A NEW GREEN BOND AUGMENTED MEASURE OF MONEY: THEORY AND PRACTICE

by

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Abstract

This thesis presents research within empirical monetary economics with a focus on the integration of monetary aggregates and their price dual with green bonds. Chapter I incorporates bonds, including green bonds, alongside traditional monetary assets in the construction of a broad Divisia monetary aggregate for the USA using the Törnqvist-Theil discrete time approximation for the continuous time Divisia Index. The user costs for calculating this index are risk-adjusted and forecasted for capital uncertain assets. The analysis reveals a significant and positive correlation between a broad Divisia monetary aggregate for the USA, including green bonds, and the detrended output gap. Consequently, the broad Divisia monetary aggregate augmented with green bonds can usefully serve as a monetary policy indicator. Green asset augmented Divisia monetary aggregates also have the potential to provide feedback to central banks on an evolving economy and aid in maintaining economic stability and addressing potential political pressures.

Chapter II contributes to the monetary vector autoregressive (VAR) literature as the first work to employ a newly green augmented Divisia price dual as a policy indicator for the USA. Three empirical results are obtained to support the use of green price dual as the policy indicator variable. First, policy shocks have significant effects on both output and price level. Second, user cost is closely correlated with the Federal Funds rate and could be an alternative for that rate as a policy indicator following the Taylor rule. The price dual is useful when the Federal Funds rate becomes stuck at its zero lower bound (ZLB) after 2008. Third, we develop a new VAR model that is not subject to the price puzzles.

Chapter III aims to examine the forecasting performance of newly constructed green-benchmarked Divisia monetary aggregates for the USA output gap. By using the green bonds as the benchmark asset, we successfully construct the green-return benchmarked and green-coupon benchmarked monetary aggregates with the rate of return on 20Y+ green bonds and the coupon rate of 20Y+ green bonds as the benchmark rate, respectively. We also construct the conventional Divisia monetary aggregate and the traditional simple sum monetary aggregate for comparison. We then employ the Markov regime switching vector autoregressive (MS-VAR) model to test the forecasting ability of these monetary aggregates in output gap. The green-benchmarked Divisia MS-VAR models are found to be superior in one-month, three-month, six-month and nine-month ahead forecasts.

Keywords: Monetary aggregation, green bonds, green price dual, aggregate demand, output forecasting, *Divisia*

DEDICATION

In memory of my late grandfather Youshan Li and late grandmother Yunhui Suo

Your everlasting love always remains with my journey …

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Introduction

1 Introduction

This thesis is set in the field of green monetary economics. In recent years, the Federal Reserve System has started to address climate change, including the creation of two internal committees focusing on the issue. It has also joined the Network for Greening the Financial System, a group of global central banks aimed at addressing the systemic risk climate change poses for the financial sector. The Federal Reserve does have narrow, but important, responsibilities regarding climate-related financial risks. These responsibilities are tightly linked to the responsibilities for bank supervision. Indeed, the public reasonably expects supervisors to require that banks understand, and appropriately manage, their material risks, including the financial risks of climate change[1]. Recent innovations in the bond markets designating funds raised by the issuance of certain bonds for specific environmental projects have led to a new source of liquidity in the economy, Zerbib (2019), and may also be bringing new savers to the financial markets. By the end of 2023, the global issuance volume of green bonds had reached USD 2.334 trillion[2]. These bonds are generally called green bonds though some are sometimes called climate change bonds. The advent of green bonds raises many issues including whether measures of liquidity or money in the economy should include data on green bonds. An important aspect of this work has been to construct a new green augmented monetary aggregate.

Central banks normally publish their economies' monetary aggregates such as M1, M2, M3 or broader monetary aggregates time series. These monetary aggregates are called simple sum aggregates, which is simply the sum of the quantities of all monetary assets in circulation. This simple sum aggregation implicitly assumes that all the component assets are perfect substitutes, which is unrealistic and theoretically flawed. Based on the microeconomic aggregation theory and index number, Barnett (1978, 1980) originated the Divisia monetary

[1] See Panel on "Central Bank Independence and the Mandate - Evolving Views" Remarks by Jerome H. Powell Chair Board of Governors of the Federal Reserve System at the Symposium on Central Bank Independence Sveriges Riksbank, Stockholm, Sweden.

[2] Source: https://www.climatebonds.net.

aggregates, which apply different weights to different assets in accordance with the degree of their contribution to the flow of monetary services in the economy.

In his seminal 1980 paper, Barnett has continually argued for as broad a Divisia monetary aggregate as possible. A rationale for this is to internalize as many asset substitution effects as possible in the aggregate. Recently revealed preference empirical results reported by Hjertstrand et al. (2016) and Binner et al. (2018), who included bonds and Hjertstrand et al. (2023), support the use of a broad monetary aggregate for the USA. In line with these research, the aim of the first chapter of this thesis is to include assets beyond data gathered by a central bank to construct a broad Divisia monetary aggregate. We do this by including various categories of bonds, including green bonds.

The mainstream approach to monetary policy is based on the New Keynesian (NK) model and is expressed in terms of interest rate rules proposed by Taylor (1993). However, the 2007 Financial Crisis and the following protracted 7-years effective-lower-bound (ELB) period highlighted the shortcomings of using the Federal Funds rate to gauge the stance of monetary policy. The price dual to an economic quantity aggregate, which is a function of all interest for all monetary components, has not been subject to a lower bound constraint. It is found that the price dual is highly correlated to the Federal Funds rate and could be useful as an indicator of the economic activity (Keating et al., 2019). Hence, the goal of the second chapter is to develop a vector autoregression (VAR) model of monetary policy shocks, which can be easily interpreted and employed after the financial crisis and in the ELB period, using a green Divisia price as the policy indicator.

The Divisia monetary aggregates are found to be superior in forecasting gross domestic product (GDP) in a handful of studies, see Barnett et al. (2016) and Barnett and Park (2023). In the analysis of the third chapter of the thesis, we employ the non-linear Markov Regime Switching VAR (MS-VAR) model proposed by Hamilton (1989) to empirically assess the out-of-sample forecasting performance of the new green-benchmarked Divisia monetary aggregates on GDP output gap (from here on, referred as output gap) for USA. In this case, we use long term green bonds as the benchmark asset to produce the green user costs for the monetary assets.

These chapters have in common that they are both empirical in their nature, and that they integrate green bonds with the Divisia monetary aggregates and concomitant prices to show the implication of monetary policy with the consideration of climate change.

1.1 Disposition

As indicated above, the thesis is divided into five parts: introduction, three chapters and conclusions. To give the reader an overview of the work, a brief introduction to each chapter is given.

The objective of chapter I is the composition and construction of a newly green augmented Divisia monetary aggregate for the USA. We chose the USA as the target country as it has one of largest and most influential green bond markets in the world and is one of the first countries to engage in green bond issuance. The data for this study covers the period from 2009Q1 to 2021Q12 as the start date is the earliest date for the green bond issuance in the USA. We start with the identification of the green augmented monetary aggregates using revealed preference tests, which include green bonds, government bonds and corporate bonds. The results show that there exists a weakly separable grouping of monetary assets and bonds that forms a monetary aggregate consistent with economy theory. This supports the inclusion of green bonds, government bonds and corporate bonds in the broad monetary aggregate. We then construct them using the Törnqvist-Theil discrete time approximation of the continuous time Divisia index. We further explore the time-series properties of the green Divisia money in IS curve[3] specifications and compare it with the green simple sum measure. Our findings suggest

[3] IS curve stands for investment-savings curve, which describe the relationship between output and interest rate.

that the green augmented Divisia monetary aggregate does provide additional information for aggregate demand in the USA, which is not presented in short-term interest rates.

In the second chapter, we turn to the analysis of the effect of green price dual on the macroeconomic variables. Different from the studies using Federal Funds rate as the policy indicator which could produce empirical puzzles, we develop a new method for identifying monetary policy shock that is free from puzzles and not subject to lower bound constraints for the USA. Following the construction of the green augmented Divisia monetary aggregates, we construct the green price dual to this green Divisia quantity aggregate and use the green price dual as the policy indicator in a structural VAR model. In comparison, we consider the alternative policy indicators, including the Federal Funds rate, the government bonds rate, the corporate bonds rate and the simple sum user cost which is the average of the interest rates of all monetary assets. In this empirical analysis, the findings indicate that the green price dual is able to measure the effects of monetary policy and is free from the price puzzles.

The third chapter focuses on the examination of the forecasting performance of the green-benchmarked Divisia monetary aggregates on the output gap for the USA based on the IS curve specification. Unlike the green augmented Divisia monetary aggregates constructed in the previous chapters, we construct a newly green-benchmarked Divisia monetary aggregate. The green-benchmarked Divisia monetary aggregate is the measure of the monetary services using long-term green bonds as the benchmark asset to produce the green user costs for monetary assets. We consider the coupon rate of 20Y+ green bonds and the rate of return on 20Y+ green bonds as the benchmark rate, which could reflect the pure green investments. We also construct the conventional Divisia monetary aggregate and the simple sum monetary aggregate for comparison. In this chapter, we use the monthly reported weekly economic index (WEI), rather than the quarterly reported GDP, as the real sector variable in the model because monthly data allow for closer tracking of the short-term economic trend fluctuations. The estimation is based on the period from 2009M1 to 2019M9 as we exclude the effect of the COVID-19 pandemic, and the out-of-sample forecasting period is from 2020M1 to 2022M12, leaving three observations for validation. For the forecasting method, we use the non-linear MS-VAR model, as it is able to capture the non-linear dynamic relationships embedded in the real-world. We also compare our results from the MS-VAR models with those from the VAR models. We consider the one-month, three-month, six-month and nine-month ahead forecasts for all models. Finally, we include the financial condition variables as controls in the models to investigate the shift of preferences for investors towards the green investments under the different economic conditions. The results indicate that the green-benchmarked Divisia MS-VAR models are the best models for the output forecasting.

The final part of this thesis concludes by pointing out the main findings and policy implications and gives suggestions for future work.

1.2 Contributions and Limitations

The contribution of this thesis is threefold.

- Firstly, it contributes to the monetary aggregation literature by extending the monetary components to include the environment-related bonds, government bonds and corporate bonds, and constructing the new green Divisia monetary aggregates and the green price dual. Our new green economic monetary aggregates successfully examine the data beyond existing central bank data for the first time. These extensions are especially useful when the money and financial markets are expanding and providing greater liquidity services, thus our work is particularly timely. The inclusion of green bonds highlights the consideration of climate change in policy implications.
- Secondly, as the first work to employ a new green augmented Divisia price dual as the policy indicator in the monetary VAR literature, this thesis highlights the informativeness

of our new green Divisia user costs as the indicator for real output and inflation in the short term by taking environmental factors into consideration.

- Thirdly, with the use of green bonds as a new benchmark asset, this thesis contributes to the existing literature by offering a new measurement of US Divisia monetary aggregates, examining the performance of green-benchmarked Divisia monetary aggregates on the economic activity and underlining its superiority through comparison with conventional Divisia monetary aggregates and their simple sum counterparts.

In the context of presenting the contributions of this research, it is appropriate to also point out the limitations of this thesis. All the work presented here is empirical in its nature; however, the empirical analysis always comes with important limitations with respect to data. For example, the availability, quality, processing and sample size of data have been questioned in empirical studies (Schennach, 2016). Additionally, the inherent unpredictable properties of the datasets can never be known beforehand. For example, the sample size utilized in this research is relatively small in comparison to other studies, a limitation primarily attributed to the constrained availability of green bonds.

Particularly, the dataset used in this research encompasses two particularly notable periods: the aftermath of the global financial crisis and the COVID-19 pandemic. These periods are marked by extraordinary economic disruptions and substantial monetary and fiscal interventions, which might limit the broader applicability of the empirical findings. The unprecedented nature of these crises, characterized by significant policy responses aimed at economic stabilization, implies that the financial market behaviours and economic indicators during these times may not represent typical conditions. Furthermore, the market for green bonds remains relatively small, constituting only a modest segment of the overall bond market. This limited market size poses challenges for empirical analysis, as the data on green bonds may not fully capture their potential long-term impacts or broader market trends. Consequently, while the study provides valuable insights into the role of green bonds during these crises, caution is advised when generalizing these results to more stable economic periods. This context highlights the necessity for ongoing research as the green bond market evolves and expands, potentially altering its influence on financial stability and economic performance.

Chapter I A Broad Green Bond Augmented USA Divisia Monetary Aggregate and Aggregate Demand

1 Introduction

The recent changes in the innovations of financial products and the deregulation of financial institutions raise a concern that the monetary aggregates by simple summation are unstable, either as an indicator of policy action or as an instrument of policy rule (Mullineux, 1996; Binner et al., 2004). One of the main conclusions often coming up following these changes is the need to redefine and remeasure money, since the simple sum aggregates as the official aggregates have an invalid assumption of perfect substitutes among monetary assets. An attempt to improve this measure of money is assigning some weights to monetary components according to their “moneyness” (Friedman and Schwartz, 1970). This method, however, fails to identify that the weighted aggregation still implies the perfect substitutability with the linear aggregation procedure.

Using microeconomic aggregation theory to define money is an appealing alternative to this linear summation. This idea originated from Barnett (1980), who treats assets as durable goods and derives the user costs of monetary assets to construct the Divisia monetary aggregates. His construction begins with the introduction of money into the consumer’s utility function, then the aggregation theory provides methods to choose which monetary assets are included and how to estimate aggregator functions. In order to avoid the problems associated with the misestimation of aggregator functions, index number theory provides a parameter-free approach to perform aggregation. In this regard, index number theory gives a class of quantity and price indexes that are calculated solely from the quantity and price data, obviating the complex procedures in parameters’ estimation. The tests for statistical properties are carried out by Fisher (1922) to evaluate the quality of statistical indexes. He discovered that the Fisher Ideal Index is the best formula, while the simple sum index is the worst. Another index, the Divisia index (Divisia, 1925), is found to have a large number of desirable properties. Applying these theories, Barnett (1980) proposed the Divisia monetary index for tracking the true flow of monetary services more accurately, which is commonly used in the monetary aggregation literature.

After the introduction of the Divisia index, a considerable literature has grown around the theme of the comparison between the simple sum aggregates and Divisia monetary aggregates as macroeconomic indicators. Many studies have shown that the differences in these two measurements can explain the "Barnett critique" – the measurement problems related to the failure of finding a significant relationship between money and macroeconomic variables. The use of Divisia monetary aggregates can solve the "Barnett critique". These studies also support that the Divisia indices contain more information than the simple sum measures. See, for example, Serletis and King (1993), Schunk (2001), Barnett et al. (2016), Barnett and Su (2015), Barnett and Tang (2016), and Tang et al. (2020), among others. Following this promising line of research, we use a superlative number to construct the aggregates we identify for the USA.

Fewer monetary studies have identified the components of an economic aggregate. Swofford and Whitney (1987, 1988, 1994), Fleissig and Whitney (2003, 2005), De Peretti (2005), Jones and De Peretti (2005), Mattson and De Peretti (2018), and Jadidzadeh and Serletis (2019) used the revealed preference approach to test the asset groupings on USA data; Elger et al. (2008) and Binner et al. (2009) also apply this method to determine the optimal level of monetary aggregation in the UK and the Euro area respectively; Binner et al. (2018) used the weak separability tests to examine what assets should compose an economic monetary aggregate for the UK and USA. In this chapter, we identify the components in economic monetary aggregates for the USA, to construct them with a superlative index and to report the empirical analysis of their time-series properties.

In recent years, there have been a number of related developments to address the problems in the construction of monetary aggregates. The one that stands out is the assumption of certain returns on assets laid out by Barnett (1978, 1980). The returns on assets are known with certainty at the start of each period under his framework. However, Rotemberg and Poterba (1987) argued that if the risky assets such as shares and bonds are added into monetary aggregates as they provide monetary services in the market, the contemporaneous interest rate risk of these assets in combination with risk aversion would make the construction of monetary aggregates more complex because the returns of risky assets are uncertain at the end of the period. This insight has sparked a new line of research in monetary aggregate literature to precisely measure money with the consideration of risk.

Barnett et al. (1991), Barnett and Zhou (1994) and Barnett (1995) firstly conduct studies on monetary aggregates under risk. They compare the tracking abilities of various index numbers to unknown risk-accounted aggregator functions. Their results show that although there is a minor difference on estimated aggregator functions with and without risk, it might be appropriate to include risky assets in monetary aggregates. Barnett et al. (1997) then continued the work on risk adjustments in monetary aggregates. Based on the consumption capital asset pricing model (CCAPM) framework, they developed a method to derive a generalised Divisia index with the risk-adjusted user costs. The risk-adjusted user cost formula depends on an expected return and an estimate of a relative risk reversion coefficient. They noticed that the risk adjustments for the assets are slight in the USA monetary aggregates. Using a more general utility function, Barnett and Wu (2004, 2005) extended the user cost risk adjustments to the case of multiple non-monetary assets and intertemporal non-separability. Their results lead to a substantial and more accurate risk adjustment compared with the risk adjustments in 1997's work. This extension is also supported by a recent work of Barnett and Su (2019) that derives the risk-adjusted user costs for credit cards augmented monetary aggregates. More recently, Binner et al. (2018) expanded the studies of Barnett and Wu (2004, 2005) by relaxing the assumption of a one-year planning horizon and introducing predicted returns on risky assets to construct risky monetary aggregates for the USA and UK. This relaxation is reasonable since the risky assets are the least liquid assets among monetary assets and thus, the investors would not change their portfolios quickly. Accordingly, the forecasted returns are more accurate than the single time ones.

No attention has been paid to the recent technological changes and innovations in the green financial sector. The green bonds, which are issued to support environment-related projects, would inject liquidity into financial markets (Zerbib, 2019). By the end of 2020, the global issuance volume of green bonds had reached USD 269 billion from the climate bonds initiative website[4]. These developments have increased the substitutability between risky assets, such as green bonds and assets more commonly included in monetary aggregates. The existing literature on the study of the construction of monetary aggregates is mainly from the perspectives of capital certain monetary assets. It is worth noting that fewer studies examined the inclusion of risky assets into monetary aggregates. As mentioned by Binner et al. (2018), risky assets contribute to the economy's liquidity in ways similar to those of money. The unanswered question in the monetary aggregates' construction is whether or not the green bond, as a new classification of risky assets, is feasible to be added into monetary aggregates under the consideration of environmental issues. In this case, the inclusion of green bonds into money measures may provide additional information. We adopt the same method as Binner et al. (2018) to account for green bonds, government bonds and corporate bonds – the assets that are similar to the ordinary bonds in their study.

In recent years, because climate-related financial risks are likely to have threatened the stability of the financial system and economic development, central banks have shown an increasing interest in seeking appropriate tools to guide the decision-making process of the monetary policy committee by considering the macroeconomic impact of climate change (Bank

[4] Source: https://www.climatebonds.net.

of England, 2018, p.3). As a vital role in conducting monetary policy shown above, the monetary aggregates therefore can be considered.

In fact, Belongia and Ireland (2016) describe how the current stance of monetary policy only accounts for its effect on interest rates but ignores the fluctuations in monetary aggregates. They suggest that a proper measure of money, such as Divisia monetary aggregates, could play an important role in the conduct of monetary policy. Many recent studies on monetary policy rules are based on small-scale macroeconomic models that include an IS curve specifying the output gap as a function of the real interest rate. In the IS equation, the real money stock or its growth rate does not appear. In the paper by Nelson (2002), he has challenged these specifications and argues that these models neglect money as an important channel of the monetary effects on output from theoretical and empirical perspectives. His findings indicate that real money growth has a sizable and significantly positive effect on aggregate demand for the USA and UK. Reimers (2002) and Binner et al. (2009) provide similar evidence for real Divisia money growth for the Euro area. We aim to test whether our green money could add additional information on aggregate demand for the USA or not.

This chapter takes the construction of green money as the research objective with the consideration of environmental bonds. We will provide evidence on the question of whether the green bonds can be added to the monetary aggregates. In this regard, we contribute to the existing literature by offering a new measurement for USA Divisia monetary aggregates, examining the performance of green Divisia monetary aggregates on economic activities and underlining their superiority through a comparison with conventional simple sum green ones. We firstly identify admissible monetary aggregates which include green bonds, government bonds and corporate bonds for the USA by using a non-parametric weak separability test. We then construct green-augmented Divisia, traditional Divisia, green-augmented simple sum and traditional simple sum monetary aggregates for admissible groupings of USA monetary assets.

Finally, we evaluate our green money as a potential indicator in the IS curve. We find that the admissible green-augmented Divisia and simple sum monetary aggregates have direct effects on aggregate demand with the IS curve specifications built by Reimers (2002). Thus, our green money does provide additional information for aggregate demand in the USA economy. Therefore, we could meet the demand of Bank of England (2018, p.5) to provide a fresh framework to consider the impact of climate change on the whole economy, and update and improve the current remit for the monetary policy committee to propose a way forward to augment its decision-making capability of balancing the relationship between climate sustainability and economic stability.

The remainder of this chapter is organized as follows. Section 2 summarizes the theoretical foundations and the development process of monetary aggregates. Our data and methodology are described in the Section 3 and 4. Section 5 presents the results from weak separability tests and the construction results. Section 6 provides the IS curve specifications, while conclusions are given in Section 7.

2 The Theoretical Foundation of Monetary Aggregates

2.1 The Microeconomic Aggregation Theory

The theory of monetary aggregation (Barnett, 1978, 1980, 1992) is based upon an optimization framework in which the monetary assets are viewed as durable goods in the individual's utility function. Assuming that the services of monetary assets and other goods enter as arguments of these in the utility function, the utility function can be written as:

$$u_t = U(m_t, z_t) \tag{A.1}$$

where m_t is a vector of the monetary assets and z_t is a vector of all other variables on time t. The utility function in equation (A.1) is assumed to be maximized subject to an income constraint of:

$$q_t' z_t + \pi_t' m_t = y_t \tag{A.2}$$

where y_t is the total income reflecting expenditures on goods and services as well as on time t; q_t denotes the vector of the prices of other variables on time t; π_t denotes the nominal user costs (or rental prices) of monetary assets on time t.

To illustrate the details of the demand for monetary services, the two-stage theory is needed to underscore, which describes a sequential expenditure allocation. In the first stage, the consumer allocates his expenditure among broad categories (i.e., monetary assets and other variables). In the second stage, the consumer decides the quantities for each component assets within each category. The consumer is guided by price indices among the two categories in the first stage, while he responds to the change in the relative prices of the monetary assets (π_i/π_j as defined above) in the second stage.

The fundamental existence condition for the decomposition of the consumer choice problem is that the consumer's preferences are weakly separable in the services of monetary assets, then we can write the utility function as:

$$u_t = U(f(m_t), z_t) \tag{A.3}$$

in which *f* is the monetary sub-utility function. The margin rate of substitution between any two monetary assets inside *f* must be independent of the quantities of z_t in order for the weak separability to exist. Under the weak separability, it is possible to consider the allocation over monetary assets in *f* alone and continue to the framework of following consumer problem: max $f(m_t)$ subject *to* $\pi_t' m_t = m_t^*$ (m_t^* denotes the total expenditure on monetary services, which is determined in the first step of the two-stage optimizing problem). Consequently, the optimal quantities of those assets can be derived given only data on quantities and prices.

2.2 The Weak Separability Tests

The weakly separable tests can be carried out either in a parametric or in a nonparametric framework. The parametric approach requires the estimation of a functional form with unknown parameters. Under this case, the structure of the preference over monetary assets and the functional form are tested, which complicates the test procedure and brings problems associated with model misspecification. In contrast, the nonparametric (NONPAR) revealed preference procedure by Varian (1982, 1983) does not require any calculations for a functional form; therefore, it avoids the associated problems with parametric approach.

The NONPAR technique, on the other hand, possesses a number of undesirable features and has been criticized in the literature. First, the procedure is non-stochastic, and a single rejection could result in a total rejection. If the data is rejected, it is impossible to test for separable subgroupings, despite the fact that this rejection is caused by some measurement errors. Second, some pessimistic results drawn from Monte Carlo experiments on weak separability tests by Barnett and Choi (1989) are biased towards rejection. Even so, this methodology is still employed in the monetary studies to identify the components of an economic aggregates, for example, Swofford and Whitney (1987, 1988, 1994), Fleissig and Whitney (2003, 2005), De Peretti (2005), Jones and De Peretti (2005), Mattson and De Peretti (2018), and Jadidzadeh and Serletis (2019) use this approach to test the asset groupings on USA data; Elger et al. (2008) and Binner et al. (2009) also apply this NONPAR reveal preference method to determine the optimal level of monetary aggregation in the UK and the Euro area respectively. Although the NONPAR approach is biased toward rejection, the problems arising from the estimations of functional forms are more serious. Hence, in order to avoid these problems in our research, we adopt the NONPAR revealed preference test in this chapter.

The Varian (1982, 1983)'s test is directly built upon the generalized axiom of revealed preference (GARP). GARP can be stated:

$If\ x^i R\ x^j\ then\ p^j x^j\ \leq p^i x^i\ for\ all\ i, j = 1, \ldots, n.$

where $p^i = (p_1^i, \ldots, p_k^i)$ be the ith observation for the prices of the k goods, $x^i = (x_1^i, \ldots, x_k^i)$ denote the corresponding quantities for the k goods and R stands for revealed preferred. If the data satisfy GARP there exists a concave, monotonic, continuous utility function that rationalizes the data.

Let $\mathbf{m}^i = (m_1^i, \ldots, m_n^i)$ denote the ith observed real quantities for a set of n monetary assets and let $\boldsymbol{\pi}^i = (\pi_1^i, \ldots, \pi_n^i)$ denote the corresponding observed nominal user costs for these assets, where $i = 1, \ldots, T$. Further, let $\mathbf{z}^i = (z_1^i, \ldots, z_k^i)$ denote the observed quantities of all other variables in the utility function (including financial assets not in $\mathbf{m}$) with the corresponding prices $\mathbf{p}^i = (p_1^i, \ldots, p_k^i)$. Varian (1983) proved that the following two conditions are equivalent:

(i) There exists a concave, monotonic, continuous weakly separable (in $\mathbf{m}$) utility function, which rationalizes the data $(\mathbf{p}^i, \mathbf{z}^i)$ and $(\boldsymbol{\pi}^i, \mathbf{m}^i)$;

(ii) There exist numbers $U^i, V^i, \lambda^i, \mu^i > 0$ $(i = 1, \ldots, T)$ such that:

$$U^i \leq U^j + \lambda^j \mathbf{p}^j(\mathbf{z}^i - \mathbf{z}^j) + \lambda^j (V^i - V^j)/\mu^j \ \forall i, j$$

$$V^i \leq V^j + \mu^j \boldsymbol{\pi}^j(\mathbf{m}^i - \mathbf{m}^j) \ \forall i, j.$$

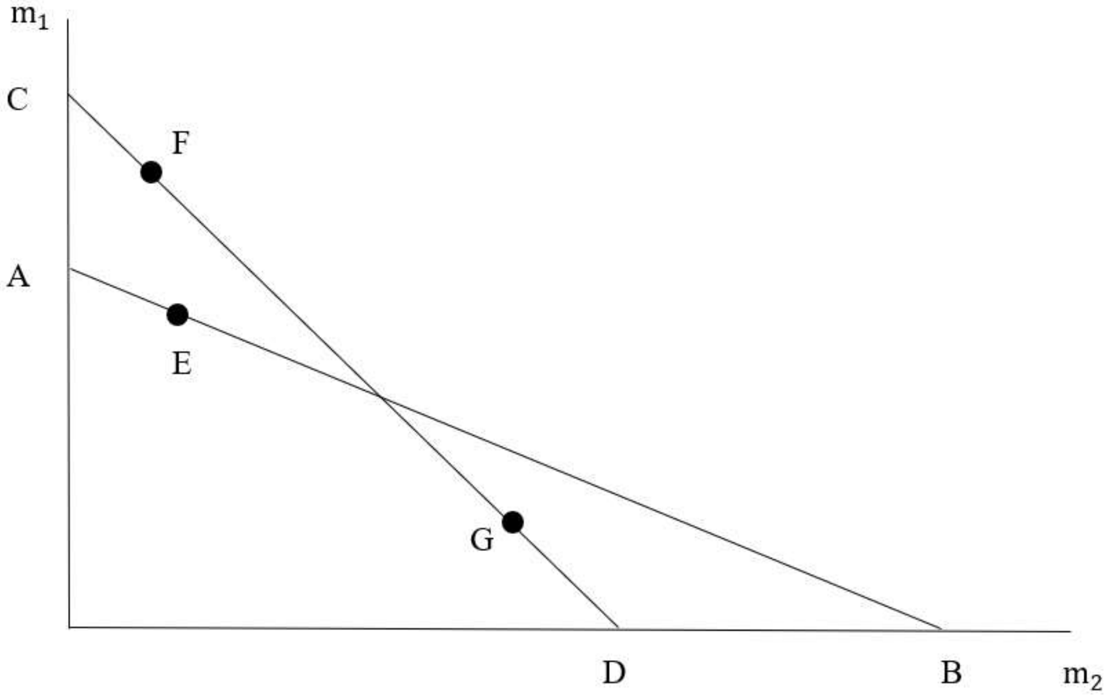
We check two necessary conditions for the weak separability. First, the combined price and quantity data for both sets of goods ($\mathbf{m}$ and $\mathbf{z}$) must satisfy GARP, otherwise the data cannot be rationalized by a well-behaved utility function, whether weak separable or otherwise. Second, the price and quantity data for the separable groups ($\boldsymbol{\pi}^i$, $\mathbf{m}^i$) must also satisfy GARP, since otherwise no feasible solution exists for the constraints in condition (ii). We will refer to those constraints as Afriat inequalities.

If these necessary conditions are satisfied, then an admissible aggregate can exist. We then check Varian's sufficient but not necessary condition. If the sufficient condition is met, then an admissible aggregate exists. We use a numerical algorithm to construct the Afriat indexes (positive numbers, V^i and μ^i, which satisfy the Afriat inequalities). Then, we replace the quantities, $\mathbf{m}^i$, with the group quantity index, V^i, and replace their user cost prices, $\boldsymbol{\pi}^i$, with the group price index, $1/\mu^i$, within the combined dataset for all goods. If the necessary conditions are satisfied and the replaced data satisfy GARP, then the dataset satisfies the necessary and sufficient conditions for the weak separability.

A violation of GARP is illustrated in Figure A.1. The consumer faces the choice of how much to consume for two goods, m_1 and m_2, given two different sets of prices. If when facing the budget AB, the consumer chooses bundle E to consume and chooses bundle G to consume if when facing the budget CD. In this situation, this consumer's preferences are inconsistent with GARP. This is a violation of GARP because the consumer chooses bundle E when bundle G is available in the budget set and turns around and chooses bundle G when bundle E is available in the budget set. Logically, bundle E cannot be preferred to bundle G, while in the later period bundle G is preferred bundle E, unless the consumer changes preferences. Thus, the consumer does not have a stable preference. As stated above, if a dataset is inconsistent with GARP, then the consumer's choices cannot be rationalized by any well-behaved utility function.

Figure A.1 also illustrates the choices by the consumer that are consistent with GARP. If when facing the budget AB, the consumer chooses bundle E to consume and chooses bundle F to consume when facing the budget CD, then the consumer's preferences are consistent with GARP. This is because when the consumer purchases bundle E, bundle F is unavailable in the budget set. When the consumer purchases bundle F, bundle E is available in the choice set meaning that bundle F is preferred to bundle E. In this case, the data are consistent with the choices in the economic theory.

Figure A.1 Examples of Choices Consistent and in Violation with GARP



2.3 The Construction of Economic Monetary Aggregates

We next turn to the aggregator function. In the preceding discussion and aggregation literature for the consumer choice problem, the quantity aggregator function is proven to be the sub-utility function solely over the monetary components. This function is defined as $f(m_t)$[5] above. We can derive the demand-function system by utilising a specified form for $f(m_t)$ and solving the decision of max $f(m_t)$. Using these derived functions and monetary data, we then can estimate the parameters of $f(m_t)$. The resulting estimated function is an economic quantity aggregate.

If we follow this approach, we need to choose a specific and differentiable function form. One problem arises that using a specific function implies a set of assumptions about the consumer's underlying structure. If we choose the weighted linear aggregator function for $f(m_t)$, for example, this linear function implies the perfect substitutes among monetary assets.

[5] The argument requires that f be homothetic. If the utility function is non-homothetic, the aggregator function is the distance function. However, the resulting index is the same in both case, so this assumption does not affect our discussions here. For details, please see Barnett, W. A., Fisher, D. and Serletis, A. 1992. Consumer theory and the demand for money. *Journal of Economic Literature,* 30**,** 2086-2119.

However, this assumption is invalid for the quantity aggregate as we discussed above. One alternative to solve this problem is to use a flexible function form to approximate the unknown monetary aggregator function. The flexible function forms could be approximate second order even higher order any unknown functional forms[6]. This approach provides a way to deal with the problems of less than perfect substitutability and of the variations in the elasticities of substitution among monetary assets in the simple summation, by using a nonlinear aggregator function. However, there is a problem that the estimated index heavily depends on the specification of the function. Different functional forms may produce various estimated aggregates. A question arises that whether it is possible to have a more direct approach to determine a quantity index only with the data on quantities and prices. With the index number theory, the current evidence available suggests it is possible.

2.3.1 Index Number Theory and Monetary Aggregation

Index number theory provides a large number of quantity and price indexes. These indexes are widely used because the inputs of the calculations are only price and quantity data, without any estimations on the underlying structure. Since the appearance of Fisher (1922)'s work on statistical index number theory, then nearly all the government statistics are based on aggregation formula from index number theory, such as the consumer price index (a Laspeyres price index), the implicit GNP deflator (a Paasche price index) and the real GNP (a Laspeyres quantity index) (Barnett et al., 1992). In addition, Fisher (1922) provides a set of tests such as the factor reversal test to assess the quality of a statistic index. The index which possesses the largest number of satisfactory properties is regarded as the best index, i.e., the Fisher Ideal Index. The Törnqvist-Theil discrete time approximation to the Divisia index is another index having a large number of these properties (Divisia, 1925).

[6] Such as the asymptotically ideal models (see Yue, P. and Fluri, R. 1991. Divisia monetary services indexes for Switzerland: are they useful for monetary targeting? *Federal Reserve Bank of St. Louis Review*, 73.).

An important characteristic of the Fisher Ideal Index is the “self-dual” (Fisher, 1922). In this case, the total expenditure on their components equals the product of the Fisher price and quantity indices; this is known as the Fisher’s factor reversal test. By contrast, the Divisia index does not pass this test. Nevertheless, even if the Divisia price index was used to calculate Divisia quantity index by dividing the total expenditure on the components over Divisia price index, the size of errors produced due to the violation of the factor reversal test is too small to ignore.

Until recently, the links between index number theory and monetary aggregation theory are developed by Diewert (1976). He claims to attach economic properties to statistical indices such that the sub-utility aggregator function can be traced without errors. He shows a variety of statistical indices are consistent with specific functional forms when used to represent the unknown economic aggregator function. These indices are denoted as exact. Diewert (1976) coined the term “superlative” to describe a class of exact index numbers that are exact for a flexible functional form – a functional form that can provide a second order approximation to an unknown aggregator function. The Divisia index is found to be exact for the linearly homogenous and flexible translog function and is, thus, superlative. Even if the homotheticity is violated, the Divisia index still can track the unknown aggregator function. This establishes its superiority for practice purposes.

A limited-use monetary aggregate is the monetary quantities index (MQ). This index is proposed by Spindt (1985) and is calculated using the formula for Fisher Ideal Index, but with monetary asset turnover rates in place of user costs. This procedure is inconsistent with aggregation theory and index number theory, and the turnover rate is not the input for Fisher Ideal Index formula. Another attempt adopted by Roper and Turnovsky (1980) to define money is emphasising the role of monetary policy. Their goal is to minimize instability in the policy target through minimizing the income variance. This approach adds difficulty in dealing with a large number of components and policy tools.

The nominal user cost π_t of the i^{th} monetary asset is given by:

$$\pi_{it} = p_t^* \frac{R_t - r_{it}}{1+ R_t} \tag{A.4}$$

where R_t is the benchmark asset rate of return measuring the maximum expected rate of return available in the economy at time *t*, and r_{it} is the own rate of return on monetary asset *i* at time *t*, p_t^* is the true cost-of-living index price at time *t*, approximated by a consumer price index. This formula measures the opportunity cost of the monetary service provided by asset i.

Given the superiority of the Divisia index theoretically, Barnett (1980), as a pioneer, connects economic aggregation theory to index number theory to derive the user cost formula in equation (A.4) and construct the Divisia monetary aggregates consistent with Diewert (1976)'s superlative index numbers. We then turn to construct an aggregate nominal service index. Under linear homogeneity, the continuous Divisia price and quantity index can be utilized to tract the aggregate, with solving the following dual differential equations with log form for the price aggregate, $\Pi_t = \Pi(\pi_t)$, and the monetary aggregates, $M_t = M(m_t)$, respectively:

$$\frac{dlog(\Pi_t)}{dt} = \sum_i s_{it} \frac{dlog(\pi_{it})}{dt} = \sum_i \frac{\pi_{it} m_{it}}{m_t^*} \frac{dlog(\pi_{it})}{dt} \tag{A.5}$$

$$\frac{dlog(M_t)}{dt} = \sum_i s_{it} \frac{dlog(m_{it})}{dt} = \sum_i \frac{\pi_{it} m_{it}}{m_t^*} \frac{dlog(m_{it})}{dt} \tag{A.6}$$

Where $m_t^* = \pi_t' m_t$ is the total expenditure on the whole portfolio's monetary assets, and $s_{it} = \frac{\pi_{it} m_{it}}{m_t^*}$ is the i^{th} asset's expenditure share or its contribution to the aggregate during period *t*.

These aggregates above satisfy the Fisher's factor reversal in continuous time. Thus, this condition can be written as:

$$\Pi_t M_t = \pi_t' m_t = m_t^* \tag{A.7}$$

Since the continuous time data is unavailable in the real-world, the discrete time representation of the Divisia index is required for the empirical research field. According to Anderson et al. (1997), Theil approximation is a second order approximation to the continuous time Divisia index. At time *t*, the discrete time representation of the Divisia price index Π_t, over the user cost, and Divisia quantity index M_t, over the monetary components, respectively, are:

$$log(\Pi_t) - log(\Pi_{t-1}) = \sum_{i=1}^{n} s_{it}^{*}(log(\pi_{it}) - log(\pi_{i,t-1})) \tag{A.8}$$

$$log(M_t) - log(M_{t-1}) = \sum_{i=1}^{n} s_{it}^{*}(log(m_{it}) - log(m_{i,t-1})) \tag{A.9}$$

where $s_{it}^{*} = \frac{1}{2}(s_{it} + s_{i,t-1})$ is the average of the current and lagged expenditure shares held by the asset *i*. Therefore, equation (A.8) and (A.9) are the weighted growth of the Π_t and M_t over the user costs and quantities of monetary components, respectively. Given the equation (A.10)[7], the Divisia monetary index in level, M_t, is:

$$\frac{M_t}{M_{t-1}} = \prod_{i=1}^{n} (\frac{m_{it}}{m_{i,t-1}})^{s_{it}^{*}} \tag{A.10}$$

The above Equation (A.10) is known as the Törnqvist-Theil Divisia monetary quantity index (Törnqvist, 1936; Theil, 1967). Dual to the aggregate quantity index, the aggregate user cost index can be directly calculated from the factor reversal equation (A.8) as:

$$\Pi_t = \frac{\pi_t' m_t}{M_t} = \frac{\sum_{i=1}^{n} \pi_{it} m_{it}}{M_t} \tag{A.11}$$

The monetary quantity Divisia index growth rate given by equation (A.10) is precise within three decimal places with the weekly or monthly monetary data, which is demonstrated by Barnett (1980), while the price aggregates from equation (A.5) and (A.11) for discrete Divisia price growth rate are not exactly the same price aggregate. However, the difference is third order and probably insignificant, especially with the component data (Barnett, 1982). In

[7] Source Barnett, W. A. 1980. Economic monetary aggregates an application of index number and aggregation theory. *Journal of Econometrics*, 14**,** 11-48. Reprinted in Barnett and Serletis (2000) as above.

this chapter, the Törnqvist-Theil discrete time approximation is applied to produce our green monetary aggregates.

2.3.2 Risk Adjustments for Divisia Monetary Aggregates

In recent years, there have been a number of related developments to address the problems in the construction of monetary aggregates. The one that stands out is the assumption of certain returns on assets laid out by Barnett (1978, 1980). The returns on assets are known with certainty at the start of each period under his framework. As explained by Barnett and Su (2018), regarding the monetary assets with relative low risk returns, risk aversion does not have significant impact on the aggregation of monetary aggregates. However, Rotemberg and Poterba (1987) argued that if the risky assets such as shares and bonds are added into monetary aggregates as they provide monetary services in the market, the contemporaneous interest rate risk of these assets in combination with risk aversion would make the construction of monetary aggregates be more complex because the returns of risky assets are uncertain at the end of the period. This insight has sparked a new line of research in monetary aggregate literature to precisely measure money with the consideration of risk.

Barnett et al. (1991), Barnett and Zhou (1994) and Barnett (1995) firstly conduct studies on monetary aggregates under risk. They compare the tracking abilities of various index numbers to unknown risk-accounted aggregator functions. Their results show that although a minor difference on estimated aggregator functions with and without risk, it might be appropriate to include risky assets in monetary aggregates. Barnett et al. (1997) then continued the work on risk adjustments in the monetary aggregates. Based on the consumption capital asset pricing model (CCAPM) framework, they developed a method to derive a generalised Divisia index with the risk-adjusted user costs, despite the CCAPM’ s limited empirical success (Elger and Binner, 2004). The CCAPM is grounded in the economic theory that the consumption decisions of agents are linked to their intertemporal choices under uncertainty.

This model allows for the incorporation of risk adjustments by considering the varying degrees of substitutability among financial assets, which is crucial for accurately measuring the true liquidity services provided by different assets. Although the CCAPM has faced challenges in empirical validation, particularly due to its assumptions about market completeness and representative agents, its theoretical framework provides a robust foundation for adjusting for risk in monetary aggregation. The empirical limitations of the CCAPM often stem from practical difficulties in measuring and quantifying the relevant variables rather than from the conceptual soundness of the model itself. By integrating risk adjustments based on the CCAPM, the Divisia index can more accurately capture the dynamic interactions between different financial assets, offering a more precise measure of the effective money supply.

The risk-adjusted user cost formula depends on an expected return and an estimate of a relative risk aversion coefficient. In this case, the user costs in equation (A.4) would be modified by applying the modified Arrow-Pratt relative risk aversion measure Z_t. The nominal adjusted user cost is given by:

$$\pi_{it}^{adj} = p_t^* \frac{E(R_t^*) - (E(r_{it}^*) - \phi_{it})}{1 + E(R_t^*)} \tag{A.12}$$

where $\phi_{it} = Z_t \, Cov\,(r_{it}^*, \frac{c_{t+1}}{c_t})$; where c is the measure of consumption and * indicates real rates. The function of ϕ_{it} is a risk adjustment to the unadjusted expected excess rate of return r_{it}^*.

Under the CCAPM framework, Barnett et al. (1997) noticed that the risk adjustments for the assets are slight in the USA monetary aggregates. This could be explained that when aggregating the data across the representative agents, the individual risks are cancelled out (Barnett et al., 1997). Another possible reason for this small adjustment is the low contemporaneous covariance between asset returns and the consumption growth in the CCAPM framework with the standard utility function (Barnett and Wu, 2005). However, this does not indicate that the risk adjustments are no longer needed. They suggested that if higher risky assets are considered, the larger risk adjustments cannot be ignored. In this chapter, the green bonds, government bonds and corporate bonds, as the substitutes of monetary assets, are assumed as risky assets; therefore, the risk adjustments may be obtained.

There are many attempts to address the problem of small risk adjustments. Using a more general utility function, Barnett and Wu (2004, 2005) extended the user cost risk adjustments to the case of multiple non-monetary assets and the intertemporal non-separability. Their results lead to a substantial and more accurate risk adjustment compared with the risk adjustments in 1997's work. This extension is also supported by a recent work of Barnett and Su (2019) that derives the risk-adjusted user costs for credit card augmented monetary aggregates. More recently, Binner et al. (2018) expanded the studies of Barnett and Wu (2004, 2005) by relaxing the assumption of a one-year planning horizon and introducing predicted returns on risky assets to construct risky monetary aggregates for the USA and UK. This relaxation is reasonable since the risky assets are the least liquid assets among monetary assets and thus, the investors would not change their portfolios quickly. Accordingly, the forecasted returns are more accurate than the single time ones. In this chapter, we adopt the same method by Binner et al. (2018) to account for green bonds, government bonds and corporate bonds – the assets that are similar to the ordinary bonds in the study of Binner et al. (2018).

3 Data

Our choice of country in this study is the USA as the USA has one of largest and most influential green bond markets in the world and is one of the first countries to engage in green bond issuance, and hence the green bond data available for this chapter will provide more robust results than can be obtained elsewhere. We utilize the quarterly USA data covering the period from the first quarter of 2009 to the fourth quarter of 2021. All data collected for this study are real per capita data, along with their corresponding nominal prices, allowing us to calculate the expenditure on each good or asset.

The goods and assets[8] considered are:

i) Gross Domestic Product (GDP),

ii) Leisure (LEIS) (this variable listed for GARP test),

iii) Currency (CUR),

iv) Demand Deposits (DD),

v) Other Liquid Deposits (OLD) (other checkable deposits + savings deposits)

vi) Retail Money Market Funds (RMMF),

vii) Small Time Deposits Total (STDT) (small time deposits commercial and small time deposits thrift),

viii) Institutional Money Market Funds (IMMF),

ix) Large Time Deposits (LTD),

x) Repurchase Agreements (RA),

xi) Commercial Paper (CP),

xii) T-Bills (TB),

xiii) Green bonds holdings (GREEN),

xiv) Government bonds holdings (GOVB),

xv) Corporate bonds holdings (CORB).

We obtained all USA non-monetary data, GDP and leisure, and associated prices listed above from the St. Louis Federal Reserve Economic Database (FRED) except work hours, which come from DataStream. From the same source we downloaded the consumer price index (CPI) and population to convert the GDP, leisure and asset series into real per capita or representative agent data. We obtained the USA capital certain nominal holdings of

[8] According to the Center for Financial Stability (CFS), other checkable deposits commercial, other checkable deposits thrift, savings deposits commercial and savings deposits thrift are aggregated into other liquid deposits (asset 6) from May 2020. Small time deposits commercial and small time deposits thrift are also aggregated into small time deposits total (asset 8) from May 2020. We follow the CFS to aggregate other checkable deposits, savings deposits and small time deposits into separate series.

components, assets iii through xii, and the associated own rates of return from the Center for Financial Stability[9] (Barnett et al., 2013). We obtained the nominal household sector holdings of green bonds, asset xiii, at market values from Bloomberg. The nominal household sector holdings of government bonds and corporate bonds, asset xiv and xv, at market value, were downloaded from FRED. We used S&P 500 green bond index and S&P 500 corporate bond index for the return on green bonds and corporate bonds, which were downloaded from DataStream. We used government-bond price index for the return on government bonds. These data were also obtained from DataStream.

The base year for the GDP is 2012 and for the CPI is 2015.

Federal Funds rates (FFR) used as the policy rate in the aggregate demand analysis was obtained from FRED.

Green bonds, as shown in Table A.1, represent a distinct category of fixed-income securities designed specifically to fund projects with positive environmental and climate impacts. These bonds are classified in Bloomberg under specific green bond sectors, which include renewable energy, energy efficiency, pollution prevention and control, sustainable water management, and green buildings, among others. The issuers of green bonds encompass a broad spectrum of entities such as sovereign governments, municipal authorities, private corporations, financial institutions, and supranational organizations, each leveraging these instruments to finance sustainability initiatives. Typically, green bonds exhibit longer maturities and lower yields compared to standard corporate bonds, reflecting the growing investor demand for sustainable investment options.

[9] Some of the USA data is available on the CFS website at the following link http://www.centerforfinancialstability.org/amfm_data.php. Some of the data are obtained in personal contact with the CFS researchers.

Table A.1 The Data Definition and Sources of Green Bonds, Corporate Bonds and Government Bonds

Assets	Definition	Sources
Green Bonds	Fixed-income instruments earmarked to raise funds for climate and environmental projects. (Only green corporate bonds included in this thesis)	Bloomberg
Corporate Bonds	Debt securities issued by corporations to raise capital. (The green corporate bonds are subtracted from corporate bonds)	FRED
Government Bonds	Debt securities issued by governments to support government spending and obligations. (Green government bonds are included)	FRED

Notes: This table illustrates the data definition and sources of green bonds, corporate bonds and government bonds in the thesis.

The significant differences between green bonds and standard corporate bonds are evident in their financial characteristics, which is displayed in Table A.2. Over our sample period, green bonds had an average term of 11.2 years, compared to 9.1 years for standard corporate bonds, indicating that green bonds typically have longer maturities. The average duration for green bonds was 7.4 years, slightly higher than the 6.2 years for standard corporate bonds, which suggests that green bonds are more sensitive to interest rate changes. In terms of financial returns, green bonds offered an average return of 2.9%, which is lower than the 3.8% average return for standard corporate bonds. This difference is also reflected in the average coupon rates and yields, with green bonds having an average coupon of 2.4% and an average yield of 2.7%, compared to 3.5% and 3.6%, respectively, for standard corporate bonds. These differences indicate that investor preferences for sustainable investments and the perceived lower risk or additional benefits associated with environmentally friendly projects.

In terms of market size, corporate green bond issuance in the USA has grown significantly, from $5 billion in 2009 to nearly $400 billion[10] in 2021, though it remains a smaller proportion compared to the overall corporate bond market. On average, corporate green bonds accounted for approximately 5% of the total corporate bond issuance from 2009 to 2021. This proportion highlights the niche, but growing segment of the bond market focused on sustainable and environmentally beneficial projects.

Table A.2 The Comparison between Green Bonds and Standard Corporate Bonds

	Green Bonds	Standard Corporate Bonds
Average Term (years)	11.2	9.1
Average Duration (years)	7.4	6.2
Average Return (%)	2.9	3.8
Average Coupon (%)	2.4	3.5
Average Yield (%)	2.7	3.6

Notes: The values of average return, average coupon and average yield are presented as percentage. All figures were obtained from Bloomberg.

4 Methodology

GDP is our real sector consumption variable and leisure is calculated from the average number of worked hours. Leisure time is 98 hours minus average work hours per week during the quarter. This figure then is multiplied by 52 weeks to arrive at an annualised quarterly number for leisure time. Available time, 98 hours per week, is based on 10 hours per day for fixed time on sleeping and eating (Swofford and Whitney, 1987).

For the bonds' assets, we use the forecasted returns. We consider the green bonds, government bonds and corporate bonds as the least liquid assets, since individual investors do not change their asset portfolio quickly and the bonds are the most expensive way to obtain medium of exchange (Binner et al., 2018). Therefore, we forecast a one-year ahead expected return on green bonds, government bonds and corporate bonds.

[10] The data on corporate green bond issuance and corporate bond issuance in the USA were obtained from Bloomberg.

The calculation of CCPAM special-case risk-adjusted real user costs for the bonds depends on the forecasted real returns on the bonds and an Arrow-Pratt measure of relative risk aversion. For the computation of the real returns, we use CPI[11] serving as a proxy for the true cost of living index to convert all returns including capital certain assets' returns into real term. We forecast a one-year ahead expected real returns on all assets by using an autoregressive model. These one-year ahead forecasts are used to construct interest rate forecasts that we use in the user cost computations. It is known that unreasonably high estimates of the degree of relative risk aversion are yielded in empirical studies, such as 25 in the study by Mehra and Prescott (1985). Since it is difficult to find an economically reasonable estimate of the degree of relative risk aversion, it becomes sensible to choose a value directly. Drake et al. (2000) use values within a range from 0 to 7. In our estimations, we take an average value of 3.5, while the value of the coefficient of relative risk aversion has only small impact on the monetary aggregates due to the low covariances between the real rates of returns on risky assets and the growth rate of real consumption. In our calculation, the estimated covariances for the green bonds, government bonds and corporate bonds are - 1.31×10^{-5} , 1.09×10^{-4} and -9.49×10^{-5}, respectively.

The benchmark rate used in the calculation of the user costs for all assets is constructed with an envelope approach. The benchmark rate is the maximum rate of return from the interest rates on the different monetary assets within our index. However, there are periods in our sample when the returns on components are equal to the benchmark rate, thus leading to zero or negative weights. Zero user costs imply that the transaction services provided by the asset are free, which is unrealistic. In order to ensure that all user costs of monetary assets and financial assets above zero, a simple way is to add a constant to the benchmark rate as arbitrary

[11] We adjust Federal Funds rates and CPI data for seasonal patterns by using X-13ARIMA-SEATS, which is the seasonal adjustment software developed and adopted by the U.S. Census Bureau.

adjustments or liquidity premium. The Divisia indices provided by the FED of St. Louis use 100 basis points (Anderson and Jones, 2011). In line with Anderson and Jones (2011), a constant of 100 basis points is necessary to obtain positive weights throughout our sample, which is the minimum value to make all user costs be positive.

GDP serves as our real sector variable in modelling aggregate demand. We denote gdp_t as the natural log of real GDP in quarter t. Output gap is measured by two different methods. First, we apply the Hodrick-Prescott (HP) filter to gdp_t considering the cyclical component generated by the filter as the output gap. As an alternative approach, we follow the methodology of Binner et al. (2009) by de-trending gdp_t through regression against a constant, t, and t^2. The resulting residuals from this regression serve as another measure of the output gap. These computations are conducted for the sample period spanning from 2010Q1 to 2021Q4. The real Federal Funds rates [12](RFFR) is used as the policy rate in the aggregate demand analysis.

Descriptive statistics of the data used for the aggregated demand analysis are presented in Table A.3. For the output gap, both measures have negative average value, which indicates a slowdown of the USA economic growth during our sample period. The average value of the real short interest rate is 2.411% and the minimum value is 0.277%. The maximum value is 9.457%. The high value 2.985 of standard deviation shows the high volatility of the real short interest rate. The average values of green simple sum real money growth and green Divisia real money growth are 0.038 and 0.026, respectively.

Table A.3 Descriptive Statistics of the Data for the Aggregate Demand Analysis

Variables	Mean	Median	Std. Dev	Max	Min
gdp_Gap_HP	-0.0002	-0.0001	0.017	0.022	-0.093

[12] We adjust Federal Funds rates for seasonal patterns by using X-13ARIMA-SEATS, which is the seasonal adjustment software developed and adopted by the U.S. Census Bureau.

gdp_Gap_QD	-0.0006	-0.0003	0.018	0.022	-0.093
$RFFR$	2.411	0.585	2.985	9.457	0.277
$\Delta_4(m-p)$_Divisia	0.038	0.032	0.032	0.127	0.003
$\Delta_4(m-p)$_Sum	0.026	0.020	0.032	0.122	-0.027

Notes: The values for real short term interest rate, RFFR, are displayed in percentage terms. Annualised real money growth is defined as $\Delta_4(m-p)_t = \Delta_4 m_t - \Delta_4 p_t$, where m_t is the natural log of the green bond augmented nominal broad Divisia monetary aggregate and its simple sum counterpart, and Δ_4 represents the difference between the current value of a variable and its fourth lag.

Having discussed the data and how and what we did, we turn to empirical results. We first present the revealed preference tests for composition of the green monetary aggregates and the comparison between the simple sum and Divisia monetary aggregates. We then report the empirical results from the IS curve specification including these green Divisia monetary aggregates.

5 Results

5.1 Weak Separability Test Results

The data of the observed prices and quantities for all goods described above are firstly tested with GARP. In Table A.4, the broad monetary aggregate given in the structure is composed of the goods from sub-utility function: V(CUR, TC, DD, OLD, RMMF, STDT, IMMF, LTD, RA, CP, GREEN, GOVB, CORB). The T-bills is not included in the tested group to avoid double counting, since the government bonds contains T-bills.

The results presented in Table A.4 show that the representative consumer for the USA chooses the consumption of bundles consistent with each other in each period. Thus, There are no violations of GARP for the full sample dataset, whether considering the sub-utility function assets or the overall dataset with a monetary aggregate of the assets in the sub-utility function substituted for the sub-utility function component assets. The data tested for the USA are found to be consistent with Varian's (1983) conditions at second stage in Section 2.

The results, therefore, imply that the data are consistent with maximization of a well-behaved utility function for the representative consumer. This means that there exists a weakly separable grouping of monetary assets and bonds that forms a monetary aggregate consistent with the economy theory. Hence, a broad monetary aggregate consistent with the economics of aggregation over goods is identified for the USA. The data meet the necessary conditions for the weakly separable utility maximization. This suggests the results on the representative consumer in the USA support the inclusion of green bonds, government bonds and corporate bonds in the broad monetary aggregate.

Table A.4 Structures and Monetary Aggregates for which Weak Separability Obtain

Structure	Weak separability
U(GDP, LEIS, V(CUR, TC, DD, OLD, RMMF, STDT, IMMF, LTD, RA, CP, GREEN, GOVB, CORB))	Y

Notes: A "Y" indicates that the asset structure is weakly separable.

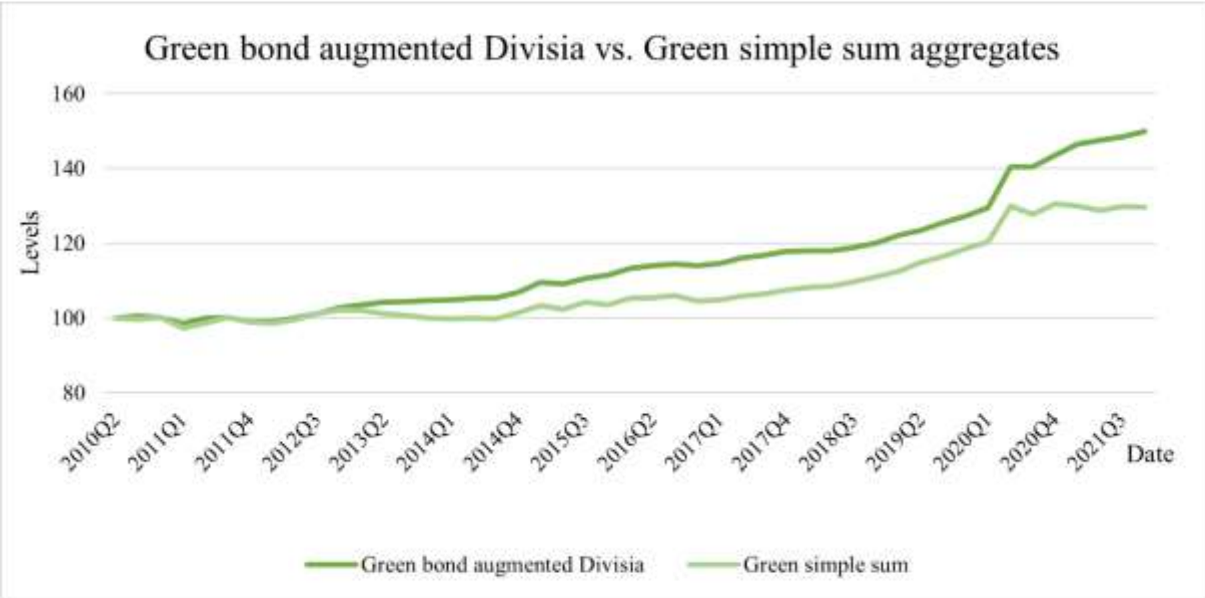
5.2 The Construction Results

We next turn to constructing the green-augmented monetary aggregates identified above. As discussed above in Section 2, a reputable index number is in Diewert's (1976) superlative class of index numbers. By using the superlative index numbers, the substitution among monetary assets is allowed as relative user costs change, thus all the components are not perfect substitutes. Thus, we use the Törnqvist-Theil discrete time approximation of the continuous time Divisia index to construct our monetary index. To compare, we also construct green-augmented simple sum monetary aggregates, traditional simple sum monetary aggregates and traditional Divisia monetary aggregates. The traditional monetary aggregates exclude the green bonds, government bonds and corporate bonds but include T-bills to be consistent.

The levels of green-augmented simple sum monetary aggregates, green-augmented Divisia monetary aggregates, traditional simple sum and Divisia are normalized to 100 at the first period (i.e., 2010Q1) to ensure their comparability. Figures A.2 and A.3 provide the levels of the green-augmented Divisia and traditional Divisia aggregates and their corresponding simple sum aggregates, respectively.

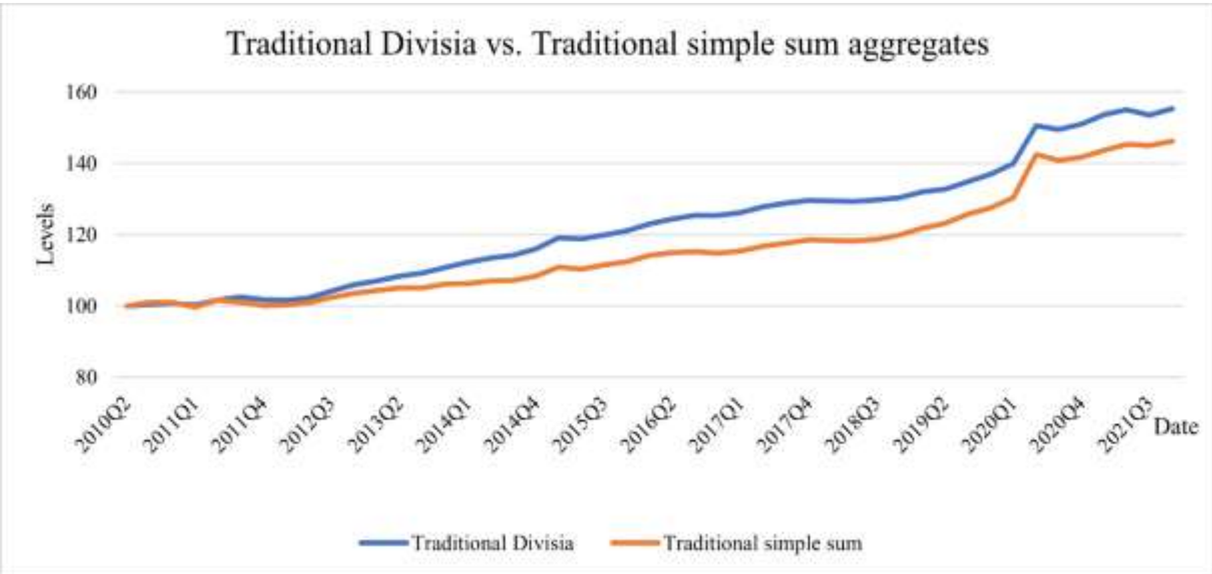
Figure A.2 shows a relatively large difference between the level of green Divisia monetary aggregates and its green simple sum counterpart. Precisely, the level of green Divisia index is greater than that of green simple sum index by 6.01% on average. Similarly, the level of traditional Divisia money is also higher than that of simple sum money by 10.70% on average given in Figure A.3. Our results are contrary to previous studies which suggested that the simple sum monetary aggregates overstate the money stock. This inconsistency may be due to the selection of different normalising date in our data sample. The variance between the simple sum and Divisia monetary aggregates in Figure A.2 and A.3 could be explained by the simple sum aggregation assigning equal weights to the components, while the Divisia measures attach different weights to the components. The lower variation in green money indicates that the degree of substitutability among the sub-components of green money is higher than that of traditional money supply. This backs up our claim above that the measurement of money matters.

Figure A.2 Levels of Green Bond Augmented Divisa Money and its Green Simple Sum Counterpart



Notes: The figure plots the levels of green bond augmented Divisia monetary aggregates and green simple sum monetary aggregates. The dark green line is the level of green augmented Divisia monetary aggregates. The light green line is the level of green simple monetary aggregates. Both are normalized to 100 in the first period (2010Q1).

Figure A.3 Levels of Traditional Divisa Money and its Traditional Simple Sum Counterpart



Notes: The figure plots the levels of traditional Divisia monetary aggregates and traditional simple sum monetary aggregates, which exclude green bonds, corporate bonds and government bonds and include T-bills. The blue line is the level of traditional Divisia monetary aggregates. The orange line is the level of traditional simple sum monetary aggregates. Both are normalized to 100 in the first period (2010Q1).

6 The IS Curve Specifications

In this section, we provide evidence to demonstrate the direct impact of our green-bond augmented Divisia monetary aggregates on the aggregate demand in the USA. We consider the following IS curve specifications, which are adapted by Reimers (2002) and Binner et al. (2009):

$$gdp\_Gap_t = \beta_0 + \beta_1 gdp\_Gap_{t-1} + \beta_2 RFFR_{t-1} + \varepsilon_t \quad \text{(A.13)}$$

$$gdp\_Gap_t = \beta_0 + \beta_1 gdp\_Gap_{t-1} + \beta_2 RFFR_{t-1} + \beta_3 \Delta_4 (m-p)_{t-1} + \varepsilon_t \quad \text{(A.14)}$$

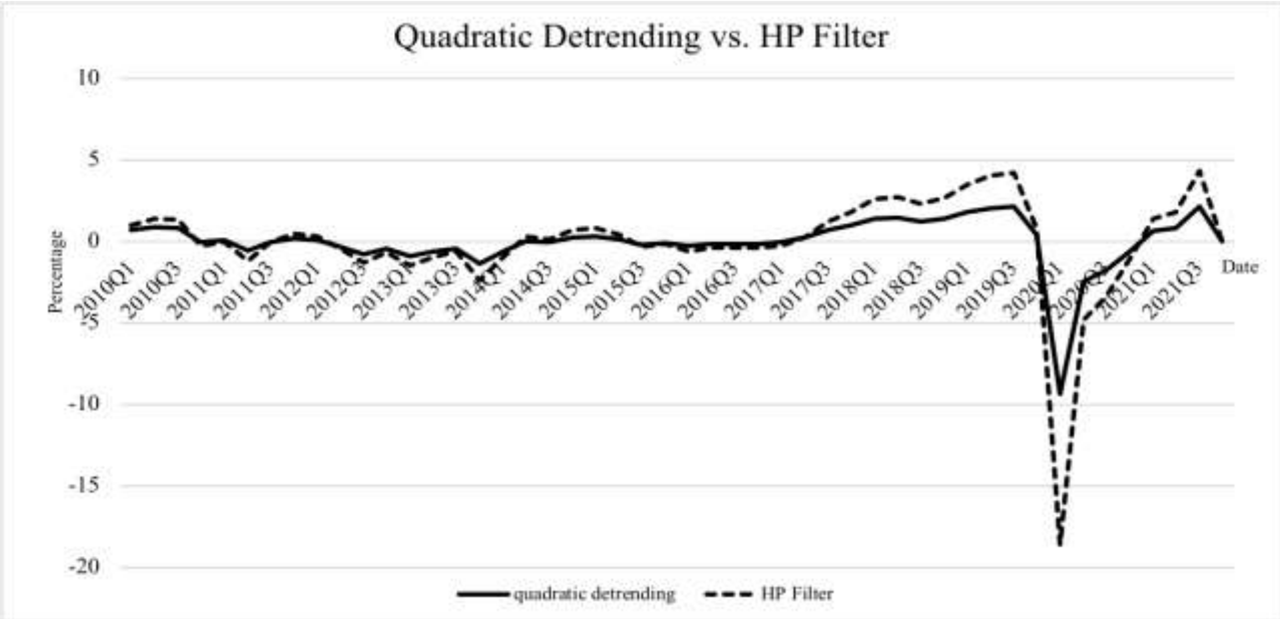
In the above equations[13], the variable gdp_Gap_t is the output gap and $RFFR_{t-1}$ is the lagged real Federal Funds short-term real interest rate. This real interest rate is defined as $RFFR_t = \sum_{j=0}^{3} FFR_{t-j} - \Delta_4 p_t$, where FFR_{t-j} is a short-term nominal interest rate (expressed as a quarterly fraction), p_t is the natural log of consumer price index, and Δ_4 takes the difference between current value of a variable and its fourth lag: $\Delta_4 x_t = x_t - x_{t-4}$. Annualised real money growth is defined as $\Delta_4 (m-p)_t = \Delta_4 m_t - \Delta_4 p_t$, where m_t is the natural log of the green bond augmented nominal broad Divisia monetary aggregate and its traditional simple sum counterpart.

The comparison of two measures of output gap is shown in Figure A.4. The solid line denotes the cyclical component from HP filter and the dashed line denotes the residuals derived from the quadratic de-trending regression (QD). Both measures are scaled by a factor of 100 to denote percentages. The graph clearly indicates that the estimated results of the two alternatives measures of output gap are very similar. The plot reports a long-lasting zero or slightly negative output gap from 2010Q2 to 2017Q3 after the financial crisis, which indicates a slow recovery for the USA economy. The actual GDP then rose above its potential GDP. However, in 2019Q4, actual GDP fell sharply below its potential due to the burst of COVID-

[13] Before the estimations, we calculate Ljung-Box Q statistic for our variable series and find that the series of real interest rate and real money growth are non-stationary, so we take second difference for real interest rate and first difference for real money growth to make it to be stationary.

19. Given these different behaviours of the output gap in our sample period, we employ Markov regime switching model by Hamilton (1989) to capture the dynamic patterns in our economic variables. The regime switching process allows us to estimate the output gap considering the possible changes in the parameters of the economy due to the COVID-19 pandemic.

Figure A.4 The Two Measures of Output Gap Used in the IS Curve Estimation



Notes: The figure plots two measures of output gap. Solid line denotes cyclical component from HP filter and dashed line denotes residuals from quadratic de-trending regression (QD). These two measures are multiplied by 100 to denote percentages.

We next turn to estimating the models with the real money growth using both green augmented Divisia and green simple sum green money for the period during 2010Q2 to 2021Q4[14]. The results from the estimated Markov Switching (MS) models for both measures of the output gap with and without a real money measure are presented in Table A.5.

In regime 1, the coefficient on lagged output gap is strongly and negatively significant in all cases, which corresponds the sudden shrink during the COVID-19 period in Figure A.5. The IS curve aggregate demand results are less plausible during the short covid shock that the FED tried to moderate with monetary policy, which is unsurprising. The unreported constant terms are negative and statistically different from zero. The coefficients on the lagged output gap are uniformly negative. This implies that a larger last period output gap during the COVID is associated with a fall in the current period output gap, if positive, and a more negative output gap when the gap is negative. The coefficient on the change in the lagged RFFR variable is

[14] We lose one observation, 2010Q1, as a result of differencing the data.

uniformly positive but less precisely estimated. This indicates that a positive change in the lagged RFFR is associated with an increase in the output gap. That means, a positive change in the change in lagged RFFR is associated with a larger positive output gap or a less negative output gap. During the short COVID period, this is not precisely estimated with the Hodrick-Prescott measure of the output gap. This finding is less consistent with the negative relation between the interest rate and output growth in theory. A possible explanation for this is that when the interest rates approach zero (around 0.07%) during the COVID-19 pandemic, the reduction on interest rate fails to stimulate the economy, therefore, the direction of the movement of interest rate and output is same.

The real green money growth term is statistically significant at 5% significant level in all cases and the coefficient on green money is positively signed, indicating that even during an aggregate supply shock, money can be useful in modelling an IS curve and in the formation of the economic policy. The broad green bond augmented Divisia monetary aggregates are more precisely estimated than their green simple sum counterparts, possibly as a result of the greater information content contained in monetary aggregates composed and constructed using a sophisticated Divisia index number formulation over goods. Thus, the growth rate of the real green monetary aggregate is a significant variable explaining the behaviour of the USA output gap, which corroborates the findings regarding the direct effect of money on aggregate demand in Binner et al. (2009).

The findings are very similar in regime 2[15]. The main difference is that the coefficient on lagged output gap is between 0.86 and 0.96 and is strongly positive in all cases. This is also evident before and after the COVID-19 period shown in Figure A.4. The change in the real interest rate is still significantly positive, indicating that the interest rate is raised for

[15] In Table A.5, the DW statistics suggest that the residuals are not autocorrelated. To test for robustness, we ran all six regressions again with an additional lag of the output gap and the real interest rate (the DW statistics range from 2.55 to 2.71). The coefficients on the real money growth terms remains significantly positive at the 1% significance level in regime 1 and 5% significance level regime 2. See Table A.6 in Appendices for details.

encouraging the economic recovery after financial crisis and COVID-19 pandemic. This finding is in line with the empirical evidence by Dotsey et al. (2003) and Lee and Werner (2022). The significant and positive coefficient on green money supports that both green Divisia index and green simple sum index have direct effects on economic growth.

Table A.5 IS Curve Estimates from the Regime Switching Specification

	Standard IS curve		Real Divisia/simple sum green money growth term included			
Regime 1: Covid Period						
Constant	-0.012***	-0.022***	-0.026***	-0.029***	-0.028***	-0.014**
	(-6.85)	(-14.80)	(-10.12)	(-8.15)	(-16.17)	(-2.26)
gdp_Gap_{t-1}	-3.681***	-3.329***	-2.076***	-2.634***	-2.031***	-3.491***
	(-53.24)	(-58.47)	(-174.94)	(-31.34)	(-241.08)	(-19.68)
$RFFR_{t-1}$	0.228***	0.202***	0.196***	0.195***	0.193***	0.233***
	(43.33)	(45.87)	(23.77)	(15.81)	(34.39)	(10.63)
$\Delta(\Delta_4(m-p))_{t-1}$	-	-	1.732***	1.225***	1.721***	0.450*
			(12.30)	(7.95)	(17.93)	(1.88)
Output gap	HP	QD	HP	HP	QD	QD
Money measure	-	-	Divisia	Simple sum	Divisia	Simple sum
DW	2.87	2.86	2.54	2.58	2.57	2.85
Regime 2: Non-Covid Period						
Constant	0.001*	0.001	0.0008	0.0009	0.0007	0.001
	(1.76)	(1.60)	(0.89)	(1.07)	(0.81)	(1.54)
gdp_Gap_{t-1}	0.864***	0.884***	0.938***	0.918***	0.953***	0.872***
	(10.25)	(11.53)	(7.92)	(8.14)	(9.40)	(11.27)
$RFFR_{t-1}$	0.007**	0.008***	0.100***	0.010***	0.010***	0.009***
	(2.50)	(2.59)	(3.57)	(3.19)	(3.75)	(3.10)
$\Delta(\Delta_4(m-p))_{t-1}$	-	-	0.113**	0.086**	0.112***	0.071*
			(2.69)	(1.07)	(2.74)	(1.86)
Output gap	HP	QD	HP	HP	QD	QD
Money measure	-	-	Divisia	Simple sum	Divisia	Simple sum
DW	2.87	2.86	2.54	2.58	2.57	2.85

Notes: 1. HP denotes cyclical component from HP filter. QD denotes residual from quadratic de-trending regression. 2. Z statistics are in parentheses. *, **, and *** represent the significant levels of 10%, 5% and 1% respectively. 3. DW is the Durbin-Watson test statistic.

Overall, our results indicate that a broad green bond Divisia monetary aggregate is found to be useful in modelling aggregate demand using an IS curve. This result is consistent with the findings of Keating et al. (2019) who find that a Divisia monetary aggregate, as opposed a traditional simple sum measure, can be a useful policy indicator.

In summary, this work contributes to and augments research in the risk adjusted Divisia monetary aggregates. We produce a new green bond augmented Divisia monetary aggregate for the USA using the Törnqvist-Theil discrete time approximation of the continuous Divisia Index. The user costs for calculating this index are risk-adjusted and forecasted in the case of non-capital certain assets, or "risky" assets. Our empirical work, based on USA aggregate demand, finds that the change in this new real green bond augmented broad Divisia aggregate is positively and significantly related to the output gap. The change in the concomitant green bond augmented simple sum aggregate over the same assets is less statistically related to the output gap, in both the short COVID-19 pandemic period and non-COVID periods.

7 Conclusions and Recommendations for Future Work

In this chapter, we construct new economically green augmented Divisia monetary aggregates and simple sum monetary aggregates for the USA by taking the environment-related bonds, government bonds and corporate bonds into consideration. Using updated data, we identified these aggregates using revealed preference tests and constructed them using the Törnqvist-Theil discrete time approximation of the continuous time Divisia index. Our new green economic monetary aggregates successfully examine the data beyond central bank data. This work is also one of only a handful of papers to emphasise the composition and construction of monetary aggregates in empirical money research. See, for example, Swofford and Whitney (1987), Schunk (2001), Binner et al. (2009) and Binner et al. (2018).

Further we explored the time-series properties of our green simple sum and Divisia money in IS curve specifications. We tested the direct effects of our aggregates on USA aggregated demand by following an approach used by Reimers (2002) and Binner et al. (2009). Our results provide evidence that both green simple sum and Divisia green money are significantly and positively correlated with economic growth from 2010Q1 to 2021Q4. These results suggest that the admissible green augmented Divisia monetary aggregates and simple sum monetary aggregates appear to provide additional information for aggregate demand in the USA, which is not presented in short-term interest rates. Far more important are our findings regarding the correlation of interest rate and output. Contrary to the findings of earlier analysis, we believe that the lagged real interest rate is characterized as positively correlated with output. These results cast doubt on the sole focus on interest rates. Interest rate reduction towards or beyond zero may even hurt economic growth. Our evidence suggests that policy makers aiming at higher economic growth could instead be looking to emphasise the role of money or propose a higher interest rate. Hence, given that the central banks' primary mandates are to ensure price stability and maximum employment[16], and its consideration of climate-related risks is focused on how these risks might impact the stability and resilience of the financial system, our new green Divisia money could satisfy the needs of the Bank of England (2018, p.5), and recognise that climate change matters to their role in the economy. Financial stability is at risk. The Bank of England has voiced its concern and is monitoring the financial sector for climate risk.

Based our findings, further research might be recommended towards the construction and composition of the green monetary aggregates applied to other monetary countries or unions. The power of green money could also be further exploited as a new monetary policy informational tool with the consideration of climate change. We also recommend that future monetary researchers examine data beyond central banks' data sets and explore the possibility that capital uncertain assets provide liquidity services.

[16] See Panel on "Central Bank Independence and the Mandate - Evolving Views" Remarks by Jerome H. Powell Chair Board of Governors of the Federal Reserve System at the Symposium on Central Bank Independence Sveriges Riksbank, Stockholm, Sweden.

Chapter II A Green User Cost Augmented Model of Monetary Policy Shocks

1 Introduction

The mainstream approach to monetary policy is based on the New Keynesian (NK) model and is expressed in terms of the interest rate rules proposed by Taylor (1993). In this approach, the Federal Funds rate could adequately describe the Federal Reserve monetary policy and unexpected increases in this interest rate decrease output and prices. In a classic monetary policy research paper, Christiano et al. (1999) (henceforth CEE) advance this consensus and provide strong support for the assumption of identifying monetary policy shocks where the central bank adjusts the Federal Funds rate in response to changes in output and prices but with a lag of these variables.

However, the 2007 Financial Crisis and the following protracted 7-years effective-lower-bound (ELB) period highlighted the shortcomings of this method that uses the Federal Funds rate alone to gauge the stance of monetary policy. The Federal Reserve, moreover, started to implement unconventional measures including the injection of novel liquidity facilities and large-scale asset purchases. These developments have sparked considerable debate with respect to the effectiveness of the interest rate as the indicator of the monetary policy. Subsequently, identification of monetary policy shocks has shifted to monetary aggregates. For example, the latest work by Keating et al. (2019) incorporated a broad monetary aggerate in monetary models within the NK framework and their empirical findings support that the quantity of money could affect real activity independently of any variations in interest rate.

There is also debate about the informational role of monetary aggregates in the macroeconomic policy and business cycle analysis. For example, analysing the Volcker disinflation, Friedman and Kuttner (1996) point out that the economy is exposed to the money market shocks by targeting broad money, which is more volatile than other types of aggregate demand shocks. Hence, reacting to fluctuations in money may be destabilizing. However, Clarida et al. (1999) state that monetary aggregates are almost immediately observable and correlated to inflation and output, and not subject to informational delays such as Federal Funds rate. In contrast, Woodford (2008) argues that the FED's ability to anchor down inflation has not been impeded by information lags.

Under the assumption of separability of consumption money in the representative agent's utility function, McCallum (2001) notes that there is no money in the standard equilibrium conditions of an NK model. He subsequently argues against this assumption since it implies that a consumer may increase his consumption independently of the amount of money withdrawn or the frequency of withdrawals. While sensitive to this argument, using post-1980 US data, Ireland (2004) supports the separability with empirical evidence and concludes that removal of money from the analysis is not problematic.

Another reason that some researchers advocate a monetary model devoid of money is the failure to properly measure the quantity of the money. Most of the research examines the role of simple-sum monetary measures rather than an index-number based measure. In summation, money is summed by the nominal values of monetary assets and perfect substitutability is required, while different yields for different assets means assets must be imperfect substitutes. With this simple-sum measure, the aggregates significantly overstate the money stock (Barnett et al., 2008). Further, Kelly et al. (2011) also find that a simple-sum aggregate masks the expected liquidity following changes in the money supply. This inappropriate measurement can relate to the failure of finding a significant relationship between money and macroeconomic variables (Chrystal and MacDonald, 1994). The preferred alternative is to measure money via Divisia index numbers, as proposed by Barnett (1978, 1980). He produces Divisia monetary aggregates using the Divisia quantity index formula with user cost prices. This alternative measure is found to be both theoretically and empirically superior to its simple-sum counterpart as an economic indicator. See, for example, Schunk

(2001), Barnett et al. (2016), Barnett and Su (2015), Barnett and Tang (2016), and Tang et al. (2020), among many others.

There is another reason for the breakdown of the strong relationship between monetary aggregates and economic activity. The erosion can be attributed primarily to an explosion of financial innovations and the mass adoption of new money markets, mutual funds and other assets in the 1980s. In a data-rich environment with a multitude of monetary instruments, a single and narrow measure of money balances such as M2 loses its appeal. Duca (1994, 2000) found that M2B (M2 plus bond funds) and M2+ (M2 plus bond and stock funds) are the better indicators of nominal GDP growth than M2 alone. Moreover, as the structure of financial innovation has expanded, the internal consistency of monetary aggregation with economic theory is growing in importance. Divisia aggregation is directly derived from economic theory and assures consistency with economic theory at all levels of aggregation. Using a superlative index number, Binner et al. (2018) construct risky Divisia monetary aggregates adding bonds, equities and unit trusts, and find statistically significant responses of GDP and price level to the risky money Divisia measure for the USA.

Quantitative Easing (QE) emerged as a significant monetary policy tool during periods when the Federal Funds Rate (FFR) was near zero, aimed at stimulating the economy by increasing the money supply. However, QE is not an ideal indicator of policy shocks. This limitation can be understood through the lens of the quantity theory of money, which is encapsulated by the equation $MV=PY$, where M is the money supply, V is the velocity of money, P is the price level, and Y is the output (Lipsey and Chrystal, 2011). The theory suggests that changes in the money supply should directly influence price levels and economic output. However, QE increases bank reserves significantly, which does not always translate into proportional increases in broader money supply metrics like M2 due to the unstable velocity of money. Consequently, QE's effect on lending and spending is inconsistent, making it an unreliable measure of monetary policy shocks.

Keynesian economics further complicates the efficacy of QE as an indicator. Keynes (1936) argued that during periods of low interest rates and economic uncertainty, increases in the money supply do not necessarily lead to increased investment or consumption, a phenomenon he described as a liquidity trap. This situation was evident during the post-2008 financial crisis when QE led to substantial increases in bank reserves without corresponding rises in consumer spending or investment, thereby failing to provide clear signals of monetary policy impacts. Although QE has played a critical role in modern monetary policy, its effectiveness as an indicator of policy shocks is limited due to its inconsistent impact on the broader money supply and economic activity. Therefore, accurate measurement of the money supply is crucial for effective monetary policy. The Divisia monetary aggregate, with its sophisticated mechanism for weighting the liquidity services provided by each individual component asset , offers a superior alternative for capturing the true measure of the money supply, thereby providing more accurate information on the quantity of money in the economy to better inform monetary policy decisions.

One raised concern with the Divisia monetary aggregate is that central banks, such as the Federal Reserve, have not shown any inclination towards using this aggregate as a policy instrument. This may call into question the interpretation of the identified monetary policy shocks (Keating et al., 2019). A rethinking and refocus on the aggregates and their user costs produces a new perspective and explanation for the macroeconomic trends; for example, Belongia and Ireland (2006) demonstrate that the own-price of money exerts a strong influence on the output. This economic price dual to an economic quantity aggregate is a function of all interest rates for all monetary components and has not been subject to a lower bound constraint. In addition, the user cost is found to be highly correlated to the Federal Funds rate prior to the zero lower bound (ZLB) and also useful after 2008 (Keating et al., 2019). Thus, the goal of this chapter is to develop a model of monetary policy shocks, which can be easily interpreted and employed after the financial crisis and in the ZLB period while taking the financial innovations into consideration. To reach this end, we develop a vector autoregression (VAR) model while maintaining the key features from the basic CEE framework, which uses a Divisia price dual as the policy indicator.

This study also relates to an emerging area, that of correcting externality by economic policy for climate change, which has been recognized as the greatest externality of today's global economy (Economides and Xepapadeas, 2018). The most straightforward policy instrument to solve environmental externalities is to simply price the environmental externality (Gillingham and Sweeney, 2010). For example, the price can be imposed directly as a pollution tax or pollution fee, with the optimal tax set at the magnitude of the externality. Or the cap-and-trade system can impose a limit for emissions, in which case the allowances under this system would produce a market-clearing price for the allowances (Dissou and Karnizova, 2016). The cap set should meet a condition that the permit price is equal to the magnitude of the externality. However, these climate change policies have been predominately a fiscal policy and little attention has been paid to the implication of monetary policy and the role of central banks. From the monetary policy perspective, in order to have an accurate price for the externality, we are therefore motivated to adopt the user cost of green bonds to assess whether the cost from investing in a green bond can somehow capture the externality in the market for the cost of the environment by causing pollution and changing the climate.

Until recently, the monetary policy has been linked to the environmental issues in macroeconomics. Researchers have started a discussion on how the environmental problems and relevant monetary policies could affect the macro economy by using a newly established environmental dynamic stochastic general equilibrium (E-DSGE) model. In their model, the monetary policy cannot be regarded as a climate policy instrument, thus they use the above fiscal instruments like carbon taxes to demonstrate how monetary policy should be adjusted under conditions of climate change. For example, Annicchiarico and Di Dio (2017) propose an NK model including pollutant emissions, abatement technology and environmental damages to examine the optimal environmental and monetary policy mix in response to productivity shocks. In their model, when the Ramsey planner controls the level of emissions and monetary policy obeys a simple monetary rule, the optimal environmental policy depends crucially on the monetary policy. If monetary policy is strongly responsive to output, then the optimal environmental policy will deliver a strong increase in the carbon price in reaction to a positive shock, delivering countercyclical emissions. Unlike Annicchiarico and Di Dio (2017), Economides and Xepapadeas (2018) build an NK model embodying a climate module and energy produced by the processing of fossil fuels and explore the effects of the introduction of a carbon tax to show how an ambitious greening policy may generate large fluctuations in the price levels. Our work differs from these studies in that including the green bonds in the basket, we construct a green Divisia price dual as a climate monetary policy tool in our VAR model to evaluate whether this price dual can be a warning sign to the economy in the short run with the limited data and short time span for green bonds.

This chapter operates on a few novel dimensions. Although we are not the first to propose incorporating the user cost of monetary aggregates in a recursive VAR, few papers have focused on empirical applications of the user cost as the policy indicator. To our knowledge, this chapter is the first work that employs a newly green augmented Divisia price dual as the policy indicator in the monetary VAR literature. Hence, we highlight the informativeness of new green Divisia user cost in nowcasting real output and inflation in the short term when taking environmental factors into consideration.

Several factors lead us to work with the user cost as our policy indicator. First, the Divisia price dual includes the assets that were especially targeted by the Federal Reserve's liquidity facilities such as repurchase agreements, institutional deposits and commercial papers. Second, the inclusion of government bonds and corporate bonds highlights the financial innovations and also avoids the puzzling responses of output, prices, interest rates, or other monetary aggregates to monetary policy shocks identified when M2 is the policy indicator. Finally, the user cost is closely correlated with the Federal Funds rate and could be an alternative for that rate as a policy indicator, following the Taylor rule. The price dual is also useful when the Federal Funds rate becomes stuck at its ZLB after 2008. Our VAR model will provide the empirical analysis and structural interpretation of the ability of Divisia price dual to identify monetary policy shocks without producing empirical puzzles.

The remainder of this chapter is organized as follows. Section 2 presents the theoretical Divisia approach to monetary aggregation and its (green augmented) Divisia price dual. Section 3 discusses a small-scale NK model with a Divisia price dual to study the effects of a monetary policy shock under an interest rate. Section 4 outlines the VAR models of monetary shocks, followed by the data in Section 5. Section 6 illustrates the construction of green augmented price dual. The empirical investigation is conducted in Section 7; in particular, we modify and extend the classic approach to identifying monetary policy shocks with Divisia price dual as the policy indicator in a VAR model. The variance decompositions are assessed in Section 8 and the robust check is given in Section 9. Section 10 concludes with an overview of the results and directions for future work.

2 Measuring the Green Divisia Price Dual of Monetary Aggregates

Empirical work in monetary economics typically uses simple aggregate measures such as M1 and M2 produced by central banks. These aggregates simply add up the nominal values of monetary assets. This methodology is referred to as simple sum aggregation and it assumes that each component is a perfect substitute for every other asset and as such, the representative consumer is assumed to have linear utility function. The perfect substitutability assumption is problematic, given that these monetary assets may have different nominal yields. This became known as the Barnett Critique after the seminar work by Barnett (1980) where he proposed an alternative methodology for an aggregate quantity of money with the invention of the Divisia index. The Divisia method relaxes the unrealistic assumption of perfect substitutability among sub-aggregates by introducing a micro-founded framework to weight the component assets with their shares of the total expenditure on monetary services. And even though the form of the sub-utility function for money holdings is unknown, the index number can track its value over time as changes in the relative prices of alternative forms of money induce substitutions. These substitutions alter the expenditure share weights of the components and the quantities held of those deposit categories within the aggregate.

No matter what type of index is chosen to measure the quantity of money, the principle of duality requires that as a matter of the internal consistency, each is paired with a precise expression for the corresponding own price of money. For the case of a linear utility function, the Leontief unit cost function is the price dual. Because of the perfect substitution assumption of simple sum aggregation which implies the equal coefficients of the linear utility function, the coefficients of the Leontief unit cost function are equal (Barnett and Serletis, 2000). In addition, because of the perfect substitutability, the agents will be expected to hold only the assets with the lowest price (Belongia and Ireland, 2006). In contrast, when the quantity of money is measured by a Divisia index, the own price of aggregate money is the share-weighted sum of each asset’s user cost. In both cases, the price dual of the monetary assets is the interest that consumers forgo to consume the services of the assets. It is assessed by the difference between the interest rate of return from holding an asset and the maximum expected benchmark rate, which is defined as the rate of return on pure investment capital providing no monetary services (Barnett, 1978).

To illustrate these points, the construction of a superlative index number starts with calculating the total expenditure on the components of the aggregate. That expenditure m_t^* can be written as $m_t^* = \sum m_{it}\pi_{it}$. m_{it} is the nominal quantity of monetary asset i at t and π_{it} is its real price. Equivalently, one can construct the expenditure magnitude by using real quantities and nominal prices. Because monetary assets are durables that do not perish during the period for use, their prices are their user costs. The formula for the real user cost of a monetary asset, derived by Barnett (1978, 1980), is given by:

$$\pi_{it} = p_t^* \frac{R_t - r_{it}}{1 + R_t} \tag{B.1}$$

where R_t is the benchmark asset rate of return measuring the maximum expected rate of return available in the economy at time *t*, and r_{it} is the own rate of return on monetary asset *i* at time *t*, p_t^* is the true cost-of-living index price at time *t*, approximated by a consumer price index.

With the user cost and quantity data, the expenditure share on asset i is $s_{it} = m_{it}\pi_{it}/m_t^*$. A Divisia quantity index in continuous time with the Törnqvist-Theil (Törnqvist, 1936; Theil, 1967) discrete time approximation computes the growth rate of the aggregate as the share-weighted average of this components and is constructed as:

$$log(M_t) - log(M_{t-1}) = \sum_{i=1}^{n} (\frac{s_{it}+s_{it-1}}{2})(log(m_{it}) - log(m_{i,t-1})) \tag{B.2}$$

while the Divisia price index Π_t in continuous time is constructed as:

$$log(\Pi_t) - log(\Pi_{t-1}) = \sum_{i=1}^{n} (\frac{s_{it}+s_{it-1}}{2})(log(\pi_{it}) - log(\pi_{i,t-1})) \tag{B.3}$$

As suggested in the introduction, analysis that remains at an M2 level of aggregation might be too narrow to shed light on the turbulence of the financial crisis. Thus, we focus on the broadest measure of money currently available referred to as Divisia M4 (DM4), which is produced by Center for Financial Stability (CFS). Our asset basket includes currency, demand deposits, other liquid deposits (the sum of other checkable deposits at commercial banks, other checkable deposits at thrift institutions, saving deposits at commercial banks and saving deposits at thrift institutions), retail money market funds, small time deposits in total (the sum of small time deposits at commercial banks and small time deposits at thrift institutions), large time deposits, repurchase agreements, commercial papers and T-bills. We exclude the travellers' checks due to the unavailable data in our sample period. In light of the financial innovations and climate change, we extend monetary aggregates to include the green bonds, government bonds and corporate bonds to construct the green augmented Divisia monetary aggregate and the green augmented Divisia price dual by using the formula (B.2) and (B.3). In this construction, we excluded T-bills in case of the double counting when including the government bonds. Because these bonds are risky assets, the risk adjustment of their user costs has to be addressed. We follow the existing risk-adjusted user cost formula established by Barnett et al. (1997). By applying the modified Arrow-Pratt relative risk aversion measure Z_t, the nominal adjusted user cost is given by:

$$\pi_{it}^{adj} = p_t^* \frac{E(R_t^*) - (E(r_{it}^*) - \phi_{it})}{1 + E(R_t^*)} \tag{B.4}$$

where $\phi_{it} = Z_t \, Cov\,(r_{it}^*, \frac{c_{t+1}}{c_t})$; where c is the measure of consumption and * indicates real rates. The function of ϕ_{it} is a risk adjustment to the unadjusted expected excess rate of return r_{it}^*.

3 Monetary Policy Shocks in a New-Keynesian Model with Divisia Price Dual

This section describes a small-scale NK model of the business cycle to assess the effects of monetary policy shocks under a price dual rule. The NK framework provides the foundation of the NK dynamic stochastic general equilibrium (DSGE) model which is the workhorse model for the analysis of monetary policy by central banks. The NK model follows the earlier rational expectations' model of Lucas Jr (1972) and Sargent and Wallace (1975), in which the role of expectations in the monetary transmission mechanism is stressed. The model builds expectations into the optimizing behaviour of households and firms through the real business cycle model (Kydland and Prescott, 1982; Kimball, 1995). The model is micro-founded and is built based on the assumptions of rational expectations of economic agents – the households, the producers/firms, and the government. The interaction of these agents in the market gives room for the market clearance and the fulfilment of the "general equilibrium" condition. Under this framework, firms are modelled as monopolistic competitors and the non-neutrality feature of monetary policy is introduced to the model by the assumption of nominal rigidity (Walsh, 2017). The basic structure of NK DSGE model consists of three blocks – a demand block, a supply block and a monetary policy block (Sbordone et al., 2010). The blocks contain three equations and three variables, and the equations can be presented as follows:

$$y_t = \beta_1 y_{t+1} - \beta_2(r_t - inf^e_{t+1}) \tag{B.5}$$

$$inf_t = \gamma_1 inf^e_{t+1} + \gamma_2 y_t \tag{B.6}$$

$$r_t = \emptyset_2 inf_{t+1} + \emptyset_3 y_t \tag{B.7}$$

In the above equations, y_t is the economy wide output, inf_t is the current inflation rate, and r_t is the nominal interest rate. From the demand block by equation (B.5), it is obvious that the current output is linked to its expected future value and to the ex-ante real interest rate. From this linkage, it indicates that when the real interest rate is temporarily high, households are willing to spend less of their current incomes and firms would rather save than invest. The link connecting the demand block to the supply block reveals that the level of activity is a key input in the determination of current inflation, together with the expectation of future inflation, which is shown in equation (B.6). The supply block shows that in prosperous times, high level of economic activities encourages firms to raise wages so as to induce employees to work longer hours. Higher wages increase marginal costs, putting pressure on prices and generating inflation. Moreover, the higher inflation is expected to be in the future, the higher is his increase in prices, thus contributing to a rise in inflation today.

As shown in equation (B.7), the monetary policy block is an interest rate rule for monetary policy that is similar to the type suggested by Taylor (1993). The termination of output and inflation from the demand and supply blocks feeds into the monetary policy block. The equation in this block describes how central banks set the nominal interest rate, usually as a function of inflation and real activity. This reflects the tendency of central banks to conduct monetary policy using the target of the short-term interest rate as opposed to any of the monetary aggregates. In the standard NK model, monetary policy works through the conventional Keynesian interest rate channel. For instance, a shock to interest rate by reducing the monetary policy rate would lower the short-term nominal interest rate, which transforms into a reduction in the real interest rate arising from costly or staggered price setting (Ireland, 2010). This reduction motives consumers to increase their current consumption or spending, which raises output and price with the adjustments after the shock. Thus, this policy rule closes the circle among demand block, supply block and monetary policy block, presenting a complete model of the relationship between three key endogenous variables: output, inflation, and nominal interest rate.

The role of expectations and the dynamic connections between blocks is highlighted in the model, in which the influence of expectations on the economy flows from monetary policy to the demand and then the supply block to determine output and inflation. This emphasises that the conduct of monetary policy has a large influence on the formation of expectations. In DSGE models, expectations are the main channel through which policy affects the economy, a feature that is consistent with the close attention paid by financial markets and the public to the pronouncements of central banks on their likely course of action. The last component of DSGE models captured is their stochastic nature. Every period, random exogenous events perturb the equilibrium conditions in each block, injecting uncertainty in the evolution of the economy and thus generating economic fluctuations.

One deviation from the standard models is found, for example, in the studies of Woodford and Walsh (2005) and Keating et al. (2019), that a broad measure of money is specified in the model as a constant elasticity of substitution (CES) aggregate currency and interest-bearing assets as in Belongia and Ireland (2014). Their adjustments allow for a more direct comparison to the monetary aggregates used as the policy indicator in the VAR model. The work by Serletis and Gogas (2014) shows that Divisia monetary aggregates hold promise as an alternative indicator of monetary policy in a VAR. We take the advantage of Divisia monetary aggregates but use the price dual as the policy indicator to avoid the interpretation problems of identifying shocks using the monetary aggregates as the policy indicator. Since the price dual or user cost is a function of all interest rates relevant to the monetary aggregates and is highly related to Federal Funds rate, the model using the user cost as the policy indicator can be described by an interest rate rule. Keating et al. (2019) show that a NK model can be closed with a specification of monetary policy that may be described by an interest rate feedback rule. Hence, we follow their specification and replace the interest rate with our user cost or price dual in our VAR model:

$$\pi_t = \rho\pi_{t-1} + (1-\rho)(\emptyset_{inf} inf_t + \emptyset_y(y_t - y_{t-1})) + u_t^{mp} \tag{B.8}$$

where u_t^{mp} is an i.i.d. monetary policy shock, ρ denotes the degree of policy inertia, and the $\emptyset$s are the standard stabilization coefficients in a Taylor-type rule.

4 The VAR Model of Monetary Policy Shocks

The theoretical model laid out is used to help formulate our new strategy for identifying monetary shocks based on the reduced-from VAR. We follow the general methodology outlined in the works by Christiano et al. (1999) and Keating et al. (2019) to obtain:

$$Z_t = B(L)u_t \tag{B.9}$$

where Z_t is an n-vector of variables; $B(L) = (I - B_1 L - \cdots - B_q L^q)^{-1}$ where the B_i are the coefficients obtained from a qth order VAR and $E(u_t u_t') = V$ is the covariance matrix of the for residuals. Correspondingly, a linear dynamic structural model yields:

$$Z_t = A(L)\varepsilon_t \tag{B.10}$$

where $A(L) = (A_0 - A_1 L - \cdots - A_q L^q)^{-1}$, assuming the dynamic structural VAR model requires q lags, and $E(\varepsilon_t \varepsilon_t') = D$ is the diagonal covariance matrix of structural shocks. The variables in the model are categorized into three groups:

$$Z_t = \begin{pmatrix} EA_t \\ imp_t \\ MI_t \end{pmatrix} \tag{B.11}$$

where EA_t represents a vector of *Economic Activity* variables, MI_t represents a vector of *Monetary Information* variables all providing informational content to the monetary policy makers such as the FED, and imp_t represents a single variable that serves as an indicator of monetary policy shocks. Each group might consist of multiple variables; however, in this work, imp_t is a single policy indicator. The structure is assumed as a block-recursive structure and takes the following form for impact matrix A_0:

$$A_0 = \begin{pmatrix} A_{11} & 0_{12} & 0_{13} \\ A_{21} & A_{22} & 0_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \tag{B.12}$$

where A_{ij} and 0_{ij} matrices of parameters and zeros, respectively. With the single policy instrument, Christiano et al. (1999) prove that a Cholesky factor of V – the covariance matrix of the system described by (B.10) – will identify the dynamic responses of all the Z variables to the monetary policy for any ordering consistent with the structural model. The ordering in (B.11) works, as will any Cholesky ordering that place all variables in the EA block ahead of imp and all variables in the MI block after imp.

This structural model assumes that monetary policy responds contemporaneously to the macroeconomic activity variables but may respond only with a lag to the monetary information.

It also assumes that the economic aggregates respond with a lag to all types of monetary variables. Throughout our analysis, we follow Keating et al. (2019) and identify monetary policy shocks using a block-recursive formulation in which EA_t consists of the real gross domestic product (GDP) and the implicit GDP price deflator. These two variables are included because of the assumption that central banks consider real output and prices when determining the stance of monetary policy. If the policy variable is the Federal Funds rate, a reaction to these two variables is consistent with a Taylor rule formulation, which is often assumed to describe the central bank's policy rule.

The monetary information block MI_t consists of money market variables that respond immediately to EA_t or imp_t but only affect these variables with a lag. In the benchmark specifications of Christiano et al. (1999), this information block includes non-borrowed reserves, total reserve and along with M1 or M2. However, the non-borrowed reserves became negative beginning from 2008. This is theoretically impossible, and it indicates significant measurement error. Therefore, we replace this variable with the monetary base which does not exhibit this strange behaviour and encompass the information that would be included in non-borrowed reserves.

Finally, we select the monetary policy instrument imp_t variable. We use Federal Funds rate and green augmented Divisia price dual as the policy indicator, respectively, to assess whether or not the green augmented price dual adds values compared to Federal Funds rate in identifying monetary policy shocks and in exerting the climate change monetary policy.

5 Data

Our choice of country in this study is the USA as the USA has one of largest and most influential green bond markets in the world and is one of the first countries to engage in green bond issuance, and hence the green bond data available for this chapter will provide more robust results than can be obtained elsewhere. For all monetary assets and economic variables, we used seasonally adjusted quarterly data covering the period 2009Q1 to 2021Q4 since 2009Q1 is the earliest available quarterly data on green bonds. Our sample period covers the period after financial crisis and the period during the COVID-19. The goods and assets[17] considered are:

i) Currency (CUR),

ii) Demand Deposits (DD),

iii) Other Liquid Deposits (OLD) (other checkable deposits + savings deposits),

iv) Retail Money Market Funds (RMMF),

v) Small Time Deposits Total (STDT) (small time deposits commercial and small time deposits thrift),

vi) Institutional Money Market Funds (IMMF),

vii) Large Time Deposits (LTD),

viii) Repurchase Agreements (RA),

ix) Commercial Paper (CP),

x) T-Bills (TB),

xi) Green bonds holdings (GREENB),

xii) Government bonds holdings (GOVB),

xiii) Corporate bonds holdings (CORB).

We obtained the USA data for nominal quantities of the monetary assets i through x, and the associated own rates of return from the Center for Financial Stability[18] (Barnett et al., 2013). We obtained the nominal household sector holdings of green bonds, asset xi, at market

[17] According to the Center for Financial Stability (CFS), other checkable deposits commercial, other checkable deposits thrift, savings deposits commercial and savings deposits thrift are aggregated into other liquid deposits (asset 6) from May 2020. Small time deposits commercial and small time deposits thrift are also aggregated into small time deposits total (asset 8) from May 2020. We follow the CFS to aggregate other checkable deposits, savings deposits and small time deposits into separate series.

[18] Some of the USA data is available on the CFS website at the following link http://www.centerforfinancialstability.org/amfm_data.php. Some of the data are obtained in personal contact with the CFS researchers.

values from Bloomberg. The nominal household sector holdings of government bonds and corporate bonds, asset xii and xiii, at market value, were downloaded from FRED. We used S&P 500 green bond index and S&P 500 corporate bond index for the return on green bonds and corporate bonds, which were downloaded from DataStream. We used government-bond price index for the return on government bonds. These data were also obtained from DataStream.

For macroeconomic variables, we obtained the GDP, the implicit GDP price deflator, Federal Funds rates[19] and price consumer index (CPI) from FRED. The base year for the GDP is 2012 and for the CPI is 2015. From the same database, the monetary base was downloaded.

6 The Construction of Green Augmented Price Dual

For the bonds' assets, we use the forecasted returns. We consider the green bonds, government bonds and corporate bonds as the least liquid assets, since individual investors do not change their asset portfolio quickly and the bonds are the most expensive way to obtain medium of exchange (Binner et al., 2018). Therefore, we forecast a one-year ahead expected return on green bonds, government bonds and corporate bonds.

The calculation of CCPAM special-case risk-adjusted real user cost for the bonds depends on the forecasted real returns on the bonds and an Arrow-Pratt measure of relative risk aversion. For the computation of the real returns, we use CPI serving as a proxy for the true cost of living index to convert all returns including capital certain assets' returns into real term. We forecast a one-year ahead expected real returns on all assets by using an autoregressive model. These one-year ahead forecasts are used to construct interest rate forecasts that we use in the user cost computations. It is known that unreasonably high estimates of the degree of relative risk aversion are yielded in empirical studies, such as 25 in the study by Mehra and

[19] We adjust Federal Funds rates and monetary base data for seasonal patterns by using X-13ARIMA-SEATS, which is the seasonal adjustment software developed and adopted by the U.S. Census Bureau.

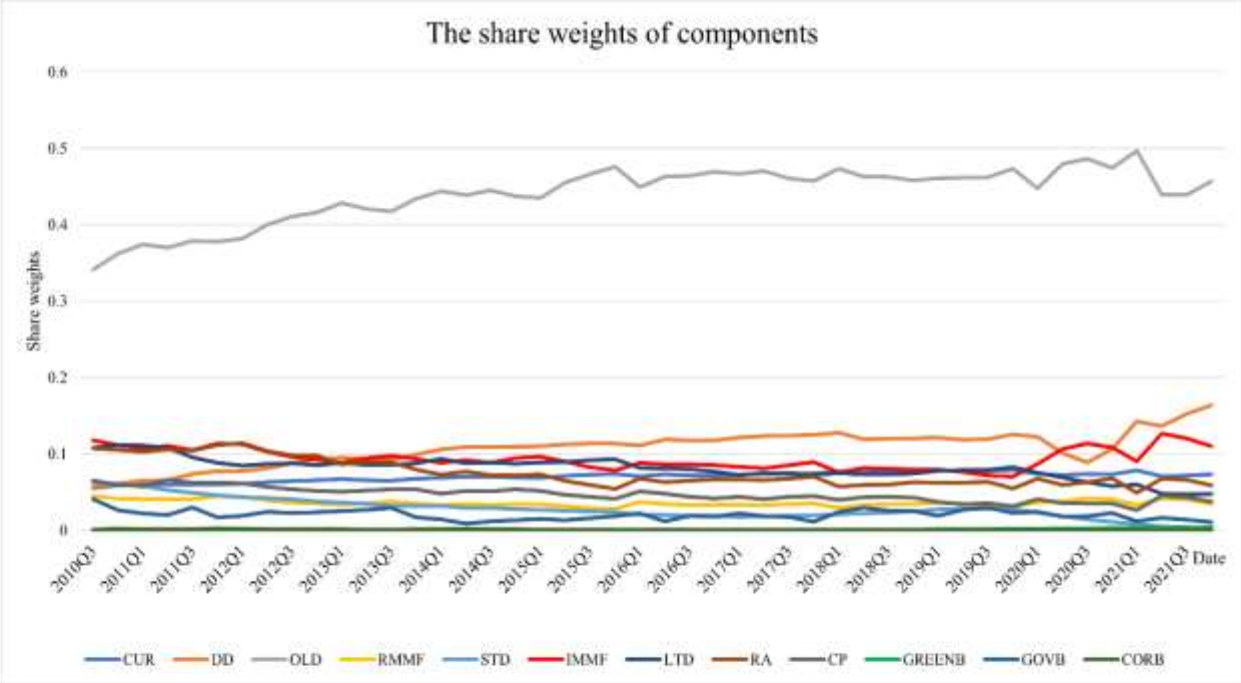
Prescott (1985). Since it is difficult to find an economically reasonable estimate of the degree of relative risk aversion, it becomes sensible to choose a value directly. Drake et al. (2000) use values within a range from 0 to 7. In our estimations, we take an average value of 3.5, while the value of the coefficient of relative risk aversion has only small impact on the monetary aggregates due to the low covariances between the real rates of returns on risky assets and the growth rate of real consumption. In our calculation, the estimated covariances for the green bonds, government bonds and corporate bonds are -1.31×10^{-5}, 1.09×10^{-4} and -9.49×10^{-5}, respectively.

The benchmark rate used in calculating user costs for all assets is constructed with an envelope approach. The benchmark rate is the maximum rate of return from the interest rates on the different monetary assets within our index. However, there are periods in our sample when the returns on components are equal to the benchmark rate, thus leading to zero or negative weights. Zero user costs imply that the transaction services provided by the asset are free, which is unrealistic. In order to ensure that all user costs of monetary assets and financial assets above zero, a simple way is to add a constant to the benchmark rate as arbitrary adjustments or liquidity premium. The Divisia indices provided by the FED of St. Louis use 100 basis points (Anderson and Jones, 2011). In line with Anderson and Jones (2011), a constant of 100 basis points is necessary to obtain positive weights throughout in our sample, which is the minimum value to make all user costs be positive.

Figure B.1 shows a large difference of the share weights between OLD and other assets. Precisely, the average share weight of OLD is 0.44 while the average share weights of other assets range from 0 to 0.1. This is due to the combination of four assets into OLD which provide more liquidity in the market. Excluding OLD, Figure B.2 plots the share weights of other components. The share weights of bonds are smaller than that of other assets because the bonds are least liquid assets and thus the user costs are small. The share weight of green bonds is quite small which is around 0.00079. This is partly because the quantity is relatively low since it started from 2009. On the other hand, green bonds, similar to other financial instruments, were affected by the COVID-19 pandemic. Initially, there was significant market volatility and uncertainty across all sectors, including green bonds. Investors were cautious and risk-averse, leading to a general slowdown in bond issuances, including green bonds. Many companies and governments were focused on managing the immediate economic and health impacts of the pandemic, diverting attention away from sustainable finance initiatives.

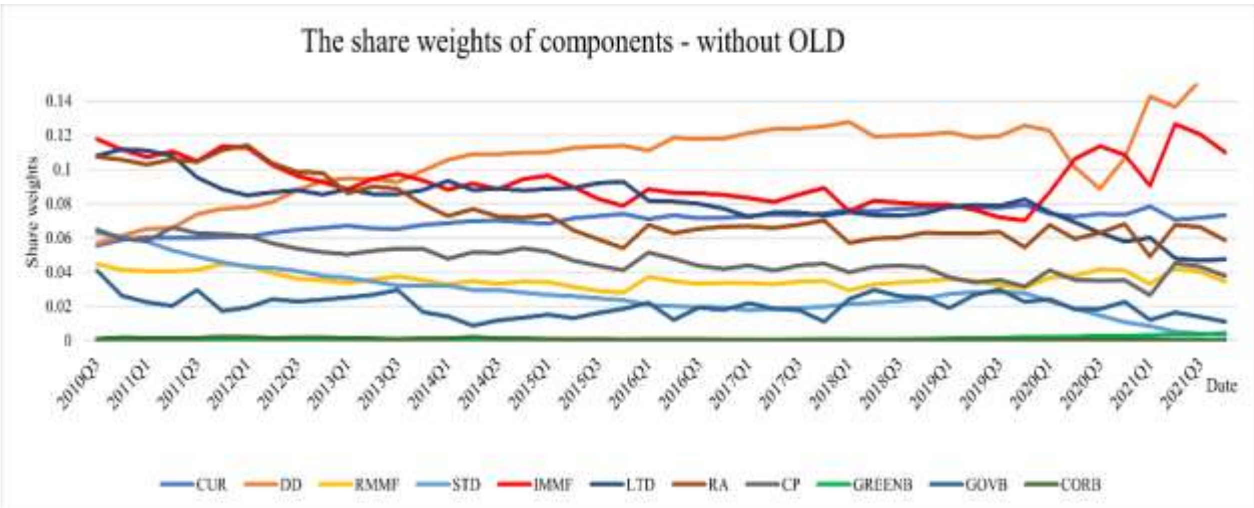
However, as the pandemic progressed and governments and central banks implemented fiscal stimulus measures such as tax incentives, there was a growing recognition for the role of sustainable investments, including green bonds, in the recovery of economy. Green bonds, which are used to finance projects with positive environmental impacts, gained attention as a means to support a green and sustainable economic recovery. The heightened focus on sustainability and the transition to a low-carbon economy are expected to drive continued growth in green bond issuances, providing investors with opportunities to support environmentally beneficial projects. The increasing trend in the share weight of green bonds (i.e., the green line) in Figure B.2 confirms that the market of green bonds is growing recently, thus, the green bonds, forming a useful way of providing liquidity services, cannot be neglected.

Figure B.1 Share Weights of All Components Contained in Green Monetary Aggregates Price Dual



Notes: The figure plots the share weights of all components. The other liquid deposits (OLD) have the highest share weights since OLD combines four assets.

Figure B.2 Share Weights of All Components Contained in Green Monetary Aggregates Price Dual-without OLD



Notes: The figure plots the share weights of all components except the other liquid deposits (OLD).

We next turn to constructing the green augmented monetary aggregates and price dual. As discussed above in Section 2, we use the Törnqvist-Theil discrete time approximation of the continuous time Divisia index to construct our monetary index. To compare, we also construct green augmented simple sum monetary aggregates, traditional simple sum monetary aggregates and traditional Divisia monetary aggregates. The traditional monetary aggregates exclude the green bonds, government bonds and corporate bonds but include T-bills to be consistent. The levels of green augmented simple sum monetary aggregates, green augmented Divisia monetary aggregates, traditional simple sum and Divisia are normalized to 100 at the first period (i.e., 2010Q1) to ensure their comparability. To be consistent with the green simple sum monetary aggregates in *MI* block, we construct green simple sum user cost by taking the average of all components' own rates.

The plot in Figure B.3 depicts the comparison between the green price dual (i.e., the green line) and the traditional price dual (i.e., the blue line). Figures B.3 indicates that the green augmented price dual has not be subject to a zero lower bound constraint, thus, it could be an alternative for Federal Funds rate, while the traditional price dual is near to zero. Furthermore, with the proper measure of money, the construction of the Divisia price dual internalises the portfolio dynamics and thus reduces the impact of COVID-19 on our analysis.

Figure B.3 Green Monetary Aggregates Price Dual and Traditional Monetary Price Dual

Notes: This figure shows the comparison between the green augmented price dual and the traditional price dual. The green line denotes the green augmented price dual, and the blue line denotes the traditional price dual which excludes the green bonds, corporate bonds and government bonds.

7 The Model Specifications of Monetary Policy Shocks

Following the study by Keating et al. (2019), we propose to use green augmented price dual as the policy indicator to identify monetary policy shocks. In this section, we show this approach is able to produce impulse responses which are free from price puzzle and have significant effects on prices compared to the model using the Federal Funds rate as the policy indicator. To compare, we additionally employ the government rate and the corporate bond rate as the policy indicator to examine whether these rates could serve as effective signals during the period of ZLB. We use both green Divisia monetary aggregates and green simple sum counterparts in *MI* block to test whether the results are different by using different measures of money. All variables except for the green augmented price dual, the Federal Funds rate, the corporate bond rate and government bond rate are transformed into their logarithmic form.

Descriptive statistics of the data used for the VAR analysis are presented in Table B.1. The mean value of GDP is 9.770, with a median almost identical at 9.768, indicating a relatively symmetric distribution. The standard deviation is low at 0.069, showing little variability. GDP deflator has an average value of 4.664, with a median slightly lower at 4.654, suggesting a small skew in the data. Notably, the average value of the green price dual is 10.101%, which is substantially higher than that of the green simple sum user cost and the Federal Funds rate. The average value of Federal Funds rate is 0.569%, which indicates it is subject to zero constraints. For the corporate bond rate and government bond rate, their standard deviations are 9.579 and 6.377, respectively, which indicates a large variability for both variables. The mean value of the monetary base is 15.075, closely matched by a median of 15.127. The maximum value recorded is 15.698, and the minimum is 14.492. Both traditional simple sum aggregates and traditional Divisia aggregates have a similar mean value, and their median and standard deviation are identical. The green simple sum aggregates have an average value of

4.686, a median of 4.656, and a notably higher standard deviation of 0.920 than that for green Divisia aggregates, indicating a considerable volatility of the green simple sum aggregates.

Table B.1 Descriptive Statistics of the Variables for the Monetary Policy Shock Analysis

Variables	Mean	Median	Std. Dev	Max	Min
GDP	9.770	9.768	0.069	9.894	9.655
GDP deflator	4.664	4.654	0.060	4.797	4.564
Green price dual	10.101	9.876	2.696	18.249	4.856
Green simple sum user cost	0.227	0.095	0.266	0.945	0.016
Federal Funds rate	0.569	0.151	0.772	2.638	0.055
Corporate bond rate	5.216	4.689	9.579	34.075	-18.194
Government bond rate	3.457	3.589	6.377	14.153	-11.935
Monetary base	15.075	15.127	0.279	15.698	14.492
Traditional simple sum aggregates	4.734	4.833	0.160	5.131	4.608
Traditional Divisia aggregates	4.834	4.833	0.160	5.131	4.608
Green simple sum aggregates	4.686	4.656	0.920	4.878	4.583
Green Divisia aggregates	4.743	4.734	0.126	5.019	4.591

Notes: The values for green price dual, green simple user cost, Federal Funds rate, government bond rate and corporate bond rate, are displayed in percentage terms. All other variables are transformed into their logarithmic form.

Since all variables involved in this chapter are time series data, the stationarity check is necessary. We use the ADF (Augmented Dickey-Fuller) test to perform unit test on variables. It can be seen from the Table B.2 that the original sequence of all variables except green price dual have unit roots, but the first differences of these variables are stationary at 5% significance level. Therefore, we use first-differenced variables to construct VAR model.

Table B.2 Units Fundamental Tests for Variables

variables	ADF	P-value	Conclusion

GDP	-0.776	0.826	nonstationary
GDP deflator	2.647	0.999	nonstationary
Green price dual	-6.075	0.000	stationary
Green simple sum user cost	-0.883	0.794	nonstationary
Federal Funds rate	-1.298	0.630	nonstationary
Corporate bond rate	-6.983	0.000	stationary
Government bond rate	-5.651	0.000	stationary
Monetary base	-0.515	0.889	nonstationary
Traditional simple sum aggregates	1.316	0.997	nonstationary
Traditional Divisia aggregates	0.696	0.990	nonstationary
Green simple sum aggregates	0.990	0.994	nonstationary
Green Divisia aggregates	2.303	0.999	nonstationary
Δ GDP	-8.723	0.000	stationary
Δ GDP deflator	-3.037	0.032	stationary
Δ Green simple sum user cost	-2.939	0.041	stationary
Δ Federal Funds rate	-4.657	0.000	stationary
Δ Monetary base	-5.043	0.000	stationary
Δ Traditional simple sum aggregates	-6.567	0.000	stationary
Δ Traditional Divisia aggregates	-6.041	0.000	stationary
Δ Green simple sum aggregates	-7.335	0.000	stationary
Δ Green Divisia aggregates	-6.094	0.000	stationary

Notes: Δ denotes the first difference. The 5% thresholds for the ADF test are 2.941 for the level and 2.944 for the first difference.

In the determination of the lag order of the VAR, we use both the Bayesian information criterion (BIC) and Akaike information criterion (AIC) to determine.

7.1 Baseline VAR Model

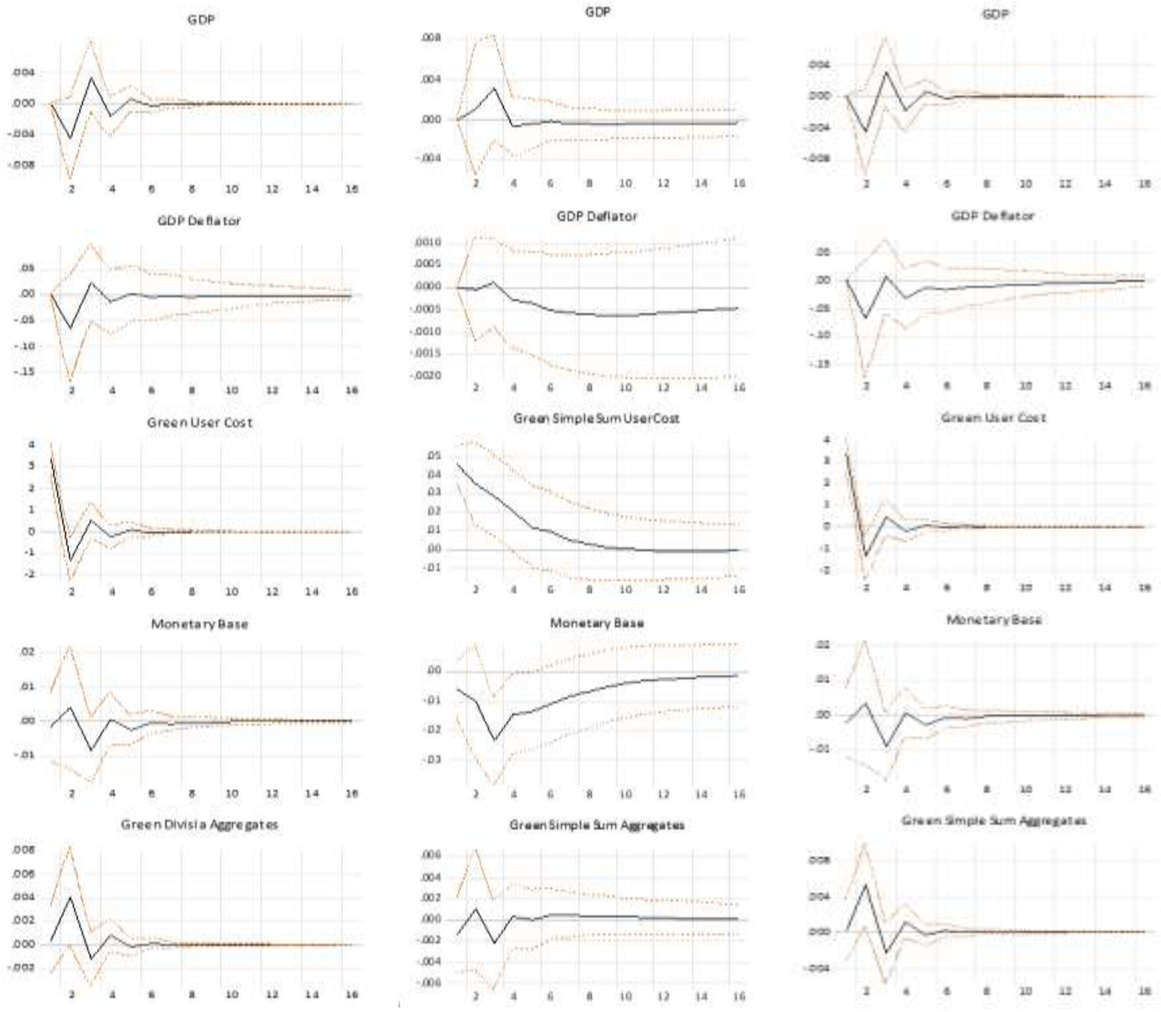
We firstly analyse impulse responses to a positive monetary policy shock over our sample period in our baseline model, which uses the green augmented price dual as the policy indicator. The AIC is minimised by an order of one for the baseline model with green Divisia monetary aggregates in *MI* block. The results are shown in the first column which use green Divisia aggregates in *MI* block in Figure B.4 along with 90% probability intervals. The impact response of real output is restricted to be zero by assumption. But in subsequent periods, output falls with a trough response and after 3 quarters increases to become positive response. This positive response only last half quarter before gradually decreasing toward zero. The negative response for real GDP has been a feature of monetary VARs. The increase in green user cost decreases the output. The unexpected positive response after 3 quarters could be explained by the behaviour of consumers. If central banks alter the price of money, individuals would be expected to reallocate their portfolios. For example, with a higher price of money, money holdings might be reduced by purchasing goods, which would stimulate the real activity, or by purchasing other financial assets, which enhance the effect of real growth since a higher asset price reduces interest rates. However, the output response is insignificant after the initial shock.

Importantly, we find no evidence of a price puzzle as the price level falls significantly around 2 years after a relatively short time lag and then increases gradually toward zero. Researchers commonly include a commodity price index on an ad hoc basis in *EA* block to solve the price puzzle problems in VAR literature. Our results based on monetary aggregates constructed in a manner consistent with economic theory of aggregation over goods are free of such puzzles. This is supported by Keating et al. (2019) that such ad hoc solutions are unnecessary when liquidity is measured properly using monetary aggregates as we use in this chapter.

The reaction of monetary base and the green Divisia aggregates are left unconstrained following the shock. The fourth plot in the left hand column shows the effect of a positive monetary shocks on monetary base. Monetary base declines in response to the positive monetary shock about 3 years before approaching to zero due to liquidity effect and this effect is significant. Further, we identify a positive response of green Divisia monetary aggregates at first, but the green Divisia aggregates soon decrease to a negative response due to liquidity effect. However, the effect is insignificant.

We then replace green Divisia monetary aggregates with green simple sum monetary aggregates in *MI* block. To be consistent with using the green simple sum monetary aggregates in monetary information block, we employ green simple sum user cost as the policy indicator, which is the average of interest rates of all monetary assets. The AIC is minimised by an order of one and the impulse responses are shown in the second column of Figure B.4. The output responses positively and significantly to a positive shock about 3 quarters and then jumps to the zero. This positive effect is inconsistent with economic theory, which is called output puzzle. The price puzzle is not found as the price declines from 3 quarters after initial shocks, but this effect is insignificant. The monetary base and green simple sum monetary aggregates significantly and negatively react to the positive monetary shock due to liquidity effect. As demonstrated by Keating et al. (2019), the elimination of measurement error arising from theoretically inferior simple sum measure of money can solve the output and price puzzles. Hence, we replace the green simple sum user cost with the more sophisticated green Divisa user cost in the second block. The AIC is minimised by an order of one and the impulse responses are shown in the third column of Figure B.4. Qualitatively, the effects are different. Output falls first and then increases to a positive response, but insignificantly, while prices respond rapidly, significantly falling more than two years after a positive monetary policy shock. This shows that our green Divisia user costs model continues to produce plausible impulse response free from price puzzles. However, the Divisia green user cost model with green simple sum monetary aggregates or green Divisia monetary aggregates in *MI* block does exhibit a liquidity puzzles. This may be explained by the indirect effects of the financial markets. The increase in the green user costs which incorporate the bonds rates would lead to a value depreciation on the financial assets and an increased liquidity for consumers. This could result in an increased spending and borrowing of the customers, thereby increasing the money supply.

Figure B.4 Impulse Responses for Baseline VAR Model



Notes: The baseline VAR model uses green Divisia user cost and green simple sum user cost as the policy indicators. The plot in the left hand column shows the impulse responses by using green user cost as the policy indicator and including green Divisia monetary aggregates in *MI* block. The plot in the middle column shows the impulses responses by using green simple sum user cost as the policy indicator and including green simple sum monetary aggregates in *MI* block. The plot in the right hand column shows the impulse responses by using green user cost as the policy indicator and including green simple sum monetary aggregates in *MI* block.

7.2 Comparisons to the Federal Funds Rate Model

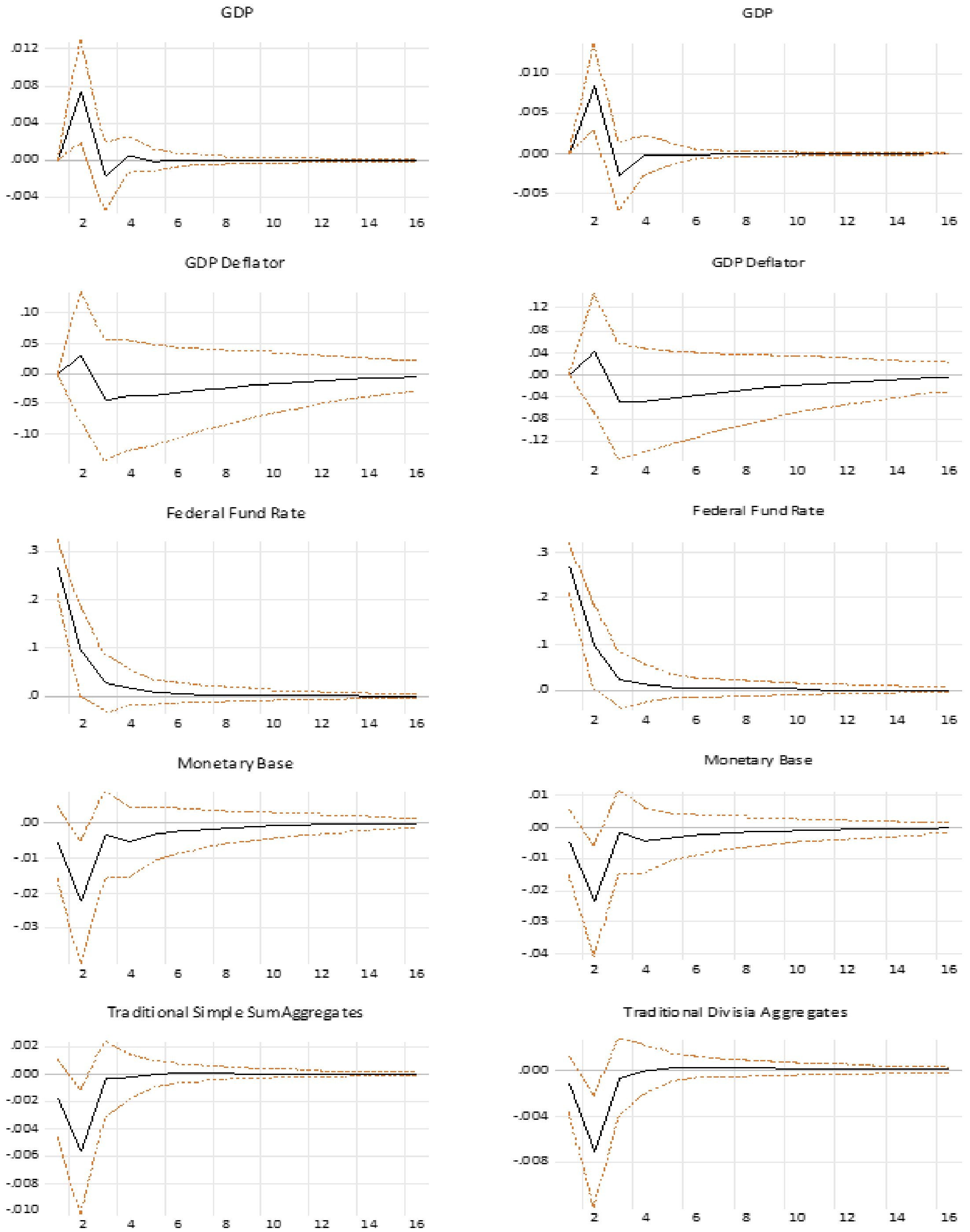
We now compare our approach to identify monetary policy shocks with the Federal Funds Rate (FF) benchmark specification of policy estimation. In the FF benchmark model, we replace the green price dual with the Federal Funds rate as the policy indicator. We also consider both Divisia monetary aggregates and simple sum counterparts in *MI* block, but we use traditional money measures which exclude green bonds, corporate bonds and corporate bonds to be consistent. The AIC is minimised by an order of two for the FF benchmark model with both traditional Divisia and traditional simple sum money measures. The impulse responses are reported in Figure B.5. The variables' responses do not share the same dynamic and statistical significance in FF benchmark model and green user cost model.

The results are shown in the first column of Figure B.5 which uses traditional simple sum monetary aggregates in *MI* block. The output responds positively to a positive monetary policy shock about 3 quarters and then jumps to zero, which means the output puzzle appears. This could be explained that when during the COVID-19 pandemic, the reduction on interest rate fails to stimulate the economy, therefore, the direction of the movement of interest rate and output is same. After financial crisis and COVID-19 pandemic, the interest rate is raised for encouraging the economic recovery. This finding is in line with the empirical evidence by Dotsey et al. (2003) and Lee and Werner (2022).

The price increases with some delay after the initial shock and declines to negative after 2 quarters, but this effect is insignificant. Unlike the model using the green user cost as the policy indicator, FF benchmark model suffers price puzzle. The responses of the variables across the lower block of the FF benchmark and green user cost model are also different. The responses of monetary base and traditional simple sum monetary aggregates are always negative and statistically significant.

We next estimate the FF benchmark model with the traditional Divisia money measure in the *MI* block and the results are shown in the second column of Figure B.5. The responses of all variables are similar to the responses in the first column of Figure B.5. The output and price generate puzzles. The responses of monetary variables are always nonpositive and significant. These results are different from the responses in green user cost model, which are free from price puzzles. Thus, we find that green price dual captures information from a portfolio of interest rates rather than one arbitrarily chosen interest rate. Therefore, the green Divisia user cost can serve as the policy indicator and has better effects on variables than when the Federal Funds rate is used.

Figure B.5 Impulse Responses for FF Benchmark VAR Model



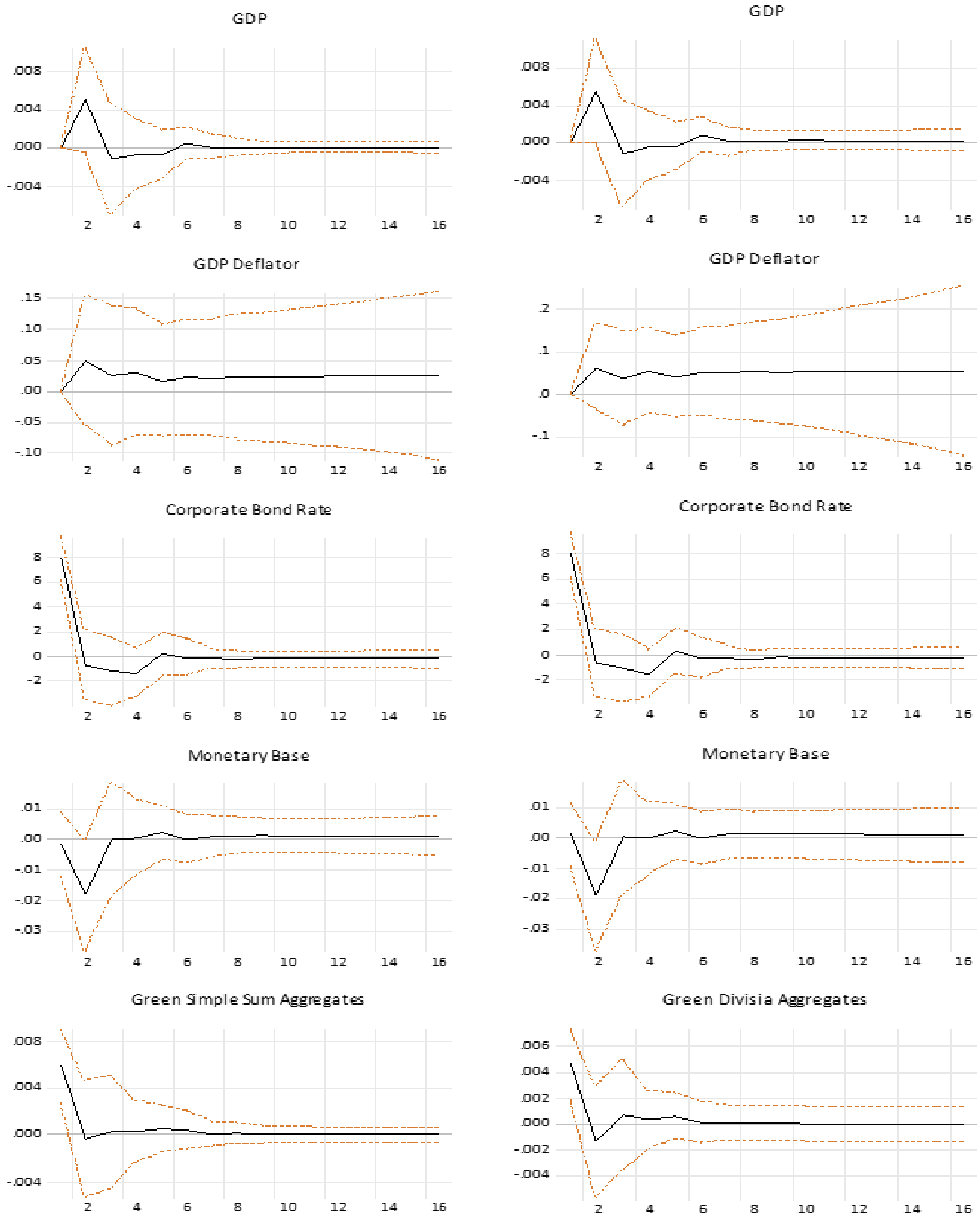
Notes: The FF benchmark VAR model uses Federal Funds rate as the policy indicator. The left hand plot shows the impulse responses by including traditional simple sum aggregates in *MI* block. The right hand plot shows the impulse responses by including traditional Divisia aggregates in *MI* block.

7.3 The Consideration of Other Bonds Rates Model

Since we extend our monetary basket to include corporate bonds and government bonds, we next consider whether the corporates bonds rate and government bonds rate could provide an alternative means as the policy indicator in our baseline model. Hence, we utilize our baseline VAR model with corporate bonds rate and government bonds rate in place of green price dual respectively. The ADF in Table B.2 indicates that both rates are level stationary at 5% significance level. The AIC is minimised by an order of one for both the corporate bonds rate model and government bonds rate model.

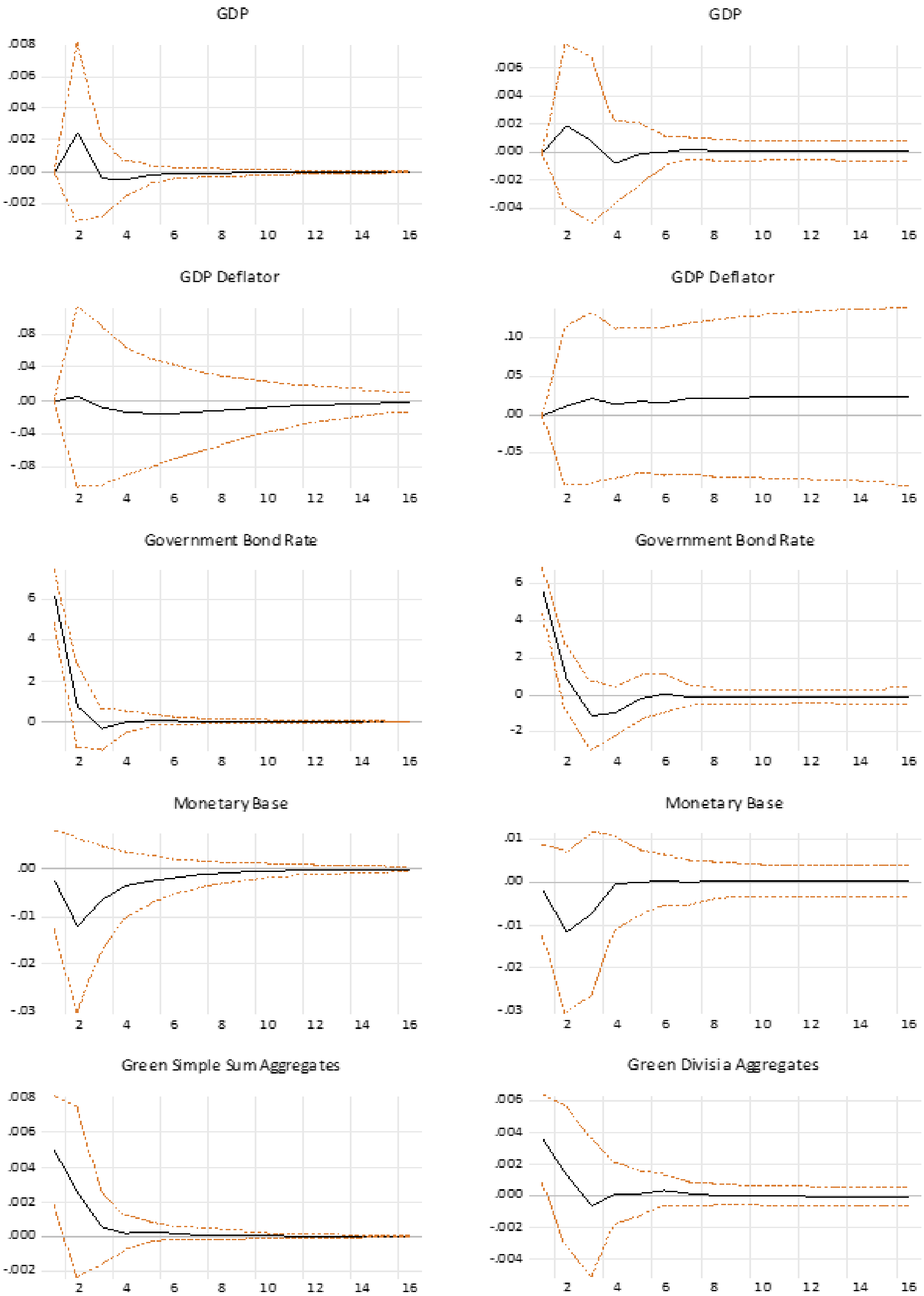
Figures B.6 and B.7 show impulse responses to an identified monetary policy shock using a corporate bond rate and a government bond rate. Both rates display a persistent and statistically significant output puzzle. The output rise for first 2 quarters and decrease to zero after 1 quarter. A persistent price puzzle is shown for all rates. Aggregate prices increase for first 2 quarters and eventually fall, but this effect is insignificant. The monetary base initially declines following an unexpected increase in the bonds rate and gradually increases towards zero about 3 quarters in corporate bonds rate model, while there is a longer delay before monetary bases increasing towards zero in government bonds rate model. This effect is statistically significant for all models. The responses of green monetary aggregates are positive and significant, which indicates a liquidity puzzle. Therefore, these puzzles emanating from the bonds rate VAR models lead to a less accommodative policy stance as an alternative for our green user costs.

Figure B.6 Impulse Responses for Corporate Bond Rate VAR Model



Notes: The corporate bond rate VAR model uses corporate bonds rate as the policy indicator. The left hand plot shows the impulse responses by including green simple sum aggregates in *MI* block. The right hand plot shows the impulse responses by including green Divisia aggregates in *MI* block.

Figure B.7 Impulse Responses for Government Bond Rate VAR Model



Notes: The government bond rate VAR model uses government bonds rate as the policy indicator. The left hand plot shows the impulse responses by including green simple sum aggregates in *MI* block. The right hand plot shows the impulse responses by including green Divisia aggregates in *MI* block.

8 The Assessment of the Effects of Monetary Policy

We then turn to our baseline VAR model to quantitatively assess the effects of monetary policy shocks. Table B.3 shows the variance decompositions of the baseline VAR model with the green Divisia monetary aggregates in *MI* block and green Divisia user cost as the policy indicator. The share of the forecast error in output explained by monetary policy shocks is 8.45% about four quarters and peaks at 8.54% only even five years out. Similarly, the monetary policy shocks explain only 1.93% of the forecast error in prices about four quarters and even a smaller share of 1.51% in five years. The similar findings are found in Table B.4 for the variance decompositions of the green simple sum user cost model with the green simple sum monetary aggregates in *MI* block. In the four quarters, the monetary policy shocks contribute 2.52% to the forecast error in output and 0.40% to that in prices. Even in the five years period, the share of forecast error in output and in prices are explained by only 2.85% and 8.53% respectively.

When we replace the green simple sum user cost with the green Divisia user cost and keep the green simple sum money in *MI* block, the contributions of the monetary policy shocks to the forecast error in output and prices have nearly quadrupled in the four quarters, with 8.26% and 2.34% respectively shown in Table B.5, which indicates the macroeconomic variables could be explained more with the green Divisia user cost as the policy indicator. This is also supported by the comparison of the variance compositions between green Divisia user cost baseline model and other model. Since the rest of models produces the puzzles, we take the average of the variance decompositions for those models. Table B.6 shows that the contribution of monetary policy shocks to the forecast error in output is 8.18% on average for other models in the four quarters, which is 3.30% lower than that for the green Divisia user cost model in Table B.3. Even in the five years period, the average share of forecast error in output for other models is lower than that for green Divisia user cost model by 2.40%. Similarly, the monetary policy shocks explain only 1.62% of the forecast error in prices on average for other models about four quarters period, which is 19.14% lower than that for green Divisia user cost model in Table B.3.

Overall, our variance decompositions show that monetary policy shocks have had positive, but negligible contribution on outputs and prices over our sample period in Table B.3, Table B.4, Table B.5 and Table B.6. These results confirm a common finding in the monetary VAR literature that monetary policy shocks are not a major driver of business cycles.

Table B.3 Variance Decompositions for Green Divisia User Cost Baseline VAR Model with Green Divisia Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	8.45	8.54	8.54
GDP deflator	1.93	1.59	1.51
Green Divisia user cost	96.74	96.49	96.40
Monetary base	2.11	2.20	2.20
Green Divisia monetary aggregates	8.65	8.66	8.65

Notes: The variance decompositions for the baseline VAR model with the green Divisia monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.4 Variance Decompositions for Green Simple Sum User Cost Baseline VAR Model

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	2.52	2.61	2.85
GDP deflator	0.40	3.79	8.52
Green simple sum user cost	60.55	58.23	58.05
Monetary base	20.50	25.74	25.86

Green simple sum monetary aggregates	2.46	2.63	2.76

Notes: The variance decompositions for the green simple sum user cost baseline VAR model with the green simple sum monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.5 Variance Decompositions for Green Divisia User Cost Baseline VAR Model with Green Simple Sum Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	8.26	8.34	8.34
GDP deflator	2.34	2.26	2.27
Green Divisia user cost	96.43	96.32	96.29
Monetary base	2.23	2.35	2.35
Green simple sum monetary aggregates	12.48	12.51	12.51

Notes: The variance decompositions for the green Divisia user cost baseline VAR model with the green simple sum monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.6 Variance Decompositions for the Average of Other VAR Model

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	8.18	8.20	8.18
GDP deflator	1.62	2.14	2.47
The policy indicator	70.77	69.13	65.65
Monetary base	8.57	8.44	8.29
Monetary aggregates	11.68	12.17	11.65

Notes: The average variance decompositions for other VAR model; all numbers are within the associated 90% probability intervals; the variance decompositions for all other models are displayed in appendices.

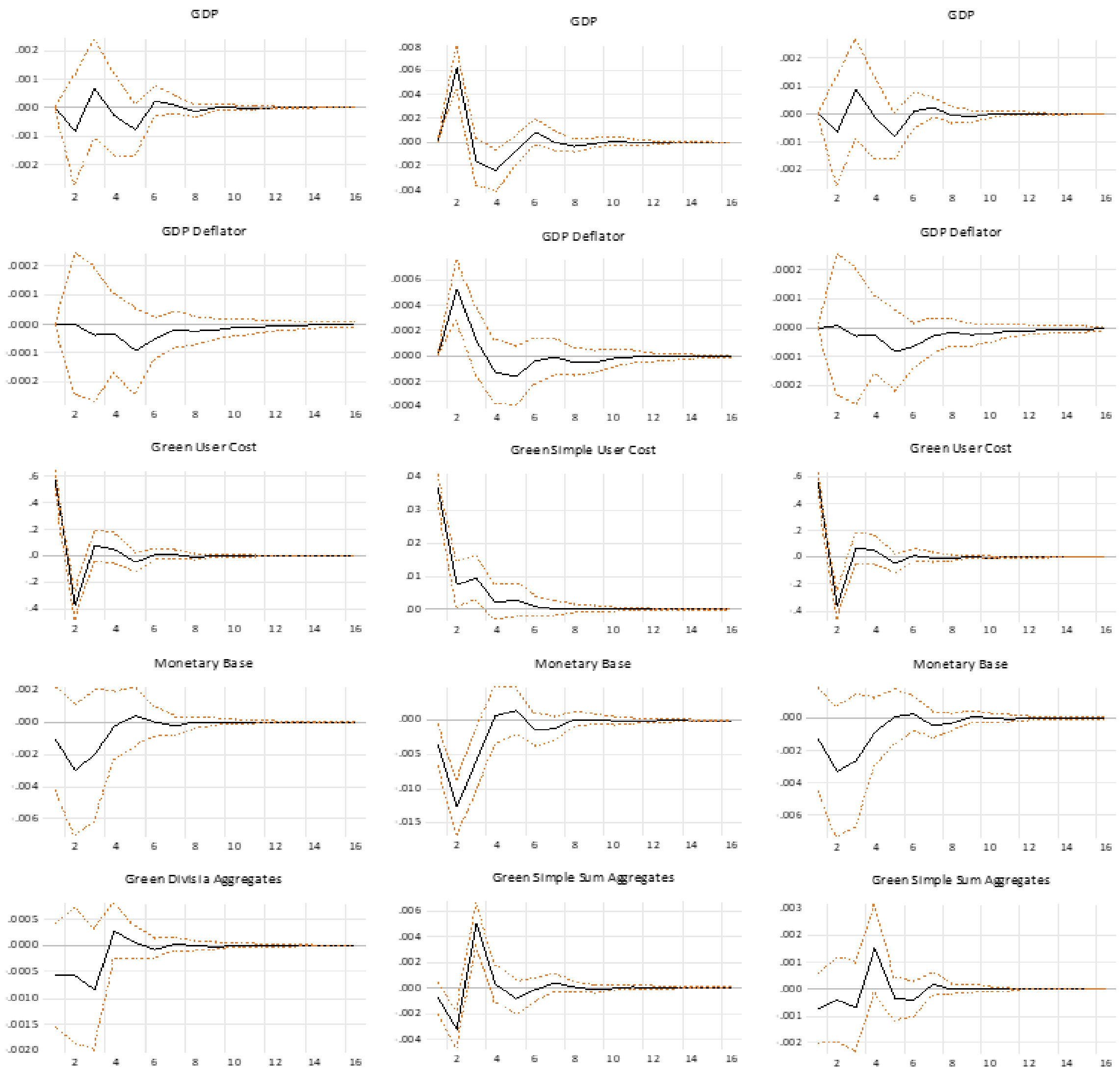
9 Robust Check by Using Monthly Data

In the recursive VAR models, the key identifying assumption is the zero restrictions on the impact response of output and prices following a monetary shock. This assumption may be more palatable at a monthly frequency than quarterly. Therefore, we impose the zero impact restrictions on monthly data to test whether the impact responses of output and prices are consistent with the results from the quarterly model. The monthly model also implies that the Federal Open Market Committee (FOMC) make changes to monetary policy 12 times per year. In reality, the FOMC has only eight regularly scheduled meetings each year, but the FED also conducted conference calls between its meeting date to make additional adjustments.

Figure B.8 shows the impulse responses from our baseline models with monthly data. Monthly nominal GDP and real GDP series are obtained from Macroeconomic Advisors which use a similar approach of informing monthly estimates with the data used by U.S. Bureau of Economic Analysis (BEA) to construct the official quarterly data. Monthly GDP deflator is calculated by monthly nominal GDP dividing real GDP.

The impact responses of output and prices in the monthly model are consistent with the results in the quarterly model. Both the sign and significance are similar in the models by using the green user cost as the policy indicators (see the plots in the left hand column and in the right hand column in Figure B.8). Only the impact response for the prices from the monthly model (the middle column in Figure B.8) by using green simple sum cost as the policy indicator is positive first and then become negative, which shows the price puzzles. This also confirms that the Divisia measure can solve the price puzzles when we replace the green simple sum user cost with the green price dual in the right hand plot in Figure 8. We conclude our findings are strongly robust to different data frequencies and the overall qualitative inference from the monthly VAR models aligns with that from the quarterly models.

Figure B.8 Impulse Responses for Monthly Baseline VAR Model



Notes: This figure plots the impulse responses to an identified monetary policy shock in our baseline VAR model using monthly data. The plot in the left hand column shows the impulse responses by using green user cost as the policy indicator and including green Divisia monetary aggregates in *MI* block. The plot in the middle column shows the impulses responses by using green simple sum user cost as the policy indicator and including green simple sum monetary aggregates in *MI* block. The plot in the right hand column shows the impulse responses by using green user cost as the policy indicator and including green simple sum monetary aggregates in *MI* block.

10 Conclusions and Recommendations for Future Work

The effects of monetary policy on aggregate economy have been emphasised in the field of monetary economics. This chapter aims to characterize these effects without producing empirical puzzles. Different from the works using Federal Funds rate as the policy indicator, we develop a new method for identifying monetary policy shocks that is free from puzzles and not subject to lower bound constraints for the USA. Drawing from the implications of a relatively standard NK model of monetary policy, we propose using green augmented price dual, which is the function of all interest rates relevant to the monetary aggregates, as the policy indicator variable. In the empirical analyses, we show that the user cost-based model is able to measure the effects of monetary policy and performs better than the Federal Funds rate model which is not free from the price puzzles common in the VAR literature. The green price dual based on monetary aggregates yields plausible results without having to adopt ad hoc solutions to price puzzles. This provides a fresh view for policy makers to employ a new green augmented price dual as the policy indicator when the Federal Funds rate is stuck at zero.

Further, we contribute to the monetary aggregation literature by extending the monetary components to include the environment-related bonds, government bonds and corporate bonds, and then construct the green Divisia monetary aggregates and the green price dual. Our new green economic monetary aggregates successfully examine the data beyond central bank data. These extensions are especially useful when the money and financial markets are expanding and providing more liquidity services, thus our work is particularly timely. The inclusion of green bonds highlights the consideration of climate change in policy implications. Thus, our green price dual could satisfy the needs of the central banks, as a monetary policy tool, to balance economic stability and green sustainability. Moreover, the user cost of green bonds is given by the opportunity costs that consumers forgo to consume the services of the green bonds. This measure could capture the externality in the market for the cost to the environment of causing pollution and changing the climate. Our evidence suggests that the policy makers aiming at providing a precise price of externality could instead look at this opportunity cost from the monetary policy perspective when green bonds are also included.

The proposed use of the Divisia green price dual in monetary policy is intended as an indicator rather than an instrument. This distinction is crucial because the Divisia green price dual serves to provide valuable insights into the inflationary pressures and the overall economic impact of green finance activities, rather than being a direct tool for implementing policy changes. As an indicator, it aids policymakers in understanding the nuanced effects of green investments and environmental sustainability initiatives on the broader economy. This approach aligns with the perspective outlined in reports by Bernanke (2024), who emphasised the importance of diverse indicators in informing monetary policy decisions, particularly in scenarios involving alternative economic conditions and shocks.

However, in this current work, we do actually go one step further and start the process of using the Divisia green price dual as a policy tool in alternative scenarios in order to help simulate the potential economic outcomes of various environmental policies and green financial investments. In this way, we provide a more comprehensive framework for policy analysis and decision-making. This method enhances the robustness and adaptability of monetary policy in addressing contemporary challenges associated with sustainable finance and environmental sustainability, and is in keeping with the studies of Keating et al. (2019) and Binner and Kelly (2017).

Based on our findings, further extensions of this current research on the construction of the price dual could be applied to consider broader assets providing liquidity services. We would also recommend that researchers employ more complex VAR models, such as the structural VAR (SVAR) model with the consideration of sign restrictions or the structural breaks in assessing the effects of monetary policy shocks or explore the possibility of how the green augmented price dual and monetary aggregates facilitate the transition to a green economy within a DSGE framework.

Chapter III Forecasting USA Output with the New Green-Benchmarked Divisia Monetary Aggregates

1 Introduction

Evaluating the current state of the economy is of a great importance to policy makers, institutions and economic agents, whose decisions are based on the assessments of the current and future economic conditions by using real time data. Hence, it is crucial to have an accurate evaluation of the current state and future path of gross domestic products (GDP) to assess fiscal sustainability. Further, as most data, such as GDP, are released with a lag, forecasting the economic conditions is an important task for central banks and other economic agents.

This chapter uses a Markov Regime Switching VAR model (MS-VAR) proposed by Hamilton (1989) to forecast the USA real GDP growth by including monetary aggregates as one of key indicators in the model. We include the monetary aggregates in the model since the changes in both quantity and price indices for monetary assets are significantly related to the changes in output (Belongia and Ireland, 2014). We expect that the inclusion of monetary aggregates would improve the accuracy of our output forecasting. In fact, a set of studies have placed the short-term interest rate at the heart of monetary policy and ignored the role of the monetary aggregates in last few decades. For example, the New Keynesian (NK) model and central banks use the interest rate as their main policy tools. In the NK model, it is assumed that the long-term effect of the money supply on economic output is neutralized, leading to the exclusion of money supply variables from its equations. On the other hand, the small-scale macroeconomic models that include an IS curve specify the output gap as a function of the real interest rate. In the IS equation, the real money stock, or its growth rate, does not appear. This theoretical stance is challenged by a recent work by Belongia and Ireland (2016). They suggest that the proper measure of money, such as Divisia monetary aggregates, could play an important role in the conduct of monetary policy. This can be exemplified by the recent shift in the USA Federal Reserve’s monetary policy to adopt quantitative easing, with its goal of affecting the supply of liquid assets. This chapter aims to examine the role that money plays in the economy if it has been measured properly.

As discussed in Chapter I, simple sum monetary aggregates, normally published by central banks, are simply the sum of the nominal value of all monetary assets. These measures, which are not based on economic aggregation or index number theory, ignore the fact that the services of different monetary assets are not perfect substitutes. Unlike the simple sum measure, Divisia monetary aggregates, originated and developed by Barnett (1978, 1980), are directly derived from economic theory and impute user costs to the marginal utilities of component assets. The resulting component growth rate weights in measuring the growth rate of the Divisia index are the component expenditure shares with user cost pricing. By applying the different weights on monetary assets based on their liquidity, the Divisia index tracks the true flow of monetary services more accurately.

Using the conventional Divisia monetary aggregates, Barnett et al. (1984) investigated the forecasting properties of the well-known, linear, fixed-coefficient money demand functions. Barnett et al. (2016) explore the performance of univariate and multivariate, linear and non-linear models in nowcasting nominal GDP growth with the inclusion of the conventional Divisia monetary aggregates. Their model, which contains information on real economic activity, inflation, interest rates and Divisia monetary aggregates, is found to be the most accurate in nominal GDP growth nowcasting. Barnett et al. (2016) extend the conventional Divisia monetary aggregate to include credit card transaction services and use this credit-card-augmented Divisia monetary aggregate as an indicator in nowcasting nominal GDP growth. Their findings also support that Divisia monetary aggregates help in nowcasting movements of GDP growth. Nowcasting is the prediction of present values, while the purpose of this chapter is to evaluate the performance of Divisia monetary aggregates as indicators in predicting future economic variables. Motivated by the NK model, which assumes that prices are sticky in the short run, we aim to forecast the USA real GDP gap rather than the nominal ones because any disturbance in economy affects the real variables more, especially the quantities, than the prices in the short run.

The forecasting performance of the conventional Divisia monetary aggregates in real GDP for USA is compared with that of their simple sum counterparts by Schunk (2001). He finds that the forecasts of USA real GDP with a four-variable vector autoregression (VAR) model are most accurate when the Divisia monetary aggregates are included. Ellington (2018) evaluates the relative empirical benefits of Divisia monetary aggregates by fitting time-varying coefficient VAR models and finds a strong relationship between Divisia money and the economic activity over the business cycle. He also finds the Divisia monetary aggregates outperform the simple sum measures in out-of-sample forecasting of economic activity. The most recent paper by Barnett and Park (2023) assesses the performance of the credit-card-augmented Divisia monetary aggregates in forecasting USA output growth at the 12-month horizon using an Autoregressive Distributed Lag model. They use three monetary aggregates: the original Divisia monetary aggregates, the credit-card-augmented Divisia monetary aggregates, and the credit-card-augmented Divisia inside money aggregates. Their results show that the credit-card-augmented Divisia indices are the best in forecasting output growth for the USA.

We investigate the green-benchmarked Divisia monetary aggregates as predictors of output. We compare the results of forecasting output using these four monetary aggregates: the conventional Divisia monetary aggregate, the green-return benchmarked Divisia monetary aggregate, the green-coupon benchmarked Divisia monetary aggregate and the traditional simple sum monetary aggregate. The green-benchmarked Divisia monetary aggregate is the measure of the monetary services using long-term green bonds as the benchmark asset to produce the green user costs for monetary assets. This green-benchmarked measure is motivated by the need of central banks to address systemic risk that climate change poses to the financial sector. Climate-related risks, both physical and transitional, can have profound impacts on financial stability[20]. By incorporating green considerations into the monetary frameworks, this green-benchmarked Divisia money measure may provide a fresh view to consider the impact of climate change on the whole economy. Additionally, the green bonds attract a specific segment of investors, who prioritize environmental considerations in their investment decisions, warranting a focused examination. These investors, both at individual and institutional levels, may demonstrate a willingness to forgo higher returns in favour of contributing to environmental sustainability. This phenomenon reflects a shift in investment paradigms where non-financial outcomes, such as environmental impact, are weighted alongside traditional financial returns. Therefore, the resulting green-benchmarked Divisia monetary aggregates from the green user cost of monetary assets, which is defined as the difference between the interest return from holding the asset and the rate of return on pure green investments, could be a monetary tool to incentivize investors to support the projects that contribute to a low-carbon economy. Since both the rate of return and the coupon rate of long-term green bonds play a crucial role in reflecting the value of green investments, we use both rates as the benchmark rate to construct the green-return benchmarked and the green-coupon benchmarked Divisia monetary aggregates respectively and then compare their forecasting performance in economic activity.

We consider using the non-linear MS-VAR estimation. The main reason for this is the failure of linear models to capture the non-linear dynamic relationships embedded in real-world data. The use of an MS framework in predicting GDP is quite well-established and a large number of studies have been published since the seminal paper by Hamilton (1989), see for

[20] See Panel on "Central Bank Independence and the Mandate - Evolving Views" Remarks by Jerome H. Powell Chair Board of Governors of the Federal Reserve System at Symposium on Central Bank Independence Sveriges Riksbank, Stockholm, Sweden.

example Barsoum and Stankiewicz (2015), Foroni et al. (2015) and Segnon et al. (2018). The underlying idea of the MS model is to include different discrete regimes that are governed by a hidden Markov chain.

The MS-VAR models are non-linear as the mean, as well as higher moments, are non-linear functions of the current state of the Markov process. The non-linear structure allows the model to properly capture the changes that characterize time series. In contrast to other non-linear or time-varying models, the MS-VAR model can endogenously estimate and forecast the probabilities of being in a given regime. All these features make the MS-VAR model appropriate for our analysis.

This chapter examines the out-of-sample forecasting performance of the green-benchmarked Divisia monetary aggregates in USA output by using the non-linear MS-VAR model as the research objective. Specifically, we consider the one-month, three-month, six-month and nine-month ahead forecasts. Note that we use monthly reported weekly economic index (WEI) as our real sector variable in the model because monthly data allow for closer tracking of the short-term economic trend fluctuations than the GDP, which is only reported on a quarterly basis. Our data sample covers the monthly period of 2009M1 to 2022M12, with the start date being determined by the data availability of the green bonds. Our estimation is based on the period from 2009M1 to 2019M9 as we exclude the effect of the COVID-19 pandemic, and the out-of-sample forecasting period is from 2020M1 to 2022M12, leaving three observations for validation. We also compare our results from the MS-VAR models with those from the VAR models to assess the improvements of output forecasting by the non-linear models. Finally, we include the financial condition variables as controls in the models to investigate the shift of preferences for investors towards the green investments under the different economic conditions.

To the best of our knowledge, this is the first attempt to integrate the green investments into the construction of the Divisia monetary aggregates and to apply MS-VAR models to predict the out-of-sample output. In this regard, we contribute to the existing literature by offering a new measurement for USA Divisia monetary aggregates, examining the performance of green-benchmarked Divisia monetary aggregates on economic activities and underlining their superiority through a comparison with conventional Divisia monetary aggregates and simple sum ones. Our findings highlight that our green-benchmarked Divisia monetary aggregates are a useful new monetary policy tool and an indicator of aggregate demand policy for the USA. We also find that the green-benchmarked Divisia monetary aggregates have superior performance on forecasting output compared to the conventional Divisia and traditional simple sum counterparts. Therefore, this new green-benchmarked Divisia monetary aggregate has the potential to assist central banks in maintaining a balance between economic stability and environmental sustainability, thus addressing potential political pressures.

We progress this chapter by turning next to the details of the construction for monetary aggregates in Section 2. Section 3 describes the out-of-sample forecasting models. Section 4 presents the data and methodology. In Section 5, we provide our construction results. Sections 6 and 7 give the estimation and forecasting results, respectively, while conclusions and suggestions for future research are contained in Section 8.

2 Divisia Monetary Aggregates

Barnett (1980) proposed that monetary aggregates be composed and constructed consistent with economic theory. To achieve this, he brought together the theory of aggregation over goods and statistical index number theory to produce Divisia monetary aggregates. The title of Barnett's seminal (1980) paper includes the terms "economic monetary aggregates" and "index number and aggregation theory." In this section we outline how we construct the conventional

Divisia monetary aggregates and the green-benchmarked Divisia monetary aggregates used for this chapter. We start with the construction of the conventional Divisia monetary aggregates.

2.1 Conventional Divisia Monetary Aggregates

Barnett (1980) proposed using the Törnqvist-Theil discrete time approximation to the Divisia index from Diewert (1976)'s class of superlative indexes. Others in the literature have used the Fisher Ideal superlative index. The difference between the two indexes is negligible, being less than the roundoff error in the components, while the Divisia index is easier to interpret than the Fisher Ideal index. The resulting monetary aggregates are strictly preferable to the simple sum monetary aggregates as the monetary components are not perfect substitutes and simple sum aggregation over imperfect substitutes is inadmissible. The aggregates used in this chapter are constructed using a Törnqvist-Theil discrete time approximation to the Divisia index.

As monetary assets are modelled as durable goods, Donovan (1978) and Barnett (1980) point out that the appropriate price is a user cost. The nominal user cost π_t of the ith monetary asset is given by

$$\pi_{it} = p_t^* \frac{R_t - r_{it}}{1 + R_t} \tag{C.1}$$

where R_t is the benchmark asset rate of return measuring the maximum expected rate of return available in the economy at time *t*, and r_{it} is the own rate of return on monetary asset *i* at time *t*, p_t^* is the true cost-of-living index price at time *t*, approximated by a consumer price index. This formula measures the opportunity cost of the monetary service provided by asset *i*.

We then turn to construct an aggregate nominal service index. Under linear homogeneity, the continuous Divisia quantity index can be utilized to tract the aggregate and the Divisia index in growth rate form in continuous time is:

$$\frac{dlog(M_t)}{dt} = \sum_i s_{it} \frac{dlog(m_{it})}{dt} = \sum_i \frac{\pi_{it} m_{it}}{m_t^*} \frac{dlog(m_{it})}{dt} \tag{C.2}$$

where $m_t^* = \pi_t' m_t$ is the total expenditure on the whole portfolio's monetary assets, and $s_{it} = \frac{\pi_{it} m_{it}}{m_t^*}$ is the ith asset's expenditure share or its contribution to the aggregate during period *t*.

Since the continuous time data is unavailable in real world, the discrete time representation of the Divisia index is required for the empirical research field. According to Anderson et al. (1997), Theil approximation is a second order approximation to the continuous time Divisia index. At time *t*, the discrete time representation of the Divisia quantity index M_t, over the monetary components, is:

$$log(M_t) - log(M_{t-1}) = \sum_{i=1}^{n} s_{it}^* (log(m_{it}) - log(m_{i,t-1})) \tag{C.3}$$

where $s_{it}^* = \frac{1}{2}(s_{it} + s_{i,t-1})$ is the average of the current and lagged expenditure shares held of asset *i*. Therefore, equation (C.3) is the weighted growth of M_t over the monetary components, and the Divisia monetary index in level, M_t, is:

$$\frac{M_t}{M_{t-1}} = \prod_{i=1}^{n} (\frac{m_{it}}{m_{i,t-1}})^{s_{it}^*} \tag{C.4}$$

2.2 Green-Benchmarked Divisia Monetary Aggregates

The green-benchmarked Divisa money is the measure of monetary services using green bonds as the benchmark asset. The optimal benchmark asset is the asset providing at least as good a store of value as the components of the money supply but having no transaction purposes (Hancock, 2005). This implies that the rate of return on such an asset should be greater than the rate of return on any components. In this chapter, we proxy the green benchmark rate with the rate of return on the long term green bonds. As the investment in green bonds have tax incentives in the form of tax exemptions or tax credits, the long-term green bonds are held to satisfy a savings motivation and reflect the pure green investment.

Based on the user cost formula derived by Barnett (1980), we define the nominal green-benchmarked user cost π_{G_t} of the ith monetary asset as:

$$\pi_{G\_it} = p_t^* \frac{R_{G\_t} - r_{it}}{1 + R_{G\_t}} \tag{C.5}$$

where R_{G_t} is the benchmark rate of return on the long term green bonds at time *t*, and r_{it} is the own rate of return on monetary asset *i* at time *t*, p_t^* is the true cost-of-living index price at time *t*, approximated by a consumer price index. This formula measures the pure green opportunity cost of the monetary service provided by asset *i*.

The green-benchmarked Divisa index in growth rate form in a discrete time is:

$$log(M_{G\_t}) - log(M_{G\_t-1}) = \sum_{i=1}^{n} s_{G\_it}^{*}(log(m_{it}) - log(m_{i,t-1})) \quad \text{(C.6)}$$

where $s_{G\_it}^{*} = \frac{1}{2}(s_{G\_it} + s_{G\_i,t-1})$ is the average of the current and lagged expenditure shares held of asset *i*. Therefore, equation (C.6) is the weighted growth of M_t over the monetary components, and the Divisia monetary index in level, M_{G_t}, is:

$$\frac{M_{G\_t}}{M_{G\_t-1}} = \prod_{i=1}^{n}(\frac{m_{it}}{m_{i,t-1}})^{s_{it}^{*}} \quad \text{(C.7)}$$

Alternatively, we consider to use the coupon rate on the long term green bonds as the benchmark rate. The coupon rate on the green bonds is the annual interest rate paid on a green bond based on its face value, which implies the coupon is the return received by the consumers to held the green bond until to maturity. Thus, the coupon rate on a long term green bond could reflect the pure return on green investments. The nominal green-benchmarked user cost π_{C_t} of the ith monetary asset as:

$$\pi_{C\_it} = p_t^{*}\frac{R_{C\_t} - r_{it}}{1 + R_{C\_t}} \quad \text{(C.8)}$$

where R_{C_t} is the benchmark coupon rate on the long-term green bonds at time *t*, and r_{it} is the own rate of return on monetary asset *i* at time *t*, p_t^* is the true cost-of-living index price at time *t*, approximated by a consumer price index. This formula measures the pure green opportunity cost of the monetary service provided by asset *i*.

3 Out-of-Sample Forecasting Models

This section provides our forecasting models, vector autoregression (VAR) model and Markov Switching vector autoregressive (MS-VAR) model. The rationale for considering the latter models is a priori belief that linear models will perform badly in our forecasting experiments. Therefore, it is logical to include a linear model as a benchmark used in our analysis. A nature choice is the VAR model, which is commonly employed in macroeconomic forecasting analysis.

3.1 Linear VAR Model

The VAR model proposed by Sims (1980) incorporates endogenous variables and lagged variables into its model structure and assumes linear relationship between variables. In the VAR model, the dynamics of x_t, a k-dimensional vector of dependent variables at time t, is defined by the following pth order autoregressive process:

$$x_t = \alpha_0 + A_1 x_{t-1} + \cdots + A_p x_{t-p} + \varepsilon_t \qquad \text{(C.9)}$$

where α_0 is a vector of intercepts, A_z, $z = 1, \dots, p$, are $k \times k$ coefficient matrices and ε_t is a white noise distributed disturbance vector. We use ordinary least squares to estimate the coefficients matrices.

3.1.1 Forecasting Method

For the VAR model, a conditional $t + 1$ forecast of x is obtained by:

$$E_t(x_{t+1}) = \alpha_0 + A_1 x_t + \cdots + A_p x_{t-p+1} \qquad \text{(C.10)}$$

for each t using the information set $X_t = \{x_t, x_{t-1}, \dots\}$. The conditional $t + 1$ forecast in (C.10) is a special case of the conditional dynamic $t + \tau$ forecast where the forecasted values of x depends on the forecast horizon and the number of included lags.

3.2 The Markov Switching VAR Model

In practice, the relationship between variables can vary across different periods due to changes in external shocks like economic situations. To extend the traditional VAR model, Hamilton (1989) introduced a nonlinear MS-VAR model, which is adept at capturing dynamic economic market states.

The MS-VAR model assumes that a discrete time-homogenous *s*-state first order Markov process governs the endogenous switches between the regimes. This assumption implies that the probability of the switch between different regimes is described by a constant transition matrix. For each regime, there is a different VAR model and only the most recent state of the Markov-process can influence the transition probabilities. We also assume that x_t can be modelled as a discrete mixture of k-variate Gaussian distributions, which implies that x_t is normal distributed conditional on the prevailing regime and the information set X_{t-1}. We then restrict the variance-covariance matrix in each regime to be constant.

An unobservable latent variable s_t represent the prevailing regime at time *t*, $s_t \in \{1,2,\dots,m\}$, where m represents the number of the possible states. The conditional probability density for the observed vector of the time series is given by:

$$p\ (x_t|X_{t-1}, s_t) = \begin{cases} f\ (x_t|X_{t-1}, \theta_1), if\ s_t = 1 \\ \dots \\ f\ (x_t|X_{t-1}, \theta_m), if\ s_t = m \end{cases} \quad \text{(C.11)}$$

where θ_m represents the parameter vector of the VAR model under the state $M = 1, \dots, m$. Then, for a given state $M = 1, \dots, m$, the VAR process is:

$$x_t = \alpha_0^{(M)} + A_1^{(M)} x_{t-1} + \cdots + \alpha_{p(M)}^{(M)} x_{t-p(M)} + \varepsilon_t^{(M)} \quad \text{(C.12)}$$

where $p(M)$ is the order of the VAR model, i.e., the number of the lags included in regime M. Further, the matrix of transition probabilities, $\mathbf{P} = \{p_{ij}\}$, where $i, j = 1, \dots, m$ is determined by the equations:

$$p_{ij} = \Pr[s_t = j|s_{t-1} = i] = \begin{bmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & p_{ij} & \vdots \\ p_{m1} & \cdots & p_{mm} \end{bmatrix} \quad \text{(C.13)}$$

We follow Hamilton (1994) to estimate the Markov switching model via maximum likelihood method. As the regimes are unobservable, s_t can be regarded as missing data. However, the probability that a given observation belongs to a particular state can be computed. With this information, we can obtain an optimal forecast probability that the next observation belongs to a particular state. Let $\Pr_t(s_t = j)$ denotes the filtered probability that $s_t = j$, given the information set X_t. Then given the information available at time *t*, $\Pr_t(s_{t+1} = j)$ denotes the forecast probabilities for time t + 1. Following Hamilton (1994), the vector of filtered probabilities are denoted by $\xi_{t|t}$ and the vector of forecast probabilities are denoted by $\xi_{t+1|t}$. The time *t* likelihood function value is computed by a by-product from the following iterations to calculate the optimal forecast probabilities:

$$\xi_{t|t} = \frac{\xi_{t|t-1} \circ \varphi_t}{\mathbf{1}'(\xi_{t|t-1} \circ \varphi_t)} \tag{C.14a}$$

$$\xi_{t+1|t} = \mathbf{P}'\xi_{t|t} \tag{C.14b}$$

where φ_t is the vector of conditional densities, $\mathbf{1}$ is a unit column vector and $\circ$ is the vector of element-by-element multiplication. Proposed by Hamilton (1989), this filter can be thought of as a non-linear Kalman filter, as the probabilities are the state-dependent variables. The filter produces an optimal inference about the predicted and filtered probabilities through one set of prediction equations and another set of filtering equations. The time *t* likelihood function is given by the denominator in (C.14a), which is the weighted sum of the conditional densities with the weights given by the forecast probabilities.

3.2.1 Forecasting Method

For the MS-VAR model, a conditional $t + 1$ forecast of x is the weighted average of the *m* different forecasted within regime means, and the weights are given by the probability of each regime to prevail in the next period, which is computed by:

$$E_t(x_{t+1}) = \sum_{j=1}^{m} \sum_{i=1}^{m} \Pr_t(s_t = j)\, p_{ij} \left(\alpha_0^{(j)} + \sum_{z=1}^{p(j)} A_z^{(j)} x_{t-z+1} \right) \tag{C.15}$$

for each *t* using the information set $X_t = \{x_t, x_{t-1}, \dots\}$. In an *m*-regime model, there are M^τ possible outcomes for each dependent variable τ periods ahead and M^τ associated probabilities. Therefore, the $t + \tau$ forecast of x is computed as the probability weighted average by traversing the non-recombining tree generated by the underlying Markov-chain along all possible paths.

The probability trees should be multiplied by the m filtered probabilities based on the formula (C.15). This indicates the uncertainty of the current prevailing regime or the unobserved Markov chain even *ex post*. From the computational perspective, there are $m \cdot 2^{\tau}$ different probabilities τ period ahead, each of which should be multiplied by the corresponding τ periods ahead forecast of x.

4 Data and Methodology

4.1 Data

Our choice of country in this study is the USA as the USA has one of largest and most influential green bond markets in the world and is one of the first countries to engage in green bond issuance, and hence the green bond data available for this study will provide more robust results than can be obtained elsewhere. We use seasonally adjusted monthly data covering the period from the January of 2009 to December of 2022. All data of monetary assets in this chapter are real quantities data with the associated nominal prices, thus the price time quantity yields expenditure on the good or service.

The goods and assets[21] considered are:

i) Currency (CUR),

ii) Demand Deposits (DD),

iii) Other Liquid Deposits (OLD) (other checkable deposits + savings deposits),

iv) Retail Money Market Funds (RMMF),

v) Small Time Deposits Total (STDT) (small time deposits commercial and small time deposits thrift),

vi) Institutional Money Market Funds (IMMF),

[21] According to the Center for Financial Stability (CFS), other checkable deposits commercial, other checkable deposits thrift, savings deposits commercial and savings deposits thrift are aggregated into other liquid deposits (asset 3) from May 2020. Small time deposits commercial and small time deposits thrift are also aggregated into small time deposits total (asset 5) from May 2020. We follow the CFS to aggregate other checkable deposits, savings deposits and small time deposits into separate series.

vii) Large Time Deposits (LTD),

viii) Repurchase Agreements (RA),

ix) Commercial Paper (CP),

x) T-Bills (TB).

In the case of assets i to x, we obtained the seasonally adjusted nominal holdings and associated own rates of return for the USA monetary components from the Center for Financial Stability[22] (CFS) (Barnett et al., 2013). We obtained the consumer price index (CPI) from St. Louis Federal Reserve Bank Economic Database (FRED) to convert the nominal holdings of all monetary assets into real term. The base year for the CPI is 2015. We used the National Rural Utilities Cooperative Finance Corporate green bond for the return on green bonds, which is the earliest issued USA green bond featuring a tenor over 20 years with the data available. This series was obtained from Bloomberg. The USA green bonds with the tenor over 20 years are used for the coupon rate on green bonds, also obtained from Bloomberg.

From same database, we downloaded the USA 10-year government bond, the yield of US 90 days T-bill, Standard and Poor's 500 index, Chicago Board Options Exchange Volatility Index (VIX), US government repo rate and reverse repo rate for our financial measures. The USA EPU index was obtained from the website of Economic Policy Uncertainty Index[23] developed by Baker et al. (2016). We obtained the National Financial Conditions Index (NFCI) and Adjusted NFCI (ANFCI) from the website[24] of the Federal Reserve Bank of Chicago. The Financial Soundness Indicator was downloaded from FRED. The Global Liquidity Indicators

[22] Some of the USA data is available on the CFS website at the following link http://www.centerforfinancialstability.org/amfm_data.php. Some of the data are obtained in personal contact with the CFS researchers.
[23] See the link: http://www.policyuncertainty.com.
[24] See the link: National Financial Conditions Index: Current Data - Federal Reserve Bank of Chicago (chicagofed.org).

(GLI)[25], defined by the Bank for International Settlements (BIS), were obtained from the website[26] of the BIS.

We obtained the weekly economic index (WEI) and the Federal Funds rate[27] (FFR) from the FRED for the aggregate demand analysis and forecasting.

4.2 Methodology

For the construction of conventional Divisia monetary aggregates, the benchmark rate used in calculating user costs for all assets is determined with an envelope approach. The benchmark rate is the maximum rate of return from the interest rates on the different monetary assets within our index. However, there are periods in our sample when the returns on components are equal to the benchmark rate, thus leading to zero or negative weights. Zero user costs imply that the transaction services provided by the asset are free, which is unrealistic. In order to ensure that all user costs of monetary assets and financial assets above zero, a simple way is to add a constant to the benchmark rate as arbitrary adjustments or liquidity premium. The Divisia indices provided by the FED of St. Louis use 100 basis points (Anderson and Jones, 2011). In line with Anderson and Jones (2011), a constant of 100 basis points is necessary to obtain positive weights throughout in our sample, which is the minimum value to make all user costs be positive.

For the construction of the green-return benchmarked Divisia monetary aggregates and the green-coupon benchmarked Divisia monetary aggregates, we consider the rate of return on green bonds and the coupon rate of green bonds as the green benchmark rate, respectively, to calculate user costs for all monetary assets. Using Kalman filter, we forecast the rate of return for green bonds to be higher than the interest rate on all monetary assets in our index to avoid

[25] Since the GLI are reported on quarterly basis, we use spline interpolation to convert quarterly data to monthly data.
[26] See the link: https://www.bis.org/statistics/gli.htm.
[27] We adjust Federal Funds rates data for seasonal patterns by using X-13ARIMA-SEATS, which is the seasonal adjustment software developed and adopted by the U.S. Census Bureau.

negative user costs. Regard to the coupon rate of green bonds, we utilise the most recent nominal interest rates with a tenor over 20 years each month to reflect the current condition of the economy. Since the green bonds with a maturity over 20 years are not issued each month, we apply Expectation-Maximization (EM) algorithm to fill the missing values. In this chapter, the coupon rate is the highest rate in each period. This means that empirically the green bonds do not provide liquidity services into the construction of green-coupon benchmarked Divisia monetary aggregates.

WEI serves as our real sector variable in modelling aggregate demand VAR. The GDP is only reported on a quarterly basis, while the WEI is an index of real economic activity using timely and high-frequency data and is standardised to align with the four-quarter GDP growth rate, thus, the WEI is more suitable for our monthly empirical analysis. We denote WEI_t as the natural log of real output in month *t*. WEI_gap is measured by the Hodrick-Prescott (HP) filter to WEI_tconsidering the cyclical component generated by the filter as the output gap. These computations are conducted for the sample period spanning from 2009M1 to 2022M4. The FFR is used as the policy rate in the aggregate demand analysis and the out-of-sample forecasting.

We follow the earlier IS curve specifications by Reimers (2002) and Binner et al. (2009) to define the variables in our empirical analysis. The real interest rate is defined as $RFFR_t = \sum_{j=0}^{11} FFR_{t-j} - \Delta_{12} p_t$, where FFR_t is a short-term nominal interest rate (expressed as a monthly fraction), p_t is the natural log of consumer price index, and Δ_{12} takes the difference between current value of a variable and its twelfth lag: $\Delta_{12} x_t = x_t - x_{t-12}$. Annualised real money growth is defined as $\Delta_{12}(m-p)_t = \Delta_{12} m_t - \Delta_{12} p_t$, where m_t is the natural log of the green-return benchmarked Divisia, green-coupon benchmarked Divisia, conventional Divisia and traditional simple sum nominal monetary aggregates.

As for the financial variables, we include the most common financial variables used in the literature like the yield spread, the stock market return, VIX and EPU. The yield spread is defined as the difference between the yield of US 10-year government bond and the yield of US 90 days T-bill. The stock market return is calculated as the month-on-month logarithmic change of the Standard and Poor's 500 index. We also use the NFCI and ANFCI constructed and maintained by the Chicago Fed to capture the state of financial markets. The ANFCI isolate a component of financial conditions uncorrelated with economic conditions to provide an update on financial conditions relative to current economic conditions. Both indices represent an extremely broad indicator of aggregate financial conditions in the USA. We then consider three liquidity measures, the repo difference, the Financial Soundness Indicator (FSI) and the Global Liquidity Indicator (GLI). The repo difference is defined as the difference between US government repo rate and reverse repo rate. The FSI is constructed by liquid assets as a percentage of short-term liabilities. The GLI, as the global financing conditions, focus on cross-border and foreign currency financing. We select the GLI for borrowers from all countries and from USA respectively. For the GLI for borrowers from all countries, it includes the borrowers from bank sector, non-bank sector and non-financial private sector, respectively, while only borrowers from non-bank sector and non-financial private sector are reported for the GLO for the borrowers from USA. All the GLI are measured by both the percentage change of GDP and the year on year change. To reduce the multicollinearity in the estimation of the models, the automatic variable selection by Least Absolute Shrinkage and Selection Operator (LASSO) is applied to eliminate irrelevant or redundant financial measures. According to Uniejewski et al. (2019), LASSO minimizes an objective function that includes a penalty proportional to the absolute values of the regression coefficients, thus shrinking some of the coefficients to exactly zero. This process effectively reduces the number of variables in the model, retaining only those that contribute most significantly to the prediction.

To implement LASSO, the first step is to standardize the data, ensuring that all financial variables are on a comparable scale. This standardization is crucial because it ensures that the penalty applied by LASSO affects all coefficients uniformly. Following standardization, cross-validation is used to determine the optimal value of the regularization parameter (λ). Cross-validation involves partitioning the data into training and validation sets, fitting the LASSO model on the training set, and evaluating its performance on the validation set to find the λ that minimizes prediction error.

In this chapter, LASSO was applied to reduce a large set of financial control variables to a more manageable set of three key predictors for GDP forecasting. By doing so, LASSO helps to prevent overfitting, a common issue when dealing with numerous predictors, where the model captures noise rather than the underlying relationship. The reduced model maintains high predictive performance while being simpler and more interpretable. The selected variables, with non-zero coefficients, form the foundation of the final model, ensuring robustness and generalizability. This methodological approach leverages the strengths of LASSO in handling high-dimensional data and improving model reliability (Tibshirani, 1996; James et al., 2013).

The three most relevant financial measures are selected based on the process above, i.e., EPU, FSI and GLI for borrowers from the USA and from non-bank sector measured by the percentage change of GDP (GUNBP).

4.3 Descriptive Statistics

Descriptive statistics of the data used for the out-of-sample forecasting are presented in Table C.1. This table summarizes the key variables in our out-of-sample forecasting models covering the sample period from 2010M4 to 2022M12[28]. We provide their mean, median, standard deviation, maximum and minimum. The WEI_gap contains negative value, indicating the slowdown of the USA economic growth during our sample period. The high value of 2.296 of

[28] The loss of the observations is due to differencing the data.

standard deviation shows the high volatility of the real economic activity. The average value of the real interest rate is 4.3% and the minimum value is -7.1%. the average values of the real money growth for all money measures are around 0.023. The values of the financial condition measures, including EPU, FSI and GUNBP, range from negative values to positive values, which implies the dynamic economic condition during our sample period.

Table C.1 Descriptive Statistics of the Data for the Out-of-Sample Forecasting

Variables	Mean	Median	Std. Dev	Max	Min
WEI_gap	2.100	2.063	2.296	10.058	-7.953
$RFFR$	0.043	0.087	0.085	0.262	-0.071
$\Delta_{12}(m-p)$_Green_Return_Divisia	0.020	0.024	0.053	0.197	-0.091
$\Delta_{12}(m-p)$_Green_Coupon_Divisia	0.019	0.024	0.051	0.191	-0.086
$\Delta_{12}(m-p)$_Divisia	0.021	0.023	0.047	0.167	-0.094
$\Delta_{12}(m-p)$_Sum	0.026	0.026	0.065	0.240	-0.091
EPU	-0.002	-0.147	1.040	4.604	-1.570
FSI	0.036	0.006	1.002	4.378	-0.883
GUNBP	-1.001	-0.473	0.991	2.014	-1.503

Notes: The values for short term interest rate, RFFR, are displayed in real term. Annualised real money growth is defined as $\Delta_{12}(m-p)_t = \Delta_{12}m_t - \Delta_{12}p_t$, where m_t is the natural log of the green-return benchmarked Divisia, green-coupon benchmarked Divisia, conventional Divisia monetary aggregates and their simple sum counterpart, and Δ_{12} represents the difference between the current value of a variable and its twelfth lag.

Having discussed the data and how and what we did, we next shift our focus to the empirical results. First, we present the green-benchmarked monetary aggregates and compare them with the traditional simple sum and the conventional Divisia money measures. Subsequently, we report the empirical results and the out-of-sample forecasting results from the VAR and MS-VAR models, which include these economic monetary aggregates, macroeconomic variables and the financial condition measures.

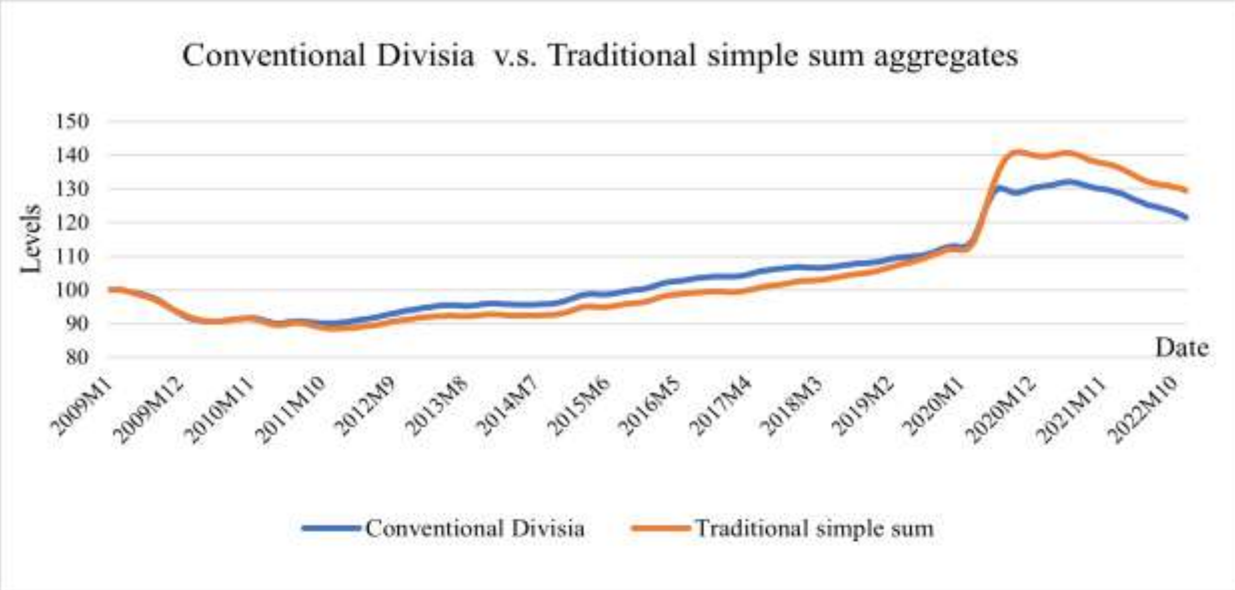
5 The Construction Results of Money Measures

As discussed in Section 2, Barnett (1980) and many times since has suggested constructing a broad monetary aggregate for analysing the USA aggregate demand. As further discussed above in Section 2 (p.105), Barnett (1980) advocated for using a reputable index number to construct Divisia monetary aggregates. Thus, we employ the Törnqvist-Theil discrete time approximation of the continuous time Divisia index over of the broad monetary assets and use a green benchmark to construct a green-benchmarked Divisia monetary aggregate. The Divisia index is one from Diewert (1976)'s superlative class of index numbers. As suggested by Barnett, utilizing a broad monetary aggregate and a superlative index number internalizes in the Divisia monetary aggregate any substitution effects among monetary assets due to changes in relative user costs. The component assets contained in the Divisia monetary aggregate are not treated as perfect substitutes.

The monetary aggregates used in this analysis are a green-return benchmarked Divisia monetary aggregate, a green-coupon benchmarked Divisia monetary aggregate and a conventional Divisia monetary aggregate. For comparison, we also present a traditional simple sum monetary aggregate using the same component assets contained in Divisia. To further ensure the Divisia monetary aggregate and the traditional simple sum aggregate are directly comparable, the levels of the green-return benchmarked Divisia monetary aggregate, the green-coupon benchmarked Divisia monetary aggregate, the conventional Divisia monetary aggregate and the simple sum measures are normalized to 100 in the first period (2009M1). Figure C.1 depicts the levels of the conventional Divisia monetary aggregate alongside its corresponding simple sum counterpart. Figure C.2 provides the levels of the green-return benchmarked Divisia monetary aggregate, the green-coupon benchmarked Divisia monetary aggregate and a conventional Divisia monetary aggregate.

Figure C.1 reveals a difference between the levels of the conventional Divisia monetary aggregate and its traditional green simple sum counterpart. Specifically, the level of conventional Divisia index is greater than that of the simple sum index by 2.51% on average before 2020M4. This result is contrary to previous studies which have suggested that the simple sum monetary aggregates overstate the money stock. This inconsistency may be due to the selection of different normalising date in our data sample. From 2020M4, the levels of both conventional Divisia and simple sum monetary aggregates has a sudden increase. This could be explained by the fact that customers tend to increase their precautionary savings and to hold more liquid assets during the COVID-19 pandemic. The conventional Divisia aggregate level is, on average, 5.69% lower than that of the traditional simple sum measure from 2020M4. This result aligns with previous studies suggesting that simple sum monetary measures tend to overstate the quantity of the money supply in an economy. The difference between the conventional Divisia monetary aggregate and the traditional simple sum measure in Figure 1 can be attributed to the differences in the weighting applied in the construction, since each measure includes exactly same constituent component assets. A simple sum over the component assets assigns equal weights to each, while Divisia monetary aggregation assigns different weights to the components according to the level of monetary services provide by each component asset. The difference in the degree of substitutability among the components of both money measures supports the idea that the more sophisticated Divisia index number construction of the monetary aggregate measures is crucial for the evaluation of the economic performance in a macroeconomic framework and for the conduct of monetary policy.

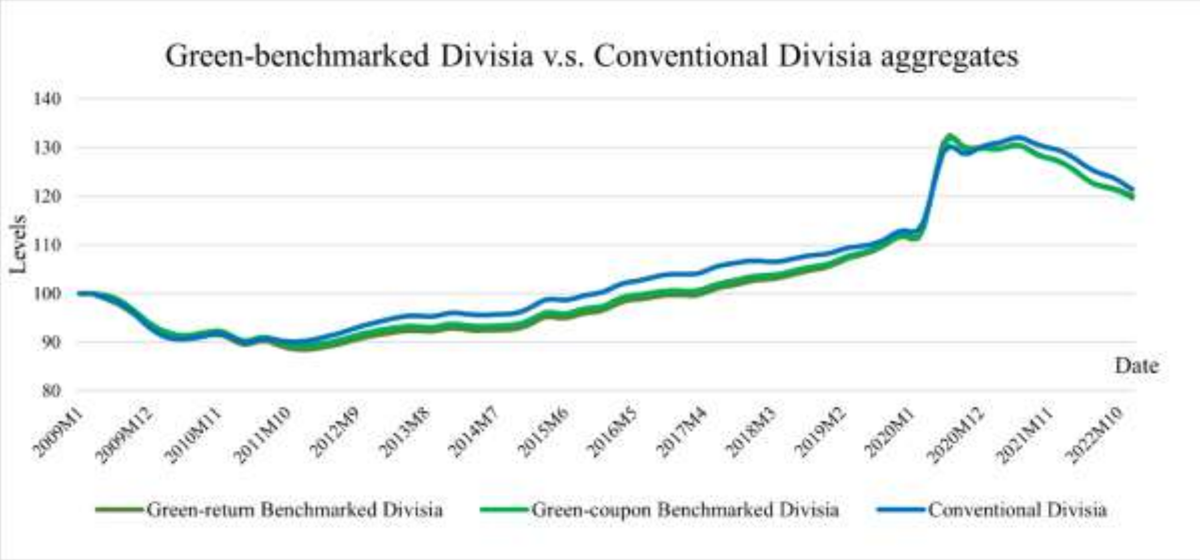
Figure C.1 Levels of Conventional Divisa Money and its Traditional Simple Sum Counterpart



Notes: The figure plots the levels of conventional Divisia monetary aggregates and traditional simple sum monetary aggregates. The blue line is the level of conventional Divisia monetary aggregates. The orange line is the level of traditional simple monetary aggregates. Both are normalized to 100 in the first period (2010Q1).

Figure C.2 illustrates the difference among the levels of the green-return benchmarked Divisia monetary aggregate, the green-coupon benchmarked Divisia monetary aggregate and its conventional Divisia counterpart. Explicitly, the green-return benchmarked Divisia monetary aggregate level and the green-coupon benchmarked Divisia monetary aggregate level are, on average, 2.15% and 1.51% respectively higher than that of the conventional Divisa measure. There is little difference between the levels of the green-return benchmarked Divisia monetary aggregate and the green-coupon benchmarked Divisia monetary aggregate, as the green-return measure is, on average, 0.62% lower than the green-coupon benchmarked measure. The lower level of the conventional Divisia money indicates that the degree of substitutability among the components of the conventional Divisia money is lower than that of both green benchmarked measures. This backs up our claim above that the measurement of money matters. Similarly, a dramatic increase in three Divisia monetary aggregates from 2020M4 reflects the flight to safety phenomenon during the COVID-19.

Figure C.2 Levels of Green-Benchmarked Divisa Money and Conventional Divisia Money



Notes: The figure plots the levels of green-benchmarked Divisia monetary aggregates and conventional Divisia monetary aggregates. The dark green line is the level of green-return benchmarked Divisia monetary aggregates. The light green line is the level of green-coupon benchmarked monetary aggregates. The blue line is the level of conventional Divisia monetary aggregates. All are normalized to 100 in the first period (2010Q1).

Having shown the difference between the constructions of the green-benchmarked Divisia monetary aggregates, the conventional Divisia monetary aggregate and the traditional simple sum monetary aggregate, we next turn to the empirical analysis.

6 Estimation Results

Before turning to a formal comparison of the forecasting performance, we briefly show the estimation results for the different models in this section. The data up to September 2019 are used for estimation and the data from January 2020 to December 2022 are used for forecasting evaluation since we exclude the COVID-19 pandemic period for estimation. The data from October 2019 to December 2019 are used for validation. We provide evidence that our green-benchmarked Divisia monetary aggregates have direct effects on aggregate demand in the USA. We follow the IS curve specifications employed by Reimers (2002) and Binner et al. (2009) to test whether our green-benchmarked Divisia monetary aggregates add more information on aggregate demand in USA by using VAR and MS-VAR models. We start with the baseline model which considers the WEI_gap, the real economic activity, and $RFFR$, the real interest rate in the models. We then include the green-return benchmarked Divisia green-coupon benchmarked Divisia, the conventional Divisia monetary aggregates and their traditional simple sum counterpart into the models to examine their effects on real economy. The variable selection for each model is summarized in Table C.2.

Table C.2 The Variable Selection by Model

	Baseline Model	Green-Return Benchmarked Divisia Model	Green-Coupon Benchmarked Divisia Model	Conventional Divisa Model	Traditional Simple Sum Model
Macroeconomic variables	WEI_gap	WEI_gap	WEI_gap	WEI_gap	WEI_gap
	$RFFR$	$RFFR$	$RFFR$	$RFFR$	$RFFR$
Monetary Measures	-	Green-Return Benchmarked Divisia Money	Green-Coupon Benchmarked Divisia Money	Conventional Divisia Money	Traditional Simple Sum Money

Since the financial conditions may impact the preference of customers to hold green bonds and also the economic activity, we introduce three financial measures discussed above, i.e., EPU, FSI, GUNBP, separately and jointly, into the estimation of models to investigate the influence of the financial conditions on the green investments and the real economic output. Table C.3 presents the variable selection for each model with the consideration of the financial conditions.

Table C.3 The Variable Selection by Model

	Baseline Model	Green-Return Benchmarked Divisia Model	Green-Coupon Benchmarked Divisia Model	Conventional Divisa Model	Traditional Simple Sum Model
Macroeconomic variables	WEI_gap	WEI_gap	WEI_gap	WEI_gap	WEI_gap
	$RFFR$	$RFFR$	$RFFR$	$RFFR$	$RFFR$
Monetary Measures	-	Green-Return Benchmarked Divisia Money	Green-Coupon Benchmarked Divisia Money	Conventional Divisia Money	Traditional Simple Sum Money

Financial Condition Measures	-	EPU/FSI/GUNBP /EPU, FSI, GUNBP	EPU/FSI/GUNBP /EPU, FSI, GUNBP	EPU/FSI/GUNBP/EPU, FSI, GUNBP	EPU/FSI/GUNBP/EPU, FSI, GUNBP

Notes: The EPU, FSI and GUNBP are added to each model separately and then all three financial condition measures are introduced into each model.

Since all variables involved in this chapter are time series data, the stationarity check is necessary. Sims et al. (1990) suggests that if the series are not difference stationary, the estimates may be inconsistent. The advantage of differencing is that the efficiency is obtained if the series are differenced stationary. We use the ADF (Augmented Dickey-Fuller) test to perform unit test on variables. It can be seen from the Table C.4 that the original sequences of all variables except $RFFR$ and GUMBP do not have unit roots, but the first differences of all variables are stationary at 10% significance level. However, given the low power of unit root tests, there is a non-trivial probability of imposing a false unit root. Thus, the inconsistency from the incorrectly differencing may outweigh the potential efficiency gained from differencing. Therefore, we use the variables in levels to construct VAR model and MS-VAR model. We use the Akaike information criterion (AIC) to determine the lag order of the VAR and the optimal lag order is 12 for all models.

Table C.4 The Unit Fundamental Test of Variables

Variables	ADF	P-value	Conclusion
WEI_gap	-2.738	0.068	stationary
$RFFR$	7.827	1.000	nonstationary
Green-Return Benchmarked Divisia aggregates	-4.710	0.000	stationary
Green-Coupon Benchmarked Divisia aggregates	-4.411	0.000	stationary
Conventional Divisia Aggregates	-6.431	0.000	stationary
Traditional simple sum aggregates	-4.509	0.000	stationary
EPU	-4.511	0.000	stationary
FSI	-3.160	0.022	stationary
GUNBP	-1.567	0.500	nonstationary
$\Delta\, WEI\_gap$	-10.847	0.000	stationary
$\Delta\, RFFR$	-4.421	0.000	stationary
ΔGreen-Return Benchmarked Divisia aggregates	-3.010	0.034	stationary

ΔGreen-Coupon Benchmarked Divisia aggregates	-3.055	0.030	stationary
Δ Conventional Divisia Aggregates	-2.630	0.087	stationary
Δ Traditional Simple Sum aggregates	-3.016	0.033	stationary
Δ EPU	-13.701	0.000	stationary
Δ FSI	-3.657	0.005	stationary
Δ GUNBP	-3.990	0.002	stationary

Notes: The 5% threshold for the ADF test is -2.889 and the 10% thresholds for the ADF test is -2.579.

6.1 VAR Estimation Results

We use the Akaike information criterion (AIC) to determine the lag order of the VAR and the optimal lag order is 12 for all models.

6.1.1 Baseline Model

The standard IS equation relates the output gap to the real interest rate. Thus, we establish the baseline VAR model with the real economic activity and real interest rate, i.e., WEI_gap and $RFFR$. The estimated result of the standard IS equation in the VAR(12) model is presented in Table C.5. As can be seen, the estimated coefficient for the constant term is significant. The first lag of WEI_gap is positively and highly significant. We also find various higher lags of output gap like the eleventh lag as well as the change in real interest rate like the fifth and twelfth lags to be significant. The R^2 of 0.81 suggests the VAR within sample fit is quite significant.

Table C.5 The VAR(12) Specification for Baseline Model

	Coefficients		
	WEI_gap	$RFFR$	
Constant			0.456*** (4.05)
L1	0.769*** (4.05)	1.519 (0.19)	
L2	-0.078 (-0.64)	-5.481 (-0.39)	
L3	0.087 (0.72)	1.045 (0.07)	
L4	0.211*	-11.340	

	(1.83)	(-0.81)
L5	-0.110	26.354*
	(-0.89)	(1.87)
L6	0.0006	-18.249
	(0.00)	(-1.27)
L7	0.017	6.470
	(0.13)	(0.45)
L8	0.082	9.527
	(0.68)	(0.66)
L9	0.021	-19.007
	(0.17)	(-1.37)
L10	0.163	7.041
	(1.38)	(0.54)
L11	-0.321***	-13.665
	(-2.75)	(-1.09)
L12	-0.051	17.405**
	(-0.54)	(2.21)

Notes: The results only present the output equation of the VAR model. In this baseline model, we include the output gap and the real interest rate. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

6.1.2 Green-Return Benchmarked Divisia Model

The IS curve which specifies the output gap as a function of the real interest rate ignores the real money stock or its growth rate as shown above. This specification has been challenged by the recent studies arguing that these models neglect money as an important channel of monetary effects on output from theoretical and empirical perspectives like Belongia and Ireland (2016). Their findings indicate that real money growth has a significantly positive effect on output gap for USA. We thus include our newly constructed green-return benchmarked Divisia monetary aggregates in the VAR model. The estimated dynamic structure of the output equation can be found in Table C.6.0. The estimate of the intercept is significant. For the lag structure, we find that the first and eleventh lags of output gap are significant as well as the fifth and twelfth lags of change in real interest rate, which is similar to the results found in the baseline model. Notably, the second, third, fourth, sixth and seventh lags of the change in green-return benchmarked Divisia monetary aggregates are significant, supporting that the monetary aggregates do have significant impact on real economy activity.

Table C.6.0 The VAR(12) Specification for Green-Return Benchmarked Divisia Model

	Coefficients			
	WEI_gap	$RFFR$	Green-Return Divisia	
Constant				0.756*** (4.21)
L1	0.652*** (6.40)	6.703 (0.87)	82.625 (1.25)	
L2	0.025 (0.22)	-10.466 (-0.79)	-470.399* (-1.79)	
L3	0.090 (0.81)	6.149 (0.46)	938.194** (2.00)	
L4	0.170 (1.62)	-17.709 (-1.32)	-715.814* (-1.67)	
L5	-0.135 (-1.24)	27.407** (2.05)	-424.854 (-1.15)	
L6	-0.012 (-0.11)	-11.971 (-0.87)	1,442.606** (2.06)	
L7	0.021 (0.19)	10.199 (0.76)	-1,228.960* (-1.74)	
L8	0.077 (0.72)	0.559 (0.04)	137.060 (0.35)	
L9	0.017 (0.16)	-16.179 (-1.25)	592.193 (1.33)	
L10	0.158 (1.51)	4.644 (0.39)	-501.854 (-1.06)	
L11	-0.274*** (-2.63)	-12.794 (-1.13)	158.691 (0.60)	
L12	-0.120 (-1.34)	13.879* (1.82)	-14.843 (-0.23)	

Notes: The results only present the output equation of the VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

We then add financial condition measures in the VAR model. Much literature has concentrated on the intensity of macro-financial linkages and the associated role of financial conditions in real economic activity. For example, Espinoza et al. (2012) suggest that tight financial and credit conditions would limit the firms' ability to expand, constraining their hiring and investment decisions. Further, the tight financial conditions prevent credit-constrained consumers from borrowing and hence consuming. Both would dampen the growth of real economic activity.

Therefore, the role of financial conditions cannot be ignored in predicting output. We introduce the selected financial measures, EPU, FSI, GUNBP, into the VAR model separately and the estimated results are shown in Table C.6.1, C.6.2 and C.6.3 respectively. The constant is significant in all three specifications. Regarding the lag structure of the output gap, the first, fourth, eleventh and twelfth lags are significant with EPU measure as well as the first, third, eighth, tenth, eleventh and twelfth lags with FSI measure and the first, tenth, eleventh and twelfth lags with GUNBP measure. The fourth and fifth lags of the change in real interest rate are significant with EPU measure and the twelfth lag with FSI and GUNBP measures. The change in green-return benchmarked Divisia monetary aggregates is only significant with EPU measure in the third, fourth and seventh lags, indicating the better performance with EPU measure in our green-return benchmarked Divisia VAR model than that with FSI and GUNBP measures. We also find the various lags of three financial measures to be significant in three VAR models respectively. The results are similar to the findings in the VAR model with three financial measures collectively, which is presented in Table C.6.4. The output gap is significant in the third, fourth, eighth, eleventh and twelfth lags and the green-return benchmarked Divisia money is significant in the first, eleventh, twelfth lags.

Table C.6.1 The VAR(12) Specification for Green-Return Benchmarked Divisia Model with EPU Included

Coefficients

	WEI_gap	$RFFR$	Green-Return Divisia	EPU	
Constant					1.594*** (6.18)
L1	0.499*** (4.92)	4.738 (0.67)	55.666 (0.86)	-0.090** (-2.19)	
L2	-0.018 (-0.16)	-0.617 (-0.05)	-363.277 (-1.41)	-0.073* (-1.75)	
L3	0.060 (0.57)	7.069 (0.58)	771.154* (1.71)	-0.051 (-1.32)	
L4	0.219** (2.31)	-21.614* (-1.76)	-705.677* (-1.74)	-0.061 (-1.59)	
L5	-0.103 (-1.04)	23.046* (1.86)	-148.718 (-0.41)	-0.013 (-0.33)	
L6	-0.016 (-0.15)	-9.586 (-0.77)	1,094.091 (1.59)	-0.013 (-0.34)	
L7	-0.025 (-0.25)	3.325 (0.26)	-1,155.189* (-1.70)	-0.104*** (-2.70)	
L8	0.093 (0.94)	6.503 (0.52)	355.675 (0.96)	0.076* (1.88)	
L9	0.004 (0.04)	-10.598 (-0.88)	408.816 (0.98)	0.004 (0.10)	
L10	0.114 (1.17)	6.588 (0.62)	-557.421 (-1.25)	-0.028 (-0.62)	
L11	-0.270*** (-2.79)	-16.321 (-1.56)	322.882 (1.28)	0.081* (1.88)	
L12	-0.308*** (-3.33)	8.954 (1.24)	-97.520 (-1.50)	0.009 (0.24)	

Notes: The results only present the output equation of the VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the EPU as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.6.2 The VAR(12) Specification for Green-Return Benchmarked Divisia Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Return Divisia	FSI	
Constant					1.224*** (3.64)

L1	0.484***	5.705	31.727	32.625***
	(4.81)	(0.79)	(0.51)	(3.58)
L2	0.046	-6.590	-223.216	-106.415***
	(0.43)	(-0.56)	(-0.92)	(-3.15)
L3	0.324***	-0.434	473.832	136.035**
	(3.04)	(-0.04)	(1.11)	(2.31)
L4	0.160	-7.078	-394.892	-33.341
	(1.52)	(-0.59)	(-1.00)	(-0.56)
L5	-0.160	19.610	-139.363	-154.477**
	(-1.48)	(1.64)	(-0.41)	(-2.55)
L6	-0.139	-10.036	614.967	268.453***
	(-1.25)	(-0.82)	(0.95)	(2.93)
L7	-0.114	9.475	-534.512	-202.375**
	(-1.03)	(0.78)	(-0.81)	(-2.15)
L8	0.178*	-1.453	101.427	19.483
	(1.69)	(-0.12)	(0.27)	(0.31)
L9	-0.014	-7.158	133.264	134.615**
	(-0.14)	(-0.63)	(0.33)	(2.33)
L10	0.209**	-0.558	-79.071	-164.797***
	(2.08)	(-0.05)	(-0.18)	(-2.78)
L11	-0.256***	-14.630	16.121	92.827***
	(-2.62)	(-1.50)	(0.06)	(2.62)
L12	-0.151*	14.766**	-11.537	-21.843**
	(-1.81)	(2.20)	(-0.18)	(-2.24)

Notes: The results only present the output equation of the VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the FSI as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.6.3 The VAR(12) Specification for Green-Return Benchmarked Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Return Divisia	GUNBP	
Constant					0.544***
					(2.74)
L1	0.516***	5.634	80.789	1.722	
	(5.38)	(0.80)	(1.07)	(1.07)	
L2	-0.006	-7.409	-297.112	-6.709	
	(-0.06)	(-0.61)	(-1.00)	(-1.11)	

L3	0.167 (1.57)	-1.157 (-0.09)	407.551 (0.78)	11.493 (1.09)
L4	0.123 (1.20)	-10.195 (-0.80)	-83.645 (-0.17)	-5.154 (-0.47)
L5	-0.073 (-0.69)	12.982 (1.03)	-454.316 (-1.01)	-17.486 (-1.48)
L6	0.085 (0.82)	-1.499 (-0.12)	504.885 (0.62)	40.242** (2.34)
L7	0.027 (0.26)	5.376 (0.44)	52.137 (0.06)	-36.491** (-2.16)
L8	0.140 (1.45)	9.329 (0.75)	-529.322 (-1.14)	4.893 (0.43)
L9	0.042 (0.44)	-17.351 (-1.45)	399.330 (0.89)	26.864** (2.24)
L10	0.197** (2.07)	7.027 (0.62)	13.815 (0.03)	-33.025*** (-2.80)
L11	-0.243*** (-2.58)	-16.240 (-1.53)	-155.725 (-0.54)	18.590*** (2.79)
L12	-0.172** (-2.01)	12.613* (1.67)	67.671 (0.88)	-4.693*** (-2.75)

Notes: The results only present the output equation of the VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.6.4 The VAR(12) Specification for Green-Return Benchmarked Divisia Model with Three Financial Measures Included

	Coefficients						
	WEI_gap	$RFFR$	Green-Return Divisia	EPU	FSI	GUNBP	
Constant							1.224*** (3.64)
L1	0.016 (0.20)	1.159 (0.14)	108.550* (1.88)	-0.084** (-2.17)	-2.221 (-0.22)	3.857* (1.79)	
L2	-0.038 (-0.40)	4.432 (0.51)	-331.880 (-1.60)	-0.111*** (-3.31)	35.392 (1.01)	-13.053* (-1.82)	
L3	0.279*** (3.15)	4.422 (0.52)	464.761 (1.35)	-0.134*** (-4.20)	-112.020** (-2.01)	21.611* (1.91)	

L4	0.226**	-10.161	-310.069	-0.180***	148.491***	-12.489
	(2.18)	(-1.06)	(-0.92)	(-5.49)	(2.75)	(-1.11)
L5	-0.104	-3.195	-80.514	-0.141***	-47.748	-20.645*
	(-1.01)	(-0.33)	(-0.23)	(-3.70)	(-0.81)	(-1.72)
L6	0.008	-6.692	351.480	-0.048	-121.460	49.196***
	(0.08)	(-0.77)	(0.67)	(-1.18)	(-1.50)	(2.90)
L7	-0.072	6.176	-351.728	-0.139***	190.434**	-37.083**
	(-0.80)	(0.66)	(-0.65)	(-3.50)	(2.27)	(-2.21)
L8	0.161**	15.408*	118.273	0.003	-96.228	-6.311
	(1.98)	(1.67)	(0.33)	(0.08)	(-1.55)	(-0.59)
L9	0.068	3.674	170.494	-0.071*	-37.329	38.297***
	(0.75)	(0.46)	(0.60)	(-1.87)	(-0.67)	(3.54)
L10	0.056	6.208	-350.644	0.023	84.035	-34.042***
	(0.66)	(0.84)	(-1.12)	(0.59)	(1.45)	(-3.05)
L11	-0.128*	-20.283**	342.729*	0.013	-55.290	13.362**
	(-1.75)	(-2.32)	(1.66)	(0.36)	(-1.51)	(2.08)
L12	-0.571***	1.390	-141.834**	-0.029	15.717	-1.634
	(-8.68)	(0.20)	(-2.15)	(-0.79)	(1.46)	(-0.95)

Notes: The results only present the output equation of the VAR model. In this green-return benchmarked Divisia model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add all three financial measures. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

6.1.3 Green-Coupon Benchmarked Divisia Model

Alternatively, we replace the green-return benchmarked Divisia monetary aggregates with the green-coupon benchmarked Divisia monetary aggregates, which use the coupon rate of the green bonds as the benchmark in the construction of Divisia monetary aggregates, in the VAR model. The estimated dynamic structure of the output equation is shown in Table C.7.0. The constant is significant. For the lag structure, the first, fourth and eleventh lags of output gap are significant as well as the fifth and twelfth lags of the change in real interest rate. In terms of the change in green-coupon benchmarked Divisia monetary aggregates, the second, third and sixth lags are significant, which also is the evidence of the additional information provided by the money. These results are quite similar to the results found in the green-return benchmarked Divisia model.

Table C.7.0 The VAR(12) Specification for Green-Coupon Benchmarked Divisia Model

	Coefficients			
	WEI_gap	$RFFR$	Green-Coupon Divisia	
Constant				0.766*** (4.22)
L1	0.651*** (6.37)	6.590 (0.85)	74.679 (1.14)	
L2	0.020 (0.18)	-10.246 (-0.77)	-434.654* (-1.67)	
L3	0.086 (0.78)	6.293 (0.47)	865.417* (1.88)	
L4	0.174* (1.65)	-17.688 (-1.32)	-647.493 (-1.53)	
L5	-0.134 (-1.22)	26.804** (1.99)	-413.932 (-1.13)	
L6	-0.020 (-0.18)	-11.809 (-0.86)	1,333.794* (1.94)	
L7	0.026 (0.23)	10.014 (0.74)	-1,100.436 (-1.59)	
L8	0.080 (0.74)	1.210 (0.09)	85.449 (0.22)	
L9	0.016 (0.15)	-16.566 (-1.28)	552.002 (1.26)	
L10	0.157 (1.50)	4.886 (0.41)	-432.690 (-0.93)	
L11	-0.275*** (-2.64)	-13.057 (-1.16)	115.971 (0.45)	
L12	-0.117 (-1.30)	13.851* (1.81)	-3.557 (-0.06)	

Notes: The results only present the output equation of the VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

For the VAR models with the financial measures incorporated, the estimated dynamic structures with EPU, FSI and GUNBP are presented in the Table C.7.1, C.7.2 and C.7.3 respectively. The constant is significant in all three specifications. Surprisingly, the money term is insignificant in all three VAR models, indicating that the green-return benchmarked

Divisia monetary aggregate performs better in the VAR model fitting than the green-coupon benchmarked Divisia monetary aggregate when the financial measures considered. For the output gap, the first, fourth, eleventh and twelfth lags are significant with EPU measure as well as the first, third, tenth, eleventh and twelfth lags with FSI measure and the first, tenth, eleventh and twelfth lags with GUNBP measure, which are similar to the results with the green-return benchmarked Divisia monetary aggregates. The significance of the change in real interest rate and financial measures is same with that of the green-return benchmarked Divisia model. When we include all three financial measures in the VAR model, the money term becomes significant in the twelfth lag as shown in Table C.7.4. The third, fourth, eighth and twelfth lags of the output gap and various lags of financial measures are significant.

Table C.7.1 The VAR(12) Specification for Green-Coupon Benchmarked Divisia Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Coupon Divisia	EPU	
Constant					1.548*** (5.96)
L1	0.514*** (5.05)	3.725 (0.52)	43.415 (0.68)	-0.086** (-2.07)	
L2	-0.018 (-0.17)	-0.251 (-0.02)	-306.912 (-1.20)	-0.075* (-1.76)	
L3	0.051 (0.49)	7.367 (0.60)	668.202 (1.49)	-0.053 (-1.35)	
L4	0.234** (2.43)	-21.457* (-1.73)	-630.910 (-1.57)	-0.057 (-1.48)	
L5	-0.107 (-1.06)	23.400* (1.87)	-105.609 (-0.29)	-0.010 (-0.25)	
L6	-0.024 (-0.23)	-9.204 (-0.73)	948.701 (1.40)	-0.013 (-0.34)	
L7	-0.021 (-0.21)	2.307 (0.18)	-1,025.929 (-1.53)	-0.102*** (-2.61)	
L8	0.087 (0.87)	7.283 (0.58)	331.521 (0.90)	0.081** (1.98)	
L9	0.008 (0.08)	-12.088 (-1.00)	350.197 (0.85)	0.008 (0.19)	

L10	0.114 (1.16)	8.045 (0.76)	-493.058 (-1.11)	-0.036 (-0.80)
L11	-0.265*** (-2.71)	-16.480 (-1.57)	291.681 (1.16)	0.081* (1.86)
L12	-0.299*** (-3.20)	8.469 (1.16)	-90.439 (-1.38)	0.010 (0.25)

Notes: The results only present the output equation of the VAR model. In this green-coupon benchmarked Divisia model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the EPU as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.7.2 The VAR(12) Specification for Green-Coupon Benchmarked Divisia Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Coupon Divisia	FSI	
Constant					1.132*** (3.40)
L1	0.491*** (4.85)	5.939 (0.81)	27.211 (0.44)	32.431*** (3.52)	
L2	0.044 (0.41)	-7.010 (-0.59)	-201.357 (-0.84)	-106.314*** (-3.12)	
L3	0.312*** (2.91)	-0.131 (-0.01)	422.885 (1.00)	136.692** (2.30)	
L4	0.163 (1.53)	-7.292 (-0.61)	-333.399 (-0.86)	-34.769 (-0.58)	
L5	-0.153 (-1.40)	19.208 (1.58)	-158.843 (-0.46)	-153.198** (-2.51)	
L6	-0.134 (-1.19)	-9.977 (-0.80)	561.463 (0.88)	266.966*** (2.90)	
L7	-0.102 (-0.91)	9.557 (0.78)	-440.038 (-0.68)	-200.269** (-2.12)	
L8	0.175 (1.64)	-0.674 (-0.06)	35.606 (0.10)	18.173 (0.29)	
L9	-0.018 (-0.18)	-8.147 (-0.71)	132.490 (0.33)	133.047** (2.29)	
L10	0.207** (2.03)	0.436 (0.04)	-35.693 (-0.08)	-161.141*** (-2.71)	
L11	-0.252** (-2.56)	-15.097 (-1.53)	-23.203 (-0.09)	90.073** (2.53)	

L12	-0.142* (-1.69)	14.401** (2.12)	2.598 (0.04)	-21.023** (-2.15)

Notes: The results only present the output equation of the VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the FSI as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.7.3 The VAR(12) Specification for Green-Coupon Benchmarked Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Coupon Divisia	GUNBP	
Constant					0.545*** (2.82)
L1	0.507*** (5.29)	6.449 (0.91)	83.207 (1.12)	1.766 (1.11)	
L2	-0.010 (-0.10)	-8.094 (-0.67)	-300.673 (-1.04)	-6.870 (-1.14)	
L3	0.164 (1.56)	-1.324 (-0.11)	394.868 (0.78)	11.700 (1.12)	
L4	0.119 (1.16)	-10.275 (-0.82)	-42.524 (-0.09)	-5.217 (-0.48)	
L5	-0.061 (-0.58)	12.420 (1.00)	-491.605 (-1.10)	-17.650 (-1.52)	
L6	0.085 (0.81)	-1.377 (-0.11)	484.346 (0.61)	40.712** (2.43)	
L7	0.026 (0.26)	6.128 (0.50)	131.786 (0.17)	-37.249** (-2.26)	
L8	0.146 (1.52)	9.294 (0.76)	-598.862 (-1.33)	5.554 (0.49)	
L9	0.039 (0.41)	-16.953 (-1.43)	398.988 (0.91)	26.819** (2.29)	
L10	0.198** (2.09)	6.281 (0.56)	65.990 (0.14)	-33.553*** (-2.95)	
L11	-0.244*** (-2.61)	-15.936 (-1.51)	-202.165 (-0.73)	19.128*** (2.99)	
L12	-0.165* (-1.93)	12.437* (1.66)	83.259 (1.12)	-4.880*** (-2.98)	

Notes: The results only present the output equation of the VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.7.4 The VAR(12) Specification for Green-Coupon Benchmarked Divisia Model with Three Financial Measures Included

	Coefficients						
	WEI_gap	$RFFR$	Green-Coupon Divisia	EPU	FSI	GUNBP	
Constant							2.645*** (4.55)
L1	0.046 (0.60)	-1.403 (-0.18)	88.762 (1.59)	-0.078** (-2.06)	-3.021 (-0.30)	3.324 (1.54)	
L2	0.008 (0.09)	3.361 (0.39)	-276.512 (-1.37)	-0.104*** (-3.08)	33.814 (0.97)	-11.442 (-1.60)	
L3	0.314*** (3.64)	5.449 (0.64)	375.204 (1.12)	-0.128*** (-3.99)	-105.475* (-1.92)	19.572* (1.74)	
L4	0.253** (2.48)	-9.881 (-1.04)	-214.259 (-0.66)	-0.172*** (-5.26)	136.829** (2.56)	-10.901 (-0.97)	
L5	-0.103 (-1.00)	-0.811 (-0.09)	-99.707 (-0.29)	-0.125*** (-3.40)	-41.174 (-0.70)	-20.919* (-1.72)	
L6	-0.020 (-0.20)	-5.242 (-0.62)	228.766 (0.45)	-0.032 (-0.79)	-114.361 (-1.42)	48.769*** (2.87)	
L7	-0.110 (-1.24)	6.277 (0.68)	-135.663 (-0.27)	-0.134*** (-3.43)	177.183** (2.13)	-37.720** (-2.26)	
L8	0.166** (2.04)	13.074 (1.45)	-26.197 (-0.08)	0.010 (0.27)	-91.580 (-1.47)	-4.817 (-0.45)	
L9	0.093 (1.06)	1.850 (0.24)	139.357 (0.50)	-0.078** (-2.03)	-29.568 (-0.53)	37.893*** (3.50)	
L10	0.100 (1.25)	5.372 (0.73)	-209.796 (-0.71)	0.011 (0.30)	72.327 (1.27)	-35.333*** (-3.19)	
L11	-0.110 (-1.51)	-20.502** (-2.37)	226.389 (1.20)	0.001 (0.02)	-47.725 (-1.31)	14.890** (2.36)	
L12	-0.576*** (-8.72)	3.733 (0.55)	-101.532* (-1.73)	-0.026 (-0.71)	14.143 (1.31)	-2.215 (-1.32)	

Notes: The results only present the output equation of the VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add all three financial measures. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

6.1.4 Conventional Divisia Model

To compare with the green benchmarked Divisia VAR models above, we also present the conventional Divisia VAR model. As shown in Table C.8.0, the constant term is significant in the output equation. For the lag structure, the results demonstrate that the first, tenth, eleventh and twelfth lags of output gap are significant as well as the fifth lag of the change in real interest rate, which are similar to the results found in the green-benchmarked Divisia models. The coefficients on the measures of the change in real conventional Divisia money growth are insignificant, which are inconsistent with the results obtained from the green-benchmarked Divisia models. This suggests that the green-benchmarked Divisia money measures are more useful in informing monetary policy than that of the conventional Divisia measure.

Table C.8.0 The VAR(12) Specification for Conventional Divisia Model

	Coefficients			
	WEI_gap	$RFFR$	Conventional Divisia	
Constant				1.139*** (5.05)
L1	0.585*** (5.85)	5.478 (0.72)	52.577 (0.77)	
L2	0.008 (0.07)	-6.913 (-0.54)	-377.016 (-1.38)	
L3	0.099 (0.91)	6.327 (0.48)	797.683 (1.62)	
L4	0.162 (1.58)	-19.086 (-1.47)	-660.444 (-1.39)	
L5	-0.133 (-1.25)	28.313** (2.18)	-221.140 (-0.53)	
L6	-0.032 (-0.29)	-11.554 (-0.86)	970.819 (1.32)	
L7	0.020 (0.19)	10.550 (0.81)	-785.541 (-1.05)	
L8	0.090 (0.87)	-4.307 (-0.34)	90.795 (0.20)	

L9	-0.005	-12.843	223.385
	(-0.05)	(-1.03)	(0.46)
L10	0.170*	2.791	-55.899
	(1.67)	(0.24)	(-0.11)
L11	-0.263***	-11.158	-84.001
	(-2.59)	(-1.02)	(-0.30)
L12	-0.170*	11.269	39.090
	(-1.91)	(1.51)	(0.58)

Notes: The results only present the output equation of the VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

We also add financial condition measures in the conventional Divisia VAR model as above. The estimated output equation of VAR models with EPU, FSI, and GUNBP are exhibited in Table C.8.1, C.8.2 and C.8.3 respectively. The constant is significant in all three equations. For the lag structure of the output gap, the first, fourth, eleventh and twelfth lags are significant with EPU measure as well as the first, third, fourth, tenth, eleventh and twelfth lags with FSI measure and the first, third, tenth, eleventh and twelfth lags with GUNBP measure. The fourth and fifth lags of the change in real interest rate is significant with EPU measure, the fifth and twelfth lags with FSI measure, and the twelfth lag with GUNBP measure. The various lags of the financial measures are significant in each VAR specification. These point estimates are remarkably similar with those estimated in green-benchmarked models. The money term is still insignificant with the financial measures included, suggesting that the conventional Divisia monetary aggregate does not contain more information than the green-return benchmarked monetary aggregate. Table C.8.4 displays the findings of the conventional Divisia model with three financial measures simultaneously. The money term becomes significant in twelfth lag. The third, fourth and twelfth lags of the output gap and various lags of financial measures are significant as same with the green-benchmarked models above.

Table C.8.1 The VAR(12) Specification for Conventional Divisia Model with EPU Included

Coefficients

	WEI_gap	$RFFR$	Conventional Divisia	EPU	
Constant					1.793*** (6.80)
L1	0.462*** (4.73)	0.651 (0.09)	55.436 (0.84)	-0.081** (-2.04)	
L2	-0.013 (-0.12)	3.059 (0.26)	-325.877 (-1.23)	-0.074* (-1.81)	
L3	0.069 (0.68)	6.782 (0.57)	642.611 (1.36)	-0.050 (-1.33)	
L4	0.231** (2.48)	-20.526* (-1.71)	-563.467 (-1.26)	-0.047 (-1.28)	
L5	-0.104 (-1.08)	24.937** (2.06)	-84.998 (-0.21)	-0.010 (-0.28)	
L6	-0.024 (-0.24)	-8.487 (-0.69)	701.655 (0.99)	0.001 (0.03)	
L7	-0.012 (-0.12)	2.919 (0.24)	-673.748 (-0.93)	-0.085** (-2.31)	
L8	0.100 (1.03)	2.673 (0.22)	191.563 (0.45)	0.095** (2.47)	
L9	0.000 (0.00)	-9.400 (-0.81)	162.845 (0.37)	0.023 (0.55)	
L10	0.113 (1.19)	4.186 (0.41)	-189.010 (-0.40)	-0.025 (-0.58)	
L11	-0.254*** (-2.67)	-14.258 (-1.41)	114.718 (0.42)	0.093** (2.22)	
L12	-0.337*** (-3.73)	5.949 (0.85)	-50.921 (-0.71)	0.019 (0.48)	

Notes: The results only present the output equation of the VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the EPU as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.8.2 The VAR(12) Specification for Conventional Divisia Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Conventional Divisia	FSI	
Constant					1.078*** (3.79)
L1	0.437***	6.353	1.861	26.817***	

	(4.38)	(0.87)	(0.03)	(2.96)
L2	0.048 (0.45)	-6.713 (-0.58)	-148.692 (-0.60)	-88.699*** (-2.67)
L3	0.324*** (3.09)	-0.952 (-0.08)	380.940 (0.86)	110.295* (1.91)
L4	0.175* (1.68)	-9.717 (-0.84)	-368.158 (-0.85)	-16.858 (-0.29)
L5	-0.115 (-1.09)	21.953* (1.87)	30.352 (0.08)	-150.438** (-2.55)
L6	-0.113 (-1.04)	-9.313 (-0.77)	215.391 (0.32)	245.124*** (2.77)
L7	-0.127 (-1.16)	13.771 (1.17)	-133.120 (-0.19)	-171.112* (-1.88)
L8	0.165 (1.60)	-6.487 (-0.56)	38.200 (0.09)	-4.376 (-0.07)
L9	-0.043 (-0.44)	-5.247 (-0.47)	-201.972 (-0.45)	138.261** (2.47)
L10	0.227** (2.31)	-3.412 (-0.34)	366.934 (0.79)	-150.531*** (-2.62)
L11	-0.220** (-2.30)	-14.014 (-1.46)	-250.905 (-0.96)	77.295** (2.24)
L12	-0.198** (-2.37)	12.535* (1.81)	59.393 (0.89)	-15.798 (-1.63)

Notes: The results only present the output equation of the VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the FSI as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.8.3 The VAR(12) Specification for Conventional Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Conventional Divisia	GUNBP	
Constant					0.912*** (4.23)
L1	0.478*** (5.14)	6.019 (0.88)	80.789 (1.07)	1.902 (1.24)	
L2	-0.013 (-0.13)	-6.893 (-0.59)	-297.112 (-1.00)	-7.519 (-1.31)	
L3	0.174*	-1.145	407.551	12.552	

	(1.70)	(-0.10)	(0.78)	(1.26)
L4	0.084 (0.85)	-9.008 (-0.73)	-83.645 (-0.17)	-4.356 (-0.42)
L5	-0.072 (-0.71)	11.612 (0.95)	-454.316 (-1.01)	-21.797** (-1.98)
L6	0.058 (0.59)	0.189 (0.02)	504.885 (0.62)	46.206*** (2.93)
L7	0.013 (0.13)	6.871 (0.58)	52.137 (0.06)	-39.936*** (-2.58)
L8	0.151 (1.63)	4.938 (0.41)	-529.322 (-1.14)	3.328 (0.32)
L9	0.016 (0.17)	-18.540 (-1.59)	399.330 (0.89)	31.534*** (2.88)
L10	0.188** (2.07)	8.227 (0.76)	13.815 (0.03)	-37.324*** (-3.47)
L11	-0.217** (-2.40)	-16.048 (-1.57)	-155.725 (-0.54)	20.776*** (3.44)
L12	-0.209** (-2.57)	13.157* (1.86)	67.671 (0.88)	-5.175*** (-3.35)

Notes: The results only present the output equation of the VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.8.4 The VAR(12) Specification for Conventional Divisia Model with Three Financial Measures Included

	Coefficients						
	WEI_gap	$RFFR$	Conventional Divisia	EPU	FSI	GUNBP	
Constant							1.875** (2.05)
L1	0.086 (1.08)	-4.345 (-0.61)	12.358 (0.19)	-0.067* (-1.70)	7.128 (0.67)	0.458 (0.16)	
L2	0.106 (1.01)	5.130 (0.59)	-46.358 (-0.22)	-0.076* (-1.75)	2.396 (0.07)	-5.011 (-0.62)	
L3	0.373*** (3.93)	3.602 (0.41)	77.815 (0.23)	-0.099** (-2.18)	-74.187 (-1.36)	15.164 (1.34)	
L4	0.218** (2.11)	-3.934 (-0.41)	-172.646 (-0.52)	-0.141*** (-3.06)	146.384*** (2.78)	-14.776 (-1.34)	

L5	-0.083	0.038	292.806	-0.083	-94.449	-13.028
	(-0.84)	(0.00)	(0.86)	(-1.63)	(-1.52)	(-1.02)
L6	-0.010	0.366	-372.488	0.028	-74.364	46.811***
	(-0.10)	(0.04)	(-0.74)	(0.52)	(-0.91)	(2.89)
L7	-0.095	4.273	233.797	-0.088*	191.440**	-43.411***
	(-0.99)	(0.46)	(0.45)	(-1.84)	(2.33)	(-2.73)
L8	0.138*	13.550	0.298	0.063	-136.785**	0.359
	(1.77)	(1.41)	(0.00)	(1.44)	(-2.07)	(0.03)
L9	0.047	-3.994	-62.663	0.012	-9.491	38.986***
	(0.59)	(-0.47)	(-0.22)	(0.25)	(-0.16)	(3.60)
L10	0.078	2.660	-73.260	0.092**	89.991	-40.881***
	(0.93)	(0.36)	(-0.24)	(1.97)	(1.61)	(-3.79)
L11	-0.090	-21.786***	208.910	0.047	-74.248**	19.639***
	(-1.20)	(-2.70)	(1.04)	(1.12)	(-2.07)	(2.86)
L12	-0.569***	1.438	-115.307*	-0.000	25.834**	-4.083*
	(-8.76)	(0.20)	(-1.84)	(-0.01)	(2.27)	(-1.85)

Notes: The results only present the output equation of the VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add all three financial measures. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

6.1.5 Traditional Simple Sum Model

For comparison, we also present a traditional simple sum VAR model which substitutes the Divisia monetary aggregates with the simple sum monetary aggregate using the same component assets contained in the Divisia. Table C.9.0 shows the estimated dynamic results of the output equation. The constant is significant. In terms of the lag structure, the first and eleventh lags of output gap are significant as well as the fifth and twelfth lags of the change in real interest rate, which is same to the results found in the green-return benchmarked Divisia model. The second, third, fourth, sixth and seventh lags of the change in the traditional simple sum monetary aggregate are significant, also supporting the active role of money in real economy activity.

Table C.9.0 The VAR(12) Specification for Traditional Simple Sum Model

Coefficients

	WEI_gap	$RFFR$	Simple Sum	
Constant				0.742*** (4.14)
L1	0.661*** (6.49)	6.329 (0.82)	85.189 (1.28)	
L2	0.022 (0.19)	-10.609 (-0.80)	-479.490* (-1.81)	
L3	0.090 (0.80)	6.472 (0.48)	955.458** (2.04)	
L4	0.172 (1.64)	-17.557 (-1.31)	-732.434* (-1.71)	
L5	-0.134 (-1.22)	27.122** (2.02)	-425.983 (-1.15)	
L6	-0.016 (-0.14)	-11.645 (-0.85)	1,469.878** (2.09)	
L7	0.018 (0.16)	10.074 (0.75)	-1,265.598* (-1.79)	
L8	0.077 (0.71)	0.969 (0.07)	151.964 (0.40)	
L9	0.018 (0.17)	-16.355 (-1.26)	609.907 (1.38)	
L10	0.158 (1.50)	4.510 (0.37)	-532.537 (-1.13)	
L11	-0.278*** (-2.66)	-13.284 (-1.17)	177.986 (0.68)	
L12	-0.114 (-1.27)	14.600* (1.91)	-19.562 (-0.30)	

Notes: The results only present the output equation of the VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

From Table C.9.1, C.9.2 and C.9.3, we find that the significance of the coefficients is also same with the that from the green-return benchmarked Divisia model when the financial measures are embedded. The intercept is significant in all three output equations. As for the lag structure of the output gap, the first, fourth, eleventh and twelfth lags are significant with EPU measure as well as the first, third, eighth, tenth, eleventh and twelfth lags with FSI measure and the first, tenth, eleventh and twelfth lags with GUNBP measure. The fourth and fifth lags of the change in real interest rate are significant with EPU measure and the twelfth lag with FSI and GUNBP measures. The change in the traditional simple sum monetary aggregates is only significant with EPU measure in third, fourth and seventh lags, indicating the better performance with EPU measure in the traditional simple sum VAR model than that with FSI and GUNBP measures. The various lags of three financial measures are significant in three VAR models respectively. Similarly, when all financial measures considered together, the significance of the estimates is same with that in the green-return benchmarked Divisia model (see Table C.9.4).

Table C.9.1 The VAR(12) Specification for Traditional Simple Sum Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	EPU	
Constant					1.549*** (6.10)
L1	0.511*** (5.04)	4.020 (0.57)	59.314 (0.91)	-0.088** (-2.15)	
L2	-0.019 (-0.18)	-0.634 (-0.05)	-374.818 (-1.45)	-0.072* (-1.73)	
L3	0.061 (0.58)	7.301 (0.60)	790.205* (1.74)	-0.050 (-1.30)	
L4	0.220** (2.30)	-21.758* (-1.77)	-718.109* (-1.78)	-0.060 (-1.55)	
L5	-0.103 (-1.04)	23.229* (1.87)	-159.283 (-0.44)	-0.011 (-0.28)	
L6	-0.018 (-0.18)	-9.229 (-0.73)	1,126.537 (1.63)	-0.013 (-0.34)	
L7	-0.028 (-0.28)	3.399 (0.27)	-1,186.441* (-1.74)	-0.105*** (-2.71)	
L8	0.093 (0.94)	6.560 (0.52)	360.986 (0.99)	0.078* (1.92)	
L9	0.005 (0.05)	-10.556 (-0.88)	427.962 (1.02)	0.005 (0.10)	

L10	0.117 (1.19)	6.336 (0.60)	-579.734 (-1.30)	-0.028 (-0.62)	
L11	-0.271*** (-2.79)	-16.578 (-1.58)	333.013 (1.33)	0.080* (1.85)	
L12	-0.297*** (-3.22)	9.745 (1.34)	-98.561 (-1.53)	0.012 (0.30)	

Notes: The results only present the output equation of the VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add the EPU as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.9.2 The VAR(12) Specification for Traditional Simple Sum Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	FSI	
Constant					1.218*** (3.76)
L1	0.492*** (4.90)	5.460 (0.76)	40.230 (0.65)	32.755*** (3.62)	
L2	0.039 (0.37)	-6.634 (-0.56)	-252.468 (-1.04)	-106.661*** (-3.17)	
L3	0.321*** (3.01)	-0.157 (-0.01)	521.488 (1.22)	136.365** (2.32)	
L4	0.160 (1.52)	-7.056 (-0.59)	-428.468 (-1.09)	-33.761 (-0.57)	
L5	-0.161 (-1.49)	19.343 (1.61)	-160.874 (-0.47)	-153.785** (-2.54)	
L6	-0.138 (-1.24)	-9.899 (-0.81)	691.835 (1.07)	268.158*** (2.93)	
L7	-0.112 (-1.01)	9.166 (0.76)	-611.196 (-0.93)	-203.504** (-2.17)	
L8	0.179* (1.70)	-0.919 (-0.08)	117.884 (0.32)	21.689 (0.34)	
L9	-0.012 (-0.12)	-7.222 (-0.63)	178.500 (0.44)	133.340** (2.31)	
L10	0.206** (2.05)	-0.359 (-0.03)	-137.157 (-0.31)	-165.433*** (-2.80)	
L11	-0.262*** (-2.68)	-15.115 (-1.55)	47.853 (0.19)	94.192*** (2.67)	
L12	-0.143*	15.325**	-18.765	-22.536**	

	(-1.71)	(2.28)	(-0.29)	(-2.32)

Notes: The results only present the output equation of the VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add the FSI as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.9.3 The VAR(12) Specification for Traditional Simple Sum Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	GUNBP	
Constant					0.542*** (2.78)
L1	0.525*** (5.44)	6.078 (0.86)	83.266 (1.09)	1.687 (1.04)	
L2	-0.009 (-0.09)	-7.992 (-0.66)	-312.410 (-1.04)	-6.644 (-1.09)	
L3	0.164 (1.54)	-0.526 (-0.04)	443.863 (0.85)	11.514 (1.09)	
L4	0.125 (1.21)	-10.524 (-0.83)	-125.677 (-0.26)	-5.371 (-0.49)	
L5	-0.069 (-0.65)	13.108 (1.04)	-449.795 (-0.99)	-17.114 (-1.44)	
L6	0.086 (0.82)	-1.914 (-0.15)	566.592 (0.69)	39.873** (2.31)	
L7	0.024 (0.24)	5.625 (0.46)	-41.512 (-0.05)	-36.288** (-2.14)	
L8	0.135 (1.39)	9.455 (0.76)	-475.564 (-1.03)	4.971 (0.43)	
L9	0.040 (0.42)	-17.208 (-1.43)	417.314 (0.92)	26.521** (2.21)	
L10	0.194** (2.02)	7.067 (0.62)	-38.740 (-0.08)	-32.625*** (-2.77)	
L11	-0.246*** (-2.60)	-16.858 (-1.57)	-118.820 (-0.41)	18.346*** (2.75)	
L12	-0.165* (-1.92)	12.674* (1.65)	57.239 (0.75)	-4.630*** (-2.70)	

Notes: The results only present the output equation of the VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add the GUNBP as the financial measure. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.9.4 The VAR(12) Specification for Traditional Simple Sum Model with Three Financial Measures Included

	Coefficients						
	WEI_gap	$RFFR$	Simple Sum	EPU	FSI	GUNBP	
Constant							3.115*** (5.40)
L1	0.019 (0.25)	1.294 (0.16)	106.890* (1.84)	-0.085** (-2.23)	-1.377 (-0.13)	3.762* (1.77)	
L2	-0.039 (-0.42)	3.356 (0.39)	-331.736 (-1.59)	-0.111*** (-3.38)	32.802 (0.93)	-12.784* (-1.79)	
L3	0.281*** (3.20)	5.417 (0.64)	470.341 (1.37)	-0.134*** (-4.27)	-108.746* (-1.95)	21.265* (1.88)	
L4	0.225** (2.14)	-10.624 (-1.11)	-310.001 (-0.93)	-0.180*** (-5.55)	146.205*** (2.71)	-12.007 (-1.07)	
L5	-0.112 (-1.08)	-2.660 (-0.28)	-98.697 (-0.28)	-0.138*** (-3.67)	-47.615 (-0.80)	-21.147* (-1.76)	
L6	0.001 (0.01)	-7.287 (-0.84)	383.333 (0.73)	-0.047 (-1.16)	-118.850 (-1.46)	49.375*** (2.90)	
L7	-0.082 (-0.90)	6.545 (0.70)	-370.861 (-0.69)	-0.141*** (-3.59)	186.721** (2.22)	-36.965** (-2.20)	
L8	0.162* (1.95)	14.849 (1.63)	113.608 (0.32)	-0.002 (-0.06)	-94.438 (-1.52)	-6.367 (-0.60)	
L9	0.066 (0.72)	3.778 (0.48)	181.162 (0.64)	-0.080** (-2.13)	-34.663 (-0.62)	38.038*** (3.51)	
L10	0.059 (0.69)	6.626 (0.90)	-350.976 (-1.13)	0.018 (0.48)	78.570 (1.35)	-33.646*** (-3.01)	
L11	-0.130* (-1.77)	-20.975** (-2.40)	333.893 (1.63)	0.009 (0.27)	-50.710 (-1.39)	13.113** (2.04)	
L12	-0.564*** (-8.53)	2.586 (0.37)	-137.174** (-2.12)	-0.031 (-0.86)	14.052 (1.31)	-1.521 (-0.89)	

Notes: The results only present the output equation of the VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add all three financial measures. L1 to L12 represents the first through the twelfth lags, respectively. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

6.2 MS-VAR Estimation Results

This section explores the non-linear interactions between the macroeconomic variables and financial conditions in the USA by assuming that all the series are regime-dependent. In our

MS-VAR framework, allowing for a different intercept in each regime and a dynamic structure across regimes could provide the best model for forecasting purposes. For the selection of optimal lags, a higher order of autoregressive parameters is introduced in our model such as 12, the forecasting performance deteriorates significantly, which may be explained by the in-sample over-fitting. Hence, we only report estimation and forecasting results for a two-regime MS-VAR(1), with a different intercept and the different dynamic structure in each regime. The lag length of 1 is the optimal lag for forecasting in our MS-VAR specifications.

6.2.1 Baseline Model

To be consistent with the linear VAR model setting, we start with the estimation of the dynamic structure of the MS-VAR model relating to output gap and the real interest rate. The estimated result of the output equation is presented in Table C.10. The point estimate of the intercept in the second regime is obviously much higher than in the first regime and both are statistically significant. The first lag of WEI_gap is positively significant in both regimes. The coefficient of the lagged change in real interest rate is negative in both regimes, but only significant in regime 2. This is in accordance with previous findings that the decrease in Federal Funds rate would stimulate the economy.

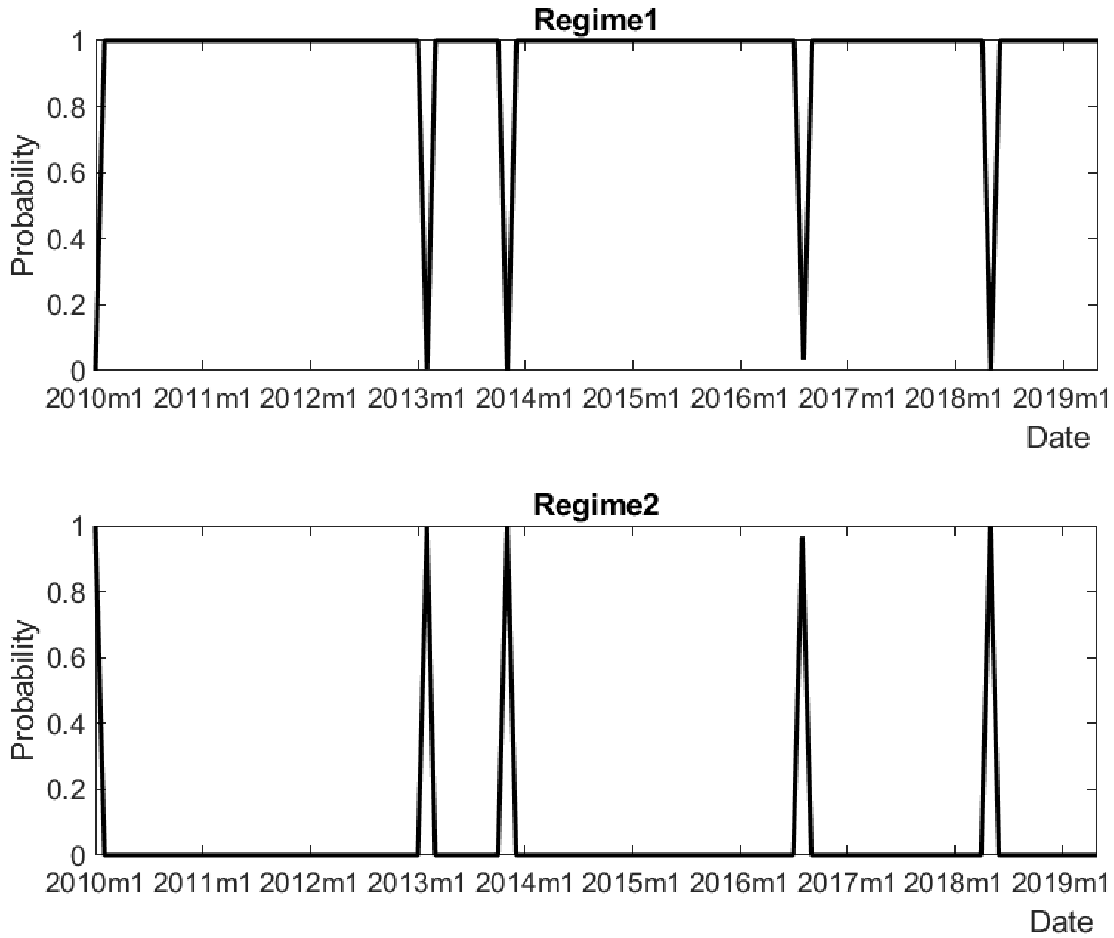
Table C.10 The MS-VAR(1) Specification for Baseline Model

	Coefficients		
	WEI_gap	$RFFR$	
Regime 1			
Constant			0.202***
			(2.69)
L1	0.889***	-0.189	
	(26.904)	(-0.669)	
Regime 2			
Constant			0.528***
			(4.33)
L1	1.015***	-0.867***	
	(242.27)	(-12.434)	

Notes: The results only present the output equation of the MS-VAR model. In this baseline model, we include the output gap and the real interest rate. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.963 and 0.000 for regime 1 and regime 2 respectively. The p_{jj} is the probability of remaining in regime j once there. The expected duration for each regime is determined by $(1 - p_{jj})^{-1}$. For the two state regime model, regime 1 is relatively persistent, with expected duration of about 27 months. Time-series plots of the filtered probabilities are also shown in Figure C.3. The attached probabilities to both regimes vary from zero to one. It can be seen that the regime 1 has dominated in our sample period, which captures the dynamic relation between the real interest rate and real economy.

Figure C.3 Filtered Probabilities of Regimes in MS-VAR Baseline Model



Notes: The plot shows the filtered probabilities of regimes in baseline model, which includes the output gap and the real interest rate in the model.

6.2.2 Green-Return Benchmarked Divisia Model

The estimated dynamic structure of the output equation for the green-return benchmarked Divisia model can be found in Table C.11.0. The constant for both regimes is significant and the value of the constant is higher in regime 2 than that in regime 1. The first lag of WEI_gap is significant in both regimes, while the coefficient of the lagged change in real interest rate is only significant in regime 2. This result is very similar to that found in the baseline model above. The significant green-return Divisia money term in both regimes supports the vital role of the money in aggregate demand, which is consistent with the linear VAR model estimated above (Table C.6.0).

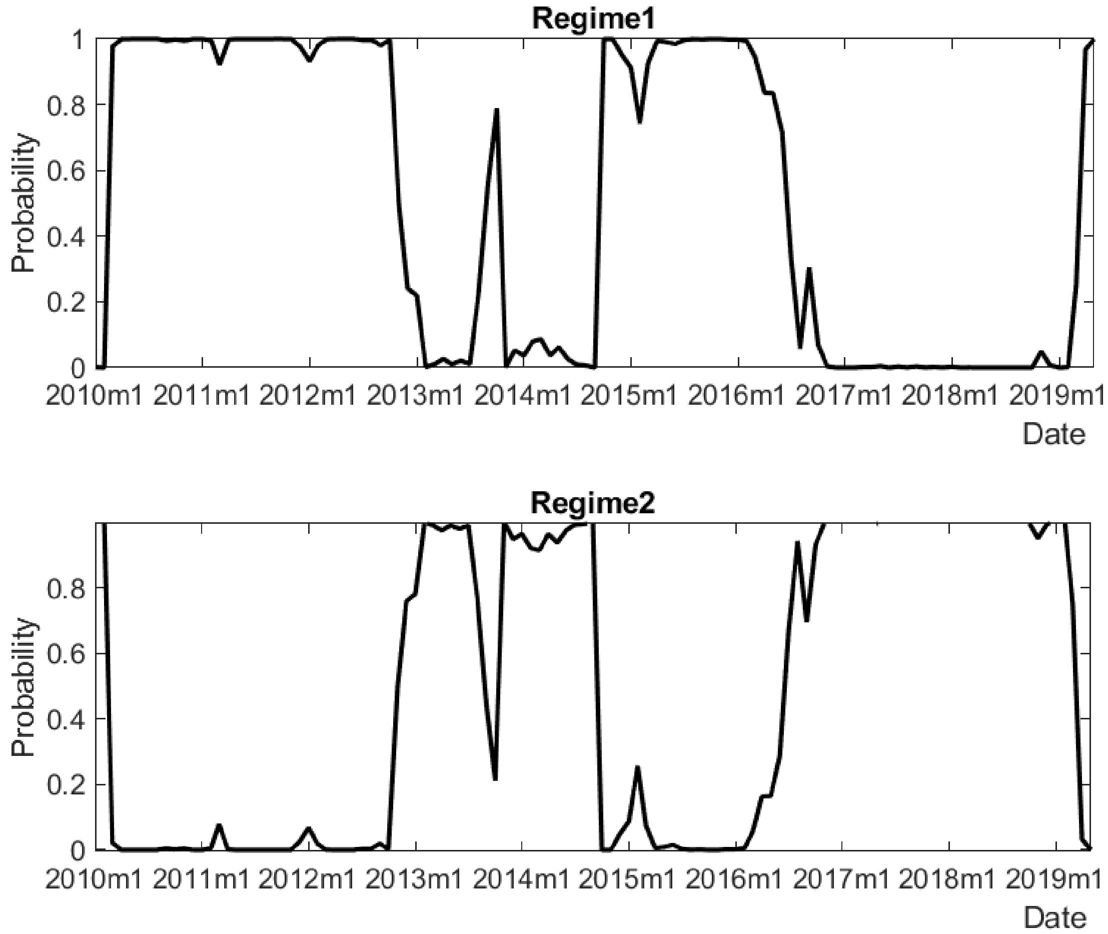
Table C.11.0 The MS-VAR(1) Specification for Green-Return Divisia Model

	Coefficients			
	WEI_gap	$RFFR$	Green-Return Divisia	
Regime 1				
Constant				0.296*** (3.33)
L1	0.827*** (19.87)	-0.050 (-0.126)	-2.056* (-1.50)	
Regime 2				
Constant				1.342*** (6.86)
L1	0.484*** (6.16)	0.693** (2.04)	-10.795*** (-6.11)	

Notes: The results only present the output equation of the MS-VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.956 and 0.944 for regime 1 and regime 2 respectively. The expected duration for each regime is about 23 months and 18 months, indicating the relative persistence of both regimes. Time-series plots of the filtered probabilities can be found in Figure C.4.0. Different from the baseline model, both regimes capture the changes in the real green-return benchmarked Divisia money, the real Federal Funds rate and the real economic activity.

Figure C.4.0 Filtered Probabilities of Regimes in Green-Return Divisia Model



Notes: The plot shows the filtered probabilities of regimes in green-return Divisia model, which includes the output gap, the real interest rate and the real green-return Divisia monetary aggregate in the model.

When the financial conditions considered, the estimated MS-VAR specifications with EPU, FSI, and GUNBP are illustrated in Table C.11.1, C.11.2 and C.11.3 respectively. The constant is significant in all cases except for the regime 1 with GUNBP measure, and the value is higher in regime 2 than that in regime 1. The lagged WEI_gap is significant in both regimes in all three specifications. The first lag of the change in real interest rate is insignificant in most cases when the financial measures are introduced in the model. The coefficient of the green-return benchmarked Divisia money is significant in all models except for the regime 1 with GUNBP measure, which indicating the better performance with EPU and FSI measures in our green-return benchmarked Divisia MS-VAR model than that GUNBP measure. We also found the insignificant values in three financial measures in the results. When three financial condition measures are incorporated, all coefficients except for that of the GUNBP in regime 2 are significant (see Table C.11.4).

Table C.11.1 The MS-VAR(1) Specification for Green-Return Divisia Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Return Divisia	EPU	
Regime 1					
Constant					0.331***
					(2.69)
L1	0.855***	0.332	-3.831**	-0.060	
	(14.56)	(0.73)	(-2.40)	(-1.55)	
Regime 2					
Constant					0.479***
					(3.32)
L1	0.756***	-0.096	-2.919**	0.036	
	(12.79)	(-0.34)	(-2.11)	(1.27)	

Notes: The results only present the output equation of the MS-VAR model. In this green-return benchmarked Divisa model, we include the output gap, real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the EPU as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.11.2 The MS-VAR(1) Specification for Green-Return Divisia Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Return Divisia	FSI	
Regime 1					
Constant					0.412***
					(2.83)
L1	0.840***	0.089	-3.017*	0.229	
	(17.60)	(0.25)	(-1.78)	(1.32)	
Regime 2					
Constant					2.084***
					(2.48)
L1	0.152**	3.553***	-17.761***	-0.062	
	(1.99)	(7.95)	(-9.47)	(-0.39)	

Notes: The results only present the output equation of the MS-VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the FSI as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.11.3 The MS-VAR(1) Specification for Green-Return Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Return Divisia	GUNBP	
Regime 1					
Constant					-0.122 (-0.89)
L1	1.125*** (16.06)	-1.487 (-1.48)	-0.465 (-0.32)	-0.184*** (-3.54)	
Regime 2					
Constant					0.489*** (4.08)
L1	0.758*** (14.83)	0.213 (0.71)	-4.298*** (-2.85)	-0.041 (-1.34)	

Notes: The results only present the output equation of the MS-VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

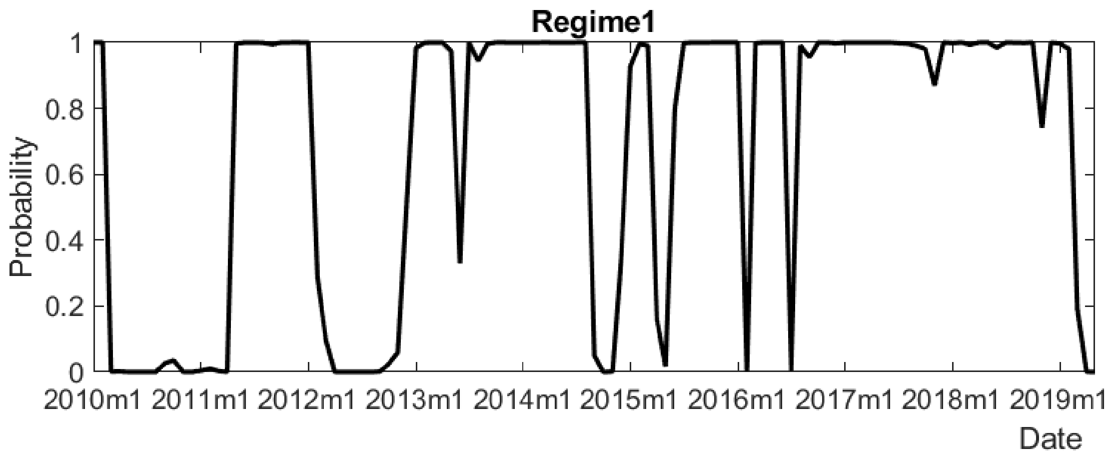
Table C.11.4 The MS-VAR(1) Specification for Green-Return Divisia Model with Three Financial Measures Included

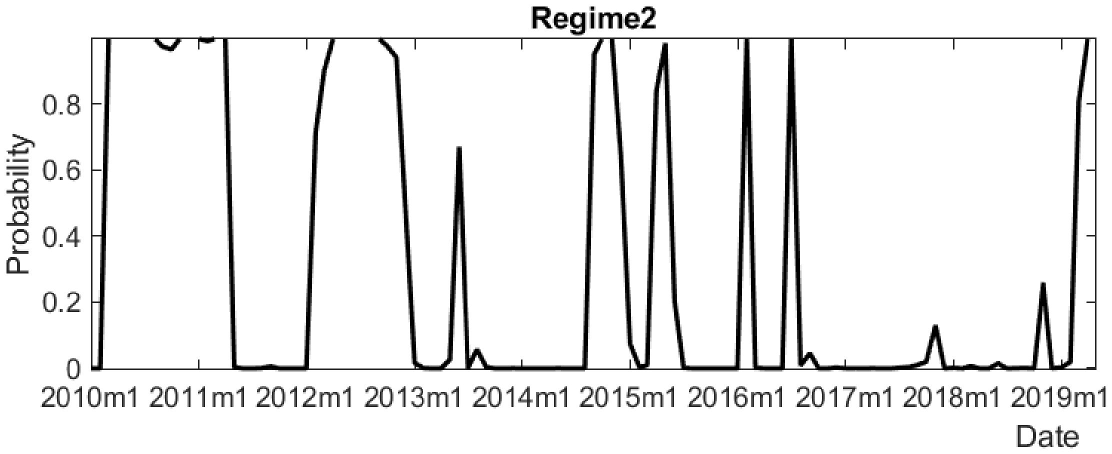
	Coefficients						
	WEI_gap	$RFFR$	Green-Return Divisia	EPU	FSI	GUNBP	
Regime 1							
Constant							1.469*** (5.83)
L1	0.691*** (12.63)	0.874** (2.26)	-11.198*** (-5.67)	0.108*** (2.66)	1.590*** (4.41)	0.189*** (2.69)	
Regime 2							
Constant							0.384*** (2.78)
L1	0.721*** (14.03)	2.127** (4.67)	-2.913** (-2.14)	-0.171*** (-5.09)	-0.342* (-1.677)	0.015 (0.34)	

Notes: The results only present the output equation of the MS-VAR model. In this green-return benchmarked Divisa model, we include the output gap, the real interest rate and the real green-return benchmarked Divisia monetary aggregate and also add all three financial measures. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.906 and 0.811 for regime 1 and regime 2 respectively with the EPU included. The expected duration for each regime is about 11 months and 6 months. For the model with the FSI measure, the estimated probabilities p_{jj} for remaining in each regime are 0.929 and 0.737 for regime 1 and regime 2 respectively, with the expected duration of about 14 months and 4 months. The estimated probabilities for the model with the GUNBP measure added are 0.621 and 0.888 for regime 1 and regime 2 respectively, and the corresponding expected duration is about 3 months and 9 months. These results suggest the less persistence in both regimes compared to the green-return benchmarked model without financial measures. If all three financial measures are involved simultaneously, the estimated probabilities for remaining in each regime are 0.885 and 0.836 for regime 1 and regime 2 respectively and the expected durations are about 9 months and 6 months. The plots of the filtered probabilities are also presented in Figure C.4.1, C.4.2, C.4.3 and C.4.4. As shown in Figure C.4.1, C.4.2 and C.4.4, regime 1 has dominated during our sample period, while regime 2 dominated in Figure C.4.3.

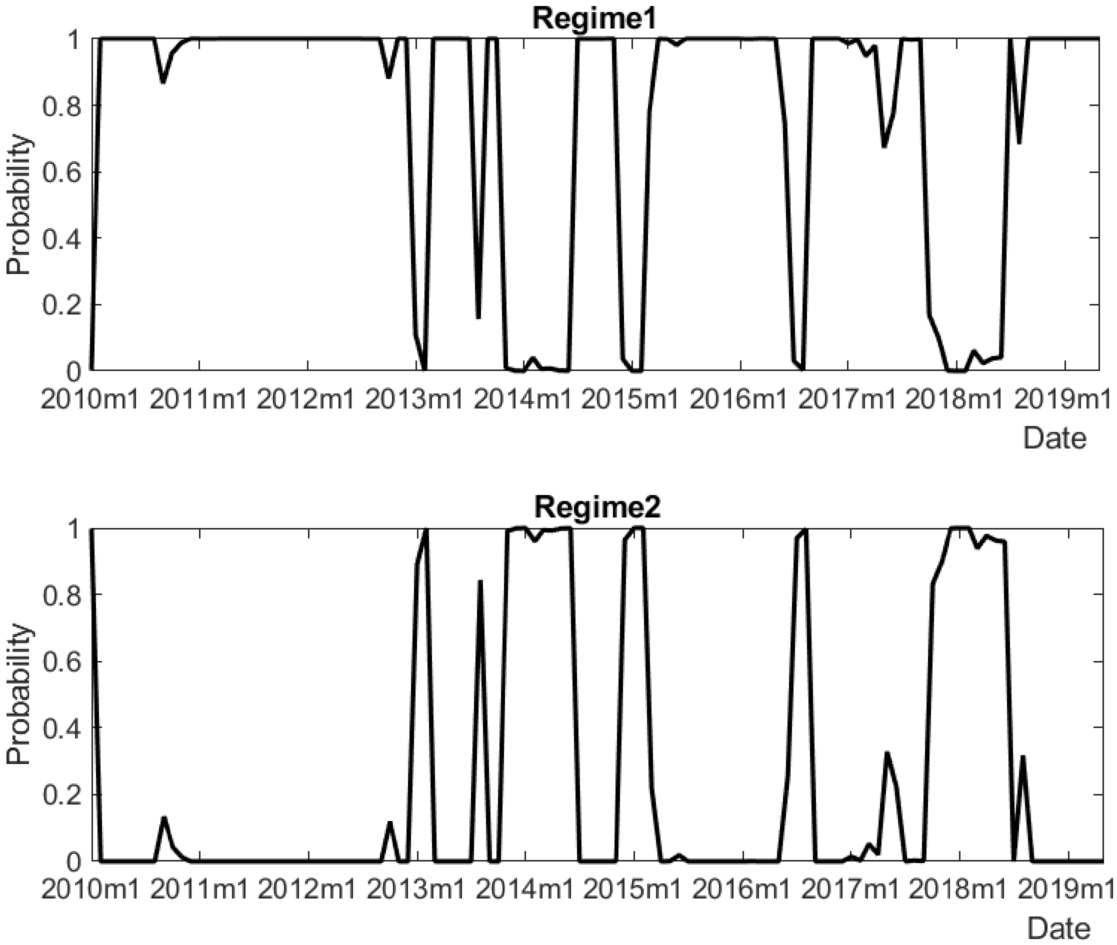
Figure C.4.1 Filtered Probabilities of Regimes in Green-Return Divisia Model with EPU Included





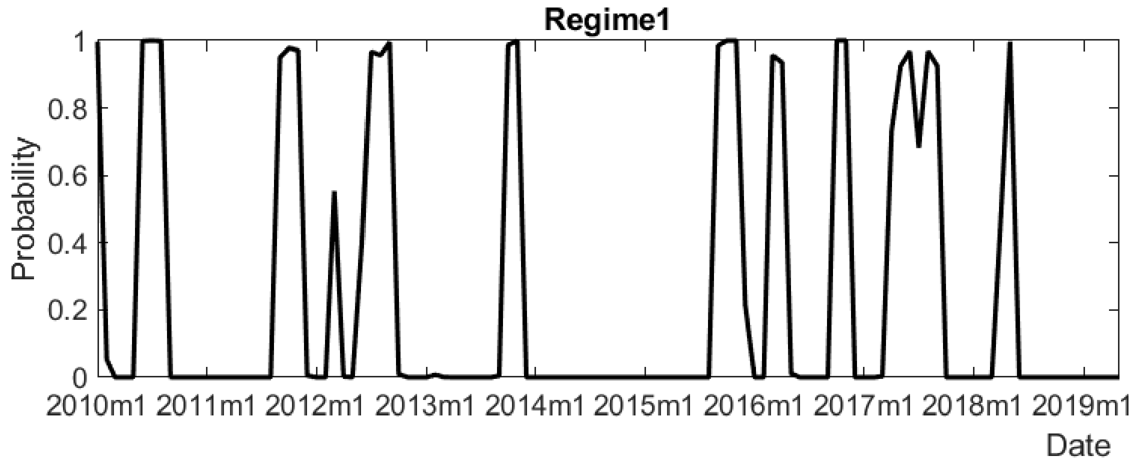
Notes: The plot shows the filtered probabilities of regimes in green-return Divisia model, which includes the output gap, the real interest rate, the real green-return Divisia monetary aggregate and the EPU in the model.

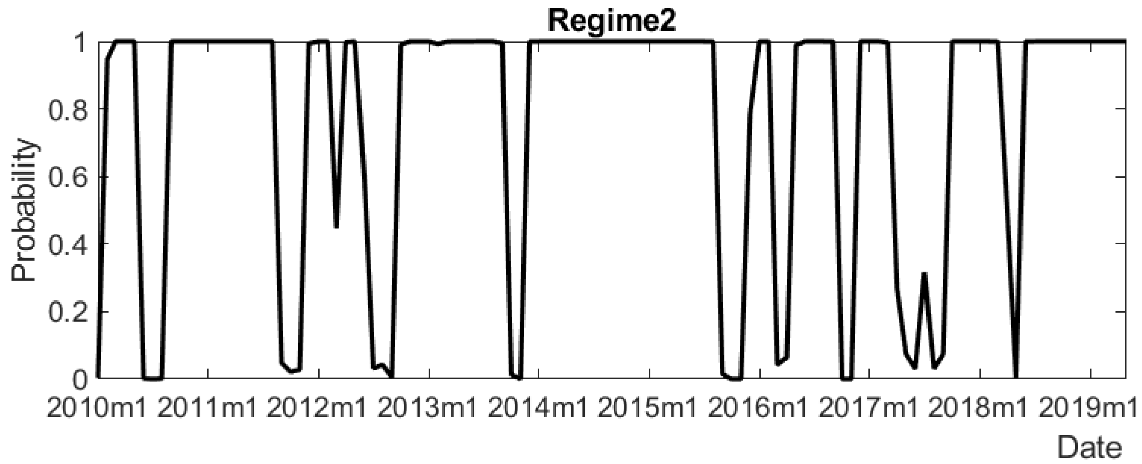
Figure C.4.2 Filtered Probabilities of Regimes in Green-Return Divisia Model with FSI Included



Notes: The plot shows the filtered probabilities of regimes in green-return Divisia model, which includes the output gap, the real interest rate, the real green-return Divisia monetary aggregate and the FSI in the model.

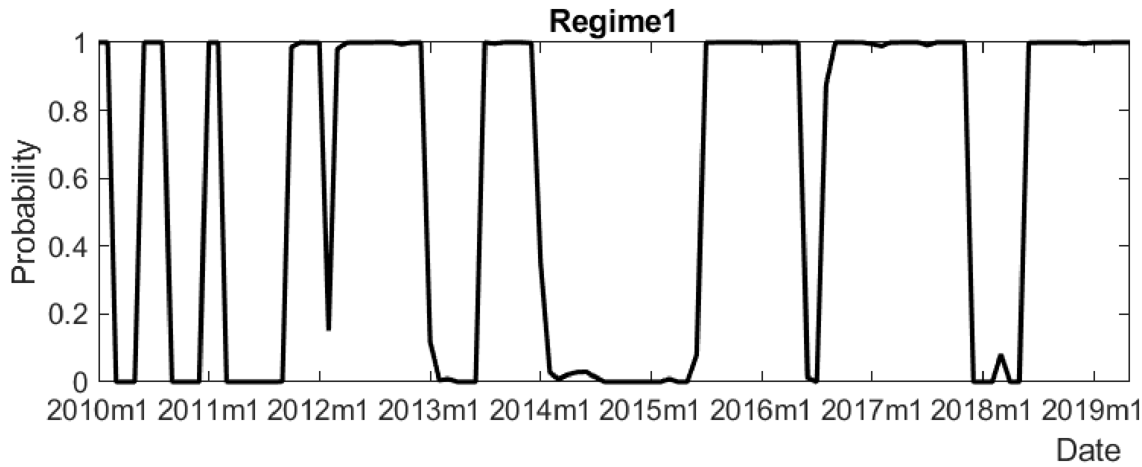
Figure C.4.3 Filtered Probabilities of Regimes in Green-Return Divisia Model with GUNBP Included

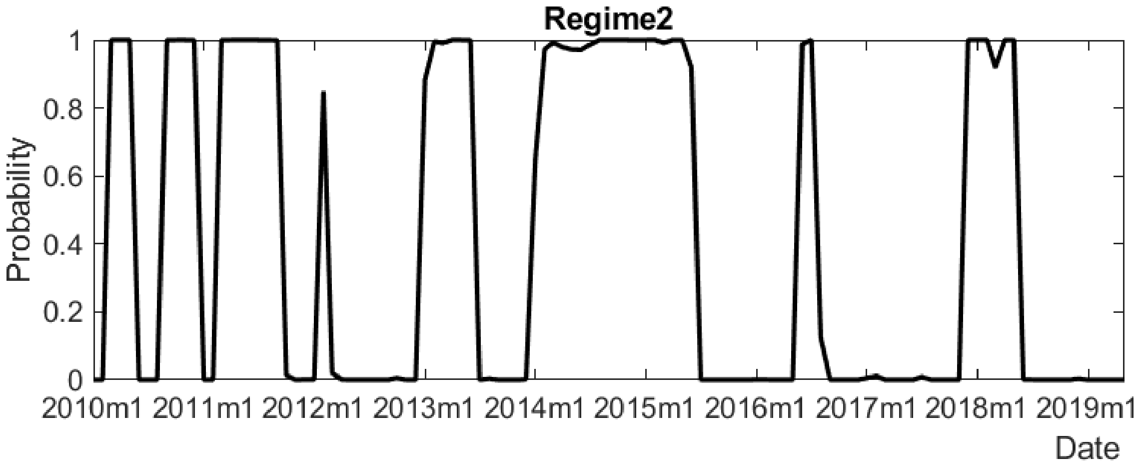




Notes: The plot shows the filtered probabilities of regimes in green-return Divisia model, which includes the output gap, the real interest rate, the real green-return Divisia monetary aggregate and the GUNBP in the model.

Figure C.4.4 Filtered Probabilities of Regimes in Green-Return Divisia Model with Three Financial Measures Included





Notes: The plot shows the filtered probabilities of regimes in green-return Divisia model, which includes the output gap, the real interest rate, the real green-return Divisia monetary aggregate and all three financial condition measures in the model.

6.2.3 Green-Coupon Benchmarked Divisia Model

The estimated results of the output equation for the green-coupon benchmarked Divisia model are demonstrated in Table C.12.0. The constant for both regimes is significant and the value of the constant is higher in regime 2 than that in regime 1. The lagged WEI_gap is significant in both regimes, while the coefficient of the lagged change in real interest rate is only significant in regime 2. These results are similar to the one obtained from the green-coupon benchmarked Divisia model in Table C.11.0. Unlike the green-return Divisia money, which is significant in both regimes, the green-coupon Divisia money term is only significant in regime 2, suggesting the superior performance of the green-return Divisia money in informing the aggregate demand.

Table C.12.0The MS-VAR(1) Specification for Green-Coupon Divisia Model

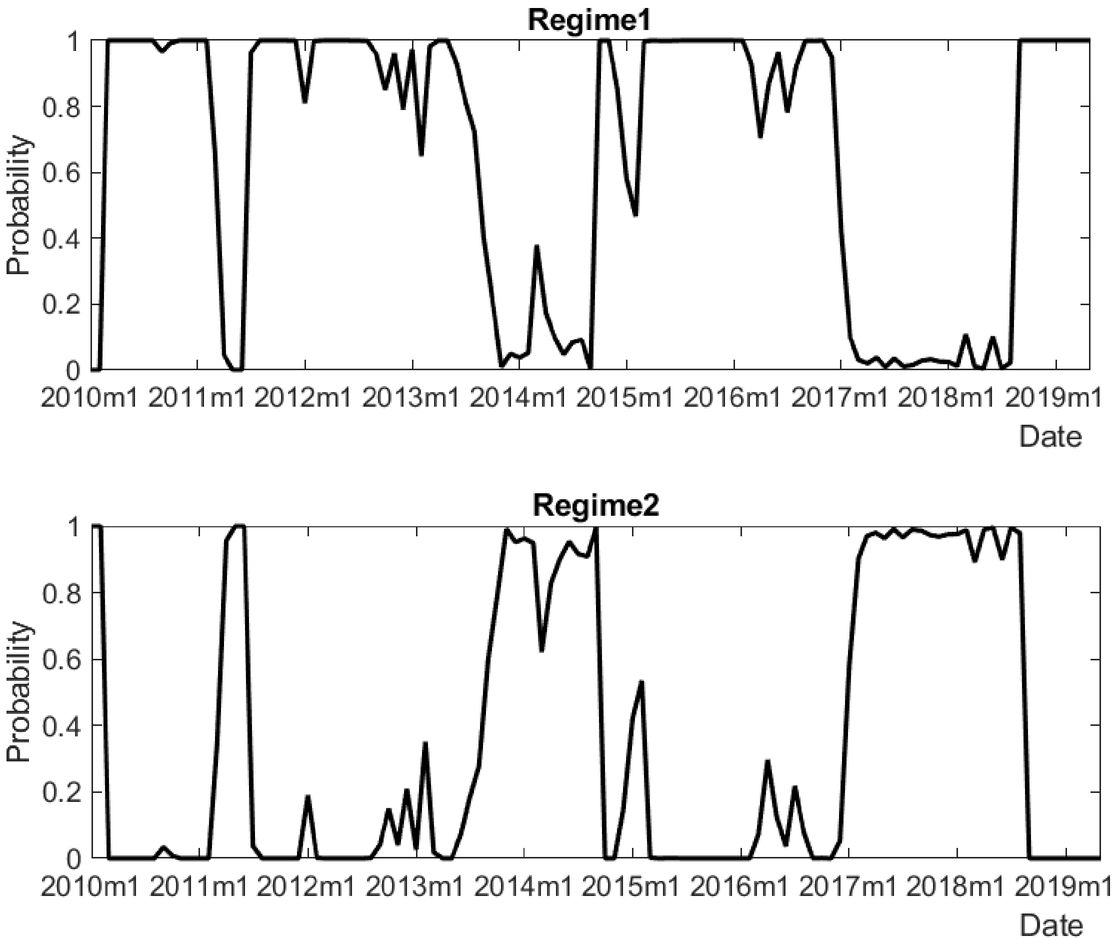
	Coefficients			
	WEI_gap	$RFFR$	Green-Coupon Divisia	
Regime 1				
Constant				0.325*** (2.93)
L1	0.829*** (16.69)	-0.226 (-0.700)	-1.222 (-0.99)	
Regime 2				
Constant				0.943*** (5.49)
L1	0.606***	1.989**	-8.900***	

	(8.37)	(4.09)	(-5.67)

Notes: The results only present the output equation of the MS-VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.949 and 0.890 for regime 1 and regime 2 respectively. The expected duration for each regime is about 20 months and 9 months. Both are less persistent compared with the expected duration when the green-return benchmarked Divisia monetary aggregate is used. Time-series plots of the filtered probabilities are also presented in Figure C.5.0. As can be seen, the regime 1 has dominated during the estimation period.

Figure C.5.0 Filtered Probabilities of Regimes in Green-Coupon Divisia Model



Notes: The plot shows the filtered probabilities of regimes in green-coupon Divisia model, which includes the output gap, the real interest rate and the real green-coupon Divisia monetary aggregate in the model.

The estimates of the extended green-coupon benchmarked Divisia MS-VAR specifications with EPU, FSI, and GUNBP are shown in Table C.12.1, C.12.2 and C.12.3 respectively. The estimate of the intercept is significant in all models, and the value is higher in regime 2 than that in regime 1 expect for the model with the GUNBP measure. The first lag of the WEI_gap is significant in both regimes, while the lagged change in real interest rate is only significant in regime 2 in all three specifications. For the green-coupon benchmarked Divisia money, the point estimate is only significant in regime 1 with the EPU and FSI measures and significant in regime 2 with the GUNBP measure. We also notice that the GUNBP is significant in both regimes, but the EPU and FSI are significant in regime 2. When we include all three measures in a MS-VAR model, all coefficient, except for that of the FSI in regime 1 and that of the green-coupon benchmarked Divisia monetary aggregate and EPU in regime 2, are significant (see Table C.12.4). All the results indicate that the green-return Divisia model fits better than the green-coupon Divisia model.

Table C.12.1 The MS-VAR(1) Specification for Green-Coupon Divisia Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Coupon Divisia	EPU	
Regime 1					
Constant					0.358***
					(2.81)
L1	0.824***	0.145	-1.815	-0.008	
	(14.91)	(0.32)	(-1.17)	(-0.24)	
Regime 2					
Constant					0.562***
					(4.16)
L1	0.747***	0.641**	-6.513***	-0.144***	
	(12.42)	(2.23)	(-4.78)	(-4.54)	

Notes: The results only present the output equation of the MS-VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the EPU as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.12.2 The MS-VAR(1) Specification for Green-Coupon Divisia Model with FSI Included

Coefficients

	WEI_gap	$RFFR$	Green-Coupon Divisia	FSI	
Regime 1					
Constant					0.528***
					(3.41)
L1	0.812***	-0.058	-2.338	0.206	
	(14.66)	(-0.11)	(-1.39)	(0.97)	
Regime 2					
Constant					0.716***
					(5.14)
L1	0.815***	1.599***	-12.793***	0.796***	
	(18.02)	(5.84)	(-7.46)	(6.23)	

Notes: The results only present the output equation of the MS-VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the FSI as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.12.3 The MS-VAR(1) Specification for Green-Coupon Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Green-Coupon Divisia	GUNBP	
Regime 1					
Constant					0.424***
					(3.36)
L1	0.814***	0.101	-5.418***	-0.107***	
	(14.65)	(0.31)	(-3.38)	(-3.09)	
Regime 2					
Constant					0.311***
					(2.81)
L1	0.819***	1.117**	0.251	0.110***	
	(16.82)	(2.48)	(0.19)	(3.12)	

Notes: The results only present the output equation of the MS-VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.12.4 The MS-VAR(1) Specification for Green-Coupon Divisia Model with Three Financial Measures Included

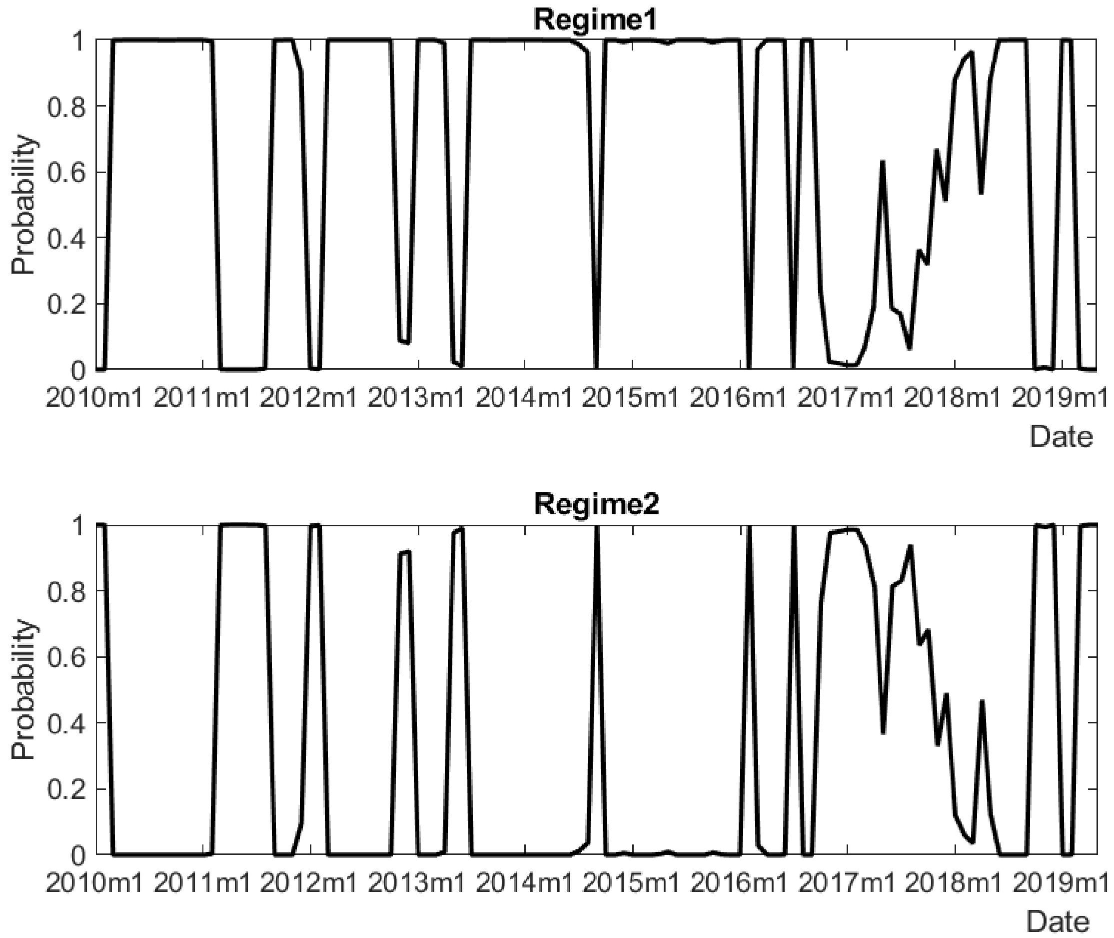
	Coefficients					
	WEI_gap	$RFFR$	Green-Coupon Divisia	EPU	FSI	GUNBP
Regime 1						

Constant							0.482***
							(2.83)
L1	0.788***	0.642**	-3.517**	-0.095***	0.292	0.095**	
	(18.50)	(2.31)	(-2.42)	(-2.61)	(1.22)	(1.99)	
Regime 2							
Constant							-0.429*
							(-1.97)
L1	0.987***	-1.15**	3.131	-0.043	-1.326***	-0.295***	
	(18.15)	(-2.29)	(1.51)	(-1.18)	(-1.68)	(0.34)	

Notes: The results only present the output equation of the MS-VAR model. In this green-coupon benchmarked Divisa model, we include the output gap, the real interest rate and the real green-coupon benchmarked Divisia monetary aggregate and also add all three financial measures. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

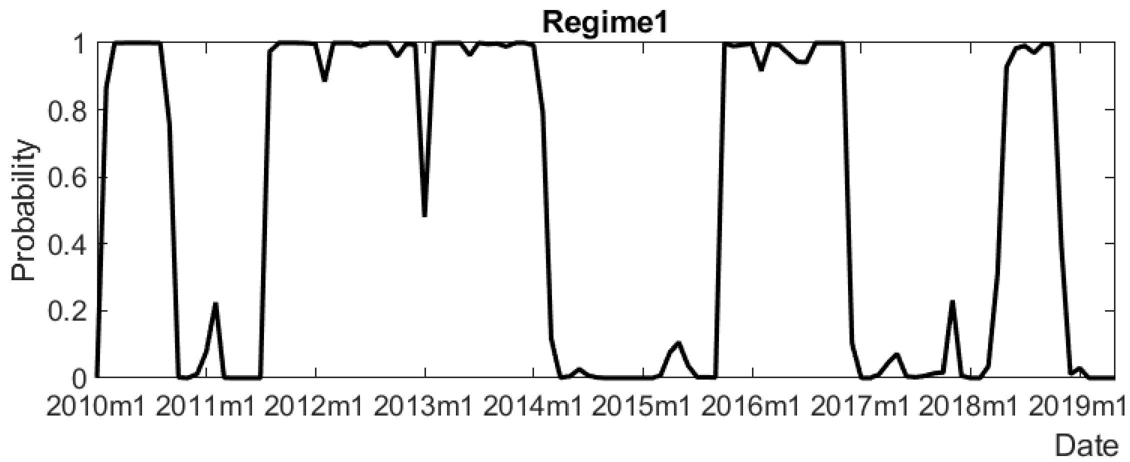
The estimated probabilities p_{jj} for remaining in each regime are 0.860 and 0.690 for regime 1 and regime 2 respectively with the EPU included. The expected duration for each regime is about 7 months and 3 months. For the model with the FSI measure, the estimated probabilities p_{jj} for remaining in each regime are 0.928 and 0.921 for regime 1 and regime 2 respectively, with the expected duration of about 14 months and 13 months. The estimated probabilities for the model with the GUNBP measure added are 0.840 and 0.771 for regime 1 and regime 2 respectively, and the corresponding expected duration is about 6 months and 4 months. These results indicate that both regimes are relatively more persistent in the model with the FSI than that with the GUNBP or the EPU measure. With all three financial measures considered at the same time, the estimated probabilities for remaining in each regime are 0.787 and 0.621 for regime 1 and regime 2 respectively and the expected durations are about 5 months and 3 months, which are less persistent than that in the green-return benchmarked Divisia model. The plots of the filtered probabilities are also given in Figure C.5.1, C.5.2, C.5.3 and C.5.4. All the figures show that the sample period is dominated by the regime 1.

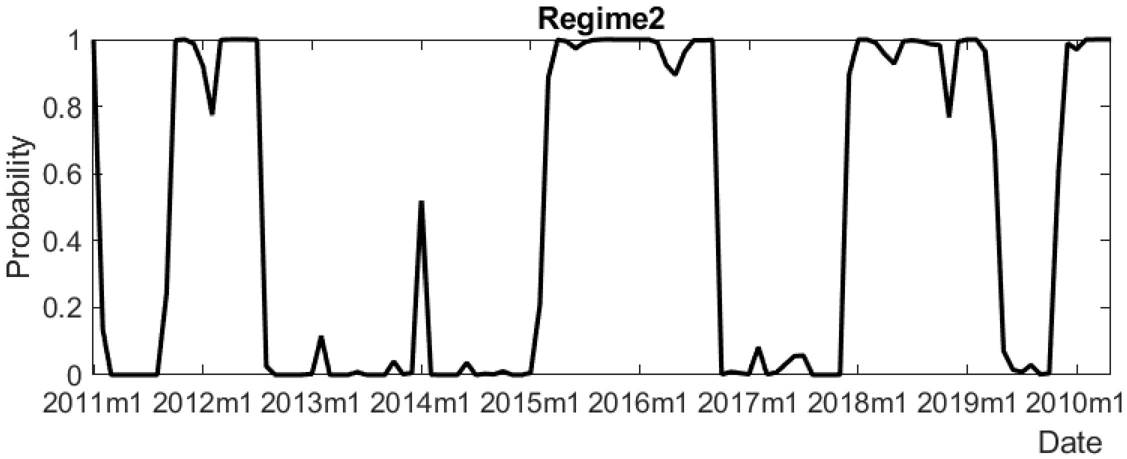
Figure C.5.1 Filtered Probabilities of Regimes in Green-Coupon Divisia Model with EPU Included



Notes: The plot shows the filtered probabilities of regimes in green-coupon Divisia model, which includes the output gap, the real interest rate, the real green-coupon Divisia monetary aggregate and the EPU in the model.

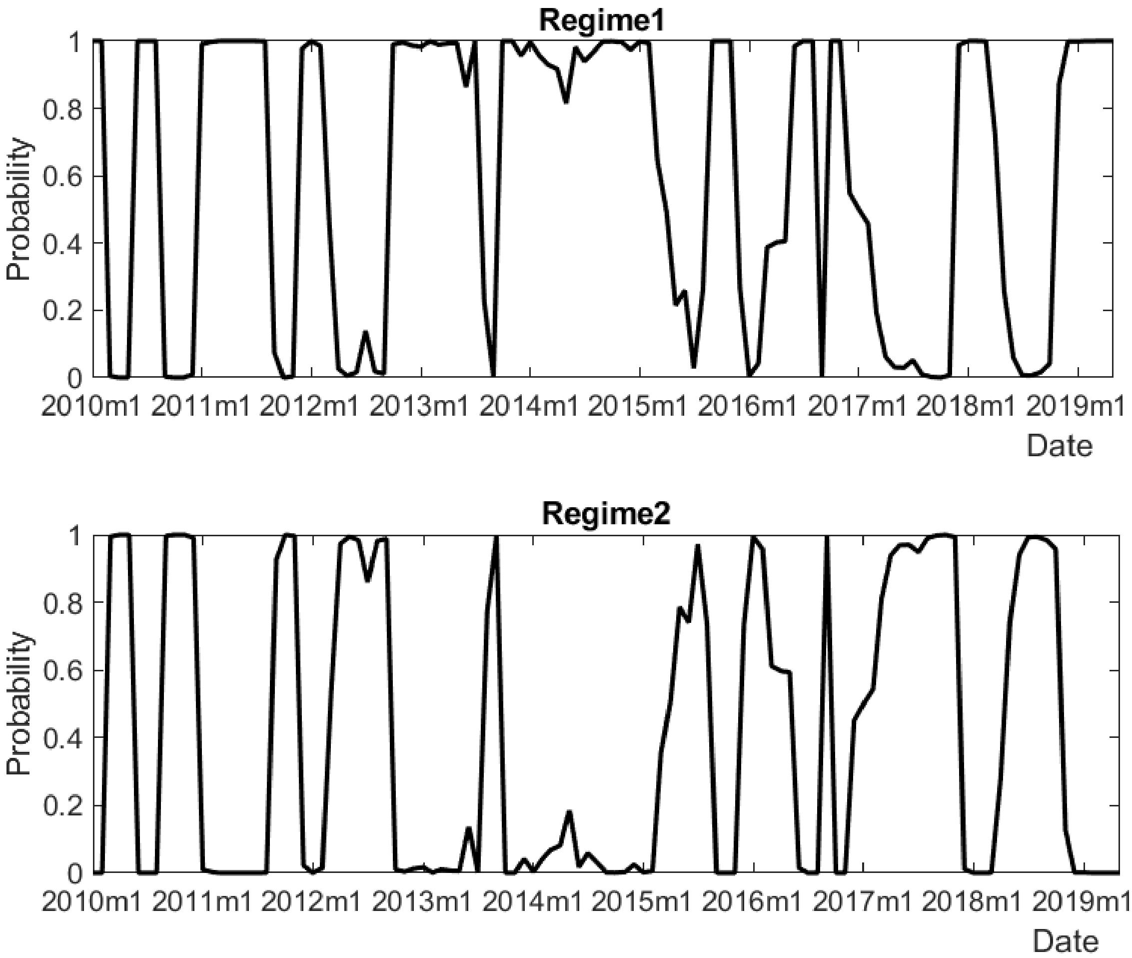
Figure C.5.2 Filtered Probabilities of Regimes in Green-Coupon Divisia Model with FSI Included





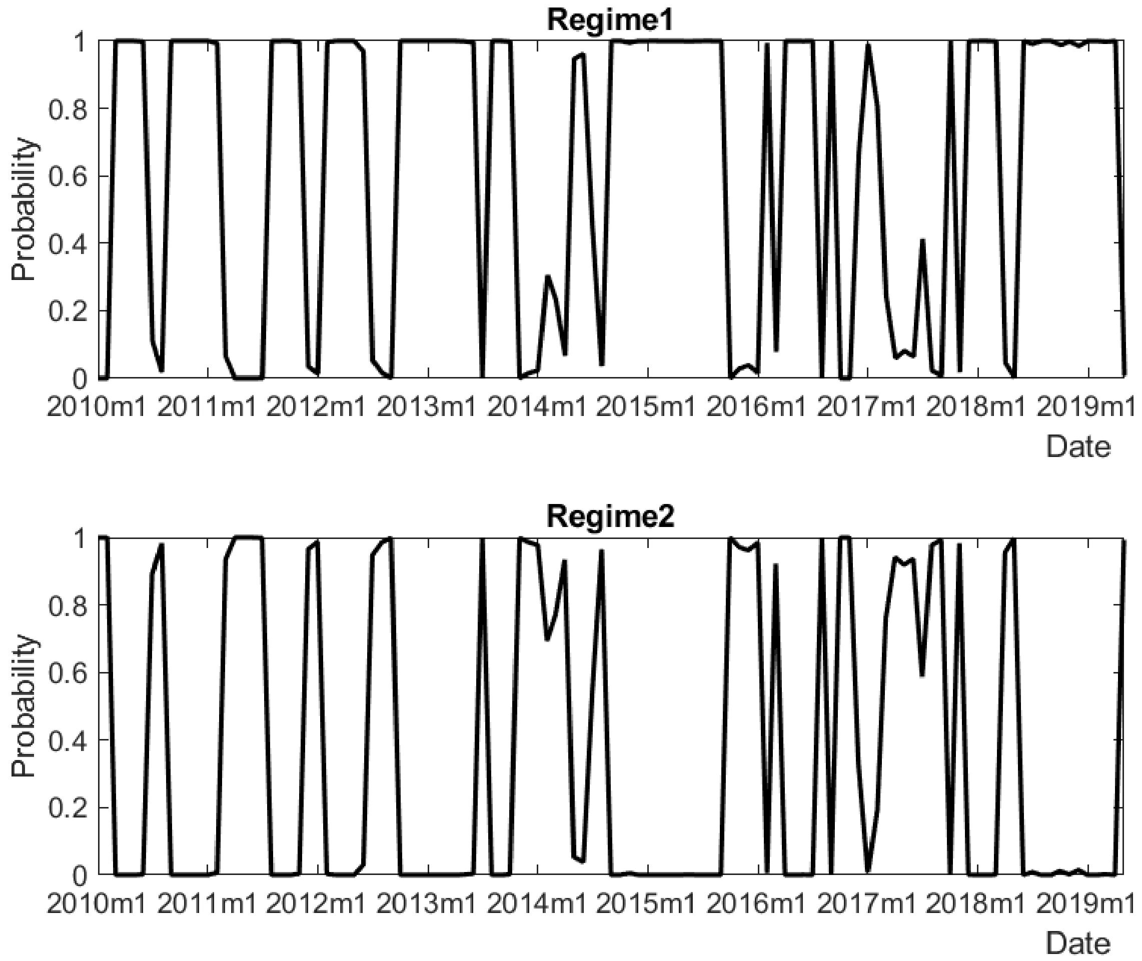
Notes: The plot shows the filtered probabilities of regimes in green-coupon Divisia model, which includes the output gap, the real interest rate, the real green-coupon Divisia monetary aggregate and the FSI in the model.

Figure C.5.3 Filtered Probabilities of Regimes in Green-Coupon Divisia Model with GUNBP Included



Notes: The plot shows the filtered probabilities of regimes in green-coupon Divisia model, which includes the output gap, the real interest rate, the real green-coupon Divisia monetary aggregate and the GUNBP in the model.

Figure C.5.4 Filtered Probabilities of Regimes in Green-Coupon Divisia Model with Three Financial Measures Included



Notes: The plot shows the filtered probabilities of regimes in green-coupon Divisia model, which includes the output gap, the real interest rate, the real green-coupon Divisia monetary aggregate and all three financial condition measures in the model.

6.2.4 Conventional Divisia Model

To compare with the green benchmarked Divisia monetary aggregate, the conventional Divisia monetary aggregate using identical components is examined in the MS-VAR model. The estimated output equation for the conventional Divisia model is displayed in Table C.13.0. The significance of the coefficients is consistent with that in the green-coupon benchmarked Divisia model. The constant for both regimes is significant, and the value of the constant is much higher in regime 2 than that in regime 1. The lagged WEI_gap is significant in both regimes, while the coefficient of the lagged change in real interest rate is only significant in regime 2. The conventional Divisia money term is only significant in regime 2, implying additional information provided by the conventional Divisia money for economy.

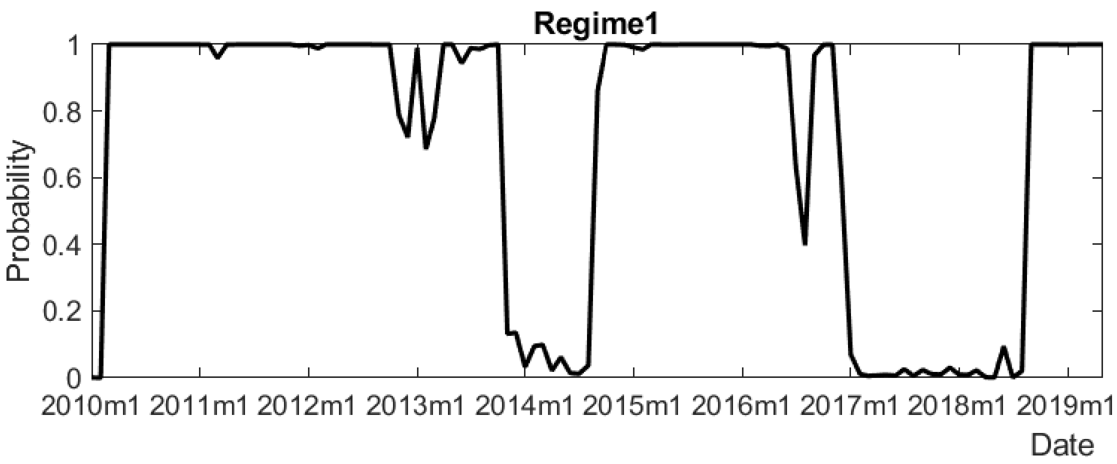
Table C.13.0 The MS-VAR(1) Specification for Conventional Divisia Model

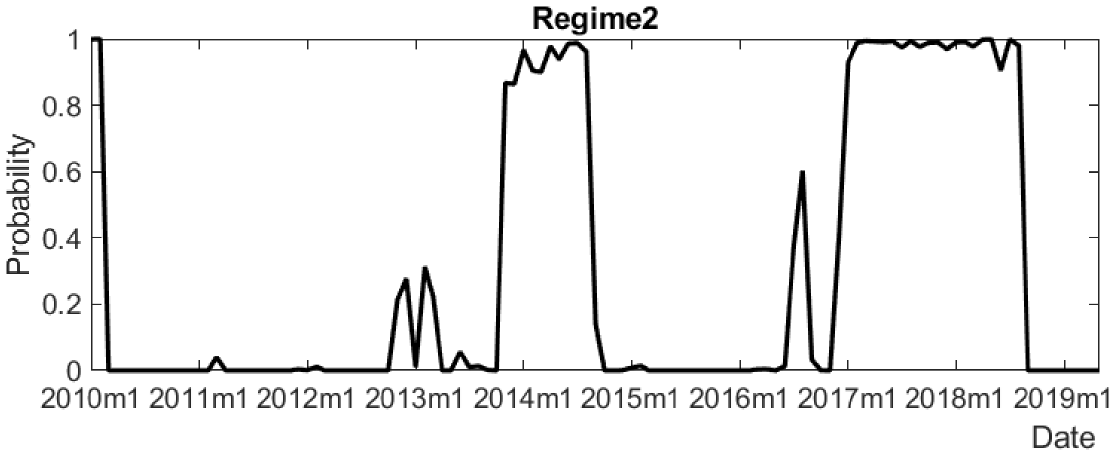
	Coefficients			
	WEI_gap	$RFFR$	Conventional Divisia	
Regime 1				
Constant				0.389***
				(3.16)
L1	0.802***	-0.287	-1.735	
	(15.41)	(-0.928)	(-0.09)	
Regime 2				
Constant				1.952***
				(10.13)
L1	0.250***	0.980***	-13.369***	
	(3.25)	(3.35)	(-9.79)	

Notes: The results only present the output equation of the MS-VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.970 and 0.898 for regime 1 and regime 2 respectively. The expected duration for each regime is about 33 months and 10 months, which indicates a higher persistence in regime 1. The plots of the filtered probabilities are also presented in Figure C.6.0, which have shown our sample period is dominated by the regime 1.

Figure C.6.0 Filtered Probabilities of Regimes in Conventional Divisia Model





Notes: The plot shows the filtered probabilities of regimes in conventional Divisia model, which includes the output gap, the real interest rate and the real conventional Divisia monetary aggregate in the model.

Table C.12.1, C.12.2 and C.12.3 show the estimated results of the conventional Divisia MS-VAR models added with EPU, FSI, and GUNBP as the financial condition indicator respectively. The constant is significant in all specifications, and the value is higher in regime 2 than that in regime 1 expect for the model with the GUNBP measure. This aligns with the findings from the green-coupon benchmarked Divisia model. The lagged WEI_gap is significant in both regimes, while the lagged change in real interest rate is only significant in regime 2 in the models with the FSI and GUNBP measures. The conventional Divisia monetary aggregate is only significant in regime 1 with the GUNBP measure and in regime 2 with the EPU and FSI measures. For the financial measures, the FSI is significant in both regimes, but the EPU and GUNBP are significant in regime 2. When we include all three measures in a MS-VAR model, all variables, except for the real Federal Funds rate and FSI in regime 1 and the conventional Divisia monetary aggregate and the constant in regime 2, are significant (see Table C.13.4). All the results suggest that the conventional Divisia monetary aggregate does add information in aggregate demand.

Table C.13.1 The MS-VAR(1) Specification for Conventional Divisia Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Conventional Divisia	EPU	
Regime 1					
Constant					0.313**

					(2.52)
L1	0.844*** (16.14)	0.096 (0.29)	-0.950 (-0.68)	0.010 (0.325)	
Regime 2					
Constant					1.653*** (14.08)
L1	0.295*** (6.12)	0.203 (1.17)	-15.394*** (-16.15)	-0.346*** (-13.78)	

Notes: The results only present the output equation of the MS-VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the EPU as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.13.2 The MS-VAR(1) Specification for Conventional Divisia Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Conventional Divisia	FSI	
Regime 1					
Constant					0.425*** (2.62)
L1	0.840*** (15.34)	-0.223 (-0.77)	-2.124 (-1.25)	0.246** (1.69)	
Regime 2					
Constant					`1.073*** (7.18)
L1	0.633*** (11.53)	2.028*** (3.57)	-8.458*** (-5.23)	0.363** (2.20)	

Notes: The results only present the output equation of the MS-VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the FSI as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.13.3 The MS-VAR(1) Specification for Conventional Divisia Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Conventional Divisia	GUNBP	
Regime 1					
Constant					0.384*** (2.77)
L1	0.829*** (14.18)	0.058 (0.18)	-3.276** (-2.20)	-0.017 (-0.50)	
Regime 2					
Constant					0.205** (2.07)
L1	0.919***	-2.177***	-0.543	-0.173***	

	(23.61)	(-7.16)	(-0.43)	(-5.49)

Notes: The results only present the output equation of the MS-VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add the GUNBP as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

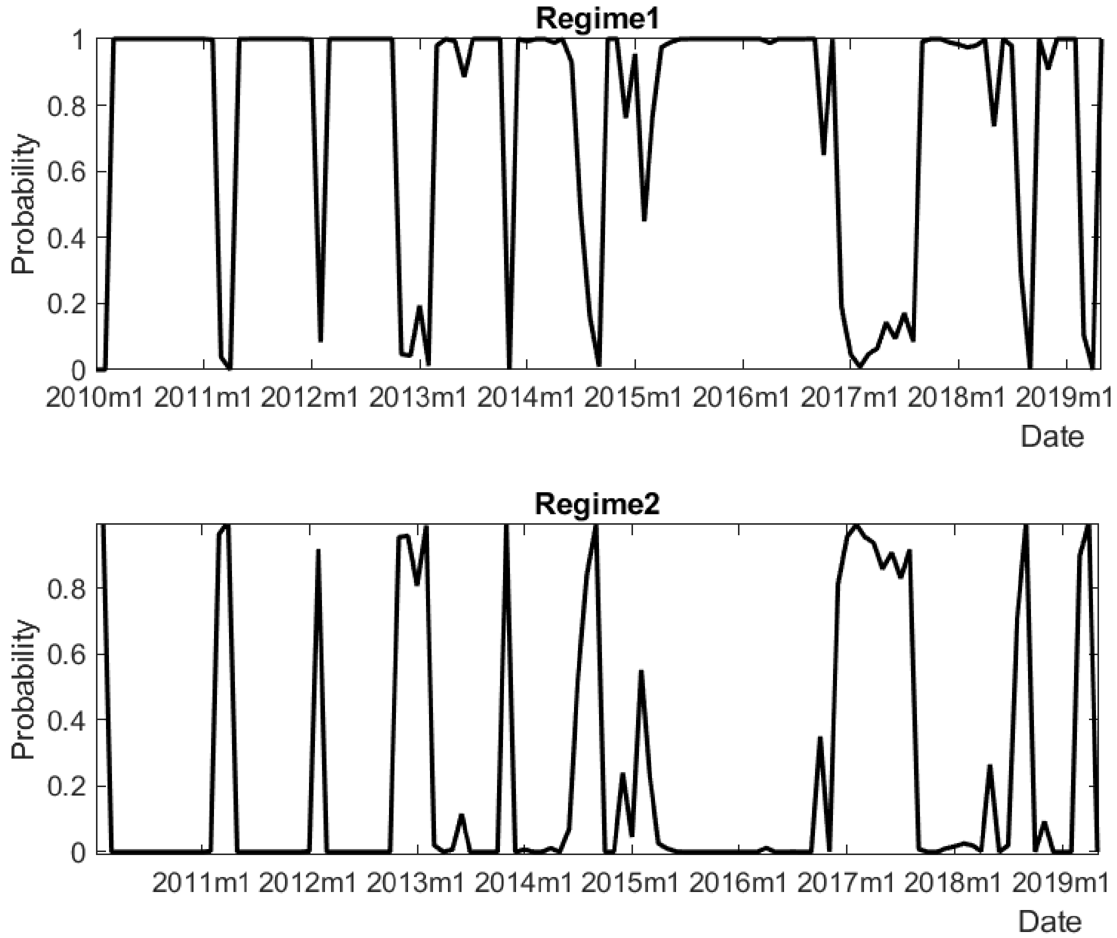
Table C.13.4 The MS-VAR(1) Specification for Conventional Divisia Model with Three Financial Measures Included

	Coefficients						
	WEI_gap	$RFFR$	Conventional Divisia	EPU	FSI	GUNBP	
Regime 1							
Constant							0.419** (2.31)
L1	0.808*** (16.52)	0.220 (0.87)	-2.821** (-2.07)	-0.098** (-2.57)	0.203 (0.91)	0.092** (1.99)	
Regime 2							
Constant							-0.362 (-1.53)
L1	0.997*** (21.17)	-1.498*** (-4.45)	2.442 (1.35)	-0.068** (-2.24)	-1.179*** (-3.15)	-0.312*** (-4.61)	

Notes: The results only present the output equation of the MS-VAR model. In this conventional Divisa model, we include the output gap, the real interest rate and the real conventional Divisia monetary aggregate and also add all three financial measures. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

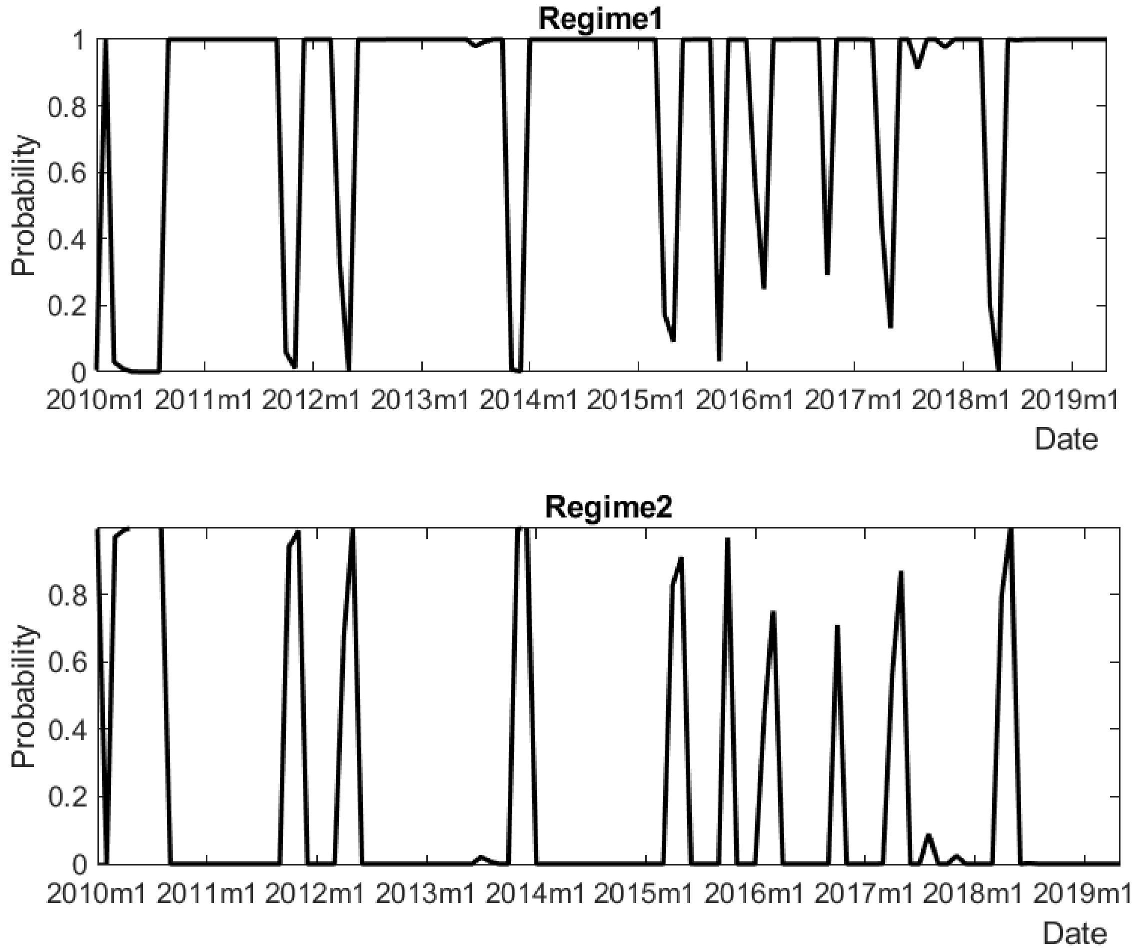
The estimated probabilities p_{jj} for remaining in each regime are 0.895 and 0.627 for regime 1 and regime 2 respectively with the EPU included. The expected duration for each regime is about 10 months and 3 months. For the model with the FSI measure, the estimated probabilities p_{jj} for remaining in each regime are 0.899 and 0.515 for regime 1 and regime 2 respectively, with the expected duration of about 10 months and 2 months. The estimated probabilities for the model with the GUNBP measure added are 0.927 and 0.558 for regime 1 and regime 2 respectively, and the corresponding expected duration is about 14 months and 2 months. These results indicate that regime 1 is more durable than regime 2 in all three cases. In a conventional Divisia MS-VAR model with all three financial measures included, the estimated probabilities for remaining in each regime are 0.773 and 0.537 for regime 1 and regime 2 respectively and the expected durations are about 4 months and 2 months. The plots of the filtered probabilities are also given in Figure C.6.1, C.6.2, C.6.3 and C.6.4, which shows regime 1 has dominated in the sample period.

Figure C.6.1 Filtered Probabilities of Regimes in Conventional Divisia Model with EPU Included



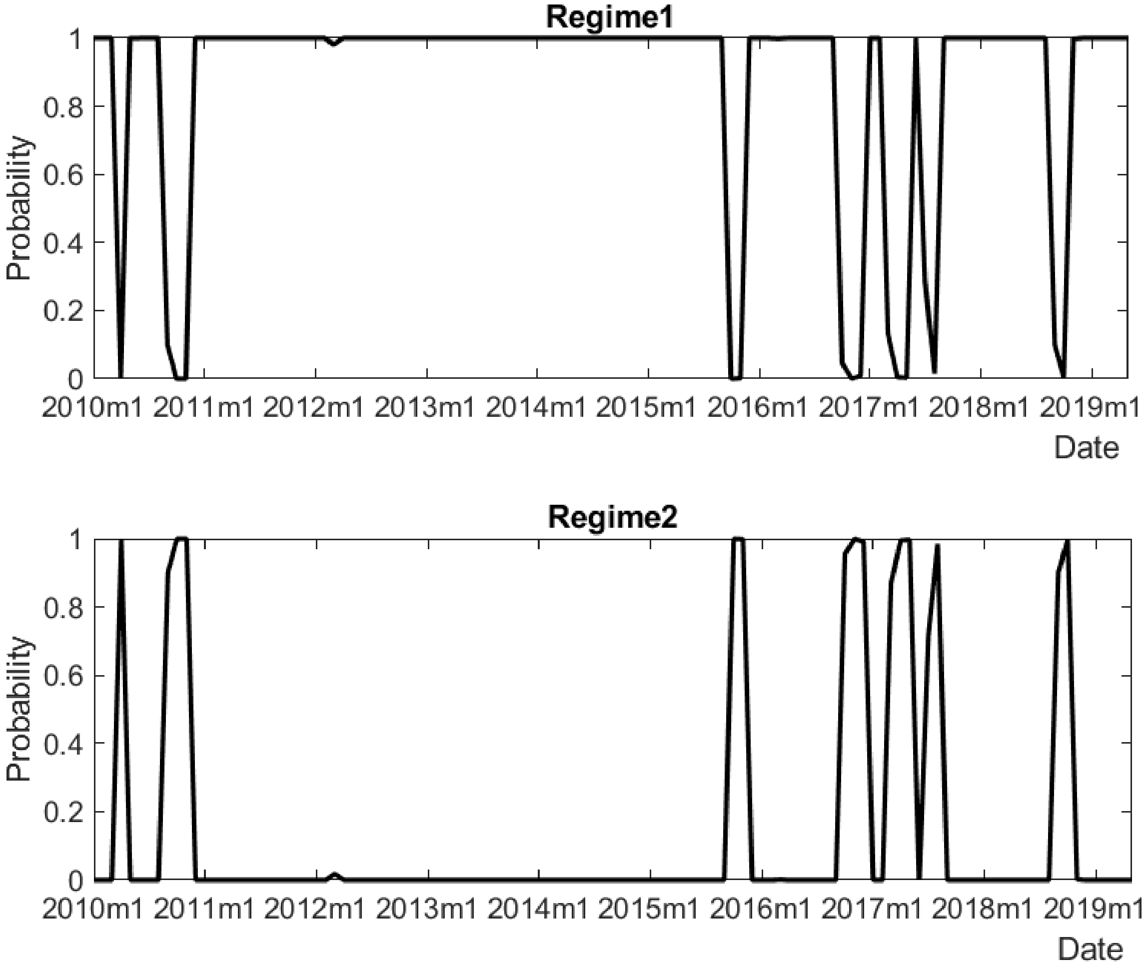
Notes: The plot shows the filtered probabilities of regimes in conventional Divisia model, which includes the output gap, the real interest rate, the real conventional Divisia monetary aggregate and the EPU in the model.

Figure C.6.2 Filtered Probabilities of Regimes in Conventional Divisia Model with FSI Included



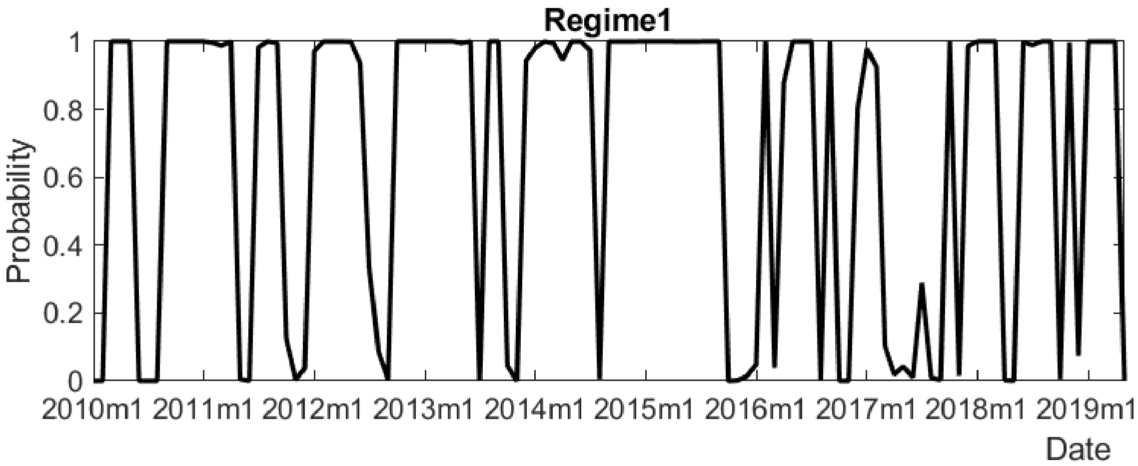
Notes: The plot shows the filtered probabilities of regimes in conventional Divisia model, which includes the output gap, the real interest rate, the real conventional Divisia monetary aggregate and the FSI in the model.

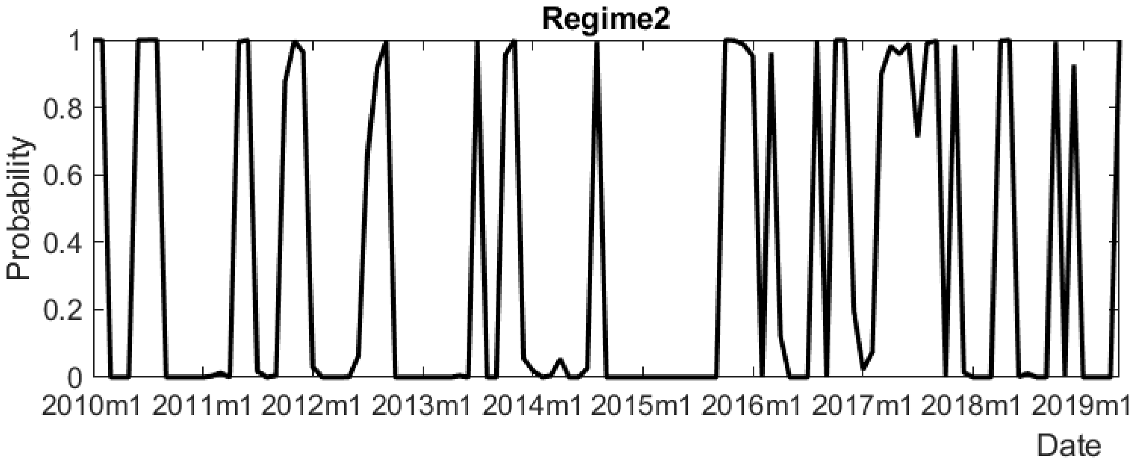
Figure C.6.3 Filtered Probabilities of Regimes in Conventional Divisia Model with GUNBP Included



Notes: The plot shows the filtered probabilities of regimes in conventional Divisia model, which includes the output gap, the real interest rate, the real conventional Divisia monetary aggregate and the GUNBP in the model.

Figure C.6.4 Filtered Probabilities of Regimes in Conventional Divisia Model with Three Financial Measures Included





Notes: The plot shows the filtered probabilities of regimes in conventional Divisia model, which includes the output gap, the real interest rate, the real conventional Divisia monetary aggregate and all three financial condition measures in the model.

6.2.5 Traditional Simple Sum Model

For comparison, we replace the Divisia money measure with the traditional simple sum money counterpart in the MS-VAR model and the estimated output equation for the traditional simple sum model is presented in Table C.14.0. Being consistent with the estimated results from the Divisia model, the constant for both regimes is significant, and the value of the constant is higher in regime 2 than that in regime 1. The first lag of WEI_gap is also significant in both regimes, while the coefficient of the lagged change in real interest rate is insignificant in both regimes. The traditional simple sum money is only significant in regime 2 compared to the significant coefficients of the green-return benchmarked money in both regimes (Table C.11.0), indicating the green-return Divisia monetary aggregate is more precisely estimated in the model than its simple sum counterpart.

Table C.14.0 The MS-VAR(1) Specification for Traditional Simple Sum Model

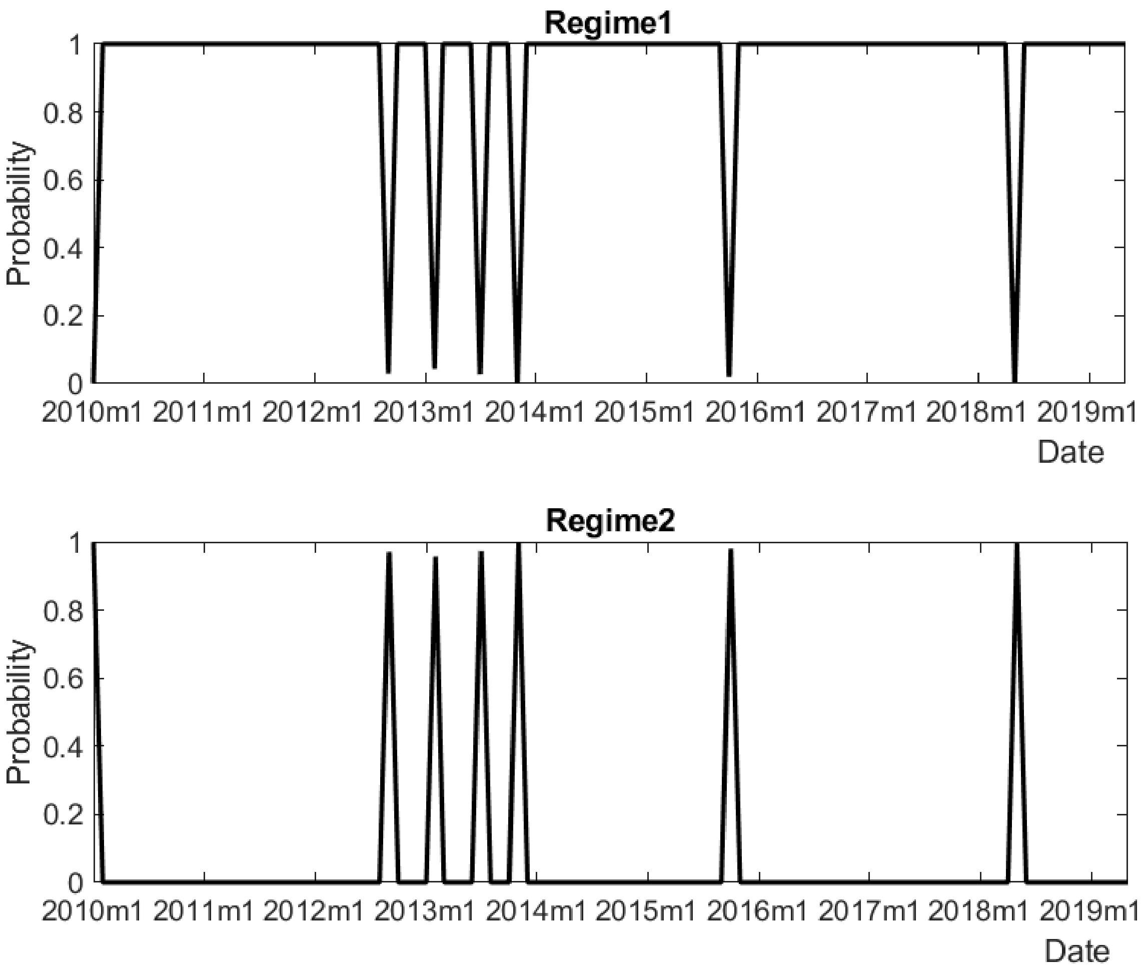
	Coefficients			
	WEI_gap	$RFFR$	Simple Sum	
Regime 1				
Constant				0.279**
				(2.57)
L1	0.851***	0.166	-1.606	
	(18.06)	(0.49)	(-1.34)	
Regime 2				

Constant				0.583*** (21.08)
L1	0.965*** (72.47)	-0.148 (-1.09)	-1.433*** (-4.86)	

Notes: The results only present the output equation of the MS-VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.944 and 0.000 for regime 1 and regime 2 respectively. The expected duration for each regime is about 28 months and 1 month, which means regime 1 is relatively persistent than regime 2. As shown in Figure C.7.0 of the plots of the filtered probabilities, the regime 1 has dominated during the period.

Figure C.7.0 Filtered Probabilities of Regimes in Traditional Simple Sum Model



Notes: The plot shows the filtered probabilities of regimes in traditional simple sum model, which includes the output gap, the real interest rate and the real traditional simple sum monetary aggregate in the model.

We also include the financial condition, i.e., EPU, FSI, and GUNBP, in the traditional simple sum MS-VAR model and the results are displayed in Table C.14.1, C.14.2 and C.14.3 respectively. The constant and the lagged WEI_gap are significant in three specifications. The first lag of the changes in real interest rate, in real simple sum money and in the financial measure is significant in regime 1 with the FSI measure and in regime 2 with EPU and GUNBP measures. When three financial condition measures are incorporated, the constant, the lagged WEI_gap and the lagged EPU are significant in both regimes, while the lagged real interest rate, the lagged simple sum money and the lagged FSI are significant only in regime 1 (see Table C.14.4).

Table C.14.1 The MS-VAR(1) Specification for Traditional Simple Sum Model with EPU Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	EPU	
Regime 1					
Constant					0.303***
					(2.67)
L1	0.841***	0.487	-1.426	-0.015	
	(16.17)	(0.97)	(-0.91)	(-0.45)	
Regime 2					
Constant					0.992***
					(6.47)
L1	0.574***	0.491*	-9.138***	-0.130***	
	(8.93)	(1.76)	(-6.33)	(-3.29)	

Notes: The results only present the output equation of the MS-VAR model. In this traditional simple sum model, we include the output gap, real interest rate and the real traditional simple sum monetary aggregate and also add the EPU as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.14.2 The MS-VAR(1) Specification for Traditional Simple Sum Model with FSI Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	FSI	
Regime 1					
Constant					0.689***
					(4.59)
L1	0.787***	0.765**	-7.345***	0.516***	

	(15.31)	(2.27)	(-4.45)	(2.84)	
Regime 2					
Constant					0.283*
					(1.68)
L1	0.839***	-0.100	0.227	-0.159	
	(14.73)	(-0.20)	(0.12)	(-0.68)	

Notes: The results only present the output equation of the MS-VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add the FSI as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.14.3 The MS-VAR(1) Specification for Traditional Simple Sum Model with GUNBP Included

	Coefficients				
	WEI_gap	$RFFR$	Simple Sum	GUNBP	
Regime 1					
Constant					0.203*
					(1.83)
L1	0.873***	0.156	-1.544	-0.0001	
	(18.55)	(0.48)	(-1.09)	(-0.003)	
Regime 2					
Constant					0.319***
					(3.14)
L1	0.950***	-0.881**	-6.195***	-0.276***	
	(18.73)	(0.43)	(-5.33)	(-9.57)	

Notes: The results only present the output equation of the MS-VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add the GUNBP as the financial measure. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

Table C.14.4 The MS-VAR(1) Specification for Traditional Simple Sum Model with Three Financial Measures Included

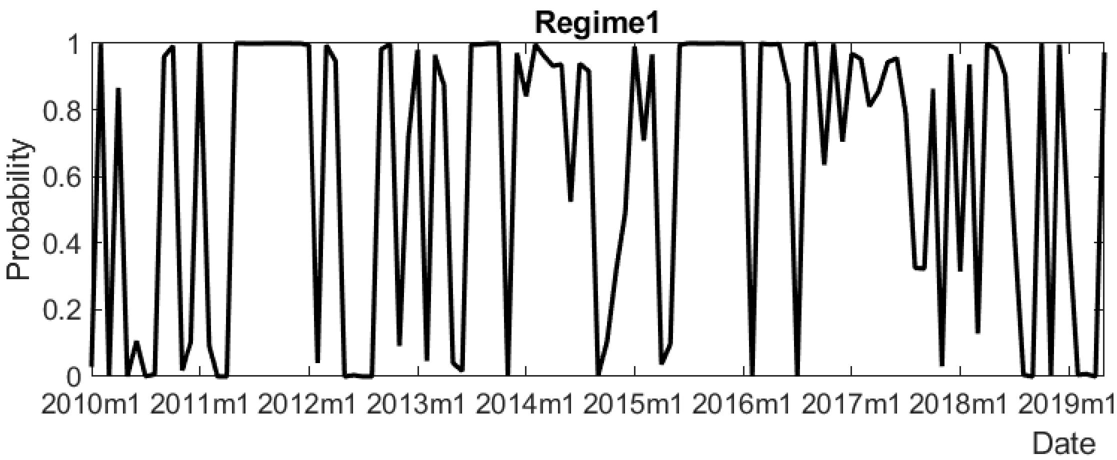
	Coefficients						
	WEI_gap	$RFFR$	Simple Sum	EPU	FSI	GUNBP	
Regime 1							
Constant							1.269***
							(4.95)
L1	0.685***	1.144**	-11.009***	0.159***	1.055***	0.055	
	(10.24)	(2.54)	(-5.09)	(3.21)	(3.32)	(0.84)	
Regime 2							
Constant							0.421***
							(2.99)

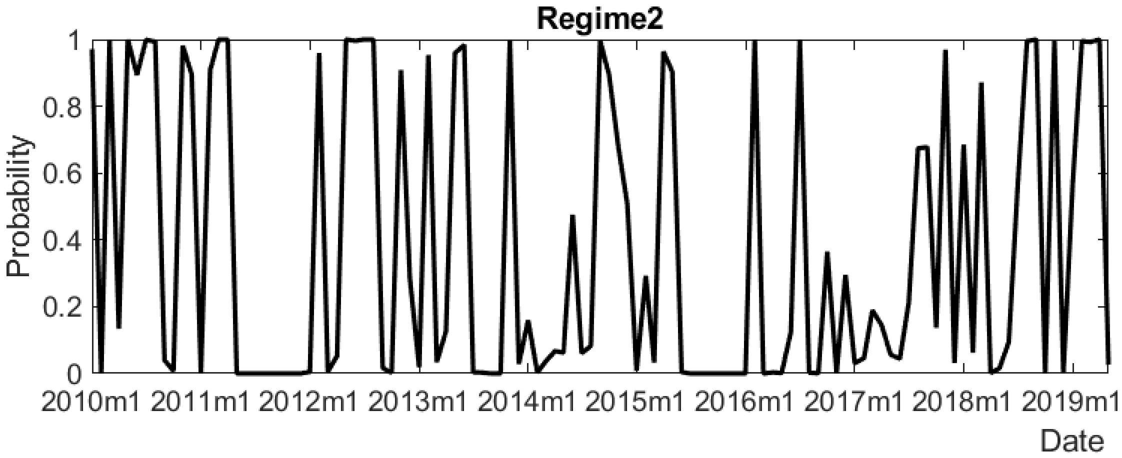
L1	0.773***	0.122	-1.876	-0.129***	-0.089	0.038
	(20.91)	(0.43)	(-1.51)	(-4.73)	(-0.39)	(0.87)

Notes: The results only present the output equation of the MS-VAR model. In this traditional simple sum model, we include the output gap, the real interest rate and the real traditional simple sum monetary aggregate and also add all three financial measures. L1 represents the first lag of variables. The notation ***, ** and * denote rejections of the null hypothesis at the 1%, 5% and 10% significance level, respectively and t-statistics are reported in the parentheses below each coefficient estimate.

The estimated probabilities p_{jj} for remaining in each regime are 0.686 and 0.461 for regime 1 and regime 2 respectively with the EPU included. The expected duration for each regime is about 3 months and 2 months. For the model with the FSI measure, the estimated probabilities p_{jj} for remaining in each regime are 0.891 and 0.791 for regime 1 and regime 2 respectively, with the expected duration of about 9 months and 5 months. The estimated probabilities for the model with the GUNBP measure added are 0.903 and 0.626 for regime 1 and regime 2 respectively, and the corresponding expected duration is about 10 months and 3 months. These results suggest the regime 1 is more persistent than the regime 2. If all three financial measures are considered simultaneously, the estimated probabilities for remaining in each regime are 0.819 and 0.772 for regime 1 and regime 2 respectively and the expected durations are about 6 months and 4 months. The plots of the filtered probabilities are also presented in Figure C.7.1, C.7.2, C.7.3 and C.7.4 and show that regime 1 has dominated during the period.

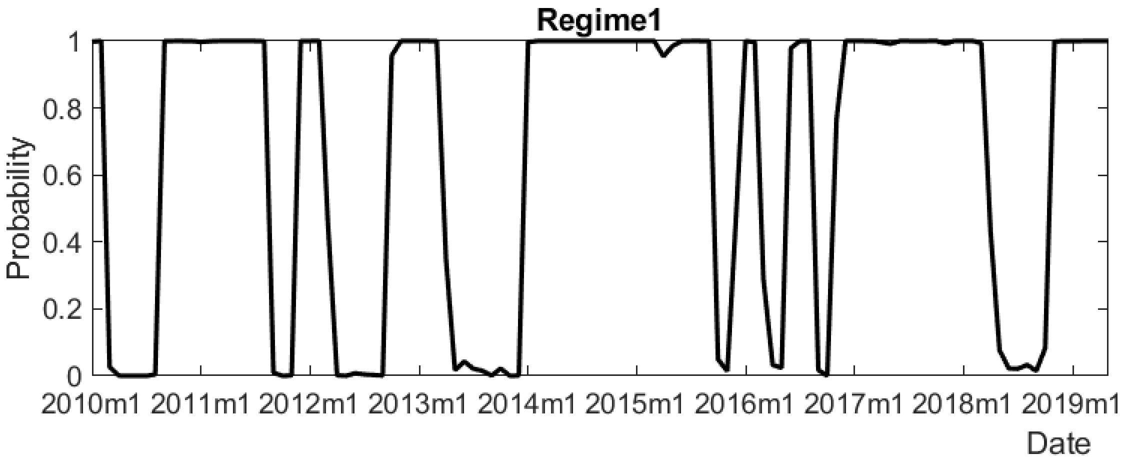
Figure C.7.1 Filtered Probabilities of Regimes in Traditional Simple Sum Model with EPU Included

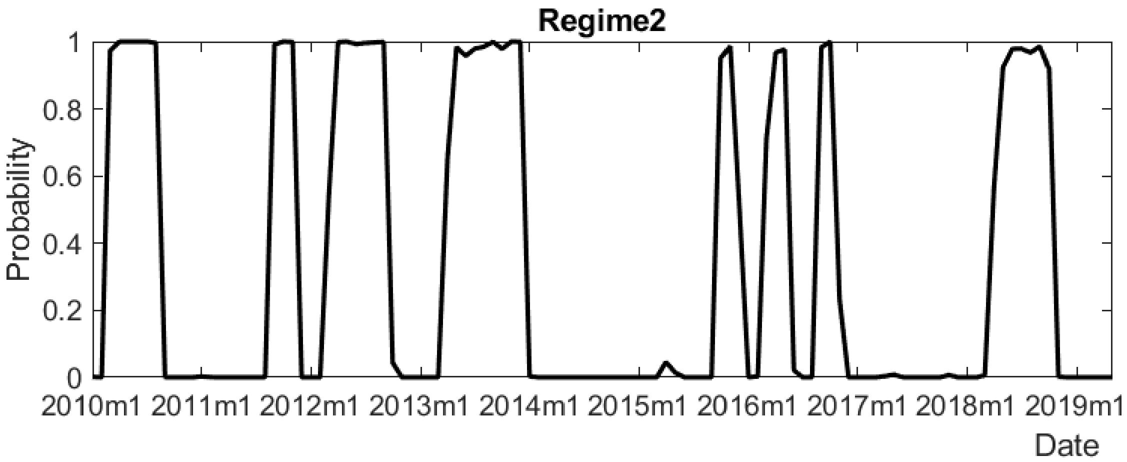




Notes: The plot shows the filtered probabilities of regimes in traditional simple sum model, which includes the output gap, the real interest rate, the real traditional simple sum monetary aggregate and the EPU in the model.

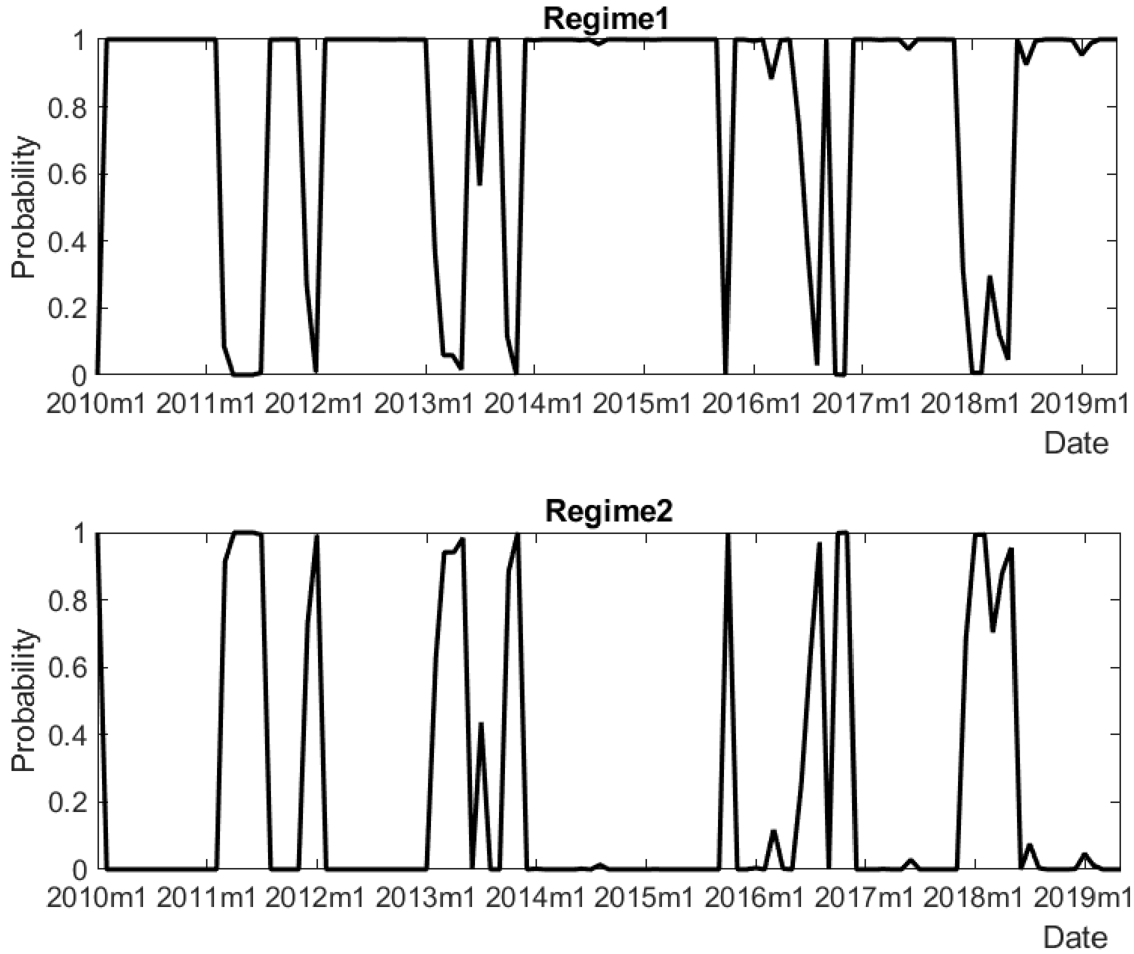
Figure C.7.2 Filtered Probabilities of Regimes in Traditional Simple Sum Model with FSI Included





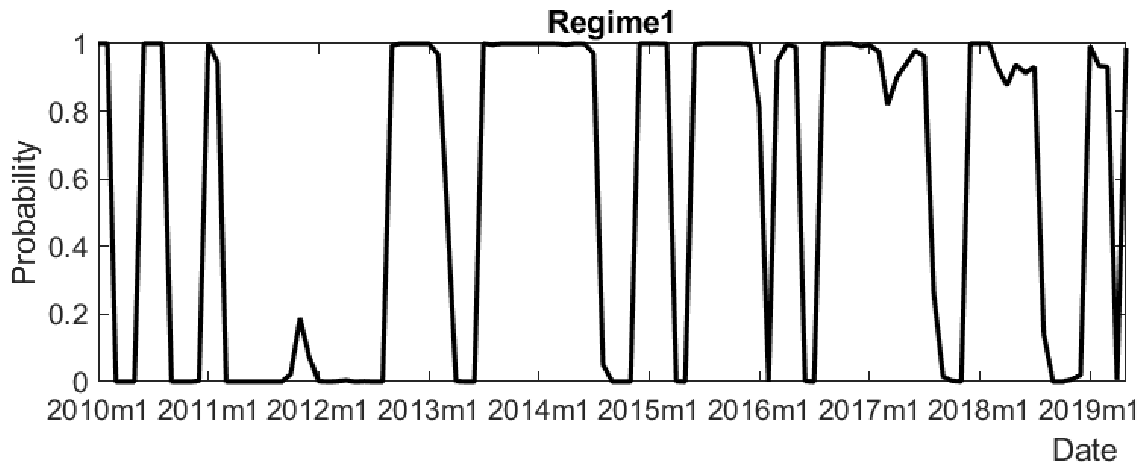
Notes: The plot shows the filtered probabilities of regimes in traditional simple sum model, which includes the output gap, the real interest rate, the real traditional simple sum monetary aggregate and the FSI in the model.

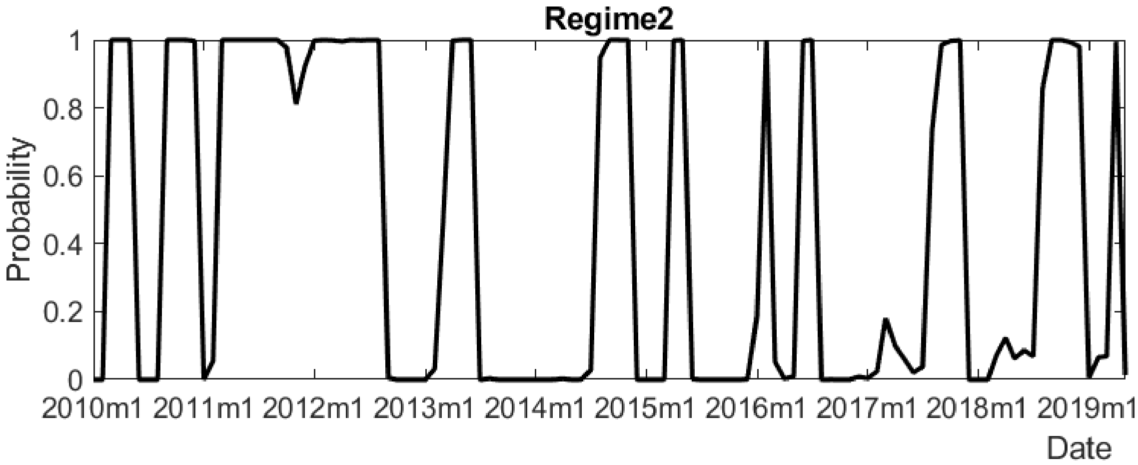
Figure C.7.3 Filtered Probabilities of Regimes in Traditional Simple Sum Model with GUNBP Included



Notes: The plot shows the filtered probabilities of regimes in traditional simple sum model, which includes the output gap, the real interest rate, the real traditional simple sum monetary aggregate and the GUNBP in the model.

Figure C.7.4 Filtered Probabilities of Regimes in Traditional Simple Sum Model with Three Financial Measures Included





Notes: The plot shows the filtered probabilities of regimes in traditional simple sum model, which includes the output gap, the real interest rate, the real traditional simple sum monetary aggregate and all three financial condition measures in the model.

In summary, this research is founded on the seminal paper by Barnett (1980), who originated the Divisia index in economic aggregation theory. This work contributes to and augments research in the Divisia monetary aggregates. We produce a new green-benchmarked Divisia monetary aggregate for the United States using the Törnqvist-Theil discrete time approximation of the continuous Divisia Index. The user costs for calculating this index employs the rate of return of the 20Y+ green bond or the coupon rate of the 20Y+ green bond, which reflects the pure green investment. Our empirical work[29], based on USA aggregate demand, finds that the change in the new green-return benchmarked Divisia aggregate is significantly related to the output gap in both VAR and MS-VAR specifications, indicating that even during an aggregate supply shock, money can be useful in modelling an IS curve and in the formation of economic policy. The green-return benchmarked Divisia monetary aggregate is more precisely estimated than the green-coupon benchmarked Divisia monetary aggregate, the conventional Divisia monetary aggregate and the simple sum monetary aggregate, possibly as a result of the greater information content contained in the green-return benchmarked monetary aggregate constructed using the rate of return of the green bond as the benchmarked

[29] All VAR models satisfy stability condition as the eigenvalues are less than one.

rate. Additionally, in all MS-VAR specifications, regime 1 exhibits predominant influence throughout the estimation period. This observation can be primarily attributed to the specific temporal boundaries of our analysis, which spans from January 2009 to September 2019. This interval is specially chosen and omits two critical global events: the Financial Crisis of 2007 and the COVID-19 pandemic as stated earlier on page 129. Consequently, the selected timeframe predominantly encapsulates a phase characterised either by economic recovery or stability. Within the framework of the MS-VAR model, such an enduring phase of stable economic conditions is systematically identified and correlated with regime 1. As a result of this correlation, regime 1 is assigned a heightened probability, leading to its dominance throughout the estimated period of January 2009 to September 2019. Even that regime 1 is predominant, the regime 2 can still identify rare but significant events, which have substantial impacts on the economy and are essential for understanding the economic fluctuations. These two economic environments cannot be captured effectively by one single regime. We therefore take advantage of the flexibility of the MSVAR model and its ability to switch between two regimes and therefore enhance the predictive accuracy of the model (Krolzig, 2013).

7 Forecast Evaluation

Turning to a formal comparison of the forecasting performance, we use two common forecast evaluation criteria: root mean squared errors (RMSE) and relative absolute errors (RAE). They are defined to be:

$$RMSE = \left[\frac{1}{K}\sum_{t=2020:01+\tau}^{2022:12}(E_{t-\tau}[WEI\_gap_t] - WEI\_gap_t)^2\right]^{1/2} \tag{C.16a}$$

$$RAE = \frac{\left[\sum_{t=2020:01+\tau}^{2022:12}(E_{t-\tau}[WEI\_gap_t] - WEI\_gap_t)^2\right]^{1/2}}{\left[\sum_{t=2020:01}^{2022:12}(WEI\_gap_t)^2\right]^{1/2}} \tag{C.16b}$$

where K is the total number of the out-of-sample forecasts and τ is the forecast horizon. In our specific study $K = 48$. To obtain the most stable and optimal forecasting performance, the horizon is the 12 in the VAR specification, while the horizon is 49 in the MS-VAR model. We complement the raw measures in (C.16a-b) by presenting ratios of the RMSE and RAE for each model to that of the best performing model. We evaluate one-month ahead, three-month ahead, six-month ahead and nine-month ahead forecasts for all VAR and MS-VAR specifications.

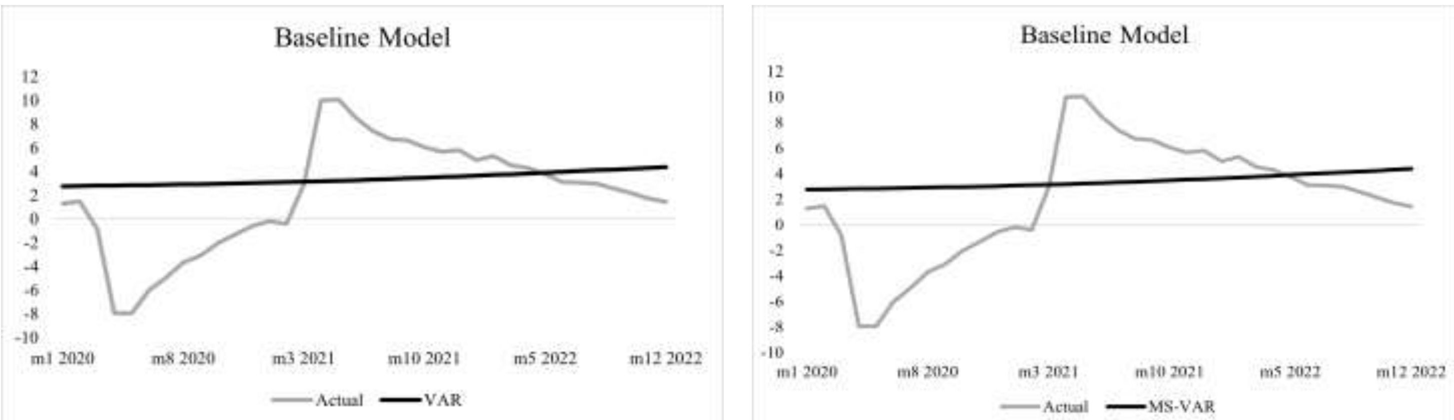
7.1 One-Month Ahead Forecasts

7.1.1 Baseline Model

We start with presenting the $t + 1$ forecasts for the baseline model as a 1×2-panel in Figure C.8. Forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The forecasts from the MS-VAR model are quite similar to those obtained from the VAR-model. We note that both forecasts tend to overshoot and undershoot large changes in the output gap since the post-COVID forecasts are based on the pre-COVID economic activity which does not account for the unprecedented disruptions to global economy due to the pandemic. Turning to RMSE and RAE, both criteria rank the different models under evaluation similarly. As is evident in Table C.15, the MS-VAR model is the better performing model, yielding slightly lower values for both RMSE and RAE than that of the VAR model.

Figure C.8 Forecasted (t + 1) and Actual Values of Output Gap from Baseline Model

VAR **MS-VAR**



Notes: The left side plot shows the forecasted (t + 1) and actual values of output gap for the baseline VAR model and the right side plots shows that for the baseline MS-VAR model.

Table C.15 Evaluation Criteria for *t* + 1 Forecasts from Baseline Model

	VAR	MS-VAR
RMSE	4.5802	4.4743
RAE	0.9156	0.8944
RMSE-ratio	102.37%	100%
RAE-ratio	102.37%	100%

Notes: The table presents the forecasting evaluation for the *t* + 1 forecasts from the baseline VAR and MS-VAR models, which includes the output gap and the real interest rate in the models.

7.1.2 Monetary Models

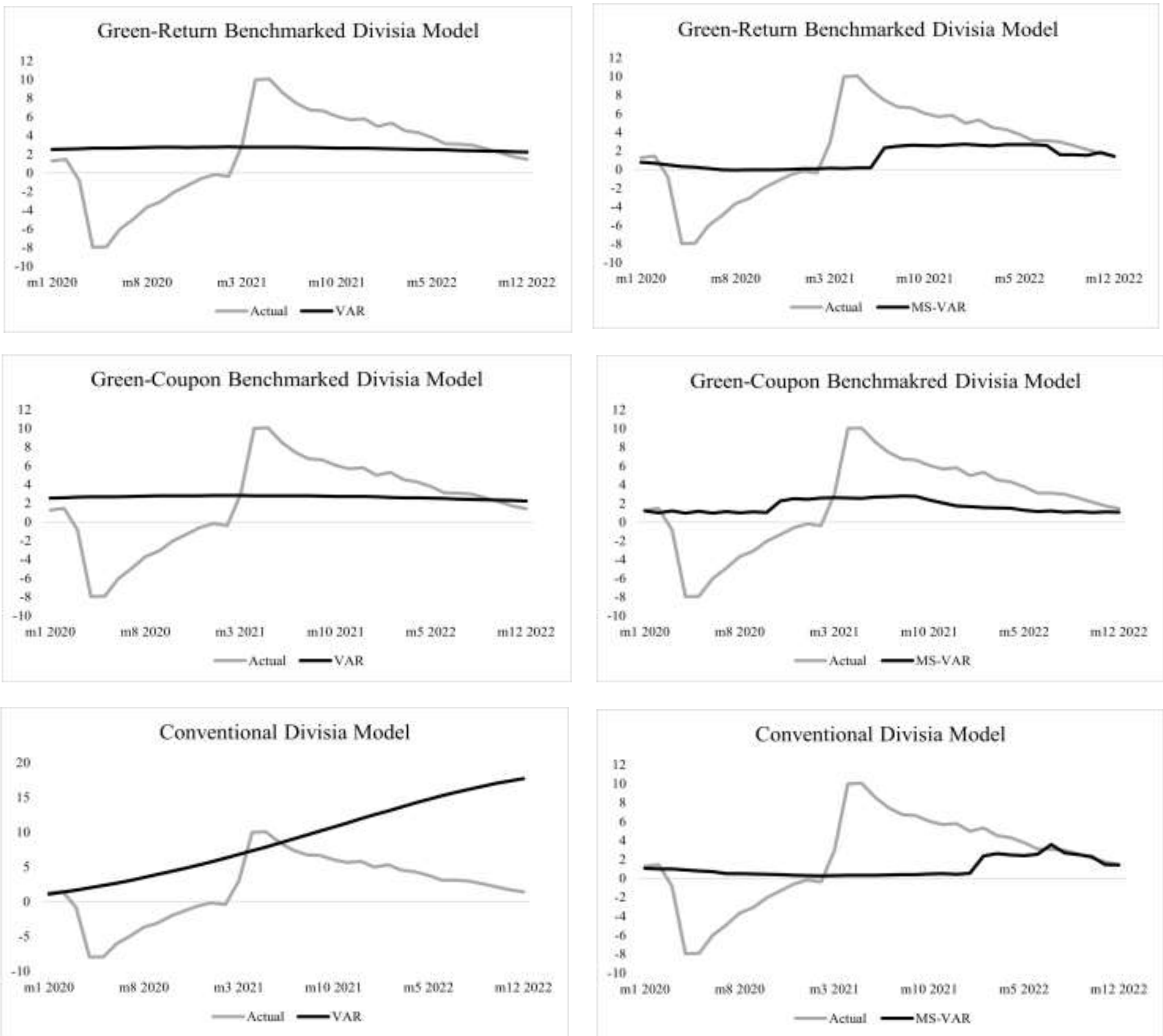
Based on the baseline model, we then add the monetary aggregates into the model to examine whether the money can improve the forecasting performance for the output gap. In Figure C.9.0, we present the $t + 1$ forecasts for the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model as a 4×2-panel. As before, forecasts (the black line) are compared to actual output gap (the grey line) in each panel. A noticeable difference between the VAR model and the MS-VAR model appears in the conventional Divisa model that the forecasts from the VAR model are biased upwards. The conventional Divisia VAR model overshoots the actual output gap. The forecasts from the MS-VAR models are similar to those obtained from the VAR-models in other three money specifications. These three monetary specifications still overshoot and undershoot the output gap, although less than that in the baseline model, which indicates the improved forecasting performance with the monetary models. This is also confirmed by RMSE and RAE criteria presented in Table C.16.0 and Table C.15, which shows that both values of RMSE and RAE of the best baseline model are higher than the values of the best model, i.e., the green-return benchmarked Divisia MS-VAR model, by 8.03% and 8.02% respectively.

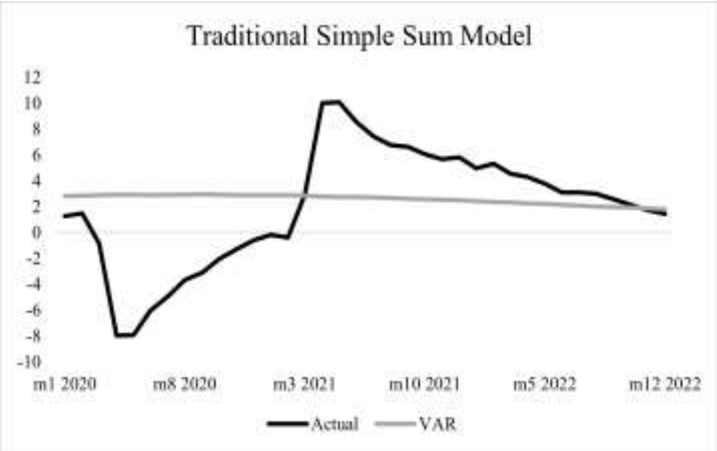
Judging from RMSE and RAE in Table C.16.0, the VAR models perform worse than the MS-VAR models in all four money specifications. This is consistent with the previous belief that the linear model performs badly in the forecasting experiment compared to the non-linear model. The green-return benchmarked Divisia MS-VAR model is the best forecaster, and the second best is the green-coupon benchmarked Divisia MS-VAR model, only 2.19%

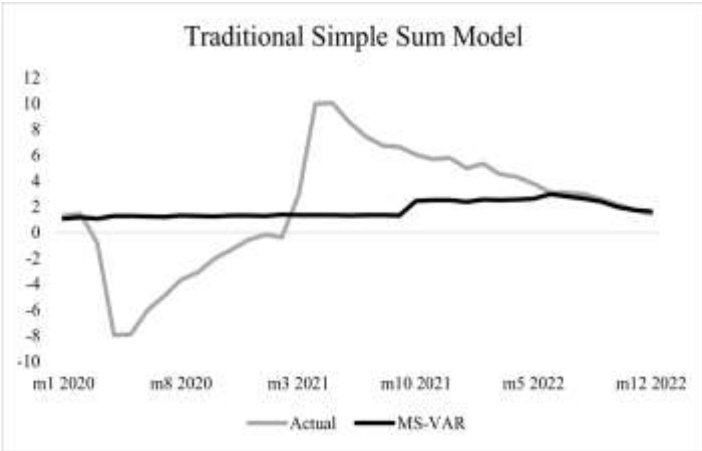
worse than the best model. The conventional Divisia VAR model is the worst performing model, yielding highest values of both RMSE and RAE than that obtained from other models. These results indicate that the newly constructed green-benchmarked Divisia monetary aggregates are found to be useful in forecasting the real economic activity and are better than the conventional Divisia monetary aggregate and the simple sum counterpart.

Figure C.9.0 Forecasted (t + 1) and Actual Values of Output Gap from Monetary Models

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 1) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models and the right column presents that by using the MS-VAR method for four money models.

Table C.16.0 Evaluation Criteria for *t* + 1 Forecasts from Monetary Models

	Green-Return Benchmarked Divisia Model		Green-Coupon Benchmarked Divisia Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.6061	4.1419	4.6086	4.2326	8.5141	4.6844	4.7319	4.3885
RAE	0.9208	0.8280	0.9213	0.8461	1.7019	0.9364	0.9459	0.8773
RMSE-ratio	111.21%	100%	111.26%	102.19%	205.57%	113.10%	114.24%	105.95%
RAE-ratio	111.21%	100%	111.27%	102.19%	205.54%	113.09%	114.24%	105.95%

Notes: The table presents the forecasting evaluation for the t + 1 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively.

7.1.3 Monetary Models with Financial Condition Measures

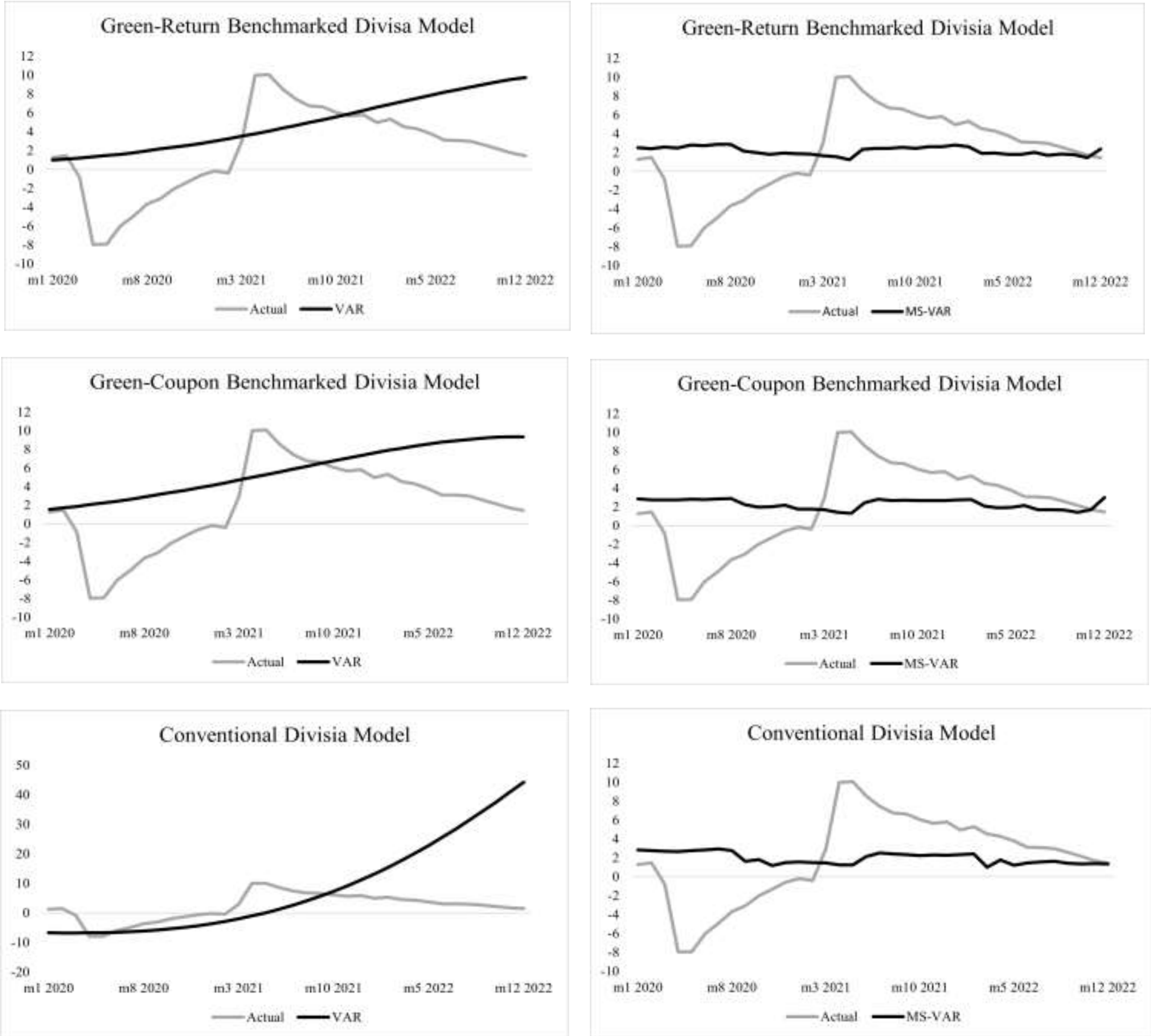
The financial conditions play a crucial role in influencing economic agents' decisions regarding investment, consumption and savings, thus affecting the economic activity (Espinoza et al., 2012). For example, tight financial conditions, characterized by high borrowing costs and stringent credit availability, could dampen economic activity, leading to a widening of the negative output gap. In addition, according to the report by Reserve Bank of India (2023), the financial conditions may alter and shape investors' preferences to the green bonds. These conditions, encompassing aspects like interest rate trends, can significantly sway investors' appetite towards or away from green financial instruments. In periods of favourable financial conditions, investors might be more inclined to allocate capital towards green investments, even at potentially lower returns, driven by broader market optimism and a lower opportunity cost of capital. Therefore, based on the four monetary models above, we use three selected financial condition measures, i.e., EPU, FSI, GUNBP, as the control variables in the models to empirically investigate whether there is a switch in investors' preference between conventional and green bonds contingent on financial conditions through the assessment of the role of the monetary aggregates in the forecasting output. In our analytical framework, we propose the individual integration of three distinct financial measures into four established monetary models. Subsequently, these measures will be collectively incorporated into the models to examine the cumulative effect. This methodological approach allows for a nuanced understanding of the impact of each financial measure in isolation, as well as in conjunction with others, within the context of the specified monetary models.

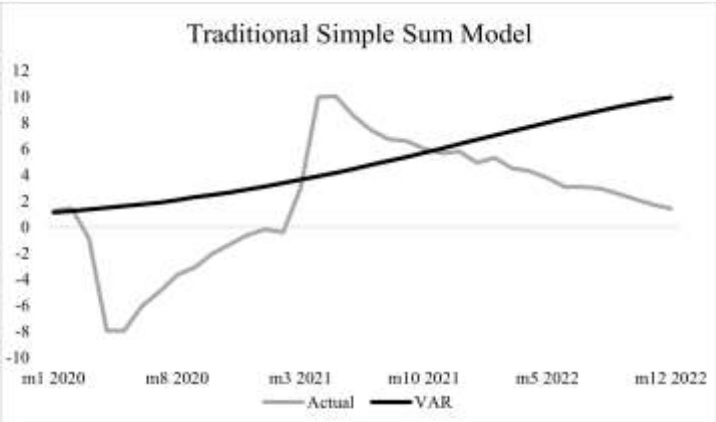
We first compare the forecasting results from four monetary models, each augmented with the inclusion of EPU. We present the $t + 1$ forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4×2-panel in Figure C.9.1. As before, the forecasts, represented by the black line, are compared to the actual output gap, depicted by the grey line, in each panel. A divergence emerges when comparing the VAR models with the MS-VAR models in all money framework. This discrepancy is characterised by an upward bias in the forecasts generated by the VAR models, indicating a systematic overestimation relative to those produced by the MS-VAR models. In contrast to the models devoid of financial measures above, all VAR models exhibits diminished performance when integrated with the EPU. This observation suggests a relative decrease in the efficacy of the VAR model in capturing the dynamics of the system when the EPU is incorporated. This is further confirmed by the comparison of RMSE and RAE criteria presented in Table C.16.1 and Table C.16.0, which reveals that both values of RMSE and RAE of the EPU incorporated monetary models are higher than those of the previously discussed monetary models without the EPU, across each respective model, by 18.75% and 18.74% on average respectively. This result indicates a relative decrease in forecasting accuracy for all VAR and MS-VAR models with EPU integration.

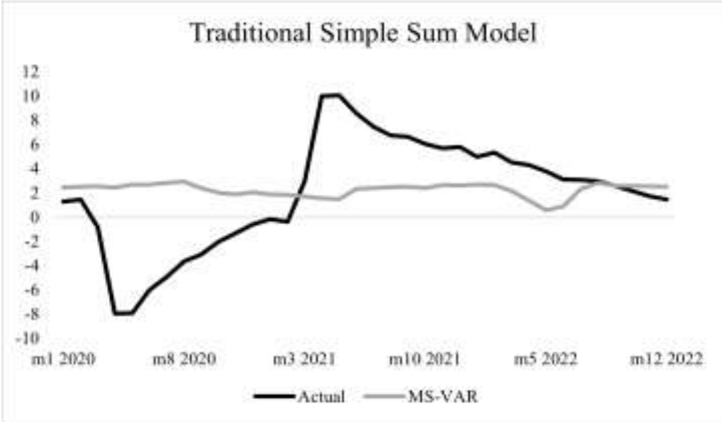
As expected, the VAR models exhibit inferior performance compared to the MS-VAR models across all four monetary specifications, evidenced by the lower values of both RMSE and RAE for the MS-VAR models than those derived from the VAR models by 22.14% and 22.13% on average in Table C.16.1. Among the models assessed, the green-return benchmarked Divisia MS-VAR model appears as the best forecaster, followed by the green-coupon benchmarked Divisia MS-VAR model, being only 0.23% less accurate than the leading model. This minor discrepancy highlights the comparative robustness of both green-benchmarked models in the context of forecasting performance. The conventional Divisia VAR model is identified as the least effective models, as it performs up to 273.56% worse than the best performing green-return benchmarked Divisia MS-VAR model. These findings are congruent with the forecasting results obtained from the monetary models without the financial measures. Given that our forecasting models rely on data excluding the COVID-19 period, the observed enhancement in the forecasting performance of the green-benchmarked Divisia monetary aggregates under a stable economic condition suggests a pivotal shift in investors' preferences towards green investments. This inference is drawn from the correlation between easing financial conditions and the relative outperformance of green-focused financial instruments, indicating a tendency among investors to favour environmentally sustainable options in periods of economic stability.

Figure C.9.1 Forecasted (t + 1) and Actual Values of Output Gap from Monetary Models with EPU Included

VAR | **MS-VAR**







Notes: The plots show the forecasted (t + 1) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with EPU added and the right column presents that by using the MS-VAR method for four money models with EPU added.

Table C.16.1 Evaluation Criteria for *t* + 1 Forecasts from Monetary Models with EPU Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.9085	4.7267	5.2412	4.7380	15.9559	4.7951	5.0065	4.7412
RAE	0.9812	0.9449	1.0477	0.9471	3.1896	0.9585	1.0008	0.9478
RMSE-ratio	103.85%	100%	110.89%	100.24%	337.57%	101.45%	105.92%	100.31%
RAE-ratio	103.84%	100%	110.88%	100.23%	337.56%	101.44%	105.92%	100.31%

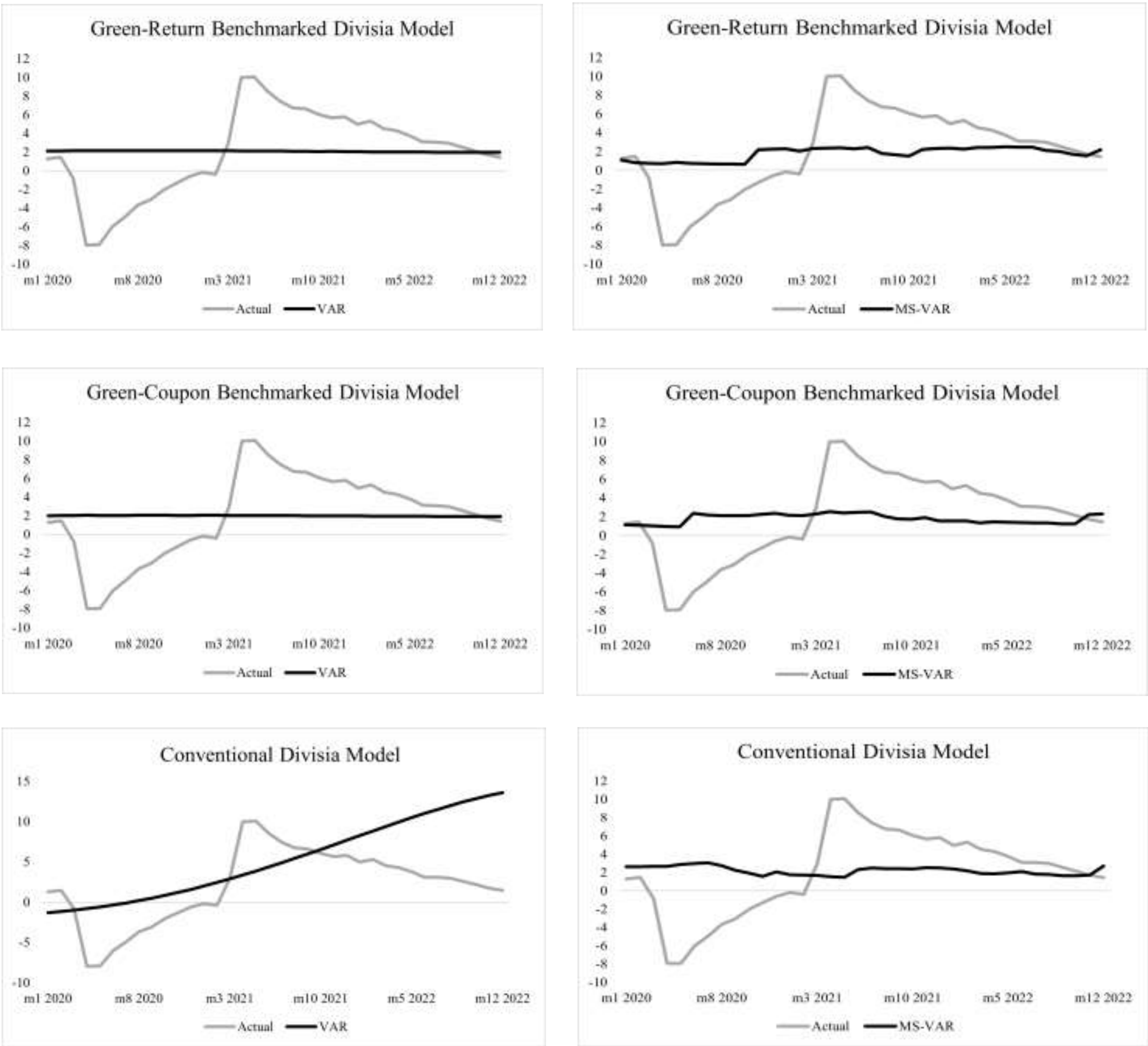
Notes: The table presents the forecasting evaluation for the t + 1 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively.

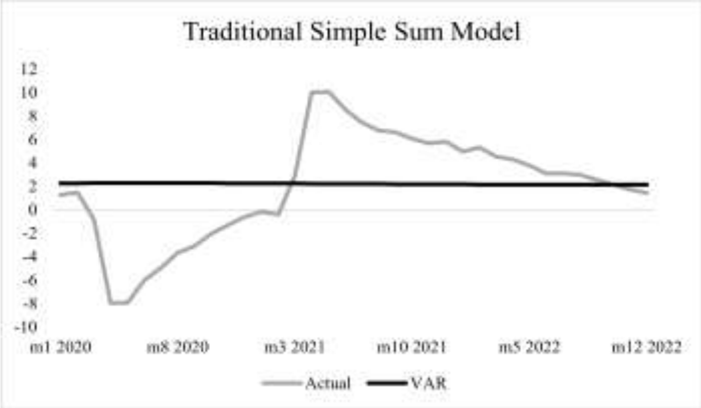
We then substitute the EPU with the FSI and conduct a comparative analysis of the forecasting outcomes from four monetary models with the financial measure incorporated. In Figure C.9.2, we present the t + 1 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. The pattern of the forecasts obtained from the conventional Divisia VAR model differs from the forecasts obtained by the other models, which appears to give an upward bias for the forecasts of the output gap. This observation diverges from the results yielded by the models incorporating the EPU but is similar to the results obtained from models without financial measures, which highlights the better forecasting performance of the models integrated with the FSI compared to those models that incorporate the EPU above. This is further substantiated through the comparison of RMSE and RAE criteria presented in Table C.16.2 and Table C.16.1. These tables show that the RMSE and RAE values for the monetary models incorporating the EPU are higher than those for the models that include the FSI, across each individual model, by 29.33% and 29.32% on average respectively, thereby illustrating the comparatively superior predictive accuracy of the latter.

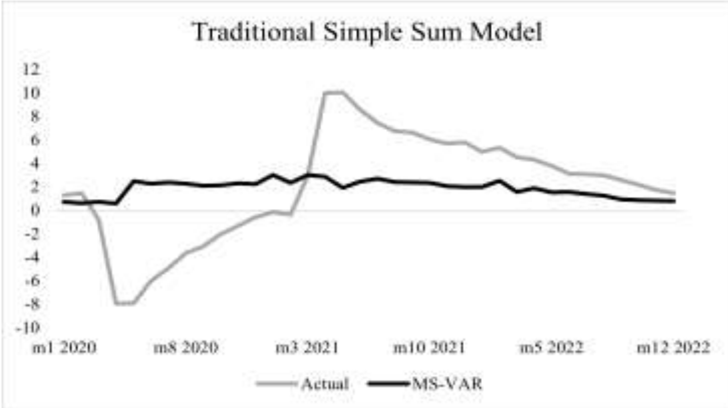
As can be seen from Table C.16.2, the values of RMSE and RAE for the MS-VAR models are notably lower than those calculated from the VAR models by 7.84% and 7.83% on average respectively, across all four monetary specifications, thereby reinforcing the superior forecasting capability of the MS-VAR model. Consistent with the previously obtained results, the green-return benchmarked Divisia MS-VAR model is the best model among the models evaluated. This is closely followed by the green-coupon benchmarked Divisia MS-VAR model, being 9.38% worse than the best model. The conventional Divisia VAR model is the worst performing model as indicated by its lowest values in both RMSE and RAE. Looking at the ratios of the forecast-evaluation criterion, it underperforms up to 39.37% compared to the best performing green-return benchmarked Divisia MS-VAR model. Incorporating the Financial Stress Index (FSI) as a financial control variable, the better forecasting capabilities of the green-benchmarked Divisia monetary aggregates, particularly under scenarios of eased financial constraints, also underscores a growing inclination among investors to prioritize environmentally sustainable investment options.

Figure C.9.2 Forecasted (t + 1) and Actual Values of Output Gap from Monetary Models with FSI Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 1) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with FSI added and the right column presents that by using the MS-VAR method for four money models with FSI added.

Table C.16.2 Evaluation Criteria for *t* + 1 Forecasts from Monetary Models with FSI Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.5963	4.1101	4.5847	4.4958	5.7285	4.7694	4.6014	4.5044
RAE	0.9188	0.8216	0.9165	0.8987	1.1451	0.9534	0.9198	0.9004
RMSE-ratio	111.83%	100%	111.55%	109.38%	139.38%	116.04%	111.95%	109.59%
RAE-ratio	111.83%	100%	111.55%	109.38%	139.37%	116.04%	111.95%	109.59%

Notes: The table presents the forecasting evaluation for the t + 1 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively.

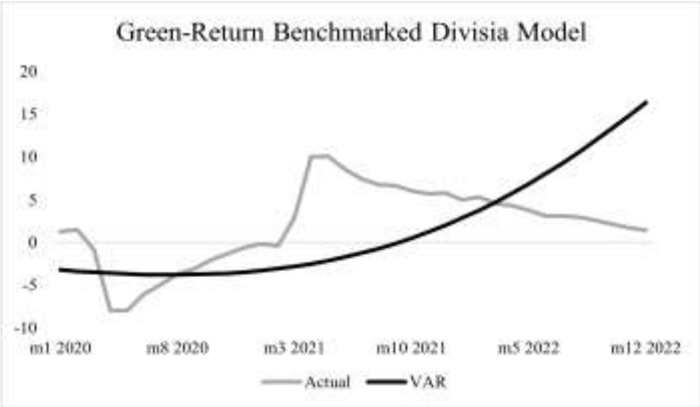
Using the GUNBP as the financial control variable, we then compare the forecasting results from four monetary models, each incorporated with this financial measure. In Figure C.9.3, we present the t + 1 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. The forecasting pattern emanating from all VAR models exhibits a distinctive divergence from those produced by the MS-VAR models, suggesting an inclination towards an upward bias in the projected estimates of the output gap. This observation is different from the results yielded by the models incorporating the FSI and the

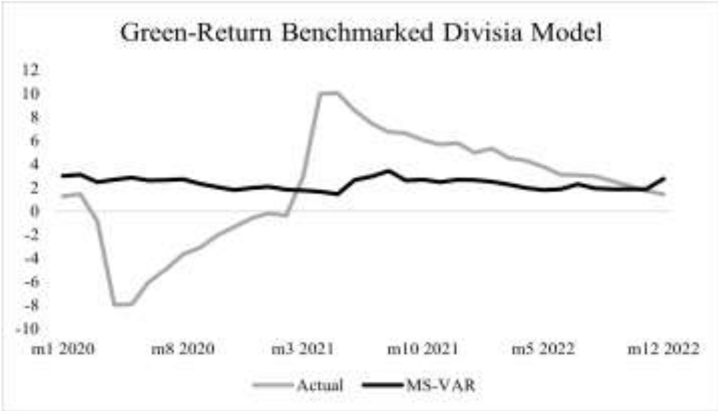
EPU, respectively. Turing next to RMSE and RAE in Table C.16.3, both values are 7.01% higher than those in the previously discussed monetary models that include the EPU as the financial measure, across each individual model. This finding demonstrates the less predictive accuracy of the models with the GUNBP incorporated.

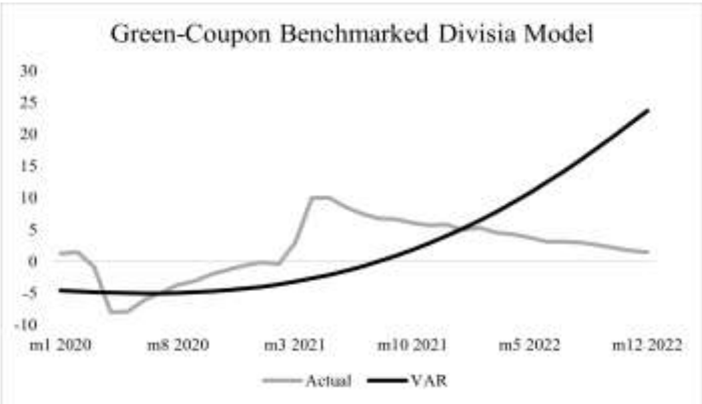
As delineated in Table C.16.3, the RMSE and RAE values for the MS-VAR models are substantially lower than those derived from the VAR models across all four monetary specifications, by 34.96% on average. This also highlights the superior forecasting capability of the MS-VAR model. In accordance with the results previously established, the green-return benchmarked Divisia MS-VAR model emerges as the best model among the evaluated models. The green-coupon benchmarked Divisia MS-VAR model and traditional simple sum MS-VAR model follow closely and exhibit a marginal decrement in the forecasting performance, being 0.43% and 0.55% less accurate, respectively, compared to the best model. The green-coupon benchmarked Divisia VAR model is identified as the model with the most suboptimal performance, as evidenced by its lowest values of both RMSE and RAE. The ratios reveal that this model's underperformance reaches up to 85.07% in comparison to the best performing green-return benchmarked Divisa MS-VAR model. Additionally, with the incorporation of the FSI as a financial control variable, the enhanced forecasting proficiency of the green-benchmarked Divisia monetary aggregates becomes apparent. This also reflects an increasing trend among investors towards favouring green investments.

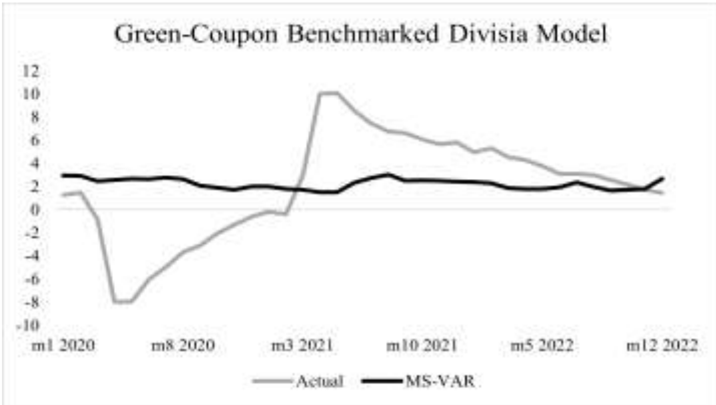
Figure C.9.3 Forecasted (t + 1) and Actual Values of Output Gap from Monetary Models with GUNBP Included

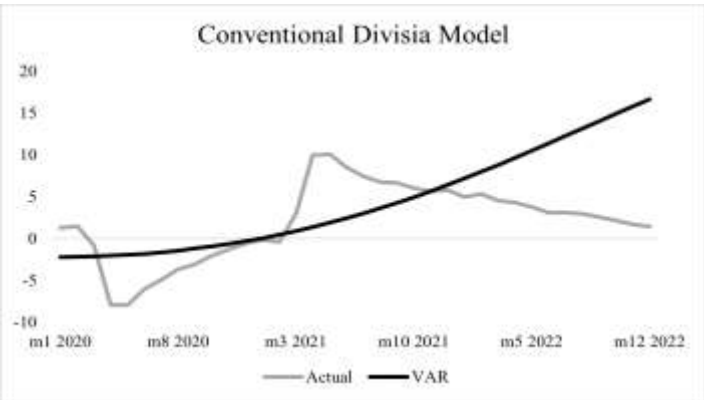
VAR **MS-VAR**

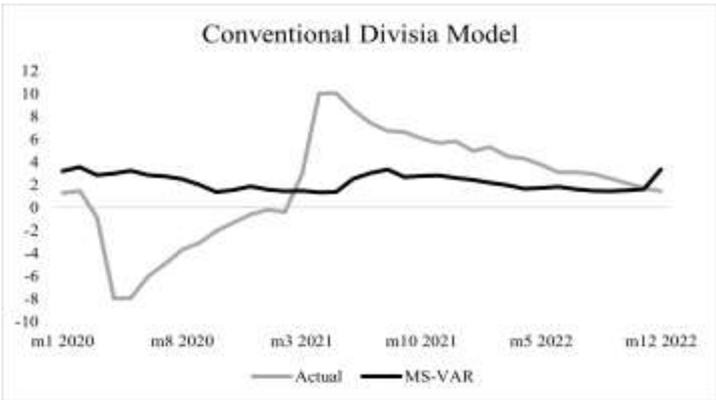


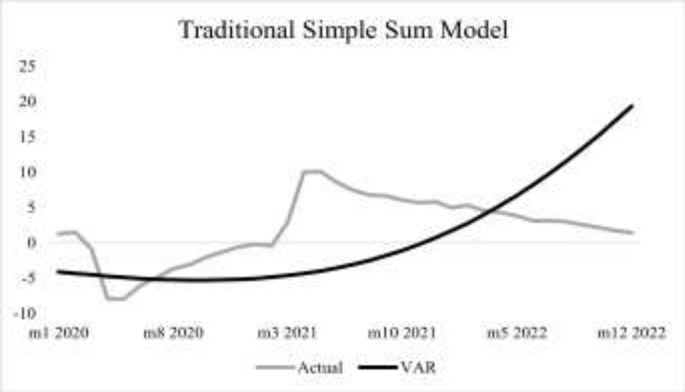


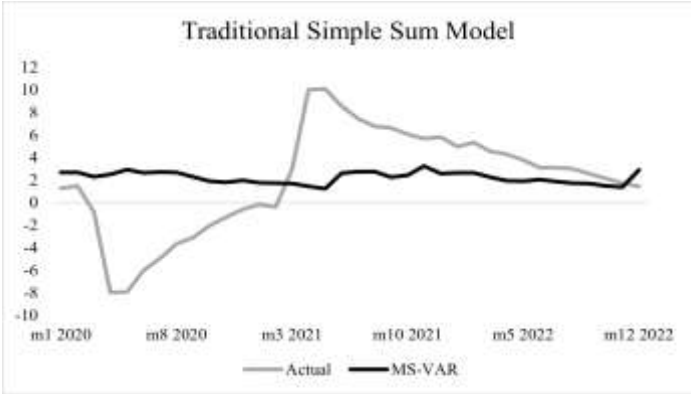












Notes: The plots show the forecasted (t + 1) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with GUNBP added and the right column presents that by using the MS-VAR method for four money models with GUNBP added.

Table C.16.3 Evaluation Criteria for t + 1 Forecasts from Monetary Models with GUNBP Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	6.5849	4.6653	8.6344	4.6855	6.3275	4.7458	7.8125	4.6908
RAE	1.3163	0.9326	1.7260	0.9366	1.2649	0.9487	1.5617	0.9377
RMSE-ratio	141.15%	100%	185.08%	100.43%	135.63%	101.73%	167.46%	100.55%
RAE-ratio	141.14%	100%	185.07%	100.43%	135.63%	101.73%	167.46%	100.55%

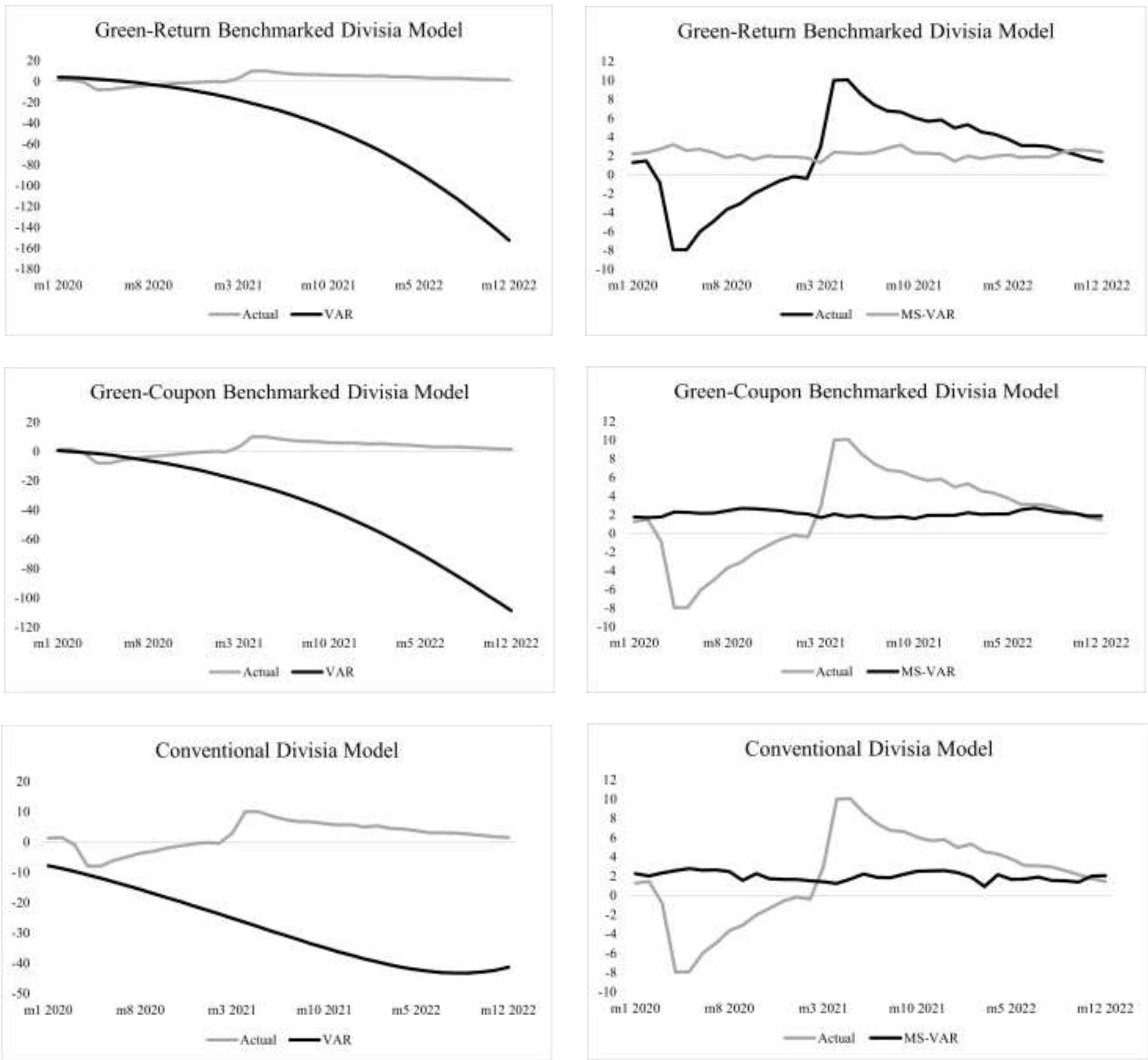
Notes: The table presents the forecasting evaluation for the t + 1 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the GUNBP in the models, respectively.

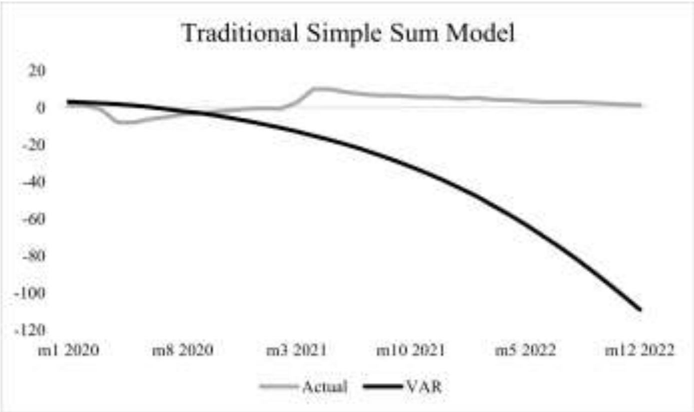
We finally integrate all three financial variables, i.e., EPU, FSI, GUNBP, into each of all four monetary models. Figure C.9.4 displays the t + 1 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. The forecasting pattern shown by all VAR specifications presents a significant deviation from those produced by the MS-VAR models. This divergence indicates a tendency towards a downward bias in the VAR models' projected estimates of the output gap, suggesting a systematic underestimation in the forecasts. This phenomenon could potentially be attributed to the presence of multicollinearity within the VAR framework, particularly when incorporating all three financial measures. The inferior performance of the VAR models is also confirmed by the values of RMSE and RAE in Table C.16.4, which reveals that the RMSE and RAE values of VAR models are markedly higher than those of MS-VAR models for each monetary model, with the discrepancy up to 1453.99%. This significant variance underscores the worse forecasting performance of the linear VAR models.

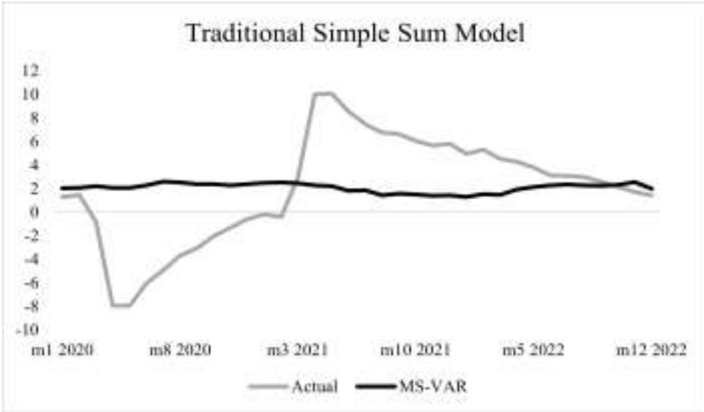
Similar to the results before, the green-return benchmarked Divisia MS-VAR model appears as the best model, yielding the lowest values in RMSE and RAE. This is followed by the green-coupon benchmarked Divisia MS-VAR model, being only 1.87% less accurate compared to the best model. The better performance of both green-benchmarked Divisia monetary aggregates when all financial measures are employed as control variables, also suggests a shift in investors' preferences towards green bonds.

Figure C.9.4 Forecasted (t + 1) and Actual Values of Output Gap from Monetary Models with Three Financial Measures Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 1) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added and the right column presents that by using the MS-VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added.

Table C.16.4 Evaluation Criteria for *t* + 1 Forecasts from Monetary Models with Three Financial Measures Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	67.3694	4.6334	53.0866	4.7206	34.2183	4.7519	49.0693	4.7694
RAE	13.4670	0.9263	10.6119	0.9436	6.8402	0.9499	9.8088	0.9534
RMSE-ratio	1453.99%	100%	1145.74%	101.88%	738.51%	102.56%	1059.03%	102.94%
RAE-ratio	1453.85%	100%	1145.62%	101.87%	738.44%	102.55%	1058.92%	102.93%

Notes: The table presents the forecasting evaluation for the t + 1 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and all three financial measures (EPU, FSI and GUNBP) in the models, respectively.

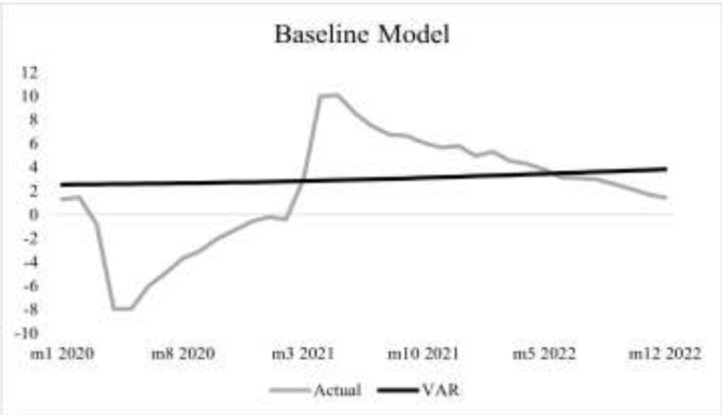
7.2 Three-Month Ahead Forecasts

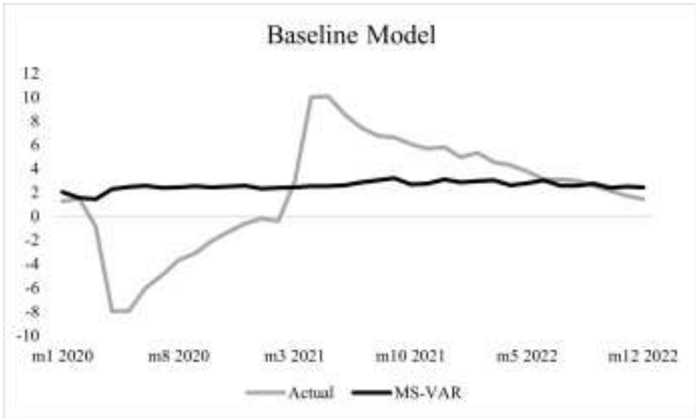
7.2.1 Baseline Model

We first present the t + 3 forecasts for the baseline model as a 1 × 2-panel in Figure C.10. Forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The forecasts generated by the MS-VAR model are quite similar to those produced by the VAR-model. Both models display a tendency to both overshoot and undershoot significant fluctuations in the output gap, a pattern that appears consistent with the one-month ahead forecasts. Turning to RMSE and RAE, both criteria reveal a parallel ranking of the models under evaluation. As indicated in Table C.17, the MS-VAR model marginally outperforms its counterpart, with a slightly lower values in both RMSE and RAE. Precisely, the difference is only 1.50% for RMSE and 1.61% for RAE.

Figure C.10 Forecasted (*t* + 3) and Actual Values of Output Gap from Baseline Model

VAR **MS-VAR**





Notes: The left side plot shows the forecasted (*t* + 3) and actual values of output gap for the baseline VAR model and the right side plots shows that for the baseline MS-VAR model.

Table C.17 Evaluation Criteria for *t* + 3 Forecasts from Baseline Model

	VAR	MS-VAR
RMSE	4.5193	4.4526
RAE	0.9043	0.8900
RMSE-ratio	101.50%	100%
RAE-ratio	101.61%	100%

Notes: The table presents the forecasting evaluation for the *t* + 3 forecasts from the baseline VAR and MS-VAR models, which includes the output gap and the real interest rate in the models.

7.2.2 Monetary Models

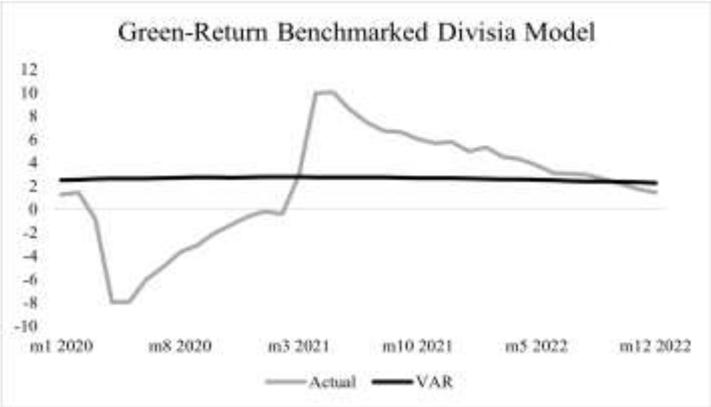
Building upon the aforementioned baseline model, we proceed to incorporate monetary aggregates into the framework to evaluate whether the inclusion of monetary variables enhances the model's *t* + 3 forecasting accuracy for the output gap. In Figure C.11.0, we present the *t* + 3 forecasts for the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model as a 4 × 2-panel. As before, forecasts (the black line) are compared to actual output gap (the grey line) in each panel. A significant difference between the VAR models and the MS-

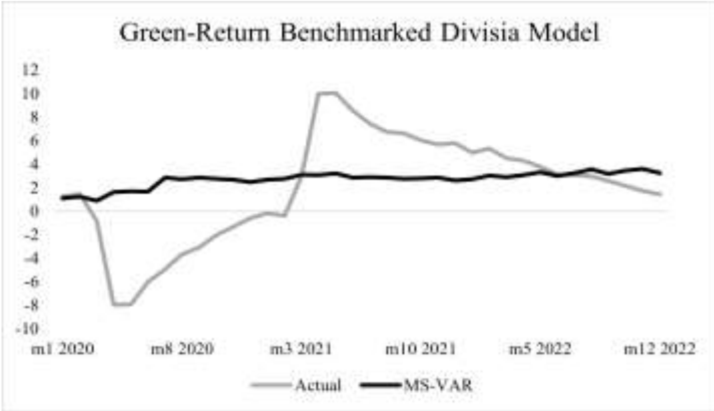
VAR models emerges in the conventional Divisa model that the forecasts generated by the VAR model exhibit an upward bias, notably overshooting the actual output gap. However, in other three monetary specifications, the forecasts derived from the MS-VAR models exhibit similarities to those from the VAR models. This tendency is similar to the pattern identified in the one-month forecasts.

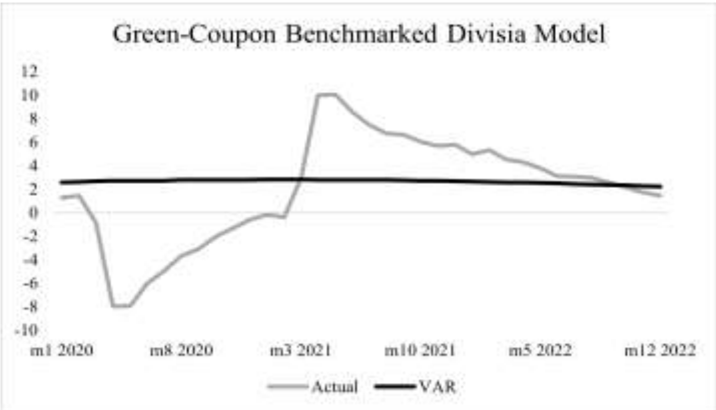
The fact that the linear VAR models performs worse than the non-linear MS-VAR models in forecasting is confirmed by the RMSE and RAE criteria for the three-month ahead forecasts presented in Table C.18.0. The VAR models demonstrate higher RMSE and RAE values relative to those of the MS-VAR models by 26.60% and 26.61% on average respectively, across each of the monetary models. Among all the MS-VAR models, the green-return benchmarked Divisia MS-VAR model is the best model, followed by the green-coupon benchmarked Divisia MS-VAR model, which shows 4.27% of both RMSE and RAE values lower that for the best model. In contrast, the traditional simple sum MS-VAR model ranks as the least effective model, yielding highest values of both RMSE and RAE in all MS-VAR specifications. This finding aligns with the previous studies that suggest Divisia monetary aggregates contain more information than their traditional simple sum counterpart, as shown by Barnett and Park (2023). Further, the observed improvement in the forecasting performance of the green-benchmarked Divisia monetary aggregates under a stable economic condition implies a shift in investors' preferences towards green investments.

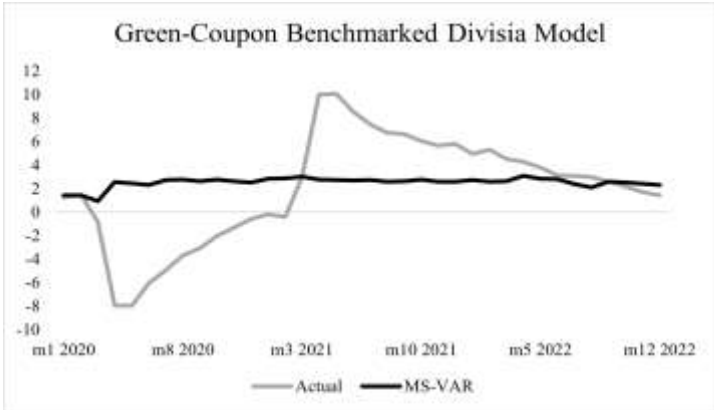
Figure C.11.0 Forecasted (t + 3) and Actual Values of Output Gap from Monetary Models

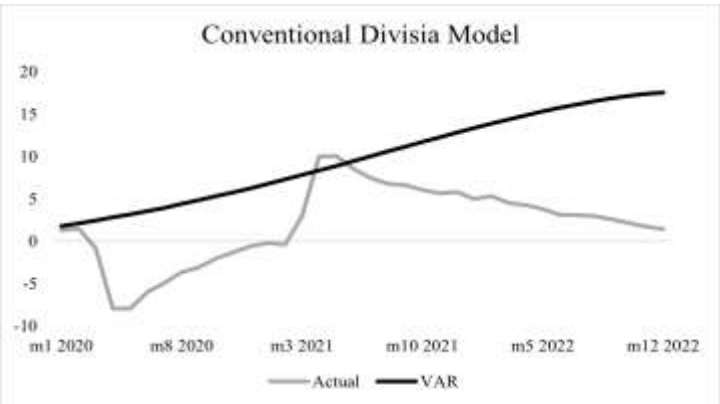
VAR **MS-VAR**

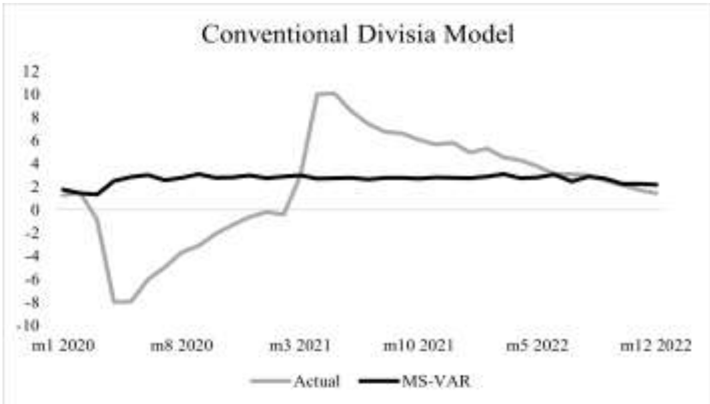


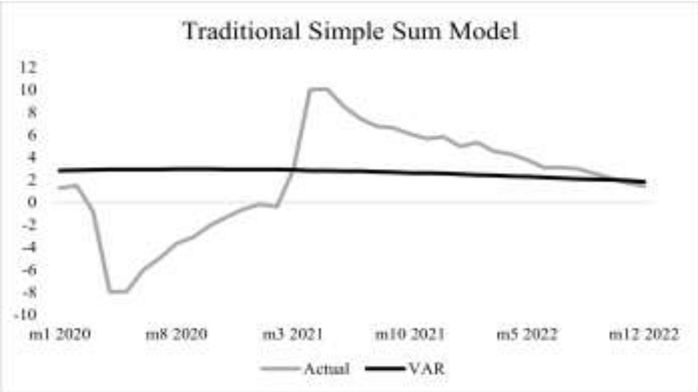


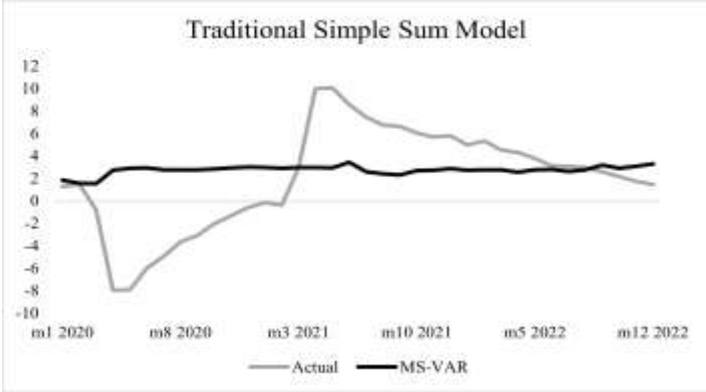












Notes: The plots show the forecasted (t + 3) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models and the right column presents that by using the MS-VAR method for four money models.

Table C.18.0 Evaluation Criteria for *t* + 3 Forecasts from Monetary Models

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.6007	4.3467	4.6170	4.5324	9.0288	4.5935	4.7227	4.6234
RAE	0.9197	0.8689	0.9229	0.9060	1.8048	0.9182	0.9441	0.9242
RMSE-ratio	105.84%	100%	106.22%	104.27%	207.72%	105.68%	108.65%	106.37%
RAE-ratio	105.85%	100%	106.21%	104.27%	205.54%	105.67%	108.65%	106.36%

Notes: The table presents the forecasting evaluation for the t + 3 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively.

7.2.3 Monetary Models with Financial Condition Measures

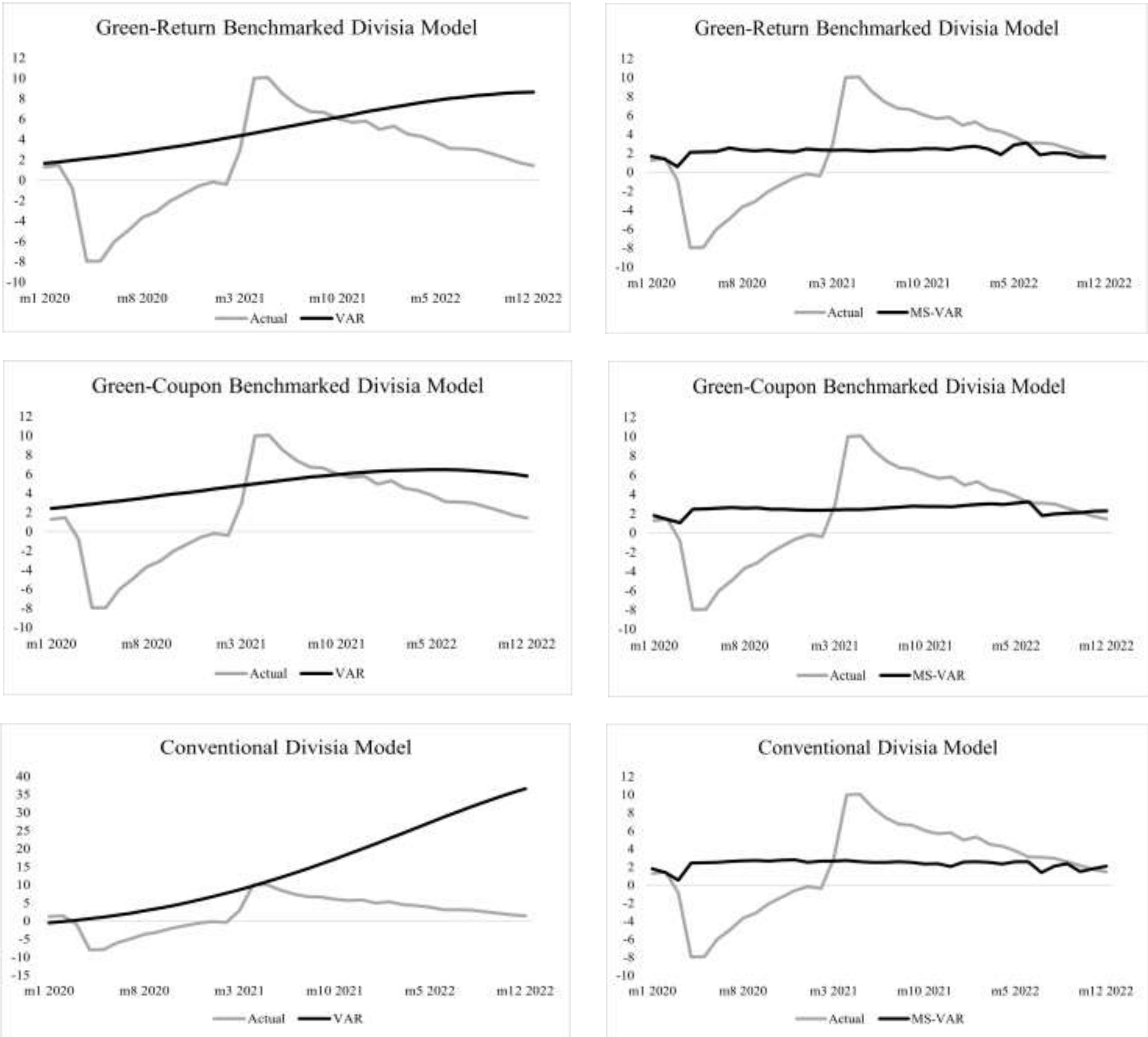
Based on the t + 3 forecasting results derived from the four monetary models above, we then incorporate the financial measures as control variables to examine the green-return benchmarked Divisia monetary aggregate maintains its status as the best forecaster in output gap prediction and also to evaluate the impact of these financial measures on the forecasting proficiency within each monetary model. In line with our prior approach, we employ the EPU, FSI and GUNBP as the financial control variables, both individually and jointly for each model specification.

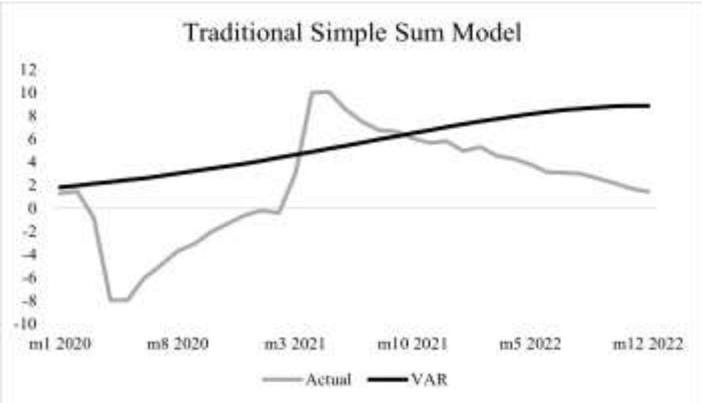
We first compare the forecasting results from four monetary models, each augmented with the inclusion of EPU. We present the $t + 3$ forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4×2-panel in Figure C.11.1. As before, the forecasts, represented by the black line, are compared to the actual output gap, depicted by the grey line, in each panel. Consistent with the tendency of the $t + 1$ forecasts, the $t + 3$ forecasts also show a divergence in the comparison between the VAR models with the MS-VAR models across all money frameworks. Specifically, the VAR models tend to produce the forecasts with an upward bias, indicating a consistent overestimation in their predictive values. Contrasting with the models that exclude financial measures, the VAR models demonstrate a reduction in performance with the incorporation of the EPU. This is further corroborated by the comparison of RMSE and RAE as presented in Table C.18.1 and C.18.0. The RMSE and RAE values for all the EPU-augmented monetary models, except the green-coupon benchmarked Divisia MS-VAR and the conventional MS-VAR models, are higher than those for the previously evaluated models that do not include the EPU by 18.89% on average, across each individual model. Such results point to a decrease in forecasting accuracy for both VAR and MS-VAR models with the integration of EPU.

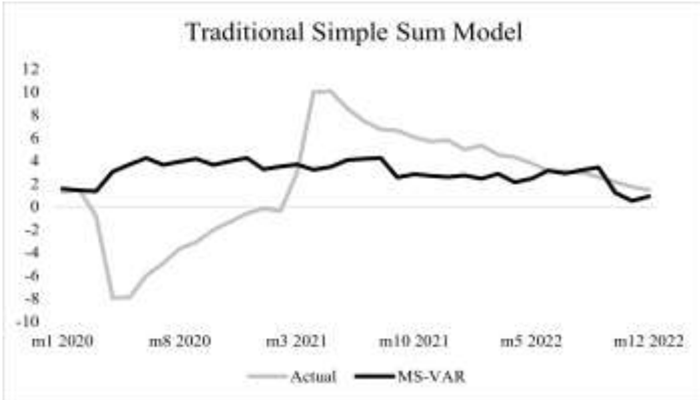
Consistently, the VAR models exhibit inferior performance compared to the MS-VAR models across all four monetary specifications. This is evidenced by the lower values of both RMSE and RAE for the MS-VAR models compared to those obtained from the VAR models by 23.19% on average in Table 18.1. The green-return benchmarked Divisia MS-VAR model is the best model, closely followed by the green-coupon benchmarked Divisia MS-VAR model, which exhibits only a marginal 0.41% decrease in forecasting accuracy. The conventional Divisia VAR model is ranked as the least effective model, underperforming by up to 366.26% compared to the highest-performing green-return benchmarked Divisia MS-VAR model. In addition, the traditional simple sum MS-VAR model ranks as the least effective model, yielding highest values of both RMSE and RAE among all MS-VAR models. This result further supports the superior performance of the Divisia monetary aggregates over their simple sum counterparts. When incorporating a financial measure as a control variable, the enhanced performance of green-benchmarked monetary aggregates additionally signals a shift in investors' preferences towards green bonds.

Figure C.11.1 Forecasted (t + 3) and Actual Values of Output Gap from Monetary Models with EPU Included

VAR | **MS-VAR**







Notes: The plots show the forecasted ($t + 3$) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with EPU added and the right column presents that by using the MS-VAR method for four money models with EPU added.

Table C.18.1 Evaluation Criteria for t + 3 Forecasts from Monetary Models with EPU Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.9889	4.5027	4.7933	4.5213	16.4918	4.5925	5.1534	4.8863
RAE	0.9973	0.9001	0.9582	0.9038	3.2967	0.9180	1.0301	0.9768
RMSE-ratio	110.78%	100%	106.45%	100.41%	366.26%	101.99%	114.45%	108.52%
RAE-ratio	110.80%	100%	106.45%	100.41%	366.26%	101.99%	114.44%	108.52%

Notes: The table presents the forecasting evaluation for the t + 3 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively.

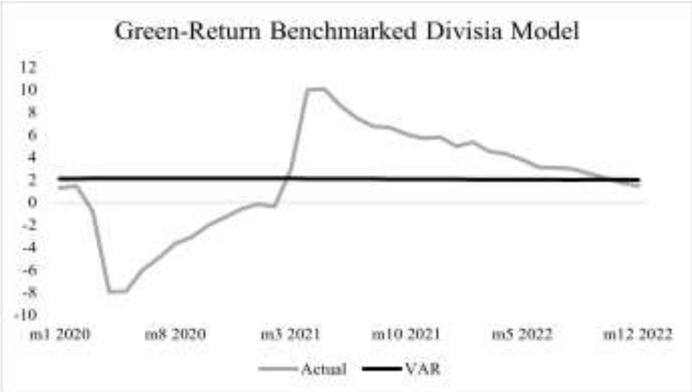
We then replace the EPU with the FSI and conduct a comparative analysis of the t + 3 forecasts from four monetary models with the financial measure incorporated. In Figure C.11.2, we present the t + 3 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. The forecasting pattern of the conventional Divisia VAR model shows a deviation from those produced by other models, suggesting an upward bias in its projection of the output gap. This observation contrasts with the outcomes derived from models that incorporate the EPU yet aligns with results from models that do not contain financial measures. from the forecasts obtained by the other models, which appears to give an upward bias for the forecasts of the output gap. These results are consistent with the one-month ahead forecasts.

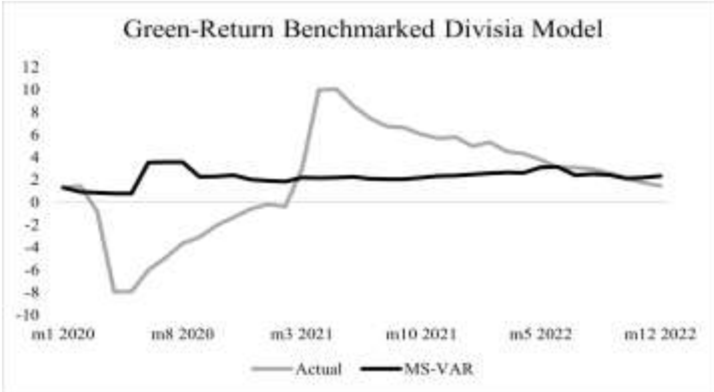
As evidenced in Table C.18.2, the values of RMSE and RAE for the MS-VAR models are slightly lower by 3.01% and 3.00% on average than those calculated from the VAR models respectively, across all four monetary specifications, which also highlights the superior forecasting performance of the non-linear MS-VAR model. The green-return benchmarked Divisia MS-VAR model, being consistent with the previous findings, is the best model. This is closely followed by the green-coupon benchmarked Divisia MS-VAR model and conventional Divisia MS-VAR model, only 0.62% and 0.66% worse than the best model, respectively. The traditional simple sum MS-VAR model is the worst MS-VAR model, suggesting the inferior performance of the simple sum monetary aggregate compared to the Divisia monetary aggregates. Moreover, the results from the monetary models with the integration of the FSI as a financial control variable reveal the increasing trend among investors towards sustainable investments.

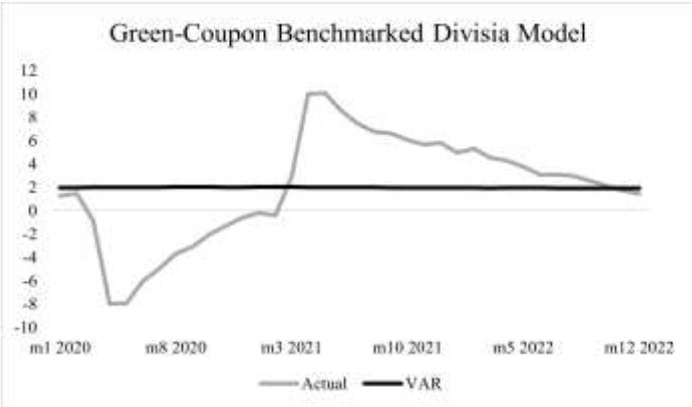
Figure C.11.2 Forecasted (*t* + 3) and Actual Values of Output Gap from Monetary Models with FSI Included

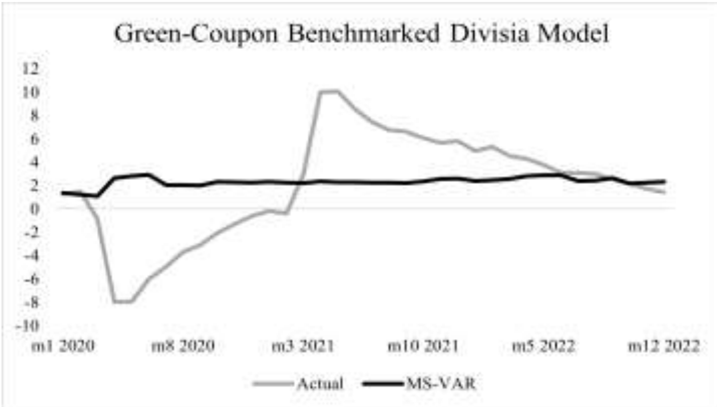
VAR

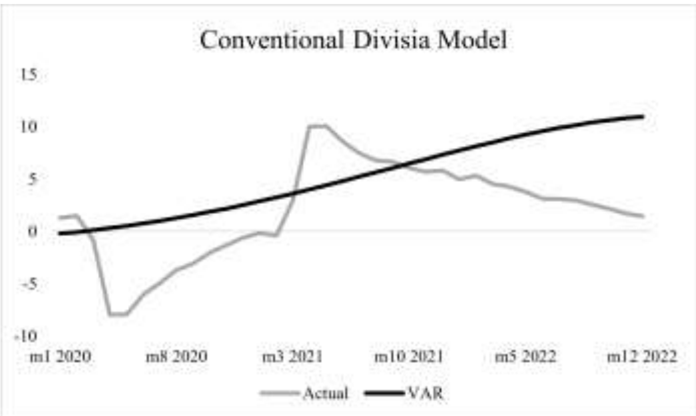
MS-VAR

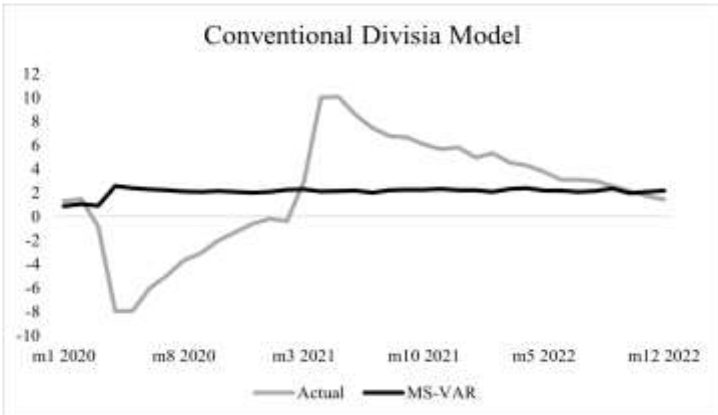


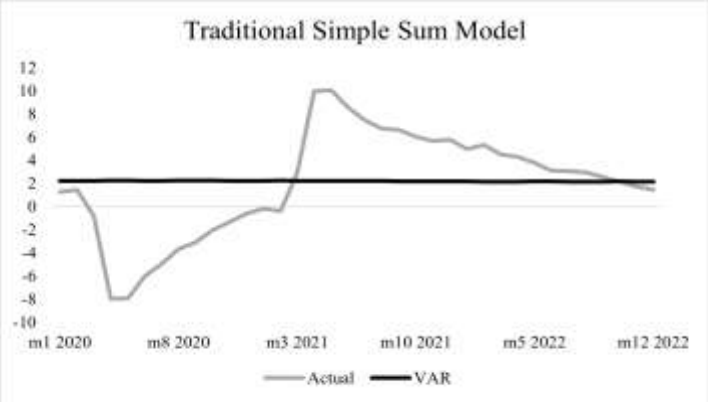


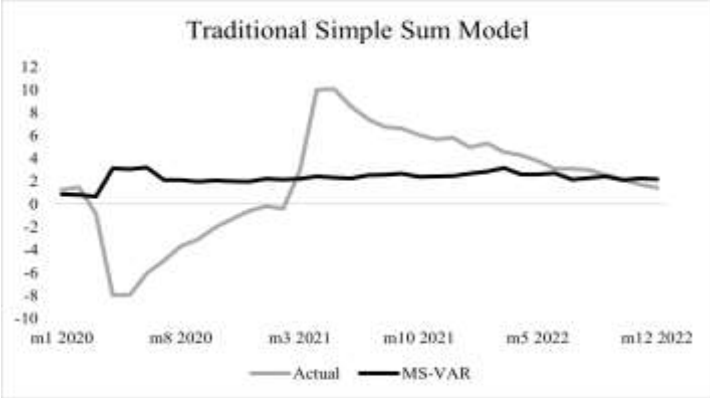












Notes: The plots show the forecasted (t + 3) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with FSI added and the right column presents that by using the MS-VAR method for four money models with FSI added.

Table C.18.2 Evaluation Criteria for t + 3 Forecasts from Monetary Models with FSI Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.5860	4.5470	4.5773	4.5753	5.1452	4.5772	4.5874	4.5834
RAE	0.9167	0.9089	0.9150	0.9146	1.0285	0.9150	0.9170	0.9162
RMSE-ratio	100.86%	100%	100.67%	100.62%	113.16%	100.66%	100.89%	100.80%
RAE-ratio	100.86%	100%	100.67%	100.63%	113.16%	100.67%	100.89%	100.80%

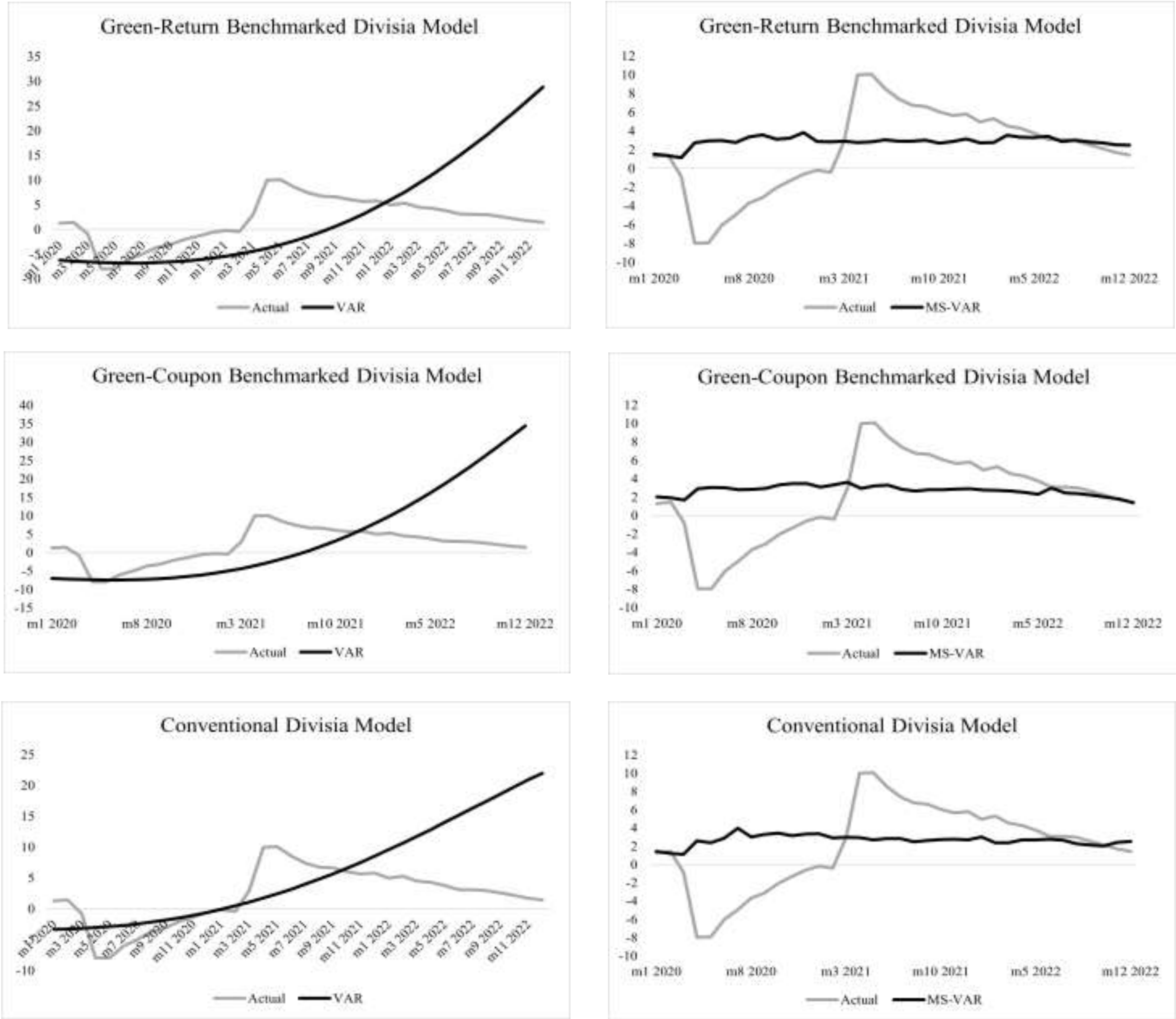
Notes: The table presents the forecasting evaluation for the t + 3 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively.

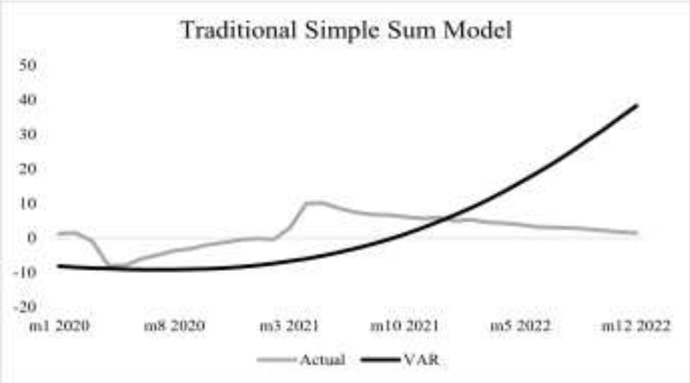
Using the GUNBP as the financial control variable, we then compare the forecasting results from four monetary models, each integrated with this financial measure. In Figure C.11.3, we present the t + 3 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. Consistent with the one-month ahead forecasts, the forecasting pattern for all VAR models displays a pronounced divergence from those produced by the MS-VAR models, indicating an upward bias in the forecasts of the output gap. This deviation contrasts with the results yielded by the models incorporating the FSI and the EPU, respectively. Turning to the RMSE and RAE in Table 18.3, both these values are higher than those in the monetary models which include the EPU or FSI as the financial measure, across each individual model. Precisely, both values of RMSE and RAE for the models incorporated with GUNBP are 49.5% higher than those for the models incorporated with EPU, and 49.57% higher than those for the models incorporated with FSI. This finding suggests that worse performance of the models with the GUNBP incorporated.

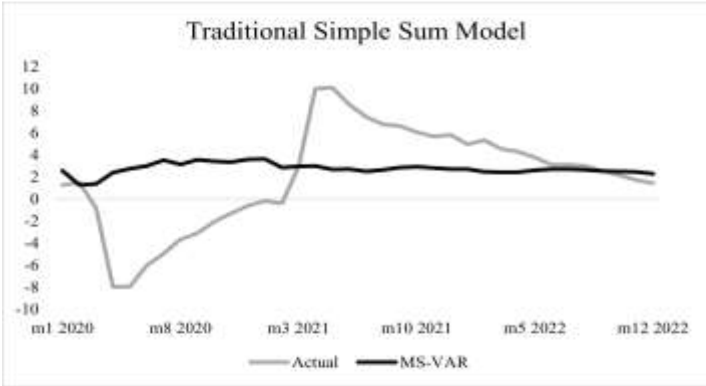
Table 18.3 clearly shows that the RMSE and RAE values for the MS-VAR models are significantly lower than those obtained from the VAR models across all four monetary specifications by 56.78% on average, which implies the superior forecasting capability of the MS-VAR model. In alignment with previous results, the green-return benchmarked Divisia MS-VAR model appears as the best model. It is closely followed by the green-coupon benchmarked Divisia MS-VAR model, the conventional Divisia MS-VAR model and traditional simple sum MS-VAR model, exhibiting only a 0.02%, 1.10% and 1.71% reduction in forecasting accuracy compared to the best model, respectively. The traditional simple sum VAR model is worst model, and it performs up to 297.66% worse than the best performing green-return benchmarked Divisa MS-VAR model. The better forecasting performance of the green-benchmarked Divisia monetary aggregates also reflects the shift of the preferences of investors towards favouring green investments under stable financial conditions.

Figure C.11.3 Forecasted (t + 3) and Actual Values of Output Gap from Monetary Models with GUNBP Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 3) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with GUNBP added and the right column presents that by using the MS-VAR method for four money models with GUNBP added.

Table C.18.3 Evaluation Criteria for t + 3 Forecasts from Monetary Models with GUNBP Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	10.5312	4.6643	12.4282	4.6654	8.2899	4.7154	13.8838	4.7440
RAE	2.1052	0.9324	2.4844	0.9326	1.6571	0.9426	2.7753	0.9483
RMSE-ratio	225.78%	100%	266.45%	100.02%	177.73%	101.10%	297.66%	101.71%
RAE-ratio	225.78%	100%	266.45%	100.02%	177.72%	101.09%	297.65%	101.71%

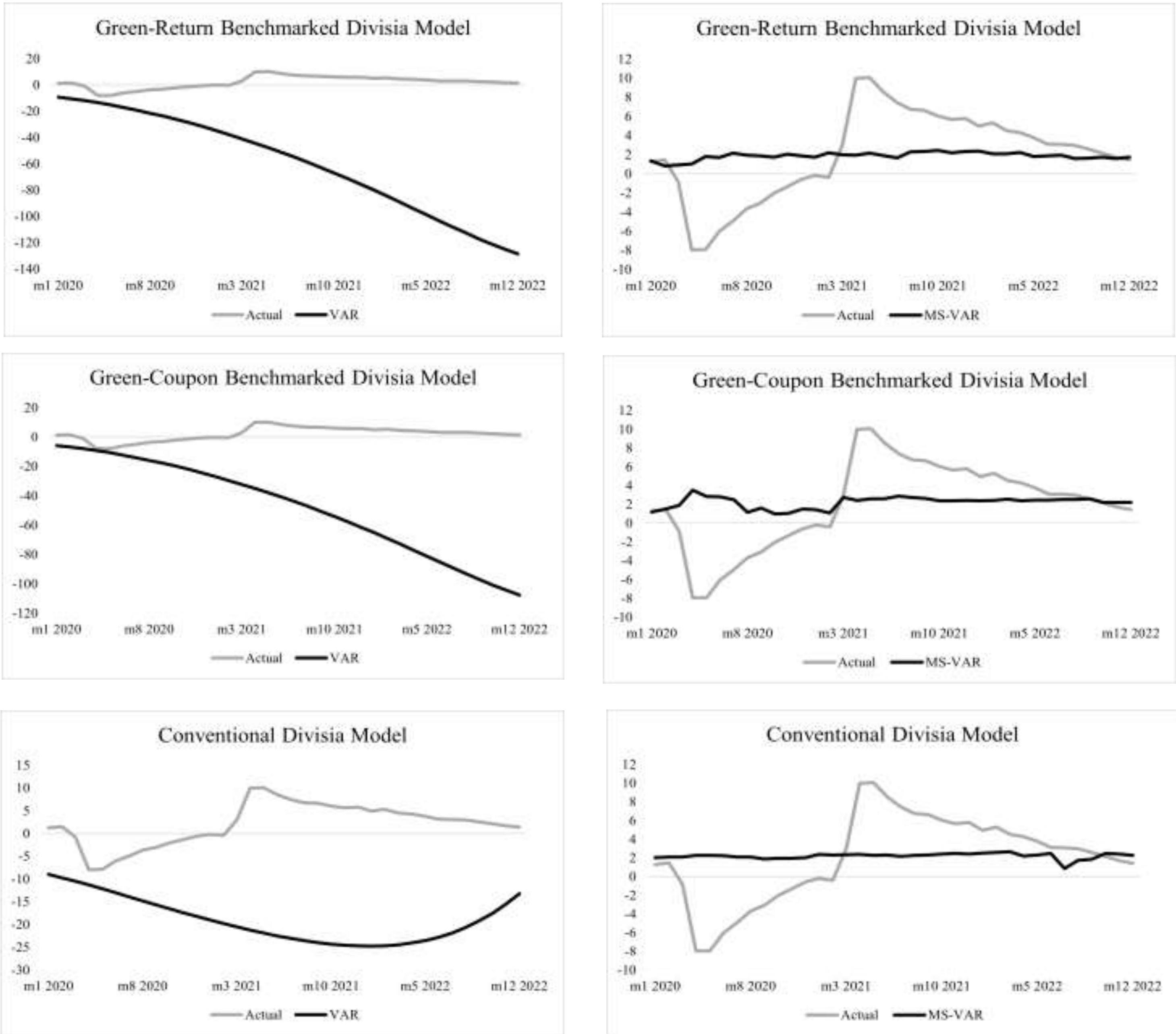
Notes: The table presents the forecasting evaluation for the t + 3 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the GUNBP in the models, respectively.

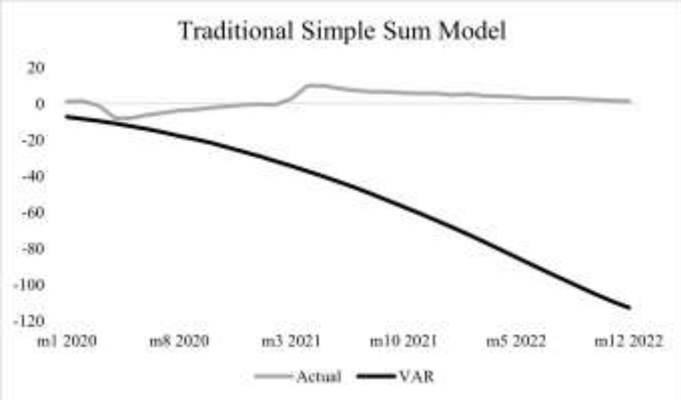
We finally include all three financial variables, i.e., EPU, FSI, GUNBP, into each of all four monetary models. Figure C.11.4 shows the t + 3 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. Consistent with the trends observed in the t + 1 forecasts, the t + 3 forecasting pattern exhibited by all VAR specifications presents a significant deviation from those generated by the MS-VAR models. This divergence indicates an inclination towards a downward bias in the projected output gap by the VAR models. The inferior performance of the VAR models is further substantiated by their significant higher values of RMSE and RAE than those for the MS-VAR model across each monetary model in Table C.18.4, with the difference up to 1653.99%. This significant variance highlights the worse forecasting performance of the linear VAR models.

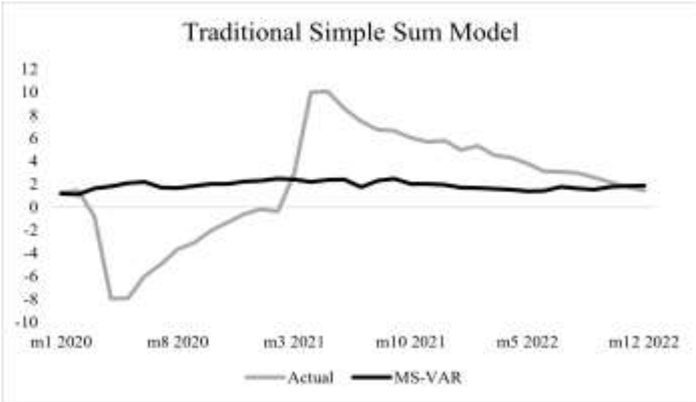
In line with the previous findings, the new green-return benchmarked Divisia MS-VAR model is the best forecasting model even when all financial measures are utilised as control variables, as it yields the lowest values in both RMSE and RAE. This is followed by the green-coupon benchmarked Divisia MS-VAR model, with only 1.17% less accurate compared to the best model.

Figure C.11.4 Forecasted ($t + 3$) and Actual Values of Output Gap from Monetary Models with Three Financial Measures Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 3) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added and the right column presents that by using the MS-VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added.

Table C.18.4 Evaluation Criteria for t + 3 Forecasts from Monetary Models with Three Financial Measures Included

	Green-Return Benchmarked Divisia Model		Green-Coupon Benchmarked Divisia Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	73.1330	4.4242	60.6168	4.4762	22.8191	4.5111	63.4016	4.5394
RAE	14.6191	0.8844	12.1172	0.8948	4.5615	0.9018	12.6738	0.9074
RMSE-ratio	1653.02%	100%	1370.12%	101.17%	515.78%	101.96%	1433.06%	102.60%
RAE-ratio	1653.00%	100%	1370.10%	101.18%	515.77%	101.97%	1433.04%	102.60%

Notes: The table presents the forecasting evaluation for the t + 3 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and all three financial measures (EPU, FSI and GUNBP) in the models, respectively.

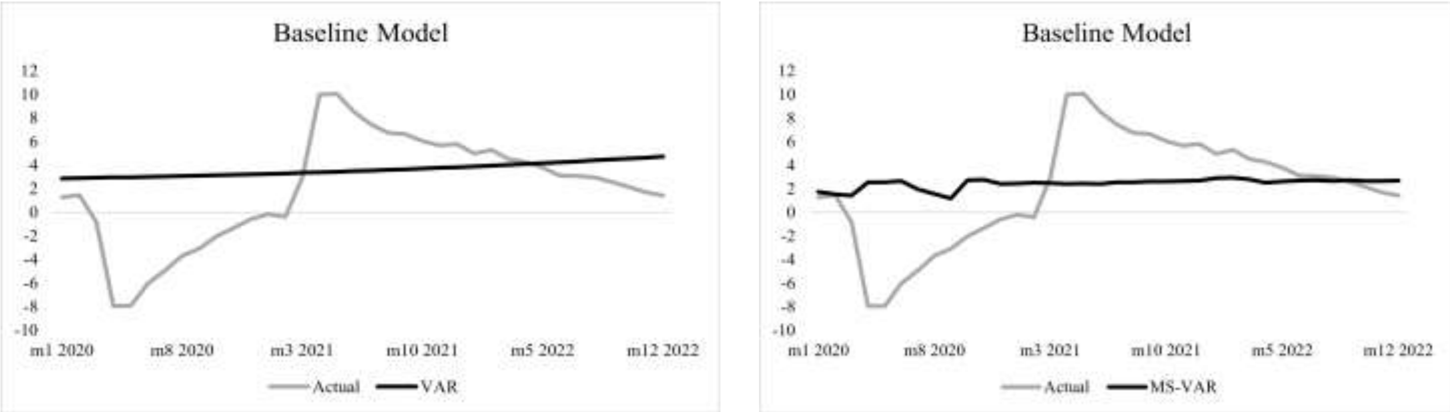
7.3 Six-Month Ahead Forecasts

7.3.1 Baseline Model

We first present the t + 6 forecasts for the baseline model as a 1 × 2-panel in Figure C.12. Forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The forecasting pattern by the MS-VAR model is similar to that by the VAR-model. Both models exhibit a tendency to both overshoot and undershoot large changes in the output gap, which aligns with the patterns observed in both one-month and three-month ahead forecasts. As delineated in Table C.19, a slightly lower value in both RMSE and RAE for the MS-VAR model than those for the VAR model suggests that the MS-VAR model outperforms its VAR counterpart in forecasting accuracy, by approximately 3.49%.

Figure C.12 Forecasted (*t* + 6) and Actual Values of Output Gap from Baseline Model

VAR **MS-VAR**



Notes: The left side plot shows the forecasted (*t* + 6) and actual values of output gap for the baseline VAR model and the right side plots shows that for the baseline MS-VAR model.

Table C.19 Evaluation Criteria for *t* + 6 Forecasts from Baseline Model

	VAR	MS-VAR
RMSE	4.6282	4.4720
RAE	0.9252	0.8939
RMSE-ratio	103.49%	100%
RAE-ratio	103.50%	100%

Notes: The table presents the forecasting evaluation for the *t* + 6 forecasts from the baseline VAR and MS-VAR models, which includes the output gap and the real interest rate in the models.

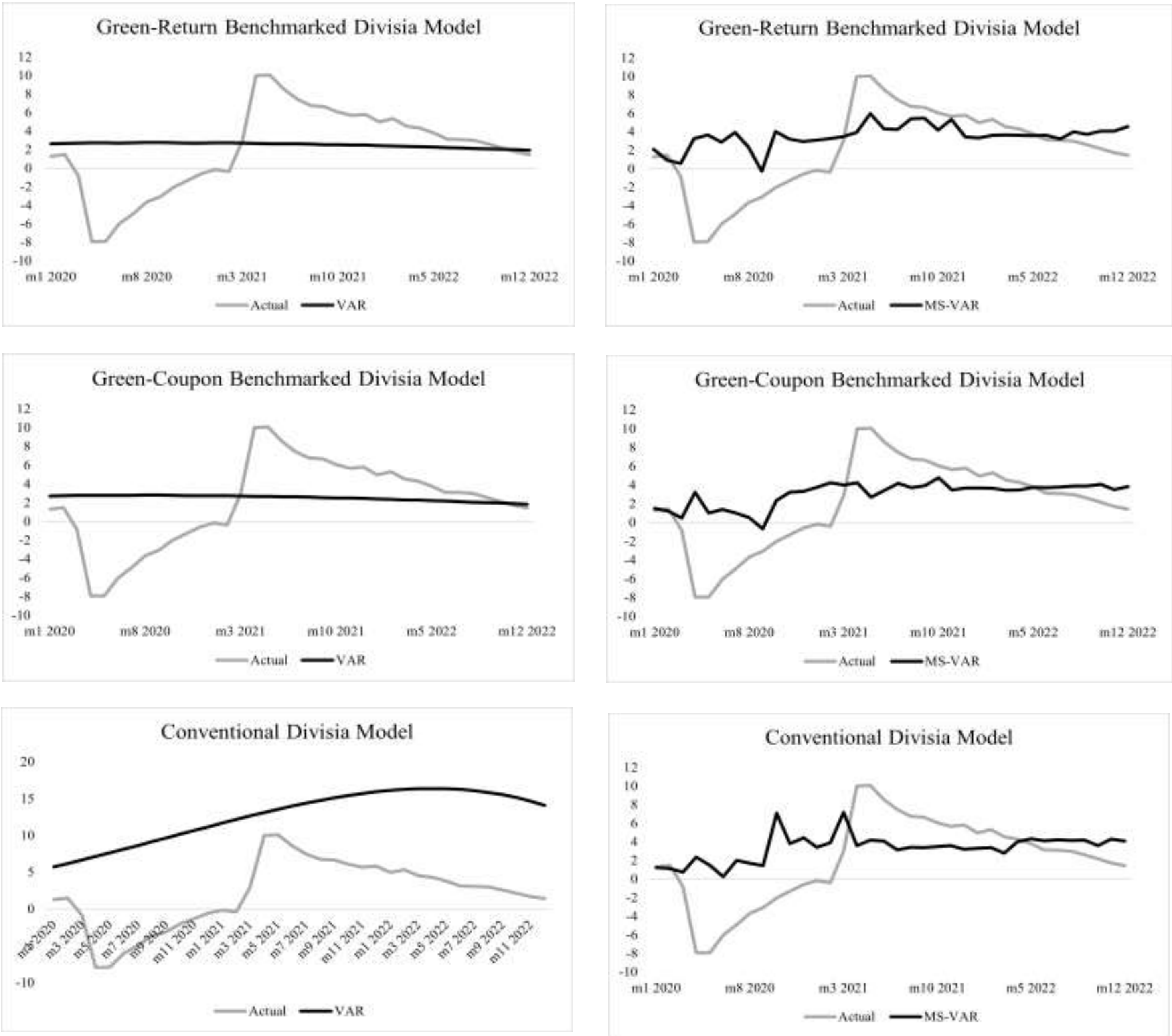
7.3.2 Monetary Models

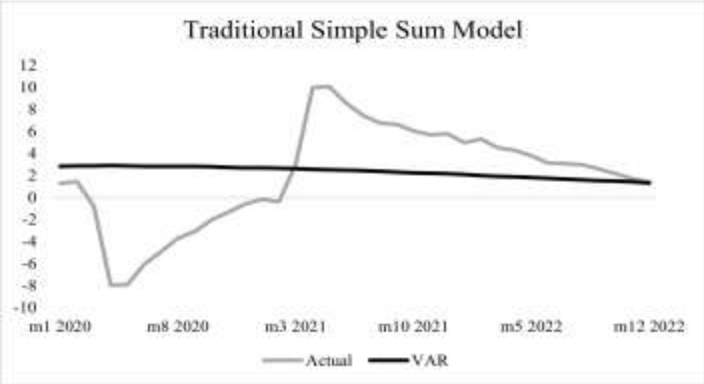
We next expand the baseline model above with the integration of monetary aggregates to assess whether the incorporation of monetary variables can improve the model's *t* + 6 forecasting accuracy for the output gap. In Figure C.13.0, we present the *t* + 6 forecasts for the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model as a 4 × 2-panel. As before, forecasts (the black line) are compared to actual output gap (the grey line) in each panel. Similar to the forecasting trend observed in both one-month and three month forecasts, a significant difference between the VAR models and the MS-VAR models appears in the conventional Divisa model that the VAR model produces a distinct upward bias. Although the forecasts from the MS-VAR models are similar to those obtained from the VAR models in other three monetary specifications, we note that there are few tendencies to overshoot large changes in output gap compared with standard VAR.

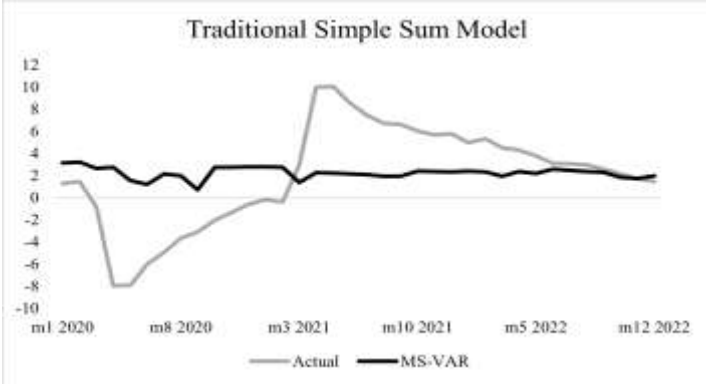
As shown in Table C.20.0, the values in both RMSE and RAE for the VAR models are higher than those for the MS-VAR models, across each monetary model, suggesting the inferior forecasting performance by the linear VAR models. Among all the MS-VAR models, the green-coupon benchmarked Divisia MS-VAR model is the best model in the six-month ahead forecasts, which is different from the results in one-month and three-month ahead forecasts. This difference may be due to the stability and predictability of the coupon rate in long term. The coupon rate, typically a fixed percentage of the bond's face value, offers a predictable and stable income stream. This fixed income characteristic is highly valued in long-term forecasting as it reduces uncertainty. In contrast, the rate of return, influenced by market dynamics, can exhibit significant variability over extended periods, leading a worse forecasting performance in longer term. The second best is the green-return benchmarked Divisia MS-VAR model followed by the conventional Divisia MS-VAR model. The traditional simple sum MS-VAR model performs worse than the other MS-VAR models. This finding suggests the superior performance of the Divisia monetary aggregates compared to their simple sum counterpart. Further, the observed improvement in the forecasting performance of the green-benchmarked Divisia monetary aggregates indicates the investors prefer to invest green bonds in stable economic conditions.

Figure C.13.0 Forecasted (*t* + 6) and Actual Values of Output Gap from Monetary Models

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 6) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models and the right column presents that by using the MS-VAR method for four money models.

Table C.20.0 Evaluation Criteria for t + 6 Forecasts from Monetary Models

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.6708	4.3435	4.6947	4.0378	11.1519	4.4159	4.8054	4.5540
RAE	0.9337	0.8683	0.9385	0.8072	2.2292	0.8827	0.9606	0.9103
RMSE-ratio	115.68%	107.57%	116.27%	100%	276.19%	109.36%	119.01%	112.78%
RAE-ratio	115.67%	107.57%	116.27%	100%	276.16%	109.35%	119.00%	112.77%

Notes: The table presents the forecasting evaluation for the t + 6 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively.

7.3.3 Monetary Models with Financial Condition Measures

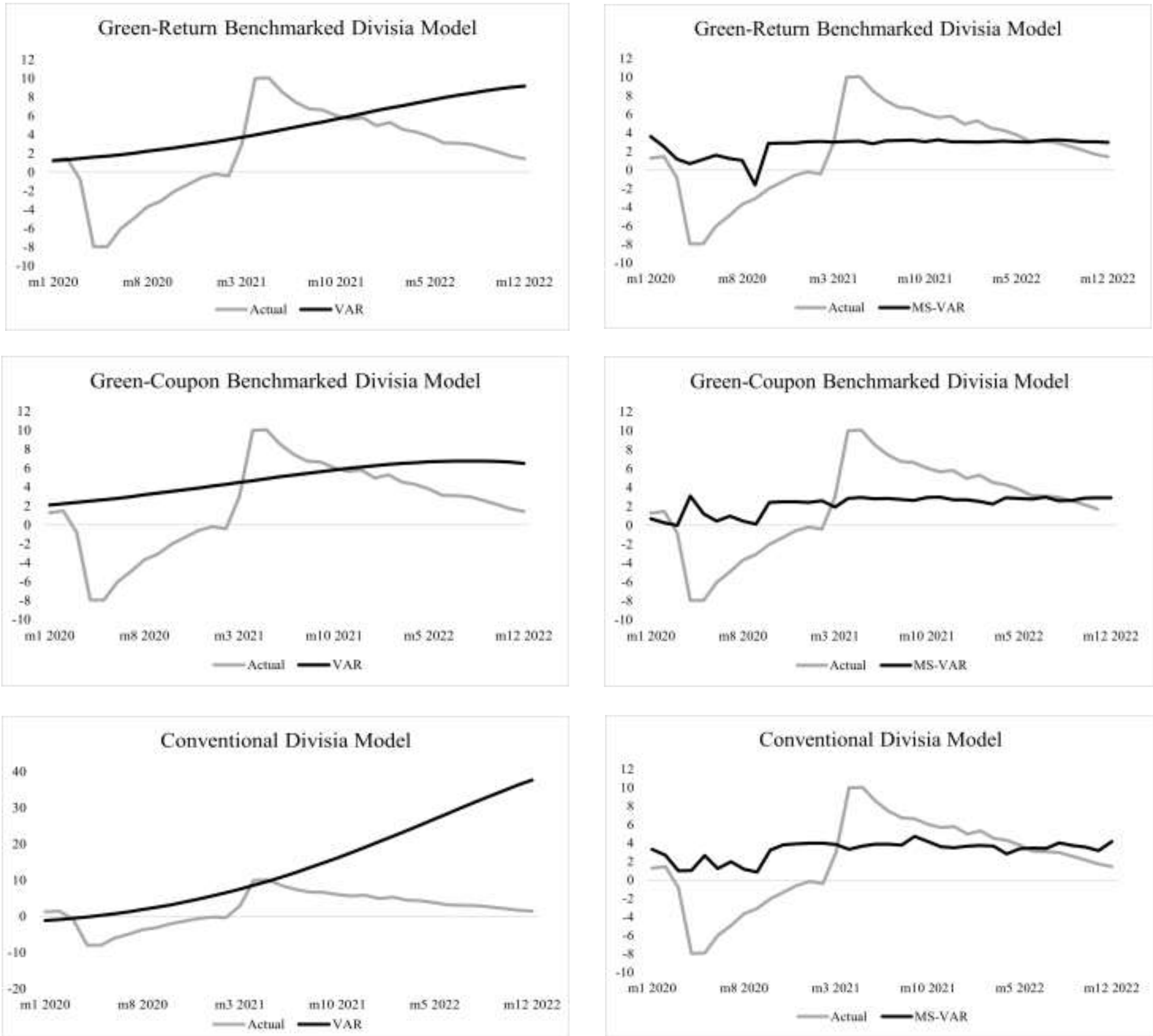
Based on the t + 6 forecasting results obtained from the four monetary models above, we then add the financial measures as control variables to evaluate whether the green-coupon benchmarked Divisia monetary aggregate sustains its superiority in the six-month ahead output gap forecasting. Using the EPU, FSI and GUNBP as the financial control variables, we also assess the influence of these financial measures on the forecasting accuracy of the monetary models, applied both individually and collectively across each model specification.

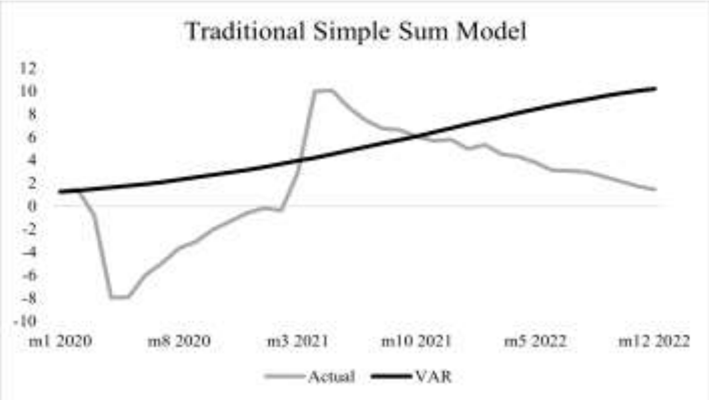
We first compare the forecasting results from the EPU-included monetary models. In Figure C.13.1, the $t + 6$ forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model are presented as a 4×2-panel. As before, the forecasts, represented by the black line, are compared to the actual output gap, depicted by the grey line, in each panel. Compared with the trends in the $t + 1$ and $t + 3$ forecasts, the $t + 6$ forecasts similarly exhibit a divergence between the VAR models with the MS-VAR models across all money frameworks. The VAR models demonstrate a tendency to generate the forecasts with an upward bias, especially in the conventional Divisia VAR model. Contrasting with the models that exclude financial measures, the VAR models show a reduction in performance with the incorporation of the EPU. This is further confirmed by the comparison of RMSE and RAE as presented in Table C.20.1 and C.20.0. The RMSE and RAE values for all the EPU-augmented VAR monetary models, are higher than those for the VAR monetary models that do not include the EPU, across each individual model, by 14.74% on average. However, the EPU-incorporated MS-VAR monetary models perform better than the MS-VAR monetary models without financial measures in six-month ahead forecasts, with the 4.56% lower values for both RMSE and RAE in EPU-incorporated MS-VAR monetary models.

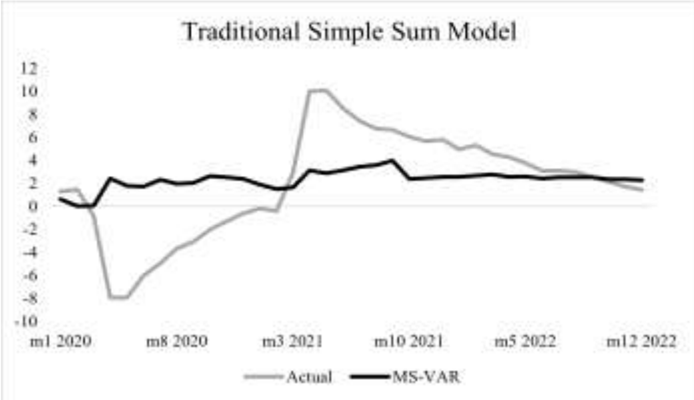
In all four monetary specifications, the VAR models consistently underperform when comparing with the MS-VAR models. This difference is reflected by the lower values of both RMSE and RAE for the MS-VAR models by 30.73% on average in Table C.20.1. The green-return benchmarked Divisia MS-VAR model is now, judging from RMSE and RAE, the best forecaster, closely followed by the green-coupon benchmarked Divisia MS-VAR model and conventional Divisia MS-VAR model, which exhibit 2.90% and 3.53% decrease in forecasting accuracy, respectively. The conventional Divisia VAR model is the worst performing model, underperforming by up to 408.26% compared to the highest-performing green-return benchmarked Divisia MS-VAR model. These results further support the superior performance of the green-benchmarked Divisia monetary aggregates over their conventional Divisia and traditional simple sum counterparts. Additionally, the better performance of green-benchmarked monetary aggregates when including the EPU as a control variable is indictive of a shifting trend in investors' preferences towards environmental investments.

Figure C.13.1 Forecasted (*t* + 6) and Actual Values of Output Gap from Monetary Models with EPU Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 6) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with EPU added and the right column presents that by using the MS-VAR method for four money models with EPU added.

Table C.20.1 Evaluation Criteria for t + 6 Forecasts from Monetary Models with EPU Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.8639	4.0058	4.7334	4.1221	16.3541	4.1471	5.1567	4.2438
RAE	0.9723	0.8008	0.9462	0.8240	3.2692	0.8290	1.0307	0.8483
RMSE-ratio	121.42%	100%	118.16%	102.90%	408.26%	103.53%	128.73%	105.94%
RAE-ratio	121.42%	100%	118.16%	102.90%	408.24%	103.52%	128.71%	105.93%

Notes: The table presents the forecasting evaluation for the t + 6 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively.

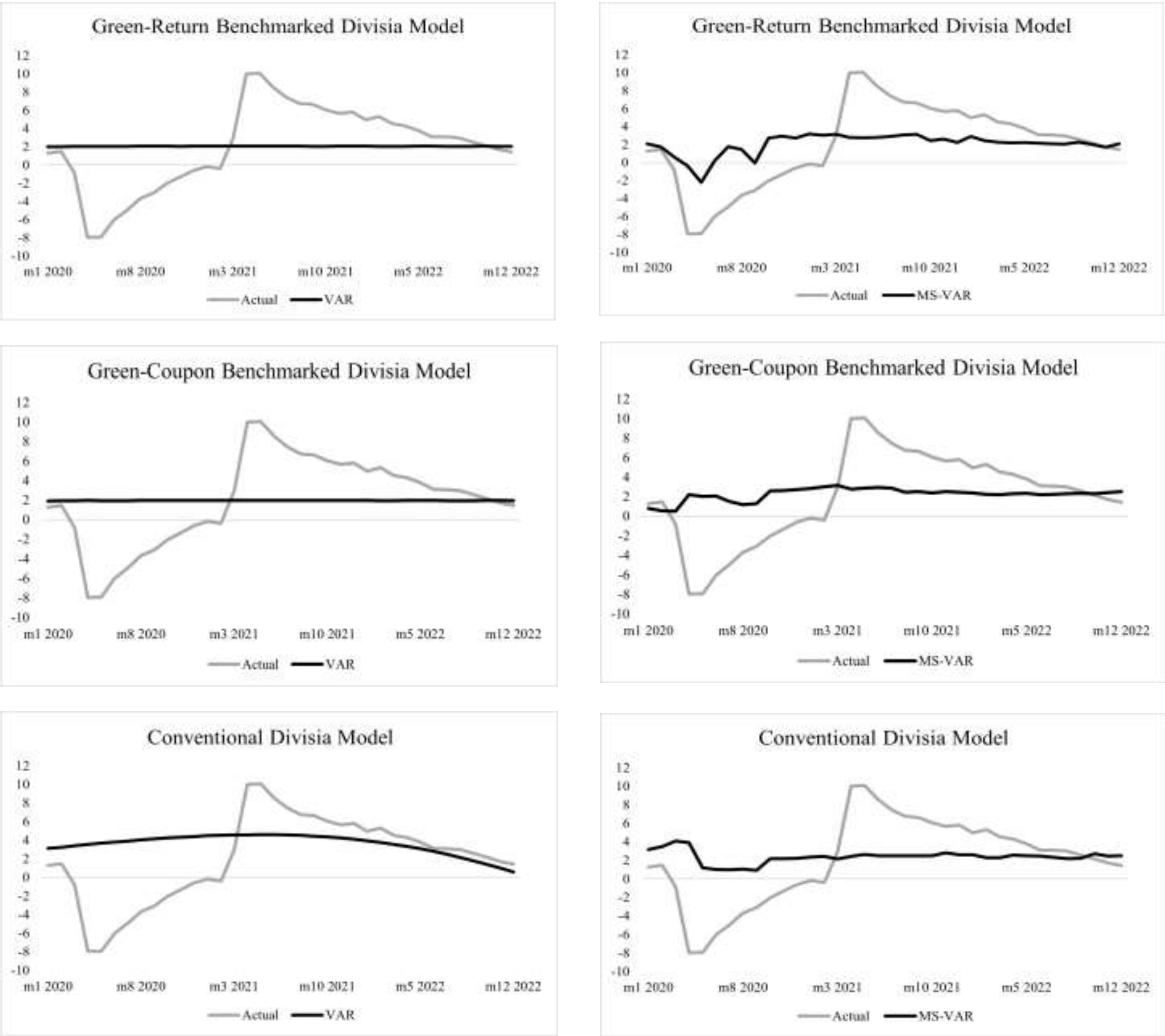
We then substitute the EPU with the FSI and compare the t + 6 forecasts for four monetary models with the financial measure incorporated. In Figure C.13.2, we present the t + 6 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. As before, the forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The forecasting patterns of all models show a high degree of similarity except for the conventional Divisia VAR model which indicates a downward bias in its projection of the output gap. These trends diverge from the results derived from models that incorporate the

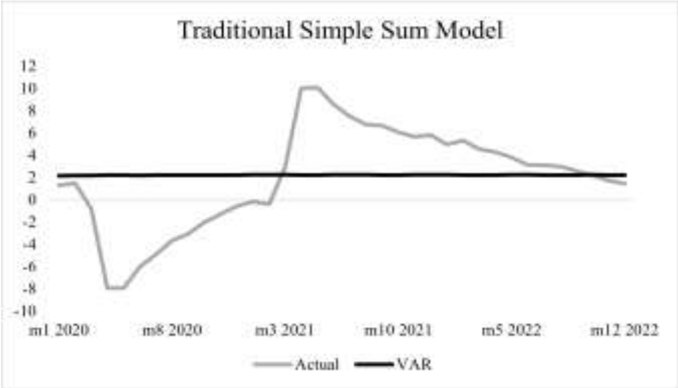
EPU yet are in accordance with the results from models that do not contain financial measures. These results are also consistent with the one-month and three-month ahead forecasts.

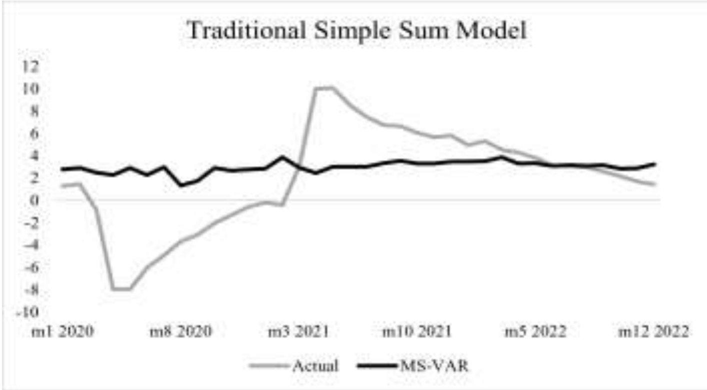
Table C.20.2 clearly illustrates that the values of RMSE and RAE for the MS-VAR models are 7.89% on average lower than those calculated from the VAR models across all four monetary specifications, indicating the superior forecasting performance of the MS-VAR models. The green-return benchmarked Divisia MS-VAR model is the best model, yielding the lowest values of both criteria. The second best is the green-coupon benchmarked Divisia MS-VAR model, which underperforms the best model by 11.86% in terms of RMSE. The conventional Divisia MS-VAR model and the traditional simple sum MS-VAR model perform approximately as well, with the conventional Divisia model yielding slightly lower values for both RMSE and RAE. With the integration of the FSI as a financial control variable, the better forecasting performance of the green-benchmarked monetary models also suggests the shift of the investors' preference towards green bonds during the stable financial conditions.

Figure C.13.2 Forecasted (t + 6) and Actual Values of Output Gap from Monetary Models with FSI Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 3) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with FSI added and the right column presents that by using the MS-VAR method for four money models with FSI added.

Table C.20.2 Evaluation Criteria for *t* + 6 Forecasts from Monetary Models with FSI Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.5620	3.8517	4.5631	4.3471	4.8065	4.4057	4.5593	4.4265
RAE	0.9119	0.7699	0.9122	0.8690	0.9608	0.8807	0.9114	0.8849
RMSE-ratio	118.44%	100%	118.47%	112.86%	124.79%	114.38%	118.37%	114.92%
RAE-ratio	118.44%	100%	118.48%	112.87%	124.79%	114.39%	118.38%	114.94%

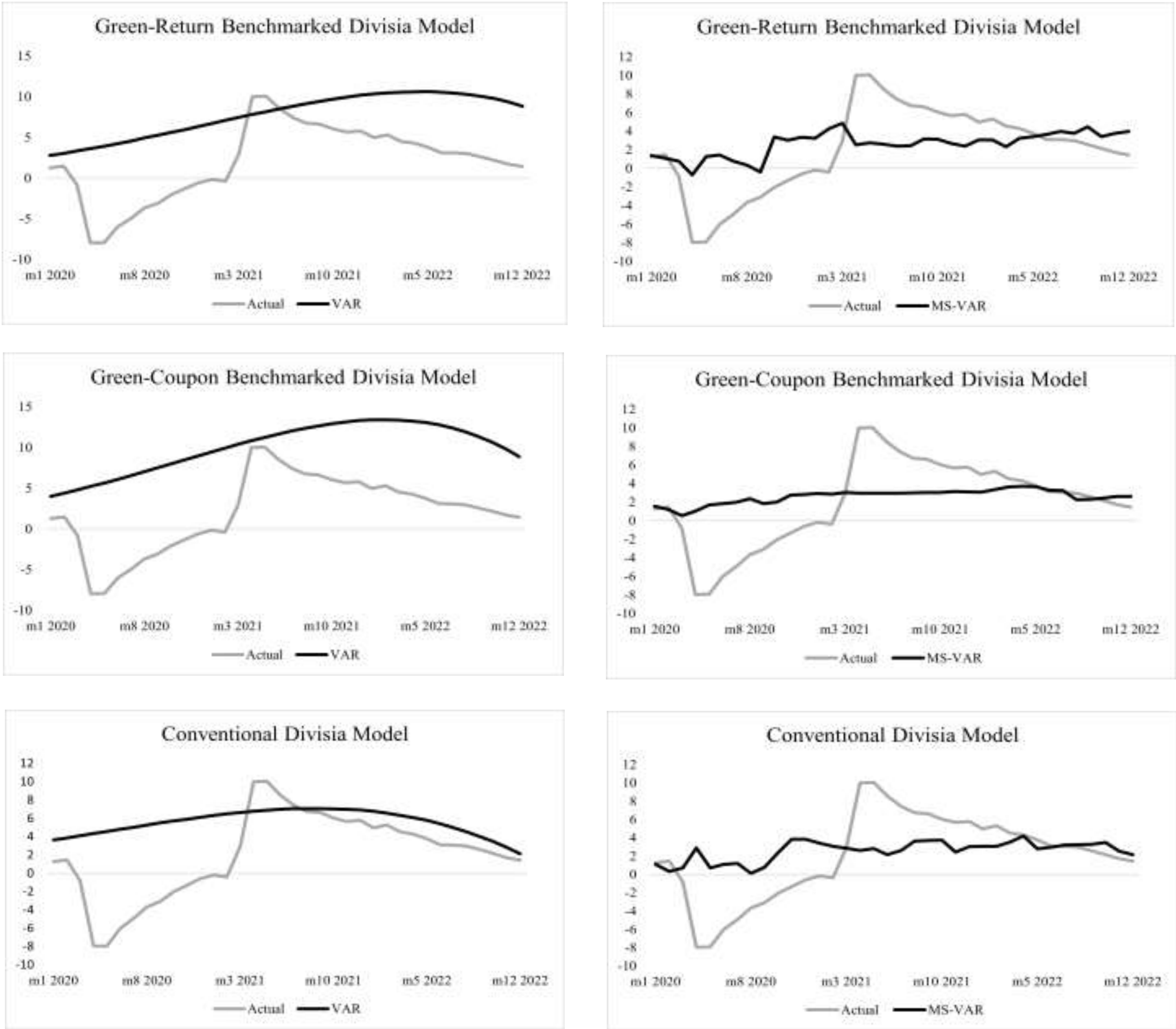
Notes: The table presents the forecasting evaluation for the t + 6 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively.

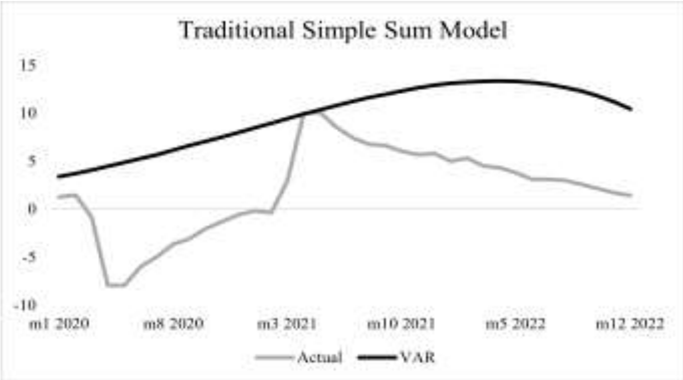
Using the GUNBP as the financial control variable, we then compare the forecasting results from four monetary models, each integrated with this financial measure. In Figure C.13.3, we present the t + 6 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. Consistent with the one-month and three-month ahead forecasts, the forecasting pattern for all VAR models shows an upward bias compared to that for the MS-VAR models across each monetary model. However, this trend is less pronounced than that in both one-month and three-month ahead forecasts. This deviation also contrasts with the results obtained from the models incorporating the FSI and the EPU, respectively.

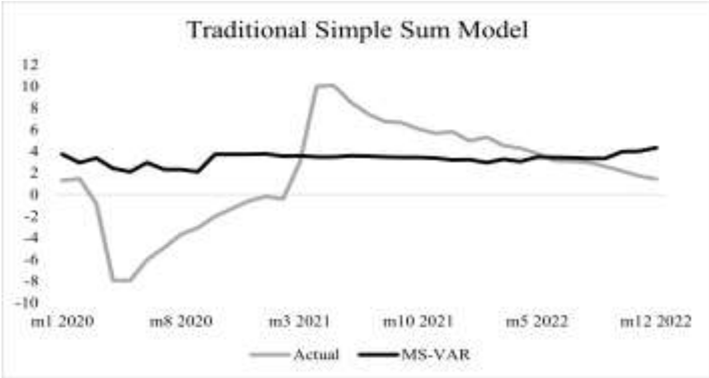
Turning to the RMSE and RAE in Table C.20.3, the values of both criteria for the MS-VAR models are 39.00% on average lower than those obtained from the VAR models across all four monetary specifications, which implies the superior forecasting performance of the MS-VAR model. The green-return benchmarked Divisia MS-VAR model, yielding the lowest values in both RMSE and RAE, is the best model, which is followed by the green-coupon benchmarked Divisia MS-VAR model, the conventional Divisia MS-VAR model and traditional simple sum MS-VAR model, exhibiting only a 0.63%, 1.38% and 9.19% reduction in forecasting accuracy compared to the best model, respectively. Aligned with the pervious findings, the superior forecasting performance of the green-benchmarked Divisia monetary aggregates also indicates a growing inclination of investors towards environmentally sustainable investment options in a stable economic condition.

Figure C.13.3 Forecasted (t + 6) and Actual Values of Output Gap from Monetary Models with GUNBP Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 6) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with GUNBP added and the right column presents that by using the MS-VAR method for four money models with GUNBP added.

Table C.20.3 Evaluation Criteria for *t* + 6 Forecasts from Monetary Models with GUNBP Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	6.5857	4.1307	8.5462	4.1568	5.3603	4.1876	8.2738	4.5103
RAE	1.3165	0.8257	1.7084	0.8309	1.0715	0.8371	1.6539	0.9016
RMSE-ratio	159.43%	100%	206.89%	100.63%	129.77%	101.38%	200.30%	109.19%
RAE-ratio	159.44%	100%	206.90%	100.63%	129.77%	101.38%	200.30%	109.19%

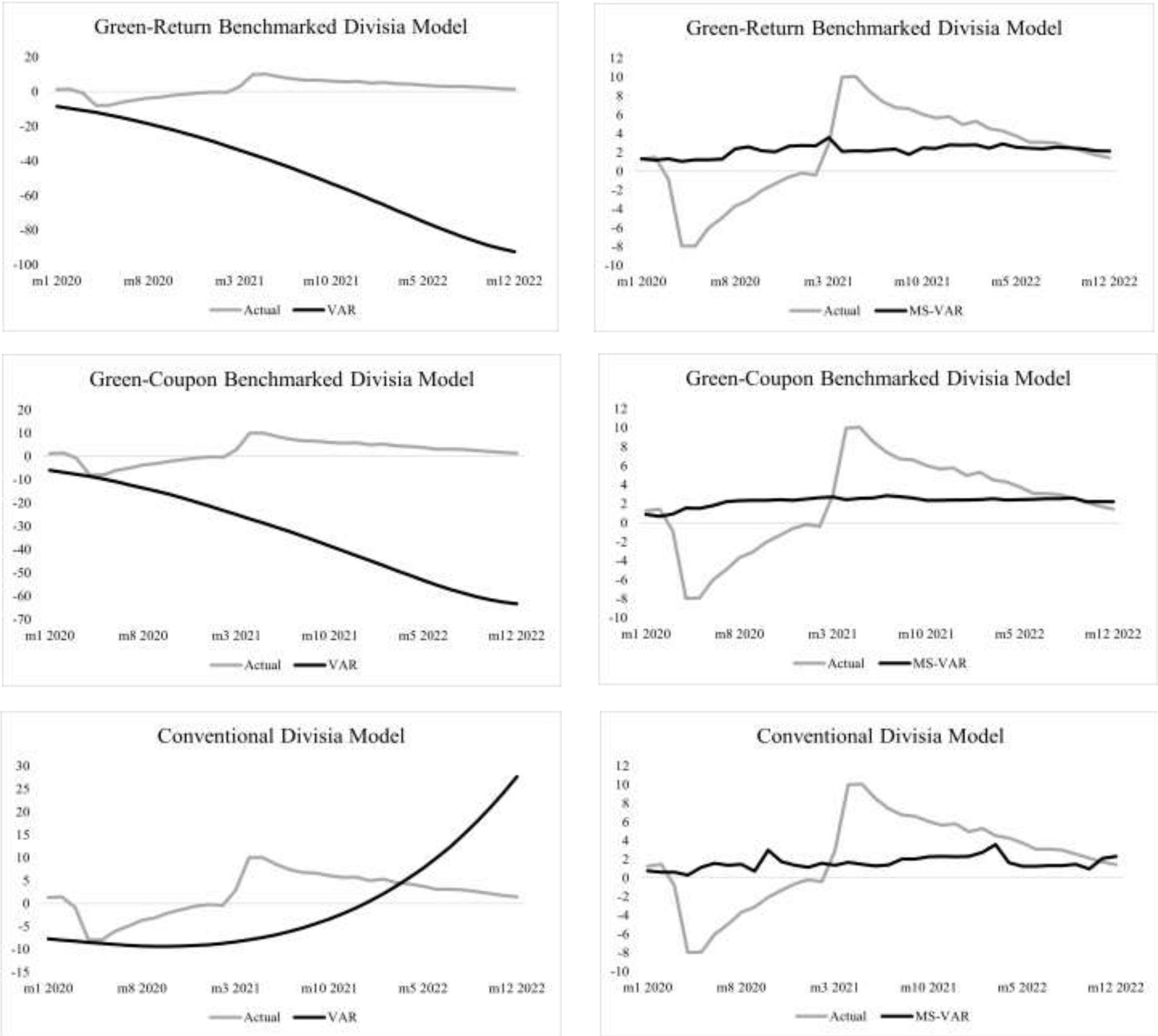
Notes: The table presents the forecasting evaluation for the t + 6 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the GUNBP in the models, respectively.

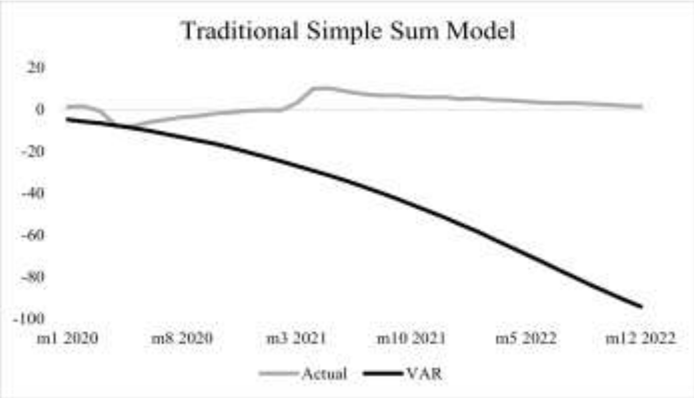
We finally include all three financial variables, i.e., EPU, FSI, GUNBP, into each of all four monetary models. Figure C.13.4 shows the t + 6 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. The t + 6 forecasting trends by all VAR specifications present a notable deviation from those by the MS-VAR models. This divergence signifies a tendency towards a downward bias in the projected output gap by the VAR models, with the exception of the conventional Divisia VAR model which, conversely, produces the forecasts with an upward bias. The suboptimal performance of the VAR models is further evidenced by their substantially higher values of RMSE and RAE than those for the MS-VAR model across each monetary model in Table C.20.4, with the difference up to 1304.16% compared to the best model.

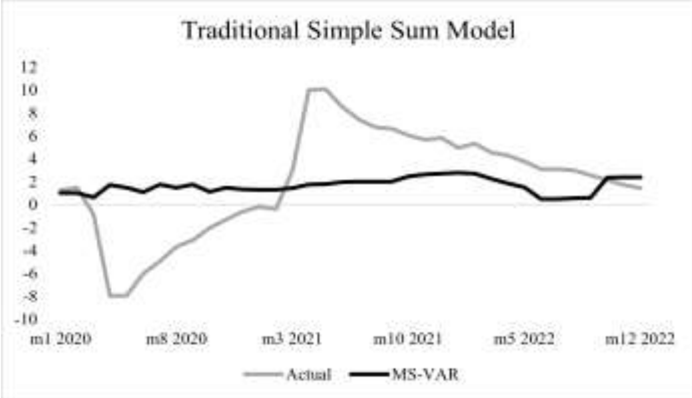
The best model, being consistent with the previous findings, is the green-return benchmarked Divisia MS-VAR model even when all financial measures are utilised as control variables, as it yields the lowest values in both RMSE and RAE. This is followed by the green-coupon benchmarked Divisia MS-VAR model, with only 0.63% less accurate compared to the leading model.

Figure C.13.4 Forecasted (t + 6) and Actual Values of Output Gap from Monetary Models with Three Financial Measures Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 6) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added and the right column presents that by using the MS-VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added.

Table C.20.4 Evaluation Criteria for t + 6 Forecasts from Monetary Models with Three Financial Measures Included

	Green-Return Benchmarked Divisia Model		Green-Coupon Benchmarked Divisia Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	56.4921	4.3317	41.0453	4.3588	11.0592	4.3885	52.3055	4.3973
RAE	11.2926	0.8659	8.2049	0.8713	2.2107	0.8773	10.4557	0.8790
RMSE-ratio	1304.16%	100%	947.56%	100.63%	255.31%	101.31%	1207.51%	101.51%
RAE-ratio	1304.15%	100%	947.56%	100.62%	255.31%	101.32%	1207.50%	101.51%

Notes: The table presents the forecasting evaluation for the t + 6 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and all three financial measures (EPU, FSI and GUNBP) in the models, respectively.

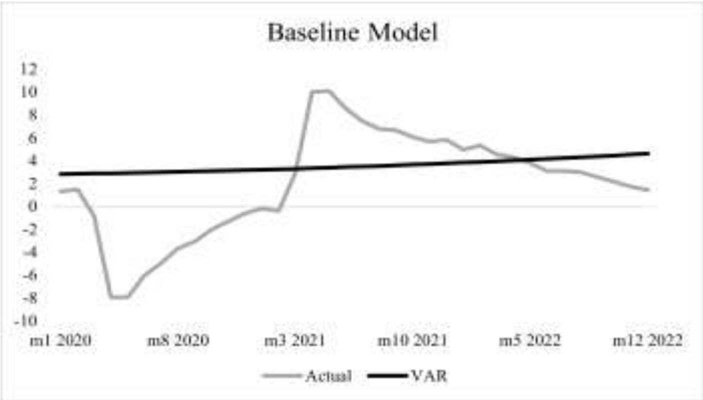
7.4 Nine-Month Ahead Forecasts

7.4.1 Baseline Model

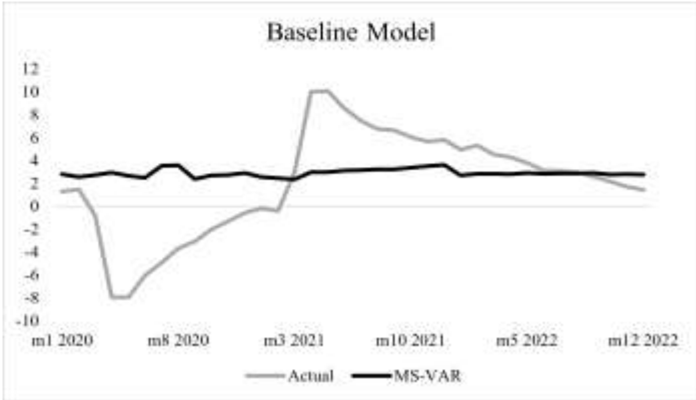
We first present the t + 9 forecasts for the baseline model as a 1 × 2-panel in Figure C.14. Forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The forecasting pattern by the MS-VAR model is similar to that by the VAR-model. Both models show a tendency to both overshoot and undershoot large changes in the output gap, which aligns with the patterns observed in both one-month, three-month and six-month ahead forecasts for the baseline model. Table C.21 illustrates that the values of both RMSE and RAE for the VAR model is marginally higher than those for the MS-VAR model, only by 0.85%, revealing a negligible difference in their nine-month ahead forecasting performance.

Figure C.14 Forecasted (*t* + 9) and Actual Values of Output Gap from Baseline Model

VAR



MS-VAR



Notes: The left side plot shows the forecasted (*t* + 9) and actual values of output gap for the baseline VAR model and the right side plots shows that for the baseline MS-VAR model.

Table C.21 Evaluation Criteria for *t* + 9 Forecasts from Baseline Model

	VAR	MS-VAR
RMSE	4.6027	4.5641
RAE	0.9201	0.9124
RMSE-ratio	100.85%	100%
RAE-ratio	100.84%	100%

Notes: The table presents the forecasting evaluation for the *t* + 9 forecasts from the baseline VAR and MS-VAR models, which includes the output gap and the real interest rate in the models.

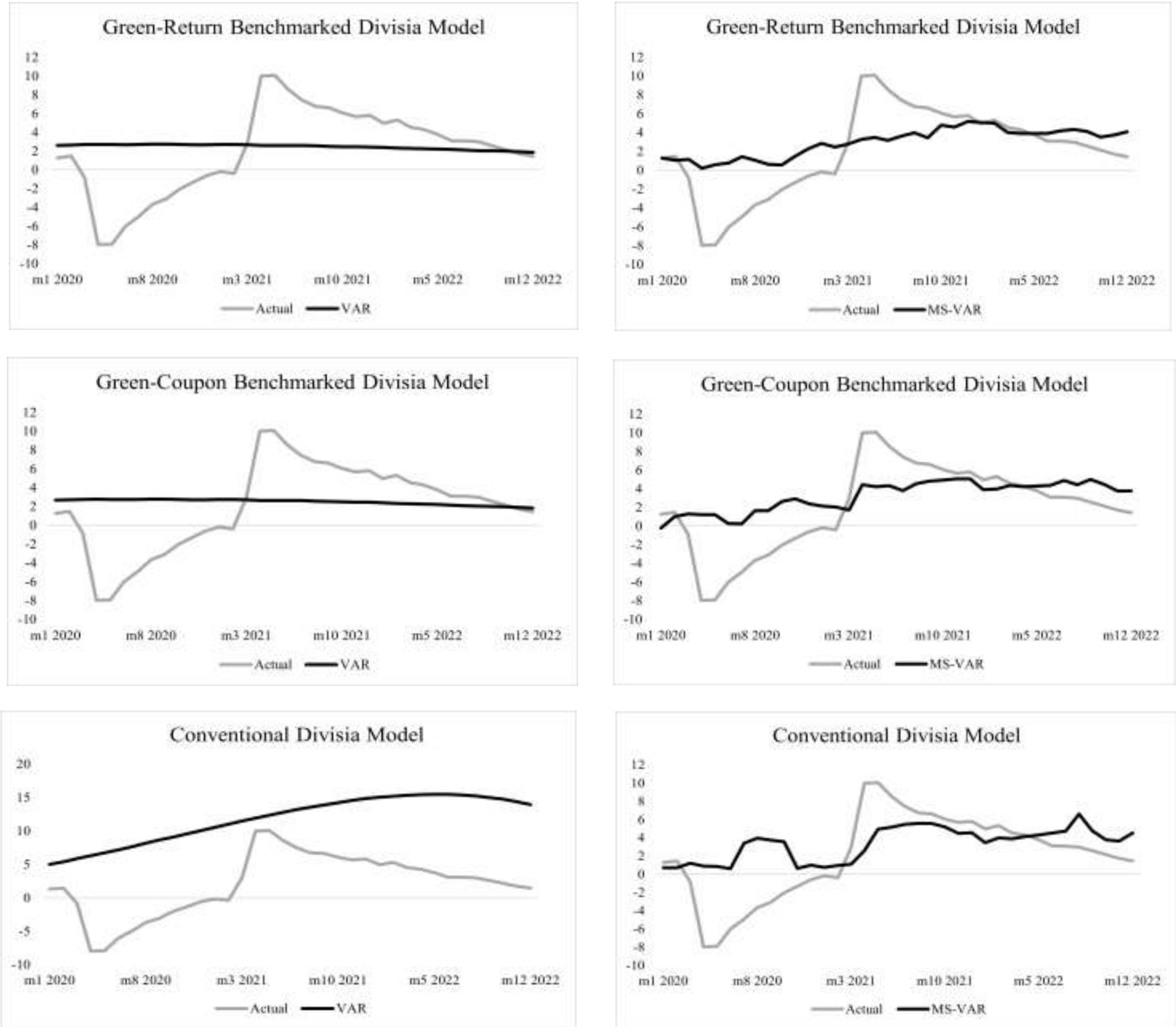
7.4.2 Monetary Models

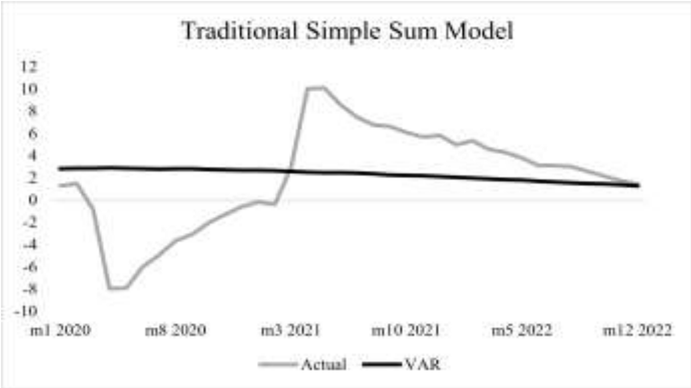
Based on the baseline model above, we then augment the model with the inclusion of monetary aggregates to evaluate whether the incorporation of monetary variables contributes to an improvement in the forecasting accuracy for the nine-month ahead output gap forecasts. In Figure C.15.0, we present the *t* + 9 forecasts for the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model as a 4 × 2-panel. As before, forecasts (the black line) are compared to actual output gap (the grey line) in each panel. A significant discrepancy is observed between the conventional Divisia VAR model and the conventional Divisia MS-VAR model. Specifically, the VAR model shows a pronounced upward bias. The forecasting pattern from the MS-VAR models is similar to those obtained from the VAR models in other three monetary specifications. However, it is notable that there are few tendencies to overshoot large changes in output gap compared with standard VAR. These results are evidenced by the lower values of both RMSE and RAE for all the MS-VAR models than those for the VAR models by 29.16% on average, across each monetary model in Table C.22.0, indicating the worse forecasting of the linear models.

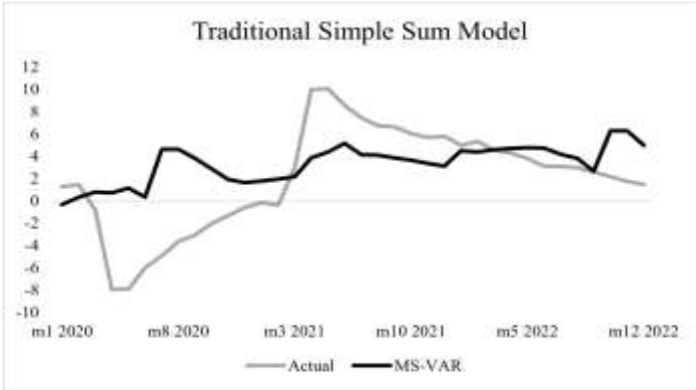
Among all the MS-VAR models, the green-return benchmarked Divisia MS-VAR model is the best model in the nine-month ahead forecasts according to the values of RMSE and RAE in Table C.22.0. The second best is the green-coupon benchmarked Divisia MS-VAR model followed by the conventional Divisia MS-VAR model. The traditional simple sum MS-VAR model performs worse than the other MS-VAR models. This observation, in alignment with three-month and six-month ahead forecasting results, suggests the superior performance of the Divisia monetary aggregates compared to their simple sum counterpart. The improved forecasting performance of the green-benchmarked Divisia monetary aggregates also implies the preferences of investors towards the green bonds in stable economic conditions. All these results are consistent with the forecasting outcomes identified in one-month and three-month forecasts.

Figure C.15.0 Forecasted (*t* + 9) and Actual Values of Output Gap from Monetary Models

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 9) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models and the right column presents that by using the MS-VAR method for four money models.

Table C.22.0 Evaluation Criteria for *t* + 9 Forecasts from Monetary Models

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.6746	3.6482	4.6901	3.6954	10.3087	3.9737	4.8095	4.2307
RAE	0.9344	0.7293	0.9375	0.7387	2.0607	0.7943	0.9614	0.8457
RMSE-ratio	128.13%	100%	128.56%	101.29%	282.57%	108.92%	131.83%	115.97%
RAE-ratio	128.12%	100%	128.55%	101.29%	282.57%	108.91%	131.83%	115.96%

Notes: The table presents the forecasting evaluation for the t + 9 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate and the real monetary aggregates in the models, respectively.

7.4.3 Monetary Models with Financial Condition Measures

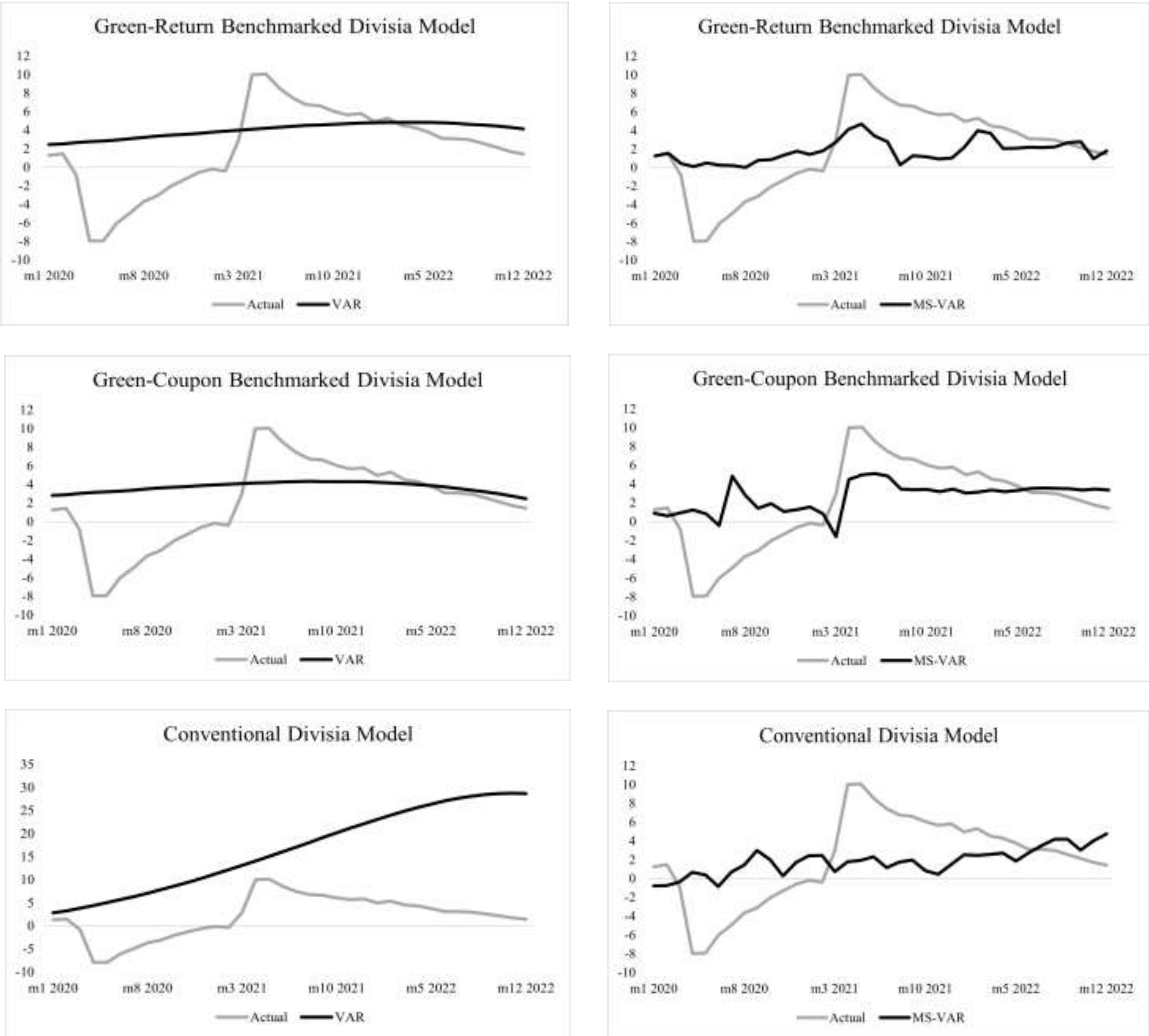
We then add the financial measures as control variables into the monetary models above to assess whether the green-coupon benchmarked Divisia monetary aggregate sustains its superiority in the nine-month ahead output gap forecasting. As before, we employ the EPU, FSI and GUNBP as the financial control variables, both individually and collectively across each model specification, and also examine the impact of these financial measures on the forecasting accuracy of the monetary models.

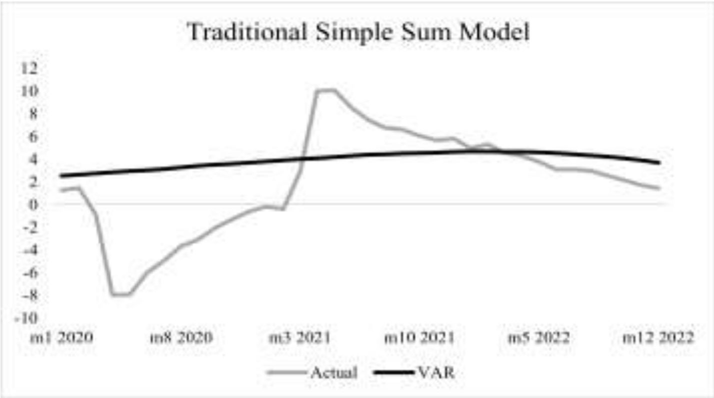
Following pervious approach, we first use the EPU as the financial measure and compare the forecasting results from the EPU-included monetary models. In Figure C.15.1, the t + 9 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model are presented as a 4 × 2-panel. As before, the forecasts, represented by the black line, are compared to the actual output gap, depicted by the grey line, in each panel. The forecasting pattern observed in t + 9 displays a divergence between the VAR models with the MS-VAR models across all money frameworks. Particularly, the forecasts produced by the conventional Divisia VAR model demonstrate an upward bias compared to those by the MS-VAR ones. These patterns are similar to those in the monetary models which do not include the financial measures.

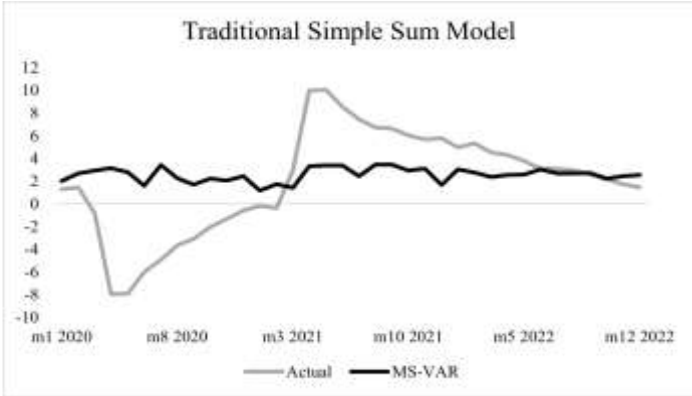
As shown in Table C.22.1, both values in RMSE and RAE for the MS-VAR models are lower than those obtained from the VAR models by 19.32% on average, across each EPU-incorporated monetary specifications, suggesting the superior performance of the non-linear models. The green-return benchmarked Divisia MS-VAR model and green-coupon benchmarked Divisia MS-VAR model are the best models. Their performance is approximately equivalent, with the green-coupon benchmarked Divisia MS-VAR model yielding a marginally higher values for both RMSE and RAE, exceeding by 0.72%. The conventional Divisia VAR model is the worst performing model, underperforming by up to 422.14% compared to the highest-performing green-return benchmarked Divisia MS-VAR model. These results further support the superior performance of the green-benchmarked Divisia monetary aggregates over their conventional Divisia and traditional simple sum counterparts. Further, the better forecasting performance of green-benchmarked monetary aggregates even when including the EPU as a control variable also indicates a shift in investors' preferences towards investments with an environmental focus.

Figure C.15.1 Forecasted (t + 9) and Actual Values of Output Gap from Monetary Models with EPU Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 9) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with EPU added and the right column presents that by using the MS-VAR method for four money models with EPU added.

Table C.22.1 Evaluation Criteria for t + 9 Forecasts from Monetary Models with EPU Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.4989	3.8449	4.5889	3.8727	16.2309	4.3951	5.1567	4.4245
RAE	0.8993	0.7686	0.9173	0.7741	3.2445	0.8786	1.0307	0.8845
RMSE-ratio	117.01%	100%	119.35%	100.72%	422.14%	114.31%	134.12%	115.07%
RAE-ratio	117.00%	100%	119.35%	100.72%	422.13%	114.32%	134.10%	115.08%

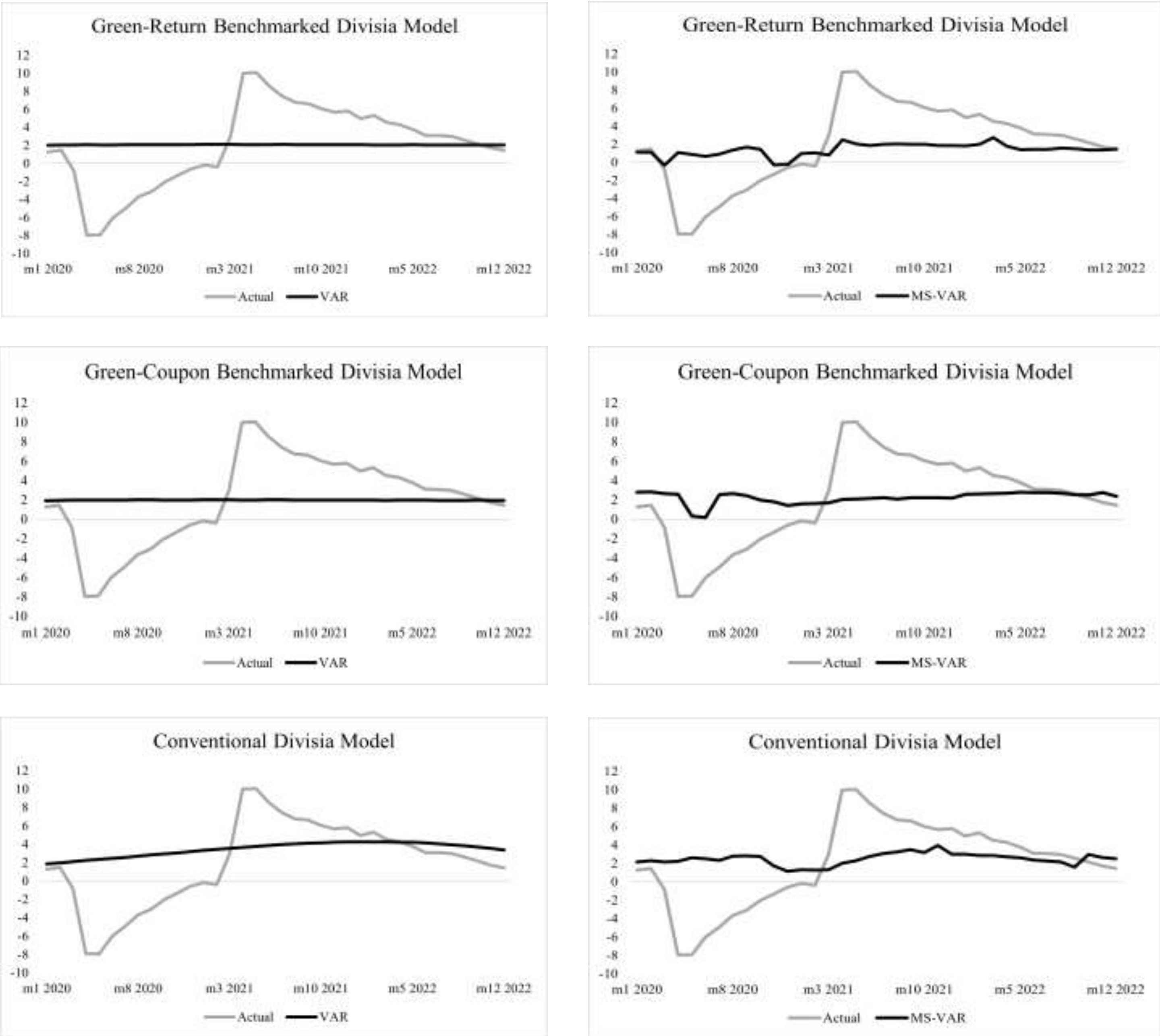
Notes: The table presents the forecasting evaluation for the t + 9 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the EPU in the models, respectively.

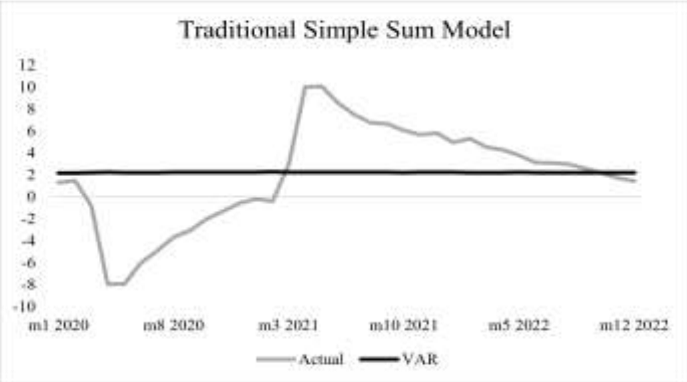
We then replace the EPU with the FSI and compare the t + 9 forecasts for four monetary models with the financial measure incorporated. In Figure C.15.2, we present the t + 9 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. As before, the forecasts (the black line) are compared to actual output gap (the grey line) in each panel. The trend of the forecasts from all models is quite similar, which is also evidenced by the slight difference in the values of both RMSE and RAE for all models in Table C.22.2. Nonetheless, the MS-VAR models still performs better than the standard VARs for each monetary specification, exhibiting marginally lower values in both RMSE and RAE, only by 3.56% on average.

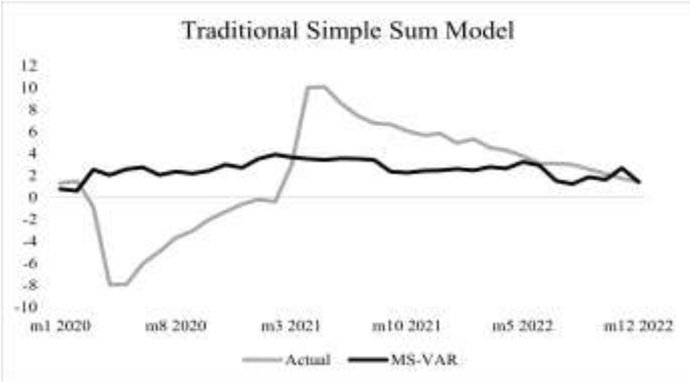
The green-return benchmarked Divisia MS-VAR model, as the best model, yields the lowest values of both criteria. The second best is the green-coupon benchmarked Divisia MS-VAR model, which underperforms the best model by 4.41% in terms of RMSE. The conventional Divisia MS-VAR model and the traditional simple sum MS-VAR model perform approximately as well, with the conventional Divisia model yielding slightly lower values for both RMSE and RAE. The superior performance of the green-benchmarked monetary models which incorporate the FSI as the financial control variable also suggests a tendency among investors to favour investing in green bonds during the stable economic periods.

Figure C.15.2 Forecasted ($t + 9$) and Actual Values of Output Gap from Monetary Models with FSI Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 9) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with FSI added and the right column presents that by using the MS-VAR method for four money models with FSI added.

Table C.22.2 Evaluation Criteria for *t* + 9 Forecasts from Monetary Models with FSI Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	4.5651	4.2149	4.5652	4.4009	4.4592	4.4419	4.5671	4.4500
RAE	0.9125	0.8425	0.9126	0.8797	0.8914	0.8879	0.9130	0.8895
RMSE-ratio	108.31%	100%	108.31%	104.41%	105.80%	105.39%	108.36%	105.58%
RAE-ratio	108.31%	100%	108.32%	104.42%	105.80%	105.39%	108.37%	105.58%

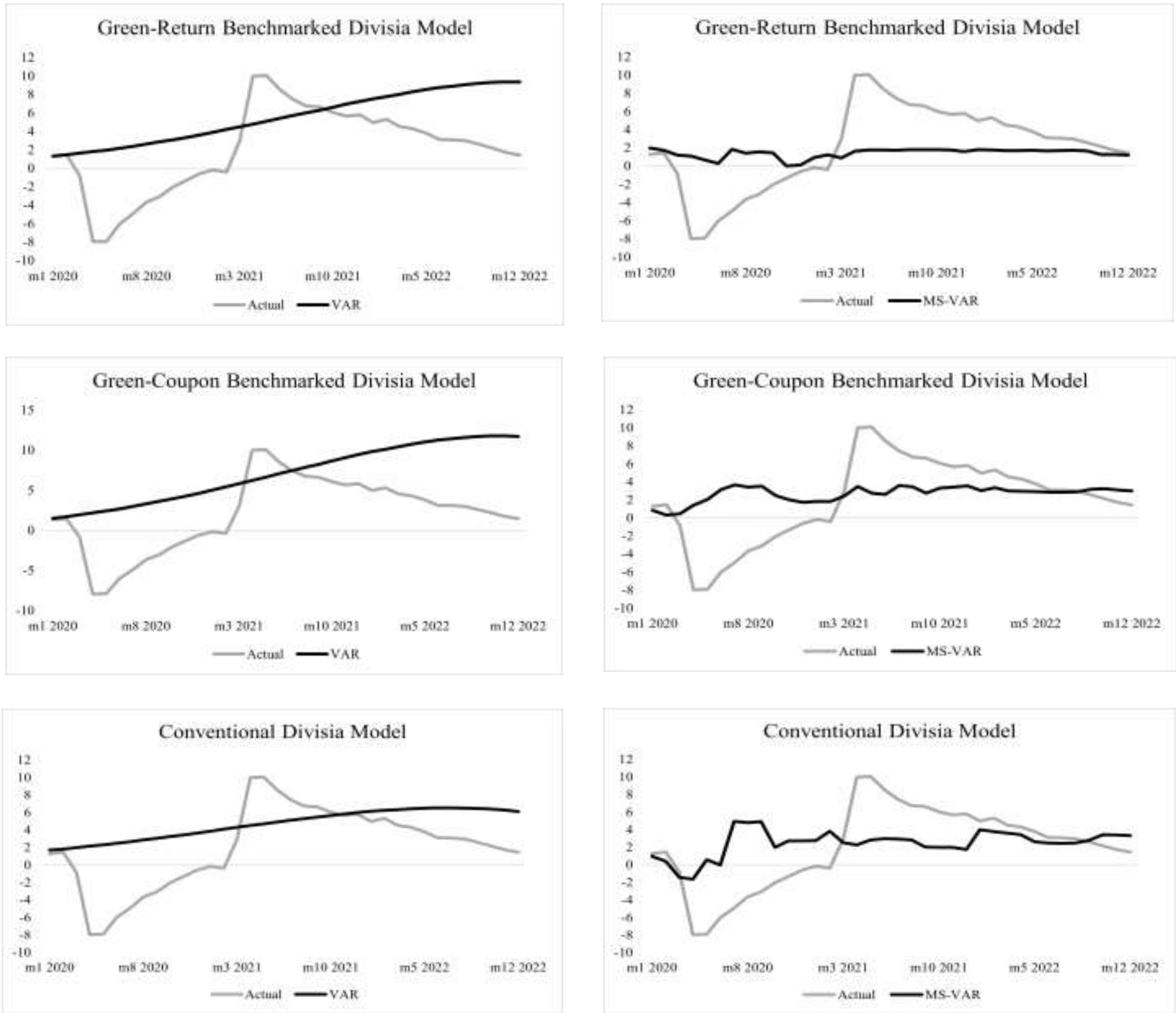
Notes: The table presents the forecasting evaluation for the t + 9 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively

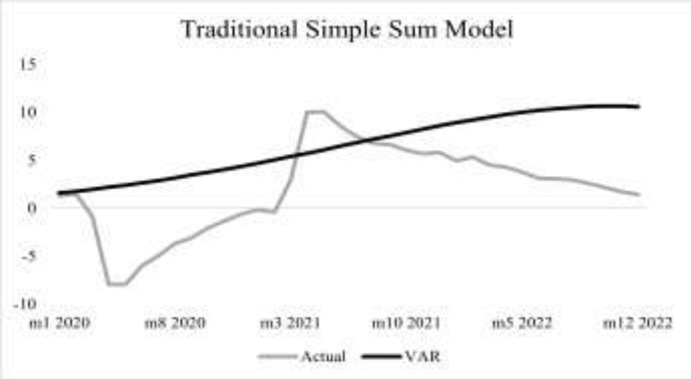
Using the GUNBP as the financial control variable, we then compare the forecasting results from four monetary models, each integrated with this financial measure. In Figure C.15.3, we present the t + 9 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. Consistent with the one-month, three-month and six-month ahead forecasts, the forecasting pattern for all VAR models shows an upward bias compared to that for the MS-VAR models across each monetary model. This deviation differs from the results obtained from the models incorporating the FSI and the EPU, respectively.

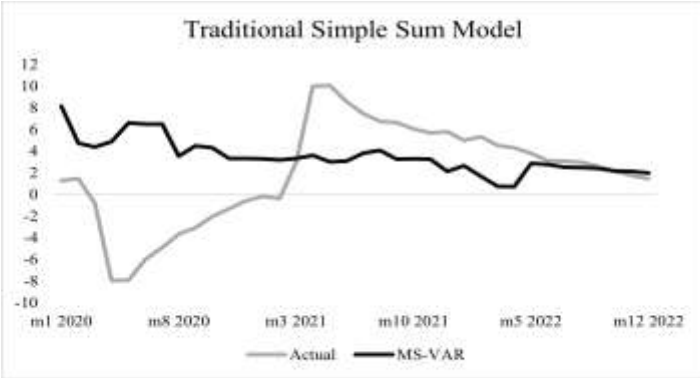
Turning to the RMSE and RAE in Table C.22.3, the values of both criteria for the MS-VAR models are lower than those obtained from the VAR models by 13.44% on average, across all four monetary specifications, suggesting the superior forecasting performance of the MS-VAR model. The green-return benchmarked Divisia MS-VAR model, yielding the lowest values in both RMSE and RAE, is the best model, which is followed by the green-coupon benchmarked Divisia MS-VAR model and the conventional Divisia MS-VAR model, underperforming the best model by 6.46% and 7.84%, respectively. The traditional simple sum Divisia MS-VAR model performs significantly worse than the other MS-VAR models. Consistent with the pervious results, the better forecasting performance of the green-benchmarked Divisia monetary aggregates further implies the investors' preferences for sustainability under stable economic conditions.

Figure C.15.3 Forecasted (*t* + 9) and Actual Values of Output Gap from Monetary Models with GUNBP Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 9) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with GUNBP added and the right column presents that by using the MS-VAR method for four money models with GUNBP added.

Table C.22.3 Evaluation Criteria for t + 9 Forecasts from Monetary Models with GUNBP Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	5.1428	4.1307	6.3319	4.3974	4.5327	4.4544	5.8460	5.7412
RAE	1.0280	0.8257	1.2657	0.8790	0.9061	0.8904	1.1686	1.1477
RMSE-ratio	124.50%	100%	153.29%	106.46%	109.73%	107.84%	141.53%	138.99%
RAE-ratio	124.50%	100%	153.29%	106.46%	109.74%	107.84%	141.53%	139.00%

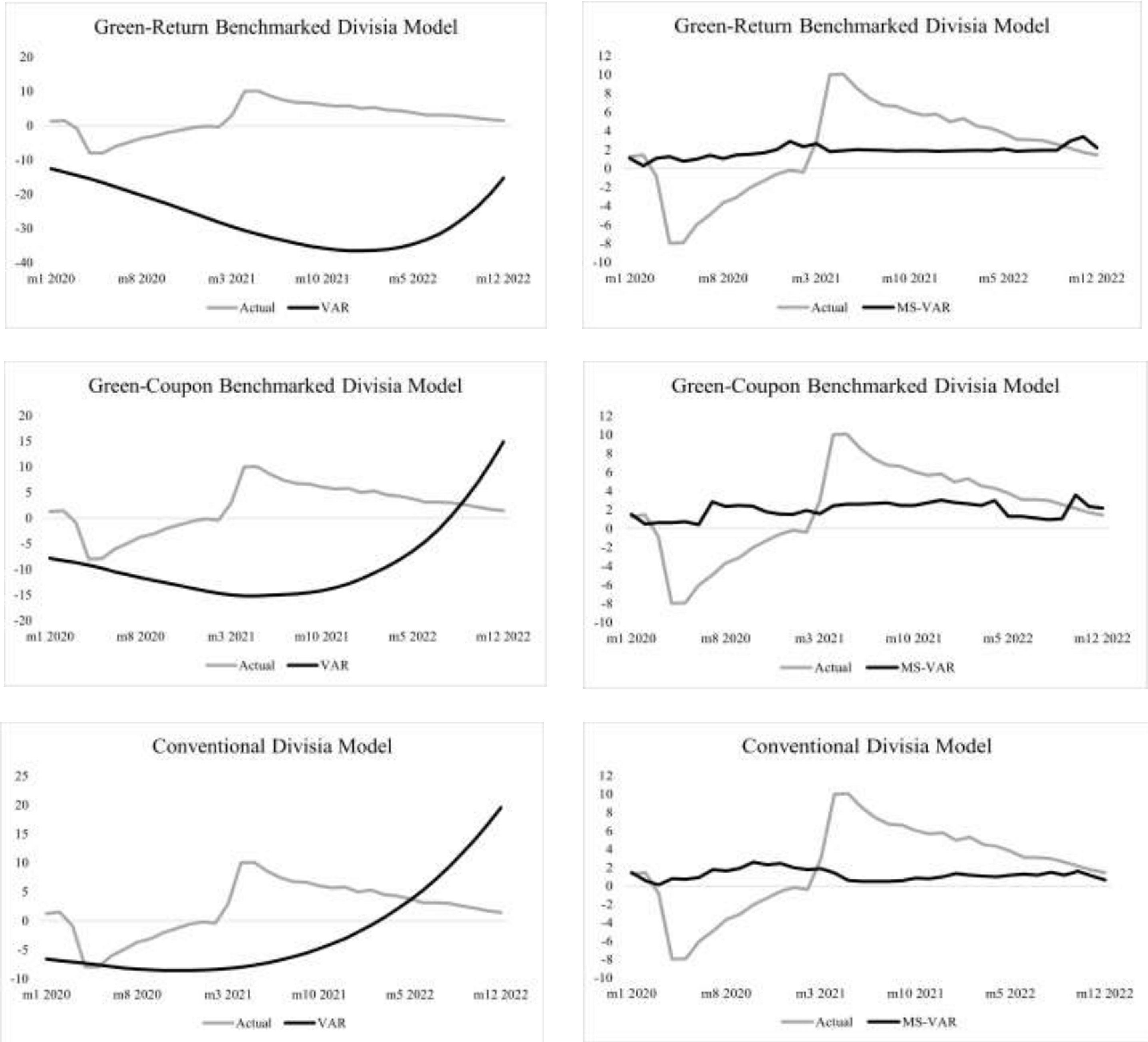
Notes: The table presents the forecasting evaluation for the t + 9 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and the GUNBP in the models, respectively.

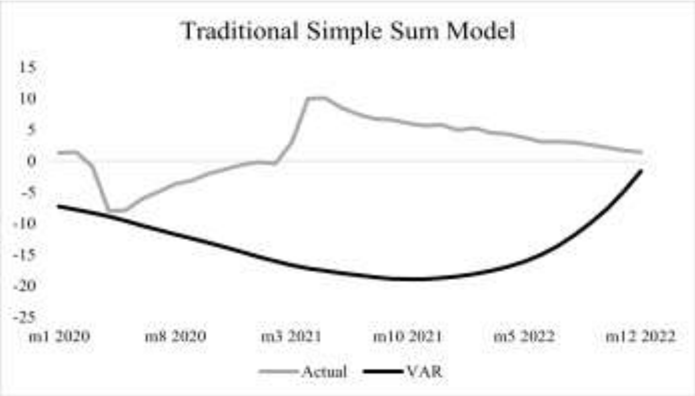
We finally include all three financial variables, i.e., EPU, FSI, GUNBP, into each of all four monetary models. Figure C.15.4 shows the t + 9 forecasts for the green-return benchmarked Divisa model, the green-coupon benchmarked Divisa model, the conventional Divisa model and the traditional simple sum model as a 4 × 2-panel. The forecasts (the black line) are compared to actual output gap (the grey line) in each panel as before. All VAR models undershoot the output gap, especially in the case of the green-return benchmarked Divisa VAR model and traditional simple sun VAR model. These forecasting patterns from the VAR models differs those from the MS-VAR models. The inferior performance of the VAR models is further confirmed by their significantly higher values of RMSE and RAE than those for the MS-VAR model across each monetary model in Table C.22.4, with the difference up to 744.34% compared to the best model.

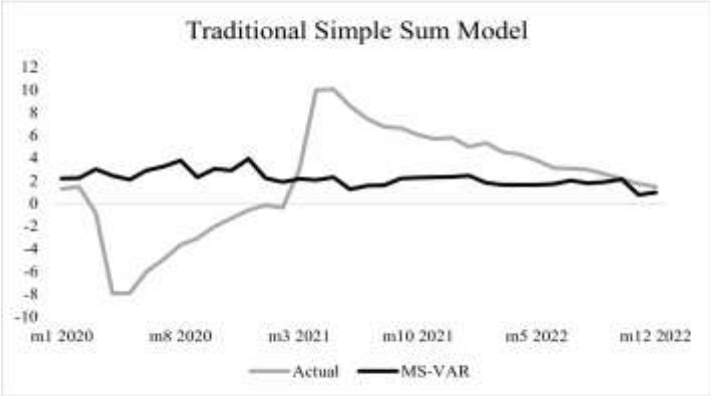
The best model is the green-coupon benchmarked Divisia MS-VAR model when all financial measures are utilised as control variables, as it yields the lowest values in both RMSE and RAE, which is inconsistent with the findings in one-month, six-month and nine-month ahead forecasts. This is followed by the green-return benchmarked Divisia MS-VAR model, with 3.65% less accurate compared to the best forecasting model.

Figure C.15.4 Forecasted (t + 9) and Actual Values of Output Gap from Monetary Models with Three Financial Measures Included

VAR **MS-VAR**







Notes: The plots show the forecasted (t + 9) and actual values of output gap based on the green-return benchmarked Divisia model, the green-coupon benchmarked Divisia model, the conventional Divisia model and the traditional simple sum model, which include the output gap, the real interest rate, the real monetary aggregates and the FSI in the models, respectively. The left column presents the forecasted and actual values by using the VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added and the right column presents that by using the MS-VAR method for four money models with all three financial measures (EPU, FSI and GUNBP) added.

Table C.22.4 Evaluation Criteria for *t* + 9 Forecasts from Monetary Models with Three Financial Measures Included

	Green-Return Benchmarked Divisa Model		Green-Coupon Benchmarked Divisa Model		Conventional Divisia Model		Traditional Simple Sum Model	
	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR	VAR	MS-VAR
RMSE	31.3561	4.3665	14.2885	4.2126	9.5540	4.8619	17.7360	4.9209
RAE	6.2680	0.8728	2.8562	0.8421	1.9098	0.9719	3.5454	0.9837
RMSE-ratio	744.34%	103.65%	339.18%	100%	226.80%	115.41%	421.02%	116.81%
RAE-ratio	744.33%	103.65%	339.18%	100%	226.79%	115.41%	421.02%	116.82%

Notes: The table presents the forecasting evaluation for the t + 9 forecasts from the green-return benchmarked VAR and MS-VAR models, the green-coupon benchmarked VAR and MS-VAR models, the conventional Divisia VAR and MS-VAR models and the traditional simple sum VAR and MS-VAR models, which include the output gap, the real interest rate, the real monetary aggregates and all three financial measures (EPU, FSI and GUNBP) in the models, respectively.

Overall, our results indicate that the green-benchmarked Divisia monetary aggregates are found to be useful in forecasting aggregate demand using a MS-VAR model and their performance are better than a traditional simple sum measure, even a conventional Divisia money. This result is consistent with the findings of Keating et al. (2019) and Barnett and Park (2023) who find that a Divisia monetary aggregate, as opposed a traditional simple sum measure, can be a useful monetary policy indicator. This highlights the potential for a broader application of green-benchmarked monetary aggregates in economic analysis and policy formulation, advocating for a more sustainable and environmentally conscious approach to economic decision-making.

Further, the observed superior forecasting performance of the green-benchmarked Divisia monetary aggregates, even when including the financial measures as the control variables, underscores the growing importance of environmental considerations in investment decisions, especially in stable financial environments. The effectiveness of green-benchmarked monetary aggregates in the forecasting of macroeconomic variables indicates that environmental factors are becoming critical in shaping financial market behaviours. This finding challenges the traditional view that financial markets are influenced solely by economic and financial indicators. It suggests that environmental sustainability is increasingly a factor in investment decisions, and this trend warrants a deeper exploration of how these considerations are factored into market analyses and investment strategies. Moreover, the specific mention of the effectiveness of these measures in stable financial environments hints at a nuanced relationship between environmental considerations and economic conditions. This raises questions about whether the influence of environmental factors varies across different economic cycles. It opens up a new area of study into how sustainability interacts with traditional economic indicators during periods of economic growth, recession, or stability. Understanding this interaction is crucial for developing more robust economic models and policies that can withstand various economic conditions.

In our analysis, we report a comprehensive set of VAR and MS-VAR results to provide a thorough examination of the relative performance of various monetary aggregates, including the green Divisia, across different models and economic conditions. By reporting multiple VAR results, we enable a detailed comparison across different monetary aggregates. This comparative approach helps identify any subtle advantages or unique characteristics of each aggregate, even if the overall performance is suboptimal. The inclusion of various models allows for robustness checks. Evaluating the consistency of results across different specifications and assumptions provides insights into the reliability and stability of our findings. Further, the comprehensive reporting ensures transparency in our research process. It allows other researchers and policymakers to understand the full scope of our analysis and replicate or critique our findings based on complete information.

Our tests of the forecasting ability of the VAR and MS-VAR models using the green Divisia monetary aggregates and other traditional aggregates, particularly over the period of the Covid-19 pandemic, revealed some critical insights and limitations. It is important to note that the green-benchmarked Divisia aggregates showed marginally better performance compared to other monetary aggregates, especially during this highly volatile period. The unprecedented economic disruptions and policy responses associated with the Covid-19 pandemic created a challenging environment for all forecasting models, leading to significant deviations from expected outcomes.

Consequently, the results should be interpreted with caution. The pandemic-induced anomalies highlight the need for more stable and predictable economic conditions to conduct more reliable and realistic testing of these forecasting models. Therefore, further evaluation during more normal times is essential to draw more definitive conclusions about the relative performance and utility of the green Divisia and other monetary aggregates in economic forecasting.

8 Conclusions and Recommendations for Future Work

In this chapter, we construct two green-benchmarked Divisia economic monetary aggregates, i.e. the green-return benchmarked Divisia monetary aggregate and the green-coupon benchmarked Divisia monetary aggregate, for the USA. We find that both green-benchmarked Divisia monetary aggregates provide additional information on USA aggregate demand during the sample period. We also compare the forecasting performance of a non-linear MS-VAR model and a benchmark linear VAR model. Our specific forecast experiment is USA output gap. Our results show that the MS-VAR model outperforms the VAR model. In terms of forecasting performance of the monetary aggregates, we find that the green-return benchmarked Divisia monetary aggregate and the green-coupon benchmarked perform approximately on a par over both forecast horizons, and both could be a valuable indicator in predicting monthly output gap. Moreover, the green-benchmarked Divisia monetary aggregates are even better than the conventional Divisia monetary aggregate for forecasting output gap.

We further include the financial condition measures as the controls to examine the forecasting performance of the monetary aggregates. Our results suggest that the green-benchmarked Divisia monetary aggregates maintain their superiority in the output gap forecasting over their conventional Divisia and simple sum counterparts, highlighting the substantive shift in investors' preferences towards green investments under stable economic conditions. This novel addition to the central bank's toolkit has the potential to provide central banks with feedback from an evolving economy to assist them in maintaining economic stability, as advocated by Bank of England (2018), thereby addressing potential political pressures. The new green monetary measures also provide a fresh view for the central banks to integrate environmental sustainability into the monetary policy framework and to guide and monitor the green financial markets.

Based our findings, further research into constructing green-benchmarked economic monetary aggregates applied to other countries or unions could be useful. We also recommend that future monetary researchers examine the performance of the proposed green-benchmarked monetary aggregates on forecasting other macroeconomic variables, such as inflation, and consider alternative models such as a Mixed Frequency Markov-Switching VAR (MF-MS-VAR) model to analyse economic data with different frequencies.

Conclusions

In the final part of this thesis, we summarise the main findings of each chapter and point out the suggestions for future research.

In the first chapter, we successfully identify and construct the green augmented Divisia monetary aggregates and green simple sum monetary aggregates for the USA by taking the environmental bonds, government bonds and corporate bonds into consideration. This work is one of only a handful of papers to emphasise the composition and construction of monetary aggregates in empirical monetary research. By testing their time-series properties, we estimate the IS curve specifications with the inclusion of the green monetary aggregates. Our results indicate that both green simple sum and Divisia green money are significantly and positively correlated with economic growth during the sample period, suggesting the additional information provided by the green augmented monetary aggregates. Contrary to earlier findings, this study suggests a positive correlation between lagged real interest rates and output. This revelation raises questions about the predominant emphasis on interest rate manipulation as a policy tool. Specifically, our findings show that reducing interest rates towards or below zero could potentially impede economic growth.

In light of this evidence, it appears that policymakers aiming to stimulate economic growth might benefit from shifting their focus towards either enhancing the role of money in the economy or considering the implementation of higher interest rates. In this context, this analysis of the green augmented Divisia monetary aggregates aligns with the objective outlined by the central banks, potentially serving as a viable monetary policy tool that maintains the dual goals of economic stability and climate sustainability.

Drawing from the implications of a relatively standard NK model of monetary policy, the second chapter advocates for the use of the green augmented price dual as a policy indicator variable in a recursive VAR model. The empirical results demonstrate that a green user cost-based model is capable of effectively measuring the impacts of monetary policy. It also shows superior performance compared to models based on the Federal Funds rate, which often encounter the 'price puzzles' commonly reported in the literature. Notably, by producing credible results without these puzzles, the green price dual offers policymakers a novel perspective. This is particularly relevant when the Federal Funds rate is at the ZLB. Additionally, the inclusion of green bonds in the monetary aggregates underscores the importance of climate change considerations in policy deliberations.

Our green price dual can also meet the needs of central banks as a tool for balancing economic stability with green sustainability. Moreover, the user cost of green bonds, defined as the opportunity costs consumers incur by utilizing green bond services, provides a measure for capturing externalities in the market. This encompasses the environmental costs of pollution and climate change. Our findings suggest that policymakers, in their pursuit of accurately pricing externalities, could benefit from considering this opportunity cost from a monetary policy standpoint. This approach offers a novel method for addressing externalities in economic policymaking.

In the third chapter, we develop two green-benchmarked Divisia economic monetary aggregates for the USA: the green-return benchmarked Divisia monetary aggregate and the green-coupon benchmarked Divisia monetary aggregate. Our analysis reveals that both green-benchmarked Divisia monetary aggregates contribute additional insights into the aggregate demand of the USA during the sample period. We also engage in a comparative evaluation of the forecasting capabilities between a non-linear MS-VAR model and a VAR model, with a specific focus on forecasting the output of the USA. The results indicate that the MS-VAR model surpasses the VAR model in terms of forecasting efficacy. Regarding the forecasting performance of the monetary aggregates, our findings suggest that the green-return benchmarked Divisia monetary aggregate and the green-coupon benchmarked Divisia monetary aggregate emerge as valuable indicators for predicting monthly output, even compared to the conventional Divisia monetary aggregate.

The incorporation of financial condition measures as control variables allowed us to assess the forecasting performance of the monetary aggregates more thoroughly. The results affirm that the green-benchmarked Divisia monetary aggregates retain their superior forecasting capabilities for output over their conventional Divisia and simple sum counterparts. This finding underscores a significant shift in investor preferences towards green investments, particularly under a stable economic condition, highlighting the importance of environmental considerations in the dynamics of financial markets. These novel measures also provide central banks with a fresh perspective on integrating environmental sustainability into the monetary policy framework. They serve as a guide and monitoring tool for the burgeoning green financial markets, thus aligning economic policy with environmental objectives for policy makers.

This thesis focuses on the output gap rather than inflation for several reasons, despite the United States' dual mandate of targeting both inflation and employment. One primary reason is the acute economic conditions prevailing during times of crisis, where the immediate concern often shifts towards mitigating output and employment shortfalls (Powell, 2020). The output gap, which measures the difference between actual and potential GDP, directly reflects the economy's underutilization of resources, including labour. Addressing the output gap can thus provide a clearer picture of economic health and prospects for the economy recovery. During our sample period, including the post-2008 recovery and the COVID-19 pandemic, the primary concern for policymakers has been stabilizing the economy and preventing severe recessions. By focusing on the output gap, we allow researchers and policymakers to prioritize measures that boost economic output and employment, which are critical for recovery. Inflation, whilst still important, often takes a secondary role in such scenarios because deflationary pressures and demand shortfalls are more pressing concerns during crisis.

Based on these findings, future research might be recommended towards the construction of the green augmented monetary aggregates, the green price dual and the green-benchmarked monetary aggregates applied to other countries. We also suggest future studies should assess the performance of these aggregates in nowcasting or forecasting additional macroeconomic variables, such as inflation. Moreover, the examination of the data beyond central banks dataset is recommended to explore the possibility of other uncertain assets providing liquidity services. Regarding the methodology, the advanced econometric models such as Mixed Frequency Markov-Switching Vector Autoregression (MF-MS-VAR), is also recommended for analysing economic data across varying frequencies.

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Appendices

Table A.6 Robust Check for IS Curve Estimates from the Regime Switching Specification

	Standard IS curve		**Real Divisia/simple sum green money growth term included**			
Regime 1: Covid Period						
Constant	-0.015***	-0.016***	0.004***	0.004***	0.002**	0.001***
	(-12.63)	(-30.20)	(4.37)	(14.89)	(2.41)	(3.75)
gdp_Gap_{t-1}	0.728***	0.740***	0.978***	0.970***	1.054***	1.023***
	(151.24)	(71.51)	(27.60)	(125.85)	(34.24)	(117.51)
gdp_Gap_{t-2}	-1.279***	-1.270***	-1.417***	-1.563***	-1.433***	-1.588***
	(-175.44)	(-89.72)	(-110.13)	(-260.35)	(-126.11)	(-224.72)
$RFFR_{t-1}$	-0.054***	-0.055***	-0.031***	-0.034***	-0.031***	-0.034***
	(-79.47)	(-58.35)	(-26.80)	(-111.76)	(-29.81)	(-117.37)
$RFFR_{t-2}$	0.069***	0.069***	0.079***	0.079***	0.077***	0.076***
	(63.24)	(173.96)	(80.41)	(315.88)	(89.86)	(227.10)
$\Delta(\Delta_4(m-p))_{t-1}$	-	-	0.672***	0.791***	0.802***	0.913***
			(9.05)	(49.07)	(12.52)	(51.07)
Output gap	HP	QD	HP	HP	QD	QD
Money measure	-	-	Divisia	Simple sum	Divisia	Simple sum
DW	2.71	2.72	2.55	2.65	2.56	2.66
Regime 2: Non-Covid Period						
Constant	0.001	0.001	-0.0003	0.0003	-0.0004	0.0003
	(1.63)	(1.35)	(-0.31)	(0.44)	(-0.35)	(0.43)
gdp_Gap_{t-1}	1.054***	1.082***	1.369***	1.257***	1.393***	1.261***
	(6.88)	(2.64)	(7.43)	(8.89)	(6.51)	(9.34)
gdp_Gap_{t-2}	-0.173**	-1.183	-0.332**	-0.294**	-0.346*	-0.294**
	(-2.02)	(-0.62)	(-2.11)	(-2.32)	(-1.81)	(-2.50)
$RFFR_{t-1}$	0.001	0.002	0.001	0.0003	0.001	0.0005
	(0.32)	(0.18)	(0.36)	(0.09)	(0.41)	(0.16)
$RFFR_{t-2}$	0.005	0.006	0.007**	0.007***	0.007**	0.007***
	(1.42)	(0.54)	(2.54)	(2.67)	(2.55)	(2.77)
$\Delta(\Delta_4(m-p))_{t-1}$	-	-	0.116***	0.070**	0.117***	0.070**

			(2.69)	(1.97)	(2.63)	(1.96)
Output gap	HP	QD	HP	HP	QD	QD
Money measure	-	-	Divisia	Simple sum	Divisia	Simple sum
DW	2.71	2.72	2.55	2.65	2.56	2.66

Notes: 1. HP denotes cyclical component from HP filter. QD denotes residual from quadratic de-trending regression. 2. Z statistics are in parentheses. *, **, and *** represent the significant levels of 10%, 5% and 1% respectively. 3. DW is the Durbin-Watson test statistic.

Table B.7 Variance Decompositions for the Federal Funds Rate VAR Model with Traditional Simple Sum Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	14.69	14.68	14.68
GDP deflator	1.67	2.67	2.96
Federal funds rate	76.05	75.80	75.76
Monetary base	13.80	13.81	13.79
Traditional simple sum monetary aggregates	14.17	14.15	14.15

Notes: The variance decompositions for the Federal Funds rate VAR model with traditional simple sum monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.8 Variance Decompositions for the Federal Funds Rate VAR Model with Traditional Divisia Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	19.06	19.06	19.06
GDP deflator	2.46	3.70	4.02
Federal funds rate	76.59	76.28	75.25
Monetary base	14.14	14.11	14.10
Traditional Divisia monetary aggregates	17.45	17.35	17.34

Notes: The variance decompositions for the Federal Funds rate VAR model with traditional Divisia monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.9 Variance Decompositions for the Corporate Bond Rate VAR Model with Green Simple Sum Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	5.86	5.88	5.80
GDP deflator	1.58	1.31	1.23
Corporate bond rate	56.86	53.71	46.38
Monetary base	6.99	6.56	5.73
Green simple sum monetary aggregates	11.15	11.16	11.12

Notes: The variance decompositions for the corporate bond rate VAR model with green simple sum monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.10 Variance Decompositions for the Corporate Bond Rate VAR Model with Green Divisia Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
GDP	6.86	6.90	6.87
GDP deflator	3.54	4.21	5.21
Corporate bond rate	58.20	56.07	51.60
Monetary base	7.65	7.39	7.35
Green Divisia monetary aggregates	9.77	9.82	9.83

Notes: The variance decompositions for the corporate bond rate VAR model with green Divisia monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.11 Variance Decompositions for the Government Bond Rate VAR Model with Green Simple Sum Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
Real GDP	1.58	1.60	1.60
GDP deflator	0.11	0.39	0.50
Government bond rate	86.29	86.08	86.05
Monetary base	4.77	4.91	4.91
Green simple sum monetary aggregates	11.41	14.42	11.42

Notes: The variance decompositions for the government bond rate VAR model with green simple sum monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.

Table B.12 Variance Decompositions for the Government Bond Rate VAR Model with Green Divisia Monetary Aggregates

Percentage of forecast error variance due to monetary policy shocks			
	Four quarters ahead	Eight quarters ahead	20 quarters ahead
Real GDP	1.05	1.06	1.06
GDP deflator	0.33	0.53	0.88
Government bond rate	70.60	66.83	58.87
Monetary base	4.05	3.87	3.84
Green Divisia monetary aggregates	6.13	6.09	6.05

Notes: The variance decompositions for the government bond rate VAR model with green Divisia monetary aggregates in *MI* block; all numbers are within the associated 90% probability intervals.