HETEROGENEOUS INFORMATION IN MARKET MICROSTRUCTURE

by

Xiangguo Zhang

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Abstract

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Department of Economics, Birmingham Business School

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This dissertation studies topics in the areas of information in financial markets. I model the competitive financial market where investors own heterogeneous information signals about trading assets.This dissertation studies the effects of strategic decisions by investors and learning on asset prices and investors' decision-making in financial markets. I mainly focus on the strategic trading decisions by investors and the impact of imposing market structures with different informational environments. Specifically, The first main chapter studies the quality aspect of information disclosure(i.e., the precision of private information), and the second main chapter analyzes the different investment ability among investors(i.e., looking-forward abilities) Finally, the third chapter analyzes the speed aspect of information, such that, how quickly an insider can process information and act on it before the information disclosure by public sections.

In the first chapter,*Market Efficiency, the Cost of Capital, and Information Disclosure Quality*. I explore the two-period noisy rational expectations claim in a model with short-horizon traders who have constant absolute risk aversion (CARA) preferences. A coherent framework is presented, showing how, when asset information is heterogeneous and includes both private and public signals, the precision of information disclosure reflects fundamentals, and liquidity trading in perfect competition. Alternatively, as a result of the noisy REE framework with linear expectations equilibrium, the results suggest that higher precision of public information disclosure even under the influence of Keynesian beauty contests still improves the financial market's efficiency level. As long as the competitive and information-related conditions are perfect, and traders aversion risk and risky asset supply have a positive relationship with the cost of capital. Crowding-out effects reduce agents' incentives to protect private information when public information disclosure is sensitive to market participants.

In the second chapter, *Level-k Reasoning with Heterogeneous Information Signals*. I study a level-k reasoning equilibrium in an asymmetric information environment populated by informed/uninformed agents. Firstly, when private information signal noise is significantly higher than public information signal noise, increasing the level of public information disclosure can lead to an increase in agent's payoff. Therefore, the social planner can improve market efficiency by disclosing more public information when the private signal is highly noisy. Secondly, consider a specification of $L0$ reasoning with uniformly randomly distribution between public and private signals; Higher-level reasoning agents can have dominant optimal strategies because of their higher prediction levels, but less sophisticated agents can also make better rational strategies under certain conditions. Specifically, this occurs when there are lower expectations of the fundamental state of the economy and a higher symmetric information environment. In these cases, less sophisticated agents may outperform higher-level reasoning agents. Finally, I analyze the Lk reasoning with full and partial disclosure of the public signal. The generalized beauty contests in Morris-Shin(M.S.) model is the particular case of partial disclosure in the Lk reasoning hypothesis when the social planner endogenously fixes the mass of the fraction.

In the third chapter, *Unobservable Information Acquisition and Insider trading*. I focus on modeling strategic communication of asymmetric information among investors and market makers, considering the costly acquisition of information. Firstly, I extend Kyle's dynamic insider trading model(Kyle, 1985) to incorporate unobservable, costly information acquisition, and the analysis reveals that there is no pure strategy equilibrium where information acquisition occurs after the transaction. This implies that insiders always deviate from the equilibrium by either entering the market earlier or later than the proposed disclosure date. Secondly, I discuss a discrete-time dynamic model of unobservable information acquisition following Caldentey and Stacchetti's (2010) study, which extends Kyle's (1985) continuous-time insider trading model by allowing discrete-time trading and unobservable information acquisition. Thirdly, as part of Kyle's (1985) contest, I consider the continuous-time dynamic insider trading model with no discounting. In this scenario, if the cost of information acquisition is high enough, it is unprofitable for insiders to acquire information. The final section discusses the implications for legal regulation of the SEC penalty of insider trading in U.S. cases, emphasizing how insiders' abnormal profits and illegal trading behaviors are determined by information disclosure.

Acknowledgement

All fantastic stories start with an occasional beginning. In the spring of four years ago, I received an email from Prof.John Fender inviting me to visit the University of Birmingham and conduct my PhD admissions interview. Two weeks later, in the John Smith building of the Department of Economics, Prof.John Fender and Dr.Pei Kuang took a two-hour interview with me. Practically, it is more of a pleasant conversation than an interview. A few days later, I received an email from Prof.John Fender: "We have agreed to admit you as a PhD student in the Department of Economics at the University of Birmingham, and Dr.Pei Kuang and I will be your supervisors. Also, we will assist you in applying for a full scholarship to the Business School. "

In fact, I studied at the Dalian University of Technology from 2012, and obtained dual bachelor's degrees(BEcon in Finance & LLB in Law) four years later when I graduated. After finishing my undergraduate study, I went to the United Kingdom for further study immediately. Three years later, I obtained dual master's degrees(MSc in International Finance and Economic Policy & MRes in Economics) from the University of Glasgow. Then, during my doctoral studies at the University of Birmingham, I completed the doctoral courses and graduation dissertation in only two years and six months and applied for graduation defence a year and a half in advance. When I look back on my ten-year undergraduate and postgraduate studies, it can be said that it was smooth sailing, without any suspense.

The premise of theoretical economics research is the creation and criticism based upon various assumptions and prerequisites. Based on the most classic rational expectations hypothesis put forward by the Chicago School of Economics, this dissertation, *Heterogeneous Information in Market Microstructure*, attempts to analyze several essential frontier issues in the field of financial economics: How does the quality of information disclosure affect market efficiency and investor behavioural decisions? If sophisticated heterogeneous investors exist in the financial market with private information signals, do investors with higher sophisticated investment ability necessarily obtain better expected payoffs than investors with low sophisticated forecasting ability? Why does insider trading in financial markets still exist even though financial regulators in various countries have formulated very strict requirements of information disclosure? How do corporate insiders play with financial regulators and arbitrage from the market? Taking the US Securities and Exchange Commission (SEC) as a legal regulation example, how does the law regulate the quality of information disclosure in financial markets and punish illegal insider trading?

When I completed the doctoral dissertation with more than 50,000 words, I realized that there were countless days and nights of reading and writing alone. Once the thesis was finally revised and submitted to my PhD defence committee, my heart was not as excited as I had imagined. Perhaps it is because the quality of my doctoral dissertation is far from the level that makes me very satisfied. During more than two years of dissertation writing, what I experienced was a long-term, rigorous and professional academic training. Countless paper rejections, conference presentations and revision comments from various parties are not only professional but also tempering my character.

I would like to thank my supervisors, Prof.John Fender and Dr.Pei Kuang, for giving me complete freedom of choice in both academic research and personal development, and also engaging me to fully explore my potential and love. In the period of writing the dissertation, I received several revision comments from many parties simultaneously. Thanks to Prof.Yongmiao Hong(School of Economics and Management, UCAS), Prof.Anna Bogomolnaia(Adam Smith Business School, University of Glasgow), Prof.Bart Taub(Adam Smith Business School, University of Glasgow), Prof.Yiquan Gu(Henley Bueiness School, University of of Reading), Prof.Aditya Goenka(Dept of Economics, University of Birmingham), Prof.Alessandra Guariglia(Dept of Economics, University of Birmingham), Prof.Patricia Chelley-Steeley (Dept of Finance, University of Birmingham), for the valuable comments and suggestions for many parts of my PhD dissertation. This research project is funded and supported by Birmingham Business School and the Econometric Society. All errors are mine.

I have written endless thoughts and memories of my ten years of undergraduate and postgraduate. As my student life draws to a close, I would like to thank my alma maters, the University of Birmingham, the University of Glasgow and the Dalian University of Technology, for their training and kindness. I will also miss and cherish my classmates and friends who support me during my student life.

In United Kingdom, the birthplace of Classical economics and modern economics, there are many shining economists and the purest theoretical economic research in the history of human thought: Adam Smith, David Ricardo, John Maynard Keynes, Friedrich Hayek, Ronald Coase, etc. There are also poems by Byron, dramas by Shakespeare, rock and roll by the Beatles, bells in Westminster, strong and malt whisky, crisp bagpipes, medieval manor castles, snow in Scotland and the magical fairy tale of children 's dreams. Finally, I would like to thank my parents, who live far away in China, for their endless thoughts and lovely support.

Awarding a doctoral degree is just a brief moment in my life. After graduation, it is still a longer and more uncertain life journey. Having the ability to study for life, choose to do what you love, and be curious about knowledge and the world is more significant than just getting a doctoral degree. More than one hundred years ago, German sociologist Max Weber mentioned in his lecture *Wissenschaft als Beruf (Science as a Vocation)*:

Das akademische Leben ist also ein wilder Hazard...Nur durch strenge Spezialisierung kann der wissenschaftliche Arbeiter tatsächlich das Vollgefühl, einmal und vielleicht nie wieder im Leben, sich zu eigen machen: hier habe ich etwas geleistet, was dauern wird.[1] (Hence academic life is a mad hazard...Only by strict specialization can the scholar become fully conscious, for once and perhaps never again in his lifetime, that he has achieved something that will endure.)

At the International Congress of Mathematicians in 2006, the hundreds of world's first-class mathematicians gathered in Madrid and announced that the genius Russian mathematician Goligri Perelman won the Fields Medal for his achievement in proving the Poincaré conjecture. However, Goligri Perelman was at his home in St. Petersburg. He refused to accept this distinguished award and said to Sir John Ball, the International Mathematical Union(IMU) president: "I have everything that I have."

[1] Weber,M. (1995). *Wissenschaft als Beruf.* Stuttgart: Reclam.

Contents

List of Figures

List of Tables

"Brevity is the soul of wit."

William Shakespeare, *Hamlet*(1602)

1 Introduction: How to Describe Information in Working of Financial Markets

1.1 An Overview

How to succinctly describe information in working of financial markets? This question would suggest that heterogeneous information comes into play a significant role in the topic of financial economics. The basic assumption of this study is based on the premise that one party in a transaction usually has better information than the other party . In reality, this information asymmetry may exist between transaction parties in various markets. For example, in labor markets, workers have more information about their abilities than employers do, and in good markets, sellers have more information about the quality of their products than buyers, corporate managers have more undisclosed corporate information than regulators and investors.

The modern financial theory relies heavily on assumptions about investors' rational strategies and market conditions. Professor Joseph E. Stiglitz, who won the Nobel Prize in Economics in 2001, classified the market failure caused by information asymmetry as a legitimate reason for the government to intervene in market economic activities. Nevertheless, it seems absurd that as Joseph E Stiglitz won the Nobel Prize in 2001, we must accept that the government should intervene in the economy. Economists have recognized, at least since Hayek (1945), that the critical role of financial markets is the aggregation and transmission of the information held by individual investors, which is original from a theory of information asymmetry proposed in the 1970s. In the early 1970s, American economists discovered that people are often in a state of information asymmetry in market transactions, and the content of information asymmetry includes *before information asymmetry* and *after information asymmetry*. Among them, the information asymmetry before the event can lead to the *adverse selection* model of hidden information, and the asymmetric information after the event can lead to the "moral hazard" model of hidden behaviour. In a nutshell, market failures will occur due to information asymmetry in market transactions.

It may be helpful to keep a classical case in great concern— *The Market for Lemons* (Akerlof,1970). In the pioneering work of information economics, George A. Akerlorf's research first discovered the adverse selection problem caused by information asymmetry in the used car market. He discussed the characteristics of the used car market and pointed out that the accurate information about the quality of used cars is asymmetrical between buyers and sellers. Because owners who sell used cars have used their cars for a while, they are well aware of the performance of used cars. However, most people who buy a used car cannot judge the quality of the car. Therefore, buyers and sellers have different information about each other; that is, there is an information asymmetry between them. Because the buyer lacks complete information, good quality cars cannot be sold at their due price, and bad cars will squeeze out good cars from the market, thus presenting a situation of adverse selection in the market. Moreover, no one is willing to sell cars with good performance, so most of the cars sent to the used car market are defective, and most of them are some grinning ”lemons”. Decades later, Joseph E. Stiglitz and Andrew Weiss proposed a classical model of information asymmetry related to banking and financial markets in their paper *Credit Rationing in Markets with Incomplete Information* (Stiglitz and Weiss, 1981). They show another case of the ”lemon market” that, as a result, low-risk businesses unwilling to pay high-interest rates are driving out risky businesses willing to borrow at higher rates without complete information.

In order to solve the problem of market failure caused by the ”lemon market”, the mandatory public information disclosure system is one of the essential means for regulators to supervise the financial market effectively. The regulatory goals of public information disclosure include: curbing unfair profit-making practices by traders using private information (Huddart, Hughes, and Levine, 2001), and improving market transparency to improve market efficiency and liquidity, thereby maintaining market fairness and justice.

The information constraints of micro-conditions have a series of significant effects on financial markets. Some researchers concentrate on micro conditions in the macroeconomy, such as market friction or market depth. The existence of a microstructural research design provides an exciting and vital perspective for explaining the ”lemon market” phenomenon in the context of a disclosure policy. Specifically, financial regulators must face several important disclosure metrics, considering how much, how often, and in what types of companies should disclose. Morris and Shin (1999, 2000, 2002) systematically study the welfare effects of public information disclosure, where agents can assess independent sources of heterogeneous information(public or private) about certain underlying states under the underlying assumptions. Moreover, from the perspective of investors and other market participants, some references such as Hellwig (1980) discuss that rational competitive expectations can lead to ”schizophrenia” problems, where informed traders abnormally price risk assets in the competition model. Kyle (1985, 1989) designs a narket microstructure model with imperfect competition conditions to solve this ”schizophrenia” problem through the Bayesian game. Specifically, in his model, informed traders can extrapolate from asset prices, including private and public information, and predict information about others in financial markets.

In traditional financial market analysis, it is generally expected that the arrival of public information, such as macroeconomic announcements, is the leading information channel of variation in the term structure of asset price. However, researchers always assert that many economic theories cannot explain the various economic phenomena in reality. Such presumptions in traditional theories do not fully consider the difference in the information held among investors and the influence of the game of this heterogeneous information on the formation of asset prices. However, the financial market has information differences and information asymmetry in reality. In general, In general, the price of an asset is sometimes the price at trading, but sometimes it is the bid or ask price (proposed price). Usually, the actual transaction price relies on the agent’s identity and the transparency of market information, whether the agent’s identity is buying or selling, and the market location of the transaction. Therefore, to describe the essence of the market more accurately, which is closer to reality, it is vital to study the microstructure of financial markets.

A key question in understanding the workings of financial markets is the nature of the interaction between the different types of informed traders and how heterogeneous information is incorporated into asset prices—specifically, considering that investors forecast investments differently, how better to describe market speculation behaviours among different types of traders? Is it plausible that insiders acquiring private information from financial markets can be better off in profits for typical trading problems in the spirit of the seminal work of Kyle’s (1985) model?

To answer the above questions, in this thesis, I first consider the economy that traders are endowed with different types of information. I address these questions using the model in the spirit of the seminal work of Grossman and Stiglitz(1980), in which traders are risk averse and trade in a market with uninformed traders and noisy supply. Second, the vast previous literature that builds on kyle's framework provides many significant insights into how informed investors are endowed with private information before trading begins, but only some papers discuss acquiring it endogenously at the time of their choosing. These questions are all situated in either micro-structure design or financial economics. I will discuss the specific models which structure the relationship between heterogeneous information and asset price in Chapter 2, 3 and 4.

Before providing the view of these chapters, the following sections introduce the research philosophy that underpins this thesis. Firstly, I introduce how I view the heterogeneous information in financial markets. Second, I discuss the economic interpretation of the regulation in financial markets, that is , why we need mandatory disclosure and law regulation for insider trading and financial arbitrage. Then, I describe the research methodology about how instrumentalist approaches and the research epistemology can better understand the financial market. Finally, I put some remarks for this thesis.

1.2 How to Define the Heterogeneous Information

This thesis is built on the hypothesis that the financial system is asymmetric and dynamic. I provide some basic concepts and theoretical backgrounds to theoretically understand how I view financial markets. In the subsections,I will sort out some basic concepts about heterogeneous information, such as information asymmetry, information disclosure quality,information constraint.

1.2.1 Information Constraint: Why Markets Need Corporate Disclosures

Regulators cannot rely on regulatory contracts that depend on the information held by only one side of the firm. Therefore, information constraints greatly limit the efficiency of government agencies in regulating enterprises.

According to different perspectives, there are different types of constraints on information asymmetry: from the perspective of the timing of information asymmetry, asymmetry may occur before the actor conducts the transaction or after the transaction. Economists traditionally divide information constraints into these two types: moral hazard and adverse selection. In the former case, the information asymmetry is ex-ante, while in the latter case, it is ex-post. Information asymmetry will lead to adverse selection through ex-ante asymmetry and moral hazard through ex-post asymmetry. Specifically, under the framework of principal-agent theory, we explain the moral hazard and adverse selection as follows:

- *Moral hazard* refers to endogenous variables that managers cannot observe. In the case of ex-post information asymmetry, one party involved in the contract changes the other party's behaviour, thereby creating a risk of harm to both parties.

  In corporate governance practice, moral hazard is mainly manifested as an enterprise arbitrarily taking activities that affect its product cost or industrial quality. It represents the working hours or the intensity of the business managers' work. For example: in corporate governance, managers may add additional costs (hiring people to lighten their workload, indifference to excessive input inventories, etc.), indulge in activities that are more beneficial to their career prospects at the expense of efficiency, postpone objectionable actions (such as laying off workers in a bad economy), high-priced purchases of raw materials and equipment. These are still examples of "negative effort".

  Another classic example is an insurance contract where the insurance company is partly or wholly liable for what people do when they get coverage. A person who defaults and causes losses are not entirely solely responsible, and insurance companies often have to bear most of the consequences. At this point, insurance companies face moral hazard. As a result, there is no incentive to maintain the integrity of the contract, so one must rely on moral self-discipline to avoid breaching the contract - the insurance company bears the risk of any loss due to behavioural changes.

- *Adverse selection* occurs when firms have more information about exogenous variables than regulators. Most observers stress the importance of adverse selection in market regulation and financial markets. Take the "lemon market" as an example, the seller knows the quality of second-hand cars, but the client does not, so the information is incomplete. The client signs the contract with the seller. A simple example is a relationship between buyers and sellers in the used car market: the seller (the agent) knows more about the quality of the car than the buyer (the customer). In this case, sellers will not trade the good car in the second-hand car market at all. However, inferior cars tend to eliminate products as the adverse selection in information asymmetry.

The existence of the moral hazard and adverse selection, and the corresponding loss of control by regulators, create the need for information search and disclosure. In most countries, companies are required to undergo regular monitoring by government regulators and are required to disclose relevant financial and operating conditions regularly.

1.2.2 Information Signals and Transmission: Michael Spence's View

In 1973, on the basis of George Akerlof's research, Michael Spence proposed a method to solve adverse selection and analyzed the transmission mechanism of information signals. His signalling theory mainly shows that the party with more information in the asymmetric information market can benefit in the market by transmitting reliable information to the party with poor information, and economic agents in the market can eliminate the negative effects of adverse selection. Economic agents take observable actions that convince the counterparty of the quality or value of their products and signalling only works if the signal senders' signal signalling cost significantly differs. A prime example is an education as a signalling tool for the labour market. If employers cannot distinguish between high and low-productivity people when recruiting new workers, the labour market will be flooded with low-productivity people.

Spence's representative work *Market Signaling: Informational Transfer in Hiring and Related Screening Processes* and *Job Market Signaling* both use people's education level as a labour market signal. Since his follow-up research involved many application cases, the research of market signal theory was extended to different markets by economists, and its importance was further verified. It is Akerlof's insight to discuss the adverse consequences of asymmetric information from the perspective of the rational agent. Regarding the "lemon market", Spence's most prominent contribution in this regard is to illustrate how rational agents in the market use signals to counteract the effects of "adverse selection." Signals here refer to observable measures taken by rational agents to convince the other side of the market of the value and quality of their products.

1.2.3 Market Transparency and the Evidence of Corporate Governance

Business evidence shows that lack of transparency is an important reason for frequent fraud scandals in capital markets and even the outbreak of financial crises. Improving marketing transparency can motivate stakeholders and discourage opportunistic behaviour and is essential to improving public and corporate governance.

For a long time, academic research on capital market transparency has mainly focused on the two aspects of corporate information disclosure environment and corporate accounting quality. For example, Beyer et al. (2010) discuss the research progress of corporate information environment from three perspectives: voluntary disclosure, mandatory disclosure, and analyst reports. Besides, Leuz and Wysocki (2016) review the progress of empirical testing of information disclosure and accounting standards-related systems and the existing problems, and pointed out that when information transparency involves multiple participants and various aspects, the complexity of capital market transparency far exceeds that of the company's information environment or the quality of company accounting information.

Forssbaeck and Oxelheim (2015) believe that in an asymmetric information environment, there is not only public information but also private information. When private information is transferred from holders to non-holders, the level of market transparency will increase. They define information holders and non-holders as information senders and receivers. Regarding stakeholders in the capital market, information senders mainly include listed companies, regulatory authorities, service departments, and investors, and information receivers mainly include external investors, regulatory authorities, etc. As far as the information content of the capital market is concerned, the company's financial information is a critical factor in conveying value and influencing decision-making. In addition, information such as stock prices, administrative penalty announcements, economic policies, and the behaviour of stakeholders will also convey the company's value. Information affects the market's judgment of the company's value and then affects the decision-making of the information receiver.

1.3 An Economic Interpretation of Legal Regulation

Information asymmetry among traders is one of the fundamental determinants of market quality. Precisely, when traders use private information to carry out arbitrage transactions, uninformed market makers can only predict investors' trading behaviour by observing the market's overall trading volume, which increases the transaction cost to compensate for the adverse selection cost. In this case, market makers can only cover the cost of adverse selection by charging a more extensive bid-ask spread, thereby reducing the market's liquidity. In response to this phenomenon, a core issue facing financial authorities is how to deal with information asymmetry and regulate insider trading by investors and companies. Policymakers in financial institutions have attempted to alleviate information asymmetry by implementing several information regulations, significantly improving the quality of information disclosure. Financial regulators have enacted laws requiring companies to disclose mandatory information to investors.

1.3.1 Asymmetric Information and Fair Disclosure

Disclosing information to the public is a way for public companies to communicate with investors and the general public. Investors and the public can obtain temporary announcements and periodical reports on listed companies through mass media announcements.

Fair disclosure in financial markets and the prohibition of unlawful insider trading are the topics of this dissertation. In many cases, financial regulations facilitate transparency by reducing the costs associated with obtaining essential information. *The RegFD of 2000* and *the Sarbanes-Oxley (SOX) of 2002* are examples of such policies. Through full and fair disclosure of materials provided by issuers (such as companies), they attempt to alleviate information asymmetries between traders. Under *the RegFD*, corporate issuers must disclose non-public information to certain entities and make it publicly available. SOX is another example. Enron, WorldCom, and Tyco's accounting scandals in the early 2000s led to the passage of *the Sarbanes-Oxley Act of 2002*, which encourages greater standardization of financial statements by publicly traded companies, which seeks to mitigate information asymmetry by improving corporate disclosure accuracy. In order to mitigate information asymmetry and improve liquidity and price efficiency, the government has implemented a number of regulations and policies designed to disseminate accurate information to all market participants. *The SFAS No.131* requires companies to disclose their major operating segments and the existence of customers that represent more than 10% of their sales revenues and the total revenues from those customers.[2] During the subprime mortgage crisis, the U.S. Congress enacted *the Dodd-Frank Act of 2010* to increase reporting requirements for loan levels. For example, the Title IX of *the Dodd-Frank Act of 2010* requires mortgage-level data to be disclosed by asset-backed securities issuers. Previously, it was possible to access such data, but it was usually more expensive and complicated. Furthermore, small and medium investors could access investment information at a reduced cost in 2009 due to a decision made by SEC corporations, as noted by Dugast and Foucault (2018). Although this standardization does not necessarily enhance the information investors have at their disposal, it makes it easier for them to access and process it.

In compliance with *the Regulation S-K*, companies are required to disclose all material contracts or agreements.[3] The most recent for the SEC to amend *Regulation S-K* was November 9, 2020. It applied to *10-Qs*, *10-Ks* and registration statements filed on or after that date. The SEC incrementally moves towards a more "principles-based, registrant-specific approach to disclosure in these amendments" from prescriptive disclosure requirements. For accounting practitioners to craft appropriate disclosure responses to the amendments, they must use experience and judgment in assessing each registrant's circumstances. As a result, it will conduct fewer line-item disclosure requirements, favouring a regime that requires each registrant to tailor disclosure based on its unique circumstances.

Rules and regulations of legal information are an important mechanism to define the rights and obligations of market stakeholders, which is more conducive to restraining the behaviour of market stakeholders and improving market efficiency and transparency. Regulations usually play a role in regulating the behaviour of the subject. For example, Watanable et al. (2019) use the European Union's Transparency Directive (TPD) to require companies to disclose financial information, an exogenous event regularly, and found that TPD is when stock price information has improved.

[2] *The FAS 131* is the abbreviation of *the FASB Statement No. 131*, *Disclosures about Segments of an Enterprise and Related Information*.

[3] *The SEC Regulation S-K* outlines the reporting requirements for various SEC filings that public companies must submit, as prescribed by *the Securities Act of 1933.*

The United States has disclosed the review of SEC comment letters in its financial reports since 2004. Duro et al. (2019) explore related issues from the perspective of regulatory enforcement. They examined quarterly report data from 1998 to 2013 and find that, after the policy change in 2004, the financial weight of the company's financial reporting decreased, the number of words in the report increased, the quality of information improved, and SEC comment letter review disclosure strengthened enforcement.

Regulations that are not directly related to information disclosure will also affect the motivation of information senders. For example, Lewellen (2022) claims that the "tax haven" policy will generally weaken the ability of shareholders to restrain managers and directors, and the quality of information disclosure will decline. The results of Hope et al. (2020) show that China's anti-corruption campaign in 2013 effectively improved the information quality of companies.

1.3.2 Insider Trading within Regulation

Typically, insider trading involves dealing in a public firm's stock by a person who, for any reason, has non-public material information about the stock. By the nature of insiders, they have privileged access to private information and use that information to trade, they are likely to benefit at the expense of uninformed outside investors. Trading by insiders can be legal or illegal, depending on the circumstances. The consequences of insider trading are severe when material information has not been made public. This capture diverges from existing theories through the concepts of insider trading and costly, unobservable information acquisition. The relevance of these concepts when discussing information design literature, based on the insider's rational strategies, and the dynamic time-continuous itself is addressed. In addition, this subsection includes a discussion on the subjects of financial economics theory and microeconomics to help place where the thesis sits in respect to these subject areas.

According to the U.S. Securities and Exchange Commission (SEC), illegal insider trading involves:*"The buying or selling a security, in breach of a fiduciary duty or other relationship of trust and confidence, based on material, non-public information about the security."* [4] Illegal insider trading includes tipping others when you

[4] U.S. Securities and Exchange Commission. See https://www.investor.gov/introduction-investing/investing-basics/glossary/insider-trading. Accessed August 18, 2020.

have any private information in your possession. Legal insider trading occurs when company directors buy or sell the stock but legally disclose their dealings. Policymakers have tried to mitigate illegal insider trading by imposing several information regulations. For example, the U.S. SEC has rules to protect investors from the effects of insider trading. Whether the company uses the non-public information does not matter how it is received. For instance, suppose someone learns the firm's non-public information from a family member and shares it with friends. All three involved could be prosecuted if the friends used insider information to profit in financial markets.

However, the fear of SEC penalties can discourage both illegal insider trading and legal and ethical information acquisition. The threat of SEC penalties can lead to a reduction in the quantity and quality of information available in the market, leading to market inefficiencies and liquidity issues(Huddart et al., 2011; Chen et al., 2018). This is because market participants may be less willing to expend resources on information acquisition and dissemination if they fear that their actions could be perceived as illegal or unethical. The key challenge is to strike a balance between deterring illegal insider trading while encouraging legal and ethical information acquisition. This requires effective enforcement mechanisms that distinguish between legal and illegal information acquisition and impose penalties that are proportionate to the severity of the offense. The penalties should also be designed in a way that maximizes their deterrence effect while minimizing their adverse impact on market efficiency and liquidity.

In summary, economic theory suggests that effective information acquisition and dissemination are essential for the proper functioning of markets. SEC penalties can deter illegal insider trading, but they can also discourage legal and ethical information acquisition, leading to market inefficiencies and liquidity issues. Therefore, it is important to strike a balance between deterring illegal behavior and encouraging legal and ethical behavior in the securities markets.

1.4 Methodology: Theoretical Modelling

This thesis provides critical insights from the previous literature on how information signals about a company's fundamental value and information disclosure precision can affect market efficiency, information production, and investor welfare and behaviour in financial markets. There are many aspects to consider the behaviours of asset traders and evaluate heterogeneous traders in affecting market efficiency. Hence, I provide some theoretical basis as an instrumentalist approach to external validity in which the presumptions both work in the model and works in reality. For my study in this thesis, I believe that a diverse set of theoretical economic models that have been sufficiently validated will provide a better understanding of the performance of financial markets in practice.

1.4.1 Rational Expectation Equilibrium(REE)

The rational expectations theory of Robert E. Lucas believes that individuals and enterprises are rational before making economic decisions and must collect and use all information to carry out economic activities. The expectations mentioned in this theory mainly refer to the estimates made by consumers and enterprises on the value of economic variables in the future. Verrecchia (2001) indicates that, relative to market-clearing prices, investors' perceptions of uncertain asset values are constantly based on Walras's general equilibrium law. Informed and uninformed investors determine the demand for an asset based on public and private information and then submit the demand curve to the market maker. By observing the level of demand, the market maker sets the asset price that balances the total supply and demand. To maximize the benefits, investors must take full advantage of all the information and knowledge they possess. Future predictions are based on comprehensive analyses. The equilibrium price is influenced by traders' resubmissions, which affects investors' perceptions and evaluations of the asset. When traders resubmit their demand curve expectations, the equilibrium price influences investors' perceptions and evaluations.

1.4.2 Level-k Reasoning

Level-k reasoning(sometimes called level-k thinking) was originally from laboratory evidence about how players model other players in behavioural economics in experiments (e.g. Stahl, 1993; Stahl and Wilson, 1994, 1995; Nagel, 1995). Level-k reasoning in financing models is a type of decision-making process that takes into account the beliefs and preferences of all parties involved in a financial transaction. It is based on the idea that each party has a different level of knowledge and understanding of the transaction, and that their decisions should be based on their own individual level of knowledge.

Compared to the REE model, the level-k reasoning model breaks Nash equilibrium rational expectations and better describes the level-k players can observe that other level-$(k-1)$ players are less complex than themselves. In such settings, it appears that some previous literature structured the non-equilibrium framework or ”p-beauty contest” on the belief of the hierarchical level thinking assumption(i.e. Camerer,Ho, and Chong, 2004; Ho, Camerer, and Weigelt, 1998; Costa-Gomes and Crawford, 2006). Behavioural experiments in theoretical economics have repeatedly demonstrated that standard equilibrium analyses based on rational expectations are often inconsistent with the actual behaviour of experimental subjects. Instead, there is laboratory evidence that level-k thinking better describes how experiment participants form their beliefs and make rational decisions. Specifically, this process makes a hypothesis that agents form higher-order beliefs—that is, beliefs about beliefs...about the behaviour of others—to some finite level-k, due to economic circumstances. Complexity, either because they believe other agents are lower-level sophisticated thinkers. Level-k reasoning assumes that people are able to reason about the beliefs of others up to a certain level of recursion. For example, a Level-2 player would assume that their opponents are able to reason about the beliefs of other players up to two levels of recursion (i.e., they can reason about what other players believe about what other players believe). In this way, higher-order beliefs are incorporated into the Level-k reasoning model.

In my third chapter, I replace rational expectations with the level-k reasoning under a beauty contest structure. In this two-stage beauty-contest structure, the social planner provides a public signal to a fraction of informed agents. The agents use their private signals and the public signal to form their beliefs about the fundamental value of the asset. The informed agents then submit their guesses to the social planner in the second stage, and the agent with the closest guess to the true fundamental value receives a prize. The goal of each agent is to choose a guess that is closer to the true fundamental value than the other agents' guesses.

In a classical case, Nagel (1995) develops the level-k theory by designing a finite-period 2/3-Beaut-Contest hierarchical experiment, which became known as the $p-$ beauty contest guessing game in experimental economics. In her hierarchical framework setup for an $N-$person game, they assume that each player in this game attempts to choose the mean of everyone's choices by N times in the closed interval $[0, 100]$. In equilibrium theory, rational players predict that even if all other players choose 100, one of the rational players should choose less than or equal to 67, which is 2/3 times 100. Then, assuming that the other competitors think similarly and that the rational thinking of all players is homogeneous, this player will again think iteratively, picking up no more than 45, which is 2/3 times the 67. Ultimately, there is only one 0 for the Nash Equilibrium of unanimously chosen choices by all rational players. This experimental evidence by Nagel (1995) using relevant guessing games have documented substantial evidence that investors have a limited degree of strategic sophistication.

Level-k thinking can be critical in a macroeconomic environment. Rational expectations require people to predict all future contingencies for any given policy. However, this requirement is very demanding since the effects of regulations are the result of complex general equilibrium (GE) relationships that link the behaviour of all agents in this framework (such as investors and firms) and policymakers. More specifically, some investors can invest time and resources in predicting the behaviour of other financial agents. These investors can respond proactively after new regulations are announced. In a nut shell, level-k thinking captures both types of behaviour: it does not assume that agents can predict all future contingencies, but at the same time, it allows some agents are more sophisticated and able to be more forward-looking than others.

In applications to financial markets, the assumption of the Level-k reasoning model may not fully capture the complexity and diversity of the decision-making processes that are involved. Financial agents may have more sophisticated models of others' beliefs and strategies than what is assumed by Level-k reasoning, and they may have access to more information than what is typically provided in experimental settings. Financial markets involve many institutional and regulatory mechanisms that can affect the behavior of agents in ways that are not easily captured by the Level-k reasoning model. For example, financial regulations, central bank policies, and market microstructure can all affect the incentives and constraints that agents face, and can influence the evolution of market prices and trading volumes. Iovino and Sergeyev (2018) use level-k thinking to critize the neutrality of central bank intervention in a macroeconomic environment. Farhi and Werning (2019) suppress monetary policy effects and use level-k thinking to reconcile forward guidance conundrums. Angeletos and Lian (2017) use level-k thinking to explain the slow decomposition of general equilibrium effects.

1.4.3 Asymmetric Trading Models

An important concept in economic theory is information asymmetry, which is difficult to estimate because private information often has unobservable characteristics. Informed investors in financial markets often have their private information to derive arbitrage profits from trading many financial assets (i.e. risk securities, corporate bonds and derivatives). This non-public information can be obtained, in part, by uninformed investors from asset prices to their advantage. Grossman(1976) and Grossman and Stiglitz(1980) initially emphasised this role of the market. In recent years, technological innovations have enhanced a new dimension to traders' access to information - speed. The goal of high-frequency traders (HFTs) is trading at the speed of light, who use sophisticated communication tools to learn information and act on it as quickly as possible. The market is more liquid as a result of their market-making behaviour. In contrast, high-frequency traders can take advantage of stale prices to pick up long-term orders before liquidity providers update them. Therefore, their speed advantage as liquidity takers puts liquidity providers at risk of adverse selection. Briefly, theoretical market microstructure lies at the intersection of two major asymmetric information models: sequential trading and multiple times trading models.

- In the sequential trading models, randomly-selected investors arrive at the market one by one, independently and anonymously at the specialist's post. In this setting, there is no need for an individual investor to consider the influence his or her actions may have on the sequential decisions of others when she only participates in the market once. This line of inquiry begins with Glosten and Milgrom (1985).

- The second sort of model usually asserts a single informed investor who can trade at multiple times, in which we can describe these as sequential strategic trading models. In this model motivation, an informed insider in sequential auctions where he revisits the market must make such calculations by taking into account others' subsequent decisions and involving these considerations into his optimal strategies. This model motivation is also sometimes described as "continuous auction," where informed traders observe a noisy signal about the fundamental value of a single asset. This line of this thought asserts with Kyle(1985).

Two of the above models are motivated by the fact that transactions reveal investors' private information. Private information and value are not excluded when all investors are the same ex-ante. It simply points out that all individual-specific variables and parameters (i.e., a value signal, the risk aversion coefficient) are identically and mutually independent, normally distributed across all participants. The core thought of an asymmetric information model is that some subset of the agents has superior private information(i.e., insider trading, level-k reasoning). Additionally, Kyle's model makes multiple fundamental assumptions about market liquidity and efficiency (Kyle, 1985). My study examines the implications of public information disclosure on trading strategies, profits allocation, and subsequent market quality (including liquidity and efficiency) based on the market microstructure theory.

1.5 Remarks

Do we want to be able to make comprehension for a set of overlapping complex mathematics assumptions that is the economy? I believe that because economic models' mathematics assumptions are a simplification of reality, they can be used to uncover the causal mechanisms behind the empirical results of economic patterns. Once the causal mechanisms are well known, these economic models can help us make better predictions about the real-world economy.

From Paul's(2006) assertion, the epistemic relativism, claimed about *epistemic justifications*, can be defined as:

"Epistemic relativism is the view that what is true, rational, or justified for one person need not be true, rational, or justified for another person. Epistemic relativism, therefore, asserts that while there are relative facts about the truth, rationality, justification, and so on, there is no perspective-independent fact of the matter."

Wherever it is used, epistemic can be traced back to the knowledge of metaphysics from ancient Greeks. Plato's dialogue *Theaetetus* concerns the nature of knowledge. In this dialogue, two philosophers, Socrates and Theaetetus, discuss knowledge in three definitions: knowledge as nothing but perception, knowledge as proper judgment, and, finally, knowledge as a proper judgment with an account. However, each of these definitions is shown to be unsatisfactory by them.

Socrates declares that Theaetetus will have benefited from discovering what he does not know and that he may be better able to approach the other topics in the future. Thousands of years after the death of Socrates that is being said, the economy, as one subject of knowledge separated from philosophy, is also incredibly complex. Also, it is impossible to derive mathematics models which are standard valid in all economic subjects theoretically. In realistic trading in financial markets, the general public's vast views that market failure and the current financial crisis triggered by the subprime debacle are evidence that mathematics or mechanics are sciences in the same sense that economics is not a science. The research reported in this thesis is proof of the contrary. It illustrates how mathematical models of theoretical economics developed from first principles before the crisis would help people understand and explain a world in crisis currently and how we got there.

Nevertheless, economics is a problematic but brevity science with many unchartered territories. Through more study of the kind reported in the results/propositions worked in this dissertation, our understanding of the operations of financial markets will be able to improve.

2 Market Efficiency, the Cost of Capital, and Information Disclosure Quality[5]

2.1 Introduction

One of the primary explanations that often emerged after the 2008 financial crisis and bankruptcy was the need to improve the quality and transparency of public disclosure. Taking the United States as an example, since *the Securities Exchange Act of 1934*, the SEC's supervision of financial market information disclosure has been strengthened many times in recent years. Several recent financial reforms in many countries and economic, regulatory entities have attempted to increase the transparency of financial assets by requiring issuers to disclose additional information to the public. These regulations, requiring asset issuers to disclose relevant information to potential traders, sound like essential measures to improve investor welfare. Since the creative assumption of the market efficiency hypothesis of capital markets in the 1980s, the incompleteness and asymmetry of market information have increasingly become the focus of financial economists. Based on a series of "trust crises" was triggered by financial scandals and the leaky information disclosure system in the U.S. asset market in previous financial crises, regulators emphasise various aspects to improve information disclosure quality and requirements.

The disclosure literature investigating information asymmetries between companies and investors has yielded many influential insights that significantly advance our understanding of corporate disclosure strategies. The premise of this view is that there are differences between investors' unequal knowledge and investors' beliefs, which may stem from unequal access to information. In a market environment with information asymmetry, lack of information can lead to adverse selection problems and market failure(Akerlof, 1970; Myers and Majluf, 1984). To enable companies to disclose more material information and improve the market's overall efficiency, regulators have introduced various mandatory disclosure requirements. In the presence of information externalities (e.g. Dye, 1990) or natural externalities (e.g. Kanodia, Singh and Spero, 2005), these disclosure provisions help improve social welfare, which is vital

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for bringing security prices closer to their fundamental values, thereby improving market efficiency. On the flip side, the disclosure also has adverse indirect effects on private information acquisition through a ”crowding out effect”. Specifically, public disclosure decreases the private information advantage controlled by informed traders. Higher firm disclosure levels reduce the incentive for insiders to pay the cost to acquire non-public information. Therefore, reduced the attraction of acquiring private information increases liquidity on the secondary market and ultimately results in higher firm values.

The cost of capital plays an important role in capital budgeting and corporate investment decisions. It affects the allocation of capital throughout the economy. However, in some empirical studies, the relationship between capital costs and information quality appears to be ambiguous(e.g., Healy and Palepu, 2001; Bertomeu and Cheynel, 2015; Leuz and Wysocki, 2016), so that it needs to be more consensus about the relationship between information quality and the cost of capital. Some previous studies (Easley and O'Hara, 2004; Hughes, Liu, and Liu, 2007), incorporate endogenous information acquisition into disclosure models, and some results demonstrate that there is no monotonic relationship between capital costs and information disclosures. Market price efficiency and information quality have a non-monotonic relationship that influences capital costs (Goldstein and Yang, 2019).

Information disclosure is crucial to the stability, development and improvement of the capital market, and the capital market is essentially a market that fully reflects the information. Due to the guiding role of information, capital flows from investors to listed companies that raise funds, thus reflecting the financing function of the capital market. The change in the disclosure information of listed companies can significantly impact investors' investment behaviours and generate numerous economic consequences. One of the significant economic consequences is influencing the cost of equity capital. Considering market microstructure and investors' interpretation of information as a starting point and assuming that information is relevant, this chapter examines the effects of information release quality and investor information explanatory capability in a perfect competition market on the cost of capital.

To sum, I address two questions. Firstly, How can disclosure quality in the financial market affect market efficiency and the cost of capital when disclosure affects the firm's investment decisions if different types of investors hold private/public information? Secondly, assuming that uninformed investors can choose their private information acquisition, in this situation, when market participants are responsible for deciding to acquire information, how do market efficiency and capital cost work? To answer these questions, I first construct the noisy REE model in which information disclosure influences a firm's investment decisions by influencing investors' valuations. Then, I examine how disclosure influences investors' investment decisions, cost of capital, and the efficiency of markets in an asymmetric market. Finally, I model the information acquisition behaviours and evaluate the crowding-out effect on cost of capital and market efficiency.

The main innovation of the study is twofold. First, I use the information precision parameters to measure the quality of disclosure information under the Keynesian beauty contest influence and extend the analysis of the basic setup model, which allows us to focus on the relationship between market efficiency and the cost of capital. Second, I assume that investors can move to acquire the extra private information after considering the cost of information acquisition. I analyze disclosure in the context of perfect competition among different types of investors to illustrate the investment effect of disclosure. Some of findings can help sort out the mixed empirical findings concerning the market efficiency, cost of capital and information disclosure quality.

Outline The reminder of this paper is as follows. Section 2 is the literature review that mainly introduces the disclosure and the effects of regulatory information by financial sectors. Section 3 presents the base model of the transfer mechanism of capital market information through the simple noisy rational expectation model and analyses the impact of information disclosure quality on investor information interpretation ability. Section 2.4 is the effects of the quality of information disclosure under the perspective of market efficiency and cost of capital. Section 2.5 studies the crowding-out effect and information acquisition in public information disclosure. Section 2.6 is the conclusion. All proofs are in the Appendix.

2.2 Related Literature

This chapter complements the literature on the nexus between the relationship effects of disclosure about market efficiency and the cost of capital in financial markets. The main goal of voluntarily disclosing information is to reduce the information asymmetries between investors and companies(i.e., Diamond and

Verrechia,1991; Easley and O'hara,2004), and decrease the uncertainty for the future payoffs(i.e., Barry and Brown,1985, Cheynel,2013).

Initially, some of the literature features the information disclosure and the crowding-out effect. Most studies, Grossman and Stiglitz (1980) structure a typical noisy rational expectations equilibrium to indicate the crowding out effect in a competitive environment. Some researchers have suggested that disseminating public information can lead to potential informed traders not being able to access private information (i.e. crowding out; Verrecchia, 1982a,1982b). A study by Bayer et al. (2010) find that voluntary and mandatory disclosure policies can shape the corporate information environment, according to mitigate information asymmetries and aid dissemination when crowding out is the sole force at work. In addition, similar to Diamond (1985), Gao and Liang (2013) show that information disclosure narrows the information gap between informed and uninformed traders, reducing investors' willingness to obtain private information. This crowding-out effect on private information increases the liquidity of company assets, ultimately prompting investors to make accurate investment decisions. In this section, I focus the literature review on the two sub-contents that this paper focuses on.

The first strand is the literature that examines the disclosure effects of regulatory information by financial sectors, which has been reviewed previously by Verecchia(2001), Kanodia(2006), and Goldstein and Yang(2017). The influence of the public information disclosure system in financial markets has been debated in previous academic research. Many economists support the public information disclosure system and agree that improving the level of public information disclosure by enterprises will reduce information asymmetry, avoid market failure, and positively impact the financial market. For reference, the earliest research, Diamond (1985) contends that public information crowded out the production of private information, reducing the degree of information asymmetry in the market, and thereby promoting market liquidity. Diamond and Verrecchia (1991) initially confirm that public information disclosure promotes market liquidity and reduces the cost of capital, thereby benefiting listed companies. Gao (2008) constructs a two-period setting with overlapping generations of investors, where higher order beliefs naturally plays a role as early traders care about short-term price and thus the higher-order beliefs of late traders. He shows that even taking into account Keynesian beauty contest incentives, more precise disclosure still improves price efficiency. According to studies by Huddart, Hughes, and Levine (2001) and Gong and Liu (2012), insider trading information accelerates asset price discovery and deepens the market depth when trading frequency increases. As a result, it decreases the expected trading profits of informed traders.

After that, there have since been several studies analyzing the impact of information regulation by financial regulators on financial markets (e.g. Dye, 2001; Verrecchia, 2001; Kanodia and Sapra, 2016). Moreover, Verrecchia (1982a, 1982b), Diamond (1985), Kim and Verrecchia (1994), Gao and Liang (2013), Colombo, Femminis, and Pavan (2014), and Goldstein and Yang (2019) emphasize the crowding out effect of fair disclosure. Some closely literature(i.e., Goldstein and Sapra, 2013;Leitner,2014) indicates that, while disclosure can enhance market discipline on efficiency, disclosure can also bring several issues, such as reducing the financial sector's regulation ability to collect information form banks.

At the same time,another strand of the literature cast doubt on the positive impact of public disclosure on the market. Originally, Fishman and Hagerty (1990) study the optimal limit of the discretion of the listed companies to disclose information. They found that only under certain limits, the restriction of the discretion of the information disclosure is conducive to improving the decision-making of the market economy. According to Lundholm's (1991) research, the costs of acquiring private information go up when firms increase information disclosure, thereby reducing the incentives for investors to seek out private information.It is important to note that, in some research papers(i.e., Allen et al., 2006; Morris and Shin,2002), public information disclosure may cause prices to be too sensitive to public information or even deviate from the actual value, resulting in price bubbles, thereby reducing social welfare. Goldstein and Sapra (2013) and Leitner (2014) study the related literature and indicate that although it can strengthen the market discipline, improving disclosure quality can create several problems in reducing the regulatory capacity of acquiring information from banks(Prescott, 2008; Leitner, 2012), decreasing the financial sector's regulation ability to learn from market prices(Bond and Goldstein,2015), or leading financial agents to allocate too much weight on public information signals(Morris and Shin, 2002: Angeletos and Pavan,2007). Some scholars argue that some exogenous factors, such as reputational concerns from media(Morrison and White, 2013; Shapiro and Skeie, 2015) and fiscal capacity (Faria-Castro et al., 2017), can influence the financial sector's regulation policies on information disclosure.

Recently, a few other growing literature discusses the critical point of the information disclosure level. This chapter also connects my model framework to a few theoretical studies that have considered the market efficiency and financial reports(i.e., Diamond, 1985; Gao and Liang, 2013; Colombo, Femminis, and Pavan ,2014; Banerjee, Davis, and Gondhi, 2018). Glodstein and Yang (2015) reveal that there is a distinction between ”good disclosure” and ”bad disclosure” based on the impact of public information on market efficiency. In addition, Gromley et al. (2018) concluded through empirical research on the information disclosure and returns of mutual funds that mandatory public information disclosure may lead to unintended consequences of declining stock market price efficiency. Chen et al. (2017) indicate that when the public information is accurate enough, the market clearing result is unique; otherwise, the market stability will be reduced, and multiple equilibria will appear. Biswas and Koufopoulos(2020) recently study a bank-entrepreneur model and find the reminiscent that there is a hill-shaped relationship curve between market power and financial market efficiency under asymmetric information. However, few of these papers highlighted the crowding-out effect with information acquisition under information disclosure, which is the focus of this chapter.

2.3 Model

Based on previous research, I rely on the Noisy Rational Expectations Equilibrium (Noisy REE) model, a workbench model for analyzing financial markets that provides the original mechanism of Hayek's (1945) view, namely prices aggregate information dispersed among asset traders. My model has traditional CARA normality: in a two-period rational expectations economy, assets are independent, all random variables are normally distributed, and short-term traders have a constant absolute risk aversion (CARA) preference. This model, pioneered by Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982a), typically introduces ”noise trading” or ”liquidity trading” to prevent market prices from fully revealing private information, and focuses on ”No Trade” issue.

2.3.1 The Setup

This section proposes a two-period, noisy, rational baseline model of expected equilibrium. To avoid difficulties, I study the disclosures of a large market in which the trading behaviour of each agent does not affect the market price. Time is discrete, and there are three dates in the economy, $t \in \{0, 1, 2\}$. In period 1, traders decide whether to become informed by acquiring information. In period 2, traders are endowed with these two asset types while trading them in a competitive market. At $t = 2$, the cash flow is realised, and all traders consume their returns realised from their portfolio. **Figure 1** shows the timeline.

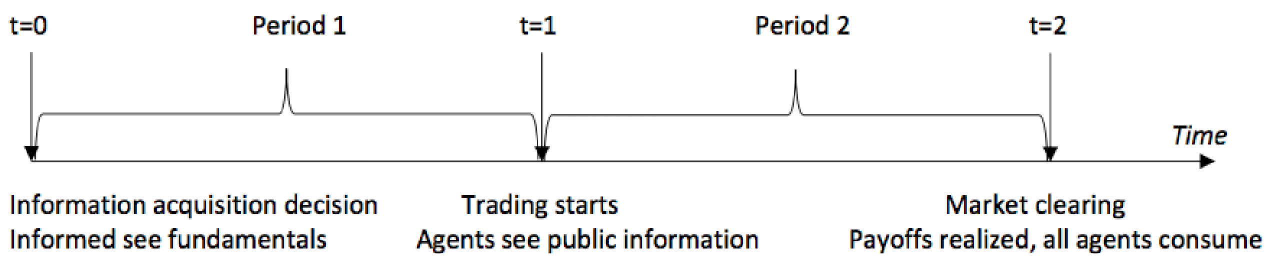

Figure 1: **Timeline of the game.**This framework has two period and three dates: $t \in \{0, 1, 2\}$. In period 1, some investors consider whether to be informed. The financial market opens, and investors start trading at $t = 1$. After period 2, final payoffs occur at $t = 2$.

Securities: Suppose two assets are available for trading in a competitive financial market: a risk-free asset (bonds) and a risky asset (stocks). The risk-free asset has a fixed constant certainty return of one unit in the first period and is in unlimited supply in financial markets. The risky asset operates on a constant aggregate supply of $X > 0$ shares at the market-clearing price $\widetilde{p}$ at $t = 2$. The returns of risky assets are normally distributed with zero mean, and ρ is the precision or the reciprocal of variance of the distribution, such as $\widetilde{v} \sim \mathcal{N}(\overline{v}, \rho_v^{-1})$.

The return of the risky assets and the signals available to different types of traders form a jointly bivariate normal distribution with the mean and covariance matrix. At $t = 0$, all traders share the same public information and the initial price of risky is p_0. At $t = 1$, the risky asset price is p_1. At $t = 2$, risky asset pays the uncertain liquidation value, denoted by $\widetilde{v}$. We induce the semi-strong pricing in a competitive risk-neutral marketing sector as $E[\widetilde{v}|\widetilde{p}] = \widetilde{p}$. Thus the expected return of the risky asset is thus $\mathbb{E}[\widetilde{v} - \widetilde{p}]$.

Noise traders, which is also called liquidity traders, trade in aggregate an exogenous random quantity $\widetilde{z}$ of asset per capita to the market as aggregate supply uncertainty, which are irrational and provide liquidity to rational traders by supplying $\widetilde{z}$ units, where $\widetilde{z} \sim \mathcal{N}(0, \rho_z^{-1})$, with $\rho_z^{-1} > 0$. Additionally, $\widetilde{z}$ is independent of other shocks in financial market(i.e. $\widetilde{v}, \widetilde{\epsilon}_i$), which implies that noise trading has no information content. The realization of a draw for an individual agent, i, is given by $\widetilde{z}_i = z_i$.

The last two agents have constant absolute risk aversion (CARA) utility with coefficient of risk aversion, denoted as $\gamma > 0$. Without loss of generality, since traders have CARA risk preferences, I normalise their initial wealth to zero. *Informed traders* can receive a private and informative signal about the dividend, and *uninformed traders* make their trade by only public information. The total mass of these two types of traders is 1, with the fraction of $\lambda \in [0, 1]$ for informed traders and $1 - \lambda$ for uninformed traders, respectively.

Before trading starts, each informed trader is given a costly private signal $\widetilde{s}_i$, which includes private information about the fundamental stochastic liquidation value $\widetilde{v}$ of the risky asset, as shown below:

$$\widetilde{s}_i = \widetilde{v} + \widetilde{\epsilon}_i, \quad \widetilde{\epsilon}_i \sim \mathcal{N}(0, \rho_\epsilon^{-1}) \tag{2.1}$$

where $\rho_\epsilon > 0$ is the precision of the private signal $\widetilde{s}_i$, which indicates that higher value of ρ_ϵ signifying that $\widetilde{s}_i$ has more information transparency related to asset stochastic liquidation values $\widetilde{v}$. $\{\widetilde{\epsilon}_i\}_{i \in [0,\lambda]}$ is the noise term and distributed independently because of the different information by different informed traders, and it is also mutually independent with $\widetilde{v}$. Meanwhile, trader i chooses a demand schedule $D_i(\cdot, \widetilde{s}_i)$ that depends on his signal $\widetilde{s}_i$, which is a vector including the numbers of shares invested in risky assets. For $i \in [0, 1]$, each agent i maximizes the exponential expected utility $U(W_i) = -exp(-\gamma W_i)$, where W_i is the final profits at $t = 2$, given as $W_{1i} = W_{0i} + (v - p)D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})$ for informed traders.

For the uninformed traders, they only make their market investment based on public signal transmitted through the asset price. Then, we can then conclude that both informed and uninformed traders can observe public information. According to the standard setup of Grossman and Stiglitz (1980) and Hellwig (1980), the standard way to introduce the public signal $\widetilde{y}$ is:

$$\widetilde{y} = \widetilde{v} + \widetilde{\eta}, \quad \widetilde{\eta} \sim \mathcal{N}(0, \rho_\eta^{-1}) \tag{2.2}$$

where the $\rho_\eta \geq 0$ represents the precision of the public signal $\widetilde{y}$. Similarly, I assume that uninformed traders are endowed with $\widetilde{y}$ of an exogenous precision parameter ρ_η, which implies that a higher ρ_η corresponds to a higher quantity level of disclosure about the asset cash flow $\widetilde{v}$, and vice versa. The public signal, $\widetilde{y}$, for instance, can be considered announcements made by the listed company regarding its financial reports and significant issues pertaining to the company (such as mergers and acquisitions, private placements, major lawsuits, arbitrations, etc.), or economic statistics published by the government, central banks, or credit rating agencies. In our assumption, asset issuers are partially disclosed for investors, so we define the interval of the exogenous precision parameter as $\rho_\eta \in [0, \overline{\rho}_\eta]$, where $\overline{\rho}_\eta$ signifies the maximum quality level of the public disclosure. If $\rho_\eta = \overline{\rho}_\eta$, it means that the the $\widetilde{y}$ is the most informative for the asset cash flow $\widetilde{v}$.

Additionally, the tilde in above equations represents the random variable from its realization. The random variables $\widetilde{v}, \widetilde{z}, \widetilde{\epsilon}_i, \widetilde{\eta}$ are all mutually independent, normally distributed random variables, with means normalized to zero. The information precision (reciprocal of variance) is assumed by $\rho_v^{-1} = Var(\widetilde{v}), \rho_z^{-1} = Var(\widetilde{z}), \rho_\epsilon^{-1} = Var(\widetilde{\epsilon})$, respectively.

2.3.2 Rational Expectations Equilibrium

As Grossman and Stiglitz (1980) illustrate, rational expectations equilibrium (REE) includes optimal strategies of agents as well as statistical behaviour of aggregate variables in a baseline asymmetric information model, in which prices are considered aggregate information. As defined in the first period, discretionary liquidity traders make market-participation decisions to maximise their participation in liquidity traders' strategies. In the following setting, market clearing is satisfied at $t = 2$ by the following conditions.

The equilibrium of noisy rational expectations in short-horizon economy requires that (a)rational asset traders invest in assets based on their public information, private information(for the informed traders), and the predicted utility; (b)the market clear when the demand for the risky asset equals the exogeneous supply X; (c) all traders make investment decisions according to the information they receive and are equally sophisticated in strategies.

Building a noisy REE in the stock market boils down to solving a price function that relies on public information $\widetilde{y}$, private information of informed traders $\widetilde{s}_i$ and noisy trades $\widetilde{z}$. According to the law of large numbers, the noise term $\widetilde{\epsilon}_i$ is contained in the private signal $\widetilde{s}_i$, so I speculate that the price $\widetilde{p}$ is determined by the triple $\{\widetilde{y}, \widetilde{v}, \widetilde{z}\}$. Following the previous noisy rational expectations literature (Grossman and Stiglitz,1980; Darrough,1993), I consider linear price function with information variables, as follows:

$$\widetilde{p} = p_0 + p_y\widetilde{y} + p_v\widetilde{v} + p_z\widetilde{z} \tag{2.3}$$

where the parameters p_0, p_y, p_v and p_z are endogenously determined. From normal distribution theory, the posterior distribution of $\widetilde{v}$ given a realization $\{\widetilde{y}, \widetilde{s}_i, \widetilde{p}\}$ is again normal. Specifically, $\widetilde{p}$ is a linear combination of normal random variable, which is jointly normal along with other stochastic variables in the economy. Given the public signal $\widetilde{y}$, the information included in the price is equal to the information signal:

$$\widetilde{s}_p \equiv \frac{\widetilde{p} - p_0 - p_y\widetilde{y}}{p_v} = \widetilde{v} + \frac{p_z}{p_v}\widetilde{z} \tag{2.4}$$

which is normally distributed, with mean $\widetilde{v}$ and endogenous precision is also the reciprocal of variance, as

$$\rho_p = \left(\frac{p_v}{p_z}\right)^2 \rho_z \tag{2.5}$$

which is the price efficiency or price informativeness that measures the extent level of aggregate information, as the standard endogenous precision indicated in noisy REE models (e.g. Kyle, 1989; Ozsoylev and Walden, 2011), conveyed in the price regarding to the asset value $\widetilde{v}$ in addition to the public signal and private information hold by informed traders. The informed traders can observe the information set $\mathcal{F}_I = \{\widetilde{y}, \widetilde{s}_i, \widetilde{p}\}$. On the belief of the existing disclosure public information $\widetilde{y}$ and costly private signal $\widetilde{s}_i$, informed traders determine the demand for securities to maximise the expected utility ($U(W_i) = -exp(-\gamma W_i)$):

$$\max_{D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})} \mathbb{E}\left[U(W_i)|\widetilde{y}, \widetilde{s}_i, \widetilde{p}\right] = -exp\left[-\gamma\left(\mathbb{E}[\widetilde{W}_{1i}|\widetilde{y}, \widetilde{s}_i, \widetilde{p}] - \frac{\gamma}{T}Var[\widetilde{W}_{1i}|\widetilde{y}, \widetilde{s}_i, \widetilde{p}]\right)\right] \tag{2.6}$$

In this case we already make a set up that $T = 2$ as the case of two-period rational expectations . Meanwhile, recalling that the budget constraint is:

$$W_{1i} = W_{0i} + (v - p)D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})$$

where W_{1i} is investor i's terminal wealth at $t = 2$ and W_{0i} is the investor i's initial wealth at $t = 1$. Thus, we can substitute the budget constraint into (2.6):

$$\mathbb{E}\left[\widetilde{W}_{1i}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] = \left(\mathbb{E}\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] - \widetilde{p}\right) D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p}) + W_{0i} \tag{2.7}$$

$$Var\left[\widetilde{W}_{1i}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] = D_I^2 Var\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] \tag{2.8}$$

Thus, this normal distribution function of asset profits characterises that the utility maximization problem can be transformed into a mean-variance optimization problem. The new investor i choose his or her demand D_I to maximise the expected utility, the objective function is:

$$\max_{D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p})} \left\{ \left(\mathbb{E}\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] - p\right) D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p}) + W_{0i} - D_I^2 \frac{\gamma}{2} Var\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] \right\} \tag{2.9}$$

Using the first-order condition, we can characterise the optimal demand takes the following form:

$$\frac{\partial \mathcal{L}}{\partial D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p})} : \left(\mathbb{E}\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] - \widetilde{p}\right) - \gamma D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p}) Var\left[\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}\right] = 0$$

$$D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p}) = \frac{\mathbb{E}(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}) - \widetilde{p}}{\gamma Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})} \tag{2.10}$$

where the $\mathbb{E}(\widetilde{v}|\cdot)$ is the conditional expectation of trader i's posterior belief about the random payoff $\widetilde{v}$, and $Var(\widetilde{v}|\cdot)$ is the variance of trader i's posterior belief about the random payoff $\widetilde{v}$, which is the same for both informed and uninformed traders as $i \in [0,1]$, respectively. The second-order condition is $-\gamma Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}) < 0$, guaranteeing that the demand D_I is the maximum solution. Then, we apply the Bayes' rule to compute the moments conditional on the information set $\mathcal{F}_I = \{\widetilde{y},\widetilde{s}_i,\widetilde{p}\}$, which shows that:

$$\mathbb{E}(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}) = \frac{\rho_v \overline{v} + \rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p}{\rho_v + \rho_\epsilon + \rho_\eta + \rho_p} \tag{2.11}$$

$$Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}) = \frac{1}{\rho_v + \rho_\epsilon + \rho_\eta + \rho_p} \tag{2.12}$$

Finally, after solving this traders' utility-maximization problem, the basic CARA-normal setup assumed here shows that the demand function of informed trader i is

$$D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p}) = \frac{\mathbb{E}(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}) - \widetilde{p}}{\gamma Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})} = \frac{\rho_v \overline{v} + \rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p - (\rho_v + \rho_\epsilon + \rho_\eta + \rho_p)\widetilde{p}}{\gamma} \tag{2.13}$$

Similarly, the uninformed traders only observe the price information $\widetilde{p}$, as $\mathcal{F}_U = \{\widetilde{y},\widetilde{p}\}$. The market-clearing price $\widetilde{p}$ is equal to the following informational signal in predicting the payoff $\widetilde{v}$, as mentioned above.

Besides, I define the uninformed trader's moments conditional on the information set $\mathcal{F}_U = \{\widetilde{y}, \widetilde{p}\}$, and calculate the conditional expectation $\mathbb{E}(\widetilde{v}|\cdot)$ with variance $Var(\widetilde{v}|\cdot)$. In order to work out the uninformed trader i's demand function $D_U(\widetilde{y}, \widetilde{p})$, we can calculate the demand function by a same method as the parts of informed traders from above, except that one can only form an expectation of the payoff $\widetilde{v}$ conditional on the public signal $\widetilde{y}$ and price $\widetilde{p}$, which the conditional expectation and the variance of trader i's posterior belief about the random payoff $\widetilde{v}$ is:

$$\mathbb{E}(\widetilde{v}|\widetilde{y}, \widetilde{p}) = \frac{\rho_v \overline{v} + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p}{\rho_v + \rho_\eta + \rho_p} \tag{2.14}$$

$$Var(\widetilde{v}|\widetilde{y}, \widetilde{p}) = \frac{1}{\rho_v + \rho_\eta + \rho_p} \tag{2.15}$$

Thus, plugging expressions (2.14) and (2.15) into the uninformed trader's demand function $D_U(\widetilde{y}, \widetilde{p})$ as:

$$D_U(\widetilde{y}, \widetilde{p}) = \frac{\mathbb{E}(\widetilde{v}|\widetilde{y}, \widetilde{p}) - \widetilde{p}}{\gamma Var(\widetilde{v}|\widetilde{y}, \widetilde{p})} = \frac{\rho_v \overline{v} + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p - (\rho_v + \rho_\eta + \rho_p)\widetilde{p}}{\gamma} \tag{2.16}$$

Now,I define the market clearing equilibrium for the asset market.

$$\int_0^\lambda D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})di + \int_\lambda^1 D_U(\widetilde{y}, \widetilde{p}) + \widetilde{z} = X \tag{2.17}$$

The left-hand side from (2.17) is the aggregate demand both from the informed traders and uninformed traders, and the right-hand side is the per-capita supply of assets perturbed by the noise trading.

To sum up, I define the equilibrium of the above mechanism design as the standard way.

Definition 2.1: *A noisy rational expectations equilibrium (REE) indicated above consists of a measurable traders with an REE price function $P(\widetilde{s}_i, \lambda) = \widetilde{p}$, $P : \mathbb{R}^2 \to \mathbb{R}$, contingent on the measurable demand functions of both informed traders $D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})$, uninformed traders $D_U(\widetilde{y}, \widetilde{p})$ and noise trading amount $\widetilde{z}$, such that:*

- *Utility Maximization Function (Informed traders): Demand is optimal for informed traders conditional upon their acquired information, as shown in equation (2.13). Besides, it can derive:*

$$D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p}) \in \arg \max_{D_I(\widetilde{y}, \widetilde{s}_i, \widetilde{p})} \mathbb{E}\Big[- exp\{-\gamma(\widetilde{v}) - \widetilde{p}))D_I\} | \widetilde{y}, \widetilde{s}_i, P(\widetilde{s}_i, \lambda) = \widetilde{p} \Big]; \tag{2.18}$$

- *Utility Maximization Function (Uninformed traders): Demand is optimal for informed traders conditional upon their acquired information, as shown in equation (2.16). Besides, we can derive:*

$$D_U(\widetilde{y},\widetilde{p}) \in \arg\max_{D_I(\widetilde{y},\widetilde{s}_i,\widetilde{p})} \mathbb{E}\Big[-exp\{-\gamma(\widetilde{v})-\widetilde{p})D_U\}|\widetilde{y},P(\widetilde{s}_i,\lambda)=\widetilde{p}\Big]; \tag{2.19}$$

- *Market clearing* Conditions: *The financial market clears at the equilibrium asset price that equates the demand of the risky asset to the supply in all pairs $(\widetilde{s}_i,\lambda)$, as in equation (2.17), I derive that:*

$$\int_0^\lambda D_I(\widetilde{y},\widetilde{s}_i,P(\widetilde{s}_i,\lambda)=\widetilde{p})di+\int_\lambda^1 D_U(\widetilde{y},P(\widetilde{s}_i,\lambda)=\widetilde{p})+\widetilde{z}=X \tag{2.20}$$

To define the equilibrium price, we substitute (2.13) and (2.16), and plug the $\widetilde{s}_p$ as shown in (2.4) into the market-clearing condition as (2.18). I solve the $\widetilde{p}$ yields that there exists a noisy rational expectations equilibrium where $\alpha_0,\alpha_y,\alpha_v,\alpha_z$, as shown in the following proposition.

Proposition 2.1: *For any $\lambda^*\in[0,1]$, there exists a unique linear rational expectations equilibrium in financial market with the pricing rule*

$$\widetilde{p}=p_0+p_y\widetilde{y}+p_v\widetilde{v}+p_z\widetilde{z}$$

where the coefficients $\alpha_0<0$, $\alpha_y>0$, $\alpha_v>0$ and $\alpha_z>0$ are parameters determined as follows:

$$p_0=\frac{\rho_v\overline{v}-\gamma X}{\rho_v+\lambda\rho_\epsilon+\rho_\eta+\rho_p} \tag{2.21}$$

$$p_y=\frac{\rho_\eta}{\rho_v+\lambda\rho_\epsilon+\rho_\eta+\rho_p} \tag{2.22}$$

$$p_v=\frac{\lambda\rho_\epsilon+\rho_p}{\rho_v+\lambda\rho_\epsilon+\rho_\eta+\rho_p} \tag{2.23}$$

$$p_z=\frac{(\lambda\rho_\epsilon/\gamma)\rho_z+\gamma}{\rho_v+\lambda\rho_\epsilon+\rho_\eta+\rho_p} \tag{2.24}$$

where ρ_v is the prior precision, $\lambda\rho_\epsilon$ is the precision of the total amount of private information holding by rational traders, ρ_η is the precision of public disclosure, with the precision of the market clearing price of the risky asset $\rho_p=(\lambda\rho_\epsilon/\gamma)^2\rho_z$.

Proof. *See* ***Appendix A.1*** .

In addition, to measure the information impacts of the information acquisition, we need to define the liquidity(noise) traders' behaviours in the financial market. Recalling that the linear price function is

$$\widetilde{p}=p_0+p_y\widetilde{y}+p_v\widetilde{v}+p_z\widetilde{z}$$

where the coefficients p_0, p_y, p_v and p_z are all endogenously determined. Specifically, parameter p_z is relevant to the liquidity trading and market depth by noise speculators, which shows that a smaller coefficient of p_z related to noise trading $\widetilde{z}$ will cause a smaller price impact, and the market is more liquid and deeper. Referred by Kyle's (1985), we can measure the size of the liquidity(noise) trading (L) in financial market formally,

$$L \equiv 1/p_z \tag{2.25}$$

From the (2.21)-(2.24) in **Proposition 2.1** , I find that the disclosure level of the financial market can increase the market liquidity, which is shown as $\partial L/\partial \rho_\eta > 0$. In other words, as the precision of disclosure quality is higher, the uncertainty of asset value will be lower; as a result, the rational (both informed and uninformed) traders will be against the noise traders and liquidity trading.

2.3.3 Discussion

In **Proposition 2.1** above, I briefly generated a two-period noisy rational expectations equilibrium, which is the standard way to speculate and verify the linear equilibrium price equation that has been widely used in differential and asymmetric information models. In contrast to previous literature (mainly Hellwig, 1980; Allen, Morris, and Shin, 2006; Gao, 2008), there are several possible extensions to Allen, Morris, and Shin (2006) and Gao (2008) on the multi-period noisy rationality of short-term traders. For example, consider the paradox noted by Grossman and Stiglitz (1980), when all private information can be acquired from the stock market-clearing price in a competitive market, and no traders have incentives to pay for the private information. In Hellwig's (1980) model, per capita supply can be observed. However, traders cannot condition their demand on contemporaneous prices. While using different information structures, Grundy and McNichols (1989) study a similar two-period noisy rational expectations equilibrium. Allen, Morris, and Shin (2006) show that the impact of rational Keynesian beauty contests increases with traders' short-term horizons.

In reality, AI and information technology advances enable investors to obtain a large amount of information and data from financial institutions at lower costs. For example, investors can easily access the company's disclosure information and industry research reports released by investment institutions (e.g., Reuters, Bloomberg) on the Internet. Investors can use this to evaluate the value of stocks and assist them in making investment decisions. Similarly, many large and influential financial technology companies (e.g., iSentium, Dataminr or Eagle Alpha) use AI algorithms to technically analyze many unstructured data(e.g., news reports, press releases, stock market announcements, tweets, satellite imagery, etc.) in financial markets, and sell these analysis reports to investors.

As information costs fall, existing models with endogenous information acquisition predict that asset prices will become more informative, either because more investors will purchase information (Grossman and Stiglitz, 1980) or because investors will acquire more precise signals (Verrecchia, 1982a). An exciting dimension that has inspired papers in this area of research is that the type of information disclosed is significant in determining whether the disclosure is desirable (Bond and Goldstein, 2015; Goldstein and Yang, 2015). Among them, I find that when improving the quality of information disclosure, the ability to interpret information will be enhanced both in informed and uninformed traders. However, the information interpretation ability of informed traders is not as high as that of uninformed traders, which reduces the degree of information asymmetry between informed and uninformed traders.

In summary, from the perspective of investors' rational expectations, I construct a model for the quality of information disclosure and the impact of different traders who have information resources on the market price under the conditions that the market is entirely competitive and information-related and show that how a unique rational expectation in the financial market exists in a market-clearing equilibrium. Additionally, by studying the previous outcomes related to traders' effects of disclosure quality, this model motivation contributes a large amount of previous literature that focuses on the market efficiency, cost of capital, information acquisition and crowding-out effects in a pure exchange economy, which will be discussed in following sections.

2.4 The Quality of Information Disclosure: Market Efficiency and the Cost of Capital

Market efficiency and the cost of capital are two significant concepts that attracted much attention in financial disclosure problem both from the previous literature and the relevant Acts[6]. Specifically, in this

[6] "fairly, orderly, efficient and efficient markets" mentioned in *Sarbanes-Oxley Act of 2002*, *Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010* & *Selective Disclosure and Insider Trading 2000*.

part, based on the additional expectations equilibrium benchmark model, I define market efficiency with Keynesian beauty contest metaphor and cost of capital as standard measures to evaluate the impact of investors' behaviours on asset prices and how investors are affected by information precision in the financial market.

2.4.1 Market Efficiency within Beauty-Contest Influence

Market efficiency, also known as price efficiency or information efficiency, concerns the extent to which market prices incorporate available information. The ability of asset prices to accurately reflect information about their intrinsic value is often seen as an essential criterion in capital markets (Hayek, 1945; Fama, 1970; Peress, 2010; Ozsoylev and Walden, 2011). Scholars further define the ability or degree to which prices reflect their underlying information as Market Efficiency. If market prices do not adequately reveal the information, there may be opportunities for arbitrage from information gathering and manipulation. An efficient information market is a market in which asset prices can quickly and reasonably disclose past and present information. As we mentioned in the previous section, Fama (1976) discusses the definition that best explains the meaning of the term *efficient* in this context, that *"An efficient capital market is a market that is efficient in processing information "*. A fundamental reason for increasing market efficiency is the belief that it is a good proxy for real efficiency, through which more price information about the underlying value can enhance the accuracy of investment decisions. Additionally, corporate governance theory believes that timely and stable information flow can reduce information asymmetry (Jensen and Meckling, 1976; Bushman et al., 2004) to improve the capability of corporate shareholders to supervise management movements (Hermalin and Weisbach, 2012). Meanwhile, the company's quality signal is communicated to existing and potential investors in the market to better distinguish between good and bad companies (Akerlof, 1970). Information disclosure can strengthen the company's position and reputation in investors' minds, stabilize investors' valuation of the company, and thus decline the company's cost of capital (Botosan, 1997) and debt costs (Sengupta, 1998).

Market efficiency should be improved by making information and financial disclosures available. The London Stock Exchange, and the Tokyo Stock Exchange, for example, provide information about trading activity and traded companies. Analysts typically cover a large portion of listed companies on these markets, and many traders and analysts participate in them. These markets are relatively efficient. By contrast, smaller securities markets may lack trading activity and material information, such as those in emerging markets. In addition, there may be significant differences in the efficiency of different types of markets (for example, some securities are traded exclusively in over-the-counter (OTC) markets, such as bonds, money market instruments, swaps, and forward contracts.) The information provided by the market makers for these markets can vary significantly in both quality and quantity over time and according to the type of information offered.

The influential metaphor of the beauty contest in financial markets describes that, because of short horizons by asset traders, they have to concern not only recognizing their expectations about the fundamentals of the asset but also recognizing about expectations of other traders' beliefs about the asset's future payoffs in the market, which is forming average expectations and higher-order expectations. In this case, asset traders will consider others' expectations of an asset's payoffs. Based on the preliminary setting from the last section, I describe a higher-order expectations equilibrium within the rational model. Similar with Morris and Shin (2002), Allen, Morris and Shin(2006), and Yang and Cai(2014), I define the conditional expectation value of the random payoff of trader i's posterior belief as $\mathbb{E}_i(\widetilde{v}|\cdot)$, and the average expectation value of the random payoff of all rational traders' posterior belief as $\overline{\mathbb{E}}(\widetilde{v}|\cdot) = \int_0^1 \mathbb{E}_i(\widetilde{v}|\cdot)di$. For higher-order beliefs, I define the k-order expectations of trader i's posterior belief as $\mathbb{E}_i^k(\widetilde{v}|\cdot)$ and the k-order average expectations of all traders posterior belief as $\overline{\mathbb{E}}_i^k(\widetilde{v}|\cdot)$. In the general perfectly competitive case, the demand function for trader i in k-order expectations is:

$$D_i(\cdot) = \frac{\mathbb{E}_i^k(\widetilde{v}|\cdot) - p_1}{\gamma Var_i^k(\widetilde{v}|\cdot)} \tag{2.26}$$

and the average demand for all traders is:

$$\overline{D}(\cdot) = \frac{\overline{\mathbb{E}}^k(\widetilde{v}|\cdot) - p_1}{\gamma Var_i^k(\widetilde{v}|\cdot)} \tag{2.27}$$

where $D_i(\cdot) \in \{D_I(F_I), D_U(F_U)\}$. The $Var_i^k(\widetilde{v}|\cdot) \in \{Var_i^k(\widetilde{v}|F_I), Var_i^k(\widetilde{v}|F_U)\}$ indicates the variance of $\mathbb{E}_i^k(\widetilde{v}|\cdot) \in \{\mathbb{E}_i^k(\widetilde{v}|F_I), \mathbb{E}_i^k(\widetilde{v}|F_U)\}$ conditional on the information set $\{F_I, F_U\}$ for rational trader i. When the trader i has both public and private information signals, using the rule of Bayesian Information Criterion(BIC), I deduce the trader i's individual expectation as follows:

$$\mathbb{E}_i(\widetilde{v}|\cdot) = \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{s}_i}{\rho_\eta + \rho_\epsilon} \tag{2.28}$$

So, I derive the average expectation value of the random payoff of all rational traders' posterior belief is

$$\overline{\mathbb{E}}(\widetilde{v}|\cdot) = \int_0^1 \mathbb{E}_i(\widetilde{v}|\cdot)di = \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{v}}{\rho_\eta + \rho_\epsilon} \tag{2.29}$$

Then, trader i's expectation of the average expectation of the random payoff $\widetilde{v}$ across agents is:

$$\begin{aligned}\mathbb{E}_i(\overline{\mathbb{E}}(\widetilde{v}|\cdot)|\cdot) &= \mathbb{E}_i\left(\frac{\rho_\eta \widetilde{y} + \rho_\epsilon \overline{\mathbb{E}}(\widetilde{v}|\cdot)}{\rho_\eta + \rho_\epsilon}|\cdot\right) \\ &= \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \left(\frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{s}_i}{\rho_\eta + \rho_\epsilon}\right)}{\rho_\eta + \rho_\epsilon} \\ &= \frac{((\rho_\eta + \rho_\epsilon)^2 - \rho_\epsilon^2)\widetilde{y} + \rho_\epsilon^2 \widetilde{s}_i}{(\rho_\eta + \rho_\epsilon)^2}\end{aligned} \tag{2.30}$$

Hence, I derive the 2-order average belief of $\widetilde{v}$ as:

$$\overline{\mathbb{E}}^2(\widetilde{v}|\cdot) = \overline{\mathbb{E}}(\overline{\mathbb{E}}(\widetilde{v}|\cdot)\cdot) = \frac{\rho_\epsilon^2}{(\rho_\eta + \rho_\epsilon)^2}\widetilde{s}_i + \left(1 - \frac{\rho_\epsilon^2}{(\rho_\eta + \rho_\epsilon)^2}\right)\widetilde{y} \tag{2.31}$$

Using the iterating operation, the general equation of k-order average belief of $\widetilde{v}$ can be derived as:

$$\overline{\mathbb{E}}^k(\widetilde{v}|\cdot) = \overline{\mathbb{E}}(\overline{\mathbb{E}}(\widetilde{v}|\cdot)\underbrace{\cdot|\cdot|\cdots|\cdot}_{\text{k-1}}) = \frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\widetilde{s}_i + \left(1 - \frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\right)\widetilde{y} \tag{2.32}$$

, and the trader i's k-order expectation and variance can be written as following:

$$\mathbb{E}_i^k(\widetilde{v}|\cdot) = \frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\widetilde{s}_i + \left(1 - \frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\right)\widetilde{y} \tag{2.33}$$

$$Var_i^k(\widetilde{v}|\cdot) = \left(\frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\right)^{2k-2} Var_i(\widetilde{v}|\cdot) = \frac{\rho_\epsilon^{2k-2}}{(\rho_\eta + \rho_\epsilon)^{2k-1}} \tag{2.34}$$

Then, considering the financial market clearing equilibrium in (2.17), where the exogenous net supply of risky asset X is fixed and equal to zero in the short term. From (2.27), the optimal price of risky asset can be given below:

$$p_1^* = \overline{\mathbb{E}}^k(\widetilde{v}|\cdot) \tag{2.35}$$

where the risky asset at market clearing price $\widetilde{p}$ is amplified by higher order beliefs. Assuming that $X = 0$ and $y = 0$, I plug the (2.32) into (2.35) and derive the optimal risky asset price as:

$$p_1^* = \frac{\rho_\epsilon^k}{(\rho_\eta + \rho_\epsilon)^k}\widetilde{v} \tag{2.36}$$

(2.36) shows the traders trade the risky asset at optimal price p_1^* at $t = 1$, which indicates that the financial market-clearing condition will reveal the average of all traders' information level.

It is important to note that the theoretical model presented by Grossman (1976) provides a simplified representation of financial markets, and it may not capture all the complexities of real-world markets. While the model suggests that the equilibrium price reflects the average of all investors' information, this assumption may not hold in practice, as investors may have varying levels of access to information, and may interpret and act on that information differently.

Regarding the statement about the path of p_1^*, it is true that (2.36) implies that the equilibrium price will float between the value of p_0 and $\widetilde{v}$. However, the statement that "the price path is always just like a random walk" is not necessarily accurate. While it is true that financial markets are characterized by a certain degree of randomness and unpredictability, there are also many factors that can influence the direction and magnitude of price movements, including economic news, political events, and changes in investor sentiment. Therefore, while the theoretical model may suggest a certain pattern of price movements, the actual behavior of prices in the market may be more complex and difficult to predict.

Based on the above k-order expectations setting conditional on beauty effects influence, I deduce the proposition of relationship between the expected return of risky assets for traders and private information precision as follows:

Proposition 2.2: *The equilibrium expected return of risky asset for investors will be lower when the asset issuers control more private information that is undisclosed in financial markets.*
Proof. *See* ***Appendix A.2****.*

In concrete practice, fair treatment of all market investors is essential to the integrity of the market and explains why regulators place so much importance on *Sarbanes-Oxley(SOX) Act of 2002* and *Dodd-Frank Wall Street Reform and Consumer Protection Act 2010*. In addition, the SEC's Fair Disclosure Regulations and *Selective Disclosure and Insider Trading 2000* also state that if issuers of securities provide nonpublic information to certain market professionals or traders, they must also Also disclose this information to the public. Some of these rules guarantee that some investors will not have the informational advantage over others and that insiders will not trade based on the private information. The civil and criminal penalties associated with violations of these rules are intended to prevent illegal insider trading, increasing the cost of insider trading, thereby promoting fair investment in the financial market.

Market efficiency captures how much information is included into the trading asset price. Following the Bayesian rule from previous literature (e.g., Vives 2008; Peress, 2010; Ozsoylev and Walden, 2011; Goldstein and Yang, 2014), I measure the market efficiency ME using the precision of the posterior about a trading asset payoff conditional on its market price. It is the reciprocal of the mean squared error (MSE) related to the assets uncertain liquidation value $\widetilde{v}$ and its stock price $\widetilde{p}$, that is:

$$ME = \frac{1}{MSE} = \frac{Cov(\widetilde{v}, \widetilde{p})}{\sqrt{Var(\widetilde{v}|\cdot)Var(\widetilde{p})}} = \frac{1}{\mathbb{E}\left[(\widetilde{v} - \widetilde{p})^2\right]} \tag{2.37}$$

where the cash flow $\widetilde{v}$ and the price $\widetilde{p}$ are normally distributed, and the market efficiency can be measured by the correlation coefficient between $\widetilde{v}$ and $\widetilde{p}$. ME is an ex ante measure and $\mathbb{E}[\cdot]$ means that the expectation is taken with respect to both supply noise and disclosure information. The derivative of $\frac{1}{MSE}$ indicates the market efficiency from the sense that it controls the precision with the fundamental information which incorporated by asset prices. This equation gives the most standard method for measuring market efficiency, which is consistent with the previous literature, such as Grossman and Stiglitz (1980), suggesting using squared correlation coefficient between price and fundamental value to measure the informativeness of the asset price.

Meanwhile, to make the numerator simpler, I define the variable $\Pi = \rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p$. From the **Proposition 2.1**, ρ_v, $\lambda\rho_\epsilon$, ρ_η, and $\rho_p = (\lambda\rho_\epsilon/\gamma)^2\rho_z$ are all precision parameters that are specific to different variables, thus we can interpret Π as the average conditional precision of asset payoff $\widetilde{v}$ or the average information interpretation ability across all traders. Then we can derive the market efficiency variable ME as:

$$ME = \frac{(\Pi)^2}{(\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z} \tag{2.38}$$

Statistically, the metric $\frac{1}{MSE}$ delivers the goodness of the risky asset price as an estimator of the fundamental value. The lower value of MSE indicates the better goodness to the fundamental value and the more efficiency level in the financial market, and vice versa. Furthermore, in asset markets as a Keynesian beauty contest, it is still a worthwhile cause to promote market efficiency through accounting public disclosure. The residual uncertainty of the fundamental $\widetilde{v}$ conditional on price $\widetilde{p}$ is low, and hence market efficiency is high. This function also indicates that the market efficiency level is highly positively related to the average conditional precision of asset payoff $\widetilde{v}$ or the average information interpretation ability across all traders.

Proposition 2.3: *Higher precision of public information improves the efficiency level in financial market, even in the presence of the Keynesian beauty contest influence.*
Proof. *See **Appendix A.3**.*

2.4.2 The Cost of Capital

By identifying and exploring possible market inefficiencies, traders incur a cost of capital that affects their interpretation of market efficiency. Throughout this section, I divide the cost of capital into two types: transaction costs and information-acquisition costs.

To exploit perceived market efficiency, trading often incurs *transaction costs*. Therefore, efficiency should be considered within the context of transaction costs. Take, for example, a violation of the principle that two identical assets should sell for the same price in different markets with different disclosure quality. Because prices appear to process information inconsistently, such a violation can be regarded as a relatively simple possible exception to market efficiency (McMillan et al., 2011). A trader can exploit this violation by simultaneously purchasing lower-priced assets and shorting higher-priced assets. It will not occur if the asymmetric level, which is often measured by the price discrepancy between the two markets, is smaller than the transaction costs involved in arbitrage for the lowest cost traders, resulting in both prices being efficient within arbitrage's bounds. Generally, these arbitrage bounds are relatively narrow in highly disclosure quality and liquid markets(e.g. U.S Treasury bills); however, they could be comprehensive in lower disclosure quality and liquid markets.

Information acquisition costs are often incurred in gathering and analyzing information. Market prices are adjusted to reflect new aggregated information based on the trades made by active traders based on information they gather and analyze. As traders place trades based on their analysis of new aggregated information, they include it in transaction prices. In the methodological view of market efficiency, active traders incur information acquisition costs, wasting money because prices already reflect all relevant information. As a result of this rigorous view of efficiency, a market is considered inefficient if active trading can recapture any part of its costs (e.g. research costs and active asset selection). A return must be provided for information acquisition; if markets are efficient, net returns for these expenses are just fair compensation for the risk incurred in equilibrium, according to Grossman and Stiglitz (1980). The modern view holds that markets are inefficient if aggressive investments pay off handsomely after deducting these costs. After deducting fees, acquiring information in an efficient market should yield returns.

Prior research has developed rational expectations equilibrium models that both public and private information can affect asset values (e.g., Easley and O'Hara, 2004; Lambert, Leuz and Verrecchia, 2012; Hughes, Liu and Liu, 2007). In this model, the return on the risky asset is $(\widetilde{v} - \widetilde{p})$. In period two, the uncertainty is resolved, and the asset price is equated to the fundamental value $\widetilde{v}$. A higher level of disclosure supposedly reduces capital costs by lowering traders' risk as shifting information from the public to the private increases the equilibrium required return, as illustrated in Easley and O'Hara's (2004) discussion.

From the **Proposition 2.1**, we can indicate that the expected mean of asset prices is $\mathbb{E}[\widetilde{p}] = \widetilde{v} - \frac{\gamma X}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}$. To make the denominator simpler, I define the variable $\Pi = \rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p$, which can be interpreted as the average conditional precision of asset payoff $\widetilde{v}$ or the average information interpretation ability across all traders, and then we get $\mathbb{E}[\widetilde{p}] = \widetilde{v} - \frac{\gamma X}{\Pi}$. Following the previous literature(e.g. Easley and O'Hara, 2004), I define the cost of capital(COC) as following the proposition.

Proposition 2.4: *Under the perfectly competitive market and information-related conditions, the cost of capital is:*

$$COC \equiv \mathbb{E}[\widetilde{v} - \widetilde{p}] = \frac{\gamma X}{\Pi} \tag{2.39}$$

where Π *measures the average risk in terms of the assets payoff that determines the cost of capital, which is multiplied by risk aversion* γ *and risk asset supply* X*, as:*

$$\Pi = \lambda \frac{1}{Var(\widetilde{v}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})} + (1-\lambda)\frac{1}{Var(\widetilde{v}|\widetilde{y}, \widetilde{p})} \tag{2.40}$$

Under the perfectly competitive and information-related conditions, the cost of capital is increasing in the risk aversion of traders and the risky asset supply, and as the disclosure level or quality of information increasing, the cost of capital will decrease, as $\frac{\partial COC}{\partial \rho_\eta} < 0$.
Proof. *See* ***Appendix A.4****.*

Additionally, I present a proposition to indicate that increasing the fraction of informed traders can decrease the cost of capital (2.40). From the **Proposition 2.4**, I derive the expression of the cost of capital as

$$COC = \frac{\gamma X}{\lambda Var(\widetilde{v}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})^{-1} + (1-\lambda) Var(\widetilde{v}|\widetilde{y}, \widetilde{p})^{-1}} \tag{2.41}$$

Not surprisingly, from (2.41), as the mass of informed traders grows, the risky asset demand is still rising due to the lower risk-aversion level and the enormous amount of private information owned by informed traders in the financial market. As I assume the total mass of informed traders is λ $(0 < \lambda < 1)$, and $1 - \lambda$ traders are uninformed, he average precision of the public disclosure is ρ_η. Increasing the precision in $\rho_e ta$ means lower information asymmetry between informed and uninformed traders and a higher level of information transparency in the financial market. In the perspective of the asset issuers, the equilibrium expected return of risky asset per share will be higher when the asset issuers have more private information. The main results are shown in the following proposition.

Proposition 2.5: *In an economy with the mass of informed traders increases, ceteris paribus, then* ρ_η *increases, the cost of capital will deceases.*
Proof. *See* ***Appendix A.5****.*

In summary, the modern view requires traders to consider transaction costs and the cost of obtaining information when assessing market efficiency. After accounting for transaction costs and information acquisition costs, the price difference must be significant enough for traders to make a profit, concluding that the difference may represent market inefficiency. Prices may not fully reflect available information but still do not provide accurate market opportunities for active traders. In such model motivation and financial markets, market efficiency and cost of capital play an important role via public information quality(precision). Taking China's stock market as an example, as the second-largest stock in the world, many stocks are very liquid. In such a large market, while the stamp tax does increase the cost of trading, it is not necessarily a reflection of market power. In fact, the Chinese government has taken steps in recent years to reduce transaction costs in the stock market, including reducing the stamp tax rate and eliminating other fees and charges. These reforms aim at increasing the efficiency and competitiveness of the market, and they suggest that a variety of factors beyond market power alone can influence transaction costs.

The Keynesian beauty contest theory is still reasonable even in behavioural pricing models; that is, the higher precision of public information, whether distracted by Keynesian beauty contest influence, can make better goodness of fit to the fundamental value, higher efficiency level in the financial market, and lower cost of capital in the perfectly competitive and information-related conditions.

2.5 Crowding-out Effect and Information Acquisition

Based on the original framework by Verrecchia (1982a, 1982b), Diamond (1985), and other scholars, this section presents the primary model of motivation for acquiring private information by market participants. The impact of public information on information production can be bidirectional because there are two competing forces. On the one hand, negative crowding out of products is documented in the literature (e.g. Diamond, 1985); higher quality disclosure can crowd out speculators' trading gains from private information, discouraging information production. In contrast, Han, Tang and Yang (2016) found a positive effect. They indicate that disclosure attracts noise trading in the baseline model with exogenous information, encouraging information production. There is some evidence that more public information can reduce trader incentives to become informed or acquire more precise information. Thus, public information crowds out private information, thereby weakening and possibly reversing the direct effect of disclosure on some market-quality variables. In this section, I demonstrate the basic argument that increasing disclosure precision can crowd out private information acquisition. This demonstrates that the impact of disclosure on market quality is more subtle once private information is endogenous. This will depend on the amount of information disclosed on information-acquisition technology, taking into account measures of market quality.

2.5.1 Crowding-out Effect on Private Information

Previous research that reproduced the crowding-out effect on private information is originally from Verrecchia (1982a) and Diamond(1985). Now, I extend the baseline noise REE framework mentioned in above section by adding the cost of information acquisition by rational traders.

Firstly, closely following the previous research(e.g. Verrecchia, 1982a; Admati and Pfleiderer, 1986; Holmström and Tirole, 1993), I designate a payment cost of information acquisition $C(\cdot)$ for rational investor i that uninformed traders can pay this information acquisition cost to become informed at $t = 0$. The payment cost $C(\cdot) > 0$ is an increasing and convex function multiplied with the precision of public (ρ_η) and private signal(ρ_ϵ), with $C(0) = C^{'}(0) = 0$. Similar to Grossman and Stiglitz(1980), both the mass λ of informed traders and the mass of $(1 - \lambda)$ of uninformed traders are ex-ante identical. Based on the demand functions (2.13) and (2.16), I design the ex-ante utility of these two types of rational traders as V_I and V_U, separately.

$$V_I = -\sqrt{\frac{Var(\widetilde{v} - \widetilde{p}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})}{Var(\widetilde{v} - \widetilde{p})}} exp\left\{\gamma C(\rho_{\epsilon i}) - \frac{(COC)^2}{2Var(\widetilde{v} - \widetilde{p})}\right\} \tag{2.42}$$

$$V_U = -\sqrt{\frac{Var(\widetilde{v} - \widetilde{p}|\widetilde{y}, \widetilde{p})}{Var(\widetilde{v} - \widetilde{p})}} exp\left\{0 - \frac{(COC)^2}{2Var(\widetilde{v} - \widetilde{p})}\right\} \tag{2.43}$$

Secondly, I add an expected net benefit of acquiring information $B(\lambda, \rho_\eta, \rho_\epsilon)$ accompanied with the precision of private signal ρ_ϵ to a potential continuum of traders. Each trader i's cost of information acquisition is $C(\rho_{\epsilon i})$. To reference the effect by Kurlat and Veldkamp (2015), the expected net benefit of information $B(\lambda, \rho_\eta, \rho_\epsilon)$ is measured by the gap of the certainty equivalent between the informed traders and uninformed traders, as $CE_I \equiv -(1/\gamma)log(-V_I)$ and $CE_U \equiv -(1/\gamma)log(-V_U)$, as follows:

$$\begin{aligned} B(\lambda; \rho_\eta, \rho_{\epsilon i}) = CE_I - CE_U &= \frac{1}{2\gamma} log\left[\frac{Var(\widetilde{v}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})}{Var(\widetilde{v}|\widetilde{y}, \widetilde{p})}\right] - C(\rho_{\epsilon i}) \\ &= \frac{1}{2\gamma} log\left[\frac{\rho_v + \rho_{\epsilon i} + \rho_\eta + \rho_p}{\rho_v + \rho_\eta + \rho_p}\right] - C(\rho_{\epsilon i}) \\ &= \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + \rho_p}\right] - C(\rho_{\epsilon i}) \end{aligned} \tag{2.44}$$

where $\rho_p = (\lambda\rho_\epsilon/\gamma)^2\rho_z$ is the same with the previous section. In (2.44), I explicitly express the net expected benefit B as a equation of $(\lambda; \rho_\eta, \rho_{\epsilon i})$ to incorporate equilibrium precision, which delivers that the net expected benefit is dominated by the mass of λ of the rational traders and the precision of private information by speculator i.

Thirdly, given the benefit equation $B(\lambda;\rho_\eta,\rho_{\epsilon i})$, traders can optimize their signal precision according to the first-order condition, given as:

$$\begin{aligned}\frac{\partial B(\lambda;\rho_\eta,\rho^*{}_\epsilon)}{\partial\rho_{\epsilon i}} &= \frac{1}{2\gamma(\rho_v+\rho^*{}_\epsilon+\rho_\eta+(\lambda\rho^*{}_\epsilon/\gamma)^2\rho_z)} - C^{'}(\rho^*_\epsilon) = 0 \\ \Rightarrow C^{'}(\rho^*_\epsilon) &= \frac{1}{2\gamma(\rho_v+\rho^*{}_\epsilon+\rho_\eta+(\lambda\rho^*{}_\epsilon/\gamma)^2\rho_z)} > 0 \end{aligned} \tag{2.45}$$

where the $\rho_{\epsilon i} = \rho^*{}_\epsilon \in (0,\overline{\rho_\epsilon}), i \in [0,1]$. Eq(2.45) defines the optimal response of the speculator i's decision that whether to pay the information acquisition cost $C(\cdot)$ to become informed in financial market. In order to find the maximization bound level of the differential and monotonically expected benefit $B(\lambda;\rho_\eta,\rho_{\epsilon i})$, I introduce a critical information acquisition value $C^*(\rho_\epsilon) \in (0,\overline{C(\rho_\epsilon)})$, with $\overline{C(\rho_\epsilon)} \in (0,+\infty)$, since the cost of acquiring informationis continuous , differential and monotone increasing.

In addition, the net expected benefit $B(\lambda;\rho_\eta,\rho_{\epsilon i})$ also determines the equilibrium mass of λ^* of informed investors as a critical value. Specifically, the mass of λ^* is linearly dependent on the cost of information acquisition, with $\lambda^* = C^*(\rho_\epsilon)/\overline{C(\rho_\epsilon)}$. Then, focusing on the relationship among net expected value, cost of information acquisition, and the mass of informed investors, I divide four possible cases to discuss the benefit of information acquisition and the crowding-out effect.

Case I: $B(\lambda;\rho_\eta,\rho_{\epsilon i}) < 0$, then the net expected benefit to become informed while paying the information acquisition is negative. Sequentially, rational traders want to be informed when no other traders are informed. The certainty equivalent of informed traders is less than that of uninformed traders. Thus, we can derive that the mass of informed traders in equilibrium λ^* are equal to zero, which means that no traders want to acquire private information, as:

$$\begin{aligned} B(\lambda;\rho_{\epsilon i}) = CE_I - CE_U &= \frac{1}{2\gamma}log\left[1+\frac{\rho_{\epsilon i}}{\rho_v+\rho_\eta+\rho_p}\right] - C(\rho_{\epsilon i}) < 0 \\ &\Rightarrow \frac{1}{2\gamma}log\left[1+\frac{\rho_{\epsilon i}}{\rho_v+\rho_\eta+(\lambda^*\rho_{\epsilon i}/\gamma)^2\rho_z}\right] < C(\rho_{\epsilon i}) \\ &\Rightarrow \frac{1}{2\gamma}log\left[1+\frac{\rho_{\epsilon i}}{\rho_v+\rho_\eta}\right] < C(\rho_{\epsilon i}) \end{aligned} \tag{2.46}$$

with $\rho_p = (\lambda\rho_\epsilon/\gamma)^2\rho_z$ and $C(\rho_{\epsilon i}) \in (C^*(\rho_\epsilon), \overline{C(\rho_\epsilon)})$.

Case II: $B(\lambda;\rho_\eta,\rho_{\epsilon i}) = 0$ with $\lambda^* \in (0,1)$. In this case the equilibrium condition for determining interior fraction of rational traders is:

$$
\begin{aligned}
B(\lambda;\rho_\eta,\rho_{\epsilon i}) = CE_I - CE_U &= \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + \rho_p}\right] - C(\rho_{\epsilon i}) = 0 \\
\Rightarrow \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + (\lambda^*\rho_{\epsilon i}/\gamma)^2\rho_z}\right] &= C(\rho_{\epsilon i}) \qquad (2.47)
\end{aligned}
$$

with the mass of informed traders in equilibrium λ^* and $C(\rho_{\epsilon i}) = C^*(\rho_\epsilon)$. As the net expected benefit $B(\lambda;\rho_\eta,\rho_{\epsilon i})$ is differential, monotonically decreasing in λ, thus there is an unique solution of the mass of informal traders λ^* in financial market equilibrium.

Case III: $B(\lambda;\rho_\eta,\rho_{\epsilon i}) > 0$ with $\lambda^* = 1$. In this case, rational traders can make benefit for being informed, and if all the rational traders are choosing to be informed($\lambda^* = 1$), then it shows:

$$
\begin{aligned}
B(\lambda;\rho_\eta,\rho_{\epsilon i}) = CE_I - CE_U &= \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + \rho_p}\right] - C(\rho_{\epsilon i}) > 0 \\
\Rightarrow \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + (\lambda^*\rho_{\epsilon i}/\gamma)^2\rho_z}\right] &> C(\rho_{\epsilon i}) \\
\Rightarrow \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + (\rho_{\epsilon i}/\gamma)^2\rho_z}\right] &> C(\rho_{\epsilon i}) \qquad (2.48)
\end{aligned}
$$

with $C(\rho_{\epsilon i}) \in (0, C^*(\rho_\epsilon))$. This case mimics the one of the research objects in Verrecchia(1982b).

Case IV: $B(\lambda;\rho_\eta,\rho_{\epsilon i}) > 0$ with $\lambda^* \in (0,1)$. Another aspect of investors is indifferent to becoming informed. In this case, I assume an endowment of the interior fraction of rational traders, which means that only one part of investors is willing to move to be informed. The rational traders are sensitive to the market information disclosure, as:

$$
\begin{aligned}
B(\lambda;\rho_\eta,\rho_{\epsilon i}) = CE_I - CE_U &= \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + \rho_p}\right] - C(\rho_{\epsilon i}) > 0 \\
\Rightarrow \frac{1}{2\gamma} log\left[1 + \frac{\rho_{\epsilon i}}{\rho_v + \rho_\eta + (\lambda^*\rho_{\epsilon i}/\gamma)^2\rho_z}\right] &> C(\rho_{\epsilon i}) \qquad (2.49)
\end{aligned}
$$

with $C(\rho_{\epsilon i}) \in (0, C^*(\rho_\epsilon))$ as well.

In summary, the above analysis from (2.46) to (2.49), it indicates that the disclosure quality of public information ρ_η is high related to the net expected benefit of becoming informed, as the public information precision increases, the benefit $B(\lambda; \rho_\eta, \rho_{\epsilon i})$ will decrease. Subsequently, the mass of informed traders λ^* in equilibrium will decrease because of the fewer benefits of information acquisition. This co-movement phenomenon is called *Crowding-out effect*. The higher precision of public disclosure will crowd out the private information production by rational market traders and subsequently cause the decrease of informed traders in financial market equilibrium. The crowding-out effect indicates a negative effect of the information disclosure in the condition that the rational traders are sensitive to the disclosure as $\lambda^* \in (0,1)$.

Proposition 2.6: *Assume that rational traders in financial markets are sensitive to the public disclosure as $\lambda^* \in (0,1)$, the increasing precision of public disclosure will dominate a crowding-out effect to private information production, which means that, the high transparency and quality of disclosure can decrease the mass of informed traders in financial market equilibrium, that is:*

$$\frac{\partial \lambda^*}{\partial \rho_\eta} < 0 \tag{2.50}$$

and

$$\lim_{\rho_\eta \to \infty} {\rho^*}_\epsilon = 0 \quad \text{and} \quad \frac{\partial {\rho^*}_\epsilon}{\partial \rho_\eta} < 0 \tag{2.51}$$

Proof. *See **Appendix A.6**.*

2.5.2 Information Acquisition

In this part, I compare the market efficiency, cost of capital, and precision of private information to reveal how information acquisition and private information quality can affect the market efficiency and the cost of capital. To demonstrate the role of disclosure that involves information acquisition, I separately measure two key coefficients associated with precision of private information signals, ρ_ϵ, and the cost of acquiring private information, $C(\cdot)$.

Specifically, I focus primarily on the case by adding an overt information-acquisition period 0, prioritising period 1 & 2. In this case, I assume uninformed traders decide to become informed at Period 1 by paying the cost of acquiring private information, $C(\cdot)$. From the last section, if an uninformed trader i

is willing to become informed and acquire private information ρ_ϵ, I define CE_I as the informed trader's expected ex-ante expected utility, where the parameter set (λ, ρ_η). Trader i only chooses $\rho_{\epsilon i}$ as their controlling variable. Like Grossman and Stiglitz's (1980) method, I calculate the gap between the CE_I and CE_U as the equilibrium outcomes of information acquisition.

In the equilibrium of information acquisition, at the condition that rational traders are indifferent between being informed and remaining uninformed, the expected utility between informed and uninformed traders is identical. In our **Case III** from the last subsection, there is a unique equilibrium outcome of the critical balance of the mass between informed traders and uninformed traders. Alternatively, suppose the cost of acquiring private information is sufficiently high. In that case, no traders have a willingness to choose to be informed, which is discussed in **Case I** from the last subsection.

Reconsidering the expressions of market efficiency and the cost of capital in the above chapter:

$$ME = \frac{(\rho_v + \lambda^*\rho_\epsilon + \rho_\eta + (\lambda^*\rho_\epsilon/\gamma)^2\rho_z)^2}{(\gamma X)^2 + (\rho_v + \rho_\eta + (\lambda^*\rho_\epsilon/\gamma)^2\rho_z) + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z}$$

$$COC = \frac{\gamma X}{\rho_v + \rho_\eta + (\lambda^*\rho_\epsilon/\gamma)^2\rho_z + \lambda^*\rho_\epsilon}$$

with $\rho_p = (\lambda\rho_\epsilon/\gamma)^2\rho_z$. Substituting the equation $\frac{\partial\rho_\eta}{\partial\rho^*{}_\epsilon} = -\frac{1}{2\gamma[C''(\rho^*{}_\epsilon)]^2} - 2\left(\frac{\lambda}{\gamma}\right)^2\rho^*{}_\epsilon\rho_z - 1 < 0$(See proof of ***Proposition 2.6***) into above the expressions of market efficiency and the cost of capital:

$$ME = \frac{(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{(\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z} \tag{2.52}$$

$$COC = \frac{\gamma X}{\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon} \tag{2.53}$$

with $\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} = \rho_v + \rho_\eta + (\lambda^*\rho_\epsilon/\gamma)^2\rho_z$. The variable $\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1}$ is the cost related factor that equals to the total precision allocated from the free public information. Given **Proposition 2.6**, that $\frac{\partial\lambda^*}{\partial\rho_\eta} < 0$, with $\lambda^* \in (0,1)$, we can conclude that because of the crowding-out effect, disclosure and information acquisition harm the market efficiency and increase the cost of capital.

Then, to make the maximization of market efficiency and the minimization of cost of capital, I compute the first-order condition of the disclosure quality, λ^*, with $\lambda^* \in (0,1)$ which shows that $\partial ME/\partial(\lambda^*) > 0$ and $\partial COC/\partial(\lambda^*) < 0$. Meanwhile, reconciling with the **Proposition 2.6** from the last subsection, when the private information can influence market efficiency and the cost of capital with a continuous and monotonic parameter. Thus, from the above analysis, we can imply the effects of information acquisition, which concludes as follows:

Proposition 2.7: *When the cost of information acquisition increases, the market efficiency will decrease and the cost of capital will increase, as* $\partial ME/\partial C(\cdot) < 0$ *and* $\partial COC/\partial C(\cdot) > 0$*. Furthermore, when the precision of the private information increases, the market efficiency will increase and the cost of capital will reduce, as* $\partial ME/\partial \rho_\epsilon > 0$ *and* $\partial COC/\partial \rho_\epsilon < 0$*.*
Proof. *See* ***Appendix A.7****.*

2.6 Conclusion

This chapter finds that the quality of information disclosure has a significant impact on market efficiency and the behaviour of investors in financial markets. Specifically, the chapter suggests that high-quality information disclosure can improve market efficiency and reduce the cost of capital. This is because high-quality information can help investors make more informed investment decisions, reducing information asymmetry between different types of traders and increasing the accuracy of market prices. We discuss the impact of information disclosure quality on different types of traders varies. Informed traders, who have a higher ability to acquire disclosed information, benefit more from high-quality information disclosure. In contrast, uninformed traders, who have a lower ability to acquire disclosed information, are less impacted by the quality of information disclosure. We also provide valuable insights into the relationship between information disclosure, market efficiency, and the behaviour of different types of traders in financial markets. The findings have important implications for policymakers and market participants, as they suggest that improving the quality of information disclosure can have significant benefits for market efficiency and the cost of capital.

Based on the basic workbench model, I use the noisy rational expectations equilibrium(noisy REE) model to analyse the market price response under aggregate information dispersion with heterogeneous risk-tolerant and short-horizon investors. Rational traders are sensitive to public disclosure. The impact of disclosure is often expressed by analyzing various measures of market quality. We focus on two critical concepts (market efficiency and cost of capital) that attracted much attention in the financial disclosure problem based on the noisy REE benchmark model framework. Market efficiency is reciprocal of the mean squared error associated with the uncertain liquidation value of an asset and its asset price, and it also conveys the goodness of risky asset prices as an estimator of functional value. The model extension results show that even in the existence of Keynesian beauty contest influence, the level of market efficiency positively correlates with the average conditional precision of risky asset returns or the gap in the information interpretation abilities between informed and uninformed traders. These results also indicate that higher precision of public information disclosure increases the efficiency level of financial markets. Meanwhile, in a perfectly competitive market and information-related conditions, the cost of capital is positively associated with the risk aversion of traders and the supply of risky assets. These results are consistent with previous studies(Easley and O'Hara, 2004; Lambert, Leuz, and Verrecchia, 2012).

Moreover, I endogenise the overt acquisition of private information by market participants by extending the baseline noisy REE model. The model motivation points out the disclosure quality of public information will dominate a crowding-out effect to the net expected benefit of becoming informed and the cost of information acquisition; , the public information precision increases, being informed will decrease because the price informed traders have a poor informational advantage. This result indicates that the high disclosure transparency can decrease the incentives of acquiring private information and profit arbitrage in asymmetric financial market equilibrium, thus narrowing the gap between informed and uninformed traders. Additionally, reducing the cost of information acquisition can also enhance market efficiency and reduce the cost of capital. This can be achieved through technological innovations or regulatory changes that lower the cost of accessing and analyzing information. Overall, the endogenous acquisition of private information model provides further insights into the relationship between information disclosure, market efficiency, and the behaviour of market participants. Policymakers and market participants should consider the impact of public and private information on market outcomes and take steps to promote transparency and reduce information asymmetry.

3 Level-k Reasoning with Heterogeneous Information Signals[7]

3.1 Introduction

Investors in financial markets are often overly optimistic about their investment strategies(see, for example, Garcia and Sangiorgi and Urošević, 2007; Bouteska and Regaieg, 2018).[8] The standard Rational Expectation Equilibrium(henceforth REE) model assumes investors have unbiased expectations and homogeneous strategies in making investment decisions. This assumption conducted by the REE model cannot explain market participants' overvaluation and overoptimistic beliefs when they decide to enter the financial market. Campbell (2018) argues that without assuming highly volatile supply shocks that are exogenous and unexplained, it is more challenging to generate the resembled trading volume using the REE model as compared with observing results. Firstly, the REE model ignores the iterated thinking process and assumes that all traders' thinking is equally sophisticated in investor behaviours, ignoring that some professionals think one step ahead compared to unprofessional investors. Secondly, the excessive trading volume observed is inconsistent with the REE model. As Hellwig (1980) points out, the ″schizophrenia″ problem of Grossman's (1976) noisy REE model in a perfectly competitive market arises from the fact that traders' individual information and the ″noise″ in the prices covariate non-negligibly, which results in a nonzero correlation between the equilibrium prices and the individual information of the traders.

An intriguing question is whether a subset of investors has superior knowledge about informed trading strategies. To solve the ″schizophrenia″ problem, I introduce the level-k reasoning model that describes investors' informed trading strategies as playing a chess game analogously.[9] In this sense, it

[7]I am grateful to John Fender, Aditya Goenka, the participants of the PhD seminar of University of Birmingham, International Conference on Derivatives and Capital Markets at Shandong University, Asian Meeting of the Econometric Society, Asian Meeting of the Econometric Society in East and South-East Asia, and various workshops for helpful comments and suggestions. All errors are mine.

[8]García, Sangiorgi and Urošević's (2007) study demonstrates that the unsophisticated, overconfident investors tend to be more active for risky opportunities in optimism periods, while such investors are less active in pessimistic periods. Empirically, Bouteska and Regaieg's (2018) study used around 6,777 quarterly observations on the population of US-insured industrial and services firms from 2006 to 2016. The robust results find that overconfidence bias is dominant to investors in the U.S.-financial market.

[9]In fact, the professional chess masters can be hired by hedge funds in Wall Street because they per-

resembles: Chess players observe chess pieces and anticipate the moves of opponents by observing chess pieces. While thinking about optimal strategies, chess players also examine their opponents' strategies and the responses they receive. A chess player who believes he has a more sophisticated mind than other players will also keep playing in these strategic environments; otherwise, he will not keep playing. The chess players' decisions metaphor suggests that the REE model cannot accurately estimate speculative behaviour in the financial market, as the REE model assumes that all investors have equally sophisticated thinking and ignores iterative thinking processes. In contrast to the REE model, the level-k reasoning model provides a better description of how level-k players perceive other level-$(k-1)$ players as not as sophisticated as themselves. There are several pieces of previous literature analyze this auction environment in which traders act sequentially, and the payoff for later traders is dependent on the actions of early traders(Hellwig and Gale, 2002; Angeletos, Lorenzoni and Panan, 2010; Goldstein, Ozdenoren and Yuan, 2011).

The core thought of the level-k reasoning model in investment behaviours is that higher level-k investors believe that all others are at least one level lower than themselves. If accurate, the level-k model would substantially fill a modelling framework gap for actions placed between irrational, random, or non-strategic investment actions and equilibrium strategies. It has been recognised as a descriptive (boundedly) rational model to account for the heterogeneous beliefs among investors and revised understanding of how an auction ought to be designed by experimental observations(Crawford et al., 2009; De Clippel et al., 2019). As Warren Buffett once said, as an investor, "*We attempt to be fearful when others are greedy and to be greedy only when others are fearful.*[10]" In reality, only a minority of investors can make positive profits on average hence investors/speculators who expect to make positive speculation profits need to take contrarian strategies possibly and always think one step forward than the market.[11]

form well in making trading decisions. For reference. See `https://dealbook.nytimes.com/2011/09/29/good-at-chess-a-hedge-fund-may-want-to-hire-you/`.

[10]See `https://www.investopedia.com/articles/investing/012116/warren-buffett-be-fearful-when-others-are-greedy.asp`. This quote vividly captures the core meaning of the level-k reasoning in making investment strategies.

[11]For example, Frey et al. (2018) examine the performance of hedge funds during bull and bear markets and find that contrarian strategies can be effective in generating positive returns even when the overall market is performing poorly. Another study by Cremers and Pareek(2019) find that the majority of actively managed mutual funds underperformed their benchmark index over a 10-year period, providing further evidence that only a minority of investors are able to consistently outperform the market.

In this chapter, I contribute to the debate on level-k reasoning and introduce the notion of dividing investors' behaviour into different levels of thinking in a beauty-contest game. A level-k model allows investors' behaviours to be rational in best responding to some heterogeneous beliefs. Particularly, to analyse the welfare effects of the enhanced role of public information through disclosures among market participants, Morris and Shin (2002) design a zero-sum game in which each agent's behaviour is the weighted average of public and private signals. As a result, rational investors will place a higher value on public signals of the same information precision when the precision of public signals is increased. As a result, if public information signals are raised with greater precision while private information signals are continuously improved, social welfare is expected to decrease. These concluding remarks propose that public information disclosure may cause a double-edged influence on public policy; that is, market participants may overreact to the public information signals, and thus any unwarranted disclosure and fake news may cause significant damage for investors' welfare in the financial market.

Outline The reminded is organised as follows. Section 3.2 discusses theoretical literature about the level-k reasoning model in beauty contest structure. Section 3.3 outlines the theoretical model for our research formally, establishes notation with unique equilibrium, and makes particular assumptions that rely on the Morris-Shin model as a particular case of my framework with the quantitative illustration. Section 3.4 provides the analytical designs and explanations for level-k reasoning. This section mainly discusses the specification of level-0 setup and the level-k reasoning strategies under full and partial disclosure conditions. Section 3.5 discuss the quantitative illustration for the level-k reasoning and optimal actions with partial informative public signals. Section 3.6 is the conclusion. Additionally, I put all proofs and some extension of Kyle(1989) model in Addendix.

3.2 Related Literature

3.2.1 Trading Strategies and Information Structures

After the financial crisis in 2007, some researchers justified the missing increase in inflation by using the level-k thinking with the introduction of bounded rationality and heterogeneous beliefs. Hong and Stein(2007) developed a mechanism framework of disagreement and heterogeneity beliefs among traders.

It shows that traders' beliefs are often a simple function of just their priors, and each of them can observe the information signals directly; this conclusion is adverse to the REE model, where each stock market trader must also update depending on the inferences about others signals and priors.

Initially, Kyle(1989) models the strategic trading in an environment of imperfect competition, while Vives (2011) and Rostek and Weretka (2012, 2015) incorporate more general information structures into the Kyle model. Several previous studies have identified the overreaction of public signals disclosure. However, they did not elicit heterogeneous noises on the fundamental asset value with different information precision. To our knowledge, in the beauty-contest game discussed by Morris and Shin (2002), Baeriswyl and Cornand (2014), Cornand and Heinemann (2015) are the first to analyse what effects the precision of the information signals have. It is worth mentioning the study by Shapiro, Shi, and Zillante (2014), which indicate that, the predictive power of the level-k reasoning approach depends on the symmetry of information and the precision of private signals. Specifically, they argue that if the private signals are precise and symmetric, then higher-level reasoning can lead to more accurate predictions of behavior. However, if the private signals are noisy or asymmetric, then higher-level reasoning may not necessarily lead to better predictions. They also note that the level-k reasoning approach may be more useful in situations where there is a high degree of homogeneity among agents' beliefs and behavior.

In other implications, Friedenberg, Kets, and Kneeland(2018) develop a novel identification strategy by the ability limitations, in which non-degenerate beliefs about rationality can be crucially determined by limitations inability rather than bounded reasoning about rationality because a player may be strategic but irrational in some circumstances. This result shows that it has crucial implications for out-of-sample predictions. In recent, Kets and Sandroni (2021) use the level-k reasoning as an exemplary method for describing what constitutes the optimal composition of teams in order to explain how agents' different culture diversity affects their reasoning and the degree of strategic uncertainty that players face. However, few discussions disentangle the specific effects of heterogeneous information precision on the agents' level-k reasoning with the beauty contest. While the role of the provision of public signals and private signals is sufficient for determining market equilibrium on the sophisticated level-k reasoning of market participants in perfect and imperfect competition, it has hardly been modelled and studied. This chapter fills these gaps.

3.2.2 Level-k Reasoning in Keynesian Beauty Contests

The experimental literature on the beauty contest game under heterogeneous information provides insights into how individuals make decisions in situations where they must infer the beliefs and actions of others based on incomplete and possibly conflicting information. These studies have important implications for understanding how people process information and make decisions in financial markets. Initially, The link between level-k and financial markets can be traced back to John Maynard Keynes's beauty contest metaphor(Keynes,1936, p. 156). Keynes' book initially likens professional investing to a newspaper contest, picking the prettiest face from a hundred photos and the faces picked by most contestants. Other contestants face the same incentives:

A "beauty contest" game simplifies the essence of Keynes's (1936) 's observation. Level-k reasoning is not mainly separate from the p-beauty contest. The game consists of a reference point, called level 0 and (finite) iterated best responses. As shown in Nagel(1995), Duffy and Nagel(1997), Costa-Gomes and Craword(2006), the level-0 reasoning, denoted as $L0$, is subjected to non-strategic actions and investors select random strategics without considering any beliefs about their opponents' actions. $L0$ is typically considered an investor's model of opponents rather than an actual investor. Level-1 investors, denoted as $L1$, anticipate this game and consider that all other investors are $L0$ sophisticated and choose the best response to this subjective belief. Similarly, Level-2 investors think that all others are $L1$ and $L0$ types, choosing best-response to this belief. Thus, a $L(k+1)$ type investor is more innovative than a Lk type in thinking about the actions of Lk types.

This chapter closely relates to Morris and Shin's (2002) model, which discuss the concept of the "beauty contest game" to study how individuals make decisions in a situation where they must infer the beliefs and actions of others based on incomplete and possibly conflicting information. Moreover, this chapter also belongs to an experimental study based on the beauty contest game with an exogenous information structure, conducted by Conrad and Heinemann (2014). Baeriswyl and Conrad (2014) discuss that partial publicity, consisting of providing a public signal with idiosyncratic noise, may effectively reduce overreaction to public disclosures. In their experiment, participants were randomly assigned to different information conditions, including full publicity, partial publicity, and no publicity. They found that partial publicity reduced the tendency for participants to overreact to public information and led to more efficient outcomes.

In addition to the above basic beauty-contest structure, some researchers subject the bound rationality to a beauty contest in a recent study. In Gill and Prowse (2016), who used repeated p-beauty contest games to investigate striking differences according to cognitive ability, subjects with higher cognitive ability responded more agreeably and were more emotionally stable to the cognitive abilities of their opponents, and earned more even when behaviour approaches the equilibrium prediction, but subjects with lower cognitive ability did not. Bosch-Rosa and Meissner (2020) designed a two-player beauty contest game that subjects can play against themselves. One of the experimental results shows that only a minority (31%) of the subjects can fully understand the one-player guessing game due to the lack of reasoning ability.

Overall, this chapter contributes a growing theoretical literature related to heterogeneous information signals and develops a practicable framework to induce beauty contest theory, Level-k reasoning model, and the competitive REE model as with the descendants of Keynes(1936), Hellwig(1980) and Grossman and Stiglitz(1980), such as Back, Cao and Willard(2000), Holden and Subrahmanyam (1992), etc. This chapter is most closely related to Morris and Shin(2002), Vives(2011), Cornand and Heinemann (2014), Shapiro, Shi and Zillante (2014), and Rostek and Weretka(2012, 2015) extend their models to incorporate more sophisticated information structures into the financial market under perfect or imperfect competitive environments with different technical frameworks to propose their results.

3.3 The Theoretical Model with Heterogeneous Information

In the level-k reasoning model, agents' actions may be heterogeneous, but they are modelled by a standard distribution based on a hierarchy of types or strategies. The model relies on a strategic behaviour game in the spirit of the beauty-contest theory mentioned in Keynes's observation in 1936. The core thought of the Keynesian beauty-contest theory is characterised by strategic complementarities in the investors' decisions. Each agent i takes her action that relates to an unknown fundamental and the average action of others $-i$(all agents except player i). This model generalises the modification of Morris and Shin's (2002)(henceforth MS) model as a preliminary basis for our analysis, to stimulate the level-k reasoning investors' optimal actions in beauty-contest under the conditions of imperfect, heterogeneous information.[12]

3.3.1 Beauty Contest Structure

I begin my analysis by examining a two-stage principal-agent game in a beauty contest framework. The social planner (i.e. authorities or central banks) decides the optimal signal precision and the limitation of information disclosure (full publicity for all agents, partial release of public information, or none release the information) that maximise social welfare before agents (i.e. investors and participants in asset market) take their investment actions. Assuming that there exists a continuum of agent i, where agent i indexed by the unit interval $[0, 1]$, and she selects an action $a_i \in \mathbb{R}$. The payoff function for agent i is:

$$u_i(a_i, \theta) \equiv -(1-\gamma)(a_i - \theta)^2 - \gamma(a_i - \bar{a}_{-i})^2 \tag{3.1}$$

where $\theta \in \mathbb{R}$ is the fundamental that has a uniform distribution and can be interpreted as the asset common value on the reals. The constant $\gamma \in [0, 1)$ is the weight assigned to the strategic component as measuring the relative importance of coordinating with decision rule, in which matching the underlying fundamentals θ versus the average action of whole population. $1 - \gamma$ is the weight on the fundamental-related utility component. Agents use γ to align their strategy complarimentries with those of others, where the larger γ reflects, the more severe is the externality.

In (3.1), the first component is the distance between agent i's action a_i, and the underlying state θ, that reflects the standard quadratic loss from mismatching the underlying fundamental θ. The second term is a Keynesian beauty contest component, a zero-sum game in which the agent who guesses closest to the target will get a non-negative reward, which refers to the difference between agent i's action and her opponents' action, $\bar{a}_{-i}$.

[12] The MS model also has been widely applied in the setting of asset pricing(Allen, Morris and Shin,2006) and experimental economics(Shapiro, Shi, and Zillante,2014).

3.3.2 Information Sources

Following the insight of the MS model, each agent i receives heterogeneous information signals that deviate from θ by independent error terms with uniform distribution on the reals. Assuming that private and public signals have the heterogeneous information precision ρ_ϵ and ρ_η respectively, each agent i observes the realisation of the owned private signal $\widetilde{s}_i$, which is :

$$\widetilde{s}_i = \theta + \widetilde{\epsilon}_i, \quad \widetilde{\epsilon}_i \sim \mathcal{N}(0, 1/\rho_\epsilon) \text{ and } \rho_\epsilon > 0 \tag{3.2}$$

where the noise term of the continuum distinct individuals for private signal, $\widetilde{\epsilon}_i$, has the normal distribution with zero mean and variance $1/\rho_\epsilon$. Private signal $\widetilde{s}_i$ is only observable by each individual agent i and cannot be observed by others.

However, public signal $\widetilde{y}$ in the sense that the actual realisation of $\widetilde{y}$ is common knowledge to all informed investors in the market, which is:

$$\widetilde{y} = \theta + \widetilde{\eta}, \quad \widetilde{\eta} \sim \mathcal{N}(0, 1/\rho_\eta) \text{ and } \rho_\eta > 0 \tag{3.3}$$

where $\widetilde{\eta}$ determines the noise for public signal, which also has normal distribution with zero mean and variance $1/\rho_\eta$. Fundamental value θ is distributed with the uniform distribution over the real line. The noise $\widetilde{\epsilon}_i$ of private signal and noise $\widetilde{\eta}$ of public signal are independent and their distribution is treated as exogenously given. In the case of partial disclosure of public signal, the signal $\widetilde{y}$ is the public for partial agents in the sense that the actual realisation of $\widetilde{y}$ is a common knowledge that can be observed among informed agents $i \in [0, \lambda]$.

Assuming that the number of agents is continuous, we can expect the fraction of agents receiving public information to be approximately equal to the number of agents, denoted as λ. I design the fraction $\lambda \equiv \frac{informed\ agents}{all\ agents}$ for the disclosure level of public signal in the economic environment. A fraction of $\lambda \in [0, 1]$ are informed traders and a fraction $1 - \lambda$ are uninformed traders.

Assumption 3.1: *(social planner) Social planner discloses public signal to some of agents with fraction λ. Informed agents $i \in [0, \lambda]$ can receive the disclosure of public signal from social planner. However, uninformed agents*

$i \in (\lambda, 1]$ do not receive public signals, and they must rely on their private signals only.

Assumption 3.2: *(uninformed agents) Uninformed agents cannot receive public signal, the expected value of fundamental θ only relies on the possesses his own private signal:*

$$\mathbb{E}_i(\theta|\widetilde{s}_i) = \widetilde{s}_i \tag{3.4}$$

, and his expected average action is:

$$\mathbb{E}_i(\bar{a}_{-i}|\widetilde{s}_i) = \widetilde{s}_i \tag{3.5}$$

Assumption 3.3: *(informed agents) The public signal is only given to informed agents. Thus, the expected value of fundamental θ conditional on his own private signal, $\widetilde{s}_i$, and public signal, $\widetilde{y}$ for the informed agent i's perfect Bayesian equilibrium action, is given by*

$$\mathbb{E}_i(\theta|\widetilde{y}, \widetilde{s}_i) = \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{s}_i}{\rho_\eta + \rho_\epsilon} \tag{3.6}$$

, and the expectation of the agent i's opponents is:

$$\mathbb{E}_i(\widetilde{s}_{-i}|\widetilde{s}_i, \widetilde{y}) = \mathbb{E}_i(\theta|\widetilde{y}, \widetilde{s}_i) = \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{s}_i}{\rho_\eta + \rho_\epsilon} \tag{3.7}$$

3.3.3 Social Welfare

Social welfare, defined as the (normalised) individual utilities in average, denoted by $W(a_i, \theta)$, can be shown as:

$$W(a_i, \theta) \equiv \frac{1}{(1-\gamma)} \int_0^1 u_i(a_i, \theta) di = -\int_0^1 (a_i - \theta)^2 di \tag{3.8}$$

So that the social planner, who focuses on social welfare, aims to keep all agents' actions close to the fundamental value, θ. Based on the average expected value of risky assets, the social planner prescribes that agents take action. On the one hand, Due to the public signal $\widetilde{\eta}$ given to all agents in the case of asymmetric disclosure choices, the social planner seeks to keep all agents' actions close to fundamental θ. Public signals would be given relatively more weight if agents considered the noise in public signals to be more valuable in predicting the actions of other agents (all agents except i) than uncorrelated private signals. On the other hand, in the case of asymmetric disclosure strategies, where informed agent discloses and uniformed agents do not. In equilibrium, prior expected welfare is calculated as follows:

$$\mathbb{E}_i(W) = -\frac{\rho_\eta + \rho_\epsilon(1-\lambda r)^2}{(\rho_\eta + \rho_\epsilon(1-\lambda r))^2} \tag{3.9}$$

which is decreasing in the precision of public signals ρ_η, if $\rho_\epsilon/\rho_\eta \leq 1/(2\lambda r - 1)(1-\lambda r)$.[13] $\mathbb{E}_i(\cdot)$ is the posterior expectation of fundamentals conditional on $\widetilde{s}_i$ and $\widetilde{y}$. In summary, I discuss the relation between social welfare and the precision of public/private signal as following:

Proposition 3.1:

- *If the degree of public disclosure published by the social planner is low, increasing the precision of public signal is beneficial for the expected social welfare.*
- *Higher precision of private signal has positive effects for the expected social welfare.*

Proof. *See* ***Appendix B.1****.*

Lemma 3.1: *The higher degree of exogenous public information published by the social planner is detrimental to social welfare in the beauty-contest framework.*

Proof. *See* ***Appendix B.1****.*

3.3.4 Market Equilibrium

I state the definition of equilibrium as the following features: (*a*) To maximise the expected social welfare, the social planner determines the precision of the public signal ρ_η and the level of public disclosure in the first stage. Then, agents choose their actions a_i in the second stage to maximize their expected utility. (*b*) The Nash equilibrium of the game is when no player is incentivized to deviate from the social planner and the continuum of agents. (*c*) Specifically, the pure symmetric market discloses public information signal across all agents by social planner such as $\lambda = 1$, while the pure asymmetric market with $\lambda = 0$ represents that the social planner would disclose no public signal in financial market and agents have to

[13] An additional explanation is shown in **Appendix B.1**.

make their actions relying on their private information only.

Agent i chooses the action a_i to maximize the utility function yields the optimal action by using the first-order condition:

$$\frac{\partial u_i(a_i, \theta)}{\partial a_i^*} = 0 \tag{3.10}$$

, thus the optimal response of agent i's action under strategic complementarities is given as:

$$a_i^* = (1-\gamma)\mathbb{E}_i[\theta] + \gamma \mathbb{E}_i[\overline{a}_{-i}] \tag{3.11}$$

where $\mathbb{E}_i[\cdot]$ is the expectation operator of agent i. Considering the non-strategic $L0$ type, traders with different levels of sophistication will form different expectations based on $\mathbb{E}_i[\overline{a}_{-i}]$ and choose an action accordingly.

In the case of partial disclosure of public signal by the social planner, I consider that only a subgroup of informed agents in the interval $i \in (0, \lambda]$ can receive both private and public signal, and the optimal action is denoted by $\overline{a}_{-i}(\widetilde{s}_i, \widetilde{y})$. Informed agents attach the optimal weight $f = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}$ assigned to signal $\widetilde{s}_i$ and $1 - f = \frac{\rho_\eta}{\rho_\eta + \rho_\epsilon}$ assigned to signal $\widetilde{y}$. For agent i's opponents, an agent $-i(-i \neq i)$ attaches weight f to his/her private signal. According to MS design, conditional expectations are a linear combination of the available information signals in the normal distribution.

Meanwhile, the subgroup of uninformed agents in the interval $i \in (\lambda, 1]$ receives just a private signal and cannot get public signal from social planner. In this subgroup, the optimal action is $a_i = \widetilde{s}_i$. As discussed in (3.11), under strategic complementarities, i's optimal response under the first-order condition is a linear function of conditional expectations. Following Morris and Shin(2002), the best strategy for an informed agent i who can receive both public and private signals is a linear strategy in the form, that is

$$a_i = f\widetilde{s}_i + (1-f)\widetilde{y} \tag{3.12}$$

, where the optimal weight f depends on an informed agent's beliefs about his/her opponents' actions. The expected estimation of the average action across all agents is expressed as

$$\mathbb{E}_i(\overline{a}_{-i}) = (1-\lambda)\mathbb{E}_i(\widetilde{s}_{-i}) + \lambda[f\mathbb{E}_i(\widetilde{s}_{-i}) + (1-f)\widetilde{y}] \tag{3.13}$$

In (3.13), the first component is the λ weighted informed agents who can receive both public and private signals, and the second component is the $1-\lambda$ weighted uninformed agents who receive just a private signal. For any agent $-i$, the average posterior expectation action conditional on $\widetilde{s}_i$ and $\widetilde{y}$ across all agents is given by

$$\begin{aligned}\mathbb{E}_i(\overline{a}_{-i}|\widetilde{s}_i,\widetilde{y}) &= \lambda[f\mathbb{E}_i(\widetilde{s}_{-i}|\widetilde{s}_i,\widetilde{y}) + (1-f)\widetilde{y}] + (1-\lambda)\mathbb{E}_i(\widetilde{s}_{-i}|\widetilde{s}_i,\widetilde{y}) \\ &= \lambda(1-f)\widetilde{y} + (\lambda f + 1 - \lambda)\mathbb{E}_i(\widetilde{s}_{-i}|\widetilde{s}_i,\widetilde{y}) \\ &= \lambda(1-f)\widetilde{y} + (\lambda f + 1 - \lambda)\frac{\rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y}}{\rho_\eta + \rho_\epsilon} \end{aligned} \tag{3.14}$$

Then, as shown in Morris and Shin(2002), the optimal action of informed agent i, for $i \in [0, \lambda]$, is a linear function that combines his/her public and private signals. Substituting (3.14) and (3.9) into (3.4), agent i's optimal action can be expressed by

$$\begin{aligned}a_i &= (1-\gamma)\mathbb{E}_i(\theta|\widetilde{s}_i,\widetilde{y}) + \gamma\mathbb{E}_i(\overline{a}_{-i}|\widetilde{s}_i,\widetilde{y}) \\ &= (1-\gamma)\frac{\rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y}}{\rho_\eta + \rho_\epsilon} + \gamma\left[\lambda(1-f)\widetilde{y} + (\lambda f + 1 - \lambda)\frac{\rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y}}{\rho_\eta + \rho_\epsilon}\right] \\ &= \frac{\rho_\epsilon(1-\gamma\lambda(1-f))}{\rho_\eta + \rho_\epsilon}\widetilde{s}_i + \frac{\rho_\eta + \rho_\epsilon\gamma\lambda(1-f)}{\rho_\eta + \rho_\epsilon}\widetilde{y} \end{aligned} \tag{3.15}$$

Comparing the coefficients in (3.12) and (3.15), we can therefore solve for the optimal weight f yields the heterogeneous information signals of this subgame,

$$f^* = \frac{\rho_\epsilon(1-\gamma\lambda(1-f^*))}{\rho_\eta + \rho_\epsilon} \Rightarrow f^* = \frac{\rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)} \tag{3.16}$$

$$1 - f^* = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)} \tag{3.17}$$

By symmetry, the optimal weight on the private and public signal is f^* and $1-f^*$, respectively. The optimal action $a_i(\widetilde{s}_i,\widetilde{y})$ of informed agent i in linear equilibrium is given by

$$a_i^*(\widetilde{s}_i,\widetilde{y}) = \underbrace{\frac{\rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}}_{f^*}\widetilde{s}_i + \underbrace{\frac{\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}}_{1-f^*}\widetilde{y} \tag{3.18}$$

Additionally, as I mentioned above, for uniformed agents who receive just a private signal, the optimal action is $a_i = \widetilde{s}_i$. For the informed agents whose optimal strategy relies on both public and private signals, the average action is $\overline{a}_{-i}(\widetilde{s}_i,\widetilde{y})$. The average action across both informed and uninformed agents is

$$\overline{a}_{-i} = \lambda\int_{i=0}^{\lambda} a_i(\widetilde{s}_i,\widetilde{y})di + (1-\lambda)\int_{\lambda}^{1}\widetilde{s}_i di = \lambda\overline{a}_{-i}(\theta,\widetilde{y}) + (1-\lambda)\theta \tag{3.19}$$

in which the average action of informed agents is

$$\overline{a}_{-i}(\theta, \widetilde{y}) = f\theta + (1-f)\widetilde{y}$$

where all informed agents choose the same f and $1-f$ in market equilibrium if the linear equilibrium is uniqueness. Substituting (3.16) and (3.17) into (3.20) to derive the average action across all agents, we finally obtain

$$\begin{aligned}
\overline{a}_{-i} &= \lambda[f\theta + (1-f)\widetilde{y}] + (1-\lambda)\theta \\
&= \lambda\left[\frac{\rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\theta + \frac{\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\widetilde{y}\right] + (1-\lambda)\theta \\
&= \frac{\rho_\eta(1-\lambda) + \rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\theta + \frac{\lambda\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\widetilde{y}
\end{aligned} \tag{3.20}$$

This average action function indicates that agents' optimal actions are distorting away from θ towards $\widetilde{y}$ in market equilibrium in this generalised beauty contest game(more discussion in Cornand and Heinemann, 2008).

3.3.5 Optimal Action

The equilibrium action defined in (3.18) and (3.20) shows the optimal action that maximises the agents' beauty-contest payoff function. So, recall the (3.1), the unconditional expected payoff function yields

$$\begin{aligned}
\mathbb{E}(u_i) = &-(1-\gamma)\left(\frac{\rho_\epsilon\widetilde{s}_i(1-\gamma\lambda) + \rho_\eta\widetilde{y}}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)} - \theta\right)^2 \\
&-\gamma\left(\frac{\rho_\epsilon\widetilde{s}_i(1-\gamma\lambda) + \rho_\eta\widetilde{y}}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)} - \frac{\rho_\eta(1-\lambda) + \rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\theta - \frac{\lambda\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\right)^2
\end{aligned} \tag{3.21}$$

Differentiating (3.21) with respect to the disclosure level λ yields

$$\frac{\partial\mathbb{E}(u_i)}{\partial\lambda} = -2(\gamma-1)\left(\frac{\rho_\epsilon\gamma\nu_3}{\nu_2^2} - \frac{\rho_\epsilon\widetilde{s}_i\gamma}{\nu_2}\right)(\theta - \frac{\nu_3}{\nu_2}) - \frac{2\gamma\nu_1(\rho_\eta(\widetilde{y}+\theta) + \rho_\epsilon\gamma(\widetilde{s}_i - \theta))}{\nu_2^2} - \frac{2\rho_\epsilon\gamma^2\nu_1^2}{\nu_2^3} \tag{3.22}$$

where:

$$\nu_1 = \rho_\epsilon(\widetilde{s}_i - \theta)(\gamma\lambda - 1) + \rho_\eta\widetilde{y}(\lambda-1) + \rho_\eta\theta(\lambda-1) < 0$$

$$\nu_2 = \rho_\eta + \rho_\epsilon(1-\gamma\lambda) > 0$$

$$\nu_3 = \rho_y\widetilde{y} - \rho_\epsilon\widetilde{s}_i(\gamma\lambda - 1) > 0$$

Proposition 3.2 *In the case where private information signal noise is sufficiently stronger than public information signal noise, the agent's payoff will be increased if the social planner increases the level of public information disclosure, that is,* $\frac{\partial \mathbb{E}(u_i)}{\partial \lambda} > 0$ *if* $\rho_\epsilon < \rho_\eta$.

Proof. *See* ***Appendix B.2****.*

Specifically, in equilibrium, we get $a_i^*(\widetilde{s}_i, \widetilde{y}) = \overline{a}_{-i}$ and no agents want to deviate. Comparing with (3.18) and (3.20), we can get $\widetilde{s}_i = \theta$ and $\lambda = 1$ if and only if $a_i^*(\widetilde{s}_i, \widetilde{y}) = \overline{a}_{-i}$. This case has two implications. First, recall (3.2), we get $\widetilde{s}_i - \theta = \widetilde{\epsilon}_i$, it indicates that, private noise is zero($\theta = 0$) as $\widetilde{s}_i = \theta$ in the equilibrium condition. Second, all agents are informed if $\lambda = 1$ is endogenously fixed. In a nut shell, our framework collapses to the static MS model, which indicates that social planner discloses public signal to all agents in a perfect competition market (the actual realization of $\widetilde{y}$ becomes a common knowledge to all agents, showing that all agents in market are informed traders). Therefore, the optimal action $a_i^*(\widetilde{s}_i, \widetilde{y})$ in equilibrium is

$$a_i^*(\widetilde{s}_i, \widetilde{y}) = \frac{\rho_\epsilon(1-\gamma)}{\rho_\eta + \rho_\epsilon(1-\gamma)}\widetilde{s}_i + \frac{\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma)}\widetilde{y}, \text{ and } \lambda = 1 \tag{3.23}$$

After demonstrating the existence of linear equilibrium of partial disclosure of public signal, I will follow this by indicating that this linear equilibrium derived above is the unique equilibrium rather than multiple. **Appendix B.3** shows the uniqueness of the linear equilibrium. The proof follows the guideline provided by Morris and Shin(2002) to build the k level of thinking expectations that rely only on the public signal by assumptions.

Many recent empirical papers have tested these results on European and U.S. financial markets. These papers provide empirical evidence to support the idea that increasing public information disclosure can be beneficial when private signal noise is larger than public signal noise. Guo, Hu, and Yague (2018) analyze the impact of public information disclosure on market liquidity and trading volume in the European corporate bond market. They find that increasing the level of public information disclosure can improve market liquidity and increase trading volume. Baghestanian, Walker, and Westerholm (2019) study the impact of public information disclosure on market efficiency in the European energy market. They suggest that increasing the level of public information disclosure can lead to a reduction in market inefficiencies and improve market outcomes for investors. Additionally, Frijns et al(2020) discuss the impact of public information disclosure on market quality and trading behavior in the U.S. equity market. They find that increasing the level of public information disclosure can lead to improvements in market quality and increased trading activity.

3.4 Limited Level-k Reasoning in Heterogeneous Information Disclosure

3.4.1 Level-0 Specification

The specification of $L0$ is crucial for the explanatory power of level-k models. I define the level-0 types as players who ignore the strategic component of their action. $L0$ represents the behavior of players who do not reason about the beliefs or strategies of others, and therefore serves as a natural reference point for higher-level players to start their reasoning. Bearing in mind that $L0$ is usually treated as an unsophisticated and non-strategic anchoring type, it represents the start point of an agent's strategic thinking.

Initially, Nagel(1995) and Stahl and Wilson(1994) define the $L0$ thinking as subjects who decide with a uniformly random distribution over all possible strategic actions. More sophisticated agents may anticipate this game, and the Lk agents play the best response to their opponents' are level $k-1$. This basic level-k model is referred to as "original level-k" in the following to differentiate it from alternative level-k models described below.

Conclusion 3.1: *The actions of* $L0$ *are randomly distributed across the public and private signals.*

As mentioned by Crawford and Iriberri(2007a, 2007b), the natural candidate for $L0$, known as the "random $L0$", assumes that $L0$'s actions are uniformly distributed between the two signals (public and private), implying that $L0$'s behavior is unsophisticated and serves as a focal point for higher-level players to coordinate their beliefs and strategies. Another $L0$ specification, known as the "non-strategic $L0$", assumes that $L0$ ignores all strategic aspects of the game and focuses solely on guessing the state. This specification implies that $L0$'s behavior is halfway between the two signals, which is different from the random $L0$ specification.

While the behavior of the random $L0$ and non-strategic $L0$ types is observationally different, they yield the same prediction for the behavior of higher types because they both imply randomly. This means that the higher-level players would still make the same predictions about the behavior of lower-level players, regardless of which $L0$ specification is used. However, it is worth noting that some researchers have argued that the non-strategic $L0$ specification may be better classified as $L1$ behavior, rather than $L0$(Costa-Gomes, Crawford and Broseta,2001). This is because the non-strategic $L0$ type is still making some strategic considerations by focusing on guessing the state, which involves taking into account the possible actions of other players. This difference is primarily a matter of semantics and does not affect the underlying predictions of the level-k model.

For $K > 2$ as hierarchical cognitive iteration, a level-k reasoning is playing the best response to level-$(k-1)$. In my setup, the optimal action $L0$ of reasoning to the uniform distribution across all reals is expressed as

$$a_i^0 = \mathbb{E}_i(\theta) \tag{3.24}$$

In this case, (3.24) is the best response for agent i who ignores the strategic component of their action. It is an optimal combination of strategies that maximizes welfare. In full disclosure of public signal($\lambda = 1$), $L0$ reasoning agent defines the actions chosen by an agent who ignores the strategic part of the payoff function.

3.4.2 Level-k Reasoning in Full Disclosure of Public Signal

Under this definition, $L1$'s behaviour favours rules of thumb as a natural initial assessment for higher-level agents to start their iterated best responses. It means to capture an agent's "first thought" about how the opponents play the strategies.[14] This $L0$ assumption is also directly similar to the $L0$-specification in our established beauty contest framework. In the full-disclosure case, as mentioned above, when $\lambda = 1$ and the social planner disclosures all public signals in symmetric, then our designed game will reduce to

[14] Crawford and Iriberri(2007b) mention that a naive agent (the level 0) might either bid any element of the strategy space $\mathbb{X}$ with equal probability (the "random" specification) or bid his true valuation (*Truthful L0 specification*), and hence define the *Random(Truthful) Lk* by iterating best responses from Random(Truthful) $L0$ in the discrete hierarchical cognitive model.

a standard beauty contest structure as Morris and Shin(2002) structured.

As I discussed, the optimal weight f_1 is weighted by the agent i's beliefs about the public and private signals. The action of a $L1$ agent can be rewritten as

$$a_i^1 = \mathbb{E}_i(\theta|\widetilde{s}_i, \widetilde{y}) = \frac{\rho_\eta \widetilde{y} + \rho_\epsilon \widetilde{s}_i}{\rho_\eta + \rho_\epsilon} = f_1 \widetilde{s}_i + (1 - f_1)\widetilde{y} \tag{3.25}$$

, where $f_1 = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}$ is the weight that a $L1$ player attaches to the private signal. Suppose that an agent j attaches weight f_1 to her private signal. Sequentially, the best response action of $L2$ reasoning is:

$$\begin{aligned} a_i^2 &= (1-\gamma)\mathbb{E}_i(\theta) + \gamma \mathbb{E}_i(\overline{a}_{-i}^1) = (1-\gamma)\mathbb{E}_i(\theta) + \gamma f_1 \mathbb{E}_i(\widetilde{s}_{-i}) + \gamma(1-f_1)\widetilde{y} \\ &= (1-\gamma(1-f_1))\mathbb{E}_i(\theta) + \gamma(1-f_1)\widetilde{y} \end{aligned} \tag{3.26}$$

So, the agent i's action in terms of $L2$ thinking is

$$a_i^2 = \underbrace{\frac{\rho_\eta \rho_\epsilon (1-\gamma) + \rho_\epsilon^2}{(\rho_\eta + \rho_\epsilon)^2}}_{f_2} \widetilde{s}_i + \underbrace{\frac{\rho_\eta^2 + \rho_\eta \rho_\epsilon (1+\gamma)}{(\rho_\eta + \rho_\epsilon)^2}}_{1-f_2} \widetilde{y} \tag{3.27}$$

, where f_2 and $1 - f_2$ are the weight on private and public signals for $L2$ reasoning, respectively.

Sequentially, I calculate the $L3$ thinking action

$$\begin{aligned} a_i^3 &= (1-\gamma)\mathbb{E}_i(\theta) + \gamma \mathbb{E}_i(\overline{a}_{-i}^2) = (1-\gamma)\mathbb{E}_i(\theta) + \gamma f_2 \mathbb{E}_i(\widetilde{s}_{-i}) + \gamma(1-f_2)\widetilde{y} \\ &= (1-\gamma 1 - f_2))\mathbb{E}_i(\theta) + \gamma(1-f_2)\widetilde{y} \end{aligned} \tag{3.28}$$

Similarly, the agent i's action in terms of $L3$ thinking is

$$a_i^3 = \underbrace{\frac{(\rho_\eta + \rho_\epsilon)^2 - \gamma \rho_\eta^2 - \gamma(1+\gamma)\rho_\eta \rho_\epsilon}{(\rho_\eta + \rho_\epsilon)^3} \rho_\epsilon}_{f_3} \widetilde{s}_i + \underbrace{\frac{(\rho_\eta + \rho_\epsilon)^2 + \gamma \rho_\eta \rho_\epsilon + \gamma \rho_\epsilon^2 (1+\gamma)}{(\rho_\eta + \rho_\epsilon)^3} \rho_\eta}_{1-f_3} \widetilde{y} \tag{3.29}$$

Therefore, in the general case of level-k reasoning, the agent i's opponents $-i(-i \neq i)$ attach the weight f_k to public signal. Using the iterated situation in above, Lk agent believes that other investors are $L(k-1)$-type sophisticated agents. The agents j's thinking can be described as

$$\mathbb{E}_i(\overline{a}_{-i}^{k-1}) = f_{k-1}\mathbb{E}_i(\widetilde{s}_{-i}) + (1 - f_{k-1})\widetilde{y} \tag{3.30}$$

, where f is mechanism of the weight to private signal that implements up to level k the social choice function so that agent i's updated estimate of the state value.

Having developed this algorithm, the actions for $L1$ and higher thinking levels in the MS structure can be calculated as follows, in which The level-k agent submits the optimal bid strategy that maximises his expected payoff against all level $k-1$ opponents(see Crawford and Iriberri(2007a,2007b) for interpretation and elaboration). Thus, suppose that all other agents $-i$ assign a weight f_{k-1} to their private signal, the optimal action of Lk agent in such condition is

$$\begin{aligned} a_i^k &= (1-\gamma)\mathbb{E}_i(\theta) + \gamma\mathbb{E}_i(\overline{a}_{-i}^{k-1}) \\ &= (1-\gamma)\mathbb{E}_i(\theta) + \gamma f_{k-1}\mathbb{E}_i(\widetilde{s}_{-i}) + \gamma(1-f_{k-1})\widetilde{y} \end{aligned} \tag{3.31}$$

By utilizing (3.9) and (3.10), we can deduce that the expected private signal of the other agents equals the expected fundamental, shown as

$$\begin{aligned} a_i^k &= (1-\gamma+\gamma f_{k-1})\mathbb{E}_i(\theta) + \gamma(1-f_{k-1})\widetilde{y} \\ &= \underbrace{f_1(1-\gamma+\gamma f_{k-1})}_{f_k}\widetilde{s}_i + \underbrace{\Big[1 - f_1(1-\gamma+\gamma f_{k-1})\Big]}_{1-f_k}\widetilde{y} \end{aligned} \tag{3.32}$$

As shown in (3.32), the weight on the private signal for in Lk reasoning in general case, denoted by f_k, is given as

$$f_k = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}(1-\gamma+\gamma f_{k-1}) \tag{3.33}$$

In this general level-k situation, an alternative level-k thinking framework where coefficient f_k is iterating in k and converges to the Nash equilibrium given by (3.31) as $k \to \infty$. Note that, while the $\lambda = 1$, in the specific level-k reasoning case, level-k investors will underweight the public signal and overweight the private signal symmetrically, as compared to the theoretical prediction. Regarding the generalized MS model, it originally indicates that the coordination motive forces agents to overweight the public signal compared with the social planner's weight fraction.

3.4.3 Level-k Reasoning in Partial Disclosure of Public Signal

In the situation of asymmetric partial disclosure, uninformed agents only receive the private information signal, so that their expected average action is $\mathbb{E}_i(\bar{a}_{-i}|\widetilde{s}_i) = \widetilde{s}_i$, as shown in (3.8). For informed agents who can receive public and private information signals, the agent i's opponent $-i$ attach weight f_k to the private signal. Meanwhile, I define the infinite number of population, of which there are λ of informed agents who can observe the semi-public signal. The proportion λ of all agents(except agent i) receive a semi-public signal $\widetilde{y}$, meanwhile the fraction of $1-\lambda$ weights the uninformed agents who just receive private signal. Hence, if agent i's reasoning is k level, the average expected action across all agents(except agent i) by this assumption is $\mathbb{E}_i(\bar{a}_{-i}^{k-1})$. Recalling (3.13), we get

$$\mathbb{E}_i(\bar{a}^{k-1}) = (1-\lambda)\mathbb{E}_i(\widetilde{s}_{-i}) + \lambda[f_{k-1}\mathbb{E}_i(\widetilde{s}_{-i}) + (1-f_{k-1})\widetilde{y}] \tag{3.34}$$

Using (3.14) and (3.15) in above model section, we derive

$$\begin{aligned}
a_i^k(\widetilde{s}_i,\widetilde{y}) &= (1-\gamma)\mathbb{E}_i(\theta) + \gamma\mathbb{E}_i(\bar{a}_{-i}^{k-1}) \\
&= (1-\gamma)\mathbb{E}_i(\widetilde{v}) + \gamma\Big[(1-\lambda)\mathbb{E}_i(\widetilde{s}_{-i}) + \lambda[f_{k-1}\mathbb{E}_i(\widetilde{s}_{-i}) + (1-f_{k-1})\widetilde{y}]\Big] \\
&= (1-\gamma\lambda+\gamma\lambda f_{k-1})\mathbb{E}_i(\widetilde{s}_{-i}) + \gamma\lambda(1-f_{k-1})\widetilde{y} \\
&= \underbrace{f_1(1-\gamma\lambda+\gamma\lambda f_{k-1})}_{f_k}\widetilde{s}_i + \underbrace{\Big[1-f_1(1-\gamma\lambda+\gamma\lambda f_{k-1})\Big]}_{1-f_k}\widetilde{y}
\end{aligned} \tag{3.35}$$

Hence, from (3.35), the weight on the private signal for in Lk reasoning in general case is

$$f_k = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}(1-\gamma\lambda+\gamma\lambda f_{k-1}) \tag{3.36}$$

where $\mathbb{E}_i(\widetilde{s}_{-i}|\widetilde{s}_i,\widetilde{y}) = \mathbb{E}_i(\theta|\widetilde{s}_i,\widetilde{y}) = f_1\widetilde{s}_i + (1-f_1)\widetilde{y}$ that we mentioned in (3.10). The equation (3.35) indicates that, the best response of Lk reasoning are distorted away from $\widetilde{s}_i$ towards $\widetilde{y}$ when increasing the precision of public information signal, ρ_η, and in the fraction of agents receiving it, λ. Furthermore, when $\lambda = 1$ the level-k reasoning action will collapse to the symmetric case as we showed in (3.35) with full public signal disclosure. In summary, assuming that either all agents or no agents disclose, the Nash equilibrium of disclosure with level-k reasoning relies on the weight of private and public signal as confirmed in the following results.

Conclusion 3.2: *The generalized beauty contests in MS model is the special case of partial disclosure model in level-k reasoning hypothesis when the social planner exogenously fix* $\lambda = 1$.

Proposition 3.3:

- *(pure asymmetric market) When the public signal is extremely imprecise or not disclosed by the social planner, or when the private signal is exact, the public signal is ignored among all agents.*
- *(pure symmetric market)All agents are informed and can receive the public signal when the public signal is highly precise and full disclosure by the social planner, or when the private signal is extremely imprecise. In this situation, the private signal will be crowded out, and all agents will disregard the private signal, choosing an average action of* $a_i = \widetilde{y}$.

Proof: *See* ***Appendix B.4****.*

Some previous studies, such as Dale and Morgan (2012) and Cornand and Heinemann (2014), have applied the static game introduced by MS structures to an experimental economic study of financial effects. Their paper provides experimental evidence that public information signals play a focal role. Compared with private information, public information outnumbers agents significantly. It was, however, found that public overreaction to public information was weaker than the theory predicted. In a cognitive hierarchy model excluding public information harmful to welfare, $L2$ thinking predicts empirical weights the best (Cornand and Heinemann, 2014). In summary, the adverse effects of public information reduce higher-order beliefs when agents are unable to form higher-order beliefs.

Furthermore, the social planner(central bank) discloses public information at maximal precision if the respective public information is not highly imprecise. Since the mid-1990s, when central bank actions and communication became more transparent, public information policy has become increasingly relevant. Two main arguments justified the increase in transparency: the first is that increased transparency has become widely accepted as adding legitimacy to the democratic legitimacy of central bank decisions; second, by increasing transparency, central banks would be better able to manage market expectations and make monetary policy more effective(Woodford, 2003; Hann et al.,2007).

As a result, our results contribute to the literature discussed above and are relevant to the optimal design of central bank communications. Multiplier effects propagate public information from informed to initially uninformed agents, thus influencing - ceteris paribus - the optimal actions among different cognitive hierarchies of agents. It indicates that, given sufficiently strong coordination incentives, multiplier effects might lower overall social welfare if agents overreact to public information accompanied by inter-temporal coordination motives.

3.5 Quantitative Illustration and Discussion

For quantitative illustration, I provide some numerical cases to **Appendix B.5** (with conditions of boundaries in limited reasoning levels) on the domain of limited average expectation and estimate the effect of different sophisticated levels of thinking iteration in the linear equilibrium of partial disclosure. These graphs report the detailed results of optimal actions varying for level-k reasonings sorted by different average expectations and external information environments. As in the continuous level-k model predictions, equilibrium bids of optimal actions increase slowly when average expectations are low but quickly when average expectations are high. Now I turn to the quantitative illustration of the level-k model to examine this prediction. I plot the value function of the optimal action in **Figure 6-11**. I am setting the parameter of different degrees of public disclosure since **Figure 6-8** and **Figure 9-11** separately show the volumes of informed agents dominated $(0.5 < \gamma\lambda < 1)$ and uninformed agents dominated $(0 < \gamma\lambda \leq 0.5)$ in the heterogeneous information market.

[Insert Figure 6-11 about here]

From the **Figure 6-11** in **Appendix B.5**, the estimation results finally obtain, of course, increasing convex curves and illustrate that, not surprisingly, the higher-level reasoning agents can make better optimal looking-forward strategies in making market predictions based on the average expected value of risky assets. When k is high enough, the equilibrium curve will become a vertical line. In general, curves of rational actions are increasing positively, and it implies that the less transparency of public signal in the market may cause the more significant distortion away from the average action level, in which the informed-, high-level reasoning agents own a strong advantage position in making optimal investment decisions. However, the lower-level thinking agents can still make better rational strategies than that of high-level thinking if the expected asset common value is lower enough and the partial disclosure of public signal by social planners is higher enough. The gap between the investment behaviours with high and low thinking-level investors will narrow along with the increasing disclosure level of the public information signal. Therefore, I summarize above discussion with the following conclusion:

Conclusion 3.3: *Although the higher-level thinking agents have dominant optimal strategies with rational expectations by using their higher sophisticated prediction levels, the less sophisticated thinking agents also can make better rational strategies, if and only if in the condition of*

- *lower expectation of fundamental state of the economy;*
- *and higher symmetric information environment.*

The argument in the above conclusion presented is that less sophisticated agents may still be able to make better rational strategies in certain conditions. Specifically, if these less sophisticated agents have a lower expectation of the fundamental state of the economy and operate in a higher symmetric information environment, they may be able to make better rational strategies despite their lower level of strategic thinking.

This argument is consistent with some research in behavioral finance, which suggests that individuals with lower levels of sophistication or expertise may sometimes outperform more sophisticated agents under certain conditions. For example, some studies have found that individual investors who trade less frequently tend to outperform those who trade more frequently, even after controlling for factors such as risk and transaction costs (e.g., Barber and Odean, 2013). Similarly, some studies have found that simpler investment strategies, such as buying and holding a diversified portfolio, can outperform more complex strategies that rely on higher levels of expertise or sophisticated models (e.g., Malkiel, 2012).Barras et al. (2010) argue that some mutual fund managers may be able to generate outperformance through luck rather than skill, and that less sophisticated investors may be more likely to invest in these lucky managers, potentially leading to better investment performance. They suggest that this may be due to less sophisticated investors being less able to distinguish between luck and skill, and therefore being more likely to invest in managers who appear to have skill but are actually just lucky.

In the recent empirical research,Song et al. (2019) find that less sophisticated investors in an emerging market tend to engage in anti-herding behavior, i.e., they tend to buy when others are selling and sell when others are buying, which can lead to better investment performance. They argue that this may be due to less sophisticated investors being less influenced by the herd mentality and therefore being better able to identify and take advantage of mispricings. Gao et al. (2021) find that less sophisticated individual investors in the Chinese stock market tend to outperform more sophisticated investors, particularly during periods of high market volatility. They argue that this may be due to less sophisticated investors being less influenced by noise in the market and therefore being better able to identify and take advantage of mispricings.

In summary, these empirical studies suggest that less sophisticated traders may be able to outperform more sophisticated traders under certain conditions, such as when the market is highly volatile or when there is a tendency towards herding behavior. However, it is important to note that the conditions under which less sophisticated traders may outperform are likely to be highly specific and context-dependent, and that there is ongoing debate among researchers about the relative importance of sophistication and other factors (such as luck) in explaining investment performance.

3.6 Concluding Remarks

This chapter develops an analytical framework that facilitates the analysis of the level-k reasoning under heterogeneous information structures on fundamental values. In the context of financial markets, the level-k reasoning model can be used to explore how different types of investors or traders may behave. For example, a level-0 trader may simply buy or sell a security based on a random guess, while a level-1 trader may base their decision on some basic analysis or information about the security. Higher-level traders may use more complex models or analysis to guide their decisions.

The chapter focuses on the level-k reasoning model in the context of financial markets, particularly in the beauty-contest MS structure. First, it shows a two-stage beauty-contest structure where includes informed and uninformed agents, as well as a social planner, participate. In this game, each agent observes their private signal, and a fraction of agents receives a public signal. The optimal actions of agents in equilibrium show that the lower the noise in the private signal and the higher the correlation with the public signal, the more significant its position is for uninformed agents. In the case where private information signal noise is sufficiently stronger than public information signal noise, the agent's payoff will be increased if the social planner increases the level of public information disclosure. This is because a stronger public signal provides more accurate information to agents and reduces the noise in the market, making it easier for agents to coordinate their actions. As a result, agents can make better decisions and earn higher payoffs.

Second, this chapter analyses the level-k reasoning in the context of three types of heterogeneous information disclosure: level-0 specification, level-k reasoning in full disclosure of public signal, and level-k reasoning in partial disclosure of public signal. The results show that the generalised beauty-contest MS model is a particular case of the partial disclosure model when the social planner fixes the public signal's disclosure rate, λ, at 1. If the social planner does not disclose any public signal, the private signal will be exact, and the public signal will lose its coordination role in a pure asymmetric market. On the other hand, if the social planner fully discloses the public signal, then the public signal will be exact, and the private signal will be disregarded by all agents in a pure symmetric market. Additionally, according to the quantitative illustration, it shows that higher-level reasoning agents can make better forward-looking strategies than less sophisticated reasoning agents. However, the less sophisticated agents can make better rational strategies if the fundamental value of the asset is low, and the information environment is more symmetric.

Finally, even though the level-k reasoning analogy has been tested and validated in several behavioural economic experiments, the analytical conclusions from this chapter and the pratice of level-k reasoning in the financial market still need to be tested by well-designed experiments and empirical data. Moreover, this chapter contributes to the theory of market microstructure according to offering some distinctive perspectives. Experienced or professional players in real-life situations may understand and use forward inductive logic more frequently. Our conclusions need to be tested in future empirical studies. Additionally, while the level-k reasoning model can help us understand how agents with different levels of strategic thinking may behave, it is not necessarily a perfect representation of how agents actually behave in real-world financial markets.

4 Unobservable Information Acquisition and Insider Trading[15]

4.1 Introduction

Insiders often find opportunities to acquire private information dynamically in an asymmetric market when the uncertainty and risk premia are high. The characteristics of information acquisition are extraordinary: costly and irreversible, in that associated costs of acquiring information cannot be fully recovered if investors exit the market. As such, incentives to acquire private information about companies' financial performances are significant to the asset prices in the market, which are vital in allocating resources, especially capital (Hayek, 1945). For concreteness, consider an example: some insiders pay the cost to investigate whether the oil price fluctuations are exposed to inventories. The public does not precisely know the U.S. commercial crude oil inventories; however, it may be disclosed at some random future time. Significantly, the value of information relies on the volatility of WTI Crude Oil price(a piece of public information disclosed in due date). If U.S. commercial crude oil inventories are relatively stable with high information transparency, whether or not the U.S. commercial crude oil inventories are hedged does not have a considerable effect on the WTI Crude Oil price. On the contrary, if the U.S. commercial crude oil inventories are unobserved and have fluctuating uncertainty, then the influence of information disclosure reflected on WTI crude oil price is much higher. As a result, acquiring information immediately before trading begins does not guarantee a profitable strategy; conversely, insiders might prefer to wait for opportunities to acquire extra private information when the uncertainty of U.S. commercial crude oil inventories is high.

Since Grossman and Stiglitz(1980) initially provided the paradigm of acquiring information at a fixed cost that shed many significant insights on various aspects of the financial markets, information acquisition has always been a core decision for investors on market microstructure.[16] Conventional wisdom mainly associates that information-acquisition activities are static and prominently observable so that

[15]I am grateful to John Fender, the participants of Asian Meeting of the Econometric Society, and various workshops for helpful comments and suggestions. All errors are mine.

[16]The standard treatment in the existing literature builds vast theoretical extensions around this chapter in different aspects, such as managerial incentives(Holmstr*ö*m and Tirole,1993), corporate financing choices(Subrahamyan and Titman,1999), and the asset management industry(Garleanu and Pedersen,2018) As of Apr 2022, this paper has more than 11790 citations in Google Scholar.

investors acquire private information to become informed before the start of trading(e.g., Admati and Pfleiderer, 1988a,1988b). However, these models largely ignore the uncertainty and dynamics of costly information acquisition, so it implies that the traditional characterisations of information acquisition in financial markets are at best inaccurate, limiting the traditional approach's applicability. Subsequently, some sophisticated investors usually make adjustments in acquiring private information dynamically to optimise their expected profits with economic conditions. On the flip side, in realistic, some essential information resources are relatively unobservable and usually unknown to the public in financial markets, so uninformed investors/clients have to pay extra costs to acquire private information such as purchasing alternative data or making insider trading from information-abundant agents in asymmetric market environments.

To better understand how the discrete/continuous-time and stochastic of information-acquisition cost influences financial markets, I describe model where the insider dynamically acquires information in response to market conditions. On the belief of Kyle's(1985) and Back(1992) framework, the motion for this chapter focuses on the case that insider's decision to acquire information cannot be observed or immediately detected by the market maker. Significantly, the core ingredient that I add is to allow rational investors to engage in secret information acquisition. The model I propose is particularly suitable for financial markets and certain types of financial agents. The critical assumption is that information processing costs of lumpy information acquisition while trading time is discrete or continuous, which delays investment decisions. In the implication of financial markets, the market maker(i.e., London Stock Exchange) can be recognised as providing liquidity for a specific risky asset. If the financial market's state is uncertain, the problem becomes significantly more challenging. Some examples Maug(1998), Kahn and Wintom(1998), and latest studies paper(e.g., Collin-Dufresne and Fos, 2016) designs a continuous-time analogue similar with various aspects to the theoretical framework in my discussion: there are some activist insiders paying cost $C(\omega)$ where ω is an effort to finally influence the terminal value of the companies.

In this chapter, I firstly study the impact of an insider's dynamic information acquisition and entry decision in discrete- and continuous-time conditions. In the section of baseline preliminaries, I design a model where an insider has access to private information about a fundamental value of an asset, based on a Brownian motion.. The insider continuously observes a signal that tracks the evolution of the asset's fundamental value, but this information is not immediately revealed to the market. Instead, the value of the asset is publicly revealed at a random time. Secondly, I discuss a discrete-time dynamic model of unobservable information acquisition following Caldentey and Stacchetti's (2010) study, which extends Kyle's (1985) continuous-time insider trading model by allowing discrete-time trading and unobservable information acquisition. In my model extension, I propose that the public announcement date is not deterministic(random deadline) but follows the exponential distribution. Thirdly, I also consider the continuous-time dynamic insider trading model without discounting in Kyle's (1985) contest. If the cost of acquiring information is high enough, insiders would not be able to acquire it profitably.

Lastly, this chapter also relates to the studies on financial market regulation and enforcement of securities laws. In reality, many countries are regulating illegal insider trading in financial markets. There are also some shreds of evidence that enforcement of insider trading legislation can effectively decrease the cost of equity in financial markets in a country(e.g., Bhattacharya and Daouk,2002). As in many other financial developed counties, taking the United States, trading securities based on private information is illegally. *Securities and Exchange Act of 1934*, Sections 16(a) and 16(b), and the relatively Security and Exchange Commission (SEC) rules and case law. For example, *Regulation Fair Disclosure(Reg F.D)* promulgated in 2000 significantly decrease the incidence of selective information disclosure in legal practice. Some empirical evidence, such as Duarte, et al.(2008), indicate that the probability of informed trade(*PIN* for Nasdaq firms raised after inducing *the Reg F.D.*. In this way, I demonstrate that accounting for the dynamic and stochastic nature of unobservable information acquisition has significant insights into several practical implications, which is central to many policy debates. How does the legislation level regulate the concreteness of the illegal insider trading of financial information in asymmetric markets? This work is also conceptually related to financial economics theory on welfare and disclosure.(e.g. Mendelson and Tunca,2004). Furthermore, the intersection of law and economics(Some evidence has shown by SEC.

Outline The reminder of this chapter proceeds as follows. Section 4.2 is the literature review. Section 4.3 gives some preliminary definitions of the Kyle-type model to discuss the fixed cost unobservable information acquisition with a random horizon and multi-dimensional Brownian motions. I derive the characteristics for discrete and continuous-time convergence and analyse its propositions. In section 4.4, following the Caldentey and Stacchetti(2010) model, I consider the case where the time is discrete, and the insider starts with delay entry in random deadline. Section 4.5 characterises the continuous-time of Kyle-type model with no discounting, studies the relationship between insider trading, and extends the model to the cases of dynamic information acquisition with unfixed/random deadline through the Markovian equilibrium. Section 4.6 discusses the U.S. regulation in legislative practices and the effects of SEC-type Penalties in terms of insider trading as an example. Section 4.7 concludes. In Appendix, I provides market implications with some hypothetical insider trading cases, which might be of independent interest, as additional auxiliary results. All constructive proofs are in the appendix.

4.2 Literature Review

In the light of the concern about the information-acquisition decision, Albert S. Kyle(*Econometrica 1985*) initially posits the discrete-time model of insider trading as the natural starting foundation of understanding how an insider strategically incorporates private information by choosing his trade size and gradually making profits maximization. His seminal contribution corresponds to whether the investment in information production is observable and investors' trading strategies on their private information affect asset prices and a series of market presumptions. Sequentially, based on the common assumption in the Kyle-type frameworks, Back(1992) solves the strategy problem of an insider trader in a continuous-time setting. After that, several studies focus this Kyle-type model and release in financial markets, building on Kyle(1985) and Back(1992), examining the effect of long-term private information on inelastic noise trading(e.g., Kim and Verrecchia,1991; Demski and Feltham,1994; Back and Baruch,2004). Discussion so far assumes that if informed traders' behaviours are unobserved, informed investors acquire more information than the observable case when the information acquisition costs are lower. On the contrary, if the information acquisition costs are high enough, informed traders may acquire less information or delay acquiring until the cost decreases. In both cases, her profits may decrease as an outcome of unobservability.

4.2.1 Insider Trading Model by Kyle(1985) and Extensions

The most active research themes in the market microstructure literature are original from a relatively small number of influential sources. Many researchers use the terms ”noise trader” or ”liquidity trader” interchangeably to think about the strategic effects of the dynamic information acquisition in financial markets, such as Glosten and Milgrom(1985) and Kyle(1985). Specifically, the critical insider trading mechanism is hard-wired in the Kyle(1985) model as a natural starting point, and it has been widely used in the market microstructure literature. Kyle (1985) examines the impact of insider trading on market efficiency in a continuous auction market. By distorting prices, insider trading reduces market efficiency, but enforcement efforts can deter insider trading and improve market efficiency. However, the effectiveness of enforcement efforts depends on regulators' ability to detect and punish insider trading. In Kyle's particular sequential auction equilibrium model, a single risky asset is switched for a riskless asset among the following three types of traders:

- *a single insider:* He monopolises information as the role of an intertemporal monopolist in assets, so he has private information access to a private observation of the *ex-post* liquidation value of the risky asset.[17] Consequentially, he considers the impact of each price transaction and future trading opportunities and uses the private/public information that can be used to obtain the maximum profit by choosing the best trading strategy.

- *random noise traders:* They are uninformed and do not own any private information. They are assumed to be a Brownian process, providing a disguise for the trading behaviours of insiders.

- *competitive risk-neutral market makers:* they set prices efficiently, relying on information they have about the investors' trading volume. It is worth noting that they only observe the total volume of trade in each trading period, but cannot observe the volume traded by the insider or noise traders individually.[18]

For the transaction process, rational insiders acquire costly private information about the fundamental's payoff. Investors can engage in either overt or secret information acquisition. Namely, the investment in information acquisition can be either publicly observable or unobservable. Kyle(1985) constructs a linear equilibrium to describe how an insider strategically and gradually makes a profit. Meanwhile, the insider's information is gradually incorporated into the announcement date until the information

[17] In the literature, the single insider can also be called informed trader or strategic trader in Kyle-type model.

[18] The U.S. Securities and Exchange Commission (SEC) defines a "market maker" as a person or company that simultaneously quotes the price of a stock to buy and sell. A Designated Primary Market maker (DPM) will take a position to buy a stock (the bid price), and the price at which the company is willing to sell it (the ask price) is called the bid-ask spread in the financial market. See https://www.sec.gov/interps/legal/mrslb9.htm.

becomes public. The rest of the model is standard: rational investors trade on private information and submit market orders to maximise their expected profits. Given the total order flows, market makers execute the orders at the conditional expected value of the asset.

Building on the idea of the original Kyle (1985) model, the contemporaneous papers(i.e., Back,1992; Taub,2018) subsequently extend in various aspects. Several papers examine the uninformed traders' choices in static or short-horizon private information conditions. For instance, Admati and Pfleiderer(1988a,1988b), Foster and Viswanathan(1990,1993) firstly extend the Kyle model to a continuous auction market with myopic agents as the essential consequence of Kyle's one-period model where agents only trade on short-lived information. Then, the standard continuous-time approach exemplified by Back(1992) set out an elliptic Partial Differential Equation(PDE for short) approach to solve the dynamic version of the Kyle-type model. His original presumption asserts that the firm's fundamental value is fixed in the time horizon, and the noise trade is the fundamental dynamic element of the model, as is following the standard(i.e., constant volatility) Brownian motion. Moreover, Madrigal(1996) constructs a two-period market with one risky asset and Kyle(1985).

Although the Kyle-type model and Glosten-Milgrom model presumes that the insider does, in reality, have material information; however, neither of them foucus on the channel through which it is acquired.[19] Besides, in my model extension, I propose that the public announcement date is not deterministic(random deadline) but follows the exponential distribution.

4.2.2 Unobservable Information Acquisition

Several works of literature under this heading also address unobserved information acquisition as an insider trading problem. This chapter partially links to the extensive literature on information acquisition topics that study the impact of the costly acquisition on asset prices and market liquidity release in finan-

[19] Different from Kyle(1985), Glosten and Milgrom(1985) put forward an abnormal formalisation of Bagehot(1971) in a formal framework. The Glosten-Milgrom framework induces bid and ask quotes in a single asset market, where insiders interact their strategies with then market maker and noise traders in the financial market. More discussion analyses that the Glosten-Milgrom model designs the convergence of the insider trading strategies that is approximately similar to the continuous-time of the Kyle-type framework(Back and Baruch,2004).

cial markets. (e.g., Grossman and Stiglitz,1980; Kyle,1985; Admati and Pfleiderer, 1988a,1988b; Back,1992; Kondor,2012; Benhabib, Liu and Wang,2019). The assumptions build that acquiring information strategies is perfectly observable in the existing literature. However, less analysis suggests that in assumptions in which investors secretly acquire information in financial markets in continuous time.[20] In this situation, the traditional models hold on overt information acquisition are likely to misunderstand the qualitative implications for market efficiency.

The severe consideration of unobservability in a game-theoretic framework can be dated back to Hart and Tirole(1990) and McAfee and Schwartz(1994). Initially, Admati and Pfleiderer (1988a,1988b) investigate the phenomenon of concentrated-trading patterns resulting from the strategic behaviour of liquidity(noise)/informed traders. Their results found a partial interpretation that discretionary noise traders with multiple trading opportunities are more inclined to trade in only one period. Centralised trading occurs because noise traders choose the period with the lowest transaction cost to conduct transactions and further reduce liquidity costs. Besides, Holden and Subrahmanyam (1992) provide an essential improvement on incorporating the competition among insider traders into the Kyle(1985) model by showing that competition among insider traders increases the release speed of information, which means that the market depth tends to be infinite. All insider information is immediately released when the trading interval tends to be infinitely small.

Sequentially, two studies highlight critical results. Hauk and Hurkens(2001) have produced a secret acquisition structure of demand information in duopoly and Cournot markets. The results demonstrate that companies excessively acquire information to compete in overt information acquisition. Mendelson and Tunca(2004) explore efficiency properties on the unobservability of investors' information acquisition, but they adopt different settings and focus on different research questions: how insider trading affects a single investor's incentives to acquire information. Their main conclusion is that, by reducing the risk they face, insider trading increases the welfare of liquidity traders.

Nevertheless, in spite of the prominently dynamic nature of acquiring information, current research

[20] These previous analyses are most closely related to trading timing, such as Admati and Pfleiderer(1988a,1988b), Ostrovsky(2012), Bouvard and Lee(2015), Dugast and Foucault(2018), Banerjee and Breon-Drish(2017,2018,2020), and so on.

mainly focus on acquiring information remains a static problem in which investors make informed decisions before the beginning of trade.[21] The second infers Kyle(1985) and Back(1992). The standard pioneered them with asymmetric information, a continuous-time setting of the Kyle-type(Kyle,1985; Back,1992) asset pricing model. In this chapter, I main focus on modelling uncertain and dynamic information acquisition based on the previous setting.

4.3 Baseline Preliminaries

My model is modified to combine discrete-time and continuous-time versions. In my extension, I introduce stochastic volatility in noise trading, as in Collion-Dufresne and Fos (2016) and Banerjee and Breon-Drish(2017,2020), in which they demonstrate that the value of information changes over time under a non-trivial but tractable and economically reasonable condition. Significantly, it is worth noting that the insider's trading strategy can involve both "continuous", where the insider makes acquisitions with a given intensity over the period, and "discrete", where the insider makes acquisitions over a countable collection of periods. **Figure 2** shows the timeline. Here I review and present some preliminary definitions and results regarding the framework.

4.3.1 Brown Motion

Stochastic processes: Define a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$, I consider a $(n+1)$-dimensional standard Brownian motion denoted by the joints distribution of random vectors $\bar{W} = (W_1, \ldots, W_n, W_Z)$, with filtering $\mathcal{F}_t^W$, independent random variables ξ and T, and an independent m-dimensional random vector N_0. Let $\mathcal{F}_t$ denote filtering $\sigma(\{\bar{W}_s\}_{\{t \geq s \geq 0\}})$-algebraic enhancement. In this regard, random time T is independently exponentially distributed with rate $\gamma > 0$, and ξ and N_0 have finite second moments. Finally, let $W = (W_1, \ldots, W_n)$ denote the first n elements of $\bar{W}$. Furthermore, I make an assumption that all mar-

[21] Kendall(2018) notice that if there is no direct cost, investors may prefer to wait for better quality insider information. Huang and Zhou(2018) discuss whether or not investors can acquire both information and speed to maintain their informational advantage from the start period. However, both set the "static acquisition /entry" decision as acquiring information prior to trading. Some existing literature set out the dynamic nature of information acquisition decisions (cf. Mendelson and Tunca,2004; Avdis,2016; Taub,2018; and references there).

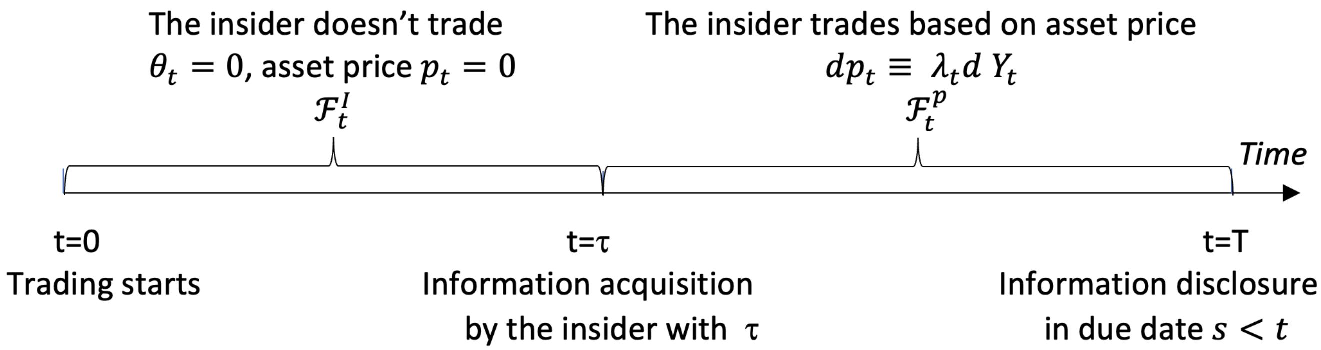

Figure 2: **Timeline of the game.** This model includes three important time points: trading starts($t = 0$), information acquisition point($t = \tau \in \mathcal{T}$), and information disclosure point($s < t$).

ket participants have a common prior to distributing returns and signals.

Note: There is a $m \geq 0$-dimensional vector consisting of publicly observable Markovian signals $N_t = (N_{1t}, \ldots, N_{mt})$, with the initial value N_0. I describe the uncertainty model, fundamental processes, and assets' terminal payoffs. Consider the above assumption and let d be a positive integer. The fundamental factor process $\boldsymbol{W}$ is a diffusion, taking values in a region $\boldsymbol{E} \subseteq \mathbb{R}^d$, with

$$dN = \mu(t, N)dt + \sum(t, N)dW_t \tag{4.1}$$

, where $\mu(t, N) = (\mu_1(t, N), \ldots, \mu_m(t, N))$ and $\sum(t, N) = (\sum_1(t, N)', \ldots, \sum_m(t, N)')$ denote the vector of drifts and matrix of diffusion coefficients(a prime "′" denotes matrix transposition). Suppose that $\mu(\cdot)$ and $\sum(\cdot)$ are such that there exists a unique strong solution to this diffusion setting of stochastic differential equations (SDEs).

4.3.2 Assets

The economy features a risk-free(safe) asset and a single risky asset. The risk-free(safe) asset is the numeraire with an interest rate normalised to zero, and its price is normalised to 1 for all periods. The single risky asset pays off an *ex post* liquidation value $V \sim \mathcal{N}(0, \sum_0)$ at random time T.[22] The T can be the date of

[22] Due to discrete-time log utility, some studies use $T = \infty$ as long as the discount rate is strictly positive, where T is independently exponentially distributed with the interest rate r. In general, this assumption does not hold for other utility functions.

an unplanned, value-related public disclosure or announcement that removes the trader's informational advantage.

As a standard, the time-t asset price of the equilibrium study is a smooth function of the cumulative order flow up to that point. So I state that:

Asset price: Given the knowledge of ξ and the history of N_t, I define the conditional expected value V of the return on the asset as of time t to be

$$\xi = f(t, \xi, N_t) \text{ where } \xi = V \tag{4.2}$$

for some function f. An information acquisition action occurs at any time in $\tau \in \mathcal{T}$, distributed as an exponential random variable with parameter r. Because there is no public signal, and the endogenous state variable accumulates the order flow $p_t = Y_t$.

Besides, note that the model I structured nests several existing frameworks from the previous literature. For concreteness, the special case of Back and Baruch(2004) and Caldentey and Stacchetti(2010) consider the asset pays off, $\xi \in \{0, 1\}$ is a binomial distribution with no observable propaganda signal , so $\xi \sim \mathcal{N}(0, \sum_0)$ thus $V_t = f(t, \xi, N_t) = \xi$, where time is continuous and there is no continuous private information flow. Different with these earlier models, my focus above is to allow insiders access to private information at a decision point in time.

4.3.3 Market Participants

The market includes three participants: an insider, the market maker, and many noise traders.

The insider: A risk-neutral insider, paying a fixed cost $C(\xi)$ to obtain a signal ξ to investigate investment opportunities from the set of entry/stopping time $\tau \in \mathcal{T}$. The insider's initial position in the risky asset is zero, given as $X_0 = 0$. Let X_t denote the trader's cumulative holdings at t, and I only consider absolute

Kyle (1985) and Back (1992) set the end date T to be fixed and normalized to 1 in the continuous-time version.

continuous trading strategies of the form,[23]

$$X_t = \int_0^t \theta_t(\cdot)dt \tag{4.3}$$

, where in the optimal trading problem. Some stochastic process $\theta(\cdot)$ (depending on V) means that the trading rate of an insider trader at time t is only a function of P_t and V.

*Market makers:*A competitive, risk-neutral *market maker* acts as a monopoly holding a publicly observable Markovian signal N_t equal to the conditional expected return given a set of public information.Market makers follow a stochastic price process and can only observe the agreement of the cumulative order flow process $Y_t = X_t + Z_t$, which is the sum of insider demand(X_t) and "noise trader" demand(Z_t). They aim to minimise their loss against the potential insider rather than maximise their profit against both the insider and the noise investor based on several justified perceptions. Instead, they have a prior that the value of risky asset $V \sim \mathcal{N}(0, \sum_0)$.

Noise traders: Noise traders whose cumulative holdings Z_t shares of the asset follows a Brownian motion with variance σ_Z^2 at time t, shown as

$$dZ_t = \sigma_Z(t, v, Z)dW_{Zt} \tag{4.4}$$

where $\sigma_Z > 0$ is a constant.

4.3.4 Lumpy Information Acquisition

Definition 4.1: *Since the information acquisition is unobservable, the public information set only contains the cumulative order flow, so $\mathcal{F}_t^P$ is an augmentation of the public information filtering $\sigma(\{Y_t\})$. The equilibrium break-even requires that market clearing price at time before information disclosure($t < T$) is*

$$P_t = \mathbb{E}[V_t|\mathcal{F}_t^P] \tag{4.5}$$

, where the initial price $P_0 \in (0,1)$ is the unconditional expectation of V.

[23]Back(1992) proposes that it is optimal for trading strategies in a model where the market maker knows that the insiders are exogeneously informed.

Note: (the entry of the insider) Let $I_t = \mathbf{1}_{\{\tau \leq t\}}$ denote whether the insider decide whether to get information at or before t.

Definition 4.2: *An equilibrium is (i) pure strategy information acquisition with optimal stopping time $\tau \in \mathcal{T}$ and insider acceptable trading strategy X_t and (ii) price process P_t satisfying (4.5). Given the acquisition strategy and price process, in general, the insider's terminal profit maximization is*

$$max_{\theta_t \in \mathcal{F}} \mathbb{E}\left[\int_0^T (f(t, \xi, N_t) - P_t)\theta_t d_t | \mathcal{F}_t^P, V \right] \tag{4.6}$$

, by satisfying the integrability condition such that[24]

$$\mathcal{F} = \{\theta_{s,t} \mathbb{E}[\int_0^T \theta_s^2 d_s] < \infty\} \tag{4.7}$$

, which shows that he trading strategy that is smooth, and satisfies the measurability restrictions.

In addition, I ask for the insider's information acquisition time, $\tau \in \mathcal{T}$, expressed as the set of $\mathcal{F}_t^P$ stopping time, where the request only depends on the public information to that time point. Let $\mathcal{F}_t^I$ denote the augmentation of filtering $\sigma(\mathcal{F}_t^P \cup \sigma(\xi))$. That is, $\mathcal{F}_t^I$ represents the set of insider information, and the post-information acquisition adapts the trading strategy before the insider information acquisition to $\mathcal{F}_t^P$ and after-the acquisition strategy to adapt to $\mathcal{F}_t^I$. Following the previous literature, I consider Markovian equilibria, where the asset price is a functional diffusion of an exogenous public signal N_t and an arbitrary (but limited time horizon) number of endogenous state variables p_t, and tracks the market maker's pair ξ beliefs.

4.3.5 Key Economic Observations

In this subsection, I present some main observations of the non-existence of equilibria with lumpy information costs. On the belief of the assumptions, our economic forces are immediate when the trading

[24]This technical assumption ensures that investors' asset demand is in equilibrium. In Kyle's (1985) model, insiders submit market orders dX_t, which are filled by market makers at $P_{t+d_t} = P_t + dP_t$. Hence, assuming a zero profit free rate, $\int_0^T (V - P_{t+d_t})X_t = \int_0^T (V - P_t)dX_t - \int_0^T dP_t dX_t$. After Back(1992), it is optimal to choose an absolutely continuous trading strategy $dX_t = \theta_t d_t$ such that the second term is zero (otherwise, the term is always negative due to price effects). Note that this requires volatility to be common sense. Otherwise, there may be multiple equilibria in which the insiders' strategies are not absolutely continuous.

starts in financial markets:

Proposition 4.1: *An pure-strategy equilibrium in which the insider obtains costly information with positive probability at the start of the trade ($t = 0$) cannot be an optimal equilibrium because there exists time $\tau \geq 0$ such that $\mathbb{P}(\tau > 0) > 0$.*

Proof: I prove this by contradiction:

- Suppose that if such an equilibrium ($t = 0$) exists at the beginning of the trade, the market maker cannot observe the insider trader's acquisition decision. The order flow is entirely uninformative about the given knowledge of the asset price ξ prior to time τ, so pricing rules cannot sensitively reflect the order flow prior to τ.

- However, since the market maker cannot directly detect the insider's decision in the $\tau \in (0, \infty)$ event, the probability of this happening is strictly positive, so he can obtain and interval $(0, \tau)$, then trade at a significant rate with zero price impact, resulting in arbitrarily better profits.

Corollary 4.1: *(Pre-emption deviation) After the market trading starts ($t > 0$), there is no equilibrium in pure strategy. It means that if there is a time $s > 0$, we have $\mathbb{P}(\tau > s) > 0$. Since the information-acquisition behavior of the insider is not observed, market makers cannot respond to insider deviations by sensitively adjusting for price effects.*

Proof: Again, by contradiction,

- Assuming such an equilibrium exists during the trading, the market maker cannot observe the insider trader's acquisition decision. Then, the order flow is entirely uninformative about the given knowledge of the asset price ξ prior to time τ. Therefore, pricing rules cannot sensitively reflect the order flow prior to τ.

- However, if $\tau > s > 0$ occurs with strictly positive probability, then insiders can trade at the arbitrarily large rate by taking information that cannot be observed in the time interval (s, τ), Thereby again, profit deviation from price has zero effect, resulting in a better profit.

The above proposition and corollary follow the reality that the insider cannot promise to obtain information costly at the detail date in future when acquisition decisions cannot be detected by market makers.Then insiders always find it profitable to deviate by pre-empting and getting information earlier. **Corollary 4.2** also indicates that information is never acquired is not an equilibrium either, which means that insider trading always exists as long as market makers are not perfectly sensitive in adjusting price immediately, shown as following:

Corollary 4.2: *(Why insider trading always exists) Insider traders follow a pure acquisition strategy, i.e. with probability 1, never acquiring information cannot be equilibrium. That is,* $\mathbb{E}(\tau = \infty) = 1$.

proof: Treat the no-entry decision as an infinite realization of stopping time, $\tau = \infty$, is the standard for the reach/stop problem.[25] Suppose there is an equilibrium in which insiders never get information cheaply, and risky asset prices are always insensitive to order flow. However, this allows insiders to deviate from the speculative equilibrium strategy by acquiring information, trading at an arbitrary large rate with zero price impact, and generating better-off profits. Because the insider can access unobserved information, market makers cannot respond to deviations by immediately adjusting for price impact.

Specifically, I discuss the delay deviation in discrete time, as the very short time interval $\Delta = t_{n+1} - t_n$. In market equilibrium, the insider's problem exhibits "trade timing indifference", defined as following:

Definition 4.3: *(trading timing indifference) If at any time t and for each time interval* $\Delta > 0$*, the expected profit of the insider varies by more than the interval* $[t, t+\Delta)$*, or zero if he does not trade within this range and then follows his conjectured equilibrium strategy afterwards. That is, if* $dX_t^s = 0$*,* $t \in [t, t+\Delta)$ *means*

$$\lim_{\Delta \to 0} \mathbb{E}_t^s [e^{-r\Delta} J^s(t+\Delta, \cdot) - J^s(t, \cdot)] = 0 \tag{4.7}$$

Therefore, the following proposition shows no equilibrium with lumpy cost characterized by indifference in transaction time and endogenous information acquisition.

Proposition 4.2: *(Delay Deviation with time difference) Fixes any time interval* $\Delta > 0$*. There is no equilibrium where (i) no difference in trading time holds, and (ii) the insider get information with positive probability* $[0, \Delta)$*, i.e. where* $\mathbb{P}(\tau \in [0, \Delta)) > 0$.

[25]See, i.e. the discussion of paragraphs with drift timing of Brownian Motion on p.6 of Karatzas and Shreve (1998), or more concretely, Shreve (2004) p. 335 discuss the exercise timing of The Brace Gatarek Musiela Model (BGM).

Proof: *See* ***Appendix C.1****.*

After that, I formalize the sufficient condition that any preliminary Markovian equilibrium must obey the trading time indifference, in which assume that :

Definition 4.4: *(Markov Equilibrium and HJB Equation) For a function g that is continuously differentiable in* t*, the price of risky assets takes* $P_t = g(t, N_t, p_t)$ *and is twice consecutively differentiable in* $\{N, p\}$*. There is a set of* $\ell > 0$ *Markovian state variables p and evolution*

$$dp = \alpha(t, N, p)dt + \Lambda(t, N, p)dW + \mathbf{1}dY \tag{4.8}$$

, where α *is the* ℓ *dimension function,* Λ *is the price-impact coefficient vector,* $\mathbf{1}$ *is the* $\ell \times 1$ *vector, so that when the cumulative order process is* $Y_t = X_t + Z_t$ *and the trading strategy* X_t *is in an acceptable form, there is a unique strong solution to this SDE.* [26] *For* $i \in support(\xi)$*, the value function* $J^i(t, N, p)$ *is continuously different in t and twice differentiable in* $\{N, p\}$ *, and satisfy the Hamilton-Jacobi-Bellman (HJB) equation*

$$\begin{aligned} 0 = sup_\theta \bigg\{ & -rJ^i + J^i_t + J^i_N \cdot \alpha + J^i_p(\alpha + \mathbf{1}\theta) + \frac{1}{2} tr(\sum \sum{}' J^i_{NN}) \\ & + \frac{1}{2} tr(J^i_{pp}(\Lambda\Lambda' + \mathbf{1}\sigma_z^2\mathbf{1}')) + tr(J_{Np}\Lambda \sum{}') + \theta(f(t, i, N_t) - p_t) \bigg\} \end{aligned} \tag{4.9}$$

and if a speculative equilibrium strategy generates the order flow, the integrability condition is

$$\mathbb{E}\bigg[\int_0^\infty e^{-2\gamma i} J'_p(i)\Lambda\Lambda' J_p(i) di\bigg] < \infty \tag{4.10}$$

$$\mathbb{E}\bigg[\int_0^\infty e^{-2\gamma i} J'_N(i) \sum \sum{}' J_N(i) di\bigg] < \infty \tag{4.11}$$

and the transversality condition

$$\lim_{t\to\infty} \mathbb{E}\Big[e^{-\gamma t} J(t, N_t, p_t)\Big] = 0 \tag{4.12}$$

Due to the linear character of θ in (4.7), θ is unconstrained, so the sum of the coefficients on θ must be

[26] Here recalls that the function $g(\cdot)$ maps the state variable of price, the overall dependence of price on order flow is $dP = \cdots + g_p \cdot \mathbf{1}dY$ and $g_p \cdot \ mathbf f 1$ is "Kyle's lambda".

the same as zero, i.e. ($\theta_t = 0$) and thus, the sum of rest terms must also be zero, as shown below

$$\begin{aligned} 0 = -rJ^i + J^i_t + J^i_N \cdot \alpha + J^i_p(\alpha + \mathbf{1}\theta) + \frac{1}{2}tr(\sum\sum' J^i_{NN}) \\ + \frac{1}{2}tr(J^i_{pp}(\Lambda\Lambda' + \mathbf{1}\sigma_z^2\mathbf{1}')) + tr(J_{Np}\Lambda\sum') + \theta(f(t,i,N_t) - p_t) \end{aligned} \tag{4.13}$$

This result is simply the expected difference in the insider's value function given the transaction rate at the beginning of the transaction($t = 0$).

Proposition 4.3: *Suppose Definition 4.4 holds. Insiders follow a smooth-trading strategy and a pure acquisition strategy, i.e. getting information immediately at the start of a deal,* $\mathbb{P}(\tau = 0) = 1$.
Proof: The implication of this argument suggests that instead of getting information immediately, the insider can wait for a short interval Δ, during which he does not trade, and then get the information expensively. If we assume that future dates are discounted, and the discounted transaction cost is Δ, then the loss of transaction gains is smaller, and the insider gain for delayed acquisition costs is strictly better.

Additionally, by working backwards, I build an overall equilibrium. If present, the insider's optimal trading strategy must be continuous (expressed in asset prices) with a value function characterized by the HJB equation. Given this trading strategy of the strategy insider, the cumulative order flow is:

$$Y_t = \int_0^t \theta_t(\cdot)dt + Z_t \tag{4.14}$$

Notice that the order rates of the informed should also depend on the market maker's pricing rule or some state variables(s). In the setting of that, from the market maker's point of perspective, the cumulative order flow has two possible interpretations due to the unobservable Z to the market makers. Further discussion in **Appendix C.2**.

Significantly, I also discussed that this equilibrium must be characterized by a path toward trading time. Note that the results below imply both pure-strategy equilibria and mixed-strategy equilibria with immediate access to information. In summarize, this establishes the following conclusions:

Conclusion 4.1: *Assume that the HJB equation holds that if there is an overall equilibrium, any optimal insider trading strategy is continuous. The value function of the insider is the solution of the following HJB equation,* *subject to the conditions of integrability and laterality. Furthermore, this equilibrium must have transaction time differences.*

Conclusion 4.2: *There is no pure strategy equilibrium that acquires information with positive probability after $t = 0$. Assuming that Markov equilibrium and HJB equation hold, thus we have,*

- *(Pre-emption Deviation) There is no equilibrium in which the insider follows a pure information acquisition strategy; they acquire information with positive probability when $t = 0$. That is, there is no time-trading indifference in pure strategy, where there is a time point $\tau > 0$ such that $\Pi(\tau > s) > 0$: the insider always deviates.*
- *There is no equilibrium where insiders follow trading strategies and mixed acquisition strategies.*

As shown above, the results in this section rule out the existence of an equilibrium where unobserved information is obtained at a fixed cost under standard regularity conditions, which indicates that the same arguments as in **Proposition 4.1** immediately imply any purely strategic equilibrium neither can involve acquisitions after $t = 0$. On the contrary, the insider can profitably deviate from any conjectural equilibrium because he can preemptively obtain information, trade with unresponsive pricing rules, and make infinite profits.

Kyle's (1985) framework is the basis for a discrete master model with period length Δ. However, in this case, if the length of this period converges to 0, the equilibrium degenerates to that of a continuous-time bound model in a random horizon. The discrete-time version of this model has a unique equilibrium that converges to a well-defined policy profile $\Delta \to 0$. The continuous-time model can be recognized as a particular representation of the discrete-time model in which agents are able to trade frequently. Therefore, the interpretation of continuous-time models is subtle and needs to be examined more closely. Therefore, in the following, I keep this explanation convenient and consider the continuous-time model as a mathematical convenience that provides a valuable tool for stochastic calculus.

4.3.6 Discussion of Framework

In this subsection, I discuss, in detail, the assumptions I make in the benchmark model, which can be categorised into the following four cases.

Allowing for stochastic variation volume of noise trading offers a efficiency and empirically relevant good reference. For example, Collin-Dufresne and Fos(2016) assert that, during high liquidity conditions (such as price impact is lower and uninformed volume is higher), the activist investor is plausibly to make the trading more aggressively. Moreover, Banerjee and Breon-Drish(2020) recently proposed in a setting with the strategics of dynamic information acquisition. In their analysis, the informed traders always deviate from any conjectured equilibrium strategies by either pre-emptively entering or delaying entry.

In reality, information acquisition decisions by large investors/companies will be highly likely disclosed and scrutinised by financial regulation institutions. Subject to regulatory reporting requirements, companies need to report and discloses trading positions, capital adequacy, shareholders changes, and other important information about trading activity at the end of each fiscal year and on some key dates. Many business changes and personnel adjustments of the board of directors, such as changes in venture capital investors, company executives and corporate operating conditions, will attract extensive attention from financial information institutions (especially financial media). Even if the mass media does not report it, other market players often have some channels to obtain this information, to obtain this information and adjust their investment strategies.

It is worth mentioning that, if there is a choice, an investment trader may be more willing to rationally analyze the degree of information advantage he has and the optimal time for him to obtain inside information and determine whether it is worthwhile to obtain relevant information before public disclosure, to Carry out arbitrage trades. No matter how the investor follows the pure strategy, the length of a period goes zero in continuous time or discrete time. In the recent academic work about actual options, Banerjee and Breon-Drish(2017) explicitly provide that there does not exist a Nash equilibrium. Recently, Xiong and Yang(2020) creatively contributed to the previous literature by providing an analogous case in financial markets' overt and secret information-acquisition behaviours. Theey set a model with static information acquisition and immediate entry, rather than keeping this choice private (which referring as "secret acquisition"). The result indicates that the insider would like to disclose the precision of their private signal better("overt acquisition"). Some shreds of evidence suggest that insider trading and the associated profitability are likely to benefit more from the higher degree of information asymmetry, restricting insider trading could improve market efficiency by motivating them to increase the quality of public information disclosure and financial reporting(e.g., Ravina and Sapienza (2010); Jayaraman,2012; Chen, et al.,2017; Zhang and Zhang,2018). The effect of information asymmetry can be either offset or reinforced by corporate governance.

Taken together, these restrictive assumptions do limitations for the model's generality. However, their relaxation does not fundamentally alter the core intuition, and the technical generalises. In Section 4.4 & 4.5, I discuss equilibrium with discrete and continuous-time in a fixed or random horizon. In my analysis, a number of the non-existence results carry over to the standard Caldentey and Stacchetti(2010) and Kyle(1985) model with a random/fixed horizon and no discounting.

4.4 Discrete-time Unobservable Information Acquisition: Caldentey and Stacchetti's(2010) Type

In this section, I discuss the discrete-time model structured by Caldentey and Stacchetti(2010), where extends Kyle's(1985) continuous-time model of insider trading by allowing for discrete-time trading and unobservable information acquisition. In this case, I describe a class of discrete choice that we shall entertain the possibility of pure/mixed strategies in information acquisition, and the discrete-time between trading rounds(Δ) is sufficiency small. In the model, there are two types of agents: informed and uninformed. Informed agents have private information about the value of a risky asset, while uninformed agents do not. In each period, agents can choose to acquire information about the asset's value at a cost or trade the asset at the prevailing market price.

4.4.1 Model Motivation

In the Caldentey and Stacchetti(2010) case, a countable collection of time is discrete and insider starts at time point $t_n = n\Delta$ with $n \geq 0, \Delta \geq 0$.[27] The payoff of risky asset $V \sim \mathcal{N}(0, \sum_0)$ after a random trading around T. Specifically, $T = \eta\Delta$ since η has a geometrically distribution for a fixed constant $\gamma > 0$, the probability of failure is $\rho = e^{-\gamma\Delta}$.

[27]These trading sessions are evenly distributed over time, e.g. daily/hourly in stock exchange. The time interval $[t_n, t_{n+1})$ is called the period n, during which the following sequence of events occurs.

Risk-neutral insider, who observes V, trades at date t_n with trading strategy x_n.

Noise traders submit trades $Z_t \sim \mathcal{N}(0, \sum_z)$ with identical and independent distribution, where $\sum_z = \sigma_z^2 \Delta$. Denote that the the time n order flow is $y_n = x_n + z_n$, as we mentioned in past subsection.

Market makers are competitive, risk-neutral, only open trades at discrete times, and set prices p_n, Δ between each round of trading, equal to the conditional expectation. $\overline{V}_n$ and $\sum_n$ represent the conditional expectation and variance of the market maker before the trading round at time t_n, respectively. Thus, $p_{n-1} = \overline{V}_n$ for all n and $p_{-1} = \mathbb{E}[V_0] = 0$ for completeness.

Following the structure of Caldentey and Stacchetti (2010), the trajectory $X = \{x_n\}$ is used for insider trading and $P = \{p_n\}$ is used for market prices. Linear Markov Equilibrium of a pair of processes (P, X), where the price at time t_n depends only on the asset price p_{n-1} and the order flow y_n in the past time, such that[28]

$$P(\overline{V}_n, y_n) = \overline{V}_n + \lambda_n y_n \tag{4.15}$$

$$X(V, \overline{V}_n) = \beta_n (V - \overline{V}_n) \tag{4.16}$$

Theorem 4.1: *(Caldentey and Stacchetti,2010) There exists the linear strategy profile (P, X), unique sequences of functions $\{\lambda_n\}, \{\beta_n\} \subset \mathbb{R}_{++}$ and the deterministic sequences $\{\alpha_n\}, \{\gamma_n\} \subset \mathbb{R}_{++}$, I describe the expected profit for insider as*

$$\Pi_n(p_{n-1}, V) = \alpha_n (V - p_{n-1})^2 + \gamma_n \quad \textit{for all } n \geq 0 \tag{4.17}$$

[28]The trajectory $\{\sum_n\}$ is *deterministic*, hereafter I drop it in (4.15) and (4.16), also in the function of Π from Caldentey and Stacchetti's (2010) orginal structure.

, given $\sum_0 > 0$, the coefficients are characterized by

$$\sum_{n+1} = \frac{\sum_n \sum_z}{\beta_n^2 \sum_n + \sum_z} \tag{4.18}$$

$$\beta_{n+1} \sum_{n+1} = \rho\beta_n \sum_n \left(\frac{\sum_z^2}{\sum_z^2 - \beta_n^4 \sum_n^2} \right) \tag{4.19}$$

$$\lambda_n = \frac{\beta_n \sum_n}{\beta_n^2 \sum_n + \sum_z} \tag{4.20}$$

$$\alpha_n = \frac{1 - \lambda_n \beta_n}{2\lambda_n} \tag{4.21}$$

$$\rho\gamma_{n+1} = \gamma_n - \frac{1 - 2\lambda_n\beta_n}{2\lambda_n(1 - \lambda_n\beta_n)} \lambda_n^2 \sum_z \tag{4.22}$$

where

$$\gamma_0 = \sum_{k=0}^{\infty} \rho^k \left(\frac{1 - 2\lambda_k\beta_k}{2\lambda_k(1 - \lambda_k\beta_k)} \right) \lambda_k^2 \sum_z \tag{4.23}$$

***Proof**: Omit. See Caldentey and Stacchetti(2010).*

4.4.2 Profit Deviation

Before the trading starts ($t = 0$), the ex-ante expected benefit of getting the insider information immediately is

$$\overline{\Pi}_0 \equiv \mathbb{E}[\Pi_0(p_{-1}, V)] - C = \alpha_0(V - p_{-1})^2 + \gamma_n - C = \alpha_0 \sum_0 + \gamma_0 - C \tag{4.24}$$

From (4.20), the expected deviation profit in delay entry is

$$\overline{\Pi}_{d0} \equiv \rho \left(\alpha_1 (\sum_0 + \lambda_0^2 \sum_z) + \gamma_1 - C \right) \tag{4.25}$$

, where $\rho = e^{-r\Delta}$. Firstly, I can obviously conclude that:

***Proposition 4.4:** In Caldentey and Stacchetti(2010) structure, the delay entry of acquiring information is always better off in profits, shown as*

$$\begin{aligned} \overline{\Pi}_{d0} - \overline{\Pi}_0 &= \rho \left(\alpha_1 (\sum_0 + \lambda_0^2 \sum_z) + \gamma_1 - C \right) - \left(\alpha_0 \sum_0 + \gamma_0 - C \right) \\ &= (\rho\alpha_1 - \alpha_0) \sum_0 + \rho\alpha_1\lambda_0^2 \sum_z + \rho\gamma_1 - \gamma_0 + (1 - \rho)C \end{aligned} \tag{4.26}$$

The following results show that this deviation for delayed entry is profitable when the discrete-time delta between trading rounds Δ is sufficiently small.

$$\frac{\overline{\Pi}_{d0} - \overline{\Pi}_0}{\Delta} = \frac{-\frac{(\beta_0^\Delta)^3 \sum_\Delta^2}{2\sigma_z^2 \Delta} - (1 - e^{-r\Delta})C}{\Delta} > 0 \tag{4.27}$$

, in which it precludes an equilibrium in which insiders follow a purely information-acquiring strategy.

Proof: *See* ***Appendix C.3****.*

The main innovation of the model is that information acquisition is unobservable. That is, there is no direct way for the market to observe whether an agent has acquired information or not. As a result, agents can strategically choose when to acquire information and when to trade, based on their private information and beliefs about other agents' information acquisition decisions. The model analyzes the equilibrium behavior of agents and market outcomes under different assumptions about the cost of information acquisition and the precision of private information signals. The results show that the equilibrium behavior of agents depends crucially on these parameters and that the market outcomes can be highly sensitive to changes in them. In summary, Caldentey and Stacchetti's (2010) model provides a rich framework for studying the strategic behavior of agents in a dynamic setting with unobservable information acquisition. The model has important implications for understanding the role of information in financial markets and the effectiveness of policies aimed at reducing information asymmetry.

4.5 Continuous-time Convergence with No Discounting: Kyle's(1985) Type

Trading costs in the asymmetric financial market are associated with whose magnitude varies over periods and space. In such environments, endogenous information acquisition has various novel effects on market liquidity dynamics and how security prices reflect fundamental information. As a standard in previous literature, I use price impact to capture the market depth and trading liquidity, measured by Kyle's λ.

4.5.1 Model Motivation

I henceforth disucss the unobservable information acquisition mainly on the Kyle's(1985) continuous-time version with no discounting, where trading takes place in a fixed time interval $[0, 1]$.

As discussed in the previous chapter, there is the same argument that any pure-strategy equilibrium cannot involve costly information acquisition after the trading starts. To prove this proposition, assume a pure-strategy equilibrium where insiders have immediate access to unobservable information at t = 0

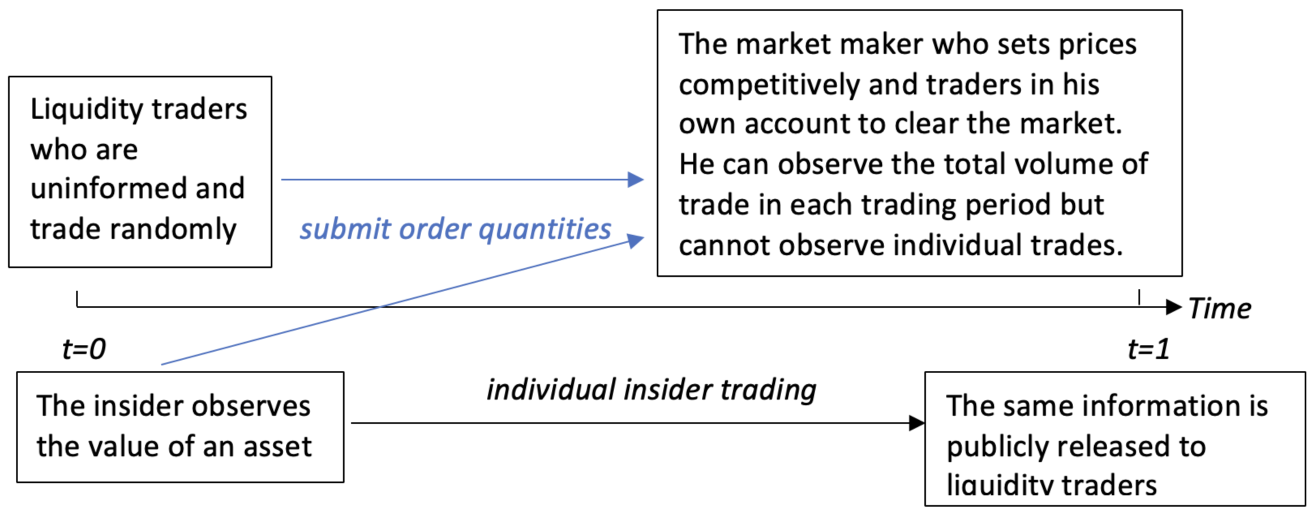

Figure 3: **Timeline of the game.**This model has one period with continuous-time version: $t \in [0, 1]$.

(start point of trade). In this case, the pricing rules and the trader's post-acquisition profits under the customarily distributed payoff function (i.e., $J(t, y)$) come from the specific cases of Kyle (1985) and Back (1992), as shown in above (see **Figure 3**).

Firstly, recall the price function in Kyle(1985),

$$P(t, y) = \lambda y \tag{4.28}$$

where $\lambda = \sqrt{\frac{\Sigma_0}{\sigma_Z^2}}$ and

$$\begin{aligned} J^v(t, y) &= \frac{1}{2}\lambda(V - P(t, y))^2 + \frac{1}{2\lambda}(1 - t) \\ &= \frac{1}{2}\sqrt{\frac{\sigma_Z^2}{\Sigma_0}}\left(V - \sqrt{\frac{\sigma_Z^2}{\Sigma_0}}y\right)^2 + \frac{1}{2}\sqrt{\sum_0 \sigma_Z^2}(1 - t) \end{aligned} \tag{4.29}$$

To conduct welfare analysis, at trading starts($t = 0$), I integrate the value function over the unconditional prior distribution of V to measure the insider's ex-ante (gross) expected trading profit from being informed, which is

$$\begin{aligned}
\bar{J}(0,0) &= \mathbb{E}[J^{v}(0,0)] \\
&= \frac{1}{2}\sqrt{\frac{\sigma_Z^2}{\sum_0}}\sum\nolimits_0 + \frac{1}{2}\sqrt{\sum\nolimits_0 \sigma_Z^2} \\
&= \sqrt{\sum\nolimits_0 \sigma_Z^2} \\
&= 1/\lambda
\end{aligned} \tag{4.30}$$

Following Back(1992) argument(in Lemma 1, p.399), he calculates the profit from this limit strategy by moving up or down the residual supply curve at period 1 to $p = v$. Constructing the insider's value function under this deviation is simple as follows

$$J^{d,U}(t,y) = J^{0}(t,y) = \frac{1}{2}\sqrt{\frac{\sum_0}{\sigma_Z^2}}y^2 + \frac{1}{2}\sqrt{\sum\nolimits_0 \sigma_Z^2}(1-t) \tag{4.31}$$

At time zero($t = 0$), derive as

$$J^{d,U}(0,0) = \frac{1}{2}\sqrt{\sum\nolimits_0 \sigma_z^2} = 1\Big/2\lambda \tag{4.32}$$

The (4.33) shows that Kyle's λ (market depth) is half of the insider's ex-ante gross profit. The equation states that this trade profit is non-zero because insiders expect future trade deviations in profit when realized noise pushes the price away from zero in the dynamic model.

From the above value functions, I conclude the proposition as follows:

Proposition 4.5: *If the security price assumption assumes that there is a smooth function between the trading period and the cumulative order flow, and* $C > 1\Big/2\lambda$, , then there is existing no pure strategy equilibrium followed by information acquisition.

Proof In the last subsection, I rule out the pure strategy equilibrium that the trader obtains through delayed entry (discussed in **Corollary 4.1**). Assuming that there is an equilibrium in which traders have immediate access to costly information ($t = 0$), Back(*The Review of Financial Studies* 5 (1992) 399)'s analysis implies that security prices and informed traders' value function must have the above form. Consider the insider's expected profit from unobservable deviations and never getting information (discussed in **Corollary 4.2**). The expected net profit for the deviation is

$$\overline{\Pi}_{d0} = J^{d0}(0,0) - (\bar{J}(0,0) - C) = C - 1\Big/2\lambda > 0 \tag{4.33}$$

, in which the insider is better off in profits by undertaking the post-entry deviation.

Conclusion 4.3: *(Higher cost→ lower deviation profit → never acquiring information) Note that the above results imply that when* $C > \frac{1}{2\lambda}$*, if the cost of information acquisition is high enough, the market equilibrium in Kyle (1985) will not be raised due to endogenous information acquisition: costly information acquisition will not happen*

The above results provide a more robust version of describing the insider's post-entry/delay-deviation decision: if the cost of acquiring private information is sufficiently high, rational insiders' best strategy is to decide *never* to acquire information. Also, **Conclusion 4.3** provides the market implication of the regulation basis of insider trading; increasing the illegal cost /penalty of insider trading can efficiently decrease the insider trading and manipulation behaviours in financial markets.

4.5.2 Kyle's λ: Market Depth and Trading Volatility

How does the information get into the risky asset prices? A priori, it would seem difficult for any insiders to acquire a non-public information advantage over other participants in the asset market given its depth and volatility. This variable, as well as market efficiency(price discovery), is central to regulatory debates(See the discussion in Easley, O'Hara, and Yang,2016). Instead, prices respond linearly to order follow. I measure market liquidity using Kyle's λ: a more liquid market is associated with a smaller λ. Intuitively, λ captures the price impact of uninformed noise trading and is thus negatively related to market depth. Kyle (1985) says that the model determines a parameter λ endogenously, which means that the magnitude of the price change is proportional to Kyle's lambda, which depends on the order flow and the model's parameters. Given its place in the model, we can interpret it as an inverse measure of liquidity. More precisely, a higher estimate of Kyle's λ, based on a regression estimate of order flow from a price change, is more likely to hold more privately informed stocks (relative to liquid trading volume). The lower is λ, the more liquid is the asset, and the greater is the volume required to move the market maker's prices. Also, the reverse of "Kyle's lambda" $1/\lambda$ measures the *depth* of the market. By calling it "liquidity", we assume that depth and liquidity go hand in hand.

Recently, Collin-Dufresne and Fos(2021) indicate that informed traders may adjust their trades based on price-impact indicators, leading to selection bias in liquidity indicator estimates. When stock price liquidity and trading volume are high and measured price impact is low, informed traders tend to trade more aggressively, and vice versa. Empirically, in the U.S stock market practice (e.g., S&P 500, Nasdaq), it is also recognised as a parameter of market liquidity and can be directly estimated through the volume required to move the security price by one dollar.

Additionally, the price-response function (Kyle λ) also relies on cumulative orders: it is small when the cumulative order is small. As accumulation increases, it peaks, then goes down to Kyle's λ, when the existence of the insider turns out to be likely (the market maker's probability estimates of the insider are convergent to 1). The concavity of λ is a constant property that explains the patience of the informed trader. This parameter is frequently referred to as "Kyle's lambda" (See Kyle(1985)) For example, the seminal model of insider trading of Kyle (1985) predicts that the regression of price change on signed order flow can estimate the "Kyle's lambda", which should be higher for stocks with more informed trading(relative to liquidity trading).

Last, market depth $\frac{1}{\lambda_t}$ is a martingale, and therefore λ_t is a submartingale concerning both the market maker and the informed traders' filtrations. With shared beliefs about the probability that the insider is better informed, the Zakai equation(Zakai,1969) solves the problems of market maker's inference, which is a non-linear filtering mathematical technique(more discussion about Zakai equation in **Appendix C.2**).

4.6 Insider Trading in Practice: Taking the U.S. SEC as An Example

The activity of insider trading in the stock market is a significant puzzle. Theoretically, in the Efficiency-Market Hypothesis, all investors are supposed to get information about publicly-traded stocks simultaneously. If that is so, why do some hedge funds spend so much time and money secretly acquiring uninformed information, such as private meetings with executive management? One apparent reason is that they find it more valuable than publicly existing information in financial markets. (e.g., recent studies by Banerjee, Davis, and Gondhi(2018) states the phenomenon that a public disclosure encourages investors to acquire instead information about others' beliefs rather than learning fundamentals; also, in Dugast and Foucault(2018), a cheaper raw(low-precision) signals can crowd out investment in the processed(low-precision) one.

In general, many regulations and rules have been designed to mitigate information asymmetry among investors in the financial markets. To better understand how the timing and uncertainty of insider trading affect financial markets, before conclusion taking the U.S. as an example, I discuss several practical implications of the findings to focus on policy regulations and penalties of insider trading in the U.S. legislation.

4.6.1 Who is Insider

The range of the insider can differ crucially under different jurisdictions, which may follow a narrow definition only and consider individuals with direct access to the information as insiders. Additionally, some may also recognise people related to corporation officials as insiders. Traditionally, an insider is an individual/agent who has access or has been given access to non-public information. The evolution of insider trading laws has attempted to see whether these laws effectively discourage insiders from trading on non-public information.

According to classification from U.S. legislation, *the Securities Exchange Act of 1934*, registered insiders(also called corporate insiders) has been held initially by Section 16(b), company's directors and officers, as well as shareholders who are directly or indirectly the beneficial owner of over 10% of any types of any equity securities (other than an exempted security) shall file the statements required by the Section 16(b) to report all their trade-in equity securities periodically to the SEC.[29] For unregistered insiders(also called outside-insiders), they are also in possession of material non-public information but are not required to report their information-acquisition behaviours to the U.S. Security and Exchange Commission(SEC). Unregistered insiders' information acquisition behaviours can be direct in their work, such as accountants, financial journalists, investment bankers, lawyers, risk arbitrages, or indirect by the intermediary of registered insiders.

In June 2010, in the famous Morrison v. National Australia Bank case,[30] the U.S. Supreme Court de-

[29] Securities and Exchange Commission. "Officers, Directors and 10% Shareholders." See https://www.sec.gov/smallbusiness/goingpublic/officersanddirectors. Accessed August 30, 2021.

[30] Morrison v. National Australia Bank Ltd., 561 U.S. 247 (2010). See https://supreme.justia.com/cases/federal/us/561/247/.

	Domestic listing	Overseas listing
Domestic transaction	Yes	Yes
Overseas transaction	Yes	No

Table 1: Application of the "transaction standard" established in the Morrison case

termined the jurisdiction over the conduct of insider trading and proposed to judge the extraterritorial application of the standard of Article 10(b) of the Anti-fraud General Clause of *the Securities Exchange Act of 1934* in the U.S.—"Trading Standards" "(Transactional test): Article 10(b) applies to the trading activities of securities listed on domestic exchanges and the domestic transactions on other securities. **Table 1** can illustrate the application method of Article 10(b) under the "Trading Standards".

The detailed rules on insider trading are complex and vary from country to country. Taking legal practice in the United States as an example, an insider of a company (manager, officer, or significant shareholder) must report his transactions in the stock of the relevant company after the fact and immediately to the SEC and the public. For example, the identification of insiders in the U.S. securities law is comprehensive in practice:

- The company's directors, supervisors, executives and their partners, and trustees buy or sell company securities after illegally obtaining the significant inside information.
- Shareholders and their partners and trustees who own more than 10% of the shares;
- Employees of service companies such as investment banking, brokerage, law, and media firms, and the responsible relevance who have family relations, friend relations or other personal relations of the persons as mentioned above;
- Government officers who obtain non-public information illegally because of their job position advantage;
- Presumed insider includes any outsider who has learned inside information through the performance of his duties;
- The information leaker and the person who knows the leak; and

Accessed January 3, 2022.

- Those who misappropriate information.

4.6.2 Regulations for Insider Trading and Penalty

Insider trading refers that insiders(most often an employee) who obtain insider information in financial markets improperly, violate laws and regulations, disclose insider information, buy or sell securities based on insider information, or provide trading advice to others. By using their particular identities or opportunities to avoid losses, insider traders obtain non-public information that helps them conduct security transactions. As a result, it violates the investing public's equal right to access information, resulting in serious adverse effects on the efficiency of information disclosure and violating the principles of "fairness, transparency, and impartiality" in the securities market. Insider trading can be legal or illegal, depending on whether the insider adheres to regulations, which means that insiders are allowed to know about developments in their companies that could potentially influence the share price, but they cannot act on that information until disclosed to the public.

Fidelity Investments manager Peter Lynch, one of the most successful American investors of all time, famously said: "*Insiders might sell their shares for any number of reasons, but they buy them for only one: they think the price will rise.* [31]" Empirically, there has also been research whether the returns of insider trading investment strategies are more profitable than uninformed. Jeng, Zeckhauser, and Metrick (2003) find that insider buying exceeds 6% of the total market share per year. It is worth noting that insider sales does not yield comparable abnormal profits. This empirical result can briefly explain why many stock investors pay close attention to the activities of company insiders.

For example, private meetings with companies are an essential private information source for investors. In reality, institutional investors who meet privately with the board of directors make more insider trading decisions. While *the Regulation Fair Disclosure(Reg F.D.)* aims to promote full and fair disclosure. This act addresses that company managers need to disclose all the selective material information publicly available and accessible to all investors. When an asset issuer discloses material non-public information to individuals who may trade relied on this private information, the equity issuer must also

[31] Investopedia: Buy Stock With Insiders: How To Track Insider Buying. See https://www.investopedia.com/articles/02/061202.asp. Accessed August 31, 2021.

publicly disclose that information to financial markets. A specific individual may be a securities market professional, such as a stock analyst or an issuer's securities holder.

In 1990, the U.S. Congress passed *the Securities Acts Amendments of 1990* , which included *the International Securities Enforcement Cooperation Act of 1990*, In *the Securities Act of 1933* and *the Securities Exchange Act of 1934(SEA)*, the main directories of the supervision cooperation between SEC and overseas regulatory agencies have been added.[32] For example, the law(*SEA, Section109b*) authorises the SEC to promulgate regulations and rules to prevent security fraud. In addition to this, the SEC, which enforces the law against stock market manipulation, prohibits insiders from disposing of shares within six months of their purchase, which means the insider is compelled to hold their shares for at least six months. The independent agency states that the information that an investor bases their trade on must be public at the time of their trade. Therefore, in most cases, an insider buying shares in their company often suggests that they believe that its share price will increase in the future.

In recent years, there have been strengthened disclosure requirements by legal regulations. Policies like *the Sarbanes-Oxley, the Regulation Fair Disclosure*, and *the Rule 10b5-1*[33] have arguable decreased the cost of information acquisition. For instance, some predictions are consistent with empirical findings and differ from existing theories. Dugast and Foucault(2018), the population size of early traders monotonically reduces(raises) in the cost the early-raw signal reduces. Similarly, Kendall(2018) indicates that investors are always more likely to "rush" (trade early) when the quality of the early signal increases.

[32]For related legislative purposes, please refer to the Senate Report No. 100-461 to accompany S. 2544, *the International Securities Enforcement Cooperation Act of 1988*,100th Congress, August 8, 1988; House Report No. 101-240 to accompany H. R. 1396, *the International Securities Enforcement Cooperation Act of 1989*, 101st Congress, September 12, 1989.

[33]In recent, SEC Proposes amendments to *the Rule 10b5-1* under *the Securities Exchange Act of 1934* has been proposed to enhance the requirements of disclosure and against insider trading. The proposal concludes the update to *the Rule 10b5-1(c)*, which strengthen the requirements to access affirmative defence to insider trading for parties that frequently have access to non-public information, including corporate officers, directors and issuers; More details see https://www.sec.gov/news/press-release/2021-256. Accessed December 31 2021.

4.6.3 The Effect of SEC-type Penalties on Insider Trading

The effect of SEC penalties on information acquisition and insider trading can be complex and depends on various factors, such as the severity of the penalty, the likelihood of being caught, and the characteristics of the market and the information itself. Assuming that the information is acquired randomly, i.e., not through insider trading, SEC penalties can still affect information acquisition in several ways. First, the fear of SEC penalties can discourage individuals and companies from engaging in activities that could be perceived as illegal or unethical, including aggressive information acquisition. This can lead to a reduction in the amount and quality of information available in the market, potentially reducing market efficiency and liquidity. Second, SEC penalties can also increase the incentives for individuals and companies to acquire information through legal and ethical means, such as conducting research and analysis or collaborating with industry experts. This can lead to an increase in the amount and quality of information available in the market, potentially enhancing market efficiency and liquidity.

The SEC has established a number of processes and procedures for receiving and investigating reports of insider trading and whistleblower tips. The SEC has established a program called the Electronic Data Gathering, Analysis, and Retrieval (EDGAR) system, which allows companies to file reports on their insider trading activities. The SEC also receives tips from whistleblowers, who are individuals who provide information about possible securities law violations. These tips can be submitted through the SEC's online whistleblower portal or by mail. Once a report of insider trading or a whistleblower tip is received, the SEC's Enforcement Division will review the information to determine whether there is evidence of a violation of the securities laws. If the Enforcement Division determines that there is evidence of a violation, it may initiate an investigation and take legal action against the individuals or companies involved. The SEC may also refer the matter to other agencies or authorities, such as the Department of Justice or state securities regulators. The following figures plot the insider trading reports and whistleblower tips by the U.K. and U.S(see **Figure 4 &5**).

In response to the SEC's regulation in insider trading, originally, Carlton and Fischel(1983) study the impact of SEC enforcement of insider trading regulations on market efficiency and the cost of capital. They argue that while enforcement efforts can deter insider trading, excessive penalties can discourage information acquisition and reduce market efficiency, so that SEC regulators need to find a balance be-

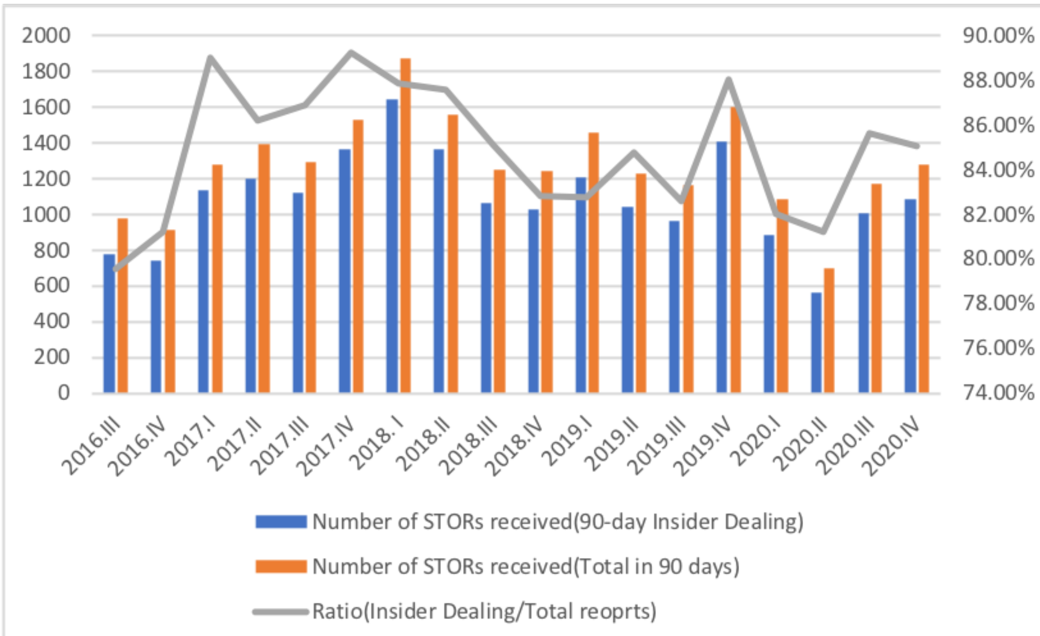

Figure 4: The total volume of insider dealing and suspicious transaction and order reports (STORs) in the U.K.

*Notes:*A new *EU Market Abuse Regulation (MAR)* took effect in 2016 that obligates professional institutions to report suspicious transactions and suspicious orders, requiring professional institutions to have reasonable grounds to suspect that a particular order or transaction is an attempt at insider trading. When the market is suspected as manipulated, it should be reported to the supervisory authority on time. Figure 8 shows the number of Suspicious Transactions and Order Reports(STORs) received since the implementation of *MAR* between the second half of 2016 and 2020. According to the FCA's disclosure, the number structure of STORs is dominated by insider trading.

Source: UK's Financial Conduct Authority(FCA), Annual Report and Accounts 2016/17,2018/19, 2019/20.

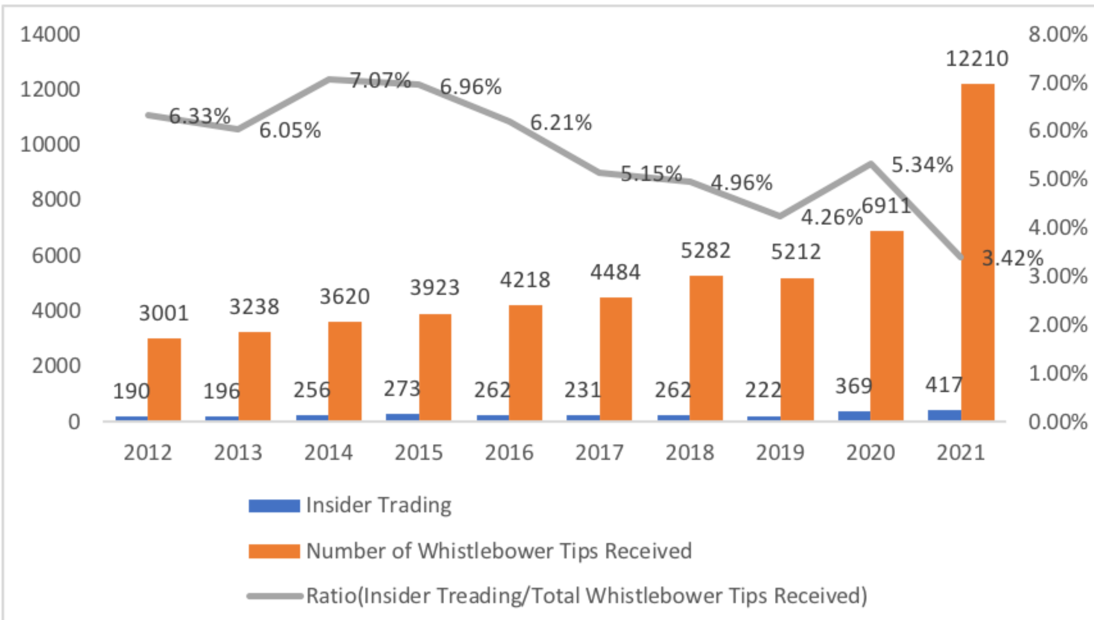

Figure 5: Whistleblower tips by allegation type comparison of fiscal years(F.Y.), 2012–2021.

*Notes:*This figure shows that the number of whistleblower tips received by the Securities and Exchange Commission (SEC) has grown by approximately 300% compared to F.Y.2012 (the first year the SEC disclosed full-year data). Since the whistleblower rules went into effect on August 12, 2011, SEC disclosed only seven weeks of whistleblower data in F.Y.2011.

Source: U.S. SEC Annual Report to Congress: Whistleblower Program, 2012-2021.

tween deterrence and promoting information acquisition to maximize market efficiency. However, Bhattacharya et al. (2003) provide evidence on the negative impact of insider trading on market efficiency and the cost of capital. The authors find that insider trading harms investors and imposes a significant cost on firms, leading to a decrease in market efficiency. They suggest that regulators need to take measures to discourage insider trading and enforce stronger penalties for violations.

Many recent empirical papers provide empirical evidence to argue the idea that the SEC regulation and the effect of the market efficiency. According to Johnson and Liberti(2016), information flow within financial conglomerates impacts market efficiency. Information sharing within conglomerates enhances market efficiency by improving price discovery and reducing informational asymmetry. SEC Regulation should encourage transparency within conglomerates to reduce information asymmetry and enhance market efficiency. Chen et al. (2018) develop a theoretical model to examine the impact of insider trading on market efficiency, and the effect of penalties on insider trading behavior. The authors find that while penalties for insider trading can deter insider trading, they can also reduce information acquisition and lead to market inefficiencies. Jachson and Moser(2019) examine the relationship between insider trading and earnings management, and the impact of SEC enforcement efforts on these practices. They find that insider trading is associated with earnings management, and that SEC enforcement efforts can reduce the incidence of both practices.

Overall, these conceptual and empirical evidence provide on the importance of transparency, information sharing, and ethical behavior in enhancing market efficiency and promoting sustainable business practices. Regulators need to take measures to discourage insider trading, encourage information sharing, and enforce stronger penalties for violations to enhance market efficiency. However, it is important to note that the effectiveness of these measures can vary depending on the specific market and regulatory environment, and further research is needed to fully understand their impact.

4.7 Conclusion

Why insider trading always exists? This question has been studied for decades but still presents challenges to regulators and researchers. Among all the insiders' decisions, information acquisition stands at the top of the list. The aggregate unobserved information-acquisition decisions further determine the quality of financial markets. To better understand theoretical model to examine the impact of the insider trading activity, I discuss the unobservable information acquisition in three-type structures: Canonical Kyle-type, Caldentey and Stacchetti(2010) type in discrete-time, and Kyle's type in continuous-time convergence.

Firstly, I propose the canonical Kyle-type framework to help explain some of these empirical findings, where market makers set prices in response to unobserved information acquisition by the insider. The analysis for unobservable information acquisition indicates that because the market maker cannot detect the insider's deviation in his acquisition strategy sensitively, there cannot exist pure/mix strategy equilibria. The results indicate that equilibrium relies on whether the market maker can observe the information acquisition. In the absence of observability, insiders tend to deviate by delaying information acquisition rather than acquiring it at $t = 0$. This is because delaying acquisition costs may result in a better payoff when their trading returns are not affected. Conversely, if the insider's information acquisition is observable and the market maker can detect the insider's profit deviations, then the insider trading equilibria can be sustained, and these deviations may no longer be profitable.

Secondly, this chapter considers the discrete-time model discussed by Caldentey and Stacchetti(2010). I extend a discrete-time dynamic model of unobservable information acquisition following Caldentey and Stacchetti's (2010) study, which relies Kyle's (1985) continuous-time insider trading model by allowing discrete-time trading and unobservable information acquisition. In my model extension, I propose that the public announcement date is not deterministic(random deadline) but follows the exponential distribution.

Thirdly, this chapter discusses another version of unobserved information acquisition, using the Kyle's(1985) type structure to discuss the continuous-time convergence without discounting. In this scenario, the high cost of information acquisition makes it unprofitable for insiders to acquire information. As a result, there will be no endogenous information acquisition, and the market equilibrium will not be affected by the presence of insider information. This implies that when $C > \frac{1}{2\lambda}$, the market may not be efficient, and there may be a significant level of information asymmetry. This highlights the importance of considering the cost of information acquisition in dynamic insider trading models and the potential limitations of insider trading regulation in improving market efficiency. This analysis suggests that the cost of information acquisition can have a significant impact on the behavior of investors and market outcomes, and it should be taken into account when designing policies and regulations aimed at improving market efficiency and reducing information asymmetry.

Finally, this chapter sheds new light on the implications of insider trading in financial markets. My analysis provides a coherent argument for understanding practical issues in the U.S. financial market, such as insider regulation of insiders with company management and legislative regulation. I discuss the typical characteristics of the insider and various types of insider trading in the U.S. financial markets. It is interesting to explore that increasing the opportunity cost of insider trading(e.g., increasing fines and penalties) reduces the incentive of insider trading. In addition, the SEC has been continuously strengthened the full and fair disclosure requirements by legal regulations. This study offers further implications for policymakers and practitioners.

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6 Appendix of Chapter 2

A.1 Proof of Proposition 2.1

Firstly, plugging the demand functions of both informed traders and uninformed traders into the market clearing equilibrium equation:

$$\int_0^{\lambda} \frac{\rho_v \overline{v} + \rho_\epsilon \widetilde{s}_i + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p - (\rho_v + \rho_\epsilon + \rho_\eta + \rho_p)\widetilde{p}}{\gamma} di + \int_{1-\lambda}^{1} \frac{\rho_v \overline{v} + \rho_\eta \widetilde{y} + \rho_p \widetilde{s}_p - (\rho_v + \rho_\eta + \rho_p)\widetilde{p}}{\gamma} + \widetilde{z} = X$$

Secondly, recalling (2.5) that the public signal $\widetilde{y}$, the information included in the price is equal to the information signal:

$$\widetilde{s}_p \equiv \frac{\widetilde{p} - p_0 - p_y \widetilde{y}}{p_v} = \widetilde{v} + \frac{p_z}{p_v} \widetilde{z}$$

I substitute this expression into the (2.3), derive the conjectured price function $\widetilde{p}$ in linear rational expectations equilibrium as:

$$\begin{aligned}
\widetilde{p} &= \frac{\rho_v \overline{v} - \gamma X}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p} + \frac{\rho_\eta}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}\widetilde{y} + \frac{\lambda\rho_\epsilon + \rho_p}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}\widetilde{v} + \frac{(\alpha_z/\alpha_v)\rho_p + \gamma}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}\widetilde{z} \\
&= \frac{1}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}\left[\rho_v \overline{v} + \rho_\eta \widetilde{y} + (\lambda\rho_\epsilon + \rho_p)\widetilde{v} + (\alpha_z/\alpha_v)\rho_p \widetilde{z} + \gamma(\widetilde{z} - X)\right] \quad \text{(A.1)}
\end{aligned}$$

Thirdly, Recalling the endogenous precision of the price $\rho_p = (p_v/p_z)^2 \rho_z$ in (2.5), I can derive that:

$$\frac{p_z}{p_v} = \frac{(p_z/p_v)\rho_p + \gamma}{\rho_p + \gamma\rho_\epsilon} \Rightarrow \frac{p_z}{p_v} = \frac{\lambda\rho_\epsilon}{\gamma} \quad \text{(A.2)}$$

Finally, after we get (A.2), it is obviously to derive that:

$$\rho_p = (p_v/p_z)^2 \rho_z = (\lambda\rho_\epsilon/\gamma)^2 \rho_z$$

Thus, comparing the conjectured price equations between (2.3) and Eq(A.1) and finally get the expressions of endogenously determined coefficients p_0, p_y, p_v and p_z, respectively.

A.2 Proof of Proposition 2.2

Recalling the optimal risky price in the condition of higher order beliefs derived from (2.37):

$$\widetilde{p}^* = \left(\frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}\right)^k \widetilde{v}$$

I rewrite this equation as the ratio between stochastic liquidation value $\widetilde{v}$ and optimal market clearing price $\widetilde{p}^*$ as:

$$\frac{\widetilde{p}^*}{\widetilde{v}} = \left(\frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}\right)^k$$

where the order of beliefs k is in positive integer set as $k \in \mathbb{N}^+$. The above equation shows that the higher order beliefs amplifies the ratio of $\widetilde{p}$ and $\widetilde{v}$, which indicates the expected return of risky assets will be higher in Keynesian-beauty-contest effect. Meanwhile, we consider the limitation of the accurate private information, that is $\rho_\epsilon \to \infty$, as follows:

$$\lim_{\rho_\epsilon \to \infty} \left(\frac{\rho_\epsilon}{\rho_\epsilon + \rho_\eta}\right)^k = 1$$

where $\rho_\epsilon \in \mathbb{R}_{++}$. So we can deduce

$$\lim_{\rho_\epsilon \to \infty} \left(\frac{\widetilde{p}}{\widetilde{v}}\right) = 1$$

$$\Rightarrow \lim_{\rho_\epsilon \to \infty} \mathbb{E}[\widetilde{v} - \widetilde{p}] = 0$$

Hence, we can proof that the expectation of return for risky asset for investors will be lower when the asset issuers have more private information that is uninformed in the financial market. At the limit of infinity of private information hold by asset issuers, the expectation of return for risky asset for investors will become zero and then no trader has willing to invest risky assets.

A.3 Proof of Proposition 2.3

Plugging (2.37) into (2.38), I define the expression of market efficiency as following:

$$ME = \frac{1}{\mathbb{E}\left[(\widetilde{v} - \widetilde{p})^2\right]} = \frac{(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)^2}{(\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z}$$

Then, I prove this proposition by using the partial derivative of ME with respect to ρ_η.

$$\frac{\partial ME}{\partial \rho_\eta} = \frac{\frac{\partial}{\partial \rho_\eta}\Big((\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)^2\Big)\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2}{\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2}$$

$$- \frac{\frac{\partial}{\partial \rho_\eta}\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)^2}{\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2}$$

$$= \frac{2(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big) - (\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)^2}{\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2} \tag{A.3}$$

Simplifying (A.3),

$$\frac{\partial ME}{\partial \rho_\eta} = \frac{2(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z - (\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)\Big)}{\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2}$$

$$= \frac{2(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)\Big((\gamma X)^2 + \lambda\rho_\epsilon + \gamma^2/\rho_z\Big)}{\Big((\gamma X)^2 + (\rho_v + \rho_\eta + \rho_p) + 2\lambda\rho_\epsilon + \gamma^2/\rho_z\Big)^2} > 0$$

Hence, the proposition that higher accurate public information increases the market efficiency level is proven. Additionally, following the Allen, Morris and Shin(AMS) model (Allen, Morris and Shin, 2006), I formulate the higher-order average beliefs by the iteration of transition matrix as:

$$\overline{\mathbb{E}}^k \begin{bmatrix} \widetilde{y} \\ \widetilde{v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{\rho_\epsilon}{\rho_\epsilon + \rho_\eta} & \frac{\rho_\epsilon}{\rho_\epsilon + \rho_\eta} \end{bmatrix}^k \begin{bmatrix} \widetilde{y} \\ \widetilde{v} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 - \left(\frac{\rho_\epsilon}{\rho_\epsilon + \rho_\eta}\right)^k & \left(\frac{\rho_\epsilon}{\rho_\epsilon + \rho_\eta}\right)^k \end{bmatrix} \begin{bmatrix} \widetilde{y} \\ \widetilde{v} \end{bmatrix}$$

$$\to \begin{bmatrix} \widetilde{y} \\ \widetilde{y} \end{bmatrix} \text{ as } k \to \infty$$

In higher order beliefs, as $k \to \infty$, then $\overline{\mathbb{E}}^k(\widetilde{v}|\cdot) \to \widetilde{y}$, where $\widetilde{y}$ is public signal. From the above matrix, I use the state space $\widetilde{y}, \widetilde{v}$ to define a two-state Markov chain. This motivation shows that when the order of beliefs raises to infinite high, the stochastic liquidation value $\widetilde{v}$ becomes less weight and finally we can ignore that. It also shows that in this Markov chain, the public signal $\widetilde{y}$ is an absorbing state, which indicating that since once the public information entering into the higher order beliefs in financial market, it will become dominant and never move out as the order of beliefs is continuously increasing in Keynesian beauty contest influence.

A.4 Proof of Proposition 2.4

Recall the average conditional precision of asset payoff, that:

$$\Pi = \rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p$$

which is consistent with the informed and uninformed traders. In order to introduce the λ mass of informed traders and $(1-\lambda)$ of uninformed traders, then I can substitute the reciprocal of Variance equations (2.12) and (2.15) into above equation, as:

$$Var(\widetilde{v}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})^{-1} = \rho_v + \rho_\epsilon + \rho_\eta + \rho_p$$

$$Var(\widetilde{v}|\widetilde{y}, \widetilde{p})^{-1} = \rho_v + \rho_\eta + \rho_p$$

Thus,

$$\begin{aligned}
\Pi &= \rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p \\
&= \lambda \frac{1}{Var(\widetilde{v}|\widetilde{y}, \widetilde{s}_i, \widetilde{p})} + (1-\lambda)\frac{1}{Var(\widetilde{v}|\widetilde{y}, \widetilde{p})}
\end{aligned}$$

As we know that the expression of the cost of capital is:

$$COC \equiv \mathbb{E}[\widetilde{v} - \widetilde{p}] = \frac{\gamma X}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}$$

The first-order condition of public disclosure precision is:

$$\begin{aligned}
\frac{\partial COC}{\partial \rho_\eta} &= \frac{\partial}{\partial \rho_\eta}\left(\frac{\gamma X}{\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p}\right) \\
&= -\frac{\gamma X}{(\rho_v + \lambda\rho_\epsilon + \rho_\eta + \rho_p)^2} < 0
\end{aligned}$$

which proves that the higher precision of public information will lower the cost of capital.

A.5 Proof of Proposition 2.5

Plugging the the information variance equations (2.12) and (2.15) into the (2.41), I rewrite the expression of the cost of capital as:

$$
\begin{aligned}
COC &= \frac{\gamma X Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})Var(\widetilde{v}|\widetilde{y},\widetilde{p})}{\lambda Var(\widetilde{v}|\widetilde{y},\widetilde{p}) + (1-\lambda)Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})} \\
&= \gamma X Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})Var(\widetilde{v}|\widetilde{y},\widetilde{p})(\lambda Var(\widetilde{v}|\widetilde{y},\widetilde{p}) + (1-\lambda)Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p}))^{-1} \\
&= \gamma X(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1}(\rho_v+\rho_\eta+\rho_p)^{-1}(\lambda(\rho_v+\rho_\eta+\rho_p)^{-1} + (1-\lambda)(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1})^{-1}
\end{aligned}
$$

The direct computation of the mass of informed traders is:

$$
\frac{\partial COC}{\partial \lambda} = \frac{-\gamma X(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1}(\rho_v+\rho_\eta+\rho_p)^{-1}}{(\lambda(\rho_v+\rho_\eta+\rho_p)^{-1}+(1-\lambda)(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1})^2}((\rho_v+\rho_\eta+\rho_p)^{-1}-(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1})
$$

Given that the information precision parameters are all strictly positive in the model setup, with $\rho_v, \rho_\eta, \rho_\epsilon, \rho_p > 0$. we have $((\rho_v+\rho_\eta+\rho_p)^{-1}-(\rho_v+\rho_\epsilon+\rho_\eta+\rho_p)^{-1}) > 0$, which implies that informed traders' posterior belief about the random payoff $\widetilde{v}$ is more homogeneous than that of uninformed traders, as $Var(\widetilde{v}|\widetilde{y},\widetilde{p}) > Var(\widetilde{v}|\widetilde{y},\widetilde{s}_i,\widetilde{p})$. Thus $\frac{\partial COC}{\partial \lambda} < 0$ is proved.

A.6 Proof of Proposition 2.6

I define that the mass of informed traders are sensitive in the interval $(0,1)$ and there is an unique optimal solution of the mass of informal traders λ^* in financial market equilibrium where we can find the maximization bound level of the differential and monotonically expected benefit $B(\lambda;\rho_\eta,\rho_{\epsilon i})$ with a critical information acquisition value $C^*(\rho_\epsilon) \in (0, \overline{C(\rho_\epsilon)})$, shown as:

$$
B(\lambda^*;\rho_\eta,\rho_\epsilon) = \frac{1}{2\gamma} log\left[1+\frac{\rho_\epsilon}{\rho_v+\rho_\eta+(\lambda^*\rho_\epsilon/\gamma)^2\rho_z}\right] - C^*(\rho_\epsilon) = 0
$$

- As we discussed in above cases, if traders are indifferent in paying information acquisition cost to become informed, that is $B(\lambda^*;\rho_\eta,\rho_\epsilon) = 0$, then we can reallocate the equation as:

$$
\begin{aligned}
&B(\lambda^*;\rho_\eta,\rho_\epsilon) = 0 \\
&\Rightarrow 2\gamma C^*(\rho_\epsilon) = log\left[1+\frac{\rho_\epsilon}{\rho_v+\rho_\eta+(\lambda^*\rho_\epsilon/\gamma)^2\rho_z}\right] \\
&\Rightarrow e^{2\gamma C^*(\rho_\epsilon)} - 1 = \frac{\rho_\epsilon}{\rho_v+\rho_\eta+(\lambda^*\rho_\epsilon/\gamma)^2\rho_z} \\
&\Rightarrow \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} = \rho_v+\rho_\eta+(\lambda^*\rho_\epsilon/\gamma)^2\rho_z
\end{aligned}
$$

Recall that the mass of λ^* is linearly depended on the cost of information acquisition, with $\lambda^* = C^*(\rho_\epsilon)/\overline{C(\rho_\epsilon)}$. Thus, we substitute $\overline{C(\rho_\epsilon)}$ into the above equation to compute the fractions of informed traders using the implicit function theorem, that is:

$$\frac{\rho_\epsilon}{e^{2\gamma\overline{C(\rho_\epsilon)}\lambda^*} - 1} = \rho_v + \rho_\eta + (\lambda^*\rho_\epsilon/\gamma)^2\rho_z$$

Finally, using the implicit function theorem, we can prove:

$$\frac{\partial\lambda^*}{\partial\rho_\eta} = -\frac{1}{\frac{2\rho_\epsilon{}^2\rho_z\lambda^*}{\gamma^2} + \frac{2\gamma\rho_\epsilon\overline{C(\rho_\epsilon)}e^{2\gamma\overline{C(\rho_\epsilon)}\lambda^*}}{(e^{2\gamma\overline{C(\rho_\epsilon)}\lambda^*}-1)^2}} < 0$$

which implies that the high transparency and quality of public information can decrease the mass of informed traders in financial market equilibrium.

- Recall the optimization of the benefit equation in the first-order condition in (2.33),

$$C'(\rho^*{}_\epsilon) = \frac{1}{2\gamma(\rho_v + \rho^*{}_\epsilon + \rho_\eta + (\lambda\rho^*{}_\epsilon/\gamma)^2\rho_z)} > 0$$
$$\Rightarrow \frac{1}{2\gamma C'(\rho^*{}_\epsilon)} = \rho_v + \rho^*{}_\epsilon + \rho_\eta + \left(\frac{\lambda\rho^*{}_\epsilon}{\gamma}\right)^2\rho_z > 0$$

where the $\rho_{\epsilon i} = \rho^*{}_\epsilon \in (0, \overline{\rho_\epsilon}), i \in [0,1]$. For the sufficiency large value of public disclosure precision ρ_η, we get the limit of $\rho_\eta \to +\infty$, where the right hand side of the above first-order condition equation tends to infinity, and the left hand side of equation $\frac{1}{2\gamma C'(\overline{\rho_\epsilon})}$ tends to infinity as well, as $C'(\overline{\rho_\epsilon}) \to 0$. Since $C'(\overline{\rho_\epsilon}) > 0$ as monotone increasing and differential, which implies $\overline{\rho_\epsilon} \to 0$, thus we can prove that $\lim_{\rho_\eta \to +\infty} \rho^*{}_\epsilon = 0$.

Next, this first-order condition equation also implicitly shows the optimal response between the $\rho^*{}_\epsilon$ and disclosure precision ρ_η. Using the implicit function theorem, I compute the second-order condition of equation as:

$$\frac{\partial\rho_\eta}{\partial\rho^*{}_\epsilon} = -\frac{1}{2\gamma[C''(\rho^*{}_\epsilon)]^2} - 2\left(\frac{\lambda}{\gamma}\right)^2\rho^*{}_\epsilon\rho_z - 1 < 0$$

which implies that when the crowding-out effect dominates the information market, the sufficiency large transparency of public information will squeeze out the private information.

A.7 Proof of Proposition 2.7

Firstly, I prove that the partial derivative of ME with respect to $C(\cdot)$ is negative, shown as:

$$\frac{\partial ME}{\partial C(\cdot)} = \frac{\frac{\partial}{\partial C(\cdot)}\left((\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2\right)\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$
$$- \frac{\frac{\partial}{\partial C(\cdot)}\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$

where

$$\frac{\partial(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\partial C(\cdot)} = -\frac{4\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)$$
$$\frac{\partial\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)}{\partial C(\cdot)} = -\frac{2\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}$$

Thus,

$$\frac{\partial ME}{\partial C(\cdot)} = \frac{-\frac{4\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right) - \frac{2\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$
$$= \frac{-\frac{4\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)\left(2(\gamma X)^2 + \frac{2\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 4\lambda^*\rho_\epsilon + 2\gamma^2/\rho_z + \frac{\rho_\epsilon}{2(e^{2\gamma C^*(\rho_\epsilon)}-1)} + \frac{1}{2}\lambda^*\rho_\epsilon\right)}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2} < 0$$

Secondly, I show that the partial derivative of COC with respect to $C(\cdot)$ is positive, shown as:

$$\frac{\partial COC}{\partial C(\cdot)} = \frac{-\gamma X}{(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}\left(-\frac{2\gamma\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}\right)$$
$$= \frac{2\gamma^2\rho_\epsilon e^{2\gamma C^*(\rho_\epsilon)}X}{(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2(e^{2\gamma C^*(\rho_\epsilon)}-1)^2} > 0$$

Thirdly, I demonstrate that the partial derivative of ME with respect to ρ_ϵ is positive, shown as:

$$\frac{\partial ME}{\partial \rho_\epsilon} = \frac{\frac{\partial}{\partial \rho_\epsilon}\left((\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2\right)\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$
$$- \frac{\frac{\partial}{\partial \rho_\epsilon}\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$

where

$$\begin{aligned}\frac{\partial(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\partial \rho_\epsilon} &= 2(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)\left(\lambda^* + \frac{e^{2\gamma C^*(\rho_\epsilon)} - 1 - 2\gamma\rho_\epsilon C'(\rho_\epsilon)e^{2\gamma C^*(\rho_\epsilon)}}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}\right)\\ &= 2(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)\left(\lambda^* + \frac{e^{2\gamma C^*(\rho_\epsilon)}(1 - 2\gamma\rho_\epsilon C'(\rho_\epsilon)) - 1}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}\right)\end{aligned}$$

and

$$\frac{\partial}{\partial \rho_\epsilon}\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right) = 2\lambda^* + \frac{e^{2\gamma C^*(\rho_\epsilon)}(1 - 2\gamma\rho_\epsilon C'(\rho_\epsilon)) - 1}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}$$

Thus,

$$\frac{\partial ME}{\partial \rho_\epsilon} = \frac{2(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)\left(\lambda^* + \frac{e^{2\gamma C^*(\rho_\epsilon)}(1-2\gamma\rho_\epsilon C'(\rho_\epsilon))-1}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}\right)\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2}$$
$$+ \frac{\left(\frac{e^{2\gamma C^*(\rho_\epsilon)}(2\gamma\rho_\epsilon C'(\rho_\epsilon)-1)+1}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2} - 2\lambda^*\right)(\lambda^*\rho_\epsilon + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1})^2}{\left((\gamma X)^2 + \frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + 2\lambda^*\rho_\epsilon + \gamma^2/\rho_z\right)^2} > 0$$

where the quadratic form is $C(\rho_{\epsilon i}) = \frac{c}{2}\rho_{\epsilon i}^2$, so $C'(\rho_\epsilon) = c\rho_\epsilon > 0$. Finally, I proof that the partial derivative of COC with respect to ρ_ϵ is negative, shown as:

$$\frac{\partial COC}{\partial \rho_\epsilon} = -\frac{\gamma X}{(\frac{\rho_\epsilon}{e^{2\gamma C^*(\rho_\epsilon)}-1} + \lambda^*\rho_\epsilon)^2}\left(\lambda^* + \frac{e^{2\gamma C^*(\rho_\epsilon)}(1 - 2\gamma\rho_\epsilon C'(\rho_\epsilon)) - 1}{(e^{2\gamma C^*(\rho_\epsilon)}-1)^2}\right) < 0$$

7 Appendix of Chapter 3

B.1 Proof of Proposition 3.1

- The impact of the precision of public information on expected welfare is

$$\frac{\partial \mathbb{E}(W)}{\partial \rho_\eta} = \lambda \frac{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)(1-2\gamma\lambda)}{(\rho_\eta + \rho_\epsilon(1-\gamma\lambda))^3} \tag{B.1}$$

  the first order condition

$$\frac{\partial \mathbb{E}(W)}{\partial \rho_\eta} \geq 0 \tag{B.2}$$

  we can get

$$\frac{\rho_\eta}{\rho_\epsilon} \geq (1-\gamma\lambda)(2\gamma\lambda - 1) \tag{B.3}$$

  Thus, we can conclude that the sign of $\frac{\mathbb{E}(W)}{\partial \rho_\eta}$ is ambiguous. Eq(B.1) shows that an increase in the precision of public signal is detrimental to social welfare if $2r\lambda > 0$. Otherwise, if if $2\gamma\lambda < 0$ private information is imprecise (small ρ_ϵ), hence the precision of public signal can increase social welfare. In summary, the social welfare trending related to the disclosure level of public information is non-monotonic. In the specific case of $\lambda = 1$, it goes back to the MS result as $\frac{\rho_\eta}{\rho_\epsilon} \geq (1-\gamma)(2\gamma - 1)$, in which indicates that the precision of public signal increases social welfare.

- The partial derivative of $\mathbb{E}(W)$ with respect to ρ_ϵ is

$$\frac{\partial \mathbb{E}(W)}{\partial \rho_\epsilon} = \lambda(1-\gamma\lambda)\frac{\rho_\eta(1+\gamma\lambda) + \rho_\epsilon(1-\gamma\lambda)^2}{(\rho_\eta + \rho_\epsilon(1-\gamma\lambda))^3} \geq 0, \text{ as } \gamma \in [0,1), \lambda \in [0,1] \tag{B.4}$$

  which indicates that increasing the private signal is always beneficial to social welfare, and completes the proof.

B.2 Proof of Proposition 3.2

I start by computing the first-order condition of the maximization problem (3.21),

$$\frac{\partial \mathbb{E}(u_i)}{\partial \lambda} = \underbrace{-2(\gamma-1)\left(\frac{\rho_\epsilon \gamma \nu_3}{\nu_2^2} - \frac{\rho_\epsilon \widetilde{s}_i \gamma}{\nu_2}\right)(\theta - \frac{\nu_3}{\nu_2})}_{part1} - \underbrace{\left[\frac{2\gamma\nu_1(\rho_\eta(\widetilde{y}+\theta) + \rho_\epsilon\gamma(\widetilde{s}_i - \theta))}{\nu_2^2} + \frac{2\rho_\epsilon\gamma^2\nu_1^2}{\nu_2^3}\right]}_{part2} < 0$$

I analyse this equation in two separate parts. First, for the interval $\forall\gamma \in [0,1), \forall\lambda \in [0,1], (\gamma\lambda - 1) < 0, \gamma - 1 < 0$. In *part 1*, we separate one part as

$$\frac{\rho_\epsilon \gamma \nu_3}{\nu_2^2} - \frac{\rho_\epsilon \widetilde{s}_i \gamma}{\nu_2} = \frac{\rho_\epsilon\gamma(\rho_y \widetilde{y} - \rho_\epsilon \widetilde{s}_i(\gamma\lambda - 1)) - \rho_\epsilon \widetilde{s}_i \gamma(\rho_\eta + \rho_\epsilon(1-\gamma\lambda))}{\nu_2^2} = \frac{\rho_\epsilon \rho_\eta \gamma(\widetilde{y} - \widetilde{s}_i)}{\nu_2^2} < 0$$

Recall (3.2) and (3.3), we can get $\forall\rho_\eta > 0, \forall\rho_\epsilon > 0, \widetilde{\epsilon}_i = \widetilde{s}_i - \theta > 0$ and $\widetilde{\eta} = \widetilde{y} - \theta > 0$, so $\widetilde{\epsilon}_i > \widetilde{\eta}$ if and only if $\widetilde{s}_i > \widetilde{y}$, it shows that,

$$\theta - \frac{\nu_3}{\nu_2} = \frac{\rho_\eta(\theta - \widetilde{y}) + \rho_\epsilon(\theta(1-\gamma\lambda) - \widetilde{s}_i(1+\gamma\lambda))}{\nu_2} < 0$$

where $\forall\gamma \in [0,1), \forall\lambda \in [0,1], \gamma\lambda - 1 < 0, 1 + \gamma\lambda > 0$. In summary, we can conclude that *part* $1 < 0$.

Second, I rewrite *part* 2 as

$$\frac{2\gamma\nu_1(\rho_\eta(\widetilde{y}+\theta) + \rho_\epsilon\gamma(\widetilde{s}_i - \theta))}{\nu_2^2} + \frac{2\rho_\epsilon\gamma^2\nu_1^2}{\nu_2^3} = \frac{2\gamma\nu_1\nu_2(\rho_\eta(\widetilde{y}+\theta) + \rho_\epsilon\gamma(\widetilde{s}_i - \theta)) + 2\rho_\epsilon\gamma^2\nu_1^2}{\nu_2^3}$$

$$= \frac{\nu_1}{\nu_2^3}\left[2\gamma\nu_2(\rho_\eta(\widetilde{y}+\theta) + \rho_\epsilon\gamma(\widetilde{s}_i - \theta)) + 2\rho_\epsilon\gamma^2(\rho_\epsilon(\widetilde{s}_i - \theta)(\gamma\lambda - 1) + \rho_\eta(\widetilde{y}+\theta)(\lambda - 1))\right]$$

where ν_2^3 is strictly positive as $\nu_2 > 0$ and $\nu_1 < 0$, so we have $\frac{\nu_1}{\nu_2^3} < 0$. Recall that $\widetilde{s}_i - \theta > 0$, hence, to complete the proof, it needs to indicate that

$$|\rho_\eta(\widetilde{y}+\theta)| > |\rho_\eta(\widetilde{y}+\theta)(\lambda - 1)| \text{ as } \forall\lambda \in [0,1]$$

$$|\rho_\epsilon\gamma(\widetilde{s}_i - \theta)| > |\rho_\epsilon(\widetilde{s}_i - \theta)(\gamma\lambda - 1)| \text{ as } \forall\gamma\lambda \in [0,1)$$

$$2\gamma\nu_2 = 2\gamma\rho_\eta + 2\gamma\rho_\epsilon(1-\gamma\lambda) > 2\rho_\epsilon\gamma^2 \tag{B.5}$$

For (B.5), I get

$$\begin{aligned} 2\gamma\rho_\eta + 2\gamma\rho_\epsilon(1-\gamma\lambda) - 2\rho_\epsilon\gamma^2 &= 2\gamma\rho_\eta + 2\gamma\rho_\epsilon - 2\gamma^2\lambda\rho_\epsilon - 2\rho_\epsilon\gamma^2 \\ &= 2\gamma(\rho_\epsilon + \rho_\eta - \gamma\lambda\rho_\epsilon - \gamma\rho_\epsilon) \\ &= 2\gamma((1-\gamma\lambda)\rho_\epsilon + \rho_\eta - \gamma\rho_\epsilon) > 0 \end{aligned}$$

where $\widetilde{\epsilon}_i > \widetilde{\eta}$ if and only if $\widetilde{s}_i > \widetilde{y}$. For the signal precision, recall that $\widetilde{\epsilon}_i \sim \mathcal{N}(0, 1/\rho_\epsilon)$ and $\widetilde{\eta} \sim \mathcal{N}(0, 1/\rho_\eta)$, $\forall\rho_\epsilon > 0, \forall\rho_\eta > 0$, it has $0 < \rho_\epsilon < \rho_\eta$ if and only if $\widetilde{s}_i > \widetilde{y}$. Therefore, *part* $2 < 0$ proofed.

In summary, as *part* $1 < 0$ and *part* $2 < 0$, we can get $\frac{\partial\mathbb{E}(u_i)}{\partial\lambda} < 0$, where $\rho_\epsilon < \rho_\eta$ is required.

B.3 Proof of Uniqueness of Linear Equilibrium

Firstly, recall the agent i's optimal action, given by the first-order condition of utility function, relies on the expected fundamental value of risky asset and the average action of all agents in market, given as

$$a_i = (1-\gamma)\mathbb{E}_i(\theta) + \gamma\mathbb{E}_i(\bar{a}_{-i})$$

As I define that, uninformed agents receive just private signal, as $a_i = \widetilde{s}_i$. However, informed agents can receive both public and private signal, thus the optimal action by informed agents is $a_i(\widetilde{s}_i, \widetilde{y}) \in \{\widetilde{s}_i, \widetilde{y}\}$. Recall the Eq(3.20), combining these two types of agents, the average action in market is denoted as

$$\overline{a}_{-i} = (1-\lambda)\int_{\lambda}^{1} \widetilde{s}_i di + \lambda \int_{i=0}^{\lambda} a_i(\widetilde{s}_i, \widetilde{y}) di = (1-\lambda)\theta + \lambda \overline{a}_{-i}(\theta, \widetilde{y})$$

Secondly, substituting the $\overline{a}$ into $\mathbb{E}_i(\overline{a})$ for the average expectation of fundamental value $\widetilde{v}$ across all agents, shown as

$$\overline{a}_{-i} = (1-\gamma)\mathbb{E}_i(\theta) + \gamma \mathbb{E}_i[\lambda \overline{a}_{-i}(\theta, \widetilde{y}) + (1-\lambda)\theta] = (1-\gamma)\mathbb{E}_i(\theta) + \gamma\lambda \mathbb{E}_i[\overline{a}_{-i}(\theta, \widetilde{y})] \tag{B.6}$$

where $\mathbb{E}_i[\overline{a}_{-i}(\theta, \widetilde{y})]$ is the average expected action by informed agents. Thirdly, the level-k action for informed agents can be shown as

$$\begin{aligned}
a_i(\widetilde{s}_i, \widetilde{y}) &= (1-\gamma\lambda)\mathbb{E}_i(\theta) + \gamma\lambda \mathbb{E}_i[\overline{a}_{-i}(\theta, \widetilde{y})] \\
&= (1-\gamma\lambda)\mathbb{E}_i(\theta) + (1-\gamma\lambda)\gamma\lambda \mathbb{E}_i[\overline{\mathbb{E}}(\theta)] + (\gamma\lambda)^2 \mathbb{E}_i[\overline{\mathbb{E}}[\overline{a}_{-i}(\theta, \widetilde{y})]] \\
&= (1-\gamma\lambda)\mathbb{E}_i(\theta) + (1-\gamma\lambda)\gamma\lambda \mathbb{E}_i[\overline{\mathbb{E}}(\theta)] + (1-\gamma\lambda)(\gamma\lambda)^2 \mathbb{E}_i[\overline{\mathbb{E}}^2(\theta)] + \cdots + \lim_{k\to\infty}(\gamma\lambda)^k \mathbb{E}_i[\overline{\mathbb{E}}^k[\overline{a}_{-i}(\theta, \widetilde{y})]] \\
&= (1-\gamma\lambda)\sum_{k=0}^{\infty}(\gamma\lambda)^k \mathbb{E}_i[\overline{\mathbb{E}}^k(\theta)] + \lim_{k\to\infty}(\gamma\lambda)^k \mathbb{E}_i[\overline{\mathbb{E}}^k[\overline{a}_{-i}(\theta, \widetilde{y})]] \\
&= (1-\gamma\lambda)\sum_{k=0}^{\infty}(\gamma\lambda)^k \mathbb{E}_i[\overline{\mathbb{E}}^k(\theta)]
\end{aligned} \tag{B.7}$$

which gives the limes term in the above equation is zero, such that

$$\lim_{k\to\infty}(\gamma\lambda)^k \mathbb{E}_i[\overline{\mathbb{E}}^k[\overline{a}_{-i}(\theta, \widetilde{y})]] = 0, \text{ as } \gamma \in [0,1), \lambda \in [0,1] \tag{B.8}$$

where k is the sophisticated level of thinking iteration. Eq(B.8) shows the thought of p-beauty contest game demonstrated by Camerer, Ho, and Chong(2004).[34] As Cornand and Heinemann(2008) explained, $\overline{\mathbb{E}}^k(\theta)$ is the average expectation of the average expectation... (k levels) fundamental value θ. Similarly, $\overline{\mathbb{E}}^k(\overline{a}_{-i})$ is the average expectation of the average expectation... (k levels) of the informed agents' average action $\overline{a}_{-i}$. $\overline{\mathbb{E}}$ is the average expectation operator across all agents such that $\mathbb{E}(\cdot) = \int_i \mathbb{E}(\cdot)di$. Morris and Shin (2002) discuss that, for any level-k thinking, $\overline{\mathbb{E}}^k(\theta) = f^k\theta + (1-f^k)\widetilde{y}$ and $\mathbb{E}_i[\overline{\mathbb{E}}^k(\theta)] = f^{k+1}\widetilde{s}_i + (1-$

[34] For $\lambda \in (0,1)$ in this case, I follow the infinitely iterating condition designed by Camerer, Ho, and Chong(2004), a agent who believes that all agents in average will convergence to the same results will reach the equilibrium zero, which is shown as $k \to +\infty$

$f)^{k+1}\widetilde{y}$ are both linear functions. Finally, substituting $\mathbb{E}_i[\overline{\mathbb{E}}^k(\theta]$ into above equation, given as

$$\begin{aligned} a_i(\widetilde{s}_i, \widetilde{y}) &= (1-\gamma\lambda)\sum_{k=0}^{\infty}(\gamma\lambda)^k[f^{k+1}\widetilde{s}_i + (1-f)^{k+1}\widetilde{y}] \\ &= \frac{\rho_\epsilon(1-\gamma\lambda)}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\widetilde{s}_i + \frac{\rho_\eta}{\rho_\eta + \rho_\epsilon(1-\gamma\lambda)}\widetilde{y} \end{aligned} \tag{B.9}$$

This equilibrium action to this infinite sum of the equations at the power $k+1$ is bounded, and it replicates that $a_i(\widetilde{s}_i, \widetilde{y})$ is the unique linear equilibrium that yields with the Section 3.3 as presented earlier and completes the proof.

B.4 Proof of Proposition 3.3

- $\rho_\eta \to 0, \lambda \to 0$ or $\rho_\epsilon \to \infty$, then (3.35) turns $f_k = 1$ and in (3.36) we get $a_i^k(\widetilde{s}_i) = \widetilde{s}_i$: in this case, when public signal is extremely imprecise or given to almost nobody, or when private signal is extremely precise, then public signal loses its coordination role and is ignored.
- $\rho_\eta \to \infty, \lambda \to 1$ or $\rho_\epsilon \to 0$, then (3.35) turns $f_k = 0$ and in (3.36) we get $a_i^k(\widetilde{y}) = \widetilde{y}$: in this case, inversely, when public signal is extremely precise or private signal extremely imprecise those who receive public signal will disregard private signal and choose $a_i = \widetilde{y}$. The others can just make strategies based on private signals, which are distributed around θ. Hence, those without public signal will choose an action θ in average.

B.5 Quantitative Illustration Graphs in Section 3.5

These graphs report the detailed results of optimal actions varying for level-k reasonings sorted by different average expectations and external information environments.

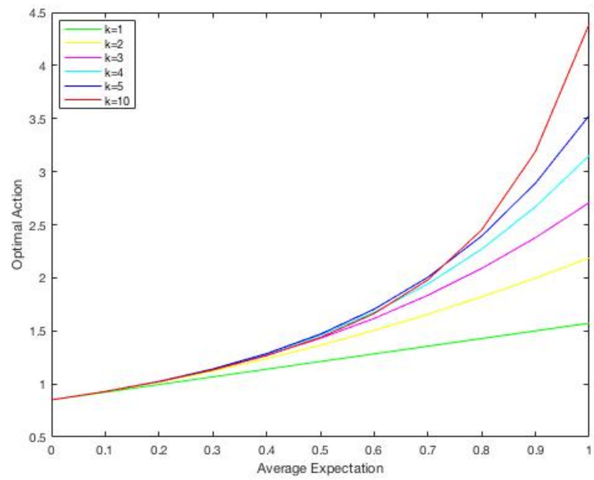

Figure 6: Simulation 1: Optimal actions for varying level-k reasoning when $r\gamma\lambda = 0.15$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.15$.

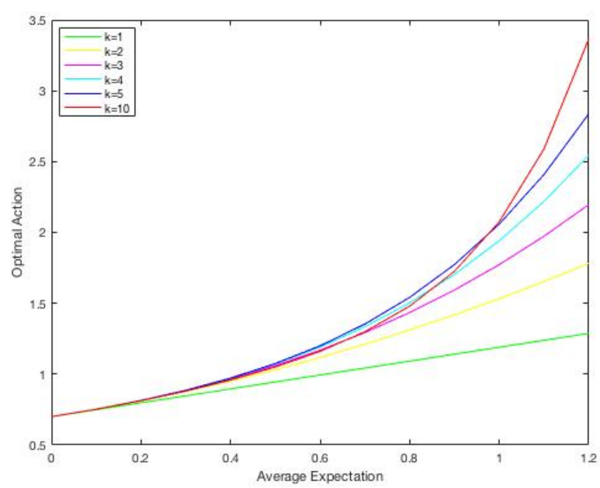

Figure 7: Simulation 2: Optimal actions for varying level-k reasoning when $\gamma\lambda = 0.3$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.3$.

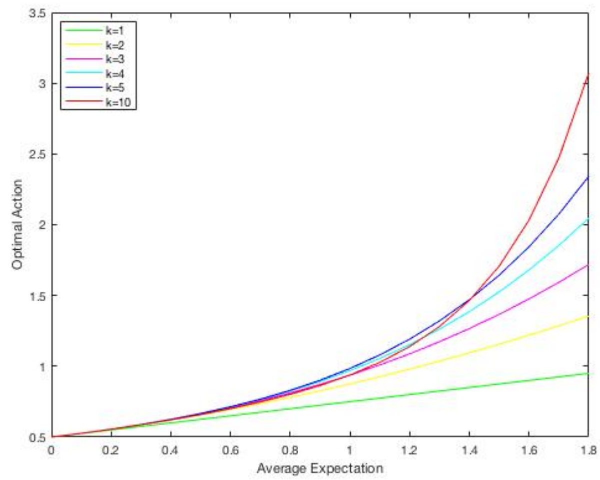

Figure 8: Simulation 3: Optimal actions for varying level-k reasoning when $\gamma\lambda = 0.5$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.5$.

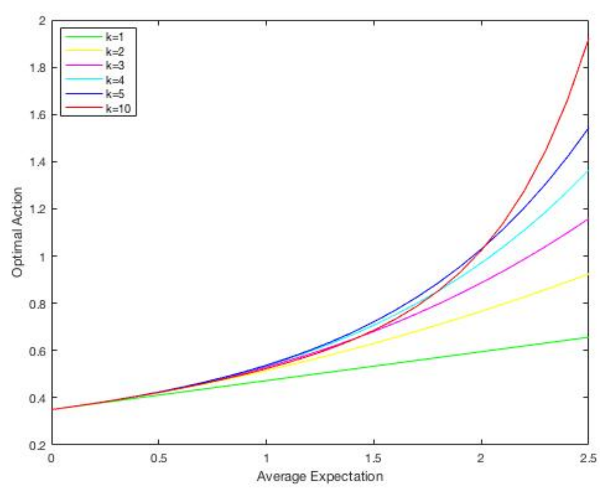

Figure 9: Simulation 4: Optimal actions for varying level-k reasoning when $\gamma\lambda = 0.65$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.65$.

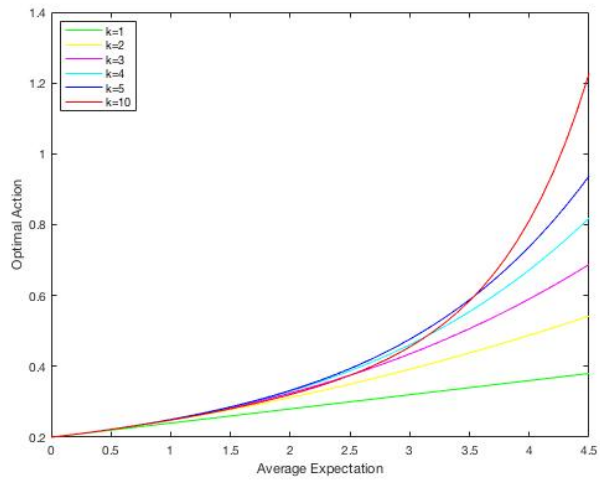

Figure 10: Simulation 5: Optimal actions for varying level-k reasoning when $\gamma\lambda = 0.8$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.8$.

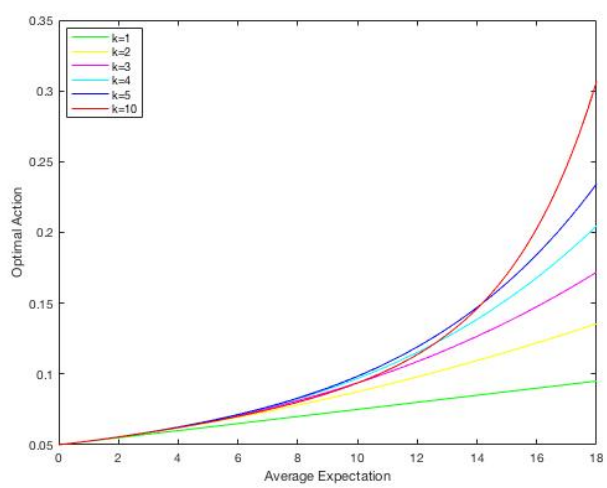

Figure 11: Simulation 6: Optimal actions for varying level-k reasoning when $\gamma\lambda = 0.95$

Notes: The illustration of limited level-k reasoning as a function of the optimal action with respect to average expectation for $k \in \{1, 2, 3, 4, 5, 10\}$ and $\gamma\lambda = 0.95$.

B.6 Recapitulation of Kyle's Model with Level-k Reasoning

The paper *Informed Speculation with Imperfect Competition* published by Albert S. Kyle in 1989 established a significant model framework by considering a Bayesian strategic game with linear market equilibrium on market microstructure. In his paper, each agent submitted a demanding schedule and informed investors to speculate on their information signals and deduce others' information signals from prices in the financial market. Firstly, we adapt Kyles's assumptions in equilibrium, where all speculators clear the market in the equivalence of the aggregate demand and aggregate supply at a market-clearing price, $\widetilde{p}$, of the single risky asset. Then, I incorporate the modification of Kyle's model extended by Alexander and Vives(2014) and Zhou(2022) into our baseline model structure to derive some propositions and lemmas of strategic sophistication in the level-k model under imperfect competitive equilibrium.

B.6.1 Extension of Kyle Model Framework

Following the Kyle's model, the environment draws noise speculators' speculating actions under imperfect competition. In a liquidity trading environment, a single risky asset is operated at an exogenous random fundamental(liquidity) value, denoted as $\widetilde{v}$, where is normally distributed with zero mean and variance, denoted as $\widetilde{v} \sim \mathcal{N}(0, \rho_v^{-1})$. Thus, the profit of risky asset is $\mathbb{E}[\widetilde{v} - \widetilde{p}]$.

Rather than the assumption of continuum of identical agents, the extension produces finite and discrete speculators in financial market, indexed as $i, j \in \{1, 2, 3, \ldots N\}$and $i \neq j$, respectively. Specifically, we focus on an speculator i receives a signal vector with noise $\widetilde{\epsilon}_i$ on the fundamental value $\widetilde{v}$, represented as the outcome of a random variable $\widetilde{s}_i$, so that each speculator i receives a private signal $\widetilde{s}_i$ following the Equation (3.5) such that

$$\widetilde{s}_i = \widetilde{v} + \widetilde{\epsilon}_i, \quad \widetilde{\epsilon}_i \sim \mathcal{N}(0, 1/\rho_\epsilon) \text{ and } \rho_\epsilon > 0$$

, where ϵ_i is an additive noise and it is independent across speculators.

Additionally, we assume the exogenous random quality of asset per capital to the market supply is uncertainty, which are irrational and provide noise(liquidity) to stock market by supplying $\widetilde{z}$ units, where follows a normal distribution $\widetilde{z} \sim \mathcal{N}(0, 1/\rho_z)$, so that ρ_z is the size of noise trading which also has an impact on the asymptotic action of the level-k reasoning strategies. Note that, all above random variables, indicated as $\{\widetilde{v}, \widetilde{z}, \widetilde{\epsilon}_1, \ldots, \widetilde{\epsilon}_N\}$, follow the joint normally and independently distribution with means and variances given by $Var[\widetilde{v}] = {\rho_v}^{-1}$, $Var[\widetilde{z}] = \rho_z^{-1}$,$Var[\widetilde{\epsilon}_i] = \rho_\epsilon^{-1}$.

After observing this private signal $\widetilde{s}_i$, speculator i selects a demand schedule $D(\widetilde{s}_i, \cdot)$ which relies on the signal $\widetilde{s}_i$ and the asset price, $\widetilde{p}$, in market clearing. Therefore, the profit function can be rewritten as

$$W_i = (\widetilde{v} - \widetilde{p})D(\widetilde{s}_i, \widetilde{p}) \tag{B.10}$$

All speculators have the constant absolute risk aversion(CARA) utility function with a risk aversion coefficient, denoted as γ.

$$u(W_i) = -exp(-\gamma W_i) \tag{B.11}$$

where W_i is realized investment profit.

As we consider that sophisticated speculators would incorporate information into their reasoning levels from price subject, I define $\mathcal{P}(\cdot)$ as the bijective function of being their models on price formation in our linear level reasoning strategies in order to map the aggregate information to asset prices. In this case, the reciprocal of bijective function $\mathcal{P}^{-1}(\widetilde{p})$ represents the signal for any asset price $\widetilde{p}$.

Sophisticated speculators compete in demand schedules and would take the maximization of his/her conditional expected utility, shown as

$$u_i[D(\widetilde{s}_i, \widetilde{p})] = \mathbb{E}[-exp(-\gamma W_i)|\widetilde{s}_i, \mathcal{P}^{-1}(\widetilde{p})] \tag{B.12}$$

Using the constant absolute risk-aversion(CARA) utility functions with these random variables, the maximization of conditional expectation utility can be rewritten as the of mean-variance format, given by

$$u_i[D(\widetilde{s}_i, \widetilde{p})] = \mathbb{E}[(\widetilde{v} - \widetilde{p})D(\widetilde{s}_i, \widetilde{p})|\widetilde{s}_i, \mathcal{P}^{-1}(\widetilde{p})] - \frac{\gamma}{2} Var[(\widetilde{v} - \widetilde{p})D(\widetilde{s}_i, \widetilde{p})|\widetilde{s}_i, \mathcal{P}^{-1}(\widetilde{p})] \tag{B.13}$$

where $\mathcal{P}(\cdot)$ is k-dependent and coefficient $\gamma > 0$. It means that level-k reasoning will be inferred in the asset price formation since signal inference from risky asset price is bounded by speculators' reasoning levels.

Moreover, the correspondences include the kinds of level speculators placed on organized exchanges. Considering Kyle(1989), the linear market equilibrium where speculator i's strategy is

$$D(\widetilde{s}_i, \widetilde{p}) = \beta_i \widetilde{s}_i - \xi_i \widetilde{p} \tag{B.14}$$

, where β_i is the marginal response to private information while ξ_i is the slope the demand function. Different from Kyle's original model, which shows that the linear strategy is $D(\widetilde{s}_i, \widetilde{p}) = \mu_i + \beta_i \widetilde{s}_i - \xi_i \widetilde{p}$, where the μ_i is ignored in our framework($\mu_i = 0$) in the condition that all speculators are informed and we define $\mathbb{E}(\widetilde{v}) = 0$. Therefore, the values of (β_i, ξ_i) will be pinned down by utility maximization. In this condition, market clearing is defined as

$$\sum_{i=1}^{N} (\beta_i \widetilde{s}_i - \xi_i \widetilde{p}) + \widetilde{z} = 0 \tag{B.15}$$

In the imperfect competition characterized by Kyle(1989), as we assumed in above section, speculator i believes others play (β_j, ξ_j), in which $\forall i \neq j$. Using (B.13) and (B.14), the asset price $\widetilde{p}$ from financial market clearing condition and indicates the signals from prices with following model

$$\mathcal{P}(\cdot) : \widetilde{p} = \frac{D(\widetilde{s}_i, \widetilde{p})}{\sum_{j \neq i} \xi_j} + \underbrace{\frac{\sum_{j \neq i} \beta_j \widetilde{s}_j}{\sum_{j \neq i} \xi_j} + \frac{\widetilde{z}}{\sum_{j \neq i} \xi_j}}_{\text{the residual supply}} \tag{B.16}$$

, where (B.16) basically shows the bijection function of asset prices as consisting of two components: the price contribution from the demand of i speculators and the residual supply.

Finally, Kyle(1989) calculates the first-order condition to maximize the utility function (3.39) directly; thus, the optimal demand of speculators at a strategic equilibrium is as follows

$$D(\widetilde{s}_i, \widetilde{p}) = \frac{\mathbb{E}[\widetilde{v} | \widetilde{s}_i, \mathcal{P}_i^{-1}(p)] - \widetilde{p}}{\delta_i + \gamma Var[\widetilde{v} - \widetilde{p} | \widetilde{s}_i, \mathcal{P}_i^{-1}(\widetilde{p})]} \tag{B.17}$$

, where δ_i in the formula represents the price impact of an marginal unit of risky asset to financial market. It will be determined endogenously and asset price changes in a way and absorbed by other speculators. The second-order condition $\gamma Var(\widetilde{v} - \widetilde{p} | \widetilde{s}_i, \widetilde{p}) > 0$ holds and it ensures that the solution to the first-order condition is a maximum. Compared with the case of imperfect competition in above subsection, $\delta = 0$ since speculators have no market power in competitive case(Alexander and Vives, 2014).[35] Furthermore,

[35]In the case of perfect competition, we assume that a demand schedule $D(\widetilde{s}_i, \widetilde{p})$ can be any convex-valued, upper-hemicontinuous correspondence mapping price $\widetilde{p}$ into non-empty subsets of the closed infinite interval $(-\infty, +\infty)$. This extension is discussed by Alexander and Vives(2014).

the market power can be calculated as

$$\delta_i = \frac{d\widetilde{p}}{dD(\widetilde{\epsilon}_i, \widetilde{p})} = \frac{1}{\sum_{j\neq i} \xi_j} \tag{B.18}$$

, in which (B.18) represents that speculator i's price impact δ_i perceived the slope of inverse supply facing other investors in imperfect competition financial market. However, in perfect competition, we get $\delta_i = 0$ since no speculator has influenced market power.

B.6.2 Extension of Level-k Reasoning Strategy in Kyle's Analogy

Firstly, using Zhou's(2022) structure design, our extension adds the entire level-k strategy to construct one with clear interpretation critically and captures the naive action in Kyle's modelled market structure. Note that in the case of anchoring in the action of $L0$ reasoning investors, we consider the naive action in the market as being ignorant of the information contained in prices compared with the standard $L0$ specification. Unlike the cognitive hierarchy level-k reasoning described in the above section, this assumption only exists in the mind of $L0$ reasoning speculators, in which the naive speculator thinks others are uninformed and naive. The market clears in equilibrium when

$$D^0(\widetilde{s}_i, \widetilde{p}) + (n-1)D_U(\widetilde{p}) + \widetilde{z} = 0 \tag{B.19}$$

, where $D_U(\widetilde{p})$ represents the demand strategy of an uninformed and naive hypothetical speculator. From (B.18), the price equation by $L0$ reasoning speculator can be denoted as

$$\mathcal{P}_0(\cdot) : \widetilde{p} = \frac{D^0(\widetilde{s}_i, \widetilde{p})}{(n-1)\xi_U} + \frac{\widetilde{z}}{(n-1)\xi_U} \tag{B.20}$$

where $\delta_0 = \frac{1}{(n-1)\xi_U}$ and $D^0(\widetilde{s}_i, \widetilde{p})$ is a linear function of $\widetilde{s}_i$ and $\widetilde{p}$. Therefore, the demand of uninformed investors, $D_U(\widetilde{p})$, is characterized as the following

$$D_U(\widetilde{p}) = \frac{\mathbb{E}(\widetilde{v}) - \widetilde{p}}{\delta_u + \gamma Var(\widetilde{v})} = -\frac{\widetilde{p}}{\delta_u + \gamma \rho_v} \tag{B.21}$$

, and the level-0 strategy is

$$D^0(\widetilde{s}_i, \widetilde{p}) = \frac{\mathbb{E}[\widetilde{v}|\widetilde{s}_i, \mathcal{P}_i^{-1}(\widetilde{p})] - \widetilde{p}}{\delta_0 + \gamma Var[\widetilde{v} - \widetilde{p}|\widetilde{s}_i, \mathcal{P}_i^{-1}(\widetilde{p})]} \tag{B.22}$$

Next, let $D^k(\widetilde{s}_i, \widetilde{p})$ denotes the optimal demand schedule with level-k reasoning and it must be consistent with the first-order condition. Thus, in this case, we get the Lk strategy as

$$D^k(\widetilde{s}_i, \widetilde{p}) = \beta_k \widetilde{s}_i - \xi_k \widetilde{p} = \frac{\mathbb{E}[\widetilde{v}|\widetilde{s}_i, \mathcal{P}_k^{-1}(\widetilde{p})] - \widetilde{p}}{\delta_k + \gamma Var[\widetilde{v} - \widetilde{p}|\widetilde{s}_i, \mathcal{P}_k^{-1}(\widetilde{p})]} \tag{B.23}$$

Following the cognitive hierarchy of the level-k reasoning, we also consider a Lk reasoning speculator i believes that all others reasoning is one round less, indexed as $L(k-1)$, with their idiosyncratic private signal, $\widetilde{s}_j$, and individual strategy, (β_{k-1}, ξ_{k-1}). Therefore, level-k reasoning speculators are sophisticated since they are relying on the information transmitted through asset price in the market clearing equilibrium, that is

$$\begin{aligned} & D^k(\widetilde{s}_i, \widetilde{p}) + \sum_{j\neq i}^{N-1} D^{k-1}(\widetilde{s}_j, \widetilde{p}) + \widetilde{z} = 0 \\ \Rightarrow & D^k(\widetilde{s}_i, \widetilde{p}) + \sum_{j\neq i}^{N-1} (\beta_{k-1}\widetilde{s}_j - \xi_{k-1}\widetilde{p}) + \widetilde{z} = 0 \end{aligned} \tag{B.24}$$

Solving for asset price $\widetilde{p}$ with level-k reasoning by (B.24), for $\xi_{k-1} \neq 0$, we get

$$\mathcal{P}_k(\cdot) : \widetilde{p} = \frac{D^k(\widetilde{s}_i, \widetilde{p})}{(N-1)\xi_{k-1}} + \underbrace{\frac{\beta_{k-1}}{\xi_{k-1}}\widetilde{s}_j + \frac{\widetilde{z}}{(N-1)\xi_0}}_{\text{residual supply}} \tag{B.25}$$

Finally, note that, the perceived price impact in level-k reasoning under imperfectly competitive equilibrium is

$$\delta_k = \frac{1}{(N-1)\xi_{k-1}} \tag{B.26}$$

Meanwhile, informed speculators make benefit from the uninformed agents in Kyle's model. Combining the level-k reasoning, speculation profits will be influenced by the market microcondition. Thus, the expected speculation profit for a level-k reasoning speculator can be derived as

$$\mathbb{E}(W_k) = \mathbb{E}[\widetilde{v} - \widetilde{p}(\beta_k\widetilde{s}_i - \xi_k\widetilde{p})] \tag{B.27}$$

8 Appendix of Chapter 4

C.1 Proof of Proposition 4.2

Proof: Assume that the insider decides to acquire information at $\tau \in \mathcal{T}$, and the gross expected profit of costly information acquisition is $\overline{J}(t, \cdot)$, shown as

$$\overline{J}(t, \cdot) = \mathbb{E}[J^i(t, \cdot)|\mathcal{F}_t^P] \tag{C.1}$$

Note that $\bar{J}(\tau,\cdot) - C \geq J^U(\tau,\cdot)$, where $J^U(\tau,\cdot)$ is the the insider's value function before acquiring information(when the insider is uninformed). Thus, we can consider the deviation of delay acquiring strategy: the insider WON'T acquire information at time $\tau \in \mathcal{T}$ in which he planned originally, do not trade in $[\tau, \tau+\Delta]$, and then acquire at $t = \tau + \Delta$ and follow the conjectured equilibrium trading strategy from that point forward. In this strategy of delay acquiring information, for this deviation, the expected profit is

$$\begin{aligned}
\overline{\Pi}_{d\tau} &\equiv e^{-r\Delta}\mathbb{E}_\tau[\bar{J}(\tau+\Delta,\cdot) - C] - J^U(\tau,\cdot) && \text{(C.2)}\\
&\geq e^{-r\Delta}\mathbb{E}_\tau[\bar{J}(\tau+\Delta,\cdot) - C] - (\bar{J}(\tau,\cdot) - C) && \text{(C.3)}\\
&= (1-e^{-r\Delta})C + \mathbb{E}_\tau[e^{-r\Delta}\bar{J}(\tau+\Delta,\cdot) - \bar{J}(\tau,\cdot)] && \text{(C.4)}\\
&= (1-e^{-r\Delta})C > 0
\end{aligned}$$

In the (C.2), we can get $\lim_{\Delta\to 0}\mathbb{E}_\tau[e^{-r\Delta}\bar{J}(\tau+\Delta,\cdot) - \bar{J}(\tau,\cdot)] = 0$, which is implied as the trade timing indifference in which we discussed in (4.8).

C.2 Further Auxiliary Results

In **Appendix C.1**, I first discuss the market maker's estimates in unobservable. That is, the insider cannot observe private information. In this setting, I apply a celebrated theorem in the filtering problem to obtain the Corollaries regarding the market maker's inference problem as follows:

Corollary 4.4: *Consider $\mu(t,\xi)$ is the estimation of the unnormalized density function with random variable V, where $V = \xi$, in which denote as the stochastic differential equation in (4.4).*[36] *Therefore, $\mu(t,\xi)$ satisfies that, the stochastic differential equation(Zakai equation) as following*

$$d\mu(t,\xi) = \frac{\theta(t,\xi)}{\sigma^2}\mu(t,\xi)dY(t), \ \ \mu(0,\xi) = f(\xi) \ \ \textit{if} \ \ t=0 \tag{C.5}$$

only has a unique solution

$$\mu(t,\xi) = f(\xi)exp\left[\frac{1}{\sigma^2}\left(\int_0^t \theta(s,\xi)dY(s) - \frac{1}{2}\int_t^\infty \theta^2(s,\xi)ds\right)\right] \tag{C.6}$$

[36] From Zakai(1969), the differential equation is a special representation of a class of stochastic differential equations(SDEs) known as *Zakai equation* in the nonlinear filtering theory, which simplifies the underlying nonlinear filtering problem by studying the unnormalized probability.

Therefore, the value estimation $J(t)$ by the Zakai filtering method is

$$J(t) \equiv \mathbb{E}[\xi|\mathcal{F}_1(t)] = \frac{\int_J \xi\mu(\xi,t)dVV}{\int_J \mu(\xi,t)dV} \tag{C.7}$$

, where satisfies $f(V) = dF(\xi)/d\xi, \forall\xi \in \mathbf{J}$ as the prior probability density function when $t = 0$.

Proof: *See Zakai(1969) and Baras(1991).*[37]

Using Ito's lemma, I can summarise the well-known results in the filtering theory, which is more familiar in the financial economics literature.

C.3 Proof of Proposition 4.4

If and only if the post-entry decision is better off, combine (4.24) and (4.25), we have

$$\begin{aligned}\overline{\Pi}_{d0} - \overline{\Pi}_0 &= \rho\left(\alpha_1(\textstyle\sum_0 + \lambda_0^2 \sum_z) + \gamma_1 - c\right) - \left(\alpha_0 \textstyle\sum_0 + \gamma_0 - c\right) \\ &= \underbrace{(\rho\alpha_1 - \alpha_0)}_{part1} \textstyle\sum_0 + \underbrace{\rho\alpha_1\lambda_0^2 \textstyle\sum_z}_{part2} + \underbrace{\rho\gamma_1 - \gamma_0}_{part3} + (1-\rho)c\end{aligned}$$

Hence, we can analyse the above four parts as follows:

Part 1: Firstly, substituting in from the difference equations for $\alpha_1 = \frac{1-\lambda_1\beta_1}{2\lambda_1}$ and $\lambda_1 = \frac{\beta_1 \sum_1}{\beta_1^2 \sum_1 + \sum_z}$, we have

$$\begin{aligned}\rho\alpha_1 - \alpha_0 &= \rho\frac{1-\lambda_1\beta_1}{2\lambda_1} - \frac{1-\lambda_0\beta_0}{2\lambda_0} \\ &= \rho\frac{1 - \frac{\beta_1^2 \sum_1}{\beta_1^2 \sum_1 + \sum_z}}{2\frac{\beta_1 \sum_1}{\beta_1^2 \sum_1 + \sum_z}} - \frac{1 - \frac{\beta_0^2 \sum_0}{\beta_0^2 \sum_0 + \sum_z}}{2\frac{\beta_0 \sum_0}{\beta_0^2 \sum_0 + \sum_z}}\end{aligned}$$

Then, simplifying this equation and substituting $\beta_1 \sum_1 = \rho\beta_0 \sum_0 \left(\frac{\sum_z^2}{\sum_z^2 - \beta_0^4 \sum_0^2}\right)$,

$$\rho\alpha_1 - \alpha_0 = \underbrace{\frac{\rho \sum_z}{2\beta_1 \sum_1}}_{\rho\alpha_1} - \underbrace{\frac{\sum_z}{2\beta_0 \sum_0}}_{\alpha_0} = \frac{\sum_z}{2\beta_0 \sum_0}\left(\frac{\sum_z^2 - \beta_0^4 \sum_0^2}{\sum_0^2} - 1\right)$$

[37] From Baras(1991), the intuition behind the likelihood ratio function (C.3) is as follows. Suppose that $H_0 = u_\delta(t) \equiv 0$

Finally, after simplifying and collecting the terms, we get

$$\rho\alpha_1 - \alpha_0 = -\frac{\beta_0^3 \sum_0}{2\sum_z} \tag{C.8}$$

Part 2: Recall $\rho\alpha_1 = \frac{\rho\sum_z}{2\beta_1\sum_1}$, we have

$$\rho\alpha_1\lambda_0^2 \sum_z = \frac{\rho\sum_z}{2\beta_1\sum_1}\lambda_0^2 \sum_z$$

then substitute the difference equation for $\beta_{n+1}\sum_{n+1} = \rho\beta_n\sum_n\left(\frac{\sum_z^2}{\sum_z^2 - \beta_n^4\sum_n^2}\right)$, we get

$$\rho\alpha_1\lambda_0^2 \sum_z = \frac{\sum_z^2 \lambda_0^2}{2\beta_0\sum_0\left(\frac{\sum_z^2}{\sum_z^2 - \beta_0^4\sum_0^2}\right)} = \frac{(\sum_z^2 - \beta_0^4\sum_0^2)\lambda_0^2}{2\beta_0\sum_0}$$

Then, substituting $\lambda_n = \frac{\beta_n\sum_n}{\beta_n^2\sum_n + \sum_z}$ and simplifying

$$\rho\alpha_1\lambda_0^2 \sum_z = \frac{(\sum_z^2 - \beta_0^4\sum_0^2)}{2\beta_0\sum_0}\frac{\beta_0\sum_0}{\beta_0^2\sum_0 + \sum_z}\lambda_0 \tag{C.9}$$

Part 3 Using the difference equation for $\rho\gamma_{n+1} = \gamma_n - \frac{1-2\lambda_n\beta_n}{2\lambda_n(1-\lambda_n\beta_n)}\lambda_n^2\sum_z$, we have

$$\rho\gamma_1 - \gamma_0 = -\frac{1-2\lambda_0\beta_0}{2\lambda_0(1-\lambda_0\beta_0)}\lambda_0^2\sum_z = -\frac{1-2\lambda_0\beta_0}{2(1-\lambda_0\beta_0)}\lambda_0\sum_z$$

Then, substituting the $\lambda_n = \frac{\beta_n\sum_n}{\beta_n^2\sum_n + \sum_z}$, we get

$$\rho\gamma_1 - \gamma_0 = -\frac{1 - 2\frac{\beta_0^2\sum_0}{\beta_0^2\sum_0 + \sum_z}}{2(1 - \frac{\beta_0^2\sum_0}{\beta_0^2\sum_0 + \sum_z})}\lambda_0\sum_z$$

Finally, simplifying this equation,

$$\rho\gamma_1 - \gamma_0 = -\frac{(\sum_z - \beta_0^2\sum_0)\lambda_0}{2} \tag{C.10}$$

To sum up, combining the (C.9) from **Part2** and (C.10) from **Part3**,

$$\rho\alpha_1\lambda_0^2\sum_z + \rho\gamma_1 - \gamma_0 = \frac{(\sum_z^2 - \beta_0^4\sum_0^2)}{2\beta_0\sum_0}\frac{\beta_0\sum_0}{\beta_0^2\sum_0 + \sum_z}\lambda_0 - \frac{(\sum_z - \beta_0^2\sum_0)\lambda_0}{2} = 0 \tag{C.11}$$

Therefore, we get the finial equation

$$\overline{\Pi}_{d0} - \overline{\Pi}_0 = -\frac{\beta_0^3\sum_0}{2\sum_z^2} + (1-\rho)c \tag{C.12}$$

If the time interval, $\Delta > 0$, is sufficiently small($\Delta \to 0$), we have

$$\lim_{\Delta \to 0} \frac{\overline{\Pi}_{d0} - \overline{\Pi}_0}{\Delta} = \lim_{\Delta \to 0} \frac{-\frac{(\beta_0^{\Delta})^3 \Sigma_0^2}{2\sigma_z^2 \Delta} + (1-\rho)c}{\Delta} > 0 \tag{C.13}$$

which establishes the result that insiders deviate from trading starts($t = 0$) always better off in profits.

C.4 Legal Practice Cases in SEC

Insider trading can be criminalized in the United States. According to SEC records, for decades, the SEC has brought illegal insider trading lawsuits against hundreds of parties. Initially, *the Insider Trading Sanctions Act*, enacted in 1984, only tripled the penalties for illegal profits or direct losses from insider trading. Subsequently, *the Insider Trading and Securities Fraud Enforcement Act* enacted in 1988 further expanded the SEC's powers and added essential elements such as strengthening cooperation with foreign financial regulators.

In 2002, the United States Congress enacted the *Public Company Accounting Reform and Investor Protection Act of 2002*(also called *the Sarbanes-Oxley Act of 2002*) to prevent recurrence of corporate scandals like those involving Enron, WorldCom, and Tyco. It further stipulates that anyone who gains benefits in the securities market through information fraud, price manipulation, or insider trading shall be charged a maximum fine of 5 million, up to 20 years of imprisonment, or both. Illegal registration accountants shall be imprisoned not more than ten years or fined.[38] At the same time, the prosecution period for securities fraud has been extended, and the prosecution period can be extended to within two years of the discovery of the illegal act or five years of the illegal activities being carried out.

There are two typical legal practice cases of insider trading reported by SEC:

- *SEC v. Shaohua (Michael) Yin, et al.*, Litigation Release No. 23747 / February 10, 2017

[38] There are some supplemental legal basis from *The Public Company Accounting Reform and Investor Protection Act of 2002, the Section 807(a), Criminal Penalties For Defrauding Shareholders Of Publicly Traded Companies*. See https://www.sox-online.com/key-sections/section-807-criminal-penalties-for-fraud/. Accessed September 1, 2021.

In an insider trading case announced by the SEC in February 2017, in the weeks leading up to the news of the acquisition, the perpetrator Shaohua (Michael) Yin used the brokerage accounts of five overseas relatives and friends that he controlled to buy a large number of DreamWorks stock(more than $56 million) before Comcast Corp announced the acquisition of DreamWorks Animation. After the disclosure of the acquisition in April, DreamWorks Animation's stock rose by 47.3%, and Shaohua (Michael) Yin made an illegal profit of US$29 million.[39]

As a result, the SEC's complaint charges that Shaohua (Michael) Yin violated anti-fraud provisions. Meanwhile, the SEC obtains a court order naming the holders of the five related brokerage accounts as relief defendants and freezing more than $29 million illegal profit in these five brokerage accounts.

- *SEC v Stanko J Grmovsek*, Litigation Release No. 21263 / October 27, 2009

Unlike common insider trading cases, which are temporary and accidental, in some cross-domestic trading cases, some actors use the convenience of providing professional services to engage in planned insider trading activities(Systematic Insider Trading).

In 2009, the SEC and the Ontario Securities Commission of Canada investigated and dealt with a cross-border transaction case. During the 14 years from 1994 to 2008, Canadian Cornblum took advantage of his work in many legal affairs companies to disclose price-sensitive information on 40 client mergers and acquisitions to his former classmate Grmovsek. Grmovsek relied on this inside information to use its shell companies in the Bahamas and the Cayman Islands and accounts in the names of relatives and friends to trade stocks, accumulating profits of more than 10 million U.S. dollars. The clues of the insider trading case were discovered by the Financial Industry Regulatory Agency (FINRA). FINRA's monitoring system identified that in some mergers and acquisitions, some large orders were exact in timing, and one of the expected points in these mergers and acquisitions was that The legal affairs company employed by Cornblum provides legal consulting services. The SEC's complaint alleges that Grobovsek violated specific legal provisions in *the Securities Exchange Act of 1934* that prohibit transactions in possession of material, non-public material information.[40]

[39] SEC charges Chinese citizens who reaped massive profits from insider trading on Comcast-Dreamworks acquisition. See https://www.sec.gov/news/pressrelease/2017-44.html. Accessed December 29, 2021.

[40] SEC Charges Canadian Citizen in $10 Million Insider Trading Scheme; See http://www.sec.gov/litigation/litreleases/2009/lr21263.htm.

As the epidemic has become increasingly severe, the American people are increasingly dissatisfied with the government's response to the COVID-19 outbreak. In March 2020, the U.S. stock market plummeted under the influence of the Covid-19 pneumonia epidemic, and the U.S. S&P index broke four times in less than two weeks (March 9-18, 2020). Stock markets volatility were creating more arbitrage opportunities of insider trading opportunities. On March 20, 2020, some U.S. media disclosed that some members of Congress had in advance put a large number of stocks or bought specific assets before the stock market crashed, based on inside information. This scandal triggered widespread criticism from all walks of life.

Accessed December 30 2021.