On Buddhist Logic

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Abstract

This thesis is the attempt to find a logical model for, and trace the history of, the *catuṣkoṭi* as it developed in the Indo-Tibetan milieu and spread, via China, to Japan. After an introduction to the history and key-concepts of Buddhist philosophy, I will finish the first chapter with some methodological considerations about the general viability of comparative philosophy. Chapter §2 is devoted to a logical analysis of the *catuṣkoṭi*. Several attempts to model this fascinating piece of Buddhist philosophy with the tools of classical logic shall be debunked. A paraconsistent alternative will be discussed but eventually dismissed. As a rejoinder, I shall propose a model for the *catuṣkoṭi* with the help of speech-acts. The remainder of this chapter will look at Chinese and Japanese forms of the *catuṣkoṭi* which I shall model in a quasi-recursive system. The third and final chapter will look at the Kyoto School's *soku-hi* dialectics which ties together the different threads of this essay. I will criticise an established, classical model of the *soku-hi* dialectics and offer an alternative with a second-order paraconsistent semantics.

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Content

Abbreviations

Buddhist Scriptures

EZ **Erdi Zhang**

Chan, W.T. (ed., tr.) (1963). *A Sourcebook in Chinese Philosophy*. Princeton University Press.

MMK **Mūlamadhyamakakārikā**

De Jong, J.W., Lindtner, C. (eds.). (2004). *Nāgārjuna's Mūlamadhyamakakārikā Prajnā Nāma*. Adyar Library.

Logical Vocabulary

B4	Four-Valued Boolean Algebra
B4Sx	Four-Valued Boolean Algebra with Status Predicated
EXP	The Principle of Explosion
FDE	First Degree Entailment
FDE(e)	Five-Valued First Degree Entailment
FDE(S)	First Degree Entailment with Status Predicated
K3	Kleene's Three Valued System
LEM	The Law of the Excluded Middle
LNC	The Law of Non-Contradiction
LP	The Logic of Paradox
SH	Soku-Hi Dialectic

§1 Buddhism and Logic

Buddhism and modern logic are intricate bedfellows. Logical tools have proven dramatically useful in excavating, making intelligible, and sometimes refuting the arguments of Buddhist philosophers. Buddhism is as little a stable monolith as Christianity, or Islam. What I talk about, when I talk about Buddhism is for the most part limited to a specific strand of Buddhism, which is Māhāyana. Hence, I will largely neglect discussing other forms of Buddhism, such as, for instance, Tiantai, Yogacara, and the whole early Abhidharma tradition. Of course, Buddhism is a religion, as a religion it is practice-based, it can be mystical and deeply enigmatic, it can be a spiritual journey. But Buddhism is also profoundly analytical, and, with respect to our twenty-first century understanding of what philosophy is, profoundly *philosophical*. Buddhism evolved and changed dramatically over the course of centuries, incorporated pre-existing beliefs and rejected others on its way from India to Japan, and beyond—a journey that started two and a half thousand years ago with the historical Buddha Gautama's sermon under the bodhi tree.

Logic, too, is hardly a stable monolith—in fact, that is one of Buddhism's fundamental doctrines, nothing is; everything is transitional. When I talk about logic, then, I talk about a tradition of formal logic kick-started by Frege and Russell. In particular, it is the tradition of non-classical logic which will be of principle interest to this project. Crudely, I aim to model some central logico-epistemic-metaphysical arguments and positions of Buddhist thinkers with the help of modern logic. Why is that an interesting and worthwhile project? What is the point and purpose of putting something into a formulaic, symbolic form—as we sometimes do with Aristotelian syllogisms?

Modern logic has the potential to render a somewhat opaque philosophy intelligible to us. Buddhist philosophy can be somewhat opaque. By 'us', I mean those who we are, for one, largely unfamiliar with the ways (classical) Buddhist philosophy is presented—in verse, aphorisms, and quasi-poetic form[1]. On the other side of the coin,

[1] Classical is not the perfect terminology here. Some Buddhologists (see, Westerhoff, 2018), use the term 'classical' to refer to the golden age of Indian Buddhist philosophy; from Abhidharma, to Yogacara, to Māhāyana up to the 6th century CE. I use 'classical' in a rather loose sense, referring to all Buddhist thought that is not a Western rendition. Contemporary Western authors thus don't count as classical. I am aware that this terminology creates unnecessary rifts (between what is Western and Non-Western, for instance). I will reflect on methodological issues of comparative philosophy below. Whether genuine comparison beyond chauvinism is at all possible, is a vexed

there is the chance to convince those critical of the assets of modern logic (or analytic philosophy, in general) that logical formalisms can be more than a mere matter of pointless symbol crunching. Those logical formalisms can help us make comprehensible those otherwise deeply enigmatic and profoundly puzzling concepts of Buddhist metaphysics, epistemology, even ethics; concepts such as *nirvana, enlightenment, nothingness*, and *compassion*. Logic, as I conceive of it here, is thus in full service to philosophical exegesis, analysis, and interpretation. Logic, as I conceive of it, is a tool to help us understand reality as the Buddhists saw it. This may sound odd, but I take logic to penetrate the phenomenology of Buddhism. I shall unpack this claim in the next section.

In recent years, Graham Priest has pioneered this way of engaging with Buddhist philosophy. I follow in his footsteps. Many of the things presented and criticised in this essay are responses to his 2010 paper 'The Logic of the *Catuṣkoṭi*' and his 2018 book-length treatment of Buddhist philosophy 'The Fifth Corner of Four'. Those publications mark a transition point in the literature on Buddhist logic. I take them to demarcate a methodological shift from pre-paraconsistent to paraconsistent engagement. Given that I shall criticise paraconsistent models, some aspects of this essay may already be considered a shift to post-paraconsistent engagement with Buddhist logic. This is, by no means, to claim that the days of paraconsistent Buddhist logic are over, before they have really begun. It is not to claim that everything that can be said about Buddhist logic in a paraconsistent vein has been said already—plenty material, indeed whole traditions of Buddhist thought, remain untapped. The shift to post-paraconsistency that I partly want to initiate here is rather supposed to plant a seed of scepticism in what otherwise looks like a perfect, happy marriage. As *prima facie* compelling as the paraconsistent engagement with Buddhist logic may be, as much as an obvious case for an application of paraconsistent logic it unarguably is, it also has its limits. I want to probe those limits, propose alternatives where I see fit. This is not *always*—sometimes a paraconsistent arrangement seems far more compelling—but *sometimes*. This boils down to that: *Just because Buddhist logic and paraconsistency seem like the perfect couple, like the Harry and Sally of comparative philosophy, it is naïve to imagine that they are immune to the occasional spat.* The occasional spat may or may not lead to divorce, depending on its

question. Since ordinary language works in a binary way, more often than not (as in 'classical' and 'non-classical'), it would be naïve to claim immunity to each and every pitfall of unwarranted dichotomy. But that is *no* excuse *not* to make an effort.

intensity. It is not my intention to discredit the paraconsistent position. On the contrary, I will defend it here and there. But I wish to shine a light on those occasional spats, as a means and matter of disenchantment.

The pre-paraconsistent corpus[2] of scholarly work on Buddhist logic is scattered, sometimes crude, but extensive. As discussed in chapter 2, pre-paraconsistent scholars of Buddhist logic have rarely been asking the right questions, or, in any case, they haven't been clear enough about the point and purpose of their engagement with Buddhist logic. Much of the pre-paraconsistent engagement with Buddhist logic looks like the clumsy attempt to understand something alien by forcefully pressing it into a familiar, Aristotelian cake mould. I am not saying that most pre-paraconsistent authors had mischievous orientalist intentions. It is simply apparent that a lot of the early (Western, academic) literature on Buddhist logic is the artefact of ebbing imperialist heydays[3].

The paraconsistent corpus of literature on Buddhist logic is rather small. A handful of papers, collected volumes, and, as of now, only one book length discussions exhaust the debate. Therefore, critical and productive engagement with the paraconsistent position is to pick low-hanging fruit. This essay is not supposed to pick all the fruit. Some are too high; others are not worth picking. We are only the start of a debate in which the modern tools of logic intermingle with classical Buddhist philosophy. We will soon have to reach for higher hanging fruit[4].

The centrepiece of the discussion, as it is in Priest's work, is the Buddhist concept of 'negation', broadly conceived. I will start with 'negation' in Madhyamaka philosophy, this is, with the work of Nāgārjuna and various commentators, and trace its development through China to Japan, where, in the form of the *soku-hi* negation of the Kyoto School, my discussion will culminate.

Very crudely, negating something but not *really* negating it is a key aspect of Buddhist logic. In fact, it is reckoned to be the way towards, and the point of Enlightenment—means are the ends, in that case. To those unfamiliar with non-classical, many-valued logic, this might sound deeply confusing already. To those familiar with paraconsistent logic, it will sound like a paragon case. From a bird's eye perspective, this dissertation seeks to model the way towards Enlightenment. It is my impression that, by

[2] By that I don't mean each and every non-paraconsistent approach. By pre-paraconsistent I mean the times roughly before paraconsistent ideas started circulating in the literature. For examples, see footnote 4.

[3] See, for instance, a seminal paper by De Vallé Poussin (1917), or Raju (1954) and Robinson (1957).

[4] The fruit of chapter §3 may already be one of those higher hanging fruits.

coming closer and closer to modern Japanese Zen, which is historically one of the latest developments of Buddhist thought and therefore ostensibly more familiar to us, today in the "West", the concept of Enlightenment becomes increasingly intelligible. This doesn't come as a surprise. When Buddhism entered Japan, it came there with roughly one and a half thousand years of philosophical baggage. This is not to say that Zen Buddhists are in any way smarter, better philosophers, or have some kind of pristine relation to the truths of their own religion. They simply had plenty of time and an abundance of philosophical resources to draw from to develop the concepts they had inherited from India to their most rigorous form.

This thesis will proceed as follows: The second section of the first chapter will be a very brief historical overview of the developments of Buddhist philosophy (limited to only those facets relevant to the subsequent discussion, deliberately neglecting vast areas of the Buddhist universe). I will then look at some key-themes of Buddhist philosophy. These are, for the most part, logico-epistemic-metaphysical constructs. Tearing the subdisciplines apart, interpreting a concept such as 'nothingness' as purely metaphysical, or purely epistemic, is to misunderstand Buddhist philosophy, or so I will argue. It is precisely to see Buddhist philosophy through the lens of todays fragmented, highly specialised, disciplinary divide within academic philosophy. There is no vindication for tearing apart disciplines in Buddhist philosophy, in which all things logical, ethical, epistemic, phenomenological, physical and metaphysical, spiritual, mystical, political and jurisdictional merged into one. I will close the first section with some methodological considerations.

Chapter two is devoted to the *catuṣkoṭi*, ultimately a view on negation and a soteriological[5] instrument to help practitioners on their way towards enlightenment. At its simplest, the *catuṣkoṭi* is the view that a given proposition can simultaneously be true, false, both true and false, and neither true nor false. This makes little sense in classical Frege-Russel logical framework. As Priest (ibid.) suggests, however, it makes perfectly good sense in a non-classical dialetheic paraconsitent logic. Dialetheism is a view about truth. It says that there are true contradictions—this is, statements of the form '$A \& \neg A$'––which, in a paraconsistent logic, do not lead to triviality via the closure of logical entailment, otherwise known as the principle of *ex contradiction quodlibet*, or simply *Explosion*[6]. I will look at classical interpretations of the *catuṣkoṭi* and arguments against

[5] I want to point at the 'logical' in soteriological here, for in chapter §2.3 it will be of relevance.

[6] For an overview, see Priest & Berto (2013).

them. I will then scrutinize dialetheic paraconsitent interpretations of the *catuṣkoṭi*, as well as arguments against them from classicists who lament the paraconsitent logician's inability to make sense of cases where classical principles like *modus ponens* are valid. One cannot just add principles like *modus ponens* as assumptions, because in the background paraconsistent logic that does not rule out their negations. Here is an example: Think of A as both true and false, and of B as strictly false. If that is the case, 'A' and 'A then B' are both true, hence, according to *modus ponens*, B must also be true, but we just said that B is strictly false. Hence, *modus ponens* fails in a paraconsistent logic. This is a problem sometimes referred to as the *Recapture Problem*. I will debunk an attempt to solve the Recapture Problem by returning to a classical interpretation, question to what extent it has been justified to speak of a Recapture Problem in the first place, and propose an alternative way of formalising the *catuṣkoṭi* with the help of (a slightly regimented) speech-act theory, leaving behind paraconsistent territory. Throughout this chapter, we will encounter several different forms of the *catuṣkoṭi*; a negative and positive formulation, one extended by a fifth, *empty*, truth value, *catuṣkoṭi* of a higher order, and a *catuṣkoṭi* that leads into recursion.

Section three is a discussion of the *soku-hi* dialectics which the Kyoto School adopted from Zen Buddhism, and ultimately, as I argue, from the *catuṣkoṭi*. In fact, the *catuṣkoṭi* is so integral to Buddhist thought that we can find it inherent in many concepts which *prima facie* don't look like they have something in common with it. By the end of chapter two, I will have shown that the Zen Koan shares a logical structure with the *catuṣkoṭi*. The *soku-hi* dialectics also shares this logical structure. I will present and criticise an augmented classical proposal for making sense of the *soku-hi* dialectics and alternatively propose a second-order paraconsistent semantics as an alternative rendition–––which is a relatively untouched territory.

We will see the *catuṣkoṭi* morphing into different forms in different historical contexts. Stylistically, the various forms the *catuṣkoṭi* are as diverse as the historical contexts from which they originated; from verse, to sermon, to quasi-poetic forms, to Heideggerian unintelligibility. What I will do, is to put them all into logical form. Logical form has the asset of great mathematical "neutrality". Of course, its neutrality can be disputed. First, the system in which formalization takes place already interferes with the "neutrality" of the formalization. Second, the formalization is already an interpretation, and therefore the interpretation of the formalizer. However, the logical form will make comparisons between distinct forms of presentation of philosophical arguments—be it in

verse, poems, metaphors or sermons—easier. That's what I mean when I talk about '"neutrality". From the position of "neutrality", differences and similarities at once become overt where erstwhile they have been concealed.

§1.1 From India to Japan

The overall argument of this dissertation is constructed such that it is necessary to make great temporal and geographical leaps. I think I should therefore begin with a brief history and geography of Buddhist philosophy. It will help the reader to, first, connect the discussion with the broader historic contexts; and second, to position those finer points within the overall argument.

Buddhism developed in India and then spread eastwards for about two-and-a-half-thousand-years. In the eighteenth-century, Buddhism also hit "the West". But it arguably took two hundred years, some would say until the 1960s, for Buddhism to gain a wider, non-academic, non-artistic recognition and momentum as a religion in its own right. As it spread eastwards, Buddhist thought hesitantly got rid of some of the more gnostic elements it adopted largely from Hinduism. The notion of rebirth is one of those. While it is still present in some form or another in most of the latest developments of Buddhist thought, its importance vanished as people became increasingly skeptical about its metaphysical truth and importance.[7] It is Priest's (2018:3) prognosis that, as Buddhism will continue to spread "westwards", the notion of rebirth will be one of its causalities. The patriarchal hierarchies within (some) Buddhist institutions will, hopefully, be one of the other casualties. In some sects of Japanese Zen and Shingon, patriarchal structures have already been abandoned.[8]

Written between the eighth and sixth century BCE, the *Upanishads* are the central texts of Hinduism, defending a metaphysics which centers around a godhead or universal soul, *brahman*, which gave birth to the myriad of things, the *atman*, each of which is part of *brahman*. Several schools of thought were founded on a rejection of *Upanishad* metaphysics of which today only Jainism and Buddhism have survived. Buddhism thus begun its existence through Siddhārtha Gautama's (the historical Buddha; ca. fifth to fourth century BCE, exact dates are unknown) forceful philosophical critique of the *Upanishad* programme[9]. Nonetheless, Buddhism doesn't reject all *Upanishad* metaphysics.[10] As mentioned above, it largely retained the notion of rebirth—up until a

[7] See, for instance, the debate on the naturalization of rebirth and karma. Lin & Yen (2015) provide a good overview.
[8] See Dumoulin (1993).
[9] See Kuznetsova, Ganeri & Chakravarthi (2012).
[10] The exact dates, here, are uncertain, but somewhere between 563 and 483 BCE seems most likely, see Mitchel (2002).

certain point, at least—and most importantly the ontological distinction of reality into the realm of enlightenment (*nirvānā*) and the realm of conventionality (*samsārā*). Very crudely, the Buddha's sermon under the bodhi tree comes down to saying that the realm of conventional is full of suffering (*dukkha*) because of its impermanence (*anitya*), but that something can be done to overcome it, if only one follows the precepts of the *Four Nobel Truth*:

> (1) Suffering, unsatisfaction and pain are inescapable characteristics of conventional reality and life in the cycle of rebirth.
> (2) Suffering is caused by desire and attachment.
> (3) The end of suffering can be attained by eliminating all of one's desire and attachment.
> (4) The *Eightfold Path* is the *how-to* manual for (3).

The *Eightfold Path* is intricate. Very crudely, it says that to end suffering (i.e. by eliminating one's desires, attachments and cravings), one ought to have:

> (1) the right view: This is, that the Buddha's insights into karma and reality are true, that there are two levels of reality, convectional and ultimate, and that in ultimate reality there is the cessation of desire.
> (2) the right intention: This is, to give up all material belongings, sensual inputs, and harmful environments. In other words, to follow a monastic life.
> (3) the right speech: This is, not to lie, be rude, or otherwise harmful with one's words.
> (4) the right actions: This is, not to harm others by physical violence or misconduct.
> (5) the right livelihood: This is, not to (indirectly) profit from harmful business.
> (6) the right effort: This is, to restrain from numbing one's senses, or do it "the easy way"
> (7) the right mindfulness: This is, to be mindful of the teachings (the *dharmas*), to never by absent-minded, to stay mentally focused on the present.
> (8) the right meditation (*samadhi*). This is, to practice the four stages of meditation (*dhyana*)[11]

[11] Which, we will see later, have a logic structure.

A written corpus of Buddhist teachings (the *Pali canon*, or *Tripitakan*) emerged roughly 600-700 years after the teachings of the historic Buddha, comprising the *Sutras*, these are teachings of the historic Buddha; the *Abhidharma*, which is the philosophical attempt to provide a quasi-mereological taxonomy for reality and its parts; and the *Vinaya*, which is concerned with the rules for monastic life. Again, about 200 years later, a new and hitherto unknown set of *Sutras* appeared and gave rise to new philosophical developments in the Indian Buddhist milieu, as well as the development of a new "kind" of Buddhism, which is *Mahāyāna*. Within the Mahāyāna context, two schools emerged; Madhyamaka, which may be classified as a metaphysical realist school, and *Yogācāra,* which is more of a transcendental idealist school—as far as those labels are applicable (see the methodological coda below)[12]. This essay will mostly be concerned with Madhyamaka sources.

Of those two, Madhyamaka philosophy is the one that traveled far and wide and had a bigger impact on the subsequent developments of Buddhist thought. While other strands of Buddhism, most notably the *Theravada* tradition moved into south-east Asia (and was more or less wiped out by the Moslem invasion to India in the first century BC), Māhāyana went into Tibet, China, Korea and Japan–that's where this essay will follow it.

The silk-route not only brought an exchange of commodities and plagues between China, India and Europa, but also the exchange of religious ideas. When Māhāyana travelled eastwards, it encountered China's distinct philosophies; Confucianism and Daoism. While it found itself to be largely compatible with Confucianism (by means of split-labour; Confucianism is for matters socio-political, Buddhism for spiritual guidance), it found itself an intricate bedfellow with Daoism. In fact, Daoism and Buddhism merged in China. While still distinct, the former was subsumed by the latter. This is probably why, today, we don't speak about Zen Daoism, but Zen Buddhism. What Daoism did to Buddhist philosophy, is to emphasize one of its already string element; its temporality[13].

The Dao itself is a temporal principle. It is the principle of flow, in accordance with which the Daoist sage seeks to live. For Buddhists, this must have been a reminder of the early concept's impermanence. Those philosophical date-nights happened between around 300 to 400 CE in today's central China, from which the *Sānlùn* school and the

[12] See Westerhoff (2018), chapter 2&3.
[13] See Hansen (2003).

Weixin school emerged, depending on weather Daoism was coupled with Madhyamaka or Yogācāra[14].

While *Weixie* never left China, the Chinese version of Madhyamaka, *Sānlùn*, influenced four new schools of distinctively Chinese Buddhism, of which perhaps only one rings a bell in the ear of most people unfamiliar with the history and kinds of Buddhism, which is *Chan*, or *Zen*, as they would say in Japan.

While the other three schools of Chinese Buddhists are philosophically intricate in their own right (*Huayan* and *Tientai*, especially)[15], and sharp boundaries between them can, at least in the formative period—this is, before Zen went to Japan—hardly be drawn, I still wish to trace only the development of Zen.

The distinctive feature of Zen Buddhism is its ecumenical view on Enlightenment. Whereas Indian Buddhism saw the road to Enlightenment as a long and strenuous one (extended over many, many cycles of rebirth), Zen Buddhists think that Enlightenment can happen spontaneously (but has to be fostered to be kept alive). If you will, Zen demystified Buddhism, brought it into contact with and service to everyday life, much more so than the secluded sects in India did. Some may say Zen's emphasis is practical, rather than theoretical, but I am not sure if this is the right way to put it. When in the thirteenth century Dogen Kiegen and Myoan Eizei imported Chan from China to Japan, they came with all the theoretical baggage of nearly two-and-a-half-thousand years of philosophical discussion[16]. When we say Zen is less theoretical than practical, we have to be aware of all the theoretical work that had been done up to the point when Zen emerged, and people were eager to try it out Enlightenment for themselves. This is why Zen got rid of a lot of, from a soteriological point of view, perhaps unnecessary theoretical baggage. This theoretical lightness is reflected in Zen architecture and shouldn't be mistaken with a hostility to theory. If the point is to reach nirvana (which most Zen Buddhists will tell you, is not the point, because it is the point) why continue, after centuries of theoretical discussion, with constantly overthinking it? Why not just try it, instead?

[14] See Garfield & Westerhoff (2015).
[15] For a good overview, see Ch'en (1964)
[16] See Dumoulin (1963).

Overview

Pre-Buddhist	**India**		**China**	**Japan**
(Before 6th century BCE)	(After 4th Century BCE)	(Around the 6th Century CE)	(After 12th Century CE)	

Upanishads --> *Pali Canon*

Abhidharma --> *Māhāyana* *Daoism*

Madhyamaka --> *Sānlùn*

Chan --> Zen --> Rinzai

Soto

As you can see from the overview, there is a break between *Sānlùn* and Chan; there is no direct line running from Madhyamaka to Zen. Part of my argument, to be discussed in §2.3, is to establish that there actually is a chain which started from the Māhāyana rejection of Abhidharma philosophy that does not, as often assumed, break somewhere between *Sānlùn* and Chan. Hence, Zen is not, as sometimes argued, a secluded, monolithic development, detached from the broader Buddhist lineage[17]. The connection between Madhyamaka and Zen is stronger than it is commonly acknowledged. From a quasi-Buddhological perspective, to support this claim is the principle aim of this essay. The essay is casted in terms of an eastwards journey. We begin the journey on the Indian subcontinent, travel over mainland China and to the Japanese archipelago.

[17] See Dumoulin (1963) an overview of the history of Zen.

§1.2 Logico-Epistemic Phenomeno-Metaphysics

I have mentioned above that within the Buddhist philosophical cosmos, sharp disciplinary lines cannot be drawn. Especially with respect to a boundary between logic and epistemology, early Buddhist philosophical texts are rather unspecific. This is why Western Buddhologists speak of a logico-epistemic philosophy in the context of early Buddhist thought[18]: On the one hand, there is the *pramāna-vāda* (Eng. doctrine of proof), the study of the nature of knowledge. On the other hand, there is the *hetu-vidya* (Eng. study of causality), a quasi-logical system of debate-rules. The boundaries between *pramāna-vāda* and *hetu-vidya* are fluid. Matilal (1998:184) elucidates this point, saying that *"[l]ogic as the study of the form of correct arguments and inference patterns, developed in India from the methodology of philosophical debate."* Matilal and Ganeri (1998) infer from this practice-guiding, normative nature of 'logic *qua* debating-rules' that "Western logic" and "Indian logic" are fundamentally distinct; the former *mathematical*, the latter *grammatical*. Whereas epistemological matters are included in the grammatical logic, say Matilal and Ganeri, they are missing, or else—if you are of a positivist inclination—purposely eliminated from mathematical logic. I have my doubts about the validity of this distinction but discussing it would exceed the scope of this essay. Where I am *d'accord* with Matilal and Ganeri, I think, is with respect to merely formal differences. Of course, two-and-a-half-thousand-year-old Buddhist logic is not presented in a way as formally rigorous as modern mathematical logic.

For the non-Buddhist *Nyāya* school of thought (between 6th century BCE and 2nd century BCE), epistemology and rules for argumentation converge in a sophisticated system of sound inference. Knowledge, according to the *Nyāya* school, is gained by a four-step inferential process connecting a thesis (*pramana*), a reason (*pratyakṣa*), a universal example (*anumāna*), and a successful cognitive comparative transfer from example to thesis (*upamāna*). Only reasoning that can follow those four steps is sound and confirms the thesis, i.e. warrants an inference from the premises to the conclusion[19]. Here is an example: The thesis, 'this water is hot', counts as knowledge, if and only if there is a reason for it, such as ‘there is steam’, an universal example-case, such as ‘wherever there is steam, there is hot water [such as in a steamy hot *onsen*]’, and a

[18] See, Matilal & Ganeri (1998).

[19] See, ibid.

successful cognitive comparative transfer between the steam of the onsen and the steam above the water, hence the thesis, can be made.

The first Buddhist thinker to become aware that within the logico-epistemic philosophy of the Hindu *Nyāya Sutra* we can distinguish between matters of knowledge and matter of reasoning was *Vasubandhu* (between 4th and 5th century BCE)[20]. For him, logical inference is the centerpiece of knowledge. Only if a correct logical inference (a *vyapti*) to a similar and a dissimilar universal example-case can be made, the case, or the premise in question will count as a valid source of knowledge (*pramana*).

So far, we have been talking about logico-epistemic constructs. Enter Nāgārjuna and his Madhyamaka school (ca. 150-250 CE), the erstwhile logico-epistemology of the early Buddhists is extended by metaphysical and phenomenological doctrines, making those disciplinary boundaries even blurrier. Because a lot in this essay will revolve around Nāgārjuna's thinking, it will pay off discussing the key themes of his *logico-epistemic-phenomeno-metaphysical* philosophy.

On the metaphysical side of things, the central concept of Nāgārjuna's and the *Madhyamaka* school's thought is that of emptiness (*śūnyatā*) and of a bicameral reality, split into conventional reality (*samvrtisatya*) and ultimate reality (*paramarthasatya*). Reality is bicameral in a metaphysically deep, perspectival sense. When reality is perceived as reality *really* is (without conceptualising it, the way we usually do), ultimate reality is perceived. When reality is not perceived as it really is, but in the way we conceptualise and superimpose our concepts on it, conventional reality is perceived. Conventional reality is the reality of everyday appearances, reality as it is with the impositions on what is *in re,* a Kantian might say. What ultimate reality is in a metaphysical sense will have to remain a covert, at least until §3, for it would lead us too far at this point. Nāgārjuna, in the *Mūlamadhyamakakārikā* 24:8-10 (henceforth, MMK), put it thus:

"The Buddha's teaching of the Dharma is based on two truths: A truth of worldly convention and an ultimate truth. Those who do not understand the distinction between these two truths do not understand the Buddha's profound truth. Without a foundation in conventional truth the significance of the ultimate cannot be taught. Without understanding the significance of the ultimate liberation cannot be achieved."

[20] See Westerhoff (2018:154-160).

If conventional reality is the opposite of ultimate reality, and yet both are like two sides of one and the same coin, does that mean that reality itself is a giant contradiction? That too shall remain a largely unanswered question until our discussion of Zen Buddhism. With respect to the contradictory nature of reality, Priest makes an insightful *a priori* point, saying that just because conventional and ultimate reality are "opposites", or perspectivally different with respect to the amount of cognitive conceptualization at play, reality doesn't need to be necessarily inherently contradictory. *"We might interpret these aspects as dispositions to be perceived in certain ways",* says Priest (2018:59). The disposition to normally be conceived as X, and the disposition to normally be conceived as ¬X are not contradictories. Hence, the Madhyamaka Buddhist has to say more to establish the inherent contradictory nature of reality.

These concepts—emptiness and two-fold *reality*—describe ontological facets, but they are also phenomenological and epistemic to the extent that they are, in all but a few Zen sects, derived at by matters of deliberate meditational insight[21]. What is the content of this concept?

Crudely, the doctrine of emptiness is a metaphysical realist claim saying that all things, not just the objects or our experience, lack inherent *substance*, or *quiddity*, or *self-being*. The Sanskrit term for substance, or quiddity, or self-being is *svahbava*. But how to properly translate the term into a Western philosophical terminology is a vexed question[22]. I take 'self-being' to be the least philosophically loaded term of those three alternatives, and so I'll stick with it. The notion of emptiness is the centerpiece of the *Prajñāpāramitā Sūtras.* The central point of their teachings is that everything is void of self-being or essence. The *Prajñāpāramitā Sūtras* are a philosophical reaction to *Abhidharma* metaphysics[23], but of uncertain origin. Knowing what the *Prajñāpāramitā Sūtras* reject, i.e. knowing what their negative views are, will make it easier to understand their positive views, such as the notion of emptiness.

The Abhidharma tradition defends a mereological claim that we, today, would call *atomism,* or maybe *simpleism*; the idea that every complex mereological whole is made from some set of smaller (unchanging and indivisible) atoms or mereological simples, the *dharmas*. The *Prajñāpāramitā Sūtras* urge us to get rid of our belief in *dharmas* for they

[21] They are also intimately linked to ethics, as from insight into the nature of emptiness, compassion is supposed to follow, but discussing it here would lead too far.
[22] See Westerhoff (2009:19-64).
[23] For a thorough Buddhological discussion, see Garfield (2016).

are philosophically untenable. Why is an atomistic mereology untenable in the eyes of an Abhidharma philosopher?

In the *Heart Sutra* (*Prajñāpāramitāhṛdaya*) we find one of the perhaps most elaborate discussions of the notion of emptiness within the whole corpus of Buddhist literature. One of the central, rather cryptical remarks is the following:

"Being is emptiness, emptiness is being. Emptiness is not separate from being; being is not separate from emptiness. Whatever is being is emptiness, whatever is emptiness is being."

The *Heart Sutra* uncovers 'emptiness' as not only a metaphysical state, but also, and maybe even first and foremost, a *Bodhisattva's* state-of-mind (i.e. the state-of-mind of the enlightened person who, guided by compassion, deliberately remains in *samsara,* instead of entering *nirvana,* until the final end of suffering (*dhukka*) for everyone). The *Prajñāpāramitā Sūtras* portrait the enlightened state of mind as coming about by and through insight into ultimate reality. Different schools and sects of Buddhism have different ideas about what it would take to gain insight into ultimate reality. We will encounter three distinct stories about how it is that a person experiences enlightenment, and we will encounter a way of modeling, with the tools of modern logic, this (meditational) process.

Let us return to Abhidharma mereology. Why is an atomistic mereology untenable, in the eyes of an Abhidharma philosopher? Nāgārjuna, an anti-Abhidharmican himself, drew heavily on the *Heart Sutra* for his most prominent work, the MMK. The source and reason for the universal lack of self-being is *dependent origination* (*pratītya-samutpāda*), reckons Nāgārjuna. In the MMK (24:18), he says; *"We state that conditioned origination is emptiness. It is mere designation depending on something, and it is the middle path"*. The idea of dependent origination is, first and foremost, a metaphysical thesis and the answer to why the atomistic mereology of the Abhidharma tradition is untenable: First, nothing has an intrinsic *dharma* (or a set of *dharmas*) for everything *is what it is,* ontologically speaking, in virtue of bearing a set of relations to something else, and eventually, by the transitivity of those relations, to everything else. And second, because the first point alone is hardly convicting, there are no *dharmas* for everything is in constant flux and change and nothing can be unchanging, not even a mereological simple.

From the concept of dependent origination, Madhyamaka philosophers infer a general point on the nature of reality: Because everything is ontologically determined by its relations to other things, reality itself is groundless. The ontological thesis of 'emptiness' thus entails a non-foundational metaphysics. In contemporary terms, the doctrine of 'emptiness' is often compared to structuralism in the philosophy of science literature[24]. Groundlessness, or non-foundationalism is the nature of ultimate reality. The underlying thought (or wishful thinking, one might say) is that by gaining insight into ultimate reality, one gains insight into the metaphysical truth that is bound to it, i.e. one gains insight into the groundlessness of reality and the universal lack of self-being within it, and thus becomes enlightened. The enlightened modus is then supposed to have ethical and maybe even socio-political ramifications—non-violence and compassion are supposed to derive from insight into the metaphysics of reality. In other words, the metaphysical insight is supposed to carry over into ethical conduct. Again, emptiness reveals itself as an epistemic *cum* metaphysical *cum* phenomenological construct[25].

Priest (2018:117) models the ontology of emptiness with a simple graphical model. Suppose that something A is constituted by its relations to X and Y. Priest would depict the situation thus:

$$X <-_a- A -_b-> Y$$

The labels on the arrows indicate those constitutive, directed relations. Since A is empty, ontologically speaking, Priest exchanges the label with some placeholder symbol 'Ø':

$$X <-_a- Ø -_b-> Y$$

Since X and Y are empty, i.e. the mere loci of a network of relations as well, they too have to be replaced by a placeholder symbol. X and Y are empty because they too are the mere loci of a set of relations to other things which are themselves the mere loci of yet another set of relations. What remains, then, is only a web of relations—only the relations remain discernible, all content disappears, and what is left is the mere ontological

[24] See, Ladyman et al. (2007), or Bliss & Priest (2018).

[25] Again, it too has ethical, aesthetic, perhaps even politics facets, but these will not be of primary interest here.

structure of A. Accprding to the doctrine of depended origination, what it is to by A, is the the locus of an ontological structure.

$$Ø <-_{c}- Ø <-_{a}- Ø -_{b}-> Ø-_{d}-> Ø$$

This has been a very crude and schematic introduction to the key-concepts of Buddhist *logico-epistemical phenomeno-metaphysics*; emptiness, the bicameral reality, and self-being (or rather the lack thereof), and I hope that those concepts will become clearer as we proceed through the subsequent chapters.[26] In chapter §2, we will discuss the argument's structure with and through which insight into ultimate reality and hence emptiness may be gained, the *catuṣkoṭi*. The argument, here, is more than a mere matter of following premises to their conclusions and expecting to thereby become enlightened. As we will discover, the *catuṣkoṭi* also depicts the phenomenological experience of the Bodhisattvas' journey towards (and through) Enlightenment. The *catuṣkoṭi* itself, or so I will argue, is one of those logico-epistemic-phenomeno-metaphysical concepts of Buddhist philosophy.

[26] Thus, when I will speak about a metaphysical concept, this is not to say it is not *also* an epistemic view—it is always equally epistemic, logical and phenomenological.

§1.3 Methodological Considerations

The methodology of this essay itself deserves some philosophical attention. Some might call the approach comparative philosophy, applying the tools of modern analytic philosophy—first and foremost, modern logic—to philosophical texts of a different socio-cultural and historical context, in this case Buddhist philosophy. But what I am trying to establish here, as do most of the authors I am quoting, is not really comparative philosophy as it is usually conceived, for it lacks something vital to comparative or cross-cultural analysis, namely a dialogical form. I do not intend to compare the logico-epistemic content of Buddhist scripture to that of a modern logic textbook, neither do I wish to bring them into dialogue—that would be an absurd task. Rather, what I wish to do is think about Buddhist philosophers and their words as if the they had available the logical and analytical tools that are available to us today. To what degree, then, is what I am doing here, and what other modern authors, most notably Graham Priest, have done, correctly characterised as comparative philosophy?

The philosophical sub-discipline (if it is one) of comparative philosophy is a relatively recent development. Only after Hegel's flippant engagement with Chinese philosophy, says Jonathan Spence (1998:71), can we speak about comparative philosophy in the West. What, of past instances of comparative philosophy, counts as a genuine act of comparative analysis has to be evaluated anew, however, and in the light of a robust definition. Just because some piece claims to be comparative, that doesn't mean it is. According to Spence (ibid), contemporary comparative philosophy is characterised by a 'serious engagement' (whatever this is) with a socio-cultural tradition alien to one's own. This way of thinking about comparative analysis has a downside: It assumes epistemic borders between traditions of thought. As a comparative *dia-logue*, it describes not only two parties talking with, about, past, or alongside each other, but also two parties that remain isolated and distinct. Comparative philosophy thereby has to presuppose some shared element, or some shared, neutral perspective from which comparative analysis becomes possible in the first place. Comparative philosophy is thus an enigma in itself: On the one hand, it presupposes unbridgeable rifts between traditions; on the other hand, it presupposes a sufficiently robust overlap between those traditions. In other words, if Eastern and Western philosophy is supposed to be "compared", there has to be some standard along which they can be compared. They have to share certain characteristics which make a comparison possible in the first place. I can compare oranges and lemons

because they are both citrus fruits. I can compare Eastern and Western philosophy, perhaps because they are both "philosophical". However, what makes the philosophical case problematic is that the possibility of comparison also presumes that there are sufficiently apparent differences between East and West. That goes against attempts to establish a continuity and unity.

Comparative Philosophy is a largely Western invention. Eastern philosophical traditions[27], on the other hand, tended to "absorb" different philosophical traditions. The historic development of Easter philosophy is a process of mingling and clashing of distinct philosophical traditions, more so than it happened among Western philosophical traditions. As Māhāyana Buddhism spread through China, for instance, it incorporated a lot of Daoist ideas, rejected some and adopted others. Zen Buddhism, where distinctive Daoist elements are still palpable today, is a product of this expansive development. As Zen entered Korea and Japan it again was deemed compatible with pre-existing religious traditions. As it entered the West, it was morphed, for better or worse, into a self-improvement technique for those stressed-out by capitalism. The history of Buddhism is itself thus a history of cross-cultural exchange. Hence, instead of comparison, Eastern philosophy sought rejection or incorporation. Since all those traditions had substantial philosophical components, the history of Buddhist philosophy is a history of comparative philosophy. There is no reason for why this expansion should come to a halt. Buddhist thought is surprisingly ecumenical.

That sensitivity to philosophical discussion and the willingness to mingle one's own beliefs with the beliefs of others—tested for its desirability and compatibility by rigorous argument—is one of the great merits of Eastern thought. Nishida Kitaro was probably the first in modern Asia to mingle East with West in his 'An Inquiry into the Good' (1911). He engaged critically with the phenomenological tradition, Husserl and most of all Heidegger, while being firmly grounded in, and practicing Māhāyana Buddhism. Subsequent generations, most notably Keiji Nishitani and Hajime Tanabe, have followed in his footsteps. We know call this distinct school of thought, the Kyoto School. The Kyoto School approach to comparative philosophy is not the approach I wish to take either. My intention is not, remaining in those bogged down categories for now, to blend East and West, Buddhist logic with Aristotelian logic. Being perennial, this is

[27] This is a term I want to avoid as much as I want to avoid the term 'Western philosophy'. It is binary, it is exclusionary, and it creates the impression of difference. I am using it solely for the expose its limits.

reluctant to accept that philosophical traditions evolve, is indeed another vice of comparative philosophy. Comparing Confucianism to Aristotelian virtue-ethics, for instance, is something else than comparing the latter to Neo-Confucianism. Philosophical traditions are neither monolithic, nor are they particularly stable. This makes every attempt of comparison highly contextual, lacking universalistic scope. I wish to go beyond that.

Approaches which seek to go beyond comparative philosophy are sometimes classified as 'world philosophy'[28]. Some aspects of my methodological approach are indeed akin to the concept of world philosophy, although not all. World philosophy is distinguished by taking into account several different philosophical traditions at once and to synthesis them into a coherent whole. The act of comparison is innate to world philosophy, too; different socio-cultural traditions are scanned for points of commonality, different strands of thought complied into a whole.

World philosophy shares with comparative philosophy a comparative core and an array of criticisms. The first thing the comparative approach has been criticised for is its alleged orientalist chauvinism. For Nussbaum (1997), there are two forms of chauvinism the comparative philosopher ought not to commit. The first is a form of descriptive chauvinism which seeks to recreate one philosophical tradition in the image of another. This is judging a philosophical text not from within the context of its origination, but from one's own positionality, assuming one can easily translate one context of interrogation to the another. The second is a form of normative chauvinism which is characterised by a suspension of charitable judgement of unfamiliar philosophical positions. It amounts to assuming, uncritically, the superiority of one's own positionality—which is a philosophical cardinal sin. This is what happens when Buddhism is wrongly discredited as new-age mysticism, as not *really* philosophical—a stubborn tendency among some Western philosophers which is changing only hesitantly. The first common misunderstanding is that philosophy has to be analytically rigorous (whatever the measures of this may be), and the second that Buddhist philosophy could *not* live up to those standards of analytical rigour. Determining what counts as 'real philosophy', and what doesn't, is an act of normative chauvinism. Whatever standards of rigour and analyticity one may want to establish, I hope the following discussion will provide ample evidence that Buddhist philosophy can live up to it.

[28] See Edelglass & Garfield (2011).

In a 1989 publication, David Wong discussed what he thinks is yet another vice common to comparative analysis: the naïve acceptance of incommensurability, i.e. the impossibility of finding appropriate overlap between traditions on the grounds of which they can be compared. In a sense, incommensurability is a rejection of comparative philosophy arrived at by taking a comparative, yet pessimistic position with regards to the possibility of comparison itself. According to Wong, some philosophical traditions are so far removed from others that not rarely one is unable to understand, let alone merit the other. Such claims of incommensurability have sparkled here and there with regards to the Buddhist concept of *sunyata* (emptiness). A common misconception is that, because epistemic or phenomenological insight into *sunyata* is bound to the intense practice of meditation, only those who actively practice can understand what Buddhism *is all about.*

We find an analogous argument in the context of intersectionality theory[29], which says that those who never had a phenomenological experience of racism, this is, never experienced racism, don't know *what racism is*, epistemically speaking. While it is true that I, as a white middle-class kid, have never experienced racism, I am well aware that it exists, that I can rationally understand what it is, where it comes from, what the societal factors are, what strategies of overcoming may be, and so on. The incommensurability argument, I think, is implausible.

How then, do I wish to proceed, methodologically, with this essay on Buddhist logic? The approach I wish to take must be designated as a *philosophy without borders*. In a sense, what I take myself to do is simply philosophy *simpliciter*. I engage with someone else's thoughts, I analyse them, I scrutinise them, I criticise them, and it is merely the case that those thoughts originated in the Buddhist milieu. The highest praise for a philosophical position is usually to challenge it, to deem it worthy of criticism, and therefore to deem it worthy of development—orientalist chauvinism is the opposite.

To be fair, I jump on a train which is already rolling. The authors I quote and whose renditions of Buddhist philosophy I criticise and/or endorse in this essay have started the train's engine a while ago. Analytical Buddhism, as I may call it, takes the modern tools of analytical philosophy to discuss and make intelligible, to us, who we are brought up with the paradigms of twentieth and twenty-first century analytic philosophy, the ideas of Buddhist thinkers. What Buddhist philosophy is, its definition, is in perpetual movement. This essay aims to go beyond comparative aspirations. The aim is not to

[29] See, for instance, Fricker (2009).

compare, but to engage, to take Buddhist philosophy (even as a non-Buddhist) seriously, and possibly to aid it to its next form. To think that a non-practicing Buddhist couldn't or shouldn't do that, is either a sign of philosophical mediocrity or dogmatic fundamentalism. People rarely care about *what you are* if your criticising the B-theory, or moral realism. People rarely care if you practice *epoché* if you are a Husserl scholar. So, why should people care about a non-Buddhist thinking about Buddhist philosophy?

A caveat: For what follows, I will, for the most part, rely on secondary literature, translations and philosophical interpretations. There is no claim on authentic Buddhological insight, scholarly precision, or interpretative authority in any of the things I present here. In fact, scholarly work on many of the original scripts I am referring to is a highly complex, quasi-archaeological activity by which original arguments are restored by looking at commentaries[30]. As Westerhoff (2009:7) says, *"our first methodological maxim when 'filling in' the missing parts of Nāgārjuna's arguments should therefore be to attempt consistency with the commentarial literature"*. For that, I am not qualified. In many cases, a charitable reading, one that makes the arguments the strongest they can be, is key here. While being charitable, those involved in this project have to be aware of their sympathies to Buddhist philosophy—otherwise, they would engage in scholarly activity, I assume. Those sympathies can betray us in philosophical argument, make us resistant to counterargument, make us less rationally engaged. Bridging this is an interesting and challenging task which requires a lot of self-reflection. It also requires the relevant language skills. Not only that, it requires a sensitivity to how the particular language behaves in philosophical contexts. It also requires abstracting away from one's familiar argumentative patterns and to suppress the urge to mindlessly pack everything into the Aristotelian mold, even if that could bring instant clarification. The life of a scholar must be the hell of a challenge. I am happy to leave all this to the experts.

[30] See, Westerhoff (2009: 7ff)

§2 The *Catuṣkoṭi* [31]

Everything is real and is not real,
Both real and not real,
Neither real nor not real.
This is Lord Buddha's teaching.
MMK (18:8)

The argument in MMK (18:8) above has the form of a *catuṣkoṭi*[32] (Sanskrit; चतुष्कोटि; Eng. Four Corners). If we look at it a little more carefully, we will see that, roughly speaking, what happens in MMK (18:8) is that a certain proposition (that everything is real) is affirmed, negated, both affirmed and negated, and neither affirmed nor negated. That's certainly puzzling. Indeed, the *catuṣkoṭi* is perhaps the most enigmatic piece of Buddhist philosophy. To understand how it works, and how to make sense of it in the first place, is one of the central aims of this essay. I will discuss several attempts of modelling the *catuṣkoṭi* in a classical logical framework, Priest's attempt to formulate it in non-classical, paraconsistent logic, and propose an alternative to both based on speech-act theory. The *catuṣkoṭi* is central to this essay because by showing how it repeatedly reappears, although in slightly altered forms, in the writings of Chinese and Japanese Buddhism, I wish to close the gap between Indian Māhāyana and Japanese Zen. This discussion fills a gap in the literature for, as Westerhoff (2009:68) writes:

"[d]espite some important work done during the last decades, a comprehensive study of the origins and development of the catuskoti from its use in the earliest Buddhist

[31] Substantial parts of this chapter have been published in Kreutz (2019a). In detail, the exposition has been shortened and altered to follow the overarching argument. The methodological maxims have been revised both in content and exposition. The list of open questions and subsequent paragraphs I took from the introduction of Kreutz (2019b), as well as the discussion of Guaratne on page 33 of this thesis. Chapters §2.1 appears in Kreutz (2019a) and has been altered only slightly where connecting the discussion to the overarching argument was necessary. Chapters §2.2 has be changed only slightly too and has appeared in its entirety in Kreutz (2019a).

[32] For what follows, I will deprive myself of the diacritical of an adequate translation from Sanskrit and write '*catuṣkoṭi*' instead.

literature to up to its later employment in the Buddhist philosophical works of Tibet, China, and Japan remains to be written."

Where the *catuṣkoṭi* originated is not entirely clear, but it is likely to be one of the things Buddhism inherited from Hinduism[33]. Any reader of Indian Buddhist philosophy will be familiar with this argumentative scheme. Within the wider Buddhist canon, the *catuṣkoṭi* has found its most prominent use in the writings of the Madhyamika school[34] of Buddhism, particularly the writings of Nāgārjuna[35].

The *catuṣkoṭi* is *prima facie* enigmatic for it clashes with the standards of classical logic, especially the principle of *tertium non datur*: Everything is either true, or false – *c'est tout*. The *catuṣkoṭi* inflates this principle (while rejecting it) to a *quintum non datur*: It states four exhaustive and mutually exclusive possibilities for any given proposition:

(1) either it holds,
(2) it does not hold,
(3) it both holds and does not hold, or
(4) it neither holds nor does not hold.

Those are the four *kotis* (which roughly translates as "corners"). What if we would like to put that into logical form? A first desideratum is this: Every model for the *catuṣkoṭi* has to maintain the *mutual exclusivity* and *exhaustive nature* of the kotis—call it the exclusivity- and exhaustivity-constraint. The reason is for why every model has to adhere to those constraints is twofold:

(I) We want to be charitable to the logical abilities to the authors who were using the *catuṣkoṭi*, (I take this to be a form of the principle of charity) and

[33] See Robinson (1967).

[34] Although, as I argue in a forthcoming article, it is neither limited to Madhyamaka, nor the Indian philosophical milieu. Just like most concepts of Buddhist thinking it has its origins in Hindu texts (or a criticism thereof) and has developed as Buddhism found its way to the East.

[35] For a general introduction to the thinking of Nāgārjuna and the history of Buddhist metaphysics and the *catuṣkoṭi*, see Priest (forthcoming).

(II) The way the *catuṣkoṭi* is commonly employed as an argument, supposed to undermine all possible ways of predicate-attribution, requires the four kotis to exhaust all logical space, and be mutually exclusive.

Point (II) refers to the reductive nature of the *catuṣkoṭi* when it is employed in the arguments of the MMK. Those arguments, which are a kind of *reductio* taken to reveal the deficiency of a given concept, are called *prasanga arguments*. The concepts the MMK seeks to debunk as deficient, employing a prasanga-style arguments, are, among others, *causation, motion, self,* and *identity.*

Take MMK (18:8), for instance. The thesis is that 'everything' is 'real'. The four corners of the *catuṣkoṭi* are supposed to exhaust all of the logical space and thus to tap every possible way "reality" can be attributed to the object which is "everything", and hence to debunk, in a way yet to be defined, the proposition that "everything is real". As Priest (2010) says:

"The central concern of the MMK is to establish that everything is empty of self-existence (svabhāva), and the ramifications of this fact. The main part of the work consists of a series of chapters which aim to establish, of all the things which one might plausibly take to have svabhāva (causes, the self, suffering, etc.), that they do not do so." Priest (2010)

To do that, i.e. to establish that everything is empty of *svabhāva*, every possible way something can *be* has to be ruled out eventually to illuminate the emptiness of its self-existence. Sounds strange? The logical models in the following sections are supposed to clarify.

As a matter of fact, the quadruple-wise exclusivity of the kotis comes as a natural consequence of wanting to establish the exhaustivity of the kotis: Only if none of the kotis is equivalent to any other koti, i.e. only if every koti establishes a distinct possibility, the logical space can be exhausted. The reasoning presupposes that the logical space consists of four distinct possibilities, i.e. the four possibilities: *(1) either it holds; (2) it does not hold, (3) it both holds and does not hold, or (4) it neither holds nor does not hold.* The task is to reflect this in our formal model. That means, if on the formal model either of those four possibilities is equivalent to any of the other possibilities (i.e. not distinct), the logical space of four possibilities cannot be exhausted. If this is the case, I will later refer to it as "collapse".

In other words, the exclusivity- and the exhaustivity-constraint determine each other and may together be taken as a minimal desideratum for a successful model of the *catuṣkoṭi*.

As Westerhoff (2009:51) mentiones, although a lot has been said about the *catuṣkoṭi* over the last decades[36], we are still awaiting a comprehensive and decisive answer to each of the following three questions:

(1) What role does the *catuṣkoṭi* play in Buddhist philosophy?
(2) What (if there is one) is the logical form of the *catuṣkoṭi*?
(3) What is the position of the *catuṣkoṭi* in the history of Buddhist philosophy?

I suspect that the source of controversy around the *catuṣkoṭi* in recent Western commentarial literature, manifested in the vast number of mutually incompatible interpretations, is grounded in a univocally piece-meal, un-unified and historically isolated approach to the subject matter.

The first point concerns, what I call, piece-meal engineering: Questions 1-3, if they have been addressed at all, have been addressed in (nearly complete) isolation. Furthermore, Western academic research has primarily been concerned with the second question; In practical terms, this means that we can see a 'logic-first approach' in the research literature on the *catuṣkoṭi*. It is, then, no surprise that the *catuṣkoṭi* has been considered perplexing, mysterious and incoherent—it simply does not fare well with classical logic, as discussed in more detail below. And as long as we are trying to fit the *catuṣkoṭi* into a preconceived Aristotelian baking mold, I predict that the *catuṣkoṭi* will remain a rebelliously mysterious piece of Buddhist literature. Instead of doing comparative philosophy in the chauvinistic way, I think we should try to be as undogmatic and charitable as possible when trying to accommodate the *catuṣkoṭi* in a model. Furthermore, the literature on the *catuṣkoṭi* has, for unknown reasons, usually tried to unhinge from its metaphysical underpinnings and to treat it as a piece of purely logical formalism[37]. Any one-sided approach of piece-meal engineering cannot but fail to do justice to a what is otherwise a logico-epistemic phenomeno-metaphysical concept.

My second point concerns the tendency to historically isolate the *catuṣkoṭi* from both its predecessor (if there are any), but more importantly from its historical successors

[36] See, for instance, Chi (1969), Murti (1955), Staal (1975), and Napper (1989),
[37] Priest/Garfield (2010) are an exception.

(of which some we will encounter throughout this essay). In fact, the attention of the recent literature has almost exclusively been on Nāgārjuna's *catuṣkoṭi*. This, however, is a drastic, self-inflicted limitation. My hypothesis is that the *catuṣkoṭi* appears in various forms throughout Buddhist philosophy, it is not unique to Madhyamaka philosophical thought.

Those charges of piece-meal engineering and historical isolation do not apply to all of the recent engagements with the *catuṣkoṭi*. Westerhoff (2009), Priest (2010a, 2018) and most commentators are the exceptions to a rule. But they surely apply to one of the most influential papers on the subject, which is Gunaratne's (1986) " The logical form of the *catuṣkoṭi*: A new solution". Here is Guaratnes agenda:

"It is thus important to examine (1) how it was that Nagarjuna came to make such extensive use of the catuṣkoṭi; (2) the logical form of Nagarjuna's catuṣkoṭi; and (3) with what purpose and in what manner this "logical apparatus" was handled by Nagarjuna in exposition of his philosophy." Gunaratne (1986: 214)

Clearly, the charges of piece-meal engineering and a misguided logic-first approach applies to claims (2) and (3), while the charge of historical isolation applies to claims (1) and (3). It is Gunaratne's approach that has dominated the recent developments within the research literature, which has therefor led into several *cul-de-sacs*. To my surprise, in the same paper we find two further proposals on how to understand the *catuṣkoṭi* which Gunaratne takes to be necessary to get *"the overall picture"*.

"(D) The catuṣkoṭi was used by him (AK: Nāgārjuna) as a dialectic which progressively leads one to truth. (E) The catuṣkoṭi was used as an instrument of meditation. It is clear that these positions need not be mutually exclusive. Limitations of space prevent any consideration of (D) and (E) here, although I think that both of these are possible interpretations of Nāgārjuna's use of the catuṣkoṭi and the consideration of them is necessary to get the overall picture of Nāgārjuna's effort in the (MMK)" Gunaratne (1986:215)

It is essentially positions (D) and (E) that I wish advocated in this essay. It is peculiar that Gunaratne, being unsatisfied with the (hitherto) noncomprehensive treatments of the *catuṣkoṭi,* did not continue research on those two possibilities especially while already

guessing them to be indispensable to attain a cohesive understanding of the *catuṣkoṭi.* The decision to disregard ontological, soteriological and historical frameworks has (unsurprisingly) led to an unrewarding treatment of the *catuṣkoṭi*, detached from its proper philosophical and historical context. This essay takes the legacy of Gunaratne's positions (D) and (E).

It is perhaps for those two reasons given above that Tillemans (1999:189) was motivated to contend that *"within Buddhist thought, the structure of argumentation that seems most resistant to our attempts at a formalization is undoubtedly the catuṣkoṭi "*—a thought which stretches as far back as to Poussin's (1917) paper (probably the earliest philosophical treatment of the *catuṣkoṭi* in the West) and established the *catuṣkoṭi*'s position as the "Buddhist tetralemma".

In seeking a comprehensive account of the *catuṣkoṭi,* this essay will be working on logical, ontological, epistemic, phenomenological, historical, and soteriological aspects *in unison*. The fact that the logical aspects of the *catuṣkoṭi* have hitherto been unhinged from their philosophical and historical fundament have prevented any fruitful modelling and obstructed further insight into the development of this fascinating piece of Buddhist philosophy. I hope that with the help of the formal modelling applied here, it will become clearer what the *catuṣkoṭi* is, and how it functions within the Buddhist canon.

§2.1 Remaining Classical

In this chapter, we will look at several attempts to model the *catuṣkoṭi* in the framework of classical logic. Every attempt will be debunked. We shall begin with a very naïve way of modelling it and continue working ourselves up to increasingly sophisticated approaches. All of them have in common the same pitfall; they fail to live up to the exclusivity- and exhaustivity-constraint. So, let us begin with a very naïve and straightforward way of putting it:

Naïve Classical *catuṣkoṭi*

(1) A

(2) $\neg A$

(3) $A \wedge \neg A$

(4) $\neg(A \vee \neg A)$

On a first look, this model is just fine. The first koti, A, introduces a proposition which is being denied in the second koti, $\neg A$. The third koti, $A \wedge \neg A$, is the conjunction of koti one and koti two. And the fourth koti is a negated disjunction, reading 'neither koti one, nor koti two'. So, why is this problematic?

The exclusivity- and exhaustivity-constraints maintain that all four kotis must be genuine possibilities. In this straightforward classical model above, however, koti three clearly entails koti one and koti two. In other words, koti three is redundant and hence not a genuine possibility. The same goes for the fourth koti if one drags the outer negation into the brackets and assumes, as it is usually the case in a classical framework, that the *Principle of Double Negation,* i.e. $(DN;\ \neg\neg A\ \dashv\vdash A)$, holds.

This classical model cannot meet the exclusivity- and exhaustivity-constraints, which are the minimal desiderata for a successful model of the *catuṣkoṭi*. In other words, the classical model *collapses* into position one and two because they are entailed by position four and three which are, by *De Morgan's Law,* equivalent. Avoiding collapse of the kotis is the principle target of any successful model for the *catuṣkoṭi*.

Tillemans (2000) tries to rescue the naïve classical model by putting the kotis behind existential, and as Priest (2018:20) rightly notes, "spurious" quantifiers. Whatever

work the quantifier is supposed to play, it has to remain a futile enterprise as long as nothing changes within the scope of the quantifier. Even in the quantified version, the third and fourth koti are equivalent and entail the first two kotis.

Quantified Classical *catuṣkoṭi*

(1) $\exists x\, A$

(2) $\exists x \neg A$

(3) $\exists x (A \wedge \neg A)$

(4) $\exists x (\neg (A \vee \neg A))$

Classical approaches have a mayor downside, one that is bound to classical negation. The common problem is that, because the negations in all four kotis are the same, and *De Morgan* holds, the kotis collapse. So, it might be obvious that one way of dealing with this situation, which is both a dead-end and strictly speaking not a classical approach, is to use *intuitionist logic* which comes with a denial of double negation. Such is Staal's (1975) suggestion. While it will rescue the exclusivity of koti four, there is, however, still the problem of koti three entailing the first two kotis. Hence, the intuitionist model for the *catuṣkoṭi* suffers not full, but partial collapse.

Intuitionist *catuṣkoṭi*

(1) A

(2) $\neg A$

(3) $A \wedge \neg A$

(4) $\neg A \wedge \neg \neg A$

Another case of partial collapse is Westerhoff's (2009:4) model. Westerhoff introduces a different kind of negation for koti four, such that koti three is no longer deducible from koti four. Still, this will be of no help as long as koti three collapses into kotis one and two. His proposal is based on the observation that Indian logic (sometimes) distinguishes internal (*prayudāsa*) from external (*prasaja*) negation[38]. Let me try to unpack that.

[38] In modern discussions on category mistakes we often find a distinction into *choice negation* and *exclusion negation*. This reflects the ancient idea of prayudasa/prasaja. See Routley (1969) as prominent example.

When we look at negations in a standard subject-predicate sentence we may notice a small but important ambiguity when it comes to what is denied in a sentence such as 'S is P'. In the Western canon, it is Aristotle how first became aware of this ambiguity, saying that there is a difference in whether one denies that 'S is P' or maintains that 'S is not P'. Aristotle distinguishes between *external negations* such as 'It is not the case that: S is P', and *internal negations*, such as 'S is not-P'. Even though it is always that case that if an internal negation of a subject-predicate sentence is true, the external negation of a subject-predicate sentence is also true, the reverse does not always hold. Hence, there is a semantic difference between external and internal negation which may be employed to distinguish the kotis.

In the Buddhist context, this distinction is grammatical: The external, *prasaja-negation* negates the verb, as in *brāhmana nasti* (Eng: This is not Brahmin); the internal, prayudāsa-*negation* connects with the noun-phrase, as in *abrāhmana asti* (Eng: This is a Non-Brahmin)[39].

An example will help make clearer the difference between the two kinds of negation. The negation of 'This bird is *not* a crane (i.e. a non-crane)' is internal since it implies that the object, i.e. 'the bird', exists, but that it is some other species than a crane, i.e. that the predicate 'crane' does not apply to the object, 'bird'. An external negation of the proposition would say '*It is not the case that* this bird is a crane'. I this case, there is no implicature to the existence of a bird, a crane, or anything whatsoever. So, the internal negation presupposes that the subject properly refers to something in the subject-position, while the external negation does not. To render this distinction robust, we can define the external negation as true *if and only if* the subject of a sentence fails to refer (for whatever reason), and the internal negation as true *if and only if* the predicate fails to apply to the subject.

Westerhoff applies this distinction to koti four and koti three to the naïve classical model, arguing that the negation before the parentheses of koti four is to be read externally, while the negations in koti two and three are not. The proposal fails because, even if koti four employs an external negation and is thus no longer equivalent to koti three, koti three still entail kotis one and two, which again leads to a partial collapse of the model, in fact the same partial collapse from which the intuitionist approach suffered. Furthermore, if there is no precise story about how the external negation of koti four functions and interacts with the proposition it negates, we are likely to end up with a flat

[39] See Westerhoff (2009).

contradiction. On one possible, and I think the intended reading of Westerhoff's proposal, koti four may be read as,

(4) $\sim(A \vee \neg A)$

where $'\sim'$ is the external negation and $'\neg'$ the internal negation. Absent a more elaborate story about how those two forms of negation interact, and especially a more elaborate story about how the external negation interferes with classical principles, we may assume that there is a reading on which $'\sim'$ denies that $A \vee \neg A$ holds, or in other words, that the *Law of the Excluded Middle* (LEM), a classical truth, is valid.

Since Westerhoff's textually grounded proposal on how to distinguish negations within the *catuṣkoṭi* leads to the negation of a classical principle, it may be warranted to look for alternatives in the realm of non-classical logic. In fact, it seems necessary given that the *catuṣkoṭi* contradicts both classical hallmarks of LEM $(A \vee \neg A)$, i.e. the *Law of the Excluded Middle*, and LNC $(\neg(A \wedge \neg A))$, i.e. the *Law of Non-Contradiction*. We can thus, without being overly reckless, say that the tools of classical logic are somewhat inadequate when sought to consult a satisfactory model for the *catuṣkoṭi*. Because the *catuṣkoṭi* seems like a paradigm case for a non-classical model in which LEM and LNC are invalid, that is what we shall turn to in the next section.

§2.2 Going Paraconsistent

The *catuṣkoṭi* is in conflict with classical logic, not only because of how 'negation' behaves in a classical framework, but also because classical logic is bivalent, this is based on the assumption that there are only two alethic values; truth and falsity. Many-valued logics extend the number of alethic values beyond the common bivalent case. Motivations for extending the number of alethic values is far and wide and differ with respect to how and by what kind of additional value the classical case ought to be extended[40]. The *catuṣkoṭi* suggests that things or propositions can be both true and false, which we may think of as an additional truth-value, and that statements or propositions can be neither true nor false, which we may also think of as an additional truth value. The former is sometimes referred to as a truth-value glut, the latter as a truth-value gap. The motivations for truth-value gluts and truth-value gaps are distinct. Let's start with the former.

A famous case is that of inconsistent rights and obligations[41]. Think of a claim that says (1) No carnivore person has the right to attend the house party, and (2) All members of High Street Book Club have the right to attend the house party. What now, if you are both carnivore and a member of the High Street Book Club. Do you have the right to attend the house party, or not? We may suppose that whoever made the claim assumed that there are no carnivore members of the High Street Book Club. But if there were, this person, it appears, both does and does not have the right to attend the house party. Whoever came up with the entrance-policy to the house party is likely to change it in the light of inconsistency, but that doesn't refute that until the policy has been rendered consistent, the contradictory nature of (1) and (2) remains. This, Priest thinks, provides sufficient reason to establish an additional alethic value, 'both true and false'. If we think of this contradiction as true, the entrance-policy is a so-called 'dialethia'—a true contradiction.

Other, paramount examples for truth-value gluts concern paradoxes of self-reference, most famously perhaps the Liar Sentence, 'this sentence is false'. If it is false,

[40] See, Priest (2001)

[41] See, Priest (ibid.126)

it is true; if it is true it is false. Hence, it is both true and false. If this reasoning is sound, its conclusion is of the form $A \,\&\, \neg A$.[42]

Truth-value gaps can be motivated by looking at future contingents, i.e. statements about future states-of-affairs. The idea is that statements such as 'The next Dalai Lama will be Chinese' are *neither true nor false* as long as its truth-value cannot be determined[43]. Of course, one could reply that we just don't know yet, but that actually, the statement is (or will be) either true or false. Aristotle gave a forceful rejoinder: On the one hand, Aristotle thinks that if it were true, in the future, that the next Dalai Lama is Chinese, then it would necessarily be true that the next Dalai Lama will be Chinese. On the other hand, if it were false that the next Dalai Lama is Chinese, then it would also necessarily be false that the next Dalai Lama will be Chinese. So, whatever the future may bring, it will happen necessarily. But, argues Aristotle, this is impossible, because what happens in the future is a matter of contingency. Hence, unless you are a very strong determinist about future possibilities, the statement that 'the next Dalai Lama will be Chinese' is neither true nor false now; this is, it is sitting in a truth-value gap[44].

To capture these additional alethic values, classical logic has to be revised by dropping one or other classical principle. A formal logical system in which EXP $(A, \neg A \models B)$, i.e. *The Principle of Explosion,* is dropped is, in modern logical terms, *paraconsistent.* The simplest paraconsistent logic is LP. A formal logical system in which LEM $(A \vee \neg A)$, i.e. *The Law of the Excluded Middle*, is dropped is, in modern logical terms, paracomplete. K3 is arguably the simplest paracomplete logic[45].

Both LP and K3 are three-valued. That means they have three truth-values, i.e. *V = {true, false, indeterminate}* where classical logic only has two, i.e. *V = {true, false}*. In K3, the indeterminate truth-value has to be read as 'neither true nor false', or simply 'neither'; in LP, it has to be read as 'both true and false', or simply 'both', which turns truth and falsity into 'true, and true only` and 'false, and false only', respectively.

Important for many-value logics is the notion of a designated value *D*, i.e. those values that are preserved in valid reasoning. In K3, we preserve only truth, this is *D = {true},* whereas in LP we preserve both truth and 'both-ness', this is *D = {true, both}.*

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[42] See, ibid.

[43] But, realistically and unfortunately, the statement seems more on the true-ish side of the specturum.

[44] Aristotle's argument can be refuted in a number of ways, see Priest (ibid. 131).

[45] For an introduction to both, see Priest (2001).

Since the fourth koti, $\neg(A \vee \neg A)$, says that A is neither true not false, it seems to be a case for K3. And since the third koti, $A \wedge \neg A$, says that A is both true and false, it seems to be a case for LP. Since our target is a comprehensive model for the whole *catuṣkoṭi*, and not just for some of its kotis, we could try combining K3 and LP. Luckily, the combination of LP and K3 is an already well-known many-valued logic called *First Degree Entailment* (FDE).

In the search for an adequate model for the *catuṣkoṭi* Priest (2010a) introduces a model for the *catuṣkoṭi* using the four-valued semantics of FDE. He expresses the four kotis with a set of status-predicates, $S = \{T, F, B, N\}$, taken to evaluate the semantic status of a proposition on a meta-level. Let T be the truth-predicate "is true", F is "is false", B is "both true and false", N is "neither true nor false". $T(A)$ is the proposition "A is true", where (A) is simply a name for A. To define $B$and N, Priest first defines F as $F(A) = T(\neg A)$. Let us call Priest's semantic for the *catuṣkoṭi* FDE(S), where the S stands for the set of status-predicates. Here is what the *catuṣkoṭi* looks like in FDE(S), which is Priest's formulation.

1) $T(A) = T(A) \wedge \neg F(A)$
2) $F(A) = \neg T(A) \wedge F(A)$
3) $B(A) = T(A) \wedge F(A)$
4) $N(A) = \neg T(A) \wedge \neg F(A)$

Taking the exclusivity-constraint into account, the *catuṣkoṭi* can be expressed in the following way:

$$Catuskoti(+)^{46}: T(A) \vee F(A) \vee B(A) \vee N(A)$$
$$Exclusivity\ Constraint: \neg(S1(A) \wedge S2(A))$$

Please note: where S1 and S2 are distinct status-predicates from the set `S`. By assigning a truth value to all the sentences of the language in which the *catuṣkoṭi* is formulated via an interpretation function v, such that $v(A)\ \epsilon\ \{t, f, b, n\}$, we generate a semantics for the logic. On this model, the four kotis may be represented by a four-valued diamond lattice, or, as it is sometimes called, a *Hasse-diagram*.

[46] The (+) indicates the positive use of the *catuṣkoṭi*. We will later encounter its negative dual.

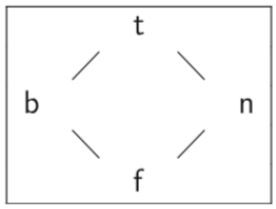


TABLE 1: The positive *catuṣkoṭi*

So far, my presentation has been rather informal, but, I think, sufficiently rigorous to get the gist of why the classical models fail, why LP and K3 can help us make sense of the third and fourth koti, and why a combination of both could be the way forward. With respect to the FDE model, however, I will have to be somewhat more formal in my presentation because of the relatively intricate counterarguments and rejoinders that are about to follow in the subsequent sections.

Let us continue this exposition of Priest's FDE-model of the *catuṣkoṭi* by defining the behaviour of the standard connectives.

- The value of disjunction, $v(\ A \vee B)$, is the *least upper bound* of $v(A)$ and $v(\ B)$.
- The value of the conjunction, $v(\ A \wedge B)$, is the *greatest lower bound* of $v(A)$ and $v(\ B)$.
- The value of negation, $v\ (\neg A)$ otherwise toggles $t\ and\ f$ but maps $\{b, n\}$ to itself.

To define validity in many-valued logic, as discussed above, we need to elaborate on the notion of a designated value, i.e. that value that is preserved in valid arguments.

- The set of designated values in FDE is $D = \{t, b\}$
- If the value of A is designated, $T(A)$ is also designated. $T(\ )$ is part of the language. If the value of A is not in D, the value of $T(A) = f$
- If A takes some value, the statement to the effect that it takes that value is designated. This can be seen by following the diagonal, top left to bottom right on the diagram below.

A	T ⟨A⟩	B ⟨A⟩	F ⟨A⟩	N ⟨A⟩
t	t or b	f	f	f or b
b	b or f	t or b	b or f	f or b
f	f	f	t or b	f or b
n	f	f	f	t

The minimal desideratum for a successful model of the *catuṣkoṭi* is to meet the exclusivity- and exhaustivity-constraint. Luckily, the truth conditions above validate the formal statements of the *catuṣkoṭi* and meet the exclusivity- and exhaustivity-constraint. To see this, take again the formulation of the *catuṣkoṭi* with status-predicates: $Catuskoti(+): T(A) \vee F(A) \vee B(A) \vee N(A)$. Whatever value a proposition takes, that value will be designated (t or b), and hence will be the disjunction.

Regarding the $Exclusivity\ Constraint$: Any statement of the form $S1(A) \wedge S2(A)$ will contain a sub-conjunction of the form $T(A) \wedge T\neg(A)$ *or* $F(A) \wedge \neg F(A)$. So, $\neg(S1(A) \wedge S2(A))$ will contain a sub-conjunction with a disjunct of the form $T(A) \vee T\neg(A)$. Either A is designated, and so is the first disjunct, or A is not designated, but then the second disjunct is. Therefore, the whole disjunction is designated.

Regarding the $Exhaustivity\ Constraint$, FDE(S) also avoids the collapse of the kotis. The fourth koti, $\neg T(A) \wedge \neg F(A)$ does not collapse into the third, $T(A) \wedge F(A)$ in FDE(S). This is because, to say that $\neg\big(T(A) \vee F(A)\big)$ is to say that A is neither true nor false, but to say that $T(A) \wedge F(A)$, is to say that A is both true and false—two very different things. Hence, we may conclude, the introductions of status-predicates into FDE avoids the collapse. The underlying reason is the following; The *catuṣkoṭi* doesn't suffer collapse since the biconditional in FDE(S), just like in LP and K3, is not contraposable, which can only be changed by adding a material conditional to the language. In other words, we don't have $\neg T(A) \rightarrow T\neg(A)$. This is because in K3, as in LP, falsity and non-truth fall apart. We will return to this thread below when I give the collapse argument against Cotnoir's four-valued quasi-classical rendition of the *catuṣkoṭi.*

A prominent characteristic of the Priest's interpretation is that it fares well with the Madhyamaka metaphysics of bicameral reality. As discussed in the first chapter, Madhyamaka philosophy promotes a distinction of reality its conventional aspect (*samvrti-satya*) and its ultimate aspect (*paramarthra-satya*).

This conventional aspect describes the world of appearances. This includes material objects, properties, causation, the self, feelings, memories, individuality, identity (etc.), and the erroneous idea that all those may have *svahbava*. Furthermore, it is the realm of thought, language, and most probably, logic. How does that resonate with the *catuṣkoṭi*? Nāgārjuna says that any claim about conventional reality has all four truth-values of the *catuṣkoṭi*; it is both true, false, both true and false, and neither true nor false. What to make of that, and whether Nāgārjuna really is proposing a position on its own here, is perhaps the pivotal point of the contemporary discourse on Nāgārjuna 's use of the *catuṣkoṭi* [47].

The ultimate aspect of reality, however, is mirrored by a rejection of all four kotis. So, whatever is or happens in ultimate reality, it is neither true, nor false, nor both true and false, nor neither true nor false. Rejecting all four kotis is sometimes called the *'fourfold negation'*. Here is the corresponding, negative *catuṣkoṭi*.

Negative *catuṣkoṭi*

(1) $\neg A$

(2) $\neg(\neg A)$

(3) $\neg(A \wedge \neg A)$

(4) $\neg(\neg(A \vee \neg A))$

In Priest (2010a), the negation before the parenthesis is expressed as a fifth possibility, resulting in a five-valued logic where the fifth value (E), which stands for *emptiness*, is equivalent to the denial of the other four values.

One might wonder whether this expanded logical machinery threatens the exhaustivity of the kotis established by appeal to FDE in the first place? Indeed, it does when one operates on the level of conventional reality only. That means, one has to bear in mind that the fifth value corresponds to the ultimate reality, the four kotis hence exhaust the logical space of the conventional reality, whereas the fifth values exhausts the logical space of ultimate reality. This is the reason why the pairwise exclusivity of the kotis isn't threatened either. In Priest's five-valued logic, the designated values remain $\{t, b\}$ since

[47] And, because the *catuṣkoṭi* is mostly (and mostly exclusively) discussed in the context of Nāgārjuna's philosophy, the pivotal point of the discourse on the *catuṣkoṭi* itself. See, again, Westerhoff (2007, 2009).

(E) means that a claim should neither be accepted nor denied. It can, therefore, not be designated.

$$Catuskoti(-)^{48}: T(A) \vee F(A) \vee B(A) \vee N(A) \vee E$$

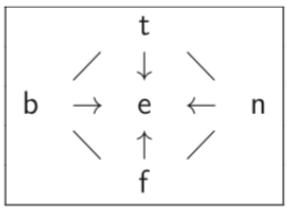
The negative *catuṣkoṭi* emphasises the view that ultimate reality is essentially ineffable. Claims about ultimate reality are ineffable since ultimate reality is inaccessible from the standpoint of conventional reality. And since our language is a language of conventional reality, truth about the fundamental reality cannot be expressed with the vocabulary of conventional language—they are asserted only for the purpose of designation.

The 'Four Extremes' (Sanskrit; चतुरन्त), a claim about the nature of being as seen from the ultimate perspective, is a particular application of the five-valued logical form of the *catuṣkoṭi* to a metaphysical thesis.

The Four Extremes

1) Being[49]
2) Non-Being
3) Being and Non-Being
4) Neither Being, nor Non-Being
5) None of the above (e)

By disproving all conceivable possibilities with (e), Nāgārjuna is thought to trace the middle path between being and non-being. Adding (e) makes our logic five-valued. The fifth value is supposed to be incompatible with the other four. Call the resulting logic FDE_e. The (quasi-)Hasse-diagram depicting this situation may look something like this:



[48] The (-) indicates the negative use of the *catuṣkoṭi*

[49] The technical term for '*being*' in Buddhist literature is 'dharma', the bare phenomena of the world. The truth-value (e) corresponds to the view that everything is empty 'sunya', empty of essence, or self-existence 'svahbava'. For more information about those terms, see Westerhoff (2007), or Priest (forthcoming).

TABLE 2: The negative *catuṣkoṭi*

But how do we proceed in our analysis now? How can a sentence expressed in conventional reality receive the value (e)?

Priest suggests a new valuation function, μ, which maps all values in the set $\{t, f, b, n\}$ to (e), such that $\mu(v(A) = e$, then $\mu(v(\neg A) = \mu\Big(v(A \wedge B) = \mu\big(v(A \vee B)\big)\Big)) = e$. This new truth-value should not be designated. If it were designated, the negative *catuṣkoṭi* would exemplify a trivialism, and Priest (2010a) holds this, for good reasons, to be untenable. That means, the connectives for (e) will put out (e) in every instance, the arrows in TABLE 2 are supposed to represent this mechanism. The negation of (e) is of course (e).

Whereas I have been talking about statements, propositions, things, or objects with respect to the positive *catuṣkoṭi*, the bearer of (e) has to be reconceptualised. We are no longer talking about statements, propositions, things, or objects, such as the positive *catuṣkoṭi* does, but we are talking about *states-of-affairs*. In other words, whereas the positive *catuṣkoṭi* is talking about statements made in and about conventional reality, the negative *catuṣkoṭi* is talking about ultimate reality *itself;* whereas we had a purely semantic thesis with the positive *catuṣkoṭi*, we now have a metaphysical[50] thesis with the negative *catuṣkoṭi*. We can now ask what the philosophical significance of the negative *catuṣkoṭi* is. An intelligible answer, I think, can only be given provided one escapes the implicit logic-first bias and the historical and contextual limitations discussed in the first part of the thesis. By this I mean: the logic should follow the phenomenon, not vice versa. The phenomenon is largely epistemic/phenomenological in terms of access, and metaphysical in terms of content. Also, but staying within the margins of Madhyamaka philosophy, it will be difficult to assess the logico-epistemic-metaphysical significance of the negative *catuṣkoṭi,* which became more overt only after Buddhism underwent some substantial developments.

Priest does escape those biases and limits, reading Nāgārjuna through the lens of Zen teaching. Priest's motivations here are rather opaque. It is not totally clear why Priest adheres to Zen teaching in this situation, given the historical and geographical divide between Nāgārjuna and Zen Buddhism. What I wish to make clear, however, is that it works, and helps to make sense of the situation. The next chapter, §2.3, addresses this

[50] This is, of course, logic-epistemic phenomeno-metaphysical, to be precise.

point by drawing explicit lines between Nāgārjuna's negative *catuṣkoṭi* and Zen teachings. So, if the reader feels a sense of bewilderment given Priest's argumentative jump, I invite the reader to return to this page after reading through §2.3.

Priest say that realizing, or rather the process of realizing, that the distinction between conventional reality and ultimate reality is itself a figment of merely conventional thought corresponds to the process of enlightenment. The background assumption is that the Bodhisattva becomes enlightened once the Bodhisattva realizes that conventional reality *fails to exist ultimately*. Moreover, the Bodhisattva has to realize that the emptiness of ultimate reality *is itself empty*—the reflective arrow in TABLE 3 shall represent this mechanism. Call it the emptiness of emptiness.

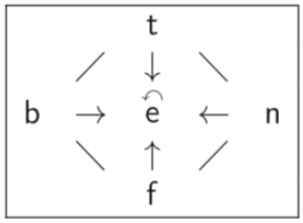


TABLE 3: The *emptiness of emptiness*

This is all very cryptic, I admit, even if the Hasse-diagrams help visualizing what is supposed to be going on inside the negative *catuṣkoṭi*. As discussed in the introduction, Madhyamaka thinkers hold that for a phenomenon to exist *at all* is for it to be empty of *svahbava*. From a soteriological/epistemic/phenomenological perspective, that means that realizing the emptiness of all phenomena is the same as realizing their (true) existence. From this point, one can jump into the abyss of enlightenment, knowing that reality will not bottom out, the Zen Teachers would say. I hope this to become clearer once we have reached §3. The soteriological end-goal of enlightenment is therefore not the extinction of self and the reality, as the negative *catuṣkoṭi* might suggest, it is rather the achievement of a deep-seated understanding of the inherent, empty nature of reality. This, in effect, also means that archiving this understanding does not change the world of appearance *in re*, but ostensibly only the appearance of the world in the eyes of the enlightened person.

Once enlightened, the soteriological utility of the negative *catuṣkoṭi* vanishes and so $\mu = e$. In other words, enlightenment stops a process of mapping all the values in the set $\{t, f, b, n\}$ to (e). But is also means that the set $\{t, f, b, n\}$, which represents conventional reality, remains what it is, namely a model of conventional reality.

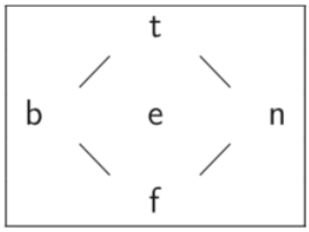


TABLE 4: Awakening

If this has been too fast to follow, I agree, and ask the reader to bear with my until the end of the next chapter when, I hope, it becomes clearer why interpreting the negative *catuṣkoṭi* as read through the lens of Zen Buddhist thought on enlightenment is in fact an ingenious move. It might feel so fast because Priest and Garfield have skipped several epochs of development of Buddhist thought, from Madhyamaka to Zen. The discussion of §2.3 will work as a magnifying glass for the epochs in-between and will hence work as a detailed argument for how and why Priest and Garfield could fast forward in the development of Buddhist thought.

Validity and FDE

Employing FDE—or FDE(S) and FDE_e, for that matter—as a model for the *catuṣkoṭi* is by no means unproblematic. First, what to make of koti (1) and koti (2) in light of a paraconsistent logic? How can we express the classical "just true" and "just false"[51]? Second, it is well known that several classical principles such as *modus ponens, modus tollens* and *hypothetical syllogism* fail in FDE. Therefore, they also fail in the Priest's model of the *catuṣkoṭi*. Yet, as Cotnoir points out, Nāgārjuna regularly employs the *modus ponens, modus tollens* and *hypothetical syllogism* throughout the MMK[52]. FDE is thus in conflict with certain parts of the MMK, even though it might be an adequate model for the *catuṣkoṭi*. This would make Nāgārjuna look either unreasonable, or else in a constant predicament. Unless we would think of Nāgārjuna as a contextual pluralist about logic,

[51] For a recent strategy on how to articulate the "just true/just false" in a paraconsistent logic, termed 'shrieking', see Beall (2013) and a response in Scharp (2018). Note that the 'just-true' problem is merely one instance of the classical recapture problem: how to make sense of cases, where the inconsistency of A and its negation is assumed?

[52] For present purposes, it will suffice to give the counterexample to one of the classical principles only. For the other counterexamples, see Cotnoir (2015).

using one framework here, another there, without the need to establish consistency between them, this constitutes a problem[53]. To give you one apparent example of a *modus ponens* in the MMK, there is this:

$$Modus\ Ponens: A \supset B, A \vDash B$$

„When there is change, there is motion. Since there is change in the moving, motion is in that which is moving." (*MMK* 2.2)

Since the material conditional in FDE is a hidden disjunction (by definition, $v(A \supset B) = v(\neg A \vee B)$), there is a counterexample to *modus ponens* in FDE. Take $v(A) = b$ and $v(B) = f$. Remember, $D = \{t, b\}$. The second premise is designated since $v(A) = b$. What about the first premise? Negation does not toggle b, so $v(\neg A) = b$. The way that disjunction is defined, the *least upper bound* of b and f is b, thus $v(\neg A \vee B) = b$. Since $v(A \supset B) = v(\neg A \vee B)$), the first premise $'v(A \supset B)'$ is also designated. By stipulation, $v(B) = f$, and is thus undesignated. Hence, we have designated premises, but an undesignated conclusion. Validity is defined as the preservation of the designated value. *Modus ponens* fails.

Similar counterexamples can be found for *modus tollens, disjunctive syllogism* and *hypothetical syllogism*, and similar instances of Nāgārjuna employing them in the MMK can be found as well.

Hence, the Priest interpretation has a serious problem: If FDE were Nāgārjuna's logic of choice, those arguments above are invalid. Or, if they were valid, the *catuṣkoṭi* would collapse unless we could find a model which is compatible with *modus ponens*. As it stands, FDE faces the classical *Recapture Problem*; how to make sense of *modus ponens* (and potentially other classical inferences; Cotnoir (2015) provides a provisional list) if they are ruled out by the background paraconsistent logic?

[53] One could, of course, think of Nāgārjuna as a logical pluralist who is using one logic hear, and another logic there, without the desire to render them jointly coherent. A problem with this approach is that Nāgārjuna (as did all the other philosophers of his time an area) did not use any "logic", *at all*. What it would mean for Nāgārjuna to be a contextual pluralist is to change his alethic commitments from context to context. What that would mean in practice, I am not sure. In any case, I see no textual evidence for contextual pluralism, and I am furthermore suspicious of the psychological viability of a contextual position. In any case, discussing this option in depth would lead us to far astray.

There are (at least) two principled ways of dealing the recapture problem: The first is to find a paraconsistent and paracomplete logic that recaptures *modus ponens*. The second approach, of which we have seen some unsatisfactory examples in §2.1, is to search for a classical semantics which meets the exclusivity- and exhaustivity-constraint.

Cotnoir's Solution to the Recapture Problem

Cotnoir's (2015) proposal for how to accommodate the weakness of FDE is upfront. He takes his inspiration from suggestions in Deguchi, Priest and Garfield (2008) who understand the positive *catuṣkoṭi* as a tool supposed to undermine the conventional reality, not as a characterization of the ontological status of the conventional perspective itself. On their reading, contradictions applied to conventional reality are merely *prima facie,* this is contradictory on a first, but not on a second look.

Cotnoir tries to evade the Recapture Problem by disambiguating the contradictions between the conventional and the ultimate perspectives. He suggests that the positive *catuṣkoṭi* blends both perspectives so that contradictions on the conventional level dissolve. To model this idea, he introduces an alternative lattice which is still four-valued, retains $\{t, b\}$ as designated values, but is formulated in plain Boolean Algebra with the effect that the crucial classical principles can accommodated.

In detail, Cotnoir takes the truth-values to be ordered pairs such that both perspectives are represented in the truth-value. The first member of the pair answers to 'Is it conventionally true?' with affirmation '1' or denial '0', the second answers to 'Is it ultimately false?' also with affirmation '1' or denial '0'. With this, Cotnoir reproduces the four corners of the *catuṣkoṭi* within a Hasse-diagrams (see TABLE 5). The possible combinations, which match the four kotis,are: (1,1), (1,0), (0,1), (0,0). (1,1), for instance, means conventionally true, and ultimately false; (0,0) is the opposite. Disjunction and conjunction are invariably defined as *least upper bound* and *greatest lower bound*. One difference is that negation not only toggles (0,1) and (1,0), but also (1,1) and (0,0). The semantics for negation thus denies the possibility of contradictions on the conventional level. Since Cotnoir retains $D = \{t, b\}$, (1,1) and (1,0) are designated. Validity ($\vDash_{B4}$) is defined in the usual designation-preserving way. The semantics generates are purely classical propositional logic, but a four-valued one. In other words, a four-valued Boolean algebra, call it B4. Given that Cotnoir's proposal is formulated in plain Boolean algebra,

it validates the *modus ponens* that FDE is too weak to validate[54]. We see this by looking at the semantics for the connectives, which are fully classical.

- Conjunction: $v(A \wedge B)$, is the greatest lower bound of $v(A)$ and $v(B)$.
- Disjunction: $v(A \vee B)$, is the least upper bound of $v(A)$ and $v(B)$.
- Negation: Where $v(A)$ = (x, y) the negation is $v(\neg A)$ = (1 – x, 1 – y).

Hence, where $v(A)$ = (1, 1), the negation $v(\neg A)$ = (0, 0) and where $v(A)$ = (1, 0), the negation $v(A)$ = (0, 1). Negation works in such a way that a proposition and its negation are never both conventionally true. As this is what produced the problem with *modus ponens* in FDE, as discussed on page 43 of this thesis, Cotnoir solves it.

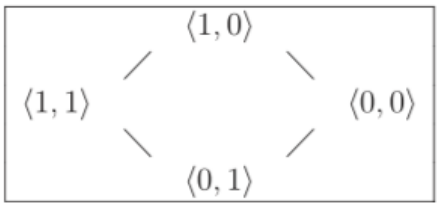


TABLE 5: Cotnoir's Lattice

Cotnoir's interpretation of the *catuṣkoṭi*, which is hinted at in Garfield and Priest (2003) and Garfield (1995), is based on the Tibetan Buddhist thinker Tsongkhapa's reading of the *catuṣkoṭi* which is recognised for its excessive engagement with the ultimate/conventional distinction. With respect to MMK 18:8, Tsongkhapa's interpretation would reads as follows:

> *Everything is real [conventionally] and is not real [ultimately].*
> *Both real [conventionally] and not real [ultimately],*
> *Neither real [ultimately] nor not real [conventionally].*
> *This is Lord Buddha's teaching.*

Tsongkhapa's excessive perspectivalism is not without criticism. See how Tillemann (1977:24) comments on this interpretation:

[54] Also, the *just-true problem* is solved since truth is conventionally identical to un-falsity, and falsity to un-truth.

„Indisputably, Tsongkhapa's interpretation offers advantages in terms of its logical clarity, but as an exegesis of Madhyamaka, his approach may seem somewhat inelegant, since it obliges us to add words almost everywhere in Madhyamaka texts. Remarkably, Tsongkhapa himself accomplishes this project down to its most minute details in his commentary on the Madhyamakakarikas—perhaps at the price of sacrificing the simplicity of Nagarjuna's language. Hence, is there a more elegant interpretation of the tetralemma...?"

The same criticism applies to Cotnoir's proposal: B4 offers a certain logical clarity on the cost of metaphysical confusion. Throwing the ultimate and conventional perspective together takes away the initial appeal and dilutes the distinction into a positive and a negative *catuṣkoṭi*. Furthermore, and most importantly, what looks like the perfect logic for Nāgārjuna cannot account for the negative *catuṣkoṭi*, utilized as a *prasanga* argument. As this is the predominant role of the *catuṣkoṭi* within the MMK, I take Cotnoir's proposal to be misguided (with respect to the MMK). To see that, remember that the B4-semantics implies that no proposition and its negation can both be ultimately not-false. But from the ultimate perspective, the *catuṣkoṭi* says that being true is not the same as being not-false. This is nicely translated into the model in FDE model, but B4 misses out on that by how negation is defined. So, this seems to be the flipside of turning away from the non-classical framework which seems to be well equipped to model logico-epistemic-metaphysical levels of, or perhaps we could say perspectives on, reality. As Cotnoir acknowledges, the B4-semantics is useful only as a model for the positive *catuṣkoṭi*. It is still FDE(e) that is the adequate logic for the negative *catuṣkoṭi*, but that again suffers from the Recapture Problem.

But let this not be the end of the story. It will get even worse for B4, which, I will argue, is an infelicitous choice even as a model for the positive *catuṣkoṭi*. This is because B4 will eventually be incapable of meeting the exclusivity- and exhaustivity-constraint.

I shall first address two concerns I have with B4 and motivate a plausible rejoinder on behalf of Cotnoir. This is not to say that those responses are the only worth considering, there may as well be better responses, maybe even responses that would suppress the collapse-argument that I will put forward. Nevertheless, I take my responses on behalf of Cotnoir to be the natural and obvious. What follows are three collapse-arguments. After the first two, I will discuss ways of escaping the collapse on behalf of

Cotnoir. The third argument, however, will be decisive, I reckon. This is, even if Cotnoir's proposal can be changed so to adapt to the first to challenges, the resulting regimented version of B4 will still succumb to the third argument.

First Concern: Classical Operators

B4 works with the classical operators, and as we saw in the discussion in §2.1, the classical operators, especially classical negation, lack the expressive power to meet the exclusivity- and exhaustivity-constraint. Since we know that adding a set of status-predicates to FDE does a satisfying work, we can try adding status-principles to B4. This is how the corners of the B4-lattice get assigned their corresponding status-predicates[55]:

B=(1,1), T=(1,0), F=(0,1), N=(0,0)

Call the resulting logic 'B4S'. The introduction of status-predicates has one major downside: The status-predicates cannot reflect the levels of reality we find in the Madhyamaka metaphysics in the way the ordered pairs do. But this is no surprise, given that we already know that B4 is not the right logic for the (ontologically-loaded) negative *catuṣkoṭi*. One way of mimicking bicameral reality in our assignments of status-predicates to B4 is this, where the indexed C reflects conventional reality, and the indexed U reflects ultimate reality[56].

$$(1,1) = (T_C, F_U)$$
$$(1,0) = (T_C, \neg F_U)$$
$$(0,1) = (F_C, F_U)$$
$$(0,0) = (F_C, \neg F_U)$$

While this assignment of status-predicates does capture the bicameral nature of Buddhism metaphysics, the problem is that, as far as common interpretations of Buddhist texts go,

[55] Compare that with TABLE 5.

[56] Thanks to Aaron Cotnoir for suggesting this assignment of status-predicates.

it is only the conventional type of truth that language can express[57]. Whatever happens in ultimate reality—and hence also any truth or falsehood about ultimate reality—remains ineffable. If this were right, then whatever kind of metalinguistic status-predicate we'd use for Cotnoir's ordered pairs, it reduces to a statement about conventional reality, and thus cannot adequately express the four corners of the *catuṣkoṭi* without collapse. Cotnoir's *catuṣkoṭi* would reduce to T_C and F_C.

One rejoinder would be to admit that we can have a status predicate for ultimate reality, but that it cannot be used to speak truly or falsely; it can only express things of the form $N_U(A)$, which is to say that from the ultimate perspective, things are ineffable.[58] The worry here is the same as above: Such a combination of status-predicates will only ever express two possibilities, (T_C, N_U) and (F_C, N_U). Hence, the only way to introduce a metalinguistic status-predicate for Cotnoir's ordered pair is by taking the ordered pair to be a conjoint claim, such as *'A is conventionally true and ultimately not false*', which is reflected in the ordered pair (1,0) and will be expressed with the status-predicate T.

Second Concern: Transparency

My second worry concerns the transparency of the truth-predicate in B4. The truth-predicate T in FDE(S) satisfies the unrestricted T-schema, it is thus transparent. A transparent truth predicate T is one that, paired with some quotation device '[]', allows for any well-formed-formula, A, for instance to be substituted by $T(A)$, and vice versa, in all extensional contexts and in all arguments without a change in validity.

$$Unrestricted\ T-Schema\ (UT): T[A] \leftrightarrow A$$

One could argue that transparency of the truth-predicate is something that Nāgārjuna would not have endorsed. Priest (2010a:24) explicitly addresses this thought, when he says that

"I do not know of any Indian textual sources that endorse this [the unrestricted T-schema]*, but it seems so natural and obvious that it is plausible that it was taken for granted. In Western philosophy, it has been problematized by paradoxes such as the Liar. But, again as far as I know, there is no awareness of these in early Indian philosophy."*

[57] See Priest (2018:64) and Westerhoff (2009:64,75)

[58] We find such a proposal in Priest (2010: 24)

The unrestricted T-Schema has been problematized by paradoxes of self-reference since, together with classical logic, via the principle of explosion, it leads to trivialism[59]. In the case of the *catuṣkoṭi*, we would suffer collapse of the kotis if our logic were classical, such as B4S, and the truth-predicate transparent, which it most probably is, given that it inherited the status-predicates from FDE(S). To see this, follow the reasoning of Priest (2012:32):

$$\begin{aligned} &Unrestricted\ T-Schema: && T[A] \leftrightarrow A \\ &Contraposition: && \neg T[A] \leftrightarrow \neg A \\ &Unrestricted\ T-Schema && T[\neg A] \leftrightarrow \neg A \\ &Transitivity && T[\neg A] \leftrightarrow \neg T[A] \end{aligned}$$

The second koti $F(A) \wedge \neg T(A)$ is be equivalent $T(\neg A) \wedge T(\neg A)$, which is $F(A)$. Since we are working in a bivalent framework, the falsity of A is equivalent to the truth of not-A. The same happens to $T(A)$. Since we express the third koti as $T(A) \wedge F(A)$, it is obvious that koti three collapses into koti one and two. Moreover, koti four is equivalent to koti three. Again, the *catuṣkoṭi* suffers complete collapse.

The price one has to pay for an unrestricted T-schema is usually the replacement of the classical biconditional with a non-contraposable non-classical one. Priest, unlike Cotnoir, avoids this by going non-classical. The biconditional in FDE(S) is not contraposable[60],

and that is why it avoids the collapse of the kotis.

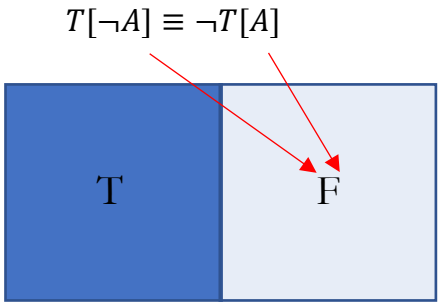
Since B4 is classical, semantically closed, and (to make sense of the exclusivity/exhaustivity constraint), inherits the transparent truth-predicate from FDE(S) it leads to trivialism. This is indeed an unacceptable consequence, but a very obvious one[61].

[59] See Priest (2006).
[60] See Priest (1987; 4, 5). A proof for the non-contrabosability of the the biconditional in FDE(S) can be found in Priest (2010a:51).
[61] Which is why we might question whether the MMK requires accepting transparent truth. Indeed, there is perhaps nothing in the MMK itself that requires transparent truth (see Priest, 2010a:24). But it is in itself a plausible idea and there is nothing in the MMK that would directly speak against it. I have to thank Aaron Cotnoir for pushing me on this point.

However, let this not be the end of the story for B4. No one says that it has to be irrevocable that an unrestricted T-schema plays havoc with classical logic. First, new developments called *Strict–Tolerant Conceptions of Truth*[62] introduce a transparent truth-predicate which does not lead to trivialism in a classical framework. Second, it might also be possible to introduce a set of status-predicates with a restricted T-schema—this is to say, with an opaque truth-predicate[63]. The advocate of the latter position would have to tell a story about paradoxes of self-reference, but I don't want to rule out such a position on *a priori* grounds. Cotnoir is not committed to transparency and so I take it that either one of those possible rejoinders can be a promising option for B4S. If one of those proposals is fruitful, B4 must not lead to trivialism, unless there were an argument about B4 collapsing even with an opaque truth-predicate.

Third Concern: Collapse

Let us assume that we have introduced a non-transparent truth-predicate by restricting the T-schema, or otherwise a transparent truth-predicate which does not lead to trivialism in a classical framework. Moreover, provided we are in classical logic, assume double negation $\neg\neg A \equiv A$ and monotonicity $T[A] \wedge T[A] \equiv T[A]$. $F[A]$ entails $\neg T[A]$. To see this, remember that by the (restricted) T-schema, the truth-condition for $T[\neg A]$ to be designated needs $\neg A$ to be designated. Since we are in classical logic it follows that A is not designated. Since $\neg T[A] \equiv F[A]$, $T[A]$ must be f, since negation toggles f and t, and $\neg T[A]$ must be t. To make this graphic, consider that if we are working in a bivalent framework $T[\neg A] and \neg T[A]$ are equivalent:



[62] See Cobreros, Egre, Ripley, van Rooij (2013).

[63] Which will also help with the problem of 'just truth' and 'just falsity'.

If this reasoning were correct, we'd again suffer collapse of the *catuṣkoṭi*.

Koti1:	$T(A) = T(A) \wedge \neg F(A)$	$\equiv$	$T(A) \wedge \neg\neg T(A)$	$\equiv$	$T(A)$
Koti2:	$F(A) = F(A) \wedge \neg T(A)$	$\equiv$	$\neg T(A) \wedge \neg T(A)$	$\equiv$	$\neg T(A)$
Koti3:	$B(A) = T(A) \wedge F(A)$	$\equiv$	$T(A) \wedge \neg T(A)$		
Koti4:	$N(A) = \neg T(A) \wedge \neg F(A)$	$\equiv$	$\neg T(A) \wedge \neg\neg T(A)$	$\equiv$	$\neg T(A) \wedge T(A)$

Here is how to unpack the argument above: First, we take the kotis as defined in Priest (2010a) on the left side of the equation sign. To formulate the 'just-truth' of the first, and the 'just-falsity' of the second koti, a conjunct is added. This is to guarantee the exclusivity of the first two kotis; $\neg F(A)$ in the case of the first koti, $\neg T(A)$ in the case of the second. Assuming double-negation, which is a law in classical logic, we can reformulate $\neg F(A)$ as $\neg\neg T(A)$, eliminating the double-negation, and we get $T(A) \wedge T(A)$. Assuming monotonicity, which is also unproblematic, we can reformulate the first koti as $T(A)$, which, for now, is just fine. For koti two we write $F(A)$ as $\neg T(A)$. This is unproblematic since negation works classically and 'just-falsity' is nothing but 'un-truth'. Again, have a look at the quadrants above. If T and F exhaust all the possibilities, F is in the right box, as is $\neg T$. Assuming monotonicity, we can formulate the second koti as $\neg T(A)$. The right side of the equation is how Cotnoir would formulate the kotis in classical bivalent logic. To continue, we may also reformulate $F(A)$ in koti three as $\neg T(A)$. The first conjunct remains unchanged. For koti four we write $\neg F(A)$, assuming double-negation, as $\neg\neg T(A)$, and if we were to eliminate the double-negation, we are left with $T(A)$. The first conjunct remains unchanged.

What should be apparent now is that the fourth koti is equivalent to the third koti. Both entail koti one and koti two; koti three and four are redundant. The classical framework of Cotnoir's proposal has the collapse of the *catuṣkoṭi* as a consequence.

Returning to FDE(S) would thus be the most obvious response. But if we were still to take Cotnoir's recapture-concerns seriously, this would mean going back to square one.

But wait! Is there really a Recapture Problem?

What if we have been barking up the wrong three all the time? If Cotnoir's proposal fails, we are back at FDE(S) and the recapture problem. But there is a way to return to FDE(S) without facing Cotnoir's problem if we stop thinking about the *catuṣkoṭi* (and its logic) as providing the basis of the relation of logical consequence for Nāgārjuna and think about the *catuṣkoṭi* simply as the basis of the Madhyamaka metaphysics[64]. Cotnoir assumes that the *catuṣkoṭi* is Nāgārjuna's basis of the relation of logical consequence. This is why FDE(S) has the weakness it has. Let this be the first way to counter Cotnoir's recapture problem.

Moreover, as we now know, taking the material conditional as the conditional is an infelicitous choice for the FDE-ist. Cotnoir himself briefly considers using the FDE-lattice (with additional resources) as a basis for a relevant conditional. The relevant conditional is known to solve some of the issues with classical inferences as it can be constructed to validate *modus ponens, modus tollens*, and *hypothetical syllogism*. Yet, the relevant conditional makes it difficult to model restricted quantification due to the strength of the conditional. Hence, Cotnoir is right to dismiss this idea. In support, he quotes a case of restricted quantification in the MMK (13:1): *"Whatever is deceptive is false. Compounded phenomena are all deceptive. Therefore, they are all false."*

One way of getting around the recapture problem is Priest's *Methodological Maxim* (1987:8.4), which, applied to FDE, suggest taking material detachment (i.e. *modus ponens*), and, more generally, classically valid inferences that are not valid in FDE as acceptable 'default inferences', calling them *quasi-valid inferences*. Is this more than just a cheap trick? As we saw in Cotnoir's counterexample to *modus ponens*, these inferences arise only when truth-value gluts or truth-value gluts occur in the reasoning. Given that, as Priest holds, in vernacular reasoning gluts and gaps are rare (this is, statistically uncommon), they come with a low *a priori probability* of obtaining. The idea is that we are justified in reasoning classically on the assumption that we are neither dealing with gluts, nor gaps. In ordinary discourse, this is certainly a plausible maxim.

Consider this example: Given some usual cat we are entitled to assume that is has four legs unless we have good *a priori* reason to think otherwise, or we (at some point) learn otherwise. The same epistemic principle applies to logical reasoning: If we reason about a situation, we are entitled to think that it is consistent until we learn otherwise. The

[64] In the same way, the PNC and PEM are metaphysical principles of Aristotle, and his account of logical consequence (syllogistic) is quite independent of this. Thanks to Graham Priest for making me aware of this.

belief that the cat has four legs is therefore fallible, as is the belief that a given situation or concept is consistent. In other words, we may have the epistemic inferences $\Sigma \models A$, but still $\Sigma \cup \Pi \not\models A$. Fallibility due to new information (Π) is, then, built into the very notion of logical consequence, a phenomenon known as "monotonicity" when part of the logic. Does this strategy work for the case of Nāgārjuna?

It does, but surprisingly not for epistemic but metaphysical reasons: Taking the underlying metaphysics and the ultimate aim of the MMK, which is to establish the lack of *svahbava,* into account, we realise that for Nāgārjuna it is the inconsistent situation which is the default position (at least from the perspective of ultimate reality).

Nāgārjuna has a maximally high *a priori probability* for dealing with inconsistent concepts in ordinary reasoning, other than the ones we find in 'ordinary' logico-mathematical contexts; change and motion, for example, are inconsistent concepts, according to the MMK. But, if both change and motion are dialethias, both premises and the conclusion in the instance of the MMK that Cotnoir thinks ground the recapture problem are designated given that $D = \{t, b\}$. MMK (2:2), this is *„When there is change, there is motion. Since there is change in the moving, motion is in that which is moving.",* which has the form of a *modus ponens* (A⊃B, A⊨B) is thus valid in FDE(S).

So, what went wrong with Cotnoir's argument? The truth-value assignment *v*(A)= b and *v*(B)= f, which makes *modus ponens* invalid in FDE(S) is an, in the context of the MMK*, impossible* truth-value assignment. On the grounds of the MMK, we can argue whether *v*(A)= b and *v*(B)= b are adequate truth-value assignments, or not. As the MMK is not trying to establish that the concepts in question are pure dialethias, but that they lack *svahbava,* the only plausible truth-value assignment for all the concepts in the MMK is (e). The above is no exception; its (ultimate) truth-value assignment must be v(A)= e and v(B)= e. Whether the modus ponens above is valid, then, hinges on the status of the fifth truth-value (e). In the semantics provided above, (e) is not a member of the set of designated values, and as validity is defined as the preservation of designated values, reasoning that includes empty concepts is invalid on Cotnoir's proposal. But it is not clear why (e) shouldn't be designated. To me, there is no evident reason why it should not be. It is now questionable whether there has ever been a recapture problem, *at all.*

A Proposal

What if we are unsatisfied with a paraconsistent and paracomplete treatment of the *catuṣkoṭi* for reasons other than the collapse argument? Let them be doctrinal, for instance. Can there really be no formulation in classical terms without a collapse? I shall now introduce an alternative model for the *catuṣkoṭi* that I, given the philosophical milieu in which the *catuṣkoṭi* has predominantly been employed, find very compelling. This is, at last more compelling than Priest's non-classical approach with respect to a fidelity to the writings and discourse style of much of the Buddhist philosophical canon.

Let me begin by repeating the minimal desiderata: What we are seeking is an adequate formulation of the *catuṣkoṭi* within a logic in which the *modus ponens* is valid, which avoids trivialism, but does not make the kotis collapse into each other. The clue is to think of negation as an illocutionary act[65].

Remember that in Westerhoff's classical model (2009: 4) we find a different type of negation for koti four such that koti three is no longer deducible from it. If we could find a way of implementing Westerhoff's analysis of negation into B4, such that koti three is no longer equivalent to koti four and neither of them entails the firth two kotis, we could resist the collapse while remaining fully classical. This is, in essence, what I wish to propose.

Let us first define the behaviour of the external (*prasaja*) negation $'\times'$ and its relation to classical negation $'\neg'$:

- $\times$ does not toggle anything
- Neither does $\neg$ entail $\times$, nor $\times$ does not entail $\neg$
- Double-negation elimination does not hold for $\times\times$ [66]
- Double negation elimination neither holds for $\times \neg$, nor $\neg \times$
- $\times$ is considered an illocutionary negation: the speech act of 'denial'[67]

While the first four points characterize the behaviour of the external negation, the last point is the most important for the following discussion. Mohanata (2009) and Westerhoff (2009) mention the concept of *prasajya pratisedha* as a non-relational negation

[65] Illocutionary negation (denial, or denegation) has been defined in Searle (1969).

[66] Denying the denial of something doesn't amount to asserting it. I am following the standard definition of external negation, found in Horn (1989).

[67] We thereby extend the notion of the presupposition-cancelling (*prasaja*) negation to a more general notion, as denial (as a speech-act) can do more than merely cancelling a proposition, e.g. express a lack of evidence or understanding. See Searle (1969: 31-33).

corresponding to the modern notion of denegation, or illocutionary negation. I follow their lead and think of *prasaja-negation* in terms of a speech act; the speech-act of 'denial', to be precise. This is a reasonable move since a denial works, just as the external negation, on the sentential level and is equally not presupposition-preserving.

It has been suggested that instead of *shrieking*, one might as well *whisper* to regain classicality in paraconsistent contexts[68]. This is to say that the glut theorist could express classicality by way of conversational implicature. Working with B4S, however, we don't need to regain classicality. Nevertheless, the core idea of the 'shrieking' approach is interesting: Propositions containing a content can be pre-fixed with an illocutionary operator.

Using the external negation as an illocutionary act lets us reject the presupposition made by the proposition that is to be negated, as discussed above with reference to Westerhoff (2009). Another interesting point corresponds to our epistemic situation. Sometimes we plainly lack the crucial evidence which would make a proposition either true or false, decisively. Considering the *catuṣkoṭi* and the way it is used as a *prasanga* argument, many of the propositions of the MMK may be of this kind.

Negation as a speech act of denial, regardless of how one accounts for it can only be restricted to the negation of propositions, as, from a semantic viewpoint, we do not normally deny any constituent smaller than a proposition in a conversation. Or else, at least if we disagree with some part of a sentence, it would not be expressed as a simple negation *qua* speech act of denial but as an expression of disagreement[69]. In my proposal I take the illocutionary act to work on the level of truth- and falsity-assignments expressing a *quasi-proposition* which reads "… the truth of A". The illocutionary act of denial would thus read '*I deny*…the truth of A, i.e. that the truth of A obtains'.

I take this to reflect the argumentative framework of the dialogues between teacher and student in which the *catuṣkoṭi* in form of a *prasanga* argument is usually employed. We may model this dialogue with the help of speech-acts:

Take the following passage which Radhakrishnan and Moore's (1957:28) translation of a conversation between Gotama and an interlocutor about the existence or nonexistence of the *Tathagata*[70] (the saint) after death. Notice that one of the most

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[68] See, *inter alia*, Shapiro (2004), Priest (2006).

[69] Thanks to Ahmad Jabbar for pressing this point.

[70] There is no universal agreement the translation of the sanskrit word. It is often thought to mean either "one who has thus gone" (*tathā-gata*) or "one who has thus come" (*tathā-āgata*). Maybe here is a 'neither' or 'both' assignment at work too. The Tathagata would thus be the one beyond

prominent features of this conversation is that every truth-assignment and falsity-assignment is followed immediately with a speech-act of denial (in other dialogues, by affirmation).

> *V1: How is it, Gotama? Does Gotama hold that the saint exists after death, and that this view alone is true, and every other false?*
> *G1: Nay, Vacca. I do not hold that the saint exists after death, and that this view alone is true, and every other false.*
> *V2: How is it, Gotama? Does Gotama hold that the saint does not exist after death, and that this view alone is true, and every other false?*
> *G2: Nay, Vacca. I do not hold that the saint does not exist after death, and that this view alone is true, and every other false.*
> *V3: How is it, Gotama? Does Gotama hold that the saint both exists and does not exist after death, and that this view alone is true, and every other false?*
> *G3: Nay, Vacca. I do not hold that the saint both exists and does not exist after death, and that this view alone is true, and every other false.*
> *V4: How is it, Gotama? Does Gotama hold that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false?*
> *G4: Nay, Vacca. I do not hold that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false.*

It is obvious then, that the Buddha does not simply give four answers to four questions. He is rather denying that he believes in any of those possibilities. This belief could be expressed doxastically, but since we are not concerned with what the Buddha believes, but rather with what the Buddha asserts or denies, I take the illocutionary route to be one that is more promising[71].

Challenges for the Speech-Act Model

coming and going, the one who has passed the world of transitory phenomena. Perhaps, the catuṣkoṭi is supposed to express that.

[71] The act of not-asserting must hence be equivalent to the act of denying.

This speech-act based proposal, which seeks to capture the conversational setting of Buddhist philosophical argumentation—which must remain in the level of a proposal, some of the technical issues are not yet sufficiently developed and have to be looked at in detail in future research—has to be assessed with respect to challenges we have encountered already—recapture and collapse—plus some that are specific to this model:

First Challenge: Priest (2010a) is right that making explicit that appeal to speech acts inside propositional contexts is pointless. The first koti formulated with the illocutionary negation of the second conjunct, $A \wedge \times \neg A$, in a logic without status-predicates doesn't make sense as there is yet no difference as to the level on which the two negations work. Taking the second koti as a single proposition, we would have a speech-act as part of the content of a proposition, but this is not how a speech-acts usually work. The same applies to the second koti. For my proposal to be successful, this has to be avoided. Let this be the first challenge.

Second Challenge: The fourth koti, formulated as $\times (A \vee \neg A)$, cannot hold. This is because A can't both hold and fail to hold at the same time. There must be an intelligible formulation for the fourth koti in B4Sx. Let this be the second challenge.

*Third Challeng*e: It would be advantageous if the proposed logic could, unlike its friend B4, accommodate the negative *catuṣkoṭi* as well.

The catuṣkoṭi in B4Sx

A logic built on B4, which has the status-predicates, and the illocutionary negation as a supplemented operator will be called B4Sx. As already mentioned, there is no direct textual evidence that there are two different negations at work in the *catuṣkoṭi*. Hence, since every negation within the *catuṣkoṭi* is potentially ambiguous, I will formulate the *catuṣkoṭi* with the help of illocutionary negations only. Regarding the fact that the most common use of the *catuṣkoṭi* is within a conversational setting, the idea of exclusively utilizing illocutionary acts in a modelling seems reasonable to me. The suggested formulation of the *catuṣkoṭi* in B4Sx is the following:

Positive catuṣkoṭi 1.0

Koti 1: $T(A) = T(A) \wedge \times F(A)$

Koti 2: $F(A) = F(A) \wedge \times T(A)$

Koti 3: $B(A) = T(A) \wedge F(A)$

Koti 4: $N(A) = \times T(A) \wedge \times F(A)$

In order to formulate the negative *catuṣkoṭi* I take the *denial of all the alternative*s to be expressible with the help of a speech-act, too. Let this speech-act be 'silence'. Or, more appropriate to the Madhyamaka context, 'ineffability', expressed by the $\otimes -operator$, which denies the existence of *svabhāva* for what is presupposed in the kotis. Call it the $sunyata - operator$.

It is important to note that 'silence' itself doesn't classify as a classical speech act in the Searleian sense, albeit as a form of communication. It is not literal silence about A, as in *not talking where talk is expected*. What the $'\otimes'$ is supposed to express instead is a statement about the ineffability of the status of (being silent about) A.

We want to apply the $\otimes -operator$ to A whenever A is not part of our language. This is, whenever *A* exceeds the conventional realm, and with it, conceptual language and thought. Significant strands of the Buddhist tradition that explicitly asserts that claims about ultimate reality are *beyond language*[72]. Madhyamaka is one of them. Hence, what a proper model needs to avoid is to make assertions about ultimate reality, and so I define $'\otimes'$ in terms of $'\times'$, denying assertions about the truth or falsehood of A. The upshot is this: By denying $T(A)$ and denying the denial of $F(A)$, one does not affirm $T(A)$ or $F(A)$ but remains silent about it.

Negative catuṣkoṭi 1.0

Koti 1: $\otimes T(A) = \times T(A) \wedge \times\times F(A)$

Koti 2: $\otimes F(A) = \times F(A) \wedge \times\times T(A)$

Koti 3: $\otimes B(A) = \times T(A) \wedge \times F(A)$

Koti 4: $\otimes N(A) = \times\times T(A) \wedge \times\times F(A)$

[72] In the Indian Buddhist tradition, there is a lively debate about the question whether such statements as "everything is empty" or "nothing exists by *svabhāva*" are correct expressions of (parts of) ultimate reality. I take it to be a plausible interpretation of the MMK and shall refrain from going into detail on this discussion. This might have the downside that my model for the *catuṣkoṭi* only works in the context of Nāgārjuna's philosophy. Even more so, saying that ultimate reality is (always) beyond language and thought, and that statements about ultimate reality are ineffable is in the context of Chinese Buddhist philosophy and its derivatives certainly contentious. Dogen, for example, literally denies the ineffability of ultimate reality, and so does much of Tientai thought. Thanks to Jan Westerhoff for pressing the former point, and an anonymous referee for pressing latter.

One important annotation on the semantics for those speech acts: Illocutionary acts are not part of the content of a sentence but indicate the force with which a sentence is uttered. It therefore makes no sense to embed a sentence with a prefixed force operator within a more complex sentence, as discussed above. However, my definition of $' \otimes '$ above is premised on regularly embedding a sentence with a prefixed force operator within a more complex sentence. We have to tweak speech-act theory a little here to make sense of this since ordinary speech-act theory is not equipped to capture conversational settings in which affirmations and denials interact with each other. We will, in the last chapter meet a different approach to the conversational nature of the MMK and many other primary sources of Buddhist philosophy.

I do this to model a dialectical conversational setting, where statements are perpetually being retracted, emphasized and synthesised. This, I think, nicely captures the settings in which Buddhist philosophical argument—especially when we look at the example above—took place.

When we are in a conversational setting it is not uncommon that things one has affirmed or denied earlier are taken back, corrected, or emphasized later. We can think of an emphasis, for instance, as a speech-act which governs another speech-acts, namely one that has been performed temporally earlier, simply by referring to it. The earlier speech-act at t1, which the manifesting speech-act at t2 is referring to is not a force in its own right anymore but has taken a propositional form which can be prefixed with a force operator (e.g. emphasis). If for instance, the temporally earlier speech-act was an imperative like 'open the window!', a temporally later speech-act could emphasize the previous speech act, as in 'I told you to open the window!', or take back and correct the earlier assertion, such as in 'I didn't mean to be bossy when I told you to open the window'.

Something like the above is, I take it, what happens in the model for the negative *catuṣkoṭi* with the $\otimes -operator$. The denial which is embedded in the complex sentence and prefixed with the illocutionary act of denial has lost its force and illocutionary power, i.e. it is illocutionarily ineffective, but it can still be referred to. If we were to formulate those ineffective illocutionary acts of denial as '$\times^{\text{ineffective}}$', and we could model the negative *catuṣkoṭi* without adhering to the use of obscure speech-acts inside speech-acts.

Negative catuṣkoṭi 2.0

Koti 1: $\otimes T(A) = \times T(A) \wedge \times\times^{\text{ineffective}} F(A)$

Koti 2: $\otimes F(A) = \times F(A) \wedge \times\times^{\text{ineffective}} T(A)$

Koti 3: $\otimes B(A) = \times T(A) \wedge \times F(A)$

Koti 4: $\otimes N(A) = \times \times^{\text{ineffective}} T(A) \wedge \times\times^{\text{ineffective}} F(A)$

It is important to note that $'\otimes'$ is not a mere rejection: It is tempting to interpret the negative *catuṣkoṭi* as a rejection of all possibilities but Nāgārjuna thinks that everything is empty of self-being (*svahbava*), which is not the same as simply denying self-being. In other words, it is not as is if every statement about ultimate reality ought to be rejected– – Nāgārjuna wasn't a mystic. See what MMK (24:18) tells us:

"Whatever is dependently co-arisen
That is explained to be emptiness.
That, being a dependent designation,
Is itself the middle way."

Interpret *"That is explained to be emptiness"* as ascribing 'emptiness' to something in a positive illocutionary act of ascribing all the possibilities of the *catuṣkoṭi* a further status, which is distinct from $T(A) \vee F(A) \vee B(A) \vee N(A)$. But to do that, all the possibilities have to be (implicitly) denied. Where Priest (2010a) adds another status-predicate (E), I interpret the affirmation of 'emptiness' as an illocutionary act which denies all $T(A), F(A), B(A)$, and $N(A)$, and thus semantically ascribes $'\otimes'$ to them. Hence, the illocutionary act, expressed by $'\otimes'$ is both an implicit denial of all the kotis as well as an explicit affirmation of emptiness. The behaviour of $'\otimes'$ is perfectly captured by a speech-act, albeit an unconventional one[73], which thus reads 'I *explicitly* affirm the emptiness (*sunyata*) of A and *implicitly* deny it's self-being (*svahbava*)'

$$Implicit\ denial\ of\ all\ kotis: \otimes \big(T(A) \vee F(A) \vee B(A) \vee N(A)\big)$$

[73] One might wonder here, if speech-acts do really work in this way. The problem is that illocutionary acts are not part of the content of a sentence, but indicate the force with which a sentence is uttered. There is, according to orthodox speech-act theory, no force that corresponds to '*explicitly* affirmation the emptiness and *implicitly* deny the self-being'. Thanks to Graham Priest for this comment. This observation is certainly correct, but as orthodox speech-act theory most probably doesn't include the illocutory forces of ancient India, and we can guess that statements about emptiness have an exceptional status, let us just think of $\otimes$ as an illocution, and let us think that there is a corresponding force.

$$\equiv$$

$$Explicit\ affirmation\ of\ Emptiness \otimes (E(A))$$

The illocutionary acts create a (non-gappy or glutty) bivalence of acceptance and denial. To make this more apparent, one could read the positive *catuṣkoṭi* alternatively with an assertion-operator $'\#'$ which, pace Searle (1969), stands for the assertive illocutionary act. $\# T(A)$ thus reads '*I assert that* $T(A)'$. The #-operator is redundant in this case, but its behaviour can nevertheless be defined:

- # doesn't toggle anything.
- # does iterate.
- Double-affirmation elimination holds for ## [74].
- $\#\times\ then\ \times$, but not $\times\#\ then\ \times$.
- # is interpreted as an illocutionary affirmation

Positive *catuṣkoṭi* 2.0
Koti 1: $T(A) = \# T(A) \wedge \#\times F(A)$
Koti 2: $F(A) = \# F(A) \wedge \#\times T(A)$
Koti 3: $B(A) = \# T(A) \wedge \# F(A))$
Koti 4: $N(A) = \times T(A) \wedge \times F(A)$

Regarding Recapture

First of all, note that B4Sx is, as we are in Four-Valued Boolean Algebra, fully classical with respect to the crucial inference rules. The illocutionary operators do not interfere with *modus ponens*, *modus tollens*, *disjunctive syllogism* or any other classical inference rule which we can find in the MMK since there is no entailment from $'\neg'$ to $'\times'$, or $'\times'$ to $'\neg'$. The model escapes Cotnoir's Recapture Problem.

Regarding the Exclusivity/Exhaustivity Constraint

[74] This might could also be regarded as re-assuring and thus increasing the degree of affirmation.

Let us now verify that B4Sx can also ground the logic for the *catuṣkoṭi* without collapse.

Concerning the first two kotis: To express the 'just-truth' and 'just-falsity' of the first two kotis its respective dual is negated by the illocutionary negation $' \times '$. Having a status-predicate governing our proposition we have left the propositional context. It would thus make sense to formulate the kotis in that way. With respect to the first koti it states the truth of A, and denies the falsity of A, which is like stating the just-truth of A. The same goes for the 'just-falsity' of koti (2). It says that the falsity of A obtains while the truth of A does not. The problem with just-truth and just-falsity problem is solved.

Concerning the last two kotis: I take $\times T(A) \wedge \times F(A)$ to be a natural formulation of the not-obtaining of neither the truth nor the falsity of A. Whereas to assert that it is not the case that *A holds and does not hold* does not make no sense, it makes good sense to assert neither the truth nor the falsity of A while making no statement about the ontological nature of A itself, whatsoever. The last koti makes good sense on this speech-act based model.

Concerning collapse: The kotis in B4Sx do not collapse. $Contraposition$ ($\times T[A] \leftrightarrow \times A$) is not valid in B4Sx. To deny the truth of A does not imply denying that A does not obtain. The same goes for the other direction. $T(\times A)$ is not equivalent to $\times T(A)$ given that $T(\times A)$, the way I have formulated it, doesn't make sense. This is because the $\times$-operator works on the level of truth-predicates only. It is also not that case that is $T(\neg A)$ equivalent to $\times T(A)$ given that $\times T(A)$ leaves open the possibility that A does obtain, while $T(\neg A)$ does not allow for this possibility. Again, this is because $' \times '$ works in the metalanguage of truth-predicates while $' \neg '$ works in the object language.

In the light of the aforementioned we can see why the *catuṣkoṭi* does not collapse in B4Sx. The first koti $T(A) \wedge \times F(A)$ is equivalent to $T(A) \wedge \times \neg T(A)$. Since double-negation elimination does not hold for $' \times \neg '$ we cannot reduce the first koti to $T(A)$. The transformation stops there. The second koti, $F(A) \wedge \times T(A)$, is equivalent to $F(A) \wedge \times \neg F(A)$. We cannot reduce this to $F(A)$ for the same reason. The transformation stops there. The fourth koti $\times T(A) \wedge \times F(A)$ is equivalent to $\times \neg F(A) \wedge \times \neg T(A)$, and the third koti $T(A) \wedge F(A)$ is equivalent to $\neg F(A) \wedge \neg T(A)$, but neither is $\times \neg F(A) \wedge \times \neg T(A)$ equivalent to, nor does it entail $\neg F(A) \wedge \neg T(A)$. The former denies the *not-obtaining* of the truth and falsity of A while the latter, which could optionally be read with an $\#$-operator, asserts the *obtaining* of both the truth and the falsity of A—two very different things. Neither from $\times \neg F(A) \wedge \times \neg T(A)$ nor from $\neg F(A) \wedge \neg T(A)$ we can

infer $T(A) \wedge \times \neg T(A)$ or $F(A) \wedge \times \neg F(A)$. Hence, the speech-act based model of the *catuṣkoṭi,* B4Sx, does not collapse!

§2.3 Contradiction and Recursion[75]

We have seen several attempts to find a model for the *catuṣkoṭi* in a classical framework, all of which had a collapse of the kotis as a consequence. We have discussed Priest's paraconsistent interpretation and two objections, the objection from just-truth/just-falsity and the collapse argument, levelled against it. Cotnoir's alternative, a four-valued Boolean Algebra, was shown not to fare better with respect to the collapse argument than the paraconsistent approach. Taking Cotnoir's concerns seriously I have introduced a speech-act based model and I have bolstered this interpretation with some textual examples. Adopting the speech-act model, both problems can successfully be avoided.

We shall now leave the Indian milieu, skip the epoch of Nāgārjuna 's commentators, and follow the footsteps of the *catuṣkoṭi* on its way towards China (500AD) where Madhyamaka Buddhism and Daoism merged into one[76]. It is in China that the role of the *catuṣkoṭi* as *upāya,* as a meditational instrument or a model for the way towards enlightenment becomes perspicuous. My hypothesis is that the school of *Sānlùn* (Ch.三論宗), which translates as "Three Treatises"[77], absorbed Madhyamaka philosophy and with it the *catuṣkoṭi*. By pointing at the structural similarities, I will show the family respeblence of *catuṣkoṭi* and what the school of *Sānlùn* calls, the *Erdi Zhang* (Ch. 二諦章). This shall be the first step in connecting Indian Madhyamaka philosophiy

[75] This chapter is an abridged version of Kreutz (2019b). Priest (2018:95-104) is putting forward a similar analysis of the *Erdi Zhang*. He is also modelling a truth-hierarchy, based on Chan's (1963) translation (which is the only useful translation to date), although with a different semantics. He is also not drawing the same conclusions from it as I do. As such, this chapter may be read as continuation of Priest's discussion.

[76] For the historical and philosophical context, see Chan (1963).

[77] The *"three treatises"*, says Rogacz (2015:130), from which the Chinese version of Madhyamaka took its name, are: *"Mūlamādhyamakakārikā – Zhōnglùn (中論), Nāgārjuna's Dvādaśanikāyaśāstra – Shíèrménlùn (十二門論) and Āryadeva's Śatakaśāstra – Bǎilùn (百論)"*. It later came to Japan under the name Sanron, but disappeared during the Nara period, or was most probably absorbed into Shingon.

with Japanese Zen. My hypothesis is that the following instance of a *catuṣkoṭi* from the MMK and the Zen *kōan* from the Wúmén Guān not only share a logical structure, they also share point and purpose which is to be a tool to reach, and model of the way towards enlightenment:

"Everything is real and is not real,	*"A Monk asked:*
Both real and not real,	*'Does a dog have Buddha-nature?'*
Neither real nor not real.	*The Zen Teacher answered:*
This is Lord Buddha's teaching."	*'Mu'"*
Mūlamadhyamakakārikā (18:8)	Wúmén Guān (1:1)

HYPOETHESIS: *Catuṣkoṭi* and *kōan* share a logical structure. From a formal perspective, the *kōan* is an abbreviation of the *catuṣkoṭi.* The logical form of both is a scaffold for *upāya* (Jap. *hôben;* Eng. skilful means) -- a soteriological instrument, intended to propel the practitioner towards *enlightenment* (Jap. *satori*).

Let us now turn to the role of the *catuṣkoṭi* in Jízàng's "*Erdi Zhang"* (Ch. 二諦章; Eng. "Essay on the Two Levels of Truth"), which commences with:

"The three kinds of 'Two Levels of Truth' all represent the principle of gradual rejection, like building a framework from the ground. Why? Ordinary people say that dharmas, as a matter of true record, possess being, without realizing that they possess nothing. Therefore, the Buddha propounded to them the doctrine that dharmas are ultimately empty and void." Chan (1963: 360)

What Jízàng describes in this opening paragraph to his most influential writing is something like a hierarchy of truth-levels built from the 'ordinary people's' idea that dharmas possess being (i.e., in Madhyamaka vocabulary, that things have *svabhāva*). In line with Madhyamaka thought he instead claims that no dharma (i.e. no phenomenon, may it be physical or psychological) possesses being (i.e. everything is devoid of *svabhāva*), and only emptiness (i.e. *sunyata*) is to be ascribed to all dharmas. We can read the 'ordinary people's' view that dharmas have self-being, taking the status-predicates into

account, as $T(B)$, where $'B'$ stands for a given proposition, and so we have something like the first koti. Its negation, therefore, is the Buddha's doctrine that dharmas are ultimately empty and void, which is our second koti, expressed as $\neg T(B)$[78].

Jízàng's following analysis, then, sustaining the thought that what is described here are the corners of the *catuṣkoṭi,* gives us a first hint at the truth of the hypothesis that the *catuṣkoṭi's* position in Buddhist philosophy is that of a scaffold for upāya (skilful means). Jízàng continues:

"When it is said that dharmas possess being, it is ordinary people who say so. This is worldly truth, the truth of ordinary people. Saints and sages, however, truly know that dharmas are empty of nature. This is absolute truth, the truth of sages. The principle is taught in order to enable people to advance from the worldly to the absolute, and to renounce the truth of ordinary people and to accept that of sages. This is the reason for clarifying the first level of twofold truth." Chan (ibid.)

We can interpret Jízàng as saying that it is an ordinary truth that dharmas have being, i.e. $T(B)$, but it's the absolute truth that they don't (i.e. a falsehood that they do). i.e. $\neg T(B)$. Still being on the first level of the twofold-truth (there are many more to follow), let me construct a table that depicts the schematic hierarchy of the levels of truth[79].

1st Level of Truth	$T(B)$	$\neg T(B)$
	Ordinary Truth	Absolute Truth

The highlighted part in the quote above is obviously referring to *upāya*, which in this case consist in the negation of the ordinary belief that dharma's have being. The negation of the ordinary belief that dharma's have being I interpret as the first step on the path towards enlightenment (an argument for that will follow below). Jízàng's essay continues:

[78] Which is equivalent to $F(B)$, i.e. the ultimate falsehood that B (that dharmas have self-being). Which is equivalent to the ultimate truth $T(\neg B)$, i.e. that ultimate truth that dharmas have no self-being. As we will see later, formulating the ideas of Jízàng with the status-predicate T (and its negation) only avoids confusion.

[79] In Nāgārjuna's words, Jízàng describes *"a truth of mundane conventions (samv̇ṛti-satya) and a truth of the ultimate (paramārtha-satya)"*, as in Westerhoff (2009:2).

"Next comes the second stage, which explains that both being and non-being belong to worldly truth, whereas non-duality (neither being nor non-being) belongs to absolute truth. It shows that being and non-being are two extremes, being the one and non-being the other. From these to permanence and non-permanence, and the cycle of life and death and Nirvana are both two extremes, they therefore constitute worldly truth, and because neither-the- absolute-nor-the-worldly, and neither-the-cycle-of-life-and-death, nor Nirvana are the Middle Path without duality, they constitute the highest truth."
Chan (ibid.)

We are being told that $T(B)$ and $\neg T(B)$ are merely worldly truth, this is, something that "ordinary people" would hold; which is no surprise as both propositions are expressed in language, and language is usually considered to be an aspect of the ordinary realm.

As we know, Madhyamaka Buddhism distinguishes between conventional (ordinary) reality, which is the realm of language, thought, the (from the ultimate perspective) erroneous view that things have own-being, and ultimate reality in which nothing has own-being and language and thought lack their descriptive power. Ordinary truth, then, are truth about the conventional reality, whereas ultimate truths are truths about ultimate reality.

To hold both $T(B)$ and $\neg T(B)$ is being grounded in duality which has to be overcome on the way towards enlightenment. But we have to be aware that, for Jízàng, $T(B)$ and $\neg T(B)$ is not a metaphysical duality given that they both correspond to different metaphysical realms. This would come down to saying that 'car A is red' and 'car B is blue' are dualities, which clearly, they are not.

The conjunction of both $T(B)$ and $\neg T(B)$ is a worldly truth. It is hence grounded in a dualistic view of reality which takes B to be conventionally false and ultimately not-false. This dualistic thinking ought to be overcome as part of the Buddhist practice. The kernel of Jízàng's treaties, then, is that overcoming dualities is the same as seeking enlightenment, and enlightenment itself is a state of non-duality[80]. In other words, dualities are not the metaphysical products of the real world, but the products of our (cognitive) apprehension of it. Dualistic thinking is thus is a conceptual error. It is Jízàng

[80] The problem with language, again, is that non-duality itself refers to a duality in that the "non" provokes an opposite concept from which non-duality is demarcated, which is duality. It is therefore better to refer to emptiness, rather than non-duality.

who hints at it and Zen Buddhism (with which we will later be concerned with) where it becomes explicit.

From a semantic point of view, since we are evaluating the status of the statements on the first level of truth, Jízàng is working on a 'second' and higher level of truth (i.e. in a meta-meta-language). Therefore, I introduce a second-level status predicate $'\mathcal{T}'$ which semantically works just like T but ranges over the first level status-predicates instead of propositions. The worldly truth on the second level, $\mathcal{T}(T(B) \wedge \neg T(B))$, is koti three, which, since ordinary and erroneous can be overcome, just like $T(B)$ ought to be overcome by negating its dual character. We thus establish koti four, which in Jízàng's framework reads $\neg\mathcal{T}(T(B) \wedge \neg T(B))$, by negating the worldly duality.

2nd Level of Truth	$\mathcal{T}(T(B) \wedge \neg T(B))$	$\neg\mathcal{T}((T(B) \wedge \neg T(B)))$
1st Level of Truth	$T(B)$	$\neg T(B)$
	Ordinary Truth	Absolute Truth

Moving this schema into a vertical position, we see the four corners of the *catuṣkoṭi* before us:

The Sānlùn catuṣkoṭi

(1) $T(B)$

(2) $\neg T(B)$

(3) $\mathcal{T}(T(B) \wedge \neg T(B))$

(4) $\neg\mathcal{T}((T(B) \wedge \neg T(B)))$

Each *koti* is established by a quasi-recursive method of conjoining ordinary and absolute truth on the nearest lower level to yield the ordinary truth on the higher level and negating the ordinary truth in the same level to yield the absolute truth.

Now we are in a predicament given that $\mathcal{T}(T(B) \wedge \neg T(B))$ and $\neg\mathcal{T}((T(B) \wedge \neg T(B)))$ are dualities, which, so says the Buddhist doctrine, shall be overcome. What does that mean for the *catuṣkoṭi*? Are there even higher levels of truth, beyond the kotis? Jízàng noticed this vexed situation himself and opts for an again higher level of truth intended to overcome the duality between duality and non-duality. But does that help? It doesn't unless it becomes a quasi-recursive move. At every level of truth, we return to a

higher-level duality which opposes the Buddhist doctrine of non-duality, and so on *ad infinitum*[81]… The production of new levels of truth will never come to a halt.

Let the following be a schematic model of the quasi-recursive machinery[82] of Jízàng's truth-hirarchy where C^x stands for conventional truth, U^x for ultimate truth, the x for the respective truth-level, > for the step from one conventional to ultimate truth "horizontally" on one level to the other via negation, and » for the "vertical" transgression to the next higher truth-level. I call this a truth-transfer (TT).

TT: $C^0>U^0$ » $C^1>U^1$ »…$C^n>U^n$ »…

It is interesting to see that TT, without being explicit on its potentially infinite nature, can already be found in some parts of the MMK, especially those where Nāgārjuna affirms all four alternatives: *"All is so, or all is not so, both so and not so, neither so nor not so. This is the Buddha's teaching."* MMK (18:8)

As Westerhoff (2009: 89) points out, the commentarial literature on this verse, especially Candrakirti, interprets this verse as referring to the graded nature of the Buddha's teachings (*anusasana*). Candrakirti's comments reflect the idea that "*all is so*" is taught to ordinary disciples, "*all is not so*" to those open to the Buddhist path while informing them about the doctrines of momentariness and impermanence, and so on.

What shall we make of TT now, knowing that the four corners of the *catuṣkoṭi* are just the beginning of an infinite and never terminating process of overcoming duality? In Garfield (2015) and Fox (1995) we find some noteworthy comments on Jízàng's levels of truth, and the observation that genuine non-duality can never be found:

"This is why I claim that either of your two truths, i.e., interdependent nature and discriminative nature; or two truth that is not two, or firmly-establish truth on the one hand, and 'Non-two and Non-non-two'; i.e., three non-nature or non-firmly-established truth on the other hand, is merely my conventional truth, whereas 'Forgetfulness of words and annihilation of thoughts' is really ultimate truth." Garfield (2015: 257)

Yet, there is also Fox elaborating on Jízàng, establishing that apparently on the fourth level,

[81] This has been recognised by Chan (1963) and Priest (2018).

[82] Not a mathematical model of the recursion, of course.

"...all of these distinctions (on the lower three truth levels) are deemed conventional, and the authentic discourse regards that any point of view cannot be said to be ultimately true, and is useful only so far as it is corrective in the above sense." Fox (1995: 87)

A sudden stop of the recursion at *whatever* level, however, seems arbitrary and unsupported by any of the other texts within Jízàng's corpus. Therefore, I don't think we should give the third or fourth level any elevated role. Let us call genuine non-duality (*forgetfulness of words and annihilation of thoughts*), or rather the experience or insight into it, N[83]. Jízàng gives no precise answer as to when insight into $'N'$ is achieved (i.e. how many levels of truth have to be crossed), sometimes its three, and sometimes four levels, but let us assume that after n-progressions through higher and higher truth-levels insight into $'N'$ may be achieved. We can add $'N'$ to our model in the following way:

n^{th} Level of Truth	N	N
…	…	…
2^{nd} Level of Truth	$\mathcal{T}(T(B)\ \&\ \neg T(B))$	$\neg\mathcal{T}((T(B)\ \&\ \neg T(B)))$
1^{st} Level of Truth	$T(B)$	$\neg T(B)$
	Ordinary Truth	Absolute Truth

$$TT(\mathbb{N}): C^0{>}U^0 \text{ » } C^1{>}U^1 \text{ » } \ldots C^n{>}U^n \text{ »} \ldots N$$

In a way, then, the recursion is a *cul-de-sac*. The soteriological "end-goal", however, is external to the recursion as there is no "bridge of logical necessity" between the recursion (i.e. everything that happens before N) and $'N'$ itself. This supports the argument that the *catuṣkoṭi* is more than a merely argumentative framework to refute philosophical enemies but a schema for the (Bodhisattva's, or Arhat's) path to enlightenment. This is coherent with the idea that the *Sānlùn catuṣkoṭi* does not defend any position on its own. One could say that what Nāgārjuna's (positive) *catuṣkoṭi* is (implicitly) defending is the Buddhist doctrine of non-duality, although the defence never fully terminates: Each level of truth,

[83]based on the Sanskrit *śunya* (Sanskrit: शून्यता), Chinese *wú* (無), or *kong* (空), all loosely translated as "emptiness" or "nothing-(ness)".

then, is *upāya* for the nearest upper level of truth, which again is *upāya* for the nearest upper level of truth… genuine non-duality, via this process, can never be achieved. So, the *Sānlùn catuṣkoṭi* doesn't defend a position on its own. It is, if we believe Rogacz (2015), Jízàng who disagrees:

"This pragmatical approach (the levels of truth) provides us to the central concept of "refutation of erroneous views as the illumination of right views", bóxiè xiànzhēn (驳谢现真), which was enunciated in the "Profound Meaning of the three Treatises". As we remember, Prāsaṅgikas claimed that Mādhyamaka is only a negative method of refuting views, but Svātantrikas believed that it has also its own, undoubted view. Although Jízàng cannot have been a witness of this dispute, he subverted the salience of this argument: refutation of erroneous views is always the illumination of right views, and vice versa. All beliefs are empty because they depend on their rejections. Two opposite beliefs (statements) share the same premises and the horizon of possible continuations. Tetralemma is transcending these artificial oppositions, such as nothingness/absolute, false/truth, samsara/nirvana, and so on. (...) In this perspective, the doctrine of emptiness seems to be the reinterpretation of the doctrine of expedient (skr. upāya, ch. fāngbiàn, 方便) means." Rogacz (2015: 232)

We have now discovered that the role of the (*Sānlùn) catuṣkoṭi* in Buddhist philosophy exceeds its practise as a purely argumentative tool. We have also witnessed how the *catuṣkoṭi* has lost its distinctive four-valued form. We also don't find the distinction into positive and negative *catuṣkoṭi* in Jízàng anymore. In a sense, the four values have melted into one. We also don't call it *catuṣkoṭi* anymore, but *bóxiè xiànzhēn*, the concept of the "*refutation of erroneous views as the illumination of right views*", (Ch. 驳谢现**真**, Eng. *Refute the Truth*). What is not clear now, however, is whether the *bóxiè xiànzhēn* (i.e. the *Sānlùn* version of the *catuṣkoṭi*) is defending a position on its own, or not.

On the one hand, we have the formalization which strongly suggests that the *bóxiè xiànzhēn* does not, and in fact cannot, defend a position on its own. We have Rogacz's interpretation of Jízàng according to which Jízàng takes the *bóxiè xiànzhēn* to defend a position on its own. I shall now put forward a different interpretation of Jízàng's commentary and argue against Rogacz.

For Jízàng, as the title of his essay suggests, the *bóxiè xiànzhēn* is not only intended to refute other philosophical positions but also as the illumination of the right view. For Rogacz's interpretation of Jízàng the recursion (i.e. the refutation of all possibilities) is equivalent to $'N'$. (i.e. "the right view") Hence, not defending any possibility *is* defending $'N'$. But how can this be right as there is no logical inference from the first position $'T(B)'$ to $'N'$, *at all*? If the formal apparatus is correct, the *bóxiè xiànzhēn* does not defend a position on its own; there is no logical connection between the kotis and $'N'$, *whatsoever*. Let us approach this problem by looking more carefully at the nature of *upāya*.

The *bóxiè xiànzhēn* understood as *upāya* alleviates the problem expressed above. The idea is simple: A means must not ultimately terminate in a goal and might only be *but one* of many means jointly necessary to reach a goal. In other words, a means doesn't have to be *on its own* sufficient to reach a goal. The recursion that is the "refutation of erroneous views" might be necessary, but doesn't have to be, *on its own*, sufficient for the 'illumination of right views'.

From the viewpoint of *upāya*, for $'N'$ (i.e. "the illumination of right views) to be brought about the 'refutation of erroneous views' has to be brought about, but yet the 'refutation of erroneous views' is *on its own* not enough to bring about $'N'$. It is thus wrong to think of the *"as"* in the "refutation of erroneous views *as* the illumination of right views" in terms of logical equivalence or a biconditional. Rogacz's use of the term *'vice versa'* is thus clearly misleading. It is the 'illumination of right views' which is always the 'refutation of erroneous views', but not *vice versa*. Hence, the *bóxiè xiànzhēn* does not defend a position on its own, neither did its predecessor, the *catuṣkoṭi*.

Yet the problems do not end here. There is, what I call, a *Halting-Problem.* The 'right view' for Jízàng is that of the middle way between dualities—a view he inherited from Nāgārjuna which we express with $'N'$. Jízàng also writes that attachment to the doctrine of emptiness, i.e. $'N'$, is misguided, hence itself ought to be overcome.

"It is like water able to extinguish the fire, if the water itself could ignite, what would be used to extinguish it? Nihilism and eternalism are like fire and emptiness can extinguish them. But if someone insists on adherence to emptiness, there is no cure which could help him" Husueh-li Cheng (1991:49)

This comment is enigmatic only as long as we think of the *bóxiè xiànzhēn* as terminating in $'N'$, as the TT(N) suggests. That would mean that with the experience of enlightenment, the practice comes to a halt (this will be made more explicit below). What the comment recommends, however, is that $'N'$, although it represents some kind of qualitative change, shouldn't be thought of as bringing the *bóxiè xiànzhēn* to a halt. In other, more polemical words, enlightenment is not 'the end of the story'. It is rather the case that assigning $'N'$ to all things is one-sided and dogmatic, and thus needs to be overcome[84].

One could argue in the following way: $'N'$ is the soteriological end-goal of Buddhist practice upon reaching it *upāya* (i.e. an ongoing refutation of erroneous views) becomes redundant. In fact, most means to reach a goal seem to be pointless once the goal is reached. The possession of a car, for examples, loses its immediate significance once one has reached one's the destination[85]. As soon as it has satisfied its immediate purposes a means has loses its value. So, here is a problem: $'N'$ both *does* and *does not* bring the *bóxiè xiànzhēn* to a halt. On the one hand, enlightenment is the soteriological end-point, on the other hand, it seems like enlightenment is just a transitional state. We either have,

TT(ℕ): $C^0>U^0$ » $C^1>U^1$ » …$C^n>U^n$ » … N

where ℕ brings the recursion to a halt. Or we have,

TT(ℕ)*: $C^0>U^0$ » $C^1>U^1$ » N » … $C^n>U^n$ » …

where the recursion does not come to a halt. According to Jízàng, as far as I can see, either is a viable option. There is, as far as I know, simply no evidence for either option in Jízàng's corpus. To dissolve this looming paradox let us now follow the development of Buddhism (and with it the *catuṣkoṭi*) further eastwards.

Jízàng is at the same time the greatest and the last philosopher of the *Sānlùn* school. Yet, the development of Māhāyana Buddhism did not stop, and with it, the

[84] Whereas the linguistics of non-duality provokes a duality, ascribing emptiness (and only this) to a phenomenon is one-sided, regardless of whether there is a dual or not. This one-sidedness is doctrinal and therefore to be rejected.

[85] If you live in a city, having to find a parking place can even render owning a car obstructive.

development of the *catuṣkoṭi* did not stop either. I want to push the idea that what is known as a *kōan* in the Chan/Zen tradition bears so many similarities to the *catuṣkoṭi* that it is difficult not to recognize a systematic connection. The situation in which Jízàng has left us is, from the perspective of Zen, no longer paradoxical, as I shall illustrate below.

A monk asks:
"Does a dog have Buddha-nature?"
The Zen Teacher says:
"Mu"
Wúmén Guān: (1)

We now find ourselves amidst the teachings of Chan/Zen Buddhism. What to make of this little dialog is difficult to say. The answer is surely baffling. The Zen Teacher's[86] answer is "Mu" (Jap. 無), which roughly translates to "nothing-(ness)". 無 turns into something intelligible only if we think a little more carefully about the question. The question goes wrong in two ways.

First, it is the *Nirvanasutra* that explicitly states that all sentient beings have Buddha-nature—it is their fundamental nature——so a dog has Buddha-nature, too. The monk, we can guess, should have known this. We may thus understand the question as ill-posed and based on an inadequate concept of being. But this is not the reason for the teacher's enigmatic answer. So, what is 無 supposed to tell us?

The question, as it is posed, is intended to be answered with an affirmation-negation linguistic device, i.e. simply with 'yes' or 'no'. But the latter presupposes that the question is well-formed (because it is asked from the perspective of conventional reality, where the content of the question is supposed to refer), which it is not, taking into account the metaphysics of *being* in Māhāyana Buddhism. Second, let me try a different approach, which needs a little bit of contextual background.

Wúmén Guān: (1) is known as 'Joshu's Dog' (Ch. 趙州狗子). It is a *kōan* (Jap. 公案). The aim of grappling with and meditating over a *kōan* is to overcome *conceptual thinking*—exactly the thinking that has instigated the monk's question. Therefore, it is not the monk's ignorance of the fact that, according to Māhāyana tradition, every sentient

[86] Chao-chou Ts'ung-shen (Jap. Joshu) is the teacher referred to.

being has Buddha-nature, but his conceptual thinking which manifests itself in a yes/no-dichotomy in which a concept either applies or does not apply that 無 is supposed to unveil. I want to argue that it is, above all, the dichotomy in Joshu's conceptual thinking that the Zen teacher's answer is pointing at. 無 is intended to hint at the monk's dualistic thinking and an invitation to question that thought process.

An answer to 'Joshu's Dog' might as well be stated in the form of a negative *catuṣkoṭi*. The act of denying all kotis comes down to denying all possible ways a dog could or could not have Buddha nature. Let us rethink the *kōan* in a Nāgārjunian manner.

In the context of the MMK, the 'Joshu's Dog' would look something like this: "*Does a dog have svabhāva?".* The Nāgārjunian answer, which we are already familiar with, is of course a denial, *qua* negative *catuṣkoṭi,* of all the possible ways a dog could have *svabhāva*. Ultimately, the dog's nature is *śūnyata* (emptiness), Nāgārjuna would say.

It is not a big conceptual leap from 'emptiness' (*śūnya,* Sanskrit: शून्यता) to 'nothing-*ness*' (Mu, Ch. 無). Given those structural similarities I want to hold that both Nāgārjuna's and Joshu's answer to the monk's question are conceptually equivalent[87]. The only profound difference is a difference in form, rather than substance: Nāgārjuna gives a profound logical apparatus with which all the possible ways a dog could have self-being, i.e. *svabhāva,* is denied. It is this having a logico-epistemic phenomeno-metaphysical apparatus in the background what is largely missing from the writings in the Zen tradition Given the historical connection and the formal similarities I still assume that the logico-epistemic phenomeno-metaphysical apparatus still resonates somewhere among other this and other implicit principles that have been inherited from the India milieu. I therefore want to put forward the thesis that the *kōan* is an abbreviated *catuṣkoṭi.*

Identifying the *kōan* with *upāya* is a relatively straight forward thing to do as it is widely agreed that the *kōan*'s role in Chan/Zen is that of a meditational object (Jap. *hôben*). We have the Rinzai school of Zen, for instance, which focusses heavily on the *kōan* as a means to gain enlightenment (Jap. satori). Zen furthermore makes a distinction

[87] I have to mitigate this parallelism: Whereas the Indian Buddhist's an-atman (no own being) and Nāgārjuna's sunyata (emptiness) are *epistemological concepts* – referring to the fact that things are only forms (as superimpositions), "Asian nothingness" (toyoteki mu)", referring to the Kyoto School's concept of nothingness through satori (and therefore the intellectualisation of Zen), refers to a lived experience of reality as a way arising naturally out of nothingness, and should therefore rather be considered a *pragmatic concept*.

into two ways in which enlightenment can occur: On the one hand, according to the Sōtō school, enlightenment comes gradually, sometimes glossed at as 'silent enlightenment', understood as a process of discovery initiated through *Zazen* (sitting meditation). On the other hand, we have the Rinzai school which advocates the concept of 'sudden enlightenment'. *Kōan* and *zazen* are both *means* of reaching enlightenment, or otherwise of following the truth-hierarchy up to $'N'$.

I shall thus conclude that *catuṣkoṭi* and *kōan* share a logical structure. From a formal perspective the *kōan* is an abbreviation of the *catuṣkoṭi.* The logical form of both is a scaffold for *upāya*, the soteriological instrument intended to propel the practitioner towards *enlightenment*

We have left Jízàng with a predicament: $'N'$ both terminates and does not terminate the *upāya*, i.e. the truth-hierarchy. We haven't yet settled whether TT(N) is the correct analysis, or whether TT(N)* is what the *bóxiè xiànzhēn* expresses. I have promised to uncover a solution to this paradox in the writings of the Zen tradition. In fact, what is left of the framework of the *catuṣkoṭi* in the Zen tradition is perhaps too sparse for it to still count as an instance of the *catuṣkoṭi*—it is drastically abbreviated. It is merely the very scaffold that has found its way into Zen literature in the form of ten pictures of a man and an ox and their corresponding verses: the *jūgyūzu* (Jap. 十牛図, Eng. Ten Ox Herding Pictures).

"Ten Bulls" by Tokuriki Tomikichiro

The pictures and verses of the *jūgyūzu* above, as it is widely recognized, are similes of the path to enlightenment, with the ox-taming being a symbol for meditation (*kōan* or zazen) practice, the ox itself being a symbol for the unsteady mind[88].

[88] The ox is a ubiquitous figure in Buddhist literature. It features in texts as early as the *Maha Gopalaka Sutra,* as a symbol for meditation (perhaps because wrestling with one's mind is as

According to D. T. Suzuki (*unpublished*), the Oxherding pictures and verses made their first appearance in China around the 12th century. The most well-known version of the jūgyūzu is the one by Chinese Chan master Kaku-an Shi-en (Ch. 廓庵師遠). It is also with Kaku-an Shi-en that the Ox Herding becomes a *jū*gyūzu (*jū,* Jap. 十, Eng. ten) instead of a *hachigyūzu* (*hachi*, Jap. 八; Eng. eight). It is the blank space at which the *hachi*gyūzu halts, whereas the *jū*gyūzu incorporates two additional pictures. This detail is of outmost significance for our ongoing discussion. To see the relations and the similarities between *bóxiè xiànzhēn* and jūgyūzu we need to know the meaning behind the pictures, and we need to refer back to the quasi-recursive schema TT, which is the scaffold of the *bóxiè xiànzhēn*.

The jūgyūzu depicts a character, a boy in the case above, and his search and taming of the ox as a metaphor for on his path to (and through) enlightenment. Hence, I have already hinted that there is a post-enlightenment state, such as TT(N)* describes it.

In the first picture, we meet the boy alone in the wilderness, lost and confused, but searching. He knows about the conventional truths and is unsatisfied with them. Let this conventional belief be $T(B)$, and his stage on the way towards enlightenment be represented by C^0.

The boy goes on and finds the traces of the ox in picture three and four, which serve as a metaphor for finding the sutras and inquiring into the Buddhist doctrines. He then learns about the erroneous nature of conventional truth and, for the first time, has a glimpse of the ultimate truth, $\neg T(B)$. Let this stage be represented by U^0. So, we have the first two kotis and the first level of Jízàng's truth-hierarchy complete.

However, the boy is unable to distinguish conventional from ultimate truth. His mind is still confused as to truth and falsehood, and so the boy believes in both $T(B)$ & $\neg T(B)$. He is not yet ready to discriminate them. This is the third koti. This stage, C^1, depicts the boy catching and taming the ox in the fourth and fifth picture.

Picture six shows the boy riding the ox. He has successfully tamed the beast and is able to let loose the line. He has realised that the duality of truth and falsehood needs to be overcome. This is stage U^1 of Jízàng's hierarchy and the fourth and final koti $\neg\mathcal{J}((T(B)\ \&\ \neg T(B)))$. As the series continues, the ox has disappeared, and the boy is back at home.

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strenuous as wrestling with an ox), and is one of the many iconographies that have survived Māhāyana's journey from India to the Japanese archipelago.

The next picture is an empty circle which shall represent $'N'$. It's the manifestation of an ineffable reality which is beyond the dualities of language and thought. This is where, for the boy, the illusion of self has vanished, and he experiences non-duality. Hence not only ox—as the ox is now useless[89]—but also the boy is gone. This represents enlightenment. This is where the *hachi*gyūzu ends. The boy has found enlightenment, and the ox, which is a metaphor for the boy's *upāya,* like Wittgenstein's ladder, has become redundant. Let the *hachi*gyūzu (OX_8) be represented by this schema which shall remind us of the *bóxiè xiànzhēn* in its TT(N)-reading:

$$OX_8\text{: } C^0 \text{»} U^0 \text{»} C^1 \text{»} U^1 \text{»} N$$

This has been the first option of two readings of the *bóxiè xiànzhēn*. It has led us to a paradox in Jízàng for which we couldn't find a solution within his own canon. If the *hachi*gyūzu is a successor of the *bóxiè xiànzhēn,* the process is supposed to terminate upon enlightenment. However, the jūgyūzu above includes two further pictures after the enlightenment. What to make of those?

The ninth picture shows a serene landscape with the boy still gone. We can interpret it as a return to the ordinary world, which, although the boy has found enlightenment, has of course not vanished (Buddhism is not Nihilism or External World Scepticism). The boy, too, has not vanished either. He returns to the world in the last picture of the jūgyūzu. It hence looks as if the journey continues, and $'N'$ has not brought the process to a halt.

The jūgyūzu and the hachigyūzu seem to say different things with respect to the endpoint of the series. But to think they are saying incompatible things is to overlook a significant detail in the iconography: the ox is missing post-enlightenment, *upāya* has become pointless. Although the boy has returned to the ordinary realm, he has not returned to a pre-enlightened state of mind which would again require the effort of overcoming dualistic thinking. For the boy, the illusion of a self and that of a world with *svabhāva* has vanished. Conventional reality is seen *as what it is*; a conceptual superimposition on ultimate reality.

[89] As the discussion on what it means to be a 'means' as already revealed; meditation is a means and not in itself sufficient, yet necessary for enlightenment Some traditions of Zen would even reject that. Formal Meditation, such as Zazen, might not be necessary, enlightenment can come unexpected. But even if formal meditation is not required, some kind of quasi-meditative process is involved.

The jūgyūzu (OX_{10}) shall be represented in the following way, where the superscript '^{N}' represents the post-enlightened perspective on conventional reality. It indicates the content of the boy's realization that the states-of-affairs in C are merely conceptual impositions on ultimate reality N:

$$OX_{10}: C^0 » U^0 » C^1 » U^1 » N » C^N$$

It is important to note that the process does not stop with C^N, but it is the post-enlightened state C^N in which the boy will remain throughout his post-enlightened life[90]. What a lucky boy.

Jízàng's paradox can be resolved if we take the distinction into pre- and post-enlightened states into account. In both cases of the *bóxiè xiànzhēn* (TT(N) and TT(N)*) the recursion stops. It is only in the latter that the post-enlightened state is added. Jízàng's comment that a dogmatic belief in $\mathbb{N}$ is erroneous and has to be overcome doesn't mean applying the quasi-recursive method of the *bóxiè xiànzhēn* to N, but to return to the starting point with a post-enlightened perspective. A famous Chan/Zen saying articulates this thought as follows:

"Thirty years ago, before I practiced Chan, I saw that mountains are mountains and rivers are rivers. However, after having achieved intimate knowledge and having gotten a way in, I saw that mountains are not mountains and rivers are not rivers. But now that I have found rest, as before, I see mountains are mountains and rivers are rivers." Qingyuan Xingsi (Wudeng Huiyuan: 1252)

Here, talk of the mind being *"still and at peace"* in Nāgārjuna (MMK dedication and 25:24) is also worth mentioning[91]. For Nāgārjuna, compulsive philosophical questioning (such as asking questions in form of the four corners) would be stopped when the mind stops grasping at philosophical theories and simply accepts those as conventional. Nāgārjuna calls this *prapañca upaśama* (Eng, "pacification (*upaśama*) of

[90] In fact, several comments of Hakuin suggest that experiences of enlightenment can be had *over and over again*, and only through ongoing practice, the post-enlightened state could be maintained and cultivated. This would question the complete repudiation of *upāya*. Still, one could guess that post-enlightened *upāya* is different from pre-enlightened *upāya*, which in consequence would not question the account given above.

[91] Thanks to an anonymous referee for making me aware that we can return to Nāgārjuna at this point.

mental proliferations (*prapañca*)"[92]). The idea of quietening the mind of philosophical questioning is indeed integrated to Madhyamaka thought. Candrakīrti commented on it in the Prasannapadā, saying that

"... for the Nobles when they see dependent-arising as it really is, that very dependent-arising is called "the calming of manifoldness (prapañca upaśama) – in the sense that there is the calming of instances of manifoldness in it. And because (it) is entirely without the misfortunes of birth, old age, death and so forth owing to the ceasing of (any) dealing with (the dichotomies): cognition and cognizables in view (of the fact) that mind and mental factors do not arise in it, it is (ultimate) welfare." McDonald (2015).

In seeking a comprehensive account of the *catuṣkoṭi,* this essay has been working on logical, ontological, historical, and soteriological aspects *in unison*. The fact that the logical aspects of the *catuṣkoṭi* have been unhinged from their ontological and pragmatic fundament has prevented any fruitful modelling and obstructed further insight into the historical development of this fascinating piece of Buddhist philosophy. I hope that with this study of the development of Madhyamaka thought and with the help of the formal modelling applied throughout the paper it is now clearer what the *catuṣkoṭi* is, and how it functions within the Buddhist canon. As the confluence of the last chapters suggests, we should not consider the *catuṣkoṭi* as exclusively a phenomenon of Madhyamaka thought, but rather think of the Madhyamaka's *catuṣkoṭi* as an instance of a much broader category, which is a schema of, and for, *upāya*, which includes the *bóxiè xiànzhe* and the jūgyūzu (and perhaps many, yet unexplored others).

The next chapter follow the Zen *kōan,* and thus the *catuṣkoṭi*, to ostensibly its latest development. While the Zen *kōan* is an abbreviated *catuṣkoṭi*, the *soku-hi* dialectics is an abbreviated *kōan.* In the hands of the Kyoto School philosophers, the *catuṣkoṭi* shrinks to the size of a single negation, the *soku-hi* negation. How to make sense of this negation is again complicated. I shall refute a classical attempt to make sense of the *soku-hi* negation and suggest an alternative with a second-order paraconsistent semantics.

[92] Thanks to Jan Westerhoff for this translation.

§3 Soku Hi

We now leave China, and the *catuṣkoṭi*, behind (but only to pick it up again), and finally to make our way to Japan. In Japan, things become more poetic—think of *Joshhu's Dog*, for instance. By looking back from Japan on the developments of Māhāyana Buddhism in China, we saw that erstwhile paradoxes can be rebutted, puzzle solved, enigmatic positions made clearer. This is by no means to say that Buddhist philosophy becomes very lucid and clear once it reaches Japan, but perhaps relatively "clearer". In this chapter we will have a look at the developments of the *catuṣkoṭi* beyond its embedment in the *koan* structure. In the hands of the Kyoto School members, the *catuṣkoṭi* is distilled into a single logical operator, a very peculiar form of negation—the *soku-hi* negation—or so I will argue.

I shall first introduce this peculiar form of negation. Following that, I will introduce some characteristics of Zen and Kyoto School philosophy which help place all things logical that are about to follow into context. That negation is part of a dialectical structure which I will try to formalize. Again, we will derive at a conflict between two fundamental principles of Buddhist philosophy, impermanence and anti-essentialism. The last chapter is an attempt to resolve this conflict by connecting the discussion with our observations on *Joshhu's Dog* and the doctrine of two-realities, conventional and ultimate.

§3.1 Zen and the Kyoto School

The philosophy of the Kyoto School members, most notably Nishitani (1900-1990), Nishida (1870-1945) and Tanabe (1885-1962), has its roots to equal extent in European philosophy, especially German Idealism, and Japanese Zen Buddhism. Chinese Chan, the predecessor of Japanese Zen, inherited this philosophical toolbox from the India milieu around the 12th century. On its way to the Japanese archipelago, the Māhāyana canon had incorporated local ideas from Daoism and Confucianism. This influence is palpable in Kyoto School philosophy, mostly because Zen amplifies two aspects of early Buddhist philosophy through the influence of Daoism and Confucianism. These are worth pointing out, as they will guide us to a better understanding of the concept of *soku-hi*.

First, as the Second Noble Truth proclaims, the world is a world of impermanence (Sanskrit: *annitya*). A similar idea is found in Daosim, expressed as the *Dao*. Zen puts an emphasis on impermanence and the continuous flux in which all things reside.

Second, the soteriological process of going towards and reaching enlightenment in Zen takes up the concept of the two truths and emphasizes the bicameral nature of reality; *heaven* (Chi. *tian*) and *earth* (Chi. *di*), as we find it in Confucianism. Only through seeing reality as two-sided and yet one, enlightenment (Jap. *satori*) can be experienced. This idea is captured in the famous Zen saying that we have already encountered during the discussion of Jízàng's paradox:

"Thirty years ago, before I practiced Chan, I saw that mountains are mountains and rivers are rivers. However, after having achieved intimate knowledge and having gotten a way in, I saw that mountains are not mountains and rivers are not rivers. But

now that I have found rest, as before, I see mountains are mountains and rivers are rivers." Wudeng Huiyuan, 1252

It was Nishida's self-proclaimed aspiration to give Zen a systematic philosophy. In the course of making this become reality, the Kyoto School philosophy, sometimes explicitly but to a large extent implicitly[93], incorporated the Māhāyana concepts.

In fact, his quest for *absolute emptiness* (Japanese. *zettai mu*), Nishida was pushing the notion of emptiness (Japanese: 無, *Mu*) to its limits. The *'zettai'* here is an emphasis on the radicalism of emptiness in this context; a context in which opposing views dissolve. It is in the concept of *absolute emptiness* that we find the notions of two-truth and non-duality again; they form an inconsistent set; that which destroys all opposite views is supposed to leave *emptiness* behind, but *emptiness* is itself opposed to *being* (something), and so *emptiness* could never be absolute, but always relative and one side of a duality. Therein lies the contradictory nature of the absolute; absolute emptiness cannot live up to its status of absolutism; it remains in a contradictory state. This contradictory state gives rise to the flux and impermanence of reality, as the contradiction can apparently never be resolved.[94] However, there is reason to doubt that the contradiction in this context is really benign.

The Madhyamaka theories of emptiness and non-duality push Nishida to his view on reality as 'absolutely contradictory self-identical' (Japanese. *zettai mujunteki jikodōitsu*). The way the Kyoto School members argue for the metaphysical thesis of 'absolutely contradictory self-identity' regularly employs a peculiar dialectics and a very peculiar negation.

Indeed, there is something slightly bizarre going on with negations in Kyoto, or so it seems. *'A is A, and yet A is immediately not A, and therefore A is A*' is a *prima facie* paradoxical inference and yet we find it repeatedly throughout the writings of the Kyoto School philosophers[95]. It is this formula that Nishitani (1983:124) expresses when he says, somewhat mysteriously, that

[93] See Heising (2010) for an extensive overview of Kyoto School philosophy and their philosophical concepts.

[94] In a Western context, this is similar to a Hegelian, materialist concept of history.

[95] This, or a very close form of inference has been identified in Suzuki (1955:119-120), Kim (1955:22), Dilworth (1987:130), and Kasulis (1998).

"the selfness of a thing cannot be expressed simply in terms of its 'one thing or another.' It is rather disclosed precisely as something that cannot be so expressed. Selfness is laid bare as something that cannot on the whole be expressed in the ordinary language of reason, nor for that matter in any language containing logical form. Should we be forced to put it into words all the same, we can only express it in terms of a paradox, such as: 'It is not this thing or that, therefore it is this thing or that.'"

This inference, call it SH, has the form of the *soku-hi dialectic* as developed and put into practice by Nishida Kitaro. The *soku-hi dialectic* gets its name from the negation employed in the inference above—the *soku-hi negation,* as I call it. It translates to 'immediate negation'; '*soku'* (Japanese: 即)[96] stands for the immediacy and '*hi'* (Japanese: 非) is a denial particle, but in this case, we can think of it as 'negation'. 'A *soku-hi* A' thus means 'A and *immediately not* A'. What is this bizarre immediate negation, how does it behave? Whence the temporal aspect of immediacy? This chapter intends to clarify and to close the bridge between India and Japan. The *soku-hi* dialectics, I want to argue, is again a slightly metamorphosed *catuṣkoṭi*.

What we would call "logic" entered Japanese philosophy comparably late in the 20th century. The "logic" that we are talking about here is also not formal, but much more (Hegelian) dialectical in nature and exposition. The Japanese term for "logic", which is "*ronri*", means as much as "the principle of discourse". That relates back to my proposal for a speech-act based model for the *catuṣkoṭi*. Hence, for the Kyoto School too, "logic" was a way of analyzing or perhaps acting in conversational settings.

Since there is no "formal base" in the writings of the Kyoto School, they argue dialectically, and they often repeat how their philosophy exceeds any language containing logical form, it is very difficult to put their logical inferences into static formalisms. The idea of "immediacy" itself is a temporal one, presumably connected to the dialectical methodology. It is a concept that can only fully unfold in a temporal setting.

Contemporary temporal logics will not fit the bill since the Kyoto School logic is not *about* time, but situated *in* time. Furthermore, "time" is not strictly speaking

[96] In contemporary Japanese, '即' translates to 'immediate'. This is the reading with which I shall work, and which I shall defend. There are other sources, especially sources in Classical Chinese, which interpret '即' as the copula 'is' which is usually omitted in Sanskrit and was added in translation.

understood as a succession of events, but as a discursive play which unfolds in time. It is thus difficult to find an adequate formal model for the SH. However, I shall try.

On the one hand, we have the apparent contradictions, on the other hand we have a sense of temporality. The latter will explain the former. In other words: Because we are working in a discursive stetting which stretches over time, there is a way to make sense of the contradictions in the SH. In order to fully unfold the mechanisms of the SH, we need to return to the discussion about external and internal negations. The Kyoto School philosophers were well-aware of the difference—not least because of the intellectual connection with Indian Buddhist philosophy[97]. It is precisely the distinct use of external negation within the writings of the Kyoto School which will play a key-role in my attempt at making sense of the SH.

[97] See Jones (2004)

§3.2 Formalizing Soku Hi

On a first, classical Fregean reading of *soku-hi* dialects (henceforth, SH), the antecedent is a conjunction of a trivial self-identity statement and a contradictory self-identity statement. The *soku-hi negation* will, from now on, be symbolized by '×'. Jones (2004:307) rightly points out that '×' is an external negation, quoting Nishida (1987:83), saying that *"[t]he logic of the existential self requires us to say that in the self's own depths there must be the fact of the self's own self-negation as constitutive of itself."* There is no predicate-negation (i.e. internal negation) at play here or elsewhere in Kyoto School philosophy.[98] Depending on how we interpret the external negation '×', A = ×A can be considered contradictory. If A = ×A ⊨ ¬(A = A), then (A = A) ∧ (A = ×A) is a contradiction from which apparently a trivial self-identity statement follows. Let this be a tentative model of the SH dialectics:

SH: $((A = A) \wedge (A = \times A)) \rightarrow A = A$

In fact, the Kyoto School philosophers acknowledge the contradictory nature of this statement—*'self-identity is a contradiction'* (Z11.174f.)[99], says Nishida—and yet they don't seem to ever feel the sting of contradiction. Why is that? Is that because of dialetheic commitments? I will argue that the reason has to be sought elsewhere.

[98] Compare, Suzuki (1955:119-120), Kim (1955:22), Dilworth (1987:130), and Kasulis (1998).
[99] Those quotes will all refer to Nishida, K. (1988). 'Collected Works, (*Nishida Kitaro¯ Zenshu*)'. Iwanami Shoten.

Kyoto School philosophy rarely takes a contradiction to be the source of major philosophical concern, in the sense of having to avoid contradictions in a theory at any cost. Tanabe (1986:13) even goes as far as saying that *"anything that can simply be reduced to the principle of identity is not a problem for philosophy. For a problem to belong to philosophy there must be something inconceivable in it."* Given Nishida's and Tanabe's statement, perhaps we can think of A = ×A as a full-fledged contradiction. The conjunction of non-contradictory self-identity and contradictory self-identity, (A = A) ∧ (A = ×A), must then express a falsity, given that the first conjunct expresses the trivial truth of self-identity and the second expresses a contradiction and the conjunction of truth and falsity yields a falsity in a classical framework. *"The dialectic of affirmation and negation, of A and yet not-A,"* says Jones (2004:303), *"is the core of the logic of soku. With it, Nishida and Nishitani relate immanence and transcendence, subject and object, self and other, life and death, mortality and immortality, relative and absolute, being and nothingness, Form and emptiness, self-identity and self-negation."*

Given that anything could be inferred from this contradiction, one could of course think of SH as an instance of *ex contradictione (falso) sequitur quodlibet*. Problem solved? No: First, there are simply no textual evidences for the use of *ex contradictione (falso)* in the writings of the Kyoto school philosophers. Second, such as with the *catuṣkoṭi*, we want our beliefs to be closed under entailment. In other words, we would want there to be a coherent story for why Kyoto School philosophers make the inferences they make. A simple *ex falso* would make it look like the SH is a random inference, and random inferences can hardly be interpreted as soteriologically valuable, or at least not compatible with the systematicity in which the SH is employed in Kyoto School writing.

It might be worth looking a little more closely at the negation itself. Jones (2004:307) argues, and I think rightly so, that the Kyoto School makes no distinction into external and internal negation. Whenever they are using negations, they are using the external negation (as in the example above). External negations deny not only predications but whole propositions. Contradictions containing an external negation are, what the Kyoto School sometimes calls, "real contradictions". The subject-canceling effect of the external negation is what is important here. As it is usually understood, it makes a proposition meaningless given that its referent/s is/are empty. From a historical perspective, since Zen is a late development of the Māhāyana tradition, the idea of reintroducing this framework into the Zen context (in which we can find some concepts of the Kyoto School) is a plausible one.

External negations are furthermore interesting because they can escape formal contradictions: Take "A = x", where 'A' is a subject and 'x' a property. The external negation denies the proposition "A = x", we get Not-"A = x". It thereby denies the existence of the subject A altogether. Both, "A = x" and Not-"A = x", taken together are not contradictory, because A has the property x, but what is *not* A (but perhaps B, or nothing at all) might not have the property x. In other words: 'A being x' and the absence of A (or B being x) are not contradictory in the relevant sense of "contradiction", says Jones. Take this example: "The flower is yellow" and yet, if the external negation cancels the subject, this is compatible with Not-"The flower is yellow" because the absence of "the flower" not having the property of yellowness is compatible with the presence of the flower having the property of yellowness. In yet other words: what creates the "classical" contradiction is the internal negation of a property, i.e. the flower being both yellow and not yellow. This, according to Jones, is a proper contradiction. We can surely debate the notion of "proper" and "improper" contradiction, but what shall matter here is merely that there is a qualitative difference. There is no apparent contradiction in the flower being yellow and the fact that the absence of the flower is not yellow, Jones (2004:308) would say. To the extent that it is a contradiction, says Jones, it is an "external contradiction". The wording here is misleading, for what he means is a unbenign contradiction, in contrast to the formal contradiction concerning what is predicated of a subject. Potentially, this is because subjects can change, but properties are static.

This gets us back to the temporal notion of "immediacy". If we think of the SH as embedded in a temporal framework, where there is continuous flux and change, the flower at t1 is not the same as the flower at t2. Its properties may have changed while the flower itself persisted through change. Without going too deep into temporal ontology, we could think of the SH as saying that at time t1, A has the property of being 'A at t1', however at a later time, t2, A no longer has the property of being 'A at t1', and still A is self-identical. That presupposes some kind of unchangeable essence which underlies changes and grounds self-identity.

What I want to propose, tentatively, is that the SH should better be understood as a shift between a self-identity statements (i.e. the consequence) and property ascriptions (i.e. the antecedence). If we look at the antecedence, $((A = A) \wedge (A = \times A))$, what we see are temporally indexed property ascriptions. It should be read as follows: At time t1, A has the property of being 'A at time t1', and immediately after t1 (i.e. t2), A no longer has the property of being 'A at time t1'. We can think about the latter as an external negation:

We negate the subject 'A at t1' at t2, for what we have at t2 is the subject 'A at t2'. The consequence, however, is a trivial self-identity statement: 'A = A'. Still, through change, A remains self-identical.

Returning to the flower as my example: the flower is always changing; it never has a property and lacks it at the same time. The flower at some later time is no longer identical to the flower at an earlier time, but that doesn't undermine the self-identity of the flower throughout change. What we have, in the terminology of contemporary philosophy of time, is a model of endurance of the subject's self-identity (the flower is always the flower), but a perdurance of its properties (yet the flower might change its colors, etc.)

Hence, I think that is a plausible observation, the Kyoto School philosophers have incorporated the idea of impermanence (*anitya*) into the SH. Everything is impermanent, but yet it remains self-identical. The contradiction resolves in a temporal ontology which is not static but allows for change through time, while the changing object, i.e. the self-identity of the changing object, persists through the change. This explains the role of *'soku-ji-sei'* (Eng. immediacy) in the *soku-hi;* A is *immediately not* A, because A, being in constant flux, can never just *be*, but is relentlessly forced into change. The 'soku' in the *soku-hi negation* thus marks, in a very intricate way, the temporal aspect of impermanence (*anitya*).

The SH should better read as follows, where the capital 'A' is the subject and the lower-case 'a' the property of 'being A'. The equation sign should be read as 'predication':

SH*: $((A^{t1} = a^{t1}) \wedge (A^{t2} = \times a^{t1})) \rightarrow A = A$

In words: A at t1 has the property of being 'A at t1' and A at t2 no longer has the property of being 'A at t1', therefore (or perhaps, in spite of that) A is self-identical. The external negation no only cancels the property, but the subject 'A at t1' at t2.

Now, one might worry how this can be squared with the Buddhist anti-essentialist doctrine of no-self (*anatman*) to which Zen Buddhism, and ultimately the Kyoto School, subscribes. According to *anatman*, there is simply nothing like 'the flower', the thing-in-itself, to begin with; all things lack a robust metaphysical substance (*svabhāva*), as already discussed in §1.

The talk about things-in-themselves suggests an essentialist substance ontology. The idea of a non-changing substance that persists throughout change is one such instance

of a thing-in-itself which would suggest a substance ontology. However, Buddhist ontology is decidedly anti-essentialist. This gets our interpretation of SH into a doctrinal conflict.

On a very naïve way of thinking about change, a substance, *S*, is that unchanging entity which underlies the change. Change occurs with respect to *S* having different incompatible properties, Φ1 and Φ2, at different times, say t1 and t2. To many, this is not an accurate account of proper change but mere Cambridge change, as it is sometimes put. *Proper* change occurs *at a moment* and so, as argued in Priest (1979), something, *S* changes by having two incompatible (contradictory) properties, Φ1 and Φ2, *at the same time*.

We have to clarify, however, that the SH is not an account of change, in the metaphysical sense, which might as well entail that something changes by having two incompatible (contradictory) properties at the same time. The SH is rather a temporally located account about the metaphysical nature of 'being' and, as I shall argue below, again another instance of soteriological importance. The objection of Cambridge change can thus not be leveled against the SH.

In the next section, I will try to square the SH's emphasis on impermanence (i.e. of a perpetual change in properties), which apparently presupposes some unchanging essence-nature (i.e. self-identity) of the thing that changes, with the fundamental Buddhist doctrine of the lack of *svabhāva.* There I will also link the SH, via our discussion of the Ox-herding metaphor in the previous chapter, to the preceding discussion of the *catuṣkoṭi* and the myriad of its incarnations. This will conclude my overarching argument that there is a lineage running from the early Abhidharma rejectionism to the Kyoto School philosophy.

§3.3 Soku-Hi and *Catuṣkoṭi*

How can we square anti-essentialism and the notion of impermanence as portrayed in the SH? The idea of perspectives on reality, the ultimate and the conventional perspective, as we saw then in *Joshu's Dog* and the discussion around it, can help. These perspectives are metaphysically incompatible. Two entities (states-of-affairs, concepts, worlds, or whatever one likes)—say perspective on reality, Φ1 and Φ2, in our case—are incompatible if and only if it is impossible for an object, *a*, to have Φ1 and Φ2 simultaneously.[100] In the given context, that means, two perspectives on reality, Φ1 and Φ2 are incompatible if and only if it is impossible for the practicing Buddhist to have both perspectives on reality at the same time.

Indeed, as the quasi-recursion discussed above suggests, the practicing path towards enlightenment is a constant back and forth between (aspects of) a conventional and an ultimate perspective on reality. They are never experienced simultaneously, however. Enlightenment itself (i.e. the ultimate perspective) is an "either-or" situation. In other words, a situation of metaphysical (or rather epistemic or phenomenological) incompatibility. The ultimate perspective is incompatible with the conventional perspective, both are incompatible with the enlightened perspective, i.e. the perspective during *satori*, as discussed by Priest and Garfield (2010). Whenever the Bodhisattva is caught in conventional reality, she doesn't have an ultimate perspective on reality, while in the state of enlightenment, the distinction between conventional and ultimate reality

[100] See Price (1990) and Dunn (1996).

breaks down. This, I take it, is the take-away message of Joshu's Dog, the *Sānlùn* hierarchy, and Priest and Garfield's interpretation of the *catuṣkoṭi,* as well as the SH.

The idea of metaphysical incompatibility is grounded in our experience of the world. In a sense, it is a phenomenological principle, such as the distinction between ultimate and conventional reality. This fares very well with Nishida's observations of the relationship between logic and world, which takes logic to *"mirror the most general structure of reality"* (Z8.274, 276: Z9.41).

It is interesting to ask what happens to the incompatibility of ultimate and conventional reality after enlightenment. As argued above, different strands of Buddhism have different answer. Some say that constant practice is required, upon enlightenment the practitioner returns into a state of conventional reality. Others think that the ultimate perspective of enlightenment is the pinnacle of phenomenological experience, which makes, once experienced, a return to conventional reality impossible. I wish to leave this as an open point, as I know it is a matter of historic and ongoing debate, as discussed in previous chapters—a debate which perhaps doesn't need to be settled. Nothing much will hang on it for the subsequent discussion.

I now want to demonstrate that the SH has the same structure as the path towards enlightenment disclosed in the *Sānlùn* hierarchy or Joshu's Dog, and since I have established a continuity between the latter and the *catuṣkoṭi,* I take this to establish a connection between the SH and the *catuṣkoṭi.*

Let me start by assigning the perspective 'conventional' (c) to the antecedent (($A^{t1} = a^{t1}$) ∧ ($A^{t2} = \times a^{t1}$)). Let us be reminded of the quote from *Wudeng Huiyuan,* where we are being told that before Zen practice, 'mountains are just mountains', while during practice 'mountains are no longer mountains'. This, again, is a phenomenological thesis which (and this is where it connects to the definition of negation as metaphysical incompatibility) reflects a practitioner's apprehension of reality. The objects of apprehension as seen from the ultimate perspective lacks a property that the object of apprehension, as seen from the conventional perspective, possess, namely a substance (*svabhāva*). This is the anti-essentialism.

Let us now assign the status 'empty' (E) to the consequence, (A = A). Following insight into the ultimate nature of reality, things are either as in the *status-quo ante*, or they remain in the enlightened modus, however one wants to interpret the 'mountains are just mountains again'—it is a post-enlightened perspective anyways. I take this to be encapsulated in the consequence of the SH. Hence, what is missing from the SH dialectics

is the moment of enlightenment itself, the *satori*. The moment of unmediated insight into ultimate reality. Other than that, it follows the familiar pattern which I have associated with *Joshu's Dog* above, and which can be traced back to the *Sānlùn* hierarchy and finally to the *catuṣkoṭi.*

One can now ruminate why it is that in the hands of the Kyoto School philosophers, the event of enlightenment no longer has a prominent place. I can only speculate. The Western influence of "enlightened" reason—as in Kant—might have made the idea of sudden enlightenment—as in Zen—look mystical. For the sake of "modernity", the Kyoto School philosophers may have wanted to "hide" the elusive notion of the actual event of enlightenment. Perhaps it was pushing against the Kyoto School's ambition to give Zen a logical structure. One can argue whether this project really succeeded, if all that the Kyoto School did was hide the soteriological importance of experiences of enlightenment. But discussing that far exceeds the scope of this discussion.

Given the structural similarities to *Wudeng Huiyuan,* I want to suggest that SH* has to be replaced with the subscripted, i.e. perspectivalised, version, SH'. I'll read the subscripts as perspectival operators, denoting mutually incompatible perspectives: C = "conventionally"; E = "empty".

SH': $((A^{t1} = a^{t1}) \wedge (A^{t2} = \times a^{t1}))_C \rightarrow (A = A)_E$

I want to invite the reader to compare this structure with my model of the Ox herding metaphor.

OX_{10}: $C^0 » U^0 » C^1 » U^1 » N » C^N$

The consequence, $(A = A)_E$, can here be understood as the post-enlightened state of C^N. In SH', we no longer have N, as discussed above. We however have parts of the *Sānlùn* hierarchy of "$C^0 » U^0$" in the form of $((A^{t1} = a^{t1}) \wedge (A^{t2} = \times a^{t1}))_C$. The first disjunct is the conventional perspective of self-identity, as in C^0, and the second disjunct is the annihilation of self-identity, as in U^0. Temporarily, or course, the second conjunct comes after the first conjunct as it requires practice.

We could, of course, then write the antecedent as $(A^{t1} = a^{t1})_C \wedge (A^{t2} = \times a^{t1})_U$, but that would hide the fact that everything before the experience of enlightenment—i.e.

unveiled insight into ultimate reality—, is still considered part of convectional reality, even if intellectual endeavors have been made to understand ultimate reality. In any case, we would need to write the equivalent of "C^0 » U^0" as follows: $((A^{t1} = a^{t1})_C \wedge (A^{t2} = \times a^{t1})_U)_C$. But that makes it difficult to distinguish between the subscript C which ranges over the first disjunct, and the subscript C which ranges over the whole disjunction. It could be a perspective of a higher level, but that might be an unnecessary complication. I therefore wish to stick to the original formulation.

We see that on this interpretation, there is there is only a conflict between impermanence and anti-essentialism in conventional reality. Indeed, if we remember how the boy struggled to tame the Ox, we can take this as a metaphor. The incompatibility of anti-essentialism and impermanence is only a phenomenon of conventional reality. The emphasis on impermanence (i.e. of a perpetual change in properties) refers to the impermanence of things in conventional reality. In conventional reality, this might as well presuppose some unchanging essence-nature (i.e. self-identity) of the thing that changes. That, however, is overcome through enlightenment, through which the fundamental Buddhist doctrine of the lack of *svabhāva* manifests itself in experience. Without *svabhāva*, permanence and impermanence no longer make sense as categories. Permanence presupposes some kind of underlying essence-nature, and impermanence does too. Again, enlightenment, or rather the post-enlightened state, is a state in which dualities dissolve.

This is how we can interpret the line 'mountains are mountains again', or the $(A = A)_E$. The "again" here is not a return to the *status-quo ante* where things have self-being, but to a place prior to the *status-quo ante* where the true nature of reality, which is characterized by a lack of *svabhāva,* manifested itself: *nothingness,* the Kyoto School philosophers would call it, I suppose.

This is important. It is neither a state of lingering insight into ultimate reality, nor a state of conventional perspective and potentially the intellectual effort of making sense of ultimate reality. The ultimate/conventional distinction itself dissolves. The self-identity statement in the consequence of the SH might thus be an expression of 'oneness'.[101] I am afraid, it has to remain as metaphorical as that, for Buddhist metaphysics will perhaps remain not only a logico-metaphysical edifice, but a logico-epistemic-phenomenon-metaphysical edifice. At least about the phenomenological aspect, I dare not to speak.

[101] See Piest (2014) for a similar take on *satori.*

More must to said about why the moment of enlightenment escaped the Kyoto School philosophers, and about why the history of the catuskoti is one of subtraction, were more and more details have been removed from the conceptual apparatus. Perhaps, this is because "the new" inherited "the old" and can implicitly refer to a whole baggage of conceptual machinery.

Synopsis

The *catuṣkoṭi* is an enigmatic piece of Buddhist philosophy. I hope that my cross-historical engagement made clearer what the *catuṣkoṭi* is, where it came from, and what role it plays in the broader Buddhist philosophical cosmos. Still, I feel like this is not the end of the story. Much more needs to be said about other cross-cultural connections, and logical modelling may help us there.

I have followed the *catuṣkoṭi* from the Indo-Tibetan milieu where we have discussed the work of Nāgārjuna, via China where I have made the connection between Madhyamaka philosophy and the *Sānlùn* School, to Japan, where, after the rise of Zen, and in the hands of the Kyoto School, the *catuṣkoṭi* shrinks to the size of the SH. The argument rests on the idea of historical transitivity: If the *catuṣkoṭi* evolved into the koan practice, and the koan schemata evolved into the *soku-hi* dialectics, then the *catuṣkoṭi* evolved into the *soku-hi* dialectics. The burden of proof is thus on those who think that there is a deep rift between *Sānlùn* and Zen to establish the opposite.

Kyoto school philosophy, just like many other contemporary "schools" of philosophy or philosophical debates, has the tendency to assume knowledge of a particular philosophical lineage—Mahayana Buddhism, in this case. A lot is therefore left unpacked. I think to have only partly uncovered this lineage. Eventually, this is just one of many more puzzle cases. Authentic buddhological investigation is necessary to bolster my claim. I am not in the position to do that. This is why I shall leave it with this: The *soku-hi* dialectic is a vastly abbreviated *catuṣkoṭi*.

Where the *catuṣkoṭi* will go in the future, of course I don't know. But as much as it did in the past, it will evolve alongside a steadily evolving religion and the highly adaptive philosophy of Buddhism.

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