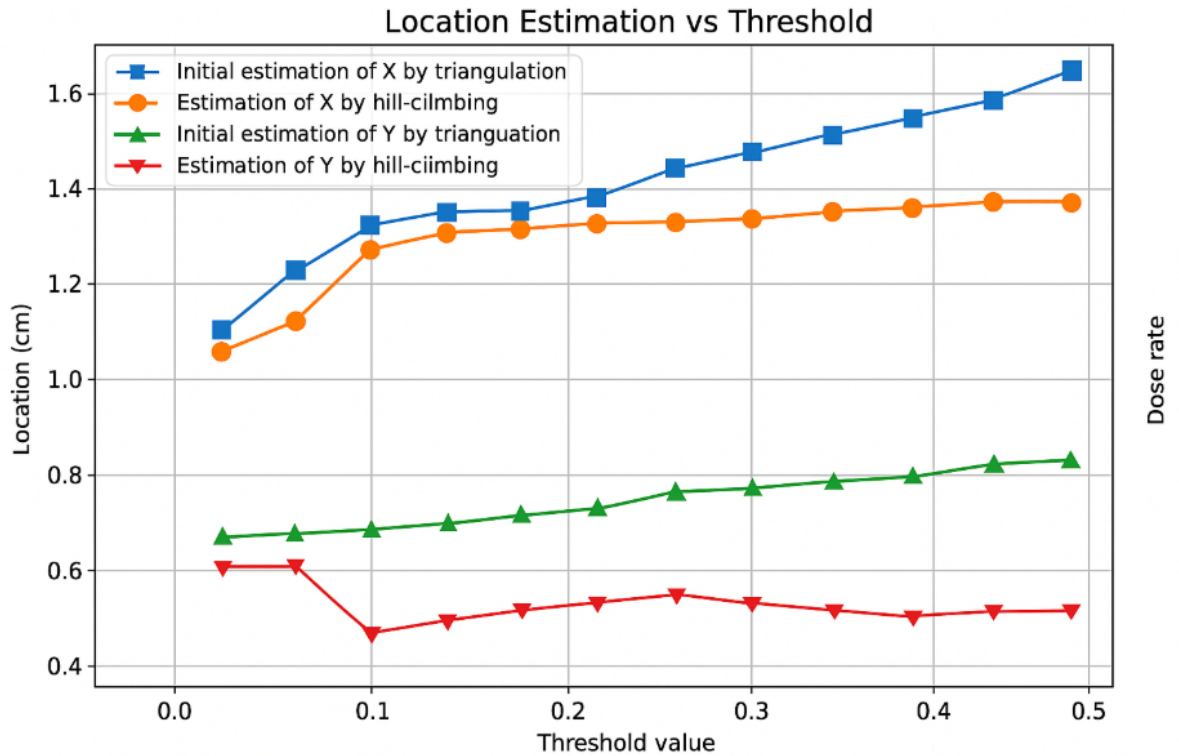


Figure 5.24: Results in X and Y axes depending on the threshold value of two methods (triangulation and hill-climbing).

It is observed that the estimates obtained with the triangulation method move away from the different actual source location points as the threshold value increases. This method is sensitive to threshold values and shows large deviations as the values increase. An increase is observed as the threshold values increase, especially in the X coordinate. At higher threshold values, ECP points are selected from points where the dose rates change less and the dose ratios change significantly, thus estimating the source location further away from the actual location.

In contrast, it is seen in [Figure 5.24](#) that the performance of the hill-climbing algorithm against threshold values is better and more stable. Despite the threshold change in the X coordinate, the estimates did not show a significant deviation and remained constant. Although local fluctuations were observed in the Y coordinate, the iterative improvement capacity of the algorithm in general performance was demonstrated. These results show that the hill-climbing method improved and stabilised the triangulation results. The optimal result was achieved when the threshold value was between 0.2 and 0.3. The triangulation and hill-climbing method was applied to find the location of the radiation source on different road routes. The results obtained for the first path are shown in [Figure 5.25](#).

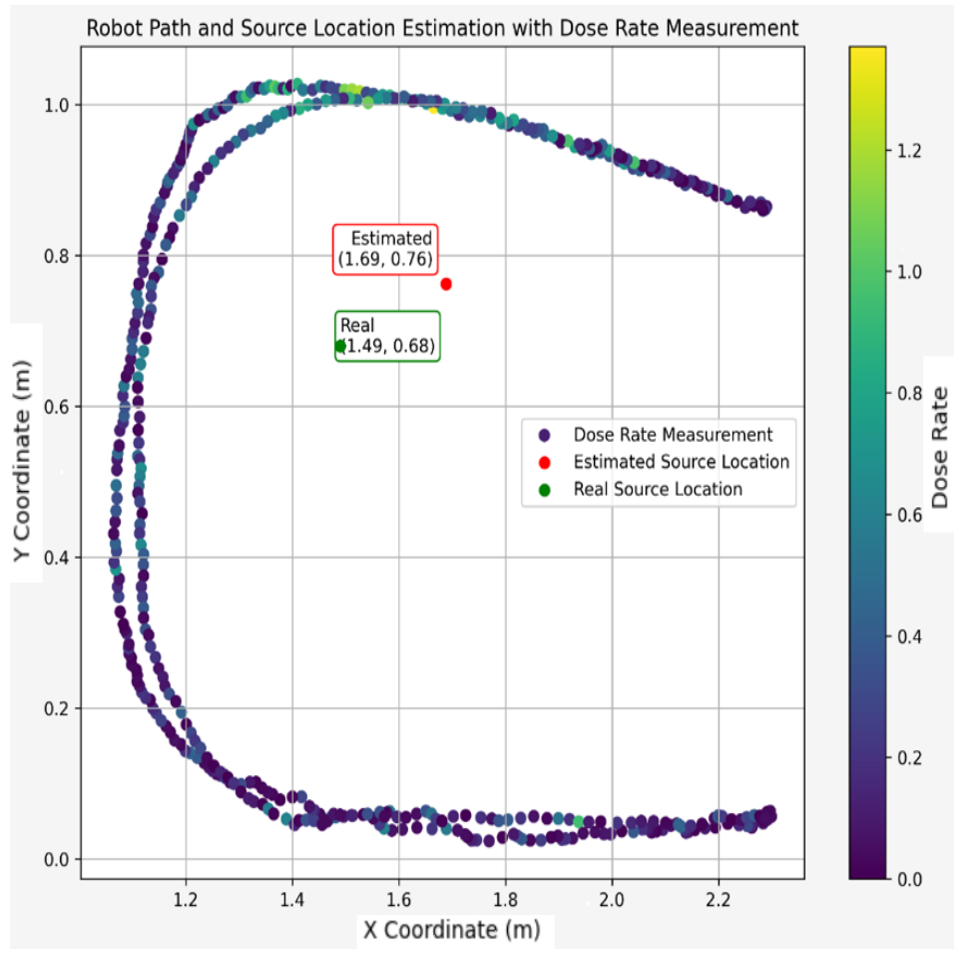
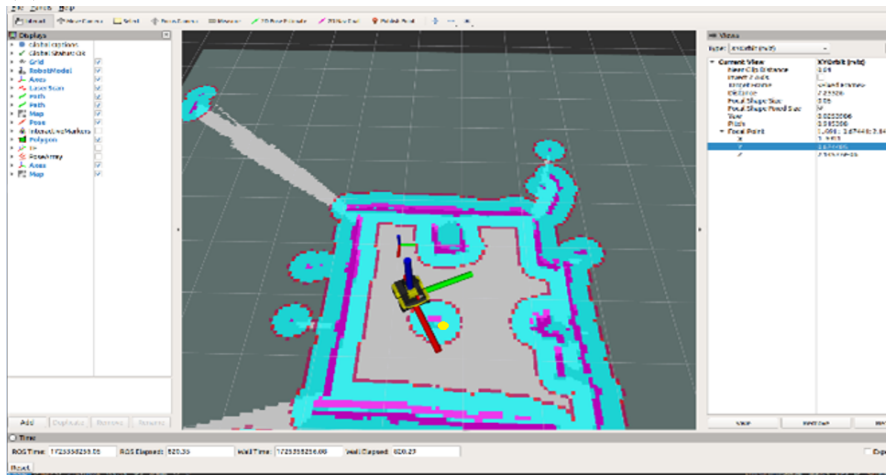


Figure 5.25: Estimated location found by Triangulation+Hill Climbing for Path 1. The dose rates measured by the robot along the semicircular path and the estimated source location are (1.69, 0.76).

In this analysed [Figure 5.25](#), the estimated source position (1.69,0.76) was found, and a significant difference was detected between the actual position. When looking at the measurement data, it is seen that the robot follows a semicircular pattern and is located between 0.9-2.2 on the X-axis and 0-2 on the Y-axis.



(a) Actual view of the robot at the estimated source location.



(b) View of the ROS/RViz interface and terminal output at the estimated source location.

Figure 5.26: Robot-Assisted Radiation source Location Detection by Combining Hill-Climbing and Triangulation Methods.

When the dose rates are examined, it is observed that the measurement of the robot near the source is limited. The predicted position was approximately 20 cm to the right and 8 cm down. The reason for this difference is that the hill-climbing algorithm could not adequately evaluate the data in the region because the robot could not measure the source close to it.

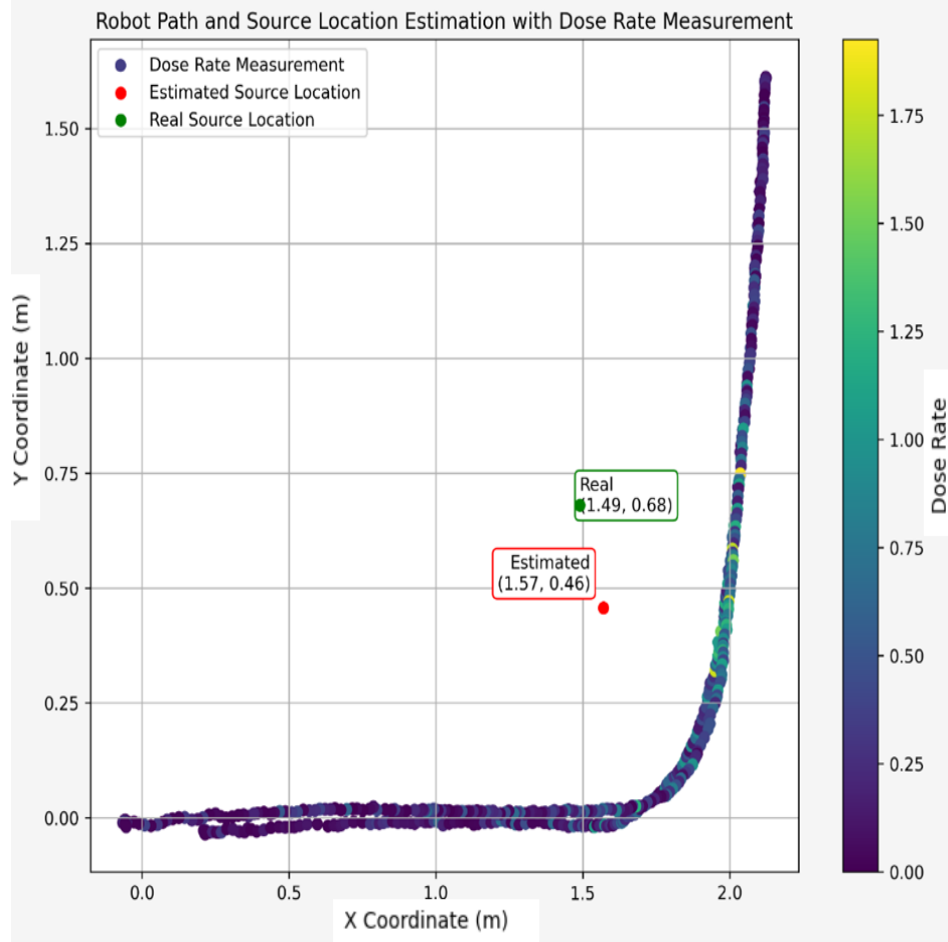


Figure 5.27: Estimated location found by Triangulation+Hill Climbing for Path 2.

Path 2 (Figure 5.27) shows an estimated source position of (1.57,0.46). As in Path 1, close error rates are found. There is a deviation of about 22 cm on

the y-axis. This is because hill-climbing is stuck at the local minimum. After all, there is not enough data on the y-axis. The asymmetric distribution of the data and the restriction of meaningful ECP pairs to regions with low dose rates caused the triangulation method to select the initial estimate incorrectly, and the hill-climbing algorithm does not have enough gradient information to correct this.

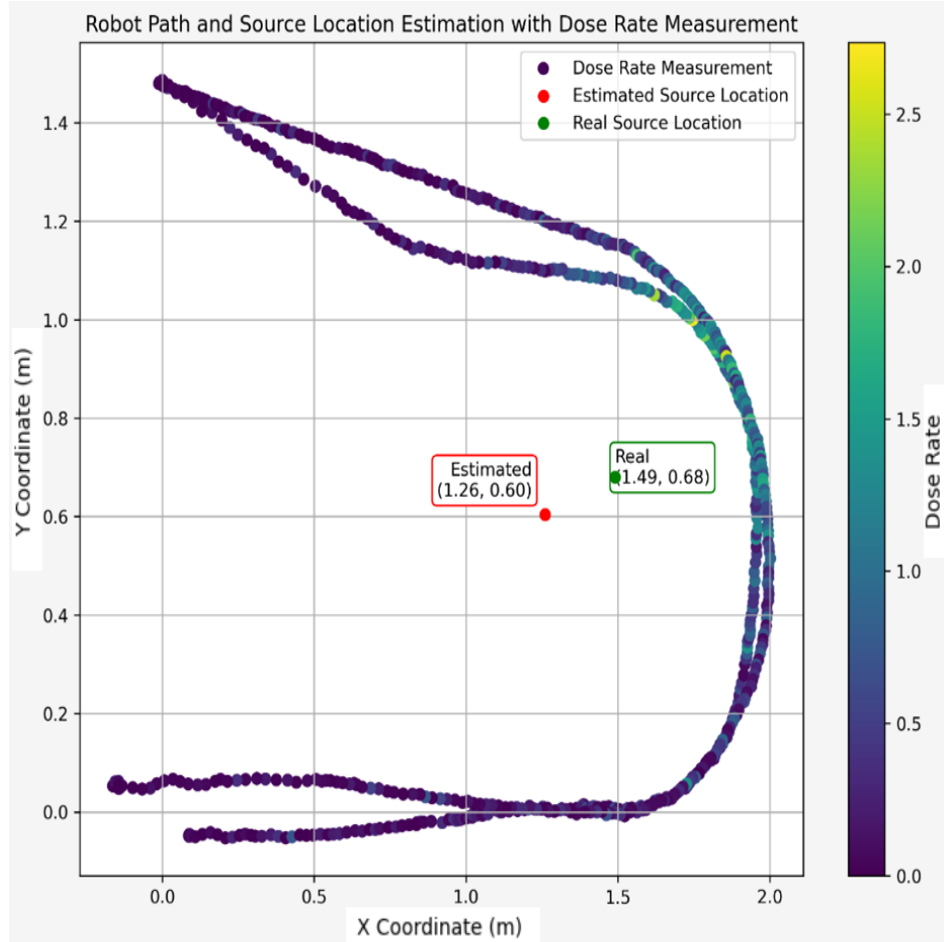


Figure 5.28: Estimated location found by Triangulation+Hill Climbing for Path 3.

As shown in Figure 5.28, the estimated position is $x=1.26$ $y=0.60$. As in the other two paths, the difference between the actual and estimated positions is

significant. Figure 5.28 shows that the ECP pairs are concentrated on the left side due to the high threshold range chosen (Figure 5.24), which leads to this estimated position. As discussed by Knoll [20], a high threshold reduces Poisson noise but leads to data loss.

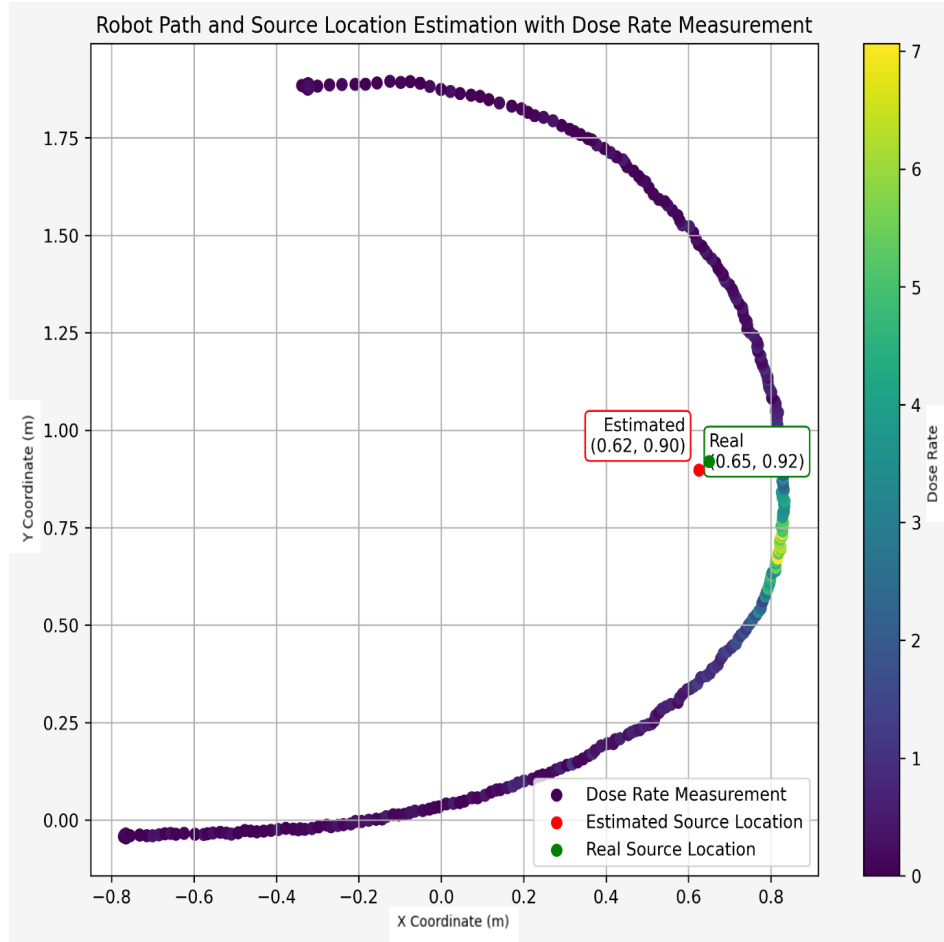


Figure 5.29: Estimated location found by Triangulation+Hill Climbing for Path 4.

Finally, the estimated source position for Path 4 is (0.62, 0.90), while the actual source position is (0.65, 0.92). There is a difference of approximately 3 cm on the x-axis and 2 cm on the y-axis, and a very close result was found. When

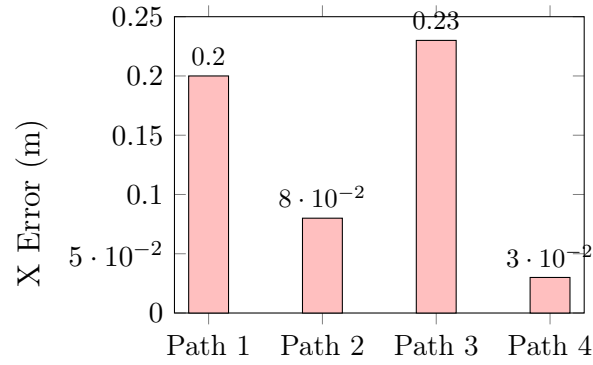
the robot receives enough data around the source, it is seen that the hill-climbing algorithm gives a good result by successfully evaluating the data. Working with high dose values leads to correctly determining the ECP citations and an estimated source close to the actual source without getting stuck in the local minimum. As Groves [78] argues, the path of the robot and the measurement points are critical. When the measurements are multidirectional and linearly independent, the Jacobian matrix reduces the number of conditions, making it more stable against source position estimation and measurement errors. When the robot moves in one direction, the measurement points may become collinear; in this case, the condition number of the Jacobian matrix increases. In this case, the initial estimate improved by the Hill-Climbing algorithm may get stuck in the local minimum. However, when the robot follows a wide arc path around the source, the measurement points are distributed linearly and independently. Thus, the number of conditions of the Jacobian matrix decreases, and the hill-climbing algorithm tends to converge to the global optimum. This study experimentally verifies the effect of measurement geometry on system performance in the navigation systems of Groves [78] in the context of radiation source localisation. The data shows that using the Triangulation+Hill Climbing method instead of Triangulation alone for radiation source location estimation provides some advantages, but the errors are still high for some paths ([Table 5.6](#)).

Table 5.6: Error Analysis Table with Absolute and Total Errors for Triangulation + Hill-Climbing Method (m)

Path	Real X	Real Y	Est. X	Est. Y	X Er- ror (m)	Y Er- ror (m)	Total Error (m)
Path 1	1.49	0.68	1.69	0.76	0.20	0.08	0.215
Path 2	1.49	0.68	1.57	0.46	0.08	0.22	0.234
Path 3	1.49	0.68	1.26	0.60	0.23	0.08	0.243
Path 4	0.65	0.92	0.62	0.90	0.03	0.02	0.036

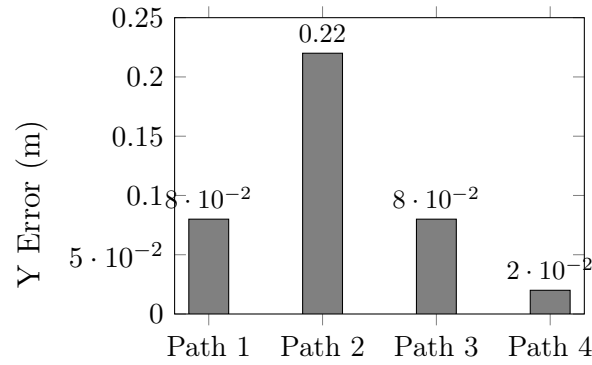
Figure 5.30 analyses the absolute errors in X and Y between the estimated and actual positions. Path 1 shows a significant error in the X direction (20 cm), while the error in the Y direction is lower (8 cm). Path 2 shows that the Y error value (22 cm) is higher than the others. In Path 3, the error on the X-axis (23 cm) indicates a large deviation. In Path 4, the X and Y errors are very low compared to the other paths, and the localisation is relatively good.

X Error Distribution (Triangulation + Hill-Climbing)



(a) X Error Distribution

Y Error Distribution (Triangulation + Hill-Climbing)



(b) Y Error Distribution

Figure 5.30: Comparison of X and Y error distributions in Triangulation + Hill-Climbing localisation.

Simulated Annealing was used in the following study stage to avoid local minimal points.

The Simulated Annealing (SA) algorithm is a meta-heuristic optimisation method developed by considering the annealing process of a metal. When a material is heated at a high temperature, followed by controlled cooling, the energy of the crystal structure is minimised. Mathematical modelling in this process aims

to reach a global minimum over the energy function. The formula

$$p = \exp\left(-\frac{\Delta E}{T}\right) \quad (5.38)$$

provides the acceptance of candidate solutions. This formula avoids local minima by accepting solutions with bad candidate (high energy) solutions with a certain probability. Thanks to this property, non-convex problems, unlike deterministic problems, are less affected by initial conditions and converge to the global minimum in the long run. The flexible structure of SA is very advantageous for radiation source localisation, especially when the measurement data is noisy and asymmetric. In addition, the triangulation (ECP method) used in the study for the initial estimates improves the quality of the optimisation result by a more efficient search of the solution space [79].

In radiation source localisation problems, the SA algorithm aims to minimise the error between calculated dose rates and model calculations and is formulated as follows:

$$E(x) = \sum_{i=1}^N (D_{\text{meas},i} - D_{\text{model},i}(x))^2. \quad (5.39)$$

Here, the vector $\mathbf{x} = (x, y)$ represents the source position, while $D_{\text{meas},i}$ and $D_{\text{model},i}(\mathbf{x})$ refer to the experimental measurements and model-based calculations, respectively.

When generating a new solution from the existing one, a new candidate

solution is obtained by adding a random variation $\Delta \mathbf{x} \sim \mathcal{N}(0, \sigma^2)$ following from a normal distribution:

$$\mathbf{x}_{\text{new}} = \mathbf{x}_{\text{current}} + \Delta \mathbf{x}. \quad (5.40)$$

For the acceptance of a new candidate solution, the Metropolis Acceptance criterion is activated, and the energy difference is calculated in the criterion:

$$\Delta E = E(\mathbf{x}_{\text{new}}) - E(\mathbf{x}_{\text{current}}), \quad (5.41)$$

If $\Delta E \leq 0$, that is, if the candidate solution is better than the current solution, the solution is definitely accepted; otherwise, the acceptance probability is determined by the formula below.

$$P(\Delta E, T) = \exp\left(-\frac{\Delta E}{T}\right). \quad (5.42)$$

T here refers to the temperature parameter, but this is not an actual physical temperature but an artificial parameter that regulates the rate at which the algorithm accepts terrible solutions in the solution space. It allows the algorithm to search for ample solution space at the beginning and to concentrate on local improvements during the cooling process. Exponential ($T_k = T_0 * \alpha^k$) or logarithmic ($T_k = \frac{T_0}{\ln(k+1)}$) cooling methods gradually reduce the temperature parameter T . Theoretically, the Geman-Geman ([80]) approximation guarantees convergence to the global optimum if the temperature is decreased logarithmically at a rate close to infinity: in practical applications, however, the guaranteed convergence is not always realised as faster cooling methods are used. In determining the location of radiation sources, the SA algorithm takes the initial estimate from the triangulation method as a reference. With the triangulation method, this initial point

x_0 is accepted as the starting point of SA. This method allows a more efficient search space exploration and a faster and safer approach to the optimum solution [79],[77]. The SA algorithm is particularly good at avoiding erroneous local optima due to Poisson noise, which is common at low dose levels. Primarily, when the measurements are asymmetrically distributed and are prone to deviate from the local minimum, SA reduces the risk of getting stuck in the local minimum by accepting bad solutions with a certain probability in the high-temperature period. This method is as in [Algorithm 5](#).

Algorithm 5 Radiation Source Localisation using Triangulation and Simulated Annealing

1: **Input:** Measurement dataset $\mathcal{D} = \{(X_i, Y_i, D_i)\}_{i=1}^N$, initial threshold T , initial SA parameters: maximum iterations $I_{\max}$, initial temperature T_0 , cooling rate α , step size s , actual source position S_{real}
 2: **Constants:** Threshold range $\{T_j\}$
 3: **procedure** FINDECPPAIRS($\mathcal{D}, T$)
 4: Initialize ECP list $\mathcal{E} \leftarrow \emptyset$
 5: **for** $i = 1$ to $N - 1$ **do**
 6: **if** $|D_i - D_{i+1}| > T$ **then**
 7: Compute midpoint:

$$(X_m, Y_m) \leftarrow \left(\frac{X_i + X_{i+1}}{2}, \frac{Y_i + Y_{i+1}}{2} \right)$$

8: Add (X_m, Y_m) to $\mathcal{E}$
 9: **end if**
 10: **end for**
 11: **return** $\mathcal{E}$
 12: **end procedure**
 13: **procedure** TRIANGULATE($\mathcal{E}$)
 14: Compute initial estimate:

$$S_{\text{init}} \leftarrow \frac{1}{|\mathcal{E}|} \sum_{(X_m, Y_m) \in \mathcal{E}} (X_m, Y_m)$$

15: **return** S_{init}
 16: **end procedure**
 17: **procedure** SIMULATEDANNEALING($S_{\text{init}}, \mathcal{D}, I_{\max}, T_0, \alpha, s$)
 18: $S \leftarrow S_{\text{init}}, T \leftarrow T_0, S_{\text{best}} \leftarrow S, E_{\text{best}} \leftarrow \infty$
 19: **for** iteration = 1 to $I_{\max}$ **do**
 20: Generate neighbor:

$$S_{\text{neighbor}} \leftarrow S + (\Delta x, \Delta y), \quad \Delta x, \Delta y \sim \text{Uniform}(-s, s)$$

21: Compute interpolated dose at S_{neighbor} :

$$D_{\text{interp}} \leftarrow \text{Interpolate}(\mathcal{D}, S_{\text{neighbor}})$$

22: Compute cost:

$$E \leftarrow \sum_{i=1}^N |D_i - D_{\text{interp}, i}|$$

23: **if** $E < E_{\text{best}}$ **or** $\exp\left(\frac{E_{\text{best}} - E}{T}\right) > \text{rand}()$ **then**
 24: $S_{\text{best}} \leftarrow S_{\text{neighbor}}, E_{\text{best}} \leftarrow E$
 25: **end if**
 26: Update temperature: $T \leftarrow T \times \alpha$
 27: Set $S \leftarrow S_{\text{best}}$
 28: **end for**
 29: **return** S_{best}
 30: **end procedure**

(continued)

```

1: procedure OPTIMISETHRESHOLD( $\mathcal{D}, \{T_j\}, S_{\text{real}}$ )
2:   Set  $T_{\text{best}} \leftarrow T_1, E_{\text{lowest}} \leftarrow \infty$ 
3:   for each threshold  $T_j \in \{T_j\}$  do
4:      $\mathcal{E} \leftarrow \text{FINDECPPAIRS}(\mathcal{D}, T_j)$ 
5:     if  $\mathcal{E}$  is empty then
6:       continue
7:     end if
8:      $S_{\text{init}} \leftarrow \text{TRIANGULATE}(\mathcal{E})$ 
9:      $S_{\text{est}} \leftarrow \text{SIMULATEDANNEALING}(S_{\text{init}}, \mathcal{D}, I_{\text{max}}, T_0, \alpha, s)$ 
10:    Compute error:

```

$$E = \sqrt{(S_{\text{est},x} - S_{\text{real},x})^2 + (S_{\text{est},y} - S_{\text{real},y})^2}$$

```

11:    if  $E < E_{\text{lowest}}$  then
12:       $E_{\text{lowest}} \leftarrow E, T_{\text{best}} \leftarrow T_j$ 
13:    end if
14:  end for
15:  return ( $T_{\text{best}}, E_{\text{lowest}}$ )
16: end procedure
17: procedure MAIN
18:   Load dataset  $\mathcal{D} = \{(X_i, Y_i, D_i)\}_{i=1}^N$  from CSV file
19:   Define threshold range  $\{T_j\}$  (e.g., 0.01, 0.02, ..., 0.99)
20:   Set actual source  $S_{\text{real}} \leftarrow (1.49, 0.68)$ 
21:    $(T_{\text{best}}, E_{\text{lowest}}) \leftarrow \text{OPTIMISETHRESHOLD}(\mathcal{D}, \{T_j\}, S_{\text{real}})$ 
22:    $\mathcal{E} \leftarrow \text{FINDECPPAIRS}(\mathcal{D}, T_{\text{best}})$ 
23:    $S_{\text{init}} \leftarrow \text{TRIANGULATE}(\mathcal{E})$ 
24:    $S_{\text{final}} \leftarrow \text{SIMULATEDANNEALING}(S_{\text{init}}, \mathcal{D}, I_{\text{max}}, T_0, \alpha, s)$ 
25:   Compute coordinate errors (in cm):

```

$$(\Delta x, \Delta y) \leftarrow 100 \times (S_{\text{final}} - S_{\text{real}})$$

```

26:   Visualize results (scatter plots, interpolation grids, etc.)
27: end procedure
28: Run: MAIN

```

When the difference between consecutive measurements exceeds a threshold value in dose measurements, these points are defined as ECP points:

$$|D_{i+1} - D_i| > \theta \Rightarrow \text{ECP points.}$$

In particular, high-dose rate transients are assumed to be close to the source.

The geometrically determined midpoints of the ECP points are used to generate a rough estimate of the source location. As in the Hill-Climbing algorithm, determining the appropriate threshold value is important for the quality of the initial value. Different threshold values are tried to minimise the Euclid distance between the actual source position and the estimated source:

$$\theta^* = \arg \min_{\theta} d(x_{\text{real}}, x_{\text{tahmin}}).$$

d is the distance between the positions. The threshold value with the lowest error determines the optimal parameter.

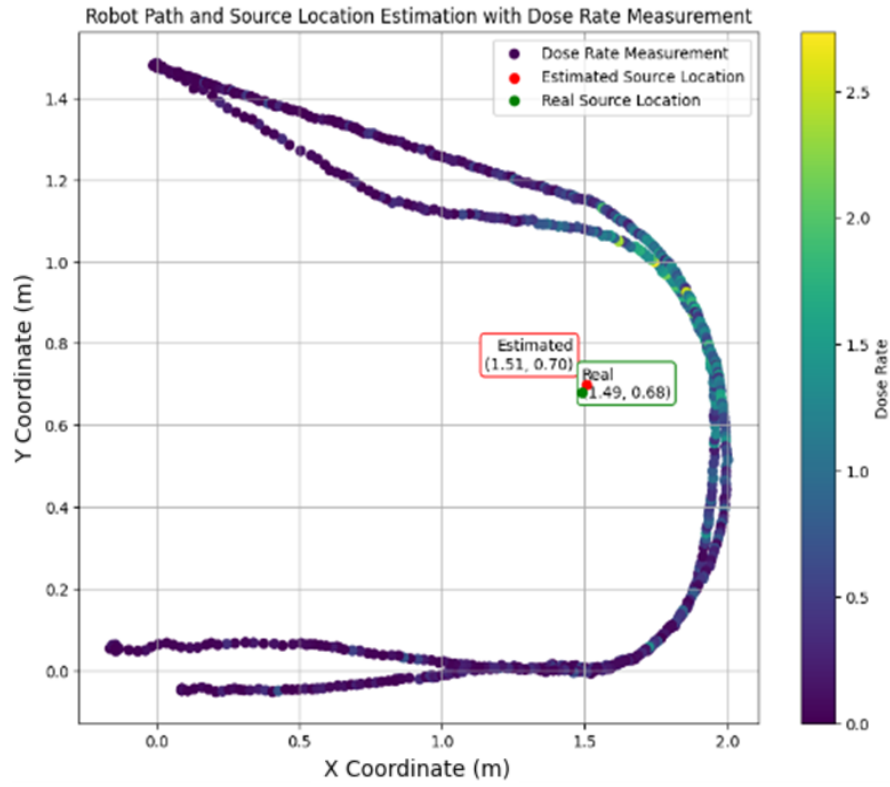
The SA algorithm optimises this initial estimate generated by triangulation. SA first performs an exhaustive search at a high initial temperature and then tries to find a better position by avoiding local minima in the solution space by lowering the temperature with a cooling programme. The `simulated_annealing()` function in the algorithm adds small random perturbations to the current position and calculates the probability of new acceptance according to the Metropolis criterion.

This mechanism is defined as:

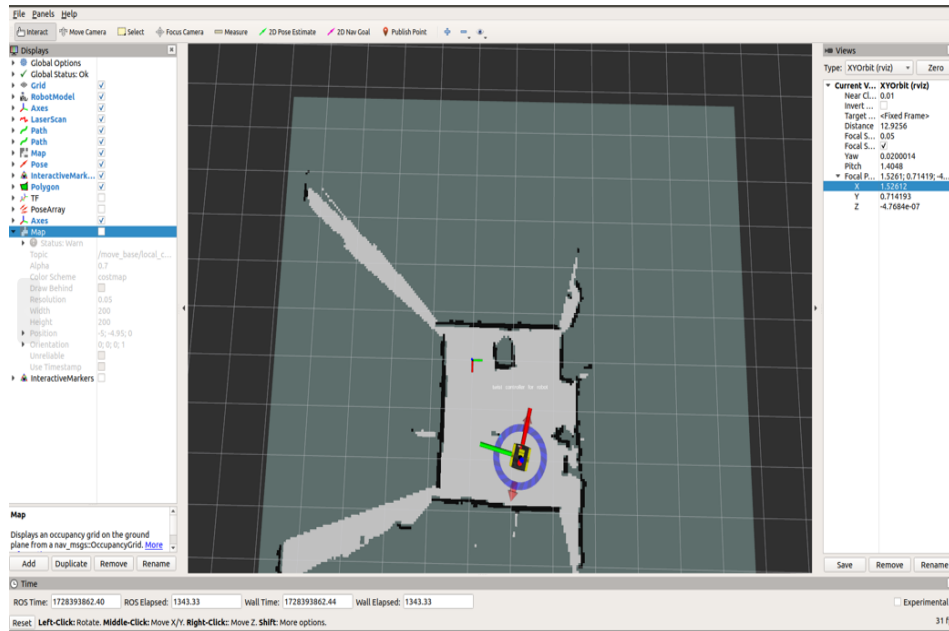
$$P(\Delta E, T) = \exp\left(-\frac{\Delta E}{T}\right).$$

Thus, the SA algorithm probabilistically filters out the statistical effects of Poisson noise and iteratively finds the location with lower error.

The initial estimates generated by triangulation were tested for different paths using the Simulated Annealing algorithm. Some of these paths are presented. In Path 1, the actual and estimated source positions are very close, as seen in [Figure 5.31](#). The difference of about 2 cm shows that the SA algorithm converges efficiently. The algorithm recognises the increase in the dose rate from the middle of the path to the right and moves around that area. The algorithm tests different locations in the solution space to get a better solution.



(a) Robot Path 1 and Source Location Predicted by SA Algorithm



(b) Motion control of the robot in the simulation environment, real-time navigation data and algorithm outputs in the ROS interface.

Figure 5.31: Robot-Assisted Radiation source Location Detection by Combining Simulated Annealing and Triangulation Methods.

The estimated position for Path 2 deviated from the actual position by about 10 cm [Figure 5.32](#). The main reason for this deviation is that the robot stayed in the low dose region ($D < 0.5 \mu\text{Sv/h}$) for a long time in the first part of the path, and the measurement geometry was asymmetric. This shows the role of the measurement geometry in the optimisation. The solution was stuck in a local minimum because the high dose rates were detected late.

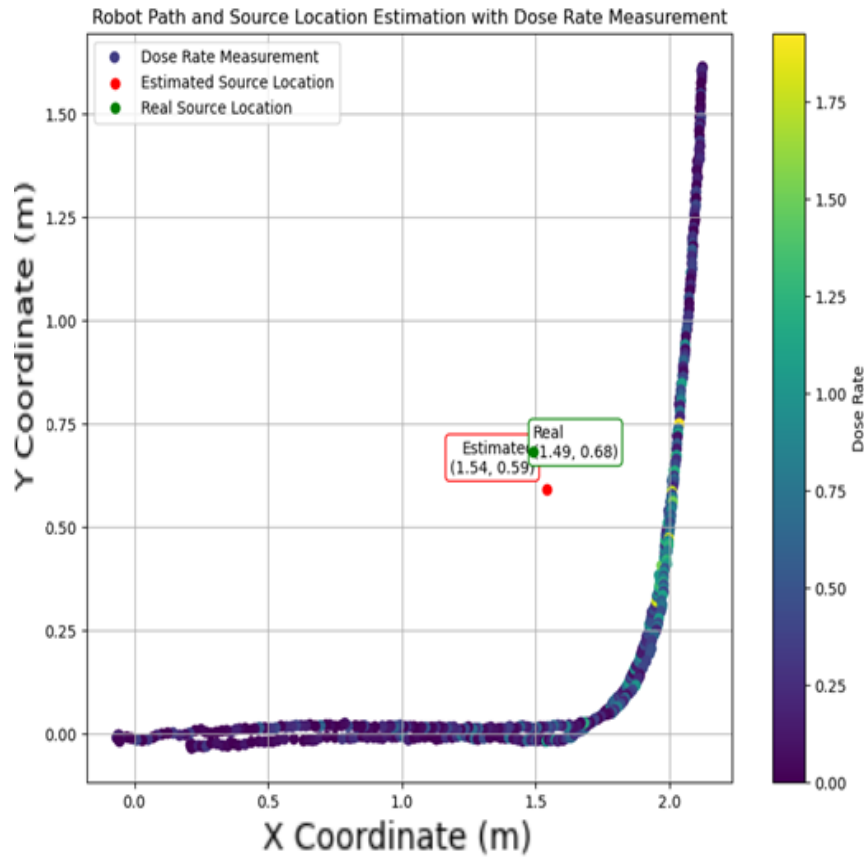


Figure 5.32: Robot Path 2 and Source Location Predicted by SA Algorithm.

For Path 3, there is a moderate accuracy between the actual and estimated

source location (7cm). The robot collected reliable data close to the source, but the reason for this deviation is the late arrival at the location where the dose rate is maximised.

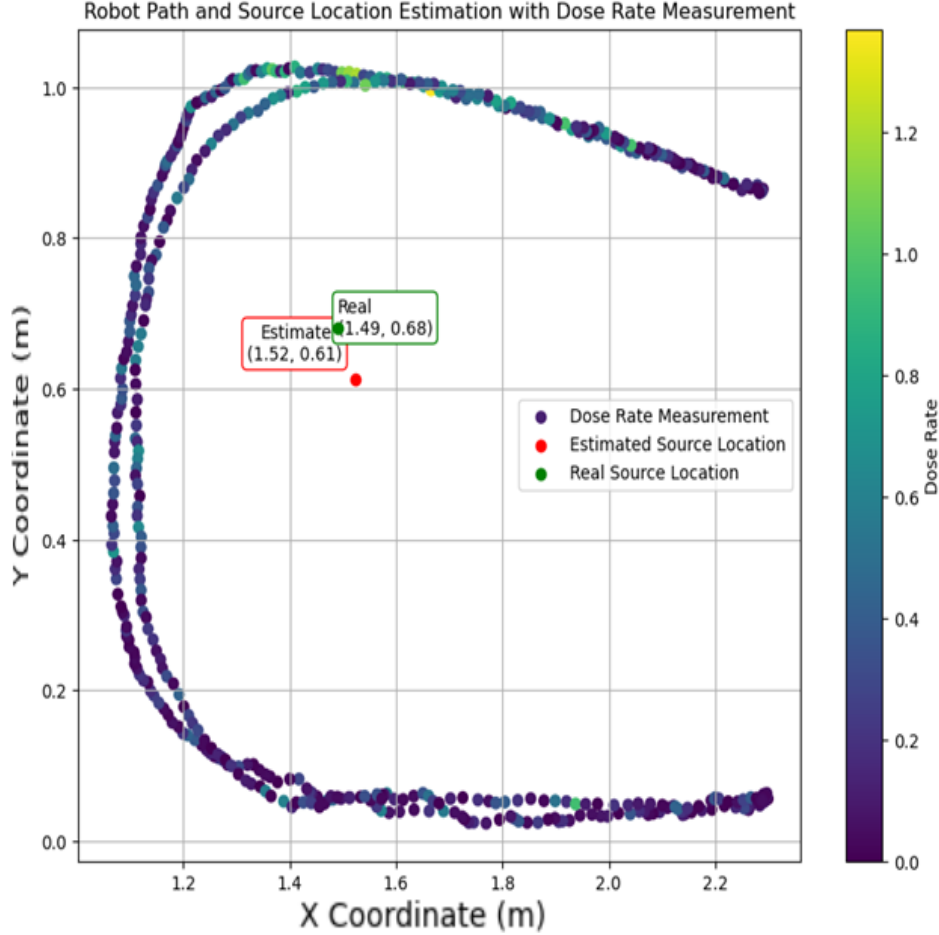


Figure 5.33: Robot Path 3 and Source Location Predicted by SA Algorithm.

The estimated position for Path 4 is determined with high accuracy, and the accuracy of the dose measurements shows that the algorithm performs a proper optimisation in the solution space. The Path 4 results strongly indicate that the algorithm can give results very close to the actual solution when the temperature

parameters and the number of iterations is optimised.

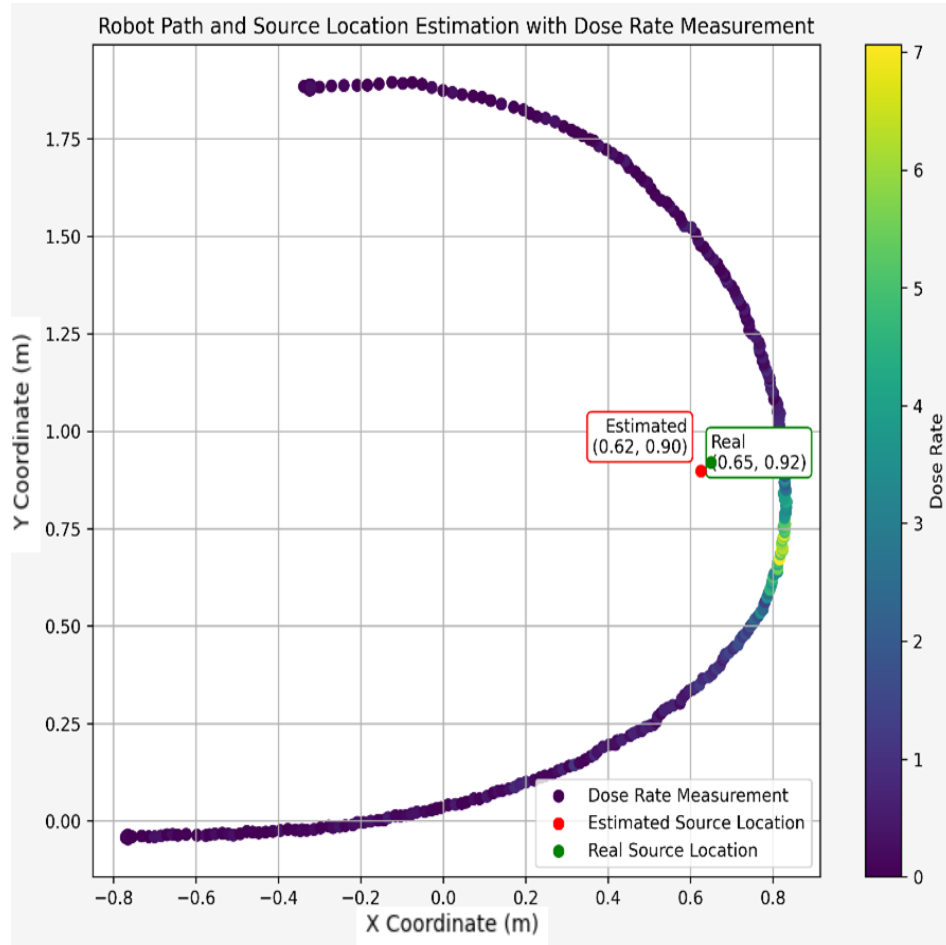


Figure 5.34: Robot Path 4 and Source Location Predicted by SA Algorithm.

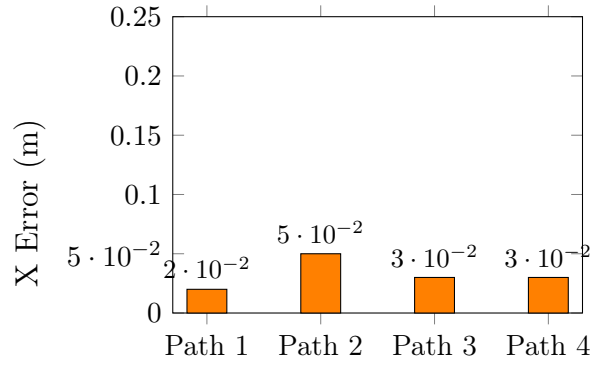
Looking at the paths, it can be seen that in general, the SA algorithm finds successful estimated source locations, especially the results for Path 1 and Path 4 are satisfactory.

Table 5.7: Error Analysis Table with Absolute and Total Errors for Triangulation + Simulated Annealing Method (m)

Path	Real X	Real Y	Est. X	Est. Y	X Er- ror (m)	Y Er- ror (m)	Total Error (m)
Path 1	1.49	0.68	1.51	0.70	0.02	0.02	0.028
Path 2	1.49	0.68	1.54	0.59	0.05	0.09	0.103
Path 3	1.49	0.68	1.52	0.61	0.03	0.07	0.076
Path 4	0.65	0.92	0.62	0.90	0.03	0.02	0.036

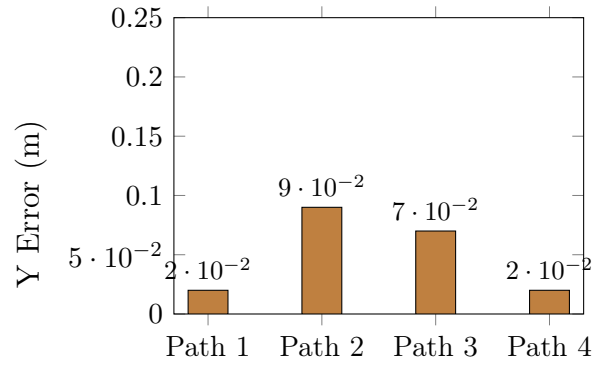
Table 5.7 shows that Path 1 shows the lowest total error at 2.8 cm. Path 2 exhibits the highest total error at 10.3 cm. Path 3 demonstrates a moderate deviation with a total error of 7.6 cm. Notably, although the starting point of Path 4 differs from the others, its estimated position is very close to the actual one, with a total error of 3.6 cm.

X Error Distribution (Triangulation + Simulated Annealing)



(a) X Error Distribution

Y Error Distribution (Triangulation + Simulated Annealing)



(b) Y Error Distribution

Figure 5.35: Comparison of X and Y error distributions in Triangulation + Simulated Annealing localisation.

Path 1 has a minimal absolute error of 2 cm in X and Y. Path 2 and Path 3 have a higher error rate in the Y-axis. Path 4 shows a more significant error on the X-axis than the Y-axis.

The Hill-Climbing algorithm is a fast and straightforward optimisation method, but gets stuck in local minima. Simulated annealing is slower but has a better chance of finding the global optimum. When the error margins of the paths

for the two methods are compared (Table 5.6 and Table 5.7), it is observed that, particularly in Path 1 and Path 2, the SA algorithm can accept suboptimal solutions with a certain probability, allowing it to escape local minima. This feature significantly reduces the margin of error. The Hill Climbing method has a deterministic structure and accepts the best solution among the neighbouring points at each step, so it gets stuck in local minima when the starting point is not chosen well and the measurement geometry is unsuitable. This is the reason for the high error seen in Path 2. The two algorithms show similar high performance in the presence of accurate and precise dose measurements. However, SA has slightly higher performance because it can perform random searches and take advantage of small details. SA has a clear advantage in complex and uncertain environments. Hill climbing is successful in simple and accurate measurements. It can be preferred when accurate data and limited time are available, while Simulated Annealing should be preferred for uncertain, complex, or noisy environments.

5.3 Comparison of Methods in Robotic, Fixed Detector and Simulation Measurement Scenarios

In the previous section, three different methods for localising radiation sources and various optimisation algorithms associated with these methods have been investigated in detail. The effectiveness of these methods has been evaluated with experimental data collected by the robot in a real environment. In this section, to isolate the effect of uncertainties due to robot movements, sensor noise and

environmental parameters and to evaluate the sensitivity of the methods more clearly, additional measurements were performed with fixed detectors in a controlled environment without the robot. In [Chapter 4](#), the data generated by the MCBEND Monte Carlo simulation is compared with the actual measurement results and found to be compatible. In this section, the performance of the methods will be examined with data collected from a single detector and simulation environment. In [Section 5.2](#), MLE, Bayesian and triangulation methods are examined by integrating them with some optimisation algorithms, and it is determined which optimisation algorithms give better results as a result of this examination. In this section, the optimisation algorithms with better results will be used. [Figure 5.36](#) shows the environment prepared for taking measurements with the detector only.

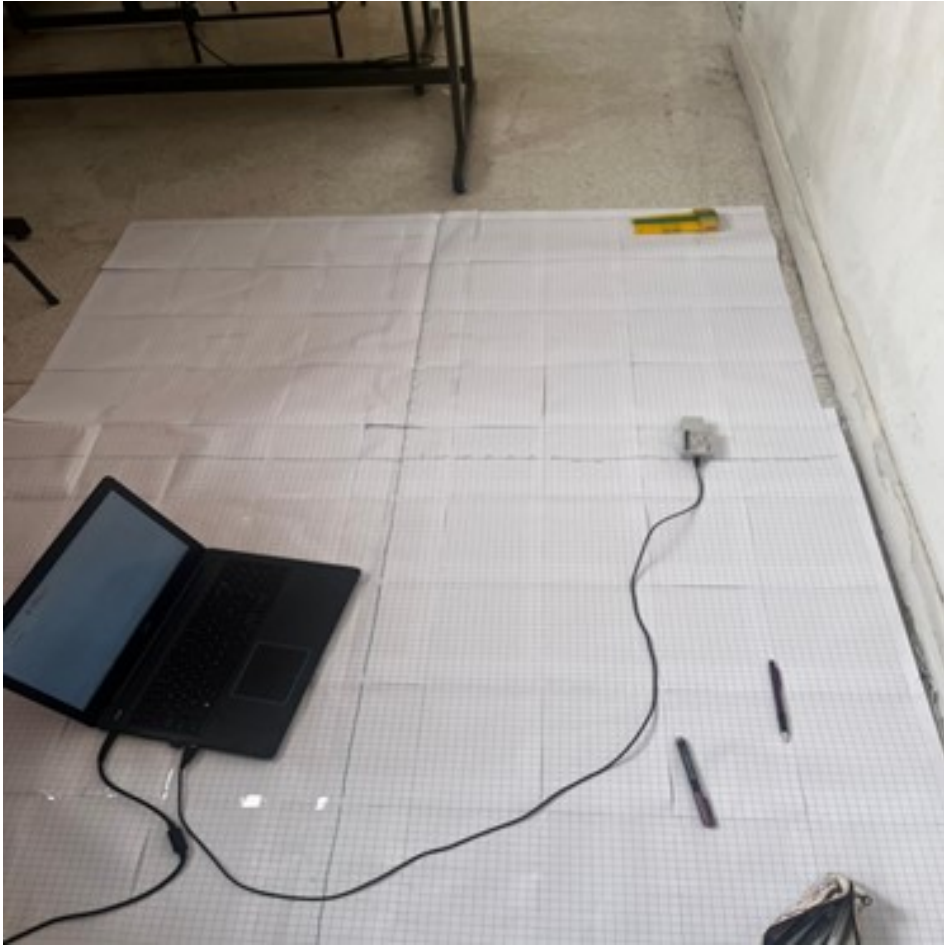


Figure 5.36: Experimental Setup for Detector-Only Measurement in Laboratory Environment.

In [Figure 5.37](#), data were collected at some locations using only the detector, and the results obtained by combining the MLE method with Monte Carlo optimisation were first examined.

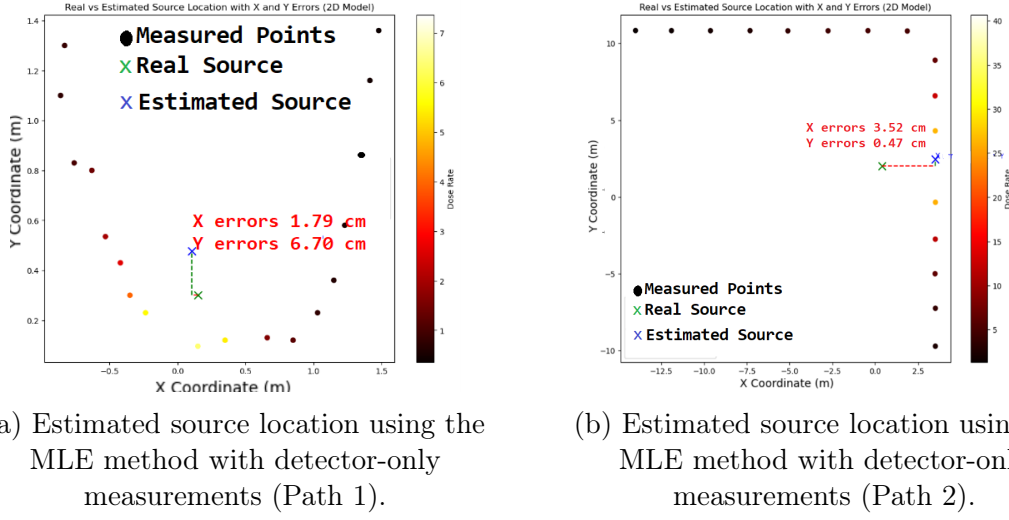
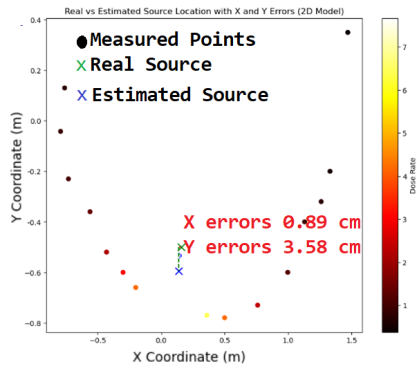


Figure 5.37: MLE method results for estimated radiation source locations obtained using only detector measurements. The plots illustrate real source positions, measurement points, estimated locations, and associated estimation errors in X and Y directions for two different experimental setups.

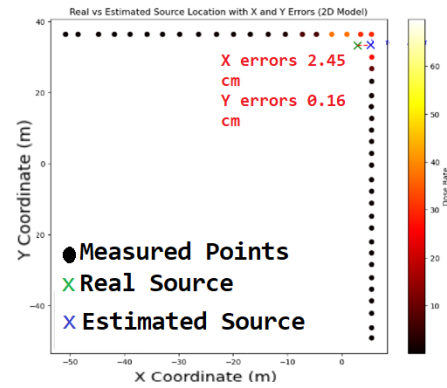
When the results are analysed, the estimated positions are very close to the actual position. The errors are reduced compared to the results obtained when the detector is moving and connected to the robot (Table 5.4). This is because the integration time at the measurement point is very short (≤ 30 s) on moving robot platforms, so the counting data subject to the Poisson process shows high variance. Due to the short integration time, a low SNR is obtained, which causes the MLE to fail to converge to the Cramér-Rao limit (the theoretically best variance that can be achieved when estimating a parameter). In addition, the physical structure of the robot body distorts the angular response of the detector, causing deviations in position estimation. The same detector was used in a fixed position to reduce the schematic errors caused by the robot movement, and the integration time was doubled. With increased integration time, the statistical uncertainty decreased by

$1/\sqrt{N}$ (N number of measurements), and the average total error decreased to 5.04 cm.

As the source distance increases, it introduces statistical uncertainty in the SLAM (simultaneous localisation and mapping) points that generate the position information of the robot using LIDAR data. Due to the longer measurement times with the manually carried detector, statistical uncertainty is reduced. In addition, in the absence of the shielding effect, distortions in the linear energy transfer are eliminated, and the effective cross-sectional area of the detector is fully utilised. Figure 5.38 shows the results obtained with the data obtained using the MCBEND programme.



(a) Estimated Source Location Using MLE Method with MCBEND Simulation (Path 1)



(b) Estimated Source Location Using MLE Method with MCBEND Simulation (Path 2)

Figure 5.38: Comparison of estimated and actual radiation source locations based on MCBEND simulations. Each plot displays measurement points, real source positions, estimated locations, and corresponding X and Y estimation errors for two distinct simulation scenarios.

MCBEND simulations are ideal environments where the source location, detector geometry, and environmental parameters are defined with absolute ac-

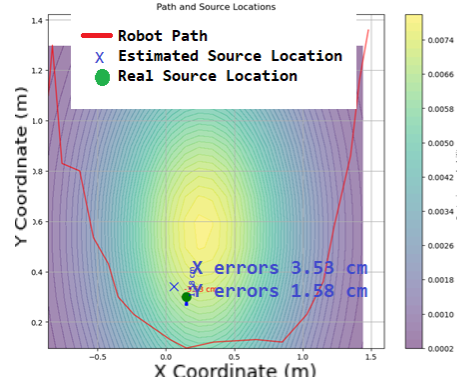
curacy. [Section 4.3](#) states that 100 million particles are sampled for MCBEND simulations, which reduces the variance and stochastic noise following the Central Limit Theorem.

Table 5.8: Average Estimation Errors under Different Conditions(MLE with Monte Carlo Optimisation)

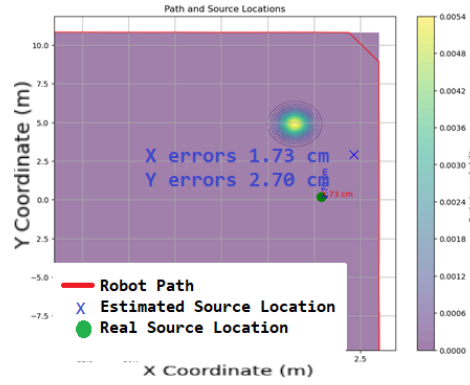
Condition	Average X Error (cm)	Average Y Error (cm)	Average Total Error (cm)
Robot	6.04	9.12	7.45
Fixed Detector	0.89	3.58	2.23
MCBEND Simulation	2.45	0.16	1.46

The measurement errors with the robot are taken from [Table 5.4](#) for [Table 5.8](#). When the robot data are compared with the measurements made with the fixed detector, the error is reduced by 70%. In the same way, the data obtained for MCBEND simulations gives 80% less error compared to the robot data and 35% improvement compared to the fixed detector. The optimisation algorithm applied in conjunction with the MLE method has limited success due to the short integration time, SLAM-based position errors, and the distortions in the angular response caused by the armouring.

Secondly, the results of the Bayesian method are analysed. The results obtained in [Figure 5.39](#) are for the fixed detector. In [Figure 5.39a](#), the estimated source position deviates from the actual source position by 3.53 cm from the X-axis and 1.58 cm from the Y-axis, while for [Figure 5.39b](#), the estimated source position is 1.73 cm from the X-axis and 2.70 cm from the Y-axis. Increasing the integration time reduces the statistical uncertainties.



(a) Estimated source location using only the detector data with the Bayesian method [path 1].



(b) Estimated source location using only the detector data with the Bayesian method [path 2].

Figure 5.39: Source localisation results using the Bayesian method based solely on detector measurements. The red line shows the robot path, the blue cross indicates the estimated source location, and the green circle represents the real source location. The background contour shows the posterior probability distribution.

The results for Figures 5.40a and 5.40b were adjusted according to the data obtained from the MCBEND simulation. The localisation errors in both the x and y axes were less than or equal to 2 cm. These results demonstrate the capability of the Bayesian approach to produce highly accurate source location predictions using only detector data.

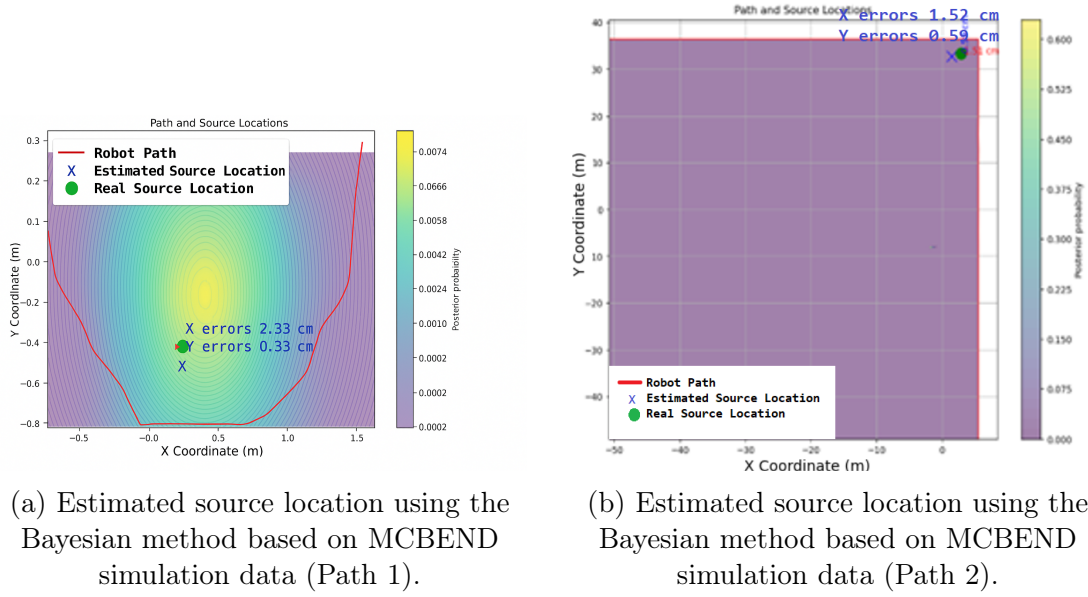


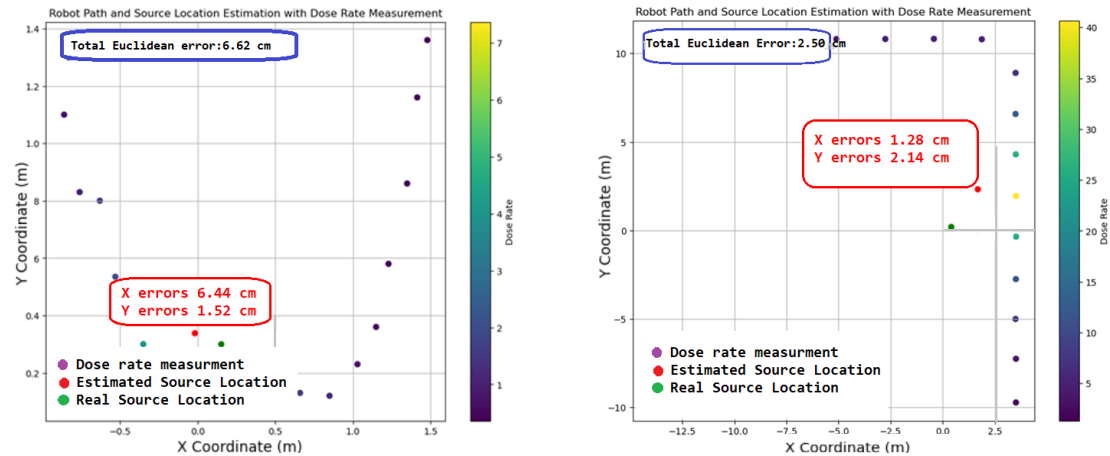
Figure 5.40: Source localisation using the Bayesian approach with only detector data derived from MCBEND simulations. The robot path is shown in red, the estimated source location is marked with a blue cross, and the true source location is shown as a green circle. The background colour map represents the posterior probability distribution.

When the measurements taken while the robot is moving are examined, it is observed that the posterior distribution is centred in Bayesian modelling, but the circumferential spread is high. However, despite the difficulties, such as the mobile robot being exposed to position changes during the measurement, high accuracy was obtained. While the error rate of the results obtained with the fixed detector decreased by 2.29% relative to those from the moving robot, the MCBEND simulation results demonstrated a greater improvement, with a reduction of approximately 24.84%. Moreover, the analyses performed using the fixed detector, especially the MCBEND simulation, yielded minimal error and produced source location estimates very close to the actual position.

Table 5.9: Average Estimation Errors using the Bayesian Method under Different Conditions

Condition	Average X Error (cm)	Average Y Error (cm)	Average Total Error (cm)
Robot	2.05	2.275	3.06
Fixed Detector	2.18	2.055	3.00
MCBEND	1.60	1.66	2.31

Finally, in the Triangulation+Simulated annealing method, the deviation in the x-axis is 6.44 cm and 1.52 cm in the y-axis in the fixed detector scenario (Figure 5.41a). In Figure 10, the errors are 1.28 cm for the x-axis and 2.14 cm for the y-axis. The higher error in the x-axis in Figure 5.41b is due to the geometrical sensitivity.

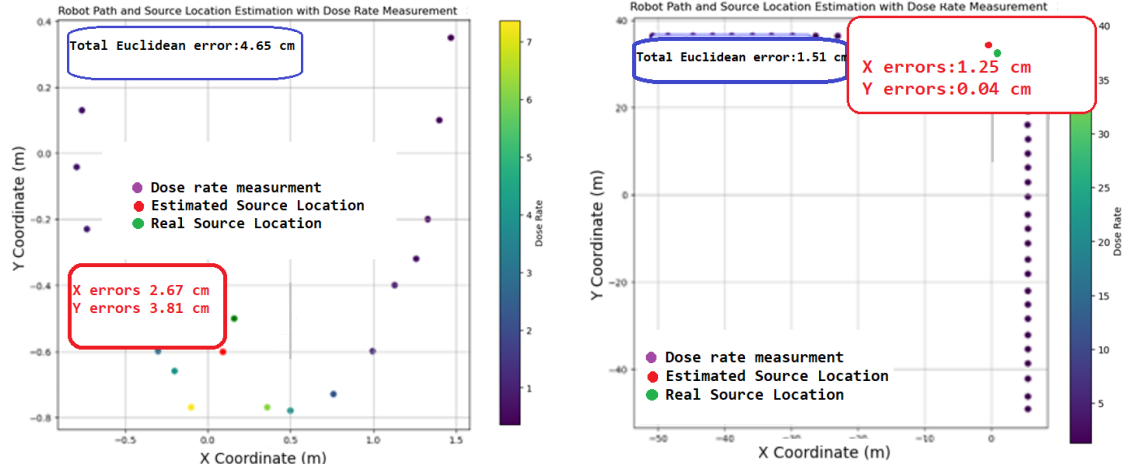


(a) Source estimation using the triangulation method with only detector measurements [path 1].

(b) Source estimation using the triangulation method with only detector measurements [path 2].

Figure 5.41: Estimated source locations using the triangulation method based solely on dose rate measurements obtained from the detector. Red points represent the estimated source positions, while green points indicate the true source locations. The background color represents the measured dose rate at each robot position. The Euclidean error between the estimated and actual source is also presented.

Millimetric errors in the detector position are affected by the triangulation method. However, when the correct geometry is provided, the errors are reduced, as shown in Figure 5.42.



(a) Source location estimation using the triangulation method with detector data generated by MCBEND simulation.

(b) Estimated source position using triangulation based solely on MCBEND-simulated dose rate measurements.

Figure 5.42: Source localisation results using the triangulation method with dose rate data obtained from MCBEND simulations. Only detector-based measurements were used for the estimations. The red marker indicates the estimated source location, and the green marker shows the true source position. The color map reflects the dose rate values recorded at each measurement point.

The results obtained from the data obtained with MCBEND are few, as with other methods.

Table 5.10: Average Estimation Errors using the Triangulation + Simulated Annealing (SA) Method under Different Conditions

Condition	Average X Error (cm)	Average Y Error (cm)	Average Total Error (cm)
Robot	3.25	5.00	5.96
Fixed Detector	3.86	1.86	4.28
MCBEND	1.96	2.33	3.04

The robot scenario has the highest error rate due to dynamic uncertainties.

The system is more stable with a fixed detector, reducing errors by 28%. The error in the MCBEND simulation under ideal conditions is reduced by 49%. This study provides a framework for radiation source localisation and demonstrates the limits of safety and accuracy of robotic systems. In the next section, the results will be evaluated through a comparison of the methods, and the most suitable method for radiation source localisation will be discussed.

5.4 Comparison of Localisation Algorithms

Using the data collected by a detector integrated into a mobile robot, three main methods for radiation source localisation and their combination with different optimisation techniques are compared. These are MLE + L-BFGS-B with deterministic optimisation, MLE+Monte Carlo based on stochastic sampling, Bayesian method combining prior knowledge and current data, and Triangulation+Hill Climbing and Triangulation+Simulated Annealing approaches using geometric estimation. The differences between these approaches are theoretical and differ when the robot is evaluated according to field data. While MLE and Bayesian methods adopt a statistical approach, Triangulation offers a geometry-based approach. Using the data collected by a detector integrated into a mobile robot, three main methods for radiation source localisation and their combination with different optimisation techniques are compared. These are MLE+L-BFGS-B with deterministic optimisation, MLE+Monte Carlo based on stochastic sampling, Bayesian method combining prior knowledge and current data, and Triangulation+Hill Climbing and Triangulation+Simulated Annealing approaches using geometric estimation.

The differences between these approaches are theoretical and differ when the robot is evaluated according to field data. On the other hand, the optimisation methods used also play a decisive role in the performance: L-BFGS-B and Hill Climbing are fast but tend to get stuck in local minima, while Monte Carlo and Simulated Annealing are more effective in the search for the global optimum. In this context, the advantages and disadvantages of each method are compared with their theoretical basis and analysis based on actual measurement data.

Evaluation of Method Accuracy Based on Localization Error

In the analysis performed with the data collected by the mobile robot, the Bayesian method was determined to be the most successful. As can be seen in [Table 5.11](#), the average margin of error is less than 3%, which is evidence that the estimated location is very close to the actual location. The fact that the performance does not deviate significantly despite the movement of the robot indicates that it remains stable under field conditions. Following the Bayesian method, Triangulation+Simulated Annealing method is in second place with an error of about 4%. Although this method yields an error that is almost twice as high as the Bayesian approach, it gives better results than the deterministic MLE methods. MLE+Monte Carlo method has a relatively high error of 6%, while MLE+L-BFGS-B method produces an error of 11.04%. Triangulation + Hill Climbing algorithm, on the other hand, tended to get stuck near the initial position due to local maxima, resulting in a higher error rate of 12%. The data obtained shows that the Bayesian approach, thanks to its statistical modelling power, finds the source location with the lowest error. Triangulation+Simulated Annealing, with its ability to find a global solution, was in second place. MLE-based algorithms

generally showed higher error rates, especially L-BFGS-B , which searches for a local solution and performs poorly. Triangulation+Hill Climbing, which has a limited search capacity, showed the lowest performance.

Table 5.11: Average Total Error by Method

Method	Average Total Error (%)
Bayesian	2.33
Triangulation + SA	4.01
MLE + MC	5.97
MLE + L-BFGS-B	11.04
Triangulation + HC	12.11

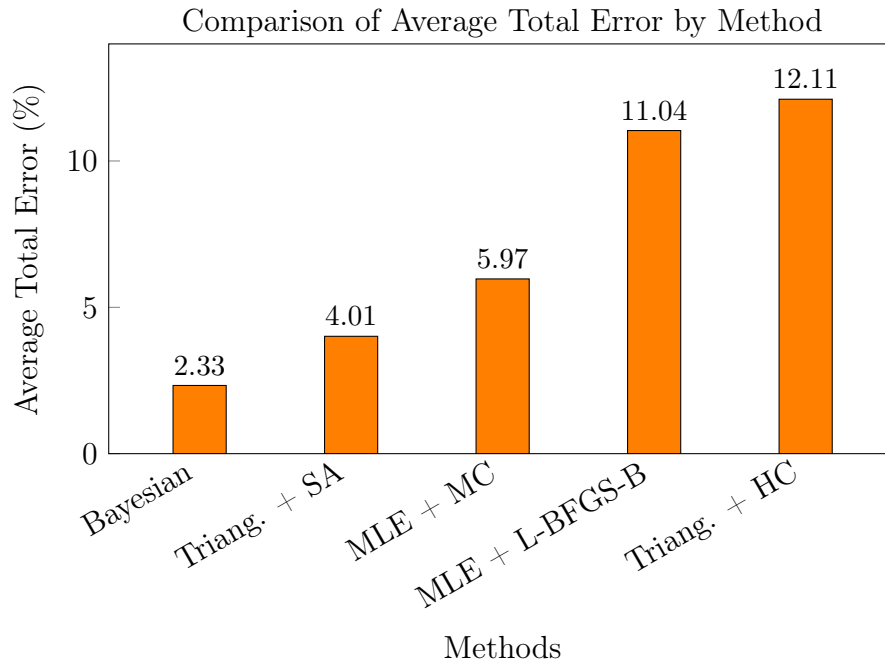


Figure 5.43: Comparison of Average Total Error by Method

Impact of Measurement Noise

Statistical noise in radiation measurements is an important factor affecting the performance of the methods in radiation source localisation. The Bayesian method is one of the most robust solutions to measurement noise. The posterior distribution, which is formed by combining the prior information with the measurement model, can maintain the probability of the correct location even under high noise conditions. Thus, it is possible to obtain reliable results even if the number of measurements is small or the data is noisy. Especially in environments with minimal noise, such as simulation, it has been observed that the estimated coherence errors obtained by the Bayesian method are less than 2 cm. MLE-based methods are directly sensitive to measurement noise, severely limiting their success under high-variance data conditions. Gradient-based optimisation algorithms such as L-BFGS-B have been unsuccessful in finding the correct direction because the noise and the solution often deviate from the actual position. The Monte Carlo +MLE approach produced more stable results by sampling different points to avoid local minima due to noise. However, its performance was slightly affected at high noise levels. As a result, both MLE methods produced higher errors than the Bayesian method. Since triangulation is based on geometry, the effect of measurement noise differs from other methods. The data from some points corresponds to a certain distance ratio; however, if there is noise in the measurements, these ratios are miscalculated. In a hill-climbing algorithm, if the initial estimate is inaccurate due to noise, it cannot correct this error when it searches for a solution in its immediate surroundings, and a spike in the estimated position is observed. On the other

hand, the Triangulation+Simulation Annealing method, even if the initial guess is terrible, can reduce the measurement lines by doing a broad search, making it more noise-tolerant. However, since the measurement noise cannot be modelled directly in triangulation-based methods, it cannot provide a noise procedure as complex as probabilistic models such as Bayesian and Maximum Likelihood Estimation. Therefore, although it yields low error values in simulation environments where noise is minimal, noise in real environments reduces the performance of the method.

Impact of Optimisation Techniques

The type of optimisation algorithms used in the MLE and triangulation methods to estimate the source location also affected the performance. The MLE+Monte Carlo approach provided closer estimates of the source location by overcoming local maxima in some cases compared to deterministic gradient-based methods such as L-BFGS-B. However, this method has a high computational cost and is useless for practical applications. Similarly, although the triangulation method seems advantageous due to its hill-climbing speed, it may give poor results due to local search because it is sensitive to a bad starting point. However, the simulated annealing algorithm provides more accurate location estimates, since it can support the search process at the global level. In contrast, when properly designed, the Bayesian approach successfully converges to the global optimum since it involves a random sampling search process. The overall results show that although deterministic methods have the advantage of speed, they are vulnerable to noise. In contrast, stochastic methods are more reliable but costly in time.

General Evaluation

The analyses performed in this study have systematically demonstrated the advantages and limitations of the algorithms applied to find five different estimated source conditions. The Bayesian approach offered the highest overall accuracy and reliability, producing low error rates and high performance under static and dynamic conditions. The ability of the Bayesian approach to incorporate measurement uncertainties into the model leads to low error rates. Triangulation+Simulated Annealing offers a more straightforward structure than the Bayesian method, but shows moderate robustness to noise. It is a strong alternative for practical applications and should be used with precision navigation systems such as SLAM, as it is affected by minor errors in robot configurations. Although theoretically sound, MLE-based methods have had limited success in field applications. While achieving more reliable results by avoiding the local optimum, Monte Carlo applications are limited by high computational time, while the Hill-Climbing method is only helpful for rough approximation.

Table 5.12: Overall Performance Summary of Localisation Methods

Criteria	Bayesian	Triang. + SA	MLE + MC	MLE + L-BFGS- B	Triang. + HC
Localization Error	3	2	2	1	1
Noise Robustness	3	2	2	1	1
Optimization Type	3	3	3	1	1
Computational Cost	2	2	1	3	3

Table 5.12 provides a comparative summary of the overall performance of

the localisation methods evaluated based on four key criteria: error rate, noise robustness, optimisation type, and computational cost. A scoring scale from 1 (poor) to 3 (strong) was used.

The Bayesian method consistently demonstrated the highest performance, achieving top scores in error rate, noise tolerance, and optimisation quality, albeit with a moderate computational cost. As a result, each method has its advantages and disadvantages. The Bayesian method should be preferred for the highest accuracy and field robustness, triangulation+SA is preferred where a certain level of accuracy is acceptable in real-time constraints, and MLE-based methods are preferred for more controlled and low-noise environments.

5.5 Estimation of Multiple Radiation Source Locations

In the presence of multiple radiation sources, the high-dimensional and complex probability space resulting from the intermixing of measurement data significantly limits the effectiveness of traditional methods, which increases the risk that global solution-seeking methods, such as MLE-based approaches, will get stuck in local minima. Theoretically, optimisation methods that can scan the entire space, such as simulated annealing, are inefficient in practice due to the slow convergence time. Although local optimisation algorithms are fast, they may give results far from the global optimum in complex problems. Therefore, choosing the starting parameter correctly for these algorithms is very important, but it is difficult to

determine the starting values in the case of multiple sources. This study applies Bayesian modelling to systematically address the computational complexity and uncertainties caused by multiple sources. Bayesian modelling offers the possibility to evaluate all parameters simultaneously in a statistical framework. In this process, the available information is defined through the prior distribution and the posterior distribution is obtained together with the measurement data. In particular, algorithms make it possible to sample from multimodal and high-dimensional probability distributions. This allows many solutions to be evaluated, i.e., the highest probability locations and the distributions of all other possible locations are analysed. This feature provides an advantage over classical methods. The ability of the Bayesian approach to flexibly model complex measurement patterns allows it to successfully represent the complex patterns caused by multiple sources [81]. Therefore, the Bayesian approach was chosen for this study because of its ability to provide reliability and uncertainty information. The experimental setup used to find locations for multiple sources is shown in [Figure 5.44](#).



Figure 5.44: Measurement system designed to determine the position of multiple radiation sources.

Since it was not possible to transport a sufficient number of radiation sources to the robotic lab, data were collected with the detector only in the setup prepared in the nuclear lab. As a continuation of this PhD study, experiments with the robot will be carried out when the appropriate resources and environment are provided for future studies. A coordinate system was prepared on paper, the detector was moved to specific points and dose rate ($\mu\text{Sv}/\text{hour}$) measurements were made at each location. The ambient conditions remained constant throughout the

experiment, and background radiation was assumed to be negligible. The measured dose rates were used as observation data for the Bayesian method. The [Algorithm 6](#) applied in this context is as follows.

Algorithm 6 Bayesian Multi-Source Localisation

```

1: Input:
2:   Detector positions  $\{(X_i, Y_i)\}_{i=1}^n$ ,
3:   Measured dose rates  $\{D_i\}_{i=1}^n$ ,
4:   Unit conversion factor  $\alpha$ ,
5:   True source coordinates  $\{(S_j^x, S_j^y)\}_{j=1}^2$ 
6: Output:
7:   Estimated coordinates  $\{(\hat{x}_j, \hat{y}_j)\}_{j=1}^2$ ,
8:   Error metrics  $\{(\Delta X_j, \Delta Y_j, \Delta d_j)\}_{j=1}^2$ 
9:
10: procedure BAYESIANLOCALISATION
11:   Data Preprocessing:
12:     Filter low-dose measurements:  $D_i > 0.5 \times 10^{-3} \mu\text{Sv/h}$ 
13:     Subsample dataset:  $n \leftarrow \min(1000, |\text{data}|)$ 
14:
15:   Model Specification:
16:     Define uniform priors for sources:
17:        $x_1, y_1, x_2, y_2 \sim \text{Uniform}(\min(X) - 1, \max(X) + 1)$ 
18:     Calculate predicted dose rates:
19:        $\forall i \in 1 : n, \mu_i = \frac{1}{(X_i - x_1)^2 + (Y_i - y_1)^2 + \epsilon} + \frac{1}{(X_i - x_2)^2 + (Y_i - y_2)^2 + \epsilon}$ 
20:     Define likelihood:  $D_i \sim \mathcal{N}(\mu_i, \sigma^2)$ 
21:     Set noise prior:  $\sigma \sim \text{HalfNormal}(0.1)$ 
22:
23:   Posterior Sampling:
24:     Initialize NUTS sampler
25:     Run MCMC: 2000 draws, 500 tuning steps, 2 chains
26:     Check convergence with  $\hat{R} < 1.01$ 
27:
28:   Source Matching:
29:     Compute posterior means:  $\{\hat{x}_j, \hat{y}_j\}_{j=1}^2$ 
30:     Solve bipartite matching:  $\min \sum_{j=1}^2 \| (S_j^x, S_j^y) - (\hat{x}_k, \hat{y}_k) \|_2$ 
31:
32:   Error Calculation:
33:     For each matched source:
34:        $\Delta X_j \leftarrow \alpha(S_j^x - \hat{x}_j)$ 
35:        $\Delta Y_j \leftarrow \alpha(S_j^y - \hat{y}_j)$ 
36:        $\Delta d_j \leftarrow \sqrt{(\Delta X_j)^2 + (\Delta Y_j)^2}$ 
37: end procedure

```

Algorithm 6, when there is no prior knowledge about the location of the sources, the location parameters for each source are given a broad and uniform distribution. Suppose the boundaries of the environment where the sources are located are known. In that case, each source location is assumed to be uniformly distributed in x and y coordinates so that the algorithm gives equal probability to all locations at the beginning. When there is more than one source, the dose rate measured by the detector is modelled as the sum of all sources. According to the inverse square distance law ($\frac{1}{r^2}$), radiation intensity decreases inversely with the square of the distance.

$$D_{\text{model},j} = \sum_{i=1}^{N_{\text{kaynak}}} \frac{K}{(\|\mathbf{s}_j - \mathbf{x}_i\| + \varepsilon)^2}, \quad (5.43)$$

Here, $\mathbf{s}_j$ denotes the position of the detector, $\mathbf{x}_i$ represents the unknown position of the source, and ε is a negligibly small positive constant introduced to prevent division by zero.

The measured dose rate $D_{\text{olcum},j}$ at detector j is assumed to follow a Gaussian distribution centered around the model-estimated dose rate $D_{\text{model},j}$, and is mathematically expressed as:

$$D_{\text{olcum},j} \sim \mathcal{N}(D_{\text{model},j}, \sigma^2), \quad (5.44)$$

In Formula 5.44, σ is treated as an unknown parameter and a half-normal distribution is usually chosen in advance for the error component. The half-normal is half of the normal distribution with only a positive degree and is an appropriate

choice for a positive quantity, such as the standard deviation. Since it is impossible for sigma to take a negative value, even with limited knowledge of sigma, the half-normal distribution gives a positive distribution with a long tail towards a more significant value, concentrating around $\sigma=0$. Therefore, this model limits the sigma to the physical instantaneous range (positive) while not restricting the magnitude of the measurement error.

Once the a priori distributions and likelihood functions are defined, the objective is to obtain the posterior distribution of the parameters. Markov Chain Monte Carlo (MCMC) methods are used for numerical sampling. The MCMC algorithm, NUTS, is an adaptive version of the Hamiltonian Monte Carlo (HMC) algorithm and is particularly known for its ability to navigate efficiently in high-dimensional continuous parameter spaces. Using derivative information, NUTS can take long steps in the posterior space without falling into random walks [82],[83]. With MCMC, after many samples are taken, it is ensured that the obtained samples come from the posterior distribution and that the chain converges. This algorithm checks convergence by running multiple chains for each parameter and comparing the results. The Gelman-Rubin statistic ($\hat{R}$) is a convergence criterion. Theoretically, an $\hat{R}$ value of 1.0 indicates that the chains have reached a common target distribution. In practice, values close enough to 1.0 are accepted, and a threshold of $\hat{R} < 1.01$ is often used as a notable convergence criterion. In this study, $\hat{R}$ values below 1.01 for each parameter indicate that the sampler was run long enough and that the posterior distribution is correctly represented. After all these checks, the samples collected are assumed to be random samples from the posterior distribution, and inferences are made from these sam-

ples [84]. Figure 5.45 shows the estimated source locations for the two scenarios in the laboratory environment and the experiment with the detector.

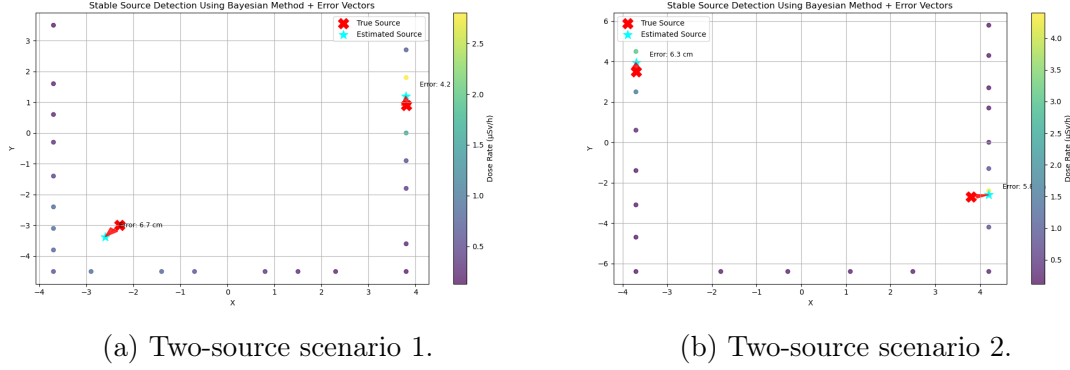


Figure 5.45: Results of multiple source location estimation by Bayesian method: (a) and (b) show the estimation results obtained for different source configurations.

Red “X” marks indicate the actual source locations, while turquoise star symbols indicate the locations found by Bayesian estimation. The map also shows the dose rate values measured by the detector at different points on a colour scale ranging from purple to green (scale in $\mu\text{Sv/h}$ on the right).

Two-source localisation results are presented in Figure 5.45. In Figure 5.45a, the estimation errors are 6.7 cm and 4.2 cm, whereas in Figure 5.45b, the corresponding errors are 6.3 cm and 5.4 cm for a different source placement. As can be seen from the Figure 5.45, the Bayesian method successfully estimated the position of both sources with reasonable error margins even with complex distributed datasets, despite the limited number of measurement points and noisy dose rate data.

Table 5.13: Position errors between the true and estimated source locations. (a) True source positions: $(-3.7, 3.5)$ and $(3.8, -2.7)$; (b) True source positions: $(-2.3, -3)$ and $(3.8, 0.9)$

(a) Position Errors – (a)		(b) Position Errors – (b)	
Source	Total Error (cm)	Source	Total Error (cm)
1	6.7	1	6.3
2	4.2	2	5.8

Table 5.13 shows the total location errors between the actual and estimated source locations for scenarios (a) and (b). Consistent with the error vectors in the figures, the total position errors for both sources are below 7 cm. These results show that the Bayesian model can produce stable and reliable estimates even in multimodal and high-dimensional probability spaces. It also supports the validity of the method in terms of providing appropriate outputs for the physical model.

5.6 Identification of Gamma Radiation Sources

Gamma spectroscopy uses the gamma radiation emitted by radioactive materials to identify the radioisotopes present in the source. The energy spectrum obtained by this method can be used as a fingerprint to identify them by the position and shape of the peaks it produces. Detailed information on this subject is given in Section 2.2. In this part of the study, the robot travels to that location once the estimated source location is found. It collects data with a CsI(Tl) detector for 5 minutes to quickly identify the gamma radiation source. The Python-based code developed for gamma spectrum analysis is integrated with radiation measurement systems to quickly and accurately identify radiation sources. The analysis code

includes the steps of background signal extraction, energy calibration, peak detection, significance evaluation, isotope match recognition, and handling of multiple source scenarios. Finally, information on the identity of the radiation source is reported. In this chapter, the analysis processes will be examined step by step, the physical principles and numerical methods in the application will be discussed in the context of the literature, and the advantages and disadvantages of the code will be discussed. The Gamma Spectrum Analyser code was developed to characterise data from relatively low-resolution systems such as the CsI(Tl) scintillator detector. Due to the low resolution, the energy peaks can be broad and indistinct, making the interpretation of complex spectra difficult. To overcome this drawback, this code combines classical methods such as peak energy matching and extensive identification with reference spectra to reduce the impact of this problem. One of the first and critical steps in the gamma spectrum analysis process is to obtain the raw data by removing the background so that the spectrum can be correctly interpreted. The sequential steps in the code are specified in [Algorithm 7](#).

Algorithm 7 Radiation Source Identification (Part 1)

```
1: Input:
2:   Raw spectrum data:  $S_{\text{raw}}[E]$   $\triangleright$  // Contains energy (keV) and count values
3:   Background data:  $B[E]$   $\triangleright$  // Measured background counts
4:   Energy range:  $[E_{\text{min}}, E_{\text{max}}]$ 
5:   Isotope library: IsoLib  $\triangleright$  // Reference energies and emission probabilities
6:   Detector resolution:  $\Delta E_{\text{res}}$ 
7:   Reference energy:  $E_0$   $\triangleright$  // For example, 661.7 keV
8:
9: procedure MAINANALYSIS
10:   // Step 1: Background Subtraction
11:    $S_{\text{net}} \leftarrow S_{\text{raw}} - B$ 
12:    $\triangleright$  //  $S_{\text{net}}(E) = S_{\text{raw}}(E) - B(E)$ 
13:
14:   // Step 2: Data Smoothing
15:    $S_{\text{smooth}} \leftarrow \text{SavitzkyGolay}(S_{\text{net}})$ 
16:    $\triangleright$  // Reduce noise using Savitzky–Golay filter
17:
18:   // Step 3: Peak Detection
19:   Peaks  $\leftarrow \text{find\_peaks}(S_{\text{smooth}})$ 
20:    $\triangleright$  // Identify local maxima in  $S_{\text{smooth}}$ 
21: end procedure
```

Algorithm 7 Radiation Source Identification (Part 2)

```

1: procedure CONTINUEANALYSIS-PART2
2:   for all  $p \in \text{Peaks}$  do
3:      $E_p \leftarrow$  Energy corresponding to peak  $p$ 
4:     if  $E_p \in [E_{\min}, E_{\max}]$  then
5:       // Step 4: Gaussian Fitting
6:        $(A, \mu, \sigma) \leftarrow \text{GaussianFit}(p)$ 
7:                                      $\triangleright$  // Fit a Gaussian:  $A \exp\left(-\frac{(E-\mu)^2}{2\sigma^2}\right)$ 
8:        $\text{FWHM} \leftarrow 2.355 \sigma$ 
9:
10:      // Step 5: Tolerance Calculation
11:       $\Delta E_{\text{tol}} \leftarrow \Delta E_{\text{res}} \sqrt{\frac{E_p}{E_0}}$ 
12:                                      $\triangleright$  // Tolerance increases with peak energy
13:
14:      // Step 6: Isotope Matching
15:       $\text{Matches} \leftarrow \{i \in \text{IsoLib} \mid |E_i - E_p| \leq \Delta E_{\text{tol}}\}$ 
16:                                      $\triangleright$  // Isotopes with reference energy  $E_i$  close to  $E_p$ 
17:      for all  $i \in \text{Matches}$  do
18:         $\text{Score}_i \leftarrow \frac{\text{Prob}_i}{1+|E_i-E_p|}$ 
19:         $\triangleright$  // Score based on emission probability and energy deviation
20:      end for
21:       $\text{TopMatches} \leftarrow \underset{i \in \text{Matches}}{\text{argmax}}(\text{Score}_i)$ 
22:    end if
23:  end for
24: end procedure
25:
26: function GAUSSIANFIT( $p$ )
27:   Input: Peak region  $p$ 
28:   Bounds:
29:
30:        $0 \leq A < \infty, \quad E_p(1 - \epsilon) \leq \mu \leq E_p(1 + \epsilon), \quad 0 \leq \sigma < \infty$ 
31:
32:   Optimization: Minimise
33:
34:       
$$\left\| S_{\text{smooth}}(E) - A \exp\left(-\frac{(E - \mu)^2}{2\sigma^2}\right) \right\|$$

35:
36:   return  $(A, \mu, \sigma)$ 
37: end function

```

Algorithm 7 Radiation Source Identification(Part 3)

```

1: procedure CONTINUEANALYSIS-PART3
2:   // Step 7: Reference Line Validation
3:   Validation  $\leftarrow$  CompareReferenceLines(TopMatches)
4:    $\triangleright$  // Check if all expected reference lines are present
5:
6:   // Step 8: Spectrum Comparison for Partial Matches
7:   if PartialMatches exist then
8:     SpectrumSimilarity  $\leftarrow$  PearsonCorrelation()
9:      $\triangleright$  // Compare measured spectrum to reference spectrum via Pearson
        correlation
10:  end if
11:
12:  // Step 9: Report Generation
13:  GenerateReport()
14:   $\triangleright$  // Compile all results into a final report
15: end procedure
16:
17: function COMPAREREFERENCELINES(TopMatches)
18:   for all  $i \in$  TopMatches do
19:     RefLines  $\leftarrow$  IsoLib[ $i$ ].lines
20:      $\triangleright$  // Retrieve reference energy lines for isotope  $i$ 
21:     MatchedCount  $\leftarrow \sum_{e \in \text{Peaks}} \mathbf{1}\{|e - \text{RefLine}| \leq \Delta E_{\text{tol}}\}$ 
22:     if MatchedCount = |RefLines| then
23:       Validation: Successful
24:     else if MatchedCount > 0 then
25:       Validation: Partially Successful
26:     else
27:       Validation: Unsuccessful
28:     end if
29:   end for
30: end function

```

The background spectrum is obtained by taking measurements with the CsI(Tl) detector for about 5 minutes without the source, and then the background spectrum can be pointwise subtracted from the source+background spectrum. The

code used in the study uses a method that scans the spectrum in a given window width, determines the minimum value in each energy region, and subtracts the background by spline interpolation over the entire spectrum. Spline interpolation models the continuous and smooth background using the minimum values determined from the spectrum to make the separation of signal and noise more precise. This approach is similar to the widely used SNIP (Sensitive Nonlinear Iterative Peak-Clipping) algorithm, which iteratively filters out slowly changing low-frequency components to reveal the underlying background beneath the peaks. The resulting background model is then subtracted from the original spectrum, resulting in a clear spectrum containing only peaks and signal components. The background correction gives the spectrum in Figure 5.46. After the background is removed, the red curve sometimes drops below zero. The spline method has small margins of error when estimating the background, and in academic studies, these are considered negative deviations [85],[86].

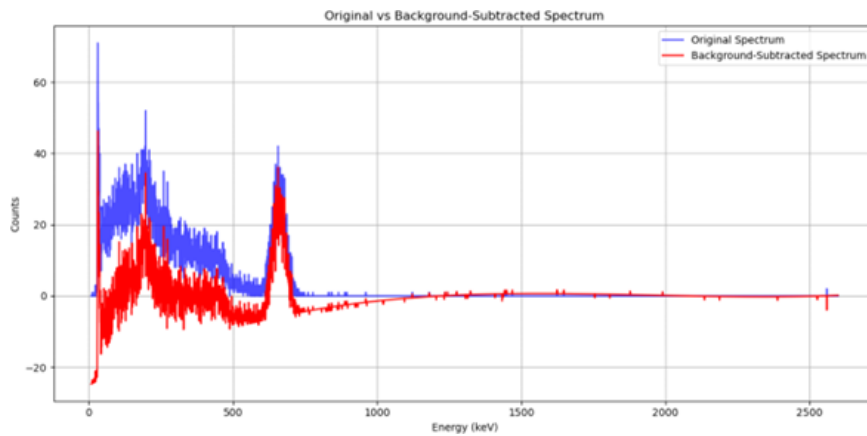


Figure 5.46: Comparison of the original spectrum (blue) and the spectrum with background subtraction using spline interpolation (red), measured with a ^{137}Cs source (116,371.64 Bq) placed 5 cm from the detector for a 5-minute acquisition.

After the background-corrected spectrum, the second step is to detect the peaks of the gamma sources. In this step, the local maxima in the spectrum are identified, and the significance of these points is statistically evaluated. The code used first reduces the noise by applying a Savitzky-Golay filter to the raw spectrum, and then the potential peak centres are revealed with the help of the zero crossings and the determined amplitude thresholds. This method has been used in different studies in the literature. For example, Lam and Zhang [87] reported that significant peaks were successfully detected after Savitzky-Golay filtering with a similar approach in their algorithm for low-resolution detectors. In peak detection, not only the peak location is determined, but also the full width at half-height (FWHM) and peak area are calculated for each peak so that the relevant data points can be fitted with Gaussian and Gaussian+tailing functions to obtain more precise peak centre area values, which are then used for subsequent isotope identification. In low-resolution systems such as the CsI(Tl) detector, the broad and asymmetric nature of the peaks necessitates a hybrid model that considers the effects of reflection and Compton continuum. The detected energy peaks are compared with the specific peak energy values of radioisotopes in a predetermined reference library and with the absolute emission probability values of these beams, which may deviate from the peak energy values measured by the detector due to the energy resolution. Therefore, the developed algorithm takes into account an uncertainty range of a few +keV for the measured energy values and performs the appropriate matching for each detected peak. In practical applications, a tolerance of $\pm 1.5 \times \text{FWHM}$ is often used to mitigate the effects of statistical fluctuations and detection imperfections [88].

If matches are achieved, the corresponding peak is recorded as a possible indication of the presence of that isotope. To test the code, measurements were first taken with the detector and gamma radiation sources in the nuclear lab were placed at different distances. ^{137}Cs , ^{60}Co and ^{152}Eu are presented as examples of the analysis performed. First, ^{137}Cs were placed 5 cm away from the detector, and the data was collected. The result of the code is presented in Table 5.14, and the spectrum is in Figure 5.47.

Table 5.14: Isotope matching summary and corrected number of expected lines

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	1	All reference lines matched	✓
^{60}Co	2	0	No match found	✗
^{22}Na	2	0	No match found	✗
^{152}Eu	8	0	No match found	✗
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

The spectrum analysis successfully detected the presence of ^{137}Cs . In particular, the 662 keV peak has been selected.

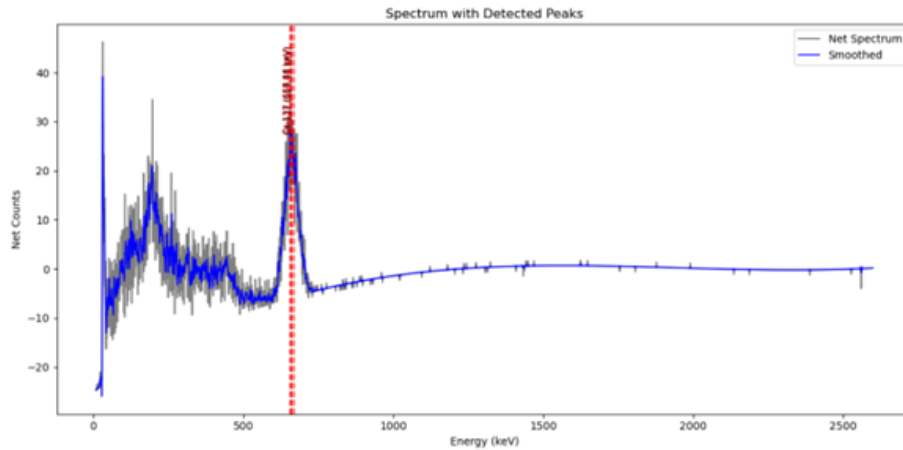


Figure 5.47: Detection of the characteristic peak (red) of the ^{137}Cs source on the net spectrum (black) obtained after background subtraction and the spectrum corrected by applying the Savitzky-Golay filter (blue)

Secondly, ^{60}Co was placed 5 cm away from the detector. The result is as in the table below:

Table 5.15: Isotope Matching Results with Corrected Expected Lines for ^{60}Co

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	0	No match found	✗
^{60}Co	2	2	All reference lines matched	✓
^{22}Na	2	0	No match found	✗
^{152}Eu	8	1	Partial match (1/8)	▲
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

The peaks of ^{60}Co are fully detected and match a peak in ^{152}Eu . At this point, the code compares with the reference spectrum loaded for ^{152}Eu (Figure 5.48).

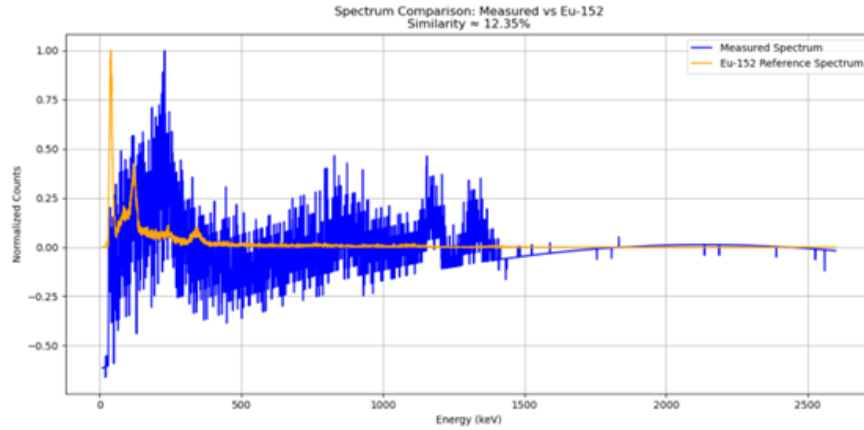


Figure 5.48: Comparison of the spectrum measured by the code (blue) and the normalized reference spectrum of ^{152}Eu (orange) (Measurement time: 5 min, Source-detector distance: 5 cm).

The function `compare_all_partial_matches` in the code compares the resulting spectrum with the reference spectra of possible source isotopes. For each isotope reference, the comparison is performed individually to determine the partial match scores. The correlation coefficient of Pearson (r), a statistical measure of similarity, is used for this purpose. The correlation coefficient of Pearson (r) ranges from -1 to 1 , where $+1$ indicates perfect linear agreement between two datasets, 0 indicates no correlation, and -1 indicates complete opposite correlation. If the reference and measurement spectra are in good agreement, the Pearson coefficient shows a high positive value; in case of disagreement, this value decreases. This method is one of the most widely used methods in the literature for automatic isotope identification and is preferred for its fast interpretation and insensitivity to normalization [89], [90].

In spectra where more than one source is present simultaneously, no reference spectrum gives a perfect correlation, so the scores for each isotope are

distributed at a moderate level between 0 and 1. The code detects even weak peaks by removing partial match scores. Figure 5.48 detects only the 121 keV peak of ^{152}Eu and shows a very weak similarity of 12.35% compared to the full spectrum. When ^{152}Eu was analysed, only the 121 keV peak was detected; the other peaks were not detected because they were weak (Table 5.16).

Table 5.16: Isotope Matching Results with Corrected Expected Lines for ^{152}Eu

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	0	No match found	✗
^{60}Co	2	0	No match found	✗
^{22}Na	2	0	No match found	✗
^{152}Eu	8	1	Partial match (1/8)	▲
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

However, compared to the reference spectrum, a similarity of 76.19% was observed (Figure 5.49).

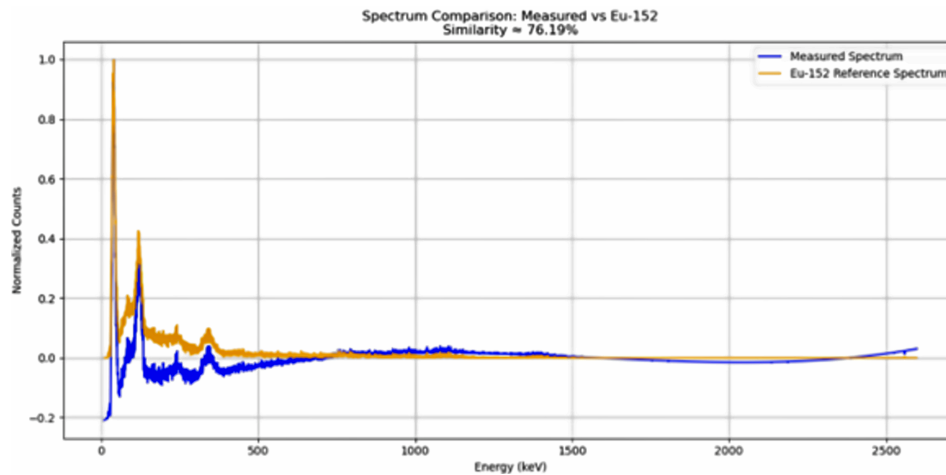


Figure 5.49: ^{152}Eu Spectrum Analysis and Reference Spectrum Comparison (Measurement time: 5 min, Source-detector distance: 5 cm).

In the code, a similarity of 85% and above is considered a strong similarity, 65% and above is considered an average similarity, and 30% and above is a weak similarity, indicating that this source could be ^{152}Eu with a moderate similarity. As the next step, the data collected by the robot were evaluated. The robot went to the estimated source location and collected data for about 5 minutes. Since the sources were weak in these experiments, 4 ^{137}Cs and 3 ^{60}Co were placed on top of each other as a single source. The robot was sent to the estimated source location in Path 1 for all algorithms, and the results are presented below. Firstly, with the Maximumlikelihood+Monte Carlo method, the total error was 4.64 cm (Table 5.4). The robot was 4.64 cm away from the source, and the results are as shown in Table 5.17:

Table 5.17: Isotope Matching Results with Corrected Expected Lines

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	1	All reference lines matched	✓
^{60}Co	2	2	All reference lines matched	✓
^{22}Na	2	0	No match found	✗
^{152}Eu	8	1	Partial match (1/8)	▲
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

The peaks of ^{137}Cs and ^{60}Co were correctly matched, and the code warned that it had found a certain correspondence with the ^{152}Eu reference spectrum. However, when it looked at the reference spectrum, it found a similarity of 19.36%. This low correlation indicates that a large part of the spectrum does not match ^{152}Eu .

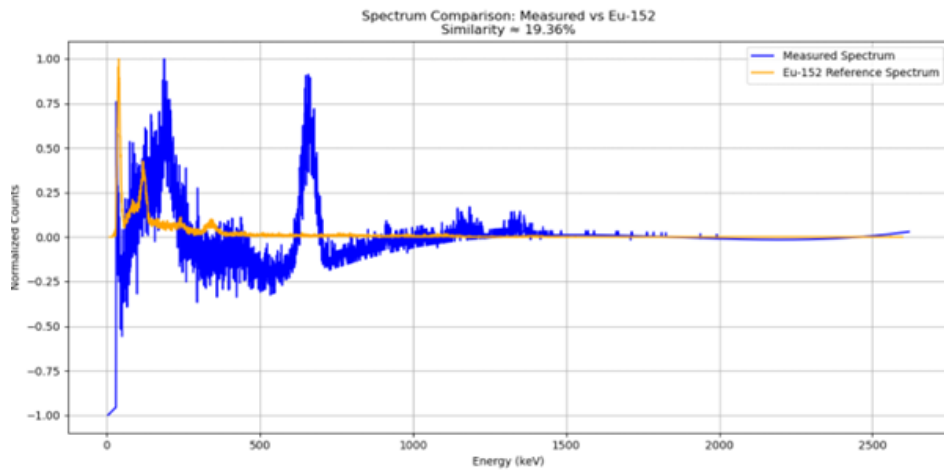


Figure 5.50: Comparison of the Measured Spectrum and the ^{152}Eu Reference Spectrum Based on the Estimated Position with the MLE+Monte Carlo Method

In the Bayesian method, the distance to the estimated source position was close to 1.51 cm (Table 5.5), so ^{137}Cs and ^{60}Co were observed quite successfully.

Table 5.18: Isotope Identification Results from Path 1 Measurements Using Bayesian Estimated Source Location (Corrected Expected Lines)

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	1	All reference lines matched	✓
^{60}Co	2	2	All reference lines matched	✓
^{22}Na	2	0	No match found	✗
^{152}Eu	8	1	Partial match (1/8)	▲
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

At 121 keV, a partial match for ^{152}Eu was again obtained, but with a very low rate (14.35%), which is very weak.

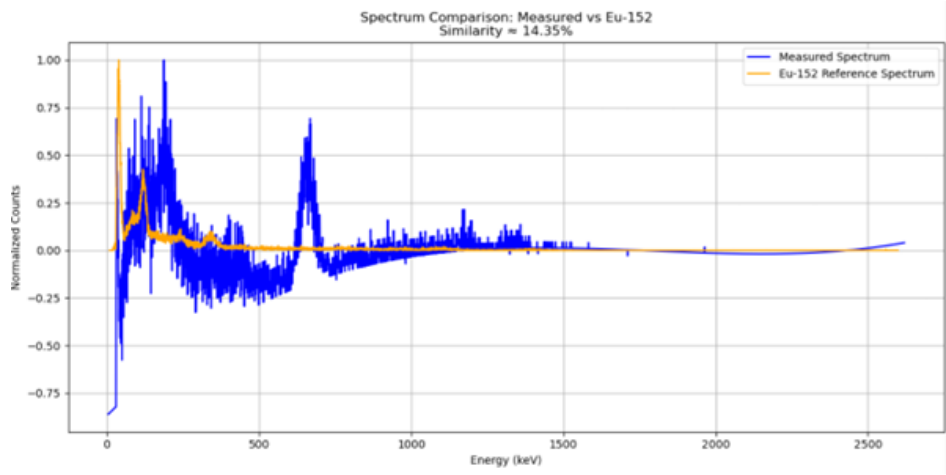


Figure 5.51: Comparison of the Measured Spectrum and the ^{152}Eu Reference Spectrum Based on the Estimated Position with the Bayesian Method

Finally, the Triangulation+Simulated annealing method showed an error of 1.73 cm (Table 5.7). The results are very similar to the Bayesian method due to the similarity of the error rates. ^{137}Cs and ^{60}Co were exactly matched, while ^{152}Eu showed a weak match of 15.32%.

When the estimated locations were close, the sources were successfully identified. As an example of weak signals and long-distance scenarios, for the MLE+L-BFGS-B method, where the error is higher, the path 4 error was taken as 11.9 cm (Table 5.3). In the results at this location, ^{137}Cs were successfully identified, while ^{60}Co was not fully detected because it was weaker.

Table 5.19: Isotope Identification Results from Path 4 Measurements Using Maximum Likelihood + L-BFGS-B Estimated Source Location (Corrected Expected Lines)

Isotope	Expected Lines	Matched	Status	Indicator
^{137}Cs	1	1	All reference lines matched	✓
^{60}Co	2	1	Partial match (1/2)	▲
^{22}Na	2	0	No match found	✗
^{152}Eu	8	0	No match found	✗
^{241}Am	1	0	No match found	✗
^{133}Ba	6	0	No match found	✗
^{44}Ti	3	0	No match found	✗
^{207}Bi	3	0	No match found	✗

A similarity of 30.38% was found for ^{60}Co . As the distance increases, the system has difficulty recognising weak sources.

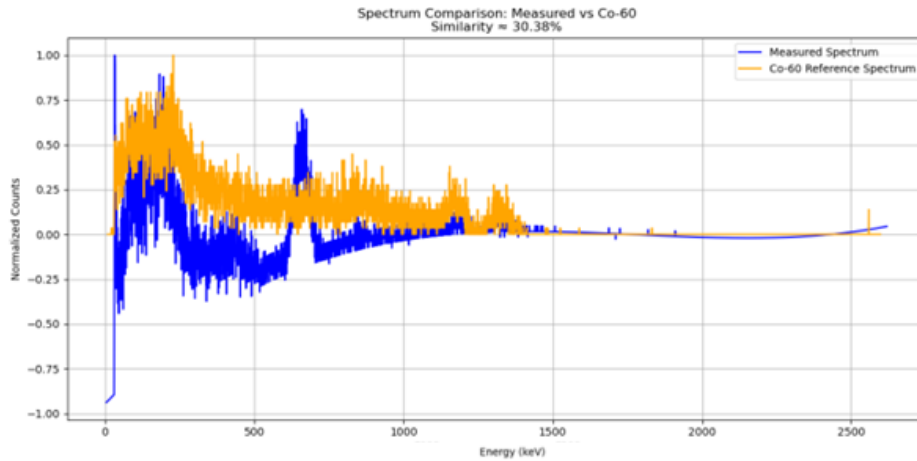


Figure 5.52: Comparison of the Measured Spectrum and the ^{60}Co Reference Spectrum Based on the Estimated Position with the MLE+L-BFGS-B Method

Experiments have shown that the developed code performs well for strong sources but has a specific error rate for weak sources. In addition, the distance to the source location limits the performance of the system.

5.7 Identification of Multiple Gamma Radiation Sources

Scenarios involving more than one radioactive isotope may be encountered in field applications. For example, different isotopes may be present simultaneously in a scenario where two radiation sources coexist. In this case, the overlapping of gamma peaks from these two sources complicates the resulting spectrum. In these cases, peaks may overlap, or signals from low-intensity sources may be lost among the dominant ones.

These are taken into account in the analysis of multiple sources. The analysis algorithm we have developed first finds out how many sources are in the system, and then code is added to identify these sources. The algorithm of the code is as in [Algorithm 8](#):

Algorithm 8 Gamma Spectrum Analysis for Multiple Radiation Source (Part 1/2)

```
1: Input:
2:   Background file: background.xlsx
3:   Measurement files: data/*.xlsx
4:   Isotope Database: List of isotopes with reference energies and tolerances
5: Output:
6:   Detected isotopes, Dose rate maps
7: procedure MAIN ANALYSIS
8:   // Phase 1: Background Processing
9:   Load  $B(E) \leftarrow \text{Read Excel}(\text{background.xlsx})$ 
10:   $B_{\text{peaks}} \leftarrow \text{find\_peaks}(B[\text{Counts}])$ 
11:  ▷ height, prominence, width
12:  // Phase 2: Measurement Loading
13:  for all file  $\in$  measurement_files do
14:     $S_{\text{raw}} \leftarrow \text{Read Excel}(\text{file})$ 
15:    Validate columns: {Energy, Counts,  $x, y$ , Dose_rate}
16:    // Background Subtraction
17:    Merge  $S_{\text{raw}}$  with  $B(E)$  on Energy
18:     $S_{\text{net}} \leftarrow S_{\text{raw}}[\text{Counts}] - B[\text{Background}]$ 
19:     $S_{\text{net}} \leftarrow \max(S_{\text{net}}, 0)$ 
20:    // Calculate Totals
21:    TotalCounts  $\leftarrow \sum S_{\text{net}}$ 
22:    Store results by  $(x, y)$  coordinates
23:  end for
24: end procedure
25: function FIND_PEAKS( $S_{\text{net}}$ )
26:   Apply Savitzky-Golay filter (window, order)
27:   Detect peaks with: height, prominence, distance
28:   Calculate FWHM using peak_widths()
29:   return {Energy, Height, FWHM}
30: end function
```

Algorithm 8 Gamma Spectrum Analysis for Multiple Radiation Source(Part 2/2)

```
1: Continued from Algorithm 8
2: procedure MAIN ANALYSIS (CONT.)
3:   // Phase 3: Isotope Detection
4:   for all  $(x, y) \in \text{coordinates}$  do
5:     // Peak Analysis
6:     PeakData  $\leftarrow$  find_peaks( $S_{\text{net}}$ )
7:     // Match Isotopes
8:     for all isotope  $\in$  IsotopeDatabase do
9:       Get reference energies  $E_{\text{ref}}$  and tolerance  $\Delta E_{\text{tol}}$  for isotope
10:      Matches  $\leftarrow$  FindEnergyMatches(PeakData,  $E_{\text{ref}}$ ,  $\Delta E_{\text{tol}}$ )
11:      // Validation Rules
12:      if Number of Matches  $<$  RequiredPeaks(isotope) then
13:        Discard
14:      else
15:        Confirm detection
16:      end if
17:    end for
18:  end for
19:  // Phase 4: Visualization
20:  PlotHeatmap(DoseRate)
21:  PlotSpectra( $S_{\text{net}}$ )
22:  GenerateDetectionReport()
23: end procedure
24: function FINDENERGYMATCHES(PeakData,  $E_{\text{ref}}$ , tol)
25:   Filter peaks where  $|E_{\text{peak}} - E_{\text{ref}}| \leq \text{tol}$ 
26:   Exclude matches near  $B_{\text{peaks}}$ 
27:   return Valid matches with prominence  $\geq 2$ 
28: end function
29: function REQUIREDPEAKS(isotope)
30:   return Minimum number of peaks required for reliable detection (depends
    on isotope)
31: end function
```

[Algorithm 8](#) first examines the spatial distribution of the dose rates measured at different points to see if there is more than one local maximum. If significant dose rate increases are observed in different regions, it analyses the spectra

at that point in detail and identifies the sources. Figure 5.53 shows how the dose rate changes according to location:

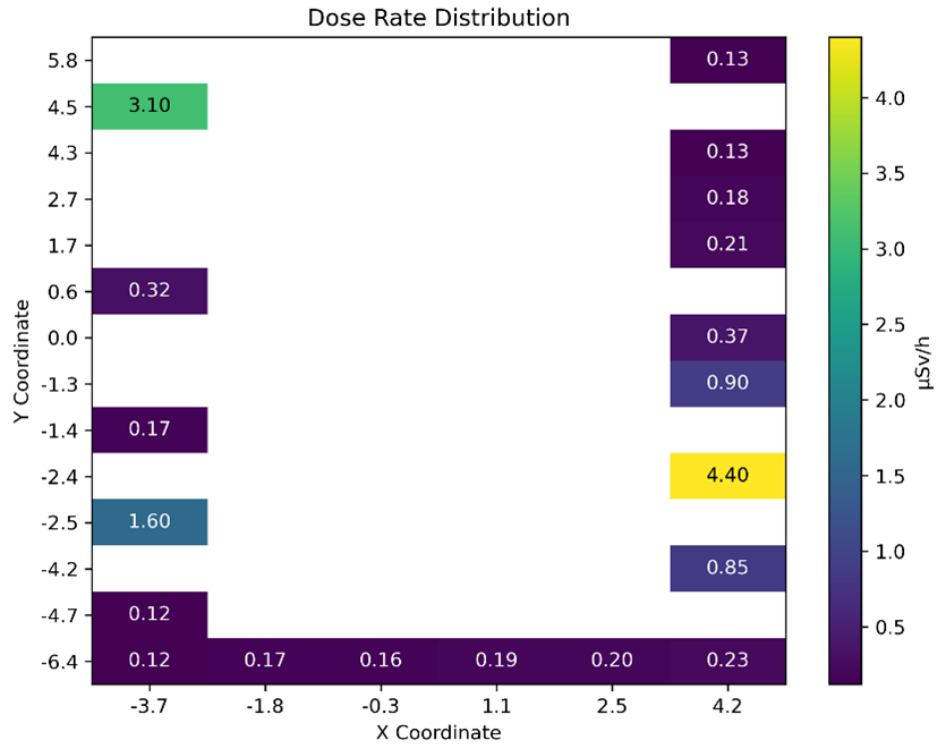


Figure 5.53: Spatial Dose Rate Distribution and Identification of Local Maxima

The spectra varying with location are as follows:

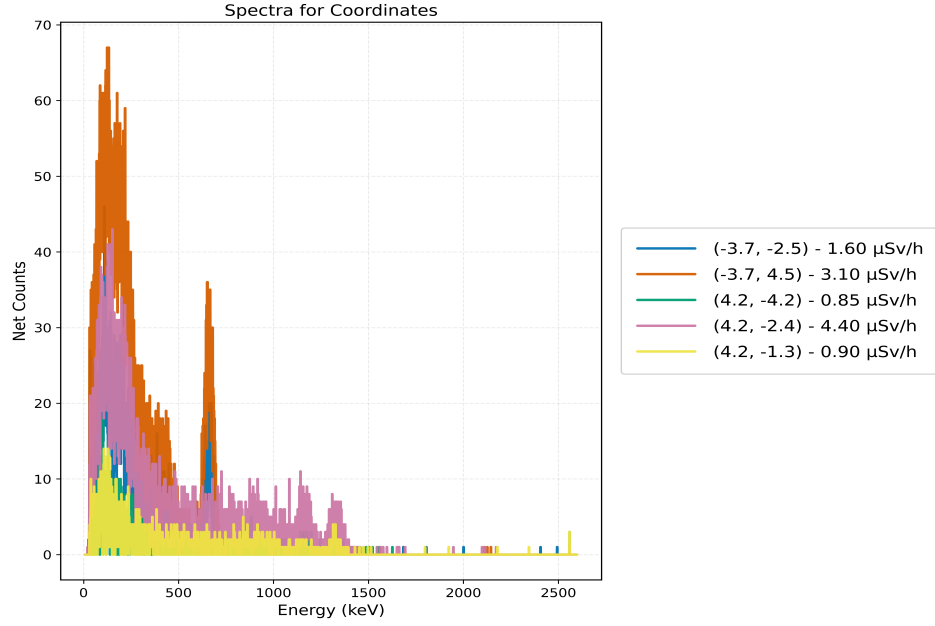


Figure 5.54: Spectra Measured at Different Coordinates

Spectra data collected at different measurement positions are shown in [Figure 5.54](#). In particular, at $(-3.7, 4.5)$, three ^{137}Cs sources are observed as a combined source, and at $(4.2, -2.4)$, four ^{60}Co sources are observed as a combined source. The algorithm analyses these local spectral densities to determine the presence of sources and perform detailed isotope identification. The code results in the following [Table 5.20](#):

Table 5.20: Spectral Analysis Results by Measurement Coordinates

Coordinate (x, y)	Detected Isotope(s)	Dose Rate ($\mu\text{Sv/h}$)
(-3.7, -6.4)	No detection	0.12
(-3.7, -4.7)	No detection	0.12
(-3.7, -2.5)	Cs-137	1.60
(-3.7, -1.4)	No detection	0.17
(-3.7, 0.6)	Cs-137	0.32
(-3.7, 4.5)	Cs-137	3.10
(-1.8, -6.4)	No detection	0.17
(-0.3, -6.4)	No detection	0.16
(1.1, -6.4)	No detection	0.19
(2.5, -6.4)	No detection	0.20
(4.2, -6.4)	No detection	0.23
(4.2, -4.2)	No detection	0.85
(4.2, -2.4)	Cs-137, Co-60	4.40
(4.2, -1.3)	No detection	0.90
(4.2, 0.0)	No detection	0.37
(4.2, 1.7)	No detection	0.21
(4.2, 2.7)	No detection	0.18
(4.2, 4.3)	No detection	0.13
(4.2, 5.8)	No detection	0.13

Following the spatial and spectral analysis, the algorithm detected two distinct radiation sources:

- Intense ^{137}Cs presence at (-3.7, 4.5)
- Intense ^{60}Co presence at (4.2, -2.4)

Detected isotopes: **Cs-137, Co-60**. Two sources are successfully detected.

^{137}Cs and ^{60}Co sources are used only once, and the result is as follows:

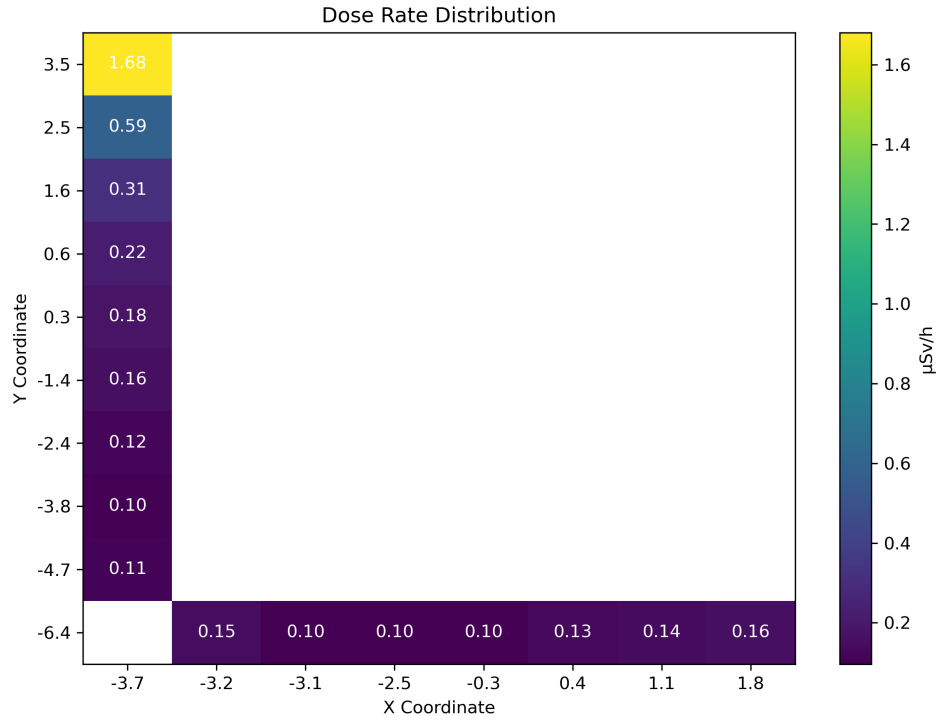


Figure 5.55: Dose Rate Distribution in Single ^{137}Cs and Single ^{60}Co Source Configuration

The dose rate distribution obtained from measurements using a single ^{137}Cs and ^{60}Co source is shown. In the figure, a significant dose increase is observed only in the region of the ^{137}Cs source. The ^{60}Co source was not detected due to low activity. The detector background noise and the short measurement time (5 minutes) made this detection difficult.

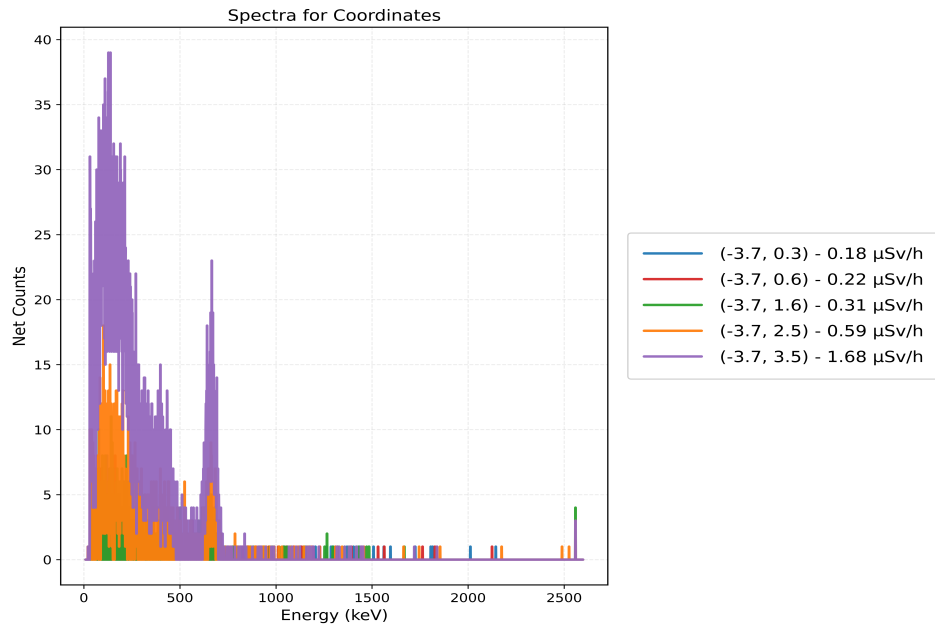


Figure 5.56: Measured Spectra for Single ^{137}Cs and Single ^{60}Co Source Configuration

Spectra obtained at different measurement locations are shown. The spectral analysis shows that only the characteristic peak of the ^{137}Cs isotope can be detected. ^{60}Co signals were lost in the detector noise. This result suggests that longer measurement times are necessary to detect low-activity sources or utilise stronger sources with higher activity levels, which are commonly encountered in real-world nuclear applications. The result of the code is reported in [Table 5.21](#).

Table 5.21: Detailed Detection Results for Single ^{137}Cs and Single ^{60}Co Source Configuration

Coordinate (x, y)	Detected Isotope(s)	Dose Rate ($\mu\text{Sv/h}$)
(-3.7, -4.7)	No detection	0.11
(-3.7, -3.8)	No detection	0.10
(-3.7, -2.4)	No detection	0.12
(-3.7, -1.4)	No detection	0.16
(-3.7, 0.3)	No detection	0.18
(-3.7, 0.6)	Cs-137	0.22
(-3.7, 1.6)	Cs-137	0.31
(-3.7, 2.5)	No detection	0.59
(-3.7, 3.5)	Cs-137	1.68
(-3.2, -6.4)	No detection	0.15
(-3.1, -6.4)	No detection	0.10
(-2.5, -6.4)	No detection	0.10
(-0.3, -6.4)	No detection	0.10
(0.4, -6.4)	No detection	0.13
(1.1, -6.4)	No detection	0.14
(1.8, -6.4)	No detection	0.16

Detected Radiation Sources: ^{137}Cs

As a result of measurements using only one ^{137}Cs source, ^{137}Cs was detected at the coordinates of interest; ^{60}Co was not detected. Due to the low dose rates, the detection success depends upon the measurement time, source strength, and detector sensitivity.

The algorithms' strengths for single and multiple sources are stability in energy calibration, speed of operation and high accuracy for single sources, while the limitations are that weak sources can be missed. These limitations are due to the physical capacity of the detector and the measurement conditions. The developed

method has shown that strong gamma sources can be identified with high accuracy, but performance degradation is expected for weak sources and at long distances. Detecting the dominant isotope in all tests is a significant achievement for the reliability of the system.

5.8 Activity Determination and Error Analysis

In the final stage of the study, the activities of the ^{137}Cs and ^{60}Co radioactive sources identified by spectrum analysis were calculated. However, before proceeding to the activity calculation, the total efficiency must be determined by correctly calculating the intrinsic efficiency and geometric efficiency values of the detector. In this context, in reference measurements made with only a fixed detector without using a mobile robot, the intrinsic efficiency $\varepsilon_{\text{int}} = 0.0225$ (%2.25) for the 661 keV peak of ^{137}Cs , 0.00711 (%0.07) and 0.00492 (%0.05) for the 1173 keV and 1332 keV peaks of ^{60}Co , respectively, were found. After calculating the intrinsic efficiency values, the sources were placed 5 cm and 10 cm away from the detector. The geometric efficiency of the source at a distance of 5 cm ($\Omega/4\pi$) is 5.38×10^{-3} , and at 10 cm it is 1.34×10^{-3} , with total efficiencies $\varepsilon_{\text{tot}} = \varepsilon_{\text{int}} \times \Omega/4\pi$. The calculations yielded an activity of 1.51×10^5 Bq (−0.3%) for ^{137}Cs and 6.46×10^4 Bq (0.0%) for ^{60}Co at a distance of 5 cm, which agrees with the actual activity values to within <0.5%. At 10 cm, the activity value for ^{137}Cs was 3.8% higher due to the wide ROI(Region of Interest). In contrast, the difference between the calculated activity value for ^{60}Co and the actual value was 0.4%. The calculated values are given in detail in the [Table 5.22](#). below.

Table 5.22: Comparison of calculated and certified source activities (with uncertainties)

Isotope	Source– detector distance	Net counts (60 s)	Calculated activity A (Bq)	Certificated activity A (Bq)	ΔA (Calc–Cert) (Bq)
^{137}Cs	5 cm	936	$(1.52 \pm 0.05) \times 10^5$	$(1.4921 \pm 0.0445) \times 10^5$	$+723 \pm 6.7 \times 10^3$
^{137}Cs	10 cm	219	$(1.57 \pm 0.11) \times 10^5$	$(1.4921 \pm 0.0445) \times 10^5$	$+5\,723 \pm 1.19 \times 10^4$
^{60}Co	5 cm	251 [†]	$(6.46 \pm 0.41) \times 10^4$	$(5.9983 \pm 0.253) \times 10^4$	$+2 \pm 4.82 \times 10^3$
^{60}Co	10 cm	63 [†]	$(6.49 \pm 0.82) \times 10^4$	$(5.9983 \pm 0.253) \times 10^4$	$+252 \pm 8.59 \times 10^3$

[†] Sum of net counts in the 1173 keV + 1332 keV full-energy peaks.

Uncertainties on calculated activities use counting statistics only ($u_A \approx A/\sqrt{N}$).

While measurements of 5 cm gave certificate values with an error of $< 0.5\%$ in both sources, the ^{137}Cs peak at 10 cm was 3.8% higher due to the wide ROI; in ^{60}Co , the uncertainty decreased to 0.4% after optimising the ROI.

The primary sources of error in determining the activity value are counting statistics, a 1 mm distance tolerance, and uncertainty in the γ -yield. The effects of these factors are summarised in Table 5.23 and Table 5.24 using measurement values for ^{137}Cs 5 cm as an example.

 Table 5.23: Uncertainty budget for the source–strength estimate (^{137}Cs , $d = 5$ cm)

Source of error	Symbol	Assumed Δ	Relative contribution
Counting statistics (peak route)	$\sigma_{\text{stat,peak}}$	$1/\sqrt{936}$	3.3 %
Counting statistics (total route)	$\sigma_{\text{stat,tot}}$	$1/\sqrt{18\,267}$	0.7 %
Distance: 50 mm $\pm$ 1 mm	σ_d	$\delta d/d = 0.02$	
		$\Rightarrow 2\delta d/d$ in $\varepsilon_{\text{geom}}$	4.0 %
γ -yield (661 keV)	σ_{I_γ}	± 0.006 on 0.851	0.7 %

Note. “Assumed Δ ” represents the assumed uncertainty or deviation for each parameter. It is used for error propagation to obtain the combined relative uncertainty shown in Table 5.24.

Table 5.24: Combined relative uncertainty for two calculation routes

Method	Combined uncertainty
Total-count route	$\sqrt{0.7^2 + 4.0^2 + 0.7^2} = \mathbf{4.2\%}$
Peak-only route	$\sqrt{3.3^2 + 4.0^2 + 0.7^2} = \mathbf{5.3\%}$

The difference between the calculated value and the certified activity value showed a deviation of less than 0.5% at approximately 5 cm. This result is consistent with the combined uncertainty results, indicating that distance and counting statistics are the primary sources of error.

In the next stage, the locations we found using the Bayesian algorithm with a mobile robot were used as a reference. For example, for measurements made from the estimated position with an approximate error of 1.5 cm found using the Bayesian method, reference measurements were performed using standard ^{137}Cs and ^{60}Co , sources with known activity, and the intrinsic efficiency coefficients corresponding to each energy peak were obtained experimentally. Based on these coefficients, the activities of the defined sources were calculated using their net peak counts. Here, unlike the first experiment, four different ^{137}Cs peak and three different ^{60}Co , sources were used. The total activity of the sources used in this experiment was calculated as 0.580 MBq. However, the activity value obtained from measurements performed using a mobile robot was 0.420 MBq, with an error rate of around 25%. This significant increase in error is due to the robot performing measurements based on the estimated source location rather than directly from the source centre. Deviations between the estimated source location of the robot and the actual source location alter the measurement geometry, directly affecting

the detected count rate and leading to systematic errors in activity calculations. Additionally, at very short distances, the detector does not comply with the $\frac{1}{d^2}$ calculation, and the detector area decreases due to finiteness and shadowing. At the same time, a very high counting rate in the same area causes pile-up and absolute coincidence summing. These three effects systematically reduce the actual count rate and distort the slope in the $\frac{1}{d^2}$ regression. In the raw data, the measured dose rate at points where $d < 0.8$ m reaches 1500–2000 cps, whereas at $d > 0.8$ m, the rate is approximately 600 cps. Here, the value of 0.8 m is used as the basis because the source-detector distance must be greater than 5–6 times ([91]) the detector diameter for the point source approach to work correctly. When the data with $d < 0.8$ m is excluded and the point source assumption is not applied to the region where it is invalid, the activity is found to be 0.507 MBq, reducing the difference from the actual activity to 12%. When different thresholds around 0.7 and 0.9 m were tested, no significant changes occurred, with the most consistent deviation of approximately -12% obtained at 0.8 m. These results reveal that the accuracy of activity determination in mobile robot-based measurements is critically related to the position accuracy in source localisation. Increasing the source localisation accuracy, especially in mobile systems, has been identified as a critical parameter that will directly improve the accuracy of activity determination. In this regard, optimisation of robotic localisation algorithms and increasing detector positioning accuracy in future studies are considered important steps that can further improve system performance.

Chapter Six

Conclusions

Recently, the use of mobile robots to protect human health and safety in nuclear facilities has increased. This thesis presents a systematic and comprehensive investigation into localising and identifying gamma radiation sources using a detector system integrated into a mobile robot. At the beginning of the study, several preparatory studies—both nuclear laboratory-based and simulation-based—were conducted to lay the groundwork for robotic applications. With the CsI(Tl) module gamma detector integrated into the mobile robot, a system has been developed for location finding, isotope identification and source activity estimation based on the data obtained from the environment. In line with this goal, different localisation algorithms have been examined together with optimisation methods in the location-finding process and analysed by integrating them into the system. In this study, as a first step, the three most frequently used algorithms for locating radiation sources in nuclear environments - Maximum Likelihood Estimation (MLE), Bayesian inference and Triangulation - are combined with various optimisation strategies in a holistic approach. The data collected by the detector

through the software infrastructure of the mobile robot has been analysed, and the performance of the algorithms under different conditions has been evaluated. As a result of the analyses, the highest accuracy and stability was achieved with the Bayesian method. Although other methods were successful in specific scenarios, they were limited, especially in low signal levels and noisy environments. The Bayesian approach was shown to outperform the other methods significantly. The work started with single-source localisation and was extended to multi-source scenarios using only detector data. In addition, the effect of robotic motion on detection performance was evaluated by comparative measurements with a fixed detector. Although the system was not as accurate as single-source detection in multiple-source environments, it performed acceptably with a margin of error of approximately 6 cm.

The second stage of the system is the rapid identification of isotopes of the gamma radiation source. The gamma spectrometry analysis algorithm was developed to perform rapid and effective radioisotope identification in the field. The analysis infrastructure was structured based on limited reference sources available at the University of Birmingham Nuclear Lab and tested in different scenarios. The system performed highly accurately in tests with particularly high activity strong sources. In scenarios involving multiple sources, both the spectrum profiles obtained at the measurement points and the dose rates were added to the analysis process to provide identification. However, when it comes to identifying weak sources, it was observed that the success of the system performance was limited due to factors such as detector sensitivity and measurement time. This situation reveals the limits of the system in terms of its applicability in the field.

In real-world scenarios, source strengths are typically higher than those employed in this study, suggesting that the current experiments represent a near worst-case scenario.

The radiation source was searched for in the robot-controlled test environment; after determining its location, the positioning process was stopped, and spectral data was collected to perform isotope identification. Thanks to the Bayesian approach, accurate estimations were made with an average of 2-3 cm positioning errors, and the detector was effectively aligned with the source. After this stage, the effect of the estimated position error on determining the source activity was analysed in detail. It was observed that the activity estimations obtained in the measurements made with the mobile robot had a higher margin of error compared to the measurements made with the fixed detector. This difference was attributed to the geometrical location sensitivity of the system and was evaluated as an important factor to be considered in practical applications. The findings obtained demonstrate the reliability and practical applicability of the methods used. In particular, it was observed that the Bayesian localisation algorithm worked with high accuracy even in environments with low signal/noise ratios; this result was consistent with theoretical inferences in the literature and was experimentally verified. The developed spectral analysis algorithm achieved success in terms of extracting meaningful results from low-resolution detector data. In multi-source cases, the approach that algorithmically analyses the change in the spectral distribution at each location provides an important contribution to solving a problem that has been addressed in the literature in a limited number of ways with an original method. However, some technical limitations were observed in the system. These

limitations include longer measurement times to detect weak sources, positioning errors on the moving platform affecting measurement accuracy, and suppression of weak sources in multi-source scenarios. These limitations should be considered in real field applications of the system and guide improvement and development studies for future studies.

In general, this thesis has introduced a mobile robot-based, low-cost original radiation detection and analysis platform that was successfully designed, implemented, and experimentally validated. The developed system has a vast potential for use in nuclear security and radiological emergency response areas. While this study presents original algorithm combinations at the academic level, it also presents an algorithmic framework that enables high-accuracy analysis with low-cost detectors in practice. The findings provide a solid basis for future research and contribute to filling methodological gaps in the existing literature.

Chapter Seven

Future work

The system developed within the scope of this thesis provides an important step in mobile robot-based radiation detection and analysis and offers many innovative research directions for future studies. Although the classical spectral analysis methods used in the thesis provided accuracy in certain scenarios, they were limited, especially for peaks at weak signal levels. For this reason, in future studies, the integration of machine learning and artificial intelligence-based algorithms into the system can shorten the analysis time and increase the accuracy.

In the future, isotope identity can be extracted directly from spectral data with deep learning-based neural network models, and even activity can be estimated. In the current literature, fuzzy logic-based algorithms and genetic algorithms approaches are shown as pioneering applications for peak analysis [90], [92]. Integrating such algorithms into the current system will increase real-time analysis capacity and significantly improve the detection of low-level radioactive sources [93],[94] .

In cases where data are limited, intelligent systems that can achieve high success with a small number of measurements can be created with methods such as transfer learning and simulation-supported data production. In this context, future studies will not only overcome the limitations of the current system but also develop more flexible, data-friendly, and field-applicable radiation monitoring systems. Another important future work area is expanding the robotic infrastructure of the developed system and increasing its autonomy. With multiple robot teams working simultaneously, large areas can be mapped quickly and in more detail. In this context, while mobile robots are used for detailed measurements, uncrewed aerial vehicles (drones) can rapidly scan from above [95]. In terms of hardware, equipping the system with different technologies, such as neutron detectors and thermal imaging systems, will expand the scope of radiation detection and enable the formation of a structure that can respond to multiple risk scenarios [96]. Thanks to all these integrations, the system will be applicable for laboratory studies, industrial applications, and real-time emergency response scenarios. The current system uses a method based on matching known peak positions in the gamma spectrum in the isotope identification process. However, advanced analysis methods are needed for more complex spectral structures and rare isotopes. With deep learning-based models, the determination of isotope profiles from raw spectral data and the accuracy of the system can be significantly improved beyond classical methods. It is aimed that the algorithms to be developed in the future will be designed in a way that can not only identify but also perform quantitative analysis, i.e. estimate the relative activity rates of more than one isotope [92]. In order to detect weak sources, methods such as statistical signal processing and advanced filtering techniques that improve the signal-to-noise ratio should

be integrated into the system. In this way, the system will appeal to a broader range of applications. Another issue that can be developed is adaptive route planning and intelligence-based search strategies that enable robots to work with more innovative search strategies. The route in the current system is made by a human steering the robot with a joystick. Current research has enabled the robot to optimise its route online with artificial intelligence-based approaches such as reinforcement learning. For example, when the robot detects high radiation, it can make more intensive measurements in that area and scan the low signal area faster, saving time. Ultimately, integrating such intelligent and adaptive strategies will enable robots to find sources faster and produce reliable radiation maps over larger areas with minimum energy and maximum efficiency [97].

In conclusion, the findings in this thesis have not only demonstrated the current success of the system but also revealed many innovative research areas for the development of radiation detection systems in the future. Machine learning-based spectral analysis, coordinated operation of multi-robot systems, development of hardware with different systems, advanced isotope recognition approaches and adaptive route planning stand out as basic development areas that can be evaluated in future studies. Thanks to the studies to be carried out in these areas, the limits of classical methods will be overcome, and beyond increasing the performance of the current system, it will pave the way for high-performance systems in radiation monitoring technologies. By combining artificial intelligence algorithms with robotic automation and advanced signal processing technologies, faster, more flexible, and more reliable innovative radiation monitoring solutions can be developed. Thus, the system will become one that protects human health, minimises

environmental risks and increases efficiency in industrial applications. Future research will transform the theoretical infrastructure of this system into applicable solutions in field conditions and make it possible to create more comprehensive usage scenarios.

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